**To link to this article** :
**DOI:**10.1063/1.2911046:
**URL** : http://dx.doi.org/10.1063/1.2911046

# Thermosolutal convection in a horizontal porous layer heated from below in the presence of a horizontal through flow

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In this paper, we study the effect of a homogeneous longitudinal through flow on the onset of convection in a horizontal porous layer saturated by a binary fluid and heated from below or above. The layer boundaries are subjected to a constant heat flux. The investigation is made by taking the Soret effect into account. It is found that in the case of positive separation ratio when the denser component moves toward the cooler wall, through flow has no effect on the stability threshold but exerts an orientating effect on the convective patterns. For negative separation ratio, a strong destabilization occurs of the spatially homogeneous state with respect to long-wave disturbances. The stability range for long-wavelength convective rolls is defined.

## I. INTRODUCTION

Thermosolutal convection in a porous medium is a classical example of the problems that reveal the interaction of different instability mechanisms. A great number of works in this field of research are devoted to investigations into the equilibrium stability of a binary mixture in a vertical temperature gradient. In this case, the instability may occur for heating both from below and from above.[1] For the case of double diffusive convection, in which inhomogeneity of the concentration field is caused by the generation of a concentration difference at the boundaries, the problem of mechanical equilibrium stability is investigated in Refs. 1–6. These works show that, as in the case of a homogeneous binary mixture, the monotonic and the oscillatory instability may arise for heating both from below and from above. For the case in which the concentration inhomogeneity occurs due to a thermodiffusion effect, stability of the mechanical equilibrium of a binary mixture in a horizontal porous layer is studied in Refs. 7 and 8 for high thermal conductivity boundaries and in Refs. 9 and 10 for low thermal conductivity boundaries (in the conditions of fixed thermal flux). A distinguishing feature of the problem with fixed thermal flux at the boundaries is that under such conditions, the long-wave instability may exist in a wide range of parameters. Through flow in the horizontal direction leads to a shift of disturbances and to a transformation of the monotonic instability into oscillatory instability. The influence of horizontal through flow on the stability of the horizontally homogeneous state at fixed temperatures at the boundaries is considered in Refs. 11 and 12 for the case of homogeneous binary fluid, in Ref. 13 for a porous medium saturated by a single-component fluid, and in Ref. 14 for a porous medium saturated with a binary fluid. Thus far, there have been no works considering the influence of horizontal through flow on the long-wave modes of instability under conditions of fixed thermal flux at the boundaries and the present study seems to be the first work on this subject.

The problem under consideration can find many industrial applications related to such processes as ingress of moisture into thermal insulation materials, spread of wastes in the soil, or food processing. The solution of this problem will help scientists to take up the challenge of radioactive and chemical waste disposal and purification of contaminated soils.

## II. MATHEMATICAL FORMULATION

We consider an isotropic and homogeneous plane horizontal porous layer heated from below and saturated with a binary mixture. There is a through flow of the binary mixture in a horizontal direction. The problem is examined by taking into account the Soret effect. Let us introduce a Cartesian rectangular coordinate system such that the $z$-axis is directed vertically upward and the $x$- and $y$-axes along the horizontal plane. The origin of the coordinates is located at equal distance from the horizontal boundaries of the layer. We assume that the Darcy law is valid and that Oberbeck–Boussinesq approximation[1] is applicable: The thermophysical properties of the binary fluid are considered constant except for the density in the buoyancy term, which linearly varies with the local temperature and mass fraction.

Thus, the governing conservation equations for mass, momentum, energy, and chemical species with the Soret effect taken into account are

$$-\frac{1}{\rho}\nabla p-\frac{\nu}{K}\vec{u}+(g\beta_T T+g\beta_C C)\vec{e}_z=0, \tag{1}$$

$$\sigma\frac{\partial T}{\partial t}+\vec{u}\cdot\nabla T=\chi_{\text{eff}}\nabla^2 T, \tag{2}$$

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$$\varepsilon\frac{\partial C}{\partial t}+\vec{u}\cdot\nabla C=D\nabla^2 C+\alpha D\nabla^2 T, \tag{3}$$

$$\nabla\cdot\vec{u}=0. \tag{4}$$

Here, $\vec{u}$ is the filtration rate, $C$ is the mass fraction of the denser component, $K$ is the coefficient of permeability, $\beta_T,\beta_C$ are volume coefficients of thermal and solutal expansions, $\sigma$ is the ratio of the heat capacity of a unit volume of porous medium saturated with a fluid to that of homogeneous fluid, $\chi_{\rm eff}$ is the effective thermal diffusivity, $\varepsilon$ is the porosity, $D$ is the mass-diffusion coefficient of the denser component diffusion, $\alpha$ is the Soret coefficient, $\vec{e}_z$ is the unit vector of the $z$-axis, and the rest of the notation is standard.

The problem is assumed to satisfy the following boundary conditions: at the solid boundaries of the layer, the normal component of the filtration velocity vanishes, the heat flux is fixed, the flux of the substance is absent, and the mass flow rate along the $x$-axis is fixed,

$$z=-h,\quad h{:}w=0,$$

$$\frac{\partial T}{\partial z}=-A,\quad \frac{\partial C}{\partial z}=-\alpha\frac{\partial T}{\partial z}, \tag{5}$$

$$\int_{-h}^{h}\langle u_x\rangle_y dz=2Uh,\quad \int_{-h}^{h}\langle u_y\rangle_x dz=0,$$

where $h$ is the half-thickness of the layer, $w$ is the vertical component of the filtration velocity, $A$ is the magnitude of the prescribed temperature gradient, $U$ is the through flow velocity, and notation $\langle\cdots\rangle_f\equiv\lim_{l\to\infty}(1/l\int_{-l/2}^{l/2}\cdots df)$, where $f=x,y$ is introduced for the procedure of averaging over the horizontal coordinates.

Note that according to Eqs. (2) and (3), the concentration disturbances are transported by the fluid flow with the velocity $u/\varepsilon$, i.e., with an average velocity of the fluid flow in pores, whereas the temperature disturbances are transported with the velocity $u/\sigma$. This difference is related to the fact that during transport of the mixture, the fluid motion occurs only in pores, and during heat transfer, the heat is transported not only through the fluid but also through the porous matrix. In a homogeneous fluid, such a difference in velocities is absent.

The through flow velocity is conveniently derived from the filtration velocity, $\vec{u}=\vec{u}'+U\vec{e}_x$, where $\vec{e}_x$ is the unit vector of the $x$-axis (in the following, the prime mark will be omitted). After such substitution, the integral condition in Eq. (5) becomes uniform, though in Eqs. (1)–(3), additional terms appear. Nonuniformity in Eq. (1) can be readily eliminated through redefining the pressure, and additional terms in Eqs. (2) and (3) can be removed by choosing an appropriate frame of reference (without further transformation of velocity). However, this cannot be simultaneously accomplished in both Eqs. (2) and (3). In the following, the reference frame moving along the $x$-axis with velocity $U/\sigma$ will be called reference frame A [in this reference frame an additional term vanishes from Eq. (2)], and the reference frame moving with velocity $U/\varepsilon$ is called reference frame B [in this reference frame, an additional term vanishes from Eq. (3)].

Let the units of length, time, velocity, temperature, concentration, and pressure be defined as follows:

$$h,\quad \frac{\varepsilon h^2}{D},\quad \frac{D}{h},\quad Ah,\quad \frac{\beta_T}{\beta_C}Ah,\quad \frac{\nu\rho D}{K}.$$

The equations and boundary conditions in dimensionless form are written as

$$-\nabla p-\vec{u}+R(T+C)\vec{e}_z=0, \tag{6}$$

$$b\frac{\partial T}{\partial t}+\vec{u}\cdot\nabla T+\mathrm{Pe}\frac{\partial T}{\partial x}=\mathrm{Le}\nabla^2 T, \tag{7}$$

$$\frac{\partial C}{\partial t}+\vec{u}\cdot\nabla C+\mathrm{Pe}\frac{\partial C}{\partial x}=\nabla^2 C-S\nabla^2 T, \tag{8}$$

$$\nabla\cdot\vec{u}=0, \tag{9}$$

$$z=-1,\quad 1{:}w=0,$$

$$\frac{\partial T}{\partial z}=-1,\quad \frac{\partial C}{\partial z}=-S, \tag{10}$$

$$\int_{-1}^{1}\langle u_x\rangle_y dz=0,\quad \int_{-1}^{1}\langle u_y\rangle_x dz=0.$$

Problem (6)–(10) is characterized by the following dimensionless parameters:

$$R=\frac{g\beta_T AKh^2}{\nu D},$$

$$\mathrm{Pe}=\frac{Uh}{D},$$

$$\mathrm{Le}=\frac{\chi}{D}, \tag{11}$$

$$S=-\alpha\frac{\beta_C}{\beta_T},$$

$$b=\frac{\sigma}{\varepsilon},$$

where Pe is the Peclet number, having the meaning of dimensionless through flow velocity, Le is the Lewis number, and $S$ is the separation ratio; the parameter $R$ is related to the Rayleigh number commonly used for a porous medium $Ra_p=4g\beta_T AKh^2/(\nu\chi)$ by the equation $R=Ra_p\mathrm{Le}/4$.

Problem (6)–(10) allows for a simple stationary solution,

$$\vec{u}=0,\quad T=-z,\quad C=-Sz \tag{12}$$

(the values of temperature and concentration at $z=0$ in the convectionless state are used as the reference values of temperature and concentration).

The solution (12) describes the state of homogeneous through flow at uniform vertical gradients of temperature and concentration. Since, in this case, the fluid motion occurs along the isotherms and isolines of concentration, the temperature and concentration fields are not disturbed. In the following sections, we will investigate the stability of this state against small disturbances and weakly nonlinear regimes.

## III. LINEAR STABILITY OF THE BASIC STATE

### A. Disturbances with finite wavelength

Let us make a linear stability analysis of the base state stability to plane disturbances with finite wavelength. By linearizing Eqs. (6)–(9) in the vicinity of the base state (12) and neglecting pressure and the horizontal velocity components, we obtain the system of equations

$$\Delta w = R\Delta_{\perp}(\vartheta + c), \tag{13}$$

$$b\frac{\partial \vartheta}{\partial t} + \mathrm{Pe}\frac{\partial \vartheta}{\partial x} = \mathrm{Le}\nabla^2\vartheta + w, \tag{14}$$

$$\frac{\partial c}{\partial t} + \mathrm{Pe}\frac{\partial c}{\partial x} = \nabla^2 c - S\nabla^2\vartheta + Sw, \tag{15}$$

with the boundary conditions

$$z = -1,1{:}w = 0, \quad \frac{\partial \vartheta}{\partial z} = 0, \quad \frac{\partial c}{\partial z} = 0. \tag{16}$$

Here, $\vartheta$ and $c$ are the disturbances of temperature and concentration, respectively, and $\Delta_{\perp}$ is the Laplace operator with respect to the horizontal coordinates.

For normal disturbances depending on $x$ and $t$ as $e^{\lambda t}e^{ikx}$ (where $\lambda$ is the complex increment and $k$ is the wave number), we obtain a system of linear differential equations with constant coefficients. The characteristic equation of this system is a bicubic one. By solving this equation, constructing the fundamental system, and satisfying boundary conditions, we obtain the cumbersome transcendental equation for $\lambda$. The condition that the real part of the increment should be equal to zero defines the boundary of linear stability of the base state. The equation for $\lambda$ has been numerically solved.

At zero value of the Soret parameter, the equation for concentration disturbances splits off and, as can be easily shown, has only decaying solutions. After substitution of $\lambda - ik\mathrm{Pe}/b$ for $\lambda$ (i.e., after passing to reference frame A) and scale transformation, the obtained problem for $w, \vartheta$ reduces to the classical problem on equilibrium stability of the porous layer heated from below. This means that at $S=0$, the critical value of the Rayleigh number is independent of the Peclet number, although the instability in the laboratory reference frame is of an oscillatory nature and the phase velocity of disturbances coincides with the transport velocity of thermal disturbances.

#### 1. Case of positive separation ratio

Now, let us discuss the results of the calculation for positive values of the separation ratio. Figures 1 and 2 show the neutral curve $R(k)$ and the dependence of phase velocity of neutral disturbances on the wave number for different values of the parameter Pe. When through flow is absent ($\mathrm{Pe}=0$), the instability is monotonic, and it is the long-wave disturbances that are most dangerous (curve 1 in Fig. 1). At nonzero values of the Peclet number, the character of the instability changes to an oscillatory one so that, in this case, at all $\mathrm{Pe} \neq 0$, the disturbances with finite wavelength are most dangerous. It should be noted that for a critical value of the parameter $R$, there is a finite limit at $k \to 0$, which does not depend on Pe but differs from the stability threshold at $\mathrm{Pe}=0$. Thus, we are dealing here with the crossover effect: The result of a sequence of limiting transitions, $\mathrm{Pe} \to 0$, $k \to 0$, depends on the order of their realization. A detailed consideration of this phenomenon will be given in the section concerning long-wave disturbances.

Actually, the phase velocity (Fig. 2) does not vary with $k$ and is very close to the value of Pe, suggesting that the oscillatory character of the instability should be attributed to the transport of solute by the base flow.

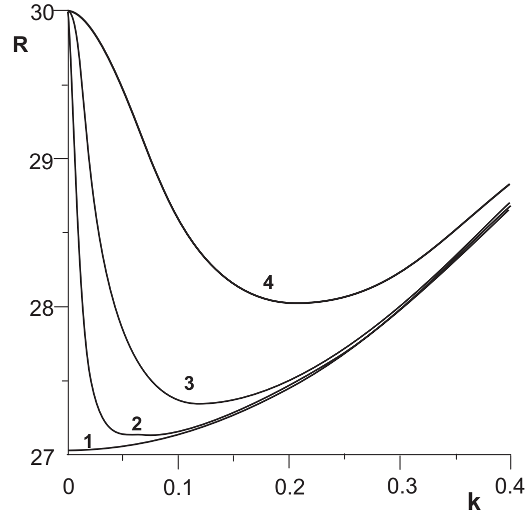


FIG. 1. Neutral curves in the case of positive separation ratio at $b=10$, $\mathrm{Le}=100$, $S=0.1$, and different values of Peclet number: (1) $\mathrm{Pe}=0$, (2) $\mathrm{Pe}=0.111$, (3) $\mathrm{Pe}=0.333$, and (4) $\mathrm{Pe}=1.11$.

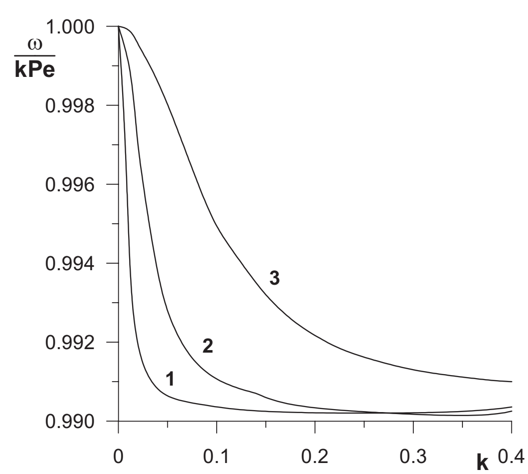


FIG. 2. Phase velocity of neutral disturbances vs wave number in the case of positive separation ratio at $b=10$, $\mathrm{Le}=100$, $S=0.1$ and different values of Peclet number: (1) $\mathrm{Pe}=0.111$, (2) $\mathrm{Pe}=0.333$, and (3) $\mathrm{Pe}=1.11$.

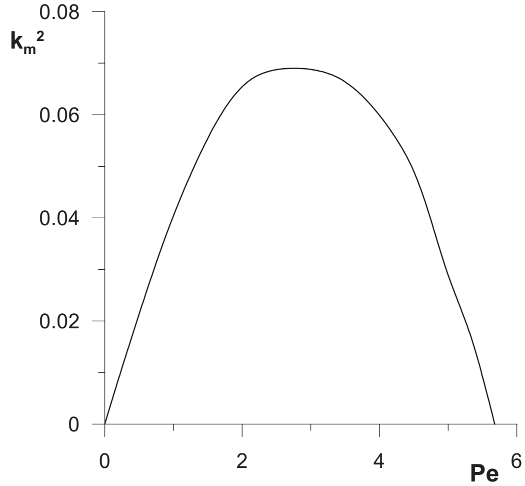


FIG. 3. Wave number of critical disturbances as a function of Peclet number in the case of positive separation ratio at $b=10$, Le=100, and $S=0.1$.

The dependence of the wave number of the most dangerous disturbances $k_m$ on the Peclet number is nonmonotonic (Fig. 3). With an increase of the through flow velocity, $k_m$ first increases, then starts to decrease, and, at some value $\mathrm{Pe}=\mathrm{Pe}_*$, goes to zero. This value of Pe will be defined later in the section dealing with long-wave asymptotics. At $\mathrm{Pe}>\mathrm{Pe}_*$, the long-wave disturbances are most dangerous.

The critical value $R_m$ of the parameter $R$, corresponding to the minimum on the neutral curve, as a function of the Peclet number is plotted in Fig. 4. As can be seen, at $\mathrm{Pe}<\mathrm{Pe}_*$, the value of $R_m$ increases with the growth of Pe; at $\mathrm{Pe}>\mathrm{Pe}_*$, the instability threshold is independent of the Peclet number.

### *2. Case of negative separation ratio*

Let us discuss the results for negative values of the separation ratio. As we know, in the absence of through flow[7–10] at negative values of the Soret coefficient, the instability can occur on heating both from below and from above. We first

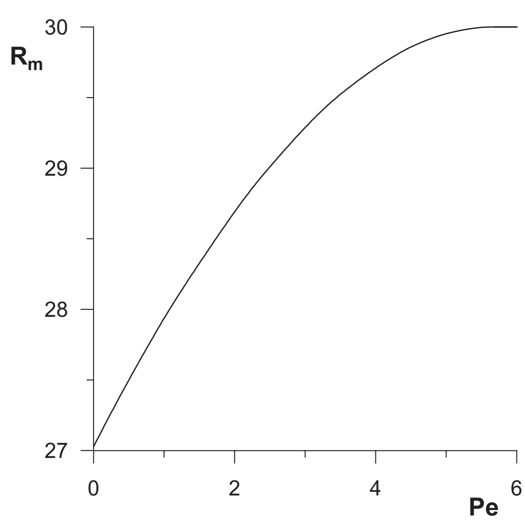


FIG. 4. Critical Rayleigh number $R_m$ as a function of Peclet number in the case of positive separation ratio at $b=10$, Le=100, and $S=0.1$.

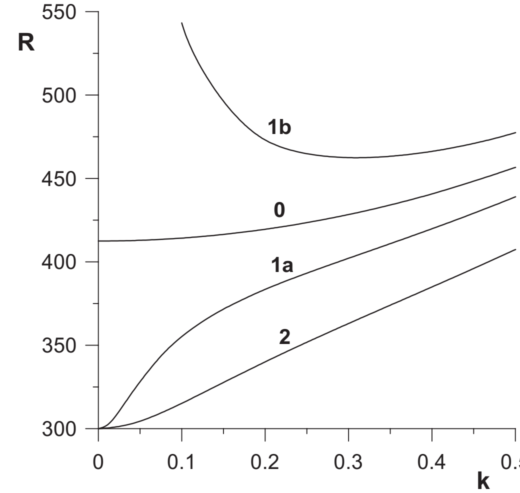


FIG. 5. Neutral curves in the case of negative separation ratio and heating from below at $b=10$, Le=100, $S=-0.02$, and different values of Peclet number: (0) Pe=0, (1a and 1b) Pe=0.333, and (2) Pe=1.11.

consider the case of heating from below ($R>0$). The neutral curves of stability for this case are given in Fig. 5. In the absence of through flow at preset values of the parameters $S$, Le, and $b$, the instability is of an oscillatory character with its lower level showing double degeneracy: the waves propagating in the negative and positive direction of the $x$-axis are equal. At the values of Pe other than zero, this equality is lost and the oscillatory level splits into two levels (see curves 1a and 1b). Note that in the laboratory reference frame, the direction of waves corresponding to the most dangerous disturbances coincides with the direction of through flow. As in the case of positive $S$, the crossover phenomenon can be observed: All neutral curves corresponding to nonzero values of the Peclet number have a limit at $k\to 0$, which does not depend on the Peclet number and differs from the long-wave limit at Pe=0. However, at $S<0$ for low levels of instability, the long-wave limit is lower than in the case of zero through flow and, as a result, at $\mathrm{Pe}\neq 0$, the instability maintains the long-wave character.

In the absence of through flow, for the most dangerous disturbances, the disturbance frequency at $k\to 0$ tends to zero according to the square law, so that the phase velocity goes to zero. At $\mathrm{Pe}\neq 0$, the phase velocity in reference frame A also goes to zero, as will be shown later. Thus, in the laboratory reference frame, the phase velocity of the most dangerous disturbances will coincide with the transport velocity of the thermal disturbances, which is equal to $\mathrm{Pe}/b$. The dependence of the phase velocity of disturbances on the wave number is shown in Fig. 6.

Thus, with heating from below and for a positive separation ratio, the effect of the instability is of a solutal nature and through flow has a stabilizing effect, whereas for negative separation ratio, the most dangerous disturbances are thermal disturbances, on which through flow exerts a destabilizing action.

Now, consider the case of heating from above. For a negative separation ratio and in the absence of through flow, heating from above may give rise to monotonic instability.

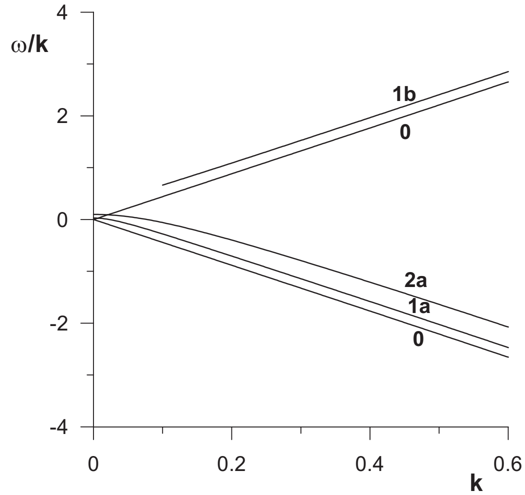


FIG. 6. Phase velocity of neutral disturbances vs wave number in the case of negative separation ratio and heating from below at $b=10$, Le=100, $S=-0.02$, and different values of Peclet number: (0) Pe=0, (1a and 1b) Pe=0.333, and (2a) Pe=1.11.

Its low level, corresponding to the Rayleigh number with the lowest absolute value, is not degenerate. Therefore, at nonzero values of the Peclet number and for a positive separation ratio, splitting of the levels is not observed but disturbances acquire an oscillatory character. The calculations show that in this case, through flow has a destabilizing effect and, at all values of the Peclet number, the long-wave disturbances are the most dangerous (see Fig. 7).

The results described refer to plane disturbances. However, it is easy to demonstrate that we can apply the analog of the Squire theorem to the problem under consideration. Indeed, in Eqs. (13)–(15), expressed in terms of disturbances of temperature, concentration, and vertical velocity component, the derivatives with respect to the horizontal coordinates enter either the Laplace operator or the combination $\mathrm{Pe}(\partial/\partial x)$. This means that the equation for the amplitudes of normal space disturbances differs from the equation for plane

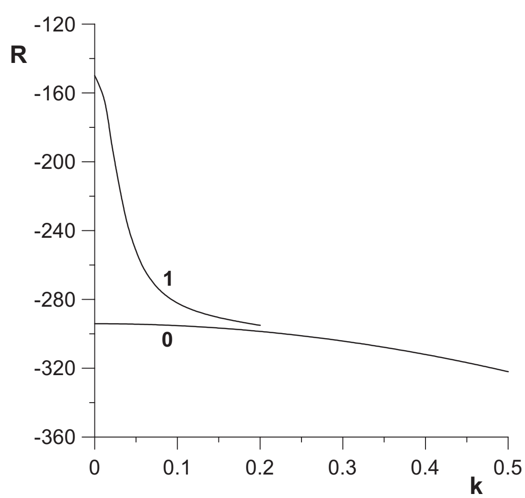


FIG. 7. Neutral curves in the case of negative separation ratio and heating from above at $b=10$, Le=100, $S=-0.02$, and different values of Peclet number: (0) Pe=0 and (1) Pe=0.333.

normal disturbances in that the Peclet number is replaced by the combination $\mathrm{Pe}\cos\varphi$, where $\varphi$ is the angle between the wave vector and the direction of through flow. Thus, the results for space disturbances coincide with the results for plane disturbances corresponding to the lower value of the Peclet number. This implies that for $S>0$ when through flow exerts a stabilizing effect on the plane disturbances, the space disturbances are more dangerous than the plane ones. From the results obtained, it might be concluded that the most dangerous perturbations are the longitudinal rolls, i.e., the convective rolls whose axes are parallel to the through flow direction. Such disturbances are not affected by through flow. Thus, in the case of positive thermal diffusion, through flow does not change the stability threshold but exerts an orientating effect on the convective patterns: the loss of problem isotropy in a horizontal plane leads to the inequality of convective rolls with different orientations. In the case of negative thermal diffusion when through flow has a destabilizing effect on the plane disturbances, these disturbances become most dangerous. Hence, it appears that through flow in this case, apart from the orientation effect (the roll axes have orthogonal orientation with respect to the direction of through flow), has a destabilizing effect.

## B. Long-wave linear theory

As follows from the above numerical results, long-wave disturbances are, in some cases, the most dangerous and, therefore, we pay particular attention to the problem of stability of the base state with respect to long-wave disturbances. The analysis is conveniently performed on the basis of the differential equations considered for the amplitudes of normal disturbances rather than on the basis of the characteristic equation. Let us consider neutral disturbances, i.e., assume that $\lambda=-i\omega$. All the unknown quantities are expressed as a wave-number power series,

$$\omega = k\omega_1 + k^2\omega_2 + \cdots , \tag{17}$$

$$R = R_0 + kR_1 + k^2R_2 + \cdots , \tag{18}$$

$$w = w_0 + kw_1 + k^2w_2 + \cdots , \tag{19}$$

$$\vartheta = \vartheta_0 + k\vartheta_1 + k^2\vartheta_2 + \cdots , \tag{20}$$

$$p = p_0 + kp_1 + k^2p_2 + \cdots , \tag{21}$$

$$c = c_0 + kc_1 + k^2c_2 + \cdots . \tag{22}$$

In these expansions, we take into account the fact that at $k=0$, the oscillatory disturbances are absent, i.e., $\omega_0=0$.

From the zero- and first-order expansions of Eqs. (13)–(15) with boundary conditions (16), we obtain

$$w_0 = 0, \quad \vartheta_0(-b\omega_1 + \mathrm{Pe}) = 0, \quad c_0(-\omega_1 + \mathrm{Pe}) = 0. \tag{23}$$

From Eq. (23), it can be seen that either $c_0=0$, $\vartheta_0\neq 0$, $\omega_1=\mathrm{Pe}/b$ (such disturbances will be called thermal) or $\vartheta_0=0$, $c_0\neq 0$, $\omega_1=\mathrm{Pe}$ (these disturbances will be called solutal).

It should be noted that in the absence of through flow, i.e., at $\mathrm{Pe}=0$, we have $\omega_1=0$; moreover, both $\vartheta_0$ and $c_0$ differ from zero. In the next order of the expansion, we obtain the following expression for $R_0$:

$$R_0 = \frac{3\mathrm{Le}}{\mathrm{Le}S + 1 + S}. \tag{24}$$

At $\mathrm{Pe}\neq 0$ in the second-order expansion, we obtain $R_0=3\mathrm{Le}$ for thermal disturbances and $R_0=3/S$ for solutal disturbances. The quantities $R_1$ and $\omega_2$ are found to be equal to zero. Thus, in the case of $S>0$ at $\mathrm{Le}S>1$, the more dangerous disturbances are the solutal ones, and at $\mathrm{Le}S<1$ the thermal ones are the more dangerous. In the case of $S<0$, the long-wave instability is caused by thermal disturbances on heating from below and by solutal disturbances on heating from above.

In the third-order expansion, we define the correction for frequency $\omega_3$,

$$\omega_3 = \frac{S\mathrm{Le}(\mathrm{Le}+1)}{\mathrm{Pe}(b-1)} \tag{25}$$

for the thermal level and

$$\omega_3 = -\frac{(S+1)}{S\mathrm{Pe}(b-1)} \tag{26}$$

for the solutal level.

In the fourth-order expansion, we define the correction for $R_2$,

$$R_2 = \frac{8}{7}\mathrm{Le} - \frac{2}{35}S\mathrm{Le}(\mathrm{Le}+1) - 3\frac{S\mathrm{Le}(\mathrm{Le}+1)(1-S\mathrm{Le})b^2}{\mathrm{Pe}^2(b-1)^2} \tag{27}$$

for thermal disturbances and

$$R_2 = \frac{8}{7S} - \frac{2}{35}\frac{S+1}{S^2\mathrm{Le}} + 3\frac{(1+S)(1-S\mathrm{Le})}{S^3\mathrm{Pe}^2(b-1)^2} \tag{28}$$

for solutal disturbances.

Let us analyze Eqs. (27) and (28) for the case of $S>0$. At small values of the Peclet number, the sign of $R_2$ is defined by the last terms in Eqs. (27) and (28). It can be seen that, at $\mathrm{Le}S>1$, $R_2$ is negative for solutal disturbances and is positive for thermal disturbances. At $\mathrm{Le}S<1$, the signs of the correction $R_2$ are opposite to those at $\mathrm{Le}S>1$. Thus, for any relation of $\mathrm{Le}S$ to 1 for a lower branch of the long-wave instability of $S>0$, the quantity $R_2$ is negative and consequently more dangerous are the disturbances with a finite wavelength. With an increase of the Peclet number, the sign of $R_2$ can change. Thus, for solutal disturbances, as is clear from Eq. (28), this occurs at

$$\mathrm{Pe}^2 = \frac{105\mathrm{Le}(S+1)(\mathrm{Le}S-1)}{2(b-1)^2S(20\mathrm{Le}S - S - 1)}. \tag{29}$$

For preset values of the parameters ($\mathrm{Le}=100$, $S=0.1$, $b=10$), formula (29) gives the value of about 5.68 for Pe. At large values of Pe, long-wave disturbances are the most dangerous, which agrees with the numerical results presented in Fig. 3.

In the case of anomalous Soret effect and $-1<S<0$, expressions (27) and (28) have determinate signs at any Peclet number, namely, $R_2>0$ for thermal disturbances and $R_2<0$ for solutal disturbances. Therefore, the most dangerous disturbances are the long-wave ones, with heating both from below and from above.

The case in which the disturbances with nonzero but small wave number are most dangerous can be described by the long-wave theory after making a number of assumptions concerning the problem parameters. These assumptions will be different for different situations according to which disturbances, thermal or solutal, are the most dangerous. For solutal disturbances, we assume that the Lewis Le and Peclet Pe numbers are large,

$$\mathrm{Le} = \frac{\mathrm{Le}_{-2}}{k^2}, \quad \mathrm{Pe} = \frac{\mathrm{Pe}_{-1}}{k}, \tag{30}$$

and for thermal disturbances, we use the assumption of small separation ratio $S$ and Peclet Pe numbers

$$S = S_2k^2, \quad \mathrm{Pe} = \mathrm{Pe}_1 k. \tag{31}$$

Let us consider the first case, in which we will use the same expansions in the small parameter $k$ as we used before, except for the series expansion with respect to frequency (17), which is now changed to the following expression:

$$\omega = k^{-1}\omega_{-1} + k\omega_1 + k^2\omega_2 + \cdots . \tag{32}$$

From the first two orders of series expansions of Eqs. (13)–(15) with the boundary conditions (16), we obtain

$$w_0 = 0, \quad \vartheta_0(-b\omega_{-1} + \mathrm{Pe}_{-1}) = 0, \quad c_0\mathrm{Pe}_{-1} = 0. \tag{33}$$

From Eq. (33), it can be seen that $c_0=0$, $\vartheta_0\neq 0$, and $\omega_{-1}=\mathrm{Pe}_{-1}/b$. From the second-order expansion, we get $R_0=3\mathrm{Le}$. The quantities $\omega_2$, $\omega_3$, and $R_1$ are found to be equal to zero. From the fourth-order expansion, we derive corrections $R_2$ and $\omega_4$,

$$R_2 = \frac{1}{7S^2}\frac{8S\mathrm{Le}_{-2}^2 + 8S(b-1)^2\mathrm{Pe}_{-1}^2 - 21\mathrm{Le}_{-2}(S+1)}{\mathrm{Le}_{-2}^2 + (b-1)^2\mathrm{Pe}_{-1}^2}, \tag{34}$$

$$\omega_4 = -\frac{(b-1)\mathrm{Pe}_{-1}(S+1)}{S[\mathrm{Le}_{-2}^2 + (b-1)^2\mathrm{Pe}_{-1}^2]}. \tag{35}$$

By combining the expressions obtained for the expansion terms $R$ and $\omega$, we arrive at the approximate formulas for the neutral curve and oscillation frequency,

$$R \approx \frac{1}{7S^2}\frac{21\mathrm{Le}(S(\mathrm{Le}-1)-1)k^2+21S(b-1)^2\mathrm{Pe}^2+8S(b-1)^2\mathrm{Pe}^2k^2+8S\mathrm{Le}^2k^4}{\mathrm{Le}^2k^2+(b-1)^2\mathrm{Pe}^2}, \tag{36}$$

$$\omega \approx \mathrm{Pe}k - \frac{(b-1)\mathrm{Pe}(S+1)}{S[\mathrm{Le}^2k^2+(b-1)^2\mathrm{Pe}^2]}k^3. \tag{37}$$

The neutral curve $R(k)$, defined by formula (36), is shown in Fig. 8. The location of the minimum for this curve is specified by the following expression:

$$k_m^2 = \frac{(b-1)|\mathrm{Pe}|}{\mathrm{Le}}\left[\sqrt{\frac{21}{8}\frac{S+1}{S\mathrm{Le}}} - \frac{(b-1)|\mathrm{Pe}|}{\mathrm{Le}}\right], \tag{38}$$

which agrees well with the numerical results obtained in the previous section in the framework of linear theory for finite-length waves (see Fig. 8).

In the case of thermal disturbances, similar expansions lead to the following expressions:

$$R \approx \frac{1}{7}\frac{21b^2\mathrm{Le}[1-S(\mathrm{Le}+1)]k^2+21(b-1)^2\mathrm{Le}\mathrm{Pe}^2+8(b-1)^2\mathrm{Pe}^2k^2+8b^2k^4}{b^2k^2+(b-1)^2\mathrm{Pe}^2}, \tag{39}$$

$$\omega = \frac{\mathrm{Pe}}{b}k - \frac{(b-1)\mathrm{Pe}\mathrm{Le}S(\mathrm{Le}+1)}{b^2[b^2k^2+(b-1)^2\mathrm{Pe}^2]}k^3. \tag{40}$$

The neutral curve $R(k)$, defined by formula (39), is given in Fig. 9. The location of the minimum is defined by

$$k_m^2 = \frac{(b-1)|\mathrm{Pe}|}{b}\left[\sqrt{\frac{21}{8}S\mathrm{Le}(\mathrm{Le}+1)} - \frac{(b-1)|\mathrm{Pe}|}{b}\right]. \tag{41}$$

As mentioned in the previous section, the limit $R(k)$ at $k\to 0$ is different for $\mathrm{Pe}=0$ and $\mathrm{Pe}\neq 0$; moreover, the difference is finite at any small values of the Peclet number. However, as follows from the numerical results presented above, the instability threshold to most dangerous perturbations with small but nonzero wave number obtained for small Pe is close to the critical value obtained for $\mathrm{Pe}=0$, i.e., there is no jump for $R_{\mathrm{min}}$.

The dependences $R_{\mathrm{m}}(\mathrm{Pe})$ and $k_{\mathrm{m}}(\mathrm{Pe})$ at small values of Peclet number can be analytically studied. For this, we assume that the parameter Pe is the quantity of the second order of smallness when the wave number is the quantity of the first order. The calculations similar to those described above lead to the following results. In the leading order of expansion, we come to the formulas (24) for $R_0$. From the next orders, we find that $R_1=0$, and for $R_2$, we obtain

$$R_2 = \frac{Ak^4+B\mathrm{Pe}^2}{k^2(S+S\mathrm{Le}+1)(S\mathrm{Le}^2+S\mathrm{Le}+b+bS)^2}, \tag{42}$$

where

$$A = 16bS\mathrm{Le}+8S^2\mathrm{Le}^2+8S^2\mathrm{Le}^4+16S^2\mathrm{Le}^3+16bS^2\mathrm{Le}^2 + 16bS\mathrm{Le}^2+16bS^2\mathrm{Le}+8S^2b^2+16Sb^2+8b^2,$$

$$B = S(-42bS+21b^2S-42bS\mathrm{Le}+21S-42b+21b^2 - 42b\mathrm{Le}+21S\mathrm{Le}+21b^2S\mathrm{Le}+21\mathrm{Le}+21b^2\mathrm{Le}+21).$$

Minimization in Eq. (42) with respect to the wave number gives

$$R_{2m} = \frac{2\mathrm{Pe}\sqrt{AB}}{(S+S\mathrm{Le}+1)(S\mathrm{Le}^2+S\mathrm{Le}+b+bS)^2}, \tag{43}$$

and for the wave number of most dangerous perturbations, we obtain

$$k_m = \left(\frac{B}{A}\right)^{1/4}\sqrt{\mathrm{Pe}}. \tag{44}$$

As one can see from Eqs. (43) and (44), $R_{2m}$ is proportional to the Peclet number and $k_m$ is proportional to $\sqrt{\mathrm{Pe}}$. For the parameter values used in the calculations, formulas (43) and (44) yield

$$R_m \approx 27.03+0.980\mathrm{Pe}, \quad k_m \approx 0.218\sqrt{\mathrm{Pe}}, \tag{45}$$

which well corresponds to the numerical data presented in Figs. 3 and 4.

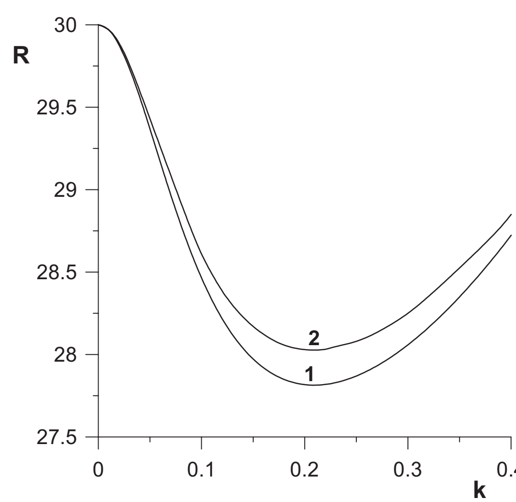


FIG. 8. Neutral curve in the case of positive separation ratio at $b=10$, $\mathrm{Le}=100$, $S=0.1$, and $\mathrm{Pe}=1.11$: (1) analytical results obtained from long-wave linear theory and (2) numerical results obtained from finite wavelength calculations.

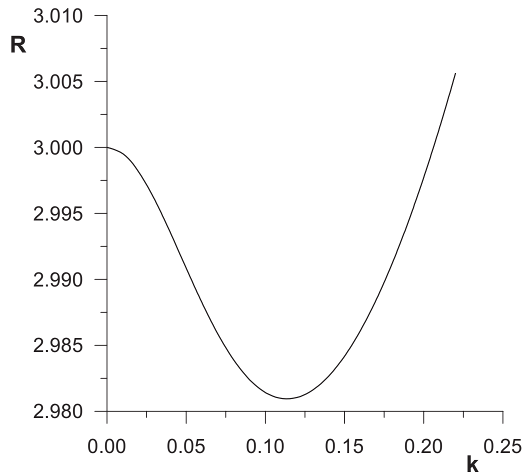


FIG. 9. Neutral curve obtained from long-wave linear theory in the case of positive separation ratio at $b=10$, Pe=1, $S=0.01$, and Pe=0.222.

## IV. WEAKLY NONLINEAR ANALYSIS

### A. Long-wave disturbances

The results of the linear theory given above allow us to define the boundary of the stability state, in which convection is absent. To specify the character of motion excitation (soft or hard excitation) and to investigate the stability of supercritical modes, it is necessary take into account the nonlinear terms in the equations of heat and admixture transfer. This analysis will be restricted to the case of long-wave disturbances. Recall that the long-wave disturbances are most dangerous at negative values of the separation ratio, and we are dealing here just with this case.

The amplitude equations describing convection in the long-wave approximation are conveniently constructed by the multiple scale method. We use an expansion in terms of the formal small parameter $\delta$,

$$\nabla_\perp = \delta \nabla_1, \tag{46}$$

$$\frac{\partial}{\partial t} = \delta \frac{\partial}{\partial t_1} + \delta^2 \frac{\partial}{\partial t_2} + \cdots, \tag{47}$$

$$R = R_0 + \delta R_1 + \delta^2 R_2 + \cdots, \tag{48}$$

$$\vec{u} = \vec{u}_0 + \delta \vec{u}_1 + \delta^2 \vec{u}_2 + \cdots, \tag{49}$$

$$\vartheta = \vartheta_0 + \delta \vartheta_1 + \delta^2 \vartheta_2 + \cdots, \tag{50}$$

$$p = p_0 + \delta p_1 + \delta^2 p_2 + \cdots, \tag{51}$$

$$c = c_0 + \delta c_1 + \delta^2 c_2 + \cdots. \tag{52}$$

By integrating the equation of heat and admixture transfer across the layer, we obtain the relations

$$\int_{-1}^{1} \left( b \frac{\partial \vartheta}{\partial t} + \vec{u} \cdot \nabla \vartheta + \mathrm{Pe} \frac{\partial \vartheta}{\partial x} - \mathrm{Le} \nabla_\perp^2 \vartheta - w \right) dz = 0, \tag{53}$$

$$\int_{-1}^{1} \left( \frac{\partial c}{\partial t} + \vec{u} \cdot \nabla c - \nabla_\perp^2 c + S \nabla_\perp^2 \vartheta - S w + \mathrm{Pe} \frac{\partial c}{\partial x} \right) dz = 0, \tag{54}$$

which play the role of the resolution condition in all orders of expansion except for the zero order.

In zero order of $\delta$, we obtain

$$w_0 = 0, \quad \vartheta_0 = \theta(t,x,y), \quad c_0 = \xi(t,x,y), \quad p_0 = R_0(\theta + \xi) z. \tag{55}$$

As in the linear theory, the disturbances are divided into two classes: Thermal and solutal. For thermal disturbances, $\xi=0$, and the main part of temperature disturbances satisfies the equation

$$b \frac{\partial \theta}{\partial t_1} = - \mathrm{Pe} \frac{\partial \theta}{\partial x_1}. \tag{56}$$

For solutal disturbances, $\theta=0$, and the main part of solutal disturbances satisfies the equation

$$\frac{\partial \xi}{\partial t_1} = - \mathrm{Pe} \frac{\partial \xi}{\partial x_1}. \tag{57}$$

Thus, in reference frame A, the thermal disturbances are quasistationary and their slow evolution is described by the times $t_2, t_3, \ldots$, whereas the solutal disturbances are quasistationary in reference frame B.

The nontrivial amplitude equations for $\theta, c$ are obtained in the fourth-order expansion in terms of $\delta$. By dropping the details of simple and cumbersome calculations, we give here only the final form of the amplitude equations for solutions homogeneous in $y$,

$$\frac{\partial \theta}{\partial t_3} = \frac{S\mathrm{Le}(\mathrm{Le}+1)}{\mathrm{Pe}(b-1)} \frac{\partial^3 \theta}{\partial x_1^3}, \tag{58}$$

$$\begin{aligned} b \frac{\partial \theta}{\partial t_4} + \frac{R_2}{3} \frac{\partial^2 \theta}{\partial x_1^2} + \gamma \frac{\partial^4 \theta}{\partial x_1^4} - \frac{6}{5} \mathrm{Le} \frac{\partial}{\partial x_1} \left( \frac{\partial \theta}{\partial x_1} \right)^3 \\ = b \left[ \frac{\partial \theta_1}{\partial t_3} - \frac{S\mathrm{Le}(\mathrm{Le}+1)}{\mathrm{Pe}(b-1)} \frac{\partial^3 \theta_1}{\partial x_1^3} \right] \end{aligned} \tag{59}$$

for thermal disturbances in reference frame A, and

$$\frac{\partial \xi}{\partial t_3} = - \frac{(S+1)}{S\mathrm{Pe}(b-1)} \frac{\partial^3 \xi}{\partial x_1^3}, \tag{60}$$

$$\begin{aligned} \frac{\partial \xi}{\partial t_4} + \frac{S R_2}{3} \frac{\partial^2 \xi}{\partial x_1^2} + \gamma_* \frac{\partial^4 \xi}{\partial x_1^4} + \frac{6}{5} \frac{1}{S^2} \frac{\partial}{\partial x_1} \left( \frac{\partial \xi}{\partial x_1} \right)^3 \\ = - \left[ \frac{\partial c_1}{\partial t_3} + \frac{S+1}{S\mathrm{Pe}(b-1)} \frac{\partial^3 c_1}{\partial x_1^3} \right] \end{aligned} \tag{61}$$

for solutal disturbances in reference frame B. Here, the following notation is used:

$$\begin{aligned} \gamma = \mathrm{Le} \Bigg\{ \frac{1}{140} \left[ \frac{160}{3} + \frac{11}{2} S(\mathrm{Le}+1) \right] \\ - S(\mathrm{Le}+1) \left[ \frac{b^2}{\mathrm{Pe}^2 (b-1)^2} (1 - S\mathrm{Le}) + \frac{7}{120} \right] \Bigg\}, \end{aligned}$$

$$\gamma_* = \frac{1}{5}\left(\frac{40}{21} + \frac{11}{56}\frac{S+1}{S\mathrm{Le}}\right) - \frac{(S+1)}{S}\left[\frac{1}{S\mathrm{Pe}^2(b-1)^2} \times (S\mathrm{Le}-1) - \frac{7}{120}\frac{1}{\mathrm{Le}}\right].$$

We shall restrict our considerations to finding solutions for thermal disturbances. The solution of Eq. (59) can be written as $\theta = \int a(k,t_4)\exp i(kx_1 - \omega t_3)dk$, where the Fourier amplitude is a function of slow time $t_4$ and $\omega$ and $k$ obey the dispersion relation, $\omega = \omega_3 k^3$. Substitution of this solution into Eq. (59) leads to the appearance of secular terms. The condition at which they are absent is derived from the equation for the Fourier amplitudes, which in the general case is rather cumbersome, and is essentially simplified in the monochromatic approximation, $\theta = a\cos(kx_1 - \omega t_3)$. Substitution of this expression into Eq. (59) leads to the equation for the amplitude $a$,

$$b\frac{\partial a}{\partial t_4} - \frac{R_2}{3}k^2 a + \gamma k^4 a + \frac{3}{10}\mathrm{Le}k^4 a^3 = 0. \tag{62}$$

At $R_2 < 3\gamma k^2$, all solutions to the amplitude equation (62) tend to zero, whereas in the case of fulfillment of the inverse inequality, i.e., above the neutral curve, all solutions tend to a stationary one,

$$a = \pm\frac{1}{k}\sqrt{\frac{10}{9}\mathrm{Le}(R_2 - 3\gamma k^2)}. \tag{63}$$

Hence, it appears that thermal disturbances are excited softly and obey an ordinary root law. The stability of these solutions against disturbances with another wave number can be analyzed in the framework of the same system of Eqs. (58) and (59). For disturbances represented as $e^{\lambda t_4}e^{i(\Omega t_3 - qx_1)}$, where $q$ is the wave number of disturbances and $\Omega$ is their frequency related to $q$ by the dispersion equation, $\Omega = \omega_3 q^3$, we obtain the following expression for the increment $\lambda$:

$$\lambda = \frac{q^2}{b}\left(2\gamma k^2 - \frac{1}{3}R_2 - \gamma q^2\right). \tag{64}$$

From Eq. (64), it follows that $\lambda < 0$ for any $q$ if $R_2 > 6\gamma k^2$. Thus, at sufficiently large supercriticality, the stationary thermal wave is stable. The stability region in the plane $R_2 - k$ is bounded from below by the line corresponding to a twofold supercriticality.

Similar results are obtained for the solutal wave. In the case considered above, the separation ratio is negative, and the plane disturbances are most dangerous. For a positive separation ratio, as we know, the most dangerous disturbances are spatial ones. However, such disturbances can be suppressed by setting vertical, closely spaced, impermeable, and thermally nonconducting boundaries to the flow region. It should be noted that in the case of homogeneous fluids, such practice is incorrect because, under the no-slip condition at the solid boundaries, the flow will change its pattern to a three-dimensional one. In porous media, realization of two-dimensional flow modes by means of setting spatial constraints is possible because the boundary conditions do not impose restrictions on the tangential velocity. Such a way of obtaining two-dimensional convective flows in a porous medium is used in Ref. 15.

### B. Disturbance with small wave number

Let us perform a weakly nonlinear analysis for this case, making the same assumptions for the problem parameters as we did in the linear long-wave theory for consideration of disturbances with a nonzero but small wave number.

Thus, for solutal disturbances, we assume that

$$\mathrm{Le} = \frac{\mathrm{Le}_{-2}}{\delta^2}, \quad \mathrm{Pe} = \frac{\mathrm{Pe}_{-1}}{\delta}. \tag{65}$$

In expansions (49)–(52), the initial terms in the series expansion with respect to the small parameter are changed in the following way:

$$\vec{u} = \delta\vec{u}_1 + \delta^2\vec{u}_2 + \cdots, \tag{66}$$

$$\vartheta = \delta\vartheta_1 + \delta^2\vartheta_2 + \cdots, \tag{67}$$

$$p = \delta p_1 + \delta^2 p_2 + \cdots, \tag{68}$$

$$c = \delta c_1 + \delta^2 c_2 + \cdots. \tag{69}$$

The low-order solution for temperature and concentration is written as (here, we present only such quantities that are important for further discussion)

$$\vartheta_1 = 0, \quad c_1 = \xi(t,x,y), \quad \vartheta_1 = 0, \quad \vartheta_1 = \theta(t,x,y). \tag{70}$$

In the fifth order, we obtain the following system of the amplitude equations in reference frame B:

$$\mathrm{Le}_{-2}\frac{\partial^2\theta}{\partial x_1^2} + (b-1)\mathrm{Pe}_{-1}\frac{\partial\theta}{\partial x_1} - \frac{1}{S}\frac{\partial^2\xi}{\partial x_1^2} = 0, \tag{71}$$

$$\frac{\partial\xi}{\partial t_4} + \frac{R_2 S}{3}\frac{\partial^2\xi}{\partial x_1^2} + \frac{8}{21}\frac{\partial^4\xi}{\partial x_1^4} + (S+1)\frac{\partial^2\theta}{\partial x_1^2} = 0. \tag{72}$$

We seek a solution to this system in the following form:

$$\xi = Fe^{i(kX_0 - \omega\tau_0)} + \mathrm{c.c.}, \quad \theta = \Phi e^{i(kX_0 - \omega\tau_0)} + \mathrm{c.c.}, \tag{73}$$

and finally obtain the dependence $R_2(k)$, which is similar to that obtained within the framework of the linear theory. The defined $R_2$ is minimized with respect to the wave number $k$ and, in the following, all calculations are carried out for this wave number. After completing all these manipulations, we get the seventh-order amplitude equation, which after transcription takes the following form:

$$\frac{\partial\psi}{\partial\tau_*} - \psi - e^{i\kappa}\frac{\partial^2\psi}{\partial X_*^2} + |\psi|^2\psi = 0, \tag{74}$$

where

$$e^{i\kappa} = \frac{q_m[5q_m^4 + p^2(3p^2 + 9q_m^2 - 2\beta)] + i(p^2 + q_m^2)(5q_m^4 + 4p^2q_m^2 + 3p^2\beta)}{\sqrt{q_m^2[5q_m^4 + p^2(3p^2 + 9q_m^2 - 2\beta)]^2 + (p^2 + q_m^2)^2(5q_m^4 + 4p^2q_m^2 + 3p^2\beta)^2}}, \tag{75}$$

$$q_m^2 = p(\sqrt{\beta} - p), \quad \beta = \frac{5}{6}\frac{S(S+1)}{\mathrm{Le}_{-2}}, \quad p = \frac{2\sqrt{5}}{\sqrt{63}}\frac{(b-1)\mathrm{Pe}_{-1}}{\mathrm{Le}_{-2}}. \tag{76}$$

This equation is a particular case of the complex Ginsburg–Landau equation and has the following quasistationary solution:

$$\psi = \Psi e^{i(qX_* - \Omega\tau_*)}, \tag{77}$$

$$\Psi = \sqrt{1 - q^2 \cos \kappa}, \tag{78}$$

$$\Omega = q^2 \sin \kappa. \tag{79}$$

The analysis of stability of this solution with respect to the disturbances represented as

$$\psi = (\Psi + \tilde{\Psi})e^{i(qX_* - \Omega\tau_*)},$$

$$\mathrm{Re}\, \tilde{\Psi} = Ae^{i(KX_* - \tilde{\Omega}\tau_*) + \sigma\tau_*} + \mathrm{c.c.}, \tag{80}$$

$$\mathrm{Im}\, \tilde{\Psi} = Be^{i(KX_* - \tilde{\Omega}\tau_*) + \sigma\tau_*} + \mathrm{c.c.}$$

shows (see Ref. 16) that for small $q$, the solutions are stable only for $q^2 \leqslant 1/3 \cos \kappa$, which can be expressed in other terms as

$$r_2 \geq 3r_0. \tag{81}$$

This is consistent with the general Eckhaus criterion.

For thermal disturbances, the results are similar except for the assumption concerning the problem parameters, which is now written as

$$S = S_2\delta^2, \quad \mathrm{Pe} = \mathrm{Pe}_1\delta. \tag{82}$$

For the stability boundary, we also obtain the Eckhaus law.

## V. CONCLUSION

The investigations made in this paper show that for a positive separation ratio, through flow has no effect on the stability threshold but exerts an orientating effect on the convective patterns: The loss of problem isotropy in a horizontal plane leads to the inequality of convective rolls with different orientations. From the results obtained, it might be concluded that instability is mainly caused by longitudinal rolls. In the case of negative separation ratio, even weak through flow of a binary mixture through the porous layer has a dramatic impact on the stability of the convectionless state. In the linear problem of stability, this manifests itself as the onset of crossover: Arbitrarily small through flow causes drastic destabilization of long-wave disturbances such that the critical Rayleigh number is shifted by a finite value. However, this instability is limited by very long waves. The existence of strong instability with respect to long-wave disturbances leads to unusual properties of the nonlinear stationary regimes. It is known[17] that in the case of long-wave instability of a horizontal layer heated from below in conditions of fixed heat flux at the boundaries, all long-wave stationary regimes are unstable. Our investigation provides conclusive evidence for the existence of stable stationary regimes in the presence of through flow.


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