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**A MORE GENERAL APPROACH TO MODELING  
EXCHANGE RATE VOLATILITY**

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**March 1997**

**A MORE GENERAL APPROACH TO MODELING**

**EXCHANGE RATE VOLATILITY**

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**Kai-Li Wang, Christopher Fawson, Christopher B. Barrett, James B. McDonald**

**ABSTRACT**

Exchange rates commonly exhibit periods of stability punctuated by infrequent, substantial adjustments. Statistically, this generates empirical distributions of exchange rate changes that have high peaks, long tails, and, sometimes, are asymmetric. Existing time-series estimation methods do not account for these characteristics satisfactorily. This paper introduces a more general GARCH model, based on the exponential generalized beta (EGB) family of distributions, which can accommodate most nonnormal characteristics of data, including leptokurtosis, skewness, and high peakedness, and yet remains tractable for estimation. Applied to daily U.S. dollar exchange rate data for six major currencies, the GARCH-EGB2 model uniformly outperforms conventional time-series models of exchange rate volatility.

# A MORE GENERAL APPROACH TO MODELING EXCHANGE RATE VOLATILITY<sup>1</sup>

## I. Introduction

Contemporary modeling of exchange rate time series makes widespread use of generalized autoregressive conditional heteroscedastic (GARCH) models.<sup>2</sup> Not only can GARCH models capture the volatility clustering often found in exchange rate series, they also accommodate a degree of leptokurtosis (i.e., thick tails) that is also an empirical regularity of exchange rate series. In recent years, however, researchers have expressed concern that the GARCH maximum-likelihood estimation based on the common assumption of conditional normality fails to capture sufficiently the leptokurtosis evident in most asset returns (Bollerslev 1987; Baillie and Bollerslev 1989; Hsieh 1989; Baillie and DeGennaro 1990; and Wang, Barrett, and Fawson 1996). This has led to a widespread adoption of conditional distributions, more general than the normal, most commonly the Student-t (Bollerslev 1987).

Increasingly, widespread adoption of the Student-t conditional distribution in GARCH modeling is, however, contrary to the intuition generated by economic theories of exchange rates. While we do not develop a formal derivation of the statistical implications of exchange rate determination models in this paper, it is nonetheless useful to consider this issue casually.<sup>3</sup> We

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<sup>2</sup>Bollerslev, Chou, and Kroner (1992) offer a good survey.

<sup>3</sup>Taylor (1995) and Obstfeld and Rogoff (1996) offer excellent, formal treatments of exchange rate determination models.

believe that selection of a conditional distribution for maximum-likelihood GARCH estimation should draw on the implications economic theory offers regarding the empirical distribution of the dependent variable, in this case, exchange rates.

What might cause the leptokurtosis commonly observed in exchange rate series and which often induces econometricians to employ leptokurtic conditional distributions (e.g., the Student-t) in estimation? Economic theories of exchange rate determination offer two likely explanations. The first is the overshooting of floating nominal exchange rates associated with monetary or fiscal shocks in the presence of sticky prices (Dornbusch 1976). The other is speculative attacks against fixed exchange rates (Krugman 1979). Both models imply infrequent, extraordinarily sharp movements in exchange rates, i.e., the sorts of movements that appear as long (i.e., fat) tails in a distribution of differenced exchange rates. Sharp exchange rate movements do not necessarily imply leptokurtosis, however, they could imply high variance in the time series. The key is that sticky prices in floating rate regimes, and especially fixed exchange rates, also generate modal daily exchange rate changes near zero (Obstfeld and Rogoff 1996). The implication is that exchange rate changes are concentrated near the mean but have long tails and hence leptokurtosis.<sup>4</sup> The choice of a Student-t conditional distribution, however, ignores the likelihood that the leptokurtosis evidenced in exchange rate series is bound up with the high peakedness of those series. The low peakedness of the Student-t signals that perhaps the time-series literature on exchange rates has gone off in an inadvisable direction. One really wants to use a conditional distribution that accommodates both long tails and high peaks.

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<sup>4</sup>An alternative way to view this, following Friedman (1953), is to recognize that profit-maximizing speculators will adjust their currency holdings in a manner that stabilizes transitory shocks to the exchange rate and accelerates movement in response to permanent shocks. If transitory shocks are far more common than permanent shocks, this will yield an empirical distribution of exchange rate changes that is high peaked and long tailed.

Moreover, the literature curiously leaps from concern with variance to concern with leptokurtosis. Theory suggests the third central moment, skewness, might also be important in exchange rate series. Skewness in exchange rate series comes from episodes of sharp depreciation (appreciation) not offset by subsequent sharp appreciation (depreciation). There are two likely reasons for such phenomena. First, permanent shocks that lead to changes in the equilibrium exchange rate may be highly asymmetric; rapid improvements in Japanese productivity over the past thirty years seem an excellent example. Second, speculative attacks against a currency tend to be one-sided (causing depreciation/devaluation). The 1992-93 European and 1994 Mexican currency crises—including attacks against the British pound and the Italian lira of particular relevance to this study—are good recent examples of such episodes. Since significant skewness is often observed in exchange rate series (Boothe and Glassman 1987; Hsieh 1988; and Peruga 1988), it would seem advisable to employ estimation methods that can accommodate skewness.<sup>5</sup>

GARCH estimation based on conditional Student-t distributions can capture the long tails evident in most exchange rate series, but fares less well in replicating their high peakedness and skewness. As a step toward resolving this deficiency, we introduce a more general GARCH model, based on the exponential generalized beta (EGB) family of distributions (McDonald and Xu 1995), which can accommodate nonnormal characteristics of data and yet remains tractable for estimation.

The plan of the paper is as follows. Section II introduces the EGB family of distributions, including the specific variant used in this paper, the exponential generalized beta of the second kind (EGB2), and then develops a GARCH-EGB2 model based on this more flexible conditional

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<sup>5</sup>Hansen's (1994) method does this, but by using a modified Student-t distribution, so that he cannot simultaneously accommodate high peakedness. Moreover, Hansen's model depends on appropriate *ex ante* lag selection.

distribution. In section III, we estimate time-series models of the U.S. dollar exchange rates for six major industrial economy currencies, using daily data. These exchange rate series all exhibit high peakedness and leptokurtosis, and several are skewed. The likelihood dominance criterion (for nonnested models), goodness-of-fit statistics, and plots of the standardized residuals all indicate that the GARCH-EGB2 model systematically outperforms Gaussian GARCH and GARCH-t models for each of the exchange rate series modeled. The concluding section summarizes our findings and highlights some implications for future research.

## II. The GARCH-EGB2 Model

McDonald and Xu (1995) introduced the five-parameter generalized beta (GB) distribution and its logarithmic transform, the exponential generalized beta (EGB) distribution. The GB includes, as special cases, many distributions commonly employed in maximum-likelihood estimation in econometrics, including the lognormal, logistic, Pareto, generalized gamma, and Burr and Weibull distributions. In this paper we use the EGB distribution because it can model both positive and negative random variables, while the GB models just positive random variables. The EGB is the natural choice for a generalized distribution in much time-series estimation, in particular when one needs to difference nonstationary variables like daily exchange rates, thereby generating stationary regressands that are negative as well as positive. When combined with a GARCH specification for conditional variance, this very flexible distribution seems to account better for the characteristics of daily exchange rate data than commonly used alternatives.

The EGB distribution is defined by the probability density function (pdf):

$$EGB(z; \delta, \sigma, c, p, q) = \frac{e^{\frac{p(z-\delta)}{\sigma}} (1 - (1-c)e^{\frac{(z-\delta)}{\sigma}})^{q-1}}{|\sigma| B(p, q) \left(1 + c e^{\frac{z-\delta}{\sigma}}\right)^{p+q}} \quad \text{for } -\infty < \frac{z-\delta}{\sigma} < \ln\left(\frac{1}{1-c}\right),$$

where  $\delta$  is a location parameter that affects the mean of the distribution,  $\sigma$  reflects the peakedness of the density function,  $p$  and  $q$  are shape parameters that together determine the skewness and kurtosis of the distribution, and  $0 \leq c \leq 1$ . Of special interest are the exponential generalized beta distributions of the first and second kind, EGB1 and EGB2, respectively, which correspond to the limiting values of  $c = 0$  and  $c = 1$  and are alternative representations of the generalized exponential and generalized logistic distributions, respectively (Johnson and Kotz 1970; and Patil, Bowell, and Ratnaparkhi 1984). The associated probability density functions are

$$EGB1(z; \delta, \sigma, p, q) = EGB(z; \delta, \sigma, c = 0, p, q) = \frac{e^{\frac{p(z-\delta)}{\sigma}} \left(1 - e^{\frac{(z-\delta)}{\sigma}}\right)^{q-1}}{|\sigma| B(p, q)}$$

$$EGB2(z; \delta, \sigma, p, q) = EGB(z; \delta, \sigma, c = 1, p, q) = \frac{e^{\frac{p(z-\delta)}{\sigma}}}{|\sigma| B(p, q) \left(1 + e^{\frac{z-\delta}{\sigma}}\right)^{p+q}}$$

Note that, unlike the more general EGB distribution, EGB1 and EGB2 do not involve a nonlinear inequality constraint for the random variable. This feature makes numerical estimation of the latter distributions simpler than for the EGB. Furthermore, while the higher order moments of the EGB involve a relatively complex, hypergeometric series, the variance, skewness, and kurtosis of EGB1 and EGB2 are relatively simple expressions. Table 1 presents equations for the variance, skewness, and kurtosis of the EGB2 distribution employed in the empirical portion of this

**Table 1. The Moments of the EGB2 Distribution**


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Mean	$\delta + \sigma[\psi(p) - \psi(q)]$
Variance	$\sigma^2[\psi'(p) + \psi'(q)]$
Skewness	$(\psi''(p) - \psi''(q)) / (\psi'(p) + \psi'(q))^{1.5}$
Kurtosis	$[(\psi'''(p) + \psi'''(q)) + 3(\psi'(p) + \psi'(q))^2] / (\psi'(p) + \psi'(q))^2$

---

Note: where  $\psi(\cdot)$ ,  $\psi'(\cdot)$ ,  $\psi''(\cdot)$ , and  $\psi'''(\cdot)$  are digamma, trigamma, tetragamma, pentagamma functions, respectively (Davis 1935).

paper. Tractability therefore favors estimating four-parameter EGB1 or EGB2 distributions over the more general EGB form as long as the  $c$  parameter lies near one or zero. In this spirit, McDonald and Xu (1995, p. 134) find that, “[t]he exponential generalized beta of the second kind (EGB2) provides the basis for partially adaptive estimation in regression and time series models to accommodate possibly thick-tailed and skewed error distributions.” Since the prevailing concern about existing GARCH modeling of exchange rate series is unsatisfactory accommodation of leptokurtosis, skewness, and high peakedness in error distributions, the EGB, or one of its two limiting distributions, seems a natural conditional distribution to employ in GARCH estimation. In order to have a probability density function without restricted support and because preliminary results of estimating both the EGB and the EGB2 models suggest  $c$  is close to the unit boundary of the parameter space for each exchange rate, with the EGB2 specification uniformly favored over the EGB by likelihood ratio tests (Table 2), we use the EGB2 in this study.

A substantial amount of recent research has found that conditional variance in asset prices, including exchange rates, is time varying. Beginning with Engle (1982), econometricians have thus

**Table 2. Comparison of EGB and EGB2 Estimation Results**

	DM	£	¥	FF	BF	IL
c	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
LLH <sup>EGB</sup>	3244.41	3122.94	3087.57	3114.19	3225.66	3109.09
LLH <sup>EGB2</sup>	3235.40	3078.89	3047.50	3097.70	3210.02	3090.04
LR	18.02*	88.10*	80.14*	32.98*	31.28*	38.10*

c is the parameter estimated from EGB distribution.

LLH<sup>EGB</sup> and LLH<sup>EGB2</sup> represent the negative of the maximal log-likelihood value of the models under the EGB and EGB2 distributions, respectively.

LR represents the likelihood ratio test statistic of GARCH-EGB2 model against the corresponding GARCH-EGB model.

\*denotes statistical significance at the 1% level.

modeled the conditional variance process of time series as potentially autocorrelated. GARCH models have been found particularly useful in explaining the behavior of monetary and financial time series (Bollerslev, Chou, and Kroner 1992), although as Hall, Miles, and Taylor (1989) note, the ARCH parameterization of the conditional variance is merely a convenient and parsimonious representation of the data; it does not have solid grounding in economic theory. Nonetheless, one major contribution of the GARCH literature is that change in asset price risk emerges predictably from a specific type of nonlinear dependence rather than depending on exogenous structural changes to the variance process.

GARCH models are commonly estimated under the assumption that the standardized residuals are normally distributed. Yet, although the unconditional distribution of a GARCH process with normal errors is leptokurtic (Engle 1982; and Bollerslev 1986)—i.e., its kurtosis is greater than 3.0, the benchmark value from the normal distribution—Gaussian GARCH models nonetheless

regularly fail to account adequately for the fat tails found in unconditional asset price distributions (Hsieh 1989; and Wang, Barrett, and Fawson 1996). As a consequence, many researchers now employ nonnormal conditional distributions, particularly the Student-t, in GARCH modeling.

Our concern about the evolution of the GARCH literature, especially as applied to exchange rate series, is that accommodation of leptokurtosis but not of the high peakedness or asymmetry commonly found in exchange rate series, for reasons discussed in the opening section, may lead to poor choice of conditional distributions for maximum-likelihood estimation. Given the problems associated with quasimaximum-likelihood GARCH estimation (Pagan and Sabau 1987; Lee and Hansen 1994; and Deb 1996), incomplete accommodation of the regular statistical characteristics of exchange rates may yield inaccurate estimates of exchange rate dynamics. We therefore develop a GARCH model based on the EGB2 distribution and study its performance versus the normal and Student-t GARCH models most commonly used in the literature.

We begin by adopting a general autoregressive moving average (ARMA) specification in the conditional mean equation with GARCH(1,1) errors.<sup>6</sup> With the right conditional distribution to describe the standardized errors,  $z_t$ , this specification can account for most of the characteristics observed in empirical financial distributions, including time-varying variance, asymmetry, thick tails, and high peakedness. Denoting a time-series dependent variable as  $y_t$ , the general form of this model is given by

ARMA(m,n) conditional mean equation:

$$\phi_m(L)y_t = \mu + \phi_n(L)\epsilon_t$$

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<sup>6</sup>The GARCH(1,1) specification we employ is generally excellent for a wide range of financial data (Bollerslev, Chou, and Kroner 1992).

GARCH(1,1) conditional variance equation:

$$\epsilon_t = h_t^{0.5} z_t$$

$$E(\epsilon_t^2 | \psi_{t-1}) = h_t = w + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1}$$

Conditional distribution:

$$\epsilon_t | \psi_{t-1} \sim D(0, h_t, \eta_t),$$

where the  $\phi(B)$  are polynomials in the lag operator of order  $p$  and  $q$ , respectively, and  $w$ ,  $\alpha_1$ , and  $\beta_1 > 0$  to ensure strictly positive conditional variance. The  $\epsilon_t$  are the residuals from the conditional mean equation and are a function of the independent and identically distributed  $z_t$ , which have zero mean and unit variance, and of the variance,  $h_t$ , conditional on the information set  $\psi_{t-1}$ . The errors follow the assumed conditional density function, further described by the parameter vector  $\eta_t$ . These are “shape” parameters,  $\eta_t = \{p, q\}$  under EGB2,  $\eta_t = \{v\}$  under the Student  $t$ -distribution, and  $\eta_t = \{\emptyset\}$  under the normal distribution. To achieve efficiency, we jointly estimate the conditional mean and conditional variance equations with the conditional distribution by full information maximum likelihood using the GAUSS constrained maximum-likelihood module.

For the standardized EGB2 distribution with the shape parameter  $p$  and  $q$ , the log-likelihood function of the GARCH-EGB2 model is

$$\text{Log}L = T[\log(\sqrt{\Omega}) - \log(B(p, q)) + p\Delta] + \sum [(p \left(\frac{\sqrt{\Omega}\epsilon_t}{h_t}\right) - \log(h_t) - (p + q)\log(1 + \exp(\frac{\sqrt{\Omega}\epsilon_t}{h_t} + \Delta))]$$

where  $\Delta = \psi(p) - \psi(q)$ ,  $\Omega = \psi'(p) + \psi'(q)$ , and  $\psi(p)$  and  $\psi'(p)$  represent digamma and trigamma functions, respectively. We show the detailed parameterization of the GARCH-EGB2 model in the technical appendix. For the Student  $t$ -distribution with  $v$  degrees of freedom, the log-likelihood function of the GARCH- $t$  model is, as presented by Bollerslev (1987), is:

$$\log L = T \left[ \log \Gamma \left( \frac{\nu+1}{2} \right) - \log \Gamma \left( \frac{\nu}{2} \right) - 0.5 \log(\nu-2) \right] - 0.5 \sum [\log h_t + (\nu+1) \log \left( 1 + \frac{\epsilon_t^2}{h_t(\nu-2)} \right)]$$

where  $\Gamma$  denotes the gamma function.

By adding just one extra parameter to be estimated, the GARCH-EGB2 model is able to account not only for the first, second, and fourth moments of the conditional distribution of the variable of interest, as do popular Gaussian GARCH and GARCH-t models, it is also able to accommodate the third moment and high peakedness. Although economic theory suggests these latter two features should be common to exchange rates, they have been largely ignored in empirical work to date. EGB2 incorporates the normal distribution as a limiting case when  $p = q$  approaches infinity. It is symmetric for  $p = q$  and is positively (negatively) skewed for  $p > q$  ( $p < q$ ) for  $\sigma > 0$ ; the skewness results reverse for  $\sigma < 0$ . The EGB2 can accommodate coefficient of skewness values between -2 and 2 and coefficient of kurtosis values up to 9 (McDonald 1991), which will suffice for most data series, in particular the exchange rate data we study in this paper.

### III. An Empirical Application to Six Daily Exchange Rates

The data are the daily noon spot U.S. dollar exchange rate (\$/local currency) for the German deutsche mark (DM), British pound (£), Japanese yen (¥), French franc (FF), Belgian franc (BF), and Italian lira (IL) over the period January 1, 1985, to November 21, 1996 (3,016 observations per series), as reported by the Exchange Rate Service of the Pacific Data Center at the University of British Columbia. To achieve stationarity, we use first-differenced exchange rate series.<sup>7</sup> With  $R > 0$  ( $R < 0$ ) indicating currency appreciation (depreciation), the data are of the form<sup>8</sup>

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<sup>7</sup>Unit root test results demonstrating each series is I(1) are available from the authors.

<sup>8</sup>There is no adjustment made for the weekend or holiday effects, so  $R$  indicates the exchange rate changes between two successive trading days.

$$R_{it} = \ln[S_{i,t} / S_{i,t-1}] * 100$$

where  $R_{it}$  = percentage change in the U.S.\$/LC exchange rate of currency  $i$  at period  $t$ ;  $S_{it}$  = foreign exchange rate of currency  $i$  at period  $t$ , expressed as U.S.\$/LC.

Table 3 presents descriptive statistics for each exchange rate series, including the coefficients of skewness<sup>9</sup> and kurtosis,<sup>10</sup> inter-percentile ranges ( $f_{0.75} - f_{0.25}$  and  $f_{0.6} - f_{0.4}$ ), the Jarque-Bera asymptotic normality test statistics, and Ljung-Box-Pierce portmanteau test statistics. The skewness properties are diverse among currencies. The yen, pound, and lira all show significant skewness. The former is likely attributable to permanent structural shocks that led to the yen's dramatic appreciation over the sample period. The negative skewness in the pound and lira series no doubt reflect the 1992 speculative attacks that knocked those currencies out of the European monetary system's exchange rate mechanism (ERM). As we will see in section IV, the GARCH-EGB2 model is especially appealing for currencies such as these, which exhibit significantly skewed percentage change distributions. The higher the coefficient of kurtosis (KUR), the less probability is concentrated around the mean, meaning that the distribution are more fat-tailed than normal distribution. For all currencies, the coefficients of kurtosis are greater than five and significantly different from the reference value of three drawn from the normal distribution. The high peakedness of each unconditional distribution is confirmed by inter-percentile ranges (e.g.,  $f_{\alpha_1} - f_{\alpha_2}$  indicates the range between the probabilities  $\alpha_1$  and  $\alpha_2$ ). Given  $\alpha_1$  and  $\alpha_2$ , the lower the value of  $f_{\alpha_1} - f_{\alpha_2}$ , the higher the peakedness of the distribution. Across all six exchange rates, the value  $f_{0.75} - f_{0.25}$  is

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<sup>9</sup>This is  $E(R_t - \mu)^3 / \sigma^3$ , where  $\mu$  is the mean and  $\sigma$  is the standard deviation.

<sup>10</sup>This is  $E(R_t - \mu)^4 / \sigma^4$ , where  $\mu$  is the mean and  $\sigma$  is the standard deviation.

**Table 3. Descriptive Statistics**

	SK	Kur	$f_{0.75}-f_{0.25}$	$f_{0.6}-f_{0.4}$	JB	Q(30)	Q <sup>2</sup> (30)
DM	-0.037 (0.045)	5.1 (0.09)	1.13	0.41	566.73*	30.03 [0.46]	391.36 [0.00]
£	-0.12 (0.045)	5.2 (0.09)	1.08	0.40	604.92*	37.98 [0.15]	451.30 [0.00]
¥	0.286 (0.045)	6.1 (0.09)	1.02	0.38	1282.9*	37.80 [0.16]	233.98 [0.00]
FF	0.02 (0.045)	6.0 (0.09)	1.14	0.39	511.89*	41.16 [0.08]	391.67 0.00]
BF	0.024 (0.045)	5.0 (0.09)	1.12	0.41	521.53*	41.69 [0.08]	370.85 [0.00]
IL	-0.616 (0.045)	8.8 (0.09)	1.14	0.41	4377.65*	33.39 [0.31]	641.75 [0.00]

SK = coefficient of skewness.

KUR = coefficient of kurtosis (the value for the normal distribution is 3.0).

The asymptotic standard errors of SK and KUR are reported in parentheses and computed as  $(6/T)^{0.5}$  and  $(24/T)^{0.5}$ , respectively.

JB = Jarque-Bera normality test statistic.

Q and Q<sup>2</sup> represent the Ljung-Box test statistics for up to 30th order serial correlation for each exchange rate series. Similar results obtain at different orders. P-values against the null hypothesis of white noise are reported in brackets.

\*denotes statistical significance at the 1% level.

uniformly less than 1.36, the reference range corresponding to the normal distribution. The unconditional distributions of these exchange rates have higher peaks than does a normal distribution around the central 50% of probability mass. The high peakedness is corroborated over the narrower interval  $f_{0.6} - f_{0.4}$ , for which all exchange rate ranges are less than 0.5, the inter-percentile value of the standard normal over its central 20% of probability mass. Given skewness, leptokurtosis, and high peakedness, it is not surprising that the null hypothesis of normality is strongly rejected by the Jarque-Bera (JB) asymptotic test for each exchange rate.

Table 3 also presents the Ljung-Box test statistics for autocorrelation in  $R_{it}$  at a lag of 30 trading days ( $Q(30)$ ), and in squared exchange rate changes ( $Q^2(30)$ ), the latter serving as a test for GARCH effects. Only the French and Belgian franc series appear autocorrelated. However, all the series exhibit serial correlation in their second moments, i.e., GARCH effects.

In summary, the descriptive statistics of Table 3 suggest the unconditional distributions of daily exchange rate changes are generally far from the traditional Gaussian assumption and also exhibit heteroscedasticity of the GARCH form.<sup>11</sup> These results are consistent with previous findings (Boothe and Glassman 1987; Hsieh 1988; and Wang, Barrett, and Fawson 1996). In particular, the empirical distributions of percentage exchange rate changes uniformly exhibit leptokurtosis and high peakedness and are often asymmetric, just as economic theory would predict.

We began estimation by identifying and estimating a common ARMA process for the stationary  $R_{it}$ . First, Box-Jenkins techniques were used to reduce the set of prospective ARMA specifications. Next, we further narrowed the pool of possible models to those having a p-value for the Ljung-Box portmanteau  $Q(30)$  statistic of greater than 0.3, a significance level clearly supporting the assumption of white noise. Finally, we chose the ARMA specification having the lowest Schwarz Bayesian criterion (SBC) value from among the candidate models having passed the Box-Jenkins and  $Q(30)$  screens. In other words, the Ljung-Box  $Q$  statistic was used to identify a few possible models and then the information criterion (SBC) selected the final ARMA specification for the conditional mean equation.

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<sup>11</sup> These characteristics are evident as well in a graphical appendix available by request from the authors.

Table 4 reports Ljung-Box portmanteau statistics for the squared standardized residual ( $z_t$ ) for all currencies under homoscedastic (HOMO), Gaussian GARCH (GARCH), Student-t GARCH (GARCH-t) and GARCH-EGB2 specifications. The p-values of the test statistics (reported in brackets) clearly suggest that each of the GARCH specifications satisfactorily eliminates the serial correlation in conditional variance found in the homoscedastic model. Accommodating volatility clustering is clearly not difficult in these exchange rate data.

While all the GARCH models appear to accommodate second-order serial correlation successfully, the issue of nonnormality remains. Skewness and excess kurtosis of the standardized residuals persist in all the Gaussian models ( $m_3^{\text{GARCH}}$  and  $m_4^{\text{GARCH}}$  in Table 5), although the leptokurtic characteristics ( $m_4^{\text{GARCH}}$ ) have generally been muted somewhat relative to the homoscedastic model ( $m_4^{\text{HOMO}}$ ).<sup>12</sup> As discussed in section II, Gaussian GARCH models inherently

**Table 4. Tests for Serially Correlated Conditional Variance**

	DM	£	¥	FF	BF	IL
HOMO	403.20 [0.00]	447.50 [0.00]	239.53 [0.00]	404.10 [0.00]	370.75 [0.00]	620.03 [0.00]
GARCH	29.28 [0.50]	24.36 [0.76]	28.72 [0.53]	25.50 [0.70]	35.82 [0.21]	23.67 [0.79]
GARCH-t	28.71 [0.53]	24.46 [0.75]	29.06 [0.52]	25.41 [0.71]	34.73 [0.25]	26.22 [0.66]
GARCH-EGB2	28.96 [0.52]	24.64 [0.74]	29.52 [0.49]	25.50 [0.70]	35.01 [0.24]	25.03 [0.72]

The figures in brackets are the p-values of the Ljung-Box Q(30) test against the null hypothesis of no serial correlation.

<sup>12</sup>Milhøj (1987), Hsieh (1989), and MaCurdy and Morgan (1987) found similar results.

**Table 5. Skewness and Kurtosis of Sample Standardized Residuals and Predicted Values**

	DM	¥	¥	FF	BF	IL
<b>HOMO</b>						
$m_3^{\text{HOMO}}$	-0.034	-0.100	0.281	0.026	0.036	-0.586
$m_4^{\text{HOMO}}$	5.054	5.053	6.099	4.943	4.966	8.550
<b>GARCH</b>						
$m_3^{\text{GARCH}}$	0.075	-0.110	0.464	0.094	0.114	-0.118
$m_4^{\text{GARCH}}$	4.419	4.365	6.154	4.350	4.402	4.745
<b>GARCH-t</b>						
$m_3^t$	0.083	-0.108	0.502	0.103	0.127	-0.160
$m_4^t$	4.451	4.345	6.373	4.398	4.462	5.172
$\phi_4^t$	4.433	4.652	15.49	4.361	4.445	4.680
$v$	6.093	5.816	4.240	6.210	6.075	5.785
<b>GARCH-EGB2</b>						
$m_3^{\text{EGB2}}$	0.083	-0.115	0.502	0.102	0.125	-0.148
$\phi_3^{\text{EGB2}}$	0.088	-0.071	0.326	0.087	0.076	-0.025
$m_4^{\text{EGB2}}$	4.451	4.395	6.355	4.394	4.457	5.065
$\phi_4^{\text{EGB2}}$	4.584	4.778	5.356	4.541	4.601	4.909

$m_3$  is the coefficient of skewness of the standardized residuals from the estimated model.

$m_4$  is the coefficient of kurtosis of the standardized residuals from the estimated model.

For each model, the asymptotic standard error of the coefficients of skewness and kurtosis are 0.045 and 0.089, respectively.

$v$  is the degree of freedom estimate from GARCH-t model.

$\phi_4^t$  is the predicted kurtosis coefficient of Student t-distribution =  $3(v-2)/(v-4)$ ,  $v > 4$ .

$\phi_3^{\text{EGB2}}$  is predicted skewness coefficient of EGB2 distribution =  $[\psi''(p) - \psi''(q)]/[\psi'(p) + \psi'(q)]^{1.5}$ .

$\phi_4^{\text{EGB2}}$  is predicted skewness coefficient of EGB2 distribution =  $\{[\psi'''(p) + \psi'''(q)] + 3[\psi'(p) + \psi'(q)]^2\}/(\psi'(p) + \psi'(q))^2$ .

capture some unconditional nonnormality but not always enough to represent exchange rate series accurately.

As a result, many applied econometricians have turned to using the Student-t conditional error distribution to account for, in particular, leptokurtosis. As measured by maximal log-likelihood values or likelihood ratio test statistics, the GARCH-t and GARCH-EGB2 models

appear uniformly superior to the Gaussian GARCH model in fitting these exchange rate series (Table 6).<sup>13</sup> Note also the low estimated values in Table 5 for the degree of freedom parameter,  $\nu$ , in each of the GARCH-t models ( $\nu \geq 30$  indicates asymptotic normality). Moreover, Table 5 shows that the conditional kurtosis values predicted by the estimated values for the shape parameters ( $\nu$  in the case of GARCH-t,  $p$ , and  $q$  for GARCH-EGB2),  $\phi_4^t$  and  $\phi_4^{EGB2}$  under GARCH-t and GARCH-EGB2 assumptions, respectively, are reasonably close to the kurtosis of the standardized residuals ( $m_4^t$  and  $m_4^{EGB2}$ ) for all series except the Japanese yen, where the GARCH-EGB2 model still performs very well (far better than the GARCH-t). This suggests that both the conditional Student-t and EGB2 distributions satisfactorily capture the leptokurtosis of exchange rate movements, permitting the applied econometrician significant gains in estimation accuracy.

**Table 6. Comparisons of Alternative Specifications**

	HOMO	GARCH		GARCH-t		GARCH-EGB2	
	LLH <sup>HOMO</sup>	LLH <sup>GARCH</sup>	LR <sup>GARCH</sup>	LLH <sup>t</sup>	LR <sup>t</sup>	LLH <sup>EGB2</sup>	LR <sup>EGB2</sup>
DM	-3342.19	-3228.52	227.34*	-3163.46	130.12*	-3161.31	134.42*
£	-3200.57	-3059.32	282.50*	-2991.03	136.58*	-2987.83	142.98*
¥	-3236.48	-3121.97	229.02*	-2970.24	303.46*	-2963.68	316.58*
FF	-3197.86	-3085.85	224.02*	-3024.20	123.30*	-3022.09	127.52*
BF	-3309.58	-3201.02	217.12*	-3137.07	127.90*	-3134.82	132.40*
IL	-3251.40	-3078.24	346.32*	-3005.76	144.96*	-3002.04	152.40*

LLH<sup>HOMO</sup>, LLH<sup>GARCH</sup>, LLH<sup>t</sup>, and LLH<sup>EGB2</sup> represent the maximal log-likelihood value of HOMO, Gaussian GARCH, GARCH-t and GARCH-EGB2 models, respectively.

LR<sup>GARCH</sup> indicates the likelihood ratio test statistic for the Gaussian GARCH models against the HOMO model.

The LR<sup>t</sup> and LR<sup>EGB2</sup> statistics are for the GARCH-t and GARCH-EGB2 against the corresponding Gaussian GARCH models, respectively.

\*denotes statistical significance at the 1% level (using the  $\chi^2(3)$ ,  $\chi^2(1)$ , and  $\chi^2(2)$  distributions for the LR<sup>GARCH</sup>, LR<sup>t</sup>, and LR<sup>EGB2</sup>, respectively).

<sup>13</sup>The Gaussian GARCH models are likewise uniformly preferable to a homoscedastic null. There are considerable gains to be had from capturing GARCH effects; our point is that there are considerable further gains to be had from accommodating nonnormal innovations.

The most important weaknesses remaining in the GARCH-t specification, however, are the apparent asymmetry of the standardized residuals ( $m_3^t \neq 0$  in Table 5) and the high peakedness observed in the data (Table 3), which the Student-t distribution will systematically fail to capture. By contrast, the estimated coefficients of skewness in the standardized residuals of the GARCH-EGB2 model ( $m_3^{\text{EGB2}}$ ) are reasonably close to the predicted coefficients implied by the estimated distribution parameters  $p$  and  $q$  ( $\phi_3^{\text{EGB2}}$ ). Unlike the conditional distributions commonly assumed in GARCH modeling—the normal and the Student-t—the more flexible EGB2 distribution appears to capture well all of the higher order moments of exchange rate series.

The superiority of the GARCH-EGB2 model in capturing the high peakedness inherent to most exchange rate series is most evident graphically. Figures 1 through 12 show paired plots of the densities of the observed standardized residuals and the corresponding predictions derived from the estimated shape parameters of GARCH-t (the top of each pair) and GARCH-EGB2 (the bottom of each pair) models. These figures clearly show that the observed standardized residuals generated by the GARCH-t model vary considerably from their assumed distribution, in particular, exhibiting high peakedness, asymmetry, or both. The empirical density plots for the standardized residuals of the GARCH-EGB2 model, by contrast, match the predicted densities closely for each exchange rate. While the GARCH-t model is only able to account for the fat tails, which make these exchange rate series nonnormal, the GARCH-EGB2 model is also able to accommodate skewness and high peakedness, which economic theory suggests are likely important features of exchange rate series.

While both the Student-t and the EGB2 nest within them the normal distribution, enabling the likelihood ratio tests used in Table 6, GARCH-t and GARCH-EGB2 are not nested within each other, so some other criterion must be used to test formally the null hypothesis that the two models

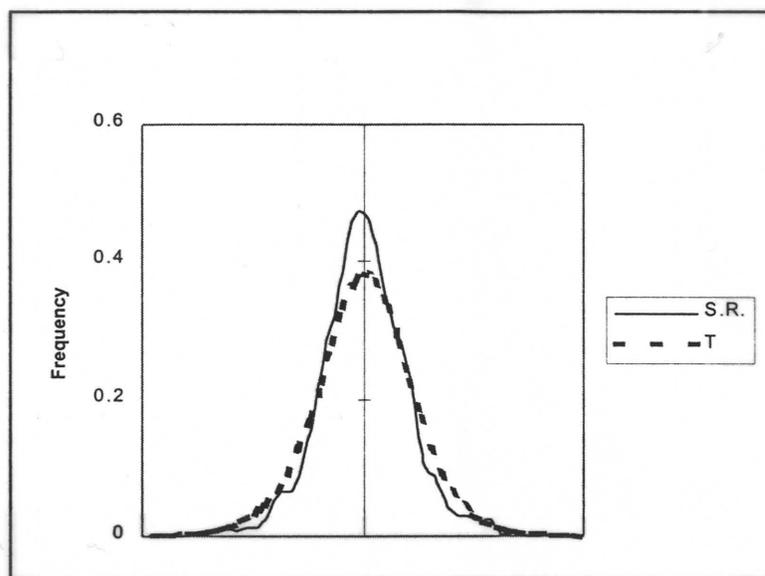


Figure 1. GARCH-t model for German DM.

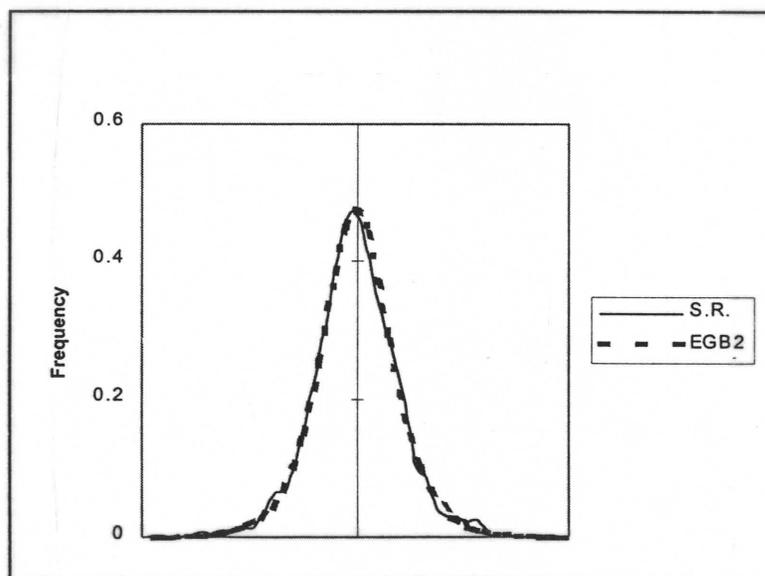


Figure 2. GARCH-EGB2 model for German DM.

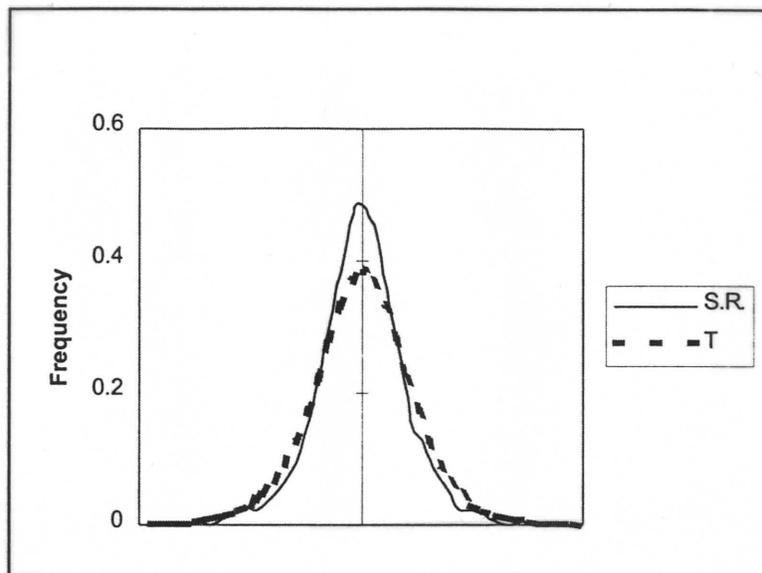


Figure 3. GARCH-t model for British pound.

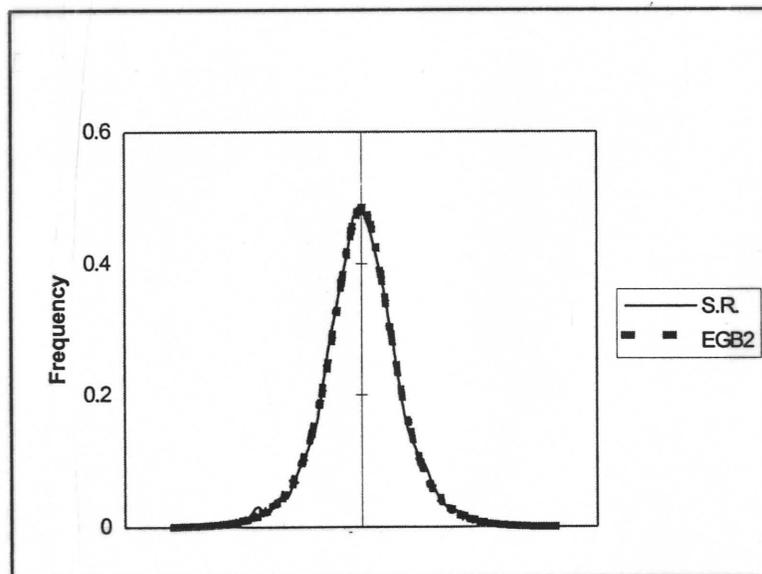


Figure 4. GARCH-EGB2 model for British pound.

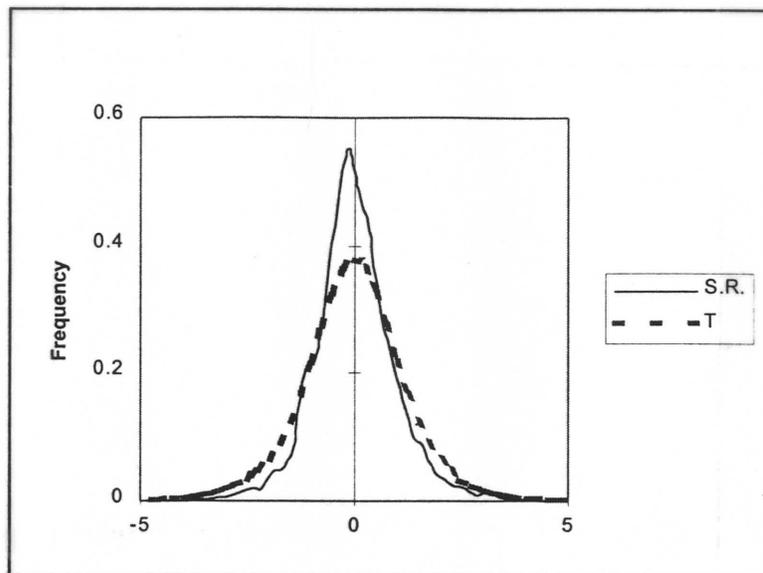


Figure 5. GARCH-t model for Japanese yen.

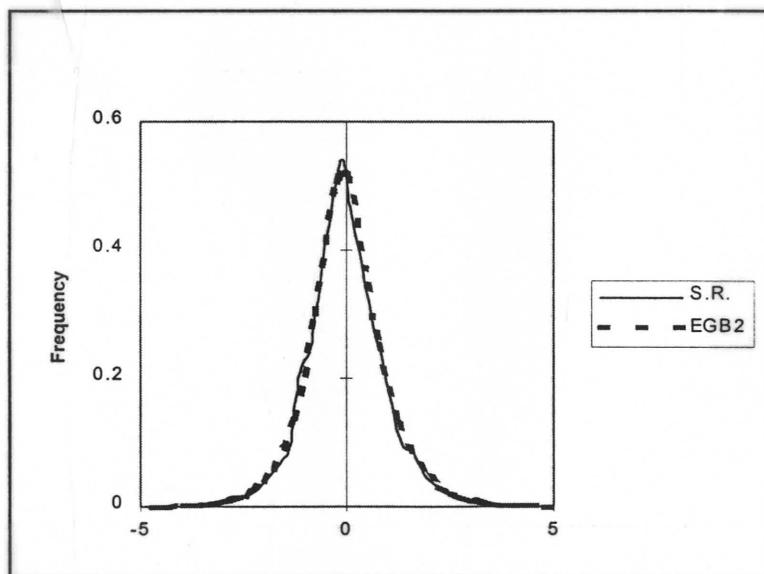


Figure 6. GARCH-EGB2 model for Japanese yen.

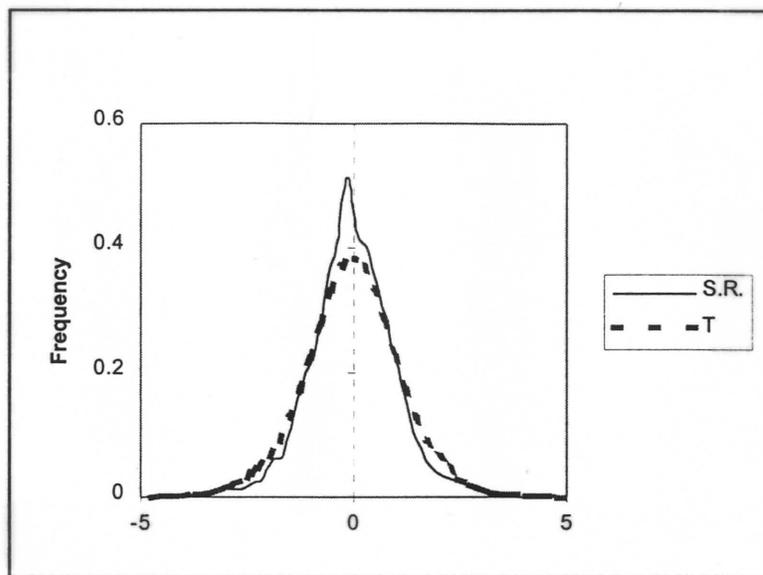


Figure 7. GARCH-t model for French franc.

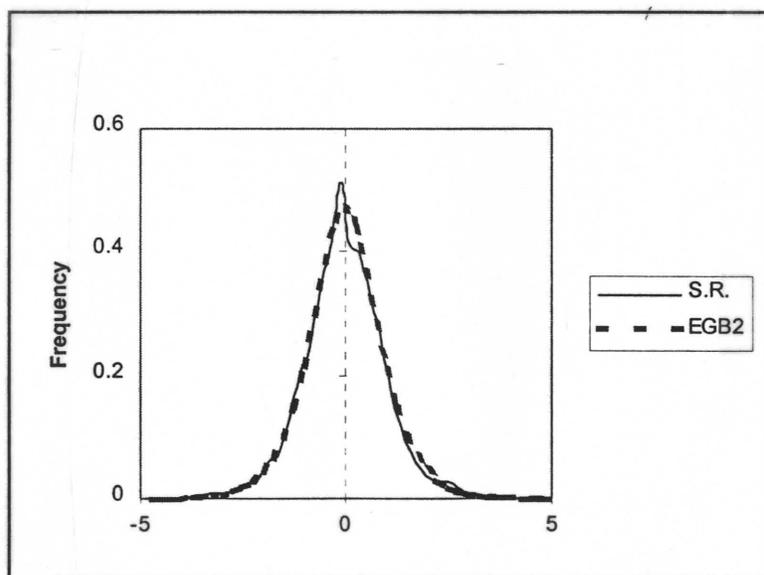


Figure 8. GARCH-EGB2 model for French franc.

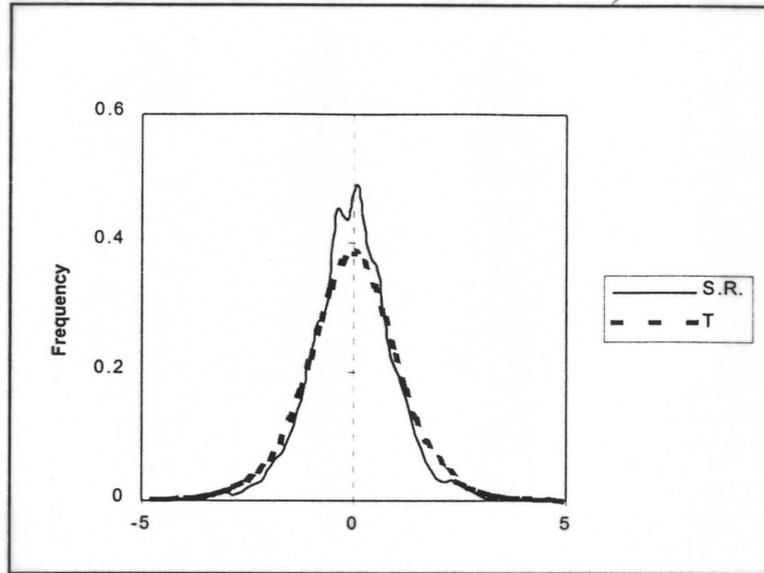


Figure 9. GARCH-t model for Belgian franc.

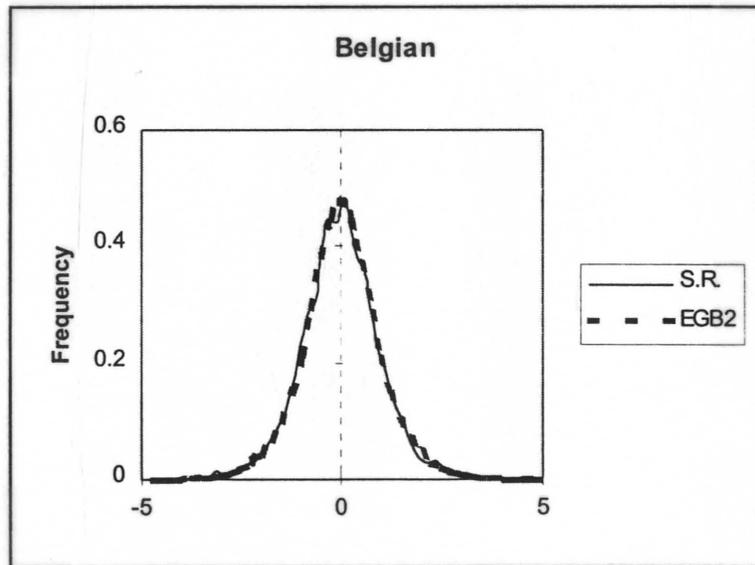


Figure 10. GARCH-EGB2 model for Belgian franc.

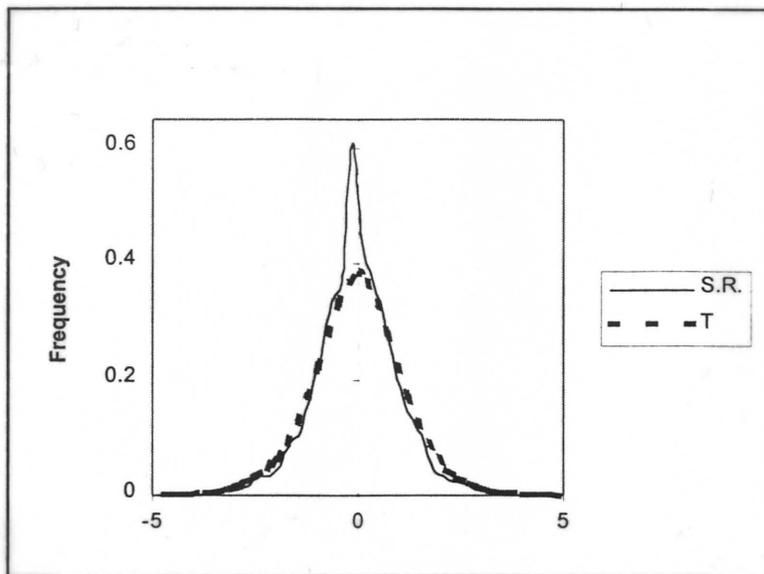


Figure 11. GARCH-t model for Italian lira.

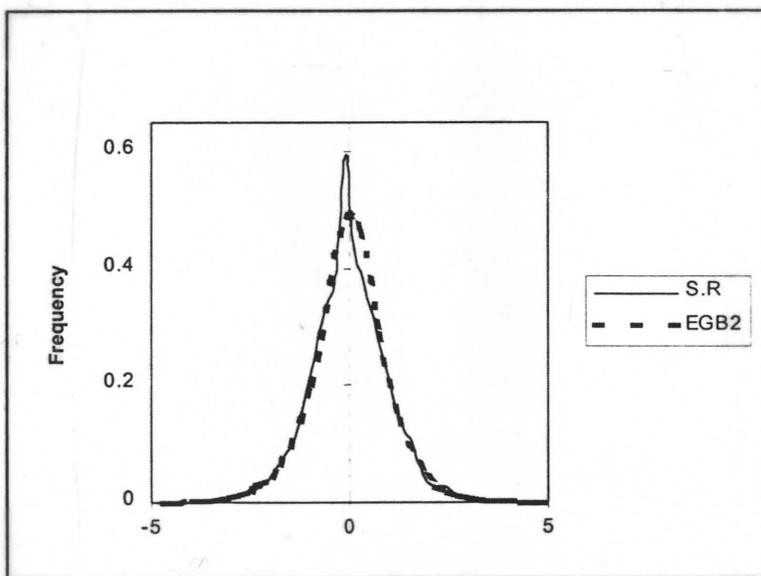


Figure 12. GARCH-EGB2 model for Italian lira.

are equivalent in these data. An appropriate option is the likelihood dominance criterion (LDC) proposed by Pollak and Wales (1991), which offers an approach to nonnested model selection consistent with the conventional inferential approach to hypothesis testing. The idea of LDC is to nest two nonnested competing models— $H_1$  and  $H_2$ —within a fictive composite and then consider a set of admissible composite parametric sizes. In most applications, the largest interesting sizes of the composite should range from one parameter more than the larger hypothesis ( $n_2 + 1$ ) to one parameter more than the sum of the number of parameters in two hypothesis ( $n_1 + n_2 + 1$ ). In this sense, the LDC model selections rules are as follows:

- (i) LDC prefers  $H_1$  to  $H_2$  if  $L_2 - L_1 < [C(n_2 + 1) - C(n_1 + 1)]/2$ ,
- (ii) LDC is indecisive between  $H_1$  and  $H_2$  if  $[C(n_2 - n_1 + 1) - C(1)]/2 > L_2 - L_1 > [C(n_2 + 1) - C(n_1 + 1)]/2$ .
- (iii) LDC prefers  $H_2$  to  $H_1$  if  $L_2 - L_1 > [C(n_2 - n_1 + 1) - C(1)]/2$ .

where  $L_1$ ,  $L_2$  denote the maximum log-likelihood values corresponding to the two models, and  $n_1$  and  $n_2$  are the numbers of parameters in  $H_1$  and  $H_2$ , respectively. LDC also assumes that  $n_1 < n_2$ .  $C(\gamma)$  is the critical values of the chi-square distribution with  $\gamma$  degrees of freedom at the prespecified significance level. In most practical situations, the LDC proves decisive for model selection.

Because GARCH-EGB2 always involves one more parameter than the GARCH-t model, the value of the criterion ( $[C(n_2 - n_1 + 1) - C(1)]/2$ ) is fixed at 1.29 for the 1% significance level. For all six exchange rates, GARCH-EGB2 dominates GARCH-t in the LDC sense (Table 7). The superiority of the GARCH-EGB2 specification is especially evident in modeling the pound, yen, and lira, each of which has a significantly skewed unconditional distribution. The returns to employing a more general conditional distribution appear greatest for asymmetric distributions, and exchange rate data commonly exhibit asymmetry.

**Table 7. Comparisons of GARCH-t and GARCH-EGB2 Models by LDC**

	$n^t$	$n^{EGB2}$	$LLH^{EGB2} - LLH^t$	$[C(n_2 + 1) - C(n_1 + 1)]/2$	$[C(n_2 - n_1 + 1) - C(1)]/2$
DM	6	7	2.15	0.81	1.29
£	6	7	3.20	0.81	1.29
¥	6	7	6.56	0.81	1.29
FF	6	7	2.11	0.81	1.29
BF	8	9	2.26	0.77	1.29
IL	8	9	3.72	0.77	1.29

$n^t$  and  $n^{EGB2}$  are the number of parameters in the GARCH-t and GARCH-EGB2 models, respectively.

$LLH^{EGB2} - LLH^t$  is the difference of log-likelihood value between the GARCH-EGB2 and GARCH-t models.

$[C(n_2 + 1) - C(n_1 + 1)]/2$  is the critical value to determine if GARCH-t is preferred to GARCH-EGB2 when  $LLH^t$  is greater than  $LLH^{EGB2}$ , whereas  $[C(n_2 - n_1 + 1) - C(1)]/2$  is used to determine if GARCH-EGB2 is preferred to GARCH-t when  $LLH^{EGB2}$  is greater than  $LLH^t$ . The critical values are derived from the  $\chi^2$  distribution evaluated at the 1% significance level.

The superiority of the GARCH-EGB2 model is further confirmed by goodness-of-fit test statistics (Table 8). These test statistics compare the frequency distribution of the residuals from the estimated models with the distribution predicted by the estimated distribution shape parameters,  $\eta_t$ . For each exchange rate, the test statistics for the GARCH-EGB2 model are far less than that from the GARCH-t model.<sup>14</sup>

Finally, Table 9 reports the parameter estimates and associated standard errors of the GARCH-EGB2 models fit to each exchange rate series. We report two standard errors for each estimated parameter: a conventional standard error and a White robust standard error (White 1982). White showed that if the model is correctly specified, the different methods to compute the

<sup>14</sup>The goodness-of-fit test has an asymptotic chi-squared distribution. For each estimated GARCH-EGB2 model, the test statistics support rejecting the hypothesis that the residuals are drawn from the assumed distribution at conventional levels of significance. However, this is common in large sample sizes, where this is a low power test.

**Table 8. Chi-Square Goodness-of-Fit Test Statistics**

Currency	GARCH-t Test Statistic	GARCH-EGB2 Test Statistic
DM	194.41	102.79
£	197.00	83.45
¥	364.63	87.04
FF	179.64	81.32
BF	194.76	99.88
IL	239.28	99.82

The test statistics are obtained by evaluating  $\sum(f_i - F_i)^2/F_i$ , where  $f_i$  is the observed count frequency of the standardized residuals,  $F_i$  is the predicted count frequency, and  $i = 1, \dots, 40$ . The  $\chi^2$  critical value at the 1% level is 63.69.

covariance matrix of the parameter will be (stochastically) the same. Our results routinely yield nearly identical standard error estimates by either method, providing informal evidence that the GARCH-EGB2 model is correctly specified.

## V. Conclusions

Although GARCH modeling based on normal or Student-t conditional distributions has been found useful in capturing the volatility clustering and leptokurtosis commonly present in asset price series, it cannot accommodate other commonly observed stylized effects in high frequency exchange rate data, notably high peakedness and skewness. Since economic theory suggests these are important statistical features, we propose a more flexible GARCH model based on the EGB2 distribution first introduced by McDonald and Xu (1995). The GARCH-EGB2 specification can

model either mesokurtic or leptokurtic data and can accommodate asymmetry, high peakedness, or both. An application to daily log price changes in six major exchange rates over the last ten years reveals the GARCH-EGB2 model significantly outperforms commonly employed specifications, as demonstrated by likelihood ratio tests against nested alternatives and by plots of standardized residuals, goodness-of-fit statistics, and the likelihood dominance criterion against nonnested alternatives. This more general GARCH-EGB2 approach can easily be applied to most financial time series. Since the improvements enjoyed due to employing a conditional EGB2 distribution are especially pronounced for unconditionally skewed data series, application to commodity price series appear especially promising.

### References

- Baillie R.T., and T. Bollerslev. "The Message in Daily Exchange Rate: A Conditional-Variance Tale." *Journal of Business and Economic Statistics* 7(1989):297-305.
- Baillie, R.T., and R.P. DeGennaro. "The Impact of Delivery Terms on Stock Return Volatility." *Journal of Financial Service Research* 3(1990):55-76.
- Bollerslev, T. "Generalized Autoregressive Conditional Heteroskedasticity." *Journal of Econometrics* 3(1986):307-27.
- Bollerslev, T. "A Conditional Heteroskedastic Time Series Model for Speculative Price and Rate of Return." *Review of Economics and Statistics* 9(1987):542-7.
- Bollerslev, T., R.Y. Chou, and K.F. Kroner. "ARCH Modeling in Finance." *Journal of Econometrics* 52(1992):5-59.
- Boothe, P., and D. Glassman. "The Statistical Distribution of Exchange Rates." *Journal of International Economics* 22(1987):297-319.
- Davis, H.T. *Tables of the Higher Mathematical Functions*. Vols. I and II. Bloomington, IN: Principia Press, 1935.
- Deb, P. "Finite Sample Properties of Maximum Likelihood and Quasi-Maximum Likelihood Estimators of EGARCH Models." *Econometric Reviews* 15(1996):51-68.
- Dornbusch, R. "Expectations and Exchange Rate Dynamics." *Journal of Political Economy* 84(1976):1161-1176.
- Engle, R.F. "Autoregressive Conditional Heteroskedasticity with the Estimates of the Variance of U.K. Inflation." *Econometrica* 50(1982):987-1008.
- Friedman, M. "The Case for Flexible Exchange Rates." *Essays in Positive Economics*. Chicago: University of Chicago Press, 1953.
- Hall, S.G., D.K. Miles, and M.P. Taylor. "Modeling Asset Prices Time-Varying Betas." *The Manchester School* 57(1989):340-56.
- Hansen, B.E. "Autoregressive Conditional Density Estimation." *International Economic Review* 35(1994):705-30.
- Hsieh, D.A. "The Statistical Properties of Daily Foreign Exchange Rates: 1974-1983." *Journal of International Economics* 24(1988):129-45.

- Hsieh, D.A. "Modeling Heteroskedasticity in Daily Foreign-Exchange Rates." *Journal of Business and Economic Statistics* 7(1989):307-17.
- Johnson, N.L., and S. Kotz. *Continuous Univariate Distributions*, Vol. 2. New York: John D. Wiley, 1970.
- Krugman, P.R. "A Model of Balance of Payments Crises." *Journal of Money, Credit and Banking* 11(1979):311-25.
- Lee, S., and B.E. Hansen. "Asymptotic Theory for the GARCH(1,1) Quasi-Maximum Likelihood Estimator." *Econometric Theory* 10(1994):29-52.
- MaCurdy, T.H., and L. Morgan. "Tests of the Martingale Hypothesis for Foreign Currency Futures with Time Varying Volatility." *International Journal of Forecasting* 3(1987):131-48.
- McDonald, J.B. "Parametric Models for Partially Adaptive Estimation with Skewed and Leptokurtic Residuals." *Economics Letters* 37(1991):273-8.
- McDonald, J.B., and Y.J. Xu. "A Generalization of the Beta Distribution with Applications." *Journal of Econometrics* 66(1995):133-52.
- Milhøj, A. "A Conditional Variance Model for Daily Deviations of an Exchange Rate." *Journal of Business and Economic Statistics* 5(1987):99-103.
- Obstfeld, M., and K. Rogoff. *Foundations of International Macroeconomics*. Cambridge: MIT Press, 1996.
- Pagan, A.R., and H. Sabau. "On the Inconsistence of the MLE in Certain Heteroskedasticity Regression Model." Monograph, 1987, University of Rochester.
- Patil, G.P., M.T. Boswell, and M.V. Ratnaparkhi. *Dictionary and Classified Bibliography of Statistical Distributions in Scientific Work: Continuous Univariate Models*. Burtonsville, MD: International Cooperative Publishing House, 1984.
- Peruga, R. "The Distributional Properties of Exchange Rate Changes Under a Peso Problem." Ph.D. dissertation, University of California, San Diego, Dept. of Economics, 1988.
- Pollak, R.A., and T.J. Wales. "The Likelihood Dominance Criterion." *Journal of Econometrics* 47(1991):227-42.
- Taylor, M.P. "The Economics of Exchange Rates." *Journal of Economic Literature* 33(1995):13-47.

Wang, K.L., C.B. Barrett, and C. Fawson. "Estimating East Asian Exchange Rates at Different Frequencies." Utah State University Economic Research Institute Study Paper #96-43, 1996.

White, H. "Maximum Likelihood Estimation of Misspecified Models." *Econometrica* 50(1982):1-25.

**Table 9. Parameter Estimates from GARCH-EGB2 Models**

	DM	£	¥	FF	BF	IL
Conditional mean equation parameters:						
C	0.021 (0.012) <sup>SE</sup> (0.012) <sup>RSE</sup>	0.020 (0.012) <sup>SE</sup> (0.011) <sup>RSE</sup>	0.023 (0.012) <sup>SE</sup> (0.012) <sup>RSE</sup>	0.020 (0.012) <sup>SE</sup> (0.012) <sup>RSE</sup>	0.024 (0.013) <sup>SE</sup> (0.013) <sup>RSE</sup>	0.014 (0.012) <sup>SE</sup> (0.012) <sup>RSE</sup>
AR(1)	0.378 (0.183) <sup>SE</sup> (0.186) <sup>RSE</sup>	0.345 (0.183) <sup>SE</sup> (0.191) <sup>RSE</sup>	0.040 (0.178) <sup>SE</sup> (0.189) <sup>RSE</sup>	0.392 (0.184) <sup>SE</sup> (0.186) <sup>RSE</sup>		
AR(3)						0.218 (0.179) <sup>SE</sup> (0.180) <sup>RSE</sup>
AR(6)					-0.248 (0.179) <sup>SE</sup> (0.183) <sup>RSE</sup>	-0.189 (0.174) <sup>SE</sup> (0.171) <sup>RSE</sup>
AR(8)					-0.319 (0.177) <sup>SE</sup> (0.179) <sup>RSE</sup>	
MA(1)					0.033 (0.019) <sup>SE</sup> (0.019) <sup>RSE</sup>	0.006 (0.019) <sup>SE</sup> (0.019) <sup>RSE</sup>
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Conditional variance equation parameters:						
w	0.015 (0.005) <sup>SE</sup> (0.006) <sup>RSE</sup>	0.01 (.) <sup>SE</sup> (.) <sup>RSE</sup>	0.011 (0.004) <sup>SE</sup> (0.006) <sup>RSE</sup>	0.016 (0.006) <sup>SE</sup> (0.007) <sup>RSE</sup>	0.015 (0.006) <sup>SE</sup> (0.007) <sup>RSE</sup>	0.022 (0.007) <sup>SE</sup> (0.008) <sup>RSE</sup>
$\alpha_1$	0.908 (0.018) <sup>SE</sup> (0.020) <sup>RSE</sup>	0.920 (0.008) <sup>SE</sup> (0.009) <sup>RSE</sup>	0.922 (0.018) <sup>SE</sup> (0.026) <sup>RSE</sup>	0.897 (0.021) <sup>SE</sup> (0.024) <sup>RSE</sup>	0.908 (0.020) <sup>SE</sup> (0.024) <sup>RSE</sup>	0.876 (0.025) <sup>SE</sup> (0.030) <sup>RSE</sup>
$\beta_1$	0.065 (0.012) <sup>SE</sup> (0.013) <sup>RSE</sup>	0.061 (0.010) <sup>SE</sup> (0.010) <sup>RSE</sup>	0.058 (0.013) <sup>SE</sup> (0.017) <sup>RSE</sup>	0.071 (0.014) <sup>SE</sup> (0.015) <sup>RSE</sup>	0.065 (0.013) <sup>SE</sup> (0.014) <sup>RSE</sup>	0.081 (0.016) <sup>SE</sup> (0.019) <sup>RSE</sup>
-----						
Distribution parameters:						
p	0.746 (0.122) <sup>SE</sup> (0.118) <sup>RSE</sup>	0.596 (0.099) <sup>SE</sup> (0.097) <sup>RSE</sup>	0.425 (0.077) <sup>SE</sup> (0.077) <sup>RSE</sup>	0.775 (0.128) <sup>SE</sup> (0.123) <sup>RSE</sup>	0.730 (0.122) <sup>SE</sup> (0.118) <sup>RSE</sup>	0.538 (0.108) <sup>SE</sup> (0.131) <sup>RSE</sup>
q	0.698 (0.112) <sup>SE</sup> (0.109) <sup>RSE</sup>	0.625 (0.106) <sup>SE</sup> (0.103) <sup>RSE</sup>	0.351 (0.061) <sup>SE</sup> (0.061) <sup>RSE</sup>	0.724 (0.117) <sup>SE</sup> (0.112) <sup>RSE</sup>	0.690 (0.113) <sup>SE</sup> (0.110) <sup>RSE</sup>	0.548 (0.112) <sup>SE</sup> (0.135) <sup>RSE</sup>

Standard errors reported in parentheses.

( )<sup>SE</sup> indicates the conventional standard error, while ( )<sup>RSE</sup> is the White robust standard error.

(.) indicates that the standard error cannot be estimated because the parameter estimate lies on the boundary of the feasible parameter space.

### Technical Appendix

Following the traditional definition of a GARCH process, suppose that

$$\epsilon_t = h_t^{0.5} z_t \quad (\text{A1})$$

where  $\{\epsilon_t\}$  is the error term sequence from the conditional mean equation and  $\{z\}$  is an i.i.d. sequence with zero mean and unit variance. Let  $h_t$  evolve according to a GARCH(1,1) process

$$h_t = \alpha_0 + \alpha_1 h_{t-1} + \beta_1 \epsilon_t^2 \quad (\text{A2})$$

If  $z_t$  is drawn from an EGB2 distribution, then the density is given by

$$EGB2(\epsilon; \delta, \sigma, p, q) = \frac{\exp\left(\frac{p(\epsilon - \delta)}{\sigma}\right)}{|\sigma| B(p, q) (1 + e^{\frac{\epsilon - \delta}{\sigma}})^{p+q}} \quad (\text{A3})$$

The mean and variance of  $z$  are then as follows:

$$\text{Var}(z) = \sigma^2 (\psi'(p) + \psi'(q)) = 1 \quad (\text{A4})$$

$$E(z) = \delta + \sigma[\psi(p) - \psi(q)] = 0 \quad (\text{A5})$$

Hence, solving for  $\sigma$  and  $\delta$  in terms of  $\Delta$  and  $\Omega$

$$\sigma = \sqrt{\frac{1}{\psi'(p) + \psi'(q)}} = \sqrt{\frac{1}{\Omega}} \quad (\text{A6})$$

$$\delta = -\sigma[\psi(p) - \psi(q)] = -\Delta \sqrt{\frac{1}{\Omega}} \quad (\text{A7})$$

where

$$\Delta = \psi(p) - \psi(q) \quad (\text{A8})$$

$$\Omega = \psi'(p) + \psi'(q) \quad (\text{A9})$$

Substituting those expressions for  $\delta$  and  $\sigma$  back into the EGB2 distribution yields an EGB2 density function with zero mean and unit variance as

$$EGB\lambda(z;p,q) = \frac{\sqrt{\Omega} \exp(p(z + \frac{\Delta}{\sqrt{\Omega}})\sqrt{\Omega})}{B(p,q)(1 + \exp((z + \frac{\Delta}{\sqrt{\Omega}})\sqrt{\Omega}))^{p+q}} \quad (A10)$$

According to the assumption (A1),

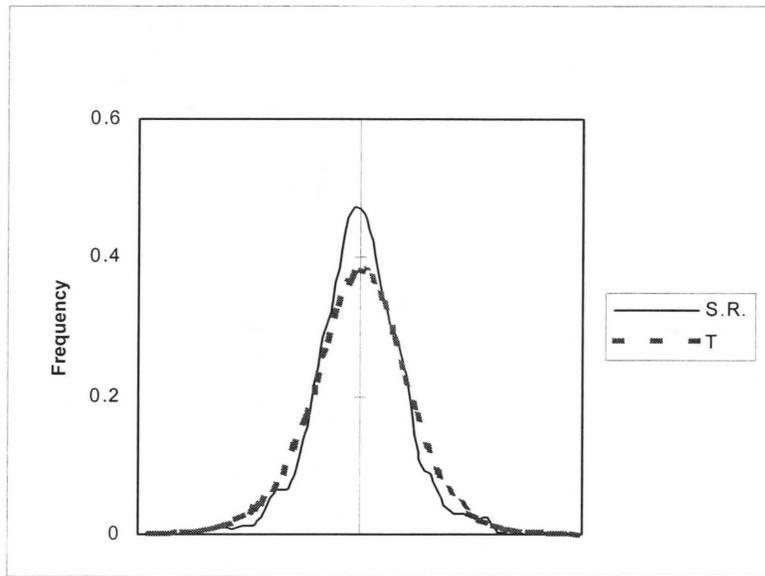
$$\epsilon_t = \frac{z_t}{\sqrt{h_t}} \quad (A11)$$

Changing the variable from  $z$  to  $\epsilon$  as follows: ( $dz = d\epsilon/h$ )

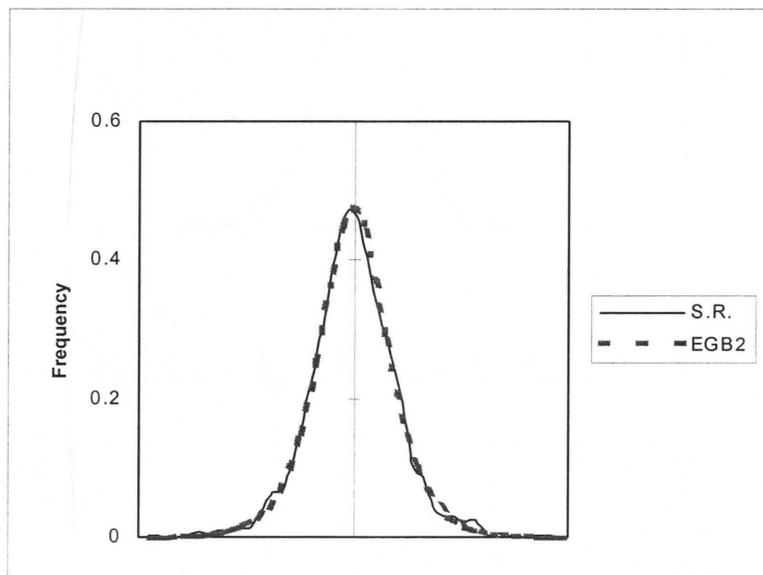
$$EGB\lambda(\epsilon;h,p,q) = \frac{\sqrt{\Omega} \exp(p(\frac{\epsilon}{\sqrt{h}} + \frac{\Delta}{\sqrt{\Omega}})\sqrt{\Omega})}{\sqrt{h} B(p,q) (1 + \exp((\frac{\epsilon}{\sqrt{h}} + \frac{\Delta}{\sqrt{\Omega}})\sqrt{\Omega}))^{p+q}} \quad (A12)$$

Algebraic manipulation then yields

$$EGB\lambda(\epsilon;h,p,q) = \frac{\sqrt{\Omega} \exp(p(\frac{\sqrt{\Omega}}{\sqrt{h}}\epsilon + \Delta))}{\sqrt{h} B(p,q) (1 + \exp(\frac{\sqrt{\Omega}}{\sqrt{h}}\epsilon + \Delta))^{p+q}} \quad (A13)$$



**Figure 1. GARCH-t model for German DM.**



**Figure 2. GARCH-EGB2 model for German DM.**

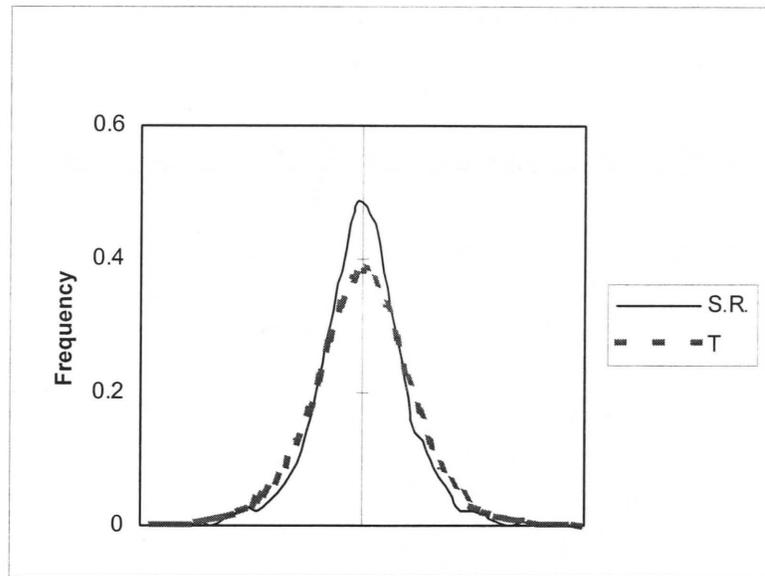


Figure 3. GARCH-t model for British pound.

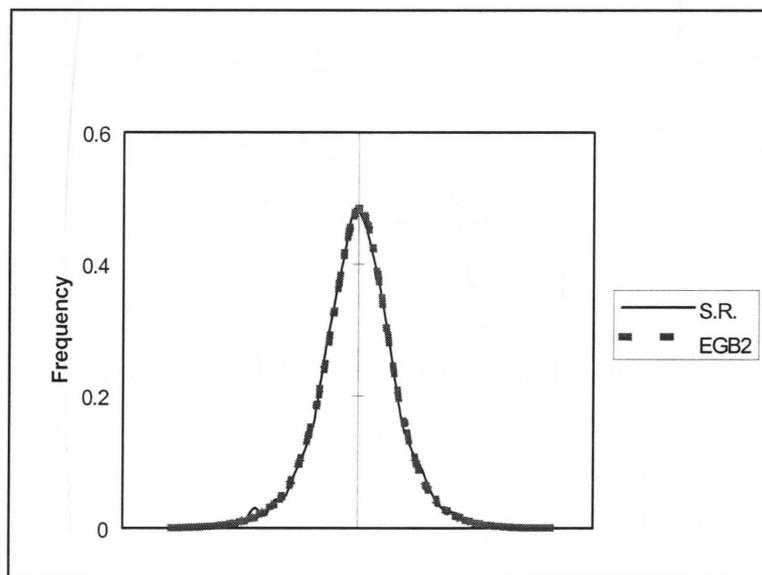


Figure 4. GARCH-EGB2 model for British pound.

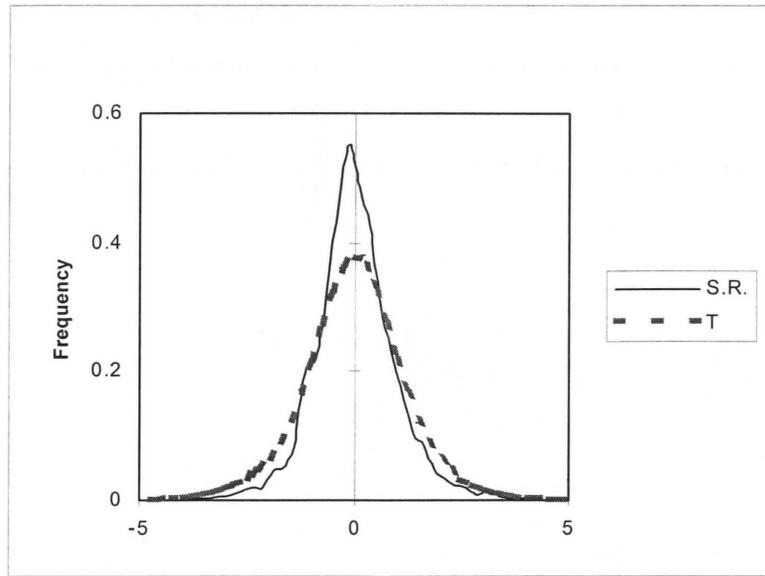


Figure 5. GARCH-t model for Japanese yen.

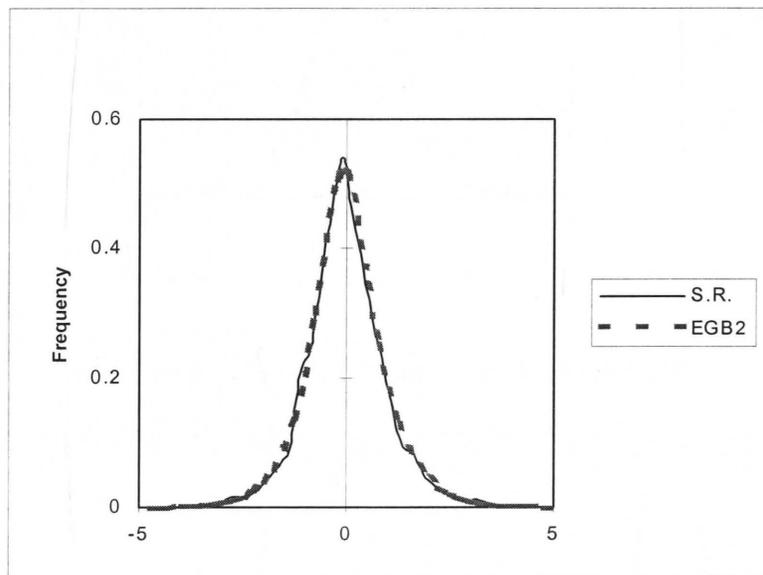


Figure 6. GARCH-EGB2 model for Japanese yen.

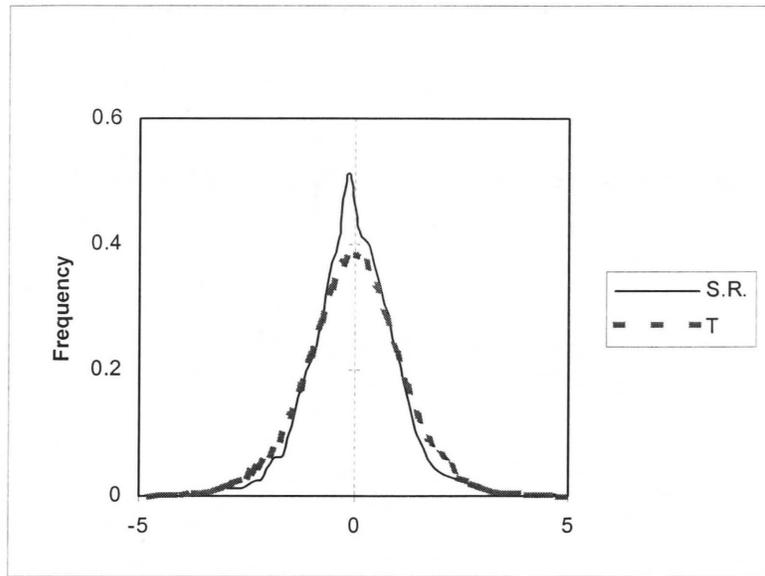


Figure 7. GARCH-t model for French franc.

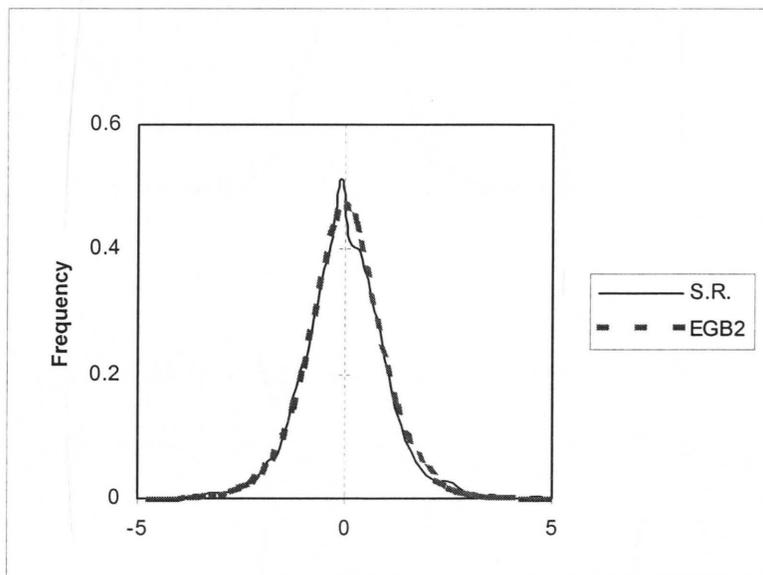


Figure 8. GARCH-EGB2 model for French franc.

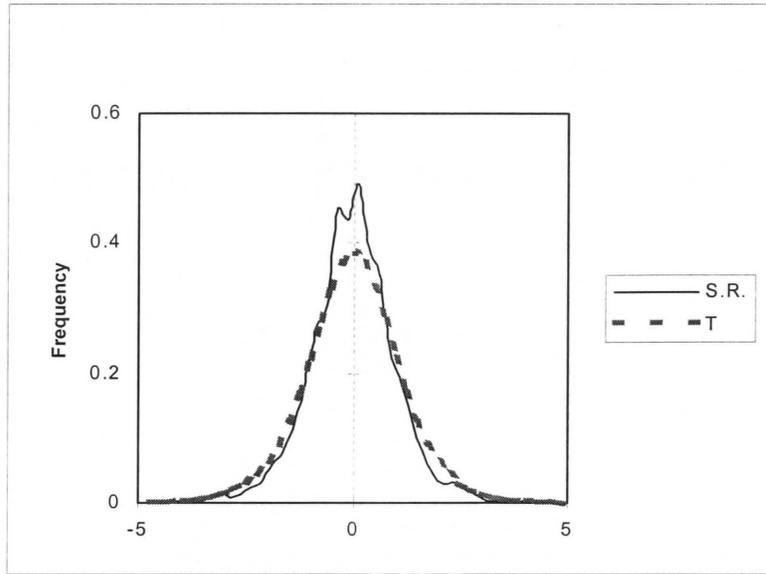


Figure 9. GARCH-t model for Belgian franc.

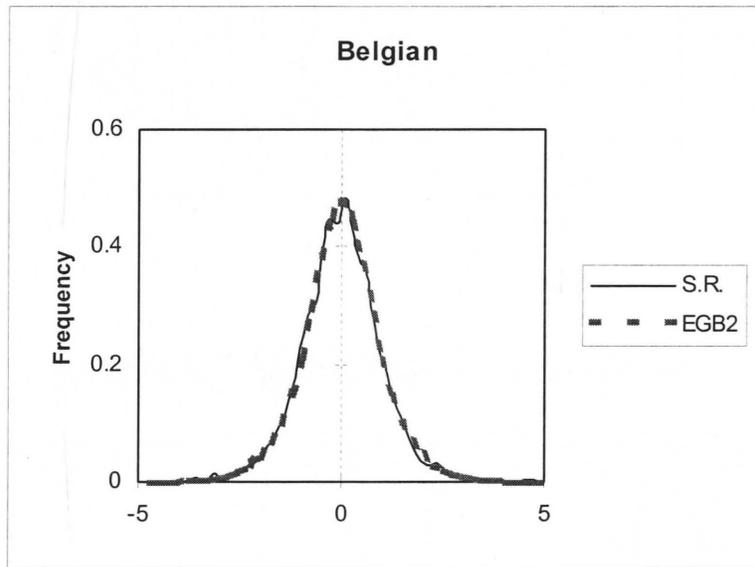


Figure 10. GARCH-EGB2 model for Belgian franc.

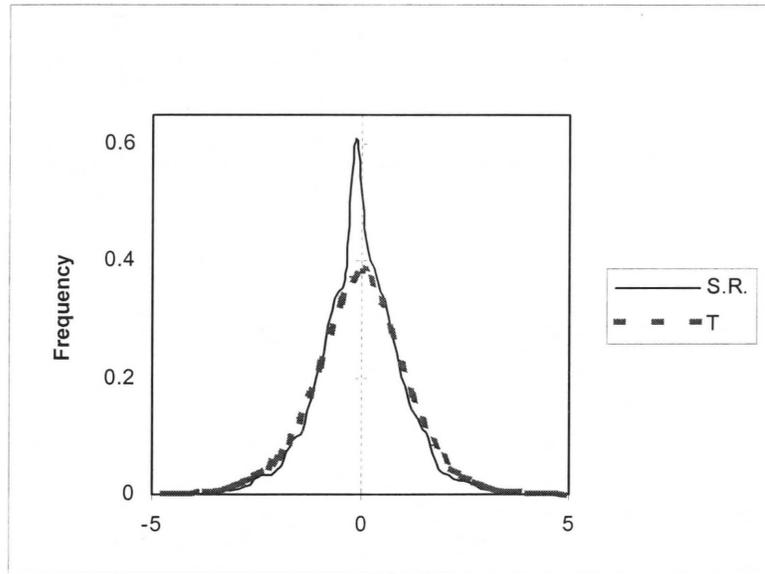


Figure 11. GARCH-t model for Italian lira.

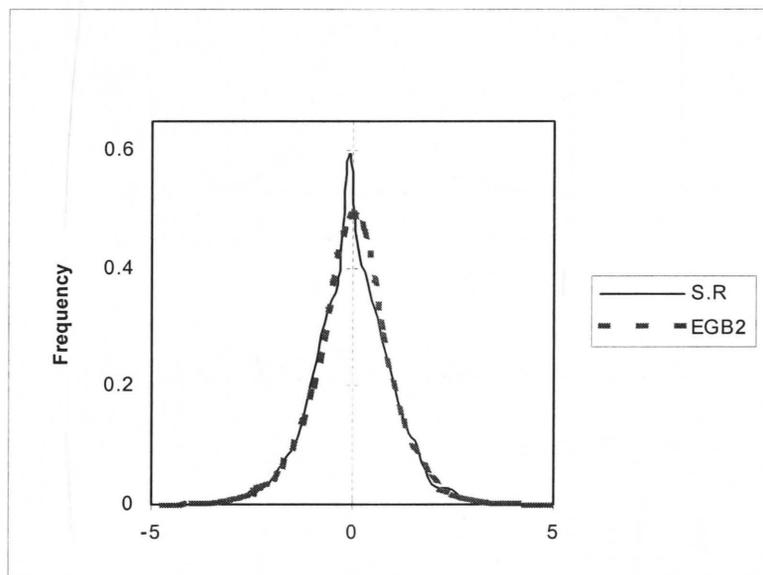


Figure 12. GARCH-EGB2 model for Italian lira.