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I. Introduction

Contemporary time-series analyses of exchange rates typically relax the constant variance assumption imposed in classical econometric modeling, allowing condition variances to change over time following ARCH (Engle 1982) or GARCH (Bollerslev 1986) specifications (Bollerslev, Chou, and Kroner 1992). Correct specification of higher-order moments of the conditional distribution is important for three reasons. First, both the efficiency and, in the case of maximum-likelihood estimation, the consistency of parameter estimates of conditional mean exchange rates require correct specification of the conditional distribution (Pagan and Sabau 1987). Second, correct specification of the conditional distribution is crucial to asset-pricing models that consider price risk. The general shape of the distribution of exchange rates matters, not just the conditional variance. If the distribution is nonnormal, conventional asset-pricing models can be inappropriate, particularly in identifying an optimal portfolio of currencies whose distributions differ by more than just location and scale parameters (Meyer 1987; Meyer and Rasche 1992). Third, forecasting accuracy turns on the underlying probability model used to describe an exchange rate (Baillie and Bollerslev 1992).

Exchange rates commonly follow nonnormal distributions, exhibiting leptokurtosis, meaning longer tails and sharper central peaks than in a normal distribution (Giddy and Dufey 1975; Burt, Kaen, and Booth 1977; Westerfied 1977; Rogalski and Vinso 1978; Friedman and Vandersteel 1982;

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Islam 1982; Bollerslev 1987; Boothe and Glassman 1987; and Hsieh 1988, 1989). This has important implications for the selection of a distribution for estimation. Most studies follow the approach proposed by Bollerslev (1987), combining GARCH modeling with a conditional t-distribution to account for both heteroskedasticity and nonnormality.

In this paper, we compare the performance of alternative exchange rate models employing different specifications of conditional variance—homoscedasticity, GARCH, LGARCH, and EGARCH—and different conditional error distributions—normal and t—at different data frequencies. We find that optimal exchange rate model specification is conditional on data frequency. Higher frequency (daily, weekly) data commonly exhibit characteristics that demand more sophisticated estimation methods and that generally vanish at lower (monthly, quarterly) frequencies.

The second significant innovation of this paper is that we study exchange rate data from the five high-performing economies of East Asia: Hong Kong, Japan, South Korea, Singapore, and Taiwan. With the exception of Japan, we have found no published paper that applies contemporary time-series econometric methods to data on these countries' (or any developing countries') currencies. This surprises us since these currencies have become quite important in the wake of those nations' remarkable economic growth over the last several decades and are widely used in international banking and trade. Indeed, Japan, Singapore, and Hong Kong are the third, fourth, and fifth most active currency trading nations in the world, respectively, and trading in their currencies is growing relatively rapidly (BIS 1996).[2] The growing importance of East Asian currencies motivates the particular application we study.

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[2]Taiwan and South Korea were not included in the BIS study.

The plan of the paper is as follows. In section II, we describe key features of the unconditional distributions of exchange rate changes in the five East Asian economies at different frequencies: daily, weekly, monthly, and quarterly. Section III discusses alternative model specifications. Section IV reports empirical estimates and compares alternative models of exchange rate changes at different frequencies. The concluding section summarizes our findings and highlights some implications for future research.

II. The Data and Descriptive Statistics

We study bilateral exchange rate data versus the U.S. dollar for five East Asian currencies (Hong Kong, Japan, Singapore, South Korea, and Taiwan) at four different observation frequencies: daily, weekly, monthly, and quarterly.[3] There are between 1,834 and 4,505 observations per series, depending on the currency involved. The Japanese data run from 2 June 1978 to 5 September 1995, Hong Kong and Singapore data cover 3 September 1984 to 5 September 1995, the Korean data are from 19 May 1986 to 5 September 1995, and the Taiwan data are from 1 April 1989 until 31 July 1995. All the exchange rate series are I(1),[4] so we work with changes in the natural logarithm of the exchange rate, with $R > 0$ ($R < 0$), indicating currency appreciation (depreciation).

$$R_{i\,t} = \ln[S_{i,\,t} / S_{i,\,t-1}] * 100$$

where $R_{i\,t}$ = percentage change in the U.S.$ exchange rate of currency i at period t, and $S_{i\,t}$ = foreign exchange rate of currency i at period t, expressed as U.S.$/Lc.

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[3]Daily data are the closing ($/local currency) spot rates. Weekly data are the Wednesday closing spot rates. Monthly data are the closing spot rates from the Wednesday of each month. Quarterly data are the closing spot rates from the last Wednesdays in November, February, May, and August. When Wednesdays were market holidays, we used Thursday data.

[4]Unit root test results, demonstrating each series is I(1), are available from the authors by request.

Table 1 presents descriptive statistics at different frequencies for each of the five R_{it}. Almost all series reveal a nominal appreciation trend, as manifest by positive mean change. The coefficient of variation (CV) declines steadily from daily to quarterly data; i.e., high frequency data are more volatile than low frequency data. The coefficient of skewness[5] (SK) indicates nonzero skewness for most of the daily and weekly data. Significantly positive unconditional skewness in high frequency data from Hong Kong, Singapore, and Japan is likely due to the remarkable appreciation of their currencies during this period. Percentage changes in exchange rates appear symmetric, however, at lower frequencies. The higher the coefficient of kurtosis[6] (KUR), the less probability it is concentrated around the mean. Excess kurtosis (leptokurtosis), relative to the Gaussian reference of 3.0, appears in all daily and weekly changes but again vanishes as frequency declines. Jarque-Bera (JB) test statistics suggest rejection of the null hypothesis of normality for all currencies at daily and weekly frequencies and for most at monthly frequencies. However, JB test statistics do not reject the normality null at quarterly frequencies for any of the exchange rates. Table 1 also presents the Ljung-Box test statistics for autocorrelation in R_{it} (Q) and in its conditional variance (Q^2), the latter serving as a test for GARCH effects.[7] There is significant serial correlation in all daily data but not at other frequencies. The Q^2 statistics likewise indicate significant GARCH effects in all the daily data, although those effects uniformly disappear at quarterly frequency in all the currencies.[8]

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[5]This is $E(R_t - \mu)^3/\sigma^3$, where μ is the mean and σ is the standard deviation.

[6]This is $E(R_t - \mu)^4/\sigma^4$, where μ is the expected mean and σ is the standard deviation.

[7]The residual comes from the simple model: $R_t = \mu + \epsilon_t$.

[8]We should note that the significant kurtosis evident in all currencies might affect the power of the Ljung-Box tests (Burt, Kaen, and Booth 1977).

Table 1. Descriptive Statistics

Country/Frequency	T	Mean	CV	SK	KUR	JB	Max	Min	Q	Q^2
Japanese (YEN)										
Daily	4505	0.02	34.01	0.33**	6.3**	2126**	4.8	-4.2	43.8**	273.0**
Weekly	922	0.10	15.80	0.45**	4.7**	143**	7.3	-6.4	28.7*	38.6**
Monthly	212	0.43	8.40	0.18	3.4	2	11.0	-11.0	9.2	14.0
Quarterly	70	1.30	5.00	-0.05	3.2	0	16.0	-16.0	2.7	4.7
Hong Kong (HK)										
Daily	2872	0.00	123.46	0.20**	11.4**	829**	0.51	-0.51	143.0**	284.0**
Weekly	573	0.00	44.05	0.48**	10.3**	1301**	0.77	-0.51	45.8**	29.0**
Monthly	132	0.01	33.89	-0.04	6.0**	50**	0.77	-0.77	31.4**	17.2
Quarterly	44	0.03	9.62	0.02	3.5	1	0.77	-0.51	27.4**	4.4
Taiwan (NT)										
Daily	1833	0.00	133.33	-2.40**	94.3**	638701**	2.3	-3.9	98.4**	40.4**
Weekly	330	0.01	89.13	0.33**	22.6**	5284**	3.8	-3.5	19.7	1.0
Monthly	75	-0.05	-22.00	-0.52	4.6**	11**	2.8	-3.9	5.5	4.8
Quarterly	27	0.07	29.41	-0.34	2.9	0	3.5	-4.8	15.9	7.7
South Korea (WON)										
Daily	2378	0.01	33.33	-1.70**	222.9**	4789181**	4.1	-4.7	261.0**	566.0**
Weekly	485	0.03	11.04	-0.03	19.7**	5590**	2.5	-2.4	210.7**	109.4**
Monthly	111	0.12	6.85	0.40*	4.8**	18**	2.7	-2.7	163.5**	26.8**
Quarterly	38	0.37	5.41	0.45	2.7	1	4.9	-3.1	43.5**	10.6
Singapore (SIN)										
Daily	2872	0.02	24.69	0.30**	13.2**	11981**	3.3	-3.8	192.0**	362.0**
Weekly	573	0.07	8.75	0.29**	6.2**	252**	4.0	-2.0	21.6	79.3**
Monthly	132	0.32	4.06	0.26	4.7**	17**	5.4	-3.4	9.2	8.1
Quarterly	44	0.99	2.99	0.13	2.9	0	7.8	5.2	10.7	2.8

CV = coefficient of variation.
SK = coefficient of skewness.
KUR = coefficient of kurtosis (3.0 for normal distribution)
JB = Jarque-Bera normality test statistic.

Q and Q^2 represent the Ljung-Box test statistics for up to 30th order serial correlation for daily data, 15th order serial correlation for weekly data, and 10th order serial correlation for both monthly and quarterly data in the residuals and squared residuals, respectively. Similar results obtain at different orders.

* and ** denote statistical significance at the 5% and 1% levels, respectively.

In summary, the descriptive statistics suggest the unconditional distributions of the daily and weekly exchange rate change data are generally far from the classic Gaussian econometric assumptions. However, normality is never rejected for quarterly data; and the normality of the monthly data is ambiguous. These results are consistent with Boothe and Glassman's (1987) findings for the British pound, Canadian dollar, German mark, and Japanese yen. We analogously found strong indications of GARCH effects in the daily and weekly data, consistent with the hypothesis that sharper variation in information flow at higher frequencies leads to GARCH properties (Bollerslev, Chou, and Kroner 1992). The important apparent differences in the unconditional distribution of exchange rate changes associated with different data frequencies suggest that the appropriate specification of exchange rate distributions may vary with data frequency.

III. Alternative Model Specifications

Since exchange rate changes tend to exhibit volatility clustering and leptokurtosis at higher frequencies, most recent empirical research employs some variant of the GARCH specification, which can (at least partly) account for both of those characteristics (Bollerslev 1986). In its original form, the GARCH(p,q) model specifies conditional variance as:

$$\epsilon_t \mid \psi_{t-1} \sim N(0, h_t),$$

$$\epsilon_t = h_t^{0.5} v_t$$

$$E(\epsilon^2_t \mid \psi_{t-1}) = h_t = w + \alpha_1 \epsilon^2_{t-1} + \ldots + \alpha_q \epsilon^2_{t-q} + \beta_1 h_{t-1} + \ldots + \beta_p h_{t-p} \qquad (1)$$

where ϵ_t is the residual from the conditional mean equation, conditional on the information set ψ_{t-1}; the v_t are independent and identically distributed with zero mean and unit variance, $w > 0$, and the

α_i and β_i are nonnegative for all i. The restrictions are imposed to ensure strictly positive conditional variance.[9]

Although the unconditional distribution of a Gaussian GARCH(p,q) process with normal errors is leptokurtic, it remains an empirical question how much leptokurtosis can be accounted for by a Gaussian GARCH specification. If the Gaussian GARCH model adequately accounts for unconditional nonnormality in the data, the standardized residuals from the estimated models, $\nu_t = \epsilon_t h_t^{-0.5}$, should follow a normal distribution. For example, Milhøj (1987) found an ARCH specification of the daily U.S.\$/SDR exchange rate drove excess kurtosis to zero. As we show in the next section, however, the Gaussian GARCH model does not always suffice.

The first of the alternative specifications we consider is the Log-GARCH (LGARCH) model, which is motivated by the nonnegativity constraints on the parameters of the GARCH model (Geweke 1986; and Pantula 1986).

$$\text{Log}(h_t) = w + \alpha_1 \log(\epsilon^2_{t-1}) + \dots + \alpha_q \log(\epsilon^2_{t-q}) + \beta_1 \log(h_{t-1}) + \dots + \beta_p \log(h_{t-p}) \qquad (2)$$

The LGARCH model is globally concave, making maximum-likelihood estimation relatively easy, and imposes fewer constraints than GARCH does. Higgins and Bera (1992) found the LGARCH model superior to an ARCH model in estimating weekly exchange rate data.

Another limitation of the GARCH model is the assumption of symmetric responses in conditional variance to positive (returns higher than expected) and negative (returns lower than expected) shocks. The linear GARCH(p,q) model cannot capture an asymmetric response pattern, since conditional variance is only a function of the past squared residuals, hence the sign of residuals has no effect on volatility. However, economic agents sometimes seem more sensitive to negative

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[9]These are sufficient, not necessary, conditions for positive conditional variance (Nelson and Cao 1992).

price changes than positive ones—so-called leverage effects—leading (Nelson 1991) to suggest a more complex form for the LGARCH model:

$$\mathrm{Log}(h_t) = w + \alpha_1\, g(v_{t-1}) + \ldots + \alpha_q g(v_{t-q}) + \beta_1 \log(h_{t-1}) + \ldots + \beta_p \log(h_{t-p}) \tag{3}$$

where $\alpha_1 = 1$

$$g(v_t) = \theta v_t + \gamma[\ |v_t| - E|v_t|\ ].$$

This conditional variance specification is known as exponential GARCH (EGARCH). The function $g(v_t)$ incorporates both a GARCH effect, $\gamma[\ |v_{t-1}| - E|v_{t-1}|\ ]$, and a leverage effect, θv_t. It is obvious that $g(v_t)$ has the slope $\theta + \gamma$ while $v_t > 0$ and $\theta - \gamma$ while $v_t < 0$, hence the possibility of an asymmetric response to positive and negative shocks. Like the symmetric LGARCH specification, the EGARCH model imposes no restrictions on parameters of the conditional variance equation to ensure positivity.

Nonetheless, mere refinement of the functional form of the conditional variance equation may not generate enough leptokurtosis or asymmetry in the unconditional distribution when the conditional distribution of v_t is assumed to be normal. One prospective solution is to adopt a different conditional distribution, in particular, one having fatter tails than the normal distribution. Hence, the suggestion by Bollerslev (1987) and Baillie and DeGennaro (1990) to treat v_t as though it is drawn from a Student t-distribution with υ degrees of freedom: $\epsilon_t \mid \psi_{t-1} \sim t(0, h_t, \nu)$. The conditional density of ϵ_t is thus

$$f(\epsilon_t \mid \psi_{t-1}) = \Gamma(.5(\nu+1))\, \Gamma(0.5\nu)^{-1}((\nu-2)\, h_t)^{-0.5}(1 + \epsilon^2_t h^{-1}_t(\nu-2)^{-1})^{-(\nu+1)/2} \tag{4}$$

where $\nu > 2$. The Student t-distribution is described by its location (mean), scale (standard deviation), and degree of freedom, which is also regarded as a parameter to measure the degree of leptokurtosis. While the degrees of freedom, ν, are significantly less than 30, the t-distribution has heavier tails than a corresponding normal distribution. This attractive feature has induced several authors to apply conditional t-distributions to models of daily exchange rates (Engle and Bollerslev 1986; Boothe and Glassman 1987; Bollerslev 1987; Baillie and Bollerslev 1989; and Hsieh 1989). None of these authors, however, have combined the functional form refinements of (2) or (3) with the more flexible t-distribution. That is an innovation we introduce in the next section.

IV. Empirical Results

In this section, we estimate alternative models distinguished by their conditional variance specifications and conditional error distributions. As discussed in section II, unconditional serial correlation in both conditional mean and conditional variance and nonnormality are generally observed in high frequency data. To account for these problems, we fit four different conditional variance specifications under both conditional normal and Student t-distributions (Table 2) to the data at each frequency. We thus estimate five different currencies at each of four different data frequencies using eight distinct models for a total of 160 different regressions.

Table 2. Alternative Models and their Acronyms

	Conditional Distribution	
Conditional Variance Specification	Normal	Student t
Homoscedastic		
$h_t = w$	HOMO	HOMO-t
GARCH (1,1)		
$h_t = w + \alpha\, e_t^2 + \beta\, h_{t-1}$	GARCH	GARCH-t
LGARCH (1,1)		
$\ln(h_t) = w + \alpha \ln(e_t^2) + \beta \ln(h_{t-1})$	LGARCH	LGARCH-t
EGARCH (1,1)		
$\ln(h_t) = w + \alpha\, g(e_{t-1}) + \beta \ln(h_{t-1})$		
$g(\eta_t) = \theta e_t + \gamma\, [\, \lvert e_t \rvert - E \lvert e_t \rvert\,]$	EGARCH	EGARCH-t

We began by identifying and estimating a common ARMA process for the stationary $R_{i\,t}$. First, Box-Jenkins techniques were used to reduce the set of prospective ARMA specifications. Next, we further narrowed the pool of possible models to those having a p-value for the Ljung-Box portmanteau Q(x) statistic of greater than 0.3, a significance level clearly supporting the assumption of white noise.[10] Finally, we chose the ARMA specification having the lowest Schwarz's Bayesian criterion (SBC) value from among the candidate models having passed the Box-Jenkins and Q(x) screens. In other words, the Ljung-Box Q statistic was used to identify a few possible models and then the information criterion (SBC) selected the final ARMA specification for the conditional mean equation.

We then estimated this ARMA conditional mean specification using each of the models identified in Table 2. The conditional mean and conditional variance equations were estimated simultaneously using maximum-likelihood.[11] Table 3 reports Ljung-Box statistics for the standardized squared residuals among Gaussian models at each data frequency. The p-values (reported in brackets) of the test statistics for the homoscedastic model clearly suggest serial correlation in conditional variance. For each data series, at least one of the GARCH specifications quite adequately accounts for these effects, thereby justifying (if ex post) selection of (1,1)-order models in Table 2. By and large, the GARCH and EGARCH specifications do a better job than LGARCH in generating white noise conditional variance for the exchange rate change series.

[10]We used x = 30 for daily data, x = 15 for weekly data, and x = 10 for monthly and quarterly data.

[11]For each model, the log-likelihood function was maximized numerically using the Berndt, Hall, Hall & Hausman (BHHH) algorithm in the constrained maximum likelihood (CML) module in GAUSS.

Table 3. Tests for Serially Correlated Conditional Variance In Gaussian Models

	Japan	Hong Kong	Taiwan	Korea	Singapore
Daily data ($Q^2(30)$):					
HOMO	274.0	311.5	49.0	218.0	354.30
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
GARCH	16.70	18.31	0.34	1.80	14.50
	[0.98]	[0.95]	[1.00]	[1.00]	[1.00]
LGARCH	103.28	30.83	2.19	94.00	128.03
	[0.00]	[0.42]	[1.00]	[0.00]	[0.00]
EGARCH	21.54	22.48	0.46	1.92	43.57*
	[0.87]	[0.83]	[1.00]	[1.00]	[0.05]*
Weekly data ($Q^2(15)$):					
HOMO	37.61	36.83	0.93	101.5	110.37
	[0.00]	[0.00]	[1.00]	[0.00]	[0.00]
GARCH	8.88	2.09	1.12	6.58	9.90
	[1.00]	[1.00]	[1.00]	[0.97]	[0.83]
LGARCH	18.90	13.01	0.86	48.04	31.40
	[0.22]	[0.60]	[1.00]	[0.00]	[0.00]
EGARCH	36.32	2.01	1.04	6.68	13.07
	[0.00]	[1.00]	[1.00]	[0.96]	[0.60]
Monthly Data ($Q^2(10)$):					
HOMO	13.28	5.21	5.07	4.32	9.48
	[0.21]	[0.88]	[0.89]	[0.93]	[0.49]
GARCH	13.34	9.48	4.75	11.95	14.45
	[0.21]	[0.49]	[0.91]	[0.29]	[0.15]
LGARCH	13.14	6.99	5.17	6.26	12.58
	[0.22]	[0.73]	[0.88]	[0.79]	[0.25]
EGARCH	13.44	43.99	5.14	0.00	8.75
	[0.20]	[0.00]	[0.88]	[1.00]	[0.56]
Quarterly Data ($Q^2(10)$):					
HOMO	3.84	5.89	7.69	5.65	5.87
	[0.95]	[0.82]	[0.66]	[0.84]	[0.83]
GARCH	3.77	10.33	7.67	5.21	4.94
	[0.96]	[0.41]	[0.66]	[0.88]	[0.90]
LGARCH	4.49	8.09	5.60	5.64	5.89
	[0.92]	[0.62]	[0.85]	[0.84]	[0.83]
EGARCH	5.70	23.24	7.66	10.88	4.67
	[0.84]	[0.01]	[0.66]	[0.36]	[0.91]

The p-value of the Ljung-Box Q(x) test (null hypothesis is no serial correlation) is in brackets.
*At a higher order, Q^2 (50) yields a p-value of 0.60.

The core results of the study are found in Table 4, which shows clearly there is considerable variation across data frequencies in which model of the eight is best, but little variation across countries at the same frequency. Table 4 presents the log-likelihood and Schwarz Bayesian criterion (SBC) statistics for each of the models. The left half of the table shows results from the Gaussian models, with the Student t models on the right half. The likelihood ratio (LR) test statistics compare each of the three heteroscedastic Gaussian models against HOMO and each of the Student t models against the equivalent Gaussian specification. This permits us to see separately the modeling benefits of (1) accommodating heteroscedasticity and (2) employing a leptokurtic conditional distribution.

All daily and most weekly GARCH models increase the log-likelihood and SBC values significantly compared to the homoscedastic specification. The LR test favors each of the three heteroscedastic specifications for all the daily and most of the weekly data, but not in most of the monthly data and never among the quarterly series. The apparent persistence of exchange rate volatility is consistent with prevailing beliefs that temporal variation in information flow is greatest at higher frequencies (Bollerslev, Chou, and Kroner 1992) as well as with the emerging theory that agents form and adjust expectations on a slower time scale than what takes place in trading (Brock and LeBaron 1994). There is rarely much difference among the three GARCH specifications tested; the value comes primarily from accommodating GARCH effects and much less from the particular GARCH specification employed.

While the heteroscedastic Gaussian models appear to accommodate GARCH effects, the issue of nonnormality remains. The skewness and kurtosis of the standardized residuals from the estimated models show asymmetry and leptokurtosis persist in all the Gaussian models estimated

Table 4. Comparisons of Alternative Specifications

Gaussian Models	Log-Likelihood	SBC	LR Test	Student t Models	Log-Likelihood	SBC	LR Test	ν
DAILY DATA:								
Japan								
HOMO	-25348	-25360		HOMO-t	-25109	-25126	478**	4.31
GARCH	-4497	-4522	41702**	GARCH-t	-4291	-4320	412**	4.57
EGARCH	-4495	-4524	41706**	***EGARCH-t***	-4284	-4317	422**	4.56
LGARCH	-4541	-4566	41614**	LGARCH-t	-4314	-4343	454**	4.38
Hong Kong								
HOMO	-8981	-8996		HOMO-t	-3955	-3975	10052**	2.00‡
GARCH	-8678	-8706	606**	GARCH-t	-2125	-2156	13106**	2.04
EGARCH	-8684	-8718	594**	***EGARCH-t***	-865	-871	15638**	2.00‡
LGARCH	-8760	-8789	442**	LGARCH-t	-6527	-6659	4466**	2.00‡
Taiwan								
HOMO	-8082	-8097		HOMO-t	-7342	-7361	1480**	2.36
GARCH	-7766	-7792	632**	***GARCH-t***	-7141	-7171	1250**	3.33
EGARCH	-7746	-7776	672**	EGARCH-t	-7150	-7184	1192**	2.53
LGARCH	-7783	-7809	598**	LGARCH-t	-7157	-7187	1252**	2.21
Korea								
HOMO	-10265	-10296		HOMO-t	-8697	-8732	3136**	2.16
GARCH	-9408	-9451	1714**	GARCH-t	-8500	-8546	1816**	3.33
EGARCH	-9262	-9216	2006**	***EGARCH-t***	-8478	-8528	1568**	2.92
LGARCH	-9445	-9488	1640**	LGARCH-t	-8537	-8584	1816**	2.57
Singapore								
HOMO	-13866	-13878		HOMO-t	-12696	-12676	2340**	2.00‡
GARCH	-13714	-13738	304**	GARCH-t	-12067	-12095	3294**	2.00‡
EGARCH	-13279	-13307	1174**	***EGARCH-t***	-11904	-11911	2750**	2.00‡
LGARCH	-13811	-13836	110**	LGARCH-t	-12886	-12913	1850**	2.23
WEEKLY DATA:								
Japan								
HOMO	-1690	-1704		HOMO-t	-1659	-1676	62**	4.71
GARCH	-1676	-1700	28**	***GARCH-t***	-1646	-1673	60**	4.57
EGARCH	-1688	-1716	4	EGARCH-t	-1657	-1687	62**	4.40
LGARCH	-1683	-1706	14**	LGARCH-t	-1652	-1679	62**	4.67
Hong Kong								
HOMO	-2138	-2148		HOMO-t	-2002	-2015	272**	2.05
GARCH	-2114	-2124	48**	***GARCH-t***	-1503	-1509	1222**	2.02
EGARCH	-2108	-2133	60**	EGARCH-t	-1790	-1819	636**	2.00‡
LGARCH	-2129	-2148	18**	LGARCH-t	-2005	-2027	248**	2.01

Table 4. (Continued)

Gaussian Models	Log-Likelihood	SBC	LR Test	Student t Models	Log-Likelihood	SBC	LR Test	ν
Taiwan								
HOMO	-225	-230		***HOMO-t***	-123	-132	204**	2.48
GARCH	-215	-230	20**	GARCH-t	-120	-138	190**	2.11
EGARCH	-206	223	38**	EGARCH-t	-120	-140	172**	2.04
LGARCH	-224	-239	2	LGARCH-t	-120	-138	208**	2.11
Korea								
HOMO	-2281	-2299		HOMO-t	-2176	-2197	210**	2.88
GARCH	-2214	-2242	134**	GARCH-t	-2164	-2195	100**	3.63
EGARCH	-2217	-2248	128**	***EGARCH-t***	-2159	-2193	116**	3.54
LGARCH	-2221	-2249	120**	LGARCH-t	-2162	-2193	118**	3.43
Singapore								
HOMO	-545	-552		HOMO-t	-520	-529	50**	4.57
GARCH	-521	-537	48**	***GARCH-t***	-508	-527	26**	5.07
EGARCH	-523	-543	44**	EGARCH-t	-517	-539	12**	4.61
LGARCH	-531	-547	28**	LGARCH-t	-513	-532	36**	2.01
MONTHLY DATA:								
Japan								
HOMO	-549	-554		HOMO-t	-548	-556	2	14.16
GARCH	-549	-562	0	GARCH-t	-548	-564	2	12.77
EGARCH	-549	-565	0	EGARCH-t	-548	-567	2	13.21
LGARCH	-548	-561	2	LGARCH-t	-548	-563	0	16.88
Hong Kong								
HOMO	-567	-574		***HOMO-t***	-559	-569	16**	3.32
GARCH	-564	-579	6	GARCH-t	-554	-572	20**	3.82
EGARCH	-559	-576	16**	EGARCH-t	-552	-572	14**	2.90
LGARCH	-563	-578	8	LGARCH-t	-559	-576	8**	2.54
Taiwan								
HOMO	-108	-113		***HOMO-t***	-106	-112	4	4.91
GARCH	-108	-119	0	GARCH-t	-105	-118	6	4.01
EGARCH	-108	-121	0	EGARCH-t	-105	-120	6	4.14
LGARCH	-108	-118	0	LGARCH-t	-104	-117	8**	3.78
Korea								
HOMO	-108	-115		***HOMO-t***	-90	-99	36**	2.77
GARCH	-102	-116	12**	GARCH-t	-84	-101	36**	2.65
EGARCH	-102	-119	12**	EGARCH-t	-83	-102	38**	2.07
LGARCH	-108	-122	0	LGARCH-t	-85	-101	46**	2.64

Table 4. (Continued)

Gaussian Models	Log-Likelihood	SBC	LR Test	Student t Models	Log-Likelihood	SBC	LR Test	ν
Singapore								
HOMO	-205	-210		***HOMO-t***	-202	-209	6	5.48
GARCH	-204	-216	2	GARCH-t	-201	-215	6	6.60
EGARCH	-203	-217	4	EGARCH-t	-200	-217	6	7.38
LGARCH	-204	-216	2	LGARCH-t	-202	-216	4	4.15
QUARTERLY DATA:								
Japan								
HOMO	-230	-232		HOMO-t	-230	-234	0	100‡
GARCH	-229	-238	2	GARCH-t	-230	-240	0	13.31
EGARCH	-227	-238	6	EGARCH-t	-227	-240	0	7.40
LGARCH	-229	-237	2	LGARCH-t	-228	-238	2	100‡
Hong Kong								
HOMO	-166	-173		HOMO-t	-166	-175	0	11.20
GARCH	-166	-178	0	GARCH-t	-166	-180	0	11.21
EGARCH	-169	-184	<0	EGARCH-t	-165	-181	8**	100‡
LGARCH	-163	-175	6	LGARCH-t	-163	-178	0	100‡
Taiwan								
HOMO	-51	-52		HOMO-t	-51	-54	0	100‡
GARCH	-51	-57	0	GARCH-t	-51	-59	0	100‡
EGARCH	-50	-58	2	EGARCH-t	-50	-59	0	100‡
LGARCH	-49	-55	4	LGARCH-t	-49	-57	0	100‡
Korea								
HOMO	-69	-72		HOMO-t	-67	-73	4	3.49
GARCH	-69	-78	0	GARCH-t	-67	-78	4	2.48
EGARCH	-65	-76	8	EGARCH-t	-61	-74	8**	8.80
LGARCH	-69	-78	0	LGARCH-t	-67	-77	4	2.61
Singapore								
HOMO	-97	-103		HOMO-t	-97	-105	0	100‡
GARCH	-97	-109	0	GARCH-t	-97	-110	0	100‡
EGARCH	-100	-113	<0	EGARCH-t	-99	-114	2	97.0
LGARCH	-97	-108	0	LGARCH-t	-96	-109	2	100‡

LR test statistic for the Gaussian models (left half of the table) is against the HOMO model, and for the Student t models (right half of the table) it is against the corresponding Gaussian specification. ** denotes statistical significance at the 1% level (using the $\chi^2(1)$, $\chi^2(3)$ and $\chi^2(4)$ distributions for the Student t, GARCH and LGARCH, and EGARCH models, respectively.

‡ denotes estimate for ν falls on the boundary [2.00, 100.0] of the parameter space.

with daily data and in most of those estimated with weekly data (Table 5).[12] Most models yield symmetric standardized residuals at monthly frequencies, however, and the leptokurtosis vanishes too by quarterly frequency. Clearly, the standardized residuals from the Gaussian models do not actually follow a normal distribution.[13] Gaussian GARCH estimation of exchange rates appears vulnerable to the problems associated with quasimaximum-likelihood GARCH estimation (Pagan and Sabau 1987; Lee and Hansen 1994; Deb 1996). As a result, many applied econometricians have turned to using the Student t conditional error distribution to account for, in particular, leptokurtosis.

We indeed find that the t-distributions uniformly provided better specification than normal distributions at daily and weekly frequencies, and most of the time at monthly frequencies (Table 4). Only at quarterly frequencies is there no apparent gain from switching to the Student t. This, of course, relates to the fact that there generally are not problems of asymmetry or leptokurtosis in the Gaussian standardized residuals at quarterly frequencies. The superiority of the t-distribution in capturing leptokurtosis is evident in both the low estimated degrees of freedom parameter (υ) and in the likelihood ratio test statistics for all the daily and weekly data. Conversely, the frequency with which the υ estimate exceeds 30, the threshold, at which the Student t and normal distributions are equivalent, is quite high in quarterly frequency data. As is evident from the LR test statistics, accommodating GARCH effects and leptokurtosis each provide the applied econometrician significant gains at daily frequency, but the returns (in terms of higher likelihood and SBC) come chiefly from moving to the Student t distribution at weekly and monthly frequencies. Simple Gaussian, homoscedastic ARMA modeling suffices at quarterly frequency.

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[12]Similar results were found by Milhøj (1987), Hsieh (1988), and McCurdy and Morgan (1987).

[13]Higgins and Bera (1992) argued that it is peakedness of the distributions that drive the kurtosis coefficients higher than that of a normal distribution and not heavy tails.

Table 5. Skewness and Kurtosis in the Standardized Residuals from Gaussian Models

	Japan	Hong Kong	Taiwan	Korea	Singapore
DAILY DATA:					
Skewness	(0.04)	(0.05)	(0.06)	(0.05)	(0.05)
HOMO	0.33	0.31	-2.74	-6.08	0.63
GARCH	0.47	0.85	-6.10	-6.48	0.62
LGARCH	0.48	0.98	-3.95	-2.50	0.73
EGARCH	0.48	0.88	-4.58	-5.29	0.67
Kurtosis	(0.07)	(0.09)	(0.12)	(0.10)	(0.09)
HOMO	6.30	11.96	96.11	188.60	12.40
GARCH	6.86	15.37	158.80	106.60	9.77
LGARCH	6.66	17.50	122.82	38.94	12.65
EGARCH	6.88	15.70	131.57	82.04	10.54
WEEKLY DATA:					
Skewness	(0.08)	(0.11)	(0.14)	(0.11)	(0.10)
HOMO	0.44	0.41	0.17	-0.73	0.28
GARCH	0.46	1.21	-1.08	-1.21	0.01
LGARCH	0.45	1.03	0.29	-0.88	0.18
EGARCH	0.43	1.25	1.13	-1.41	0.02
Kurtosis	(0.16)	(0.21)	(0.27)	(0.23)	(0.21)
HOMO	4.79	10.70	21.44	24.75	6.00
GARCH	4.79	16.78	19.31	12.13	4.22
LGARCH	4.72	15.39	21.71	13.21	4.64
EGARCH	4.82	17.15	17.10	14.05	4.27
MONTHLY DATA:					
Skewness	(0.17)	(0.21)	(0.28)	(0.23)	(0.22)
HOMO	0.13	-0.04	-0.53	-0.06	0.32
GARCH	0.17	0.90	-0.56	0.12	-0.06
EGARCH	0.17	-0.16	-0.38	0.12	0.21
LGARCH	0.14	0.8	-0.33	-0.67	0.23
Kurtosis	(0.34)	(0.43)	(0.57)	(0.47)	(0.44)
HOMO	3.43	5.57	4.65	7.97	4.70
GARCH	3.48	5.79	4.80	7.46	3.65
EGARCH	3.47	5.48	4.02	7.22	3.83
LGARCH	3.37	6.5	4.43	6.28	3.65

Table 5. (Continued)

	Japan	Hong Kong	Taiwan	Korea	Singapore
QUARTERLY DATA:					
Skewness	(0.29)	(0.39)	(0.50)	(0.41)	(0.38)
HOMO	-0.05	0.68	-0.32	0.20	0.15
GARCH	0.11	0.42	-0.32	0.16	0.16
EGARCH	-0.01	2.5	-0.12	0.29	0.22
LGARCH	-0.04	0.51	0.18	0.2	0.24
Kurtosis	(0.58)	(0.79)	(1.00)	(0.82)	(0.76)
HOMO	3.16	3.02	2.76	3.81	2.59
GARCH	3.06	2.55	2.76	3.82	2.60
EGARCH	2.96	8.49	2.32	3.22	2.73
LGARCH	2.93	2.46	2.22	3.81	2.77

The asymptotic standard error of skewness and kurtosis coefficients, reported in parentheses, are computed as $(6/T)^{0.5}$ and $(24/T)^{0.5}$, respectively.

Table 6 reports parameter estimates and associated standard errors for the best model we found for each exchange rate at each frequency (using the SBC for selection). The most complex model, EGARCH-t, dominates the other models in the daily data. At weekly frequency, the asymmetric leverage effects and greater nonlinearity captured by the EGARCH specification are less prevalent, and the GARCH-t model is generally best. At monthly frequency, volatility persistence is no longer pronounced, so the HOMO-t model typically offers the best specification. Finally, at quarterly frequency, classic homoscedastic ARMA estimation, the HOMO model, uniformly maximizes the log-likelihood and SBC for each series. At least in the East Asian exchange rate data, appropriate econometric modeling techniques are plainly dependent on data frequency.

Table 6. Best Model Parameter Estimates by Currency and Frequency

	Japan	HongKong	Taiwan	Korea	Singapore
Daily Data:					
Best model:	**EGARCH-t**	**EGARCH-t**	**GARCH-t**	**EGARCH-t**	**EGARCH-t**
Conditional mean equation parameters					
C	0.013(0.010)	-0.000(.)	0.01(0.209)	-0.021(0.158)	0.000(.)
AR(1)			0.459(0.246)		-0.001(.)
AR(2)			-1.005(0.220)		
AR(4)		0.049(0.000)			
AR(5)			0.360(0.173)		
AR(9)	0.314(0.141)				
AR(10)	0.589(0.132)			0.446(0.124)	
AR(14)				0.357(0.101)	
AR(18)				0.175(0.087)	
AR(19)				0.344(0.104)	
AR(20)				0.434(0.088)	
MA(1)				0.251(0.019)	
Conditional variance equation parameters					
EGARCH					
w	-0.072(0.009)	-0.523(0.025)		0.514(0.064)	4.671(0.000)
α	0.897(0.009)	0.786 (0.000)		0.908(0.011)	0.125(0.028)
θ	0.027(0.008)	-0.009(0.000)		0.068(0.028)	-0.391(0.020)
γ	0.223(0.015)	0.150 (0.004)		0.463(0.041)	1.211(0.000)
GARCH					
w			25.225(3.500)		
α1			0.528(0.034)		
β1			0.471(0.034)		
Distribution parameters					
NU	4.561(0.338)	2.000(.)	3.327(0.164)	2.922(0.166)	2.000(.)
Weekly Data:					
Best model:	**GARCH-t**	**GARCH-t**	**HOMO-t**	**EGARCH-t**	**GARCH-t**
Conditional mean equation parameters					
C	0.003(0.043)	0.000(0.008)	0.012(0.015)	-0.552(0.903)	0.084(0.023)
AR(1)	0.063(0.034)		0.094(0.028)		-0.134(0.042)
AR(2)	0.104(0.031)				
AR(3)	0.067(0.033)			1.724(0.415)	
AR(4)				1.266(0.400)	
AR(5)				1.179(0.350)	
AR(6)				1.092(0.330)	
AR(8)				0.810(0.312)	
MA(1)		-0.065(0.012)			

Table 6. Best Model Parameter Estimates by Currency and Frequency

	Japan	HongKong	Taiwan	Korea	Singapore
Weekly Data: (Cont'd)					
Conditional variance equation parameters					
GARCH					
w	0.297(0.138)	0.001(.)			0.033(0.020)
$\alpha 1$	0.112(0.040)	0.016(0.002)			0.094(0.038)
$\beta 1$	0.780(0.077)	0.985(0.002)			0.830(0.074)
EGARCH					
w				1.542(0.520)	
$\alpha 1$				0.772(0.077)	
θ				0.036(0.07)	
γ				0.525(0.121)	
Distribution parameters					
NU	4.571(0.863)	2.026(0.002)	2.483(0.078)	3.543(0.567)	5.074(1.389)
Monthly Data:					
Best model:	**HOMO**	**HOMO-t**	**HOMO-t**	**HOMO-t**	**HOMO-t**
Conditional mean equation parameters					
C	0.401(0.247)	2.555(2.305)	0.013(0.109)	0.002(0.033)	0.390(0.104)
AR(1)		-8.022(1.365)	0.109(0.091)	0.820(0.063)	
AR(5)					-0.150(0.075)
AR(6)	-0.165(0.072)				
MA(1)		0.719(0.164)		-0.239(0.098)	
Distribution parameters					
NU		3.319(0.466)	4.909(2.092)	2.768(0.240)	5.477(2.085)
Quarterly Data:					
Best model:	**HOMO**	**HOMO**	**HOMO**	**HOMO**	**HOMO**
Conditional mean equation parameters					
C	1.197(0.777)	3.731(1.684)	0.015(0.409)	0.059(0.347)	0.731(0.263)
AR(1)				0.596(0.172)	0.335(0.218)
AR(2)		-4.781(1.185)			
MA(1)		-0.473(0.171)			-0.872(0.139)

Standard errors reported in parentheses.
The lower bound of NU is set to 2.000.
(.) means the standard error cannot be estimated because the parameter estimate lies on the boundary of the feasible parameter space.

VI. Conclusions

The task of the applied econometrician is to identify a model that adequately yet parsimoniously describes the conditional distribution of the economic variable under study. The frequency of observation is an important component of the definition of a dependent variable represented in time series. In this study, we have shown data frequency to have important effects on specification strategies for exchange rate analysis.

Our empirical study of five East Asian currencies shows high-frequency (e.g., daily) data are characterized by considerable volatility clustering, leverage effects, and nonnormal error distributions. These characteristics demand more sophisticated econometric modeling techniques than conventional Gaussian GARCH models. Of the eight specifications we fit, the EGARCH-t performs best with daily frequency data because it accommodates more nonlinearity, asymmetry, and leptokurtosis than the alternatives considered. But these confounding characteristics of the data generally vanish as the differencing interval increases, i.e., in lower frequency data. Thus, while EGARCH-t models seem best able to represent daily exchange rate generating processes, GARCH-t (i.e., a specification that does not accommodate leverage effects) works best at weekly frequency, a homoscedastic Student t model is generally best at monthly frequencies, and a straight homoscedastic Gaussian model is uniformly best at quarterly frequencies. The optimal specification for East Asian exchange rate series seems reasonably uniform across currencies but varies predictably with data frequency.

An important remaining question is whether a conditional t-distribution really accounts fully for the asymmetry and leptokurtosis evident in the standardized residuals of Gaussian GARCH models. At daily and weekly frequencies, it appears they do not. Plots of the standardized residuals from the optimal t-distribution specifications still show skewness and high peakedness, and χ^2 tests of the goodness-of-fit routinely reject the null hypothesis that the standardized regression residuals are drawn from a Student t distribution of the estimated degrees of freedom (υ) (Table 7). Even at monthly frequency, some exchange rate series still deviate significantly from the assumed Student t conditional normalized error distribution. This implies that GARCH-t or EGARCH-t estimation of high frequency exchange rate data is still only QMLE, with the potential finite sample estimation problems that entails (Lee and Hansen 1994; and Deb 1996). Identification of a distribution that can accommodate these stylized characteristics of exchange rate data and yet remain tractable for estimation is a subject ripe for further research.

Table 7. χ^2 Goodness-of-Fit Test Statistics for Best Models

Country	Best Model	Null Distribution	Test Statistic	p-Value
Daily Data:				
Japan	EGARCH-t	Student t (4.56)	358.8	0.00
Hong Kong	EGARCH-t	Student t (2.00)	3350.8	0.00
Taiwan	GARCH-t	Student t (3.33)	489.4	0.00
Korea	EGARCH-t	Student t (2.92)	452.5	0.00
Singapore	EGARCH-t	Student t (2.00)	2601.1	0.00
Weekly Data:				
Japan	GARCH-t	Student t (4.57)	77.8	0.00
Hong Kong	GARCH-t	Student t (2.02)	794.7	0.00
Taiwan	EGARCH-t	Student t (2.04)	127.0	0.00
Korea	EGARCH-t	Student t (3.54)	77.3	0.00
Singapore	GARCH-t	Student t (5.07)	320.7	0.00
Monthly Data:				
Japan	HOMO	Normal	13.3	0.51
Hong Kong	HOMO-t	Student t (3.32)	44.5	0.00
Taiwan	HOMO-t	Student t (4.91)	9.8	0.63
Korea	HOMO-t	Student t (2.77)	43.1	0.00
Singapore	HOMO-t	Student t (5.48)	9.7	0.78
Quarterly Data:				
Japan	HOMO	Normal	7.2	0.93
Hong Kong	HOMO	Normal	18.2	0.11
Taiwan	HOMO	Normal	3.4	0.97
Korea	HOMO	Normal	12.7	0.31
Singapore	HOMO	Normal	9.3	0.81

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