11-29-2013

Forward Model for Temperature Derivation from Atmospheric Lidar

Jaren Hobbs
Utah State University

USU PHYSICS 4900

Forward Model for Temperature Derivation from Atmospheric Lidar

Principle Investigator Professor Vincent Wickwar
Atmospheric Lidar Observatory
Center for Atmospheric And Space Sciences
Utah State University

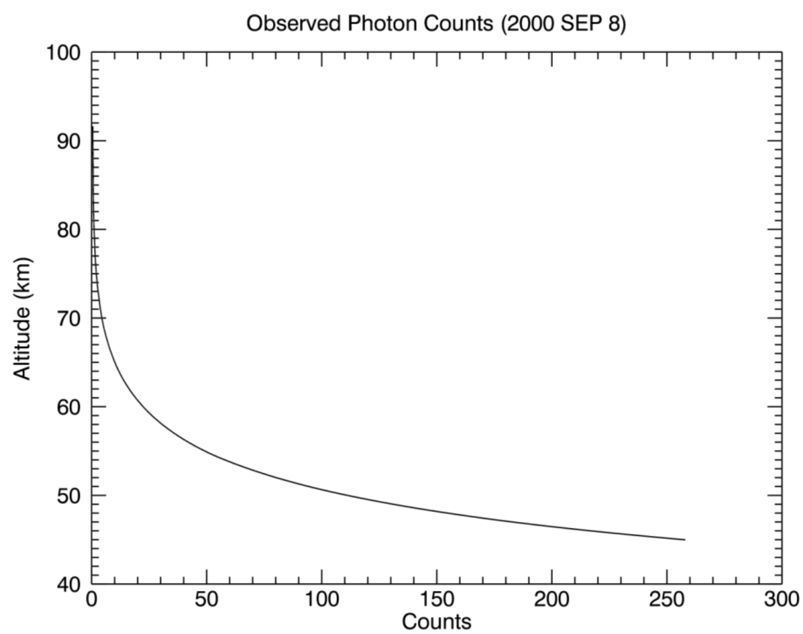
Jaren Hobbs

11/29/2013

Atmospheric Lidar takes advantage of Rayleigh backscattering to create a relative density profile of the atmosphere. The method for temperature derivation is based on the work of Chanin and Hauchecorne (CH). Beginning with an initial temperature, and utilizing the ideal gas law, a downward integration procedure is applied to create a temperature profile from the density profile down to forty-five kilometers. Since this initial temperature is only a best guess, the temperatures towards the top of the profile may not be accurate. However, so long as the guess is reasonable, within perhaps a fifty Kelvin margin (though hopefully not so much), multiple guesses seem to converge after working down fifteen or so kilometers.

The Khanna method attempts to reclaim the uppermost data by applying a forward model using the CH temperatures and working back up from the bottom. The forward model takes a temperature profile and derives a density profile. The Khanna method adjust the CH temperatures until the derived densities converge with the observed densities.

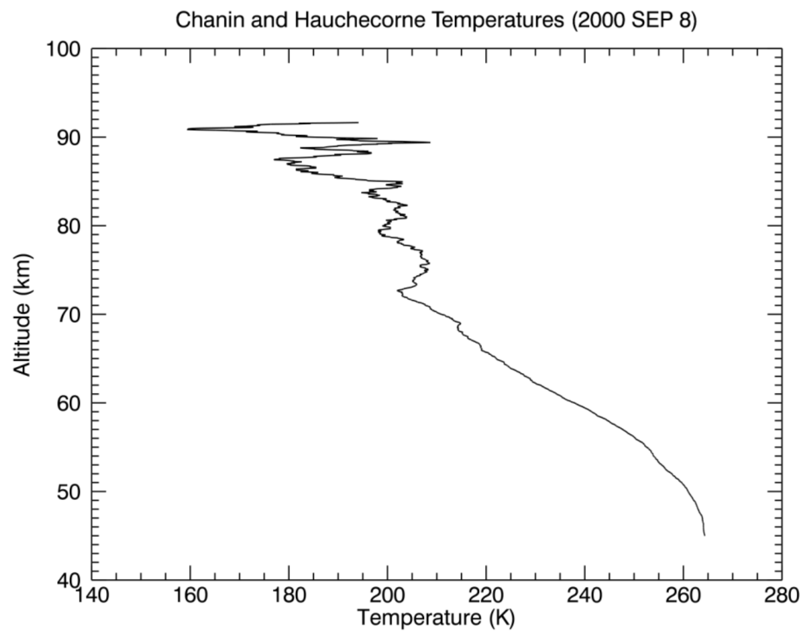
The Lidar system uses a neodymium-doped yttrium aluminum garnet laser which produces 1065nm infrared pulses. The pulses last eight nanoseconds, and are frequency doubled to 532nm, resulting in thirty 2.4 meter long pulses per minute. Light is scattered off nitrogen and oxygen in the atmosphere, some of this is backscattered and collected at the observatory using large mirrors to direct the light into a forty-four centimeter telescope. The light is then focused and directed to a photomultiplier tube, for two minutes the photomultiplier is sampled every 250 nanoseconds resulting in a profile with a measurement resolution of 37.5 meters; 14,000 or 14,007 data points per two minute profile. These profiles are reduced to temperature by an IDL program written by Josh Herron.



Figure[1]

[1] A graph of observed photons counts, after background adjustment, against altitude, from September 8th, 2000.

Herron's program performs a boxcar average on the data over three kilometers. Upper portions of the data are averaged, where backscattering is assumed to have insignificant contribution; this ambient background is then subtracted from the data. The composition of the atmosphere is taken to be relatively constant, and so the photon counts at each altitude bin may be converted into a relative density profile, normalized to 1 at forty-five kilometers. From here the CH technique is applied to produce a temperature profile. Herron's processed data were saved into IDL structures for later access.



Figure[2]

The forward model in the Khanna method takes the lowest observed photon count, and at each altitude scales that value with an expression based on the temperature profile.

$$S(r) = S(r_0)\frac{T(h_0)}{T(h)}\frac{r_0^2}{r^2}\exp\left\{-\frac{1}{k}\int_{h_0}^{h}\frac{m(h')g(h')}{T(h')}dh'\right\}$$

Here, the variables denoted with a '***0***' indicate values at the lowest altitude. This expression will provide a derived photon count at one altitude, ***r***. ***S*** indicates photon (signal) counts; ***T***, temperatures; ***h*** is an altitude related to ***r*** by $\boldsymbol{h} = \boldsymbol{r} + 1466$; ***k*** is Boltzmann's constant; ***m*** is the mean molecular mass (which is taken as constant); and ***g*** is gravity (for which a detailed altitude dependent profile is provided). Initially a single derived photon count profile is generated from the entire CH temperature profile. This is then checked against the observed counts for convergence.

---

[2] A temperature profile pulled from Herron's processed Lidar data, from September 8th, 2000.

Since the atmosphere is not a series of independent points, there must be some means of gauging the effects of one temperature difference varying the derived counts for the entire profile. This is accomplished using a covariance matrix, this matrix represents the way in which atmospheric densities at each altitude are dependent upon the densities at all other altitudes. This is a square matrix of ***M***x***M***, where ***M*** is the number of data points in one, two minute, profile for the whole altitude range. The matrix is symmetrical, $\sigma_{ij}^2 = \sigma_{ji}^2$, and each element is given by the following equation:

$$\sigma_{ij}^2 = \frac{1}{M}\sum_{k=1}^{M}\left(S_{ik} - \overline{S}_i\right)\left(S_{jk} - \overline{S}_j\right)$$

S is again the photon signal count, and ***M*** the total number of data points per profile. Indices ***i*** and ***j*** refer to element positions in the covariance matrix; ***k*** is only a summation index, the summation is performed in its entirety for each element. Although, programmatically, this is not a difficult calculation, running this for the entire covariance matrix is the most time consuming and resource intensive portion of the Khanna method. The covariance matrix is then inverted for use in a χ^2 cost function.

$$\chi^2 = \begin{bmatrix} S_{SL} - S_{SL} \\ \vdots \\ S_{SH} - S_{SH} \end{bmatrix}^T \mathbf{COV}^{-1} \begin{bmatrix} S_{SL} - S_{SL} \\ \vdots \\ S_{SH} - S_{SH} \end{bmatrix} = [\mathbf{S} - \mathbf{S}]^T \mathbf{COV}^{-1} [\mathbf{S} - \mathbf{S}]$$

The function will help estimate the degree of convergence between the observed and derived signal profiles. A vector array is created from the differences between these two profiles, subtracting the observed count at each point $\boldsymbol{S}$ from the derived count **S**, **[S - *S*]** represents this vector, and $[\mathbf{S} - \boldsymbol{S}]^\tau$ its transpose. When applied in the Khanna method, χ^2 does not directly represent convergence, rather it calculated after each temperature adjustment and compared to an earlier calculated χ^2. Convergence is said to occur when the fractional difference $(\chi_{t-1}^2 - \chi_t^2)/\chi_t^2$ is less than some stop value. This has been problematic, and will be addressed later on.

Khanna employs what is called a grid search method for the temperature adjustments. The temperature is adjusted, from the bottom up, for an entire profile by a set amount, ΔT_t . After a full pass through the profile, the index ***t*** is advanced, and the value of ΔT_t is reduced. At each altitude, during a full pass, the program does not advance to the next altitude until χ^2 is minimized, this is not the convergence check. After each adjustment at a single altitude, χ^2 is recalculated and compared the last χ_0^2, if it is less, then χ^2 becomes the new χ_0^2, the adjust is saved, and further adjustment is made.

For temperature adjustments, the program first subtracts ΔT_t from the CH temperature at that altitude, if this results in an increase in χ^2, then the program will add to the CH temperature and check χ^2 again. Once the direction of change is determined, if any change is necessary, then the program will continue to change the temperature until χ^2 increases again. At which point the change resulting in an increase is discarded, and the routine advances to the next altitude. After a full profile is processed for all altitudes, a convergence check is made, and the program either exits, or advances ***t*** and starts again at the bottom.

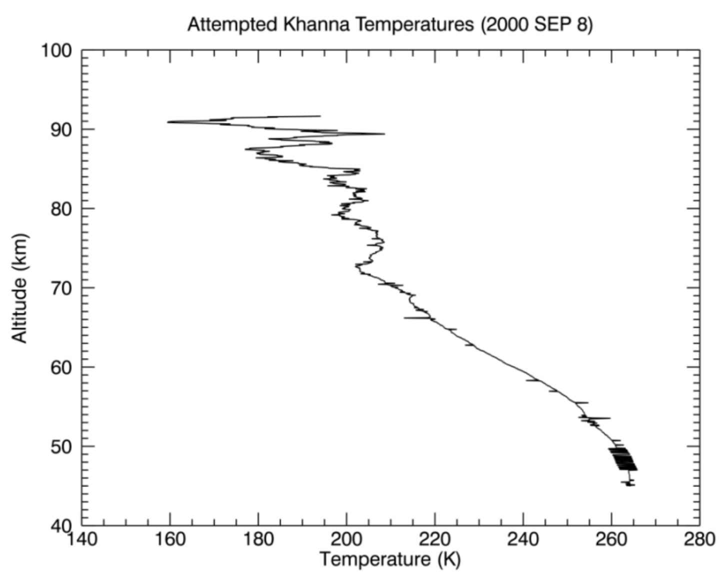
The method of checking convergence, however, has a flaw. As of yet, no work around or better method has been found. If a full iteration $\boldsymbol{t}$ is completed, and no temperature adjustments are made, then $(\chi_{t-1}^2 - \chi_t^2)/\chi_t^2 = 0$ and the program exits. If this occurs at on the second pass, that doesn't mean the count profiles have converged, it just means too large a change has been made. However, if it happens on the tenth pass, convergence may have occurred. So the test cannot require the check to be less than a stop value and greater than zero, as a legitimate case may occur where no further temperature adjustments are necessary.

Also, as described by Khanna, the cost function itself has only a local minimum near the convergence point, it may have a more extreme minimum elsewhere. If this local minimum is jumped over by an adjustment, then the program may run off the temperature by hundreds or thousands of Kelvin until a new minimum is found. To avoid this Khanna recommends inserting a boundary condition on the temperature adjustments. As a formula for this boundary was not proffered, some experimentation with the calculations was necessary. This is what was settled upon:

$$\left|\mathbf{T}_{\mathbf{j}} - \boldsymbol{T}_{\boldsymbol{j}}\right| > \frac{\boldsymbol{t}}{2}\ln{(j+1)}$$

If this condition is met, then the change is discarded and the altitude index, $\boldsymbol{j}$, is advanced. Here $\mathbf{T}$ is the CH temperature and $\boldsymbol{T}$ is the adjusted temperature, each at altitude $\boldsymbol{j}$. While the inclusion of this condition did help somewhat, no apparent convergence could be achieved.

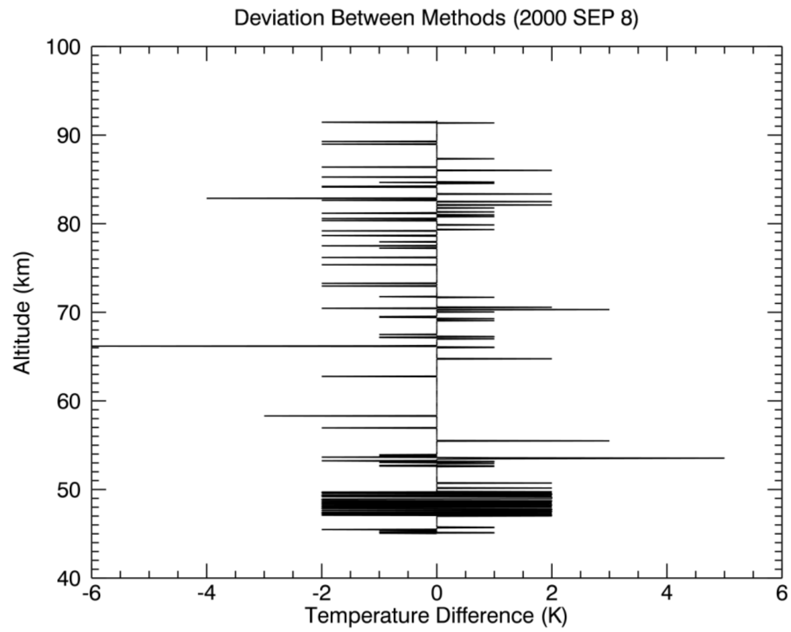
In principle, this method should show excellent agreement with the CH method at low altitudes, and some variation at higher altitudes. And the final signal counts produced should be very nearly the same as the observed counts, for all altitudes. This is not the case with this attempt to emulate Khanna's method.



Figure[3]

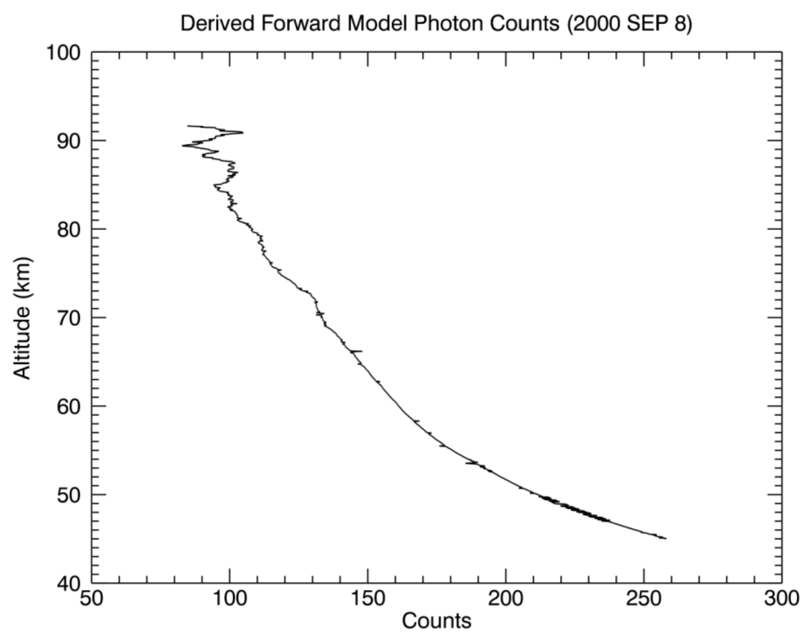
[3] A full temperature profile after the Khanna emulation (KE) processing.

Sharp, dramatic temperature differences occur at some points, and altitudes just above show no change. This is contrary to Khanna's findings, and observations of atmospheric thermodynamics.



Figure[4]

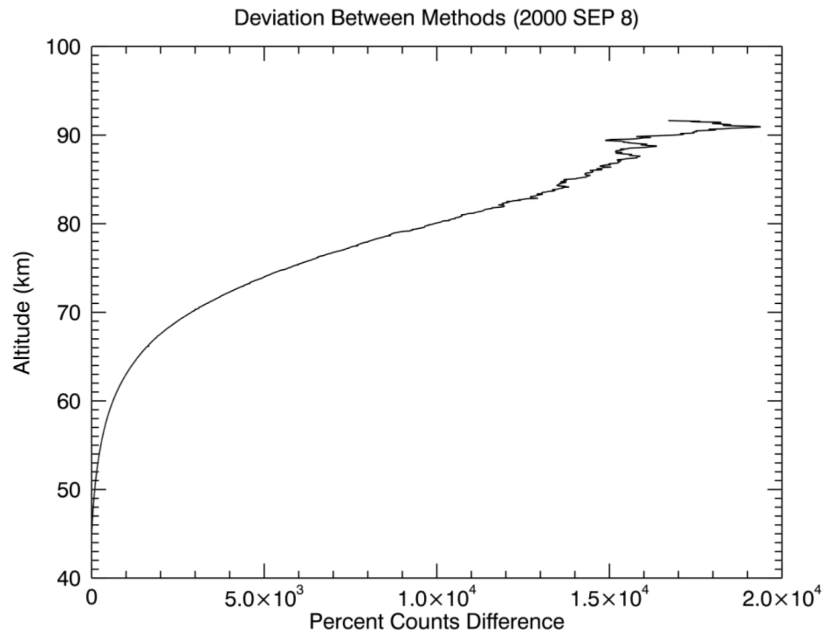
This can be more clearly seen when only the calculated differences are displayed.



Figure[5]

[4] A graph showing the difference between KE and CH temperatures.
[5] Derived photon counts from the KE forward model, after completed processing.

The derived counts profile, from a completed processing, also show inconsistent results. The photon counts should show an exponential decay with increasing altitude.



Figure[6]

This discrepancy is most obvious when in terms of the percent differences, near ninety kilometers, the derived counts are nearly 20,000% greater than the observed count rates.

Although Khanna's method shows very promising results in the published thesis paper, some detailed analysis of this attempt to emulate the work are necessary before any useful data may be derived. Some variations on the iteration order have been attempted, including making only single temperature adjustments at each altitude during a full profile pass, then decreasing the temperature adjustment value and repeating until convergence. This did not significantly change the nature of the results: sharp dramatic changes. It seems the problem may be either with the covariance matrix, or the cost check. However, attempts to remove these lead to order of magnitude differences in temperature.

Khanna, J. (2011). *Atmospheric Temperature Retrievals From Lidar Measurements Using Techniques of Non-Linear Mathematical Inversion.*

Lidar Description. (n.d.). Retrieved from USU Physics: http://www.usu.edu/alo/aboutlidar.htm

Wickwar, V. B. (2013). *Implementing the Khanna et al. Lidar Temperature Analysis.*

[6] The difference between observed photon counts and KE counts, expressed percentages of the observed counts.