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Introduction

Forest lands provide numerous things adding to the social, cultural, and economic aspects of life for many people including fuel, water, forage, stabilization of shifting sands, protection of catchment areas, soil erosion and flood control, watershed, habitat for wildlife, and sites for outdoor recreation. Because of their large area and wide geographic dispersion, they are also important in maintaining the natural environment. They are the source of timber, an important industry in many parts of the world. Products made from trees affect everyone, including those who may never have the opportunity to enjoy the natural beauty of a forest or to participate in forest-based recreation.

The continued economic viability of forests has generated concern for several reasons. Forecasts of rapid depletion, multiple-use conflicts, and increasing environmental restrictions have made modern forest management a controversial public policy issue in many parts of the world. India is no exception.[1]

The total area of lands classified as forests in India is about 24 percent of the geographical area. Forests and forest products provide jobs for only 0.2 percent of the working population but account for 1.5 percent of the national income. This contribution has been rising at the annual rate of nearly 15 percent per year compared to a 3 percent rate of growth for total national income (Kulkarni, 1970). Again, the addition of non-timber benefits of forests would increase the contribution of forests and forest products.

With over a hundred years' history of forestry practice, India nevertheless stands classified on the world map of forest resources as

belonging to a "deficit" zone. The nearly 1/7th of the world's population that lives in this country has hardly 1/55th of the world's forest area to depend upon. Available forests in India are not yet fully productive. With the rapid pace of industrialization and the rising standard of living, the requirements of forests and forest products in this country are steadily mounting. Furthermore, the sort of rural economy that exists in India is so intricately tied into local forestry that attempts to segregate the two create serious problems, both social and economic (Kulkarni, 1970). Thus, the presence of and issues involved in a multidimensional natural resource like forestry in the socio-economic sphere of India can hardly be ignored. This paper addresses one such issue.

Forest management involves the simultaneous management of multiple-use resources because, timber is only one of many outputs produced from a forest land and represents one of the earliest cases of formal application of economic principles to resource management.

One of the major policy questions which has dominated forest resource economics literature is: When should timber be harvested? In an economic context, any time sequence for harvesting constitutes a rotation policy; a sequence that maximizes the discounted total net benefits is an optimal rotation policy.

Theoretical Setting

Determining the optimal rotation period may be regarded as an expression of a basic economic problem. Fundamentally, it is a problem in capital theory and asset replacement. Growing forest stock represents the accumulation of forest capital. During the transition

from seedlings to maturity, trees serve as both inventory and capital. Thus, the question of how much capital to invest for how long is critical for timber production economics (Gregory, 1972; Perrin, 1972; and Hyde, 1980). This, in turn, necessarily involves other basic economic issues. What, if anything, does a firm (e.g., in the U.S.) or a public forest land manager (e.g., in India) attempt to maximize over time? What is the logical financial objective in managing a forest?

Over time, several different objectives have been proposed for determining optimality. These are discussed in Gaffney (1960), Bentley and Teeguarden (1965), Gregory (1972), and Samuelson (1976). Their arguments show an overall preference for the maximum net present value (NPV) rule. Samuelson (1976) argues that correct capital theoretic analysis requires that the primary objective should be to maximize the NPV of revenues obtainable from all the infinite sequence of harvests which can be obtained from the forest land. This view, known in the forestry literature as the "soil expectation value" (SE) approach, was advocated originally by Faustmann (1849).

The Faustmann model has played a key role in forest economics. It has become the keystone of the currently held view regarding timber rotation under a criterion of financial maturity (Samuelson, 1976).

Faustmann introduced the simple and deterministic competitive economic model, with the objective of maximizing the present value V(t) of perpetual returns to the fixed factor of production, an acre of timber land. The total value, V(t), is the sum of revenues minus costs. Revenue is the expected price, p, times the volume harvested, $Q(t_1)$, discounted from the time of harves, t_1, to the initial moment of land availability, by the opportunity cost of capital, r. Since, in this

model, trees grow naturally without silviclultural inputs, harvest volume continues to be a function only of time and there are no costs other than opportunity costs of capital (r) and land (R). The cost of land is the economic rent, R, discounted over the duration of the timber production period. If timber production constitutes the best use of the land, then substituting a perpetual timber production term for the rent term should allow the problem to be stated as:

$$V(t) = \max_{t_n} \; p \sum_{n=1}^{\infty} Q(t_n) e^{-r \sum_{i=1}^{n} t_i} . \qquad (1)$$

Because all the parameters continue unchanged from one production period to the next, an identical problem confronts the forest manager following each harvest. Therefore, each succeeding production period is of the same length ($t_i = t_j \; \forall \; i, \; j$) and equation (1) is usually simplified as

$$V(t) = \max_{t} \; p.Q(t)e^{-rt} \; (1-e^{-rt})^{-1}. \qquad (2)$$

This form is familiar to the foresters as the Faustmann equation and rV(t) represents the "soil expectation value" (SE). Samuelson (1976) proved that the single rotation model with land rental payments and the perpetual timber production model possess identical optimality conditions.

The necessary and sufficient conditions for a maximum derived from equation (2) are

$$Q_t = rQ(1-e^{-rt})^{-1} \qquad (3)$$

$$Q_{tt} < rQ_t \qquad (4)$$

where the subscripts indicate derivative of the function with respect to the subscript. Timber is "financially mature" when its natural growth rate is $r(1-e^{-rt})^{-1}$, which is equal to the opportunity cost of capital adjusted upward to compensate for the implicit land rent. The greater the cost of capital, the shorter the production or rotation period.

It can be shown that the optimal economic production period is shorter than the optimal biological production period when the cost of capital r is positive. For smaller costs of capital, the value-maximizing harvest age increases until it converges with the volume-maximizing age (Hyde, 1980).

Modified Faustmann models within static deterministic framework

Within the static Faustmann framework, several articles have recently appeared indicating alternative solutions under different and sometimes less restrictive assumptions (Clark, 1976; Walter, 1980; Hyde, 1980; Nautiyal and Fowler, 1980; Heaps, 1981; McConnell et al., 1983; Chang, 1981 and 1983; Nautiyal, 1983; Hardie et al, 1984). Individually, each provides valuable ingredients toward generalization. Each extends and modifies the basic Faustmann formulation.

However, the optimum rotation problem viewed by these authors is an optimum timber management problem abstracting from the important multiple-use characteristics of forest land. Samuelson (1976) took note of the problem and Hartman (1976) and Strang (1983) developed a generalized Faustmann model by incorporating benefits associated with the forest resource besides timbering. The stock of standing forest resource provides other benefits to society, such as water, hiking, flood control, and wildlife. The flow of these services is an increasing

function of the age of the forest. In order to simplify the model somewhat, these may collectively be viewed as "recreation" benefits (Hartman, 1976). This formal recognition of recreational services leads to a longer optimal rotation.

On examining the model of Hartman and Strang, and with their help, we obtain some new results:

1. A finite optimal harvesting date may not exist. In this case the forest is intended to provide only recreational services.

2. If there is a finite optimal rotation, it may imply harvesting after the forest has reached its maximum growth and has started to decline.

3. If by mistake we have delayed harvesting past the optimal date, then the correct decision may switch to leaving the forest in tact. This is in contrast to the usual result of clear-cutting as soon as the mistake is realized.

Dynamic treatment

The literature discussed to this point strongly depends on long-run predictions of future prices, costs, and discount rates. These elements are observed during a single moment in time. However, they change over time and can be properly captured only within a dynamic framework. Anderson (1976), Clark (1976), Heaps and Neher (1979), and Berck (1981) have extended previous analyses by providing a dynamic treatment of forest harvesting. The authors have utilized optimal control theory (the maximum principle). Some interesting suggestions for coping with the optimum rotation question have evolved from these studies. Anderson's steady-state control solution, in particular, is identical

with the Faustmann rotation model, lending support to the latter as appropriate not only for private timber management decisions but also for public policy where the goal of the planner is the maximization of discounted net social welfare from timber production over an infinite planning horizon.

Treatment of uncertainty

All the analyses mentioned so far assume a deterministic world. In reality, of course, current and future prices of timber are uncertain as are the effects of environmental changes on resource stocks and the amount of the resource available for extraction.

Norstrom (1975) using a Markov model for price fluctuations demonstrated that for a single production process with either uncertain output volumes or uncertain output prices, longer rotations and larger harvests are optimal. Recently, the optimal rotation period when the risk of unpredictable destruction (e.g. by fire, insects, flood, and storm) is present has been considered by Martell (1980), Routledge (1980), and Reed (1984). Martell and Routledge solved the problem in discrete time. Using Poisson stochastic process Reed formulated and solved the problem in continuous time, deriving a modified form of the Faustmann formula.

Additional Dimensions[2]

(A) Costs: Existing literature dealing with the problem of determining the optimal rotation period for a forest stand under conditions of certainty as well as uncertainty lacks generality with respect to the costs of providing benefits from a multiple use forest.

Hartman (1976), Strang (1983), and Berck (1981) addressed this situation by introducing the consumptive value of standing forest in their models. Yet in doing so, they have ignored the costs involved in providing and making these consumptive values accessible to potential users.

One way to partially bridge this gap is to incorporate into the model the costs associated with regeneration of the tree population and associated maintenance, and the costs associated with providing recreational services. This is absolutely necessary if the required management decision is based on net values (Hyde, 1980). While regeneration costs have been accounted for in part by some authors, recreation costs in the context of the rotation problem have received little attention. Thus, in such a framework, the objective functions to be maximized are to be expressed in terms of a forest that provides net values (as opposed to gross values) when standing as well as when harvested.

Let $R_t = R(t)$, be the optimal quasi-rent stream flowing from providing recreational services. Quasi-rent, as defined here, is the difference between the present value of revenue from recreational services and the present value of the variable costs associated with providing recreational services such as road development and maintenance, campground preparation and clean-up, wild life habitat improvement programs, etc. The quasi-rent function is so derived that it gives the maximum quasi-rent obtainable at each point in time from operating a standing forest. It is based upon the underlying optimal combination of inputs and output (recreational services). The quasi-rent function may be used for analyzing the rotation length without the explicit introduction of value of recreational services and costs. R(t)

is strictly concave with respect to time (Fig. 2). The forest stand is regenerated in an initially barren land at time t=0 at a fixed regeneration and maintenance cost, C_o^R. The stumpage value (net of harvesting costs) of the tree stock in a competitive market at time t = T, G_T, is a function of the age of the forest, such that $G_T = G(T)$. Due to its underlying biological characteristics $G'(T) \gtreqless 0$, (Figure 1). It is plausible to assume that both R(t) and G(t) are bounded and continuous.

Given that the forest operator plans for an infinite horizon and an infinite chain of identical forests succeeding one another, the objective function, in this more generalized model, to the maximized is given by

$$V(T) = \frac{\int_0^T R(t)e^{-rt}dt - C_o^R + G(T)e^{-rT}}{1 - e^{-rT}}$$

$$= \frac{V_1(T)}{1-e^{-rT}} \qquad (5)$$

Assumptions made about R, G, and r imply that function V is bounded and continuous. Thus, it can be shown that V(T) attains a maximum on $[0, \infty]$ for some $T \leqq T_o$. This implies that the maximum net return is obtained at a finite rotation age (as opposed to Hartman-Strang never to cut solution), though there may be more than one local maximum. For a single rotation, the first order condition for the optimum implies $R(t) + G'(t) = rG(t)$ and is shown in Figure 3 (the subscript H stands for Hartman-Strang specifications). It can also be shown that, depending on the values different components of costs, the finite rotation period indicated by the solution of this model may be identical to, shorter or

longer than that indicated by the Hartman-Strang finite solution. The difference between the per year flow of marginal variable costs of recreational services and the present value of average costs per year of the regenerated forest stand over the period t = 0 to t = T, appears to be the crucial factor.

(B) Optimal control solution when standing forest has value: Optimal control (maximum principle) of Pontiyagin et. al. (1964) has emerged as a very powerful modern analytical tool of research for dynamic optimization problems. The optimal rotation rule when forest lands posses recreation value besides timber value can also be derived analytically by utilizing the steady-state properties of an optimal-control (maximum principle) framework. But no such attempt has yet been made.

Let us consider a synchronized forest of even-aged stands. It is hypothesized that the stock of the standing forest resource provides benefits to society but the private resource owner may ignore this flow of services related to the stock of the resource. The model outlined below is, thus, a normative model that will permit us to derive rules characterizing optimum behavior from a social viewpoint. It is then examined to what extent a competitive decision characterized by a Faustmann-type decision rule is likely to behave in this way.

In the present model, the forest resource is controlled by a hypothetical social manager/planner whose primary function is to manage the natural resource commodity, timber. It is assumed that the manager -chooses the rate of harvest in each period to maximize the social utility of the discounted stream of net benefits from the resource over an infinite planning horizon.

The following assumptions and relations are maintained in the development of the model:

Let $X = X(t)$, a scalar, be the stock of the harvestable population of trees in a forest at time t. Let its growth be described by the differential equation $dX/dt = \dot{X}(t) = g[X(t)] - h(t)$, where $g[X(t)]$ is a concave function representing the natural growth rate for the resource population. The variable $h = h(t)$ is the rate of harvesting at time t. Let $F = F[X(t)]$ be the value of recreational services that the stock of standing trees (the resource population) provides to society. The function F is assumed to be concave and twice differentiable.

Let $c = c[h(t), X(t)]$, where c is the (total) cost of harvesting. Cost is assumed to be negatively related to stock ($\partial c/\partial x < 0$). It is also assumed that $\partial c/\partial h \geq 0$. The costs directly associated with the harvest rate h(t) are composed of the opportunity costs of inputs and the loss of recreational services that will be assumed to be related to the remaining undisturbed stock of the standing forest. The costs indirectly associated with h(t) are those imposed on the future as a result of using some of the timber stock.

The social benefits (SB) associated with a rate of natural resource (forest) commodity (timber) utilization (harvesting) of h(t) can be represented by the area under the timber demand curve up to the harvest rate h(t), plus the value of recreational services related to the undisturbed stock, X(t), such that $SB(t) = \int_0^h D(\theta)d\theta + F[X(t)] = U(h) + F[X(t)]$. The planner's/social manager's object is to

$$\text{Max } W = \int_0^\infty [U(h) - c(h,X) + F(X)]e^{-rt}dt \qquad (6)$$

subject to

$$\dot{X} = g[X(t)] - h(t)$$

$$X \geq 0;\ h\ [0,\ h_{max}] \qquad (7)$$

In (6) W is the discounted "social" value of the perpetual stream of net benefits over time and is assumed to be convex from above. Equations (6) and (7) comprise a problem in optimal control theory, with the control variable being h(t) and the state variable being X(t). The equation of motion specifying the rate of change of X(t) is (7).

It can be demonstrated that an optimal control model is consistent with the Faustmann framework for maximizing the NPV of a series of rotation cycles of identical length even when the value of recreational services and the regeneration costs are added to the model. Forest managers utilize the Faustmann framework to maximize the discounted net return of forested land when the forest provides timber value, if harvested, and a flow of value of recreational services, if standing, provided they take account of the flow of positive externality flowing from the stock of biomass. In the process, the managers follow an infinite chain of harvests, the steady-state characteristics of which are equivalent to the steady-state rule that would be adopted by a manager/planner maximizing social welfare in the context of equations (6) and (7).

(C) Uncertainty and risk: As noted earlier, traditionally, the problem of determining optimal forest rotation has been treated within the framework of deterministic models. The more generalized deterministic model (incorporating both the benefits and costs of the recreational services and replanting costs) presented above can be

further extended by incorporating at least two aspects of stochashic environment separately: (1) An uncertain stumpage price when forest owner is risk averse; (2) Risk of unpredictable catastrophe making stock of resource biomass (tree population) uncertain.

As for situation (1), uncertainty in stumpage price results in a V that is stochastic. Hence, the manager must select the best of the available probability distributions for V, which are called random prospects. If we assume that the manager's behavior in solving this problem conforms to the Von Neumann-Morgenstern axioms, then it can be inferred that the preference ordering for various random prospects can be represented by a utility function U[V(t)] and that the best prospect is found by maximizing the expected value of utility.[3]

For a forest manager with a planning horizon running through an infinite sequence of identical harvest cycles the objective function to be maximized turns out to be

$$W(T) = E\{ U[V_1(T)/1-e^{-rt}]\} \tag{8}$$

where $r > o$ is the riskless interest rate. The forest manager's attitude towards risk in resource return is represented by the form of the U[V(T)]. Strict concavity in the utility function implies risk aversion. The choice of the particular form is based on its risk characteristics in terms of the measures of risk aversion developed by Arrow (1971) and Pratt (1964). In the analysis here, utility is represented by a concave, continuous, and twice differentiable function of discounted net returns, U[V(T)], where

$$U'[V(T)] > 0,\ U''[V(T)] < 0 \tag{9}$$

so that the forest manager is assumed to be risk averse.

The expected utility of discounted net returns from an infinite chain of cycles can be written as

$$E\{U[V_1(T)]/1-e^{-rt}\} = \int U[\int_0^T e^{-rt}R(t)dt + e^{-rt}G(T) - C_0^R]f[G(T)]\, dG(T)/1-e^{-rt} \qquad (10)$$

where the first integration is over the range of G(T).

Solution of (10) shows that the optimal rotation period will be longer than that under conditions of certainty. It can also be shown that the period will be lengthened with increasing risk and shortened with increasing expected stumpage price under nonincreasing absolute risk aversion of the forest manager.

Situation (2) considers the possibility of unpredictable destruction of a forest stand by natural causes (e.g., forest fire, storm, flood, disease, and insect plagues) and its impact on the rotation decisions. It is assumed that natural catastrophes occur in an age-independent homogeneous Poisson process.[4] Two cases are considered: when catastrophes result in total destruction of the forest stand, and when destruction through loss agent is only partial. It is assumed that the objective of the forest operator is to maximize discounted expected return from the forest. In effect it is assumed that the forest operator is risk neutral.

It can be shown that risk of catastrophic destruction of biomass whether total or partial will lead to a rotation period dependent on the value of the average rate of occurrence of catastrophes (λ). However, the conclusion that the rotation period will be shorter than that suggested by the simple Faustmann rule, is shown to hold unambiguously.

As $\lambda \to 0$, the rotation period tends to coincide with the generalized Faustmann rotation period. With higher values of λ the rotation length tends to be shorter. $\lambda > 0$ shortens the rotation length in two ways: one through its impact as a risk-permium and the other through its impact on both the stumpage value and on the net value of recreational services.

Scope for Further Research

The theoretical generality obtained thus far need to be empirically tested, not only to verify the theoretical results but also to extend the theories leading to more definitive conclusions.

The optimal control formulation discussed here regards recreational benefits as a positive stock externality assumed to be ignored by a private forest manager. But the current trend towards creating and providing recreational facilities by private forest operators (e.g., in the U.S.) needs to be captured in such a dynamic model where production of recreational services is an activity having both benefits and costs associated with it.

The whole problem of uncertainty needs to be treated in a more general and, preferably, dynamic framework incorporating all major sources of uncertainty.

Even within a partial-equilibrium framework impact of uncertainty related to demand for recreational services and prices of inputs and the impact of risk of age dependent natural catastrophes in presence of net recreational values need further investigation. The latter, furthermore, needs to incorporate the more plausible assumption of risk aversion as a behavior towards risk.

The economics of optimum forest rotation in the context of multiple-use characteristics of forests needs deeper probe. If timber production for commercial use is the primary objective of management of a forest, non-timber benefits may be treated as stock-externalities. On the other hand, sometime in some locations the primary objective of public forest management may be to provide non-timber benefits per se to the society. In either case, while benefits like recreation (as the term connotes) can be provided as private goods (as in the U.S.), many other multiple benefits epitomized by ecological and environmental impacts of forestry, essentially assume the nature of public goods. They generally, can not be withheld from one individual without withholding from all and thus, must be supplied communally. In the context of countries like India, this public goods characteristic of non-timber benefits (including recreation) is definitely very significant. Optimal provision of these public goods may, thus, necessitate the intervention of the government. In fact, in India, as much as 92.3 percent of the total forest area is owned by the government. Determining the optimum forest rotation in the context of optimal provision of public goods flowing from forests, provides ample area of further investigation--theoretical as well as empirical.

FOOTNOTES

[1]See, e.g., the editorial comments in The Statesman Weekly, "As the population grows and, with it, the number of cattle, the temptation to cut down forests becomes irresistible. The demand for more land for cultivation and grazing, as well as for more wood for fuel, house construction, furniture and industry can mean wanton damage:..." (1985)

[2]For the details of the formulations, derivations and analyses of the following discussions see Bhattacharyya (1985)

[3]See Sandmo (1971)

[4]See Ross (1983)

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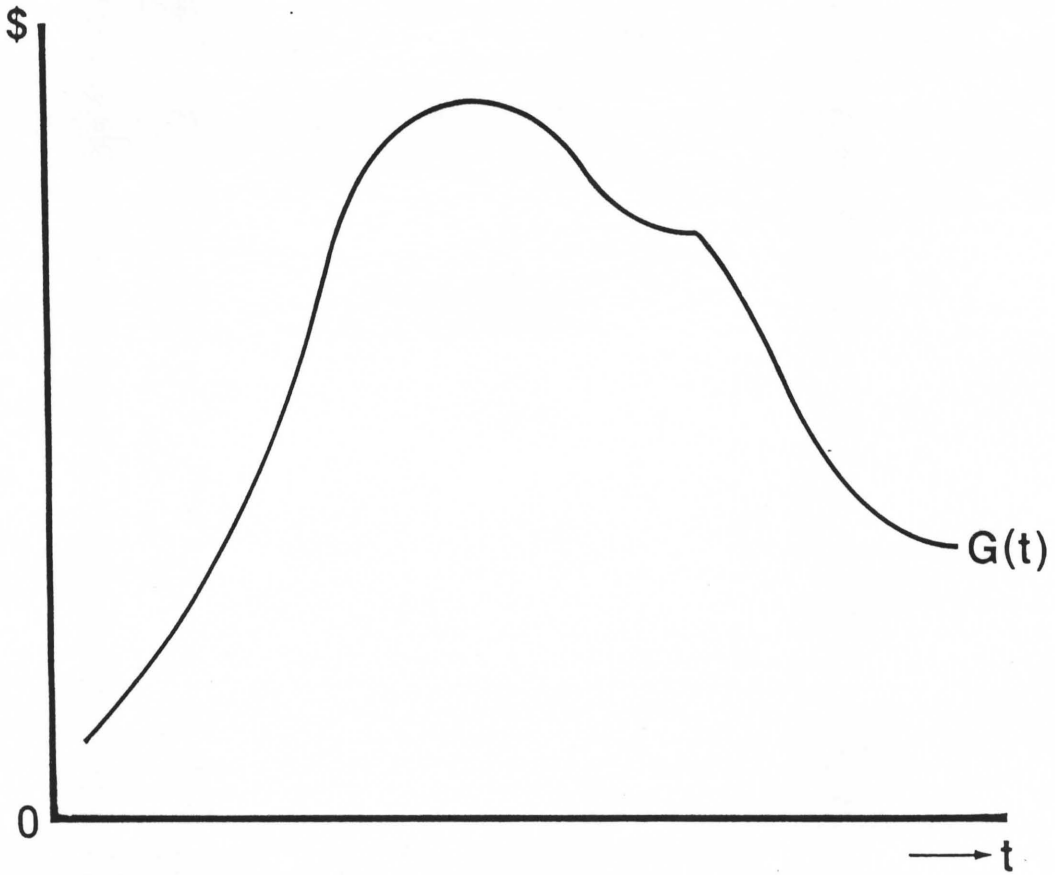

Figure 1. Stumpage Value Growth Curve.

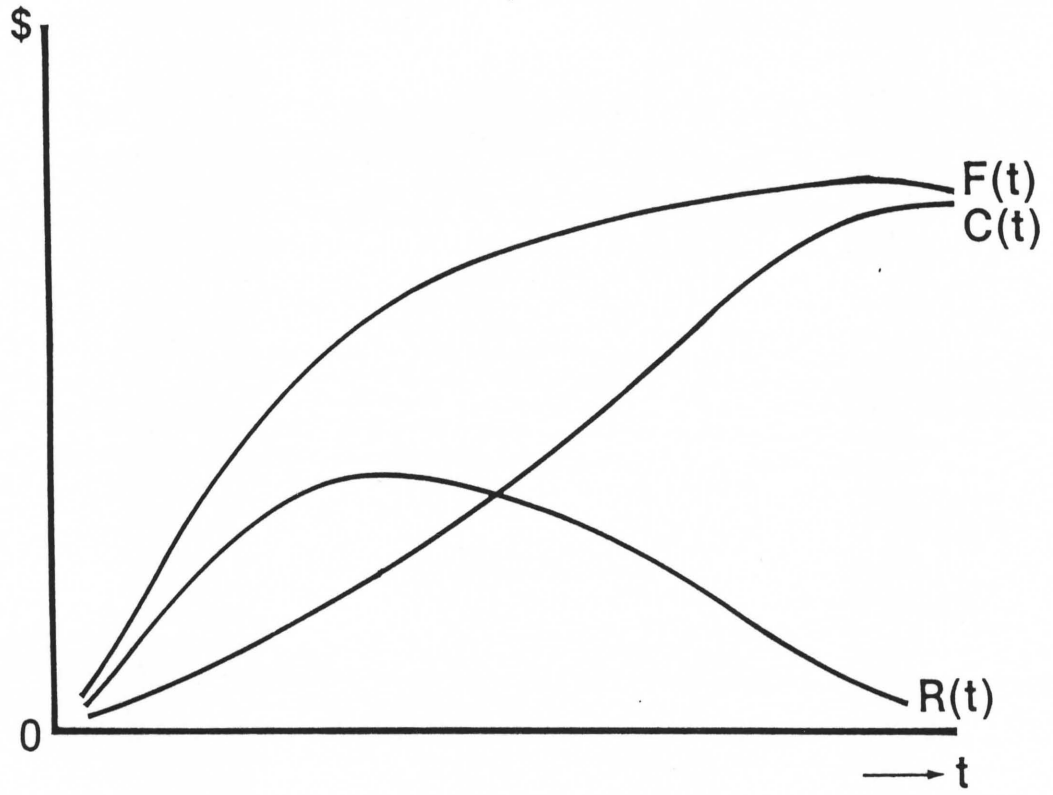

Figure 2. Value (F(t)), Net Value (R(t)), and Cost (C(t)) Curves of Recreational Services.

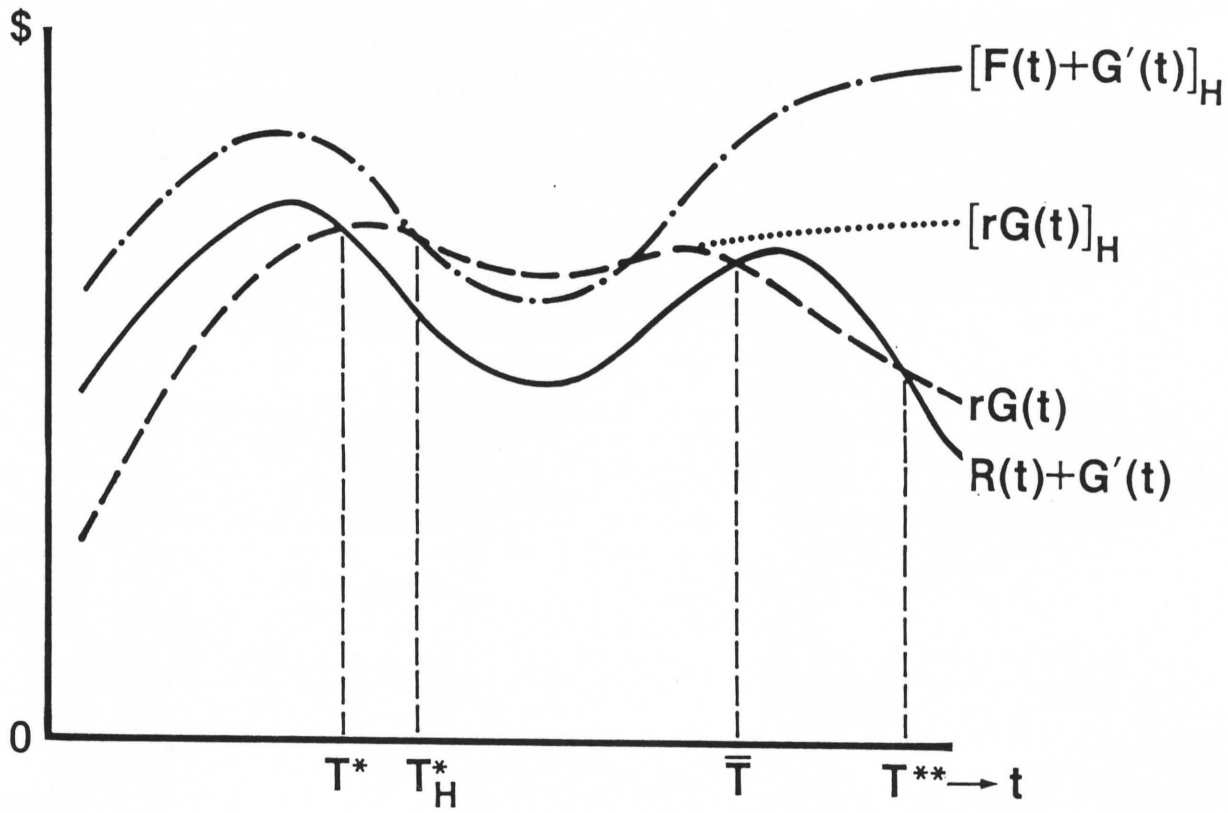

Figure 3. Marginal Benefits and Marginal Costs of Not Harvesting Under Alternative Assumptions.