Modeling of Spectrum Demands through Hybrids of Analytic Hierarchy Process and Integer Programming

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***Abstract*—As wireless technology advances, demand for spectrum bandwidth increases and thus, spectrum has become scarce. As a scarce resource, spectrum bandwidths need to be efficiently allocated to potential service providers. Hence, this paper first reviews current approaches on how spectrum bandwidths are being allocated to the service providers. Then we present an efficient integrated approach in allocating spectrum volumes, whereby the approaches of Analytic Hierarchy Process and Integer Programming are integrated and applied to produce systematic and consistent allocation results. The integrated approach is able to cater multi-criteria problems through determination of suitable weights and computations, which exhibits a more efficient alternative as compared to the existing approaches. The illustrations revealed that the integrated approach indeed has the potential to be implemented and gives an alternative to the current existing approaches.**

***Index Terms*—Spectrum allocation problem, integer programming, analytic hierarchy process**

I. Introduction

Resources are important things or elements to be allocated. Resource allocation involves assigning the available resources in an economic way and at the same time fulfilling certain organizational constraints [1]. The importance of resource allocation is evidenced by several resource allocation problems in the real world that were being studied by previous researchers. Several examples are emergency resource allocation in relation to missile allocation [2], resource allocation for emergency response after earthquake disaster [3], donors allocation [4], and multiple emergency resources allocation which include fire engines, fire trucks and ambulance [5].

In the manufacturing context, resource allocation strategies involve problems such as shelf allocation [6] and warehouse allocation [7]. In telecommunication domain, resources allocation dealt with a number of processes in a network of processors [8], strategies for real-time services [9], channel allocation ([10]; [11]; [12]), and spectrum demand allocation [13]. Other resource allocation works are such as resource allocation to optimize ship berthing [14] , airport slot allocation [15], and agriculture land allocation [16], to name a few.

A. Background of the Problem

Telecommunication industry is a fast-moving industry that may generate surprisingly large revenues through the management of spectrum as the main resource and thus, the focus of this resource allocation research. Spectrum refers to a collection of various types of electromagnetic radiations of different wavelengths [17]. The allocation process is the main efficiency issue in spectrum management. Allocation involves designating bands of spectrum for specific types of services or classes of users, such as designating certain bands for commercial use and others for government use.

With the advancement of a variety of mobile radio communication services [18], the introduction of new mobile radio communication services one after another [18], and also the upgrading of future generations of wireless technologies [19], the demand for spectrum increases. As a result, the available spectrum bands become scarce [20].

The scarcity of spectrum may also due to today's telecommunication trend, where current available telecommunication service providers are demanding for much spectrum simultaneously. At the same time, when a new service provider comes into the market, a lack of spectrum will occur as demands may outweigh the supply due to increased competitions.

B. Research Motivation

In order to develop and implement better wireless communication technologies, the spectrum or more efficient usage of available spectrum is needed. Hence, efficient utilization of the scarcely available radio spectrum becomes a fundamental problem as agreed by [18], [21] and [22]. Therefore, appropriate planning is required to maximize the efficient use of spectrum resources by planning the size of spectrum that needs to be allocated. Then, the regulating body (or regulator) will need to turn to an efficient approach or technique on spectrum allocation.

Moreover, finding the right approach for the optimal allocation of spectrum is crucial for several reasons as discussed below:

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1) Because frequencies differ in what they can do, there is a problem of matching them to particular uses [23].

2) Spectrum can be in short supply because there may be more potential users of particular frequencies than available spectrum. There is therefore a need for rationing its use and giving priority to more important applications [24].

3) Because spectrum is renewable and cannot be stored, there is no reason to hoard it for later use [25].

4) Because spectrum is location specific, it can only be used to provide services in a given territory [23].

Hence, the objectives of the paper are to address the issue and present an efficient approach in planning and allocating the spectrum in a beneficial way. Subsequently, the paper discusses the issues on current spectrum allocation approaches and some relevant works. It continues with the discussions on the proposed efficient approach, its results, analysis and the concluding remarks.

II. Current Spectrum Allocation Approach

It has been found that in current related literature ([13]; [26]) several approaches of assigning or awarding the spectrum licenses to service providers were being exercised such as the beauty contest, lottery and auction.

A beauty contest involves the allocation of spectrum by the regulating body with the proposed outlines and criteria to be followed in the selection process of the service provider. Beauty contest is also called comparative process, tender or bureaucratic method, in which the qualifications of each of the competing spectrum applicants are formally compared based on established and national criteria. Typically, these criteria might include population to be served, quality of service, and speed of service implementation. The regulating body then determines the best qualified applicant to use the spectrum and awards the license [27].

In a similar spectrum allocation problem in Sweden, [28] conducted a study using the beauty contest approach for the 3G licensing process. However, [29] argued that beauty contests are extremely time-consuming, wasteful and lack of transparency. Moreover, the method can be resource intensive, may not assign spectrum to those who value it most highly, and may not generate any revenues unless license fees and/or application fees are charged. Additionally, this comparative process is often decided on the basis of minor differences among applicants, and may cause the decision to be contested by unsuccessful applicants [27]. Based on some of these reasons, [29] is in favor of lottery which is a qualitative approach involving some kind of element of fate or luck. Reference [29] agreed that the lottery approach could be successful in assigning licenses quickly since the prospect of a windfall gain may attract a large number of applicants.

Specifically, in a lottery, licensees are selected at random from among all competing spectrum applicants [30]. Lotteries can decrease some aspects of the administrative burden entailed in comparative hearings, such as legal expenses, but may create a different kind of administrative burden by encouraging more applications to be filed. In addition, lotteries do not assign spectrum to those who value it most highly, except by chance, lead to significant transaction costs, and again generate no revenues, unless fees are attached to the license assigned by lottery or an entry fee to participate in the lottery is charged. In some instances, lottery winners in many cases transfer their spectrum rights to other parties, thus capturing the resource rents for themselves. Thus lotteries, without significant application fees or other measures that guarantee the applicants' intent to provide spectrum services, tend to encourage speculation.

On the other hand, auctions represent a new form of license mechanism where the applicants determine the spectrum value to be charged. An auction is a multi-criteria decision making approach in which a regulating body could adopt to select the best service providers among the competing ones. The auction approach has its advantages. For one, it is a transparent exercise which involves multiple criteria normally set by the spectrum regulator. It has the tendency to assign the spectrum to those who will be able to demonstrate the best usage of the spectrum.

In the process, licenses are awarded through bidding among competing spectrum applicants. Thus, auctions award licenses to those who value them most highly, while simultaneously generating revenues for the spectrum authority which can be used to offset distorted taxation. Other advantages of the auction approach are that it holds potential for an accurate reflection of the value of the spectrum and it imposes costs on those who directly benefit from spectrum use. Auction approaches may significantly decrease the administrative costs and time associated with the spectrum assignment process and therefore improve overall administrative efficiency in contrast to beauty contest [30]. Most economists claim that auction is better in contrast to the beauty contest because it offers a more market-oriented, objective and transparent approach for awarding spectrum licenses ([26]; [28]; [30]; [31]). Spectrum license auctions are widely recognized by economists as more efficient than lotteries or beauty contest to allocate exclusive right to spectrum [32].

However, one of the drawbacks of auction is that auctions may increase the price that telephone-service customers ultimately pay [30]. Moreover, the higher fees needed to apply for a license may discourage participation, especially by new entrants [28]. Besides that, auction also needs more times and involves more cost before the results are obtained. This will be a liability to the company. Reference [29] also notes that there are some problems associated with the auction approach. One of them relates to the simultaneous ascending auction's vulnerability to revenue-reducing strategies in situations where competition is weak. Bidders have an incentive to reduce their demands in order to keep prices low, and to use bid signaling strategies to coordinate on a split of the licenses.

In the case with an unrestricted spectrum market, auctions may raise competitive concerns if not combined with an active competition policy and limits on how much spectrum an entity may purchase. Market forces do not ensure economic efficiency or maximize consumer welfare in markets that are not competitive because a dominant service provider or a group of providers have market power. In addition, auctions may fail to adequately provide certain socially desirable services or distribute licenses to certain groups, such as small businesses (if that is an objective). Also, auctions may be inefficient or impractical for certain services or situations. One case is where there is no competition for spectrum. A second case is where providers of socially desirable spectrum-using services such as national defense or scientific research may have difficulties in placing a financial value on spectrum, which could lead to those services being under-provided to society if all providers of spectrum-using services faced auctions. Lastly, a disadvantage of this approach is that revenues are uncertain, and may exceed or fall short of what is needed to adequately fund spectrum management. Spectrum managers could attempt to ensure that revenues would be sufficient by establishing minimum bid amounts; however, if these amounts were set too high, no bids would be received [33].

Judging from the three approaches as discussed, it is understood that only the service providers are evaluated in deciding whether they are qualified to be awarded the spectrum volumes. The evaluation process involves in the three approaches being discussed does take certain determining criteria into consideration, but only subjective judgments were being exercised. When there are competing interests for specific spectrum, the regulating body must determine which use or uses of the spectrum will best serve the public interest.

These criteria for allocating spectrum are such as economic efficiency, promotion of competition, fairness, revenue maximization, fulfillment of specific requirements, encouraging innovation and investment in the telecommunication sector [13], and encouraging green technology [42] as elaborated below:

1) Economic efficiency – to allocate the licenses to the players who will use them most efficiently, i.e. to be able to generate most value ([34]; [35]).
2) Promotion of competition – to provide a sound competitive market structure as an outcome of the spectrum allocation process ([35]; [36]).
3) Fairness – to ensure a transparent and objective process of allocation, so that applicants know in advance the basis upon which they will compete [37].
4) Revenue maximization – to maximize revenue to the government from the process [38].
5) Specific requirements regarding geographic coverage, obligations relating to the speed and cost of rollout, and obligations relating to quality of service [35].
6) Encouraging innovation and investment in the telecommunication sector ([13]; [36]).
7) Encouraging green technology [29].

When making decision on which best service provider to be awarded the spectrum volumes using any of the qualitative approaches, it is rather difficult to combine and consider all those criteria simultaneously. Therefore, an efficient approach or technique is seek for as an alternative, which is a mathematical programming based approach since it can handle problems with multiple objectives and multiple criteria or constraints.

III. Some Related Work

As there is no previous work on spectrum allocation using any efficient approach such as the mathematical programming (MP) techniques (to our knowledge), some similar work on allocating scarce resources like energy and land is referred to. The work by [16] dealt with allocation of land areas for development using a multi-objective integer programming (MIP) model. The criteria considered in this allocation problem are cost, proximity to desirable and undesirable land features, and the shape of area, which were successfully applied for a development area in Tennessee.

When there is a need to quantify subjective judgments which involve multi-criteria decision making, the Analytic Hierarchy Process (AHP) is an effective method [39]. In the work of [40], they combined AHP with integer programming (IP) to aid the land use planning. The AHP was applied first in order to obtain the relative importance weights of alternative interest groups. Then, the weights were utilized as weighting factors in the objective function of the IP model, which aims at maximizing the consensus among the interest groups.

In a similar approach, [41] integrated AHP with a Linear Programming (LP) technique on an allocation problem for alternative material handling devices. The criteria involved are cost, benefit, and compatibility of each device with respect to each manufacturing cell, whereby the objective is to select a set of devices with the maximum weights.

IV. Research Process

In the current allocation process, certain criteria were considered throughout the process but in a subjective manner until the final decision is made. The regulating body may have different preferences or judgments for different licensees in relation to each of these criteria when exercising the process of spectrum allocation. Hence, the integrated approach is turned to as an alternative to solve problem of allocating the spectrum to service providers by considering all related criteria so that the allocation can be more efficiently made. The proposed approach quantifies qualitative judgments into the integrated computation of AHP and IP.

A case problem was chosen to exhibit the feasibility of the proposed integrated approach but the criteria considered in this problem were not quite clear. Thus, based on the case and enhancement from the literature, we conclude that seven criteria and four service providers are to be considered in the proposed decision making process of allocating spectrum. A service provider or licensee is the company which provides the telecommunication service. For the purpose the research, we named the four licensees as A, B, C, and D.

These criteria are economic efficiency, promotion of competition, fairness, revenue maximization, quality of service, encouragement for innovation and investment, and encouragement for green technology. The first sixth criteria as discussed above were all obtained from reviews of relevant literature. However, the seventh criterion was decided based on discussion with a number of experts in the regulating body, which actually has never been applied before but would be good to include since everyone should support the existence of green environment. Therefore, this is a new criterion being considered, which results in the enhanced integrated model. For details of each criterion please refer to [42].

Currently, allocation of spectrum bandwidths involves various ranges or sizes of the bandwidth. However, for the purpose of illustrating the proposed approach, only spectrum in Global System for Mobile Communication (GSM) in 2G is considered. All of these licensees provide 2G telecommunication, which can be categorized into two types. The first type is called Type I, which is for bandwidth in the range of 800-900MHz. The second type is called Type II with the bandwidth in the range of 1800-1900MHz. The model can be extended to any similar situation in other categories if needed. All data regarding the allocation of spectrum was obtained from the regulating body and as similar to the real situation where possible. The proposed integrated model is constructed based on the following procedures as exhibited in Fig. 1. However, when adopting the MP approaches, different but similar integrated models need to be constructed for each type of the bandwidth range. We thus illustrate the integrated model for Type I only for discussion in this paper as it is similar.

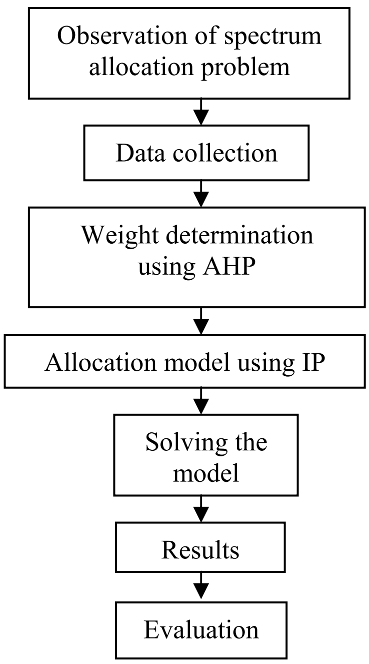

Fig. 1. Structure chart of work stages in spectrum allocation

V. The Proposed Integrated Model

The proposed integrated model allows the AHP component to be embedded in the IP model. The function of AHP is to obtain the weights for each criterion and licensee accordingly, which then were used as the coefficients in the IP formulations.

A. The IP Model

The modeling components are described below which include definition of variables, parameters, objective function, and constraints function. The IP model is constructed such that the expected volumes of all licensees are fulfilled.

x_{ij} = number of spectrum packets from bandwidth type i allocated to licensee j;
i = type of bandwidth in 2G, $i = 1, \ldots, I$;
j = type of licensee, $j = 1, 2, \ldots, J$;
k = type of criteria, $k = 1, 2, \ldots, K$;
p = number of available packets, where each packet is 2.5 MHz in width;
α_k = weight for criteria k;
C_{ijk} = weight for licensee j allocated to bandwidth of type i with respect to criteria k;
R_{ij} = required number of packets for bandwidth type i requested by licensee j;
E_{ij} = expected number of packets for bandwidth type i requested by licensee j.

The objective function (1) maximizes Z, the total efficiency that are obtained based on each of the specific criteria, i.e. economic efficiency, promotion of competition, fairness, revenue maximization, quality of service, encouragement for innovation and investment, and encouragement for green technology.

$$\text{Maximize } Z = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \alpha_k C_{ijk} x_{ij} \quad (1)$$

Subject to

$$\sum_{j=1}^{J} x_{ij} \le p, \forall_i, i = 1, 2, \ldots, I \quad (2)$$

$$x_{ij} \ge R_{ij}; \quad (3)$$

$$x_{ij} \le E_{ij}; \quad (4)$$

$$x_{ij} \ge 0 \text{ for each } i, j. \quad (5)$$

Constraint (2) enforces that the licensee j be allocated each type of spectrum bandwidth i which cannot exceed the available number of packets p or supply volumes of spectrum.

This supply volume should be revised from time to time by the regulating body and must take into consideration the volume to be reserved for other services. A market mechanism would almost certainly result in the relevant spectrum being used for other, more commercial purposes. On the other hand, political and security concerns may well result in an over-allocation of spectrum to these services, and the use of the spectrum by public service operators may become very wasteful if there is no mechanism to enforce the most efficient use of frequencies. Thus, P must be determined efficiently.

Constraint (3) means that the allocation of spectrum bandwidth type i to licensee j must be at least equal to the required number of packets. The required volume is the minimum spectrum requirement that is needed for the licensee to be operable and needs to be granted. Failing to secure this required volume would result in the licensee not being able to function. On the other hand, constraint (4) refers to the allocation of spectrum bandwidth type i to licensee j that could fulfill the expected number of packets. This expected number is the spectrum volume level that would guarantee the licensee to operate at the utmost best, but may not necessarily be fulfilled in the solution search by the model. Both constraints (3) and (4) are demand constraints. Finally, constraint (5) ensures the non-negativity compliance.

B. The AHP Component

The first step in AHP is to develop a hierarchy structure, which consists of three main levels, i.e., goal setting, selection criteria and followed by decision alternatives. The decision alternatives and selection criteria should be clearly determined in order to get a clear picture of the problem. There are two sets of weight needed for this case problem, which are weights for criteria and weights for licensees.

Weight is the relative importance granted to elements of criteria and licensees based on the judgments of the decision makers, who are the members of the regulating body. The numerical values to relate to these weights were obtained based on the AHP scale of 1 – 9 through a series of pair-wise comparisons. The process continued to the synthesization stage and then to the consistency test. In the synthesization stage, the values were transformed into comparison matrices which were then computed through a normalization process to obtain weights and priorities for each of the criteria. A consistency test is important in the AHP which measure the degree of inconsistency in pair-wise comparisons. Please refer to [43] for details of the technique. These consistent weights were then used as coefficients in the objective function (1).

VI. Results And Analysis

The initial solutions to the integrated model for bandwidth of Type I are in term of weights or relative importance, which were obtained through a series of hierarchical process and computations of the AHP. Prior ratings for pair-wise comparison purposes were simulated in order for the process to take place. The efficiency weights or relative importance of each of the seven criteria is as shown in Table 1. Based on simulated decision makers' judgments, licensee A is the best in term of economic efficiency, while licensee B is the best in term of promotion of competition, fairness, revenue maximization and also encouragement for green technology. On the other hand, licensee D is the best in term quality of service and encouragement for innovation and investment.

TABLE 1: Weights or Relative Importance of Each Criterion

Criterion	Weight
Economic efficiency	0.3504
Promotion of competition	0.0462
Fairness	0.1590
Revenue maximization	0.0318
Quality of service	0.0696
Encouragement for innovation and investment	0.1056
Encouragement for green technology	0.2375

Through another set of computations of the AHP, we obtained the coefficient values or efficiency weights for each licensee in relation to each criterion, which are presented in Table 2. For example, in term of economic efficiency, licensee A scores first, licensee C scores second, licensee B scores third, and licensee D scores last. The description continues similarly with the rest of the criteria.

TABLE 2: Overall Weights for Each Licensee Based ON Criteria in Type I Bandwidth

Criteria	A	B	C	D
Economic efficiency	0.49	0.13	0.31	0.08
Promotion of competition	0.08	0.54	0.14	0.23
Fairness	0.24	0.55	0.16	0.05
Revenue maximization	0.28	0.47	0.17	0.07
Quality of service	0.26	0.07	0.11	0.56
Encouragement for innovation and investment	0.28	0.10	0.16	0.47
Encouragement for green technology	0.28	0.47	0.10	0.16

Subsequently, in implementing the proposed integrated model, we specify all the required parameters that are $I = 1$ (i.e. for Type I only), $J = 4$, $K = 7$, and $p = 40$ as currently observed. The values for parameters R and E for all i and j are as given in Table 3.

TABLE 3: Supply Volume, Required and Expected Demands of Spectrum for Type I Bandwidth

Licensee	Type I	
	Required demand, R	Expected demand, E
A	14	15
B	10	14
C	7	14
D	8	13
Supply volume	40	

After all parameters have been inserted, the integrated IP model was run using LINGO 8.0 optimization software. The results or outputs of the proposed model are shown in relation to each decision variable, which represents the number of spectrum packets that has been successfully allocated to each relevant licensee as presented in Table 4. The optimal total efficiency scored when adopting the proposed model is 21.0835.

TABLE 4: Results of the Proposed Integrated Model

Decision Alternatives	**Number of spectrum packets**
Licensee A	15
Licensee B	10
Licensee C	7
Licensee D	8

All results analyzed are for bandwidths of Type I, where we can conclude that all licensees are able obtain their required demands for the spectrum. However, licensee A is granted with its expected demand (i.e., 15) in which it can operate at its utmost best, while licensees B, C, and D are only allocated with the minimum required demands, i.e. 10, 7, and 8 respectively.

VII. Discussion and Conclusions

We have successfully developed an integrated model of AHP and IP to allocate spectrum volumes to all potential licensees as based on certain criteria, thus presenting it as the efficient approach. The integrated model is a much more efficient alternative as compared to the existing approaches. Moreover, we have succeeded in identifying a new and important criterion, i.e. encouraging green technology, when conducting the related research. Thus, this resulted in the enhanced integrated model. Based on the illustrations given we can see that ILP can easily be developed. The advantage of this integrated approach is that it is able to combine subjective judgments computation with a mathematically formulated approach to produce a systematic and consistent result. However, these relevant judgments need careful evaluation by the regulating body as the authoritative decision maker, in which the judgments are transformed into weights. Some weights can be judged explicitly based on exact values, whereas some other criteria might have to be judged based on perceptions.

In addition, some elements of the integrated model can be manipulated and potential results can be evaluated such that further insights obtained can aid in meaningful decision makings. The beauty about applying IP is that several what-if analyses can be done easily using LINGO 8.0 software or any other optimization software available in the market. One potential further evaluation that can be done is when the objective of the regulating body is slightly changed to accommodate different demand constraints, which requires minor model modification. Other example is one can see the effect of changing the spectrum availability on the final spectrum allocation. Also, if we change the weight combinations of the factors, the spectrum allocation result would also change.

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