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The technology and cost structure

of a natural gas pipeline:

Insights for costs and rate-of-return regulation

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Abstract

This note details a complete microeconomic characterization of the physical relationships between input use and the level of output of a simple point-to-point gas pipeline system and uses it to contribute to the public policy discussions pertaining to the economic regulation of natural gas pipelines. We show that the engineering equations governing the design and operations of that infrastructure can be approximated by a single production equation of the Cobb-Douglas type. We use that result to inform three public policy debates. First, we prove that the long-run cost function of the infrastructure formally verifies the condition for a natural monopoly, thereby justifying the need of regulatory intervention in that industry. Second, we examine the conditions for cost-recovery in the short-run and contribute to the emerging European discussions on the implementation of short-run marginal cost pricing on interconnector pipelines. Lastly, we analyze the performance of rate-of-return regulation in that industry and inform the regulatory policy debates on the selection of an appropriate authorized rate of return. We highlight that, contrary to popular belief, the socially desirable rate of return can be larger than the market price of capital for that industry.

Keywords: Production function; Natural gas pipeline; Regulation.

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1. Introduction

The last 30 years have seen an enduring interest in the construction of large-scale natural gas pipelines across the globe. Though an emerging literature has studied the market effects of a new pipeline project,[1] the examination of the technology and costs of these capital-intensive infrastructures has attracted less attention. Yet, that analysis is critically needed to inform policy development and decisions. Even in countries where liberalization reforms have been implemented, natural gas pipelines remain regulated (von Hirschhausen, 2008) and authorities must frequently deal with project-specific requests for adjustments within the regulatory framework.[2]

So far, two different methodological approaches have been considered to investigate the technology. The first is rooted in engineering and can be traced back to Chenery (1949). It aims at numerically determining the least-cost design of a given infrastructure using optimization techniques (Kabirian and Hemmati, 2007; Ruan et al., 2009; André and Bonnans, 2011). This approach is widely applied by planners and development agencies to assess the cost of a specific project (Yépez, 2008). Yet, because of its sophistication and its numerical nature, it is seldom considered in regulatory policy debates (Massol, 2011). The second approach involves the econometric estimation of a flexible functional form – usually a translog specification – to obtain an approximate cost function. This method has become popular in Northern America either to estimate the industry cost function using cross-section datasets (Ellig and Giberson, 1993) or to model the cost function of a single firm using a time series approach (Gordon et al., 2003). So far, data availability issues have hampered the application of this empirical approach in Continental Europe and Asia.

This research note develops a third approach: it proves that a production function of the Cobb-Douglas type captures the physical relationship between input use and the level of output of a simple point-to-point pipeline infrastructure. More precisely, we show how that micro-founded model of the

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[1] *Among others, Newbery (1987) assesses the trade opportunities generated by a new pipeline, Hubert and Ikonnikova (2011) evaluate the impacts on the relative bargaining powers of exporting and transit countries, and Rupérez Micola and Bunn (2007) and Massol and Banal-Estañol (2016) investigated the relation between pipeline utilization and the degree of spatial market integration between interconnected markets.*

[2] *For example, the augmented rate-of-return that was allocated to two new pipeline projects in France during the years 2009–16: the pipeline connecting the new Dunkerque LNG terminal to the national transportation network and the North-South Eridan project (CRE, 2012).*

technology naturally emerges from the engineering equations governing the design of that infrastructure. One of the great merits of that approach is that it greatly facilitates the application of the standard theory of production to characterize the microeconomics of a natural gas pipeline system.

To explore the policy implications, we use that production function to successively examine the properties of the cost function in the long and in the short run. We also compare the market outcomes obtained under three alternative conditions of industrial organization (unregulated private monopoly, average-cost pricing, and rate-of-return regulation). Our results: (i) indicate the presence of pronounced increasing returns to scale in the long run; (ii) confirm the natural monopolistic nature of a gas pipeline system and the need for regulatory intervention; (iii) clarify the conditions for cost-recovery if short-run marginal cost pricing is imposed on such infrastructure; (iv) quantify the performance of rate-of-return regulation in that industry, and (v) reveal that the socially desirable rate of return is not necessarily equal to the market price of capital in this case.

2. Theoretical model of the technology

We consider a simple point-to-point pipeline infrastructure that consists of a compressor station injecting a pressurized flow of natural gas Q into a pipeline to transport it across a given distance l.

Following Chenery (1949) and Yépez (2008), designing such a system imposes to determine the value of three engineering variables: the compressor horsepower H, the inside diameter of the pipe D and τ the pipe thickness. These variables must verify three engineering equations presented in Table 1 (first column). The compressor equation gives the power required to compress the gas flow from a given inlet pressure p_0 to a predefined outlet pressure $p_0 + \Delta p$ where Δp is the net pressure rise. The Weymouth equation models the pressure drop between the inlet pressure $p_0 + \Delta p$ measured after the compressor station, and the outlet one p_1, which is assumed to be equal to p_0. Lastly, concerns about the mechanical stability of the pipe impose a relation between the thickness τ and the inside diameter D.

Table 1. Engineering equations

Exact engineering equations	Approximate engineering equations
Compressor equation: [a] $$H = c_1 \cdot \left[\left(\frac{p_0 + \Delta p}{p_0} \right)^b - 1 \right] Q$$	Approximate compressor equation: [a] $$H = c_1 b \frac{\Delta p}{p_0} Q$$
Weymouth flow equation: [b] $$Q = \frac{c_2}{\sqrt{l}} D^{8/3} \sqrt{\left(p_0 + \Delta p \right)^2 - p_1^2}$$	Approximate flow equation: [b] $$Q = \frac{c_2 p_0 \sqrt{2}}{\sqrt{l}} D^{8/3} \sqrt{\frac{\Delta p}{p_0}}$$
Mechanical stability equation: [c] $$\tau = c_3 D$$	Mechanical stability equation: [c] $$\tau = c_3 D$$

Notes: [a] [b] the positive constant parameters c_1, c_2 and b (with $b<1$) are detailed in Yépez (2008) for the USCS unit system. Elevation changes along the pipeline are neglected in the flow equation. [c] This equation follows the industry-standard practice and assumes that the pipe thickness equals a predetermined fraction c_3 of the inside diameter (e.g., $c_3 =0.9\%$ in Ruan et al. (2009 – p. 3044)).

We now combine these equations to construct an approximate production function. To our knowledge, the pressure rise Δp usually ranges between 1% and 30% of p_0, which leads to the first-order approximations detailed in Table 1 (second column). Combining them, one can eliminate the relative pressure rise $\Delta p / p_0$ and obtain the following relation between the output Q and two engineering variables H and D:

$$Q = \sqrt[3]{\frac{2 \left(c_2 p_0 \right)^2}{c_1 b l}} D^{16/9} H^{1/3} . \tag{1}$$

This relation can be reformulated as a production function that gives the output as a function of two inputs: energy and capital. First, we let E denote the total amount of energy consumed by the infrastructure to power the compressor. By definition, the total amount of energy E is directly proportional to the horsepower H. Second, we let K denote the replacement value of the pipeline. We assume that the capital stock K is directly proportional to the pipeline total weight of steel S and let P_S denote the unit cost of steel per unit of weight. Hence, $K = P_S S$. The total weight of steel S required to build that pipeline is obtained by multiplying the volume of steel in an open cylinder by the weight of steel per unit of volume W_S:

$$S = l\pi \left[\left(\frac{D}{2} + \tau \right)^2 - \left(\frac{D}{2} \right)^2 \right] W_S , \tag{2}$$

where $\pi \approx 3.1416$ is the mathematical constant. Combining that equation with the mechanical stability equation in Table 1, the amount of capital expenditure related to the pipeline is as follows:

$$K = P_S l\pi D^2 \left[c_3 + c_3{}^2 \right] W_S . \tag{3}$$

This equation shows that the pipeline diameter is directly proportional to the square root of K, the amount of capital invested in the pipeline. So, the engineering equation (1) can readily be rewritten as a production function: $Q = B\, K^{8/9} E^{1/3}$, where B is a constant. To simplify, we rescale the output by dividing it by B and use this rescaled output thereafter. So, the Cobb-Douglas production function of a gas pipeline is:

$$Q^\beta = K^\alpha E^{1-\alpha} , \tag{4}$$

where the capital exponent parameter is $\alpha = 8/11$ and $\beta = 9/11$ is the inverse of the degree to which output is homogeneous in capital and energy. As $\beta < 1$, the technology exhibits increasing returns to scale.

3. Results and policy implications

In this section, we show how the technological model above can be applied to derive several policy-relevant insights. Since natural gas pipelines are deemed as natural monopolies, we first examine whether that reputation is supported by the properties of the long-run cost function. We then examine the short-run cost function to assess the performance of short-run marginal cost pricing. Lastly, we assess the performance of rate-of-return regulation for that industry.

3.1 Long-run cost

We let e denote the market price of the energy input and r the market price of capital faced by the firm. From the cost-minimizing combination of inputs needed to transport the output Q, one can derive the long-run total cost function (Cf., Appendix A):

$$C(Q) = \frac{r^\alpha e^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}} Q^\beta . \tag{5}$$

Three insights can be drawn from that specification. Firstly, the elasticity of the long-run cost with respect to output is $\beta = 9/11$ and lower than one. The cost function (5) also validates the empirical remarks in Chenery (1952) and Massol (2011) who suggested that this elasticity is almost constant over most of the output range. Secondly, the ratio of the long-run marginal cost to the long-run average cost is constant and also equals β. As $\beta < 1$, setting the price equal to the long-run marginal cost systematically yields a negative profit. Lastly, one can note that the univariate cost function (5) is concave and thus strictly subadditive (Sharkey, 1982 - Proposition 4.1). This property has important policy implications: it attests that a point-to-point gas pipeline system verifies the technological condition for a natural monopoly. As this particular industry structure may lead to a variety of economic performance problems (such as excessive prices, production inefficiencies, and costly duplication of facilities), the implementation of price and entry regulation of some form can be justified to mitigate the social cost of these market failures (Joskow, 2007).

3.2   Short-run cost

We now examine how cost varies in the short-run. We consider an existing infrastructure that has been designed to transport the output Q_0 at minimum long-run cost by installing the amount of capital stock K_0. The short-run total cost function is obtained by holding K_0 constant and varying the output Q. Introducing the variable input requirements function $E(Q, K_0) = {}^{1-\alpha}\sqrt{K_0^{-\alpha} Q^\beta}$ that gives the amount of energy needed to transport Q along that pipeline, the short-run total cost function is:

$$SRTC^{K_0}(Q) = rK_0 + eK_0^{\frac{-\alpha}{1-\alpha}} Q^{\frac{\beta}{1-\alpha}}. \tag{6}$$

The technical discussion presented in Appendix B confirms that the short-run average cost $SRAC^{K_0}$ curve is U-shaped and attains its minimum at $Q = \underline{Q}$, where $\underline{Q}$ is the unique output at which the short-run marginal cost curve intersects the $SRAC^{K_0}$ one. Solving, one can show that the output ratio $\underline{Q}/Q_0$ verifies:

$$\frac{\underline{Q}}{Q_0} = \left[\frac{\alpha}{\beta + \alpha - 1} \right]^{\frac{1-\alpha}{\beta}} = \sqrt[3]{\frac{4}{3}} \approx 1.1006. \tag{7}$$

It should be noted that this ratio is entirely determined by the technological parameters α and β and does not depend on the input prices or the capital stock K_0.

At the output level $Q = Q_0$, the short-run marginal cost is lower than the short-run average cost and expanding the output to $\underline{Q} = \sqrt[3]{4/3}Q_0$ occasions a reduction in the short-run average cost.

It follows that, for any output Q with $Q < \underline{Q}$, imposing the pipeline operator to charge a price equal to the short-run marginal does not allow that firm to break even. This last finding can usefully inform the contemporary European policy debates pertaining to the regular revision of the European Gas Target Model (ACER, 2015). In a recent policy proposal, Hecking (2015) advocates the application of short-run marginal-cost pricing for cross-border interconnector pipelines in Europe. Compared to the current ad-hoc pricing system, one of the main merits of this pricing arrangement is to favor an efficient use of these infrastructures in the short-run. Yet, it should be stressed that the capital costs bulk large as a percentage of the total cost of a gas pipeline system. Therefore, its application on an existing interconnector may generate a cost-recovery issue if the output is lower than the level $\underline{Q}$.[3] For new interconnector projects, this pricing scheme, when considered alone, can deter investment. It could thus adversely impact the feasibility of a series of major European projects aimed at fostering market integration across the continent (such as the MidCat project proposed to connect the Iberian peninsula with France and the rest of Europe). This also confirms the need to combine marginal-cost pricing of interconnectors with other cost-recovery instruments such as network pricing to recover the remaining costs.

3.3 Rate-of-return regulation

The analysis above indicates that a pipeline has elements of a natural monopoly. As rate-of-return regulation[4] remains a prominent instrument used by numerous authorities internationally (including the U.S., Belgium, and South-Africa), we now explore what insights our characterization of the technology can provide to regulators and practitioners.

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[3] *Arguably, a share of these capital costs could be considered as sunk which could trigger a discussion as to whether these costs have to be recouped or not.*

[4] *This form of regulation sees costs as exogenous and observable and forms prices on the basis of observed variable costs and an authorized rate of return on invested capital s based on an assessment of the risk-based cost of capital.*

Following the literature (Klevorick, 1971: Callen et al., 1976), we assume the isoelastic inverse demand function $P(Q) = A\,Q^{-\varepsilon}$, where $1/\varepsilon$ is the absolute price elasticity with $\varepsilon < 1$ (so that the total revenue obtained by a firm producing zero output is zero) and $1 - \beta < \varepsilon$ (to verify the second-order condition for a maximum in the regulated firm's optimization problem), and let s denote the allowed rate of return set by the regulatory authority. For concision, the solution of the profit-maximization problem of a regulated firm whose accounting profit (i.e., the total revenue $P(Q)Q$ minus $eE(Q,K)$ the cost of the variable input) cannot exceed the allowed return on invested capital sK is reviewed in a supporting technical appendix.

Callen et al. (1976) examine the problem of a regulator that sets the allowed rate of return s at the level s_R that maximizes the net social welfare given the regulated firm's reaction to that rate. They formally prove that this socially desirable rate is:

$$s_R = \max\left(r, \frac{\left[\beta - (1-\varepsilon)(1-\alpha)\right]^2}{\alpha\left[\beta - (1-\alpha)(1-\varepsilon)^2\right]} r \right). \tag{8}$$

We can use the values of α and β above to highlight two interesting results pertaining to the application of rate-of-return regulation in the gas pipeline sector. First, it is straightforward to verify that, whenever the demand parameter ε is in the open interval $\left(\left(2 + 4\sqrt{3}\right)/11, 1 \right)$, the condition $\left[\beta - (1-\varepsilon)(1-\alpha)\right]^2 > \alpha\left[\beta - (1-\alpha)(1-\varepsilon)^2\right]$ holds which indicates that the socially desirable rate of return is $s_R = \left[\beta - (1-\varepsilon)(1-\alpha)\right]^2 r / \left(\alpha\left[\beta - (1-\alpha)(1-\varepsilon)^2\right]\right)$ and thus verifies $s_R > r$. Hence, if the absolute price elasticity is low and in the range $1 < 1/\varepsilon < 1.232$, setting the allowed rate-of-return as close as possible to the market price of capital does not maximize the net social welfare. This is a noteworthy finding that contradicts a popular belief. Second, we can observe that the ratio s_R/r is bounded as the relation $(s_R/r) < (\beta/\alpha)$ holds for any value of ε in the assumed range $1 - \beta < \varepsilon < 1$. This remark provides useful operational guidance for the selection of a rate of return: if the regulator has zero information on the value of the price elasticity of the demand and thus cannot exactly evaluate s_R, it should not implement a rate of return that is larger than $\beta r/\alpha$, that is $\beta/\alpha = 9/8 = 1.125$ times the market price of capital r.

It is also instructive to evaluate the relative performance of rate-of-return regulation in the gas pipeline sector by comparing the market outcomes (subscribed with R) with the ones obtained in case of either a standard (unregulated) private monopoly (subscribed with M) or a benevolent social planner that maximizes the net social welfare while providing zero economic profit to the pipeline operator[5] (subscribed with a as it sets the output at the level at which price equals the long-run average cost). To ease the comparisons, we simply tabulate the ratios presented in Callen et al. (1976) for a range of possible values for the demand elasticity (Cf., Table 2). These ratios are also detailed in the technical appendix (cf., Table TA-3) and respectively compare:

- the output levels decided by: a private monopoly Q_M, a social planner applying the average-cost-pricing rule Q_a and a regulated monopoly Q_R;

- the cost C_R incurred by the regulated firm subject to rate-of-return regulation and the cost $C(Q_R)$ that would have been incurred by a cost-minimizing firm producing the same output Q_R;

- the gain in net social welfare resulting from the regulation of a private monopoly $(W_R - W_M)$ and the gain in net social welfare $(W_a - W_M)$ that would be obtained by a social planner applying the average-cost-pricing rule to a previously monopolistic industry.

These ratios are invariant with the relative input prices and are entirely determined by: the demand and technology parameters, and the ratio s/r that relates s the allowed rate of return set by the regulator to r the market price of capital (Callen et al., 1976).

To begin with, we examine the case presented in Table 2 – Panel A of a regulatory agency that implements the socially desirable rate of return s_R in (8). If the absolute price elasticity of the demand is less than 1.30, we observe that: (i) the output level Q_R is substantially lower than the value Q_a obtained under the ideal case of a benevolent social planner imposing the long-run average cost pricing rule (it hardly attains the three quarters of that value); and (ii) the magnitude of the extra-cost caused by

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[5] *Recall that marginal cost pricing would lead to a negative profit. This case thus corresponds to the second-best solution examined by Boiteux (1956) whereby the firm is instructed to act so as to maximize the social welfare while balancing its budget.*

the overcapitalization effect pointed in Averch and Johnson (1962)[6] can be important (i.e., the cost increase is larger than 20% of the long-run total cost and attains 378.9% in case of a price elasticity equal to 1.001). That said, it is worth noting that despite these two adverse effects, the application of rate-of-return regulation on an unregulated monopolistic operator induces a very large rise in the pipeline output level (cf., the large values of the output ratio Q_R/Q_M). Overall, that form of regulation generates substantial welfare gains: the net increase in social welfare $(W_R - W_M)$ attains more than 70% of the difference $(W_a - W_M)$ that measures the gains obtained under the theoretical benchmark of a benevolent social planner applying average-cost-pricing (i.e., the second best solution).

As regulatory agencies seldom have complete knowledge of the price elasticity of demand needed to evaluate the socially desirable rate of return s_R, Table 2 – Panel B then examines the performance of rate-of-return regulation when the regulator simply sets $s = \beta r/\alpha$. By construction, the gains in social welfare values are lower than the ones detailed in Panel A. Yet, we observe that the differences remain tolerable whenever the absolute price elasticity is less than 1.50, which is likely to be the case in the natural gas pipeline industry. Hence, this form of regulation remains a powerful regulatory instrument even when the regulator simply sets the allowed rate of return s within the range $r \leq s \leq \beta r/\alpha$.

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[6] *The analysis of rate-of-return regulation in Averch and Johnson (1962) highlighted the tendency of the regulated firm to engage in excessive amounts of capital accumulation (ratebase) to expand its potential for profits. It should be noted that Averch and Johnson focused on the telephone industry which, at that time, mainly used two inputs: capital and labor. The Averch-Johnson effect is thus usually invoked as a tradeoff between capital and labor. In the present paper, the tradeoff is between capital and energy.*

Table 2. Output, cost, and welfare ratios for alternative demand elasticities

		$1/\varepsilon$						
		1.001	1.05	1.10	1.20	1.30	1.50	2.00
PANEL A The socially desirable case $s = s_R$	Ratio rate of return/price of capital $\dfrac{s}{r} = \dfrac{s_R}{r}$	1.124	1.090	1.061	1.013	1.000	1.000	1.000
	Output ratios $\dfrac{Q_R}{Q_a}$	0.127	0.448	0.551	0.678	0.746	0.815	0.895
	$\dfrac{Q_R}{Q_M}$	468.211	17.955	11.313	7.795	6.438	5.196	4.207
	Cost ratio $\dfrac{C_R}{C(Q_R)}$	4.789	1.705	1.456	1.273	1.187	1.104	1.036
	Welfare ratio $\dfrac{(W_R - W_M)}{(W_a - W_M)}$	0.729	0.724	0.725	0.735	0.750	0.781	0.850
PANEL B The case of a rate of return set at the upper bound $s = \beta r/\alpha$	Ratio rate of return/price of capital $\dfrac{s}{r} = \dfrac{\beta}{\alpha}$	1.125 (+0.07%)	1.125 (+3.17%)	1.125 (+6.05%)	1.125 (+11.07%)	1.125 (+12.50%)	1.125 (+12.50%)	1.125 (+12.50%)
	Output ratios $\dfrac{Q_R}{Q_a}$	0.127 (-0.06%)	0.435 (-2.91%)	0.520 (-5.71%)	0.603 (-11.06%)	0.645 (-13.57%)	0.683 (-16.19%)	0.684 (-23.60%)
	$\dfrac{Q_R}{Q_M}$	467.934 (-0.06%)	17.433 (-2.91%)	10.667 (-5.71%)	6.933 (-11.06%)	5.564 (-13.57%)	4.355 (-16.19%)	3.214 (-23.60%)
	Cost ratio $\dfrac{C_R}{C(Q_R)}$	4.788 (-0.02%)	1.691 (-0.79%)	1.436 (-1.40%)	1.244 (-2.22%)	1.161 (-2.23%)	1.084 (-1.82%)	1.024 (-1.14%)
	Welfare ratio $\dfrac{(W_R - W_M)}{(W_a - W_M)}$	0.729 (+0.00%)	0.724 (-0.02%)	0.724 (-0.13%)	0.730 (-0.65%)	0.738 (-1.60%)	0.748 (-4.23%)	0.731 (-13.94%)

Notes: In Panel B, the numbers in parentheses indicate the relative change (in percent) with respect to the ideal case of a regulator capable to set the regulated rate of return at the value s_R in equation (8).

4. Conclusion

The analysis presented in this concise paper shows how the complex engineering equations governing the functioning of a pipeline system can be combined in a single production equation of the Cobb-Douglas type that is commonly applied in microeconomics.

This characterization of the technology of a natural gas pipeline allows us to highlight the following points that should be pertinent to researchers and policymakers interested in understanding the economics of natural gas pipelines.

- First, the analysis assesses the magnitude of the long-run economies of scale that exists on point-to-point pipeline systems, thereby confirming the natural monopolistic nature of this infrastructure and justifying the need to implement price and entry regulation of some form in the industry.

- Second, in the short-run, the analysis reveals that it is possible to monotonically lower the average transportation cost incurred on an existing pipeline infrastructure by expanding the output up to a threshold level that represents about 110% of the output that was considered at the time of the construction of that infrastructure. This finding has important implications for the applicability of short-run marginal-cost pricing, confirming that this pricing scheme cannot allow recovery of the capital costs incurred by the pipeline operator if output is lower than that threshold level.

- Lastly, this paper combines the technological analysis with the standard industrial organization literature to contribute to the understanding of the performance of rate-of-return regulation in the pipeline industry. It first reveals that, contrary to popular intuition, the rate of return that maximizes net social welfare can be larger than the market price of capital when the price elasticity of demand is low. To assist regulators, the analysis also provides a ceiling value for that socially desirable rate of return. Then, it also assesses the magnitude of the Averch-Johnson distortions on both the output and the cost of the regulated firm. Despite these distortions, the application of this basic form of economic regulation remains a valuable instrument to protect the community from monopolistic exploitation.

While the present discussion is centered on the case of a simple point-to-point natural gas transportation infrastructure, it suggests several possibly fruitful directions for future research. First, future works could extend the analysis to the case of more complex natural gas trunkline systems forming a meshed network. Second, future research could explore whether this methodology could be adapted and combined with the recent engineering literature on either hydrogen pipelines (André et al., 2013) or CO_2 pipelines (Massol et al., 2015) to inform the burgeoning policy discussions on the regulation of these future low-carbon technologies. Lastly, one may conceivably explore whether an

adaptation is possible for the case of the natural gas distribution networks. At first sight, this might be feasible for the specific case of natural gas distribution networks equipped with local compressor stations but a series of issues have to be examined including: the possibly different flow equation governing the movement of natural gas into small diameter pipes, the role of specific cost drivers (e.g., to dig a trench) and the reintroduction of labor as a production factor.

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Appendix A – The long-run cost function

The long-run total cost function C to transport the output Q is the solution of the cost-minimization problem:

$$\underset{K,E}{\text{Min}} \quad C(Q) = r\,K + e\,E \qquad (A.1)$$

$$\text{s.t.} \quad Q^{\beta} = K^{\alpha} E^{1-\alpha} \qquad (A.2)$$

The first-order conditions for optimality indicate that the marginal rate of technical substitution of E for K has to equate the ratio of the input prices:

$$\frac{(1-\alpha)}{\alpha} \frac{K}{E} = \frac{e}{r}.$$

(A.3)

Using the variable input requirements function $E(Q,K) = \sqrt[1-\alpha]{K^{-\alpha}Q^{\beta}}$ that gives the amount of energy needed to transport Q along that pipeline, one can rearrange (A.3) to define a function that gives the long-run cost-minimizing amount of capital stock needed to transport the output Q:

$$K(Q) = \left(\frac{e\alpha}{r(1-\alpha)}\right)^{1-\alpha} Q^{\beta},$$

(A.4)

The long-run total cost function is $C(Q) = rK(Q) + eE(Q, K(Q))$ and thus:

$$C(Q) = \frac{r^{\alpha}e^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} Q^{\beta}.$$

(A.5)

Appendix B – Short-run costs

A review of short-run cost concepts

Assuming a fixed amount of capital input K, the short-run total cost function is:

$$SRTC^{K}(Q) = rK + eE(Q, K),$$

(B.1)

where $E(Q,K) = \sqrt[1-\alpha]{K^{-\alpha}Q^{\beta}}$ is the variable input requirements function. As $\beta > 1-\alpha$ for the gas pipeline, this function is monotonically increasing and convex.

The short-run marginal cost function is:

$$SRMC^{K}(Q) = eE_{Q}(Q, K).$$

(B.2)

where $E_{Q}(Q,K)$ denote the derivative of the input requirement function with respect to the output variable.

The short-run average cost function is:

$$SRAC^K(Q) = \frac{rK}{Q} + e\frac{E(Q,K)}{Q}. \tag{B.3}$$

With $\alpha = 8/11$ and $\beta = 9/11$, this twice-differentiable function verifies $\lim_{Q \to 0^+} SRAC^K(Q) = +\infty$, $\lim_{Q \to +\infty} SRAC^K(Q) = +\infty$ and is strictly convex.[7] Hence, the short-run average cost curve has the usual U shape. Because of the strict convexity, the short-run average cost function has a unique minimum. At that output level, the short-run average cost equals the short-run marginal cost.[8] We let $\underline{Q}$ denote the output at which the short-run average cost is minimal, i.e. $\underline{Q} = \underset{Q>0}{\text{ArgMin}} \ SRAC^K(Q)$. For any output Q lower (respectively larger) than $\underline{Q}$, the short-run average cost $SRAC^K(Q)$ is larger (respectively lower) than the short-run marginal cost $SRMC^K(Q)$.

Discussion

We now consider the infrastructure that has been optimally designed to transport the output Q_0 at minimum long-run cost by installing the amount of capital stock $K_0 = K(Q_0)$, and aim at comparing the design output Q_0 and the average-cost-minimizing output $\underline{Q}$ on that specific pipeline system.

Recall that $\underline{Q}$ is such that the short-run average cost $SRAC^{K_0}(\underline{Q})$ equals the short-run marginal cost $SRMC^{K_0}(\underline{Q})$, that is:

$$\frac{rK_0}{\underline{Q}} + e\frac{E(\underline{Q},K_0)}{\underline{Q}} = eE_Q(\underline{Q},K_0). \tag{B.4}$$

Using $E(Q,K) = \sqrt[1-\alpha]{K^{-\alpha}Q^\beta}$ and simplifying, one obtains:

$$\underline{Q} = \left(\frac{r(1-\alpha)}{e(\beta+\alpha-1)}\right)^{\frac{1-\alpha}{\beta}} K_0^{\frac{1}{\beta}}. \tag{B.5}$$

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[7] *Remark that its second derivative equals* $2\left(rKQ^{-3} + eK^{-8/3}\right)$ *which is positive for any* $Q > 0$.

[8] *Proof: The gradient of* $SRAC^K$ *w.r.t.* Q *equals* $\left[-SRAC^K(Q) + eE_Q(Q,K)\right]/Q$, *that is using* (B.2) $\left[-SRAC^K(Q) + SRMC^K(Q)\right]/Q$.

Using (A.4), one can directly obtains the design output Q_0 as a function of the capital stock that has been installed:

$$Q_0 = \left(\frac{r(1-\alpha)}{e\alpha} \right)^{\frac{1-\alpha}{\beta}} K_0^{\frac{1}{\beta}} . \tag{B.6}$$

Equations (B.5) and (B.6) together indicate that the ratio $\underline{Q}/Q_0$ is entirely determined by the technological parameters α and β :

$$\frac{\underline{Q}}{Q_0} = \left[\frac{\alpha}{\beta+\alpha-1} \right]^{\frac{1-\alpha}{\beta}} . \tag{B.7}$$

With $\alpha = 8/11$ and $\beta = 9/11$, this ratio indicates that $\underline{Q} = \sqrt[3]{4/3}Q_0 \approx 1.1006Q_0$. It should be noted that for any output lower than $\underline{Q}$, the short-run average cost is larger than the short-run marginal cost.

Technical Appendix

"The technology and cost structure
of a natural gas pipeline:
Insights for costs and rate-of-return regulation"

Florian PERROTTON   **Olivier MASSOL**

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This technical appendix is organized as follows. Section 1 summarizes the assumptions and introduces the notation. Section 2 reviews the standard cases of a monopoly and a social planner. Section 3 examines the case of rate-of-return regulation and gives a concise presentation of Klevorick (1971) and Callen et al. (1976) who were the first to analytically examine the economics of rate-of-return regulation for a Cobb-Douglas technology. Section 4 details the ratios presented in the paper.

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1. Assumptions and notations

<u>Technology</u>

We consider the simple point-to-point pipeline infrastructure studied in our paper and assume the Cobb-Douglas production function: $Q^{\beta} = K^{\alpha}E^{1-\alpha}$, where $\alpha = 8/11$ is the capital exponent parameter and $\beta = 9/11$ is the scale coefficient.

From that production function, one can define $E(Q,K) = \sqrt[1-\alpha]{K^{-\alpha}Q^{\beta}}$ the variable input requirements function that gives the amount of energy needed to transport the output Q on a pipeline infrastructure that has a given fixed amount of capital input K. We let $E_Q(Q,K)$ (respectively, $E_K(Q,K)$) denote the derivative of the input requirement function with respect to the output (respectively, the capital) variable. With our technology parameters, $E_Q(Q,K) > 0$ and $E_K(Q,K) < 0$.

<u>Input prices</u>

We let e denote the market price of the energy input and r denote the market cost of capital faced by the firm.

<u>Cost function</u>

Following the argumentation presented in Appendix A of the paper, the long-run cost-minimizing amount of capital stock needed to transport the flow Q is:

$$K(Q) = \left(\frac{e\alpha}{r(1-\alpha)}\right)^{1-\alpha} Q^{\beta}, \tag{1}$$

The long-run total cost function is $C(Q) = rK(Q) + eE(Q, K(Q))$ and thus:

$$C(Q) = \frac{r^{\alpha} e^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} Q^{\beta}. \tag{2}$$

<u>Demand</u>

The inverse demand function is: $P(Q) = A\,Q^{-\varepsilon}$, where A is a constant and $1/\varepsilon$ is the absolute value of the price elasticity of demand. Here, it is assumed that: $\varepsilon < 1$ so that the total revenue obtained by a monopolist producing zero output is zero and that $\varepsilon > 1-\beta$ so that the demand schedule always intersects the marginal cost schedule from above.[1]

For notational convenience, we follow Callen et al. (1976) and introduce three parameters: (i) $\gamma \equiv \beta + \varepsilon - 1$, (ii) $\delta \equiv e\beta/[A(1-\varepsilon)(1-\alpha)]$, and (iii) $\eta \equiv \beta - (1-\varepsilon)(1-\alpha)$.

2. The cases of a monopoly and of a social planner

This section briefly reviews the standard outcomes obtained under two polar cases: (i) the profit-maximizing unregulated monopoly that charges a non-discriminatory price; and (ii) the hypothetical case of a welfare-maximizing social planner that behaves so as to maximize the sum of the producers' and consumers' surpluses (i.e., the net social welfare) while ensuring that the firm obtains zero economic profit. The latter case mimics the situation studied in Boiteux (1956).[2]

These two cases can be modeled using the optimization problems presented in Table TA-1. For concision, we omit the straightforward derivations of the first-order conditions and simply report the optimal decisions.

Note that in both cases: (i) the optimal amount of capital stock equals the cost-minimizing amount, that is, $K_M = K(Q_M)$ and $K_a = K(Q_a)$; and (ii) production is cost efficient as the equations $C(Q_M) = rK_M + eE(Q_M, K_M)$ and $C(Q_a) = rK_a + eE(Q_a, K_a)$ hold. Note also that, for the social planner, substitution of the optimal decisions Q_a and K_a in the zero profit condition (5) gives $P(Q_a)Q_a - C(Q_a) = 0$ which means that the output is set at a level such that the price equals the long-run average cost.

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[1] *These restrictions together impose that $1/\varepsilon$ is in the range (1,5.5) which is not a concern in our application.*

[2] *For concision, we omit the first-best solution that consists of solely maximizing the sum of the producers' and consumers' surpluses without paying attention to the firm's profitability. As this first-best solution entails establishing an output level for which price equals the long-run marginal cost, it compels the pipeline operator to operate at a loss, which is not realistic.*

Table TA-1. The optimal decisions taken by a profit-maximizing unregulated monopoly and a welfare-maximizing social planner providing zero profit to the firm

	The unregulated monopoly	The welfare-maximizing planner that provides zero-profits to the firm
Optimization program	$\underset{K,Q}{\text{Max}}\ \Pi_M(Q) = P(Q)Q - rK - eE(Q,K)$ (3)	$\underset{K,Q}{\text{Max}}\ W(Q) = \int_0^Q P(q)dq - rK - eE(Q,K)$ (4) s.t. $P(Q)Q - rK - eE(Q,K) = 0$ (5)
Solution: Output	$Q_M = \left[\dfrac{A(1-\varepsilon)}{\beta} \left(\dfrac{\alpha}{r} \right)^{\alpha} \left(\dfrac{1-\alpha}{e} \right)^{1-\alpha} \right]^{\frac{1}{\gamma}}$ (6)	$Q_a = \left[A \left(\dfrac{\alpha}{r} \right)^{\alpha} \left(\dfrac{1-\alpha}{e} \right)^{1-\alpha} \right]^{\frac{1}{\gamma}}$ (7)
Capital	$K_M = \left(\dfrac{e\alpha}{r(1-\alpha)} \right)^{1-\alpha} (Q_M)^{\beta}$ (8)	$K_a = \left(\dfrac{e\alpha}{r(1-\alpha)} \right)^{1-\alpha} (Q_a)^{\beta}$ (9)

Note: The objective function (3) is the firm's profit, i.e.: the difference between the total revenue $P(Q)Q$ and the sum of the capital cost rK and the energy cost $eE(Q,K)$. The objective function (4) is the net social welfare defined as the sum of the consumer surplus $\int_0^Q P(q)dq - P(Q)Q$ and the producer's surplus $P(Q)Q - rK - eE(Q,K)$. The constraint (5) states that the firm is compelled to obtain zero economic profit.

Callen et al. (1976) define s_M the monopolist's rate of return on invested capital which is the ratio of: the accounting profit derived from the production of the output Q_M (that is: $P(Q_M)Q_M - eE(Q_M, K_M)$), and K_M the profit-maximizing capital stock: $s_M \equiv \left[\beta/(1-\varepsilon) - (1-\alpha) \right] r/\alpha$.

3. Rate-of-return regulation

We now assume that the infrastructure is provided by a private monopoly that is subject to rate-of-return regulation. This section briefly presents the theoretical literature on rate-of-return regulation for the special case of a Cobb-Douglas technology (Klevorick, 1971; Callen et al., 1976). It first reviews the behavior of the regulated monopoly before discussing the identification of a socially desirable rate of return.

3.1 The behavior of the regulated monopoly

The regulated monopoly is allowed to earn a fixed and exogenously-determined rate of return s that is lower than the rate of return s_M obtained by an unregulated monopolist (i.e., $s < s_M$).

The rate-of-return constraint stipulates that the monopoly's accounting profit (i.e., the total revenue $P(Q)Q$ minus $eE(Q,K)$ the cost of the variable input) cannot exceed the allowed return on invested capital sK. As the condition $s < s_M$ holds, the rate-of-return constraint is binding:

$$P(Q)Q - e\,E(Q,K) = s\,K, \qquad\qquad (10)$$

The regulated firm is allowed to choose any combination of inputs (K and E) and output (Q) that jointly verifies the production function equation, and the rate-of-return constraint. Assuming profit maximization, the behavior of the regulated monopoly is thus determined by the following program:

$$\underset{K,Q}{\text{Max}} \quad \Pi(Q) = P(Q)Q - rK - eE(Q,K) \tag{11}$$

$$\text{s.t.} \quad P(Q)Q - eE(Q,K) = sK$$

$$K \geq 0 \ , \ Q \geq 0 \, .$$

If the allowed rate of return is lower than the market cost of capital (i.e., $s < r$), profit maximization involves a corner solution: the firm's optimal decision is to withdraw from the market.

One must thus concentrate on the situation $s \geq r$. As shown in Klevorick (1971), the firm's optimal decisions must jointly verify the rate-of-return constraint (10) and the condition:

$$(s-r)\left[P'(Q)Q + P(Q) - eE_Q(Q,K)\right] = 0, \tag{12}$$

One can first examine the case $s > r$ where the allowed rate of return is larger than the market price of capital. The condition (12) indicates that the marginal revenue $P'(Q)Q + P(Q)$ must equal the regulated marginal cost $eE_Q(Q,K)$ which is the marginal cost of producing an additional unit of output when K is set at the level required to satisfy the rate-of-return constraint (10). Using that condition and the rate-of-return constraint (10), Callen et al., (1976) obtain the optimal decisions (K_R, Q_R) for a Cobb-Douglas production function and then evaluate: C_R the cost incurred by the regulated operator and W_R the net social welfare. Their results are summarized in Table TA-2.

Table TA-2. The optimal decisions taken by a regulated monopoly (case $s > r$)

Output	$Q_R = \left[\dfrac{A\delta - e}{s\delta^{1/\alpha}}\right]^{\alpha/\gamma}$	(13)
Capital	$K_R = \delta^{(1-\alpha)/\alpha} Q_R^{\,\eta/\alpha}$	(14)
Cost	$C_R = r\delta^{(1-\alpha)/\alpha} Q_R^{\,\eta/\alpha} + \dfrac{e}{\delta} Q_R^{\,1-\varepsilon}$	(15)
Net social welfare	$W_R = \dfrac{1}{1-\varepsilon} P(Q_R)Q_R - C_R$	(16)

In the specific case $s = r$, the allowed rate of return equals the market price of capital and the regulated firm is constrained to make at most zero economic profit. Klevorick (1971) highlights that the behavior of the regulated monopoly is indeterminate: the three combinations $(0,0)$, (K_a, Q_a), and

(K_R, Q_R) evaluated with $s = r$ yield zero economic profit. To avoid that indeterminacy, we assume hereafter that the rate effectively implemented by the regulatory authority will be no less than r plus an infinitesimally small and positive increment. This rule imposes the choice of the combination (K_R, Q_R).

3.2 The socially desirable rate of return

Klevorick (1971) and Callen et al. (1976) both examine the determination by a regulator of the fair rate of return s that maximizes the net social welfare given the regulated firm's reactions to that rate. They consider the two-level optimization problem:

$$\underset{s}{\text{Max}} \quad W(s) = \int_0^Q P(q)\,dq \ - \ r\,K \ - \ e\,E(Q,K) \tag{17}$$

$$\text{s.t.} \quad \left[\begin{array}{l} \underset{K,Q}{\text{Max}} \quad \Pi(Q) = P(Q)Q \ - \ r\,K \ - \ e\,E(Q,K) \\[2mm] \quad \text{s.t.} \quad P(Q)Q \ - \ e\,E(Q,K) \ = \ s\,K \\[2mm] \quad \quad \quad K \geq 0 \ , \ Q \geq 0 . \end{array} \right.$$

We let s_R denote the solution to that program. The discussion above has shown that for a given rate of return s with $s_M > s > r$, the unique solution to the lower-level problem is the pair (K_R, Q_R) defined in Table TA-2 which is parameterized by s. Callen et al. (1976) thus reformulate the problem as a single-variable optimization problem:[3]

$$\underset{s}{\text{Max}} \quad W(s) = \int_0^{Q_R(s)} P(q)\,dq \ - \ r\,K_R(s) \ - \ e\,E\big(Q_R(s), K_R(s)\big) . \tag{18}$$

The first-order condition for optimality yields the optimum value of the allowable rate of return s_R:

$$s_R = \frac{\eta^2 r}{\alpha\left[\beta - (1-\alpha)(1-\varepsilon)^2\right]} . \tag{19}$$

Note that, by assumption, the condition $0 < \varepsilon < 1$ holds, so the socially desirable rate of return s_R is lower than s_M the one obtained by the unregulated monopolist.

The rate s_R in (19) is valid if and only if, it verifies $s_R > r$, that is, if the elasticity and technological parameters are such that $\eta^2 > \alpha\left[\beta - (1-\alpha)(1-\varepsilon)^2\right]$. If that is not the case, the authority's best decision is to set s_R equal to r (plus an infinitesimally small and positive increment).

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[3] *Note that this reformulation is rendered possible by their derivation of an analytical solution of the lower-level problem for the specific case of a Cobb-Douglas specification for the production function.*

4. Static comparisons

To assess the performance of rate-of-return regulation, Callen et al. (1976) propose a series of ratios that are detailed in Table TA-3. These ratios are entirely determined by the ratio s/r, the demand elasticity and the technology parameters.

These ratios respectively compare:

- the output levels decided by: a private monopoly Q_M, a social planner applying the average-cost-pricing rule Q_a and a regulated monopoly Q_R;
- the cost C_R incurred by the regulated firm and $C(Q_R)$ the cost that would have been incurred by a cost-minimizing firm producing the same output Q_R to assess the magnitude of the cost-increases caused by the Averch-Johnson effect (Averch and Johnson, 1962).
- the gain in net social welfare resulting from the regulation of a private monopoly $(W_R - W_M)$ versus $(W_a - W_M)$ the gain in net social welfare resulting from the implementation of a social planner applying the average-cost-pricing rule in a monopolistically-controlled industry.

Table TA-3. The performance ratios

Output	$$\frac{Q_R}{Q_a} = \left(\frac{r}{s} \frac{\eta}{(1-\varepsilon)\alpha} \right)^{\alpha/\gamma} \left(\frac{(1-\varepsilon)}{\beta} \right)^{\frac{1}{\gamma}}$$
Output	$$\frac{Q_R}{Q_M} = \left(\frac{r}{s} \frac{\eta}{(1-\varepsilon)\alpha} \right)^{\alpha/\gamma}$$
Cost	$$\frac{C_R}{C(Q_R)} = (1-\alpha)\left[\frac{s}{r} \frac{(1-\varepsilon)\alpha}{\eta} \right]^{\alpha} + \alpha\left[\frac{r}{s} \frac{\eta}{(1-\varepsilon)\alpha} \right]^{1-\alpha}$$
Net social welfare	$$\frac{(W_R - W_M)}{(W_a - W_M)} = \frac{A}{B}$$ Where $$A \equiv \left\{ \begin{array}{l} \left[\frac{1}{1-\varepsilon}\left(\frac{Q_R}{Q_M} \right)^{1-\varepsilon}\left(\frac{1-\varepsilon}{\beta} \right)^{\frac{1-\varepsilon}{\gamma}} - \frac{C_R}{C(Q_R)}\left(\frac{Q_R}{Q_M} \right)^{\beta}\left(\frac{1-\varepsilon}{\beta} \right)^{\frac{\beta}{\gamma}} \right] \\ -\left[\frac{1}{1-\varepsilon}\left(\frac{1-\varepsilon}{\beta} \right)^{\frac{1-\varepsilon}{\gamma}} - \left(\frac{1-\varepsilon}{\beta} \right)^{\frac{\beta}{\gamma}} \right] \end{array} \right\}$$ and $$B \equiv \frac{\varepsilon}{1-\varepsilon} - \left[\frac{1}{1-\varepsilon}\left(\frac{1-\varepsilon}{\beta} \right)^{\frac{1-\varepsilon}{\gamma}} - \left(\frac{1-\varepsilon}{\beta} \right)^{\frac{\beta}{\gamma}} \right]$$

<u>Note:</u> *As the derivation of the ratio $(W_R - W_M)/(W_a - W_M)$ is not detailed in Callen et al. (1976), we briefly explain how it can be reconstructed. The net social welfare W_M and W_a are obtained using the formula: $W = \left[A/(1-\varepsilon)\right]Q^{1-\varepsilon} - C(Q)$.*

Recall that Q_a is the output such that price equals the average cost: $A(Q_a)^{1-\varepsilon} = C(Q_a)$. So, the net social welfare is: $W_a = P(Q_a)Q_a\left[\varepsilon/(1-\varepsilon)\right]$.

Remarking that $Q_M = \sqrt[\gamma]{((1-\varepsilon)/\beta)}Q_a$ and using the relation $A(Q_a)^{1-\varepsilon} = C(Q_a)$, the net social welfare obtained in case of a monopoly is:

$$W_M = P(Q_a)Q_a \cdot \left(\frac{1}{1-\varepsilon}\left(\frac{1-\varepsilon}{\beta}\right)^{\frac{1-\varepsilon}{\gamma}} - \left(\frac{1-\varepsilon}{\beta}\right)^{\frac{\beta}{\gamma}} \right), \tag{20}$$

Under rate-of-return regulation, the net social welfare W_R is defined in (16) and can be rearranged as follows:

$$W_R = \frac{A}{1-\varepsilon}\left(\frac{Q_R}{Q_M}\times\frac{Q_M}{Q_a}\times Q_a\right)^{1-\varepsilon} - \frac{C_R}{C(Q_R)}C\left(\frac{Q_R}{Q_M}\times\frac{Q_M}{Q_a}\times Q_a\right). \tag{21}$$

As the output Q_a is such that $A(Q_a)^{1-\varepsilon} = C(Q_a)$, the net social welfare W_R can be rewritten so as to be directly proportional to the total revenue $P(Q_a)Q_a$ obtained by the firm if average cost pricing is implemented:

$$W_R = P(Q_a)Q_a \cdot \left[\frac{1}{1-\varepsilon}\left(\frac{Q_R}{Q_M}\right)^{1-\varepsilon}\left(\frac{1-\varepsilon}{\beta}\right)^{\frac{1-\varepsilon}{\gamma}} - \frac{C_R}{C(Q_R)}\left(\frac{Q_R}{Q_M}\right)^{\beta}\left(\frac{1-\varepsilon}{\beta}\right)^{\frac{\beta}{\gamma}} \right]. \tag{22}$$

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