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A BIOECONOMIC MODEL OF THE GREAT

SALT LAKE WATERSHED

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A BIOECONOMIC MODEL OF THE GREAT SALT LAKE WATERSHED David Finnoff and Arthur J. Caplan

ABSTRACT

We present a computable general equilibrium model of the interface between the Great Salt Lake (GSL) ecosystem and the regional economy that impacts the ecosystem. With respect to the ecosystem, the model treats the various representative species as net-energy maximizers and bases population dynamics on the period-by-period sizes of surplus net energy. Energy markets—where predators and prey exchange biomass—determine equilibrium energy prices. With respect to the regional economy, we model five production sectors (at the aggregate industry level)—brine cyst harvesters, the mineral-extraction industry, agriculture, recreation, and a composite-good industry—as well as the household sector. By performing dynamic simulations of the joint ecosystem-regional economy model, we isolate the effects of period-byperiod stochastic changes in salinity levels and an initial shock to species-population levels on the ecological and economic variables of the model.

JEL Classification: C68, D58, Q57

Key words: net energy, biomass demand and supply, regional economy, Great Salt Lake

A BIOECONOMIC MODEL OF THE GREAT SALT LAKE WATERSHED¹

1. Introduction

Management of the Great Salt Lake (GSL) watershed has evolved sporadically during the past 150 years—from a state provision in 1850 appropriating two islands in the lake for herding purposes, to a state-sponsored study in 1958 focusing solely on the need for extensive diking to control the lake's intermittent flood levels, to a plan published by the Utah Department of Natural Resources (DNR) in 2000 promoting *inter alia* the maintenance of the lake and its marshes as a critical waterfowl flyway system (DNR, 2000 and Adler, 1999). As Adler (1999) points out, this planning process is perhaps best described as a slow evolution toward a broadly focused, multiple-use plan that nevertheless lacks sufficient research and monitoring, has often been institutionally disjointed, and is inappropriately tethered to the lake's official meander line rather than its watershed. Echoing this sentiment, the DNR has noted that "managers [of the GSL] do not fully understand how reductions in inflows and other water and land uses [within the watershed] will affect population dynamics and species interactions" (DNR, 2000, p. 48). How these effects in turn feedback through the regional economy is even less understood.

This paper demonstrates a new integrated ecological/economic, or bioeconomic, model of the GSL watershed that (a) accounts for the basic ecological relationships and human activities that interact within the lake's watershed and (b) enables the measurement of ecosystem externalities that might occur as a result of "shocks" within the watershed and ecosystem, thereby identifying the degree to which certain species may be threatened. In other words, the

¹The authors thank the Utah Agricultural Experiment Station for the multi-year funding used to complete this project. We also thank participants at the 2004 Utah State University Spring Runoff Conference for their insightful comments.

paper demonstrates one approach that the DNR might use to help it understand the interrelationships between human activity and biological interactions within the GSL watershed and how these interrelationships impact the vulnerability of any given species. The model, patterned closely after Finnoff and Tschirhart (2003 and 2004) (henceforth FT), is based on individual-species behavior directing aggregate outcomes in a multiple-species food web. In effect, a general-equilibrium ecosystem model (GEEM) is combined with a computable general-equilibrium (CGE) model of the regional economy, providing a tight integration of the GSL's ecology and economy.

Our model extends FT in three important respects. First, it incorporates stochastic ecological parameters, such as the salinity and nitrogen balances of the lake at any given point in time. Second, unlike FT's application to a marine ecosystem, the application here is to an inland water body where water in- and outflows are crucial to the health of the ecosystem and economy. The model is therefore an initial attempt at capturing the multi-dimensional effects of human intervention in an ecosystem, rather than solely through the harvesting of a focal species as in the case of a marine environment.² Third, commercial harvesting of the focal species—brine shrimp—is not of the species itself, but rather of its eggs. Thus, harvesting impacts the species' population dynamics in a unique way, which has been heretofore unexplored in the literature.

We find evidence that the GSL ecosystem is stable at current levels of human intervention and at the current level of government regulation of the brine-shrimp industry. However, our results illustrate the extent to which unintended ecological and economic consequences may occur as humans intentionally interact with the ecosystem. We demonstrate these consequences through a simple simulation exercise that is initiated by a series of one-time

²This statement of course abstracts from the effects of global climate change on marine environments.

species-population shocks to the ecosystem in concert with period-by-period stochastic shocks to the lake's salinity and nitrogen balances. The primary goal of this exercise is to demonstrate how the model can be used for management purposes, in order to better understand the specific ways in which water- and land-use changes within the watershed affect population dynamics and species interactions within the GSL ecosystem.

The next section describes the GSL ecosystem and presents the simple ecology underlying the GEEM sub-model. Here, we not only discuss the basic food web within which the various species interact, but also the constrained optimization problems solved by each species on an individual basis. Section 3 describes the CGE regional-economy sub-model and the mechanisms through which humans impact the GSL. In this section, we portray the household- and industry-level optimization problems that motivate these impacts. Section 4 presents results from a simple dynamic simulation of the joint GEEM-CGE bioeconomic model in which the various species encounter alternate 10% positive and negative one-time population shocks. Lake salinity and nitrogen balances are assumed to encounter periodic random shocks drawn from a normal distribution. Section 5 concludes with a summary of our findings and a discussion of future research avenues. The specific parameter values and functional forms used to calibrate our bioeconomic model to the steady-state equilibrium are provided in a technical appendix.

2. The Great Salt Lake Ecosystem

Figure 1 displays a map of the GSL. The ecosystem is estimated to be 3,011 square miles in area, approximately half of which is encompassed by the lake's meander line, while the land area that actually contributes water to the lake (i.e., its watershed) is an estimated 22,000 square

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miles (Adler, 1999; Aldrich and Paul, 2002).³ In size, therefore, the GSL is the largest saline and terminal lake in North America and the fourth largest in the world (Adler, 1999; Arnow and Stephens, 1990). The lake is located within five Utah counties and three-quarters of the state's wetlands are located along its shores (Adler, 1999). More than 50% of the state's 1.8 million people live within 20 miles of its meander line and adjacent wetlands (DNR, 2000).

[INSERT FIGURE 1 HERE]

Approximately 66% of total inflow to the lake is in the form of surface water; the three largest sources being the Bear, Weber, and Jordan Rivers.⁴ These rivers flow into the southern arm of the lake (primarily Gilbert Bay), but incoming freshwater is prevented from mixing with the lake's northern arm (Gunnison Bay) due to a massive east-west railroad causeway constructed in 1902. As a result, only the southern arm maintains a salinity balance conducive to brine-shrimp (the focal species) reproduction. Along with mineral extraction (primarily salt and magnesium), wildlife viewing, and waterfowl hunting, the brine-shrimp industry accounts for a predominant share of the lake's economic value.

There is an extensive literature on the GSL's unique biology and limnology.⁵ Aside from gleaning what statistics are available from this literature for model-calibration purposes, three universally acknowledged characteristics of the GSL have guided the formulation of our

³Historically, the lake has reached an area of approximately 2,300 square miles during flood stage (DNR, 2000).

⁴The remaining inflows are direct precipitation (31%) and groundwater (3%). Estimates of the average annual surface-water inflow from the Bear, Weber, and Jordan Rivers range between 1.9 and 3.7 million acre feet (Stephens and Birdsey, Jr., 2002; DNR, 2000).

⁵Adler (1999), DNR (2000), Gwynn (2002), and Westby (2002) provide extensive overviews of the GSL ecosystem and economy. Gliwicz, et al. (1995), Wurtzbaugh (1995), Belovsky (1996), Belovsky and Mellison (1997 and 1998), Stephens (1997a, 1997b, and 1999), Belovsky, et al. (1999), and Stephens and Birdsey (2002) provide in-depth information on the GSL food web. See Montague, et al. (1982) for further information on the brine shrimp-algae dynamics; Wurtsbaugh and Berry (1990), Rushforth and Felix (1982), Stephens and Gillespie (1976), and Van Auken and McNulty (1973) for further information on the environmental factors affecting algae growth; Wurtsbaugh (1992) for information on the corixid bug; and Cooper, et al. (1984) and Cardell (2001) for information on the eared grebe, one of the lake's most prolific waterbirds.

bioeconomic model. First, while the diversity of species in the lake itself is considered quite low, its biological productivity is extremely high.⁶ In other words, the GSL's food web is simple and capable of supporting dense species populations. Second, the reproductive capability of the brine shrimp is highly sensitive to the lake's salinity and nitrogen balances. These balances are in turn sensitive to anthropogenic activity within the lake's boundaries itself (e.g., through mineral extraction) as well as within the watershed (e.g., through agricultural production and urbanization). Thus, the brine shrimp population contends not only with these indirect impacts, but also with the direct impacts of commercial harvesting and predation by waterfowl and shorebirds. Third, the GSL ecosystem is considered a critical waterfowl flyway system, reflected by its designation as one of 19 habitat sites in the Western Shorebird Reserve Network (Adler, 1999). An estimated 5 to 10 million waterfowl and shorebirds (representing 257 different species) annually utilize the ecosystem's resources for migration and nesting purposes. As a result, wildlife viewing is emerging as one of the GSL's most lucrative industries.

Figure 2 presents a schematic of the ecosystem's basic food web, based primarily on a synthesis of Gliwicz, et al. (1995), Wurtzbaugh (1995), Belovsky (1996), Belovsky and Mellison (1997 and 1998), Stephens (1997a, 1997b, and 1999), Belovsky, et al. (1999), DNR (2000), and Stephens and Birdsey (2002). As with all ecosystems, the sun is the primary energy source. Green algae (*Dunaliella*) obtain energy (and thus biomass) from the sun and are regulated primarily by the inflow of fresh water and nutrient loadings, which determine the lake's nitrogen and salinity balances.⁷ The growth in green algae is believed to be parabolic with respect to salinity level and increasing with respect to the nitrogen balance.

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⁶Although biological diversity is quite low in the lake itself, diversity in the associated wetland, riparian, and terrestrial ecosystems is quite high (Arnow and Stephens, 1990).

⁷Several larger diatom species of algae compete with *Dunaliella* for space at various times of the year. Since brine shrimp are unable to digest these larger-diatom species, we abstract from their existence in the lake.

[INSERT FIGURE 2 HERE]

Both brine flies (*Ephydra cinerea*) and brine shrimp (*Artemia fransiscana*) prey on algae and in turn are preyed upon by waterfowl and shorebirds (designated simply as waterbirds). Corixid bugs (*Trichocorixa verticalis*) also prey on the brine shrimp. Most importantly from an economic standpoint, the brine shrimp produce hard-cased eggs, or cysts, that over-winter to produce the next generation of shrimp.⁸ Cysts are harvested in the fall (under relatively strict governmental regulation) and sold primarily as high-grade prawn feed to large-scale operations in Southeast Asia and Latin America (Isaacson, et al. 2002). Estimated market value of the GSL cysts averaged approximately \$30 million from 1992-1997 and has fluctuated between \$75 million and \$150 million since then (Isaacson, et al., 2002 and *The Salt Lake Tribune*, 2001).

The GSL is similarly rich in mineral deposits, particularly salt and magnesium chloride. Approximately three million tons of mineral products are extracted from the lake annually by six companies, averaging roughly \$220 to \$300 million in aggregate market value (DNR, 2000; Isaacson, et al., 2002; Adler, 1999). Mineral production at its current level results in 95,000-180,000 acre feet of water diverted per year, although if used to their fullest extent, perfected water rights would allow approximately 360,000 acre feet diverted per year (DNR, 2000).

As in FT, our analysis of the GSL ecosystem occurs at the micro level—individual organism behavior drives ecosystem behavior.⁹ The analysis exploits three themes fundamental

⁸As described in Belovsky (1996), Belovsky and Mellison (1997 and 1998), and Belovsky, et al. (1999), the brine-shrimp population dynamics are quite complex. We abstract from this complexity by assuming that over-wintering cysts produce one generation of shrimp in the beginning of the next period (e.g., the spring), which then produce the subsequent generation of cysts at the end of that period (e.g., the fall).

⁹Hannon (1973 and 1976) was the first to adopt the energy maximization approach for a single organism in a partial equilibrium framework.

to economics—rational behavior, efficiency, and equilibrium.¹⁰ Simultaneous solutions for equilibrium "energy prices" and biomass quantities evolve in "energy markets" as a result of the predator-prey interactions between representative "demanders" for and "suppliers" of biomass. Representative organisms are assumed to maximize their energy flow subject to limiting resources, respiration requirements, predator-prey relationships, etc. Maximization yields the organism biomass demands for and supplies to other organisms in the food web. In a general equilibrium, demands and supplies are equal at the species' level. In an economic system, longrun general equilibrium is obtained through entry and exit of firms as they respond to changes in profits. Analogously, long-run general equilibrium is obtained in the ecosystem when species populations are adjusted upward (downward) in response to positive (negative) surplus net energies at the species level.¹¹ To facilitate our discussion of the GSL food web, we assign numbers to each species included in Figure 2 (including the sun) according to Table 1.

[INSERT TABLE 1 HERE]

As mentioned above, each representative organism of a given species i (i = 1,...,6, $i \neq 3$) is a net-energy maximizer, where net energy is defined as the difference between energy inflows and outflows.¹² Consider, for example, an adult brine shrimp's net-energy maximization problem,¹³

$$\begin{array}{l} Max \\ \left\{x_{21}\right\} \ R_{2} = \left[e_{1} - e_{21}\right]x_{21} - e_{2}\left[\left[1 + t_{25}e_{52}\right]y_{25}\left(x_{21}\right) + \left[1 + t_{26}e_{62}\right]y_{26}\left(x_{21}\right)\right] - f_{2}\left(x_{21}\right) - \beta_{2}, \quad (1) \end{array}$$

¹⁰The general equilibrium framework was first proposed by Crocker and Tschirhart (1992).

¹¹Net energy at the species level is simply an aggregation of the individual organisms' net energies across the entire population of organisms.

¹²Because brine-shrimp cysts are in egg form they are assumed completely constrained in net-energy production.

¹³Square brackets indicate that the terms inside the brackets are multiplied by those on the outside. Curved brackets indicate that the variable on the outside of the brackets is a function of the variable(s) included on the inside.

where R_2 is a brine shrimp's net energy measured in power units (e.g., watts or kilocalories) per unit of time, constant e_i is the energy embodied in a unit of species i's biomass (kilocalories/kilogram), and variable e_{ji} is the energy (kilocalories/kilogram) that a member of species *j* must expend to locate, capture, and handle a unit of species *i*'s biomass (i.e., it is the given energy price species *j* "pays" for preying on a unit of species *i*'s biomass, j > i). For example, e_{21} is the energy a shrimp expends in preying on a unit of algae, e_{52} is the energy a waterbird expends in preying on a biomass unit of shrimp, etc. Within the ecosystem energy prices are endogenous, determined by demand and supply interactions explained below.

Variable x_{21} is the biomass (in kilograms/time) transferred to, or demanded by, a shrimp from algae and variable y_{ij} is the biomass transferred from, or supplied by, a member of species *i* to the population of species *j*. For example, y_{25} is the biomass supplied by a shrimp to the waterbird population and y_{26} is the biomass supplied by a shrimp to corixid bugs. Note that y_{25} and y_{26} are strictly increasing, concave functions of x_{21} , implying that as a brine shrimp demands more algae biomass it in turn supplies more of its biomass to its predators.¹⁴

Variable f_2 represents the respiratory energy expended by a shrimp in reproduction, defecation, defense of territory, etc. Since f_2 depends on energy intake, it too is a strictly increasing, concave function of x_{21} . On the other hand, β_2 is a shrimp's constant rate of basal metabolism, which is independent of energy intake. Finally, constant t_{ij} is a "tax rate" on each member of species *i* to account for the energy it expends to avoid being captured by members of species *j*, e.g., t_{25} is a shrimp's tax rate for avoiding waterbirds and t_{26} is its tax rate for avoiding corixid bugs. In the case of brine shrimp, this energy disbursement would be due to schooling

¹⁴Note that because we are modeling a representative individual of each species, we assume without loss of generality that when the individual supplies biomass to its predators it is not completely extinguished. As a result, to obtain an aggregate species-level amount of biomass supplied to predators we simply multiply the representative individual's biomass supply by the total number of individuals in the species. The functionality of this assumption will become clearer in our discussion below.

effort, etc. Note that the total tax paid by a member of species *i* is assumed to increase in the effort expended by members of species *j* to capture it. For example, as waterbirds expend more effort to capture a given supply of shrimp the per-unit energy price of shrimp (e_{52}) increases, thus increasing the tax paid by any given shrimp (in terms of a greater amount of energy that the shrimp expends avoiding capture).

To summarize equation (1), the first term $([e_1 - e_{21}]x_{21})$ represents a shrimp's energy intake, while the sum of the last three terms $(e_2[[1+t_{25}e_{52}]y_{25}(x_{21})+[1+t_{26}e_{62}]y_{26}(x_{21})]$, $f_2(x_{21})$, and β_2) represent energy outflow. The *R* functions for each representative species are similarly described in Table 2. Note that for algae we assume the salinity and nitrogen balances (represented by the cumulative variable S_a and measured as a deviation from the steady-state level) directly affects variable respiration f_i . This seems to reflect general findings in Wurtsbaugh and Berry (1990), Rushforth and Felix (1982), Stephens and Gillespie (1976), and Van Auken and McNulty (1973).¹⁵

[INSERT TABLE 2 HERE]

In solving its net-energy maximization problem (as in equation (1)), each organism sets the marginal energy received from preying on a lower species equal to the sum of (a) the marginal energy lost from being preyed upon by a higher species and (b) marginal respiration. The resulting first-order conditions can be solved for the vector of six equilibrium demands x_{ji} , where each x_{ji} is a function of *inter alia* the entire vector of energy prices e_{ji} .¹⁶ These demand

¹⁵Given that very little is currently known about how the salinity and nitrogen balances interact to affect algae reproduction, no consensus has yet to emerge from the literature.

¹⁶This assumes that the second-order conditions for a maximum are satisfied (see Tschirhart, 2000). Recall that waterbirds have two separate demands - one for brine shrimp (x_{52}) , the other for brine flies (x_{54}) .

expressions may then be substituted into their corresponding y_{ij} supply functions to obtain a vector of traditional supplies as functions of e_{ii} .¹⁷

A short-run equilibrium (within a reproductive period) emerges satisfying two properties. First, aggregate demand and supply are equated in each of six biomass markets (i.e., between each predator-prey pair), resulting in species biomass levels that are consistent with their corresponding market-clearing energy prices. For example, in the brine shrimp-algae biomass market the market-equilibrium condition is expressed as $N_2 x_{21} = N_1 y_{12}$, where N_1 and N_2 represent algae and brine shrimp population levels, respectively. In this case, $N_2 x_{21}$ represents brine shrimp aggregate demand for algae biomass and $N_1 y_{12}$ represents algae aggregate supply of biomass to brine shrimp. The resulting algae biomass level consumed by brine shrimp is consistent with the equilibrium energy price e_{21} .¹⁸ Second, each species population is constant. A representative organism and its species may have nonzero net energy at its maximum, however, a nonzero net energy leads to population changes in the long run (across reproductive periods). Positive(negative) net energy implies greater(lesser) fitness, thus inducing a population increase(decrease).¹⁹ Populations adjust toward a long-run equilibrium in which all individuals have zero net energy and the short-run equilibrium conditions hold (analogous to a competitive economy where the number of firms in an industry changes in accordance with positive or negative profits).

¹⁷Under appropriate assumptions, both x_{ji} and y_{ij} are downward sloping in their corresponding e_{ji} . Note that we do not include a "solar supply" equation for algae, as it is assumed that the sun is an unconstrained resource.

¹⁸In the solar market, where algae "prey" upon the sun, the sun's energy supply is assumed limitless. However, the physical space occupied by the algae is finite, equal to an area represented by A, which is measured in biomass units. If the algae do not fill A, e.g., because of the effects on their variable respiration of a salinity or nitrogen imbalance, then there is no competition for sunlight. As a result, $A > N_I x_{10}$ and $e_{10} = 0$. In a competitive state, which is assumed to exist in this model, space A is filled with algae biomass, thus $A = N_I x_{10}$ and $e_{10} > 0$.

¹⁹See FT for further intuition about the population-adjustment process.

The bioeconomic model ultimately captures this population adjustment through an equation relating next period's population to this period's population of species *i*. To begin, consider how population changes for a top predator such as waterbirds.²⁰ In the steady state it must be the case that births equal deaths. If s_5 is the lifespan of a representative waterbird, then the total number of waterbird births and deaths must be N_{5}/s_5 , with per-capita steady-state birth and death rates of $1/s_5$. Letting the representative waterbird's maximized net energy be given by $R_5(x_{52}, x_{54}; N^t) = R_5^*$, where (a) x_{52} and x_{54} are optimum biomass demands as functions of equilibrium energy prices e_{52} and e_{54} and (b) N^t is a vector of all species' populations, it must be the case that $R_5^* = 0$ in the steady state.

Reproduction requires energy, which is subsumed in the functional form of f_5 . Let v_5^{ss} be a waterbird's steady-state variable respiration and $\rho_5 v_5^{ss}$ be the proportion of this variable respiration devoted to reproduction. Thus, in a steady state the energy given by $\rho_5 v_5^{ss}$ over all members of the waterbird species yields the number of births that exactly offset deaths, i.e., *Births* = *Deaths* $\Box N_5^t [\rho_5 v_5^{ss}] = N_5^t / s_5$, where $\rho_5 = 1/[v_5^{ss} s_5]$ converts reproductive energy into individuals. If the waterbird species is not in the steady state, then $R_5^* \neq 0$ and an individual's variable respiration is v_5 . Assuming that the proportion of R_5^* available for reproduction is the same as that from v_5 , the total energy now available for reproduction is $\rho_5 [R_5^* + v_5]$. Further assuming that reproduction is linear in available energy, it follows from $N_5^t [\rho_5 v_5^{ss}] = N_5^t / s_5$ that $\rho_5 [R_5^* + v_5]$ yields a per-capita birth rate of $[R_5^* + v_5]/[s_5 v_5^{ss}]$. Finally, assuming that the

²⁰A top predator is easier to work with because there are no predation terms in its net-energy expression (such as $e_2 \left[\left[1 + t_{25}e_{52} \right] y_{25} \left(x_{21} \right) + \left[1 + t_{26}e_{62} \right] y_{26} \left(x_{21} \right) \right]$ in (1) for brine shrimp).

death rate is independent of energy available for reproduction, the waterbird population adjustment equation may be written as,

$$N_5^{t+1} = N_5^t + \frac{N_5^t}{s_5} \left[\frac{R_5^* + v_5}{v_5^{ss}} - 1 \right].$$
 (2)

Note that (2) reduces to $N_5^{t+1} = N_5^t$ in the steady state and that $R_5^* > (<)0$ is sufficient for the waterbird population to increase (decrease), i.e., $N_5^{t+1} > (<)N_5^t$, when $v_5 > (<)v_5^{ss}$. Further, because biomass demands depend on the period-*t* populations of each species, the population adjustment for species *i* indirectly depends on the populations of all other species.

If the species is not a top predator and is prey for another species, then in the steady state births equal the sum of deaths and individuals lost to predation. Using brine flies as an example, recall that each individual fly loses $y_{45}(x_{41})$ per period to waterbirds. The summation of all individual losses to predation yields total brine-fly biomass lost to predation, and total biomass divided by an individual brine fly's weight, w_4 , in turn yields the number of individuals lost to predation, i.e., $N_4^t y_{45} (x_{41})/w_4$. Therefore, the steady-state number of births (from respiration energy) equals the sum of deaths from predation and natural mortality net of losses to predation, i.e., $N_4^t \rho_4 v_4^{ss} = N_4^t \left[\left[y_{45} (x_{41}^{ss})/w_4 \right] \left[1 - 1/s_4 \right] + 1/s_4 \right]$, resulting in $\rho_4 = \frac{\left[\left[y_{45} (x_{41}^{ss})/w_4 \right] \left[1 - 1/s_4 \right] + 1/s_4 \right]}{v_4^{ss}}$. Again, assuming that equal proportions of R_4^* and f_4 are available for reproduction, the non-steady-state population-update equation for brine flies

becomes,

$$N_{4}^{t+1} = N_{4}^{t} \left[1 + \left[\frac{\left[\left[y_{45} \left(x_{41}^{ss} \right) / w_{4} \right] \left[1 - 1 / s_{4} \right] + 1 / s_{4} \right]}{v_{4}^{ss}} \right] \left[R_{4}^{*} + v_{4} \right] - \left[\left[y_{45} \left(x_{41}^{ss} \right) / w_{4} \right] \left[1 - 1 / s_{4} \right] + 1 / s_{4} \right] \right], \quad (3)$$

where in the steady state (3) reduces to $N_4^{t+1} = N_4^t$. The population-updating equations for each representative species are provided in Table 3.

[INSERT TABLE 3 HERE]

The brine-shrimp and cyst population-updating equations require further discussion. The cyst population in period t+1, i.e., N_3^{t+1} , equals period-t brine-shrimp births (the first term) net of the period-t cyst harvest, H_3^t (discussed further in Section 3). This updating equation abstracts from two facets of brine-shrimp and cyst population dynamics. First, two to three generations of brine shrimp are typically reproduced by ovoviviparity in a single season (from spring to midsummer) prior to the reproduction of a single generation of cysts by oviparity at the end of the season (late summer to late fall). We have avoided modeling the process of ovoviviparity reproduction, as this would unnecessarily complicate the model. Second, a fraction of unharvested cysts do not survive the winter to hatch into next season's first generation of brine shrimp due inter alia to being washed up on shore and desiccating. However, scant empirical information about the over-wintering process is presently available, thus precluding us from explicitly accounting for over-wintering survivability in our cyst population-updating equation. With respect to the brine-shrimp population-updating equation, we assume that period t+1's population, i.e., N_2^{t+1} , is the sum of N_3^{t+1} and the brine-shrimp steady-state population, N_2^{ss} . This is an 'accounting convention' that maintains a consistency between our estimate of the steadystate population and subsequent populations that arise in future periods through model simulation.

3. The GSL Regional Economy

Our CGE model of the GSL regional economy closely follows FT. The FT approach is "myopically dynamic" in that it consists of a sequence of static optimizations resulting in a dynamic equilibrium where the sequences are linked through the evolution of factor stocks and household saving.²¹ Households are intertemporal utility-maximizers making savings decisions (for the purpose of future consumption) based on myopic expectations about future prices. Current consumption is over regionally produced goods and imports of a composite good (discussed below), given prices. The savings process consists of households instantaneously purchasing investment goods with their savings to augment their capital endowments for future periods. Balanced growth is assumed to occur when the capital stock and labor force grow at the same rate. Finally, incomes are endogenously derived from (a) the sale/rental to firms of the household's (homogeneous and perfectly mobile between domestic industries) labor, land, and capital endowments (the latter being partially determined by savings) and (b) government revenue obtained through the sale of water rights to the mineral-extraction and agricultural industries (discussed below). The resulting household product demands and factor supplies satisfy the neoclassical tenants of non-negativity, continuity, and dependence solely on relative prices.

Production in the regional economy occurs at a high level of aggregation in five (singleproduct) production sectors: the brine-shrimp fishery, recreation/wildlife-viewing, a composite good, and the mineral-extraction and agricultural sectors. Individual firms, for simplicity aggregated at the industry level, are myopic, static profit maximizers operating under constant returns-to-scale and given prices. They purchase labor and capital from households to produce differentiated output that is allocated between domestic and export markets given endogenous

²¹The FT approach is in turn predicated on Ballard, et al. (1985), de Melo and Tarr (1992), and Burniaux, et al. (1991).

domestic and export prices and Armington (1969) imperfect-substitution possibilities between the two markets. In addition to labor and capital, agricultural and mineral-extraction firms purchase water allocations determined by the DNR, agricultural firms also purchase land from households, and cyst-harvesting firms purchase certificates of registration (i.e., quotas) from the DNR.²² Although each sector engages in production for both domestic and export markets, only the composite-good sector is assumed to compete with an imperfectly substitutable import, which is resold to the households. Similar to the households, firms' factor demands and output supplies satisfy the neoclassical tenants of equality between output prices and unit costs and between given factor prices and marginal value products.

The economic system is in general equilibrium when all individuals of all sectors optimize, there exists a set of prices and output levels consistent with zero profits for all firms, and all markets clear. Given the set of market-clearing prices, consumer expenditure exhausts current disposable income to maintain Walras Law, and trade balances in the current account. The final requirement of the static single-period economic model is that it replicates an assumed equilibrium benchmark dataset through model parameterization known as calibration. When the parameterized model is run with the benchmark dataset a general equilibrium for the economy is obtained. The benchmark dataset is presented in the technical appendix.

3.1. The Brine-Shrimp Fishery

The brine-shrimp fishery is modeled as a single, vertically-integrated industry, assumed to encapsulate cyst harvesting, processing, and marketing. As the fishery is heavily regulated by

²²For simplicity, we assume that the firms do not produce intermediate goods for sale as inputs across or within industries. Mineral-extraction and cyst-harvesting firms pay royalties to the government based on quantities sold, which we subsume in output prices. While revenues from the sale of water allocations are redistributed to households, revenues from the sale of certificates of registration are retained by the DNR in order to fund its regulatory activities.

the DNR, we derive a "regulated open-access equilibrium" following FT. The regulatory instrument is total allowable catch (TAC).²³ Given its *TAC*, the fishery makes a three-tiered decision each period concerning (a) the amounts of labor, L_f , and capital, K_f , to demand from the household sector in order to harvest cysts at level Q_f , (b) the proportion of Q_f that is exported, and (c) the amount of investment (I_f) to "supply" to the domestic household sector.²⁴ We assume a Cobb-Douglas production function for Q_f according to,

$$Q_f = d_f L_f^{a_f} K_f^{b_f} N_3^{c_f}$$
(4)

where parameters a_f , b_f , and c_f are each less than one and $a_f + b_f + c_f \le 1$, d_f is a Hicks-neutral technology parameter, and N_3 is the cyst population as defined above. Given economy-wide wage (w) and rental (r), rates, the cost-minimizing ratio of L_f and K_f is therefore obtained from,

$$\frac{w}{r} = \frac{a_f K_f}{b_f L_f}.$$
(5)

The fishery also abides by the following conditions,

$$Q_f = TAC(N_3) = q_f^d \left(p_f^d, p_f^e \right) + q_f^e \left(p_f^d, p_f^e \right)$$
(6a)

$$I_f = a_f^I Q_f \tag{6b}$$

$$P_f Q_f = w L_f + r K_f + COR \tag{6c}$$

where a_f^I is a (given) proportionality factor equal to the ratio of the steady-state levels of I_f and domestic quantity of cysts supplied (q_f^d) , q_f^e is quantity of cysts exported, p_f^d and p_f^e are the respective per-unit prices of q_f^d and q_f^e , P_f is the composite price of p_f^d and p_f^e , and *COR* is the

²³The DNR regularly measures brine-shrimp density in the lake during the cyst-harvesting season. Whenever the estimated density falls below a threshold of 21 cysts per liter, cyst harvesting is suspended until the density increases beyond the threshold (DNR, 2000). Therefore, the DNR implicitly determines a TAC each season.

 $^{^{24}}Q_f$ is the same as H'_3 from Section 2. For simplicity, we assume that I_f is measured in cyst units.

fixed certificate-of-registration cost.²⁵ Equation (6a) states that the cyst harvest (a) equals the total allowable catch for each period, which is ultimately a function of the cyst population in that period, and (b) is divided between the domestic and export markets. This division of Q_f between q_f^d and q_f^e is determined by a (constrained) revenue maximization problem based on the Armington (1969) assumption of imperfect substitutability, which, similar to (5), defines the optimal ratio of q_f^d and q_f^e as a ratio of p_f^d and p_f^e . Equation (6b) states that the effective proportion of the industry "supplied" to households in the form of investment (which in turn adds to the value of the household-sector's capital endowment) is a constant proportion of Q_f , and (6c) is a zero-profit condition.

3.2. The Mineral-Extraction Sector

Similar to the brine-shrimp fishery, we assume a Cobb-Douglas production function for the total quantity of minerals extracted, Q_m , according to,

$$Q_m = d_m L_m^{a_m} K_m^{b_m} W_m^{c_m}, (7)$$

where parameters a_f , b_f , c_f , and d_f and variables L_m and K_m are defined analogously to the brineshrimp fishery's, optimal ratio of L_m and K_m is determined analogously to (5), and W_m is a fixed water allocation determined by the DNR. The mineral-extraction sector likewise abides by equations analogous to (6a)-(6c), obviously without a regulatory limit such as *TAC* in (6a) and with $P_W W_m$ replacing *COR* in (6c), where the per-unit price of water P_W is set equal to zero.²⁶

²⁵Thus, $q_f^d - I_f$ equals the quantity of cysts sold in the domestic commodity market. For the model simulations performed in Section 4 we assume that cysts are sold solely in the export market, which reflects the fact that GSL cysts are currently marketed exclusively outside of the GSL watershed. Thus, $q_f^d = I_f$ and p_f^d is undefined for this problem.

²⁶Under Utah water law, the mineral-extraction and agricultural industries have historically been provided with free water allocations. The model can easily be modified if the law is changed in the future such that these industries are required to pay for their water allocations on a per-unit or block basis.

3.3. The Agricultural Sector

Similar to the mineral-extraction sector, we assume a Cobb-Douglas production function for the total quantity of agricultural goods produced, Q_a , according to,

$$Q_a = d_a L_a^{a_a} K_a^{b_a} W_a^{c_a} T_a^{g_a} , (8)$$

where parameters a_a , b_a , c_a , and d_a and variables L_a , K_a , and W_a are defined analogously to the mineral-extraction industry's. The variable T_a is the amount of land devoted to agricultural production, with parameter $g_a > 0$ such that $a_a + b_a + c_a + g_a \le 1$. Because T_a is effectively a choice variable of the industry, the optimal ratios of L_a , K_a , and T_a require the simultaneous solution of two conditions – one analogous to (5) and the other,

$$\frac{P_T}{r} = \frac{g_a K_a}{b_a T_a} \tag{9}$$

where P_T is the per-unit price of land.

The agricultural sector likewise abides by equations analogous to (6a)-(6c), again without a regulatory limit such as *TAC* in (6a) and with $P_W W_a$ replacing *COR* in (6c), where the per-unit price of water P_W is set equal to zero. Also, the total cost of land, i.e., $P_T T_a$, is included on the right-hand side of analogous (6c).

3.4. The Recreation/Wildlife-Viewing Sector

Similar to the previous sectors, we assume a Cobb-Douglas production function for the total quantity of recreation and wildlife viewing, Q_R , according to,

$$Q_{R} = d_{R} L_{R}^{a_{R}} K_{R}^{b_{R}} N_{5}^{c_{R}}, \qquad (10)$$

where parameters a_R , b_R , c_R , and d_R and variables L_R and K_R , are defined analogously to the brine-shrimp fishery's. An analogous equation (5) determines the cost-minimizing ratio of L_R

and K_R , and analogous equations (6a)-(6c), obviously without a regulatory limit such as *TAC* in (6a) and without a fixed cost such as *COR* in (6c), are also satisfied.

3.5. The Composite-Good Sector

The composite-good sector is modeled slightly differently from the previous sectors, due in part to the role of imports in this sector. Following FT, we assume that imports of the composite good is the residual difference between what domestic households demand overall and what is produced by the domestic composite-good sector. Thus, the domestic composite-good sector effectively imports the foreign-produced composite-good for re-sale to households at zero cost and mark-up. Similar to the other sectors, the composite-good sector abides by conditions analogous to (6a) for determining the proportions of domestic production allocated to the domestic and export markets, (6b) for determining the proportion of domestic production supplied to households in the form of investment, and (6c) for zero profits.

We assume a constant-elasticity-of-substitution (CES) cost function to ultimately determine the total quantity of the domestically produced composite good, Q_C , according to,

$$TC_{c} = \frac{Q_{C}}{\phi_{C}} \left[\delta_{C} w^{(1-\sigma_{C})} + \left[1 - \delta_{C} \right] r^{(1-\sigma_{C})} \right]^{\frac{1}{(1-\sigma_{C})}}$$
(11)

where TC_C is total cost of production, ϕ_C is an efficiency parameter, $0 < \delta_C < 1$ is a distribution parameter, and σ_C is the partial elasticity of substitution. Application of Shepard's Lemma to (11) results in the sector factor demand functions for labor (L_C) and capital (K_C), respectively,

$$L_{C} = \left(\frac{Q_{C}}{\phi_{C}}\right)^{(1-\sigma_{C})} \left[TC_{C}\left[\frac{\delta_{C}}{w}\right]\right]^{\sigma_{C}}$$
(12a)
$$K_{C} = \left(\frac{Q_{C}}{\phi_{C}}\right)^{(1-\sigma_{C})} \left[TC_{C}\left[\frac{(1-\delta_{C})}{r}\right]\right]^{\sigma_{C}}.$$
(12b)

Rather than derive Q_C from the (dual) production function associated with (11), we obtain its value directly from the household sector's utility-maximization problem, to which we now turn.

3.6. The Household Sector

The household sector consumes goods from the five producing sectors and saves for future consumption.²⁷ Following FT, the sector derives gross income (*Y*) from the sale/rental of its current labor and capital endowment (described further in Section 3.7). Given *Y*, household behavior is modeled according to a tri-level nesting structure. In the top nest, an allocation is made between composite consumption today (C_T) and composite future consumption (C_F) given composite prices P_{C_T} and P_{C_F} , respectively. In terms of household inter-temporal behavior, C_F is funded through the stock of household savings (*S*), costing, or valued at, P_S per unit. Savings decisions are based on expected increments to a stream of consumption in future periods (e.g., C_I, C_2, \ldots .), with C_F being a composite measure. Consumer expectations of future consumption are assumed to be myopic, in that current prices, P_{C_T} , are expected to remain constant in all future periods, i.e., from the household's standpoint $P_{C_T} = P_{C_F}$ in each period. P_SS is used to purchase investment goods I (e.g., I_f in (6b) from the brine-shrimp fishery), which add to the stock of household capital to be used for future consumption.

The transformation of household savings into capital services is governed by the identity $P_S S = r\gamma S$, where γ is the initial real rate of return associated with the benchmark value of r (i.e., the proportion of savings translated into capital services in future periods). Household income derived from capital services (i.e., $r\gamma S$) in turn allows future consumption according to the

²⁷By "goods" we mean both the output and the investments "supplied" by the sectors, e.g., as depicted in (6a) and (6b) for the brine-shrimp fishery.

identity $P_{C_T}C_F = r\gamma S$. Manipulation of this expression equates the value of savings to the present value of expected future consumption, i.e.,

$$P_S S = \frac{P_S P_{C_T}}{r \gamma} C_F \,. \tag{13}$$

Therefore, the household's top-nest utility-maximization problem in any given period is,

$$\underset{\left\{C_{T},C_{F},S\right\}}{Max} \quad U_{TN} = \left[\alpha_{TN}^{\left(\frac{1}{\sigma_{TN}}\right)}C_{T}^{\left(\frac{(\sigma_{TN}-1)}{\sigma_{TN}}\right)} + \left[1-\alpha_{TN}\right]^{\left(\frac{1}{\sigma_{TN}}\right)}C_{F}^{\left(\frac{(\sigma_{TN}-1)}{\sigma_{TN}}\right)}\right]^{\left(\frac{\sigma_{TN}}{(\sigma_{TN}-1)}\right)}$$
(14)

subject to,

$$Y = P_{C_T} C_T + \frac{P_S P_{C_T}}{r \gamma} C_F$$
(15a)

$$P_{C_T} = \left[\left[1 - \beta_{TN} \right] \overline{P}^{(1 - \nu_{TN})} + \beta_{TN} P_R^{(1 - \nu_{TN})} \right]^{\left[\frac{1}{(1 - \nu_{TN})} \right]}$$
(15b)

where $0 < \alpha_{TN} < 1$ and $0 < \beta_{TN} < 1$ are distribution parameters; σ_{TN} and ν_{TN} are partial elasticities of substitution; \overline{P} is a per-unit composite price of the mineral-extraction, agricultural, and composite goods; and P_R is the per-unit price of the recreation/wildlife-viewing good. Equation (14) indicates that utility in the top nest is determined by a CES function defined over current and future composite consumption. Equation (15a) is the household budget constraint and (15b) indicates that P_{C_T} is a CES weighted average of (a) a composite price of the mineralextraction, agricultural, and composite goods and (b) the price of the recreation/wildlife-viewing good.

From this problem, the first-order optimality condition is obtained for the stock of household savings,

$$S = \frac{\left[1 - \alpha_{TN}\right]Y}{\left(P_{S}^{\sigma_{TN}}\left[\frac{P_{C_{T}}}{r\gamma}\right]^{(\sigma_{TN}-1)}\left[\alpha_{TN}P_{C_{T}}^{(1-\sigma_{TN})} + \left[1 - \alpha_{TN}\right]\left[\frac{P_{S}P_{C_{T}}}{r\gamma}\right]^{(1-\sigma_{TN})}\right]\right)}$$
(16)

In the second nest, income for current consumption (C_T) is divided between expenditures on recreation (X_R) at price P_R and the composite consumption commodity (\overline{X}) that encompasses the mineral-extraction, agricultural, and composite goods at price \overline{P} . Similar to its top next problem, the household sector's second-nest utility-maximization problem is,

$$\begin{cases} Max \\ \{X_R, \overline{X}\} \end{cases} \quad U_{SN} = \left[\alpha_{SN}^{\left(\frac{1}{\sigma_{SN}}\right)} X_R^{\left(\frac{(\sigma_{SN}-1)}{\sigma_{SN}}\right)} + \left[1 - \alpha_{SN}\right]^{\left(\frac{1}{\sigma_{SN}}\right)} \overline{X}^{\left(\frac{(\sigma_{SN}-1)}{\sigma_{SN}}\right)} \right]^{\left(\frac{\sigma_{SN}}{(\sigma_{SN}-1)}\right)}$$
(17)

subject to,

$$Y - P_s S = \overline{PX} + P_R X_R \tag{18}$$

where $0 < \alpha_{SN} < 1$ is a distribution parameter and σ_{SN} is a partial elasticity of substitution. The respective first-order optimality conditions for the recreation/wildlife-viewing and composite consumption commodities are,

$$X_{R} = \frac{\left(\alpha_{SN} \left[Y - P_{S}S\right]\right)}{\left(P_{R}^{\sigma_{SN}} \left[\alpha_{SN} P_{R}^{(1-\sigma_{SN})} + \left[1 - \alpha_{SN}\right]\overline{P}^{(1-\sigma_{SN})}\right]\right)}$$
(19a)

$$\overline{X} = \frac{\left(\left[1 - \alpha_{SN} \right] \left[Y - P_S S \right] \right)}{\left(\overline{P}^{\sigma_{SN}} \left[\alpha_{SN} P_R^{(1 - \sigma_{SN})} + \left[1 - \alpha_{SN} \right] \overline{P}^{(1 - \sigma_{SN})} \right] \right)}.$$
(19b)

Finally, in the third nest income is divided between expenditures on the mineralextraction (X_m) , agricultural (X_a) , and composite (X_c) goods at prices P_m , P_a , and P_c , respectively. Following Ballard, et al. (1985), the household sector's third-nest subutility-maximization problem is,

$$\frac{Max}{\{X_m, X_a, X_c\}} \quad \overline{\mathbf{X}} = X_m^{\lambda_m} X_a^{\lambda_a} X_c^{\lambda_c} \tag{20}$$

subject to,

$$Y - P_S S - P_R X_R = P_m X_m + P_a X_a + P_c X_c$$
⁽²¹⁾

where λ_i , i = m, a, c are the Cobb-Douglas expenditure shares. The respective first-order optimality conditions for this problem are,

$$X_i = \frac{\lambda_i \left[Y - P_S S - P_R X_R \right]}{P_i}, \quad i = m, a, c.$$

$$(22)$$

Combining equations (20)-(22) results in the following definition of \overline{P} ,

$$\overline{P} = \coprod_{i} \left(\frac{P_{i}}{\lambda_{i}} \right)^{\lambda_{i}}, \ i = m, a, c .$$
(23)

As Ballard, et al. (1985) point out, an especially convenient property of this kind of Cobb-Douglas price index is that the composite price can be calculated without knowing X_{i} , i = m, a, c, thus simplifying our calculations considerably.

3.7. The Market-Clearing and Endowment-Updating Equations

We begin this section by defining the composite price indices for each of the production sectors, reflecting the fact that the indices are weighted averages of domestic and foreign prices. In the composite-good sector, the domestic price faced by households (P_c) is,

$$P_{c} = \frac{\left(p_{c}^{d}q_{c}^{d} + p_{c}^{IM}q_{c}^{IM}\right)}{\left(q_{c}^{d} + q_{c}^{IM}\right)}$$
(24)

where p_c^d and p_c^M are the domestically determined and exogenous import prices, respectively, and q_c^d and q_c^M are corresponding quantities. As shown in FT, the prices p_c^d and p_c^M are taken by the household sector in a CES cost-minimization problem which determines its optimal mix of the domestically produced and imported composite goods according to,

$$\frac{q_c^d}{q_c^{IM}} = \left(\left[\frac{p_c^{IM}}{p_c^d} \right] \left[\frac{1 - \delta^c}{\delta^c} \right] \right)^{\sigma^c}$$
(25)

where δ^{e} is a distributional share parameter in a CES transformation function and σ^{e} is an associated elasticity of transformation between domestically produced and imported composite goods.²⁸

The domestic prices faced by each production sector (and, except for the composite good, by the household sector) are,

$$P_{i} = \frac{\left(p_{i}^{d}q_{i}^{d} + p_{i}^{e}q_{i}^{e}\right)}{Q_{i}}, \quad i = f, m, a, R$$
(26a)

$$P_{c}^{'} = \frac{\left(p_{c}^{d}q_{c}^{d} + p_{c}^{e}q_{c}^{e}\right)}{Q_{c}}.$$
(26b)

where, again, $Q_i = q_i^d + q_i^f$, i = f, *m*, *a*, *R*, *c* and the remaining variables were defined previously in this section. The domestic market-clearing conditions are,

$$q_i^d = I_i + X_i, \ i = f, m, a, R, c.,$$
 (27)

which are used to determine X_{i} , i = f, m, a, R, c.²⁹

In terms of the household sector's income balance, the following condition holds by definition,

$$Y = w \sum_{i} L_{i} + r \sum_{i} K_{i} + P_{T} T_{a} + P_{W} \sum_{j} W_{j} + COR, \quad i = f, m, a, R, c, \quad j = m, a.$$
(28)

²⁸Note that this condition presupposes market clearing.

²⁹Note that the p_i^d , i = f, m, a, R, c, are determined by $Q_i = q_i^d + q_i^f$, i = f, m, a, R, c.

and with respect to the economy's balance of payments we have the identity,

$$P_{S}S - \sum_{i} p_{i}^{d}I_{i} = p_{c}^{IM}q_{c}^{IM} - \sum_{i} p_{i}^{e}q_{i}^{e}, \quad i = f, m, a, R, c,$$
⁽²⁹⁾

where the left-hand side represents the savings-investment balance and the right-hand side represents the current-account balance.

Finally, household endowments of capital, labor, land, and water are updated per period according to the following series of equations,³⁰

$$\sum_{i} K_{i}^{t+1} = \sum_{i} K_{i}^{t} + \gamma S^{t}, \quad i = f, m, a, R, c$$
(30a)

$$\sum_{i} L_{i}^{t+1} = (1+n) \sum_{i} L_{i}^{t}, \quad i = f, m, a, R, c$$
(30b)

$$\sum_{j} W_{j}^{t+1} = \sum_{j} W_{j}^{t}, \ j = m, a$$
(30c)

$$T_a^{t+1} = T_a^t \,. \tag{30d}$$

Equation (30a) states that the capital endowment in period t+1 equals the capital endowment in period t plus the (real) growth in period t's "stock" of savings, where again γ represents the proportion of savings translated into capital services. Equation (30b) states that the labor endowment in period t+1 equals the labor endowment in period t plus the growth in labor at the

rate *n*. Rate $n = \frac{\gamma S^{ss}}{\sum_{i} K_{i}^{ss}}$, i = f, m, a, R, c, i.e., the rate at which labor would have to grow in the

steady state to ensure that the capital-labor ratio remains constant, where the superscript *ss* indicates a steady-state level.

4. Simulation Results

Our simulation results are based on initial one-time shocks to the species' steady-state population levels. Specifically, we shock the populations of algae, brine flies, and corixid bugs

³⁰Of course price changes through time also impact the value of these endowments.

downward by 10% each, and simultaneously shock the populations of brine shrimp and waterbirds upward by 10% each. These arbitrary shocks are merely to demonstrate how the bioeconomic model updates these population levels, as well as the remaining ecological and economic variables, over time (for the next 100 periods). In addition to these one-time shocks, recall that the salinity level is also being "hit" with period-by-period random shocks (see the Technical Appendix for further details).

Figures 3 and 4 present the simulation results for the algae and brine cyst populations.³¹ Beginning with the algae population note that following the population shocks, algae density (the blue(pink) line without(with) random salinity shocks) returns rather smoothly to a steady state after approximately 10 periods. This steady state is significantly beneath the predicted steady state without human interventions (green line). For this exercise, ecological and economic steady states were determined in isolation from one another, although the data used to calibrate the model is drawn from a point in time where the two sub-models are obviously interacting. Thus, the distance between the green and the blue/pink lines can also be interpreted as the model's degree of error in calibrating the joint model to an overall steady state. In our future work, where we calibrate the model using more actual data, we expect this error to shrink. In the meantime, the steady state established by the blue/pink line is a better approximation to the actual joint steady state of the GSL and the regional economy.

[INSERT FIGURE 3 HERE]

A similar smooth transition to the steady state for the brine cyst population is depicted in Figure 4. Note, however, that the transition occurs more rapidly, after approximately five years. This result is driven by the fact that as the algae population falls and the brine shrimp population

³¹Due to space restrictions, we are unable to present the simulation results for the other species. These are available upon request from the authors.

increases, the brine shrimp must compete for a diminished food source that now carries with it an increased energy price. The representative brine shrimp "responds" to this higher energy price by reducing the variable respiration available for reproduction and devoting more energy to searching for algae biomass. As a result, not only does the representative brine shrimp reduce its production of brine cysts, but the brine-shrimp population itself shrinks relatively quickly back to its steady-state level. These two reactions lead to a decrease in the brine cyst population, but as the brine shrimp population quickly recovers, so too does the cyst population.

[INSERT FIGURE 4 HERE]

With respect to the path of the regional economy, Figures 5 and 6 present the transition paths for the brine-shrimp fishery capital stock and household demand for recreation/wildlifeviewing. In Figure 5, the brine-shrimp fishery responds to the initial positive(negative) shock to the brine-shrimp(algae) population (which translates into an initial decrease in the cyst population) by decreasing its capital stock. This implies an initial shift of investment out of the brine-shrimp industry. However, following the recovery of the cyst population to its steady-state level, the capital stock returns to its steady-state level rather quickly. Also evidenced in Figure 5 is the slight oscillation of the capital stock around its steady-state value over time, which similarly mirrors the slight oscillations evident in the cyst and brine-shrimp populations over time as well.

[INSERT FIGURE 5 HERE]

With respect to household demand for recreation/wildlife-viewing, we note that the household sector's demand rises steadily over time from its steady-state value of \$16 million per period. Two forces account for this steady increase in demand. First, the initial increase in the waterbird population provides an initial *ceteris paribus* positive shock to the household sector's

utility. Given that recreation/wildlife-viewing is a normal good, this provides a *ceteris paribus* boost to household demand. Further, since it is assumed that labor supply increases at a constant rate of return (n), household-sector income increases overtime as well. This increase has a multiplier effect on income, since savings increases with income, enabling a higher level of future consumption of all commodities.

[INSERT FIGURE 6 HERE]

5. Conclusions

This paper demonstrates a new technique for modeling the "bioeconomics" of a watershed, in particular the Great Salt Lake (GSL) watershed located in north-central Utah. The bioeconomic model accounts for the basic ecological relationships and human activities that interact within the lake's watershed and enables the measurement of ecosystem externalities that might occur as a result of "shocks" within the watershed and ecosystem, thereby identifying the degree to which certain species may be threatened. Since the regional economy is premised on a household-sector utility-maximization problem, the model is ultimately capable of estimating compensating-variation welfare measures for threatened species that account for the full breadth of interdependencies that exist within the watershed.

This capability should prove useful to regulatory authorities such as the Utah Department of Natural Resources in helping its scientists and policy makers better understand the interrelationships that exist between human activity and biological interactions within the GSL watershed and how these interrelationships impact the vulnerability of any given species. Of course, in order to be truly effective in guiding public policies concerning the GSL ecosystem, the bioeconomic model will ultimately need to be linked with models of the watershed's hydrology and regional economy, particularly that of the Wasatch Front. In addition, much of the ecological data that is currently used to calibrate the model's steady state needs to be updated. These "needs" form the basis of future research avenues; avenues which will enable regional planners to better forecast the effects of economic growth on the GSL ecosystem and to weigh the benefits and costs associated with various aspects of this growth.

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Technical Appendix

We begin with the specific functional forms used for the species' biomass-supply functions appearing in Tables 2 and 3 and the variable-respiration functions appearing in Table 2. The first column in Table A1 presents the functional forms for supply functions y_{12} , y_{14} , y_{25} , y_{26} , and y_{45} , respectively, while the second column presents the variable-respiration functions f_1 , f_2 , f_4 , f_5 , and f_6 . Specific values for the parameters δ_{ij}^{GSL} , α_{ij}^{GSL} , γ_i^{GSL} , r_i^{GSL} , and r_{ji}^{GSL} are provided below in Table A2.

Table A1. Species' Biomass-Supply and Variable-Respiration Functions

$$\begin{split} \delta_{12}^{GSL} x_{10}^{\alpha_{12}^{GSL}} & r_{1}^{GSL} \left[\left(S_{a} - S_{a}^{ss} \right)^{2} + 1 \right] x_{10}^{\gamma_{1}^{GSL}} \\ \delta_{14}^{GSL} x_{10}^{\alpha_{14}^{GSL}} & r_{2}^{GSL} x_{21}^{\gamma_{2}^{GSL}} \\ \delta_{25}^{GSL} x_{21}^{\alpha_{25}^{GSL}} & r_{4}^{GSL} x_{41}^{\gamma_{4}^{GSL}} \\ \delta_{26}^{GSL} x_{21}^{\alpha_{26}^{GSL}} & r_{5}^{GSL} \left[x_{52} + x_{54} \right] + 0.5 r_{5}^{GSL} \left[r_{54}^{GSL} x_{52} x_{54} + x_{54}^{\gamma_{5}^{GSL}} + r_{54}^{GSL} x_{54}^{\gamma_{5}^{GSL}} \right] \\ \delta_{45}^{GSL} x_{41}^{\alpha_{45}^{GSL}} & r_{6}^{GSL} x_{62}^{\gamma_{6}^{GSL}} \end{split}$$

The variable-respiration functions for algae and waterbirds requires further explanation. For algae, $S_a^{ss} = 1$ is the steady-state level of salinity and $S_a = s_m + s_a$ is the salinity level in any given time period, where $s_m = \frac{W_m}{W_m + W_a} + \varepsilon_m$ and $s_a = \frac{W_a}{W_m + W_a} + \varepsilon_a$ and ε_m and ε_a are independently distributed normal with means zero and standard deviations equal to 0.1. In other words, the overall salinity level as it impacts algae respiration is a sum of the salinity effects from the mineral-extraction and agricultural sectors, where the salinity effects are in turn weighted averages of the water used in the respective sectors plus independent stochastic shocks

$\frac{\text{Embodied Energies}}{e_0 = 1500} \\ e_1 = 1300 \\ e_2 = 1000 \\ e_4 = 500$	$\frac{Taxes}{t_{12} = 0.0000688}$ $t_{14} = 0.0000131$ $t_{45} = 0.0115926$ $t_{26} = 0.0004738$ $t_{25} = 0.0090030$	$\frac{r \text{ parameters}}{r_1^{GSL} = 3989}$ $r_2^{GSL} = 932$ $r_4^{GSL} = 1018$ $r_5^{GSL} = 186$ $r_{54}^{GSL} = 0.018$ $r_c^{GSL} = 2399$
$\frac{\text{Species Populations}}{N_1 = 409,139,538}$ $N_2 = 386,392$ $N_4 = 200,000$ $N_5 = 200$ $N_6 = 100,000$	$\frac{\text{Alpha Parameters}}{\alpha_{12}^{GSL} = \alpha_{14}^{GSL} = \alpha_{25}^{GSL}} = \alpha_{26}^{GSL} = \alpha_{45}^{GSL} = 0.5$	$\frac{\text{Beta Parameters}^{*}}{\beta_{1} = 1.381}$ $\beta_{2} = 86.111$ $\beta_{4} = 53.478$ $\beta_{5} = 47175.134$ $\beta_{6} = 3.460$
$\frac{\text{Biomass Demands}}{x_{10} = 0.007}$ $x_{21} = 0.656$ $x_{41} = 0.401$ $x_{52} = 156$ $x_{54} = 104$ $x_{62} = 0.016$ $A = N_1 x_{10}$	$\frac{\text{Delta Parameters}^{*}}{\delta_{12}^{GSL} = 0.0074}$ $\delta_{14}^{GSL} = 0.0023$ $\delta_{25}^{GSL} = 0.1000$ $\delta_{26}^{GSL} = 0.0053$ $\delta_{45}^{GSL} = 0.1647$	$\frac{\text{Species Life Spans}}{s_1 = s_2 = s_4 = s_6 = 5}$ $s_5 = 15$
$\frac{\text{Biomass Energy Prices}}{e_{10} = 0.09 N_1^{0.4}}$ $e_{21} = 0.09e_1$ $e_{41} = 0.15e_1$ $e_{52} = 0.09e_2$ $e_{54} = 0.09e_4$ $e_{62} = 0.09e_2$	$\frac{Gamma Parameters}{\gamma_1^{GSL} = \gamma_2^{GSL} = \gamma_4^{GSL}}$ $= \gamma_5^{GSL} = \gamma_6^{GSL} = 1.3$	Species Weights $w_1 = 0.007$ $w_2 = 0.776$ $w_4 = 0.887$

Table A2. Ecological Parameter and Steady-State Variable Values

that follow mean-zero normal distributions. For waterbirds, variable respiration is a polynomial function that accounts for available prey-substitution possibilities between brine shrimp and brine flies.

Table A2 contains the parameter values and steady-state values of the model's ecological variables. The superscript * indicates that the parameter or variable value is determined as an outcome of the calibration process. As mentioned in the text, those values not determined as an outcome of the calibration process were obtained from the ecological literature cited throughout Section 2.

With respect to the regional economy, we first account for the functional forms expressed in (6a) for each production sector. Following FT, these functional forms are presented as the first-order conditions that determine sector-level exports through constrained maximization of sector-level CES revenue functions,

$$\frac{q_i^d}{q_i^e} = \left(\frac{\left[1 - \delta_i\right]p_i^e}{\delta_i p_i^d}\right)^{\sigma_i} \quad i = f, m, a, R, c.$$
(A1)

There is no corresponding first-order condition determining the importation of the composite good since it is assumed that imports are the residual of the household sector's overall demand in excess of domestic production.

We next create a per-sector aggregated social accounting matrix based on data obtained from IMPLAN, thus ensuring cross-sector account balances.³² Table A3 contains the parameter values and steady-state values of the model's economic variables, presented by sector. Note that all non-composite output and input prices are normalized to one in the steady state (except for $\gamma = 0.04$ and $P_W = 0$), implying that the physical quantities are also the dollar values in the steady state. The superscript * indicates that the parameter or variable value is determined as an outcome of the calibration process.

³²Insert IMPLAN version # here.

Brine-Shrimp	Mineral Ext.	Agriculture	Composite	Recreation	Household
$q_{f}^{d} = 0.5$ $q_{f}^{e} = 9.5$ $I_{f} = 0.5$ $a_{f}^{I} = 0.05^{*}$ $L_{f} = 4.96$ $K_{f} = 4.96$ $a_{f} = b_{f} = 0.375^{*}$ $c_{f} = 0.25$ $d_{f} = 0.121^{*}$ $\delta_{f} = 0.258^{*}$ $\sigma_{f} = 2.79$ $COR = 0.08$ $TAC = 0.0000001N_{3}^{ss}$	$q_{m}^{d} = 5.1$ $q_{m}^{e} = 94.9$ $I_{m} = 5$ $a_{m}^{I} = 0.05^{*}$ $L_{m} = 45$ $K_{m} = 50$ $W_{m} = 5$ $a_{m} = 0.337^{*}$ $b_{m} = 0.375^{*}$ $c_{m} = 0.25$ $d_{m} = 4.268^{*}$ $\delta_{m} = 0.26^{*}$ $\sigma_{m} = 2.79$	$q_{a}^{d} = 87.5$ $q_{a}^{e} = 62.5$ $I_{a} = 7.5$ $a_{a}^{I} = 0.05^{*}$ $L_{a} = 40$ $K_{a} = 40$ $W_{a} = 10$ $T_{a} = 60$ $a_{a} = 0.2^{*}$ $b_{a} = 0.2^{*}$ $b_{a} = 0.2^{*}$ $d_{a} = 5.647^{*}$ $g_{a} = 0.3^{*}$ $\delta_{a} = 0.522^{*}$ $\sigma_{a} = 3.9$	$q_{c}^{d} = 370.9$ $q_{c}^{e} = 10$ $q_{c}^{IM} = 180.9$ $I_{c} = 10$ $a_{c}^{I} = 0.05^{*}$ $L_{c} = 100$ $K_{c} = 100$ $K_{c} = 2^{*}$ $\delta_{c} = 0.5^{*}$ $\sigma_{c} = 0.867^{*}$ $\delta_{c} = 0.785^{*}$ $\sigma_{c} = 2.79$ $\delta^{e} = 0.488^{*}$ $\sigma^{c} = 2.12$	$q_{R}^{d} = 16$ $q_{R}^{e} = 4$ $I_{R} = 1$ $a_{R}^{I} = 0.05^{*}$ $L_{R} = 10$ $K_{R} = 10$ $a_{R} = 0.375^{*}$ $b_{R} = 0.375^{*}$ $c_{R} = 0.25$ $d_{R} = 0.945^{*}$ $\delta_{R} = 0.622^{*}$ $\sigma_{R} = 2.79$	$C_{T} = 456$ $C_{F} = 24$ S = 24 Y = 480 $\beta_{TN} = \alpha_{SN} = 0.035^{*}$ $\nu_{TN} = \sigma_{SN} = 0.867$ $\alpha_{TN} = 0.731^{*}$ $\sigma_{TN} = 1.6$ $\lambda_{m} = 0.0002^{*}$ $\lambda_{a} = 0.181^{*}$ $\lambda_{c} = 0.818^{*}$

Table A3. Economic Parameter and Steady-State Variable Values



Figure 1. Map of the Great Salt Lake.



Figure 2. The Great Salt Lake ecosystem.



Figure 3. Algae population following initial negative shock.



Figure 4. Brine cyst population following initial population shocks.



Figure 5. The brine-shrimp fishery capital stock following the initial population shocks.



Figure 6. Household recreation/wildlife-viewing demand following the initial population shocks.

Species Number	Species Name
0	Sun
1	Algae
2	Brine Shrimp
3	Cysts
4	Brine Flies
5	Waterbirds
6	Corixids

Table 1. Species Identification

Table 2. Species' Net-Energy Functions

Species Number	<u>Net Energies (R_i)</u>
1	$R_{1} = \left[e_{0} - e_{10}\right]x_{10} - e_{1}\left[\left[1 + t_{12}e_{21}\right]y_{12}\left(x_{10}\right) + \left[1 + t_{14}e_{41}\right]y_{14}\left(x_{10}\right)\right] - f_{1}\left(x_{10}, S_{a}\right) - \beta_{1}\left(x_{10}, S_{a}\right)$
2	$R_{2} = \left[e_{1} - e_{21}\right]x_{21} - e_{2}\left[\left[1 + t_{25}e_{52}\right]y_{25}\left(x_{21}\right) + \left[1 + t_{26}e_{62}\right]y_{26}\left(x_{21}\right)\right] - f_{2}\left(x_{21}\right) - \beta_{2}\left(x_{21}\right) - \beta_{2}\left(x_{21}\right$
3	N/A [*]
4	$R_{4} = \left[e_{1} - e_{41}\right]x_{41} - e_{4}\left[1 + t_{45}e_{54}\right]y_{45}\left(x_{41}\right) - f_{4}\left(x_{41}\right) - \beta_{4}$
5	$R_{5} = [e_{2} - e_{52}]x_{52} + [e_{4} - e_{54}]x_{54} - f_{5}(x_{52}, x_{54}) - \beta_{5}$
6	$R_{5} = \left[e_{2} - e_{62}\right] x_{62} - f_{6}\left(x_{62}\right) - \beta_{6}$

* Because brine-shrimp cysts are in egg form they are assumed completely constrained in net-energy production.

Table 3. Species' Population-Updating Equations

 Species Number
 Population-Updating Equations

 1
 $N_4^{t+1} = N_4^t \left[1 + \left[\frac{\left[\left[y_{45} \left(x_{41}^{st} \right) / w_4 \right] \left[1 - 1 / s_4 \right] + 1 / s_4 \right]}{v_4^{st}} \right] \left[R_4^* + v_4 \right] - \left[\left[y_{45} \left(x_{41}^{st} \right) / w_4 \right] \left[1 - 1 / s_4 \right] + 1 / s_4 \right] \right] \right]$

 2
 $N_2^{t+1} = N_2^{st} + N_3^{st+1}$

 3
 $N_3^{t+1} = N_2^t \left[\frac{\left[\left[\left[y_{25} \left(x_{52}^{st} \right) + y_{25} \left(x_{52}^{st} \right) \right] / w_2 \right] \left[1 - 1 / s_2 \right] + 1 / s_2 \right]}{v_2^{st}} \right] \left[R_2^* + v_2 \right] - H_3^t$

 4
 $N_4^{t+1} = N_4^t \left[1 + \left[\frac{\left[\left[y_{25} \left(x_{51}^{st} \right) / w_4 \right] \left[1 - 1 / s_4 \right] + 1 / s_4 \right]}{v_4^{st}} \right] \left[R_4^* + v_4 \right] - \left[\left[y_{45} \left(x_{41}^{st} \right) / w_4 \right] \left[1 - 1 / s_4 \right] + 1 / s_4 \right] \right]$

 5
 $N_5^{t+1} = N_5^t + \frac{N_5^t}{s_5} \left[\frac{R_5^* + v_5}{v_5^{st}} - 1 \right]$ $N_4^{t+1} = N_5^t \left[\frac{R_5^* + v_5}{v_5^{st}} - 1 \right]$ $N_4^{t+1} = N_5^t \left[\frac{R_5^* + v_5}{v_5^{st}} - 1 \right]$ $N_5^{t+1} = N_5^t \left[\frac{R_5^t + v_5}{v_5^{st}} - 1 \right]$ $N_5^{t+1} = N_5^t \left[\frac{R_5^t + v_5}{v_5^{st}} - 1 \right]$ $N_5^{t+1} = N_5^t \left[\frac{R_5^t + v_5}{v_5^{st}} - 1 \right]$ $N_5^{t+1} = N_5^t \left[\frac{R_5^t + v_5}{v_5^{st}} - 1 \right]$ $N_5^{t+1} = N_5^t \left[\frac{R_5^t + v_5}{v_5^{st}} - 1 \right]$ $N_5^{t+1} = N_5^t \left[\frac{R_5^t + v_5}{v_5^{st}} - 1 \right]$ $N_5^{t+1} = N_5^t \left[\frac{R_5^t + v_5}{v_5^{st}} - 1 \right]$ $N_5^{t+1} = N_5^t \left[\frac{R_5^t + v_5}{v_5^{st}} - 1 \right]$ $N_5^{t+1} = N_5^t \left[\frac{R_5^t + v_5}{v_5^{st}} - 1 \right]$ $N_5^{t+1} = N_5^t \left[\frac{R_5^t + v_5}{v_5^{st}} - 1 \right]$ $N_5^{t+1} = N_5^t \left[\frac{R_5^t + v_5}{v_5^{st}} - 1 \right]$ $N_5^{t+1} = N_5^t \left[\frac{R_5^t + V_5}{v_5^{st}} - 1 \right]$ $N_$

$$N_6^{t+1} = N_6^t + \frac{N_6^t}{s_6} \left[\frac{R_6^* + v_6}{v_6^{ss}} - 1 \right]$$

6

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