Abstract
Many manufacturers, including Lenovo, Sony, Procter \& Gamble, and Buckle, have adopted differentiated distribution channels to market vertically differentiated products. However, there is scant literature addressing the issue of quality differentiation in the presence of differentiated distribution channel policies. To fill this void, we examine whether (how) differentiated channel policies affect manufacturers' quality differentiation and all parties' performance. Specifically, we consider a manufacturer who produces two vertically differentiated products (high- and low-tier ) together, but with two marketing options: (1) distributing both products through one retailer (Model O, One-channel policy), or (2) providing high-quality products through one channel but low-tier products through another (Model T, Two-channel policy). Our results show that the manufacturer is more likely to decrease the level of quality differentiation in Model T than in Model O. Moreover, contrary to popular belief, we show that "quality distortion" is not limited to low-tier products but can occur with high-tier products. Among other results, we find that the one-channel policy benefits the retailer but hurts both the manufacturer and the total supply chain. To test the robustness of the results, we also comment on how the additional horizontal consumer heterogeneity affects our results and the implications of the competition at the manufacturer level.

Keywords: Manufacturing/Marketing interface; Quality segmentation; Channel policy; Game theory

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## 1. Introduction

In the past two decades, with improving living standards and accelerating globalisation of economies, consumer demands have become more diversified and personalised (Ma et al. 2012). To cater to a broader and more heterogeneous mix of consumer groups, manufacturers increasingly design product lines by segmenting their markets in terms of quality attributes (Desai 2001). For example, Lenovo offers different sizes of memory for its laptops, SONY makes different screen sizes for its TVs, and Toyota provides cars ranging from the small Tercel to the full-size Avalon.

Although quality differentiation is a fundamental goal in creating a competitive advantage for a firm (Meulenbroeks 1998), a range of operational management issues arise when delivering quality segmentation solutions (Desai et al. 2001). The dominant concern is the risk of a cannibalisation problem in designing product lines (Pelegrin et al. 2016). For example, in 2010, when Apple intended to extend its product line from Macintosh to the iPad, it was particularly worried about the potential for cannibalisation of Macintosh sales by the iPad. Similarly, the subsequent launch of the iPad Mini sparked a widespread discussion on how this new, smaller iPad may cannibalise sales for the company's existing tablet computers (Barnato 2012). When confronting such "serious concerns", the CEO of Apple, Tim Cook, was inclined to accept it: "I see cannibalisation as a huge opportunity for us, we know that iPad will cannibalise some Macs. That doesn't worry us" (Seward 2013).

Quality segmentation strategies apply not only in a manufacturer's product lines' design, but also in its marketing channel decisions (Zhang and Cao 2014, Handley and Gray 2015). Manufacturers consider the many possible combinations of marketing channel design elements and quality segmentation. For example, to mitigate the potential cannibalisation problems between high- and low-value segments, many manufacturers adopt a "two-channel policy", selling their high-tier products in a high-end store and their low-tier products in a low-end store. For example, Procter \& Gamble (P\&G) provides "Olay" for low-end users through supermarkets and "SK-II" for high-end users through specially designed cabinets in department stores and shopping malls. The underlying rationale behind the above channel decisions is as follows. A two-channel policy enables a firm to segment heterogeneous consumers better and mitigates the potential cannibalisation problems; therefore, a two-channel policy should be optimal for multiproduct manufacturers (Zhang and Cao 2014). Although simple and useful, this
perspective ignores a key point: such a two-channel policy results in more competition between downstream stores, which might only care about their own interests and independently seek to maximise their own profit. Some manufacturers then adopt a "one-channel policy" that reduces competition by selling all products in one store or chain. For example, in the skin-care and cosmetics industry, Johnson \& Johnson (J\&J) launched its skincare lines "Clean \& Clear", "Neutrogena" and "Johnson's baby care" under one channel (Palsule-Desai et al. 2015). Differentiated channel policies can also be observed in a variety of industries; for example, Buckle (apparel), Conn's (electronics and appliances), and Tiffany \& Co. (jewellery) adopt a one-channel policy. Conversely, Sterling Jewelers (jewellery), Matai Inc. and Gap Inc. (apparel) adopt a two-channel policy.

The above discussion raises the fundamental question addressed in this paper -- whether (how) differentiated channel policies affect manufacturers' quality differentiation and all parties' performance. In practice, to deal with such a manufacturing/marketing problem, a multiproduct manufacturer needs to grow sales while simultaneously developing operational models of quality segmentation. More specifically, from the manufacturing interface, the manufacturer can match a broader mix of consumer groups by adopting quality differentiation strategies. However, such quality differentiation strategies usually raise the concern that the lower-margin products may cannibalise the sales of higher-margin products (Parlakturk 2012, Yan et al. 2015). In contrast, from the marketing perspective, the manufacturer can limit cannibalization problem by providing high-tier products through one channel and low-tier ones through another; however, multi-product manufacturers have to carefully consider the problem of competition between downstream stores, because consumers can self-select the products they want to purchase (Desai 2001).

In this paper, we address the above mentioned question from a manufacturing \& marketing perspective and derive theoretical implications for two possible configurations. A multiproduct manufacturer that produces two types of products (high- and low-tier) together has two options for marketing: (1) marketing both products through one retailer (Model O, the one-channel policy), or (2) providing high-quality products through one retailer and low-tier products through another retailer (Model T, the two-channel policy). Using both models, we explore the relationship between three interrelated decisions regarding the manufacturer's product lines' design and distribution channel decisions:
(1) How do the manufacturer's quality decisions vary under differentiated channel policies? (2) Which scenario is beneficial for the manufacturer, the retailer(s) and the supply chain: selling differentiated products under one channel or two? (3) What is the effect of channel structure on the equilibrium?

There is a considerable body of literature addressing the quality segmentation confronting heterogeneous consumers who differ in their willingness to pay for quality (see, Qi et al. (2015) and references therein). However, these studies do not consider the horizontal interactions between downstream intermediaries in marketing on a manufacturer's quality differentiation decisions. We fill this gap by highlighting the fact that, when implementing workable quality segmentation, a multiproduct manufacturer needs to trade off marketing channel design elements and quality segmentation emphasis. Conversely, despite numerous researchers studying channel policy from a marketing perspective (see, Zhang and Cao (2014) and references therein), previous studies traditionally assume that quality is exogenous and little is known about how channel policy affects a manufacturer's manufacturing management and quality segmentation. We therefore provide an alternative approach that is also somewhat complementary, to highlight how the manufacturer's quality decisions vary under differentiated channel policies.

Our results show that a manufacturer is more likely to reduce the level of quality differentiation under the two-channel policy than the one-channel policy. Furthermore, we find that "quality distortion" is not limited to low-tier products, as previously reported, but can occur with high-tier products. The direction of the high-quality distortions is always downward. In addition, our results reveal that the one-channel policy benefits the retailer but hurts both the manufacturer and the total supply chain. We then extend both models to a market where consumers are two-dimensionally heterogeneous and/or the manufacturers compete with each other, these two extensions further reveal that all results are robust regardless of whether there is a customer search problem and the competition at manufacturers level or not.

The remainder of the paper is organized as follows. Section 2 reviews the related literature and explains our contributions in more detail. Section 3 introduces notations and outlines our two models. Section 4 reports our main findings. Section 5 presents two possible model generalizations. Section 6 concludes the paper.

## 2. Relevant literature

Most research addressing quality segmentation in manufacturing has taken one of two approaches. The first is an emphasis on quality differentiation under the assumption that product quality is exogenous. Mussa and Rosen (1978) first considered a monopolist selecting quality positions when serving a market with consumers that have heterogeneous valuations for quality. Recently, Zhao et al. (2009) examined the choice of a channel structure in which decisions regarding vertical integration or decentralisation influence firms' quality and price strategies. More recent work by Lee et al. (2013) estimates a general model that summarises the linkages among the factors shaping optimal channel structure decisions in a multi-brand, multi-outlet market. Subsequently, Xiao et al. (2014) indicated that, if the reservation price in the indirect channel is sufficiently low, then adding the direct channel raises the unit wholesale price and retail price in the indirect channel. In contrast to these studies, in both of our models we consider that quality is an endogenous decision made by the manufacturer.

There are also many studies, beginning with Spengler (1950), that assume that product quality is endogenous and that customers have heterogeneous preferences for quality. Rhee (1996) notes that manufacturers should offer a product of similar quality when consumer heterogeneity is not sufficient; otherwise, offering identical qualities is optimal. Ha et al. (2016) show that a manufacturer offering differentiated products through two channels prefers to sell its high-tier product through a direct channel. Several other papers have studied endogenous quality in supply chain coordination (e.g., Bacchiega and Bonroy (2015), Yang et al. (2015), brand value (e.g., Choi and Coughlan (2006) and Davcik and Sharma (2015)), and product line design (e.g., Desai (2001)). This paper follows this stream of research by treating product quality as a decision variable for the manufacturer, but differs in an important way: we examine the strategic consequences of cannibalisation and competition under manufacturing/marketing trade-offs. That is, we highlight whether (how) differentiated channel policies affect manufacturers' quality differentiation and all parties' performance, which has been overlooked by previous researchers.

Although most research on quality segmentation has not considered the role of marketing channel structures, there are a few notable exceptions. In particular, Villas-Boas (1998) establishes that channel decentralisation drives a manufacturer to downward quality distortion for low-value consumers. In contrast, Chung and Lee (2014) show
that channel decentralisation does not necessarily lead to quality distortion with lowend products, but that this can occur with high-end products. Shi et al. (2013) find that the effect of channel decentralisation on product quality depends on the type of consumer heterogeneity and its distribution in a market. However, as a set, these papers do not consider the horizontal interactions among downstream intermediaries in marketing on a manufacturer's quality differentiation decisions, which is a focus of our paper. These previous studies provided the inspiration for us to explore this theme.

The final related stream of literature has studied channel policies in marketing. Jeuland and Shugan (1983) consider the channel coordination problem with a manufacturer distributing its products through a one-channel policy. Cachon and Lariviere (2005) study revenue-sharing contracts with revenues determined by each retailer's purchase quantity and price, and demonstrate that revenue sharing can coordinate a supply chain with a one-channel policy. Geylani et al. (2007) illustrate a strategic manufacturer's response to a two-channel policy (i.e., a dominant and a weak retailer) for the sale of a single product. Liu et al. (2013) evaluate the implications of advertising strategies for overall supply chain efficiency and consumer welfare, in the context of a manufacturer selling to consumers through a one-channel policy. Zhang and Cao (2014) investigate the case in which a multi-product retail firm facing deterministic demand distributes two vertically differentiated products and chooses one or two stores (channels) at which to sell them. Glock and Kim (2015) study a single-vendor multi-retailer supply chain and consider the effect of decreasing the competition between marketing channels by forward integration. To our knowledge, previous studies of channel policy have not examined the manufacturing/marketing trade-offs. We therefore provide an alternative approach that is also somewhat complementary, to highlight how a manufacturer's quality decisions vary under differentiated channel policies.

## 3. Model description and equilibrium analysis

### 3.1. Model setup

We consider a supply chain consisting of a manufacturer and one and/or two retailer(s). The manufacturer provides two different quality products: high- and low-tier. She ${ }^{1}$ then has two differentiated channel policies with which to market the products: (1)

[^1]distributing both high- and low-tier products through one channel, i.e., the one-channel policy (Model O); or (2) selling the high-tier products through one store and the low-tier products through another, i.e., the two-channel policy (Model T).

We assume the timing in both models is as follows: first, the manufacturer decides on the optimal quality levels $\left(u_{h}, u_{l}\right)$ and the wholesale prices $\left(w_{h}, w_{l}\right)$ for both products. Observing the manufacturer's optimal strategies on quality and wholesale prices, the retailer(s) then chooses the optimal units $\left(q_{h}, q_{l}\right)$ to be sold to consumers. Our assumptions regarding the manufacturer, retailer(s), consumer preferences, and decision-making framework are as follows.

### 3.1.1. Manufacturer

The manufacturer's problem is to choose the optimal quality levels for both products and the wholesale prices to maximise her profit. As in Ha et al. (2016), we assume that the manufacturer's unit cost for producing a product with quality $u$ is $k u^{2}$. Since $u_{h}>u_{l}>0$, the unit cost for producing a high-tier product $\left(u_{h}\right)$ is higher than that for a low-tier product, that is, $k u_{h}^{2}>k u_{l}^{2}>0$.

### 3.1.2. Retailer

The retailer is a profit maximiser who is responsible for the optimal units for both products $\left(q_{h}, q_{l}\right)$, where $q_{h}$ is the quantity of high-tier products, and $q_{l}$ is the quantity of low-tier products. Marketing high-tier products is usually accompanied by more promoters, luxurious decorations, and more exclusive shelves, while these costs are lower for a retailer who distributes low-tier products, we therefore distinguish the cost of selling high- and low-tier products with an assumption of $c_{h}=c>c_{l}=0 .{ }^{2}$ Such a premium has been widely adopted in the literature in marketing to reflect the level of competition between both channels (e.g., Arya et al. 2007, Ha et al. 2016, Yan et al. 2018).

### 3.1.3. Consumers

Consistent with Li et al. (2014) and Qi et al. (2015), we consider a market, with size normalised to 1 , that consists of consumers whose heterogeneous preferences for quality are uniformly distributed over $[0,1]$. Then, the consumer's utility can be defined as $U(u, p, \theta)=\theta u-p$. Without loss of generality, let $u_{h}>u_{l}$, we can derive the inverse

[^2]demand functions for high- and low-tier products from the consumer utility functions as follows: ${ }^{3}$
\[

$$
\begin{align*}
p_{h} & =u_{h}-u_{h} q_{h}-u_{l} q_{l}  \tag{1}\\
p_{l} & =u_{l}\left(1-q_{h}-q_{l}\right)
\end{align*}
$$
\]

### 3.2. Equilibrium analysis

Based on the inverse demand functions in equation (1), we can now consider our two models-Model O and Model T-in which $\pi_{a}^{b}$ represents the profit for player $a$ under $b$ channel policy, where subscript $a \in\{m, r, s\}$ denotes the manufacturer, the retailer, and the supply chain, respectively; and superscript $b \in\{O, T\}$ denotes Model O and Model T, respectively.

### 3.2.1. Quality differentiation under one-channel policy (Model O)

In Model O, all products are sold through one store. The retailer chooses the optimal outputs of high- and low-tier products $\left(q_{h}, q_{l}\right)$ to maximise his profit. That is, taking the wholesale prices of high- and low-tier products $\left(w_{h}, w_{l}\right)$ as given, the retailer's problem is:

$$
\begin{equation*}
\max _{q_{h}, q_{l}} \pi_{r}^{O}=\left(p_{h}-w_{h}-c\right) q_{h}+\left(p_{l}-w_{l}\right) q_{l} \tag{2}
\end{equation*}
$$

where the first term is the retailer's revenue from selling high-tier products, the second term is the retailer's income from marketing low-tier products, and the remaining two terms are the retailer's cost of wholesaling high- and low-tier products.

Anticipating the retailer's response to the wholesale prices she sets, the manufacturer chooses the wholesale prices $\left(w_{h}, w_{l}\right)$ and quality levels $\left(u_{h}, u_{l}\right)$ to maximise her profit:

$$
\begin{equation*}
\max _{w_{h}, w_{l}, u_{h}, u_{l}} \pi_{m}^{O}=\left(w_{h}-k u_{h}^{2}\right) q_{h}+\left(w_{l}-k u_{l}^{2}\right) q_{l} \tag{3}
\end{equation*}
$$

Backward induction is employed to determine the subgame perfect equilibrium in each model. Specifically, we first determine the retailer's optimal quantities from (2) and then substitute them into (3), which provides the equilibrium wholesale prices and quality levels. The following proposition summarises both players' optimal decisions in Model O. ${ }^{4}$

[^3]Proposition 1. In Model $O$, the equilibrium quantities, wholesale prices, quality levels, and profits can be summarized as follows:
$u_{h}^{O^{*}}=\frac{2 \sqrt{1-20 k c}+12 k c-2}{k(2 \sqrt{1-20 k c}-2)}$,
$u_{l}^{O^{*}}=\frac{3-2 \sqrt{1-20 k c}}{5 k}$,
$w_{h}^{O^{*}}=\frac{72 k c^{2}}{(2-2 \sqrt{1-20 k c})^{2}}-\frac{18 c}{2-2 \sqrt{1-20 k c}}+\frac{1}{k}-\frac{c}{2}$,
$w_{l}^{O^{*}}=\frac{14-11 \sqrt{1-20 k c}-40 k c}{25 k}$
$q_{h}^{O^{*}}=\frac{\sqrt{1-20 k c}+12 k c-1}{2-2 \sqrt{1-20 k c}}$,
$q_{l}^{O^{*}}=\frac{(16 k c-5) \sqrt{1-20 k c}-64 k c+5}{2(2 \sqrt{1-20 k c}-3)(\sqrt{1-20 k c}-1)}$,
$\pi_{m}^{O^{*}}=\frac{2 c\left((8 k c-1) \sqrt{1-20 k c}+40 k^{2} c^{2}-18 k c+1\right)}{\left(1-\sqrt{1-20 k c^{3}}\right)}$,
$\pi_{r}^{O^{*}}=\frac{5 c\left(\left(16 k^{2} c^{2}-12 k c+1\right) \sqrt{1-20 k c}-88 k^{2} c^{2}+22 k c-1\right)}{(1-\sqrt{1-20 k c})^{3}(2 \sqrt{1-20 k c}-3)}$,
$\pi_{s}^{O^{*}}=\frac{15 c\left(\left(16 k^{2} c^{2}-12 k c+1\right) \sqrt{11-20 k c}-88 k^{2} c^{2}+22 k c-1\right)}{(1-\sqrt{1-20 k c})^{3}(2 \sqrt{1-20 k c}-3)}$.
Proposition 1 is partly consistent with previous studies (e.g., Chung and Lee (2014) $)^{5}$ and provides a baseline for subsequent analysis to focus on the key drivers underlying the effects of different channel structures on product line design. In that regard, the first variation we consider is the case of the two different quality products being distributed through differentiated stores (i.e., Model T), ceteris paribus.

### 3.2.2. Quality differentiation under two-channel policy (Model T)

In Model T, the manufacturer can reach consumers by adopting a two-channel policy, in which high-tier products are distributed through one channel and low-tier products are sold by another. More specifically, Retailer One chooses his output of high-tier products $\left(q_{h}\right)$ and Retailer Two chooses his output of low-tier products $\left(q_{l}\right)$.

$$
\left.\begin{array}{rl}
\max _{q_{h}} & \pi_{r 1}^{T} \\
\max _{q_{l}} & \pi_{r 2}^{T} \tag{4}
\end{array}=\left(p_{h}-w_{h}-c\right) q_{h}-w_{l}\right) q_{l}-1 .
$$

Anticipating the retailer's optimal strategies, the manufacturer chooses the optimal wholesale prices $\left(w_{h}, w_{l}\right)$ and quality levels $\left(u_{h}, u_{l}\right)$ to maximise her profit, that is:

$$
\begin{equation*}
\max _{w_{h}, w_{l}, u_{h}, u_{l}} \pi_{m}^{T}=\left(w_{h}-k u_{h}^{2}\right) q_{h}+\left(w_{l}-k u_{l}^{2}\right) q_{l} \tag{5}
\end{equation*}
$$

[^4]As before, we can obtain the following equilibrium quantities, wholesale prices, quality level and profits using backward induction:

Proposition 2. In Model T, the equilibrium quantities, wholesale prices, quality levels, and profits, respectively, are:
$u_{h}^{T^{*}}=\frac{9+3 \sqrt{9+92 k c}}{46 k}$,
$u_{l}^{T^{*}}=\frac{6 \sqrt{9+92 k c}+92 k c+18}{23 k(3+\sqrt{9+92 k c})}$,
$w_{h}^{T^{*}}=\frac{24 \sqrt{9+92 k c}-161 k c+72}{529 k}$,
$w_{l}^{T^{*}}=\frac{(9+3 \sqrt{9+92 k c}+46 k c)(87+29 \sqrt{9+92 k c}+92 k c)}{529 k(3+\sqrt{9+92 k c})^{2}}$
$q_{h}^{T^{*}}=\frac{(15-138 k c) \sqrt{9+92 k c}-184 k c+45}{138 \sqrt{9+92 k c} 2116 k c+414}$,
$q_{l}^{T^{*}}=\frac{3+\sqrt{9+92 k c}}{46}$,
$\pi_{m}^{T^{*}}=\frac{184 k^{3} c^{3}-18 k^{2} c^{2}+\frac{108}{23} k c+\left(4 k^{2} c^{2}-\frac{18}{23} k c+\frac{243}{529}\right) \sqrt{9+92 k c}+\frac{729}{529}}{k(9+3 \sqrt{9+92 k c}+46 k c)(3+\sqrt{9+92 k c})}$,
$\pi_{r 1}^{T^{*}}=\frac{3((138 k c-15) \sqrt{9+92 k c}+184 k c-45)\left((299 k c-45) \sqrt{9+92 k c}+6348 k^{2} c^{2}+207 k c-135\right)}{48688(3 \sqrt{9+92 k c}+46 k c+9)^{2}}$,
$\pi_{r 2}^{T^{*}}=\frac{(23 k c+9) \sqrt{9+92 k c}+207 k c+27}{12167 k}$,
$\pi_{s}^{T^{*}}=\frac{\left[\begin{array}{l}\left(15817100 k^{3} c^{3}-185679 k^{2} c^{2}+462024 k c+136323\right) \sqrt{9+92 k c}+167904600 k^{4} c^{4} \\ +53802474 k^{3} c^{3}+1185489 k^{2} c^{2}+3476358 k c+408969\end{array}\right]}{12167 k(9+3 \sqrt{9+92 k c}+46 k c)^{2}(3+\sqrt{9+92 k c})}$.
From Proposition 2, compared with proposition 1, we find that the quantities of both products have increased (i.e., $q_{l}^{T^{*}}>q_{l}^{O^{*}}, q_{h}^{T^{*}}>q_{h}^{O^{*}}$ ). Possible explanations for this observation are as follows. Both our models face the classic double marginalisation prob$l e m^{6}$ because they consist of an upstream agent (manufacturer) and downstream agents (retailers). However, in Model T, the manufacturer distributes products through two competitive retailers, a strategy that can mitigate the adverse effects of double marginalisation. As a result, compared with Model O, the units of both products increase in Model T; that is, $q_{l}^{T^{*}}>q_{l}^{O^{*}}, q_{h}^{T^{*}}>q_{h}^{O^{*}}$.

## 4. Results and implications

To ensure the comparison of the interior point solutions to both models, as in Gilbert and Cvsa (2003), Savaskan et al. (2004) and Yan et al. (2015), we derive the following assumption: in both models, the cost of selling a high-tier product is not sufficiently large; that is, $0<c<\min \left(\frac{1}{36 k}, 1\right)$. As in the rest of the subsection, we consider only the intersection of the two models.

[^5]
### 4.1. Effect of differentiated channel policies on quality segmentation

Based on Propositions 1 and 2, we derive some interesting insights into the two models. We now address the question posed at the beginning of this paper: How do the manufacturer's quality decisions vary under differentiated channel policies? We answer this question as follows:

Remark 1. Compared with Model O, the levels of quality differentiation in Model T decrease, that is, $u_{h}^{T^{*}}-u_{l}^{T^{*}}<u_{h}^{O^{*}}-u_{l}^{O^{*}}$.

A major concern of this paper is to examine the strategic consequences of cannibalisation and competition under the manufacturing/marketing trade-offs. Remark 1 reveals that, when confronted by two competitive retailers, the optimal policy for the manufacturer is more likely to reduce the difference between both products than to increase it. This argument is contrary to the conventional wisdom that, under a competitive situation, a firm needs to "distort" product quality levels away from each other to mitigate the cannibalisation problem between product lines (e.g., Mussa and Rosen (1978), Desai (2001) and Ha et al. (2016)).

This can be interpreted as follows. Note that the monopoly manufacturer can interact with two competitive retailers in Model T. Intuitively, as the competition between the retailers increases, the profitability of the supplier increases (Kopalle et al. 2009; Biswas et al. 2016). Taking this reasoning one step further, to introduce more intense downstream competition, as described in Remark 1, the manufacturer is more likely to increase the substitutability of products, which leads to a more intense cannibalisation problem. Conversely, in Model O, all products are distributed by a monopoly retailer; thus, if the manufacturer creates a more intense cannibalisation problem, both the monopoly retailer and the manufacturer will suffer from the increased substitutability of both products.

The common conclusion of previous research in this area (e.g., Villas-Boas (1998), Desai et al. (2001) and Qi et al. (2016)) is that, in general, exaggerated product differentiation in a product line is created by downward quality distortion of the lowtier product, while the high-tier product is immune to quality distortion. However, it is not clear whether this conclusion will hold if the manufacturer confronts a retailer (or retailers) who has a potential flexibility to choose different channel polices. In particular, we formulate the following remark:

Remark 2. Compared with Model O, the manufacturer always downwardly distorts the high-tier products in Model T; however, the quality distortion of low-tier products may be downward or upward.

Remark 1 shows that, compared to that in Model O, the optimal policy of the manufacturer would reduce the difference between the two products in Model T. Remark 2 further indicates that the competition between downstream agents may affect both the high-tier and low-tier products: On the one hand, in a high-valuation market, the optimal quality of high-tier products in Model T is always lower than that in Model O . On the other hand, in a high-valuation market, when $c>\frac{162}{10000 k}$, the optimal quality of low-tier products in Model T is always lower than that in Model O ; otherwise, the opposite is true. Taken together, these two remarks suggest that, when confronting the competition between downstream agents, the manufacturer is more likely to reduce the difference between the two products by unduly downwardly distorting the quality of the high-tier products; however, she may downwardly or upwardly distort the quality of the low-tier products.

As mentioned earlier, selling products through a two-channel policy, in which two downstream agents independently seek to maximise their own profit, results in stronger competition than in Model O. If the high-tier products were not counterbalanced by setting a lower price through downwardly distorting quality, then the cannibalisation from low-tier products would unduly reduce the demand for the high-tier products and thereby reduce the profits. Thus, although the downward quality distortion for hightier products reduces the marginal revenue from them, it increases profits by supporting their substantial demand through offering lower prices. Note that the manufacturer's profits come from two sources: selling high- and low-tier products. When the selling cost disadvantage for high-tier products is sufficiently pronounced (i.e., $c>\frac{162}{10000 k}$ ), the manufacturer's profitability from high-tier products decreases. Thus, in order to earn more profits, the manufacturer has little concern about cannibalisation from the lowtier products and would increase the availability of low-tier products by downwardly distorting their quality. However, when the selling cost disadvantage for high-tier products is not pronounced (i.e., $c<\frac{162}{10000 k}$ ), the manufacturer is greatly concerned about cannibalisation from the low-tier products. To avoid reducing the marginal revenue from high-tier products, the manufacturer would upwardly distort low-tier products, resulting in a lower cannibalisation problem from those low-tier products.

Conventional wisdom also suggests that an exaggerated product differentiation accompanies the downward quality distortion of a low-tier product, while the high-tier product is immune to quality distortion. In particular, Villas-Boas (1998) concluded that, in general, the downward quality distortion of a low-tier product becomes magnified, leading to quality degradation and increased differentiation in the product line. However, Remark 2 reveals that, when confronting competing downstream agents, a manufacturer is more likely to reduce the quality difference by unduly downwardly distorting the quality of the high-tier products. Although a similar modelling approach is adopted in Villas-Boas (1998), our model differs due to its focus on whether (how) differentiated channel policies affect manufacturers' quality differentiation and all parties' performance. It is also inconsistent with the results of Chung and Lee (2014), who show that channel decentralisation does not necessarily lead to quality distortion of low-tier products, but that this can happen to high-tier products.

### 4.2. Effect of differentiated channel policies on profitability

We can now address the second question posed at the beginning of this paper: Which scenario is beneficial for the manufacturer, the retailer(s) and the supply chain: selling differentiated products under one channel or two? Based on Propositions 1 and 2, we are able to summarise several key differences between the two models:

Remark 3. i) The manufacturer is always better off in Model $T$ than in Model $O$; that is, $\pi_{m}^{T^{*}}>\pi_{m}^{O^{*}}$;
ii) The retailer is usually worse off in Model $T$ than in Model $O$; that is, $\pi_{r}^{T^{*}}<\pi_{r}^{O^{*}}$;
iii) The profit of the total supply chain in Model $T$ is higher than that in Model $O$; that is, $\pi_{s}^{T^{*}}>\pi_{s}^{O^{*}}$.

Remark 3i) shows that the manufacturer always benefits from the two-channel policy because two factors provide her with greater profits in Model T. First, as the number of retailers increases (from one retailer in Model O to two retailers in Model T), the competition between downstream agents becomes fiercer; consequently, both retailers are more likely to offer a lower price but larger quantities than those in Model O. Thus, consistent with Remark 3i) shown, as the competition between downstream agents (retailers) increases, the profitability of the supplier (manufacturer) increases. Second, as described in Remark 1, under the two-channel policy in Model T, the manufacturer can derive more revenue from retailer competition by decreasing the level of quality
differentiation. As a result, the manufacturer can obtain even higher profits from the two-channel policy than from the one-channel policy.

Not surprisingly, the profits of the retailer are always lower in Model T than in Model O. Interestingly, however, Remark 3ii) is inconsistent with the results of Zhang and Cao (2014); they treat quality as an exogenous variable, whereas we consider quality as an endogenous decision made by the manufacturer. Moreover, they only address different channel policies from the retailers' perspective, and pay little attention to how different channel policies can affect the manufacturer's quality differentiation decisions.

To explain the variation in the supply chain profit, we first note that allowing retailers to compete with each other in Model T can mitigate the traditional double marginalisation problem in the supply chain. Not surprisingly, Remark 3iii) reveals that, although the retailer suffers more in Model T , the profits of the total supply chain are always greater in Model T than in Model O. On the one hand, as described in Remark 3i), as the competition between downstream agents (retailers) increases, the profitability of the supplier (manufacturer) increases. On the other hand, the competition between retailers can enhance the supply chain profit even when it reduces both retailers' profits (see Remark 3ii)), due to mitigation of the traditional double marginalisation problem in the supply chain when the two retailers compete.

### 4.3. The role of competition between downstream agents

We distinguish between the cost of selling high- and low-tier products with an assumption of $c_{h}=c>c_{l}=0$. Such a premium has been widely adopted in the literature to reflect the level of the competition between two channels (Arya et al. 2007, Ha et al. 2016, Yan et al. 2018). We can now highlight the role of competition between downstream agents by considering the effect of differentiated selling costs on the equilibrium in both the models below.

Remark 4. i) As the selling cost of high-tier products (c) increases, the levels of quality differentiation in Model $T$ become smaller relative to those in Model $O$; that is, $\partial\left[\left(u_{h}^{T^{*}}-u_{l}^{T^{*}}\right)-\left(u_{h}^{O^{*}}-u_{l}^{O^{*}}\right)\right] / \partial c>0 ;$
ii) The difference in the retailer's profit between the two models is the highest for the medium selling cost of $c_{\Delta}$; that is, when $c<c_{\Delta}, \partial\left(\pi_{R}^{T^{*}}-\pi_{R}^{O^{*}}\right) / \partial c>0$, otherwise, the opposite is true;
iii) As the cost of selling high-tier products (c) increases, the difference in the profitability for the manufacturer and the supply chain between the two models decrease; that $i s, \partial\left(\pi_{M}^{T^{*}}-\pi_{M}^{O^{*}}\right) / \partial c<0, \partial\left(\pi_{S}^{T^{*}}-\pi_{S}^{O^{*}}\right) / \partial c<0$.

Remark 4i) suggests that the quality differentiation in both models decreases with the cost of selling high-tier products. Recall that an increase in the cost of selling high-tier products means that retailers have a greater disadvantage in marketing hightier products, which can reduce the competition between high- and low-tier products. Note that increased competition among retailers contributes to the profitability of the manufacturer. Hence, in Model T, as the disadvantage from selling high-tier products increases, the manufacturer tries to increase the difference between the products. However, in Model O, when confronting a monopolist retailer who distributes both products together, as the disadvantage of selling high-tier products increases, the manufacturer is more likely to reduce the difference between the products.

Remark 4ii) shows that the cost of selling high-tier products plays an interesting and intuitive role in the retailer's profits: in addition to cannibalisation of high-tier products by low-tier ones, as the cost of selling high-tier products decreases, the competition between the two channels intensifies and causes the profitability of both retailers to decline. Conversely, the cost of selling high-tier products increases and the retail cost disadvantage for the high-end store is too great, which causes the high-end store to derive less revenue from high-tier products and results in the retailer's profitability to decrease. Therefore, the difference between the two models in the retailer's profit is highest for a medium sale cost of $c_{\Delta}$.

As Remark 4iii) shows, the difference in profits for the manufacturer and the total supply chain reduces between the two models. This can be interpreted as follows: as mentioned earlier, an increase in the cost of selling high-tier products can mitigate the competition between downstream agents. More specifically, in Model T, high-tier products and low-tier products are distributed through two independent retailers who do not care about the other's profitability. However, in Model O, all products are distributed by a monopoly retailer who cares greatly about the cannibalisation problem between the two products. Thus, as Remark 4iii) indicates, an increased cost of selling high-tier products has a greater impact on the profitability of both the manufacturer and industry in Model O than in Model T.

### 4.4. Numerical analysis

In our analysis to this stage, we have used the game theoretical method to address how differentiated channel policies in marketing affect a manufacturer's design of product lines and the profitability of all parties. To confirm our results, we now undertake an
extensive numerical analysis.
In our both of our models, the manufacturer's optimal decisions depend on a fundamental question: whether (how) differentiated channel policies affect manufacturers' quality differentiation and all parties' performance. To address the effects of differentiated channel policies, we will focus our numerical examples on how the nature of competition between downstream agents, $c$, affects the equilibrium of both models. Without loss of generality, in all numerical experiments, we would let $k=0.02$. Recall that, to ensure the comparison of the interior point solutions to both models, we set $0<c<\min \left(\frac{1}{36 k}, 1\right)$; that is, in all numerical examples, we restrict that $0<c<1$. All figures are obtained from numerical simulation in Matlab 2014.

In the first analysis, we confirm that the optimal quality chosen and the difference in quality segmentation under the differentiated channel policies are consistent with Remarks 1 and 2. More specifically, on the one hand, by comparing ( $u_{h}^{T^{*}}-u_{l}^{T^{*}}$ ) and $\left(u_{h}^{O^{*}}-u_{l}^{O^{*}}\right)$ in Figure 1(a), we can conclude that $u_{h}^{T^{*}}-u_{l}^{T^{*}}<u_{h}^{O^{*}}-u_{l}^{O^{*}}$. That is, as Remark 1 shows, the levels of quality differentiation in Model T decrease compared with Model O. On the other hand, Figure 1(a) shows that, for any cost of marketing a hightier product $c, u_{h}^{T^{*}}$ is always lower than $u_{h}^{O^{*}}$; this means that the manufacturer always downwardly distorts the high-tier products in Model T relative to Model O. However, the quality distortion of the low-tier products is illustrated by $u_{l}^{T^{*}}$ and $u_{l}^{O^{*}}$ in Figure 1(a). More specifically, as Remark 2 shows, there exists a threshold, $c=0.81$, above which the optimal quality of low-tier products in Model T is always lower than that in Model O. This means that, when $c>0.81$, the manufacturer always downwardly distorts the low-tier products in Model T relative to Model O; otherwise, the opposite is true. Additionally, based on Figure 1(a), the quality of all products in both models decreases with the competition between the downstream agents.

In the second study, to check on the robustness of Remark 3 on the competition between downstream agents, we performed a numerical analysis of the effect of differentiated channel policies on all parties' profitability. To avoid unnecessary complication, we again assume that $k=0.02$ and $0<c<\min \left(\frac{1}{36 k}, 1\right)$. From Figure 1(b) we conclude that, as the selling cost of high-tier products (c) increases, the manufacturer's profits decrease in both models. Furthermore, as Remark 2i) shows, for any selling cost of $c$, the manufacturer's profit is always higher in Model T than in Model O. We see a similar effect: as the selling cost of high-tier products $(c)$ increases, the retailer's profits
in both models decrease (see Figure 1(c)). However, we can observe that, as Remark 2ii) shows, for any selling cost of $c$, the retailer's profit is always lower in Model T than in Model O. From Figure 1(d), we find that, for any selling cost of $c$, the profit of the total supply chain is higher in Model T than in Model O. That is, compared to Model O, the manufacturer's profit in Model T is sufficiently large to "compensate" for the profit "loss" of the retailer.

(a) Effect of channel policies on $u^{*}$

(c) Effect of channel policies on $\pi_{r}^{b^{*}}$

(b) Effect of channel policies on $\pi_{m}^{b^{*}}$

(d) Effect of channel policies on $\pi_{s}^{b^{*}}$

Figure 1: Effect of differentiated channel policies on equilibrium.

To further explore the implications of differentiated channel policies on the equilibrium in both models, we now demonstrate numerically how our results are affected by competition between downstream agents. More specifically, from Figure 2(a), as the selling cost, $c$, decreases (meaning that competition increases), in Model T, the manufacturer tries to increase the difference between the two products. However, in Model O, when confronting a monopolist retailer who distributes both products together, as the selling cost, $c$, decreases, the manufacturer is more likely to reduce the difference between the products; this is to maximise his own profit and to mitigate the canni-
balisation between both products. Figure 2(b) illustrates that, as Remark 4 ii) and iii) shown, the difference in the retailer's profit between the two models is the highest for the medium selling cost of $c_{\Delta}$. However, the difference in the profitability for the manufacturer and the supply chain between the two models decrease with the cost of $c$.

(a) Effect of $c$ on $u_{h}^{b^{*}}-u_{l}^{b^{*}}$

(b) Effect of $c$ on $\pi_{a}^{T^{*}}-\pi_{a}^{O^{*}}$

Figure 2: Effect of $c$ on equilibrium.

## 5. Model Generalizations ${ }^{7}$

In this section, we analyze two relevant extensions and discuss: 1) How does the additional horizontal heterogeneous in their search costs, transaction costs, or brand loyalty for differentiated channels affect the equilibrium decisions (see §5.1); 2) What is the implications of the competition between manufacturers. (see §5.2)

### 5.1. Two-dimensional consumer heterogeneity

In the previous sections, we considered a market where all consumers are only vertically heterogeneous with respect to their willingness to pay for differentiated quality products. Although this is consistent with previous literature on quality segmentation (e.g., Desai et al. (2001), Choudhary et al. (2005) and Ha et al. (2016)), in reality, the manufacturer may adopt differentiated channel policies in terms of market segmentation, with a correlation between the consumers' values and search costs. To capture this possibility, we incorporate the additional horizontal heterogeneous behavior in our framework implies that consumers utility as being two-dimensionally heterogeneous with both the

[^6]vertical dimension (in their willingness to pay for differentiated quality products) and horizontal dimension (in their search costs, transaction costs, or brand loyalty for differentiated channels). In accordance with previous studies involving two-dimensional consumer heterogeneity (Desai et al. 2001, Tyagi 2004, Shi et al. 2013), we assume that consumer utility is defined as $U(u, p, \theta, t, x)=\theta u-p-t x$, where consumers are horizontally heterogeneous along transaction costs in $x$, which follows a general distribution over a $[0,1]$ line segment representing a linear market (Hotelling 1929). Like Tyagi (2004) and Shi et al. (2013), we can derive the inverse demand functions for highand low-tier products from the consumer utility functions as follows:
\[

$$
\begin{align*}
& p_{h}=u_{h}-u_{h} q_{h}-u_{l} q_{l}-t x  \tag{6}\\
& p_{l}=u_{l}\left(1-q_{h}-q_{l}\right)-t(1-x)
\end{align*}
$$
\]

We can use backward induction to solve both models and obtain the following result.
Remark 5. If consumers are consumers are two-dimensionally heterogeneous with one vertical dimension and one horizontal dimension, then:
i) The manufacturer is more likely to reduce the product quality distortion in Model $T$ than in Model $O$; i.e., $u_{h}^{T^{*}}-u_{l}^{T^{*}}<u_{h}^{O^{*}}-u_{l}^{O^{*}}$; furthermore, $\partial\left(\left(u_{h}^{T^{*}}-u_{l}^{T^{*}}\right)-\left(u_{h}^{O^{*}}-u_{l}^{O^{*}}\right)\right) / \partial x<0$ and achieves minimum at $x_{\Delta}$;
ii) Both the industry and the manufacturer are better off in Model $T$ than in Model O, i.e., $\pi_{m}^{T^{*}}>\pi_{m}^{O^{*}}, \pi_{s}^{T^{*}}>\pi_{s}^{O^{*}}$, while the opposite is true for the retailer, i.e., $\pi_{r}^{T^{*}}<\pi_{r}^{O^{*}}$; furthermore, $\partial\left(\pi_{m}^{T^{*}}-\pi_{m}^{O^{*}}\right) / \partial x>0 ; \partial\left(\pi_{r}^{T^{*}}-\pi_{r}^{O^{*}}\right) / \partial x<0$; and $\partial\left(\pi_{s}^{T^{*}}-\pi_{s}^{O^{*}}\right) / \partial x>0$.

Remark 5 indicates how the transaction costs for different channels impacts on the manufacturer's quality segmentation under differentiated channel policies. It also reveals that Remarks 1-4, which indicate that a range of operational management issues arise for manufacturers when all consumers are only vertically heterogeneous on differentiated quality products, can be extended to a market where consumer utility is two-dimensionally heterogeneous in the vertical dimension (in their willingness to pay for differentiated quality products) and the horizontal dimension (in their search costs, transaction costs, or brand loyalty for differentiated channels).

Next, we go a step further to reveal all possible outcomes in the numerical experiments. First, from Figure 3 (a) we observe that $\left(u_{h}^{T^{*}}-u_{l}^{T^{*}}\right)-\left(u_{h}^{O^{*}}-u_{l}^{O^{*}}\right)<0$. Thus, we can conclude that Remark 1, which indicates that the levels of quality differentiation decline in Model T relative to Model O , is robust, regardless of whether there is a customer search problem and/or transaction costs between different channels. Furthermore, the difference in the levels of quality differentiation under both models is a
concave function for the transaction costs $x$, and reaches its maximum at $x_{\Delta}$. Second, Figure 3(b) shows that the manufacturer's profit is always higher under Model T than under Model O . This difference increases with the transaction costs $x$; that is, $\partial\left(\pi_{m}^{T^{*}}-\pi_{m}^{O^{*}}\right) / \partial x>0$. Third, Figure 3(b) shows that, from the retailer's perspective, selling differentiated quality products through two channels can still lead to a loss in profitability; that is, $\pi_{r}^{T^{*}}<\pi_{r}^{O^{*}}$. This is consistent with Remark 3ii). Finally, Figure 3(b) shows that selling differentiated quality products through two channels can still lead to a higher profit for the supply chain; that is, $\pi_{s}^{T^{*}}>\pi_{s}^{O^{*}}$. This is consistent with Remark 3iii).


Figure 3: Variations in equilibrium.

### 5.2. Manufacturer-level competition

Our analysis until now has assumed that the manufacturer is the monopoly supplier in the market. This is inconsistent with the practice where multiple manufacturers compete with each other to distribute products through a common retailer in the same market. Thus, in this subsection, we consider the scenario in which two manufacturers compete with each other for providing differentiated products. Comparing these results from those in the preceding section allows us to focus specifically on the implications of competition at the manufacturer level.

Let $q_{i}$, and $Q_{i}$, be the units of products made by two manufacturers, where $i=h, l$ denotes the type of product (high- or low-quality, respectively) of manufacturer 1 or 2 . Then, following (McGuire and Staelin 1983, Lal 1990, Desai and Purohit 1999), each firm's demand functions are given by:

Focal Firm:

$$
\begin{align*}
& p_{h}=u_{h}-u_{h}\left(q_{h}+e Q_{h}\right)-u_{l}\left(q_{l}+e Q_{l}\right)  \tag{7}\\
& p_{l}=u_{l}\left(1-q_{h}-e Q_{h}-q_{l}-e Q_{l}\right)
\end{align*}
$$

Competitor:

$$
\begin{align*}
& P_{h}=u_{h}-u_{h}\left(Q_{h}+e q_{h}\right)-u_{l}\left(Q_{l}+e q_{l}\right)  \tag{8}\\
& P_{l}=u_{l}\left(1-Q_{h}-e q_{h}-Q_{l}-e q_{l}\right)
\end{align*}
$$

Where $0<e<1$ represents the degree of competition between the two manufacturers The higher the value of $e$, the more intense is the competition between them.

Solving both competitors' problems with backward induction, we can obtain several interesting characteristics under competition at the manufacturer level.

Remark 6. If manufacturers compete with each other in a market, then:
i) Compared with Model $O$, the levels of quality differentiation in Model $T$ decrease, that is, $u_{h}^{T^{*}}-u_{l}^{T^{*}}<u_{h}^{O^{*}}-u_{l}^{O^{*}}$;
ii) Both manufacturers are always better off in Model $T$ than in Model O, i.e., $\pi_{m}^{T^{*}}>\pi_{m}^{O^{*}}, \Pi_{m}^{T^{*}}>\Pi_{m}^{O^{*}}$, though their profits in both models decrease with the level of competition, i.e. $\partial \pi_{m}^{j^{*}} / \partial e<0, \partial \Pi_{m}^{j^{*}} / \partial e<0$;
iii) The retailer is always worse off in Model $T$ than in Model $O$, i.e., $\pi_{r}^{T^{*}}<\pi_{r}^{O^{*}}$, though its profits in both models increase with the level of competition, i.e. $\partial \pi_{r}^{j^{*}} / \partial e>0$;
iv) Iff $e<e_{\Delta}$, the profit of the total supply chain in Model $T$ is higher than that in Model O, i.e., $\pi_{s}^{T^{*}}>\pi_{s}^{O^{*}}$ and $\partial \pi_{s}^{j^{*}} / \partial e<0$.

By comparing the equilibrium decisions in Model O and Model T , we can obtain that Remark 6 counterparts of our main results in the preceding sections (see, e.g., $u_{h}^{T^{*}}-u_{l}^{T^{*}}<u_{h}^{O^{*}}-u_{l}^{O^{*}}, \pi_{m}^{T^{*}}>\pi_{m}^{O^{*}}, \pi_{r}^{T^{*}}<\pi_{r}^{O^{*}}$, and $\left.\Pi_{m}^{T^{*}}>\Pi_{m}^{O^{*}}\right)$. That is, the above results are valid regardless of whether the manufacturer has monopolistic position or not. We further find that, first, compared with Model T, Model O, creating lower profitability for both manufacturers (see, Figure 4 b), is quite consistent with traditional wisdom: As the competition between the manufacturers becomes fiercer, the prices of both products decrease; consequently, both manufacturers are more likely to be hurt in their profitability. Second, the competition between upstream agents (manufacturers) induces the downstream agents (retailers) to restore their monopoly position. Remark 6iii) confirms this conventional wisdom: as the competition between the manufacturers increases, the retailers' profits in both models increase (see, Figure 4 c). Finally, the supply chain' profits in both models would decrease with the competition at the manufacturers' level (see, Figure 4 d).


Figure 4: Effect of $e$ on equilibrium.

## 6. Discussion and managerial implications

During the past two decades, consumer demands have become more diversified and personalized (Ma et al. 2012), to cater to a broader (more heterogeneous) mix of consumer groups, many manufacturers have responded by offering product lines with differentiated quality. Although, there is a considerable body of research on product lines design, most of extent research is focused on quality segmentation from the manufacturing interface and did not include market-related factors, such as the differentiated distribution channel policies. Conversely, in spite the fact that many manufacturers, including Lenovo, Sony, Procter \& Gamble and Buckle, have adopted differentiated channel policies through which to market products of different quality, little is known about whether (how) differentiated channel policies affect manufacturers' quality differentiation and all parties' performance.

To gain additional insight into quality segmentation in the impact of market-related factors, such as differentiated distribution channels, we develop two channel models for a
manufacturer who produces two types of products (high- and low-tier products) together but with two options for marketing them: (1) marketing both products through one retailer (one-channel policy) or (2) providing high-tier products through one retailer but low-tier products through another (two-channel policy). Our main analysis and discussion is of interest to product and marketing managers, as quality segmentation is characterized by a close relationship with differentiated distribution channels. We discuss managerial implications of our key results and make suggestions for further research below.

First, our study suggests that the manufacturer is more likely to decrease the level of quality differentiation in Model T than in Model O. That is, our first result points to the fact that cannibalization in product lines design is not an "evil" to prevent, but an effective strategy that leads financial growth. This is no surprise, on the one hand, as previous research has argued that, as the competition among the retailers increases, the profitability of the supplier increases. Taking the reasoning one step further, we demonstrate that the manufacturer is more likely to increase the substitutability of products, which leads a more intense competition between downstream agents. On the other hand, although many believe that the cannibalization is detrimental to manufacturer, and, thus, should be prevented through a selection with multi-distribution channels, our results are in line with the work of Nijssen (1999), who provided empirical support for this theoretical result when they conducted a survey of 95 product and marketing managers from 21 fast-moving consumer goods companies. In particular, they argued that the manufacturer would prefer to line extensions involve cannibalization problems due to "cannibalization is very much positive related to a line extension's success".

Second, our analysis reveals that "quality distortion" is not limited to low-tier products, but can occur with high-tier products, an argument supported by Robertson (1998) who showed that, although the taste of consumers have dramatically improved, rather releasing those products with radical innovation, many firms are more likely distort downward the quality of high-tier products by sharing components in commonality with those low-tier ones. For example, Toyota motor offered several model of Lexus (high-tier products) based on the same platform and engine as that of the Camry line (low-tier ones). Similarly, the premium Honda Acura car is nothing but "Honda Accord: same perfume, different bottle" (Desai et al. 2001). Similar case also appears in a variety of industries, such as Mobile Phones, Personal Computers, and Electronics and Appli-
ances, where high-tier products usually share basic-common with the existing low-tier units.

Finally, it should be noted that, we have shown a conflict internal to the supply chain between the upstream agents (i.e., manufacturers) and downstream agents (i.e., retailers): The two-channel policy benefits the manufacturer but hurts the retailer. During the 1980s, in order to generate asymmetric bargaining power, manufacturers used distributing quality differentiated products through multi-channels to create an advantage of sharing revenue from the sale process (Aaker et al. 1994). However, the situation has now changed. In particular, the retailing industry today is increasingly dominated by centrally managed "power retailers" who are more sophisticated and manage their product categories more efficiently (Raju and Zhang 2005). As a result, how to coordinate such a channel and help all parties support Model T is particular important for product and marketing managers. ${ }^{8}$

We acknowledge that our analysis is subject to three limitations. First, we assume a monopoly manufacturer who acts as the Stackelberg leader, future research can relax such assumptions by highlighting power structure on the retail service. Second, our model assumes that both players can make decisions under the condition of complete information; in reality, information can be incomplete. ${ }^{9}$ Third, it can also empirically test some of our predictions regarding quality differentiation.

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## Appendices

## A. Derivation the inverse demand functions

We normalized market size to 1 . That is, we assume that consumers' types are distributed uniformly in the interval $[0,1]$ where a consumer of type $\theta \in[0,1]$ has a willingness-to-pay of $u_{h} \theta$ for a high-tier product. Given this assumption, the consumer utility function would be $U_{h}=u_{h} \theta-p_{h}$, where $U_{h}$ represents the consumer's utility for a high-tier product and $p_{h}$ is the price paid for it. Similarly, the consumer utility function for the low-tier product would be $U_{l}=u_{l} \theta-p_{l}$.

Since $u_{h}>u_{l}$, as shown in Figure 5, the utility that each consumer derives from purchasing a product is given by the difference of their valuation and the price. From these two utility functions, we can find that if $U_{l}=u_{l} \theta-p_{l}=0$, a consumer is indifferent between buying a low-tier product and not buying. Therefore, the consumers with $\theta>p_{l} / u_{l}$ would buy the low-tier product. And, when $U_{h}=u_{h} \theta-p_{h}=u_{l} \theta-p_{l}=U_{l}$, a consumer would be indifferent between buying a high-tier product and buying a low-tier one. Hence, the consumers with $\theta>\left(p_{h}-p_{l}\right) /\left(u_{h}-u_{l}\right)$ prefer to the high-tier product than the low-tier one. Based on the net utilities at two different points, we can derive the inverse demand functions in Equation (1).

## B. Proof of Proposition 1

Plugging (1) into the retailer's profit (2), the problem of the retailer is given by: $\max _{q_{h}^{O}, q_{l}^{O}}\left(u_{h}-u_{h} q_{h}-u_{l} q_{l}-w_{h}-c\right) q_{h}+\left(u_{l}-u_{l} q_{h}-u_{l} q_{l}-w_{l}\right) q_{l} . \pi_{r}^{O}$ is jointly concave in $\left(q_{h}, q_{l}\right)$. Thus there is a unique global optimal $\left(q_{h}^{O^{*}}, q_{l}^{O^{*}}\right)$. By applying FOCs to it with respect to $q, q_{l}$, we can obtain $q_{h}^{O^{*}}=\frac{u_{h}+w_{l}-w_{h}-c-u_{l}}{2\left(u_{h}-u_{l}\right)}, q_{l}^{O^{*}}=\frac{u_{h} w_{l}-u_{l} w_{h}-u_{l} c}{2 u_{l}\left(u_{h}-u_{l}\right)}$

Plugging (1), $q_{h}^{O^{*}}$ and $q_{l}^{O^{*}}$ into the manufacturer's profit (3) and $\pi_{m}^{O}$ is jointly concave in $\left(w_{h}, w_{l}\right)$. Thus there is a unique global optimal $\left(w_{h}^{O^{*}}, w_{l}^{O^{*}}\right)$. Solving the first-order condition yields $w_{h}^{O^{*}}=\frac{k u_{h}^{2}+u_{h}-c}{2}, w_{l}^{O^{*}}=\frac{u_{l}+k u_{l}^{2}}{2}$.

Plugging (1), $w_{h}^{O^{*}}$ and $w_{l}^{O^{*}}$ into the manufacturer's profit (3) and we can find that $\pi_{m}^{O}$ is jointly concave in $\left(u_{h}^{O}, u_{l}^{O}\right)$, iff $\frac{2+k u_{l}^{O}-\sqrt{4+4 k k_{l}^{O}-15 k^{2} u_{l}^{O^{2}}}}{8 k}<u_{h}<\frac{2+k u_{l}^{O}+\sqrt{4+4 k u_{l}^{O}-15 k^{2} u_{l}^{O^{2}}}}{8 k}$.

Solving the first-order condition yields one root of $u_{h}^{O^{*}}=\frac{2 \sqrt{1-20 k c}+12 k c-2}{k(2 \sqrt{1-20 k c}-2)}, u_{l}^{O^{*}}=$ $\frac{3-2 \sqrt{1-20 k c}}{5 k}$ is sufficient to above conditions, that is, it will be a maximum point.

Substituting $u_{h}^{O^{*}}, u_{l}^{O^{*}}$ into $w_{h}^{O^{*}}, w_{l}^{O^{*}}, q_{h}^{O^{*}}, q_{l}^{O^{*}}$, (2), (3) and the total profit of the supply chain provides the equilibrium outcomes in Model O.

## C. Proof of Proposition 2

Plugging (1) into the retailer's profit (4), the problem of the retailer is given by:


Figure 5: Consumer state space and corresponding utilities
$\max _{q_{h}^{T}}\left(u_{h}-u_{h} q_{h}-u_{l} q_{l}-w_{h}\right) q_{h}, \max _{q_{l}^{T}}\left(u_{l}\left(1-q_{h}-q_{l}\right)-w_{l}\right) q_{l}$ since $\pi_{R 1}^{T}, \pi_{R 2}^{T}$ is concave of $q_{h}, q_{l}$, respectively. By applying FOCs to it with respect to $q_{h}, q_{l}$, we can obtain $q_{h}^{T *}=\frac{2 u_{h}+w-l-u_{l}-2 w_{h}-2 c}{4 u_{h}-u_{l}}, q_{l}^{T^{*}}=\frac{u_{h} u_{l}-2 u_{h} w_{l}+u_{l} w_{h}+u_{l} c}{u_{l}\left(4 u_{h}-u_{l}\right)}$.

Plugging (1), $q_{h}^{T^{*}}$ and $q_{l}^{T^{*}}$ into the manufacturer's profit (5) and $\pi_{M}^{T}$ is jointly concave in $\left(w_{h}, w_{l}\right)$. Thus there is a unique global optimal $\left(w_{h}^{T^{*}}, w_{l}^{T^{*}}\right)$. Solving the first-order condition yields $w_{h}^{T^{*}}=\frac{k u_{h}^{2}+u_{h}-c}{2}, w_{l}^{T^{*}}=\frac{u_{l}+k u_{l}^{2}}{2}$

Plugging (1), $w_{h}^{T^{*}}, w_{l}^{T^{*}}, q_{h}^{T^{*}}$ and $q_{l}^{T^{*}}$ into the manufacturer's profit (5) and solving the first-order condition yields one root of $u_{h}^{T^{*}}=\frac{9+3 \sqrt{9+92 k c}}{46 k}, u_{l}^{T^{*}}=\frac{6 \sqrt{9+92 k c}+92 k c+18}{23 k(3+\sqrt{9+92 k c})}$ is sufficient to be a maximum point.

Substituting $u_{h}^{T^{*}}, u_{l}^{T^{*}}$ into $q_{h}^{T^{*}}, q_{l}^{T^{*}}, w_{h}^{T^{*}}, w_{l}^{T^{*}}$, (4), (5), and the total profit of the supply chain provides the equilibrium outcomes in Model T.

## D. Proof of remark 1

Comparing $0 \leq q_{h}^{O}, 0 \leq q_{l}^{O}, 0 \leq q_{h}^{T}$, and $0 \leq q_{l}^{T}$, we find that, when $0<c \leq \frac{1}{36 k}$, all are satisfied.

To prove $u_{h}^{T^{*}}-u_{l}^{T^{*}}<u_{h}^{O^{*}}-u_{l}^{O^{*}}$, we have to show that $\frac{9+3 \sqrt{9+92 k c}}{46 k}-\frac{12 \sqrt{9+92 k c}+184 k c+36}{138 k+46 k \sqrt{9+92 k c}}<$
$\frac{2 \sqrt{1-20 k c}+12 k c-2}{k(2 \sqrt{1-20 k c}-2)}-\frac{3-2 \sqrt{1-20 k c}}{5 k} \Leftrightarrow \frac{3 \sqrt{9+92 k c}+46 k c+9}{23 k(3+\sqrt{9+92 k c})}-\frac{2 c}{1-\sqrt{1-20 k c}}<0$
After simplification, this reduces to $0<c<\frac{2}{49 k}$, for the sales of both products to be positive, $0<c<\frac{1}{36 k}$ with this restriction, $u_{h}^{T^{*}}-u_{l}^{T^{*}}<u_{h}^{O^{*}}-u_{l}^{O^{*}}$ is always holds.

## E. Proof of remark 2

To prove $u_{h}^{T^{*}}<u_{h}^{O^{*}}$, we have to show that $\frac{9+3 \sqrt{9+92 k c}}{46 k}<\frac{2 \sqrt{1-20 k c}+12 k c-2}{k(2 \sqrt{1-20 k c}-2)}$
After simplification, this reduces to $0<c<\frac{1}{20 k}$. Since $\frac{1}{36 k}<\frac{1}{20 k}$, that is to say for any $0<c<\frac{1}{36 k}, u_{h}^{T^{*}}<u_{h}^{O^{*}}$ is always holds.

Similarly, simplifying $u_{l}^{T^{*}}-u_{l}^{O^{*}}$, we can obtain that $\frac{460 k c-117-39 \sqrt{9+92 k c}+138 \sqrt{1-20 k c}+46 \sqrt{9+92 k c} \sqrt{1-20 k c}}{115 k(3+\sqrt{9+92 k c})}$.

We can easy find that $u_{l}^{T^{*}}>u_{l}^{O^{*}}$, iff $c<\frac{864 \sqrt{2}-611}{37636 k}$; otherwise, $u_{l}^{T^{*}}<u_{l}^{O^{*}}$. That is, if $c<\frac{864 \sqrt{2}-611}{37636 k}$, the manufacturer would downward distorts the low-tier products in Model T ; otherwise, the quality distortion of low-tier products would be upward.

## F. Proof of remark 3

(i) To prove $\pi_{m}^{T^{*}}>\pi_{m}^{O^{*}}$, we have to show that $\frac{184 k^{3} c^{3}-18 k^{2} c^{2}+\frac{108}{23} k c+\left(4 k^{2} c^{2}-\frac{18}{23} k c+\frac{243}{529}\right) \sqrt{9+92 k c}+\frac{729}{529}}{\mathrm{k}(9+3 \sqrt{9+92 k c}+46 k c)(3+\sqrt{9+92 k c})}>\frac{2 c\left((8 k c-1) \sqrt{1-20 k c}+40 k^{2} c^{2}-18 k c+1\right)}{\left(1-\sqrt{1-20 k c)^{3}}\right.}$.

For the sales of both products to be positive $0<c<\frac{1}{36 k}$. With this restriction, $\pi_{m}^{T^{*}}>\pi_{m}^{O^{*}}$ is always holds.
(ii) To prove $\pi_{r}^{T^{*}}<\pi_{r}^{O^{*}}$, we have to show that
$\frac{3((138 k c-15) \sqrt{9+92 k c}+184 k c-45)\left((299 k c-45) \sqrt{9+92 k c}+6348 k^{2} c^{2}+207 k c-135\right)}{48668 k(3 \sqrt{9+92 k c}+46 k c+9)^{2}}+\frac{(23 k c+9) \sqrt{9+92 k c}+207 k c+27}{12167 k}$ $<\frac{5 c\left(\left(16 k^{2} c^{2}-12 k c+1\right) \sqrt{1-20 k c}-88 k^{2} c^{2}+22 k c-1\right)}{(1-\sqrt{1-20 k c})^{3}(2 \sqrt{1-20 k c}-3)}$ for the sales of both products to be positive $0<$ $c<\frac{1}{36 k}$. With this restriction, $\pi_{r}^{T^{*}}<\pi_{r}^{O^{*}}$ is always holds.
(iii) To prove $\pi_{s}^{T^{*}}>\pi_{s}^{O^{*}}$, we have to show that
$\frac{\left[\begin{array}{l}\left(15817100 k^{3} c^{3}-185679 k^{2} c^{2}+462024 k c+136323\right) \sqrt{9+92 k c} \\ +167904600 k^{4} c^{4}+53802474 k^{3} c^{3}+1185489 k^{2} c^{2}+3476358 k c+408969\end{array}\right]}{12167 k(9+3 \sqrt{9+92 k c}+46 k c)^{2}(3+\sqrt{9+92 k c})}$
$>\frac{15 c\left(\left(16 k^{2} c^{2}-12 k c+1\right) \sqrt{1-20 k c}-88 k^{2} c^{2}+22 k c-1\right)}{(1-\sqrt{1-20 k c})^{3}(2 \sqrt{1-20 k c}-3)}$
for the sales of both products to be positive $0<c<\frac{1}{36 k}$. With this restriction, $\pi_{s}^{T^{*}}>\pi_{s}^{O^{*}}$ is always holds.

## G. Proof of remark 4

(i) Based on Remark 1, we can find that $\partial\left[\left(u_{h}^{T^{*}}-u_{l}^{T^{*}}\right)-\left(u_{h}^{O^{*}}-u_{l}^{O^{*}}\right)\right] / \partial c=\frac{\frac{6 k}{\sqrt{9+92 k c}}+2 k}{k(3+\sqrt{9+92 k c})}-$ $\frac{18+6 \sqrt{9+92 k c}+92 k c}{(3+\sqrt{9+92 k c})^{2} \sqrt{9+92 k c}}+\frac{20 k c}{(\sqrt{1-20 k c}-1)^{2} \sqrt{1-20 k c}}+\frac{2}{\sqrt{1-20 k c}-1}$. Because, $0<c<\frac{1}{36 k}, k>0$, thus, $\partial\left[\left(u_{h}^{T^{*}}-u_{l}^{T^{*}}\right)-\left(u_{h}^{O^{*}}-u_{l}^{O^{*}}\right)\right] / \partial c>0$
(ii) Based on Remark 3, we can find that $\partial\left(\pi_{r}^{T^{*}}-\pi_{r}^{O^{*}}\right) / \partial c=$
$8311163436 k^{5} c^{5} \sqrt{(9+92 k c)(1-20 k c)}-223074 \sqrt{(9+92 k c)(1-20 k c)}-669222 \sqrt{1-20 k c}$ $+889015736 k^{3} c^{3} \sqrt{9+92 k c}+10783314 k^{2} c^{2} \sqrt{9+92 k c}+4539564 k c \sqrt{(9+92 k c)(1-20 k c)}$ $-128909140704 k^{5} c^{5}+2284281248 k^{6} c^{6} \sqrt{(9+92 k c)(1-20 k c)}+21063218392 k^{6} c^{6} \sqrt{9+92 k c}$ $+30703152594 k^{5} c^{5} \sqrt{1-20 k c}+12357778560 k^{7} c^{7} \sqrt{9+92 k c}-22635927986 k^{5} c^{5} \sqrt{9+92 k c}$ $+85365326712 k^{6} c^{6} \sqrt{1-20 k c}-6770304 k c \sqrt{9+92 k c}-795511741 k^{4} c^{4} \sqrt{(1-20 k c)(9+92 k c)}$ $+669222-16890444 k c+10198224 k c \sqrt{1-20 k c}+148723722 k^{2} c^{2} \sqrt{1-20 k c}-98188858032 k^{6} c^{6}$ $-10375631775 k^{4} c^{4} \sqrt{1-20 k c}-2545454689 k^{4} c^{4} \sqrt{9+92 k c}-80202582 k^{2} c^{2}+331510842240 k^{7} c^{7}$ $+3142364196 k^{3} c^{3}-1479826776 k^{3} c^{3} \sqrt{1-20 k c}+223074 \sqrt{9+92 k c}-15581546880 k^{7} c^{7} \sqrt{1-20 k c}$ $-538988276 k^{3} c^{3} \sqrt{(9+92 k c)(1-20 k c)}+23458626 k^{2} c^{2} \sqrt{(9+92 k c)(1-20 k c)}+3931271205 k^{4} c^{4}$

Solving $\partial\left(\pi_{r}^{T^{*}}-\pi_{r}^{O^{*}}\right) / \partial c=0$ yields $c_{\Delta}=\frac{119}{10000 k} . \quad$ When $0<c<\frac{119}{10000 k}, k>0$, $\partial\left(\pi_{r}^{T^{*}}-\pi_{r}^{O^{*}}\right) / \partial c>0 ;$ otherwise, $\partial\left(\pi_{r}^{T^{*}}-\pi_{r}^{O^{*}}\right) / \partial c<0$. Thus, $c_{\Delta}=\frac{119}{10000 k}$ will be a maximum point of $\partial\left(\pi_{r}^{T^{*}}-\pi_{r}^{O^{*}}\right) / \partial c$.
(iii) Based on Remark 3, we can find that $\partial\left(\pi_{m}^{T^{*}}-\pi_{m}^{O^{*}}\right) / \partial c=$ $\underline{552 k^{3} c^{2}+\left(-36 c+8 c \sqrt{9+92 k c}+\frac{184 c^{2} k}{\sqrt{9+92 k c}}\right) k^{2}+\left(-\frac{18}{23} \sqrt{9+92 k c}-\frac{36 k c}{\sqrt{9+92 k c}}+\frac{108}{23}\right) k+\frac{486 k}{23 \sqrt{9+92 k c}}}$
$\mathrm{k}(9+3 \sqrt{9+92 \mathrm{kc}}+46 \mathrm{kc})(3+\sqrt{9+92 \mathrm{kc}})$
$-\frac{\left(184 k^{3} c^{3}+\left(-18 c^{2}+4 c^{2} \sqrt{9+92 k c}\right) k^{2}+\left(-\frac{18}{23} c \sqrt{9+92 k c}+\frac{108}{23} c\right) k+\frac{243}{529} \sqrt{9+92 k c}+\frac{729}{529}\right)\left(\frac{138 k}{\sqrt{9+92 k c}}+46 k\right)}{\mathrm{k}(9+3 \sqrt{9+92 \mathrm{kc}}+46 \mathrm{kc})^{2}(3+\sqrt{9+92 \mathrm{kc}})}$
$-\frac{46\left(184 k^{3} c^{3}+\left(-18 c^{2}+4 c^{2} \sqrt{9+92 k c}\right) k^{2}+\left(-\frac{18}{23} c \sqrt{9+92 k c}+\frac{108}{23} c\right) k+\frac{243}{529} \sqrt{9+92 k c}+\frac{729}{529}\right)}{(9+3 \sqrt{9+92 \mathrm{kc}}+46 \mathrm{kc})(3+\sqrt{9+92 \mathrm{kc}})^{2} \sqrt{9+92 k c}}$
$+\frac{2\left(-\sqrt{1-20 k c}+8 k c \sqrt{1-20 k c}+40 k^{2} c^{2}+1-18 k c\right)+2 c\left(\frac{10 k}{\sqrt{1-20 k c}}+8 k \sqrt{1-20 k c}-\frac{80 k^{2} c}{\sqrt{1-20 k c}}+80 k^{2} c-18 k\right)}{(-1+\sqrt{1-20 k c})^{3}}$
$+\frac{60 c\left(-\sqrt{1-20 k c}+8 k c \sqrt{1-20 k c}+40 k^{2} c^{2}+1-18 k c\right) k}{(-1+\sqrt{1-20 k c})^{4} \sqrt{1-20 k c}}$
Because, $0<c<\frac{1}{36 k}, k>0$, thus, $\partial\left(\pi_{m}^{T^{*}}-\pi_{m}^{O^{*}}\right) / \partial c<0$.
Based on Remark 3, we can find that $\partial\left(\pi_{s}^{T^{*}}-\pi_{s}^{O^{*}}\right) / \partial c=$
$\frac{5026443387276561}{17592186044416} k+\frac{2266748183490475}{4398046511104} \frac{k}{\sqrt{9+92 k c}}+\frac{40176}{23} \frac{k^{2} c}{\sqrt{9+92 k c}}+\frac{20088}{529} k \sqrt{(9+92 k c}+\frac{4482}{23} k^{2} c$
$+13266 k^{3} c^{2}-\frac{702 k^{3} c^{2}}{\sqrt{9+92 k c}}-\frac{702}{23} k^{2} c \sqrt{9+92 k c}+55200 k^{4} c^{3}+\frac{59800 k^{4} c^{3}}{\sqrt{9+92 k c}}+3900 k^{3} c^{2} \sqrt{9+92 k c}$
$k(3+\sqrt{9+92 k c})(9+3 \sqrt{9+92 k c}+46 k c)^{2}$
$\left(\frac{138 \mathrm{k}}{\sqrt{9+92 \mathrm{kc}}}+46 \mathrm{k}\right)\left[\begin{array}{l}\frac{5026443387276561}{879609302208} \mathrm{kc}+\frac{6307473206233365}{281474976710656} \sqrt{9+92 \mathrm{kc}}+\frac{40176}{529} \mathrm{kc} \sqrt{9+92 \mathrm{kc}}+\frac{4482}{23} k^{2} c^{2} \\ +\frac{2365302552337887}{35184372088832}-\frac{702}{23} k^{2} c^{2} \sqrt{9+92 k c}+27600 k^{4} c^{4}+2600 k^{3} c^{3} \sqrt{9+92 k c} \\ +8844 k^{3} c^{3}\end{array}\right]$ $k(3+\sqrt{9+92 k c})(9+3 \sqrt{9+92 k c}+46 k c)^{3}$
$\frac{115608197907360903}{879609302208} k c+\frac{145071883743390395}{281474976710656} \sqrt{9+92 k c}+\frac{40176}{23} k c \sqrt{9+92 k c}+4482 k^{2} c^{2}$
$+\frac{54401956403771401}{35184372088832}-702 k^{2} c^{2} \sqrt{9+92 k c}+634800 k^{4} c^{4}+59800 k^{3} c^{3} \sqrt{9+92 k c}$
$+203412 k^{3} c^{3}$
$\sqrt{9+92 k c}(3+\sqrt{9+92 k c})^{2}(9+3 \sqrt{9+92 k c}+46 k c)^{2}$
$+\frac{\left[\begin{array}{l}-15+15 \sqrt{1-20 k c}+330 k c-180 k c \sqrt{1-20 k c}-1320 k^{2} c^{2}+240 k^{2} c^{2} \sqrt{1-20 k c}+15 c \\ \left(\frac{120 k^{2} c}{\sqrt{1-20 k c}}-12 k \sqrt{1-20 k c}-\frac{10 k}{\sqrt{1-20 k c}}+22 k-176 k^{2} c+32 k^{2} c \sqrt{1-20 k c}-\frac{160 k^{3} c^{2}}{\sqrt{1-20 k c}}\right)\end{array}\right]}{(\sqrt{1-20 k c}-1)^{3}(2 \sqrt{1-20 k c}-3)}$
$+\frac{450 k c\left(-1+\sqrt{1-20 k c}+22 k c-12 k c \sqrt{1-20 k c}-88 k^{2} c^{2}+16 k^{2} c^{2} \sqrt{1-20 k c}\right)}{}$
$+\frac{300 k c\left(-1+\sqrt{1-20 k c}+22 k c-12 k c \sqrt{1-20 k c}-88 k^{2} c^{2}+16 k^{2} c^{2} \sqrt{1-20 k c}\right)}{\sqrt{1-20 k c}(2 \sqrt{1-20 k c}-3)^{2}(\sqrt{1-20 k c}-1)^{3}}$
Because, $0<c<\frac{1}{36 k}, k>0$, thus, $\partial\left(\pi_{s}^{T^{*}}-\pi_{s}^{O^{*}}\right) / \partial c<0$.

## H. Proof of Remark 5

In Model O, all products are sold through one store, and the retailer therefore chooses his optimal outputs of high- and low-tier products $\left(q_{h}, q_{l}\right)$ to maximise $\max _{q_{h}, q_{l}} \pi_{r}^{O}=\left(p_{h}-\right.$ $\left.w_{h}\right) q_{h}+\left(p_{l}-w_{l}\right) q_{l}{ }^{10}$, to establish optimal quantities as $q_{h}=\frac{1+u_{h}-x-w_{h}+w_{l}-u_{l}-x}{2\left(u_{h}-u_{l}\right)}$ and $q_{l}=\frac{x u_{l}+u_{l} w_{h}-u_{h}(1-x)-w_{l} u_{h}}{2 u_{l}\left(u_{h}-u_{l}\right)}$. Substituting these into Equation (3) and solving the FOCs provides $w_{h}=\frac{k u_{h}^{2}+u_{h}-x}{2}$ and $w_{l}=\frac{k u_{l}^{2}+u_{l}-(1-x)}{2}$, respectively. In the last stage, the manufacturer's problem is to design product qualities to maximise the profit in Equation (3); accordingly, we can determine that $u_{h}=\frac{3 k+\sqrt{5-4 \sqrt{1+12 k(1-x)}+48 k(1-x)-84 k x}}{6 k}$ and $u_{l}=$ $\frac{1+\sqrt{1+12 k(1-x)}}{6 k}$.

In Model T, Retailer One chooses his output of high-tier products $\left(q_{h}\right)$ to maximise $\max _{q_{h}} \pi_{r 1}^{T}=\left(p_{h}-w_{h}\right) q_{h}$, while, Retailer Two chooses his output of low-tier products $\left(q_{r}\right)$ to maximise $\max _{q_{l}} \pi_{r 2}^{T}=\left(p_{l}-w_{l}\right) q_{l}$, to establish optimal quantities as $q_{h}=\frac{1+2 u_{h}-3 x-2 w_{h}-u_{l}+w_{l}}{4 u_{h}-u_{l}}$ and $q_{l}=\frac{u_{h} u_{l}+x u_{l}+u_{l} w_{h}-2 u_{h}(1-x)-2 u_{h} w_{l}}{u_{l}\left(4 u_{h}-u_{l}\right)}$. Substituting these into Equation (5) and solving the FOCs provides $w_{h}=\frac{k u_{h}^{2}+u_{h}-x}{2}$ and $w_{l}=\frac{k u_{l}^{2}+u_{l}-1+x}{2}$, respectively. The manufacturer's problem is then to design product qualities to maximise Equation (3), which provides $u_{h}=\frac{3+\sqrt{7-2 \sqrt{1+12 k(1-x)}-60 k x+24 k}}{6 k}$ and $u_{l}=\frac{1+\sqrt{1+12 k x}+\sqrt{2+2 \sqrt{1+12 k x}+48 k x-36 k}}{6 k}$.

As before, we can obtain the equilibrium outcomes using backward induction, in particular,
$u_{h}^{O^{*}}=\frac{3+A}{16 k}, u_{l}^{O^{*}}=\frac{1+B}{16 k}$
$\pi_{m}^{O^{*}}=\frac{\left[\begin{array}{l}1-12 k+18 k x B+6 k x A+144 k^{2} x B+72 k^{2} x^{2} A-72 k^{2} B+288 k^{2} \\ +12 k x-72 k^{2} x^{2} B-144 k^{2} x A-12 k A B+B+A B-6 k A-18 k B \\ +A+72 k^{2} A+12 k x A B+288 k^{2} x^{2}-576 k^{2} x\end{array}\right]}{54 k(1+B)(2-B+A)}$
$\pi_{r}^{O^{*}}=\frac{\left[\begin{array}{l}1-12 k+18 k x B+6 k x A+144 k^{2} x B+72 k^{2} x^{2} A-72 k^{2} B \\ +288 k^{2}+12 k x-72 k^{2} x B-144 k^{2} x A-12 k B A-576 k^{2} x \\ +B+A B-6 k A-18 k B+A+72 k^{2} A+12 k x A B+288 k^{2} x^{2}\end{array}\right]}{108 k(1+B)(2-B+A)}$
$u_{h}^{T^{*}}=\frac{3+C}{6 k}, u_{l}^{T^{*}}=\frac{1+D+E}{6 k}$

[^8]\[

$$
\begin{aligned}
& \pi_{m}^{T^{*}}=\frac{\left[\begin{array}{l}
5-216 k+3 E+12 k x C D E-360 k^{2}+180 k x-B+216 k^{2} x^{2} E-720 k^{2} x D \\
-216 k^{2} x C+360 k^{2} x^{2} C+72 k C D+12 k D E-102 k x E+78 k x C+C D E+2 C \\
+432 k^{2} x^{2} D+5 D+720 k^{2} x^{2}+72 k^{2} x-30 k D B-12 k B E-24 k x B-D E B \\
-36 k x C E+24 k x D E+2 C D+24 k x B E+42 k x B D+72 k^{2} E+288 k^{2} D-B D \\
+216 k D+2 D E-90 k C+6 k E+C E-288 k^{2} x E-276 k x D-96 k x C D+24 k B
\end{array}\right]}{54 k(1+D+E)(11-D-E+4 C)} \\
& \pi_{r 1}^{T^{*}}=\frac{\left[\begin{array}{l}
72 k-3-108 k x-6 B+C+6 E+3 D+C D+2 C E-8 C+24 k x C \\
-3 D E-(7-2 C D E)(1+36 k x+2 B-24 k-D-2 E+D E)
\end{array}\right]}{216 k(11-D-E+4 C)^{2}} \\
& \pi_{r 2}^{T^{*}}=\frac{\left[\begin{array}{l}
84 k x-5-12 k+12 k x D+12 k x E+2 C D E+6 D E-12 k D+B \\
+24 k x C-2 C D+B E+B D-5 D-11 E-2 C-4 C E-12 k E
\end{array}\right]}{216 k(11-D-E+4 C)} \\
& \text { where } A=\sqrt{5-4 \sqrt{1+12 k-12 k x}+48 k-84 k x,} \\
& B=\sqrt{1+12 k-12 k x,} \\
& C=\sqrt{7-2 \sqrt{1+12 k-12 k x}-60 k x+24 k}, \\
& D=\sqrt{1+12 k x,} \\
& E=\sqrt{2+2 \sqrt{1+12 k x}+48 k x-36 k} .
\end{aligned}
$$
\]

Note that, to ensure all parameters and variables in this subsection must satisfy non-negativity constraints, we need $\frac{1670 k^{3}-2000 k^{2}+1183 k+200}{1000} \leq c<\frac{47760 k^{3}+340 k^{2}+418 k-15}{100000 k^{3}}$

The procedure for the proof of Remark 5 is similar to that of Remark 4 in $\S 4.3$. Thus the details are omitted here.

## H. Proof of Remark 6

In Model O, all products are sold through one store, and the retailer therefore chooses his optimal outputs of high- and low-tier products $\left(q_{h}, q_{l}, Q_{h}, Q_{l}\right)$ to maximise $\max _{h_{h}, q_{l}, Q_{h}, Q_{l}} \pi_{r}^{O}=\left(p_{h}-w_{h}\right) q_{h}+\left(p_{l}-w_{l}\right) q_{l}+\left(P_{h}-W_{h}\right) Q_{h}+\left(P_{l}-W_{l}\right) Q_{l}$, to establish optimal quantities as $q_{h}=Q_{h}=\frac{e W_{l}-e W_{h}+u_{l}-w_{l}-u_{h}-u_{l} e+u_{h} e+w_{h}}{2\left(u_{l}-u_{l} e^{2}+u_{h} e^{2}-u_{h}\right)}$ and $q_{l}=Q_{l}=\frac{e u_{l} W_{h}-u_{h} e W_{l}+u_{h} w_{l}-u_{l} w_{h}}{2 u_{l}\left(u_{l}-u_{l} e^{2}+u_{h} e^{2}-u_{h}\right)}$. Substituting these into Equation (3) and the similar expression for the competitor. Solving the FOCs provides $w_{h}=W_{h}=\frac{u_{h}\left(u_{h} k-e+1\right)}{2-e}$ and $w_{l}=W_{l}=\frac{u_{l}\left(u_{l} k-e+1\right)}{2-e}$, respectively. In the last stage, the manufacturer's problem is to design product qualities to maximise the profit in Equation (3); accordingly, we can determine that $u_{h}=\frac{2}{5 k}$ and $u_{l}=\frac{1}{5 k}$.

In Model T, Retailer One chooses his output of high-tier products ( $q_{h}$ and $Q_{h}$ ) to maximise $\max _{q_{h}, Q_{h}} \pi_{r 1}^{T}=\left(p_{h}-w_{h}\right) q_{h}+\left(P_{h}-W_{h}\right) Q_{h}$, while, Retailer Two chooses his output of low-tier products $\left(q_{l}\right.$ and $\left.Q_{l}\right)$ to maximise $\max _{q_{l}, Q_{l}} \pi_{r 2}^{T}=\left(p_{l}-w_{l}\right) q_{l}+\left(P_{l}-W_{l}\right) Q_{l}$, to establish optimal quantities as $q_{h}=Q_{h}=\frac{2 u_{h} e-2 u_{h}+2 w_{h}-w_{l}-u_{l} e+u_{l}-2 e W_{h}+e W_{l}}{4 u_{h} e^{2}-4 u_{h}+u_{l}-u_{l} e^{2}}$ and $q_{l}=$
$Q_{l}=\frac{u_{l} W_{h}-2 e W_{l} u_{h}+u_{l} u_{h} e-u_{l} w_{h}-u_{l} u_{h}+2 w_{l} u_{h}}{u_{l}\left(4 u_{h} e^{2}-4 u_{h}+u_{l}-u_{l} e^{2}\right)}$. Substituting these into Equation (5) and the similar expression for the competitor. Solving the FOCs provides $w_{h}=W_{h}=\frac{u_{h}\left(u_{h} k-e+1\right)}{2-e}$ and $w_{l}=W_{l}=\frac{u_{l}\left(1+u_{l} k-e\right)}{2-e}$, respectively. In the last stage, the manufacturer's problem is to design product qualities to maximise the profit in Equation (3); accordingly, we can determine that $u_{h}=\frac{9}{23 k}$ and $u_{l}=\frac{6}{23 k}$.

The details are omitted here and all equilibrium decisions and profits in the following Table.

| Equilibrium Decisions in Model O | Equilibrium Decisions in Model T |
| :--- | :--- |
| $u_{h}^{O^{*}}=\frac{2 k}{5}$ | $u_{h}^{T^{*}}=\frac{9}{23 k}$ |
| $u_{l}^{O^{*}}=\frac{k}{5}$ | $u_{l}^{T^{*}}=\frac{6}{23 k}$ |
| $w_{h}^{O^{*}}=W_{h}^{O^{*}}=\frac{2(7-5 e)}{25 \mathrm{k}(2-e)}$ | $w_{h}^{T^{*}}=W_{h}^{O^{*}}=\frac{9(32-23 e)}{529 \mathrm{k}(2-e)}$ |
| $w_{l}^{O^{*}}=W_{l}^{O^{*}}=\frac{(6-5 e)}{25 \mathrm{k}(2-e)}$ | $w_{l}^{T^{*}}=W_{l}^{O^{*}}=\frac{6(29-23 \mathrm{e}}{529 \mathrm{k}(2-e)}$ |
| $q_{h}^{O^{*}}=Q_{h}^{O^{*}}=\frac{e+1}{5(2-e)}$ | $q_{h}^{T^{*}}=Q_{h}^{O^{*}}=\frac{5(e+1)}{23(2-e)}$ |
| $q_{l}^{O^{*}}=Q_{l}^{O^{*}}=\frac{e+1}{5(2-e)}$ | $q_{l}^{T^{*}}=Q_{l}^{O^{*}}=\frac{6(e+1)}{23(2-e)}$ |
| $\pi_{m}^{O^{*}}=\Pi_{m}^{O^{*}}=\frac{2(1-e)}{25(e+1)(2-e)^{2} k}$ | $\pi_{m}^{T^{*}}=\Pi_{m}^{O^{*}}=\frac{54(1-e)}{529(e+1)(2-e)^{2} k}$ |
| $\pi_{r}^{O^{*}}=\frac{2}{25(e+1)(2-e)^{2} k}$ | $\pi_{r 1}^{T^{*}}=\frac{450}{12167(e+1)(2-e)^{2} k}$ |
|  | $\pi_{r 2}^{T^{*}}=\frac{432}{12167(e+1)(2-e)^{2} k}$ |

The procedure for the proof of Remark 6 is similar to that of Remark 4 in $\S 4.3$. Thus the details are omitted here.


[^0]:    *Corresponding author
    Email address: yanwei@uestc.edu.cn (Wei Yan)

[^1]:    ${ }^{1}$ Throughout this article, we use the feminine pronoun to refer to the manufacturer and the masculine pronoun to refer to the retailer.

[^2]:    ${ }^{2}$ we thank an anonymous reviewer for pointing this out.

[^3]:    ${ }^{3}$ See Appendix for the detailed derivation. We thank an anonymous reviewer for suggesting to list the detailed derivation.
    ${ }^{4}$ For clarity, all proofs are provided in the appendix.

[^4]:    ${ }^{5}$ This determination differs from those of Chung and Lee (2004), which is a key difference that we believe stems from our model's focus on different channel policies and competition between retailers rather than a channel composed of one manufacturer and one retailer, which is either vertically integrated or decentralised.

[^5]:    ${ }^{6}$ All channel members independently seek to maximize their own profit, resulting in higher retail prices and lower sales quantities and profits than in a vertically integrated channel (Spengler 1950).

[^6]:    ${ }^{7}$ We thank an anonymous reviewer for suggesting these two possible model extensions.

[^7]:    ${ }^{8}$ We refer interested readers to Seifbarghy et al. (2015) for a complete discussion.
    ${ }^{9}$ Zhang and Cao (2014), for example, show that when product quality is not readily observable to all consumers, a one-roof policy facilitates more efficient signalling and results in greater profit than a two-roof policy.

[^8]:    ${ }^{10}$ To enable clear analysis of the effect of transaction cost, we assume that the retailer's unit marketing costs for high- and low-tier products are identical, i.e., $c_{h}=c_{l}=c$, and normalised to zero, i.e., $c=0$.

