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Optimised Sequential Experimental Design for Geoelectrical Resistivity Monitoring Surveys

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SUMMARY

Sequential experimental design methods use previous data and results to guide the choice and design of future experiments. This paper describes the application of a sequential design technique to produce optimal resistivity imaging surveys for time-lapse geoelectrical monitoring experiments. These survey designs are time-dependent, and are optimised to focus a greater degree of the image resolution on the regions of the subsurface that are actively changing than static optimised surveys that do not change over time. The sequential design method is applied to a synthetic 2.5D monitoring experiment comprising a well-defined cylindrical target moving along a trajectory that changes its depth and lateral position. The data are simulated to be as realistic as possible, incorporating survey design constraints for a real resistivity monitoring system and realistic levels and distributions of random noise, in order to match a forthcoming experimental test of the method. The results of the simulations indicate that sequentially designed optimal surveys yield an increase in image quality over and above that produced by using a static (time-independent) optimised survey.

Optimal design algorithms for geoelectrical resistivity imaging surveys have been developed that produce greater image resolution than standard surveys (e.g. dipole-dipole) for the same measurement time and cost. Such surveys have been applied to image geological structures e.g. a landslide (Wilkinson et al. 2012a). However, there is considerable interest in using resistivity imaging in a monitoring context as well as for one-off surveys. This raises the possibility that information from previous images could be used to guide the design of future surveys, a process known as sequential experimental design (Guest and Curtis, 2009). Although the optimal design of geoelectrical surveys depends only very weakly on the resistivity structure of the subsurface (Wilkinson et al. 2012b), it would still be possible to focus the resolution of future surveys on regions of the subsurface that have been identified in previous images as being of interest (i.e. regions where significant changes are taking place). In this paper, we describe a proof-of-principle demonstration of a sequential optimised survey design scheme for geoelectrical monitoring. The study uses synthetic data, contaminated by realistic levels of random noise, to simulate a laboratory demonstration of this monitoring concept.

Method & Results

A linearised estimate of the model resolution matrix for the inverse resistivity problem is given by $\mathbf{R} \approx (\mathbf{G}^T\mathbf{G}+\mathbf{C})^{-1}\mathbf{G}^T\mathbf{G}$, where $\mathbf{G}$ is the Jacobian matrix comprising the logarithmic sensitivities of the measurements to changes in the model cell resistivities and $\mathbf{C}$ is the constraint matrix. The principal diagonal of $\mathbf{R}$ gives an estimate of R, the model resolution of the cells, where $R = 0$ is unresolved and $R = 1$ is perfectly resolved. All the permitted unique four-electrode measurements, subject to upper limits on geometric factor and chosen to avoid unstable configurations (Wilkinson et al 2012a), form the comprehensive measurement set. The model resolution for this set over a homogeneous subsurface is shown in Fig. 1(a), calculated for a linear array of 28 electrodes spaced at 0.05m intervals. We assess the quality of a given survey design by its relative model resolution $R_r = R_{survey}/R_{compr}$. To generate a survey, the CR-method uses a locally-optimal iterative design scheme (Wilkinson et al 2012a). It maximises a weighted average of the relative resolution across the image of m cells, $S = \Sigma wR_r/m$, by direct calculation of the changes in R_r when given measurements are added to the survey. In contrast to our previous studies, here we allow each cell to have a different weight w in the sum, to enable the resolution of the image to be focussed on regions of interest. Figures 1(b) and 1(c) show the relative model resolution distributions for a dipole-dipole survey with $a = 1\text{-}4$ and $n = 1\text{-}10$ and an optimised survey, designed with uniform weighting across the model space. Both surveys would require the same number of multi-channel commands (82), and hence the same duration and power, to execute on a ten-channel BGS ALERT system (Wilkinson et al. 2012a). These surveys were applied to monitor the position of a simulated moving target shown in Fig. 1(d). The target is a non-conducting infinite cylinder, arranged with its long axis perpendicular to the imaging plane. The cylinder has a diameter of 0.15m, and starts with its centre at (0.075,-0.135), shown by the dotted circle, and finishes at (1.275,-0.257), showed by the filled circle, moving at a uniform rate along the dashed line over 20 time steps. The data were modelled using Res2DMod with a fine model grid (0.0125m horizontal and vertical spacing) and inverted on a coarser grid with Res2DInv using an L_2 model constraint and an L_1 data constraint (Loke et al.
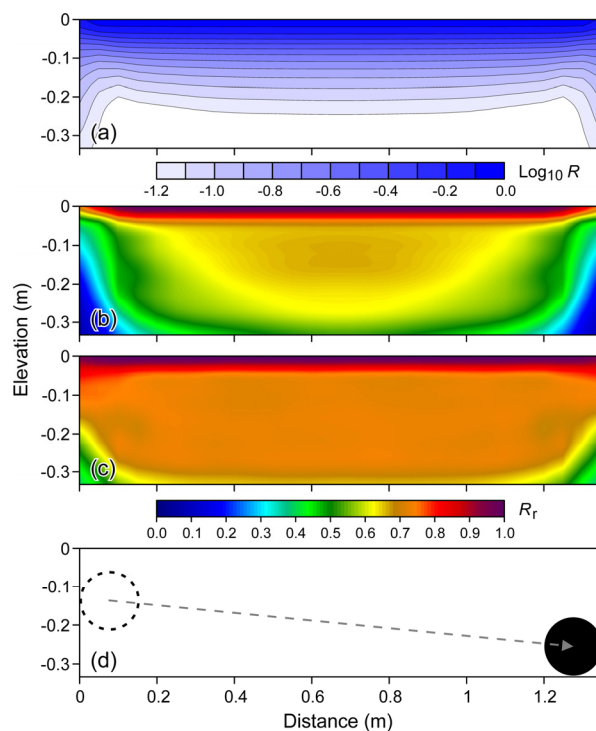

Figure 1(a) *Comprehensive model resolution* R. ***(b)*** *&* ***(c)*** *Relative model resolution* R$_r$ *for the dipole-dipole and optimised surveys.* ***(d)*** *Geometry and track of target.*

2003). The simulated data were contaminated with Gaussian random noise with a geometric factor (K) dependent standard deviation of (0.584 + 0.0076 K) % (Freidel, 2003), which was chosen to match the typical noise levels observed in our laboratory imaging setup.

The purpose of this study is to compare the results of sequentially designed surveys with the standard dipole-dipole survey and the static optimised survey. It is important to distinguish between iterative optimal design techniques for non-linear problems, which are often described as "sequential" (Guest and Curtis, 2009), and sequential experimental design methods which depend explicitly on previous data. In these methods, it is assumed that there is a set of available experiments that may be conducted. After each stage of observation, a decision is made as to which (if any) experiment will be performed next. By attempting to choose the most informative experiment at each stage, superior results can potentially be obtained in comparison to repeating the same experiment at each stage. In geophysical monitoring, one could, for example design a survey to be optimal given estimates from previous t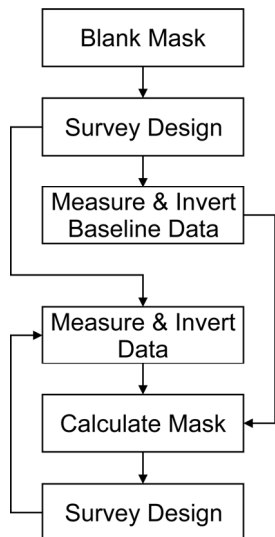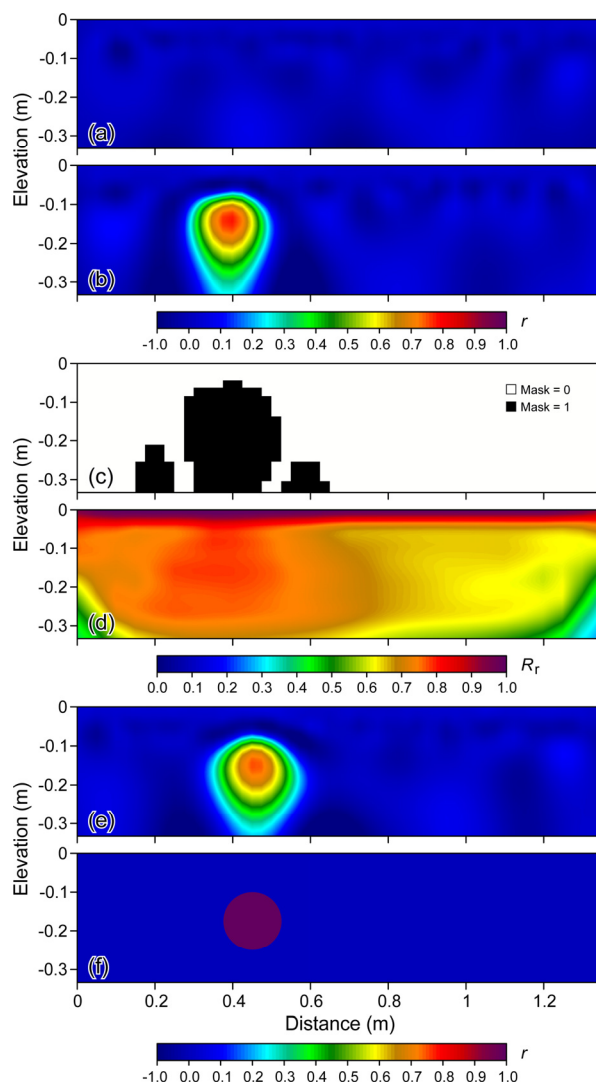ime steps of the noise conditions or the physical structure of the subsurface (Wilkinson et al 2012a; 2012b). But in this study, we chose to use results from previous time steps to determine the regions of the image that are exhibiting the greatest changes, and to focus the resolution of the next optimised survey onto these regions. A flowchart of the algorithm that we used is shown in Fig. 2. At each stage, the survey is designed using a "change mask" to determine the cell weights w. This mask is 0 for cells that have not significantly changed in the last time step, and 1 for cells that have. Appropriate weights w for cells with mask values of 0 and 1 were found empirically to be 0.05 and 1 respectively. For the first time step, every cell in the "blank" mask was set to 1. The survey designed using this mask was used to generate data for the baseline image (with no target present) and for the image at the first time step. Surveys at subsequent time steps were designed using masks generated by comparing the image at the previous time step to the baseline image. A chi-squared test was applied to each cell along with its eight immediately adjacent neighbours (Radke 2005) to determine whether the sum of the observed changes in these cells was significant compared to an expected degree of background variation (determined empirically to be ~7%). The significance level for the test was 0.1%, so typically only one cell in 1000 would be misidentified. The survey thus designed from the previous image and the baseline would then be used to generate the data for the image at the next time step. This process would then be iterated until the end of the monitoring sequence.

Figure 2 *Sequential design process.*

Figure 3(a) *Baseline image (in terms of resistivity reflection coefficient* r*). (b) Previous image. (c) Calculated change mask. (d) Relative model resolution of sequential survey design. (e) Sequential survey image of target shown in (f).*
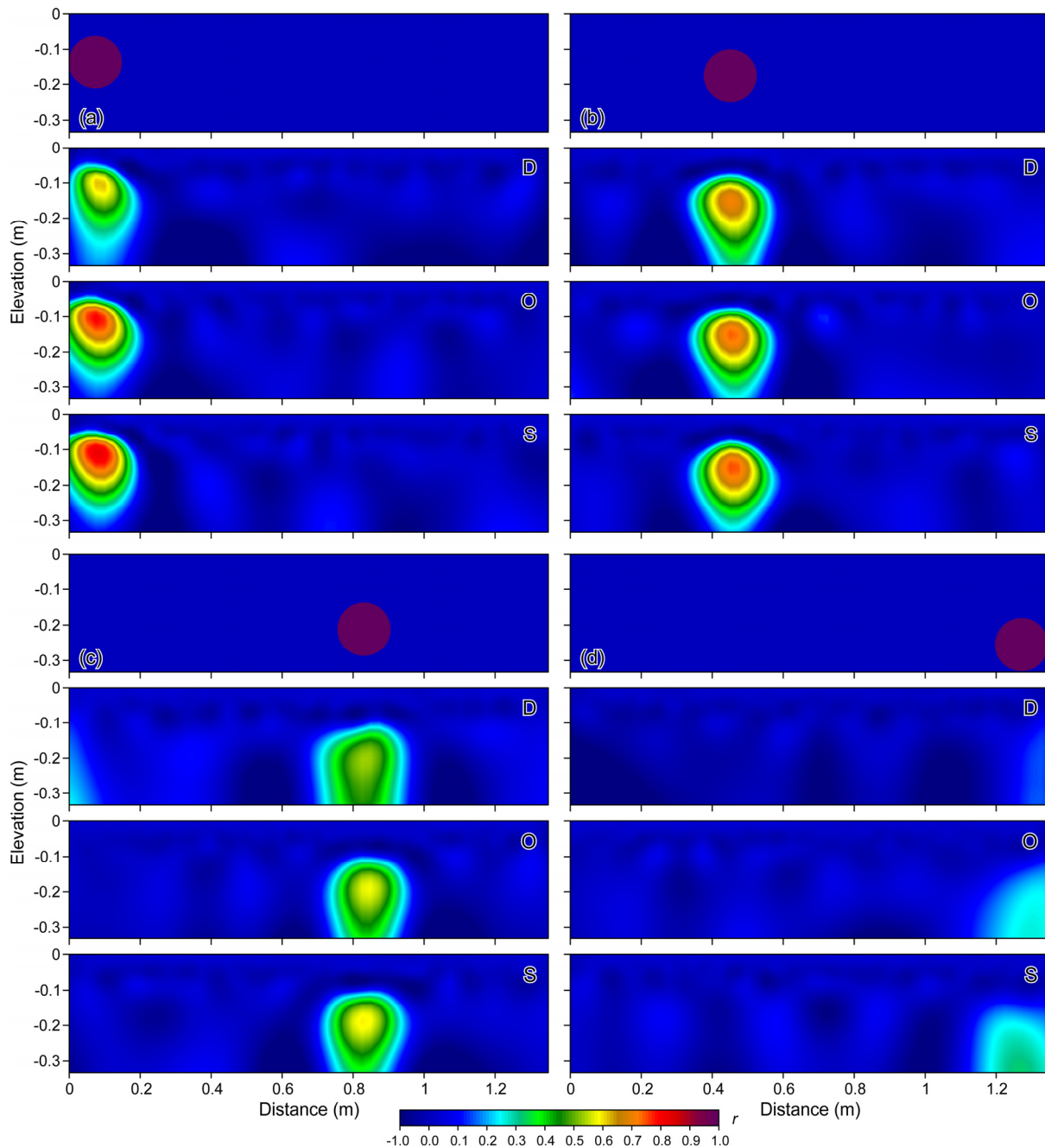
Figure 4 *Target position, Dipole-Dipole (D), Optimised (O) and Sequential (S) images at time steps (a) 0, (b) 6, (c) 12, and (d) 19.*

The elements of the sequential design process are illustrated for time step 6 in Fig. 3. Figures 3(a) and 3(b) show the inverted images of the baseline and the target at time step 5 respectively. They are plotted in terms of a reflection coefficient $r = (\rho - \rho_0) / (\rho + \rho_0)$, where ρ is the resistivity and $\rho_0 = 14$ Ωm is the background resistivity. On this scale, the perfect image would consist of a background with $r = 0$ and a cylindrical target with $r = 1$. The change mask calculated from these two images is shown in Fig. 3(c), and the relative model resolution of the sequential optimised survey designed to focus on this mask is shown in Fig. 3(d). Comparing this with Fig. 1(c) shows that the sequential optimised survey has greater model resolution than the static optimised survey in the vicinity of the changed regions, but that this comes at the expense of decreased resolution away from these regions. Note however that the resolution is still everywhere greater than the equivalent dipole-dipole survey (Fig. 1(b)). The resulting image and the actual location and geometry of the target at time step 6 are shown in Figs. 3(e) and 3(f) respectively.
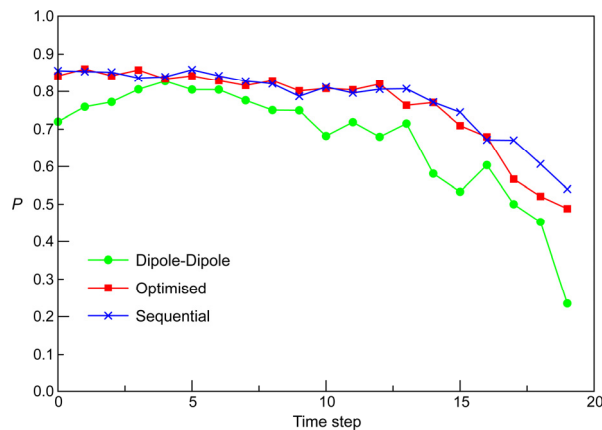
Figure 5 *Pearson correlation coefficients,* P*, between inverted images and target for Dipole-Dipole, Optimised and Sequential survey designs.*

The results of the three types of monitoring survey (dipole-dipole "D", static optimised "O", and sequential optimised "S") are shown in Fig. 4 at four separate time steps. As expected it can be seen that the image resolution decreases as the depth of the target increases. By qualitatively comparing the recovered contrast and shape of the target, it is clear that the both types of optimised survey (static and sequential) produce better images than the dipole-dipole survey. It also seems that the sequential surveys produce slightly better images than the static optimised surveys, although the improvement is not as pronounced. To quantify the levels of improvement, Fig. 5 shows the Pearson correlation coefficients between the inverted images and the target. Over the twenty time-steps the average increase between the dipole-dipole and static optimised images is $\Delta P_{DO} = 0.091$. The average extra improvement gained from using the sequential design method, i.e. the increase between the static and sequential optimised images, is $\Delta P_{OS} = 0.016$.

Conclusions

We have used simulated data to provide a proof-of-concept demonstration of using sequential optimised survey design to improve image quality in geoelectrical monitoring experiments. Measured by correlation coefficients, in this example sequential optimisation produced an extra 17% increase in image quality compared to the use of a static optimised design. Although this is a simple example with a well-defined target geometry, the results ought to be applicable to any similar resistivity monitoring experiment providing that the data can be inverted and the survey design calculated in a time less than the data acquisition interval (i.e. more quickly than the characteristic timescales of the processes being monitored).

Acknowledgements

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