Pricing in Two-Sided Markets and Social Networks

A.D. MDLXII

Elias Carroni

Università degli Studi di Sassari

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Abstract

This work analyzes the problem of optimal pricing policies in markets characterized by cross-group network externalities (two-sided markets) and in the presence of information held by companies about the social network of consumers. The thesis is divided in three chapters, each one readable as a distinct paper.

The first chapter aims at providing a comprehensive overview of the state of the art in the literature of two-sided markets, i.e. markets characterized by the presence of platforms serving different groups of consumers linked with each other by network externalities. In particular, chapter 1 explains the main trade-offs and issues raised in the literature concerning different market structures (monopoly vs. oligopoly), exclusivity of the service (*multi-* vs. *single-homing*), price instruments (membership vs. transaction fees), type of externality (*inter-* vs. *intra-group*), interest of customers in quality and type of price discrimination (*cross-* vs. *within-group*).

Chapter 2 is closely related to the first and aims at investigating about a very recent direction of the research in two-sided market. In particular, it provides an analysis of the practice of firms to offer different prices to consumers according to the past purchase behavior (*BBPD*) in the context of two-sided markets. In a two period model, two platforms compete for heterogeneous firms and end-users. Our contribution is that we allow platforms to discriminate prices on the users' side according to their past purchase behavior. The main findings are two. In the second period game with market shares taken as given, each platform may find it optimal either to offer discounts to rivals' users or to reward loyalty, depending on the number of users attracted in the past. Moreover,

switching towards both platforms occurs if and only if the inherited market partition is symmetric enough. Making the first period game endogenous, BBPD affects both *ex-ante* and *ex-post* competition. *Ex-post* competition is strengthened compared to the regime in which a uniform price is charged in users' side. *Ex-ante* competition is relaxed (intensified) if users are the low (high) value group. The overall effect on inter-temporal profits of platforms is negative, confirming the previous results of *BBPD* literature.

Chapter 3 changes completely topic compared to the first two. Specifically, it models the strategy of a monopolist that offers rewards to current clients in order to induce them to activate their social network and convince peers to buy from the company. In presence of heterogeneous search costs and reservation prices, this network-activation reward program may serve to expand the client base through a flow of information from informed to uninformed consumers. The offer of the monopolist affects individual incentives of aware people to share information, determining a minimal degree condition for investment. The optimal unitary reward balances the information spread effect (i.e. more receivers) and the crowding effect (i.e. less individual incentives) of an increase in the number of speakers. The monopolist always finds it profitable to use the bonus. Nevertheless, its introduction has ambiguous effects on the price and profits, depending on the process of spread of information and, in turn, on the network structure.

Keywords: Behavior Based Price Discrimination, Two-Sided Markets, Social Networks, Monopoly Pricing, Network-Based Pricing, Search Costs.

To Franca & Bobore,

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Contents

[0]This Chapter is part of a joint work with Simone Righi.

1

Competition and Pricing in Two-Sided Markets

1.1 Introduction

Two sided-markets theory is a relatively recent field of economic research referred to markets characterized by the presence of cross-group network externalities. In general, we refer to two-sided markets to describe industries in which the benefit for an agent belonging to a given group (side) depends on the number of agents in the other group. For this interaction to occur, it is required the birth of a platform able to internalize the externalities.

To understand the importance of this theory, it is worth noticing how economy is nowadays often characterized by platforms that enable the interaction between different types of economic agents: Operative Systems (OS), Credit Cards and dating club are, among others, well-known examples of two-sided platforms.

In the contexts just mentioned, the point of the economic game is always the interaction between two sides and the presence of an intermediary:

(i) an OS represents a platform by means of which users and application developers can interact with each other, because the former can use the application developed by the latter;

(ii) a transaction between merchants and cardholders represents an interaction, which is facilitated by Visa or MasterCard. As in point *(i)*, the more the

merchants accepting a card, the more consumers have a benefit from owning a card and, equivalently, a merchant's benefit from accepting the card is increasing in the number of cardholders;

(iii) the access to a dating club (platform) gives a bigger benefit to an agent of one type (a man), if the number on the other side (women) is higher.

Similar examples are: yellow pages, newspapers, network television and websites for advertising; game consoles for the players-developers interaction; dating club, marriage agencies, job agencies and real estate agencies for the matching search; shopping malls and localized markets for what concerns physical marketplaces.

The heterogeneity of these examples in terms of types of externalities, platforms' ownership, pricing rule have made it difficult to provide a generalized and unified two-sided markets theory. Early literature has indeed mostly followed an industry specific approach and, when more general, has made a systematic change in assumptions and settings in order to better fit the different instances.

Important industry specific works are Ferrando et al. (2008) on media industry, Nocke et al. (2007) on the shopping malls, Wright (2003), Rochet and Tirole (2002) on the payment system, Evans et al. (2005) on software platforms. More general approaches can be founded in Armstrong (2006),Caillaud and Jullien (2003) and Rochet and Tirole (2003, 2006). The first paper studies the competition in two sided-market, distinguishing various cases of market structure and relationship between agents and platform. The second one deals with intermediation and matchmaking services, mentioning from time to time examples in the new-economy intermediation (e-commerce) or in dating club contest. In their model, agents in one side of the market search for a perfect matching partner on the other side. Finally, Rochet and Tirole study the behavior of a platform serving the two sides, by proposing four different market structures (platform as a monopolist, platform as a Ramsey planner, competition between platforms and no for-profit platform). Moreover, they provide different models fitting various industries (i.e. internet, software platforms, video-games, medias and payment system), pointing out how most of the markets exhibiting network externalities can be considered two-sided.

More recently, two sides logic has been extended to markets not considered in the early literature. The most important examples are two recent papers by Bardey et al. (2009) and Bardey and Rochet (2009), which apply this theory on health and education sector. The first paper introduces a new concept of common network externalities and fits both industries. The second one draws on US private health care system, explaining how competition between health plans works.[1] Since a policy-holder joining a given health program can choose to be treated by physician affiliated to the same plan, two-sided logic can be used. The health plan in itself can be viewed as a platform, which aims at developing a network attracting physicians on one side and policy-holders on the other side.

The most important issue when analyzing two-sided platforms turns out to be the pricing rule. When platforms are run by for-profit intermediaries, these latter have to choose how to charge each side of the market in order to maximize profits. The most important and peculiar novelty introduced by the two-sidedness of the market is that any pricing rule must take into account not only the elasticity of demand in each side as usual, but also how a price cut in one side affects other side's participation. This will lead to the principal difference that pricing in presence of cross-group externalities shows compared to the price setting in one-sided markets, which is the observed inter-side price discrimination.

Depending on the industry, price may either alternatively or contemporaneously consist in membership or usage fees. The first ones are referred to fees charged by intermediaries for the access to the platform, regardless the actual interaction with the other side. With usage fees we mean instead all charges linked to the transaction with the other group. We have several examples of these price instruments. Nocke et al. (2007) on shopping malls notice that platforms usually charge both sides with membership fees: shop owners pay the only access to the platform, while consumers tend to be subsidized with free parking, cheaper car wash etc. Ferrando et al. (2008) analyze the media market and point out how readers only buy the newspaper (access fee) whereas advertiser are usually charged on per-reader basis. In the video-game industry, developers are are provided with the development kit (in fact, a subsidy for accessing the platform) and pay a fee per copy sold, while users only pay for the console. Differently, the similar industry of OS exhibits only membership fees

[1] A health plan is simply a program allowing patients to access medical services at a lower prices, through an insurance mechanism. An individual who decides to insure himself is partially or completely covered once an illness occurs

in each side. In the credit card industry, card-holders are charged for the access to the system (yearly fee payed to Visa or MasterCard) while merchants are charged on the transaction.[2]

Talking about platforms' competition, two cases may arise, as an agent either joins only one (i.e. single-homing) or several platforms (multi-homing). A comprehensive list of examples is available in Evans and Schmalensee (2013), which analyzes in detail the presence of multi-homing in several industries. In particular, there are clear examples of single-homing (e.g. most of the times, PC users work only on one operative system) and of multi-homing (e.g. advertisers buy their ad-spaces in more than one newspaper), whereas some cases' categorizations is debated. For instance, in video-games markets, Evans reports that in 2002 households that already own one console, on average own 1.4 consoles. In a model of Hagiu (2009b) fitting this market, the assumption is that users single-homing. Belleflamme and Peitz (2010b) point out how nowadays some gamers play on more than one console, and then we cannot consider this side as an example of single-homing. On the empirical side, while Evans and Schmalensee (2013) report that Electronic Arts, a leading game developer, recently released its games for the Nintendo, Microsoft and Sony consoles, Clements and Ohashi (2005), looking at the past in video-games industries, report that only the 17% of titles in their sample were available on multiple platforms.

To analyze as completely as possible this broad literature, this paper is organized in sections which consider separately the most important theoretical issues in this field of research. We start in section 1.2 with the *chicken-and-egg* problem and its effects on the price structure. The analysis thus enters into details in the subsequent sections. Section 1.3 gives a theoretical explanation of the price structure, considering the benchmark case of a monopoly platform. In section 1.4 competition between platforms is introduced, considering the different settings provided by the literature: in particular, we focus on platform differentiation (sub-section 1.4.1) and on multi-homing (1.4.2). Section 1.5 is aims at presenting the different kinds of

[2] The case of payment system is much more complicated than other examples because of the presence of the interchange fee, which is an percentage amount of the underlying transaction that a merchant's bank (the "acquiring bank") pays to a customer's bank (the "issuing bank"), whenever a merchants accepts a card. A higher interchange fee implies profits of the platform to be made relatively on merchants' side.

externalities and price discrimination that have been used in the literature. Finally, 1.6 is devoted to the conclusion and works as a link with Chapter 2 introducing the concept of intra-side price discrimination as an important point in the research agenda of this literature.

1.2 Chicken & Egg Problem

A for-profit intermediary serving two groups of agents needs to decide how to charge each side of the market in order to develop a business. To get the "two sides on board", scholars agree on the possibility for a platform to implement a simple strategy called *Divide & Conquer* (Caillaud and Jullien (2003)), consisting in the offer of an attractive low price to one side (divide), together with a compensation charging the other side with a high price (conquer). More specifically, intermediaries subsidize the low value group and tax the high value group.[1] The expression high (low) value group is referred to the side that is relatively more (less) interested to reach the other side, i.e. more (less) willing to pay to join the platform. To clarify this, consider the dating club example as a matchmaking service that allows women and men to meet. Clearly, a critic mass of men and women is required in order to increase the probability to find a matching partner for both types of agents. Since men are the most interested in meeting the other group, their willingness to pay is greater. In other words, they represent the high value group and pay a positive fee, basically subsidizing women entrance.

As pointed out by the early literature, two-sided markets are characterized by a pricing structure favorable to one side relative to the other. One group is subsidized or, at least, charged at the marginal cost. In response to this very attractive price, a large number of people access the platform, with a consequent increase in the willingness to pay of the other group. This allows to charge more the latter, thus restoring the loss made in the first group. Rochet and Tirole (2003) provide a wide list of markets, specifying which side between the two is subsidized and which one is charged more. In their jargon, one side is the break-even or subsidized segment, while the other side is the profit making or subsidizing segment of the market. For

[1] In this way, the losses in one side are smaller or at most equal than the gains coming from the other side.

illustrative purposes, we report this list in table 1.1.

The choice of ”how to charge who” of the platform materializes in the allocation between the two sides of the aggregate price level or, equivalently, the pricing structure, which in turns is the concept used by Rochet and Tirole (2006) to define the two-sidedness of a market. According to their paper, a market is said to be two-sided when the platform can affect the volume of transactions by charging more to one side of the market and reducing the price paid by the other side by an equal amount. In other words, price structure matters being not neutral and, accordingly, platforms design it so as to bring both sides on board. This non-neutrality of the price structure is case of failure of the Coase theorem, stating the optimal allocation in presence of externalities.

A very important example to understand what non neutral price structure means is the credit card industry, analyzed by Rochet and Tirole (2002), in which often merchants are not allowed to set different prices to purchases paid by cash and by card (no-surcharge rule). If this rule did not apply, then a price structure unfavorable for merchants relatively to consumers will have the only effect to a passage of this increase in price for merchants to consumers by means of a higher retail price. The no-surcharge rule entails the impossibility for shop owners to transfer the increase in the fee to the other side, making the price structure clearly non neutral. This reasoning is even stronger when the interaction/transaction between the two sides does not involve any monetary transfer, as the passing through of an increased cost is by definition unfeasible. Take for example the case of a phone call: if the cost of the call becomes higher, the caller cannot pass through this increase to the receiver.

1.3 Benchmark Case of a Monopoly Platform

As discussed above, a two-sided market is characterized by the presence of three types of agents: agents belonging to side 1 (e.g. men, video-game developers, advertisers, merchants), to side 2 (respectively women, gamers, readers of a newspaper, cardholders) and a (some) platform(s). Formally, the presence of indirect cross-group network externalities means that the utility that an agent type $i = \{1, 2\}$ obtains by joining a given platform depends on the number of agents type $j \neq i$

Table 1.1: Examples of two-sided markets. Source Rochet and Tirole (2003) and Evans and Schmalensee (2013)

INDUSTRY	PLATFORM	SUBSIDIZED GROUP	PROFIT-MAKING GROUP
Software	OS	Software Developers (Development kit, Support, Functionality etc.)	Users
Software	Videogames console	Users	Games developers
Software	Browser	Users	Web servers
Media	Newspaper	Readers	Advertisers
Media	Free Tv Channel	Viewers	Advertisers
Media	Portals & Web Pages	Web "surfers"	Advertisers
Advertisements	Yellow Pages	Consumers (Free Parking, Cheap gas etc.)	Firms
Payment Card System	Credit Cards	Cardholders	Merchants
Commerce	Shopping Malls	Consumers	Shops

joining the same platform.

The most general and simple frameworks are presented in the works of Rochet and Tirole (2003, 2006), Armstrong (2006) and Caillaud and Jullien (2003). Rochet and Tirole (2003) and Armstrong (2006) presented models in which platforms charge only a membership fee, whileRochet and Tirole (2006) Caillaud and Jullien (2003) use both types of prices following a more general approach. To better explain how a two-sided market model works, we refer to Armstrong's contribution, which is basically built on two main assumptions. On the one hand, the utility for an agent belonging to group i is assumed to be linear in the number of people in the other group he can interact with. On the other hand, agents are all equal within groups. Defining n_j are the number of side j people joining the platform, the utility function of an agent belonging to side i is described by the following:

$$u_i = \begin{cases} \alpha_i n_j - p_i & \text{if he joins the platform} \\ \\ 0 & \text{if does not} \end{cases} \tag{1.1}$$

where the parameter α_i represents the strength of network externalities for agents belonging to side i. To get the demand faced by this monopoly platform, notice how the group i agents are more willing to join the platform the higher the utility is, i.e. $n_i = D(u_i)$. Assuming a constant marginal costs c_1 (respectively c_2) to serve a consumer in side 1 (2), then the platform owner sets p_1 and p_2 aiming at maximizing the following profit:

$$\max_{p_1,p_2} \pi = n_1(p_1 - c_1) + n_2(p_2 - c_2) \tag{1.2}$$

When prices are set, some level of utility is indirectly "offered" to people who join the platform. Therefore, we can safely express prices in function of the utility simply defining the implicit price given utility u_i, $p_i(u_i) = \alpha_i n_j - u_i$. This amount is nothing more than the price that the monopolist should offer in order to make people in side i receive a utility u_i. Plugging into the profit function, platform's maximization problem can be thus expressed over $u_1$and u_2 as follows:

$$\max_{u_1,u_2} \sum_{i=1}^{2} \left[\alpha_i n_j - u_i - c_i\right] D(u_i) \text{ where } j \neq i \tag{1.3}$$

Defining $D'(u_i)$ as the derivative of the number of agents joining the platform in side i with respect to the level of utility, the first order conditions are simply given by:

$$\begin{cases} [\alpha_1 n_2 - c_1]\, D'(u_1) - u_1 D'(u_1) - D(u_1) + \alpha_2 n_2 D'(u_1) = 0 \\ \\ [\alpha_2 n_1 - c_2]\, D'(u_2) - u_2 D'(u_2) - D(u_2) + \alpha_1 n_1 D'(u_2) = 0 \end{cases} \tag{1.4}$$

Simply plugging definition of utilities provided in 1.1, we can easily compute prices satisfying these first order conditions, which represent the classical price setting rule in two-sided markets:

$$\underbrace{\frac{p_i - c_1}{p_i} = \frac{D(u_i)}{p_i D'(u_i)}}_{\substack{\text{Lerner pricing rule} \\ \text{one-sided market}}} - \frac{\alpha_j n_j}{p_i} \quad \text{with } i \in \{1,2\} \text{ and } i \neq j \tag{1.5}$$

The formula above is similar to the usual monopoly price computed according to Lerner rule $\frac{p-c}{p} = -\frac{D}{pD'} = -\frac{1}{\eta}$, which tells that the mark up for a monopolist optimally setting price is simply equal to the negative inverse of price elasticity of demand it faces. The last term shifts downwards the price, taking into account how a price cut in one side entails an increase in other side's demand. This describes the typical effect of cross-group externalities on prices observed in the real markets. Back to the example of clubs, men are charged more because they exhibit a higher α, as they are relatively strongly interested in meeting women.

This results implies two obvious corollaries. On the one hand, agents in group i may be subsidized, with price below the marginal cost, when side j's are interested enough in meeting them (α_j sufficiently high). Indeed, women often enter clubs for free, being subsidized by men. On the other hand, profits of the platform are relatively made in the side exhibiting stronger network externalities. Indeed, take the maximized profit of the monopolist:

$$\pi = \underbrace{\left[\frac{D(u_1)}{D'(u_1)} - \alpha_2 n_2\right] n_1}_{\substack{\text{profits made in side 1} \\ \equiv \pi_1}} + \underbrace{\left[\frac{D(u_2)}{D'(u_2)} - \alpha_1 n_1\right] n_2}_{\substack{\text{profits made in side 2} \\ \equiv \pi_2}} \tag{1.6}$$

As expressed above, the first element represents the amount of profits made up by the platform in side 1 while the second one is the remaining part of the profit coming from the other side. We can thus define the following ratio:

$$\pi_{1,2} = \frac{\pi_1}{\pi_2} = \frac{\left[\frac{D(u_1)}{D'(u_1)} - \alpha_2 n_2\right] n_1}{\left[\frac{D(u_2)}{D'(u_2)} - \alpha_1 n_1\right] n_2} \tag{1.7}$$

as the relative size of profits made in side 1. Clearly, this value in increasing in α_1, meaning that the stronger externalities in side 1, the higher the share of profits made by the platform at the expenses of this side.

This is what is observed in real world. Media (newspapers, TVs) make profits on advertisers, clearly the most interest to reaching the other side (readers, viewers). Shopping malls provide consumers with a number of benefits as free parking, cheap gas etc., while their profits are made on the shops' side. Evans et al. (2005) report that console producers tend to subsidize users and charge the copy sold by developers, while operating systems do exactly the opposite (they offer the development kit to developers and charge only the users' side). This difference has been an important input for the works of Hagiu (2006, 2009b) fitting these markets. As pointed out by Evans and Schmalensee (2013), given the particular features of two-sided markets, the antitrust and regulation policies might suffer a misrepresentation problem. In particular, when talking about price (e.g. predatory pricing rules) the two-sidedness of the market has to be taken into account. The price applied in one side alone nothing says about the competitive behavior of firms operating in these markets.[1]

More specifically, Wright (2004) provides an exhaustive map of all the critics that can be addressed to regulation and antitrust institutions for their decisions. In particular, he notices how these institutions often use one-side logic analyzing industries characterized by cross-group externalities. This approach bring them to conclude that a high price-cost margin indicates market power, a price below marginal cost indicates predation and an increase in competition necessarily results in a more efficient structure of prices and in a more balanced price structure. All these conclusions do not fit two-sided markets: the price structure of a platform has to take into account network externalities and, as we show in section 1.4, competition might have effects on prices appreciably different from the one-side markets's

[1] A classical example in the literature is the case of video-game consoles, sometimes offered to users at prices below or just above the marginal price. The same Evans and Schmalensee (2013) mention the example of Microsoft which sold its X-box at a price below the marginal cost.

case. In particular, Wright tells about some regulatory investigations into credit card schemes in Australia and the United Kingdom and he highlights how big mistakes in the policies are due to these arguments about prices and links between prices and competition.

The monopoly platform with cross-group externalities is just the beginning of a large literature. First of all, platform competition is treated in different ways by scholars (competition in prices, horizontal differentiation of the platforms, exclusivity vs multi-homing). Moreover, considering different types of network externalities (intra-group competition), preferences for platforms variety, quality on the other side, quality of the platform, intra-side price discrimination complicate end enrich this scenario. All these issues are presented in the remainder.

1.4 Competition in Two-Sided Markets

Competition between platforms has to take into account a couple of features. First, it makes sense to reckon some level of platform differentiation. For example, a reader (viewer) of a newspaper (TV-channel) shows often a strong connection with a specific platform, because of political opinions, favorite programs etc. It is plausible to think that a computer user tends to have a relationship with an operative system, since a change of operative system involves the learning of new work methods and procedures. Another example is the physical distance in the case of shopping malls: a consumer wanting to go shopping is more likely to go the closest shop. These preferences about platforms among customers make platforms enjoy some degree of market power on each side.

Moreover, when platforms are competing, agents may opt for joining more than one platform. For instance, merchants usually accept both Visa and MasterCard, application developers offer their programs to all OS, retail chains locate in competing shopping malls etc. This scenario is the so-called multi-homing: literature studies cases in which only one or both sides multi-home and situations in which all sides choose exclusively a platform (single-homing). Competition and pricing structure noticeably differ in each of these cases.

1.4.1 Homogeneous Agents vs Platform Differentiation

When agents are homogeneous, they all are willing to switch from one platform to another in response to the even slightest price cut. Caillaud and Jullien (2003) study the competition between matchmaking service fitting the internet informational intermediation. Agents are assumed to be unable to find a matching partner alone, and the platform offer the service of an intermediation increasing the likelihood of a good match. This probability represents the quality of the intermediation service, i.e. no mistakes in the collection of informations and in the data processing. Agents are assumed to be homogeneous ex-ante in each side, i.e. all should make the same decisions at equilibrium and there is not platform differentiation: platforms compete only in prices.

Platforms use both access and transaction fees, assuming that the transaction can be observed and, consequently, charged by the intermediary. Platform's objective is to maximize profits by attracting agents in both sides and charge them. The paper argues that the unique equilibrium involves the presence of a dominant incumbent subsidizing the access and charging the maximal transaction fee, in order to get the overall surplus created by the transaction. The intuition behind this result is the following: since a potential entrant could implement the same strategy, the only way to prevent entry is to make this strategy of stealing agents in both sides unprofitable. Only one platform exist at equilibrium, but the market is highly contestable, profits very close to zero as in all classical Bertrand results. The point is that if platforms compete only in prices and the service offered is exclusive,[1] then only one platform serves the market. In this case, network externalities imply concentration: since the more agents in one side, the more intense other side's participation, only the platform able to attract the largest numbers in both sides survives in the market. Since agents are homogeneous, all take the same decisions, i.e. join the same platform. Moreover, the strong pressure on prices makes the market highly contestable.

In order to avoid this zero-profit monopoly result, scholars agree on platforms'

[1] The case of localized markets, like flea or farmer market, and the case of some employment agencies for temporary work are the only examples that can justify a so strong assumption. Nevertheless, even if multi-homing is very common, exclusivity serves to isolate specific mechanisms, according to the objective that a single work aims at.

horizontal differentiation. In particular, Rochet and Tirole (2002, 2003, 2006) and Armstrong (2006) propose a setting with platforms located at the end-points of a Hotelling segment, agents' locations drawn from a distribution and presence of transportation costs. This has become the usual manner to describe platform competition in a two-sided market model. Following the same notation used in the monopoly case, the assumption are that platform $j \in \{A, B\}$ are located at the endpoints of a Hotelling unit segment, while consumers locations x are drawn from a uniform distribution. Side i agents bear a transportation cost t_i and thus the utility of a side i agent located at x will be:

$$u_i^j = \alpha_i n_{-i}^j - p_i^j - |x - x^j| t_i \tag{1.8}$$

where $i \in \{1, 2\}$,$-i \neq i$ and $x^A = 0, x^B = 1$. The usual way to deal with this kind of competition is to concentrate the analysis to the symmetric market sharing equilibria, that is the situation in which network effects are not so big to have a monopolistic platform. The necessary and sufficient condition for a market sharing equilibrium is in this context given by the following condition:

$$4t_1t_2 \geq (\alpha_1 + \alpha_2)^2 \tag{1.9}$$

Side i agent is indifferent between joining platform A and B if he is located at the $\bar{x}_i$ such that:

$$\bar{x}_i = \frac{1}{2} + \frac{\alpha_i \left(n_{-i}^A - n_{-i}^B\right)}{2t_i} \tag{1.10}$$

Since total population is normalized to one, all agents on the left of this critical threshold should join platform A, whereas all the remaining will join platform B. Accordingly, the number of A joiners in side i is given by $n_i^A = \bar{x}_i$, while $n_i^B = 1 - \bar{x}_i$.

In the symmetric equilibrium, the number of people joining platform j in side i will be given by:

$$\begin{cases} n_1^j = \frac{1}{2} + \frac{\alpha_1(p_1^{-j} - p_1^j) + t_2(p_1^{-j} - p_1^j)}{2(t_1t_2 - \alpha_1\alpha_2)} \\ \\ n_1^j = \frac{1}{2} + \frac{\alpha_1(p_1^{-j} - p_1^j) + t_2(p_1^{-j} - p_1^j)}{2(t_1t_2 - \alpha_1\alpha_2)} \end{cases} \tag{1.11}$$

We assume in the remainder of the this section that both platforms bear the same marginal costs, defined as c_1 and c_2 as in the monopoly case. Accordingly, platform j solves the following maximization problem:

$$\max_{p_i^j} \sum_{i\in\{1,2\}} (p_i^j - c_i)n_i^j \tag{1.12}$$

This objective function of the monopolist above is concave in prices if the condition required in 1.9 holds. Therefore, first order conditions are sufficient for optimality. As a consequence, the prices charged by both platform in side i will be equal to:

$$p_i = c_i + t_i - \alpha_{-i} \tag{1.13}$$

The interpretation is the same provided for monopolistic prices. The price charged to group i is composed by the marginal cost c_i, shifted upward by the market power exerted on this side represented by the transportation cost t_i and shifted downward by the network externalities. Indeed, the last term α_{-i} represents the gain in side $-i \neq i$ participation due to a price cut in side 1. The main conclusion is that, in the case of exclusive services, platform competition may arise only if the network effects are lower than preferences over platforms.

The assumption of heterogeneous consumers has become an important instrument used in most studies, at least in one side of the market. Ferrando et al. (2008) work on media industries considers heterogeneous readers (justified by political and cultural preferences on a newspaper). Bardey et al. (2009) on competition among health plans consider heterogeneous physicians and homogeneous patients (here heterogeneity is just to simplify the analysis and to make platforms not to compete for the same physicians). Hagiu (2006, 2009a) provide models fitting video-games and operating systems proposing a heterogeneity in users and homogeneity in developers. Bardey et al. (2009) on health and education sectors consider heterogeneity in both sides as well as Armstrong and Wright (2007), Rochet and Tirole (2003, 2006) in their more general approaches.

In credit card industries, heterogeneity of agents and platform differentiation have been neglected. In the most important works of Rochet and Tirole (2002), Guthrie and Wright (2007) and Wright (2003), both cardholders and merchants take their decision only according to prices they face.

1.4.2 Multi-homing vs single-homing

As mentioned in the introduction, some degree multi-homingsingle-homing is often observed in the real world. Games and software are available in more than one console OS, respectively; advertisement are present in more than one newspaper or TV-channel; merchants usually accept and often consumers hold more than one card. Except the very particular case of localized market, it is usual that one side or both decide to multi-home.

According to the decisions in each side, we can distinguish three cases:

(i) both sides *single-home*. Classic examples are localized markets, as flea or farmer markets: here single-homing in both sides is due to the fact that agents cannot be in more than one place at the same moment.

(ii) One side *single-homes* and the other *multi-homes*. This case is the most common, e.g. applications are developed for more than one OS, while users tend to use join only one platform. It is the case called by Armstrong and Wright (2007) *competitive bottleneck* scenario.

(iii) Both sides *multi-home*. This is common as well, but less appealing for its economic relevance about network externalities: as pointed out by Armstrong (2006), if one side *multi-homes*, it does not make sense that the other do the same, since it can reach the other side single-homing and paying only once the price. According to the empirical examples provided by Evans and Schmalensee (2013), this case can be noticed in particular network televisions and credit card industry.[1]

single-homing is often due to some exogenous reasons, which determine an impossibility of multi-homing. In particular, single-homing occurs in the case of indivisibility and limited resources. This can be clarified looking at localized markets and in general any case of physical platforms: these market-places allow buyers and sellers to interact with themselves only if both are present in the same place simultaneously.[2]

[1] Rochet and Tirole (2002, 2003) consider the case of multi-homing in each side in credit cards market.

[2] These considerations are always true for what concerns buyers. Differently, sellers can be

The occurrence of *multi-* vs. single-homing may be due to a decision of platforms as well as to consumers' preferences. Indeed, firstly platforms may find it profitable either to allow multi-homing or to impose exclusive contracts. On the other hand, provided that customers can choose, they might decide to *multi-home* if they are mainly concerned in meeting the other side of the market or *single-home* if their are mostly interested in platform services.

Platforms' decision The choice between allowing multi-homing and imposing single-homing is one of the most important strategic decisions of a platform, as pointed out by Caillaud and Jullien (2003). When the service is exclusive, they conclude that an incumbent can always find a divide-and-conquer strategy in order to reach a dominant firm equilibrium: the entrant stays out of the market and, given the competition in prices, the incumbent receives no profits. When multi-homing is instead allowed, the contestability of the market turns out to be partial and the entry barriers weaker. Allowing multi-homing can be indeed a solution to the strong price competition in presence of cross-group externalities. In this way, two platforms can operate in the market and their profits are higher than in the case of exclusivity with a unique firm serving the market.

The most important effect of the introduction of multi-homing is the tendency of an increase in the price concerning the multi-homing side, due to this mitigation of the price competition. In Caillaud and Jullien (2003) analysis, three different equilibria could arise: the dominant firm equilibrium, the global multi-homing equilibrium in which both sides multi-home and the market-sharing in which one side *multi-home* and the other *single-home*. The latter case is very interesting because it involves an equilibrium price structure particularly unfavorable to the side that opts for multi-homing. This is exactly the *competitive bottleneck* scenario described by Armstrong and Wright (2007).

Platforms' strategy is the following: attract the largest number of single-homing agents though subsidization of them and compensate these losses by charging a high price to the multi-homing side. The competitive bottleneck described by armstrong

sorted in two categories. If the shop is own managed, the unique physical shop cannot be active in more than one platform simultaneously. Oppositely, chain stores sell their products in more the one shopping mall.

and right is referred to the fact that the high price induces a suboptimal participation in the multi-homing side.

This link between multi-homing and pricing is directly visible in the price structure of platforms such as Yellow Pages and media in general, payment system and shopping malls. In the first case multi-homing is more likely to occur in the advertisers' side and indeed the price structure is unfavorable to this side. In the second case, merchants usually find it profitable to accept all cards, while a consumer in general has no incentives to hold more than one card, if nothing else because it entails a duplication of the early fees without adding any additional benefit. Unsurprisingly, card associations make on merchants. Similarly, shops' owners are more likely to *multi-home* and they basically subsidize consumers, who enjoy free services offered by the shopping malls.

In these contexts, a platform might find it profitable to offer exclusive contracts in order to lessen the competitive bottleneck. For example, a platform may offer a kind of menu pricing according to which non-exclusive prices are substantially high while exclusive prices are slightly lower then the ones chosen by the rival. In this way, this platform would attract the whole multi-homing side and thus extract the surplus of the single-homing side. A similar case are pay-TVs, in which TV stipulate exclusive contracts with contents' producers and this choice has the effect of a surplus extraction in viewers' side. Therefore, the choice to allow multi-homing or not is crucial for the platform because of these two opposite possible effects of multi-homing on profits.

In a recent paper of Belleflamme and Peitz (2010b), it is explained how multi-homing and single-homing have different effects on incentives to investments in the sellers' side. The main idea is to model the possibility for differentiated sellers (monopolistic) to invest in cost reduction and quality improvements,[1] when they need to join a platform to reach buyers. The point is whether for-profit platform(s) reduce the incentives relative to a free platform, considering that in the for-profit case part of the surplus created by the transaction buyer-seller is captured by the

[1] We consider for simplicity only the cases in which an investment involves an improvement in the utility of the buyers. For completeness, the paper proposes two other types of investment, i.e. investment in consumers' targeting which improves the possibility of price discrimination and investment in expansion of demand. In these cases the results are noticeably different and more complicated.

platform. Results depend on which side of the market *multi-homes*: when sellers *single-home*, incentives to investment are stronger in for-profit platform settings, while they are weaker otherwise. The logic behind is that in the latter case, the price charged to sellers tends to increase. In this way, the benefit of an investment can be better captured by the platform by charging higher access fees. It clearly lessens the incentive of investments. Oppositely, if buyers are allowed to *multi-home*, competition for sellers turns out to be stronger, so to cause a downward pressure on the price charged to them. Accordingly, platforms find it more difficult to extract surplus and in turns sellers are more incentivized to invest.

Agents' decisions. Provided that multi-homing is allowed, it is possible to concentrate the attention on agents' decisions. The point here becomes the understanding of whether and why one side would choose to *multi-home* and how this choice affects pricing and platforms' profit composition. Particularly suitable to explain the causes of multi-homing is the work of Armstrong (2006), which mainly concludes that if network externalities are strong enough (in particular stronger than the platform differentiation) then multi-homing occurs.

The choice of a side between *single-* and multi-homing is driven by the preferences of agents. Intuitively, the stronger platforms differentiation, the higher the cost for joining each platform in term of transportation cost. This means that when this cost becomes higher than a certain level, agents are not willing to bear it twice and, thus, they only join the closer platform.

Oppositely, if agents put a big weight on reaching the other side (i.e. strong network externalities), then these agents are more likely to *multi-home*. This two opposite forces drive the choice of the agents in a given side: in particular, if the network externality is stronger than the duplication of costs given by the choice to join both platforms, then multi-homing will occur. In this setting, the utility function of side i agent becomes:

$$u_i = \begin{cases} \alpha_i \left(n_{-i}^A + N_{-i}\right) - p_i^A - xt_i = u_i^A & \text{if joins platform } A \\ \alpha_i \left(n_{-i}^B + N_{-i}\right) - p_i^B - (1-x)t_i = u_i^B & \text{if joins platform } B \\ \alpha_i \left(n_{-i}^A + n_{-i}^B + N_{-i}\right) - p_i^A - p_i^B - t_i = u_i^M & \text{if joins multi-homes} \end{cases} \tag{1.14}$$

Where n_{-i}^j represents the number of agents in the other side who decide to join only platform j, whereas N_{-i} indicates the number of multi-homing agents. If side

i agents opt for multi-homing, the utility function must take into account both platforms prices and the transportation cost is paid for the whole unit distance.[1]

Armstrong and Wright (2007) demonstrate that if transportation costs are high enough compared to the network externality parameter[2] we are back to the case of single-homing in both sides, even though agents are allowed to *multi-home*. Indeed, if the assumption $t_i > \alpha_i$ for $i \in \{1, 2\}$ holds, then multi-homing can never be a profitable decision. Indeed, let us consider the indifferent agent, by definition the most likely to prefer to *multi-home*.[3] This agent is the one located at x such that $u_i^A = u_i^B$. Considering the utility evaluated at this point, the gain in utility enjoyed by choosing to join both rather than only one platform is given by the following expression:

$$u_i^M - u_i(\bar{x}_i) = \frac{1}{2}\left[\alpha_i(n_{-i}^A + n_{-i}^B) - p_i^A - p_i^B - t_i\right] \tag{1.15}$$

The difference above turns out to be negative if transportation costs are high enough.

In their analysis, Armstrong and Wright (2007) provide the very interesting case of the already mentioned *competitive bottleneck*, in which platforms are differentiated only in one side, i.e. $t_1 > \alpha_1$ and $t_2 = 0$. Under this assumption, they demonstrate that equilibria with side 1 single-homing and side 2 multi-homing arise. In the context of this manuscript, what matters is that the platform will set prices charging more the side that *multi-homes* (side 2). The idea behind is always the same: the higher network externalities exhibited by one side, the more this side will be charged. Moreover, when one side exhibits stronger network externalities with respect to platform differentiation, competition in this side is weakened because a price cut in this side has less effect in strategies of stealing demand from the rival. In a competitive bottleneck setting in which side 2 multi-homes and side 1 single-homes, side 1's participation is subsidized by side 2. In this way, platforms can attract a large number of agents in the side exhibiting strong platform differentiation and charge higher fees to the multi-homing side.

[1] In the case of multi-homing, agents bear the duplication of costs, both prices and transportation costs in the Hotelling specification.

[2] Or, equivalently, in presence of strong platform differentiation in both sides

[3] Indeed, for this agent the difference between bearing costs twice and once is minimal, as the cost of joining only one platform is at its maximal level

The problem that might arise is that price charged to side 2 can be so big as there is a too low participation of side 2's agents. As explained in the previous paragraph, a possible solution to this problem is to propose exclusive contracts to the party who would *multi-home* if this was allowed.

1.5 Externalities and Price Discrimination.

1.5.1 Inter-group Externalities

The general framework presented in the previous section and followed by the early literature in two-sided markets is based on the simple assumption that the benefit to join a platform in one side is increasing in the number of agents in the other side. Since the objective is to reach an interaction with the other side, the more agents are present on one side, the higher the utility of the other side. The group having stronger interests to reach the other side is charged more by the platform, since the larger part of surplus created by the transaction between the two parties is caught by this group.

In communications industries, advertisers are clearly the high value group for platforms, as they care about having their ads read by the highest number of media customers. On the other side, though, it is not clear whether a heavier presence of ads may benefit or hurts readers and viewers. In particular, it is widely accepted the assumption that TV-viewers are reluctant to advertisement whereas the case of readers is debated. Ferrando et al. (2008) provide a theoretical study of the media industries considering the population of readers-viewers as split in ad-liking and ad-adverse ones. They propose a model where two contents providers compete both for news and advertising market. As their main aim is to explain the observed concentration in press market, they focus on the possibility of only one content provider surviving in the market. Concentration is explained by the eviction of one platform in the case of a population of reader characterized by a majority of ad-liking. Differently, in case of ad-averse readers' preponderance, the choice to exit of the market by one or both platform is voluntary. The intuition behind is that when readers are mostly ad-liking, positive inter-group externalities are stronger than negative ones and then we observe a concentration à la Caillaud and Jullien (2003). On the other hand, when ad-averse readers are more, the negative network externality prevails and then one of the two platforms may not find it profitable to enter advertisements' market. In all other industries described in the literature, externalities are positive.

1.5.2 Intra-Group Externalities

Intra-group externalities are considered as the only kind of externality in the general approaches of Caillaud and Jullien (2003), Rochet and Tirole (2003, 2006) and Armstrong (2006). Considering both sides of the market as end-users, these works are able to isolate the cross group externalities and deal in a theoretical manner with the problems of pricing and platform competition. Nevertheless, in real world environments a side of the market can well be composed by agents competing with each other. In this case of rivalry, we talk about intra-group negative externalities.[1]

Several industry specific works treat one side as rival. Rochet and Tirole (2002), Guthrie and Wright (2007) and Wright (2003) on the payment system treat merchants as competing: the crucial point is that the more is competition in this side, the more that side will be charged by the platform.[2] Hagiu (2006, 2009a) develop a model of developers-users fitting video-games and OS in which developer profits are decreasing with the number of other programs available on the platforms. In Nocke et al. (2007) on shopping malls, sellers compete for differentiated products. Kurucu (2007) presents a matchmaking model fitting labor search agencies assuming firms to compete for the most qualified workers.

A more general approach is proposed by Belleflamme and Toulemonde (2009), who investigate about the possibility for a for-profit platform to divert agents from a pre-existing platform by implementing some divide-and-conquer strategy in presence of negative intra-group externalities (rivalry) and positive inter-group. They find that divide and conquer strategies are feasible if intra-group network externalities are strong enough but not too strong relatively to the inter-group ones. In Belleflamme and Peitz (2007) sellers offer perfectly differentiated products on the platform (no specific assumption of competition). Nevertheless, their profits

[1] See Belleflamme and Toulemonde (2009).

[2] In the case of card association, as pointed out in the introduction, the pricing structure depends on the interchange fee which the banks agree on. It complicates the matter because we can have not only competition between merchants but also between banks.

decrease with the number of sellers joining the platform.

Mixing negative intra-group and positive inter-group externalities yields a trade-off for the platform when deciding how many agents to attract in each side. On one hand, positive *inter-group* externalities would induce platforms to attract the largest number of agents, since the more are agents in one side, the more is the surplus (and then, the willingness to pay) in the other side. On the other hand, intra-platform competition entails that the demand in the rival side would be lower as the number of agents in the same side increases. Therefore, the total effect on the pricing structure should depend on the strength of each kind of externality. In a paper about software industries, Hagiu (2009b) provides a model in which developers compete with one another to develop differentiated products for a platform joined by users that prefer variety of applications. Their main finding is that the more competitive developers' market (lower market power), the more platform's profits are made on users. Moreover, a stronger preference for variety (which results in a jump up of single producers' market power) implies profits to be relatively made on developers. As competition makes developers less willing to pay a price cut is needed to attract the largest number of them.

1.5.3 Quality

Other considerations have concerned the literature, in particular quality. More specifically, the literature looked at quality from two different perspectives:

(i) **Quality of platform's service.**

Recent papers (Bardey et al., 2009; Bardey and Rochet, 2009) study competition between platforms in health care and education sector. Schools and hospitals can be viewed as two-sided since they try to attract patients/pupils in one side and physicians/teachers on the other side. The first is an application of the two-sided logic in a model where two different health plans compete for policy-holders on one side and for physicians on the other side, assuming heterogeneity of the policy-holders in risk of illness. Riskier policy-holders are more willing to pay and have a stronger preference for diversity of physicians. This idea is to model the coexistence in the market of two different health plans: one offering more diversity and attracting riskier policy-holders, the other offering less diversity and attracting lower risk policy-holders. Since a riskier policy-holder exhibits a higher willingness to pay, the attraction physicians improves the service of the platform. Nevertheless, it also induce riskier policy holders to join the health plan, involving an upward pressure on prices for patients.

The second paper generalizes the first considering the concept of common network externality, i.e. both groups benefit from an increase in the size of one group and from a decrease in the size of the other. In particular they argue that in education and health, both sides evaluate a sort of quality index defined by the ratios #physicians/#patients and #teachers/#pupils. It is used as an indicator of quality because an increase of this ratio allows patients/pupils to be better followed up and physicians-teachers to work better. In this realistic case, the common network externality is homogeneous of degree zero: the paper demonstrates that common network externalities have no impact on equilibrium profits of the platform. In particular, the increase in price in one side is entirely shifted to the other side.

(ii) **Quality of the other side.**

One may argue that agents are interested not only in the number of agents joining the platform in the other side, but also in their quality. In particular, Damiano and Li (2007); Damiano and Hao (2008) and Hagiu (2009a), have pointed out how, when quality matters, the positive externalities are somehow mitigated, as the platform finds it profitable to allow entry only of high quality consumers. In other words, it implies the implementation of an exclusion policy by the platform. The phenomenon of exclusion is evident in the governance rules of romantic matchmaking internet sites and video-games consoles.

In the first case, some romantic matchmaking sites like eHarmony carefully screen and reject a fraction of applicants; Damiano and Hao (2008); Damiano and Li (2007) deal with the informational problem of dating online, in which agents often misrepresent their profile. They point out how these informational problems might decrease the demand for dating online, because of its perception among the public. It reduces the quality of Internet search and matching and, in fact, it prevents many lonely people from fully utilizing the online dating services. Internet dating agencies rely on individual users to report information about themselves truthfully and have little resource or capability of directly validating the information, then price discrimination can be used to make the reported information credible and to improve match quality.

Price discrimination as a mechanism of self-selection is evident in traditional meeting places: important examples are night clubs catering people with more expensive tastes, which charge more agents for the access. In order to model this phenomenon, an important (and strong) assumption is that agents in each side can be vertically differentiated according to a one-dimensional characteristic perceived as quality by the other side. To emphasize this aspect, the authors focus only on this possibility, neglecting both positive *inter-group* externalities and platform competition.

According to the example, the platform does not observe types but it can use prices to sort out high types from low types in each side. It might choose to launch two different market places, choosing two different prices for each side. The idea is to induce agents in both sides to choose the market place conceived for them relying on a self-selection mechanism, which improves the quality of the matchmaking service, since low (high) types in each side should choose the same market place of low (high) types of the other side. The choice of prices must fulfill incentive compatibility constraints, inducing low type and high type individuals to choose the "correct" market place.

A recent paper of Hagiu (2009a) is motivated by the observation that Microsoft, Nintendo and Sony insert security chips in their consoles in order to exclude low quality game developers. It goes further Damiano and Hao (2008), as it takes into account both cross externalities and platforms' competition. For these reasons, this work can have a wider application and it is more specifically linked with the early literature of two-sided markets. However, this work finds more fitting application in software industries, in particular video-games. The crucial assumption is that users' side is interested in the average quality and in the number of applications available on the platforms. Preference for quality results in the incentives for the platform to exclude some - low quality - developers. Differently from Damiano and Hao (2008), Hagiu focuses a non-pricing instrument to exclude, i.e. a minimum quality standard. Indeed, while in matchmaking contexts the intermediary lacks (truthful) information about quality, a console producer is able to observe directly the quality of game proposed by developers and then it can directly discriminate choosing other-than-prices tools.

Taking into account quality involves some additional costs for platforms, which have *(i)* to evaluate the quality of agents and *(ii)* to regulate the interplay between the two sides. Case *(i)* is referred to the fact that platform does not precisely know the quality of agents who are joining. The solution to this informational problem proposed by Damiano and Hao (2008) is the use of sorting prices. Case (ii) refers to situations in which this quality is known by the platform. In this case, exclusion is a solution to regulate the interplay between the two sides. The imposition of minimum quality standards proposed by Hagiu is only one of a set of possible governance rules that platforms can use (and, in fact, use).

1.5.4 Intra-Side Price Discrimination.

In the real world, the analyzed inter-side price discrimination often goes together with an intra-side price discrimination. For example, platforms may have some information and use it setting different prices to different types subgroups of consumers within the same side. This discriminatory strategies are very common in markets typically said to be two-sided, as for example media markets, as well documented in Gil and Riera-Crichton (2012) and Asplund et al. (2002).

Gil and Riera-Crichton (2012) is specifically addressed to two-sided markets and study the existing relation between price discrimination and competition, using data of Spanish local TVs industry. Taking advantage of years of changes in regulation and consequent changes in the market structure, their main result is that price discrimination is less likely to be used as the number of TVs competing in the market increases. Moreover, the consumers' surplus may actually decrease if the products are enough differentiated, widely differing from the typical one-sided oligopolistic in which customers are better-off with price discrimination.

Asplund et al. (2002) provide an empirical investigation that uses Swedish data about regional newspapers. Disregarding the two-sidedness of the market, they consider only the readers' side and find that the more common type of price discrimination is third degree and it is more frequent in more competitive environments and inversely related with the newspaper's market share.

Angelucci et al. (2013) investigate both theoretically and empirically the determinants of second-degree price discrimination in two-sided markets. Their main concern is to study how incentives to engage in price discrimination on the readers' side is affected by the revenues made up on advertisers' side. In their proposal, readers are heterogeneous in preferences and face uncertainty about future benefits of reading whereas advertisers exhibit different outside options, taste for subscribers and taste for occasional buyers. Using french data, they test their model and find evidence of increased price discrimination as a result of a drop in advertisement revenues.

Two recent papers are devoted to the theoretical analysis of *intra-group* price discrimination. Liu and Serfes (2013) deal with the introduction of perfect price discrimination. Their model is a one period Hotelling model in which the initial market share of the two platforms is exogenous and platforms know the precise location of each customer, so to be able to engage in perfect price discrimination in both sides.

The main finding is that intra-side price discrimination involves a negative effect as well as a positive effect on platforms' profits. The negative effect is the usual flexibility in prices, according to which two different prices simply reflect the difference in transportation costs. On the other hand, they demonstrate how price discrimination might be a tool to neutralize cross-group externalities in equilibrium, with a positive effect on average prices in each side and on profits. Their main conclusion is that price discrimination in two-sided markets may soften the severity of competition. The *prisoners' dilemma* result of the price discrimination in competitive environments is still possible, but the possibility of an increase in firms' profit cannot be excluded. In particular, they find that for network externalities strong enough relative to the marginal cost, intra-side perfect price discrimination increase firms' profits compared to the case of the intra side uniform price.

Böhme (2012) focuses instead on second-degree price discrimination in the case of a monopoly platform. In a positive analysis, the papers shows that many of the results are basically equivalent to the correspondent one-sided market case, but the maximization problem of the monopolist is much more complex in presence of cross-group externalities. This is because the reduction of low-demand agents in side 1[1] determines a negative impact on the demand of side 2 and, in turns, on the willingness to pay of high-demand side 1 consumers.

1.6 Conclusion

The aim of this paper was to analyze competition and pricing in two-sided markets, providing an extensive review of the literature in this field of research. In order to do that, we have presented the most important theoretical issues, focusing our attention on the main differences that these markets highlight in comparison with a one sided market and on the main features of the different two-sided industries.

The two-sided market literature has grown very quickly in the last years. Starting from the first general approaches of Armstrong (2006), Caillaud and Jullien (2003) and Rochet and Tirole (2003, 2006), this theory has been applied in several markets and, given the different features of each market, we observe a bunch of possible combinations about prices (membership and transaction fees), competition between platforms (single-homing vs multi-homing), preferences of agents (platform differentiation, negative intra-group externalities) and price discrimination (*within-group* vs. *cross-group*).

Early literature focuses in particular on the pricing structure in presence of inter-group externalities. Except media case, in which readers of a newspaper or viewers of a TV channel might dislike advertising, the assumption is that the benefit from joining a platform for one side is increasing in the number of agents joining in the other side.

An important effect of *inter-group* externalities is the tendency of the market to be concentrated and of the platforms to be large. Since an intermediary who wants to develop a platform needs to attract a large number of agents in each side, the optimal strategy is to find some prices in order to achieve this objective.

[1] Which is unambiguously profit enhancing in a one-sided market.

Price structure is relatively disadvantageous for the side which exhibiting stronger externalities. These considerations about imbalance of prices between the two sides holds and they are strengthened when we allow multi-homing: in this sense, multi-homing of one side has the effect of a weak price competition in that side, with a consequent upward pressure on price.

The tendency to attract a large number of agents in each side is the most important and general conclusion in the first approaches. Nevertheless, platforms' decisions turn out to be more complicated when we consider different aspects on preferences of the agents. In particular, an intermediary running a two-sided platform might face some kinds of trade-offs when choosing how many agents to attract in each side, because of an intra-platform competition (rivalry) and because of interests on quality that agents may have. The first case is referred to situations in which the utility of agents in one side is increasing with the number of agents joining the platform on the other side, but decreasing with the number of competitors.

In this case, attracting a larger number has a positive effect on the demand on the other side, but a negative effect on the demand on rivals' side. *Intra-group* competition is present in industry specific works of Rochet and Tirole (2002), Guthrie and Wright (2007) and Wright (2003) on the payment system in which merchants are competing; of Hagiu (2006, 2009b) on video-games and operating systems in which developers' profits are decreasing with the number of developers; of Nocke et al. (2007) on shopping malls, in which sellers compete for differentiated products. The main conclusion is that the more agents in the rival side compete with one another, the more profits of the platform are relatively made in the other side. It means that competition tends to make rival agents less willing to pay and then we need a price cut in their side to attract the largest number of them.

Another important issue treated in the literature is quality. Bardey et al. (2009); Bardey and Rochet (2009) deal with the interest of the two sides on quality of platform's service. These works are referred to two particular industries not considered in the early two-sided literature, i.e. health and education. In this industries, each side is interested in the number of agents in the other side (positive inter-group externalities), but also in the ratio #teachers/#pupils and #physicians/#patients. This kind of externality (called common externality) works as a quality index of the platform's service and it change the price structure in the sense that there is a shift of prices from pupils' (patients') to teachers'(physicians) side, without any effect on platforms' profits.

A different concept of quality is discussed by Hagiu (2009a) and Damiano and Li (2007); Damiano and Hao (2008). They deal with the problem of platform's decisions in cases in which agents in one side (or both) are interested in the quality of the other side, typical problem of matchmaking services (Damiano and Li (2007); Damiano and Hao (2008)) and video-games consoles (Hagiu (2009a)). In both markets, these preference for quality induce platforms to engage exclusion with pricing (romantic matchmaking) or other-than-pricing instruments (video-games).

Intra-group price discrimination captured my attention as a possible research agenda of two-sided market literature. As recently pointed out by both theoretical and empirical papers, we can observe indeed different prices charged to agents belonging to the same side. Taking this into account enriches the results both of traditional two-sided markets and price discrimination in one-sided markets literatures.

According to this new tendency, Chapter 2 is addressed to the theoretical analysis of a particular kind of price discrimination that is often used *within side*, which is called by the literature *behavior based price discrimination* (BBPD). We will then postpone the discussion of $BBPD$ in two-sided markets to the following chapter, which can be actually read as a distinct paper belonging to this growing line of research.

2

Behavior Based Price Discrimination with Cross Group Externalities

2.1 Introduction

When a firm knows the identity of its customers, it may decide to charge new customers with a lower price in order to increase its demand.

As pointed out by Taylor (2003), price discrimination based on past purchases, called behavior based price discrimination (BBPD), according to which firms offer discounted prices to new customers inducing them to switch, is very common in subscription markets. This is because transactions are never anonymous: once a customer signs the subscription, a firm knows whether she is one of the current customers or not. According to that, it may have an incentive to propose low introductory prices to customers who did not buy its product in the past.

Discounts take different forms such as low introductory prices, trial memberships and free installations. As mentioned in Caillaud and Nijs (2011), a new subscriber for 3 months to the French newspaper "Le Monde", pays 50 euros whereas a previous customer is charged 131.30 euros. A similar strategy is the free trial membership to the program Amazon Prime offered by Amazon,[1] which offers free streaming

[1]From Amazon website "Amazon Prime members in the U.S. can enjoy instant videos: unlimited, commercial-free, instant streaming of thousands of movies and TV shows through Amazon Instant Video at no additional cost. Members who own Kindle devices can also choose from thou-

contents, magazines and books for Kindle's owners. Moreover, first subscriptions to credit cards and TVs/internet services are often offered for free.[1]

Firms selling products as credit cards, magazines and newspapers, satellite TVs, internet access and e-book readers have the common feature that subscribers are not the only customers. Indeed, these firms compete also for another side of the market, e.g. merchants (credit cards), advertisers (media), content providers (e-book readers and internet). These firms are two-sided platforms that serve and allow the interaction between different groups of customers linked to each other by cross-group externalities. Indeed, when a cardholder decides whether to hold a card or not, his utility is increasing in the number of shops in which she can use it. Shops (merchants) represent the other side of the market and in turn they are more willing to pay to hold a card reader as the number of card users increases.

The same interaction arises in the other mentioned markets: medias (magazines, newspapers) allow the interaction between readers and advertisers; satellite TVs/internet providers between viewers/surfers and content providers. For what concerns Kindle Owners' Lending Library and Amazon Instant Video, Amazon is nothing else than a platform that facilitates the interaction between readers/viewers and content providers. Publishers are interested in selling their books to a large number of readers who, in turn, are interested in the variety of contents. Thus, the utility that a reader obtains from subscription increases in the number of publishers, and the utility that a publisher receives to have his book available on Kindle Library is increasing in the number of Amazon Prime subscribers.

Because of the externalities, one of the distinctive features of these markets is the pricing rule, which is different from the general rule that applies in a one-side framework (i.e. market without externalities), both for a monopolistic and a competitive environment. Think for example to Amazon Prime program: because of the cross-group externalities between readers and publishers, the subscription fee charged to the readers affects not only the demand in this group, but also the

sands of books - including more than 100 current and former New York Times Bestsellers - to borrow and read for free, as frequently as a book a month with no due dates, from the Kindle Owners' Lending Library. Eligible customers can try out a membership by starting a free trial"

[1] Taylor (2003) also mentions a 1998 *Wall Street Journal*'s article by Bailey and Kilman reported that "the 60% of all Visa and MasterCard solicitations include a "teaser" (low introductory rate) on balances transferred from a card issued by another bank".

willingness to pay of publishers to have their books available on Amazon Kindle.

This is the basic reason for which we observe different prices for different sides of the market (cross-group price discrimination): price charged to each group of agents depends on the cross externalities, so that a group whose participation entails a large participation of the other group will be charged less.[1]

This idea is very clear when we look at medias: since advertisers are only interested in reaching a high number of readers/viewers while the other way around it is not (necessarily) true,[2] they are charged more and most of medias' profits are made on ads.

According to this discussion, in many subscription markets two kinds of strategies are used by competing platforms: the mentioned cross-group price discrimination typical of a two-sided market and the within-group BBPD in subscribers' side. These strategies have a common feature: platforms have some information about the characteristics of various groups of customers and exploit this information setting targeted prices to each group.

However, the type of information required to implement these strategies is fundamentally different. On one hand, to engage in cross-group price discrimination, platforms simply sort customers according to their externalities. On the other hand, within-group BBPD requires platforms to know the identity and the behavior of customers.

This paper provides a two-sided market analysis to address to the following question: "What does it change if platforms are allowed to offer different prices to subscribers according to their past purchase behavior?". Specifically, the aim is to investigate about the effects on prices and platforms' profits when within-group BBPD can be implemented because subscribers are identified.

In order to answer to this question, we provide a two period model in which platforms compete in a Hotelling fashion for two different groups of agents, users (subscribers in the examples above) and firms.

[1] This result is firstly due to Caillaud and Jullien (2003), which calls this price strategy divide and conquer, and has became the reference point for the succeeding studies on pricing in two-sided markets.

[2] In particular, the industry specific work of Ferrando et al. (2008) on media market studies provide a two sided model in which a proportion viewers/readers is ad-lovers and the remaining part is composed by ad-averse people.

In the first period platforms set prices and then each firm and user decide which platform to join. In the second period, platforms come across a new information of the identity of old and new users, which they may exploit by discriminating prices between the two groups

We will answer to the following questions:

(i) Taking the first period outcome as given, how do externalities change the actual switching when we consider only the second period competition? In this case our benchmark is the one-sided market case.

(ii) Making agents perfectly able to anticipate the second period equilibrium, how do the interplay BBPD-externalities affect prices and profits? In this case our benchmark is the case of within group uniform price.

Related literature

This paper is naturally linked to the two-sided market literature, initially formalized by Rochet and Tirole (2003), Armstrong (2006) and Caillaud and Jullien (2003). The main result around which this literature is built on is the cross-group price discrimination, which follows the concept of *Divide & Conquer* firstly proposed by Caillaud and Jullien (2003). To develop a business, a platform has to attract a large number of customers on one side, even subsidizing them (divide) and after restore its losses charging a relatively high price to the other side (conquer).

As in Rochet and Tirole (2003), Armstrong (2006) we use a Hotelling model, to capture the idea that customers are not indifferent about joining one platform or another but can be horizontally ordered according to their preferences. The model focuses on the simplest case in which platforms charge only a price independent of the number of interactions with the other side[1] and customers can join at most one platform.[2]

[1] Literature distinguishes between subscription fee and usage fee. In the analysis of the media market of Ferrando et al. (2008) is pointed out how, while readers are charged with the price of the newspaper, advertiser are charged on per readers basis. In this case we can see an access fee in one side and a transaction fee on the other side.

[2] As a matter of fact, literature points out how often at least one side decides to multi-home, i.e. to join more than one platform. Armstrong (2006) and Armstrong and Wright (2007) provide an analysis on the reasons and on the effects of multi-homing in platforms competition.

On the other hand our paper is strongly related with BBPD literature, which starts with Villas-Boas (1999) and Fudenberg and Tirole (2000). The main finding of this literature is that BBPD is detrimental for firms, which compete fiercely in prices and face a *prisoners' dilemma* problem.

In particular, our model is built on Fudenberg and Tirole (2000), which provide a Hotelling model played twice, allowing firms to know whether a customer in the second period is new (weak market) or he was already buying from the same firm (strong market). They establish that offering discounted prices to new customers is an equilibrium phenomenon that involves a decrease in prices and in profits for firms with consequential increase in the total surplus of consumers.

Villas-Boas (1999) makes the same analysis but in infinite time with overlapping generations of consumers while Chen and Zhang (2009) and Esteves (2010) present models with different distributions of consumers types. Except the work of Chen and Zhang, the literature agrees on the result that customers' recognition and consequent price discrimination hurt firms compared to a situation in which the targeted pricing is not possible. Even if a firm alone would prefer to obtain the information (and so benefit from the surplus extraction), if both get it then a market stealing effect tends to prevail.

Liu and Serfes (2013) is close to our paper in that both of us analyze within-group price discrimination. In particular, platforms are allowed to engage in perfect price discrimination within each side. Their main finding is that discrimination might be a tool to neutralize cross-group externalities with a positive effect on prices and platforms' profits. There are two main differences with our work. First of all, they only consider one period, keeping the past behavior of consumers and market's shares as given. Second, they analyze the case of perfect price discrimination, while we focus on a more realistic discrimination based on past purchase behavior.

The remainder of the paper is organized as follows. In section 2.2 we introduce the model. In section 2.3 we analyze the model considering first period competition as exogenous, then we add first period competition in section 2.4. In section 2.5 we compare our results with the case in which $BBPD$ is not (or cannot be) used before providing conclusions in section 2.6.

2.2 The model

Two competing platforms $j = A, B$ aim to sell a service to two different groups of customers. For the sake of expositive clarity, we name end-users the customers on one side and firms on the other side. With end-users' side we simply refer to the group of final consumers, which can be books/newspapers' readers, TV's viewers or cardholders. We call firms' side, instead, the group of customers composed by advertisers, contents' providers or merchants.

Both end-users (side or group E) and firms (side or group F) are heterogeneous according to their locations: they are assumed to be uniformly distributed along a unit segment. In turn, platforms' locations are kept fixed at the end-points of this segment, i.e. platform A's location is $x^A = 0$, while platform B is located in $x^B = 1$.

Both sides of the market exhibit linear utilities from joining a platform. An agent located at $x \in [0, 1]$ bears a constant transportation cost t per unit of distance covered to reach the location of each platform.[1] Moreover, the benefit that an agent receives from joining a given platform depends on the number of agents joining the same platform on the other side.

Specifically, utility is assumed to be linear in other side's participation. The parameters α_F and α_E are assumed to be both in the interval $(0, 1)$ and have to be interpreted as the extra-benefit for a firm (respectively and end-user) when an additional user (firm) joins the same platform.

We have two periods $\tau = 1, 2$. In each period platforms set prices. After having observed the prices offered, firms and users decide which platform to join. Defining $p^j_{i\tau}$ as the price set by platform j to side i in time τ, the utility for an agent belonging to side i located at x joining platform j in time τ is simply:

$$U^j_{i\tau}(x) = u + \alpha_i n^j_{i'\tau} - p^j_{i\tau} - \left|x - x^j\right| t \text{ where } i' \neq i \tag{2.1}$$

where $n^j_{i'\tau}$ is the total number of other side's agents joining platform j. u is a constant representing the standalone utility, i.e. the utility that an agent benefits

[1] Throughout the paper, the transportation cost is assumed to be the same for both sides. This assumption is quite arguable, but since the intuition behind the results provided in the paper remains the same even if we consider two different transportation costs, we use only one transportation cost in order to keep notation as simple as possible.

from joining a platform, regardless which platform he is joining.[2]

Platforms seek to maximize inter-temporal profits, bearing unitary cost normalized to 0 for both sides. The profit of a firm in time τ is simply given by the sum of the products between the price charged to each group (or sub-group, as we will see afterwards) and the number of joiners belonging to the same group. Thus, the profit of platform j in time τ when charging prices $p^j_{E\tau}$ and $p^j_{F\tau}$ to each side is indicated in equation by the following:

$$\pi^j_\tau = \sum_{i=E,F} p^j_{i\tau} n_{i\tau} \tag{2.2}$$

Platforms set prices in each time period in order to maximize the sum of inter-temporal profits,[1] given by:

$$\Pi^j = \pi^j_1 + \pi^j_2 \tag{2.3}$$

Three main assumptions are used throughout the paper: demand is fully served, transportation cost is big enough so to have single-homing in both sides and time profit functions are concave. Using a formal jargon, we write down the assumptions $A1$, $A2$ and $A3$.

ASSUMPTION A1 (Market fully served): u big enough.

If the standalone utility u is big enough, every agent prefers to join at least one platform instead of joining none. In this way we insure that the market is fully covered. For simplicity, an agent who does not join any platform is assumed to receive no utility.

[2] Here it is assumed to be the same for both sides of the market just to keep notation as simple as possible but it is not crucial in the analysis of the model.

[1] We assume without loss of generality the absence of any discounting.

ASSUMPTION A2 (Single-homing): $t > \alpha_E$ and $t > \alpha_F$.

As shown in Armstrong and Wright (2007), this assumption means that the transportation cost is high enough to have that each agent joins at most one platform. In words, agents are interested in reaching the other side, but not so much to decide to join both platforms and bear price and transportation cost twice. Following the usual phrasing of two-sided markets literature, they opt for single-homing instead of multi-homing.

ASSUMPTION A3 (Concavity): $t^2 > 2\left(\alpha_E + \alpha_F\right)^2$.

This is simply the assumption we need for the profit functions to be concave. Proof is provided in Appendix 3.6.1.

According to the examples mentioned in the introduction, we allow platforms to discriminate prices in the users' side according to past purchase behavior: in the second period, discounted prices can be offered to consumers who did not subscribe in the first period. Since our objective is to analyze subscription markets in which BBPD is used, we focus only on price discrimination in the users' side, the only one for which we have evidence about BBPD.[1]

Formally, platform j in period 2 offers this pair of prices for users

$p_{E2}^{jj} \equiv$ price chosen by platform j for users who have already bought from it in period 1 (agents who are loyal to j)

$p_{E2}^{jj'} \equiv$ price chosen by platform j for users who have bought from platform $j' \neq j$ in period 1 (i.e. agents who are supposed to switch from j' to j)

Potentially, given this pair of prices, some agents remain loyal to one platform while some others may decide to switch to the rival, because of a lower introductory price offered by the latter. Definig n_{E1} as the inherited number of users who have joined platform A in period one and x_2^A (respectively x_2^B) as the location of the user indifferent between switching and subscribe again to platform A (respectively platform B).

[1] As a matter of fact, some price discrimination may be used also in firms' side, but we keep this possibility out of our analysis.

Thus, the number of switchers from platform A to platform B will be $n_{E2}^{BA} = \max\left\{n_{E1} - x_2^A, 0\right\}$ and the switchers going in the opposite direction are $n_{E2}^{AB} = \max\left\{x_2^B - n_{E1}, 0\right\}$. Alike, the number of users loyal to A is $n_{E2}^{AA} = \min\left\{x_2^A, n_{E1}\right\}$ while the loyal to B are $n_{E2}^{BB} = \min\left\{1 - x_2^B, 1 - n_{E1}\right\}$.

According to that, the profit function in time $\tau = 2$ turns out to be slightly different from the one defined in (2.2). Namely, it takes the following form:

$$\pi_{\tau=2}^{j} = \begin{cases} p_{E2}^{AA} n_{E2}^{AA} + p_{E2}^{AB} n_{E2}^{AB} + p_{F2}^{A} n_{F2}^{A} & \text{if j=A} \\ \\ p_{E2}^{BB} n_{E2}^{BB} + p_{E2}^{BA} n_{E2}^{BA} + p_{F2}^{B} n_{F2}^{B} & \text{if j=B} \end{cases} \tag{2.4}$$

In the next section, we discuss the second period equilibrium when what occurred in the first is kept exogenous. We distinguish between the case of switching occurring in both directions (sub-section 2.3.1) from the one in which switching may occur only towards one of the two platforms (sub-section 2.3.2) and we provide the conditions on first period outcome under which we switch from one case to the other. Subsequently, we assume that agents are able to anticipate the second period result and then solve the whole two-period game by finding out time 1 prices.

2.3 Time 2 competition game.

In this section, we analyze the decisions of players in the second period, treating the initial subscription to each platform as exogenous. The equilibria of this game depend on what occurred in the first period.

Platforms set prices in each side. In firms' side, the price is uniform, i.e. each firm joining the same platform is going to pay the same price. For what concerns users, we allow platforms to set different prices according to their past (observed) purchase behavior. Before setting their prices, platforms form expectations about the participation of both sides of the market, since the utility functions are common knowledge.

After having observed the price offers of platforms, users decide whether to confirm or not the decision they have taken in the past. If they join the same platform as in the first period, they are said to be loyalists, while if they change, they are switchers. Their decision is also taken looking at prices for the firms' side. According to the prices they observe, users have expectations about participation on the other side. The firms' participation as well as the prices they have to pay will determine the number of loyalists and switchers.

Firms' decisions follow exactly the same reasoning. They observe prices they have to pay in each case and then form expectations about the participation of users.

At equilibrium, each customer joins the platform which gives him the highest utility. Anticipating the participation and the choices of the rival, each platform maximizes profits choosing prices.

Users Who is going to switch and who is going to stay? Users simply compare utility they get joining each platform and decide upon which platform to join given offered prices and given the expected number of firms' contents available in both platforms. As explained in section 2.2 for given prices offered by platforms, we can find where the indifferent users between switching and staying are located.

Consider an end-user who has bought from platform A in period 1. He is indifferent between buying again from platform A (paying price charged by A to its old customers, p_{E2}^{AA}) and switching to B (and paying price chosen by B for new customers, p_{E2}^{BA}) if his location x_2^A is such that given the prices charged in A turf he attains the same utility from joining one of the two platforms. This cut-off location is given by what follows

$$x_2^A = \frac{1}{2} + \frac{\alpha_E n_{F2}^A - \alpha_E n_{F2}^B + p_{E2}^{BA} - p_{E2}^{AA}}{2t} \tag{2.5}$$

The same reasoning is followed to define x_2^B, threshold representing the agent who has decided to join platform B in the past and now is indifferent between choose to buy again from B (and pay p_{E2}^{BB}) and switch to platform A (paying price p_{E2}^{AB}), i.e:

$$x_2^B = \frac{1}{2} + \frac{\alpha_E n_{F2}^A - \alpha_E n_{F2}^B + p_{E2}^{BB} - p_{E2}^{AB}}{2t} \tag{2.6}$$

According to the fact that the number of users loyal to A is given by n_{E2}^{AA} while the number of switchers to A is n_{E2}^{AB}, the total number of users joining platform A in time 2 will be:

$$n_{E2}^A = n_{E2}^{AA} + n_{E2}^{AB} = \min\left\{x_2^A, n_{E1}\right\} + \max\left\{x_2^B - n_{E1}, 0\right\} \tag{2.7}$$

Different cases may arise. If $n_{E1} \in \left(x_2^A, x_2^B\right)$, then switching to both directions occurs and (2.7) becomes $n_{E2}^A = x_2^A + x_2^B - n_{E1}$. Depending on n_{E1}, switching to both directions may be not the case. If $x_2^A > n_{E1}$, then there are no switchers from A to B and $n_{E2}^A = x_2^B$; if $x_2^B < n_{E1}$, then there are no switchers from B to A and then $n_{E2}^A = x_2^A$.[1] Moreover, according to assumption $A1$, $n_{E2}^B = 1 - n_{E2}^A$.

Firms Firms take their decision following basically the same reasoning as users. They observe prices offered by both platforms and according to how many users they expect to subscribe to each platform, they decide which platform to join.

In order to define this indifferent firm, we directly consider that the total number of users joining each platform is the sum of loyalists and switchers as described in equation (2.7). We define this cut-off location as n_{F2}, which turns out to be what follows, simply by equalizing the utility firms obtain from joining each platform

$$n_{F2} = \frac{1}{2} + \frac{\alpha_F}{t}\left(n_{E2}^{AA} + n_{E2}^{AB} - \frac{1}{2}\right) + \frac{1}{2t}\left(p_{F2}^B - p_{F2}^A\right) \tag{2.8}$$

According to this threshold and assumption $A1$, the number of firms joining platform A (respectively B) is simply $n_{F2}^A = n_{F2}$ (resp. $n_{F2}^B = 1 - n_{F2}$).

Platforms act to maximize profits, knowing what is the respectively market share inherited from period 1. As already said, there is a priori uncertainty about the fact that the strategy of setting introductory prices can be useful for platforms. Namely, if the inherited number of subscribers is very high, it may be too costly for a platform to attract the small residual number of users (and consequently of firms, because of externalities) who have subscribed to the rival in period 1. Moreover, having a high inherited number of subscribers makes it more difficult for a platform to retain old customers.

Due to these reasonings, inherited market split should be symmetric enough in order for two-direction switching to occur,[2] while a unbalanced market implies that

[1] As it will be explained afterwards, it cannot exist any situation in which switching does not occur at all, i.e. $x_2^B < n_{E1} < x_2^A$. In the equilibrium, at least in one direction, some end-users are going to change platform.

[2] In the analysis of their two periods model of BBPD in a one-sided market, Fudenberg and Tirole (2000) use exactly this assumption to solve backward the model (see page 639: "We will show that, provided that $|\theta^*|$ is not too large, the second-period equilibrium has this form: Both firms

switching occurs from the "strong" to the "weak" platform.

In what follows we directly investigate under which conditions on first period equilibrium two-direction (TDS) and/or one-direction (ODS) switching may arise at equilibrium.

2.3.1 Two Directions Switching (TDS)

In this section we analyze the case of two-direction switching (TDS), i.e. the inherited market split is located in such a way that x_2^A can be actually higher as well as x_2^B lower than n_{E1}. As illustrated in figure 2.1, the users' side would be partitioned in four sub-segments : segment of old consumers who remain in the network A, consumers who switch from A to B, consumers who switch from B to A and old consumers who remain in the network A.

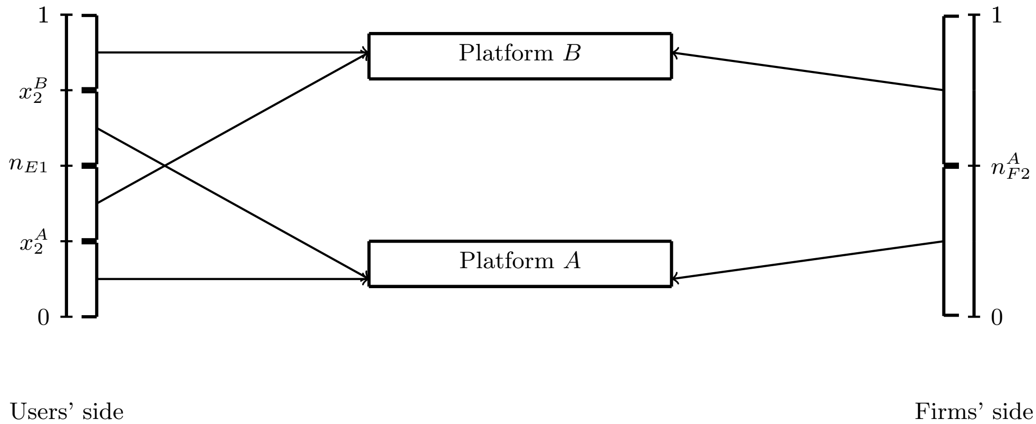

Figure 2.1: Two-directions Switching.

Formally, we analyze the case in which the number of switchers is positive for both platforms, i.e. $n_{E2}^{AA} = x_2^A$ and $n_{E2}^{AB} = x_2^B - n_{E1}$ (and symmetrically $n_{E2}^{BB} = 1 - x_2^B$ and $n_{E2}^{AB} = n_{E1} - x_2^A$). Plugging into equation (2.8) and putting together with (2.5) and (2.6), we get

$$\begin{aligned} x_2^A = & \tfrac{t^2-\alpha_E\alpha_F}{kt}\left(p_{E2}^{BA} - p_{E2}^{AA}\right) + \tfrac{\alpha_E\alpha_F}{kt}\left(p_{E2}^{BB} - p_{E2}^{AB}\right) \\ & + \tfrac{\alpha_E}{k}\left(p_{F2}^{B} - p_{F2}^{A}\right) + \tfrac{t^2-\alpha_E\alpha_F(1+2n_{E1})}{k} \end{aligned} \tag{2.9}$$

poach some of their rival's first-period customers, so that some consumers do switch providers")

$$x_2^B = \frac{t^2-\alpha_E\alpha_F}{kt}\left(p_{E2}^{BB} - p_{E2}^{AB}\right) + \frac{\alpha_E\alpha_F}{kt}\left(p_{E2}^{BA} - p_{E2}^{AA}\right) \\ + \frac{\alpha_E}{k}\left(p_{F2}^{B} - p_{F2}^{A}\right) + \frac{t^2-\alpha_E\alpha_F(1+2n_{E1})}{k} \tag{2.10}$$

$$n_{F2} = \frac{1}{2} + \frac{\alpha_F t}{k}\left[1 - 2n_{E1}\right] + \frac{\alpha_F}{k}\left(p_{E2}^{BB} + p_{E2}^{BA}\right) \\ - \frac{\alpha_F}{k}\left(p_{E2}^{AA} + p_{E2}^{AB}\right) + \frac{t}{k}\left(p_{F2}^{B} - p_{F2}^{A}\right) \tag{2.11}$$

where $k \equiv 2t^2 - 4\alpha_E\alpha_F > 0$ by the concavity conditions stated in assumption $A3$.

Looking at the negative marginal effect of n_{E1} on x_2^A and on x_2^B, the probability for a platform of retaining old users (and for the rival of attracting new ones) is clealry decreasing in the number of subscribers that this platform inherits from the past.

Moreover, all cut-offs depend on all six prices charged by the two platforms. While the dependence on other side's prices is nothing surprising,[1] slightly more puzzling is that the prices charged by the two platforms to A's (B's) inherited subscribers affect the switching behavior in B's (A's) turf. This is a feedback effect of externalities: since the competition in $j^{'}$'s turf affects the participation of firms on the other side, it indirectly affects the utility of agents in j's turf when they take their subscription decisions.

Since platforms expect TDS to occur, they take into account the thresholds in (2.9), (2.10), (2.11), the demand for platform A is composed by three segments. The first one is referred to users with locations going from 0 to x_2^A and the price charged is p_{E2}^{AA}. The second is referred to users' locations in the interval with length $x_2^B - n_{E1}$ and the price charged is p_{E2}^{AB}. The last group is given by firms located in the interval $[0, n_{F2}]$, which pay price p_{F2}^A. Thus, platform A solve the following maximization problem:

$$\max_{p_{E2}^{AA}, p_{E2}^{AB}, p_{F2}^{A}} p_{E2}^{AA} x_2^A + p_{E2}^{AB}\left(x_2^B - n_{E1}\right) + p_{F2}^{A} n_{F2}^{A}$$

Using the first order conditions of the maximization problem, we obtain the best response function of platform A, represented by prices $p_{E2}^{AA}, p_{E2}^{AB}, p_{F2}^{A}$ in function of prices charged by the rival platform, which are relegated to Appendix 3.6.2.

[1] The first implication prices of the presence of cross group externalities is that customers care not only about price and location but also about the number of agents joining in the other side. The pricing rule takes it into account.

Solving the system of the best responses, we obtain the following equilibrium prices:

$$\begin{array}{ll} p_{F2}^{A\,*} = & t-\alpha_E+\frac{t^2(2n_{E1}-1)(\alpha_E-\alpha_F)}{\Omega} \\ p_{F2}^{B\,*} = & t-\alpha_E-\frac{t^2(2n_{E1}-1)(\alpha_E-\alpha_F)}{\Omega} \\ p_{E2}^{AA*} = & \frac{5}{12}t-\alpha_F+\frac{1}{2}tn_{E1}+\frac{3t(2n_{E1}-1)\Psi}{4\Omega} \\ p_{E2}^{AB*} = & \frac{13}{12}t-\alpha_F-\frac{3}{2}tn_{E1}+\frac{3t(2n_{E1}-1)\Psi}{4\Omega} \\ p_{E2}^{BA*} = & -\frac{5}{12}t-\alpha_F+\frac{3}{2}tn_{E1}-\frac{3t(2n_{E1}-1)\Psi}{4\Omega} \\ p_{E2}^{BB*} = & \frac{11}{12}t-\alpha_F-\frac{1}{2}tn_{E1}-\frac{3t(2n_{E1}-1)\Psi}{4\Omega} \end{array} \tag{2.12}$$

Where $\Omega \equiv 9t^2-2\left(2\alpha_E+\alpha_F\right)\left(\alpha_E+2\alpha_F\right)>0$, $\Psi=3t^2-2\alpha_E\left(2\alpha_E+\alpha_F\right)>0$ and $\Omega>\Psi$ by concavity condition (Assumption $A3$).

In the analysis of this sub-section, platforms expect TDS to occur. It means that for the prices in (2.12) to be consistent with these expectations, we need that $x_2^A<n_{E1}<x_2^B$. Given the equilibrium prices in (2.12), the indifferent user between switching to B and subscribe again with A turns out to be:

$$x_2^A=\frac{1}{12}+\frac{n_{E1}}{2}+\frac{9t^2(1-2n_{E1})}{12\Omega} \tag{2.13}$$

while the indifferent user between switching to A and subscribe again to B turns out to be:

$$x_2^B=\frac{5}{12}+\frac{n_{E1}}{2}+\frac{9t^2(1-2n_{E1})}{12\Omega} \tag{2.14}$$

As hinted at the beginning of the current section, TDS can be the case only if there is enough symmetry in the inherited number of subscribers: only if a platform has a high enough number of subscribers, the rival can steal some of them. Indeed, the threshold in (2.13) is below n_{E1} if and only if:

$$n_{E1}>\bar{n}\equiv\frac{1}{6}+\frac{t^2}{\Omega+3t^2} \tag{2.15}$$

Thus, if the number of subscribers to A is lower than the threshold above, then only switching towards A can occur at equilibrium (and we move to the ODS case of the next section). For the same reasons, only switching towards B happens whenever $n_{E1}>1-\bar{n}$.

The likelihood of TDS equilibrium to arise depends on the strength of externalities in both sides, through the effect of α_E and α_F have on the term Ω. Since the externality parameters are bounded by 1 (from above) and 0 (from below) n_{E1} always lays on the interval $(\frac{1}{4}, \frac{t^2-1}{4t^3-6})$.

Specifically, the higher the externalities are, the narrower the interval of inherited number of users allowing TDS. Indeed, since the term Ω is clearly decreasing in both α_E and α_F, $\bar{n}$ moves up as externalities increase, meaning that the presence of externalities reduces the actual possibilities of two-direction switching compared to the one-sided market case. In particular, in the limit case in which externalities are equal to 0 in both sides, we come back to the same possibilities of switching of one-sided market.[1]

In proposition 1, the conditions for being in the TDS case are summarized in the following proposition.

Proposition 1. *(TDS) Consider two symmetric two-sided platforms competing along a unit Hotelling segment for firms on one side and users on the other. Suppose that n_{E1} users have already subscribed to platform A in the past, then at equilibrium:*

1. *TDS may occur if and only if the number of users who have subscribed to both platforms lays on the interval $(\bar{n}, 1-\bar{n})$.*

2. *The presence of externalities reduces the length of the interval of inherited market split compatible with TDS compared with the case of a one-sided market.*

As a matter of fact, proposition 1 implies that when we move from the interval in point 1 (i.e. when the inherited number of subscribers is not symmetric enough), the profit functions considered so far are not coherent with what would happen at equilibrium. Indeed for the prices in equation (2.12) to be an equilibrium, platforms should believe bi-directional switching to occur. However, when n_{E1} is too close to one of the two end-points platforms should believe that bi-directional switching is not a possible outcome. These expectations entail that they maximize a different profit function, as we are going to see in the next sub-section.

[1] In an unpublished paper Gehrig et al. (2006) provide an analysis of the BBPD with inherited market shares and finding how this $\bar{n}$ is equal to $\frac{1}{4}$ in a one-sided market. They use this cut-off in order to switch from the case of weak dominance to the one of strong dominance of the purchase history of consumers, which coincides with a passage from TDS to a ODS towards the *dominated* firm.

Once we have ensured under which conditions TDS arises at equilibrium, we can provide other features of the equlibrium users' and firms' participation and of prices. An important and slightly surprising result is that, whenever a platform starts period 2 with a relatively small number of subscribers, then it will attract more than the half of the subscribers in the second period. Roughly speaking, it overturns the first period result becoming the most present platform in users' side.

To see why, consider the case in which n_{E1} is slightly below $\frac{1}{2}$. In this case, two-direction switching occurs so that A loses some past subscribers and gains some others. According to (2.13) and (2.14), its new total number of subscribers will be $n_{E2}^A = x_2^A + x_2^B - n_{E1} = 1/2 + \left(9t^2(1-2n_{E1})\right)/6\Omega > 1/2$ as long as $n_{E1} < 1/2$.

Similarly, the number of firms joining platform A depends as well on the number of previous subscribers. In particular, the number of firms joining a given platform is decreasing in the number of previous subscribers has from the past. Indeed, consider as an example platform A number of firms joining at equilibrium, n_{F2}^A (notice that the number of firms joining network B are $1 - n_{F2}^A$):

$$n_{F2}^A = \frac{1}{2} + \frac{t(1-2n_{E1})(2\alpha_E + \alpha_F)}{2\Omega} \tag{2.16}$$

The number of firms joining platform A is exactly $\frac{1}{2}$ when the initial number of subscribers is perfectly split. When instead, the initial number of subscribers from period is unbalanced towards platform A, the number of firms joining platform A in period 2 will be lower.

This is an indirect effect of externalities: a high number of previous subscribers makes it less likely to retain loyal subscribers as well as to attract new of them. Moreover, the number of users that switch to the rival is not compensated by the number of new users attracted. Since each user carries an externality to firms' side, the number of firms tends to decrease as the number of previous users increases.

For what concerns equilibrium prices, we should distinguish between firms, old users and switchers. In the firms' side, the price charged by each platform may depend either positively or negatively on the number of users who already subscribe to a given platform. This result is clearer once we understand what is going on in the users' side pricing and participation behavior. It depends on whether users are the group exhibiting stronger or weaker externalities.

Prices charged to old users depend positively on the inherited market share while prices for switchers go exactly to the opposite direction. Thus we can find the minimal market share of period 1 for which a given platform offers a discounted price to new customers. Below this threshold the price will be lower for old customers.

Consider platform A's optimal equilibrium prices in users' side, p_{E2}^{AA} and p_{E2}^{AB}. Discounted prices to rival's customers are offered as long as $p_{E2}^{AA} > p_{E2}^{AB}$ or simply if $n_{E1} > \frac{1}{3}$. Thus platform A finds it optimal to offer two different prices to old and new users, with a discount for the latter, only if his inherited market share is at least $\frac{1}{3}$. The opposite argument holds when we are below $\frac{1}{3}$ and symmetrically this cut-off value turns out to be $\frac{2}{3}$ for platform B. The results obtained so far are summarized in the following lemma.

Lemma 2. *(Price behavior and switching behavior under TDS) Consider two symmetric two-sided platforms competing along a unit Hotelling segment for firms on one side and users on the other. Suppose that TDS occurs according to proposition 1, then at equilibrium:*

1. *the price charged to the firms' side may be decreasing or increasing in n_{E1}.*
2. *if the number of users who subscribed in the past is relatively unbalanced towards a platform ($n_{E1} \in \left\{0, \frac{1}{3}\right)$ or $n_{E1} \in \left(\frac{2}{3}, 1\right)$), then the platform with a low number of old customers rewards loyalty while the rival offers discounted prices to new ones.*
3. *if the number of users who have subscribed to both platforms is close enough to the half of the whole population($n_{E1} \in \left(\frac{1}{3}, \frac{2}{3}\right)$), then both platforms offer discounted prices to rival previous users.*
4. *Inherited market leadership is overturned in both sides.*

This result is the same in the one-sided market case of Fudenberg and Tirole (2000) and simply comes from the fact that when a platform is dominant in the users' side, it should charge rivals users with a very low price to be attractive, as these agents are very far away from its location. Oppositely, the "weak" platform is more worried about being able to retain old users knows that many of the rival's previous subscribers have a relative preference towards it. According to that, this weak platform rewards loyalty and charges higher prices to new users.

To complete this section, equilibrium profits may be potentially different between platforms. Which is the platform that receives the higher profit depends on the inherited number of subscribers, which affects switching in users side and, in turn, number of firms and total profits. We report below time 2's equilibrium profits when n_{E1} is kept as given:

$$\begin{aligned}\pi_2^A(n_{E1}) = {} & \frac{90t^3(1+2(n_{E1}-1)n_{E1})-18(t-\alpha_E-\alpha_F)(2\alpha_E+\alpha_F)(\alpha_E+2\alpha_F)}{18\Omega} \\ & + \frac{t\left[81t^2+18t((n_{E1}-1)\alpha_E)+(n_{E1}-5)\alpha_F\right]}{18\Omega} \\ & - \frac{(2\alpha_E+\alpha_F)\left(\alpha_E\left(22+36n_{E1}^2-42n_{E1}\right)\right)+\alpha_F\left(35-66n_{E1}+72n_{E1}^2\right)\big)}{18\Omega}\end{aligned} \tag{2.17}$$

$$\begin{aligned}\pi_2^B(n_{E1}) = {} & \frac{90t^3(1+2(n_{E1}-1)n_{E1})-18(t-\alpha_E-\alpha_F)(2\alpha_E+\alpha_F)(\alpha_E+2\alpha_F)}{18\Omega} \\ & + \frac{t\left[81t^2+18t((n_{E1}-5)\alpha_E)+(n_{E1}-4)\alpha_F\right]}{18\Omega} \\ & - \frac{(2\alpha_E+\alpha_F)\left(\alpha_E\left(16+18n_{E1}^2-30n_{E1}\right)\right)+\alpha_F\left(41-78n_{E1}+72n_{E1}^2\right)\big)}{18\Omega}\end{aligned} \tag{2.18}$$

2.3.2 One-direction switching (ODS)

Let now consider the case in which n_{E1} is relatively close to 0, meaning that most of the users have subscribed to platform B in the past. The competition for the residual number of subscribers is very unbalanced in favor of platform A, since these users are very close to its location. It means that even if platform B was very aggressive in pricing A's previous subscribers, it would be unlikely to have switching from A to B.

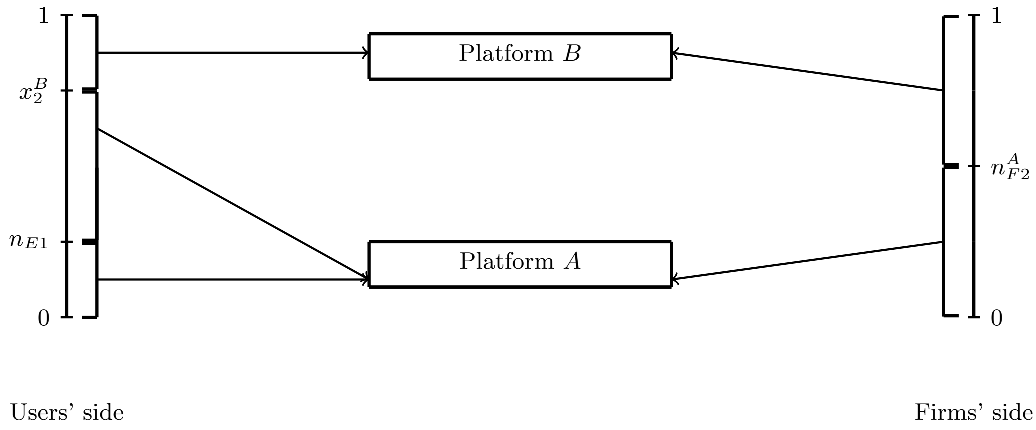

Figure 2.2: Switching only to A.

Suppose that this is the case as showed in Figure 2, so that only switching to A may occur. It means that n_{E1} is so low that $x_2^A \geq n_{E1}$, i.e. $n_{E2}^{AA} = n_{E1}$ and $n_{E2}^{AB} = 0$. Thus the sum $n_2^{AA} + n_2^{AB}$ to plug into equation (2.8) reduces to x_2^B and we end up with the following thresholds.

$$x_2^B = \frac{1}{2} + \frac{t(p_{E2}^{BB} - p_{E2}^{AB}) + \alpha_E(p_{F2}^B - p_{F2}^A)}{\Omega} \tag{2.19}$$

$$n_{F2} = \frac{1}{2} + \frac{\alpha_F(p_{E2}^{BB} - p_{E2}^{AB}) + t(p_{F2}^B - p_{F2}^A)}{\Omega} \tag{2.20}$$

$$x_2^A = \frac{1}{2} - \frac{\alpha_E}{2t} + \frac{p_{E2}^{BA} - p_{E2}^{AA}}{2t} + \frac{\alpha_E\alpha_F(p_{E2}^{BB} - p_{E2}^{AB}) + \alpha_E t(p_{F2}^B - p_{F2}^A)}{t\Omega} \tag{2.21}$$

where $\Omega \equiv 2(t^2 - \alpha_E\alpha_F)$

The first thing to notice is how x_2^B and n_{F2} do not depend on the prices charged to the A's inherited users, i.e. p_{E2}^{AA} and p_{E2}^{BA}. This is because these prices have no effect on the behavior of any user who subscribed to platform A in time 1 (there is no switching to B): we are assuming (imposing) that n_{E1} is such that the competition for A's previous subscribers cannot occur, since A will keep all of them.

The just mentioned inertia of A's customers implies that firms on the other side are not interested in those prices (since the participation of A's previous users remains the same) and consequently no feedback effect can arise on the other users.

Platform A sets prices for old and new users, knowing that no user will switch to the rival platform, i.e. x_2^A is not "well located" as in Figure 1 but it is represented by Figure 2. Formally, the platforms' problems become constrained maximization problem with $x_2^B \geq n_{E1}$ and $x_2^A \geq n_{E1}$, which give the following Lagrangian functions:

$$\mathcal{L}^A = p_{E2}^{AA}n_{E1} + p_{E2}^{AB}(x_2^B - n_{E1}) + p_{F2}^A n_{F2}^A + \lambda_1\left[x_2^B - n_{E1}\right] + \lambda_2\left[x_2^A - n_{E1}\right]$$

for platform A and

$$\mathcal{L}^B = p_{E2}^{BB}(1 - x_2^B) + p_{F2}^B n_{F2}^B + \gamma_1\left[x_2^B - n_{E1}\right] + \gamma_2\left[x_2^A - n_{E1}\right]$$

for platform B. In proposition 3, the conditions for being in the ODS case are summarized.

Proposition 3. *(ODS) Consider two symmetric two-sided platforms competing along a unit Hotelling segment for firms on one side and users on the other. Suppose that n_{E1} users have already subscribed to platform A in the past, then at equilibrium:*

1. *ODS to may occur if and only if the number of users who have subscribed to platform A lays on the interval $(0, \bar{\bar{n}})$*
2. *The presence of externalities reduces the length of the interval of inherited market split compatible with TDS compared with the case of a one-sided market.*

Thus, as already discussed, we can conclude that one-direction switching to A is possible only if the market partition inherited from the first period is unbalanced enough towards platform B. Moreover, we find that the minimal market share of A needed to have switching towards B is lower than the the maximal market share platform A can enjoy so to avoid switching to B. Indeed:

$$\bar{\bar{n}} - \bar{n} = \frac{t^2}{6t^2 - (2\alpha_E + \alpha_F)(\alpha_E + 2\alpha_F)} > 0 \tag{2.22}$$

2.4 Period One Endogenous.

This section is devoted to the analysis of the first period decisions. Specifically, we consider the second period equilibrium and we endogenize the number of subscribers n_{E1}, which depends on price competition in time 1.

We assume customers in both sides to be myopic. An agent is said to be myopic if, when taking decisions in period t, he only looks at that period outcomes, regardless the effects on time $t + k$'s utility. In our setting, a myopic customer simply decides which platform to join in time 1 according to the utility he gets in time 1, without taking into account that tomorrow he could switch to the rival, possibly enjoying a discounted price.

Following the notation of the second period game, we denote by n_{E1}(respectively n_{F1}) the user (resp. firm) indifferent between joining platform A and joining platform B. This thresholds are defined by:

$$n_{E1} = \frac{1}{2} + \frac{\alpha_E}{2t}\left(n_{F1}^A - n_{F1}^B\right) + \frac{1}{2t}(p_{E1}^B - p_{E1}^A) \tag{2.23}$$

$$n_{F1} = \frac{1}{2} + \frac{\alpha_F}{2t}\left(n_{E1}^A - n_{E1}^B\right) + \frac{1}{2t}\left(p_{F1}^B - p_{F1}^A\right) \tag{2.24}$$

Because of assumption $A1$, $n_{E1}^A = n_{E1}$, $n_{E1}^B = 1 - n_{E1}$ and $n_{F1}^A = n_{F1}$, $n_{F1}^B = 1 - n_{F1}$ are the total numbers of customers joining each platform in side E and F respectively. Putting together (2.23) and (2.24), we obtain the number of customers in each side depending only on prices:

$$n_{E1} = \frac{1}{2} + \frac{\alpha_E \left(p_{F1}^B - p_{F1}^A\right) + t(p_{E1}^B - p_{E1}^A)}{t^2 - \alpha_E \alpha_F} \tag{2.25}$$

$$n_{F1} = \frac{1}{2} + \frac{\alpha_F \left(p_{E1}^B - p_{E1}^A\right) + t(p_{F1}^B - p_{F1}^A)}{t^2 - \alpha_E \alpha_F} \tag{2.26}$$

As said before, first period prices chosen by platforms have an effect not only on current profits but also on second period profits, since the market share of period 1 determines whether platforms choose to offer discounted prices to rivals' previous subscribers as well as whether switching may actually occur. Indeed, having a high number of previous subscribers today reduces the possibilities both to steal customers from the rival and to retain old customers overcoming the poaching attempted by the rival.

We consider only the maximization problem of platform A, since the problem is symmetric for B. Platform A sets prices for firms and users in order to maximize inter-temporal profits. Formally:

$$\max_{p_{E1}, p_{F1}} p_{E1} n_{E1} + p_{F1} n_{F1} + \delta \pi_2^A (n_{E1}(p_{E1}, p_{F1}, q_{E1}, q_{F1}))$$

From the first order conditions of this problem, we simply obtain the following first period equilibrium prices.

$$p_{E1}^A = p_{E1}^B = t - \alpha_F + \delta \frac{t(3t - 2\alpha_E - \alpha_F)(\alpha_E - \alpha_F)}{3\Omega} \tag{2.27}$$

$$p_{F1}^A = p_{F1}^B = t - \alpha_E \tag{2.28}$$

Because of the symmetry, in each side the price charged by both platforms turns out to be the same at equilibrium. Firms pay the same price that they would have paid if the price in users' side had been uniform in the second period. In users' side, the effect of BBPD is captured by the third term in (2.27). Comments on these results are provided in the following section.

Looking to the segments of the demands, since equilibrium prices are equal for both platforms, we obtain a perfectly split market in both sides, i.e. $n_{E1} = n_{F1} = \frac{1}{2}$.

Bringing this result to period two equilibrium, both platforms charge lower prices for new users and bi-directional switching occurs. Indeed, substituting $n_{E1} = \frac{1}{2}$ in the second period prices, platforms converge exactly to the following equilibrium prices:

$$\begin{aligned} p_{E2}^{AA} &= p_{E2}^{BB} = \tfrac{2t}{3} - \alpha_F \\ p_{E2}^{AB} &= p_{E2}^{BA} = \tfrac{t}{3} - \alpha_F \\ p_{F2}^{A} &= p_{F2}^{B} = t - \alpha_E \end{aligned} \tag{2.29}$$

These prices imply bi-directional switching and, since both platforms charge exactly the same prices, the number of users switching from A to B is the same as the number of switchers from B to A. In particular, users laying on the interval $\left(\frac{1}{2}, \frac{2}{3}\right)$ switch from platform B to platform A and agents in $\left(\frac{1}{3}, \frac{1}{2}\right)$ switch towards the opposite direction.

Therefore $x_2^A = \frac{1}{3}$ and $x_2^B = \frac{2}{3}$. In firms' side, nothing changes: since the total number of agents joining each platform in time 2 is still $\frac{1}{2}$, no switching in this side may happen and so $n_{F2}^A = n_{F2}^B = \frac{1}{2}$. Finally, the inter-temporal equilibrium profits for both firms are given by:

$$\Pi^A = \Pi^B = \Pi = \frac{9(2t-\alpha_E-\alpha_F)\Omega+(45(t)^2t-18(t-\alpha_E-\alpha_F)(2\alpha_E+\alpha_F)(\alpha_E+2\alpha_F)}{18\Omega} + \frac{t(81(t)^2-18t(4\alpha_E+5\alpha_F)-(2\alpha_E+\alpha_F)(13\alpha_E+17\alpha_F)))\delta)}{18\Omega} \tag{2.30}$$

2.5 Discussion

The main purpose of this paper was to understand which are the effects of the combination of cross-group externalities with the implementation of a within-group price discrimination strategy on competition and platforms' profits. In particular, in a two period model in which second period strategies differ from and depend on first period ones, both ex-ante and ex-post competition may be affected because of the interplay between externalities and price strategies.

Indeed, externalities affect ex-ante competition because prices should take into account that bringing one firm (or one user) in the platform means also bringing some users (firms) and, consequently, they affect the possibility to poach users tomorrow (ex-post competition).

On the other hand, discrimination between old and new users affects ex-post competition because platforms compete fiercely to steal rival's users and to retain their own. In turn, when platforms compete ex-ante they take into account the possible poaching tomorrow.

The way to investigate about these effects is to compare the results with the benchmark case in which platforms do not (or cannot) offer any differentiated price to users according to the past purchases. If it were the case, since platforms do not have any information about past purchase behavior, they cannot distinguish between old and new users. Thus, they can only engage across sides but not within-side price discrimination. It means that $p_{E2}^{AA} = p_{E2}^{AB} = p_{E2}^{A}$ and $p_{E2}^{BB} = p_{E2}^{BA} = p_{E2}^{B}$.

The final result is that we have exactly the same Hotelling competition game played twice, so that prices chosen in time 1 are kept in time 2. Since platforms have no more information in time 2, prices set in time 1 are optimal also in time 2, so that nothing changes at equilibrium. Moreover, equilibrium prices are the same for both platforms because of the symmetry of the model. Solving the maximization problem of the function defined in (2.2) choosing prices, equilibrium prices are indeed $\bar{p}_E = t - \alpha_F$ for end-users and $\bar{p}_F = t - \alpha_E$ for firms, with a consequent level of profits:

$$\bar{\Pi}^A = \bar{\Pi}^B = \bar{\Pi} = [2t - (\alpha_E + \alpha_F)] \frac{(1+\delta)}{2} \tag{2.31}$$

Equilibrium prices simply reflect the presence of externalities. As an example, when a user subscribes for the service offered by the platform, he is not only "bringing" himself to the platform but also a number of firms, which join the platform because of his presence. In particular, each user who joins the platform carries α_F firms so that he is rewarded for that. Thus, a profit maximizer platform may well set a negative price on users' side and recoup the loss made in this side by charging a high price to firms, which willingness to pay is high when a lot of users subscribe. Hereafter, this case is used as a benchmark because it allows to see which are the effects of the the low introductory subscription fees charged by the platforms on equilibrium prices and profits.

The results are summarized in the following proposition and we comment them with some intuitions in the course of this section.

Proposition 4. *Consider the case in which 2 symmetric two-sided platforms compete in a two-period Hotelling segment for users on one side and firms on the other side. Suppose that two regimes can arise: within-group uniform price regime and BBPD regime. Under assumptions A1, A2 and A3, if the customers are myopic, then:*

1. *first and second period prices in firms' side are the same under both regimes.*
2. *second period prices for users are lower under the BBPD regime.*
3. *Depending from the externalities, first period prices for users are either lower or higher under the BBPD regime.*
4. *inter-temporal profits are unambiguously lower under the BBPD regime*

Proof. See Appendix 3.6.4. ■

To understand what is going on in the model, it is worth to spend a few words on the pricing rule in two sided markets. As already hinted in section 1.3, cross-group externalities involve price discrimination among sides because each agent on one side is rewarded according to the number of agents on the other side he indirectly attracts by himself joining a platform. This reward is simply given by the externality parameter of the other side. Indeed, α_F (respectively α_E) represents the number of firms (users) that follow the joining decision of a user (firm). Because of that, a user (firm) will pay a price lower than the transportation cost by an amount α_F (α_E).

The main conclusion is that the price rule followed by the platforms is such that the low value group (the side exhibiting lower α) is a loss leader or break even segment. This group is subsidized (or at least it enjoys a lower price) in order to attract the high value group (the side with α relatively high), which becomes the profit making segment for platforms. The key to compete in these markets is to charge one group with a very low price and recoup the losses by charging a high price to the side more interested in the interaction. The first quite intuitive result is that within-group price discrimination in users' side does not affect equilibrium prices charged to firms, both in time one and two.[1] In time one, the result is quite

[1] This result holds once we assume that asymmetric equilibria in the first period do not arise. We follow the idea of Fudenberg and Tirole (2000), who consider only the cases in which the market is symmetric enough in their backward reasoning.

obvious, since users are equally split between the two platforms. Thus, the price of both platforms should be the same as if we were in the benchmark model, since the strategy used is the same and the externality effect remains the same (users split between platforms exactly in number $\frac{1}{2}$ for each).

Moreover, the outcome is not so different in the second period. The number of users who switch from A to B is exactly the same as the number of users moving towards the opposite direction, keeping the total number of users joining each platform equal. Intuitively, the identity of users subscribing to a given platform changes, but the total number (what matters when pricing firms) remains the same. For these simple reasons, prices in firms' side are the same in both regimes and reflect the general two-sided markets' price rule just described above.

More interesting and puzzling are the effects on competition in the users' side. In this side, ex-ante and ex-post competition are basically driven by two different effects, the *poaching* effect and the *externality* effect.

(i) Poaching. With poaching effect we refer to the fact that platforms compete fiercely in the second period, lowering prices to steal consumers from the rival's inherited turf and/or to retain old consumers in their own turf. This strategy has a clear ex-post effect, while the impact on ex-ante competition is ambiguous.

Ex-post competition is very strong, since the incentives to steal some users to the rival as well as the fear to lose some others make prices go down, both for new and old users. This effect on prices and then competition is simply measured by the differences $t - \frac{t}{3}$ for switchers and $t - \frac{2t}{3}$ for loyalists that we can observe comparing $\bar{p}_E$ with prices for users in (2.29). Since the level of prices is lower both for loyalists and switchers (and equal in the firms' side) under within-group price discrimination and the total number of agents in each side remains the same, second period profits will be lower.

As stated above, the effect of this strategy on ex-ante competition is ambiguous. Indeed, being aggressive in the first period price competition entails two different effects. On one hand, if a platform is aggressive in pricing users, then it can enjoy a high number of subscribers and consequently either attract a relatively high number of firms or charge them more.

On the other hand, this approach has some drawbacks: it reduces the likelihood both to attract new users and retain old users tomorrow. In particular, from the analysis of the second period switching and price behavior, we have an overturn in the relative advantage in the number of subscribers. This negative effect on future profits makes the competition ex-ante less strong.

Which one of the two forces on ex-ante competition prevails depends crucially on the externalities of each side of the market, as we explain in point *(ii)*.

(ii) Externality. With the externality effect we refer to the fact that a price cut in one side of the market involves an effect on the number of joiners on the other side. In ex-post competition, it is clear that externalities have exactly the same effects both in the BBPD regime and the benchmark case. They only reduce the price for users by an amount equal to the externality parameter of the firms' side.

The effect on ex-ante competition depends on whether the users are the low or the high value group. Looking at the equilibrium prices for users in (2.27) and comparing them with the benchmark case, we can conclude that if users are the high value group, first period prices are higher under within-group discrimination regime and then ex-ante competition is relaxed. On the other hand, when users are the low value group, competition is intensified in the first period.

One question spontaneously arises: why does the optimal first period price differ in the two regimes? The key is the balance between pros and cons of being aggressive in the first period competition discussed in point *(i)*. Suppose that one platform set the price as in the benchmark case. If users care relatively much about the interaction with firms (more than how much firms care), the rival would have an incentive to set a higher price because the gain tomorrow in future subscriptions is higher than the loss today. On the other hand, if firms are the high value group, platforms tend to be more aggressive.

The intuition behind is strongly linked to the concept of subsidizing/subsidized segment typical of a two sided market: suppose that users are the high value group, i.e. $\alpha_E > \alpha_F$. In this case, users are attracted basically offering price cuts to firms in order to have a critic mass of them and increase the willingness to pay of users. Then, the basic strategy for a platform is to charge more users than firms.

Moreover, the strategy of within-group price discrimination in time 2 pushes platforms to charge users even more. The most part of platforms' profits are made on users. In this side, platforms have another possibility to attract them in the future.

Thus, externalities change the competition in the first period when platforms use BBPD in users' side. Suppose that one platform sets the benchmark price in $\bar{p}_E$ in users' side. Then, the rival best response turns out to be: *(i)* set a higher price for users (which do not impose a big loss on firms' participation) with a lower number of subscribers but higher margins, *(ii)* set different prices and overturn the result in the second period.

On the other hand, if users are the low value group, the main aim of platforms is to attract a big number of users, subsidizing them and make profits on the firms' side. Here, the incentive to attract a very high number of users is stronger than before, since attracting a high number of users also means to be very strong in the firms segment. Thus, price competition between platforms becomes aggressive.

Putting together these conclusions about prices, we can infer the effects on platforms' profits. Ex-post competition clearly increases and profits decrease, since prices go down for all users and nothing changes in the firms' side. This is the same result that is common in the literature of BBPD, e.g. Fudenberg and Tirole (2000). Ex-ante, two different instances may arise.

If price discrimination is used in the high value group, then competition is strengthened and first period profits are higher when platforms discriminate prices compared to the one that would have been attained under a within-group uniform price.

When users are very interested in reaching firms, cross-group externalities emphasize the negative effects on inter-temporal profits of BBPD, decreasing also first period profits. If platforms discriminate among agents belonging to the low value group instead, cross-group externalities mitigate the negative effects on inter-temporal profits of BBPD, relaxing the first period competition. Although, even if the first period competition is relaxed, the negative total effect on profits is confirmed.

2.6 Conclusions

We have provided a model of two-sided platforms which compete in two periods for firms on one side and users on the other side of the market. We allow platforms to discriminate prices among users, according to the fact that BBPD is often used in subscription markets. The main finding of the analysis is that cross-group externalities do involve some effects on prices and competition when platforms discriminate prices in users' side.

First of all, when we consider the second period keeping the first period inherited market share as given, externalities have an effect on the concrete possibility for two-direction switching to occur. In particular we can find that the stronger externalities, the more likely the case of two-direction switching, since the minimal (maximal) market share for two-directions switching to occur depends negatively (positively) on the externalities.

The natural next step of the analysis should be to investigate about under which conditions on parameters switching occurs one-direction towards one of the two platform and to compare these scenarios with the one of two-directions switching. In presence of externalities, the inter-temporal equilibrium found so far may be not unique, i.e. asymmetric equilibria may arise.

In the case analyzed so far, when we consider the two-period model as a whole, prices turn out to be the same for both platforms and both users and firms equally split between the two competing networks.

Along the equilibrium path, platforms face a prisoners' dilemma in the second period. Each one alone has the incentive to offer discounted prices to rival's previous users but if both of them do it, then they are both worse off. The general level of prices goes down under discrimination and each platform steals the same number of agents, leaving platforms with a lower level of profits in the second period.

In the first period, the general level of prices under discrimination may be either below or above the within-group uniform price regime. In other words, the combination of cross-group externalities and within-group price discrimination may either mitigate or intensify ex-ante competition.

Specifically, ex-ante competition is intensified when users are the low value group, mitigated otherwise. In the markets we have in mind, often users represent the the low value group. Medias market is the most telling example: advertisers care a lot about how many readers/viewers subscribe to a newspaper/TV while on the opposite direction the externality is clearly lower. For these reasons, medias make profits on advertisers, while end-users are subsidized. What the model predicts is that BBPD intensifies ex-ante competition, hurting firms and benefiting consumers even more in the first period.

Ambiguous in terms of which side exhibits the strongest externality is the instance of the subscription program offered by Amazon, which provides contents (video and reading) to the subscribers and access to demand for providers. Since subscribers are interested in contents, the effect of an additional provider entering the platform is substantial, because the number of videos and e-books available increases. If the externality of subscribers is higher than the one of providers, according to our model ex-ante competition is relaxed when platforms use within-group BBPD.

The weaknesses of this first attempt to study within-group price discrimination in a multi-period setting define natural future lines of research.

On one hand, in the group of users inherited leadership is always reversed in the second period when the inherited market is not perfectly split. This result is quite surprising in a two-sided market context, in which the early stages of competition are very critical. Indeed, attracting a high number of customers in both groups at the very beginning is often decisive to keep a strong market position in the future.

On the other hand, externalities do not play any role other than the usual "reward" for each side for the agents joining the platforms on the other side, which is nothing but the usual pricing rule in two-sided markets. No any other effect of externalities arises in second period prices, keeping the results of BBPD in one-sided markets.

It is reasonable to believe that seeking for asymmetric equilibria with switchers going only towards one direction may be a way to have different results. In particular, if these kinds of equilibria exist, externalities may well entail different consequences on second period profits.

Another important assumption which the paper is based on is that both firms and users are allowed to join at most one platform (single-homing). In fact, it is common that at least one side decides to bear price and transportation cost twice in order to be present in both platforms.

As pointed out by Armstrong (2006), when agents are mostly interested in the interaction with the other group rather than the product offered by the platforms themselves,[1] they may take the decision to join both platforms in order to meet the other side. The main result is that multi-homing relaxes price competition between platforms, because the multi-homing side exhibits a lower elasticity to price. In our setting, considering multi-homing firms may change the results on platforms' profits, overturning the results of the standard one-sided BBPD literature. The intuition behind is simply that platforms could recover the losses in the users' side by charging higher prices to firms, which are strongly interested in reaching the whole population of users. Accordingly, we may eventually end up with situations in which BBPD in the users' side and the consequent increase in competition in this side is simply financed by firms.

The last point to notice is that the only case of myopic customers is analyzed in our model. As shown by Fudenberg and Tirole (2000) and Villas-Boas (1999) in one-sided markets, if customers are assumed to be forward looking ex-ante competition is relaxed. This result depends on the fact that ex-ante customers' elasticity is reduced because they know that tomorrow they can switch, enjoying a discounted price. In our setting, which already assumes deep rationality, it can be interesting to see how ex-ante competition is affected by the fact that both firms and end-users expect platforms to use within-group price discrimination and take it into account when taking their ex-ante decisions.

[1] Think for example to medias, which offer products (contents) that are basically non-differentiated in the eyes of advertisers.

3

Pricing in Social Networks under Limited Information

3.1 Introduction

Consumers are never perfectly aware about the different purchase options they have. As pointed out by influential papers in the 60s,[1] the access to information about the existence of a product and its characteristics is a crucial point when observing consumers decisions. This lack of information is an issue for consumers as well as for producers, that in turn need their product to be known to sell it. The traditional solution that producers opted for is advertisement in its informative view as in Nelson (1974) which, however, requires heavy sunk investments in exchange for uncertain outcome.

In the modern economy, consumers' and producers' access to information has changed considerably and towards different directions. On the one hand, new technologies have substantially improved the possibility for consumers to acquire information, but nevertheless they are required to have more skills to use the information they have and to understand more and more sophisticated products. Namely, consumers suffer an information overload Zandt (2004) and only some of them actually have the ability to face this complexity. On the other hand, the improved knowl-

[0]This Chapter is part of a joint work with Simone Righi.

[1]Examples of this interest towards information and consumers behavior are Stigler (1961), which main focus is on the search cost that consumers bear to discover prices and Nelson (1970), interested in the difficulties that consumers have to actually evaluate the quality of a product

edge of producers of the social ties among consumers have increased the interest to exploit client's social network in order to generate business.[1] The use of consumers' network is an alternative solution to the informative problem, and it is more effective than advertising as consumers are usually embedded with a considerable component of trust.[2] Moreover, the decrease in costs of communicating to a large number of subject, brought upon by modern technologies (such as social networks, emails, messages), improves the effectiveness of this kind of programs, increasing their profitability.

In wanting to make use of the social network, consumers that are popular and able to acquire information often become the target of companies strategies. In particular, advantageous deals in the form of a reduced price or a gift are proposed to old buyers who support the firm to extend its clients base by convincing others to buy. Old buyers are clearly aware about the existence and characteristics of the product (as they experienced it in the past) and the more popular of them are more likely to be effective in helping firms to enlarge the demand.

This use of network-based pricing is increasingly observable in several markets, taking different shapes. An important example are the online storage services such as iCloud and Dropbox, which offer free storage space to clients that convince their friends to subscribe their services. According to Huston (2010), founder and CEO of Dropbox, their referral program, run in 2009, extended their client basis of 60% and referral was responsible of 35% of daily new signups. Similar is the case of money transfers systems such as Paypal and UWC.[3] In these instances, the enterprise can even decide to give a monetary prizes for each new customer brought in the costumer base through external knowledge. In the same context, also more traditional banks recently began to offer more advantageous conditions (in the form of higher interest rates on the deposit or lowered service's fee) for each new customer that an old client manages to bring into the bank.

This paper is a first attempt at modeling theoretically the strategic decisions of a monopolist choosing to discriminate the price according to the ability of a

[1] This is confirmed by the recent development of companies (Anafore, ReferTo, NextBee) specialized in offering technical and consultancy services for the implementation of referral programs

[2] In an empirical paper of Schmitt et al. (2011) it is well documented that referred customers tend to be both more profitable and more loyal than customers acquired through other channels.

[3] http://www.uwcfs.com/en/faq/other-services/referral-program

consumer to induce others to buy. In our setup, the reduction in price takes the form of a monetary gift offered by the monopolist for each new customer brought into the clients base in a context where the population of potential buyers exhibits heterogeneous search costs. In our framework, the search cost is interpreted as the time and skills that an agent needs to dedicate for the acquirement of information.[1]

We directly investigate the effect of the introduction of the network-based reward on the flow of information when the monopolist has a very limited knowledge of the social network. The monopolist's offer creates some incentives for old consumers to communicate with uninformed peers about the existence of company's product thus reducing their informational problem. The objective of our analysis is twofold. On the one hand, we aim at characterizing the optimal unitary reward chosen by the monopolist and its dependence on the characteristics of the population and the social network structure. In aggregate terms, this reward will entail some implications for the spread of information about a product on a social network. On the other hand, we are interested in the effect that the introduction of this reward have on the general level of prices and profits of the monopolist. To reach this goal, we make comparisons with the case of no reward (or uniform pricing) to analyze the level of prices and profits.

The remaining part of this paper is divided as follow. After a discussing the related literature in Section 3.2, we discuss the mathematical aspects of the model in Section 3.3. Then, in Section 3.4, we solve it and we discuss the results' implications. Finally we conclude in Section 3.5.

3.2 Related literature

It is now well known in economic theory that the solipsistic view of the consumer, which characterized the discipline in the past, can be relaxed considering the consumer as a member of a social group, that influences and is influenced by his behavior through local interactions. Economic theory introduced the concept of network while discussing economic interactions in a variety of fields. As pointed out in the

[1] For example, young people tend to have more time to spend searching for information about technological products or services than old people. Moreover, they own stronger skills to obtain and interpret information about prices and characteristics of these products. In our interpretation, young people would have a lower search cost.

comprehensive analysis of Jackson (2005) networks influences agents economic behaviour in fields such as decentralized financial markets, labour markets, criminal behaviour and spread of information and diseases.

In recent years the attention of industrial economists shifted from the network externalities approach, following the tradition of Katz and Shapiro (1985), to a new focus on the direct study of the effects of social interaction on the behavior of economic agents. In the traditional approach, consumers' own valuation of a good depends on the number of peers consuming the same good. The new tendency is to move the analysis a step further, linking the externality to a subset of neighbors rather than to the population overall. This new tradition is clearly exemplified by Sundararajan (2006) which proposes a model of network adoption where the externalities are local and consumers have incomplete information about adoption complementarities between all other agents. Following the same idea of locality, Banerji and Dutta (2009) find out the possible emergence of local monopolies even if homogenous firms compete only in prices. However, the focus of those papers remains on the study of consumption externalities in the new framework. Our approach is different both with respect assumptions and objectives of the research as we take the existence of a structure of social interactions as given and we study how the latter can be exploited by a monopolist to increase the consumers' base.

More specifically related to our work is the recent strand of the literature dealing with the issue of pricing in social networks, which take directly into account the topology of social relationships. Sääskilahti (2007) studies uniform monopoly pricing introducing network topology in a model of network adoption. The paper demonstrates that taking into account local interactions reduces the traditional network size effect tin the monopolist's ability to extract surplus, thus concluding that rents and total surplus are exaggerated considering only the size of the network. In Ghiglino and Goyal (2010), consumers compare their consumption with that of their neighbors, suffering a negative consumption externality. They characterize prices and allocations and demonstrate that identical consumers located make different consumption decisions when they are located in different positions in the social network (e.g. have different centrality). Bloch and Quérou (2013) study the optimal monopoly pricing in a context in which the producer is able to perfectly identify the network centrality of consumers and chooses a target price for each of them on the base of this variable. Their main result is that, if consumers benefit from neighbor consumption (network externality) then pricing decisions are indifferent to consumer's centrality. However, when the consumers compare their price with those received by their social neighborhood then the producer has incentives to charge higher prices to central nodes. In this manuscript, on the one side we relax the informational requirements of the producer to use network based marketing strategies and, on the other side, we consider a sequential setting instead of simultaneous consumption decision for all consumers.

Similarly, in a setup that typically fits communication markets, Shi (2003) studies the pricing strategy of a monopolist that sells a network good. His main finding is that the strength of network ties can be used as to discriminate prices among consumers. A crucial assumption of this paper is that, two or more clients must consume the network good together in order to enjoy discounted prices proposed by a monopolist, an assumption that we relax completely (thus eliminating the coordination problem involved). Shi's main result is that producer's pricing choices depend on the composition of client's ego network. He proposes discount to clients on communications with strong ties (friends and family) in order to profit (imposing higher prices) from his weaker links. While the setup of Shi (2003) uses the locality of the network only to allow consumers to cooperate (consume together the good) in our setup the locality of network structure has both positive (a more dense network implies stronger incentives for informed consumer to spread information) and negative (more dense networks also intensify the competition for being the person referred by the consumer) effects.

In the setting we propose, the monopolist sets the prices and, after some consumers buy the product, a "bring-a-friend" discount is introduced. The timing of our paper differs from Bloch and Quérou (2013) and is similar to the one proposed by similar papers as Hartline et al. (2008) and Arthur et al. (2009). In their papers, they study the monopoly pricing in an environment in which myopic consumers take their decisions according to the number of people that bought the good in the past. In our proposal, myopic consumers decide whether to buy the product or not without any direct externality from consumption.

The paper is also related to the marketing literature studying referral bonuses. Two papers are worth mentioning from a theoretical point of view: Biyalogorsky et al. (2001) and Kornish and Li (2010). The first one defines a customer as delighted when he is willing to recommend a product; in this setup the reward to optimally enlarge the client basis is positively correlated with the share of delighted consumers. Kornish and Li (2010) focus instead on the impact of referrals on customers evaluations (namely, reservation price) of the product in a setting of asymmetric information in which agents put a value on friend's utility. They find that the higher is the interest on friend's payoff, the higher the optimal referral bonus should be. Both these paper, however, disregard the effects on strategic interaction of social networks among consumer. The empirical literature on the subject is represented by Leskovec et al. (2007), which study the adoption of a referral market strategy by an online retailer and discuss the product categories for which this strategy works better.

3.3 The model

We consider a setup where a monopolist seeks to sell a product to a large, but finite, population $N = \{1, 2, \ldots, i, \ldots, n\}$ of agents. Consumers differ according to their willingness to pay and their search cost. The utility function for an agent i from buying the product at price p is defined as:

$$u_i = r_i - p - s_i \tag{3.1}$$

The reservation price r is distributed according to a c.d.f. G on the support $[0, 1]$, while the search cost s is a binary variable. A proportion $1 - \beta$ of consumers exhibits a low search cost normalized to 0, while the remaining β have a high search cost s_H which, by assumption, is larger than the maximal willingness to pay 1. These latter consumers would never get informed and thus never buy unless they passively receive the information from some external source. The heterogeneity in search costs captures the different consumers' skills to access and use the available informational tools.

Interactions and communication among consumers are restricted by an existing social network structure, which we consider as given. In particular, each agent i has a finite number of neighbors $K_i \subseteq N$ to interact with. The degree k_i (the number of neighbors) is just the cardinality of K_i. We further assume the consumer's social network to be undirected, in the sense that if node i is linked to node j, then j is in turn linked to i. The degree of the agents is distributed according to some p.d.f. $f(k)$, which has to be interpreted as the fraction of agents having k neighbors. In other terms, selecting a random agent from the social network, the probability that she has exactly k neighbors is $f(k)$. This general formulation allows us to provide results for any interaction structure. Moreover, it is possible to substitute $f(k)$ with specific networks and compare results across different topologies.

We consider a two period model where the supply side of the market is constituted by a monopolist which aims at maximizing the sum of inter-temporal profits.[1] Defining $D_1(p)$ as the demand in the first period and assuming a marginal cost normalized to 0, the expected profit obtained charging price p will be given by:

$$\pi_1 = p\mathbb{E}(D_1(p)) \tag{3.2}$$

Adding to the formulation in Equation 3.2, in the second period of our model, we allow the monopolist to offer rewards to old customers through a "bring a friend" program. Namely, the monopolist knows the distribution of the degrees in the social network and, accordingly, offers a gift to the old consumers who inform their friends about the existence of the product and convince them to buy. The rationale of this offer is to eliminate the high search costs that prevent some of the potential consumers from buying. This gift takes the form of a unitary amount b for each referral. Since each new consumer corresponds to one reward b given to some old customer the margin in the second period is given by $(p-b)$. Thus, defining $D_2(p, b)$ as the demand in the second period coming from new consumers, the expected profit π_2 turns out to be:

$$\pi_2 = (p - b)\mathbb{E}(D_2(p, b)) \tag{3.3}$$

The dependence of D_2 on p and b takes into account, on the one hand the willingness to buy of customers given price p (fixed in period one), and on the other hand the probability of getting informed about the product, which in turn depends on the incentives to speak given to customers in D_1 through b. To enjoy rewards, old buyers need to contact their social network which implies a costly investment of a fixed amount C.[2]

[1] Future gains are discounted by a factor δ, normalized to one without loss of generality.

[2] The choice of studying the case of fixed cost has been made in order to capture the idea that

It is important to discuss the informational structure of the model as it constitutes a peculiar feature of our study. Specifically, the information available to agents about the idiosyncratic characteristics of all the others is summarized in the Assumption 5.

Assumption 5. *The distribution of the variables r_i, k_i and s_i are common knowledge and independent from each other. Agents do not possess additional private information.*

Assumption 5 implies that consumers cannot condition their decisions on their local social neighborhood and the monopolist is not able to base his choice upon individual characteristics of consumers.

Our game is played in two periods and it is solved by backward induction. Each time period, in itself, is a sequential game in which the monopolist chooses first and consumers react. In period 1 the monopolist sets a price p (Period 1.A) and consumers, after having observed it, decide whether to purchase the good (Period 1.B). In the second period, the monopolist introduces the reward b (Period 2.A) and the first period buyers decide upon the possibility of contacting their friends (Period 2.B). Given the total investment of old consumers, information about the existence of the product may reach some potential new buyers. If reached, each customer purchases if his reservation price is sufficiently high (Period 2.C).

3.4 Results

We now proceed to solve our model by studying the decisions of the agents, from the last to the first, and assuming that what happened before is taken as given.

Period 2.C - Purchase decisions of uninformed consumers. In the last step, consumers with high search cost decide upon purchase. Some of them may receive the information through old buyers making their search cost drop to zero. We define ρ as the probability for an agent of receiving the information at least once. From the point of view of the single agent ρ is function of the number of social ties he has k and of the number of first period consumers that invest in social network,

the emergence of the online social networks and the use of e-mails tends to make the difference in the number of people contacted negligible in terms of total cost.

which we define as D_1^{Inv}. Indeed, the more friends one person has, the more likely it is that at least one of them decides to invest and to speak with him about the product. Moreover, as the number of investors increases, the odds for each single neighbor to be an investor are higher. Since the agents who did not receive the information are stuck with a high search cost and cannot buy, the second period demand is composed by the fraction of newly informed agents exhibiting reservation price $r_i > p$. Given the degree distribution $f(k)$ and the probability of receiving the information $\rho(k, D_1^{Inv})$ we can derive the new expected demand in the second period as:

$$\mathbb{E}(D_2) = \beta(1 - G(p))\bar{\rho}n \tag{3.4}$$

where $\bar{\rho} = \sum_{k=1}^{n-1} \rho(k, D_1^{Inv}) f(k)$ represents the average probability of receiving the information about the existence of the product and then $\bar{\rho}n$ is the total number of receivers in the population.

2.B - Investment decisions of old buyers. After having observed the reward offered by the monopolist, old buyers take their decision about the investment in the social network considering the expected purchase behaviour of the agents they inform. The two alternatives are either to bear a cost and inform their friends (thus possibly getting rewards) or to give up the benefit enjoying no extra utility. Defining $B(b, k_i)$ as the total number of rewards received by an agent with degree k_i given the unitary reward b, the expect utility of informed agent i is:

$$\mathbb{E}(u_i) = \begin{cases} \mathbb{E}(B(b, k_i))b - C & \text{if } i \text{ invests} \\ 0 & \text{if } i \text{ does not invests.} \end{cases} \tag{3.5}$$

According to Equation 3.5, each agent invests if the amount he expect to receive $\mathbb{E}(B(b, k_i))b$ is bigger than the cost C. While the cost of activating the social network is assumed to be fixed, the expected benefit requires a more precise analysis. Indeed, this amount is composed by two elements: the total number of rewards the informed agent expects to get and the unitary bonus offered by the monopolist for each friend brought in the customers base. The first element, $\mathbb{E}(B(b, k_i))$, is agent specific, as it depends on the number of uninformed people that agent i is actually able to contact. This in turns is clearly an increasing function of his degree k_i. It follows that, for given b, the degree of an agent affects positively also the total amount that this agent expects to receive. Taking as given the degree of agent i, the unitary benefit b affects instead both elements of the total monetary reward. Clearly, as the unitary amount increases, so does the total amount that each agent expects to receive from speaking. Nevertheless, b also increases the incentives to invest for all agents, making the expectations about the total number of rewards $\mathbb{E}(B(b,k_i))$ change downwards due to a crowding effect.

So we assume that:

Assumption 6. *The number of expected rewards $\mathbb{E}(B(b,k_i))$ is decreasing in b for each degree level. Moreover, $\lim_{b\to 0}\frac{\partial E(B)}{\partial b}=0$ and $\lim_{b\to 1}\frac{\partial E(B)}{\partial b}=-1$.*

The assumption about the limit values of the derivative $\frac{\partial E(B)}{\partial b}$ is made in order to avoid the degenerate cases in which the crowding effect created by b is so strong to exceed its positive effect, thus leading to an unrealistic decrease of the total expected benefit of an agent i as b increases.

Given the presence of a fixed cost C, the actual investors will be those for which $\mathbb{E}[B]\,b \geq C$. Since $\mathbb{E}[B]$ is monotonically increasing in k, this implies that there exists some $\underline{k}$ s.t. all agents i with $k_i \geq \underline{k}$ invest. Simply by equating benefits and cost, we find the critical degree:

$$\mathbb{E}\left[B(b,\underline{k})\right] b = C \tag{3.6}$$

Since the LHS of Equation 3.6 is increasing in b and $\underline{k}$ while the RHS is constant then $\underline{k}$ must be decreasing in b to maintain the equality. In economic terms this relationship indicates that offering an increased b creates stronger incentives for informed agents to invest given their degree. The fixed cost plays the opposite role.

Knowing the existence of $\underline{k}$, we can now compute the average probability of receiving the information in the population, which in turns requires the derivation of this probability for each k, namely $\rho(k, D_1^{inv})$. A degree k uninformed agent knows, on average, $(1-\beta)(1-G(p))k$ old - informed - buyers. Among them only the ones with $k \geq \underline{k}$ invest, i.e. a proportion $\sum_{k\geq\underline{k}} f(k)$. In expected terms the probability of receiving the information from each single friend turns out to be equal to the share of investors in the total population $\frac{D_1^{inv}}{n}$. Thus, the probability of receiving the information from at least one among k friends is:

$$\rho(k, D_1^{inv}) = 1 - \left[1 - \frac{D_1^{inv}}{n}\right]^k \tag{3.7}$$

where $D_1^{inv} = n(1-\beta)(1-G(p)) \sum_{k \geq \underline{k}} f(k)$.

Summing over all ks the expression in Equation 3.7 we find explicitly $\bar{\rho}$. This can be plugged in Equation 3.4 obtaining the expected number of new consumers buying the product in period 2.

At equilibrium the number of new consumers at period 2 must be equal to the total number of benefits given away by the monopolist. The latter is simply the number of potential investors given by $n(1-G(p))(1-\beta)$ times the average number of benefits. Rearranging terms of this equilibrium condition:

$$\sum_{k \geq \underline{k}} f(k)\mathbb{E}(B(k,b)) = \frac{\beta}{1-\beta}\bar{\rho} \tag{3.8}$$

A naturally corollary of the equilibrium condition in Equation 3.8 is that the average expected benefit increases in b. In principle b entails two different effects on $\mathbb{E}(B)$. On the one hand it decreases the expected number of benefits obtainable for each degree level, making each term in the sum of average expected benefit lower. On the other hand, it decreases the investment threshold thus increasing the number of elements in the sum. At equilibrium, the second effect always dominates the first. This means that, from an individual point of view, b decreases the expectations about the number of rewards, but at the aggregate level, it increases the total expected number of rewards issued by the monopolist.

The derivations obtained so far, allows us to have a first set of results, regarding the behaviour of agent when monopolist's choices are taken as given, which are summarized in Proposition 7:

Proposition 7. *$\bar{\rho}$ is decreasing in the fixed cost of investment C and in price p while it is increasing in the unitary benefit b chosen by the monopolist.*

The proportion of uninformed agents (β) has an ambiguous effect on the investment threshold $\underline{k}$, depending on the balance between the inverse of the share of informed consumers ($\frac{1}{1-\beta}$) and the elasticity of the spread of information $\bar{\rho}$ on β, while its effect on $\bar{\rho}$ is unambiguously negative.

Proof. See Appendix 3.7.1. ■

Decreasing $\underline{k}$ implies increasing the share of investors. The natural consequence is that, as the number of investors grows, so does the probability of receiving the information for each individual and thus its average. Through this channel and considering how C, b and p affects the incentives to speak the effect on this variables on $\bar{\rho}$ are easily derived. Indeed, since an increase in C or p (respectively a decrease in b) has a negative effect on the incentives to invest, $\underline{k}$ increases in them and, consequently, $\bar{\rho}$ decreases.

β has two opposite effects on the aggregate demand in the second period. Indeed, it increases the number of potential buyers but at the same time it decreases the probability that information reaches each of them. Which of this two effects is the strongest, depends on the reaction of information to a change in β measured by the elasticity. From the single agent point of view, an increase in β leads to change in the expectation on the total mass of benefits to be shared with other investors. The sign of this change depends on the strength of the reaction of $\bar{\rho}$ with respect to β. If the latter is strong enough, then the expectations will be updated downward and with them also the individual expectations of reward. This, in turns, reduces the incentives to invest for each degree level and thus the minimal $\underline{k}$ increases. The opposite is true when the response of information in β is not strong enough, compared with the inverse of the share of informed consumers.

2.A - Monopolist's choice of b. The monopolist, anticipating consumers decisions, faces a tradeoff. On the one hand, offering a bonus clearly reduces the margins that the monopolist can attain on the single new buyer. Indeed, the reward b works as a cost, since for each new buyer, the monopolist gives an amount b to one old buyer. On the other hand, the dimension of the unitary reward has a positive effect on the demand for the good (as it helps reducing the informational problem) as summarized in Lemma 8.

Lemma 8. *The demand faced by the monopolist in the second period is increasing in the unitary reward b.*

Proof. Let compute the sign of $\frac{\partial \mathbb{E}(D_2)}{\partial b}$. Since $\frac{\partial \rho(k)}{\partial b} = \frac{\partial \rho(k)}{\partial \underline{k}} \frac{\partial \underline{k}}{\partial b} > 0$ as proven in

Proposition 7, by simple computation we get:

$$\frac{\partial \mathbb{E}(D_2)}{\partial b} = \beta\,(1 - G(p))\,n \left(\sum_{k=1}^{n} \frac{\partial \rho(k)}{\partial b} f(k) \right) > 0 \tag{3.9}$$

■

Differently from the usual maximization problem, in this model the monopolist maximizes choosing b in a context where the margins are decreasing in this variable and the demand increasing in it:

$$\max_b (p - b)\mathbb{E}(D_2(b)) \tag{3.10}$$

Which yields the following modified Lerner rule:

$$\frac{p - b^*}{b^*} = \frac{1}{\eta_{\bar{\rho},b^*}} \tag{3.11}$$

where $\eta_{\bar{\rho},b^*}$ is the elasticity of the average probability of receiving the information to the unitary bonus computed at the optimal point. From this maximization problem yields the following proposition:

Proposition 9. *For any price charged in the first period, the monopolist always finds it profitable to run the program setting a unitary reward b^*.*

For the comparative statics on b^* two cases arise.

(i) If the optimal b^* lies in a interval with decreasing elasticity of $\bar{\rho}$ with respect to b, then b^* is increasing in the price p, in investment cost C and in the proportion of uninformed agents β.

(ii) If the optimal b^* lies in a interval with increasing elasticity of $\bar{\rho}$ with respect to b, then b^* is decreasing in the price p, in investment cost C and in the proportion of uninformed agents β. Unless:

- *(ii.1) either the elasticity is too small. In which case the results are the same as in (i) for all variables.*
- *(ii.2) or the elasticity is big enough. And then then b^* increases in p.*

Proof. See Appendix 3.7.2 ■

When setting b the monopolist faces a trade-off. Indeed, by increasing the unitary gift he obtains two effects. The demand increases and in this additional demand the monopolist makes a margin. The total additional profit represents the *marginal gain* of increasing b. At the same time, an increase in b decreases the margin of an equivalent amount for each new agent which is expected to buy. This reduction of the total profit represents the *marginal loss* of increasing b. The optimal b describes a situation in which these two opposite forces perfectly offset. In this setting the effects on the demand only take the form of effects in $\bar{\rho}$ so that the two terms can be use interchangeably.

The parameters of the model change the equilibrium situation by affecting the incentives. In particular, while C and β only affect the demand faced by the monopolist, p also raises the margins.

As we have seen, an increase in C or β has a negative effect on the spread of information in the network and thus the average probability of getting informed for uninformed people. This means that, for any p and b set by the monopolist, the number of people that are going to buy is lower with higher values of these parameters. This effect is certain, while the effect that this has on the elasticity in the Lerner rule depends on the relationship between the latter and the reward.

To any couple of values of b and $\bar{\rho}$ corresponds one elasticity. When we refer to the case of increasing elasticity we are considering that, for high levels of $\bar{\rho}$ and b together (since $\bar{\rho}$ is increasing in b) the elasticity is higher. Thus, the second order effects of b on $\bar{\rho}$ (increasing or decreasing elasticity) are going to determine the effect of the parameters on the elasticity.

Following this reasoning, if the elasticity is decreasing, then an increase of C or β makes the elasticity larger. The opposite is true when the elasticity is increasing.

Assume that the optimal b is such that C or β increase the elasticity of information to b. This increase depends on the reduced incentive that decreases the information and thus the demand. In this context the marginal loss is reduced unambiguously more than the marginal gain. Consequently the monopolist is profitable to increase b, up to the point in which the two are again equal.

More interesting is the case in which C or β decreases the elasticity of information to b. This may be due either to a decrease of the response of the information to an increase in b or to an increase in the demand level. Since C reduces the incentives to speak and the spread of information, the second effect is excluded allowing us to conclude that the marginal effect of b on $\bar{\rho}$ becomes lower. Consequently the marginal gain shrinks as margins are fixed. Moreover, its reduction is stronger than the reduction in demand in order to have the desired effect on elasticity. Since the reduction in demand represent the marginal loss of increasing b, in general we can conclude that the latter becomes bigger than the marginal gain inducing the monopolist to decrease the reward.

The only limit case in which this is not true is the one in which even if the response of the demand to an increase in b is very low, the margins are so high, and the demand so low, that the monopolist finds it profitable to increase b.

A similar reasoning applies for p, with the exception that p does not only affect incentives and thus demand but also, directly, the margins. In particular p affects the margins the response of the demand and the level of demand. When the p makes the demand less elastic the negative effect on the level of the demand always overcome the combination of the other two. This pushed b to increase to balance.

When, instead the spread of information turns out to be more elastic as effect of an increase in p then the opposite is true for intermediates level of elasticity. The limit case that we have discussed for β and C, recourse itself here. However, in this case the conditions for his appearance are less tight as it appears both for very low and very high elasticity. The limit case of low elasticity, leading to an increase in b is similar to one studied for β and C with the addition that with an increase in p we also have higher margins, thus increasing more the the marginal gain compared to the previous case. A similar reasoning applies when we have a very high elasticity.

In the first period, the monopolist sets the price in order to maximize the sum of inter-temporal profits. In doing so, it knows the distribution of willingnesses to pay and search costs and internalizes customers decisions in the second period. Moreover, consumers will observe the price and decide whether to buy the product.

Period 1.B - Purchase decisions of consumers. After having observed the price p charged by the monopolist, agent i decides whether to buy the product. The utility that he enjoys from the purchase is $u_i = r_i - p - s_i$ and 0 otherwise. Since s_H is assumed to be larger than the greatest possible willingness to pay only agents with low search cost can buy. Thus only a proportion $1-\beta$ of the the population is eligible to buy.

Once an agent with no search cost gets informed, his decision depends on his preferences. Specifically, he buys only if the price set by the monopolist is lower than his reservation price (i.e. if $r_i > p$). As we already defined, the probability for the willingness to pay to be larger than p is indicated by $(1 - G(p))$ allowing us to conclude that the total number of buyers at price p, is:

$$\mathbb{E}(D_1(p)) = (1-\beta)(1-G(p))n \tag{3.12}$$

The remaining part of the population is composed by βn agents who are uninformed and $(1-\beta)G(P)n$ who are informed but not interested to buy at price p.

Period 1.A - Monopolist sets p. Anticipating what will occur in the second period and having expectations about the purchase decisions of the present period, the monopolist sets the price so to maximize its inter-temporal profits as defined in Equations 3.2 and 3.3:

$$\pi = \pi_1 + \pi_2(b^*) = n\,(1-G(p))\,(1-\beta)p + (p - b^*(p))\mathbb{E}(D_2(b^*(p), p)) \tag{3.13}$$

After some rearrangement, the first order condition of the maximization problem yields the optimal price p^* which is the one such that:

$$\begin{array}{c} (1-\beta)(G'(p^*))p^* + \beta\left[\bar{\rho}(G'(p^*)) - (1-G(p^*))\,\frac{\partial\bar{\rho}}{\partial p^*}\right](p^* - b^*) \\ = \\ (1-\beta)\,(1-G(p^*)) + \beta\bar{\rho}\,(1-G(p^*)) - \beta\,(1-G(p^*))\,\bar{\rho}\frac{\partial b^*}{\partial p^*} \end{array} \tag{3.14}$$

The LHS of Equation 3.14 summarizes all the marginal losses in profits given by an increase in p while the RHS represents the marginal gains.

The first term on the LHS can be interpreted as the marginal loss that an increase in p yields the number of informed consumers that buy in the first period.

The second term is instead referred to the second period and can be decomposed in two different components. The first one is a direct effect, due to the fact that less people are willing to buy at the increased price. The second one is indirect. On average less people receive the information in the second period both because there are both less potential investors (first period buyers) and less incentives to invest (because of the direct effect).

The first two term on the RHS are referred to the increase in total margin obtained respectively in the first (from informed people) and the second period (from initially uninformed people).

Finally, the last term represents the marginal increase in b (a loss for the monopolist) that should be provided in order to retain the same demand from uninformed consumers, which are less willing to buy because of the increased price. Indeed, when the price increases less people are willing to buy at the new price. This constitutes a disincentive for potential investors because there are less rewards to be gained. If the objective of the monopolist is to keep the demand at the same level, then the increase in b should more than compensate this effect so to improve the informational process. The level of demand remains the same but the composition changes: more people receive the information but, among them, a lower share will buy.

We compare the results obtained above with the benchmark case in which the monopolist does not run the reward program. In this case only the first period informed agents can be attracted and the maximization problem reduces to:

$$\max_p np(1-\beta)(1-G(p)) \tag{3.15}$$

Consequently the first order conditions yields the optimal price p^{**}, which is the one such that:

$$p^{**}(1-\beta)(G'(p^{**})) = (1-\beta)(1-G(p^{**})) \tag{3.16}$$

Analyzing the difference between 3.16 and 3.14 we have:

Proposition 10. *The price p^*, optimal when the program is run, is higher than the price p^{**}, optimal when the program is not run, if and only if:*

$$\eta_{p-b,p} > -(\eta_{(1-G),p} + \eta_{\bar{\rho},p}), \tag{3.17}$$

lower otherwise. The terms in this inequality are the elasticities with respect to p of the variables in the subscript.

Proof. See Appendix 3.7.3. ■

In our model, the effect of price on monopolist's profit is different from the usual price setting because of the introduction of the program in the second period. Increasing the prices not only has the classical effect on the first period, but also two effects on the second period. What is peculiar of our model in the second period is the double effect both on the margins and on the demand.

Indeed, the increase in margins can be higher or lower than usual because we have an effect on the unitary reward that will be chosen. Moreover, an increase in p decreases the number of people willing to buy and reduces also the number of receivers of information. The effects on margins is the marginal gain of choosing a price higher than in the benchmark case (described by the LHS of Equation 3.17), while the effects on demand is the marginal loss (represented by the RHS of Equation 3.17). Clearly when the first dominates the price set with the program is higher than in the benchmark case.

This is because in such condition the net marginal gain is positive in the second period and thus negative in the first (to maintain the balance on the first order conditions). The first period price is comparable (actually the same thing) than the one of the benchmark case. It means that if we were in the benchmark case p^* would be such that the marginal loss would be higher than the marginal gain (and thus it would have been necessary to reduce the price to balance the correspondent first order condition).

3.5 Conclusions

In this paper we considered a setup in which a monopolist tries to reduce the search cost affecting part of his potential client base using a referral program. His aim is to incentivize a mobilization of the current customer base, creating a flow of communication from informed to uninformed consumers on a social network, leading to an expansion of the total number of buyers. The reward program consists in offering to informed consumers a bonus for each new consumer convinced to buy. The incentives created by the offer are clearly stronger the more one person is connected and lead to the emergence of a minimal degree above which an agent invests in communicating with peers. This leads to the quite realistic result that the equilibrium investors are only those with relatively high degrees and thus only a limited fraction of agents gets the discounts.

We confirm that centrality matters when pricing is done on social networks as in Bloch and Quérou (2013). The main difference is that, in our setup, the monopolist has only limited information about the topology of the network (degree distribution) while they assume the producer to know perfectly all nodal characteristics of each single agent. Different informational assumptions lead our results in opposite directions. In Bloch and Quérou (2013), where the authors use the concept of reference prices, central agents turn out to be charged more while, in this manuscript, being central is advantageous as it allows to receive discounted prices (in terms of reward).

The offer of the monopolist produces two competing effects. On the one hand, it creates incentives for informed people to invest in their social network and transmit information about the existence of the product. On the other hand, it also reduces the total amount of rewards that each agent expects to receive because of a crowding effect emerging as more people invest.

The balance combination of these two effects leads to different responses of the optimal reward to exogenous changes of incentives. When the crowding effect is more important the marginal effect of the reward is lower and thus decreased incentives lead to decreased rewards and vice-versa when the information effect dominates.

Investing agents ignite a process of spread of information. The efficiency of this process strictly depends on the type of network we have (how, is subject of our current investigation) and on relative share of uninformed consumers. The determination of whom is better (or worse) off due to the existence of the program is strictly linked to the efficiency of this process. This is to say that what matters is the amount of information at the beginning and how well it circulates.

Indeed, the reward program has different effects on the different categories of agents. Uninformed agents receive transfers from investing consumers and are thus unambiguously better off. When the price is decreased with respect to the benchmark case all consumers are weakly better off while the situation of the monopolist is ambiguous. If we have a large share of uninformed consumers this makes the second period more important than the first. In this case we have that raising b would increase margins of a relatively small amount that may not be enough to compensate the loss in first period margins so to put the monopolist in a problem of time inconsistency. In this case the exploitation of the social network can be detrimental by the monopolist. When instead the price increases clearly the monopolist is better off while the informed agents who do not invest are surely worse off (they pay a higher price). In this case the ambiguity goes on the effect that running the program has on informed investors. Their position is indeed very ambiguous. In expected terms however, it is more probable that for more connected agents the increase in price is compensated by the received gifts.

While it is reasonable to assume the independence between the willingness to pay of one agent and his degree, one could challenge our assumption that search costs are independent with centrality and reservation prices. Indeed, one could consider the case in which a more central node may have lower search cost due to his popularity. The only channel through which this may happen is that they receive the information through their social network. But, the communication among agents is the core of this paper and initially uninformed people with many connections will be more likely to see their search cost drop to zero in the second period.

The study of a monopoly is a starting point to understand the effects on pricing of network's exploitation under limited information, but most markets where such programs are run are, up to some degree, oligopolistic. Consequently, our current research endeavors are focused on extending our setup to an imperfect competition environment, where firms compete on prices. We expect that, increasing the competitive pressure, would push producers to offer higher rewards (thus extending the share of consumers interested in activating their social network). In such models the informational problem described here could be accompanied by a problem of switching costs, that may induce producers to offer rewards to switchers as wells as to those who convince them to buy. An alternative is to think of competition in the context of an entry model. Here, the challenge would be to understand whether the referral program is a way to prevent entrance for the incumbent or a way to steal a part of his market for the entrant.

Mathematical Appendices

3.6 Appendices to Chapter 2

3.6.1 Concavity conditions

Under ASSUMPTION $A.3$, the profits functions are strictly concave. **Proof.** Let compute the first derivatives of the profit w.r.t. prices and the Hessian matrix.

$$H = \begin{pmatrix} \frac{\partial^2 \pi_2^A}{\partial p_{E2}^{AA} \partial p_{E2}^{AA}} & \frac{\partial^2 \pi_2^A}{\partial p_{E2}^{AA} \partial p_{E2}^{AB}} & \frac{\partial^2 \pi_2^A}{\partial p_{E2}^{AA} \partial p_{F2}^{A}} \\ \frac{\partial^2 \pi_2^A}{\partial p_{E2}^{AB} \partial p_{E2}^{AA}} & \frac{\partial^2 \pi_2^A}{\partial p_{E2}^{AB} \partial p_{E2}^{AB}} & \frac{\partial^2 \pi_2^A}{\partial p_{E2}^{AB} \partial p_{F2}^{A}} \\ \frac{\partial^2 \pi_2^A}{\partial p_{F2}^{A} \partial p_{E2}^{AA}} & \frac{\partial^2 \pi_2^A}{\partial p_{F2}^{A} \partial p_{E2}^{AB}} & \frac{\partial^2 \pi_2^A}{\partial p_{F2}^{A} \partial p_{F2}^{A}} \end{pmatrix} = \begin{pmatrix} \frac{t^2-\alpha_E\alpha_F}{t(2t^2-4\alpha_E\alpha_F)} & -\frac{\alpha_E\alpha_F}{t(2t^2-4\alpha_E\alpha_F)} & -\frac{\alpha_E+\alpha_F}{2t^2-4\alpha_E\alpha_F} \\ \frac{\alpha_E\alpha_F}{t(2t^2-4\alpha_E\alpha_F)} & -\frac{t^2-\alpha_E\alpha_F}{t(2t^2-4\alpha_E\alpha_F)} & -\frac{\alpha_E+\alpha_F}{2t^2-4\alpha_E\alpha_F} \\ \frac{\alpha_E+\alpha_F}{2t^2-4\alpha_E\alpha_F} & -\frac{\alpha_E+\alpha_F}{2t^2-4\alpha_E\alpha_F} & -\frac{t}{2t^2-4\alpha_E\alpha_F} \end{pmatrix}$$

Conditions for strict concavity

1. Fundamental principal minor of order 1 should be negative

$$-\frac{t^2-\alpha_E\alpha_F}{t\left(2t^2-4\alpha_E\alpha_F\right)} < 0 \text{ if } t^2 > 2\alpha_E\alpha_F \text{ and } t > 0$$

2. Fundamental principal minor of order 2

$$\begin{vmatrix} -\frac{t^2-\alpha_E\alpha_F}{t(2t^2-4\alpha_E\alpha_F)} & -\frac{\alpha_E\alpha_F}{t(2t^2-4\alpha_E\alpha_F)} \\ -\frac{\alpha_E\alpha_F}{t(2t^2-4\alpha_E\alpha_F)} & -\frac{t^2-\alpha_E\alpha_F}{t(2t^2-4\alpha_E\alpha_F)} \end{vmatrix} = \left(\frac{t^2-\alpha_E\alpha_F}{t(2t^2-4\alpha_E\alpha_F)}\right)^2 - \left(\frac{\alpha_E\alpha_F}{t(2t^2-4\alpha_E\alpha_F)}\right)^2 > 0$$

Satisfied if and only if $t^2 - 2\alpha_E\alpha_F > 0$

3. Fundamental principal minor of order 3 (Hessian matrix)

$$\begin{vmatrix} -\frac{t^2-\alpha_E\alpha_F}{t(2t^2-4\alpha_E\alpha_F)} & -\frac{\alpha_E\alpha_F}{t(2t^2-4\alpha_E\alpha_F)} & -\frac{\alpha_E+\alpha_F}{2t^2-4\alpha_E\alpha_F} \\ -\frac{\alpha_E\alpha_F}{t(2t^2-4\alpha_E\alpha_F)} & -\frac{t^2-\alpha_E\alpha_F}{t(2t^2-4\alpha_E\alpha_F)} & -\frac{\alpha_E+\alpha_F}{2t^2-4\alpha_E\alpha_F} \\ -\frac{\alpha_E+\alpha_F}{2t^2-4\alpha_E\alpha_F} & -\frac{\alpha_E+\alpha_F}{2t^2-4\alpha_E\alpha_F} & -\frac{t}{2t^2-4\alpha_E\alpha_F} \end{vmatrix} = \frac{2\left(\alpha_E+\alpha_F\right)^2 - t^2}{8t\left(t^2-\alpha_E\alpha_F\right)} < 0$$

Satisfied if and only if ASSUMPTION A3: $t^2 > 2\left(\alpha_E+\alpha_F\right)^2$ holds.

∎

3.6.2 Second period maximization problem under TDS

Platform A solves the following maximization problem:

$$\max_{p_{E2}^{AA}, p_{E2}^{AB}, p_{F2}^{A}} p_{E2}^{AA} x_2^A + p_{E2}^{AB}\left(x_2^B - n_{E1}\right) + p_{F2}^{A} n_{F2}^{A} \tag{3.18}$$

Using the first order conditions of this problem, we obtain the best response function of platform A, represented by prices $p_{E2}^{AA}, p_{E2}^{AB}, p_{F2}^{A}$ in function of prices charged by the rival platform:

$$\begin{aligned} p_{E2}^{AA}(p_{E2}^{BB}, p_{E2}^{BA}, p_{F2}^{B}) &= \tfrac{4t^3-2t^2(\alpha_E+\alpha_F)+(\alpha_E+\alpha_F)(p_{E2}^{BB}\alpha_E-p_{E2}^{BB}\alpha_F+4\alpha_E\alpha_F)}{8t^2-4(\alpha_E+\alpha_F)^2} \\ &+\tfrac{2t\left(\alpha_E(p_{F2}^{B}-n_{E1}\alpha_E)-(p_{F2}^{B}+3\alpha_E)\alpha_F+(-1+n_{E1})\alpha_F^2\right)+p_{E2}^{BA}\left(4t^2-(\alpha_E+\alpha_F)(\alpha_E+3\alpha_F)\right)}{8t^2-4(\alpha_E+\alpha_F)^2} \end{aligned} \tag{3.19}$$

$$\begin{aligned} p_{E2}^{AB}(p_{E2}^{BB}, p_{E2}^{BA}, p_{F2}^{B}) &= \tfrac{t^3(4-8n_{E1})-2t^2(\alpha_E+\alpha_F)+(\alpha_E+\alpha_F)(p_{E2}^{BA}(\alpha_E-\alpha_F)+4\alpha_E\alpha_F)}{8t^2-4(\alpha_E+\alpha_F)^2} \\ &+\tfrac{2t(p_{F2}^{B}(\alpha_E-\alpha_F)-\alpha_F(3\alpha_E+\alpha_F)+n_{E1}(\alpha_E+\alpha_F)(\alpha_E+3\alpha_F))}{8t^2-4(\alpha_E+\alpha_F)^2} \end{aligned} \tag{3.20}$$

$$\begin{aligned} p_{F2}^{A}(p_{E2}^{BB}, p_{E2}^{BA}, p_{F2}^{B}) &= \tfrac{2t^3-(p_{E2}^{BA}+p_{E2}^{BB})t\alpha_E+t(p_{E2}^{BA}+p_{E2}^{BB}-4\alpha_E)\alpha_F+2\alpha_E\alpha_F(\alpha_E+\alpha_F)}{8t^2-4(\alpha_E+\alpha_F)^2} \\ &+\tfrac{2t^2((-1+n_{E1})\alpha_E-n_{E1}\alpha_F)+2p_{F2}^{B}\left(t^2-\alpha_E(\alpha_E+\alpha_F)\right)}{4t^2-2(\alpha_E+\alpha_F)^2} \end{aligned} \tag{3.21}$$

3.6.3 Proof of Proposition 3

Proof. Platform A solves the following maximization problem:

$$\begin{gathered} \max_{p_{E2}^{AA}, p_{E2}^{AB}, p_{F2}^{A}} p_{E2}^{AA} n_{E1} + p_{E2}^{AB}(x_2^B - n_{E1}) + p_{F2}^{A} n_{F2}^{A} \\ \text{s.t. } x_2^B \geq n_{E1} \\ x_2^A \geq n_{E1} \end{gathered} \tag{3.22}$$

Thus, the Lagrangian will be:

$$\mathcal{L}^A = p_{E2}^{AA} n_{E1} + p_{E2}^{AB}(x_2^B - n_{E1}) + p_{F2}^{A} n_{F2}^{A} + \lambda_1\left[x_2^B - n_{E1}\right] + \lambda_2\left[x_2^A - n_{E1}\right]$$

At the optimum, the Kuhn Tucker conditions have to be satisfied:

$$\mathcal{L}^A_{p^{AA}_{E2}} = 0 \implies n_{E1} - \tfrac{1}{2t}\lambda_2 = 0$$

$$\mathcal{L}^A_{p^{AB}_{E2}} = 0 \implies x^B_2 - n_{E1} - \tfrac{t}{\Omega}p^{AB}_{E2} - \tfrac{\alpha_F}{\Omega}p^A_{F2} - \tfrac{t}{\Omega}\lambda_1 - \tfrac{\alpha_E\alpha_F}{t\Omega}\lambda_2 = 0$$

$$\mathcal{L}^A_{p^A_{F2}} = 0 \implies -\tfrac{\alpha_E}{\Omega}p^{AB}_{E2} + n^A_{F2} - \tfrac{t}{\Omega}p^A_{F2} - \tfrac{\alpha_E}{\Omega}\lambda_1 - \tfrac{\alpha_E}{\Omega}\lambda_2 = 0$$

$$\lambda_1 \geq 0,\ x^B_2 - n_{E1} \geq 0,\ \lambda_1\left[x^B_2 - n_{E1}\right] = 0$$

$$\lambda_2 \geq 0,\ x^A_2 - n_{E1} \geq 0,\ \lambda_2\left[x^A_2 - n_{E1}\right] = 0$$

From the derivative of the Lagrangian w.r.t. p^{AA}_{E2}, the second Lagrange multiplier turns out to be $\lambda_2 = 2tn_{E1}$, which implies the second constraint to be binding. It means that when no switching to B arises, then at the optimum, prices set by platform A are such that the threshold x^A_2 is located exactly in the same point where the indifferent user in time 1 was. On the other hand, the second constraint should be non-binding to have switching to B, i.e. $\lambda_1 = 0$. The system reduces to:

$$\mathcal{L}_{p^{AB}_{E2}} = 0 \implies x^B_2 - \tfrac{t^2-2\alpha_E\alpha_F}{\Omega}2n_{E1} - \tfrac{t}{\Omega}p^{AB}_{E2} - \tfrac{\alpha_F}{\Omega}p^A_{F2} = 0$$

$$\mathcal{L}_{p^A_{F2}} = 0 \implies -\tfrac{\alpha_E}{\Omega}p^{AB}_{E2} + n^A_{F2} - \tfrac{t}{\Omega}p^A_{F2} - \tfrac{2t\alpha_E}{\Omega}n_{E1} = 0$$

$$x^A_2 = n_{E1}$$

Solving this reduced system for prices, we obtain the following best response prices of platform A:

$$\begin{aligned} p^{AB}_{E2} = &\ \tfrac{2t^2-\alpha_F(\alpha_E+\alpha_F)}{4t^2-(\alpha_E+\alpha_F)^2}p^{BB}_{E2} + \tfrac{(\alpha_E-\alpha_F)t}{4t^2-(\alpha_E+\alpha_F)^2}p^B_{F2} \\ &+\tfrac{t^3+2\alpha_E t(\alpha_E+3\alpha_F)}{4t^2-(\alpha_E+\alpha_F)^2}n_{E1} - \tfrac{2t(1+\alpha_E\alpha_F)}{4t^2-(\alpha_E+\alpha_F)^2} \end{aligned} \tag{3.23}$$

$$\begin{aligned} p^A_{F2} = &\ \tfrac{2t^2-\alpha_E(\alpha_E+\alpha_F)}{4t^2-(\alpha_E+\alpha_F)^2}p^B_{F2} + \tfrac{(\alpha_E-\alpha_F)t}{4t^2-(\alpha_E+\alpha_F)^2}p^{BB}_{E2} \\ &-\alpha_E\tfrac{3t^2-2\alpha_F(\alpha_E+\alpha_F)}{4t^2-(\alpha_E+\alpha_F)^2}n_{E1} - \tfrac{(\alpha_E+\alpha_F)(t^2-\alpha_E\alpha_F)}{4t^2-(\alpha_E+\alpha_F)^2} \end{aligned} \tag{3.24}$$

$$p^{AA}_{E2} = p^{BA}_{E2} + t - \alpha_E + \frac{2\alpha_E\alpha_F}{\Omega}(p^{BB}_{E2} - p^{AB}_{E2}) + \frac{2t\alpha_E}{\Omega}(p^B_{F2} - p^A_{F2}) - 2tn_{E1} \tag{3.25}$$

Following the same logic, platform B maximization problem has the following Lagrangian:

$$\mathcal{L}^B = p^{BB}_{E2}(1 - x^B_2) + p^B_{F2}n^B_{F2} + \gamma_1\left[x^B_2 - n_{E1}\right] + \gamma_2\left[x^A_2 - n_{E1}\right]$$

So as before, the Kuhn Tucker conditions require:

$$\mathcal{L}^B_{p^{BB}_{E2}} = 0 \implies 1 - x^B_2 - \tfrac{t}{\Omega} p^{BB}_{E2} - \tfrac{\alpha_F}{\Omega} p^B_{F2} + \gamma_1 \tfrac{t}{\Omega} + \gamma_2 \tfrac{\alpha_E \alpha_F}{t\Omega} = 0$$

$$\mathcal{L}^B_{p^{BA}_{E2}} = 0 \implies \tfrac{\gamma_2}{2t} = 0 \implies \gamma_2 = 0$$

$$\mathcal{L}^B_{p^B_{F2}} = 0 \implies -\tfrac{\alpha_E}{\Omega} p^{BB}_{E2} - \tfrac{t}{\Omega} p^B_{F2} + n^B_{F2} + \gamma_1 \tfrac{\alpha_E}{\Omega} + \gamma_2 \tfrac{\alpha_E}{\Omega} = 0$$

$$\gamma_1 \geq 0,\ x^B_2 - n_{E1} \geq 0,\ \gamma_1 \left[x^B_2 - n_{E1}\right] = 0$$

$$\gamma_2 \geq 0,\ x^A_2 - n_{E1} \geq 0,\ \gamma_2 \left[x^A_2 - n_{E1}\right] = 0$$

Solving the system for prices, the best response of platform B turns out to be:

$$p^{BB}_{E2} = \frac{2t^2 - \alpha_F(\alpha_E + \alpha_F)}{4t^2 - (\alpha_E + \alpha_F)^2} p^{AB}_{E2} + \frac{(\alpha_E - \alpha_F)t}{4t^2 - (\alpha_E + \alpha_F)^2} p^A_{F2} + \frac{2t(t^2 - \alpha_E \alpha_F)}{4t^2 - (\alpha_E + \alpha_F)^2} \tag{3.26}$$

$$p^B_{F2} = \frac{2t^2 - \alpha_E(\alpha_E + \alpha_F)}{4t^2 - (\alpha_E + \alpha_F)^2} p^A_{F2} + \frac{(\alpha_E - \alpha_F)t}{4t^2 - (\alpha_E + \alpha_F)^2} p^{AB}_{E2} - \frac{(t^2 - \alpha_E \alpha_F)(\alpha_E + \alpha_F)}{4t^2 - (\alpha_E + \alpha_F)} \tag{3.27}$$

$$p^{BA}_{E2} \in \mathbb{C} \tag{3.28}$$

where $\mathbb{C} = p \in \mathbb{R} : x^A_2(p) \geq n_{E1}$. According to these best responses, our aim is to find for which prices and for which inherited market shares an equilibrium with switching only towards A exists. Since any price p compatible with the constraint can be candidate as an equilibrium price, we first of all analyze for which prices we actually have an equilibrium of the type *ODS* (One Direction Switching) to A only . Assume that this equilibrium exists and let $p^{BA}_a = p$, where p is some price exogenous for firm A that belongs to set C. Plugging this price in the best responses, we obtain this equilibrium prices:

$$\begin{aligned}
p^{AA}_{E2} &= t - 2tn_{E1} + p + \tfrac{2tn_{E1}\alpha_E(2\alpha_E + \alpha_F)}{9t^2 - (2\alpha_E + \alpha_F)(\alpha_E + 2\alpha_F)} \\
p^{BA}_{E2} &= p \\
p^{AB}_{E2} &= t - tn_{E1} - \alpha_F - \tfrac{tn_{E1}\left(3t^2 - \alpha_E(2\alpha_E + \alpha_F)\right)}{9t^2 - (2\alpha_E + \alpha_F)(\alpha_E + 2\alpha_F)} \\
p^{BB}_{E2} &= t - tn_{E1} - \alpha_F + \tfrac{tn_{E1}\left(3t^2 - \alpha_E(2\alpha_E + \alpha_F)\right)}{9t^2 - (2\alpha_E + \alpha_F)(\alpha_E + 2\alpha_F)} \\
p^A_{F2} &= t - \alpha_E + \tfrac{2t^2 n_{E1}(-\alpha_E + \alpha_F)}{9t^2 - (2\alpha_E + \alpha_F)(\alpha_E + 2\alpha_F)} \\
p^B_{F2} &= t - \alpha_E + \tfrac{2t^2 n_{E1}(\alpha_E - \alpha_F)}{9t^2 - (2\alpha_E + \alpha_F)(\alpha_E + 2\alpha_F)}
\end{aligned} \tag{3.29}$$

which yield $x_B = \frac{1}{2} + \frac{3t^2 n_{E1}}{9t^2-(2\alpha_E+\alpha_F)(\alpha_E+2\alpha_F)}$ and $n_{F2} = \frac{1}{2} + \frac{t n_{E1}(2\alpha_E+\alpha_F)}{9t^2-(2\alpha_E+\alpha_F)(\alpha_E+2\alpha_F)}$. Now, our aim is to analyze the possible deviations of both firms in order to find price p_a^{BA} and to guarantee that this is actually an equilibrium.

Deviations of Platform B**.** Assume an equilibrium with the prices defined in (3.29) exists: thus platform B should prefer not to attract A's consumers over the deviation to try to poach some A's inherited clients.

(i) Platform B accepts not to attract A's users. It means that its profit is given by what follows:

$$\begin{aligned}\Pi^B = &\frac{2t^3(9+2(-3+n_{E1})n_{E1})+(\alpha_E+\alpha_F)(2\alpha_E+\alpha_F)(\alpha_E+2\alpha_F)}{2(9t^2-(2\alpha_E+\alpha_F)(\alpha_E+2\alpha_F))}\\ &+\frac{2t(2\alpha_E+\alpha_F)((n_{E1}-1)\alpha_E-t^2((9+2n_{E1})\alpha_E+(9-2n_{E1})\alpha_F)+(n_{E1}-2)\alpha_F}{2(9t^2-(2\alpha_E+\alpha_F)(\alpha_E+2\alpha_F))}\end{aligned} \tag{3.30}$$

(ii) Firm B deviates and tries to poach A's customers, knowing that A set prices according to equation (3.29). Thus, firm B sets price p_{E2}^{BA}, p_{E2}^{BB}, p_{F2}^{B} to maximize the profit of deviation Π_d^B:

$$\Pi_d^B = p_{E2}^{BB}(1-x_B) + p_{E2}^{BA}(n_{E1}-x_A) + p_{F2}^{B}(1-n_{F2}) \tag{3.31}$$

where x_A, x_B and n_{F2} are the ones referring to the two-direction switching case with all prices of platform A defined in (3.29) substituted in the formulas. Thus, the first conditions of this maximization problem yield the following results:

$$\begin{aligned}p_{E2}^{BA} = &\frac{t^3(8n_{E1}-4)-p_{E2}^{AB}\alpha_E^2+2\alpha_E(p_{E2}^{AB}+\alpha_E)\alpha_F-(p_{E2}^{AB}-2\alpha_E)\alpha_F^2-2t^2(\alpha_E+\alpha_F)}{8t^2-4\left(\alpha_E^2+\alpha_F^2\right)}\\ &+\frac{2t\left(\alpha_E(p_{F2}^{A}-\alpha_E n_{E1})+p_{F2}^{A}\alpha_F+(2-3n_{E1})\alpha_F^2\right)+p_{E2}^{AA}\left(4t^2-\alpha_E^2-3\alpha_F^2\right)}{8t^2-4\left(\alpha_E^2+\alpha_F^2\right)}\end{aligned} \tag{3.32}$$

$$\begin{aligned}p_{E2}^{BB} = &\frac{4p_{E2}^{AB}t^2+4t^3-2p_{F2}^{A}t\alpha_E+2t^2\alpha_E-p_{E2}^{AA}\alpha_E^2-p_{E2}^{AB}\alpha_E^2-2tn_{E1}\alpha_E^2}{8t^2-4\left(\alpha_E^2+\alpha_F^2\right)}\\ &\frac{-2\left(p_{F2}^{A}t+t^2+2t(n_{E1}-1)\alpha_E+(p_{E2}^{AA}-\alpha_E)\alpha_E\right)\alpha_F-(p_{E2}^{AA}+3p_{E2}^{AB}+2(tn_{E1}+\alpha_E))\alpha_F^2}{8t^2-4\left(\alpha_E^2+\alpha_F^2\right)}\end{aligned} \tag{3.33}$$

$$\begin{aligned}p_{F2}^{B} = &\frac{t\left(2t^2(p_{F2}^{A}+t)+t(-p_{E2}^{AA}+p_{E2}^{AB}-2t(-1+n_{E1}))\alpha_E-2p_{F2}^{A}\alpha_E^2\right)+(p_{E2}^{AA}+p_{E2}^{AB}}{4t^3-2t\left(\alpha_E^2+\alpha_F^2\right)}\\ &+\frac{2t(-1+n_{E1}))(t-\alpha_E)(t+\alpha_E)\alpha_F(p_{E2}^{AA}-p_{E2}^{AB}+2t(-1+n_{E1})-2\alpha_E)\alpha_E\alpha_F^2}{4t^3-2t\left(\alpha_E^2+\alpha_F^2\right)}\end{aligned} \tag{3.34}$$

where prices of platform A p_{E2}^{AA}, p_{E2}^{AB} and p_{F2}^{A} are the ones defined in Equation (3.29). Plugging these prices in the cutoffs and in the profit function of deviation, we get the difference between profit of deviation Π_d^B and profit Π^B:

$$\Pi_d^B - \Pi^B = \frac{\left(4t^2 - (\alpha_E + \alpha_F)^2\right)\left((p + \alpha_F)A + 2tn_{E1}\alpha_F(2\alpha_E + \alpha_F)\right)^2}{16t\left(2t^2 - (\alpha_E + \alpha_F)^2\right)A^2} \tag{3.35}$$

where $A = \left(9t^2 - (2\alpha_E + \alpha_F)(\alpha_E + 2\alpha_F)\right) > 0$ This difference is always positive unless

$$p = -\alpha_F - \frac{2tn_{E1}\alpha_F(2\alpha_E + \alpha_F)}{A} \tag{3.36}$$

According to this discussion about deviation, an equilibrium with switching only to A is possible only if platform A believes that platform B sets his price to new users equal to the one described in equation (3.36) and platform B actually does it. Since this price is the one that makes platform B indifferent between deviating and adopting this strategy, any other price cannot be compatible with one direction switching to A. This possible equilibrium is represented by the following prices:

$$\begin{aligned} p_{E2}^{AA} &= t - \alpha_F + 2tn_{E1} + \tfrac{2tn_{E1}(\alpha_E - \alpha_F)(2\alpha_E + \alpha_F)}{9t^2 - (2\alpha_E + \alpha_F)(\alpha_E + 2\alpha_F)} \\ p_{E2}^{BA} &= -\alpha_F - \tfrac{2tn_{E1}\alpha_F(2\alpha_E + \alpha_F)}{9t^2 - (2\alpha_E + \alpha_F)(\alpha_E + 2\alpha_F)} \\ p_{E2}^{AB} &= t - tn_{E1} - \alpha_F - \tfrac{tn_{E1}\left(3t^2 - \alpha_E(2\alpha_E + \alpha_F)\right)}{9t^2 - (2\alpha_E + \alpha_F)(\alpha_E + 2\alpha_F)} \\ p_{E2}^{BB} &= t - tn_{E1} - \alpha_F + \tfrac{tn_{E1}\left(3t^2 - \alpha_E(2\alpha_E + \alpha_F)\right)}{9t^2 - (2\alpha_E + \alpha_F)(\alpha_E + 2\alpha_F)} \\ p_{F2}^{A} &= t - \alpha_E + \tfrac{2t^2 n_{E1}(-\alpha_E + \alpha_F)}{9t^2 - (2\alpha_E + \alpha_F)(\alpha_E + 2\alpha_F)} \\ p_{F2}^{B} &= t - \alpha_E + \tfrac{2t^2 n_{E1}(\alpha_E - \alpha_F)}{9t^2 - (2\alpha_E + \alpha_F)(\alpha_E + 2\alpha_F)} \end{aligned} \tag{3.37}$$

And the corresponding profits are respectively for platform A and platform B:

$$\begin{aligned} \Pi^A = & \tfrac{1}{2}\left(2t - tn_{E1} - 4tn_{E1}^2 - \alpha_E - \alpha_F\right) \\ & + \tfrac{tn_{E1}\left(t^2(3 + 16n_{E1}) + 2t(\alpha_E - \alpha_F) - \alpha_E(2\alpha_E + \alpha_F)\right)}{2(9t^2 - (2\alpha_E + \alpha_F)(\alpha_E + 2\alpha_F))} \end{aligned} \tag{3.38}$$

$$\begin{aligned} \Pi^B = & \tfrac{2t^3(9 + 2(-3 + n_{E1})n_{E1}) + (\alpha_E + \alpha_F)(2\alpha_E + \alpha_F)(\alpha_E + 2\alpha_F)}{2(9t^2 - (2\alpha_E + \alpha_F)(\alpha_E + 2\alpha_F))} \\ & - \tfrac{t^2((9 + 2n_{E1})\alpha_E + (9 - 2n_{E1})\alpha_F) + 2t(2\alpha_E + \alpha_F)((1 - n_{E1})\alpha_E + (2 - n_{E1})\alpha_F)}{2(9t^2 - (2\alpha_E + \alpha_F)(\alpha_E + 2\alpha_F))} \end{aligned} \tag{3.39}$$

Deviations of platform A. Now assume that the scenario described by prices in (3.37) is actually an equilibrium. Following the same reasoning used for platform B, A may find profitable to change prices (in particular for inherited old users, p_{E2}^{AA})

allowing for some switching to B. According to what we found so far as a candidate equilibrium, we should compare the following two alternatives:

(i) Platform A does not allow B to attract any user. It means that platform A profit is simply the one that results from it behaving as described in the previous paragraph, with Π^A as in (3.38).

(ii) Firm A deviates and allows some switching to B.Under which conditions on n_{E1} does platform A find it profitable to set different prices when platform B behaves according to an equilibrium with switching only to A ? It means that given prices $p_{E2}^{BA}, p_{E2}^{BB}, p_{F2}^{B}$ charged by platform B according to the candidate equilibrium described in (3.37), platform A solves the maximization problem considering $n_{E1} \in (x_2^A, x_2^B)$. Formally A solves:

$$\max_{p_{E2}^{AA}, p_{E2}^{AB}, p_{F2}^{A}} \Pi_d^A = \max_{p_{E2}^{AA}, p_{E2}^{AB}, p_{F2}^{A}} p^{AA} x_A + p_{E2}^{AB} \left(x_2^B - n_{E1} \right) + p_{F2}^{A} n_{F2} \tag{3.40}$$

where x_A, x_B and n_{F2} are the ones referring to the TDS case with all prices of platform B defined in (3.37) substituted in the formulas.

The gain of deviation obviously depends as well on the location of n_{E1}, specifically two cases may arise:

$n_{E1} > \bar{\bar{n}}$ then A always deviates. It should charge too low prices to old customers to retain them, then it prefers to allow some switching to B and change prices.

$n_{E1} \leq \bar{\bar{n}}$ then no deviation can be profitable. Indeed, since TDS is not possible and prices in (3.37) are optimal when ODS is assumed to be the case. In particular, using the triple $p_{E2}^{AA}, p_{E2}^{AB}, p_{F2}^{A}$ solving the maximization problem and plugging in x_2^A, we find $x_2^A > n_{E1}$, i.e. one direction switching to A only.

where

$$\bar{\bar{n}} = \frac{9t^2 - 2\alpha_E^2 - 5\alpha_E\alpha_F - 2\alpha_F^2}{36t^2 - 12\alpha_E^2 - 30\alpha_E\alpha_F - 12\alpha_F^2} \tag{3.41}$$

■

3.6.4 Proof of Proposition 4

Proof. It is easy to prove the first two results simply comparing prices in (2.28) and (2.29) with $\bar{p}_F$ and $\bar{p}_E$. In particular, second period prices in in side E become lower than under *within group* uniform price regime when platform discriminate among users. This is true both for new customers (for who discriminatory prices are lower by an amount $\frac{2t}{3}$) and for old customers (for who the difference is $\frac{t}{3}$). For the third result, look at first period prices under *BBPD* regime in (2.27) and first period prices under uniform price. First period prices in side E are higher under *BBPD* regime if and only if $\delta\frac{t(3t-2\alpha_E-\alpha_F)(\alpha_E-\alpha_F)}{3\Omega} > 0$, or simply if

$$(3t - 2\alpha_E - \alpha_F)(\alpha_E - \alpha_F) > 0$$

provided that $\delta\frac{t}{3\Omega} > 0$ by concavity conditions and $3t > 2\alpha_E + \alpha_F$ by assumption A2.This inequality is verified only if $\alpha_E > \alpha_F$, otherwise we have exactly the opposite result, i.e. first period prices for users are lower under the BBPD regime. The fourth point of proposition is proved as follows. The difference between price discrimination profits in (2.30) and benchmark profits in (2.31) is given by

$$\Pi - \bar{\Pi} = \frac{\delta t(-36t^2 + 9t(\alpha_E - \alpha_F) + (2\alpha_E + \alpha_F)(5\alpha_E + 19\alpha_F))}{18\Omega} \tag{3.42}$$

Since denominator is positive by concavity conditions, the difference in (3.42) is positive if and only if:

$$9t(\alpha_E - \alpha_F) - 36t^2 + (2\alpha_E + \alpha_F)(5\alpha_E + 19\alpha_F) > 0 \tag{3.43}$$

First thing to notice is that LHS is decreasing in t, thus a lower t makes it more likely to be fulfilled. It means that if condition (3.43) is not fulfilled even when we consider transportation cost high just enough to satisfy assumptions $A1$ and $A2$, then it is never satisfiable. We consider two cases. If $\alpha_E > \alpha_F$, then the minimum value of t^2 is $2(\alpha_E + \alpha_F)^2 + \epsilon$ while $t > \alpha_E + \epsilon$. Substituting these lowest possible values we obtain

$$-61\alpha_E^2 - 53\alpha_F^2 - 102\alpha_E\alpha_F < 0$$

The same result arises when $\alpha_F > \alpha_E$, which implies that $t > \alpha_E + \epsilon$. Substituting these lowest possible values we obtain

$$-62\alpha_E^2 - 54\alpha_F^2 - 101\alpha_E\alpha_F < 0$$

Thus, condition (3.43) cannot be satisfied under assumption $A1$ and $A2$, profits are lower under the BBPD regime. ■

3.7 Appendices to Chapter 3

3.7.1 Proof of Proposition 7

Proof. To obtain the results of the Proposition we simply need to derive $\bar{\rho}$ with respect to the parameters of the model. From Equation 3.7 there is no direct effect of C and b on $\bar{\rho}$, the only effect is through $\underline{k}$. Thus $\frac{\partial \bar{\rho}}{\partial b} = \frac{\partial \bar{\rho}}{\partial \underline{k}} \frac{\partial \underline{k}}{\partial b}$ and $\frac{\partial \bar{\rho}}{\partial C} = \frac{\partial \bar{\rho}}{\partial \underline{k}} \frac{\partial \underline{k}}{\partial C}$. The first terms can be computed from Equation 3.7 as:

$$\frac{\partial \rho(k)}{\partial \underline{k}} = k \left[1 - (1-\beta)(1-G(p)) \sum_{k \geq \underline{k}} f(k)\right]^{k-1} (1-\beta)(1-G(p))(-f(\underline{k})) < 0 \tag{3.44}$$

Notice that the derivative of $\frac{\partial \bar{\rho}}{\partial \underline{k}} = \sum_{k=1}^{n} \frac{\partial \rho(k)}{\partial \underline{k}} f(k)$. From the assumptions we know the signs of the second terms, and thus of the effects, in particular:

- $\frac{\partial \underline{k}}{\partial C} > 0 \longrightarrow \frac{\partial \bar{\rho}}{\partial C} < 0$
- $\frac{\partial \underline{k}}{\partial b} < 0 \longrightarrow \frac{\partial \bar{\rho}}{\partial b} > 0$

The result of Proposition 7 with respect to p is instead calculated as:

$$\begin{aligned}\frac{\partial \rho(k)}{\partial p} = \quad & k(1-\beta)\left[1 - (1-\beta)(1-G(p)) \sum_{k \geq \underline{k}} f(k)\right]^{k-1} \\ & \left(\underbrace{\sum_{k \geq \underline{k}} f(k)(-G'(p))}_{<0}\right) < 0\end{aligned} \tag{3.45}$$

Finally, we can calculate the derivative for β:

$$\begin{aligned}\frac{\partial \rho(k)}{\partial \beta} = \quad & k(1-G(p))\left[1 - (1-\beta)(1-G(p)) \sum_{k \geq \underline{k}} f(k)\right]^{k-1} \\ & \left(\underbrace{(1-\beta)\frac{\partial \sum_{k \geq \underline{k}} f(k)}{\partial \beta}}_{ambiguous} - \underbrace{\sum_{k \geq \underline{k}} f(k)}_{>0}\right)\end{aligned} \tag{3.46}$$

The sign of Equation 3.46 is clearly ambiguous. However, multiplying by:

$$\frac{\beta}{(1-\beta)\sum\limits_{k\geq\underline{k}} f(k)} \frac{\left[1-(1-\beta)\left(1-G(p)\right)\sum\limits_{k\geq\underline{k}} f(k)\right]^{1-k}}{k\left(1-G(p)\right)} \tag{3.47}$$

and rearranging terms, we can obtain the following condition for positiveness of the derivative:

$$\eta_{\sum\limits_{k\geq\underline{k}} f(k),\beta} > \frac{\beta}{1-\beta} \tag{3.48}$$

According to the equilibrium condition stated in Equation 3.8, define:

$$\phi = \frac{1}{b}\sum_{k\geq\underline{k}} f(k)\mathbb{E}(B(k,b)) - \frac{\beta}{1-\beta}\bar{\rho} \tag{3.49}$$

In order to compute the derivative of $\frac{\partial \underline{k}}{\partial \beta}$ we use the total derivation of ϕ w.r.t. both variables and equate to zero, so that $\frac{d\underline{k}}{d\beta} = -\frac{\partial\phi/\partial\beta}{\partial\phi/\partial\underline{k}}$. Computing:

$$\frac{\partial\phi}{\partial\underline{k}} = \underbrace{\frac{1}{b}\frac{\partial\sum_{k\geq\underline{k}} f(k)\mathbb{E}(B(k,b))}{\partial\underline{k}}}_{>0} - \underbrace{\frac{\beta}{1-\beta}\frac{\partial\bar{\rho}}{\partial\underline{k}}}_{<0} > 0 \tag{3.50}$$

The first term is indeed positive because the average expected benefit can only increasing in the lower bound of the sum, while the other term's sign derives from Equation 3.44. On the other side:

$$\frac{\partial\phi}{\partial\beta} = -\frac{\frac{\partial\bar{\rho}}{\partial\beta}\beta(1-\beta)+\bar{\rho}}{(1-\beta)^2} \tag{3.51}$$

Interpreting the sign of this equation we find the condition such that $\frac{\partial\underline{k}}{\partial\beta} > 0$:

$$\frac{1}{1-\beta} > -\frac{\partial\bar{\rho}}{\partial\beta}\frac{\beta}{\bar{\rho}} \tag{3.52}$$

Now we are able to proof that $\frac{\partial\underline{k}}{\partial\beta}$ is ambiguous (depends on the satisfaction of Equation 3.52), while the $\frac{\partial\bar{\rho}}{\partial\beta}$ is unambiguously negative. We do this in 2 steps:

1. Let's assume that Condition in Equation 3.52 is satisfied. Then, since $\frac{\partial\underline{k}}{\partial\beta} > 0$, the share of investors $\sum_{k\geq\underline{k}} f(k)$ is decreasing in β. Substituting this result in Equation 3.46, the condition in Equation 3.48 can never be satisfied.

2. Let's assume that Condition in Equation 3.52 is reversed. Then, since $\frac{\partial \underline{k}}{\partial \beta} < 0$, the share of investors $\sum_{k \geq \underline{k}} f(k)$ is increasing in β.Substituting this result in Equation 3.46, two sub cases arise:

   - the condition in Equation 3.48 is satisfied. It implies that $\frac{\partial \bar{\rho}}{\partial \beta} > 0$. This leads to a contradiction. Indeed, the LHS of Equation 3.52 becomes negative implying that $\frac{\beta}{1-\beta} > -\frac{\partial \bar{\rho}}{\partial \beta}\frac{\beta^2}{\bar{\rho}}$.
   - The condition in Equation 3.48 is reversed. In this case both $\frac{\partial \bar{\rho}}{\partial \beta}$ and $\frac{\partial \underline{k}}{\partial \beta}$ are negative.

■

3.7.2 Proof of Proposition 9

Proof.

Interior solution. Proving that it is always profitable to run the program can be stated formally:

$$\forall p \in (0,1]\ \exists\ b \in (0,p) \text{ s.t. } \frac{p-b^*}{b^*} \geq \frac{1}{\eta_{\bar{\rho},b^*}} \tag{3.53}$$

For any p, take any b equal to $p-\epsilon$ with ϵ arbitrarily close to 0. We can always find an ϵ such that the equation above is true, since $\eta_{\bar{\rho},b^*} > 0$ as proven in Proposition 7. Moreover, b^* is always strictly lower than p otherwise the profit would be zero. This is equivalent to prove that there exists an interior solution to the maximization problem in Equation 3.10.

Comparative statics on the FoC. Define function ϕ as follows:

$$\phi = \frac{p-b^*}{b^*} - \frac{1}{\eta_{\bar{\rho},b^*}} \tag{3.54}$$

Take the derivatives of ϕ w.r.t. b, p, C, β.

$$\frac{\partial \phi}{\partial b} = -\frac{p}{b^2} + \frac{\partial \eta_{\bar{\rho},b}}{\partial b}\frac{1}{(\eta_{\bar{\rho},b})^2} \tag{3.55}$$

$$\frac{\partial \phi}{\partial p} = \frac{1}{b} + \frac{\partial \eta_{\bar{\rho},b}}{\partial p}\frac{1}{(\eta_{\bar{\rho},b})^2} \tag{3.56}$$

$$\frac{\partial \phi}{\partial C} = \frac{\partial \eta_{\bar{\rho},b}}{\partial C} \frac{1}{(\eta_{\bar{\rho},b})^2} \tag{3.57}$$

$$\frac{\partial \phi}{\partial \beta} = \frac{\partial \eta_{\bar{\rho},b}}{\partial \beta} \frac{1}{(\eta_{\bar{\rho},b})^2} \tag{3.58}$$

The signs of the partial derivatives above, depend crucially on the sign of the derivative of $\eta_{\bar{\rho},b^*}$ with respect to all variables.

Partial Derivatives of Elasticity $\eta_{\bar{\rho},b^*}$. Two cases need to be studied:

1. Assume that the optimal point b^* lies in an interval in which the elasticity is decreasing, i.e. $\frac{\partial \eta_{\bar{\rho},b}}{\partial b} < 0$. Then , the partial derivatives of the elasticity $\eta_{\bar{\rho},b}$ w.r.t. b, p, C and β can be rewritten in such a way to easily study their signs:

$$\underbrace{\frac{\partial \eta_{\bar{\rho},b}}{\partial b}}_{<0} = \frac{\partial \eta_{\bar{\rho},b}}{\partial \bar{\rho}} \underbrace{\frac{\partial \bar{\rho}}{\partial b}}_{>0} \implies \frac{\partial \eta_{\bar{\rho},b}}{\partial \bar{\rho}} < 0$$

Thus:

$$\frac{\partial \eta_{\bar{\rho},b}}{\partial p} = \underbrace{\frac{\partial \eta_{\bar{\rho},b}}{\partial \bar{\rho}}}_{<0} \underbrace{\frac{\partial \bar{\rho}}{\partial p}}_{<0} > 0$$

$$\frac{\partial \eta_{\bar{\rho},b}}{\partial C} = \underbrace{\frac{\partial \eta_{\bar{\rho},b}}{\partial \bar{\rho}}}_{<0} \underbrace{\frac{\partial \bar{\rho}}{\partial C}}_{<0} > 0$$

$$\frac{\partial \eta_{\bar{\rho},b}}{\partial \beta} = \underbrace{\frac{\partial \eta_{\bar{\rho},b}}{\partial \bar{\rho}}}_{<0} \underbrace{\frac{\partial \bar{\rho}}{\partial \beta}}_{<0} > 0$$

Plugging these results into the Equations 3.55, 3.56, 3.57 and 3.58 we find that $\frac{\partial \phi}{\partial p} > 0$, $\frac{\partial \phi}{\partial b} < 0$, $\frac{\partial \phi}{\partial C} > 0$ and $\frac{\partial \phi}{\partial \beta} > 0$. It follows that:

$$\frac{db}{dp} = -\frac{\frac{\partial \phi}{\partial p}}{\frac{\partial \phi}{\partial b}} > 0 \tag{3.59}$$

$$\frac{db}{d\beta} = -\frac{\frac{\partial \phi}{\partial \beta}}{\frac{\partial \phi}{\partial b}} > 0 \tag{3.60}$$

$$\frac{db}{dC} = -\frac{\frac{\partial \phi}{\partial C}}{\frac{\partial \phi}{\partial b}} > 0 \tag{3.61}$$

2. Assume that the optimal point b^* lies in an interval in which the elasticity is increasing, i.e. $\frac{\partial \eta_{\bar{\rho},b}}{\partial b} > 0$. As in Case 1 above, the partial derivatives of the elasticity $\eta_{\bar{\rho},b}$ w.r.t. b, p, C and β can be rewritten as:

$$\underbrace{\frac{\partial \eta_{\bar{\rho},b}}{\partial b}}_{>0} = \frac{\partial \eta_{\bar{\rho},b}}{\partial \bar{\rho}} \underbrace{\frac{\partial \bar{\rho}}{\partial b}}_{>0} \implies \frac{\partial \eta_{\bar{\rho},b}}{\partial \bar{\rho}} > 0$$

Thus:

$$\frac{\partial \eta_{\bar{\rho},b}}{\partial p} = \underbrace{\frac{\partial \eta_{\bar{\rho},b}}{\partial \bar{\rho}}}_{>0} \underbrace{\frac{\partial \bar{\rho}}{\partial p}}_{<0} < 0$$

$$\frac{\partial \eta_{\bar{\rho},b}}{\partial C} = \underbrace{\frac{\partial \eta_{\bar{\rho},b}}{\partial \bar{\rho}}}_{>0} \underbrace{\frac{\partial \bar{\rho}}{\partial C}}_{<0} < 0$$

$$\frac{\partial \eta_{\bar{\rho},b}}{\partial \beta} = \underbrace{\frac{\partial \eta_{\bar{\rho},b}}{\partial \bar{\rho}}}_{>0} \underbrace{\frac{\partial \bar{\rho}}{\partial \beta}}_{<0} < 0$$

Plugging these results into the Equations 3.55, 3.56, 3.57 and 3.58 we find:

2.A. If $\eta_{\bar{\rho},b^*} < \sqrt{\frac{\partial \eta_{\bar{\rho},b^*}}{\partial b} \frac{b^2}{p}} \equiv \bar{\eta}$ then $\frac{\partial \phi}{\partial b} > 0$, $\frac{\partial \phi}{\partial C} < 0$ and $\frac{\partial \phi}{\partial \beta} < 0$. Thus we can conclude that:

$$\frac{db}{d\beta} = -\frac{\frac{\partial \phi}{\partial \beta}}{\frac{\partial \phi}{\partial b}} > 0 \tag{3.62}$$

$$\frac{db}{dC} = -\frac{\frac{\partial \phi}{\partial C}}{\frac{\partial \phi}{\partial b}} > 0 \tag{3.63}$$

The signs are reversed if $\eta_{\bar{\rho},b^*} < \bar{\eta}$.

2.B. If $\eta_{\bar{\rho},b^*} < \sqrt{-\frac{\partial \eta_{\bar{\rho},b^*}}{\partial p} b} \equiv \hat{\eta}$, then $\frac{\partial \phi}{\partial p} < 0$ and thus we can conclude that if either $(\eta_{\bar{\rho},b^*}) < \min\{\bar{\eta}, \hat{\eta}\}$ or $\eta_{\bar{\rho},b^*} > \max\{\bar{\eta}, \hat{\eta}\}$:

$$\frac{db}{dp} = -\frac{\frac{\partial \phi}{\partial p}}{\frac{\partial \phi}{\partial b}} > 0 \tag{3.64}$$

The signs are reversed if $\min\{\bar{\eta}, \hat{\eta}\} < \eta_{\bar{\rho},b^*} < \max\{\bar{\eta}, \hat{\eta}\}$.

■

3.7.3 Proof of Proposition 10

Proof. The LHS (respectively RHS) of the FOC in Equation 3.16 is equal to the first term of the LHS (respectively RHS) of Equation 3.14. Indeed, these are the effects of the price on profits without considering the population of new entrants in the second period. Thus what makes p^* different from p^{**} are the terms referred to the second period demand (which depends on the spread of information in the network). Focusing only on these terms, if the negative effect of an increase in p in the LHS of Equation 3.14 is higher than the positive effect on the RHS, the marginal loss from the first period in p^* should be lower than the corresponding marginal benefits, thus the price p^* should be lower than the benchmark case p^{**}. Formally, taking the difference of these remaining effects and multiplying by $\frac{1}{(1-G(p^*))\bar{\rho}b^*}$ after some rearrangement we get that, if:

$$\eta_{p-b,p} < -(\eta_{(1-G),p} + \eta_{\bar{\rho},p}) \tag{3.65}$$

then the price running the program is lower than the one in the benchmark case (i.e.: $p^* < p^{**}$), higher otherwise.

■

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