VATT PUBLICATIONS

66

Essays on Auction Mechanisms and Information in Regulating Pollution

Kimmo Ollikka

Valtion taloudellinen tutkimuskeskus
Government Institute for Economic Research
Helsinki 2014

ISBN 978-952-274-109-7 (nid.)
ISBN 978-952-274-110-3 (PDF)

ISSN 0788-4990 (nid.)
ISSN 1795-3332 (PDF)

Valtion taloudellinen tutkimuskeskus
Government Institute for Economic Research
Arkadiankatu 7, 00100 Helsinki, Finland

Edita Prima Oy
Helsinki, April 2014

Cover design: Niilas Nordenswan

Essays on Auction Mechanisms and Information in Regulating Pollution

Kimmo Ollikka

Department of Economics and Management
Faculty of Agriculture and Forestry
University of Helsinki

Academic Dissertation:

To be presented, by the permission of the Faculty of Agriculture and Forestry of the University of Helsinki, for public examination in the Auditorium of the University Museum Arppeanum, Snellmaninkatu 3, Helsinki, on April 15th 2014, at 12 noon.

Supervisors: Professor Markku Ollikainen
Department of Economics and Management
University of Helsinki, Finland

Professor Matti Liski
Department of Economics
Aalto University, Helsinki, Finland

Professor Adriaan Perrels
Finnish Meteorological Institute
Helsinki, Finland

Preliminary examiners: Professor Hannu Vartiainen
Helsinki Center of Economic Research (HECER)
University of Helsinki, Finland

Professor Andrew Yates
Department of Economics
University of North Carolina, Chapel Hill, USA

Public examiner: Professor Juan-Pablo Montero
Department of Economics
Pontificia Universidad Católica de Chile, Santiago, Chile

Custos: Professor Markku Ollikainen
Department of Economics and Management
University of Helsinki, Finland

Abstract

Environmental regulation often has to be designed using asymmetric and incomplete information. Polluting firms, for instance, are normally privately better informed than the regulator with regard to the costs of reducing their emissions. However, even regulated firms may not have accurate information about their own abatement costs. The regulator is eager to know this private information in order to implement the most efficient environmental policy given the information at hand. In this thesis, I study, among other things, how auction mechanisms can be used to incentivize firms to reveal their private information to the regulator.

One of the central questions in pollution control theory is whether a price instrument like an emission tax or a quantity instrument like tradeable permits is better in environmental or climate policy. In climate policy, emissions trading programs have been more popular both in Europe and the U.S. Also, auctions and in particular uniform price auction formats have been used as an initial allocation method in many trading programs.

In the first two essays of this thesis, I study two-stage mechanisms for controlling pollution. In the first stage, the regulator conducts a generalized multi-unit Vickrey auction in order to allocate emission permits to firms. More importantly, the auction mechanism aims to collect private information from regulated firms. In the second stage, the regulator implements a range of environmental policy instruments, in the light of the information from the auction.

In the first essay, the regulator uses either a constant price regulation or a program of tradeable permits with a fixed supply of permits. I show that firms have less incentive to bid sincerely in an auction when using a tax instrument compared to emissions trading.

In the second essay, the regulator implements a tradeable permits program in the second stage, where the permit supply is elastic in price. Moreover, the permit market suffers some frictions, which increase the costs of trading. I derive incentive compatibility conditions for firms to bid sincerely in the first-stage auction given the regulation in the second stage and the various information structures.

In the third essay, I compare the Vickrey and uniform price auction formats in allocations of emission allowances without an allowance resale market. Firms may collude and thus coordinate their bidding behavior in auctions. The Vickrey auction is efficient but the revenues decrease the more firms collude. However, the efficiency and revenues of uniform price auctions depend heavily on the coalition game and the structure of the market.

Tiivistelmä

Ympäristöpolitiikan ohjauskeinot on usein suunniteltava ilman täydellistä tietämystä päästöjen vähentämisen kustannuksista tai hyödyistä. Vaikka saastuttavien yritysten käsitys mahdollisista päästövähennysteknologioistaan voi olla epävarmaa, saattaa yrityksillä olla viranomaista parempi ymmärrys niiden kustannuksista. Viranomainen haluaisi saada yritysten tiedon käyttöönsä suunnitellakseen ohjauskeinot paremmin. Tässä väitöskirjassa tutkin muun muassa, miten huutokauppamekanismeja voidaan hyödyntää yritysten palkitsemiseksi, jotta ne paljastaisivat totuudenmukaisesti tietämyksensä viranomaiselle.

Yksi keskeisimmistä ympäristökontrollin teoriaan liittyvistä kysymyksistä on perinteisesti ollut, tulisiko saastuttamista ohjata hintainstrumentilla kuten veroilla vai määräinstrumentilla kuten kaupattavilla päästöoikeuksilla. Ilmastopolitiikassa päästöoikeuksien kauppaohjelmat ovat olleet suositumpia niin Euroopassa kuin Yhdysvalloissa. Huutokauppaa on sovellettu monessa kauppaohjelmassa päästöoikeuksien alkujakomenetelmänä.

Väitöskirjan kahdessa ensimmäisessä esseessä tutkin kaksivaiheista ympäristöohjausta. Ensimmäisessä vaiheessa viranomainen huutokauppaa päästöoikeuksia saastuttaville yrityksille hyödyntäen Vickrey huutokauppaa. Huutokauppamekanismin avulla viranomainen oppii yritysten puhdistuskustannuksista. Ohjausmekanismin toisessa vaiheessa viranomainen asettaa yrityksille erilaisia ympäristöpolitiikan ohjauskeinoja hyödyntäen oppimaansa.

Ensimmäisessä esseessä viranomainen valitsee joko kiinteän hintaohjauksen tai päästöoikeuksien kaupan, jossa markkinoille jaettavien päästöoikeuksien määrä on kiinteä. Osoitan, että yritysten halukkuus paljastaa tietonsa totuudenmukaisesti huutokaupassa on rajoittuneempaa, kun käytössä on vero-ohjaus, kuin jos varsinaiseksi ohjauskeinoksi valitaan päästökauppa.

Toisessa esseessä viranomainen valitsee toisen vaiheen ohjauskeinoksi päästöoikeuksien kaupan, jossa päästöoikeuksien tarjonta on joustava hinnan suhteen. Lisäksi päästöoikeusmarkkinoiden toimintaan liittyy kaupankäynnin kustannuksia lisäävää kitkaa. Johdan ehdot tietorakenteelle, jolloin yritykset paljastavat tietonsa totuudenmukaisesti huutokaupassa.

Kolmannessa esseessä vertailen Vickrey huutokauppaa ja mm. EU:n päästökaupassa sovellettua yhtenäishinnoittelun huutokauppaa, kun yritykset eivät voi käydä kauppaa päästöoikeuksien jälkimarkkinoilla. Yritykset voivat kuitenkin koordinoida käyttäytymistään päästöoikeuksien huutokaupassa. Vickrey huutokauppa jakaa päästöoikeudet tehokkaasti, mutta huutokaupan tuotot alenevat yritysten koordinoidessa käyttäytymistään. Yhtenäishinnoittelun huutokaupan tulokset ovat riippuvaisia markkinarakenteesta ja koalitionmuodostuksen luonteesta.

Acknowledgments

I would like to thank many people who have helped me while working with this dissertation. First of all, I would like to thank my three thesis supervisors. I am deeply grateful to Markku Ollikainen for guiding me all the way through my master's and doctoral studies. Markku's passion and enthusiasm for environmental economics have inspired me and many other students and have greatly motivated us in our work. I owe a great debt of gratitude to Markku. I thank Adriaan Perrels for his comments on the thesis and also for many interesting and stimulating discussions. Most importantly, this thesis would never have been completed without the contribution of Matti Liski. I cannot thank Matti enough for his encouragement, guidance and clear economic insight he has provided to me. I am also grateful for all the impressive ideas Matti has shared with me. Maybe someday I will write a paper about a man, a painting, and a closet. There is no doubt, it will be a breakthrough.

I thank the preliminary examiners Hannu Vartiainen and Andrew Yates for their many insightful comments and suggestions. They improved the work significantly. I would like to express my sincere appreciation to Juan-Pablo Montero for agreeing to be my public examiner. I have long admired the work of professor Montero. His work inspired me to start working with auction mechanisms.

During my doctoral studies, I have worked in the Finnish Environment Institute SYKE, the Department of Economics and Management at the University of Helsinki, and currently the Government Institute for Economic Research VATT. I would like to take the opportunity to thank all the friends and colleagues I have been so fortunate to study, teach and work with. Unfortunately, I am here able to thank by name only those who have most directly contributed my dissertation.

I have written this thesis while working in VATT. I thank Anni Huhtala for her wise advice and her patience and understanding. Marita Laukkanen has helped me improve my writing in many ways. I thank her for that. I also thank my co-author Janne Tukiainen. After all, our joint paper was not included in this dissertation. However, it is closely related to these essays and, during the hilarious working process, Janne has taught me most of what I know about auction theory.

I thank the academic staff in Environmental Economics at the Department of Economics and Management: Marko Lindroos, Chiara Lombardini, and Pauli Lappi. Thank you for your inspiring company. Especially, I would like to offer my thanks to my fellow students Piia Aatola and Anna Sahari not only for their valuable comments on the essays but also for their support and friendship during all these years. Of course, very special thanks go to Antti

Iho. I feel very fortunate that I have had the opportunity to study and work with Antti. I have learned a great deal from Antti and I hope that our cooperation and companionship will continue in the years to come.

I gratefully acknowledge the financial support from Helsinki University Centre for Environment HENVI (the research programme Global Environmental Change), the Academy of Finland (BEET, project no. 124480), Tekes - the Finnish Funding Agency for Innovation (ClimBus / POMAR), the Fortum Foundation, and the Maj and Tor Nessling Foundation. I thank Andrew Lightfoot for proof-reading and correcting my numerous mistakes in English, and Sari Virtanen for the final read-through and check.

Finally, I owe my deepest gratitude to my parents Irmeli and Seppo. I have always been able to trust that you are there for me and nowadays for my family. Thank you.

Paula, Nuutti ja Topias, olette rakkaita!

Hämeenlinna, March 2014

Kimmo Ollikka

Contents

Chapter 1

Introduction

1.1 Background

In a competitive economy, where all goods and services are private, all goods and services have markets, all markets clear, all producers and consumers are price-takers and have complete information, there are no externalities, and consumers' preferences and producers' production functions satisfy certain conditions, a market mechanism with an appropriate price vector results in a Pareto optimal (efficient) allocation of resources. This is the first fundamental theorem of welfare economics. The second fundamental theorem of welfare economics says that any desired Pareto optimal allocation can be achieved by introducing appropriate lump-sum transfers. Once such transfers are instituted, the competitive market mechanism will take care of the efficient, first-best allocation. (E.g. Mas-Colell et al. 1995.)

In practice, the market mechanism is never completely perfect, and the famous "invisible hand" does not lead to the Pareto optimal allocation. In this thesis, I consider a number of market failures and examine regulatory mechanisms to correct them. In particular, I study the interactions of the pollution externality problem together with asymmetric and incomplete information. I examine how auction mechanisms can be employed to manage market failures, and particularly the problem of asymmetric information.

Pollution is a traditional example of a negative externality. Producers (or firms) produce valuable goods and services for consumers. Often, as a by-product, firms produce bads such as pollution. That is to say, firms use e.g. clean air or water as inputs in their production. Clean air and water are common resources. They are owned by all the agents in the economy and, at the same time, by none of the agents. Thus the property rights to these common resources are not clearly defined and it is impossible for agents to negotiate the use of these resources. Without any intervention by the social planner, pollution is not internalized into the pricing system. When the bads are external to the economic system, producers do not take them into account in their production decisions.

Information is complete when all the information affecting the values of goods and bads is completely known by all the agents in the economy. Under these conditions, i.e. when the pollution externality is the only market failure, the social planner can correct the pricing system and fully internalize the externality problem. The social planner may use various regulatory instruments to achieve the Pareto optimal allocation. However, the distribution of wealth may vary depending on the instrument used. On the other hand, if the relevant information is not available, the intervention is not first-best and the instruments may also differ in their efficiency properties.

Furthermore, information may also be distributed asymmetrically between economic agents and the social planner. Firstly, if producers or consumers know more about the externality problem than the social planner does, then incentive mechanisms to reveal this private information to the social planner are called for. In this thesis I examine the performance of auction mechanisms in revealing the private information of firms about their emission reduction costs. Secondly, the market mechanism may fail if the information is distributed asymmetrically between market participants. Also, in this case, auction mechanisms may improve the functioning of markets by providing more accurate and more evenly distributed information and bringing the allocation of resources closer to the Pareto optimal allocation.

The study also covers two other types of market failures: transaction costs and market power. The perfect market hypothesis assumes that transactions between economic agents are costless. However, when the number of agents is large, it may be a time-consuming task for an agent to find someone who is willing to trade products with him. Moreover, even if the market participants are matched, the bargaining process and decision-making may be costly for them. Also, monitoring pollution and enforcement of regulations may cause costs for the social planner and economic agents. (E.g. Hahn and Stavins 2011.)

Economic agents have no market power if they assume that their actions have no impact on market prices. They take prices as given. This is a somewhat contradictory assumption, because in the general equilibrium theory, under certain conditions, every action affects everything in the economy. The price-taking assumption is based on the large number of both producers and consumers in the economy. Thus the effect of one agent is negligible on the equilibrium outcome. In contrast, if an agent notices that he can influence the equilibrium price by his production or consumption decisions, it will steer the equilibrium away from the competitive outcome and the equilibrium price will not reveal the true costs of (marginal) production or the true value of (marginal) consumption. The equilibrium allocation is not efficient and there are gains from trade that are not realized in the economy.

This thesis is a collection of three independent essays and an introduction. The first two essays, in Chapters 2 and 3, study pollution regulation under incomplete and asymmetric information. The third essay, in Chapter 4, examines market power and, in particular, the collusive behavior of firms in emission permit auctions.

This chapter is an introduction. It is organized as follows. In the next two sections, I shortly review the literature on pollution regulation (Section 1.2) and auction mechanisms (Section 1.3), in the light of the above market failures. In Section 1.4, I introduce the affine linear model, which is used in the two first essays. I also explain how this information structure reflects the problem of climate change. In the last section, I summarize the essays and explain how they contribute to the literature on pollution regulation.

1.2 Regulating pollution

Pigou (1920) was the first to address how the pollution externality could be internalized into the economic system. Levying, for instance, a uniform emission tax on polluting firms, equaling the marginal damage of pollution, would provide the right incentives for firms to reduce emissions. Each firm would find it profitable to reduce its emissions to a level where the marginal abatement cost is equal to the Pigouvian tax. Coase (1960) challenged Pigou's view. Coase states that when transaction costs are zero and property rights are well defined, the Pigouvian solutions are unnecessary and government actions are not needed. Economic agents will find the most efficient solution by bargaining and the original distribution of property rights between economic agents will not disturb this efficient solution. This idea is known as the so-called Coase Theorem, formulated by Stigler (1966). However, as Coase himself said, this was not the actual message of the original paper:

> I tend to regard the Coase Theorem as a stepping stone on the way to an analysis of an economy with positive transaction costs. [...] My conclusion; let us study the world of positive transaction costs. (Coase 1992.)[1]

Nevertheless, the basic idea of another and nowadays relatively popular environmental regulatory instrument is based on the Coase Theorem. Namely, the first ideas of emissions trading were formulated by Crocker (1966) and Dales (1968) to regulate air and water pollution, respectively. Briefly, in a cap-and-trade emissions trading program, the regulator first announces the total amount of emissions permitted for regulated firms. This is the emissions cap. Second, the regulator allocates pollution permits to firms up to the announced emissions cap by using some initial allocation mechanism.[2] Third, firms are not allowed to pollute more emissions than they have permits in aggregate, but they are free to trade permits among themselves in the markets. Thus the emissions of a particular firm may exceed its initial allocation, but not its final permit holding. The emissions trading program provides a cost-efficient solution to pollution control if the marginal abatement costs are equal among regulated firms in equilibrium. In the spirit of the Coase Theorem, Montgomery (1972) proved that tradeable permits would indeed provide a cost-efficient solution under competitive market conditions without any transaction costs. In addition, the solu-

[1] This is the lecture by Ronald Coase in memory of Alfred Nobel, December 9, 1991.

[2] The initial allocation of permits can be free using some grandfathering or benchmarking rules, or it can be conducted by an auction. In the literature, tradeable permits are also called allowances, licenses, quotas or rights. I use the term permit in Chapters 2 and 3 and the term allowance in Chapter 4. Note that in the European Union Emissions Trading System (EU ETS) permits stand for administrative permissions for given installations emitting greenhouse gases, whereas allowances stand for tradeable pollution rights.

tion is independent of the initial allocation of permits. Hahn and Stavins (2011) call this the independence property.

If the externality problem was the only market failure, the regulator could guarantee an efficient solution either by levying a Pigouvian tax or a system of tradeable permits or some other regulatory instrument, such as non-tradeable permits or emission reduction subsidies. In this thesis I consider only the first two: an emission tax and tradeable permits. The reason is two-fold. First, there is an ongoing policy debate as to whether taxes or tradeable permits are preferable in environmental policy and in particular in climate policy. Second, since Pigou (1920) and Coase (1960), the academic discussion about the relative merits of price regulation (e.g. taxes) and quantity regulation (e.g. tradeable permits) has broadened in many respects. However, contrary to the advice of Ronald Coase, the academic discussion about prices versus quantities was not initially extended to questions of transaction costs. Instead, incomplete information was shown to have an impact on the relative merits of prices and quantities.

1.2.1 Incomplete information

Weitzman (1974) derives a rule for the choice between price and quantity controls, when abatement costs and the damage caused by pollution are uncertain.[3] Weitzman uses first-order linear approximations of the marginal abatement costs and a marginal damage function and assumes that the uncertainty is captured entirely by the constant terms of these linear functions.[4] If the abatement costs and the pollution damage are not correlated, the rule is simple. The regulator should control the quantity of pollution and use quantity instruments if the marginal benefits of pollution reduction increase more rapidly than the marginal costs of reduction. On the other hand, the price instrument provides a lower expected welfare loss if the slope of the marginal abatement costs is steeper than the slope of the marginal damage function. The reason for this is intuitive. If the aggregate marginal abatement costs are greater than expected, the equilibrium emissions will exceed the optimal level under a uniform tax and will be below the optimal level under quantity control and vice versa, if the marginal abatement costs are lower than expected. Hence, if the slope of the aggregate marginal abatement costs is steeper (flatter) than the slope of the marginal damage, the closer (further) the resulting emissions will be from the optimal level under a tax as compared to quantity control.

[3]Weitzman (1974) formulates the model as a general planning problem. However, he uses the problem of air pollution as a possible example of the formulation.

[4]See Malcomson (1978) for a critique of the linear approximations.

However, emission reduction costs and benefits may have a statistical dependence on each other (see Stavins 1996). Under these circumstances, a positive correlation between emission reduction costs and benefits will favor quantity control and a negative correlation will favor price regulation as compared to a regime of statistical independence between emission reduction benefits and costs. If the marginal abatement costs are not as expected, the positive correlation will move the optimal emission level towards the expected level and thus towards the emissions cap of the quantity control.

It is important to emphasize some relevant points related to Weitzman's (1974) model. First, Weitzman assumes that as much information as it is feasible to gather has already been obtained by the regulator when designing and implementing the policy instruments. However, regulation policy will have been set before the uncertainty about abatement costs or pollution damage has been resolved. After implementation, firms will acquire more information about their true abatement costs and react to the new knowledge. Second, two policy alternatives are constant: the pollution tax is uniform and set at the level of the expected first-best price, and the total allowable pollution in the quantity control is set at the expected first-best level. These two will not adjust to any changes in abatement costs or pollution damage. Third, even though Weitzman does not consider emissions trading in his original paper, quantity control can easily be extended to the case of tradeable permits. Since Weitzman's original contribution, the literature on prices versus quantities has extended to compare tax and tradeable permits e.g. in cases of stock pollution (e.g. Hoel and Karp 2001, 2002, Newell and Pizer 2003, Karp and Zhang 2012), incomplete enforcement (e.g. Montero 2002), banking of permits (e.g. Fell et al. 2012), technology choice (e.g. Krysiak 2008) or multiple pollutants (e.g. Ambec and Coria 2013). In these papers, firms are assumed to be price-takers in the emissions permit market if the trading of permits is allowed.[5]

The price or quantity control scenarios in Weitzman (1974) can be improved by making the regulatory schemes non-constant. Weitzman (1978) himself proposes a tax regulation, where the marginal tax rate is a linear function of firms' emissions. Roberts and Spence (1976), on the other hand, introduce a hybrid scheme. In their hybrid regulation, the aggregate supply of pollution permits is not constant. In the simplest case, supply is represented by a step function where the equilibrium price of the market (with a fixed supply of pollution permits) is constrained by two additional price instruments: a price floor and a price cap. If the price of pollution permits falls to the price floor due to lower than expected abatement costs, the regulator will buy back permits from the firms at the floor price. If the abatement costs are higher than expected and thus the equilibrium price becomes too high, the regulator

[5] Firms are also assumed to be risk-neutral. See Ben-David et al. (2000) or Baldursson and von der Fehr (2004) for an analysis of risk-aversive firms under a tradeable permit regulatory regime.

will set a fee for pollution, i.e. it sells permits to firms at the price cap. Furthermore, adding subsequent steps into the supply function and increasing the number of steps to the limit results in a continuous permit supply function (Roberts and Spence 1976, Appendix). The optimal aggregate permit supply function is equal to the expected pollution damage function. Moreover, if the pollution is uniformly mixed and the permit market is perfect, tradeable permits with a non-constant permit supply will perform better than non-constant taxes (Kennedy et al. 2010, Yates 2012).

1.2.2 Asymmetric information

It is realistic to assume that, at the implementation stage of pollution regulation, firms' knowledge about their abatement costs is better than the social planner's information. Even if regulated firms are uncertain about their future costs of emission reductions, they have more accurate information about their production technologies, possibilities to reduce emissions, knowledge of the price formation of essential inputs and outputs and so forth. In addition, firms conduct R&D activities and they have strong incentives not to reveal information about their own innovation processes outside the company. At the same time, such information is valuable to the regulator if it improves the efficiency of regulations. Firms are, however, not willing to reveal such information sincerely. Depending on the planned regulatory scheme, firms may have incentives either to overestimate or to underestimate their uncertain abatement costs. Lewis (1996) provides a review of this topic. She points out that in most instances pure forms of marketable permits or emission taxes are insufficient regulatory instruments when economic agents are asymmetrically informed.

Kwerel (1977) was one of the first to introduce an incentive mechanism for the disclosure of firms' information in pollution regulation. The incentive mechanism of Kwerel has two building blocks. The regulator 1) issues a fixed amount of tradeable pollution permits[6] denoted by L, and 2) sets a subsidy e per permits in excess of emissions produced by firms. Hence the regulator commits to buy back those permits which are not used at price e. In addition, before the implementation of the regulation, there is one round of communication between regulated firms and the regulator. Firms are asked to report their clean-up costs to the regulator. Prior to reporting, the regulator announces that it will set the parameters L and e as follows. The expected marginal damage equals the reported aggregate marginal clean-up costs at pollution level L. Moreover, subsidy e equals the level of these marginal functions evaluated at L. Kwerel argues that in a competitive permit market, reporting

[6] Kwerel uses the term transferable licenses.

sincerely to the regulator is a Nash equilibrium of this game. That is, if every other firm is reporting sincerely, it is also the best response for each firm to report sincerely.

Kwerel, however, says nothing about how the regulator initially distributes permits to firms. Montero (2008) argues that, depending on the initial allocation method, firms may find more profitable strategies compared to the sincere reporting strategies in Kwerel's scheme. If permits are allocated for free and if firms are able to coordinate their reporting strategies, regulation becomes inefficient. Also, if permits are allocated in a uniform price auction and firms use low-price equilibrium strategies in the auction, the allocation will not be efficient.

Vickrey (1961), Clarke (1971) and Groves (1973) provide an efficient (VCG) mechanism for the provision of public goods, where agents are privately informed about the costs of their actions. The VCG mechanism implements efficient allocation in dominant strategies.[7] That is, whatever other firms report to the regulator about their emission reduction costs, it is a dominant strategy for each firm to report its costs sincerely to the regulator. The intuition of the mechanism is explained in later sections. Dasgupta, Hammond and Maskin (1980) use the VCG mechanism to implement a tax regulation for privately informed firms (DHM tax mechanism). Montero (2008) describes a simple auction mechanism where a discriminatory Vickrey pricing rule is used to induce firms to bid sincerely. Both mechanisms, the DHM tax mechanism and Montero's auction mechanism, allocate emission permits efficiently among regulated firms, given the increasing expected marginal damage of pollution. Montero (2008) argues, however, that these mechanisms differ in two important ways. First, the DHM tax mechanism fails to allocate permits efficiently when the supply of permits is fixed. Second, collusive actions may distort the first-best property of the DHM tax mechanism. Montero, in contrast, shows that the efficient allocation of the VCG auction mechanism is not distorted by the inelastic supply of permits or the collusive actions of firms. Thus, even if firms are able to coordinate their bidding strategies prior to the auction, the mechanism assigns an efficient amount of permits to colluding firms. If a coalition agrees on the efficient distribution of permits within the coalition, then the allocation is efficient. Finally, it is important to note that in Kwerel (1977), Dasgupta et al. (1980) and Montero (2008), agents' values are private and polluting firms know their abatement costs exactly.

In addition to the problem of asymmetric information between the social planner and firms, Coasian bargaining may not work as intended if the information is asymmetric between firms. Then the otherwise perfect market may fail to assign objects efficiently. To give an intuition of this, consider the following simple example.[8] Suppose that two firms are trading a single

[7]The VCG mechanism is a multi-unit extension of a single-unit Vickrey auction (i.e. a single-unit second-price auction). However, the term Vickrey auction is used occasionally in a multi-unit context.

[8]Chatterjee and Samuelson (1983) provide a more general analysis of this example.

pollution permit bilaterally. Suppose also that the marginal value of the permit is v_S for the seller and v_B for the buyer, where $v_B > v_S$. Hence, there are gains from trade, because any price between v_S and v_B would not make the firms worse off and at least one of the firms better off. With complete information, the firms would bargain about the price and it would be in both firms' interest to get the trade done. If the seller has all the bargaining power, it can reap all the gains from the trade and the resulting equilibrium price, for instance, in a take-it-or-leave-it game[9] would be $p = v_B$. The trading is efficient, because the object goes to the agent who values it most. However, if the firms do not have exact information about the value of the trading partner, the trade may not occur. For instance, in the take-it-or-leave-it game, suppose that $f(v_B)$ is the density of the buyer's valuation with a support $v_B \in [a, b]$, where $v_S < b$. Hence, v_B is a random draw from the distribution $F(v_B)$. Suppose also that the true value v_B is known by the buyer, whereas the seller knows only the distribution $F(v_B)$. The seller maximizes its expected gains from trade $U_S(p) = \int_p^b (p - v_S) f(v_B) \, dv_B$ with respect to the offer price $p \geq v_S$. Then the optimal offer price by the seller satisfies $p = v_S + \frac{1-F(p)}{f(p)}$. If the buyer's true value is less than the offered price, i.e. $p > v_B$, no trade is done even if $v_B > v_S$.

In fact, Myerson and Satterthwaite (1983) show that generally there is no Bayesian incentive compatible and individually rational allocation mechanism that can guarantee efficient allocation in bilateral trading. This is an important result. Moreover, the Vickrey-Clarke-Groves mechanism would provide efficient allocation (with private values), but one relevant problem of the VCG mechanism is that it is not budget-balanced. Hence, in order to guarantee efficient allocation under asymmetric information between traders, there should be a coordinator or broker to provide extra funding. This is one of the central reasons why auction mechanisms are needed. In auctions, where all the agents are on the demand side, efficient allocation can be achieved by collecting money from the bidders and thus the budget is unbalanced to the regulator's benefit. Moreover, revenues from the auction can be used in other sectors of the economy.[10]

1.2.3 Transaction costs

Following the so-called Coase Theorem, Stavins (1995) was the first to show how the costs of trading may influence the equilibrium of the emissions permit market. Transaction costs may affect the cost-efficiency of the market and the independence property of the initial allocation of permits. Transaction costs may be borne from various sources. Stavins (1995) identifies

[9]The take-it-or-leave-it game is also called the ultimatum game.

[10]See also Lewis (1996).

three potential sources in the permit markets: 1) search and information, 2) bargaining and decision, and 3) monitoring and enforcement. Furthermore, whether the marginal transaction costs are increasing, constant or decreasing has different implications for the independence property (see Stavins 1995 for a discussion of the sources of different types of transaction costs). Stavins shows that if the marginal transaction costs are constant, the final allocation of allowances is independent of the pre-trade allocation. The cost-efficiency, however, is not achieved unless the pre-trade allocation is already Pareto optimal. The gains from trade are decreased due to the transaction costs and not all, otherwise beneficial, trades are conducted. With increasing marginal transaction costs, the closer the pre-trade allocation is to the Pareto optimal allocation, the closer the equilibrium allocation is to the efficient solution. Hence the independence property fails to hold. With decreasing marginal transaction costs, there are scale economies from trading and the shift in the pre-trade allocation away from efficient allocation results in an equilibrium outcome which is closer to the efficient solution than the post-trading outcome without the shift. An intuitively similar result with decreasing marginal transaction costs is provided by Liski (2001), who examines a case where transaction costs are a function of market size. In thick markets, transaction costs are presumably lower than in thin markets and transaction costs vanish if the pre-trade allocation of permits is significantly different from the efficient allocation.[11]

1.2.4 Market power

Traditionally, oligopolistic competition, i.e. competition between strategic agents, is modeled by quantity competition à la Cournot or by price competition à la Bertrand. In both models, the equilibrium is close to the competitive equilibrium when the number of agents increases. The first contribution concerning market power in emissions trading markets was made by Hahn (1984), who considers one dominant firm in the permit market. If the initial allocation of permits is not at the efficient level, the dominant firm manipulates the permit market price and the equilibrium is not efficient. If the dominant firm is on the supply side of the market, it drives the price up by reducing sales of permits and if it is on the demand side, it steers the price downwards by reducing purchases of permits. However, this market power vanishes if the allocation of the dominant firm is at the competitive equilibrium at the outset. Misiolek and Elder (1989) extend the dominant firm model to cover output markets. Furthermore, models of market power in emission permit markets are extended to a dynamic set-up by Liski and Montero (e.g. 2006, 2011) and to an oligopolistic setting by e.g. von der Fehr

[11] See also Montero (1998), who examines the combined effect of transaction costs and uncertainty on trade approval.

(1993) and recently by Malueg and Yates (2009) and Lappi (2012). Contrary to previous studies of Cournot competition, in the model of Malueg and Yates firms compete using linear trading schedules and all the players act strategically in the permit market.[12]

I, however, consider oligopolistic competition in multi-unit auctions. In auction models, oligopolistic agents compete with price-quantity pairs, i.e. with supply[13] or demand schedules as in Malueg and Yates (2009). In these models, strategic bidding does not always result in a competitive outcome even if the number of bidders increases to the limit (see Wilson 1979, Back and Zender 1993). Another and related aspect of strategic bidding is collusion, which is a central concept of the oligopoly theory (Vives 1999). I consider collusive behavior in auctions, where firms coordinate their bidding strategies. Collusion may affect the efficiency and revenues of the auction. These issues are discussed in more detail in the next section.

1.3 Auction mechanisms

In auctions of emission permits, the seller has multiple homogenous units to sell and bidders want multiple units. Most of the theoretical literature on auctions concerns single-unit auctions. The theory of multi-unit auctions is much less developed than single-unit auction theory. Next, I shortly review some central results of single-unit auction theory and then introduce and discuss the properties of some of the most popular multi-unit auction mechanisms.

1.3.1 Single-unit auctions

The benchmark model of auction theory is the independent private values (IPV) model of a single unit. The risk-neutral seller has a single object to sell to a number of n risk-neutral bidders. Bidders have values for the object, $v_1, \ldots, v_n$, identically and independently distributed with a cumulative distribution function $F(v)$.

There are four traditional single-unit auction designs. In a first-price sealed bid auction, bidders submit their bids simultaneously to the auctioneer. The bidder with the highest bid wins the object and pays her bid. In a second-price sealed bid auction (or Vickrey auction) she pays the second-highest bid. Two most common open (or dynamic) auction designs are the ascending-bid auction (English auction) and the descending-bid auction (Dutch auction). For instance, in a typical English auction, the auctioneer first announces a starting or reserve

[12] E.g. Montero (2009) reviews the literature on market power in pollution permit markets.

[13] Firms compete with supply schedules in a procurement setting.

price, and bidders start to bid with increasing bids. The auction continues until only one bidder remains. The bidder with the highest bid wins and pays her bid for the object. In a descending auction, the auctioneer starts at a high price and lowers the price until one of the bidders calls that she is willing to buy the object at the current price.

In the IPV model, the second-price auction and the ascending auction are (almost) strategically equivalent. In the second-price auction it is a dominant strategy for each bidder to bid her true value of the object. Hence, the payment is the value for the object of the highest loser, and the winner's bid does not affect this payment. Expected profits are maximized when bidding is truthfull. In the ascending auction, bidders remain until the price exceeds their values. Hence the auction stops when the price (incrementally) exceeds the second-highest value. The first-price auction and the Dutch auction are strategically equivalent in the IPV model. In the first-price auction, for example, bidders shade their bids, in order to maximize their expected revenues, conditional on their information about their own value.

One of the most famous results in auction theory is the revenue equivalence theorem (Vickrey 1961, Myerson 1981, Riley and Samuelson 1981). The revenue equivalence theorem states that given the IPV model, any auction design in which i) the bidder with the highest value wins, and ii) the bidder with the lowest value gets zero pay-off yields the same expected revenue for the seller. All the aforementioned standard auctions are thus revenue-equivalent. There is voluminous literature on auction theory examining various aspects of single-unit auctions, whilst relaxing the assumptions of the benchmark model (e.g. Milgrom 2004 provides an excellent survey of the literature).

When bidders' values are not private or values are affiliated, the revenue equivalence breaks down. When bidders have private but affiliated values, the high value of one bidder makes high values of other bidders more likely. Bidders' valuations may also be uncertain, and expected valuations may depend not only on each bidder's own information but also on other bidders' information. Suppose that each bidder receives a private signal of the object's value to her. Bidders' values are interdependent if signals of other bidders also affect this valuation. Bidders' values are common if they all have the same (but uncertain) valuation of the object. Moreover, signals are affiliated if a high signal of one bidder makes high values of other bidders' signals more likely. Milgrom and Weber (1982) show that, with affiliated (and either private, interdependent or common) values, the English auction is better than the second-price auction in terms of expected revenues. In addition, the Dutch auction and the first-price auction are strategically equivalent and they generate lower expected revenues than the second-price auction and the English auction.[14] This result is related to the winner's

[14]Milgrom and Weber (1982) also derive many other important results concerning e.g. seller's information,

curse. Winning the object is bad news, because it reveals that other bidders value the object less, which implies that the object is of low value for the winner too. Information of the other bidders is (partly) revealed in the ascending auction, which alleviates the winner's curse. However, in the first-price auction bidders shade their bids more due to the winner's curse, when values are affiliated.

In many auctions the main objective of the seller is to maximize revenues. An auction design is said to be optimal if it represents the revenue-maximizing mechanism. According to the "optimal auctions" literature, the revenue-maximizing assignment rule is based on virtual valuations and not on true valuations of bidders (Myerson 1981). Suppose that bidder i's valuation for the object is v_i and this is drawn from the distribution $F_i(v)$ with a density $f_i(v)$. Then the virtual valuation (or marginal revenue) of bidder i is

$$MR_i = v_i - \frac{1 - F_i(v)}{f_i(v)}. \tag{1.1}$$

The revenue-maximizing rule may assign the good to a bidder who does not value it most. Values are said to be regular if the virtual value is monotonically increasing in v_i. Then the revenue-maximizing mechanism is also efficient. Besides, the seller may increase the expected revenue by setting a reserve price such that $MR_i = v_s$, where v_s is the value of the object for the seller. The revenue-maximizing seller does not assign the object at all if bids are below the reserve price, even if $v_i > v_s$ for some i. Thus, the gains from trade will not necessarily be realized.

Maskin and Riley (2000) relax the assumption of the identical distribution of bidder values and examine a model of asymmetric bidders. Suppose that there are two bidders: strong (s) and weak (w). The supports of their value distributions are $v_i \in [\beta_i, \alpha_i]$. Moreover, the distribution of the strong bidder's valuation first-order stochastically dominates that of the weak bidder's distribution: $F_s(v) > F_w(v)$ for all $v \in [\beta_w, \alpha_s]$. Maskin and Riley show that in a first-price auction, the weak bidder bids more aggressively than the strong bidder with the same value v. Thus the strong bidder may lose the auction even if she had a greater valuation. This will not happen in a second-price auction. Furthermore, strong bidders favor second-price auctions whereas weak bidders favor first-price auctions. Which auction design guarantees greater expected revenues depends on the shapes and supports of the distribution functions. However, the first-price auction may often be more profitable, while it favors weak bidders. This is related to the result of revenue-maximizing auctions, which favor weak bidders with greater marginal revenues (Milgrom 2004, 153).

---

reserve pricing and entry fees.

The revenue and efficiency results may also break down if bidders are able to coordinate their bids prior to the auction. McAfee and McMillan (1992) is a seminal contribution on collusion and bidding rings in single-unit auctions. Bidding rings, or cartels, may agree that no bidder bids more than the reserve price in the auction. After the auction the object is allocated between the members using some cartel mechanism. However, cartels face several problems. First, what is the mechanism to divide the spoils of the cartel agreement? Second, while cartels are illegal, and side payments are in most cases impossible, the cartel agreement must be self-enforcing. Third, collusion and thus low prices may induce other firms to enter the market. Fourth, the regulator has strong incentives to destroy cartels, which makes cartel agreements harder to sustain. Thus McAfee and McMillan show that weak cartels, i.e. cartels whose members are unable to make side payments among themselves, cannot do any better in first-price auctions than to randomize the allocation among their members. Any other allocation method is ex ante weakly dominated for all bidders by random allocation. However, if side payments are possible, it is possible to attain the optimal cartel agreement: the member with the highest valuation is assigned the object and new entrants are excluded.

1.3.2 Multi-unit auctions

The two most common multi-unit auction mechanisms are the discriminatory price auction, also known as the "pay-as-bid" auction, and the uniform price auction.[15] In an auction with fixed supply, bidders submit non-increasing bid functions. The auctioneer aggregates the bid functions and clears the auction. The clearing price is the price at which the aggregate demand intersects the supply. All bids above or equal to the clearing price are accepted as winning bids.[16] In a uniform price auction, each bidder pays the market clearing price for every unit she wins. In a discriminatory price auction, bidders pay their bids for all the units they have won in the auction. In both uniform price and discriminatory price auctions, strategic bidders tend to reduce their demand in order to decrease the price and raise the profits from the auction. This might result in an inefficient allocation of auctioned goods and the allocation may differ between the two auction formats. Hence the weak form of revenue equivalence does not hold (see Ausubel et al. 2013). In addition, bid-shading affects the revenues collected by the auctioneer.

With private values, the Vickrey-Clarke-Groves mechanism (or the Vickrey auction) pro-

[15] The discriminatory price auction is often incorrectly thought of as a multi-unit extension of the single-unit first-price auction and the uniform price auction as a multi-unit extension of the single-unit second-price auction (Ausubel et al. 2013).

[16] If the clearing price is the first rejected bid, then all bids above the clearing price are winning bids. If there is excess demand at the clearing price, then some rationing rules are needed.

vides efficient allocation in multi-unit auctions. Instead of the clearing price, bidders pay the opportunity cost of each unit they win in a Vickrey auction. Despite its many useful theoretical properties, the VCG mechanism is rarely used in practice. The reasons why it is used so rarely include, for instance, possibility of complex bidding strategies, low seller revenues, non-monotonic payment functions, and vulnerability to collusion or to the use of multiple bidding identities by a single bidder. Ausubel and Milgrom (2006) and Milgrom (2004) discuss the reasons in more detail.

The literature on multi-unit auctions generally focuses on a comparison of uniform price and discriminatory price mechanisms in terms of efficiency and revenues. The challenge in theoretical models of these two mechanisms is that analytical equilibrium characterizations are difficult or impossible even in the case of symmetric independent private values (e.g. Hortaçsu 2011, Ausubel et al. 2013). This can be seen from the first-order conditions of the bidder's maximization problem under the Vickrey auction (VA), the uniform price auction (UPA) and the discriminatory price auction (DPA) (e.g. Hortaçsu 2011, Wilson 1979):

$$VA: \quad v_i\left(D_i\left(p\right)\right) = p, \tag{1.2}$$

$$UPA: \quad v_i\left(D_i\left(p\right)\right) = p - D_i\left(p\right)\frac{H_q}{H_p}, \tag{1.3}$$

$$DPA: \quad v_i\left(D_i\left(p\right)\right) = p + \frac{H}{H_p}, \tag{1.4}$$

where $v_i\left(q_i\right)$ is bidder i's marginal value function, $D_i\left(p\right)$ is the bid function and $H\left(p, D_i\left(p\right)\right)$ is the probability distribution of the market clearing price, i.e. the probability that the market clearing price p is not higher than the bid for unit $D_i\left(p\right)$.

In Vickrey auctions bidders are price-takers, whereas in uniform price and discriminatory price auctions the last terms in the right-hand sides of the first order conditions are the bid-shading factors. In many cases, it is very difficult to evaluate analytically the probability distribution H (see Hortaçsu 2011). What is more, there are typically multiple equilibria in these models (e.g. Klemperer and Meyer 1989, Wang and Zender 2002).[17] Hence any comparison between the uniform and discriminatory price auction formats is more an empirical question (Ausubel et al. 2013). Indeed, there is a growing empirical literature on multi-unit auctions where different mechanisms are used for selling, for instance, treasury bills and bonds (e.g. Hortaçsu and McAdams 2010, Kastl 2011) or electricity (e.g. Hortaçsu

[17] Ollikka and Tukiainen (2013) derive approximations of equilibrium strategies in the uniform price, discriminatory price and Vickrey auction formats. Their model is applied in the setting of central bank liquidity auctions. To my knowledge, there are no other theoretical models of these auction mechanisms, where bidders' values are asymmetric and interdependent.

and Puller 2008, Wolak 2003).

Nowadays, auction mechanisms are used in many emissions trading programs to allocate emission permits to regulated firms. The uniform price format is used, for instance, in the European Union Emissions Trading System (EU ETS), in California's Cap-and-Trade Program and in the U.S.'s Regional Greenhouse Gas Initiative (RGGI). Thus far, however, the literature on multi-unit emission permit auctions is relatively scarce (see Cramton and Kerr 2002, Lopomo et al. 2011).

In this thesis I examine two multi-unit auction designs. I study the Vickrey auction because of its efficiency properties. In addition, equilibrium characterizations are possible in the Vickrey auction even if bidders' values are interdependent. The uniform price auction is studied because it is the most widely used format in emission permit auctions.

Vickrey auction

Ausubel and Milgrom (2006) provide a good introduction to the Vickrey-Clarke-Groves mechanisms. In a private values setting, bidders pay the opportunity cost of their participation in the mechanism. This is clearly seen in the single-unit second price auction (the Vickrey auction), where bidders bid their values and the winning bidder pays the second-highest bid.

In pollution permit auctions, as in Montero (2008), the direct VCG mechanism can be interpreted as follows. Suppose that $U_i(q_i)$ is firm i's gross value for its pollution q_i, i.e. the value of the avoided abatement costs from zero emissions, and $DF(Q)$ is the damage function of total pollution $Q = \sum_{i=1}^{n} q_i$, where n is the number of polluting firms. It is assumed that these functions are non-decreasing in pollution, i.e. $U_i'(q_i) \geq 0$ and $DF'(Q) \geq 0$. Each firm knows its own value of pollution, but the pollution damage function is common knowledge. In the direct VCG mechanism, each bidder submits a report of its value function $\hat{U}_i(q_i)$ to the regulator. (In the equilibrium bidders are truthful and hence $\hat{U}_i(q_i) = U_i(q_i)$.) Given these reports, the regulator computes the welfare-maximizing allocation of emissions (permits):

$$\mathbf{q}^{\star} \in \arg\max_{\mathbf{q}} \left\{ \sum_{i=1}^{n} \hat{U}_i(q_i) - DF\left(\sum_{i=1}^{n} q_i\right) \right\}.$$

Next, suppose that

$$\bar{\mathbf{q}}_{-\mathbf{i}} \in \arg\max_{\mathbf{q}_{-i}} \left\{ \sum_{j \neq i}^{n} \hat{U}_j(q_j) - DF\left(\sum_{j \neq i}^{n} q_j\right) \right\}$$

is the welfare-maximizing allocation of emissions without firm i's participation. Suppose, for simplicity, that there are unique interior solutions to these problems, i.e. $\mathbf{q}^\star = (q_1^\star, \ldots, q_n^\star)$ and $\bar{\mathbf{q}}_{-\mathbf{i}} = (\bar{q}_1, \ldots, \bar{q}_{i-1}, \bar{q}_{i+1}, \ldots, \bar{q}_n)$, where $q_i^\star > 0$ and $\bar{q}_j > 0$ for all i, j. The VCG payment of firm i is

$$\begin{aligned} R_i &= \left[\sum_{j\neq i}^{n} \hat{U}_j\left(\bar{q}_j\right) - DF\left(\sum_{j\neq i}^{n} \bar{q}_j\right)\right] - \left[\sum_{j\neq i}^{n} \hat{U}_j\left(q_j^\star\right) - DF\left(\sum_{i=1}^{n} q_i^\star\right)\right] \\ &= \underbrace{\sum_{j\neq i}^{n} \hat{U}_j\left(\bar{q}_j\right) - \sum_{j\neq i}^{n} \hat{U}_j\left(q_j^\star\right)}_{PE} + \underbrace{DF\left(\sum_{i=1}^{n} q_i^\star\right) - DF\left(\sum_{j\neq i}^{n} \bar{q}_j\right)}_{PO}. \end{aligned} \tag{1.5}$$

The VCG payment includes two parts: the pollution externality and the pecuniary externality. The pecuniary externality is defined as $PE \equiv \sum_{j\neq i}^{n} \hat{U}_j\left(\bar{q}_j\right) - \sum_{j\neq i}^{n} \hat{U}_j\left(q_j^\star\right)$. This is the value of those units to other firms, which are not assigned them due to firm i's participation. The pollution externality is the extra damage of increased pollution due to firm i's participation: $PO \equiv DF\left(\sum_{i=1}^{n} q_i^\star\right) - DF\left(\sum_{j\neq i}^{n} \bar{q}_j\right)$. These externalities are both non-negative, while $\bar{q}_j \geq q_j^\star$ for all j, but $\sum_{i=1}^{n} q_i^\star \geq \sum_{j\neq i}^{n} \bar{q}_j$. The reported function $\hat{U}_i\left(q_i\right)$ does not affect the payment schedule R_i otherwise than determining the allocation $\mathbf{q}^\star$. Only the reports submitted by the other firms directly affect the payment schedule of firm i.

Montero (2008) provides an indirect interpretation of the same mechanism, where firms submit bid functions to the regulator. After the auction is cleared, the firms first pay the clearing price for all the units they have won. In addition, the firms receive paybacks from the regulator, which are determined by the bid functions of the other firms. Due to the paybacks, the final payment of firm i is equal to (1.5). The VCG payment rule induces firms to bid sincerely in the auction in dominant strategies and the allocation is efficient. Hence the private values paradigm provides some convenient properties for the VCG mechanism. Due to the dominant strategy property, firms do not have to know anything about other firms' values. In addition, under some continuity assumptions, the VCG mechanism is the only mechanism that can implement efficient outcomes in dominant strategies (Green and Laffont 1979, Holmström 1979). Besides, of the set of efficient mechanisms, the VCG mechanism is also the revenue-maximizing mechanism (e.g. Krishna and Perry 2000, Ausubel and Cramton 1999).

When agents' values have common value components, things get more complicated. Jehiel and Moldovanu (2001) show that with interdependent or common values generally no mechanism is able to implement efficient allocation. However, Dasgupta and Maskin (2000) and Ausubel and Cramton (2004) show that ex-post efficient implementation can be achieved if agents' valuations satisfy certain conditions. Suppose that both the abatement costs and pollution damage are uncertain, but prior to the auction firms receive signals defined by a vector $\mathbf{s} = (s_1, \ldots, s_n)$. The signals reflect the firms' true valuations $U_i(q_i)$. The values are interdependent if firm i's expected marginal valuation $v_i(q_i; \mathbf{s}) \equiv E\left[\left.\frac{dU_i(q_i)}{dq_i}\right| \mathbf{s}\right]$ depends on the amount of emissions (or permits) q_i and its own signal s_i, but also on other firms' signals $\mathbf{s}_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n)$. Now, the signals should be one-dimensional and the expected marginal value functions should satisfy the following three assumptions:

1. Continuity: $v_i(q_i; \mathbf{s})$ is jointly continuous in $(\mathbf{s}, q_i)$.
2. Value monotonicity: $v_i(q_i; \mathbf{s})$ is non-negative, and $\frac{\partial v_i(q_i;\mathbf{s})}{\partial s_i} > 0$ and $\frac{\partial v_i(q_i;\mathbf{s})}{\partial q_i} \leq 0$.
3. Single-crossing: Let $\mathbf{s}'$ denote a signal vector $\mathbf{s}' = (s_i', \mathbf{s}_{-i})$ and $\mathbf{s} = (s_i, \mathbf{s}_{-i})$. Then $v_i(q_i; \mathbf{s})$ has a single-crossing property if for all i, $j \neq i$, q_i, q_j, $\mathbf{s}_{-i}$ and $s_i' > s_i$:

$$v_i(q_i; \mathbf{s}) > v_j(q_j; \mathbf{s}) \Rightarrow v_i(q_i; \mathbf{s}') > v_j(q_j; \mathbf{s}')$$

and

$$v_i(q_i; \mathbf{s}') < v_j(q_j; \mathbf{s}') \Rightarrow v_i(q_i; \mathbf{s}) < v_j(q_j; \mathbf{s}).$$

Ausubel and Cramton (2004) prove that truthful bidding is the ex-post equilibrium in the Vickrey auction with reserve pricing. This holds for any monotonic aggregate quantity rule $\bar{Q}(\mathbf{s})$ and associated monotonic efficient assignment rule $q_i^e(\mathbf{s})$, and for any value function satisfying continuity, value monotonicity and the single-crossing property. In addition, a permit resale market does not distort the equilibrium of the Vickrey auction if all the gains from trade are realized in the resale market.

The generalized Vickrey auction is defined as follows (see Ausubel and Cramton 2004). First, the monotonic efficient assignment rule $q_i^e(\mathbf{s})$ is defined by

$$v_i(q_i^e(\mathbf{s}); \mathbf{s}) \begin{cases} \leq v_{-i}\left(q_{-i}^e(\mathbf{s}); \mathbf{s}\right), & \text{if } q_i^e(\mathbf{s}) = 0 \\ = v_{-i}\left(q_{-i}^e(\mathbf{s}); \mathbf{s}\right), & \text{if } 0 < q_i^e(\mathbf{s}) < \bar{Q}(\mathbf{s}) \\ \geq v_{-i}\left(q_{-i}^e(\mathbf{s}); \mathbf{s}\right), & \text{if } q_i^e(\mathbf{s}) = \bar{Q}(\mathbf{s}). \end{cases} \tag{1.6}$$

Second, the aggregate quantity rule $\bar{Q}(\mathbf{s})$ is determined by

$$\bar{Q}(\mathbf{s}) = \begin{cases} y^{-1}\left(v_{-i}\left(q_{-i}^e(\mathbf{s}); \mathbf{s}\right); \mathbf{s}\right), & \text{if } q_i^e(\mathbf{s}) = 0 \\ y^{-1}\left(v_i\left(q_i^e(\mathbf{s}); \mathbf{s}\right); \mathbf{s}\right), & \text{if } q_i^e(\mathbf{s}) > 0, \end{cases} \tag{1.7}$$

where $y(Q;\mathbf{s}) \equiv E\left[\left.\frac{dDF(Q)}{dQ}\right|\mathbf{s}\right]$ is the conditional expected marginal damage of total pollution $Q = \sum_{i=1}^{n} q_i$.

Third, the Vickrey payment rule is

$$R_i(\mathbf{s}) = \int_0^{q_i^e(\mathbf{s})} v_i\left(x; \hat{s}_i(\mathbf{s}_{-i}, x), \mathbf{s}_{-i}\right) dx., \tag{1.8}$$

where signal $\hat{s}_i$ is the lowest possible signal for which firm i would have won unit x given other bidders' (true) signals $\mathbf{s}_{-i}$:

$$\hat{s}_i(x, \mathbf{s}_{-i}) = \inf_{s_i}\left\{ s_i \middle| q_i^e(s_i, \mathbf{s}_{-i}) \geq x \right\}. \tag{1.9}$$

Thus, the marginal payment for unit x is the expected marginal valuation of firm i evaluated at x, if firm i had received and reported the lowest possible signal $\hat{s}_i$ such that $x = \hat{q}_i^e(\hat{s}_i, \mathbf{s}_{-i})$. Note that by the efficient assignment rule $v_i\left(\hat{q}_i^e(\hat{s}_i, \mathbf{s}_{-i}); \hat{s}_i, \mathbf{s}_{-i}\right) = v_{-i}\left(\hat{q}_i^e(\hat{s}_i, \mathbf{s}_{-i}); \hat{s}_i, \mathbf{s}_{-i}\right)$, where $\hat{q}_i^e(\hat{s}_i, \mathbf{s}_{-i})$ is the efficient allocation given the signal vector $\hat{\mathbf{s}} = (\hat{s}_i, \mathbf{s}_{-i})$. The marginal payment is thus based on valuations conditional on $\hat{\mathbf{s}}$ and not on true signals $\mathbf{s}$. Hence, with interdependent values the payment is not the full externality cost, in contrast to the pure private values case. The payment does not include the informational externality of signal s_i to other bidders' values and to the damage of pollution.

Ausubel and Cramton (2004) also show that in the case of independent signals and when the seller has no value for the objects on sale, the Vickrey auction with reserve pricing attains the upper bound for revenues in a resale-constrained auction program. Thus, when agents are able to trade units freely after the auction mechanism, the best the auctioneer can do with respect to efficiency and revenues is to conduct a Vickrey auction with a reserve price.[18]

Uniform price auction

In the uniform price auction with the fixed supply and private values[19], the first-order condition from (1.3) is written as (e.g. Holmberg 2009)

$$v_i(D_i(p)) = p - \frac{D_i(p)}{D'_{-i}(p)}, \tag{1.10}$$

where $D'_{-i}(p) \leq 0$ is a price derivative of the aggregate demand of every other bidder at p. Because the total supply is fixed, $D'_{-i}(p)$ is thus equal to the (negative) price derivative of

[18] In the first two essays of the thesis, bidders' values are interdependent. However, the signals are not independent and I am not able to derive any results using the revenue equivalence theorem (see Section 1.4).

[19] This is the setting in the third essay of this thesis.

the residual supply. There are a number of notable points to be made from the first-order condition (1.10). First, compared to the competitive equilibrium, where bidders act as price-takers, bid-shading results in a lower clearing price. Second, bidders do not shade their bids for the first units. If the clearing price is so high that $D_i(p)$ tends to zero, bidders act as price-takers, i.e. $D_i(p) = v_i^{-1}(p)$. Third, bidders shade their bids more, the relatively larger they are. Large bidders have larger $D_i(p)$ and they face a more elastic residual supply, i.e. a smaller $\left|D'_{-i}(p)\right|$, than small bidders. This makes the shading factor higher for large bidders. Fourth, due to the differing shading characteristics, large (small) bidders tend to receive less (more) units than in the efficient outcome. Fifth, the number of possible equilibria is infinite and even very low price equilibria are possible, as shown by Wilson (1979). However, underpricing can be reduced by adjusting the inelastic supply after the submission of bids (Back and Zender 2001, McAdams 2007), making the supply elastic (LiCalzi and Pavan 2005) or forcing the bid functions to be discrete (Kremer and Nyborg 2004).

Equation (1.10) constitutes a system of n differential equations. Solving it analytically is a very demanding task. Holmberg (2008) derives a unique solution to this problem with a procurement auction model for when firms compete with supply functions. However, the solution requires a set of assumptions: the perfectly inelastic demand is uncertain, there is a price cap, firms are symmetric, firms' production capacities are constrained, and the capacity constraints bind with positive probability (see also Rudkevich et al. 1998, Anderson and Philpott 2002, Keloharju et al. 2005). Holmberg (2009), on the other hand, derives a numerical solution to an otherwise similar model but in the case of asymmetric firms. Moreover, assuming a linear model and linear bid schedules simplifies the model and offers tractable solutions (e.g. Green 1996, Baldick et al. 2004, Ausubel et al. 2013).

The Vickrey auction is shown to be vulnerable to collusion. In a uniform pricing format, collusion has been studied in laboratory experiments by e.g. Goswami et al. (1996) and Burtraw et al. (2009). In infinitely repeated uniform price auctions, Fabra (2003) and Dechenaux and Kovenock (2007), for instance, examine how perfect collusion can be sustained among the capacity-constrained firms.[20]

1.4 Information

Information plays a key role in pollution regulation in many respects, as we saw in previous sections. One of the most significant, and perhaps the greatest, current environmental problem is climate change. Climate change is an extremely complex, multi-level and dynamic

[20] Fabra (2003) also studies discriminatory price auctions.

problem associated with great uncertainty. Auction mechanisms are one possible tool to overcome some of the issues related to the incomplete and asymmetric information in pollution regulation. However, auction mechanisms with multiple units are relatively complicated to model, even in the simplest possible setting. In this thesis, I try to simplify the complex information structure with a simple static representation similar to Vives (2010, 2011)[21] In the following, I explain how the static affine linear model reflects the dynamic problem and preserves the main informational characteristics of climate change.[22] Note, however, that these models are relatively general and can be applied in numerous types of environmental and other problems.

The recent report by the Intergovernmental Panel on Climate Change (IPCC)[23] affirms, once again, that the climate is warming and that global warming is due to increased concentrations of anthropogenic greenhouse gases in the atmosphere. To what extent high greenhouse gas concentrations increase global temperatures is very uncertain. The resulting damage caused by a rise in global temperatures is even more uncertain, whether this is due to rising sea levels, ocean acidification, extreme weather events, floods, droughts, changes in ecosystems or any other possible impact. Tol (2009) surveys the literature on the economic effects of climate change. Estimates of the total effects vary from a 2.3% increase in global GDP due to a $1.0°C$ increase in global temperature (Tol 2005) to a 4.8% reduction in global GDP due to a $3.0°C$ temperature increase (Nordhaus 1994). However, the estimated impacts of climate change vary heavily between regions or economic sectors. In particular, low-income countries are the most vulnerable to climate change, such as countries in Africa and Asia. If the global temperature rise is only modest (such as $1.0°C$), the positive effects for high-income countries may offset the damage for more vulnerable regions. However, it seems more probable that the temperature rise will be more severe and that we will face notable reductions in global GDP due to global warming.

The climate change impacts can also be expressed as the net present value of incremental damage due to a small increase in greenhouse gas emissions. This is the social cost of carbon, or the marginal damage of pollution. In the studies analyzed by Tol (2009), the average estimate of the social cost of carbon is approximately US$29 per tonne of CO_2 (i.e. $105 per tonne of carbon), but the range of the estimates is very large. Including uncertainty in the models tends to increase and equity weighting tends to reduce the estimates. Most importantly, the appropriate discount rate is the major open issue concerning the social cost

[21] The affine linear model is applied in the first two essays. This gives a simple interdependent values model. In the third essay, the firms' marginal valuations are private.

[22] The climate change problem is a dynamic stock pollution problem. This is studied e.g. by Hoel and Karp (2001, 2002), Newell and Pizer (2003), Karp and Zhang (2005, 2012).

[23] The Working Group I contribution to the IPCC's Fifth Assessment Report (AR5).

of carbon. (Nordhaus 2011.) In a recent paper, Anthoff and Tol (2013) discuss these issues in more detail.

Nevertheless, I model the marginal (net present) damage of pollution as a linear function:[24]

$$MDF\,(Q;\gamma) = \gamma + \delta Q, \tag{1.11}$$

where Q can be interpreted as the total greenhouse gas pollution of the next 50 years. Pollution is uniformly mixed. Thus pollution levels depend only on the total emissions levels; the locations of emission sources are not relevant. Following Weitzman (1974), the damage function is linearized around the first-best pollution level $Q^{\star}$. This gives the parameters of the linear function: $\gamma \equiv MDF\,(Q^{\star}) - \delta Q^{\star}$ and $\delta \equiv \frac{dMDF(Q)}{dQ}\Big|_{Q^{\star}}$. For simplicity and in order to guarantee tractable solutions, the slope parameter $\delta \geq 0$ is assumed to be common knowledge. Thus the uncertainty of pollution damage is captured by the damage parameter γ. It is normally distributed with a mean and a variance given by $\gamma \sim N(\bar{\gamma}, \sigma_{\gamma}^2)$.

In order to mitigate the damage of global warming, greenhouse gas emissions should be reduced substantially in coming decades. Depending on the ambition of climate policy, this requires a considerable technological shift in electricity generation from fossil fuels to carbon-free technologies such as wind and solar power. Currently, these technologies are more costly than e.g. coal or gas plants. With appropriate climate policies new carbon-free technologies will be competitive with conventional technologies. This, however, is a very uncertain process. The implementation costs and the learning rates for the new technologies, i.e. the reductions in costs as a function of installed capacity, vary widely inside and between different sets of renewable energy technologies, among others (see Fischedick et al. 2011).

The firms I model in the essays can be interpreted as electricity companies. A typical electricity company has different plants in its generation portfolio. The generating mix contains different shares of e.g. gas, oil, coal and nuclear power, and renewable energy. In order to reduce emissions, the company must invest in more efficient fossil fuel plants or carbon-free technologies. However, the investment costs and the future maintenance costs for different technologies are uncertain. This cost uncertainty arises from several factors, such as the development and learning effects of new technologies, the relative costs of primary fuels, local weather conditions, economic growth, the demand for electricity or future climate policies. Nevertheless, the more firms have to reduce emissions the more they have to pay and the more valuable emission permits become.

As is well known, technological change is a complex dynamic problem. To avoid complex

[24] See e.g. Weitzman (2010) for discussion of the specification of the damage function.

details, I simplify the technological description considerably by making it static. Suppose that the emission reduction activities and investment decisions of a single firm are independent of each other. In that case the net present value of the future emission reduction path is simple to calculate and the problem can be solved as a static problem. This is, of course, a very significant simplification.

Hence, in the models, the linearized marginal (net present) value of pollution for firm i is written as

$$u_i(q_i; \theta_i) = \theta_i - \beta q_i, \tag{1.12}$$

where the cost parameter $\theta_i \equiv u_i(q_i^\star) - \beta q_i^\star$ and the slope parameter $\beta \equiv -\frac{du_i(q_i)}{dq_i}\Big|_{q_i^\star} \geq 0$ are defined by the first-best level of pollution $q_i^\star$. The slope parameter β is constant and common knowledge to all the firms and to the regulator. The uncertainty is, again, captured by the cost parameter θ_i. In the models, the private cost parameter θ_i is initially uncertain to firm i (and to other firms and the regulator). I assume ex-ante symmetry between firms, and hence the cost parameters share the same prior distribution, $\theta_i \sim N(\bar{\theta}, \sigma_\theta^2)$. However, the firms are not identical and they have some private information about their reduction costs. The firms have different generation portfolios and each firm thus has a noisy signal of its own cost parameter, $s_i = \theta_i + \varepsilon_i$. The noise terms are identically and independently distributed with a normal distribution around zero, i.e. $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$.

The affine linear model is assumed to entail two important correlations associated with functions (1.11) and (1.12). First, due to the similar set of units in their generation portfolios and similar investment possibilities, it is reasonable to assume that emission reduction costs are correlated between firms. I assume symmetric correlation between firms, hence the cost parameters θ_i and θ_i $(i \neq j)$ have a covariance $cov[\theta_i, \theta_j] = \rho\sigma_\theta^2$.

Second, the correlation between the emission reduction costs and benefits, i.e. the avoided damage of pollution, has an important role in the models. This statistical dependence is discussed by Stavins (1996). He states that with uniformly mixed pollution, correlation between the benefits and costs of emission reductions is not likely. However, climate change is a problem with a very long time horizon. Global warming impacts economic growth as well as local weather conditions, among other things. The nature of the statistical dependence between pollution damage and emission reduction costs is not clear. On the one hand, the relative costs of wind or wave energy, for example, can be reduced locally due to higher wind speeds. On the other hand, decreased economic growth may affect the financing costs or availability and costs of other resources. This may increase the costs of emission reductions in the long run. In the models, I assume that the possible dependence between environmental damage and the cost of emissions reductions is the same for all regulated companies. The correlation between the damage parameter γ and the average cost parameter $\theta_m = \sum_{i=1}^{n} \theta_i$ is given by $cov[\gamma, \theta_m] = \sigma_{\gamma\theta}$.

With linear functions and normal random parameters, the conditional expectations of uncertain variables are affine functions. Thus it is easy to calculate the expected value of θ_i conditional on s_i and s_m:

$$E[\theta_i | s_i, s_m] = A\bar{\theta} + Bs_i + Cns_m, \tag{1.13}$$

where $A \equiv A(\xi)$, $B \equiv B(\xi)$ and $C \equiv C(\xi)$ are functions of the information structure, $\xi \equiv (n, \sigma_\varepsilon^2, \sigma_\theta^2, \rho)$. Respectively, the conditional expectation of the damage parameter is written as

$$E[\gamma | s_i, s_m] = \bar{\gamma} + Z\left(ns_m - n\bar{\theta}\right), \tag{1.14}$$

where $Z \equiv Z(n, \sigma_\varepsilon^2, \sigma_\theta^2, \rho, \sigma_{\gamma\theta})$. This property is very useful in the analysis. With this construction, the signals are one-dimensional and the expected marginal value functions $v_i(q_i; \mathbf{s}) = E[\theta_i | s_i, s_m] - \beta q_i$ satisfy the continuity, value monotonicity and single-crossing properties. In addition, the aggregate quantity rule defined in (1.7) may derive from (1.11) and (1.14). Thus the affine linear model provides a convenient and simple set-up for extending the analysis of optimal pollution regulation, where regulated firms have private, yet uncertain, information about their emission reduction costs.

1.5 Summaries of the essays

1.5.1 Prices vs. quantities when information is incomplete and asymmetric

I examine the Weitzman (1974) prices versus quantities model by comparing a uniform Piqouvian tax and a program of tradeable permits where the permit market is assumed to perform perfectly. I extend the information structure of Weitzman by allowing firms to have private information about their uncertain abatement costs. Moreover, the abatement costs are symmetrically correlated between firms. I also allow the emission reduction benefits and costs to be correlated. The model is linear and the information structure is affine. Hence, firms' values for pollution are interdependent, but the marginal value functions, i.e. the marginal values of avoided abatement activities, satisfy the continuity, value monotonicity and single-crossing properties.

In the absence of regulation, firms will not initiate emission reduction activities. Once regulation is implemented, firms update their production processes, install new and cleaner technologies and gradually learn their costs of abatement. Hence, at the outset there are uncertainties related both to the level of abatement costs and when the true costs are revealed. However, firms are better informed than the regulator, which is an important modification to the Weitzman model. Moreover, the regulator's strategies are constrained in two important ways in this paper. First, the regulator can only choose between a uniform tax and tradeable permits with a fixed total quantity. Second, the initial allocation is the only stage at which it can influence the rules of the regulation. Moreover, there is only one round of communication between firms and the regulator and it is also conducted during the process of the initial allocation.

I propose the following two-stage regulation. The second stage is the standard prices versus quantities setting, where the regulator implements either a uniform tax or a program of tradeable permits. However, the information on which the policy parameters, i.e. the level of tax or the total amount of permits, are based, is gathered in the first stage of the regulation. In the first stage, the regulator conducts a generalized Vickrey auction. The main task of the auction is to collect private information from regulated firms. In addition, the first-stage auction mechanism serves as an initial allocation method for pollution permits. Permits are also allocated in the case of the price regulation. Under the tax (or subsidy) regulation, firms may buy more permits from the regulator or sell permits back to the regulator at a given price in the second stage. In the quantity regulation, on the other hand, the emissions cap is fixed, but firms may trade permits among themselves.

The main contribution of the paper is to investigate whether the Vickrey pricing rule induces firms to bid sincerely in an auction and reveal their information to the regulator when the auction is followed by either a price or quantity regulation. I show that the Vickrey auction is incentive compatible when followed by a constant quantity regulation, whenever the positive correlation between emissions reduction benefits and costs is not too high. However, if constant price regulation is used in the second stage, firms do not have incentives to bid sincerely in the Vickrey auction, unless the correlation between emissions reduction benefits and costs is relatively high and negative. This, however, is not expected in most pollution problem cases, as discussed by Stavins (1996). Hence, if the information is valuable to the regulator and if the regulator is able to implement an incentive mechanism to collect the private information of firms, tradeable permits are a more preferable instrument relative to taxes when compared with the Weitzman model.

In addition, the solution concept of the generalized Vickrey auction is a Bayes Nash equilibrium. It is thus not possible to implement the efficient mechanism in dominant strategies, when bidders' values are interdependent. However, even if the dominant strategy implementation is not possible, I show that collusion does not distort the outcomes of the ex-post efficient allocation and incentive compatibility of the Vickrey auction and thus the results are in line with Montero (2008).

1.5.2 Learning through one round of communication in regulating the commons when markets are imperfect

I apply the same information and regulation structure as in the first essay. Polluting firms are privately better informed than the regulator and firms' abatement costs are uncertain but correlated. In addition, the pollution damage is uncertain and can be correlated with the abatement costs. The regulator implements a two-stage regulation using the generalized Vickrey auction in the first stage.

The essay provides two important extensions. Firstly, for the second-stage regulation, the regulator implements a quantity regulation, but now the emissions cap is not fixed. Instead, the supply of pollution permits is dictated by the non-constant permit supply schedules for each firm. Secondly, I relax the assumption of a perfect permit market. However, I do not specify the sources of market imperfections. Imperfections can arise from transaction costs, asymmetric information between bidders or any other market friction. Taking these frictions seriously would make the modeling extremely difficult. Hence I make a rough simplification and assume that the marginal cost of trading for each firm is a linear function of the amount the firm trades in the permit market with other firms. This simplification provides a tractable solution, but it does not change the intuition of the results.

Even if trading between firms is costly, firms are able to trade permits with the regulator without any extra costs in the model. Furthermore, the non-constant permit supply schedules take into account the frictions of the permit market. If the frictions are modest, the regulation in the second stage is close to the non-constant permit regulation of Roberts and Spence (1976) and if the frictions are very great, the optimal second stage regulation is Weitzman's (1978) non-constant tax regulation.

The main contributions of the essay are two-fold. First, I study the incentive compatibility conditions of the first-stage auction mechanism followed by the permit resale market in the regulation stage. I show that given the affine linear structure of the model, the best strategy is to bid sincerely in the Vickrey auction if every other firm bids sincerely, unless the (negative or positive) correlation between the aggregate abatement costs and damage of pollution is relatively high.

Second, I address the role of firms' private information when the permit market suffers from frictions. Given a perfectly competitive permit market, the regulator would be able to obtain a solution that maximizes the expected social welfare even without the knowledge of firms' private information. This is Roberts and Spence's (1976) non-constant permit regulation, where the regulator is concerned about the aggregate pollution level and thus the aggregate supply schedule of permits needs to be set equal to the expected marginal damage function. The permit market solves the asymmetries between firms. With an imperfect permit market, the regulator needs to take into account both the aggregate pollution and the distribution and the initial allocation of permits among regulated firms. The private information of firms is then valuable to the regulator.

1.5.3 Collusion in emission allowance auctions

Assuming that firms' values are private and their knowledge about their own abatement costs is complete, I compare two auction designs for allocating the emission allowances: the Vickrey auction and the uniform price auction. Even though the Vickrey auction is an efficient mechanism, it is vulnerable to collusion, as shown by Montero (2008). Collusion reduces the auction revenues. In contrast, uniform price auctions are not necessarily efficient. This gives an interesting set-up for revenue and efficiency comparisons.

In the model, there are no secondary markets and the market consists of two parts: a competitive fringe and a number of strategic firms. The fringe firms behave as price-takers in the auction and thus the fringe balances the market. Hence I do not have to consider the low price equilibria of Wilson (1979). To analyze these two auction designs, I postulate a linear-quadratic demand function equilibrium with a fixed supply of emission allowances. Moreover, strategic firms may collude prior to an auction. Hence I link the auction model to a coalition-formation game. I apply a partition function approach (e.g. Yi 2003), where the coalition formation is conducted in two stages. In the first stage, strategic firms decide whether to participate in coalitions and in the second stage the coalitions play a non-cooperative auction game against each other. This is the main contribution of the essay.

Montero (2008) shows that Vickrey auctions provide an efficient allocation of allowances even if firms collude. However, all the strategic firms have strong incentives to form one big coalition in Vickrey auctions. Thus the revenue loss increases when the market share of strategic firms increases.

In contrast, uniform price auctions create a coalition game with positive externalities. The more concentrated the coalition structure is, the better off the coalition outsiders are. In such games, large coalitions are hard to sustain, because firms have strong incentives to deviate. I examine three examples of the coalition formation game in a uniform price auction: a cartel game with either myopic or farsighted firms and an open membership game with multiple coalitions. The stable coalition structure and hence the efficiency and revenues of uniform price auctions depend heavily on the coalition game and the structure of the market.

Bibliography

[1] Ambec, S., and Coria, J., 2013. Prices vs Quantities with Multiple Pollutants. Journal of Environmental Economics and Management 66, 123-140.

[2] Anderson, E.J., and Philpott, A., 2002. Using Supply Functions for Offering Generation into an Electricity Market. Operations Research 50, 477-489.

[3] Anthoff, D., and Tol, R.S.J., 2013. The Uncertainty about the Social Cost of Carbon: A Decomposition Analysis Using Fund. Climatic Change 117, 515-530.

[4] Ausubel, L.M., and Cramton, P., 1999. The Optimality of Being Efficient. Working Paper, University of Maryland.

[5] Ausubel, L.M., and Cramton, P., 2004. Vickrey Auctions with Reserve Pricing. Economic Theory 23, 493-505.

[6] Ausubel, L.M., Cramton, P., Pycia, M., Rostek, M., and Weretka, M., 2013. Demand Reduction, Inefficiency and Revenues in Multi-unit Auctions. Mimeo.

[7] Ausubel, L.M., and Milgrom, P., 2006. The Lovely but Lonely Vickrey Auction. In Cramton, P., Shoham, Y., and Steinberg, R. (eds). Combinatorial Auctions. MIT Press.

[8] Back, K., and Zender, J.F., 1993. Auctions of Divisible Goods: On the Rationale for the Treasury Experiment. Review of Financial Studies 6, 733-764.

[9] Back, K., and Zender, J.F., 2001. Auctions of Divisible Goods with Endogenous Supply. Economics Letters 73, 29-34.

[10] Baldick, R., Grant, R., and Kahn, E., 2004. Theory and Application of Linear Supply Function Equilibrium in Electricity Markets. Journal of Regulatory Economics 25(2), 143-167.

[11] Baldursson, F.M., and von der Fehr, N.-H.M., 2004. Price Volatility and Risk Exposure: On Market-based Environmental Policy Instruments. Journal of Environmental Economics and Management 48(1), 682-704.

[12] Ben-David, S., Brookshire, S.D., Burness, S., McKee, M., and Schmidt, C., 2000. Attitudes Toward Risk and Compliance in Emission Permit Markets. Land Economics 76(4), 590-600.

[13] Burtraw, D., Goeree, J., Holt, C. A., Myers, E., Palmer, K., and Shobe, W., 2009. Collusion in Auctions for Emission Permits: An Experimental Analysis. Journal of Policy Analysis and Management 28, 672-691.

[14] Chatterjee, K., and Samuelson, W., 1983. Bargaining under Incomplete Information. Operations Research 31(5), 835-851.

[15] Clarke, E., 1971. Multipart Pricing of Public Goods. Public Choice 2, 19-33.

[16] Coase, R., 1960. The Problem of Social Cost. The Journal of Law and Economics 3, 1-44.

[17] Coase, R., 1992. The Institutional Structure of Production. American Economic Review 82(4), 713-719.

[18] Cramton, P., and Kerr, S., 2002. Tradeable Carbon Permit Auctions: How and Why to Auction and Not Grandfather. Energy Policy 30(4), 333-345.

[19] Crocker, T.D., 1966. The Structuring of Atmospheric Pollution Control Systems. In Wolozin, H. (ed.) The Economics of Air Pollution. New York: W. W. Norton & Co., 61-86.

[20] Dales, J.H., 1968. Pollution, Property and Prices. Toronto: University Press.

[21] Dasgupta, P., Hammond, P., and Maskin, E., 1980. On Imperfect Information and Optimal Pollution Control. Review of Economic Studies 47(5), 857-860.

[22] Dasgupta, P., and Maskin, E., 2000. Efficient Auctions. Quarterly Journal of Economics 115(2), 341-388.

[23] Dechenaux, E., and Kovenock, D., 2007. Tacit Collusion and Capacity Withholding in Repeated Uniform Price Auctions. RAND Journal of Economics 38(4), 1044-1069.

[24] Fabra, N., 2003. Tacit Collusion in Repeated Auctions: Uniform versus Discriminatory. Journal of Industrial Economics 51, 271-293.

[25] Fell, H., MacKenzie, I.A., and Pizer, W.A., 2012. Prices versus Quantities versus Bankable Quantities. Resource and Energy Economics 34, 607-623.

[26] Fischedick, M., Schaeffer, R., Adedoyin, A., Akai, M., Bruckner, T., Clarke, L., Krey, V., Savolainen, I., Teske, S., Ürge-Vorsatz, D., and Wright, R., 2011. Mitigation Potential and Costs. In Edenhofer, O., Pichs-Madruga, R., Sokona, Y., Seyboth, K., Matschoss, P., Kadner, S., Zwickel, T., Eickemeier, P., Hansen, G., Schlömer, S., and von Stechow, C. (eds.) IPCC Special Report on Renewable Energy Sources and Climate Change Mitigation. Cambridge University Press.

[27] Goswami, G., Noe, T.H., and Rebello, M.J., 1996. Collusion in Uniform-Price Auctions: Experimental Evidence and Implications for Treasury Auctions. Review of Financial Studies 9, 757-785.

[28] Green, R.J., 1996. Increasing Competition in the British Electricity Spot Market. Journal of Industrial Economics 44(2), 205-216.

[29] Green, J., and Laffont, J-J., 1979. Incentives in Public Decision Making. North Holland: Amsterdam.

[30] Groves, T., 1973. Incentives in Teams. Econometrica 41, 617-631.

[31] Hahn, R.W., 1984. Market Power and Transferable Property Rights. Quarterly Journal of Economics 99, 753-65.

[32] Hahn, R.W., and Stavins, R.N., 2011. The Effect of Allowance Allocations on Cap-and-Trade System Performance. Journal of Law and Economics 54(4), S267-S294.

[33] Hoel, M., and Karp, L., 2001. Taxes versus Quotas for a Stock Pollutant with Multiplicative Uncertainty. Journal of Public Economics 82, 91-114.

[34] Hoel, M., and Karp, L., 2002. Taxes versus Quantities for a Stock Pollutant. Resource and Energy Economics 24(4), 367-384.

[35] Holmberg, P., 2008. Unique Supply Function Equilibrium with Capacity Constraints. Energy Economics 30(1), 148-172.

[36] Holmberg, P., 2009. Numerical Calculation of an Asymmetric Supply Function Equilibrium with Capacity Constraints. European Journal of Operational Research 199, 285-295.

[37] Holmström, B., 1979. Groves Schemes on Restricted Domains. Econometrica 47, 1137-1144.

[38] Hortaçsu, A., 2011. Recent Progress in the Empirical Analysis of Multi-unit Auctions. International Journal of Industrial Organization 29, 345-349.

[39] Hortaçsu, A., and McAdams, D., 2010. Mechanism Choice and Strategic Bidding in Divisible Good Auctions: An Empirical Analysis of the Turkish Treasury Auction Market. Journal of Political Economy 118(5), 833-865.

[40] Hortaçsu, A., and Puller, S.L., 2008. Understanding Strategic Bidding in Multi-unit Auctions: A Case Study of the Texas Electricity Spot Market. RAND Journal of Economics 39, 86-114.

[41] Jehiel, P., and Moldovanu, B., 2001. Efficient Design with Interdependent Valuations. Econometrica 69(5), 1237-1259.

[42] Karp, L. and Zhang, J., 2005. Regulation of Stock Externalities with Correlated Abatement Costs. Environmental and Resource Economics 32, 273-299.

[43] Karp, L. and Zhang, J., 2012. Taxes versus Quantities for a Stock Pollutant with Endogenous Abatement Costs and Asymmetric Information. Economic Theory 49, 371-409.

[44] Kastl, J., 2011. Discrete Bids and Empirical Inference in Divisible Good Auctions. Review of Economic Studies 78, 978-1014.

[45] Keloharju, M., Nyborg, K.G., and Rydqvist, K., 2005. Strategic Behavior and Underpricing in Uniform Price Auctions: Evidence from Finnish Treasury Auctions. The Journal of Finance 60(4), 1865-1902.

[46] Kennedy, P., Laplante, B. and Whittington, D., 2010. Simple Pricing Schemes for Pollution Control under Asymmetric Information. The B.E. Journal of Economic Analysis & Policy 10(1), Article 12.

[47] Klemperer, P.D., and Meyer, M.A., 1989. Supply Function Equilibria in Oligopoly under Uncertainty. Econometrica 57(6), 1243-1277.

[48] Kremer, I., and Nyborg, K.G., 2004. Underpricing and Market Power in Uniform Price Auctions. The Review of Financial Studies 17(3), 849-877.

[49] Krishna, V., and Perry, M., 2000. Efficient Mechanism Design. Working Paper, Pennsylvania State University.

[50] Krysiak, F.C., 2008. Prices vs. Quantities: The Effects on Technology Choice. Journal of Public Economics 92, 1275-1287.

[51] Kwerel, E., 1977. To Tell the Truth: Imperfect Information and Optimal Pollution Control. Review of Economic Studies 44(3), 595-601.

[52] Lappi, P., 2012. Duopolists in Output and Permit Markets: Interaction and Regulation. Strategic Behavior and the Environment 2(3), 279-293.

[53] Lewis, T.R., 1996. Protecting the Environment when Costs and Benefits Are Privately Known. RAND Journal of Economics 27(4), 819-847.

[54] LiCalzi, M., and Pavan, A., 2005. Tilting the Supply Schedule to Enhance Competition in Uniform-price Auctions. European Economic Review 49, 227-250.

[55] Liski, M., 2001. Thin versus Thick CO2 Market. Journal of Environmental Economics and Management 41, 295-311.

[56] Liski, M., and Montero, J.-P., 2006. On Pollution Permit Banking and Market Power. Journal of Regulatory Economics 29, 283-302.

[57] Liski, M., and Montero, J.-P., 2011. Market Power in an Exhaustible Resource Market: The Case of Storable Pollution Permits. The Economic Journal 121(551), 116-144.

[58] Lopomo, G., Marx, L.M., McAdams, D., and Murray, B.C. 2011. Carbon Allowance Auction Design: An Assessment of Options for the United States. Review of Environmental Economics and Policy 5(1), 25-43.

[59] Malcomson, J.M. 1978. Prices vs. Quantities: A Critical Note of the Use of Approximations. Review of Economic Studies 45(1), 203-207.

[60] Malueg, D.A., and Yates, A.J., 2009. Bilateral Oligopoly, Private Information, and Pollution Permit Markets. Environmental and Resource Economics 43, 553-572.

[61] Maskin, E., and Riley, J., 2000. Asymmetric Auctions. Review of Economic Studies 67, 413-438.

[62] Mas-Colell, A., Whinston, M.D., and Green, J.R., 1995. Microeconomic Theory. Oxford University Press.

[63] McAdams, D., 2007. Adjustable Supply in Uniform Price Auctions: Non-commitment as a Strategic Tool. Economics Letters 95, 48-53.

[64] McAfee, P.R., and McMillan, J., 1992. Bidding Rings. The American Economic Review 82(3), 579-599.

[65] Milgrom, P., 2004. Putting Auction Theory to Work. Cambridge: Cambridge University Press.

[66] Milgrom, P., and Weber, R., 1982. A Theory of Auctions and Competitive Bidding. Econometrica 50, 1089-1122.

[67] Misiolek, W.S., and Elder, H.W., 1989. Exclusionary Manipulation of Markets for Pollution Rights. Journal of Environmental Economics and Management 16, 156-166.

[68] Montero, J.-P., 1998. Marketable Pollution Permits with Uncertainty and Transaction Costs. Resource and Energy Economics 20, 27-50.

[69] Montero, J.-P., 2002. Prices versus Quantities with Incomplete Enforcement. Journal of Public Economics 85(3), 435-454.

[70] Montero, J.-P., 2008. A Simple Auction Mechanism for the Optimal Allocation of the Commons. American Economic Review 98(1), 496-518.

[71] Montero, J.-P., 2009. Market Power in Pollution Permit Markets. Energy Journal 30, 115-142.

[72] Montgomery, D.W., 1972. Markets in Licenses and Efficient Pollution Control Programs. Journal of Economic Theory 5, 395-418.

[73] Myerson, R.B, 1981. Optimal Auction Design. Mathematics of Operations Research 6, 58-73.

[74] Myerson, R.B., and Satterthwaite, M.A., 1983. Efficient Mechanisms for Bilateral Trading. Journal of Economic Theory 29, 265-281.

[75] Newell, R.G., and Pizer, W.A., 2003. Regulating Stock Externalities under Uncertainty. Journal of Environmental Economics and Management 45, 416-432.

[76] Nordhaus, W.D., 1994. Expert Opinion on Climate Change. American Scientist 82(1), 45-51.

[77] Nordhaus, W.D., 2011. Estimates of the Social Cost of Carbon: Background and Results from the RICE-2011 Model. Discussion Paper 1826, Cowles Foundation, New Haven.

[78] Ollikka, K., and Tukiainen, J., 2013. Central bank liquidity auction mechanism design and the interbank market. Bank of Finland Research Discussion Paper No. 21/2013.

[79] Pigou, A., 1920. The Economics of Welfare. London: Macmillan and Co.

[80] Riley, J., and Samuelson, W., 1981. Optimal Auctions. American Economic Review 71, 381-392.

[81] Roberts, M.J., and Spence, M., 1976. Effluent Charges and Licenses under Uncertainty. Journal of Public Economics 5(3-4), 193-208.

[82] Rudkevich, A., Duckworth, M., and Rosen, R., 1998. Modeling Electricity Pricing in a Deregulated Generation Industry: The Potential for Oligopoly Pricing in Poolco. The Energy Journal 19, 19-48.

[83] Stavins, R.N., 1995. Transaction Costs and Tradeable Permits. Journal of Environmental Economics and Management 29, 133-148.

[84] Stavins, R.N., 1996. Correlated Uncertainty and Policy Instrument Choice. Journal of Environmental Economics and Management 30, 218-232.

[85] Stigler, G.J., 1966. The Theory of Price. 3rd ed. New York: Macmillan.

[86] Tol, R.S.J., 2005. Emission Abatement versus Development as Strategies to Reduce Vulnerability to Climate Change: An Application of FUND. Environment and Development Economics 10(5), 615-629.

[87] Tol, R.S.J., 2009. The Economic Effects of Climate Change. Journal of Economic Perspectives 23(2), 29-51.

[88] Vickrey, W., 1961. Counterspeculation, Auctions and Competitive Sealed Tenders. Journal of Finance 16, 8-37.

[89] Vives, X., 1999. Oligopoly Pricing: Old Ideas and New Tools. MIT Press.

[90] Vives, X., 2010. Asset Auctions, Information, and Liquidity. Journal of the European Economic Association 8(2-3), 467-477.

[91] Vives, X., 2011. Strategic Supply Function Competition with Private Information. Econometrica 79(6), 1919-1966.

[92] von der Fehr, N.-H., 1993. Tradeable Emission Rights and Strategic Interaction. Environmental and Resource Economics 3, 129-151.

[93] Wang, J.J.D., and Zender, J.F., 2002. Auctioning Divisible Goods. Economic Theory 19, 673-705.

[94] Weitzman, M.L., 1974. Prices vs. Quantities. Review of Economic Studies 41(4), 477-491.

[95] Weitzman, M.L., 1978. Optimal Rewards for Economic Regulation. American Economic Review 68, 683-691.

[96] Weitzman, M.L., 2010. What is the "Damages Function" for Global Warming - and What Difference Might it Make? Climate Change Economics 1(1), 57-69.

[97] Wilson, R., 1979. Auctions of Shares. Quarterly Journal of Economics 94, 675-689.

[98] Wolak, F., 2003. Identification and Estimation of Cost Functions Using Observed Bid Data: An Application to Electricity. In Dewatripont, M., Hansen, L.P., and Turnovsky, S.J. (eds). Advances in Econometrics: Theory and Applications, Eighth World Congress, Vol. II. Cambridge, UK: Cambridge University Press, 133-169.

[99] Yates, A.J., 2012. On a Fundamental Advantage of Permits Over Taxes for the Control of Pollution. Environmental and Resource Economics 51, 583-598.

[100] Yi, S., 2003. Endogenous Formation of Economic Coalitions: A Survey of the Partition Function Approach. In Carraro, C. (ed.) The Endogenous Formation of Economic Coalitions. Edward Elgar, Cheltenham, UK, 80-127.

Chapter 2

Prices vs. quantities when information is incomplete and asymmetric

Abstract

I extend the Weitzman (1974) model by allowing firms to have private information about their uncertain abatement costs. The abatement costs are correlated between firms. I propose a two-stage regulation. In the first stage, the regulator conducts a generalized Vickrey-Clarke-Groves (VCG) mechanism. The first-stage auction mechanism serves as an initial allocation method of pollution permits and also collects private information from regulated firms. In the second stage, the regulator implements either a constant price or a constant quantity regulation. In the constant price regulation, a uniform tax rate is set at the level of the expected first-best price. The constant quantity regulation is implemented through a tradeable permit program, where the supply of permits is fixed at the level of the expected first-best aggregate pollution. I show, using an affine linear model, that the VCG mechanism is incentive compatible when followed by the constant quantity regulation, whenever the positive correlation between emissions reduction benefits and costs is not too high. However, if the constant price regulation is used in the second stage, the information mechanism is incentive compatible only if the negative correlation between emissions reduction benefits and costs is relatively high.

2.1 Introduction

When regulating harmful pollution, the relevant information about emission reduction costs is often in the hands of regulated firms. Thus, in order to implement efficient regulation, the regulatory authority often communicates with regulated firms to get them to reveal their private information. However, it is not in the firms' interest to tell the regulator their information truthfully, if this is expected to increase their costs. Instead, firms might overestimate or underestimate their costs of abatement activities depending on their beliefs about the type of the future regulation. For instance, if the authority is planning to implement a uniform tax on pollution, it is in the firms' interest to underestimate their costs, and thus have the tax as low as possible. If, on the other hand, the regulator is planning to put up a program of tradeable emission permits with a fixed supply of permits, firms may find it profitable to overestimate their expected costs in order to get the regulator to issue more permits to the market. Signs of this kind of behavior have been seen, for example, in the first phases of the US Acid Rain Program and the EU Emissions Trading System. In both of these programs the initial allocation of permits was generous and the resulting equilibrium prices fell much lower than was initially expected (e.g. McAllister 2009). Hence, if the information provided by the regulated firms is biased or incomplete, the regulation is not as efficient as it could be.

This paper extends, at least to my knowledge, the previous literature on regulating pollution in two respects. First, I address the role of firms' private information in more detail. I assume that regulated firms do not have accurate information about their own abatement costs. Nevertheless, their information is more accurate than that of the regulator. In addition, the firms' private information is correlated. Second, I combine the traditional prices versus quantities regulation model of Weitzman (1974) with the information mechanism. It is of great interest to find a mechanism that gives incentives to regulated firms to reveal their private information truthfully to the regulator. Hence, I present the following two-stage regulation. In the first stage, the regulator conducts a generalized Vickrey-Clarke-Groves (VCG) mechanism, which allocates emission permits initially to regulated firms. The main goal of the auction mechanism, however, is to collect the firms' private information. In the second stage, the regulator implements either a constant price or a constant quantity regulation. In the constant price regulation, a uniform tax/subsidy rate is set at the level of the expected first-best price. During the regulation period, firms are able to trade permits with the regulator at this price. The constant quantity regulation is implemented through a tradeable permit program, where the supply of permits is fixed to the level of the expected first-best aggregate pollution. In the quantity regulation, firms are free to trade permits with each other. Even if the first-stage information mechanism is ex-post efficient without any secondary market, regulated firms may not be willing to reveal their information truthfully when the VCG mechanism is followed by the regulation stage. Hence, the main contribution of this paper is to derive conditions for an incentive compatible VCG mechanism, when it is followed by either a constant price or a constant quantity regulation. In addition, I show when firms' private information is most valuable to the regulator and, thus, when the information mechanism improves the outcomes of the regulation.

Weitzman (1974) derives a rule for choosing between a constant price and quantity regulation, when a social planner is "at the decision node where as much information as is feasible to gather has already been obtained by one means or another and an operational plan must be decided on the basis of the available current knowledge". Given that the permit market is perfect, Weitzman shows that the choice between a price or quantity regulation depends on the relationship between the slope of the aggregate marginal abatement costs and the slope of the marginal pollution damage. For example, if the slope of aggregate marginal abatement costs is greater than the slope of the marginal damage, then the constant price regulation should be used. In addition, if the marginal abatement costs are correlated with the marginal pollution damage, then the positive correlation increases the comparative advantage of the quantity regulation. In this paper, I study the two constant regulations because these are the two most recommended instruments that are also used in practice.[1] For instance, there is a wide and ongoing debate as to whether to use a tax or an emissions trading system in climate policy (e.g. Newell and Pizer 2003, Karp and Zhang 2005, Metcalf 2007, Stavins 2007).[2]

If regulated firms have more accurate information about the costs of emission reductions than the regulator has, then the regulator is eager to communicate with the firms before

[1]Roberts and Spence (1976), Weitzman (1978), Kennedy et al. (2010) and Yates (2012), for instance, provide models with non-constant regulatory schemes. In a non-constant tax regulation, the tax rate varies with the quantity of emissions and in a non-constant quantity regulation the permit supply is defined as a function of price. With a similar information structure to this chapter, I examine in Chapter 3 a two-stage regulation with a non-constant quantity regulation in the second stage. In addition, in Chapter 3, I relax the assumption of a perfect permit market.

[2]The climate change problem is a dynamic stock pollution problem. In a problem of stock pollution, Newell and Pizer (2003) and Karp and Zhang (2005), for instance, build dynamic models with autocorrelated abatement costs. Using estimates of marginal abatement costs, marginal damage and other relevant parameters in their models, they argue that taxes dominate quota regulation in the climate change problem. Moreover, in Karp and Zhang (2005), the regulator learns from firms' reactions in different periods and adjusts regulation based on the new information (feed-back policy). However, in this paper the model is static. During one period the regulation is fixed by assumption and the regulation period of this paper can be interpreted as a single period of a dynamic model. Also the timing of information is different. In the model of this paper, the regulator is eager to learn the private information of firms at the beginning of the regulation period in order to improve the regulation for the same period, whereas in Karp and Zhang (2005) the new information is used for adjusting the regulation in subsequent periods.

implementing any regulation. Kwerel (1977) gives an example of a regulatory scheme with one round of communication between the regulator and the regulated firms. In Kwerel's model firms know their own abatement costs exactly. Kwerel proposes a simple subsidy and license (permit) scheme, where polluting firms first submit reports on their abatement costs to the regulator. Based on the firms' reports, the regulator then allocates pollution licenses to the firms and sets the price of a subsidy at which it will buy pack any licenses that are in excess of the firms' emissions. Kwerel's scheme implements the first-best in Nash equilibrium. In other words, given that other firms report their private information truthfully to the regulator, it is also the best response for each firm to report truthfully. However, Kwerel's scheme works only if all licenses are auctioned off with an uniform-price design and if the uniform-price auction is competitive (Montero 2008). If licenses are allocated for free, there are more profitable equilibrium strategies than those proposed by Kwerel. When using these profitable deviation strategies, firms over-report their demand for licenses to the maximum extent. Also, if licenses are allocated using a uniform-price auction, bid-shading and the resulting low-price auction equilibrium provide incentives for over-reporting and make the regulation a money-making machine for firms.

Montero (2008) provides an efficient mechanism for the commons problem by applying the Vickrey-Clarke-Groves (VCG) pricing rule (Vickrey 1961, Clarke 1971, Groves 1973).[3] Montero examines an indirect implementation of the VCG mechanism by proposing a simple sealed-bid auction mechanism for emissions permits with endogenous (non-constant) supply.[4] In Montero, each firm is certain about its abatement costs. In other words, firms have pure private values of the emission permits. In a pure private values case, the VCG mechanism implements efficient allocation in dominant strategies. Montero also shows that the Vickrey auction implements the first-best outcome, even if firms collude and coordinate their bids in the auction.

I follow the previous literature (e.g. Weitzman 1974, 1978) and linearize the unknown functions around the first-best. In addition, I apply the affine information structure from Vives (2010, 2011). The model is thus the symmetric case of Weitzman (1974, 1978). Moreover, I assume that prior distributions, the number of regulated firms and the functional forms of the costs and benefits of emission reductions are common knowledge. With the quantity regulation, I further assume that the second-stage market is perfect. Hence, trading is efficient, firms do not have market power and the market does not suffer from any kind of frictions. Pollution is assumed to be uniformly mixed.

[3]The VCG mechanism is a multi-unit extension of a single-unit Vickrey auction. In this paper these are used as synonyms.

[4]Dasgupta et al. (1980) propose a tax scheme applying a direct VCG mechanism.

However, it is important to recognize the two important extensions that I make to the Weitzman (1974) model. First, I take one step backwards and assume that regulated firms have some private information about their own abatement costs. Reducing emissions is costly for firms. They have to install new and cleaner technology, modify their production processes, use more expensive inputs or perhaps even reduce production to some extent. Above all, polluting firms have more accurate information about their expected abatement costs than the regulator. This information is valuable but not available, a priori, to the regulator.

Second, regulated firms do not have complete information about their true abatement costs. Without any regulation, firms are not willing to reduce their emissions from the business-as-usual level, and they have only limited understanding of the future costs of emissions reductions. Once the regulation is implemented and firms start to invest in new technologies, firms gradually learn their true abatement costs. However, the exact timing of the learning process is not known beforehand. Moreover, if all regulated firms choose their abatement technologies from a similar technology set, uncertain costs are correlated between firms. Hence, the values that firms place on emission permits are interdependent. This complicates the auction mechanism implemented in the first stage of the regulation. In Montero (2008), values are private and the Vickrey auction provides the first-best solution. Unfortunately, with common or interdependent values, the efficiency property of VCG mechanisms (or any other mechanism) is not generally sustained (Jehiel and Moldovanu 2001). However, with suitable conditions, such as when bidders' uncertainty is one-dimensional and marginal values have single-crossing properties, a generalized Vickrey auction is ex-post efficient (Dasgupta and Maskin 2000, Ausubel and Cramton 2004). This holds at least in the absence of a secondary market for permits.

Furthermore, I allow the uncertain benefits and costs of emissions reductions to be correlated (see also Stavins 1996). The relative magnitude of this correlation determines whether the Vickrey auction is incentive compatible, when it is followed by one of the two constant regulations. I show that it is optimal to bid sincerely in a Vickrey auction if every other bidder is bidding sincerely and if the auction is followed by a constant quantity regulation, whenever the positive correlation between emissions reduction benefits and costs is not too high. However, using the constant price regulation in the second stage, the Vickrey auction is incentive compatible only if the negative correlation between aggregate abatement costs and pollution damage is relatively high. Moreover, if there is no statistical dependence between the marginal benefits and marginal costs of environmental protection, I derive a modified rule for choosing between the one-stage price regulation (uniform tax without any information mechanism) and the two-stage quantity regulation (a tradeable emission permit program with the Vickrey auction as an initial allocation mechanism). The comparative advantage of the two-stage quantity regulation increases if the private information of firms becomes more accurate, if the correlation between firms' abatement costs increases and if the regulated firms are more heterogeneous. In addition, I show that when the incentive compatibility conditions are satisfied, collusive actions do not distort the ex-post efficiency of the Vickrey auction.

In Section 2.2, I introduce the two-stage regulation and the affine linear model. I solve the problem backwards. Hence in Section 2.3, I first derive the expected deadweight losses of the constant price regulation and the constant quantity regulation and compare them given the information of the regulator at the time of implementing the second-stage regulation. I derive the value of information in different information structures. In Section 2.4, I describe the Vickrey payment rule of the information stage, and derive the incentive compatibility conditions of the Vickrey auction. A modified Weitzman rule is introduced in Section 2.5 and Section 2.6 provides a robustness check for collusion. Section 2.7 concludes.

2.2 Model

Consider $n \geq 2$ risk-neutral firms indexed with $i = 1, ..., n$.[5] As a by-product of the normal production of goods and services, firms pollute. Pollution is denoted by a vector $\mathbf{q} = (q_1, \ldots, q_n)$ and $Q = \sum_{i=1}^{n} q_i$ denotes the aggregate pollution. Without any regulation, the pollution of firm i is at the business-as-usual level, q_i^{bau}. Reducing emissions below the business-as-usual level is costly for firms.

Let $AC_i\left(z_i; \theta_i\right)$ denote the abatement costs of firm i, where $z_i\left(q_i\right) = q_i^{bau} - q_i \geq 0$ is the amount of abatement and θ_i is a firm-specific cost parameter. Put the other way round, the gross value of the avoided abatement costs of firm i, that is the gross value of pollution, writes $U_i\left(q_i; \theta_i\right) = AC_i\left(q_i^{bau}; \theta_i\right) - AC_i\left(z_i\left(q_i\right); \theta_i\right)$. Hence the marginal abatement costs, or the marginal value of the avoided abatement costs, is denoted by $U_i' = \frac{dU_i}{dq_i} = u_i\left(q_i; \theta_i\right)$. The cost parameter θ_i defines the level of the marginal abatement costs of firm i such that $u_i\left(q_i; \theta_i\right) \geq u_i\left(q_i; \theta_i'\right)$ if $\theta_i \geq \theta_i'$ for all $0 \leq q_i \leq q_i^{bau}$ and i. Moreover, it is reasonable to assume that with closely related industrial firms, emission reduction technologies are related and costs are thus correlated. In particular, I assume that cost parameters are correlated between firms.

[5] I relax this assumption later and let there be one cartel with all firms as members or only one big firm, and thus $n = 1$.

Furthermore, pollution causes damage to the environment and the damage function is denoted by $DF(Q;\gamma)$. The marginal damage function is denoted by $DF' = \frac{dDF}{dQ} = MDF(Q;\gamma)$ and γ is a damage parameter such that $MDF(Q;\gamma) \geq MDF(Q;\gamma')$ if $\gamma \geq \gamma'$ for all $0 \leq Q \leq \sum_{i=1}^{n} q_i^{bau}$. Pollution is assumed to be uniformly mixed. The standard assumptions hold, $U_i' > 0$, $U_i'' \leq 0$, and $DF' > 0$, $DF'' \geq 0$.

The first-best solution maximizes the social welfare with respect to the pollution vector $\mathbf{q}$:

$$\max_{\mathbf{q}} W(\mathbf{q}) = \sum_{i=1}^{n} U_i(q_i;\theta_i) - DF\left(\sum_{i=1}^{n} q_i;\gamma\right). \tag{2.1}$$

Let us assume that this problem has an interior solution denoted by a vector $\mathbf{q}^\star = (q_1^\star, \ldots, q_n^\star)$ where $0 < q_i^\star < q_i^{bau}$ for all i and $Q^\star = \sum_{i=1}^{n} q_i^\star$. In the first-best, the pollution of each firm is at the level where the marginal value of pollution equals the first-best price and, in addition, the first-best price equals the value of the marginal damage function:

$$u_i(q_i^\star;\theta_i) = p^\star = MDF(Q^\star;\gamma).$$

Unfortunately, the first-best solution is unknown due to the uncertain information about the true damage function and the true emission reduction costs. The cost parameters $\theta = (\theta_1, \ldots, \theta_n)$ are unknown to all at the outset, but each firm receives a private signal of its own cost parameter and firms learn their true abatement costs during the regulation period. In order to maximize the expected welfare, the regulator implements regulation r. Firms react to this regulation. Let $\pi_{i,r}(\cdot)$ denote the profit function of firm i. The profit-maximizing solution of firm i, after the revelation of θ_i, is thus denoted by $q_{i,r} = \arg\max \pi_{i,r}(q_i;\theta_i)$. Hence, $\mathbf{q}_r = (q_{1,r}, \ldots, q_{n,r})$ denotes the vector of profit maximizing pollution levels and $Q_r = \sum_{i=1}^{n} q_{i,r}$ is the total level of pollution. The problem of the regulator is then to choose a regulation r which maximizes the expected welfare given the reactions of firms to the regulation:

$$\begin{aligned} \max_{r} E[W(\mathbf{q}_r)] &= E\left[\sum_{i=1}^{n} U_i(q_{i,r};\theta_i) - DF(Q_r;\gamma)\right] \qquad (2.2) \\ s.t. & \\ q_{i,r} &= \arg\max \pi_{i,r}(q_i;\theta_i). \end{aligned}$$

In the next two sections I first describe the two-stage regulation model where the regulator can communicate with regulated firms before implementing the actual regulation. This communication is conducted by an information mechanism. Furthermore, I assume that every firm and the regulator know the primitives of the model: the prior distributions of uncertain variables, the number of regulated firms and the functional forms of the costs and benefits of emission reductions. In particular, I apply an affine linear model, which is also introduced below.

2.2.1 Two-stage regulation

With the possibility of communication, the model has two stages. In the first stage the regulator implements an information mechanism, which aims to reveal firms' private information to the regulator. In addition, the first-stage mechanism serves as an initial allocation method for permits. This is done utilizing a generalized Vickrey auction and I denote this stage the information stage. Before conducting the auction mechanism, the regulator informs all firms about the rules of the auction and the regulation period. Then, according to the auction mechanism, firms submit reports (bidding schedules) to the regulator. The regulator's objective is to get firms bid sincerely in the auction. Thus, the following definition is the core concept in this paper.

Definition 2.1. *The Vickrey auction in the information stage is said to be incentive compatible (IC) if bidding sincerely in the auction is the best response to other bidders' strategies when they too bid sincerely.*

After the auction, the regulator allocates pollution permits to firms and collects the auction payments from them. The payments and the allocation rule are determined by the firms' reports. The auction mechanism is a time-consuming procedure. The regulator is not able to conduct an auction at any point of time. I thus assume that after an auction there is a relatively long time period when firms take different actions: trade inputs and outputs of production, make decisions about reducing emissions and, most importantly, learn. This period is the second stage of the model and it is called the regulation period.

Using the information from the first stage, the regulator sets up a regulation in the beginning of the regulation period. If the price regulation is chosen, then the regulator establishes a uniform tax/subsidy for firms. The tax (subsidy) specifies the price at which firms are able to buy (sell) emission permits from (to) the regulator during the regulation stage. Hence, permit transactions are conducted on two occasions. First, firms have to buy pollution permits based on their reports after the information stage. Second, firms may update their holdings of emissions permits during the regulation period.

Instead, if a quantity regulation is applied in the regulation period, firms may trade pollution rights with each other. Firms may need this opportunity when they learn their true abatement costs. However, the total amount of permits is fixed after the information mechanism. With the quantity regulation, I assume that the second-stage market is perfect. The assumption of perfect competition is a natural first step for the resale market. Suppose that the initial allocation is ex-post efficient. Then the initial allocation after the information stage is fairly close to the first-best and firms' ability to use market power is limited. Nevertheless, the results would change if there were trade frictions in the secondary market. However, I ignore all market imperfections in the resale market.[6]

At the end of the regulation period, each firm is obligated to hold an amount of permits equalling the emissions it had in the regulation period. I assume that the penalty for being non-compliant is very high and firms do not violate the compliance rule on purpose. To recap, the timing of the regulation is the following.

- t_0: All agents (regulated firms and the regulator) learn the distribution functions of the uncertain parameters and the functional forms of firms' abatement costs and pollution damage. Each firm receives a noisy signal about its own abatement costs.

- t_1: The first stage - the information stage. The regulator conducts an auction, in which emission permits are initially allocated to firms. The regulator announces both the rules of the auction mechanism and the rules of the regulation during the regulation period. In the auction, each firm i simultaneously submits a demand schedule to the regulator. The regulator sets the clearing price and the total quantity of permits to be allocated. Then it distributes permits to firms and collects auction payments from firms. The regulation period starts after the auction.

- $t_1 - t_2$: The second stage - the regulation stage. In the quantity regulation, the aggregate supply of permits is fixed at the level of the initial allocation. Firms are, however, allowed to trade permits with each other. In contrast, if the price regulation is used, firms may purchase more permits from the regulator or sell permits back to the regulator at the announced uniform price, which is set at the level of the first-stage auction clearing price. Firms learn their cost parameters during the regulation period.

- t_2: All firms have learned their cost parameters. The time point t_2 is not known to any firm or to the regulator at the outset. The true damage of pollution is not revealed.

[6]In Appendix 2.E, I derive the results of the model when the permit market suffers from market frictions and run some numerical simulations. See Chapter 3 for a more general analysis when the permit market is not perfect.

2.2.2 Affine linear model

In the following, I introduce the affine linear model and derive the linearized version of the regulator's problem (2.2). In particular, all the unknown marginal functions are linearized around the first-best (see Weitzman 1974, 1978). In addition, all the random variables are normally distributed (see Vives 2010, 2011).[7] To put it more formally, the affine linear model is defined as follows.

Definition 2.2. *The affine linear model is defined by equations (2.4) - (2.16), where the distribution functions of the uncertain variables and the functional forms of the abatement costs and pollution damage are common knowledge and the permit market is perfect.*

Each firm has the following quadratic approximation of its abatement cost function evaluated around the first-best:

$$U_i\left(q_i;\theta_i\right) = U_i\left(q_i^\star;\theta_i\right) + U_i'\left(q_i^\star;\theta_i\right)\left(q_i - q_i^\star\right) + \frac{1}{2}U_i''\left(q_i^\star;\theta_i\right)\left(q_i - q_i^\star\right)^2. \tag{2.3}$$

The linearized marginal value function of firm i is thus written as

$$u_i\left(q_i;\theta_i\right) = \theta_i - \beta q_i, \tag{2.4}$$

where the intercept, i.e. the cost parameter, is $\theta_i \equiv U_i'\left(q_i^\star;\theta_i\right) - U_i''\left(q_i^\star;\theta_i\right)q_i^\star > 0$ and the slope $\beta \equiv -U_i''\left(q_i^\star;\theta_i\right) \geq 0$. The slope parameter β is constant and common knowledge to all firms and to the regulator.[8] However, the cost parameter θ_i is initially uncertain to firm i (and to other firms and the regulator). The cost parameters share the same prior distribution, $\theta_i \sim N\left(\bar{\theta},\sigma_\theta^2\right)$ and these cost parameters are symmetrically correlated between firms with a covariance, $cov\left[\theta_i,\theta_j\right] = \rho\sigma_\theta^2$. The average cost parameter $\theta_m = \frac{1}{n}\sum_{i=1}^{n}\theta_i$ has an expected value $E\left[\theta_m\right] = \bar{\theta}$ and a variance $var\left[\theta_m\right] = \frac{1}{n}\left(1+(n-1)\rho\right)\sigma_\theta^2$.

At the outset, each firm receives a noisy signal of its own cost parameter, $s_i = \theta_i + \varepsilon_i$. The noise terms are i.i.d., with a normal distribution around zero, $\varepsilon_i \sim N\left(0,\sigma_\varepsilon^2\right)$. Let $\mathbf{s} = (s_1,\ldots,s_n)$ denote the signal vector and $\mathbf{s}_{-i} = (s_1,\ldots,s_{i-1},s_{i+1},\ldots,s_n)$ is the signal vector of every other firm but firm i. The average signal is denoted by $s_m = \frac{1}{n}\sum_{i=1}^{n}s_i$ and it

[7] Using normally distributed random variables is convenient for modeling purposes but lacks reality. The support of the normal random variable is $[-\infty,\infty]$. Thus, given the assumptions of the model, there is a positive probability that a firm's marginal value of pollution permits is highly negative and decreasing in the firm's pollution. This is not realistic, nor is it realistic that the marginal damage function of pollution is negative. Hence I assume throughout the paper that all the parameter values of the model are such that all unrealistic events are highly unprobable, and thus can be ignored.

[8] The assumption of a constant slope makes the model a bit more easy to solve. The common knowledge assumption is, however, more restrictive. Without this assumption, the model would not be tractable.

has an expected value $E[s_m] = \bar{\theta}$ and a variance $var[s_m] = \frac{1}{n}(\sigma_\varepsilon^2 + (1+(n-1)\rho)\sigma_\theta^2)$.[9] I assume that the expected values are interdependent and hence $\sigma_\varepsilon^2 > 0$ and $0 < \rho < 1$. I do not consider negative correlation between marginal values.[10]

Firms update their beliefs about cost parameters given the information they have. In addition to the firm's own signal, the clearing price of the first-stage auction reveals information about the signals of other firms. Indeed, given that the auction mechanism is incentive compatible (Definition 2.1) and thus all firms bid sincerely, then under the affine linear model (Definition 2.2) the clearing price p is sufficient statistics for s_m. This entails that $E[\theta_i | s_m]$ is informationally equivalent to $E[\theta_i | p]$. Furthermore, due to the symmetric correlation between firms' cost parameters, $E[\theta_i | \mathbf{s}]$ is informationally equivalent to $E[\theta_i | s_m]$, and thus $E[\theta_i | \mathbf{s}] = E[\theta_i | s_m] = E[\theta_i | p]$. I explain this mechanism later. Now, given that firm i knows both its own signal s_i and the average signal s_m (or the whole signal vector $\mathbf{s}$, or the clearing price of the incentive compatible Vickrey auction), then the conditional expected value of θ_i writes as (see Appendix 2.A and e.g. DeGroot 1970, Vives 2011)

$$E[\theta_i | \mathbf{s}] = A\bar{\theta} + Bs_i + Cns_m, \tag{2.5}$$

where[11]

$$\begin{aligned} A &= \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + (1+(n-1)\rho)\sigma_\theta^2} \\ B &= \frac{(1-\rho)\sigma_\theta^2}{\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2} \\ C &= \frac{\rho\sigma_\theta^2\sigma_\varepsilon^2}{(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2)(\sigma_\varepsilon^2 + (1+(n-1)\rho)\sigma_\theta^2)}. \end{aligned}$$

The variance of θ_i conditional on $\mathbf{s}$ is, respectively,

$$var[\theta_i | \mathbf{s}] = (B+C)\sigma_\varepsilon^2. \tag{2.6}$$

Furthermore, the conditional expected value and the variance of the average cost parameter writes as

$$E[\theta_m | \mathbf{s}] = \bar{\theta} + (1-A)\left(s_m - \bar{\theta}\right), \tag{2.7}$$

[9] Note also that $var[s_m] = cov[s_i, s_m]$.

[10] The model would have independent private values if $\rho = 0$. The pure common values case is when the value parameters are perfectly correlated and thus $\rho = 1$.

[11] The expected value of θ_i conditional only on signal s_i is $E[\theta_i | s_i] = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\theta^2}\bar{\theta} + \frac{\sigma_\theta^2}{\sigma_\varepsilon^2 + \sigma_\theta^2}s_i$. Note also that $A + B + nC = 1$ and thus $1 - A = B + nC$.

$$var\left[\theta_m|\,\mathbf{s}\right] = A\cdot var\left[\theta_m\right] = \left(\frac{B}{n}+C\right)\sigma_\varepsilon^2. \tag{2.8}$$

Note that after the revelation of the average signal s_m, there would still be some uncertainty before firm i learns its true cost parameter θ_i. The remaining uncertainty of firm i is $\varepsilon_i^s = \theta_i - E\left[\theta_i|\,\mathbf{s}\right]$. This is a normally distributed random variable with the parameters $\varepsilon_i^s \sim N\left(0,(B+C)\,\sigma_\varepsilon^2\right)$ and a covariance, $cov\left[\varepsilon_i^s,\varepsilon_j^s\right] = C\sigma_\varepsilon^2$. The distribution of the remaining aggregate uncertainty, $n\varepsilon_m^s = \sum_{i=1}^n \varepsilon_i^s$, has the parameters $n\varepsilon_m^s \sim N\left(0,(B+nC)\,n\sigma_\varepsilon^2\right)$.

Hence (2.4) and (2.5) yield the expected marginal value function conditional on $\mathbf{s}$:

$$v_i\left(q_i;\mathbf{s}\right) = E\left[u_i\left(q_i\right)|\,\mathbf{s}\right] = A\bar{\theta} + Bs_i + Cns_m - \beta q_i. \tag{2.9}$$

Respectively, the second-order approximation of the damage function writes as

$$DF\left(Q;\gamma\right) = DF\left(Q^\star;\gamma\right) + DF'\left(Q^\star;\gamma\right)\left(Q-Q^\star\right) + \frac{1}{2}DF''\left(Q^\star;\gamma\right)\left(Q-Q^\star\right)^2. \tag{2.10}$$

This gives the marginal damage function:

$$MDF\left(Q;\gamma\right) = \gamma + \delta Q, \tag{2.11}$$

where $\gamma \equiv DF'\left(Q^\star;\gamma\right) - DF''\left(Q^\star;\gamma\right)Q^\star$ and $\delta \equiv DF''\left(Q^\star;\gamma\right) \geq 0$. Again, the slope parameter δ is assumed to be common knowledge. The uncertainty of the damage function is captured by the damage parameter γ. This is assumed to be a normally distributed random variable, $\gamma \sim N(\bar{\gamma},\sigma_\gamma^2)$. Furthermore, γ is correlated with θ_m and I denote the covariance by $cov\left[\gamma,\theta_m\right] = \sigma_{\gamma\theta}$.[12] Hence, the expected marginal damage function conditional on the sum of all signals $ns_m = \sum\limits_{i=1}^{n} s_i$ is written as

$$\begin{aligned} y\left(Q;\mathbf{s}\right) &= E\left[MDF\left(Q\right)|\,\mathbf{s}\right] \\ &= E\left[\gamma|\,\mathbf{s}\right] + \delta Q, \end{aligned} \tag{2.12}$$

where

$$E\left[\gamma|\,\mathbf{s}\right] = \bar{\gamma} + Z\left(ns_m - n\bar{\theta}\right), \tag{2.13}$$

and

$$Z = \frac{cov\left[\gamma, ns_m\right]}{var\left[ns_m\right]} = \frac{\sigma_{\gamma\theta}}{\sigma_\varepsilon^2 + \left(1+(n-1)\,\rho\right)\sigma_\theta^2}. \tag{2.14}$$

I assume that the true damage parameter is not revealed during the regulation period. The

[12] The correlation between γ and θ_m is $\rho_{\gamma\theta} = \frac{\sigma_{\gamma\theta}}{\sqrt{var[\theta_m]}\sqrt{var[\gamma]}}$ and thus $\sigma_{\gamma\theta} = \sqrt{\frac{1}{n}\left(1-\rho+n\rho\right)}\sigma_\theta\sigma_\gamma\rho_{\gamma\theta}$.

remaining uncertainty related to the damage parameter, after the revelation of signals, is $\varepsilon_\gamma^s = \gamma - \bar{\gamma} - Z\left(ns_m - n\bar{\theta}\right)$ and the conditional variance of γ and ε_γ^s is

$$var\left[\gamma|\,\mathbf{s}\right] = var\left[\varepsilon_\gamma^s\right] = \sigma_\gamma^2 - nZ\sigma_{\gamma\theta}. \tag{2.15}$$

Note also that the covariance between the average cost and damage parameters conditional on signal vector $\mathbf{s}$ is simply

$$cov\left[\gamma, \theta_m|\,\mathbf{s}\right] = A\sigma_{\gamma\theta}. \tag{2.16}$$

The covariance between abatement costs and pollution damage plays an important role in this paper. In particular, it defines the conditions under which sincere bidding in the information stage of the two-stage regulation is incentive compatible. I show later in Proposition 2.1 that given the affine linear model, the information mechanism is incentive compatible when followed by a constant quantity regulation, whenever the positive correlation between emissions reduction benefits and costs is not too high:

$$\frac{\sigma_{\gamma\theta}}{var\left[\theta_m\right]} \leq 1. \tag{2.17}$$

In particular, (2.17) ensures that the aggregate quantity rule of the first-stage auction is weakly increasing in each bidder's signal. Note that $1 - A - nZ = \frac{var[\theta_m]-\sigma_{\gamma\theta}}{var[s_m]}$ and thus (2.17) is equivalent to $nZ \leq 1 - A$. Furthermore, in Proposition 2.2, I show that, when a constant price regulation is used in the second stage, the information mechanism is incentive compatible only if the negative correlation between emissions reduction benefits and costs is relatively high:

$$\frac{\sigma_{\gamma\theta}}{var\left[\theta_m\right]} \leq -\frac{n\delta}{\beta}. \tag{2.18}$$

This is equivalent to $nZ \leq -\frac{n\delta}{\beta}\left(1 - A\right)$.

Finally, given the affine linear model, the pollution of firm i in the first-best is $q_i^\star = \frac{1}{\beta}\left(\theta_i - p^\star\right)$ where $p^\star$ is the first-best price:

$$p^\star = \frac{n\delta\theta_m + \beta\gamma}{\beta + n\delta}. \tag{2.19}$$

Furthermore, the welfare-maximizing aggregate pollution level may write

$$Q^\star = \frac{n\left(\theta_m - \gamma\right)}{\beta + n\delta}. \tag{2.20}$$

Hence the second-best regulation minimizes the following approximation of the expected deadweight loss equation:

$$\begin{aligned}\min_r E\left[DWL_r\right] &= E\left[W\left(\mathbf{q}^{\star}\right) - W\left(\mathbf{q}_r\right)\right] \\ &\approx E\left[\sum_i^n \theta_i\left(q_i^{\star} - q_{i,r}\right) - \frac{\beta}{2}\sum_i^n\left(q_i^{\star 2} - q_{i,r}^2\right) - \gamma\left(Q^{\star} - Q_r\right) - \frac{\delta}{2}\left(Q^{\star 2} - Q_r^2\right)\right].\end{aligned} \tag{2.21}$$

This problem is equivalent to (2.2). In this paper I consider two alternative constant regulations. The subscript $r = p$ denotes that the regulator implements a uniform tax/subsidy and thus uses a constant price regulation. On the other hand, if $r = q$ then the constant quantity regulation through tradeable emission permits is chosen for the regulation period.

2.3 Regulation stage

In this section, I examine Weitzman's (1974) prices vs. quantities comparison under the affine linear model. I compare the constant price and quantity regulations, given the regulator's information. In particular, let the information parameter $I = 0$ denote that the regulator knows only the prior information and thus the regulation is implemented in the absence of the information mechanism. In contrast, $I = \mathbf{s}$ denotes that the regulator has complete information about the private information of firms. The problem of the regulator is to choose $r = p, q$ that minimizes the expected deadweight loss from (2.21) given the responses of regulated firms.

2.3.1 Prices

With the price instrument, the regulator sets a tax/subsidy at the level of the expected first-best price. When the regulator knows only the prior information ($I = 0$), the expected first-best price is

$$\bar{p}\left(0\right) = \frac{n\delta\bar{\theta} + \beta\bar{\gamma}}{\beta + n\delta}. \tag{2.22}$$

Respectively, if the regulator has complete information about the signal vector ($I = \mathbf{s}$), the expected first-best price writes as

$$\begin{aligned}\bar{p}\left(\mathbf{s}\right) &= \frac{n\delta\bar{\theta}_m\left(\mathbf{s}\right) + \beta\bar{\gamma}\left(\mathbf{s}\right)}{\beta + n\delta} \\ &= \bar{p}\left(0\right) + \frac{n\delta\left(1 - A\right) + \beta n Z}{\beta + n\delta}\left(s_m - \bar{\theta}\right),\end{aligned} \tag{2.23}$$

where the last line comes from inserting $\bar{\theta}_m(\mathbf{s}) = E[\theta_m|\mathbf{s}]$ from (2.7) and $\bar{\gamma}(\mathbf{s}) = E[\gamma|\mathbf{s}]$ from (2.13).

In the beginning of the regulation period, the regulator allocates permits to firms according to the expected first-best emissions conditional on the regulator's information I. For firm i this gives $\bar{q}_i(I) = \frac{1}{\beta}\left(\bar{\theta}_i(I) - \bar{p}(I)\right)$, where $\bar{p}(I)$ is the expected first-best price from (2.22) or (2.23), and $\bar{\theta}_i(I) = E[\theta_i|I]$. With the affine information structure, $\bar{\theta}_i(0) = \bar{\theta}$ for all firms, and $\bar{\theta}_i(\mathbf{s}) = E[\theta_i|\mathbf{s}]$ from (2.5). For the average firm, indexed with m and receiving signal s_m, $\bar{q}_m(I) = \frac{1}{n}\bar{Q}(I)$, where $\bar{Q}(I) = \sum_{i=1}^{n}\bar{q}_i(I)$ is the total emissions cap in the beginning of the regulation period. If the regulator has no information about firms' signals, all firms receive an equal amount of permits:

$$\begin{aligned} \bar{q}_i(0) &= \bar{q} \\ &= \frac{\bar{\theta} - \bar{\gamma}}{\beta + n\delta}. \end{aligned} \tag{2.24}$$

On the other hand, when the regulator knows the signal vector $\mathbf{s}$, it can implement the ex-post efficient allocation denoted by $\mathbf{q}^e(\mathbf{s}) = (q_1^e(\mathbf{s}), \ldots, q_n^e(\mathbf{s})) \equiv E[q^\star|I]$:

$$\begin{aligned} \bar{q}_i(\mathbf{s}) &= q_i^e(\mathbf{s}) \\ &= \underbrace{\bar{q} + \left(\frac{1 - A - nZ}{\beta + n\delta}\right)\left(s_m - \bar{\theta}\right)}_{\bar{q}_m(\mathbf{s})} + \frac{B}{\beta}(s_i - s_m). \end{aligned} \tag{2.25}$$

Firms maximize their profits given the announced tax (or subsidy):

$$\max_{q_i} \pi_{i,p}(q_i;\theta_i) = \int_{\bar{q}_i(I)}^{q_i} \{u_i(x;\theta_i) - \bar{p}(I)\}\, dx. \tag{2.26}$$

In equilibrium, firms equate their marginal abatement costs with the tax (or subsidy). In Appendix 2.B, I derive the outcomes of the price and quantity regulation (see also Weitzman 1974, and Stavins 1996). The profit-maximizing solution of firm i for equilibrium pollution yields:

$$q_{i,p} = \bar{q}_i(I) + \frac{1}{\beta}\left(\theta_i - \bar{\theta}_i(I)\right). \tag{2.27}$$

However, while the price is fixed, the realized aggregate emissions may be over or under the first-best emissions after the revelation of the true abatement costs. The expected deadweight loss of the price regulation is thus

$$E\left[DWL_p(I)\right]=\frac{1}{2}\left(\frac{1}{\frac{\beta}{n}+\delta}\right)\left\{\left(\frac{n\delta}{\beta}\right)^2 var\left[\theta_m|\,I\right]+2\left(\frac{n\delta}{\beta}\right)cov\left[\theta_m,\gamma|\,I\right]+var\left[\gamma|\,I\right]\right\}. \tag{2.28}$$

2.3.2 Quantities

The initial allocation of the quantity regulation is given by the expected first-best quantities from (2.24) or (2.25). During the regulation stage, firms trade permits with each other. Thus, when the market is competitive, the maximization problem of firm i writes as

$$\max_{q_i}\ \pi_{i,q}(q_i;\theta_i)=\int_{\bar{q}_i(I)}^{q_i}\left\{u_i(x;\theta_i)-p_q\right\}dx. \tag{2.29}$$

In the competitive equilibrium, firms equate their marginal abatement costs with the permit market price p_q. The equilibrium price is defined by the market-clearing rule (see again Appendix 2.B):

$$p_q=\bar{p}(I)+\theta_m-\bar{\theta}_m(I). \tag{2.30}$$

However, the total allocation of permits does not adjust to any changes in abatement costs. The equilibrium pollution of firm i is

$$q_{i,q}=\bar{q}_i(I)+\frac{1}{\beta}\left\{\left(\theta_i-\bar{\theta}_i(I)\right)-\left(\theta_m-\bar{\theta}_m(I)\right)\right\}. \tag{2.31}$$

The expected deadweight loss, due to the non-adjustable total supply of permits, is written as

$$E\left[DWL_q(I)\right]=\frac{1}{2}\left(\frac{1}{\frac{\beta}{n}+\delta}\right)\left\{var\left[\theta_m|\,I\right]-2cov\left[\theta_m,\gamma|\,I\right]+var\left[\gamma|\,I\right]\right\}. \tag{2.32}$$

2.3.3 Prices vs. quantities

Equations (2.28) and (2.32) give Weitzman's (1974) prices vs. quantities comparison. The comparative advantage of the constant price regulation over the quantity regulation has an expected value

$$\begin{aligned}\Delta_{pq}(I) &= E\left[DWL_q(I)-DWL_p(I)\right] \\ &= \frac{n}{2\beta}var\left[\theta_m|\,I\right]\left\{1-\frac{n\delta}{\beta}-2\frac{\sigma_{\gamma\theta}}{var[\theta_m]}\right\}.\end{aligned} \tag{2.33}$$

Equation (2.33) defines the Weitzman rule. The constant price regulation should be chosen if $\Delta_{pq}(I) > 0$ and the constant quantity regulation if $\Delta_{pq}(I) < 0$.

Lemma 2.1. (Weitzman rule) *Consider the affine linear model with constant price and quantity regulations. In expected terms, when the abatement costs and pollution damage are not correlated ($\sigma_{\gamma\theta} = 0$), the constant price regulation is more favorable if the slope of the aggregate marginal abatement cost function is steeper (less price-elastic) than the marginal damage function, that is if $\frac{\beta}{n} > \delta$. Conversely, if the slope of the marginal damage is steeper, i.e. if $\delta > \frac{\beta}{n}$, then the regulator should use the constant quantity regulation. Furthermore, if the marginal abatement costs are correlated with the marginal damage of pollution, then the positive correlation ($\sigma_{\gamma\theta} > 0$) increases the comparative advantage of the constant quantity regulation (see also Stavins, 1996).*

Proof. See (2.33).

The critical value of the ratio between the slopes at which both instruments have equal expected welfare, i.e. $\kappa \equiv \left(\frac{n\delta}{\beta} : \Delta_{pq} = 0\right)$, is given by

$$\kappa = 1 - 2\frac{\sigma_{\gamma\theta}}{var[\theta_m]}.$$

It is also easy to see from (2.33) that the regulator should always use quantities if the positive covariance between abatement costs and emission reduction benefits, in relative terms, is high enough. In particular, this is the case whenever $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} > \frac{1}{2}$ and thus $\Delta_{pq}(I)$ is then always non-positive, while $\frac{n\delta}{\beta} \geq 0$. Respectively, the constant price regulation should be used whenever $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} < \frac{1}{2}\left(1 - \frac{n\delta}{\beta}\right)$. I examine next the role of the regulator's information. The results are collected in the following lemmas.

Lemma 2.2. *Given the affine linear model, the choice between constant price and quantity regulations is independent of the regulator's knowledge of firms' private information.*

Proof. The choice between constant price and quantity regulations depends on the sign of $\Delta_{pq}(I)$. Furthermore, $\text{sign}\left[\Delta_{pq}(I)\right] = \text{sign}\left[1 - \frac{n\delta}{\beta} - 2\frac{\sigma_{\gamma\theta}}{var[\theta_m]}\right]$. This is independent of the regulator's information I. ■

Lemma 2.3. *Given the affine linear model, the difference between the expected deadweight losses of constant price and quantity regulations is decreasing in the regulator's information I if $\sigma_\theta^2 > 0$.*

Proof. Note that $var\left[\theta_m | \mathbf{s}\right] = A \cdot var\left[\theta_m\right]$. From (2.33) it is then clear that $\left|\Delta_{pq}(\mathbf{s})\right| < \left|\Delta_{pq}(0)\right|$ if $A < 1$. This, on the other hand, is true whenever $\sigma_\theta^2 > 0$. ■

Lemma 2.4. *Given the affine linear model, the private information of firms is valueless to the regulator if* $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} = 1$.

Proof. According to the Weitzman rule (2.33), the regulator should use constant quantity regulation, whenever $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} > \frac{1}{2}$. Furthermore, from (2.32) we may derive the value of firms' private information to the regulator when using a constant quantity regulation:

$$\begin{aligned}\Delta_{q,I} &= E\left[DWL_q(0) - DWL_q(\mathbf{s})\right] \\ &= \frac{1}{2}\left(\frac{1}{\frac{\beta}{n}+\delta}\right)\frac{(var[\theta_m])^2}{var[s_m]}\left(1 - \frac{\sigma_{\gamma\theta}}{var[\theta_m]}\right)^2.\end{aligned}$$

This is zero when $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} = 1 > \frac{1}{2}$. ■

Lemma 2.5. *Given the affine linear model, the private information of firms is valueless to the regulator if* $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} = -\frac{n\delta}{\beta}$.

Proof. According to the Weitzman rule (2.33) the regulator should use constant price regulation, whenever $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} < \frac{1}{2}\left(1 - \frac{n\delta}{\beta}\right)$. From (2.28) we may derive the value of firms' private information to the regulator when using a constant price regulation:

$$\begin{aligned}\Delta_{p,I} &= E\left[DWL_p(0) - DWL_p(\mathbf{s})\right] \\ &= \frac{1}{2}\left(\frac{1}{\frac{\beta}{n}+\delta}\right)\frac{(var[\theta_m])^2}{var[s_m]}\left(\frac{n\delta}{\beta} + \frac{\sigma_{\gamma\theta}}{var[\theta_m]}\right)^2.\end{aligned}$$

This is zero when $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} = -\frac{n\delta}{\beta} < \frac{1}{2}\left(1 - \frac{n\delta}{\beta}\right)$. ■

Lemmas 2.4 and 2.5 imply that the aggregate initial allocation $\bar{Q}(I)$ is independent of information I when $\sigma_{\gamma\theta} = var[\theta_m]$ and, respectively, the level of uniform tax $\bar{p}(I)$ is independent of I when $\sigma_{\gamma\theta} = -\frac{n\delta}{\beta} var[\theta_m]$. These are also easily derived from (2.23) and (2.25). In other cases the private information of firms is valuable to the regulator. However, given the regulations being considered, the natural next question is: Are there mechanisms that give incentives for firms to reveal their private information to the regulator? This is examined in the next section.

2.4 Information stage

In this section, I examine a mechanism which aims to reveal the private information of firms to the regulator. The regulator is then able to implement ex-post efficient allocation of emissions permits in the beginning of the regulation stage and thus improve the outcome of the chosen regulation. Due to the correlated cost parameters, firm j's signal affects firm i's expected marginal value function. Firms' expected values are thus interdependent. In general, this poses a problem of finding a mechanism that is able to implement efficient allocation (Jehiel and Moldovanu 2001). Hence, in order to achieve ex-post efficiency in the auction in the information stage, some additional assumptions about expected marginal value functions are needed (Dasgupta and Maskin 2000, Ausubel and Cramton 2004). Following Ausubel and Cramton, the expected marginal value functions should satisfy the following three assumptions:

1. Continuity: $v_i(q_i; \mathbf{s})$ is jointly continuous in $(\mathbf{s}, q_i)$.
2. Value monotonicity: $v_i(q_i; \mathbf{s})$ is non-negative, and $\frac{\partial v_i(q_i;\mathbf{s})}{\partial s_i} > 0$ and $\frac{\partial v_i(q_i;\mathbf{s})}{\partial q_i} \leq 0$.
3. Single-crossing: Let $\mathbf{s}'$ denote a signal vector $\mathbf{s}' = (s_i', \mathbf{s}_{-i})$ and $\mathbf{s} = (s_i, \mathbf{s}_{-i})$. Then $v_i(q_i; \mathbf{s})$ has a single-crossing property, if for all i, $j \neq i$, q_i, q_j, $\mathbf{s}_{-i}$ and $s_i' > s_i$,

$$v_i(q_i; \mathbf{s}) > v_j(q_j; \mathbf{s}) \Rightarrow v_i(q_i; \mathbf{s}') > v_j(q_j; \mathbf{s}')$$

and

$$v_i(q_i; \mathbf{s}') < v_j(q_j; \mathbf{s}') \Rightarrow v_i(q_i; \mathbf{s}) < v_j(q_j; \mathbf{s}).$$

It is easy to see that (2.9) satisfies all these conditions. Continuity is just a regular assumption that guarantees an unambiguous solution to the first stage auction. Value monotonicity implies that firms are naturally ordered with respect to their signals, and that firms' demand curves in the Vickrey auction are weakly downward-sloping. Single-crossing means that an increase in signal s_i increases firm i's expected marginal value more than any other firm's marginal value for a given quantity. Furthermore, if a fixed quantity is assigned efficiently among the firms in the auction, then firm i's quantity q_i is weakly increasing in signal s_i. (Ausubel and Cramton 2004.)

Single-crossing also implies that signal s_i does not affect the natural order of firms other than i. This means that if firms other than i are ordered by a vector

$$O_{-i}(x; \mathbf{s}) \equiv (v_1(x; \mathbf{s}), \ldots, v_{i-1}(x; \mathbf{s}), v_{i+1}(x; \mathbf{s}), \ldots, v_n(x; \mathbf{s}))$$

such that $v_j(x; \mathbf{s}) \geq v_k(x; \mathbf{s})$ for every x and $j < k$, then signal s_i does not affect the order of vector $O_{-i}(x; \mathbf{s})$.

Next I first describe the generalized VCG mechanism and then I study the incentive compatibility conditions of the VCG mechanism conducted in the information stage, when followed by the two possible constant regulations.

2.4.1 Vickrey auction

According to the revelation principle, for each indirect Bayesian mechanism there is a payoff-equivalent direct revelation mechanism (e.g. Myerson 1981). I first describe the direct and then the indirect interpretation of the same VCG mechanism using the affine linear model. In the direct mechanism, the regulator requests reports from firms on their payoff-relevant parameters unknown to the regulator. In our model the reports include signals. The regulator also informs firms about the allocation and payment rules, which are determined by the reports and are the basis of the regulation in the second stage. In their Theorem 1, Ausubel and Cramton (2004) prove that for any value function satisfying continuity, value monotonicity and the single-crossing property, the Vickrey auction with reserve pricing has truthful bidding as an ex-post equilibrium for any monotonic aggregate quantity rule $\bar{Q}(\mathbf{s})$ and associated monotonic efficient assignment rule $q_i^e(\mathbf{s})$.

Firstly, following Ausubel and Cramton (2004), the monotonic efficient assignment rule $q_i^e(\mathbf{s})$ is defined by

$$v_i\left(q_i^e(\mathbf{s});\mathbf{s}\right)\begin{cases}\leq v_{-i}\left(q_{-i}^e(\mathbf{s});\mathbf{s}\right), & \text{if } q_i^e(\mathbf{s})=0\\ = v_{-i}\left(q_{-i}^e(\mathbf{s});\mathbf{s}\right), & \text{if } 0<q_i^e(\mathbf{s})<\bar{Q}(\mathbf{s})\\ \geq v_{-i}\left(q_{-i}^e(\mathbf{s});\mathbf{s}\right), & \text{if } q_i^e(\mathbf{s})=\bar{Q}(\mathbf{s}).\end{cases} \tag{2.34}$$

Secondly, the Vickrey auction is defined as a mechanism with the payment rule

$$R_i(\mathbf{s})=\int_0^{q_i^e(\mathbf{s})} v_i\left(x;\hat{s}_i\left(\mathbf{s}_{-i},x\right),\mathbf{s}_{-i}\right)dx. \tag{2.35}$$

In (2.35), signal $\hat{s}_i$ is the lowest possible signal for which firm i would have won the unit x given other bidders' signals $\mathbf{s}_{-i}$:

$$\hat{s}_i\left(\mathbf{s}_{-i},x\right)=\inf_{s_i}\left\{s_i \middle| q_i^e\left(s_i,\mathbf{s}_{-i}\right)\geq x\right\}. \tag{2.36}$$

Thus the marginal payment of unit x is the expected marginal value of firm i evaluated at x, if firm i would have received and reported the lowest possible signal $\hat{s}_i$ such that $x=q_i^e\left(\hat{s}_i,\mathbf{s}_{-i}\right)$.

Finally, reserve pricing is defined by a monotonic aggregate quantity rule $\bar{Q}(\mathbf{s})$ which is weakly increasing in each bidder's signal. Due to this and the single-crossing property, it is possible to distribute the total quantity efficiently and each firm's allocation is weakly increasing in its signal. Ausubel and Cramton (2004) also assume independent types, which is a requirement for their general revenue equivalence theorem. With the affine information structure, signals are not independent. However, this is not an issue, while revenue extraction is not a central question in this model. The primary objective of the regulator is to maximize the expected social welfare and not to extract the maximum amount of revenue.[13] The analysis of this paper is based on "ex-post" arguments which do not require any assumptions about the distribution of signals, as noted also by Ausubel and Cramton (2004). The aggregate quantity rule $\bar{Q}(\mathbf{s})$ is determined by

$$\bar{Q}(\mathbf{s}) = \begin{cases} y^{-1}\left(v_{-i}\left(q_{-i}^{e}(\mathbf{s});\mathbf{s}\right);\mathbf{s}\right), & \text{if } q_i^e(\mathbf{s}) = 0 \\ y^{-1}\left(v_i\left(q_i^e(\mathbf{s});\mathbf{s}\right);\mathbf{s}\right), & \text{if } q_i^e(\mathbf{s}) > 0. \end{cases} \tag{2.37}$$

In addition, Ausubel and Cramton (2004) show that even if an equilibrium in an auction without a resale is typically not an equilibrium in an auction followed by a resale market, a resale market does not distort the equilibrium of the Vickrey auction. In Theorem 2, they state that if the Vickrey auction with reserve pricing is followed by any resale process that is coalitionally-rational against individual bidders, truthful bidding remains the ex-post equilibrium. Hence, given that other bidders give truthful reports, the sum of i) the expected payoff in the Vickrey auction when misreporting and, ii) all the gains from trade in the resale market due to misreporting is lower than the payoff when reporting truthfully in the first place. In this section I examine whether these incentive compatibility conditions of the information mechanism are satisfied in the regulation model under consideration.

To give more intuition on the information mechanism I next describe the indirect interpretation of the VCG mechanism introduced in equations (2.34) - (2.37). Moreover, I apply the affine linear model and derive the equilibrium of the auction game. The following auction mechanism is similar to the indirect VCG mechanism of Montero (2008), who provides more detailed analysis in a pure private values environment.

In the auction mechanism, instead of signals, firms report bid functions to the regulator. The regulator collects all bid schedules, determines the clearing price at which the total demand equals supply and allocates units to firms that have submitted winning bids, i.e. bids above the clearing price. Let $D_i(p; s_i', s_i)$ be the bid function of firm i, when it bids according to signal s_i' when its true signal is s_i, and where p is the price. Suppose, for a moment, that every firm bids sincerely and I thus write $D_i(p; s_i, s_i) \equiv D_i(p; s_i)$. Later I

[13] When bidders are not symmetric, the revenue-optimizing seller may either misassign or withhold goods. According to the "optimal auctions" literature, the revenue-maximizing assignment rule is based on the virtual values and not on the marginal values of bidders and the rule may assign goods in hands that do not value them most. Besides, the seller may also increase the expected revenues by setting a reserve price and not assigning units at all if bids are below the reserve price. Ausubel and Cramton (2004) show that in the case of independent types and when the seller places no value on the objects on sale, the Vickrey auction with reserve pricing attains the upper bound for revenues in the resale-constrained auction program. Thus, when agents are able to trade units freely after the auction mechanism, the best the auctioneer can do with respect to efficiency and revenues is to conduct a Vickrey auction with a reserve price.

will relax this assumption and derive conditions under which it is profitable for a firm to bid sincerely when other firms are bidding sincerely in a Vickrey auction. This constitutes an ex-post efficient Bayes Nash equilibrium. To simplify the analysis, I assume perfectly divisible units and hence no rationing rules are needed. Total demand in the auction is $D(p;\mathbf{s}) = D_i(p;s_i) + D_{-i}(p;\mathbf{s}_{-i})$ where $D_{-i}(p;\mathbf{s}_{-i})$ is the demand of every other bidder but bidder i.

The price-elastic supply of pollution permits is simply

$$\begin{aligned} Q_S(p;\mathbf{s}) &= y^{-1}(p;\mathbf{s}) \\ &= \frac{1}{\delta}\left(p - \bar{\gamma} - nZ\left(s_m - \bar{\theta}\right)\right). \end{aligned} \tag{2.38}$$

Let $p_v(\mathbf{s})$ denote the clearing price in the Vickrey auction. Given that the Vickrey auction is ex-post efficient, it must hold from (2.23) that $p_v(\mathbf{s}) = \bar{p}(\mathbf{s})$. Then the aggregate quantity rule $\bar{Q}(\mathbf{s})$ is weakly increasing in each bidder's signal if

$$\frac{d\bar{Q}(\mathbf{s})}{ds_i} = \frac{1}{\delta}\left(\frac{\delta(1-A) + \beta Z}{\beta + n\delta} - Z\right) \geq 0 \Leftrightarrow var[\theta_m] \geq \sigma_{\gamma\theta}.$$

Note that this gives condition (2.17). Furthermore, the residual supply for bidder i is $RS_i(p;\mathbf{s}_{-i}) = Q_S(p;\mathbf{s}) - D_{-i}(p;\mathbf{s}_{-i})$ and the inverse demand function is written $P_i(q_i;s_i) \equiv D_i^{-1}(q_i;s_i)$. In the Vickrey auction, in addition to the clearing price and the allocation of permits, the regulator determines paybacks for each firm. Hence, the final payment that firms have to pay for the units received is not the clearing price. Instead, the share of the paybacks is defined by

$$\alpha_i = 1 - \frac{\int_0^{q_i} RS_i^{-1}(x;\mathbf{s}_{-i})\,dx}{RS_i^{-1}(q_i;\mathbf{s}_{-i})\,q_i}. \tag{2.39}$$

While $p = RS_i^{-1}(q_i;\mathbf{s}_{-i})$ in the equilibrium, the payment of bidder i in the auction writes as

$$\begin{aligned} R_{i,v} &= (1-\alpha_i)\,pq_i \\ &= \int_0^{q_i} RS_i^{-1}(x;\mathbf{s}_{-i})\,dx. \end{aligned} \tag{2.40}$$

Each firm faces a payment schedule where the marginal payment is given by the inverse residual supply function. Note that $R_{i,v}$ depends on signal s_i only through the end point q_i. Hence the payback mechanism makes bidders bid their expected marginal value functions, conditional on the aggregate information. That information is incorporated in the clearing price of the auction. The payback function is determined by the strategies of all other bidders but bidder i. With sincere bidding, (2.40) is equivalent to (2.35).

Given that bidders act sincerely and the expected marginal value function is linear in signals and in quantity q_i, firms utilize linear strategies defined by

$$D_i(p; s_i) = a + bs_i - cp, \tag{2.41}$$

where a, b and c are some positive constants. The total demand for pollution rights in the auction may then write

$$D(p; \mathbf{s}) = na + nbs_m - ncp. \tag{2.42}$$

Knowing the form of the bidding strategies of other agents, firm i observes the clearing price $p = p_v(\mathbf{s})$ after the auction but before the auction conditions its bidding strategy on ns_m. Furthermore, using (2.38) and (2.42) yields

$$ns_m = \left(\frac{1}{b + \frac{Z}{\delta}}\right)\left(\frac{1}{\delta}\left(p - \bar{\gamma} + Zn\bar{\theta}\right) - na + ncp\right). \tag{2.43}$$

With linear strategies and normal random variables the clearing price is sufficient statistics for ns_m and hence $E[\theta_i|\mathbf{s}]$ is informationally equivalent to $E[\theta_i|s_i, p]$ (Vives 2011). The conditional expectation of θ_i, derived in equation (2.5), can then plug into the first-order condition of the considered maximization problem. Note that the expected efficiency makes the profits of the second stage random such that the expected value is $E[\pi_{i,\tau}(q_i, h_i; \theta_i)] = 0$. Consider for a moment that this holds. The first-order condition when bidding sincerely and thus in the price-taking equilibrium of the auction is

$$E[\theta_i|s_i, p] - \beta q_i - p = 0. \tag{2.44}$$

Furthermore, plugging equation (2.5) and (2.43) into the first-order equation (2.44) gives

$$q_i = \frac{1}{\beta}\left\{A\bar{\theta} + Bs_i + C\left(\frac{1}{b + \frac{Z}{\delta}}\right)\left(\frac{1}{\delta}\left(p - \bar{\gamma} + Zn\bar{\theta}\right) - na + ncp\right) - p\right\}. \tag{2.45}$$

Equating this with the strategy $D_i(p; s_i) = a + bs_i - cp$ and solving the system, we get the linear Bayesian demand function equilibrium strategy, where

$$a = \frac{1}{\beta}\left(\frac{1}{B+nC+\frac{\beta}{\delta}Z}\right)\left\{\left(AB+\frac{\beta}{\delta}(A+nC)Z\right)\bar{\theta}-\frac{\beta}{\delta}C\bar{\gamma}\right\} \tag{2.46}$$

$$b = \frac{1}{\beta}B \tag{2.47}$$

$$c = \frac{1}{\beta}\left(\frac{B-\frac{\beta}{\delta}C+\frac{\beta}{\delta}Z}{B+nC+\frac{\beta}{\delta}Z}\right). \tag{2.48}$$

From (2.38) and (2.42), the equilibrium price is then given by

$$p_v(\mathbf{s}) = \frac{n\delta a+\bar{\gamma}+n\delta b\bar{\theta}+n(\delta b+Z)\left(s_m-\bar{\theta}\right)}{n\delta c+1}. \tag{2.49}$$

I show in Appendix 2.C that plugging (2.46) - (2.48) into (2.49) yields $p_v(\mathbf{s}) = \bar{p}(\mathbf{s})$ and $D_i(p_v(\mathbf{s});s_i) = \bar{q}_i(\mathbf{s}) = q_i^e(\mathbf{s})$, where $\bar{p}(\mathbf{s})$ and $q_i^e(\mathbf{s})$ are given by (2.23) and (2.25).

The equilibrium of the Vickrey auction is described in Figure 2.1. It is updated from Figure 2 in Montero (2008). The curves on the left side of Figure 2.1 describe the maximization problem of firm i, whereas the curves on the right side of the figure the market as a whole. The curve $\hat{y}(Q;s_i',\mathbf{s}_{-i})$ plots the equilibrium values of the marginal damage function $y\left(\bar{Q}(s_i',\mathbf{s}_{-i});s_i',\mathbf{s}_{-i}\right)$ for different signal values s_i', where s_i' is the report of firm i's signal, i.e. the signal on which its bid function is based, when its true signal is s_i. I have assumed that $\sigma_{\gamma\theta}>0$ and thus $\hat{y}(Q;s_i',\mathbf{s}_{-i})$ has a greater slope than $y(Q;\mathbf{s})$.

The Vickrey auction without a resale process is incentive compatible if $RS_i^{-1}(q_i;\mathbf{s}_{-i}) < v_i(q_i;\mathbf{s})$ when $q_i < \bar{q}_i(\mathbf{s})$ and $RS_i^{-1}(q_i;\mathbf{s}_{-i}) > v_i(q_i;\mathbf{s})$ when $q_i > \bar{q}_i(\mathbf{s})$. This requires that the slope of the inverse residual supply function denoted by τ_v is greater than the slope of the expected marginal value function, i.e. $\tau_v > -\beta$, which holds when $var[\theta_m] > \sigma_{\gamma\theta}$ (see equations (2.53) and (2.54) below). The total payment $R_{i,v}(\mathbf{s})$ is defined by the area under the $RS_i^{-1}(q_i;\mathbf{s}_{-i})$ curve. Respectively, firm i's profit in the Vickrey auction given the signal vector $\mathbf{s}$, denoted by $\pi_{i,v}(\mathbf{s})$, is the area between the $v_i(q_i;\mathbf{s})$ and $RS_i^{-1}(q_i;\mathbf{s}_{-i})$ curves from zero to the allocated quantity $\bar{q}_i(\mathbf{s})$. The report s_i' affects the bid function $P_i(q_i;s_i)$ and the quantity allocated to firm i, but not the $v_i(q_i;\mathbf{s})$ or $RS_i^{-1}(q_i;\mathbf{s}_{-i})$ functions. If firm i submitted a bid function according to the signal $s_i' < s_i$ with $P_i(q_i;s_i')$ thus lying below the sincere bid function $P_i(q_i;s_i)$, the clearing price of the auction would be lower than $p_v(\mathbf{s})$ and firm i would receive less quantity $\bar{q}_i(\mathbf{s}') < \bar{q}_i(\mathbf{s})$. Hence, firm i would lose some of its expected profits, while $v_i(q_i;\mathbf{s}) \geq RS_i^{-1}(q_i;\mathbf{s}_{-i})$ when $q_i \in [\bar{q}_i(\mathbf{s}'),\bar{q}_i(\mathbf{s})]$. A similar argument applies when $s_i' > s_i$. Then $\bar{q}_i(\mathbf{s}') > \bar{q}_i(\mathbf{s})$ but $v_i(q_i;\mathbf{s}) \leq RS_i^{-1}(q_i;\mathbf{s}_{-i})$ when $q_i \in [\bar{q}_i(\mathbf{s}),\bar{q}_i(\mathbf{s}')]$.

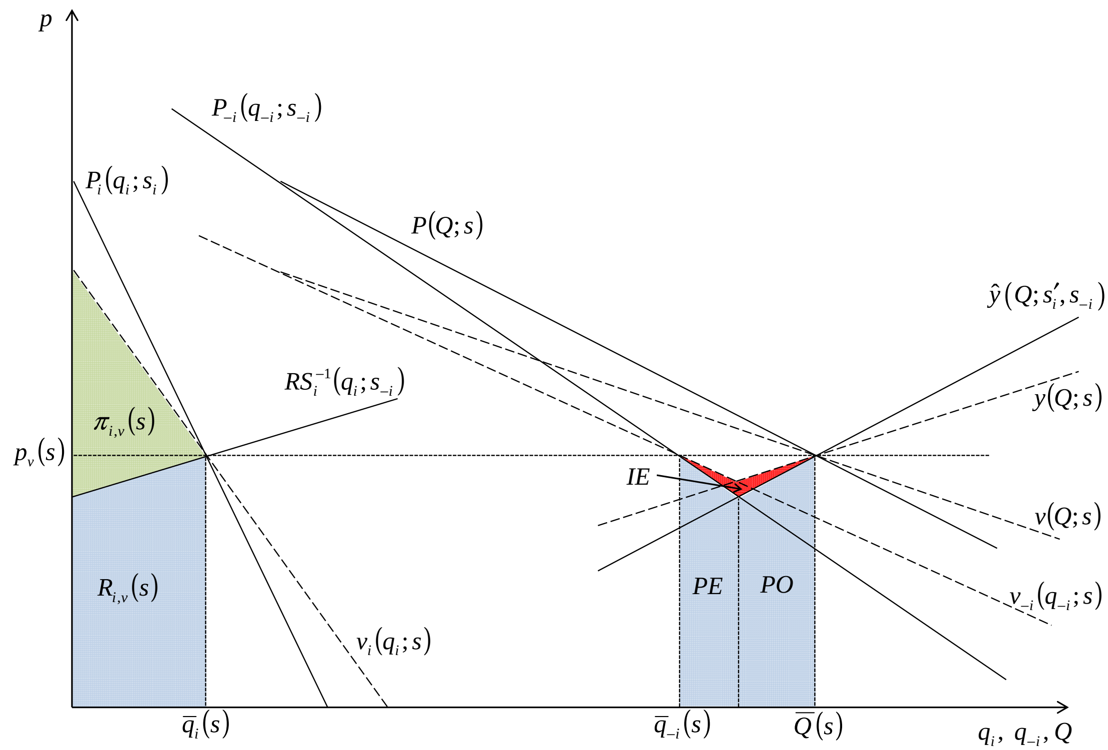

Figure 2.1: Equilibrium of the Vickrey auction.

Thus it is optimal to act sincerely in an auction without a resale market if every other firm bid sincerely.

With pure private values, the marginal payment at each quantity in the Vickrey auction is equal to the opportunity cost of that particular unit. When firm i participates in the auction, it increases the total amount of pollution permits and decreases the amount of pollution rights assigned to other firms (at least when $-\frac{n\delta}{\beta} \leq \frac{\sigma_{\gamma\theta}}{var[\theta_m]} \leq 1$). With pure private values it is a dominant strategy to bid truthfully in the Vickrey auction and hence $P_i(q_i;s_i) = v_i(q_i;s_i) = u_i(q_i;\theta_i)$. The total payment $R_{i,v}$ is then the sum of the pecuniary externality to other firms (the area PE) and the pollution externality of increased pollution (PO). The pecuniary externality is defined as the value of those units to other bidders, and which are not assigned to them due to firm i's participation. However, with interdependent values the payment is not the full externality cost, in contrast to the pure private values case with a similar payment rule (see Montero 2008). The payment does not include the informational externality (IE) of signal s_i to other bidders' values and to the damage of pollution.

2.4.2 Incentive compatibility

In this section I examine whether sincere bidding in the Vickrey auction is incentive compatible if it is followed by one of the constant regulations. Given that every other firm is bidding sincerely in the Vickrey auction, it is in firm i's interest to bid truthfully if the expected loss in the Vickrey auction when deviating from a sincere bidding strategy is greater than the expected benefit in the regulation stage from a deviation strategy.

Recall that the auction payment $R_{i,v}(\mathbf{s}) = \int_0^{\bar{q}_i(\mathbf{s})} RS_i^{-1}(x; \mathbf{s}_{-i})\, dx$ depends only on signal s_i through its end point $\bar{q}_i(\mathbf{s})$. Let $\tilde{s}_{-i} = \frac{1}{n-1}\sum_{j \neq i} s_j$ denote the average signal of every other firm but firm i, and suppose that other firms bid sincerely in the auction. Consider for a moment that firm i receives a signal $\hat{s}_i$. Then it is easy to derive the clearing price p_v as a function of $\hat{\mathbf{s}} = (\hat{s}_i, \mathbf{s}_{-i})$ from (2.49):

$$p_v(\hat{\mathbf{s}}) = \frac{n\delta a + \bar{\gamma} - nZ\bar{\theta} + (n-1)(\delta b + Z)\tilde{s}_{-i}}{n\delta c + 1} + \frac{\delta b + Z}{n\delta c + 1}\hat{s}_i, \tag{2.50}$$

where $\frac{\delta b + Z}{n\delta c + 1} = \frac{\delta(1-A) + \beta Z}{\beta + n\delta}$. Using this and the equilibrium condition

$$\bar{q}_i(\hat{s}_i, \mathbf{s}_{-i}) = Q_S(p_v(\hat{\mathbf{s}}); \hat{s}_i, \mathbf{s}_{-i}) - D_{-i}(p_v(\hat{\mathbf{s}}); \mathbf{s}_{-i}),$$

it is easy to see that given sincere bidding:

$$\begin{aligned} \hat{s}_i(\mathbf{s}_{-i}, q_i) &= \left(\frac{nc\delta + 1}{(nc\delta + 1)b - c(b\delta + Z)}\right) \\ &\quad \times \left\{\frac{-a + c\bar{\gamma} - ncZ\bar{\theta} + c(n-1)(b\delta + Z)\tilde{s}_{-i}}{nc\delta + 1} + q_i\right\}. \end{aligned} \tag{2.51}$$

Plugging (2.51) into (2.50), the inverse residual supply may write $RS_i^{-1}(q_i; \mathbf{s}_{-i}) = \Omega_i(\mathbf{s}_{-i}) + \tau_v q_i$, which is independent of s_i. However, again using the fact that the inverse residual supply function goes through the equilibrium point $(p_v(\mathbf{s}), q_i^e(\mathbf{s}))$, yields

$$RS_i^{-1}(q_i; \mathbf{s}_{-i}) = p_v(\mathbf{s}) + \tau_v(q_i - q_i^e(\mathbf{s})), \tag{2.52}$$

where the slope is given by

$$\begin{aligned} \tau_v &= \frac{\delta b + Z}{(nc\delta + 1)b - c(\delta b + Z)} \\ &= \frac{\beta\sigma_{\gamma\theta} + n\delta \cdot var[\theta_m]}{\left(1 + \frac{n\delta}{\beta}\right)(n-1)B \cdot var[s_m] + var[\theta_m] - \sigma_{\gamma\theta}}. \end{aligned} \tag{2.53}$$

Firstly, note that in the absence of the second stage, sincere bidding in the Vickrey auction would be incentive compatible if $RS_i^{-1}(q_i;\mathbf{s}_{-i}) < v_i(q_i;\mathbf{s})$ when $q_i < \bar{q}_i(\mathbf{s})$ and $RS_i^{-1}(q_i;\mathbf{s}_{-i}) > v_i(q_i;\mathbf{s})$ when $q_i > \bar{q}_i(\mathbf{s})$. This holds while at the auction equilibrium $RS_i^{-1}(\bar{q}_i(\mathbf{s});\mathbf{s}_{-i}) = v_i(\bar{q}_i(\mathbf{s});\mathbf{s})$ and from (2.53) we get $\tau_v \geq -\beta$, whenever

$$\left(1+\frac{n\delta}{\beta}\right)(n-1)\,B\cdot var\,[s_m] + var\,[\theta_m] - \sigma_{\gamma\theta} \geq 0. \tag{2.54}$$

This, on the other hand, is fulfilled whenever the aggregate quantity rule $\bar{Q}(\mathbf{s})$ is weakly increasing in each bidder's signal and thus if $var\,[\theta_m] \geq \sigma_{\gamma\theta}$, which gives (2.17). Note, however, that (2.17) is too restrictive and $\bar{Q}(\mathbf{s})$ needs not to be increasing in s_i in order for the Vickrey auction without a resale market to be incentive compatible. For the Vickrey auction without a second stage to be incentive compatible only requires that the equilibrium allocation $\bar{q}_i(\mathbf{s})$ is increasing in s_i, which is guaranteed by (2.54). I show in Corollary 2.1 that (2.17) may be relaxed even with the second-stage constant quantity regulation, unless $n=1$.

Furthermore, suppose next that firm i bids according to signal s_i' when its true signal is s_i, and I thus denote $\mathbf{s}' = (s_i', \mathbf{s}_{-i})$. Hence, firm i uses a deviation strategy $D_i(p;s_i',s_i) = a + bs_i' - cp$.[14] Given fixed $\mathbf{s}_{-i}$, the initial allocation of permits to firm i reduces to

$$\begin{aligned}
\bar{q}_i(\mathbf{s}') &= a + bs_i' - cp_v(\mathbf{s}') \\
&= q_i^e(\mathbf{s}) - \frac{1}{\beta}\left((n-1)\,B + \frac{1-A-nZ}{1+\frac{n\delta}{\beta}}\right)\frac{1}{n}(s_i - s_i') \\
&= q_i^e(\mathbf{s}) - \left(\frac{\left(1+\frac{n\delta}{\beta}\right)(n-1)\,B\cdot var\,[s_m] + var\,[\theta_m] - \sigma_{\gamma\theta}}{(\beta+n\delta)\,var\,[s_m]}\right)\frac{1}{n}(s_i - s_i').
\end{aligned}$$

From this it is easy to see that $\frac{d\bar{q}_i(\mathbf{s}')}{ds_i'} > 0$ if (2.54) holds. The profit in the auction with the deviation strategy writes $\pi_{i,v}(s_i';s_i,\mathbf{s}_{-i})$ and the loss in the Vickrey auction is thus

$$\begin{aligned}
L_{i,v}(s_i';s_i,\mathbf{s}_{-i}) &= \pi_{i,v}(s_i;s_i,\mathbf{s}_{-i}) - \pi_{i,v}(s_i';s_i,\mathbf{s}_{-i}) \\
&= \int_{\bar{q}_i(\mathbf{s}')}^{q_i^e(\mathbf{s})} \left\{v_i(x;\mathbf{s}) - RS_i^{-1}(x;\mathbf{s}_{-i})\right\} dx.
\end{aligned} \tag{2.55}$$

Respectively, the expected profit in the secondary market due to the deviation strategy writes

[14] Alternatively, firm i may use any bid function that goes through the point $(p_v(\mathbf{s}'), \bar{q}_i(\mathbf{s}'))$.

as

$$\pi_{i,r}\left(s_i';s_i,\mathbf{s}_{-i}\right)=\begin{cases}\int_{\bar{q}_i(\mathbf{s}')}^{q_{i,p}(\mathbf{s}')}\left\{v_i\left(x;\mathbf{s}\right)-\bar{p}\left(\mathbf{s}'\right)\right\}dx, & \text{if } r=p\\ \int_{\bar{q}_i(\mathbf{s}')}^{q_{i,q}(\mathbf{s}')}\left\{v_i\left(x;\mathbf{s}\right)-p_q\left(\mathbf{s}'\right)\right\}dx, & \text{if } r=q\end{cases}, \tag{2.56}$$

where $\bar{p}(\mathbf{s}')$ is the tax/subsidy under the price regulation, $p_q(\mathbf{s}')$ the expected equilibrium price of the secondary market under the quantity regulation, and $q_{i,p}(\mathbf{s}')$ and $q_{i,q}(\mathbf{s}')$ are the corresponding expected equilibrium quantities given the initial allocation according to $\mathbf{s}'$. Due to the ex-post efficiency of the Vickrey auction, the expected profits of firm i in the second stage when bidding sincerely in the auction are zero, $\pi_{i,r}(s_i;s_i,\mathbf{s}_{-i})=0$. Hence the auction mechanism of the first stage is incentive compatible if

$$\begin{aligned}\Delta_{IC} &= \pi_{i,v}\left(s_i;s_i,\mathbf{s}_{-i}\right)+\pi_{i,r}\left(s_i;s_i,\mathbf{s}_{-i}\right)-\pi_{i,v}\left(s_i';s_i,\mathbf{s}_{-i}\right)-\pi_{i,r}\left(s_i';s_i,\mathbf{s}_{-i}\right) \\ &= L_{i,v}\left(s_i';s_i,\mathbf{s}_{-i}\right)-\pi_{i,r}\left(s_i';s_i,\mathbf{s}_{-i}\right) \\ &\geq 0.\end{aligned} \tag{2.57}$$

The main results of this paper are provided in the following propositions. Proofs can be found in Appendix 2.D.

Proposition 2.1. *Given the affine linear model, the information mechanism of the two-stage regulation with the constant quantity regulation in the second stage is incentive compatible whenever* $\frac{\sigma_{\gamma\theta}}{var[\theta_m]}\leq 1$, *if the resale market is coalitionally-rational against individual bidders.*

Proof. See Appendix 2.D.

Corollary 2.1. *Given the affine linear model, the information mechanism without the second stage, and thus without any resale market, is incentive compatible if*

$$\frac{\sigma_{\gamma\theta}}{var\left[\theta_m\right]}\leq 1+\left(1+\frac{n\delta}{\beta}\right)(n-1)\left(\frac{B}{B+nC}\right).$$

Moreover, the information mechanism of the two-stage regulation with the constant quantity regulation in the second stage and when the resale market is coalitionally-rational against individual bidders is incentive compatible if

$$\frac{\sigma_{\gamma\theta}}{var\left[\theta_m\right]}\leq 1+\left(1+\frac{n\delta}{\beta}\right)(n-1)\left(\frac{B}{B+nC}\right)\left(\frac{C}{B+C}\right).$$

Proof. See Appendix 2.D.

Corollary 2.2. *Given the affine linear model, the information mechanism of the two-stage regulation with the constant quantity regulation in the second stage is incentive compatible* *if firms pay (receive) the equilibrium price for all permits they buy (sell) in the second-stage resale market, and if* $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} \leq 1$.

Proof. See Appendix 2.D.

Proposition 2.2. *Given the affine linear model, the information mechanism of the two-stage regulation with the constant price regulation in the second stage is incentive compatible only if* $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} < -\frac{n\delta}{\beta}$.

Proof. See Appendix 2.D.

2.5 Prices vs. quantities revisited

In this section I combine the results from Sections 2.3 and 2.4. In particular, I compare four different regulatory instruments: constant price and constant quantity regulations with and without the Vickrey auction. For simplicity, I denote by the one-stage price regulation (abbreviated $P1$) the regulation where a uniform tax is set using only the prior information and thus without any information mechanism. The two-stage price regulation ($P2$) denotes a regulation with a Vickrey auction in the first stage and a constant price regulation in the second stage. One-stage and two-stage quantity regulations ($Q1$ and $Q2$) are defined respectively. Hence in the following I discuss which of these four regulations should be used in different settings. The results are described in Table 2.1 and Figure 2.2.

Table 2.1: Prices vs. quantities with and without a Vickrey auction

Information structure	**Regulation**	**Details**
$\frac{\sigma_{\gamma\theta}}{var[\theta_m]} < -\frac{n\delta}{\beta}$	$P2$	Equation (2.33), Proposition 2.2
$-\frac{n\delta}{\beta} < \frac{\sigma_{\gamma\theta}}{var[\theta_m]} < \frac{1}{2}\left(1 - \frac{n\delta}{\beta}\right)$	$P1$ / $Q2$	Equation (2.58), Proposition 2.1, Proposition 2.2
$\frac{1}{2}\left(1 - \frac{n\delta}{\beta}\right) < \frac{\sigma_{\gamma\theta}}{var[\theta_m]} < 1$	$Q2$	Equation(2.33), Proposition 2.1
$1 < \frac{\sigma_{\gamma\theta}}{var[\theta_m]} < 1 + \left(1 + \frac{n\delta}{\beta}\right)(n-1)\left(\frac{B}{B+nC}\right)\left(\frac{C}{B+C}\right)$	$Q2$ / $Q1$	Equation (2.33), Corollary 2.1, Lemma 2.4
$\frac{\sigma_{\gamma\theta}}{var[\theta_m]} > 1 + \left(1 + \frac{n\delta}{\beta}\right)(n-1)\left(\frac{B}{B+nC}\right)\left(\frac{C}{B+C}\right)$	$Q1$	Equation (2.33), Corollary 2.1

In Figure 2.2, $Q2$ marks the area where the inverse residual supply function of the Vickrey auction for firm i should lie when the two-stage quantity regulation maximizes the expected

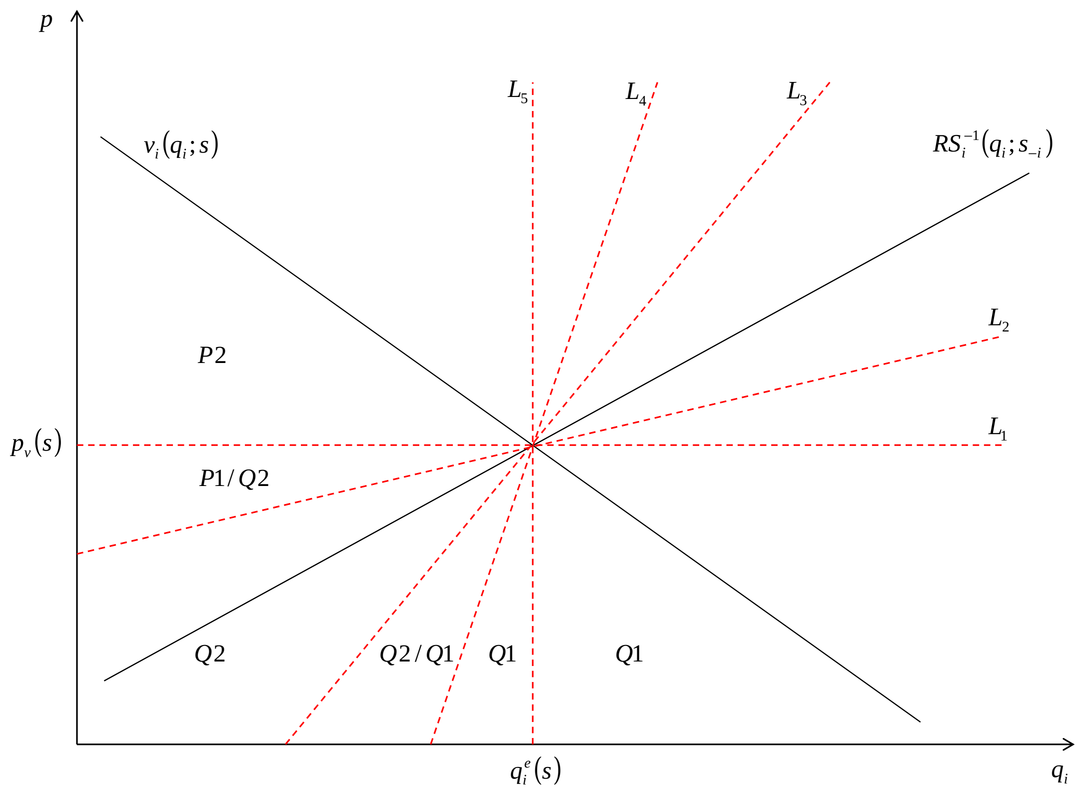

Figure 2.2: Inverse residual supply function of the Vickrey auction.

welfare from the set of four regulatory alternatives. Moreover, Figure 2.2 shows the following threshold curves:

$$\begin{aligned}
L_1 &\equiv \left(RS_i^{-1}(q_i;\mathbf{s}_{-i}) \middle| \frac{\sigma_{\gamma\theta}}{var\left[\theta_m\right]} = -\frac{n\delta}{\beta} \right), \\
L_2 &\equiv \left(RS_i^{-1}(q_i;\mathbf{s}_{-i}) \middle| \frac{\sigma_{\gamma\theta}}{var\left[\theta_m\right]} = \frac{1}{2}\left(1-\frac{n\delta}{\beta}\right) \right), \\
L_3 &\equiv \left(RS_i^{-1}(q_i;\mathbf{s}_{-i}) \middle| \frac{\sigma_{\gamma\theta}}{var\left[\theta_m\right]} = 1 \right), \\
L_4 &\equiv \left(RS_i^{-1}(q_i;\mathbf{s}_{-i}) \middle| \frac{\sigma_{\gamma\theta}}{var\left[\theta_m\right]} = 1+\left(1+\frac{n\delta}{\beta}\right)(n-1)\left(\frac{B}{B+nC}\right)\left(\frac{C}{B+C}\right) \right), \\
L_5 &\equiv \left(RS_i^{-1}(q_i;\mathbf{s}_{-i}) \middle| \frac{\sigma_{\gamma\theta}}{var\left[\theta_m\right]} = 1+\left(1+\frac{n\delta}{\beta}\right)(n-1)\left(\frac{B}{B+nC}\right) \right).
\end{aligned}$$

First, according to the Weitzman rule (2.33) and Proposition 2.1, when $\frac{1}{2}\left(1-\frac{n\delta}{\beta}\right) \leq \frac{\sigma_{\gamma\theta}}{var[\theta_m]} < 1$ the regulator should use the two-stage quantity regulation ($Q2$). This is the case in Figure 2.2 where $RS_i^{-1}(q_i;\mathbf{s}_{-i})$ is between curves L_2 and L_3.

Second, according to the Weitzman rule (2.33) and Proposition 2.2, the regulator should use the two-stage price regulation ($P2$), whenever $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} < -\frac{n\delta}{\beta}$. In Figure 2.2, $RS_i^{-1}(q_i; \mathbf{s}_{-i})$ would then lie below the $v_i(q_i; \mathbf{s})$ and above L_1 curves for $q_i \leq q_i^e(\mathbf{s})$. Note also that when $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} = -\frac{n\delta}{\beta}$ the private information of firms is of no value to the regulator (Lemma 2.5).

Third, when $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} \geq 1$, according to the Weitzman rule (2.33), the regulator should use the constant quantity regulation. Then $RS_i^{-1}(q_i; \mathbf{s}_{-i})$ would have a steeper slope than curve L_3 in Figure 2.2. Even if $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} \geq 1$, there are some parameter values which yield $\Delta_{q,IC} > 0$ and the Vickrey auction followed by the constant quantity regulation would be incentive compatible. These are examined in Corollary 2.1. In Figure 2.2, $RS_i^{-1}(q_i; \mathbf{s}_{-i})$ would then lie between curves L_3 and L_4. However, then $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} \approx 1$ and, according to Lemma 2.4, the value of firms' private information to the regulator is relatively low. Furthermore, when $RS_i^{-1}(q_i; \mathbf{s}_{-i})$ is between curves L_4 and L_5, the Vickrey auction is incentive compatible without the second stage but it is not incentive compatible when followed by the second-stage regulation. Hence, to conclude, and using a simple rule, whenever $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} \geq 1$, the regulator should use the one-stage quantity regulation ($Q1$).

Fourth, what should the choice be when $-\frac{n\delta}{\beta} \leq \frac{\sigma_{\gamma\theta}}{var[\theta_m]} < \frac{1}{2}\left(1 - \frac{n\delta}{\beta}\right)$ and when $RS_i^{-1}(q_i; \mathbf{s}_{-i})$ is between L_1 and L_2. In those cases the Weitzman rule says that the constant price regulation performs better. On the other hand, the regulator is not able to get firms to reveal their private information in the first stage if the price regulation is used in the regulation period. Instead, using the quantity regulation in the second stage provides ex-post efficient allocation of permits after the information stage. Thus, the choice between the two-stage quantity regulation and the one-stage price regulation depends on the modified Weitzman rule:

$$\begin{aligned} \Delta_{pq}^{Mod} &= E\left[DWL_q(\mathbf{s}) - DWL_p(0)\right] \\ &= \frac{1}{2}\left(\frac{1}{\frac{\beta}{n} + \delta}\right) var\left[\theta_m\right] \left\{A - \left(\frac{n\delta}{\beta}\right)^2 - \left[2A + nZ + 2\left(\frac{n\delta}{\beta}\right)\right] \frac{\sigma_{\gamma\theta}}{var\left[\theta_m\right]}\right\}. \end{aligned} \tag{2.58}$$

For example, consider the most likely case where the abatement costs and pollution damage are not correlated ($\sigma_{\gamma\theta} = 0$). Then the one-stage price regulation is better if $\Delta_{pq}^{Mod} > 0$ and thus if

$$\frac{n\delta}{\beta} < \sqrt{\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + (1 + (n-1)\rho)\sigma_\theta^2}}.$$

Hence, the less noisy the signals are (low σ_ε^2), the more correlated abatement costs are between firms (high ρ) and the more heterogeneous firms are (high σ_θ^2), the more is gained from the information stage and the steeper the slope of the aggregate marginal abatement cost function should be relative to the slope of the marginal damage function, in order for the regulator to consider the one-stage constant price regulation. If $\sigma_\varepsilon^2 = 0$, firms have private values and the Vickrey auction provides the second-best solution. However, if the signals are extremely noisy ($\sigma_\varepsilon^2 \to \infty$), firms do not have any better information about their abatement costs than the regulator has and the model reduces to the original prices vs. quantities comparison, where the information mechanism provides no additional value to the regulator.

Finally, the role of correlation between the costs and benefits of emission reductions does not change from (2.33) when $-\frac{n\delta}{\beta} \leq \frac{\sigma_{\gamma\theta}}{var[\theta_m]}$. The positive correlation ($\sigma_{\gamma\theta} > 0$) increases the comparative advantage of the two-stage quantity regulation. The last term of (2.58), i.e. $\left[2A + nZ + 2\left(\frac{n\delta}{\beta}\right)\right] \frac{\sigma_{\gamma\theta}}{var[\theta_m]}$, is positive when $\sigma_{\gamma\theta} > 0$, and negative when $-\frac{n\delta}{\beta} var\left[\theta_m\right] \leq \sigma_{\gamma\theta} < 0$.[15]

2.6 Collusion

Montero (2008) shows that, with pure private values, the auction mechanism implements the first-best even if firms collude and thus coordinate their bids in the Vickrey auction. With pure private values, the VCG mechanism implements the first-best in dominant strategies. With interdependent values, a dominant strategy implementation is not possible. The equilibrium concept is Bayes Nash equilibrium. However, ex-post efficiency is attained even if firms collude. This is a characteristic which is not supported by the scheme of Kwerel (1977), for example.

Suppose that all firms meet before the auction and decide to coordinate their bidding schedules in the auction and their actions in the regulation stage. In addition, they agree on the procedure for sharing the cartel profits after the auction. Hence the cartel faces both an external coordination problem of submitting bids in the Vickrey auction but also an internal mechanism design problem. Montero (2008) explains how the incentive compatible cartel mechanism induces firms to reveal their individual demand curves truthfully to the cartel organization. Moreover, Montero shows that the optimal collusive agreement for a cartel of $m \leq n$ firms is to submit only one serious bid in the Vickrey auction with the true aggregate demand curve of the cartel. This bid is submitted by one cartel member while all the other members submit empty demand schedules.

I first consider the external coordination problem with interdependent values and assume that the cartel members can agree on the efficient cartel mechanism after the auction. Hence, if there is only one cartel with all firms as members, with the affine linear model this means

[15] This is clear while $\left[2A + nZ + 2\left(\frac{n\delta}{\beta}\right)\right] \frac{\sigma_{\gamma\theta}}{var[\theta_m]}\Big|_{\sigma_{\gamma\theta} = -\frac{n\delta}{\beta} var[\theta_m]} = -\left[2A + \frac{n\delta}{\beta}\left(1 + A\right)\right] \frac{n\delta}{\beta} < 0$.

that the serious bid is based on the average signal. Next I show, using the equations of the direct VCG mechanism (2.34) - (2.37), why a bid function based on the sincere report $s'_m = s_m$ is optimal for the cartel.

After all firms have shared their private information sincerely inside the cartel, the aggregate expected marginal value function, and thus the expected marginal value function of the cartel writes as

$$v_c(Q; s_m) = \bar{\theta} + (1-A)\left(s_m - \bar{\theta}\right) - \frac{\beta}{n}Q. \tag{2.59}$$

Consider first the information mechanism without any second-stage regulation. Suppose that the bid function the cartel submits in the Vickrey auction is based on signal s'_m when the true average signal is s_m. Hence, without any other firm, the inverse bid function of the cartel is

$$P_c(Q; s'_m, s_m) = \bar{\theta} + (1-A)\left(s'_m - \bar{\theta}\right) - \frac{\beta}{n}Q. \tag{2.60}$$

The regulator assumes that this is the sincere bid. Hence from (2.37), the aggregate quantity rule writes as

$$\begin{aligned} \bar{Q}(s'_m) &= y^{-1}\left(P_c(Q; s'_m, s_m); s'_m\right) \\ &= \frac{\bar{\theta} - \bar{\gamma} + (1 - A - nZ)\left(s'_m - \bar{\theta}\right)}{\delta + \frac{\beta}{n}}. \end{aligned} \tag{2.61}$$

Note that taking an inverse of the aggregate quantity rule yields the lowest possible signal $\hat{s}_m$, which gives a quantity Q to the cartel:

$$\hat{s}_m(Q) = \bar{\theta} + \left(\frac{\bar{\gamma} - \bar{\theta}}{1 - A - nZ}\right) + \left(\frac{\delta + \frac{\beta}{n}}{1 - A - nZ}\right)Q. \tag{2.62}$$

From (2.35) we may derive the Vickrey payment of the cartel with a report s'_m:

$$\begin{aligned} R_{c,v}(s'_m) &= \int_0^{\bar{Q}(s'_m)} v_c\left(X; \hat{s}_m(X)\right) dX \\ &= \int_0^{\bar{Q}(s'_m)} \left\{\bar{\theta} + (1-A)\left(\hat{s}_m(X) - \bar{\theta}\right) - \frac{\beta}{n}X\right\} dX. \end{aligned} \tag{2.63}$$

Hence the cartel payoff in the Vickrey auction with a report s'_m, when the true average signal is s_m, writes as

$$\pi_{c,v}\left(s'_m; s_m\right) = \int_0^{\bar{Q}(s'_m)} v_c\left(X; s_m\right) - v_c\left(X; \hat{s}_m\left(X\right)\right) dX = (1-A)\int_0^{\bar{Q}(s'_m)} \left(s_m - \hat{s}_m\left(X\right)\right) dX. \tag{2.64}$$

By definition $s_m = \hat{s}_m\left(\bar{Q}\left(s_m\right)\right)$. If $1-A-nZ>0$ and thus $\frac{\sigma_{\gamma\theta}}{var[\theta_m]}<1$, the aggregate quantity is increasing in report s'_m. Then $s_m > \hat{s}_m\left(\bar{Q}\left(s'_m\right)\right)$ for all $s'_m < s_m$ and $s_m < \hat{s}_m\left(\bar{Q}\left(s'_m\right)\right)$ for all $s'_m > s_m$. It is then clear that the sincere report $s'_m = s_m$ maximizes the cartel profits $\pi_{c,v}\left(s'_m; s_m\right)$.

From the regulator's perspective there is only one firm, and thus there is no need for the second-stage permit market if the quantity regulation is chosen. If the cartel can agree on the efficient allocation, the Vickrey auction provides ex-post efficient allocation, even if firms collude.

Instead, with a constant price regulation, the regulator implements a tax/subsidy rate $\bar{p}\left(s'_m\right) = P_c\left(\bar{Q}\left(s'_m\right); s'_m, s_m\right)$ for the regulation period. Note that the inverse (residual) supply for the cartel may write

$$RS_c^{-1}\left(Q\right) = y\left(Q; \hat{s}_m\left(Q\right)\right) = v_c\left(Q; \hat{s}_m\left(Q\right)\right) = \frac{(1-A)\bar{\gamma} - nZ\bar{\theta}}{1-A-nZ} + \underbrace{\left(\frac{\beta nZ + n\delta\left(1-A\right)}{1-A-nZ}\right)}_{\equiv\tau^{RS}}\frac{1}{n}Q, \tag{2.65}$$

where τ^{RS} is the optimal slope of the Roberts and Spence (1976) non-constant quantity regulation with a perfect permit market (see Chapter 3). Thus, if $-\frac{n\delta}{\beta} > \frac{\sigma_{\gamma\theta}}{var[\theta_m]} > 1$, the inverse supply schedule is an increasing function and deviating from sincere bidding is profitable. Giving a report $s'_m < s_m$ decreases the initial allocation of permits but also the second-stage tax. After the auction the cartel can buy permits from the regulator at a lower rate than in the Vickrey auction. Respectively, with a report $s'_m > s_m$ the cartel receives too many permits in the auction compared to the efficient allocation, but it can sell permits back to the regulator at a rate exceeding the Vickrey price. Hence a deviation strategy $P_c\left(Q; s'_m, s_m\right)$ is profitable to the cartel. In the case of a second-stage price regulation, sincere bidding is incentive compatible for the cartel only if $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} \leq -\frac{n\delta}{\beta}$, and thus when $\tau^{RS} \leq 0$ (Proposition 2.2).

Finally, there must be a optimal cartel mechanism if a quantity regulation is chosen for the second stage and if $\tau_v > 0$ and thus if $-\frac{n\delta}{\beta} > \frac{\sigma_{\gamma\theta}}{var[\theta_m]} > 1$. The intuition is clear. Then the average price of permits in the Vickrey auction is decreasing in bidder size. If all firms join one big cartel, i.e. a grand coalition, the total Vickrey payoff of the cartel is greater than the cumulative payoffs of individual firms when participating in the auction individually. The difference between these two gives the cartel profits. The cartel mechanism must be designed such that each member receives (in expectation) an equal number of permits to what it would have obtained individually after a Vickrey auction without a cartel agreement. In addition, cartel members can agree on how to share the cartel profits such that each member's cartel payoff in the auction is the sum of its share of the cartel profits and the individual auction profits that it would have received if all firms had participated in the auction individually. This makes each member better off when joining a cartel and reporting sincerely inside the cartel (see Montero 2008). However, when $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} < -\frac{n\delta}{\beta}$ the inverse residual supply functions are decreasing ($\tau_v < 0$) and the average price of permits in the Vickrey auction is increasing in bidder size. Thus collusive actions are then not profitable to firms.

To conclude, when $-\frac{n\delta}{\beta} > \frac{\sigma_{\gamma\theta}}{var[\theta_m]} > 1$, firms are able to agree on the cartel mechanism, given that the second-stage regulation is based on quantities. Even if firms coordinate their bids in the Vickrey auction, the aggregate allocation is at the ex-post optimal level. Furthermore, if the cartel mechanism is efficient, the outcome after the Vickrey auction is also ex-post efficient. On the other hand, when $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} < -\frac{n\delta}{\beta}$, firms are better off when they participate in the Vickrey auction individually. Thus possible collusive actions do not distort the ex-post efficient properties of the Vickrey auction.

2.7 Conclusions

This paper considers a two-stage regulation for regulating pollution. The analysis extends the Weitzman (1974) prices vs. quantities model to an environment where firms are better informed about their abatement costs than the regulator. Thus, the private information of firms is valuable to the regulator when designing environmental policy. However, firms themselves do not know the abatement costs exactly. Also, abatement costs are correlated between firms, which makes the model one of interdependent values. In the first stage the regulator conducts an auction mechanism to allocate emission permits to regulated firms. Applying the Vickrey pricing rule, the initial allocation is ex-post efficient, conditional on sincere bidding being incentive compatibile. During the actual regulation stage two constant regulations are considered.

With a constant price regulation a uniform price is set at the level of the expected first-best price. During the regulation stage, firms are able to update their emissions permit assets by trading permits with the regulator at a predefined price. At the end of the regulation period, firms' permit accounts must contain a number of permits equal to their emissions in the regulation period. Hence the marginal cost of abatement is fixed after the information stage, but emissions vary. With a constant quantity regulation, the aggregate supply of permits is fixed, but firms are able to trade permits with each other after the initial allocation. However, the equilibrium price is uncertain when implementing the regulation.

Knowing these features, regulated firms have limited incentives to share their information sincerely with the regulator. If firms may influence the price that they have to pay for permits, they certainly will try to influence it, if this is expected to be profitable for them. Hence finding an incentive mechanism which induces firms to report their information sincerely to the regulator is of great importance.

I have shown that with a constant quantity regulation the generalized Vickrey auction implements the ex-post efficient allocation of permits and thus incentivizes firms to report their private signals sincerely to the regulator. This is not the case, only if the positive correlation between pollution damage and abatement costs is high. Moreover, the mechanism remains ex-post efficient even if firms coordinate their bids in the auction. With the constant price regulation, on the other hand, the truth-telling property of the Vickrey auction is not sustained. Firms' ability to influence the second-stage tax is too attractive. Only if the expected price is decreasing in the firms' reports, and thus when the pollution damage and abatement costs are highly correlated with a negative sign, is the Vickrey auction incentive compatible. Then lying and colluding in the Vickrey auction causes harm for an individual firm, if all other firms are sincere.

Bibliography

[1] Ausubel, L., and Cramton, P., 2004. Vickrey Auctions with Reserve Pricing. Economic Theory 23, 493-505.

[2] Clarke, E., 1971. Multipart Pricing of Public Goods. Public Choice 2, 19-33.

[3] Dasgupta, P., Hammond, P., and Maskin, E., 1980. On Imperfect Information and Optimal Pollution Control. Review of Economic Studies 47(5), 857-860.

[4] Dasgupta, P., and Maskin, E., 2000. Efficient Auctions. Quarterly Journal of Economics 115(2), 341-388.

[5] DeGroot, M.H., 1970. Optimal Statistical Decisions. New York: McGraw-Hill.

[6] Groves, T., 1973. Incentives in Teams. Econometrica 41, 617-631.

[7] Jehiel, P., and Moldovanu, B., 2001. Efficient Design with Interdependent Valuations. Econometrica 69(5), 1237-1259.

[8] Karp, L., and Zhang, J., 2005. Regulation of Stock Externalities with Correlated Abatement Costs. Environmental and Resource Economics 32, 273-299.

[9] Kennedy, P., Laplante, B., and Whittington, D., 2010. Simple Pricing Schemes for Pollution Control under Asymmetric Information. The B.E. Journal of Economic Analysis & Policy 10(1), Article 12.

[10] Kwerel, E., 1977. To Tell the Truth: Imperfect Information and Optimal Pollution Control. Review of Economic Studies 44(3), 595-601.

[11] McAllister, L.K., 2009. The Overallocation Problem in Cap-and-Trade: Moving Toward Stringency. Columbia Journal of Environmental Law 43, 395-445.

[12] Metcalf, G.E., 2007. A Proposal for a U.S. Carbon Tax Swap: An Equitable Tax Reform to Address Global Climate Change. The Hamilton Project, Washington, DC.

[13] Montero, J.-P., 2008. A Simple Auction Mechanism for the Optimal Allocation of the Commons. American Economic Review 98(1), 496-518.

[14] Myerson, R.B, 1981. Optimal Auction Design. Mathematics of Operations Research 6, 58-73.

[15] Newell, R.G., and Pizer, W.A., 2003. Regulating Stock Externalities under Uncertainty. Journal of Environmental Economics and Management 45, 416-432.

[16] Roberts, M.J., and Spence, M., 1976. Effluent Charges and Licenses under Uncertainty. Journal of Public Economics 5(3-4), 193-208.

[17] Stavins, R.N., 1996. Correlated Uncertainty and Policy Instrument Choice. Journal of Environmental Economics and Management 30, 218-232.

[18] Stavins, R.N. 2007. Proposal for a U.S. Cap-and-Trade System to Address Global Climate Change: A Sensible and Practical Approach to Reduce Greenhouse Gas Emissions. The Hamilton Project, Washington, DC.

[19] Vickrey, W., 1961. Counterspeculation, Auctions and Competitive Sealed Tenders. Journal of Finance 16, 8-37.

[20] Vives, X., 2010. Asset Auctions, Information, and Liquidity. Journal of the European Economic Association 8(2-3), 467-477.

[21] Vives, X., 2011. Strategic Supply Function Competition with Private Information. Econometrica 79(6), 1919-1966.

[22] Weitzman, M.L., 1974. Prices vs. Quantities. Review of Economic Studies 41(4), 477-491.

[23] Weitzman, M.L., 1978. Optimal Rewards for Economic Regulation. American Economic Review 68, 683-691.

[24] Yates, A.J., 2012. On a Fundamental Advantage of Permits Over Taxes for the Control of Pollution. Environmental and Resource Economics 51, 583-598.

Appendices

2.A Conditional expectation of the value parameter θ_i

According to the information structure introduced in Section 2.2, consider the following multivariate normal random variable $X_i = (X_1, X_{2.1}, X_{2.2}) = (\theta_i, s_i, s_m)$ with a mean vector

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_{2.1} \\ \mu_{2.2} \end{bmatrix} = \begin{bmatrix} E[\theta_i] \\ E[s_i] \\ E[s_m] \end{bmatrix} = \begin{bmatrix} \bar{\theta} \\ \bar{\theta} \\ \bar{\theta} \end{bmatrix},$$

and a covariance matrix

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix},$$

where

$$\begin{aligned}
\Sigma_{11} &= var[\theta_i] = \sigma_\theta^2 \\
\Sigma_{12} &= \Sigma_{21}^T = \begin{bmatrix} cov[\theta_i, s_i] \\ cov[\theta_i, s_m] \end{bmatrix}^T = \begin{bmatrix} \sigma_\theta^2 \\ \frac{1}{n}\left(\sigma_\theta^2 + (n-1)\rho\sigma_\theta^2\right) \end{bmatrix}^T \equiv \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix}^T \\
\Sigma_{22} &= \begin{bmatrix} var[s_i] & cov[s_i, s_m] \\ cov[s_i, s_m] & var[s_m] \end{bmatrix} \\
&= \begin{bmatrix} \sigma_\theta^2 + \sigma_\varepsilon^2 & \frac{1}{n}\left(\sigma_\theta^2 + \sigma_\varepsilon^2 + (n-1)\rho\sigma_\theta^2\right) \\ \frac{1}{n}\left(\sigma_\theta^2 + \sigma_\varepsilon^2 + (n-1)\rho\sigma_\theta^2\right) & \frac{1}{n}\left(\sigma_\theta^2 + \sigma_\varepsilon^2 + (n-1)\rho\sigma_\theta^2\right) \end{bmatrix} \\
&\equiv \begin{bmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{bmatrix}.
\end{aligned}$$

The inverse of Σ_{22} is

$$\Sigma_{22}^{-1} = \frac{1}{\det(\Sigma_{22})} \begin{bmatrix} \det(\Delta_{22}) & -\det(\Delta_{21}) \\ -\det(\Delta_{12}) & \det(\Delta_{11}) \end{bmatrix} = \frac{1}{\Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21}} \begin{bmatrix} \Delta_{22} & -\Delta_{21} \\ -\Delta_{12} & \Delta_{11} \end{bmatrix}.$$

The conditional distribution of the random variable $(\theta_i | s_i, s_m)$ has an expected value (DeGroot 1970)

$$\begin{aligned}
E\left[\theta_i|s_i,s_m\right] &= \mu_1+\Sigma_{12}\Sigma_{22}^{-1}\left[\begin{array}{c} s_i-\mu_{2.1} \\ s_m-\mu_{2.2}\end{array}\right] \\
&= \left(1-\frac{\delta_1\Delta_{22}-\delta_2\Delta_{12}+\delta_2\Delta_{11}-\delta_1\Delta_{21}}{\Delta_{11}\Delta_{22}-\Delta_{12}\Delta_{21}}\right)\bar{\theta} \\
&\quad +\frac{\delta_1\Delta_{22}-\delta_2\Delta_{12}}{\Delta_{11}\Delta_{22}-\Delta_{12}\Delta_{21}}s_i \\
&\quad +\frac{\delta_2\Delta_{11}-\delta_1\Delta_{21}}{\Delta_{11}\Delta_{22}-\Delta_{12}\Delta_{21}}s_m,
\end{aligned}$$

where

$$\begin{aligned}
\Delta_{11}\Delta_{22}-\Delta_{12}\Delta_{21} &= \frac{(n-1)}{n^2}\left(\sigma_\theta^2+\sigma_\varepsilon^2+(n-1)\,\rho\sigma_\theta^2\right)\left(\sigma_\theta^2+\sigma_\varepsilon^2-\rho\sigma_\theta^2\right), \\
\delta_1\Delta_{22}-\delta_2\Delta_{12} &= \frac{(n-1)}{n^2}(1-\rho)\,\sigma_\theta^2\left(\sigma_\theta^2+\sigma_\varepsilon^2+(n-1)\,\rho\sigma_\theta^2\right), \\
\delta_2\Delta_{11}-\delta_1\Delta_{21} &= \frac{(n-1)}{n^2}n\rho\sigma_\theta^2\sigma_\varepsilon^2.
\end{aligned}$$

Using these, and after some calculations, the expected value of θ_i conditional on the signal vector $\mathbf{s}$ can be written as

$$E\left[\theta_i|s_i,s_m\right]=A\bar{\theta}+Bs_i+Cns_m,$$

where,

$$\begin{aligned}
A &= \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2+(1-\rho+n\rho)\,\sigma_\theta^2} \\
B &= \frac{(1-\rho)\,\sigma_\theta^2}{\sigma_\varepsilon^2+(1-\rho)\,\sigma_\theta^2} \\
C &= \frac{\rho\sigma_\theta^2\sigma_\varepsilon^2}{\left(\sigma_\varepsilon^2+(1-\rho+n\rho)\,\sigma_\theta^2\right)\left(\sigma_\varepsilon^2+(1-\rho)\,\sigma_\theta^2\right)}.
\end{aligned}$$

Moreover, the conditional variance is

$$
\begin{aligned}
var\left[\theta_i|s_i,s_m\right] &= \Sigma_{11}-\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} \\
&= \Sigma_{11}-\delta_1\frac{\delta_1\Delta_{22}-\delta_2\Delta_{21}}{\Delta_{11}\Delta_{22}-\Delta_{12}\Delta_{21}}-\delta_2\frac{\delta_2\Delta_{11}-\delta_1\Delta_{12}}{\Delta_{11}\Delta_{22}-\Delta_{12}\Delta_{21}} \\
&= \sigma_\theta^2-\sigma_\theta^2 B-\left(1+(n-1)\rho\right)\sigma_\theta^2 C \\
&= \sigma_\theta^2\left(1-B\right)\left(1-\rho+\rho A\right) \\
&= \sigma_\theta^2\left(1-B\right)\left(1-\rho\right)+\sigma_\theta^2\left(1-B\right)\rho A \\
&= \left(B+C\right)\sigma_\varepsilon^2.
\end{aligned}
$$

Furthermore, let the remaining uncertainty be denoted by $\varepsilon_i^s=\theta_i-E\left[\theta_i|s_i,s_m\right]$. It is a normal random variable with zero expected value and a variance $var\left[\varepsilon_i^s\right]=var\left[\theta_i|s_i,s_m\right]$. Furthermore, the covariance between the remaining uncertainties may write

$$
\begin{aligned}
cov\left[\varepsilon_i^s,\varepsilon_j^s\right] &= E\left[\left\{\left(\theta_i-\bar\theta\right)-B\left(s_i-\bar\theta\right)-Cn\left(s_m-\bar\theta\right)\right\}\right. \\
&\qquad \left.\times\left\{\left(\theta_j-\bar\theta\right)-B\left(s_j-\bar\theta\right)-Cn\left(s_m-\bar\theta\right)\right\}\right] \\
&= E\left[\left(\theta_i-\bar\theta\right)\left(\theta_j-\bar\theta\right)+B^2\left(s_i-\bar\theta\right)\left(s_j-\bar\theta\right)+C^2n^2\left(s_m-\bar\theta\right)\right] \\
&\quad -2E\left[B\left(\theta_i-\bar\theta\right)\left(s_j-\bar\theta\right)+Cn\left(\theta_i-\bar\theta\right)\left(s_m-\bar\theta\right)-BCn\left(s_i-\bar\theta\right)\left(s_m-\bar\theta\right)\right] \\
&= \rho\sigma_\theta^2+B^2\rho\sigma_\theta^2+C^2n^2\cdot var\left[s_m\right]-2B\rho\sigma_\theta^2-2C\cdot var\left[\theta_m\right]+2BCn\cdot var\left[s_m\right] \\
&= var\left[\varepsilon_i^s\right]-\left(1-B\right)^2\sigma_\theta^2\left(1-\rho\right)-B^2\sigma_\varepsilon^2 \\
&= var\left[\varepsilon_i^s\right]-B\sigma_\varepsilon^2 \\
&= C\sigma_\varepsilon^2.
\end{aligned}
$$

The aggregate uncertainty is given by (note that $B+nC=1-A=\frac{var[\theta_m]}{var[s_m]}$)

$$
n\varepsilon_m^s=\sum_{i=1}^{n}\varepsilon_i^s=\left(\theta_m-\bar\theta\right)-\left(1-A\right)\left(s_m-\bar\theta\right).
$$

It has a normal distribution with zero expected value and a variance

$$
\begin{aligned}
var\left[n\varepsilon_m^s\right] &= E\left[\left(\left(\theta_m-\bar\theta\right)-\left(1-A\right)\left(s_m-\bar\theta\right)\right)^2\right] \\
&= E\left[\left(\theta_m-\bar\theta\right)^2+\left(1-A\right)^2\left(s_m-\bar\theta\right)^2-2\left(1-A\right)\left(\theta_m-\bar\theta\right)\left(s_m-\bar\theta\right)\right] \\
&= var\left[\theta_m\right]+\left(1-A\right)^2var\left[s_m\right]-2\left(1-A\right)cov\left[\theta_m,s_m\right] \\
&= A^2var\left[\theta_m\right]+\left(1-A\right)^2var\left[\varepsilon_m\right] \\
&= \left(1-A\right)n\sigma_\varepsilon^2.
\end{aligned}
$$

The damage parameter γ is assumed to be a normally distributed random variable, $\gamma \sim N(\bar{\gamma}, \sigma_\gamma^2)$. Furthermore, the damage and abatement costs are correlated and the covariance between γ and θ_m is denoted by $cov[\gamma, \theta_m] = \sigma_{\gamma\theta}$. The expected damage parameter conditional on the sum of all signals, $ns_m = \sum_{i=1}^{n} s_i$, writes as

$$E[\gamma|\, s_m] = \bar{\gamma} + Z\left(ns_m - n\bar{\theta}\right),$$

where

$$Z = \frac{cov[\gamma, ns_m]}{var[ns_m]} = \frac{\sigma_{\gamma\theta}}{\sigma_\varepsilon^2 + (1 + (n-1)\rho)\,\sigma_\theta^2}.$$

The remaining uncertainty related to the damage parameter γ is thus $\varepsilon_\gamma^s = \gamma - \bar{\gamma} - Z\left(ns_m - n\bar{\theta}\right)$ and the conditional variance of γ is

$$\begin{aligned}
var[\gamma|\,\mathbf{s}] &= E\left[\left(\varepsilon_\gamma^s\right)^2\right] \\
&= E\left[(\gamma - \bar{\gamma})^2 + (nZ)^2\left(s_m - \bar{\theta}\right)^2 - 2nZ(\gamma - \bar{\gamma})\left(\theta_m - \bar{\theta} + \varepsilon_m\right)\right] \\
&= \sigma_\gamma^2 + (nZ)^2\, var[s_m] - 2nZ\sigma_{\gamma\theta} \\
&= \sigma_\gamma^2 - nZ\sigma_{\gamma\theta}.
\end{aligned}$$

The covariance between the average cost and benefit parameters of emissions reductions, conditional on the signal vector $\mathbf{s}$, is simply

$$\begin{aligned}
cov[\gamma, \theta_m|\,\mathbf{s}] &= E\left[\left(A\left(\theta_m - \bar{\theta}\right) - (1-A)\,\varepsilon_m\right)\left((\gamma - \bar{\gamma}) - nZ\left(\theta_m - \bar{\theta}\right) - nZ\varepsilon_m\right)\right] \\
&= E\left[A\left(\theta_m - \bar{\theta}\right)(\gamma - \bar{\gamma}) - AnZ\left(\theta_m - \bar{\theta}\right)^2 + (1-A)\,nZ\varepsilon_m^2\right] \\
&\quad + \underbrace{E\left[-(1-A)(\gamma - \bar{\gamma})\,\varepsilon_m + (1-A)\,nZ\left(\theta_m - \bar{\theta}\right)\varepsilon_m - AnZ\left(\theta_m - \bar{\theta}\right)\varepsilon_m\right]}_{=0} \\
&= A\sigma_{\gamma\theta} - nZA \cdot var[\theta_m] + (1-A)\,nZ \cdot var[\varepsilon_m] \\
&= A\sigma_{\gamma\theta}.
\end{aligned}$$

2.B Regulation stage - prices vs. quantities

In order to derive the prices vs. quantities comparison (Weitzman 1974), I utilize a general non-constant regulation model. Constant price and quantity regulations are the two extremes of this general model (see also Weitzman 1978). With a non-constant regulation, the supply of permits for each firm in the regulation stage is given by a linear permit schedule:

$$T_i(q_i; I) = \bar{p}(I) + \tau\left(q_i - \bar{q}_i(I)\right), \tag{2.66}$$

where $\bar{p}(I) = E[p^{\star}| I]$ and $\bar{q}_i(I) = E[q_i^{\star}| I]$ are the expected first-best outcomes conditional on information I and τ is the slope of the permit supply schedules. Due to the model being symmetric, i.e. constant β and symmetric correlation of abatement costs ρ, the slope τ is the same for each firm. Then, in the beginning of the regulation period, the regulator allocates pollution permits to firms according to $\bar{q}(I) = (\bar{q}_1(I), ..., \bar{q}_n(I))$. However, each firm may purchase (or sell back) permits from (to) the regulator according to the price schedule (2.66). In addition, firms are free to trade permits with each other in the secondary market.

The two alternative constant regulations may derive from (2.66) in the following way. Using $\tau = 0$, the permit supply schedule reduces to the constant tax (or subsidy) $T_i(q_i; I) = \bar{p}(I)$. Also, this defines the equilibrium price of the secondary market and firms have no incentives to trade permits with each other. On the other hand, if $\tau \to \infty$, the regulation is a constant quantity regulation where the aggregate supply of permits is fixed at $\bar{Q}(I) = \sum_{i=1}^{n} \bar{q}_i(I)$. Equation (2.66) then only defines the initial allocation of permits. I first derive the solution with a general model and then describe the solution with the two regulatory extremes: a constant price regulation ($\tau = 0$) and a constant quantity regulation ($\tau \to \infty$).

With the non-constant regulation model, the net purchases of firm i from the regulator, denoted by h_i, is a sum of the initial allocation $\bar{q}_i(I)$ and transactions with the regulator in the regulation period. Then the amount of trading in the permit markets is simply $\Delta q_i = |q_i - h_i| \geq 0$. Firm i buys permits in the resale market if $q_i > h_i$ and sells permits if $h_i > q_i$.

The problem of the regulator is to choose the τ that minimizes the expected deadweight loss from (2.21) given the responses of regulated firms. Firm i maximizes its profits with respect to pollution q_i and purchases from the regulator h_i given the supply schedule $T_i(q_i; I)$:

$$\begin{aligned} \max_{q_i, h_i} \pi_{i,\tau}(q_i, h_i; \theta_i) &= \int_{\bar{q}_i(I)}^{q_i} u_i(x; \theta_i)\, dx - \int_{\bar{q}_i(I)}^{h_i} T_i(x; I)\, dx \\ &\quad + p_\tau \Delta q_i \left\{ \mathbf{1}_{\{h_i > q_i\}} - \mathbf{1}_{\{q_i > h_i\}} \right\}. \end{aligned} \tag{2.67}$$

In (2.67), p_τ is the equilibrium price of the secondary market and $\mathbf{1}_{\{.\}}$ is an indicator function with the value 1, if its argument is true, and otherwise 0. The first-order condition with respect to pollution is given by

$$\begin{aligned} 0 &= u_i(q_i; \theta_i) + p_\tau \frac{d(h_i - q_i)}{dq_i} \mathbf{1}_{\{h_i > q_i\}} - p_\tau \frac{d(q_i - h_i)}{dq_i} \mathbf{1}_{\{q_i > h_i\}} \\ &= u_i(q_i; \theta_i) - p_\tau, \end{aligned} \tag{2.68}$$

and with respect to permit purchases from the regulator by

$$\begin{aligned} 0 &= -T_i\left(h_i; I\right) + p_\tau \frac{d\left(h_i - q_i\right)}{dh_i} \mathbf{1}_{\{h_i > q_i\}} - p_\tau \frac{d\left(q_i - h_i\right)}{dh_i} \mathbf{1}_{\{q_i > h_i\}} \\ &= -T_i\left(h_i; I\right) + p_\tau. \end{aligned} \tag{2.69}$$

Let $q_{i,\tau}$ and $h_{i,\tau}$ denote the profit-maximizing pollution level and purchases from the regulator of firm i given τ. The first-order conditions imply that firms equate their marginal value functions with the marginal costs of purchasing and hence from (2.68) and (2.69):

$$\tau h_{i,\tau} + \beta q_{i,\tau} = \left(\beta + \tau\right) \bar{q}_i\left(I\right) + \left(\theta_i - \bar{\theta}_i\left(I\right)\right). \tag{2.70}$$

Further, market clearing implies that $\sum_{i=1}^n h_{i,\tau} = \sum_{i=1}^n q_{i,\tau} = Q_\tau$ and summing equations (2.70) from 1 to n gives the total level of pollution

$$Q_\tau = \bar{Q}\left(I\right) + \frac{n\left(\theta_m - \bar{\theta}_m\left(I\right)\right)}{\beta + \tau}. \tag{2.71}$$

For the average firm we get

$$q_{m,\tau} = h_{m,\tau} = \bar{q}_m\left(I\right) + \frac{\theta_m - \bar{\theta}_m\left(I\right)}{\beta + \tau}. \tag{2.72}$$

Plugging this into the first-order condition gives the equilibrium price:

$$p_\tau = \bar{p}\left(I\right) + \left(\frac{\tau}{\beta + \tau}\right)\left(\theta_m - \bar{\theta}_m\left(I\right)\right). \tag{2.73}$$

Moreover, firm i's equilibrium outcomes may derive from (2.68), (2.69), (2.70) and (2.73):

$$q_{i,\tau} = \bar{q}_i\left(I\right) + \frac{1}{\beta}\left\{\left(\theta_i - \bar{\theta}_i\left(I\right)\right) - \left(\frac{\tau}{\beta + \tau}\right)\left(\theta_m - \bar{\theta}_m\left(I\right)\right)\right\}, \tag{2.74}$$

$$h_{i,\tau} = \bar{q}_i\left(I\right) + \left(\frac{1}{\beta + \tau}\right)\left(\theta_m - \bar{\theta}_m\left(I\right)\right). \tag{2.75}$$

Plugging (2.74) into the deadweight loss equation (2.21) yields, after a few lines of simple calculations,

$$DWL_{\tau}(I) = \frac{1}{2}\left(\frac{1}{\frac{\beta}{n}+\delta}\right)\left\{\left(\frac{n\delta-\tau}{\beta+\tau}\right)^{2}\left(\theta_m-\bar{\theta}_m(I)\right)^2+\left(\gamma-\bar{\gamma}(I)\right)^2\right\} + \left(\frac{1}{\frac{\beta}{n}+\delta}\right)\left(\frac{n\delta-\tau}{\beta+\tau}\right)\left(\theta_m-\bar{\theta}_m(I)\right)\left(\gamma-\bar{\gamma}(I)\right), \tag{2.76}$$

and thus the expected value of (2.76) is

$$E\left[DWL_{\tau}(I)\right] = \frac{1}{2}\left(\frac{1}{\frac{\beta}{n}+\delta}\right)\left\{\left(\frac{n\delta-\tau}{\beta+\tau}\right)^{2} var\left[\theta_m|\, I\right]+var\left[\gamma|\, I\right]\right\} + \left(\frac{1}{\frac{\beta}{n}+\delta}\right)\left(\frac{n\delta-\tau}{\beta+\tau}\right) cov\left[\theta_m,\gamma|\, I\right]. \tag{2.77}$$

Recall that the regulator chooses only between constant regulations.[16] If the regulator uses a constant pigovian tax/subsidy, i.e. $\tau=0$ and thus $T_i(q_i;I)=\bar{p}(I)$, the expected deadweight loss writes as

$$\begin{aligned} E\left[DWL_p(I)\right] &= E\left[DWL_{\tau}(I)|\,\tau=0\right] \\ &= \frac{1}{2}\left(\frac{1}{\frac{\beta}{n}+\delta}\right)\left\{\left(\frac{n\delta}{\beta}\right)^2 var\left[\theta_m|\, I\right]+2\left(\frac{n\delta}{\beta}\right)cov\left[\theta_m,\gamma|\, I\right]+var\left[\gamma|\, I\right]\right\}. \end{aligned} \tag{2.78}$$

Respectively, with a constant quantity regulation ($\tau\to\infty$) the expected deadweight loss is

$$\begin{aligned} E\left[DWL_q(I)\right] &= E\left[DWL_{\tau}(I)|\,\tau\to\infty\right] \\ &= \frac{1}{2}\left(\frac{1}{\frac{\beta}{n}+\delta}\right)\left\{var\left[\theta_m|\, I\right]-2cov\left[\theta_m,\gamma|\, I\right]+var\left[\gamma|\, I\right]\right\}. \end{aligned} \tag{2.79}$$

2.C Linear equilibrium strategy of the generalized VCG mechanism

From Appendix 2.A, the expected value of θ_i conditional on s_i and s_m is $E\left[\theta_i|\mathbf{s}\right]=A\bar{\theta}+Bs_i+Cns_m$. If there is correlation between marginal abatement costs and pollution damage ($\sigma_{\gamma\theta}\neq 0$), the total supply of pollution rights writes as $Q_S=\frac{1}{\delta}\left(p-\bar{\gamma}-nZ\left(s_m-\bar{\theta}\right)\right)$. Thus,

[16]Note that the optimal slope of the general model is $\tau^{\star}=\frac{\beta\sigma_{\gamma\theta}+n\delta\frac{1}{n}(1+(n-1)\rho)\sigma_{\theta}^2}{\frac{1}{n}(1+(n-1)\rho)\sigma_{\theta}^2-\sigma_{\gamma\theta}}$, which gives $E\left[DWL_{\tau}^{\star}\right]=\frac{1}{2}\left(\frac{1}{\frac{\beta}{n}+\delta}\right)var\left[\gamma|\,\theta_m\right]$. With the symmetric affine linear model, $\tau^{\star}$ and $E\left[DWL_{\tau}^{\star}\right]$ are independent of information I, when the permit market is perfect. For a more general analysis, see Chapter 3.

in equilibrium we have

$$ns_m = \frac{1}{b+\frac{Z}{\delta}}\left(\frac{1}{\delta}\left(p-\bar{\gamma}+Zn\bar{\theta}\right)-na+ncp\right).$$

The first-order condition of the Vickrey auction is written as

$$\begin{aligned}
E\left[\theta_i|s_i,s_m\right]-\beta q_i &= p\\
\beta q_i &= A\bar{\theta}+Bs_i+C\left(\frac{1}{b+\frac{1}{\delta}Z}\right)\left(\frac{1}{\delta}\left(p-\bar{\gamma}+Zn\bar{\theta}\right)-na+ncp\right)-p\\
q_i &= \frac{Ab\bar{\theta}-\frac{1}{\delta}\left(C\bar{\gamma}-\left(A+nC\right)Z\bar{\theta}\right)-Cna}{\beta\left(b+\frac{1}{\delta}Z\right)}+\frac{B}{\beta}s_i\\
&\quad -\frac{\left(b-Cnc-\frac{1}{\delta}\left(C-Z\right)\right)}{\beta\left(b+\frac{1}{\delta}Z\right)}p.
\end{aligned}$$

Plugging in $q_i = D_i\left(p;s_i\right) = a+bs_i-cp$ and solving for parameters a, b and c results in

$$\begin{aligned}
a &= \frac{1}{\beta}\left(\frac{BA\bar{\theta}}{B+nC+\frac{\beta}{\delta}Z}\right)-\frac{1}{\delta}\left(\frac{C\bar{\gamma}-\left(A+nC\right)Z\bar{\theta}}{B+nC+\frac{\beta}{\delta}Z}\right)\\
b &= \frac{1}{\beta}B\\
c &= \frac{1}{\beta}\left(\frac{B}{B+nC+\frac{\beta}{\delta}Z}\right)-\frac{1}{\delta}\left(\frac{C-Z}{B+nC+\frac{\beta}{\delta}Z}\right).
\end{aligned}$$

The clearing price of the Vickrey auction in the information stage is thus

$$
\begin{aligned}
p_v(\mathbf{s}) &= \frac{n\delta a + n\delta b s_m + \bar{\gamma} + nZ\left(s_m - \bar{\theta}\right)}{n\delta c + 1} \\
&= \frac{n\delta\left\{\frac{1}{\beta}BA\bar{\theta} - \frac{1}{\delta}\left(C\bar{\gamma} - (A + nC)Z\bar{\theta}\right)\right\}}{n\delta\left(\frac{1}{\beta}B - \frac{1}{\delta}C + \frac{1}{\delta}Z\right) + B + nC + \frac{\beta}{\delta}Z} \\
&\quad + \frac{\left(B + nC + \frac{\beta}{\delta}Z\right)\left(\frac{n\delta}{\beta}Bs_m + \bar{\gamma} + nZ\left(s_m - \bar{\theta}\right)\right)}{n\delta\left(\frac{1}{\beta}B - \frac{1}{\delta}C + \frac{1}{\delta}Z\right) + B + nC + \frac{\beta}{\delta}Z} \\
&= \frac{\left(B + \frac{\beta}{\delta}Z\right)\left(\frac{n\delta}{\beta}A\bar{\theta} + \bar{\gamma} + nZ\left(s_m - \bar{\theta}\right)\right) + nC\left(nZ\bar{\theta} - \bar{\gamma}\right)}{\left(1 + \frac{n\delta}{\beta}\right)\left(B + \frac{\beta}{\delta}Z\right)} \\
&\quad + \frac{\left(B + \frac{\beta}{\delta}Z\right)\frac{n\delta}{\beta}(B + nC)s_m + nC\left(\bar{\gamma} - nZ\bar{\theta}\right)}{\left(1 + \frac{n\delta}{\beta}\right)\left(B + \frac{\beta}{\delta}Z\right)} \\
&= \frac{\frac{n\delta}{\beta}A\bar{\theta} + \bar{\gamma} + nZ\left(s_m - \bar{\theta}\right) + \frac{n\delta}{\beta}(1 - A)s_m}{\left(1 + \frac{n\delta}{\beta}\right)} \\
&= \frac{n\delta\bar{\theta} + \beta\bar{\gamma} + (n\delta(1 - A) + \beta nZ)\left(s_m - \bar{\theta}\right)}{\beta + n\delta}.
\end{aligned}
$$

This is equal to $\bar{p}(\mathbf{s})$ in (2.23). Respectively, the total level of allocated permits is

$$
\begin{aligned}
Q_v &= \frac{1}{\delta}\left(p_v - \bar{\gamma} - Z\left(ns_m - n\bar{\theta}\right)\right) \\
&= \frac{\bar{\theta} + \frac{\beta}{n\delta}\bar{\gamma} + \left(1 - A + \frac{\beta}{n\delta}nZ\right)\left(s_m - \bar{\theta}\right) - \left(\frac{\beta}{n\delta} + 1\right)\bar{\gamma} - \left(\frac{\beta}{n\delta} + 1\right)nZ\left(s_m - \bar{\theta}\right)}{\frac{\beta}{n} + \delta} \\
&= \frac{\bar{\theta} - \bar{\gamma} + (1 - A - nZ)\left(s_m - \bar{\theta}\right)}{\frac{\beta}{n} + \delta} \\
&= \bar{Q}(\mathbf{s}).
\end{aligned}
$$

Furthermore, $\bar{q}_m(\mathbf{s}) = \frac{1}{n}\bar{Q}(\mathbf{s})$ and this gives for an individual firm $\bar{q}_i(\mathbf{s}) = \bar{q}_m(\mathbf{s}) + b(s_i - s_m) = \bar{q}_m(\mathbf{s}) + \frac{B}{\beta}(s_i - s_m)$, which is equal to (2.25).

2.D Proofs of Section 2.4

Proof of Proposition 2.1

Suppose that other firms bid sincerely in the information stage and firm i bids according to signal s_i' when its true signal is s_i. Suppose also that $s_i' < s_i$. The expected equilibrium outcome $q_{i,q}(\mathbf{s}')$ given strategies according to $\mathbf{s}' = (s_i', \mathbf{s}_{-i})$ in the information stage and the constant quantity regulation in the second stage may derive from (2.31). Using $\bar{\theta}_i(\mathbf{s}) - \bar{\theta}_i(\mathbf{s}') = (B + C)(s_i - s_i')$ and $\bar{\theta}_m(\mathbf{s}) - \bar{\theta}_m(\mathbf{s}') = \left(\frac{B}{n} + C\right)(s_i - s_i')$ yields

$$\begin{aligned} q_{i,q}(\mathbf{s}') &= \bar{q}_i(\mathbf{s}') + \frac{1}{\beta}\left(\frac{n-1}{n}\right) B (s_i - s_i'), \\ &= q_i^e(\mathbf{s}) - \left(\frac{1 - A - nZ}{\beta + n\delta}\right)\frac{1}{n}(s_i - s_i'). \end{aligned}$$

Furthermore, the expected equilibrium price of the secondary market writes from (2.30):

$$\begin{aligned} p_q(\mathbf{s}') &= \underbrace{p_v(\mathbf{s}) - \frac{\delta(1 - A) + \beta Z}{\beta + n\delta}(s_i - s_i')}_{p_v(\mathbf{s}')} + \left(\frac{B}{n} + C\right)(s_i - s_i') \\ &= p_v(\mathbf{s}) + \left(\frac{1 - A - nZ}{1 + \frac{n\delta}{\beta}}\right)\frac{1}{n}(s_i - s_i'). \end{aligned}$$

If the resale market is coalitionally-rational against individual bidders, firm i can reap all the gains from trade in the second-stage resale market, but not more. If this is the case, instead of the equilibrium price $p_q(\mathbf{s}')$, it pays for permits according to the expected inverse residual supply function

$$\overline{RS}_i^{-1}(q_i, s_i'; \mathbf{s}) = p_q(\mathbf{s}') + \left(\frac{\beta}{n-1}\right)(q_i - q_{i,q}(\mathbf{s}')).$$

The profit of the second stage for firm i due to the deviation strategy is then from (2.56):

$$\pi_{i,q}(s_i'; s_i, \mathbf{s}_{-i}) = \int_{\bar{q}_i(\mathbf{s}')}^{q_{i,q}(\mathbf{s}')} \left\{ v_i(x; \mathbf{s}) - \overline{RS}_i^{-1}(x, s_i'; \mathbf{s}) \right\} dx.$$

Suppose that $var[\theta_m] \geq \sigma_{\gamma\theta}$ holds and thus $1 - A \geq nZ$. This implies that $\bar{q}_i(\mathbf{s}') < q_{i,q}(\mathbf{s}') <$

$q_i^e(\mathbf{s})$ and $p_q(\mathbf{s}') > p_v(\mathbf{s})$ when $\mathbf{s}' = (s_i', \mathbf{s}_{-i})$ with $s_i' < s_i$. These give for the IC condition:

$$\Delta_{q,IC} = \int_{\bar{q}_i(\mathbf{s}')}^{q_{i,q}(\mathbf{s}')} \left\{ \overline{RS}_i^{-1}(x, s_i'; \mathbf{s}) - RS_i^{-1}(x; \mathbf{s}_{-i}) \right\} dx + \int_{q_{i,q}(\mathbf{s}')}^{q_i^e(\mathbf{s})} \left\{ v_i(x; \mathbf{s}) - RS_i^{-1}(x; \mathbf{s}_{-i}) \right\} dx. \tag{2.80}$$

Given that (2.54) holds, $v_i(q_i; \mathbf{s}) \geq RS_i^{-1}(q_i; \mathbf{s}_{-i})$ for all $q_i \leq q_i^e(\mathbf{s})$. Thus, the second integral of (2.80) is positive. Furthermore, if $\overline{RS}_i^{-1}(q_i, s_i'; \mathbf{s}) \geq RS_i^{-1}(q_i; \mathbf{s}_{-i})$ at both extreme points of the first integral in (2.80), then $\Delta_{q,IC} \geq 0$. At $q_{i,q}(\mathbf{s}')$, equilibrium conditions guarantee that $\overline{RS}_i^{-1}(q_{i,q}(\mathbf{s}'), s_i'; \mathbf{s}) = v_i(q_{i,q}(\mathbf{s}'); \mathbf{s}) \geq RS_i^{-1}(q_{i,q}(\mathbf{s}'); \mathbf{s}_{-i})$. At $\bar{q}_i(\mathbf{s}')$, on the other hand, $RS_i^{-1}(\bar{q}_i(\mathbf{s}'); \mathbf{s}_{-i}) = p_v(\mathbf{s}')$ and

$$\begin{aligned} \overline{RS}_i^{-1}(\bar{q}_i(\mathbf{s}'), s_i'; \mathbf{s}) &= p_q(\mathbf{s}') + \left(\frac{\beta}{n-1}\right)(\bar{q}_i(\mathbf{s}') - q_{i,q}(\mathbf{s}')) \\ &= p_v(\mathbf{s}') + C(s_i - s_i'). \end{aligned}$$

Thus if $s_i' \leq s_i$ then $\overline{RS}_i^{-1}(q_i, s_i'; \mathbf{s}) \geq RS_i^{-1}(q_i; \mathbf{s}_{-i})$ for an interval $q_i \in [\bar{q}_i(\mathbf{s}'), q_{i,q}(\mathbf{s}')]$ and $\Delta_{q,IC} \geq 0$. Similar arguments hold if $s_i' > s_i$. ■

Proof of Corollary 2.1

According to Proposition 2.1, the information mechanism is incentive compatible if $var[\theta_m] \geq \sigma_{\gamma\theta}$ and thus if $1 - A \geq nZ$. Suppose instead that $nZ > 1 - A$, which implies that $\bar{Q}(\mathbf{s}') > \bar{Q}(\mathbf{s})$ and hence with constant quantities in the second stage $\bar{q}_i(\mathbf{s}') < q_i^e(\mathbf{s}) < q_{i,q}(\mathbf{s}')$ and $p_q(\mathbf{s}') < p_v(\mathbf{s})$ when $\mathbf{s}' = (s_i', \mathbf{s}_{-i})$ with $s_i' < s_i$. Thus it is easy to see that

$$v_i(q_i; \mathbf{s}) - RS_i^{-1}(q_i; \mathbf{s}_{-i}) = (\beta + \tau_v) q_i^e(\mathbf{s}) - (\beta + \tau_v) q_i$$

and

$$v_i(q_i; \mathbf{s}) - \overline{RS}_i^{-1}(q_i, s_i'; \mathbf{s}) = \left(\frac{n}{n-1}\right)\beta q_{i,q}(\mathbf{s}') - \left(\frac{n}{n-1}\right)\beta q_i.$$

Thus, with the second-stage quantity regulation and when the resale market is coalitionally-rational against individual bidders, the IC condition may write

$$\begin{aligned}
\Delta_{q,IC} &= \int_{\bar{q}_i(\mathbf{s}')}^{q_i^e(\mathbf{s})} \left\{ v_i(x;\mathbf{s}) - RS_i^{-1}(x;\mathbf{s}_{-i}) \right\} dx - \int_{\bar{q}_i(\mathbf{s}')}^{q_{i,q}(\mathbf{s}')} \left\{ v_i(x;\mathbf{s}) - \overline{RS}_i^{-1}(x, s_i'; \mathbf{s}) \right\} dx \\
&= (\beta + \tau_v) \int_{\bar{q}_i(\mathbf{s}')}^{q_i^e(\mathbf{s})} \{ q_i^e(\mathbf{s}) - x \} dx - \left(\frac{n}{n-1} \right) \beta \int_{\bar{q}_i(\mathbf{s}')}^{q_{i,q}(\mathbf{s}')} \{ q_{i,q}(\mathbf{s}') - x \} dx \\
&= \frac{1}{2} (\beta + \tau_v) \left(q_i^e(\mathbf{s}) - \bar{q}_i(\mathbf{s}') \right)^2 - \frac{1}{2} \left(\frac{n}{n-1} \right) \beta \left(q_{i,q}(\mathbf{s}') - \bar{q}_i(\mathbf{s}') \right)^2 \\
&= \frac{1}{2} (\beta + \tau_v) \left(\frac{1}{\beta} \left((n-1) B + \frac{1 - A - nZ}{1 + \frac{n\delta}{\beta}} \right) \frac{1}{n} (s_i - s_i') \right)^2 \\
&\quad - \frac{1}{2} \left(\frac{n}{n-1} \right) \beta \left(\frac{1}{\beta} \left(\frac{n-1}{n} \right) B (s_i - s_i') \right)^2 \\
&= \frac{1}{2\beta} \left(\frac{n-1}{n} \right) (s_i - s_i')^2 B^2 \\
&\quad \times \left(\left(1 + \frac{\tau_v}{\beta} \right) \left(\frac{n-1}{n} \right) \left(1 + \frac{1 - A - nZ}{\left(1 + \frac{n\delta}{\beta}\right)(n-1) B} \right)^2 - 1 \right).
\end{aligned}$$

Using (2.53) we get

$$\left(1 + \frac{\tau_v}{\beta} \right) = \left(1 + \frac{n\delta}{\beta} \right) \left(\frac{(n-1) B \cdot var[s_m] + var[\theta_m]}{\left(1 + \frac{n\delta}{\beta}\right)(n-1) B \cdot var[s_m] + var[\theta_m] - \sigma_{\gamma\theta}} \right).$$

Hence, while $1 - A - nZ = \frac{var[\theta_m] - \sigma_{\gamma\theta}}{var[s_m]}$, the Vickrey auction of the first stage with constant quantities in the second stage is incentive compatible and $\Delta_{q,IC} \geq 0$ if

$$\begin{aligned}
\sigma_{\gamma\theta} &\leq var[\theta_m] \\
&\quad + \left(1 + \frac{n\delta}{\beta} \right) (n-1) B \cdot var[s_m] \left(1 - \left(\frac{n}{n-1} \right) \frac{(n-1) B \cdot var[s_m]}{(n-1) B \cdot var[s_m] + var[\theta_m]} \right) \\
\frac{\sigma_{\gamma\theta}}{var[\theta_m]} &\leq 1 + \left(1 + \frac{n\delta}{\beta} \right) (n-1) \left(\frac{B}{B + nC} \right) \left(\frac{C}{B + C} \right).
\end{aligned}$$

Respectively, without any resale market,

$$\begin{aligned}\Delta_{q,IC} &= \int_{\bar{q}_i(\mathbf{s}')}^{q_i^e(\mathbf{s})} \left\{v_i\left(x;\mathbf{s}\right) - RS_i^{-1}\left(x;\mathbf{s}\right)\right\} dx \\ &= \frac{1}{2\beta}\left(\frac{1}{n}\left(s_i - s_i'\right)\right)^2 \left(\frac{(n-1)\,B\cdot var\left[s_m\right] + var\left[\theta_m\right]}{\left(1+\frac{n\delta}{\beta}\right)\left(var\left[s_m\right]\right)^2}\right) \\ &\quad \times \left(\left(1+\frac{n\delta}{\beta}\right)(n-1)\,B\cdot var\left[s_m\right] + var\left[\theta_m\right] - \sigma_{\gamma\theta}\right).\end{aligned}$$

Thus, without the second-stage regulation, $\Delta_{q,IC} \geq 0$ if

$$\frac{\sigma_{\gamma\theta}}{var\left[\theta_m\right]} \leq 1 + \left(1+\frac{n\delta}{\beta}\right)(n-1)\left(\frac{B}{B+nC}\right). \blacksquare$$

Proof of Corollary 2.2

Suppose that other firms bid sincerely in the information stage and firm i bids according to signal s_i' when its true signal is s_i. Suppose also that $s_i' < s_i$. Then $\overline{RS}_i^{-1}\left(q_i, s_i'; \mathbf{s}\right) \leq p_q\left(\mathbf{s}'\right)$ and thus $p_q\left(\mathbf{s}'\right) \geq RS_i^{-1}\left(q_i; \mathbf{s}_{-i}\right)$ for $q_i \in \left[\bar{q}_i\left(\mathbf{s}'\right), q_{i,q}\left(\mathbf{s}'\right)\right]$. Hence $\Delta_{q,IC} > 0$ if $\overline{RS}_i^{-1}\left(q_i, s_i'; \mathbf{s}\right)$ is replaced by $p_q\left(\mathbf{s}'\right)$ in (2.80). Similar arguments hold if $s_i' > s_i$. $\blacksquare$

Proof of Proposition 2.2

Suppose that other firms bid sincerely in the information stage and firm i bids according to signal s_i' when its true signal is s_i. Suppose also that $s_i' < s_i$. Then, given a signal vector $\mathbf{s}' = \left(s_i', \mathbf{s}_{-i}\right)$, the constant price regulation in the second stage is defined by $\bar{p}\left(\mathbf{s}'\right) = p_v\left(\mathbf{s}'\right)$. Moreover, the expected profit of the second stage for firm i reduces to

$$\pi_{i,p}\left(s_i'; s_i, \mathbf{s}_{-i}\right) = \int_{\bar{q}_i(\mathbf{s}')}^{q_{i,p}(\mathbf{s}')} \left\{v_i\left(x;\mathbf{s}\right) - p_v\left(\mathbf{s}'\right)\right\} dx.$$

This on the other hand yields for the IC condition

$$\Delta_{p,IC} = -\int_{\bar{q}_i(\mathbf{s}')}^{q_i^e(\mathbf{s})} \left\{RS_i^{-1}\left(x;\mathbf{s}_{-i}\right) - p_v\left(\mathbf{s}'\right)\right\} dx - \int_{q_i^e(\mathbf{s})}^{q_{i,p}(\mathbf{s}')} \left\{v_i\left(x;\mathbf{s}\right) - p_v\left(\mathbf{s}'\right)\right\} dx. \qquad (2.81)$$

From the equilibrium conditions it is known that $RS_i^{-1}\left(\bar{q}_i\left(\mathbf{s}'\right); \mathbf{s}_{-i}\right) = p_v\left(\mathbf{s}'\right)$ and $v_i\left(q_{i,p}\left(\mathbf{s}'\right); \mathbf{s}\right) = p_v\left(\mathbf{s}'\right)$. Suppose that $\delta\left(1-A\right) + \beta Z > 0$ and thus $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} > -\frac{n\delta}{\beta}$. Hence, $p_v\left(\mathbf{s}'\right) \leq p_v\left(\mathbf{s}\right)$ when $s_i' < s_i$. This also implies that $\bar{q}_i\left(\mathbf{s}'\right) < q_i^e(\mathbf{s}) < q_{i,p}\left(\mathbf{s}'\right)$. Also, $RS_i^{-1}(q_i; \mathbf{s}_{-i})$ must then be a non-decreasing function and $RS_i^{-1}(q_i;\mathbf{s}_{-i}) \geq p_v(\mathbf{s}')$ for all $\bar{q}_i(\mathbf{s}') \leq q_i \leq q_i^e(\mathbf{s})$. While $v_i(q_i;\mathbf{s})$ is a non-increasing function in q_i, both integrals in (2.81) are non-negative and thus $\Delta_{p,IC} < 0$.

However, whenever $p_v(\mathbf{s}') \geq p_v(\mathbf{s})$ the opposite is true and $\Delta_{p,IC} \geq 0$. If $p_v(\mathbf{s}') \geq p_v(\mathbf{s})$, it must hold that $q_{i,p}(\mathbf{s}') < q_i^e(\mathbf{s})$ due to non-increasing $v_i(q_i;\mathbf{s})$ and (2.81) may write

$$\Delta_{p,IC} = \int_{\bar{q}_i(\mathbf{s}')}^{q_{i,p}(\mathbf{s}')} \left\{p_v(\mathbf{s}') - RS_i^{-1}(x;\mathbf{s}_{-i})\right\} dx + \int_{q_{i,p}(\mathbf{s}')}^{q_i^e(\mathbf{s})} \left\{v_i(x;\mathbf{s}) - RS_i^{-1}(x;\mathbf{s}_{-i})\right\} dx. \quad (2.82)$$

Furthermore, $RS_i^{-1}(q_i;\mathbf{s}_{-i})$ must now be a non-increasing function and $p_v(\mathbf{s}') \geq RS_i^{-1}(q_i;\mathbf{s}_{-i})$ for all $\bar{q}_i(\mathbf{s}') \leq q_i \leq q_i^e(\mathbf{s})$. Hence the first integral must be non-negative. Furthermore, $v_i(q_i;\mathbf{s}) \geq RS_i^{-1}(q_i;\mathbf{s}_{-i})$ for all $q_i \leq q_i^e(\mathbf{s})$, given that $var[\theta_m] \geq \sigma_{\gamma\theta}$, and also the second integral must be non-negative. Hence $\Delta_{p,IC} \geq 0$ and the Vickrey auction followed by the constant price regulation is incentive compatible only if $var[\theta_m] \geq \sigma_{\gamma\theta}$ and $\frac{dRS_i^{-1}(q_i;\mathbf{s}_{-i})}{dq_i} \leq 0$. Together these require that

$$\frac{\sigma_{\gamma\theta}}{var[\theta_m]} \leq -\frac{n\delta}{\beta}.$$

Similar arguments hold if $s_i' > s_i$. ■

2.E Imperfect permit market

In this section I relax the assumption of a perfect permit market. However, I keep the model relatively general and just assume that the market suffers from trade frictions which make trading costly for firms. The trading cost function is denoted by $TC(\Delta q_i)$, where $\Delta q_i = |q_i - h_i| \geq 0$ is the amount of trading in the permit market of firm i. Further, I assume quadratic trading costs:

$$TC(\Delta q_i) = \frac{1}{2}\omega(\Delta q_i)^2, \quad (2.83)$$

where $\omega > 0$ denotes the coefficient of market performance. For simplicity, I assume that ω is common knowledge and exogenous to the other parameters of the model. Trading costs do not change the results of the constant price regulation, but the maximization problem of the constant quantity regulation turns into:

$$\begin{aligned} \max_{q_i,h_i} \pi_{i,q}(q_i,h_i;\theta_i) &= \int_{\bar{q}_i(I)}^{q_i} u_i(x;\theta_i)\,dx \quad (2.84) \\ &+ \int_0^{\Delta q_i} \{p_q - \omega x\}\,dx\mathbf{1}_{\{h_i>q_i\}} - \int_0^{\Delta q_i} \{p_q + \omega x\}\,dx\mathbf{1}_{\{q_i>h_i\}}. \end{aligned}$$

Taking the first-order conditions and solving the system gives the equilibrium price from (2.73)

$$p_q = \bar{p}(I) + \theta_m - \bar{\theta}_m(I). \tag{2.85}$$

In the equilibrium, firm i's pollution is

$$q_{i,q} = \bar{q}_i(I) + \left(\frac{1}{\beta+\omega}\right)\left\{\left(\theta_i - \bar{\theta}_i(I)\right) - \left(\theta_m - \bar{\theta}_m(I)\right)\right\}. \tag{2.86}$$

Hence, with a constant quantity regulation, the expected deadweight loss writes as (see Chapter 3 for derivation)

$$\begin{aligned} E\left[DWL_q(I,\omega)\right] &= \frac{1}{2}\left(\frac{1}{\frac{\beta}{n}+\delta}\right)\left\{var\left[\theta_m|\,I\right] - 2cov\left[\theta_m,\gamma|\,I\right] + var\left[\gamma|\,I\right]\right\} \\ &\quad \underbrace{+\frac{n}{2\beta}\left(\frac{1}{1+\frac{\beta}{\omega}}\right)^2\left\{var\left[\theta_i|\,I\right] - var\left[\theta_m|\,I\right]\right\}.}_{EDWL_\omega(I)} \end{aligned} \tag{2.87}$$

When the frictions in the permit market increase, the second term of (2.87) increases and the quantity regulation becomes a less attractive regulatory instrument. Note also that

$$\begin{aligned} var\left[\theta_i|\,0\right] - var\left[\theta_m|\,0\right] &\geq var\left[\theta_i|\,\mathbf{s}\right] - var\left[\theta_m|\,\mathbf{s}\right] \\ \left(\frac{n-1}{n}\right)(1-\rho)\,\sigma_\theta^2 &\geq (1-B)\left(\frac{n-1}{n}\right)(1-\rho)\,\sigma_\theta^2 \\ B &\geq 0. \end{aligned}$$

Hence, the less noisy the signals are the more is gained from the information mechanism even if the permit market is imperfect. This is clear while the allocation is then close to the first-best and the resale market is not needed. The modified Weitzman rule with linear marginal trading costs in the permit market is written as

$$\begin{aligned} \Delta_{pq,\omega}^{Mod} &= \frac{1}{2}\left(\frac{1}{\frac{\beta}{n}+\delta}\right)\left\{\left[A - \left(\frac{n\delta}{\beta}\right)^2\right] var\left[\theta_m\right] - \left[2A + nZ + 2\left(\frac{n\delta}{\beta}\right)\right]\sigma_{\gamma\theta}\right\} \\ &\quad -\frac{1}{2\beta}\left(\frac{1}{1+\frac{\beta}{\omega}}\right)^2 (1-B)(n-1)(1-\rho)\,\sigma_\theta^2. \end{aligned} \tag{2.88}$$

Below I have run numerical simulations. In the simulations I vary the standard deviation of the signal noise (σ_ε) and the correlation coefficient of the aggregate abatement costs and pollution damage ($\rho_{\gamma\theta}$). The other parameters of the affine linear model are kept fixed. The fixed parameter values are presented in Table 2.2 and the values of the two variating variables in Table 2.3.

Table 2.2: Simulations - fixed parameter values

Variable		Value
n	Number of firms	10
σ_θ	Standard deviation of the abatement cost parameter	10
ρ	Correlation coefficient of the abatement costs	0.5
β/n	Slope of the aggregate abatement costs	1
σ_γ	Standard deviation of the damage parameter	10
δ	Slope of the pollution damage	1

Table 2.3: Simulations - values of variating variables

Variable			σ_ε Standard deviation of the signal noise		
		Value	5	10	20
$\rho_{\gamma\theta}$	Correlation coefficient	-0.2	Simulation 1	Simulation 2	Simulation 3
	of abatement costs	0	Simulation 4	Simulation 5	Simulation 6
	and pollution damage	0.2	Simulation 7	Simulation 8	Simulation 9

Figure 2.3 presents the expected deadweight losses of the two constant regulations. The dashed lines are constant price regulations and the solid lines constant quantity regulations when the regulator knows the private information of firms ($I = \mathbf{s}$, blue curves) and when the regulation is implemented using only prior information ($I = 0$, red curves). Given the structure of the model, the price regulation with $I = \mathbf{s}$ cannot be attained due to the incentive compatibility conditions and the dashed blue curve $p(s)$ denoting the two-stage price regulation is thus thinner in Figure 2.3 than the other curves.

From the simulations, the two-stage quantity regulation (solid blue line) performs worse than the one-stage price regulation (dashed red line) when the correlation between abatement costs and pollution damage is negative (the upper panels), signal noise is high (panels on the right) and when the permit market frictions increase. The two extremes are Simulations 3 and 7. Moreover, the value of information increases in precision of signals. In all simulations $-\frac{n\delta}{\beta} < \frac{\sigma_{\gamma\theta}}{var[\theta_m]} < 1$. Hence, according to Lemma 2.4, the difference between the welfares of quantity regulations with and without private information of firms is decreasing in $\sigma_{\gamma\theta}$. Respectively, the welfare difference of price regulations due to private information is increasing in $\sigma_{\gamma\theta}$ (Lemma 2.5).

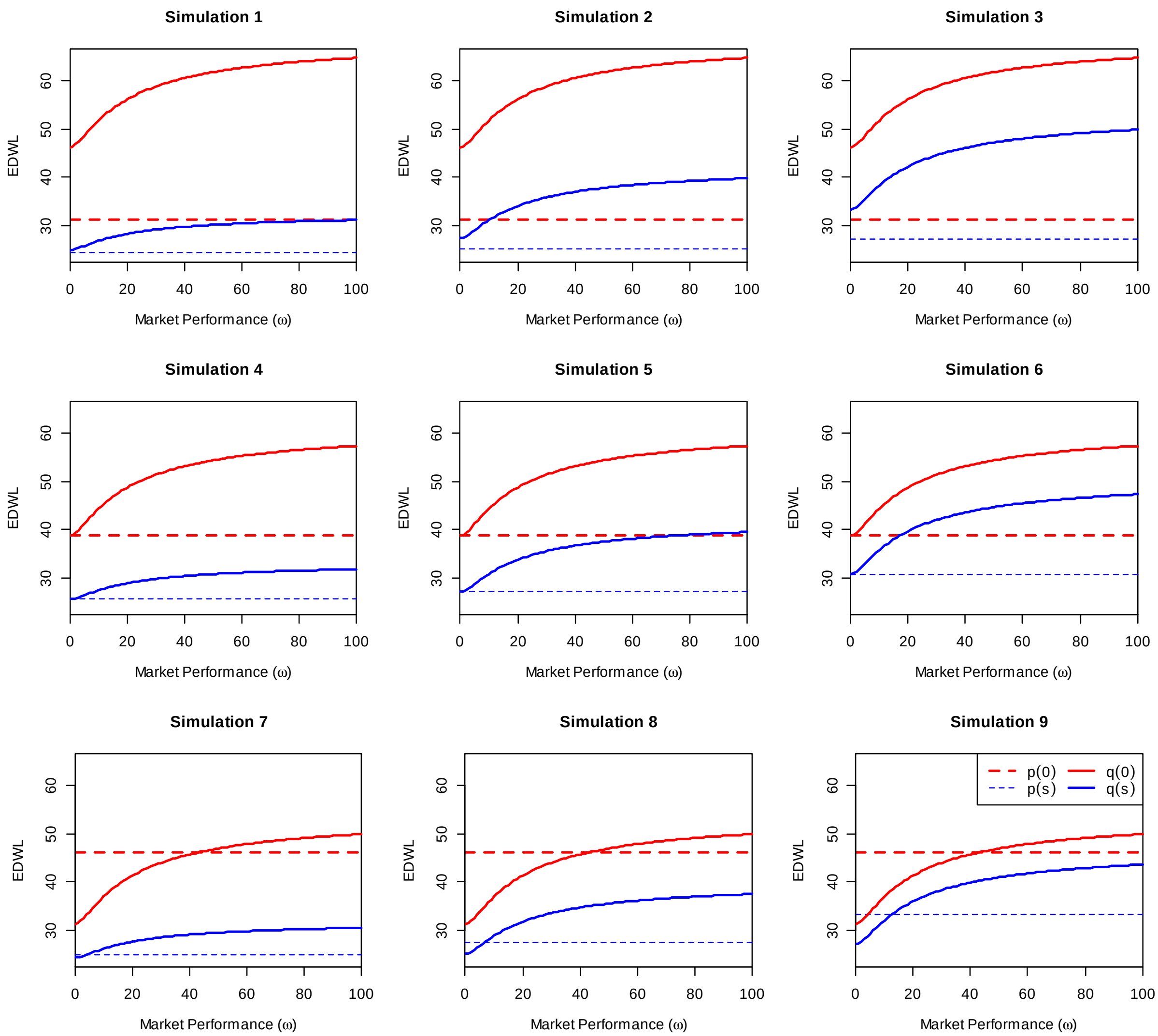

Figure 2.3: Expected deadweight losses of the constant price regulation $(p\,(I))$ and the constant quantity regulation $(q\,(I))$ when the coefficient of market performance ω increases. The red curves describe the cases when the regulator knows only the prior information $(I = 0)$ and the blue curves the cases when the regulator knows the private information of firms $(I = \mathbf{s})$. The parameter values of the various simulations are presented in Tables 2.2 and 2.3.

Chapter 3

Learning through one round of communication in regulating the commons when markets are imperfect

Abstract

When regulating pollution, regulation has often to be designed using only asymmetric and incomplete information. Even if polluting firms are privately better informed than the regulator, they may not have accurate information about their own emission abatement costs. If the regulator is planning to implement a program of tradeable emissions permits and if the permit market is perfect, the regulator is able to obtain a solution that maximizes the expected social welfare without the private information of firms. However, this private information is valuable to the regulator if the permit market is not perfect. This paper presents a two-stage regulation when the permit market suffers from market imperfections. In the first stage, the regulator conducts a generalized Vickrey-Clarke-Groves (VCG) mechanism. The main goal of the first-stage auction mechanism is to collect private information from regulated firms. In addition, the first stage serves as an initial permit allocation method. The Vickrey payment rule rewards firms for revealing their information sincerely to the regulator. In the second stage, given the information on the expected costs of reducing emissions, the regulator implements a quantity regulation, where non-constant permit supply schedules take into account the frictions of the permit market. I study the incentive compatibility conditions of the first stage auction mechanism followed by a resale market for permits in the regulation stage. I show that given the affine linear structure of the model, the best response is to bid sincerely in the Vickrey auction if every other firm is bidding sincerely, unless the (negative or positive) correlation between aggregate abatement costs and pollution damage is relatively high.

3.1 Introduction

In many commons problems, regulation has to be designed using asymmetric and incomplete information. Polluting firms, for instance, are normally privately better informed than the regulator with regard to the costs of reducing emissions. When a new regulation is being implemented, even regulated firms may not have accurate information about their own abatement costs. To comply with the new regulation the production processes need to be revised or new, and perhaps still immature, technologies implemented. Moreover, if all the regulated firms have similar sets of possible abatement technologies, uncertain costs may be correlated between firms. Once the regulation is implemented and firms start to invest in new technologies, the uncertainty about the costs will gradually vanish. From the regulator's point of view, however, it may not be possible to wait for the revelation of uncertain reduction costs. Without any regulation, firms are not willing to install new technologies and they do not learn the true costs of emission reductions. The choice of the regulatory instrument has to be made under incomplete information. Suppose, for instance, that the regulator is planning to implement a program of tradeable emissions permits. When the market for emission permits is new, the market may not perform perfectly. Trading may be costly due to searching or other transaction costs, trading may not be efficient due to asymmetric information between trading partners, some of the traders may have market power, or there may be policy failure due to overlapping environmental regulation. If the permit market was perfectly competitive without any trading frictions, the regulator would be able to obtain a solution that maximizes the expected social welfare without the private information of firms. Then the regulator would have to care only about the aggregate pollution level and thus the aggregate supply schedule of permits needs to be set equal to the expected marginal damage function. However, when the permit market is not perfect, it is not only the aggregate pollution but also the distribution and the initial allocation of permits among regulated firms which is important in terms of the expected welfare. Most importantly, the private information of firms is then valuable to the regulator.

For concreteness, consider electricity companies emitting greenhouse gases. In recent years, these companies have been regulated, or are expected to be regulated, by new climate policy instruments, such as the EU Emissions Trading System (EU ETS). The generation mix of a typical company contains varying shares of gas, oil, coal and nuclear power, and renewable energy. Depending on the production portfolio and investment cycles, companies have different investment plans for the future. If the regulatory requirement changes the investment plan of a company, the company faces additional costs.[1] Uncertainty about these abate-

[1]This is of course a dynamic problem, whereas the model I consider is a static one. However, consider

ment costs arises from several factors, such as the development and learning effects of new technologies, the relative costs of primary fuels, local weather conditions, economic growth or future climate policies. However, due to similar sets of generating units in production portfolios and possible new technologies, the effects of these factors are fairly similar for all companies. Hence, the assumption of uncertain and correlated abatement costs is reasonable. Furthermore, the implementation and design of the EU ETS has been criticized due to numerous flaws. One of the most crucial is the overallocation of EU emissions allowances in the first two phases of the EU ETS. This was due to the limited information about firms' abatement costs and business-as-usual emissions before Phase I (2005-2007) but also due to the non-adjustable supply of allowances during the recent economic downturn in Phase II (2008-2012).

This paper addresses the role of firms' private information when a new regulatory instrument for pollution is designed and implemented. To my knowledge, the previous literature does not distinguish clearly whether firms learn their abatement costs before or after the regulation is designed and implemented and how the information structure affects the optimal regulation. In particular, I ask the following questions. How should the regulation be designed when the permit market is imperfect, abatement costs are uncertain, and firms are better informed than the regulator at the outset? Under what conditions is the private information of regulated firms valuable to the regulator and are there any mechanisms that provide incentives for regulated firms to reveal their information to the regulator?

To answer these questions, I propose a two-stage regulation. In the first stage, the regulator conducts an auction mechanism. The main goal of the auction is to collect private information from regulated firms. In addition, the auction mechanism serves as an initial allocation method for pollution permits. In the second stage, given the information on the expected abatement costs, the regulator implements a quantity regulation, where permit supply schedules adjust to the abatement cost shocks and take into account the frictions of the permit market.

There is a broad literature on regulating externalities under incomplete information. In a seminal paper, Weitzman (1974) defines a rule for choosing between price and quantity regulation. With a price regulation, a uniform tax rate is set at the constant level, regardless of the quantity of emissions. Respectively, in a quantity regulation, which could be implemented through a tradeable permit program, the supply of permits is constant regardless

a program of tradeable permits. If all permits that will be issued to the market are eligible for the whole regulation period (banking of permits is allowed) and if emission reduction activities are independent of each other, it is then possible to simplify the analysis to the static case. Then we just calculate the net present value of every single abatement activity conducted at different points in time.

of the price. These two simple regulation alternatives may be called "constant" regulations. Roberts and Spence (1976), Weitzman (1978) and, more recently, Kennedy et al. (2010) and Yates (2012) provide examples of non-constant regulatory schemes that improve the outcomes relative to constant regulations. In a non-constant tax regulation, the tax rate varies with the quantity of emissions. In a non-constant quantity regulation, the permit supply is defined as a function of price. Kennedy et al. and Yates show that, when the permit market is perfect, non-constant permits with free trading lead unambiguously to lower total expected social costs than non-constant taxes. The intuition is clear. In equilibrium, when the aggregate permit supply equals the expected marginal damage and when the permit market is perfect, the marginal abatement costs are equal across firms, and they are also equal to the expected marginal damage. Thus the expected welfare is maximized. However, when using non-constant taxes, the marginal abatement costs may not be equal across firms. On the other hand, when the permit market does not function properly or is absent, optimal permit supply schedules will coincide with optimal non-constant taxes.

All the aforementioned papers use a similar model structure in many respects. Abatement costs and pollution damage are quadratic and their functional forms are common knowledge. Apart from Yates (2012), pollution is assumed to be uniformly mixed. Intercepts of marginal abatement costs and marginal damage are uncertain to the regulator but the slopes are common knowledge.[2] The regulator knows only the distribution functions of the unknown parameters of the abatement costs, whereas firms know their abatement costs exactly. Firms may learn their abatement costs either before or after the regulation is designed. However, the timing of the learning is not in the focus in these papers. Finally, if the quantity regulation is considered, the permit market is assumed to be perfect.

The second strand of literature this paper is related to considers multi-unit auctions and, in particular, Vickrey-Clarke-Groves (VCG) mechanisms[3] (Vickrey 1961, Clarke 1971, Groves 1973). In a pollution regulation setup, Dasgupta et al. (1980) and Montero (2008) provide efficient mechanisms applying the VCG pricing rule. Both Dasgupta et al. and Montero consider the model of pure private values, i.e. each firm has complete information about its own abatement costs. Dasgupta et al. propose a tax scheme applying a direct VCG mechanism and Montero examines indirect implementation of a VCG mechanism by proposing a simple sealed-bid auction mechanism for emissions permits with endogenous (non-constant) supply. With pure private values, the VCG mechanism implements efficient allocation of permits in dominant strategies. Thus it is in each firm's private interest to reveal its true

[2]In Kennedy et al. (2010) and Yates (2012) the damage function is not uncertain.

[3]The VCG mechanism is a multi-unit extension of a single-unit Vickrey auction. In this paper these are used as synonyms.

information, whatever the other firms do. Due to the Vickrey pricing rule, each firm pays exactly the externality it imposes on other agents. In the case of pollution regulation, the payment includes the pecuniary externality to other firms and the residual damage of the firm's own emissions.

In this paper, the model structure is different in three important ways. First, I relax the assumption of a perfect permit market. However, I keep the model relatively general in this respect and do not specify the source or nature of the market imperfections. However, these market frictions affect symmetrically both the demand and the supply side of the permit market. The frictions are modeled as linear marginal trading cost functions. If the frictions are modest, the regulation in the second stage is close to the Roberts and Spence (1976) non-constant permit regulation and if the frictions are very large, the optimal second-stage regulation is Weitzman's (1978) non-constant tax regulation.[4]

Second, I assume that firms are better informed than the regulator about abatement costs when the regulation is designed and implemented. In the absence of frictions, the regulator does not need the private information of firms in order to implement the optimal second-best regulation. However, when market frictions are present, the expected welfare loss is reduced due to better information from the first-stage auction mechanism. Then the private information of firms is valuable to the regulator when designing the regulation.

Third, I assume that regulated firms do not have complete information about their own abatement costs before the regulation is implemented. Each firm has only a noisy estimate of its own abatement costs. Furthermore, the abatement costs between firms are correlated. Due to these uncertain and correlated abatement costs, the firms' expected marginal valuations of permits in the first-stage auction mechanism are interdependent. Without any regulation, firms do not reduce emissions and the true abatement costs will never be revealed. Besides, it is not known to any firm or to the regulator when the true costs will be revealed to firms, even when firms start to abate their emissions. Thus it is not possible for the regulator to wait for the values to become private. Unfortunately, the efficiency property of the VCG mechanisms is not generally sustained when agents have common or interdependent values (Jehiel and Moldovanu, 2001). However, when the bidders' private information can be summarized by one-dimensional signals and the marginal value functions satisfy continuity, value monotonicity and the single-crossing properties, then a generalized Vickrey auction without a resale market is ex-post efficient (Dasgupta and Maskin 2000, Ausubel and Cramton 2004).

Moreover, I allow the uncertain benefits and costs of emissions reductions to be correlated.

[4] In Chapter 2, I examine a two-stage regulation with either a constant quantity regulation or a constant price regulation in the second stage. However, in Chapter 2 I assume a perfect permit market. Otherwise the models have a similar structure.

Stavins (1996) presents examples of statistical dependence between the marginal benefits and marginal costs of environmental protection.[5] I show that, if the correlation between the aggregate abatement costs and pollution damage is not too high (negatively or positively), it is the best response to bid sincerely in the Vickrey auction if every other firm is bidding sincerely. Hence the Vickrey auction is incentive compatible and ex-post efficient even if the auction is followed by the regulation stage and thus the resale market of permits.

In Section 3.2, I introduce a two-stage regulation model with trade frictions and the affine linear structure of the model. The model is solved backwards. In Section 3.3, I describe a non-constant quantity regulation in the second stage. The Vickrey auction in the first stage is elaborated in Section 3.4. Section 3.5 concludes.

3.2 Model

The model consists of $n \geq 2$ risk-neutral polluting firms indexed with $i = 1, ..., n$.[6] Pollution is denoted by a vector $\mathbf{q} = (q_1, \ldots, q_n)$ and the aggregate pollution is given by $Q = \sum_{i=1}^{n} q_i$. In order to reduce emissions, firms have to install new and cleaner technology, change production processes, use more expensive inputs or perhaps even reduce production to some extent. Hence, firm i's value of its pollution q_i is based on the avoided costs of reducing emissions from the business-as-usual level of pollution, q_i^{bau}. The gross value of avoided abatement costs of firm i, i.e. the gross value of pollution, is denoted by $U_i(q_i; \theta_i)$, where the firm-specific cost paramater is θ_i. Let $\theta = (\theta_1, \ldots, \theta_n)$ denote the vector of cost parameters. I assume that these cost parameters are correlated. $U_i' = \frac{dU_i}{dq_i} = u_i(q_i; \theta_i)$ is the marginal value of avoided abatement costs (or the marginal abatement cost function). Furthermore, the pollution damage function is $DF(Q; \gamma)$. $DF' = \frac{dDF}{dQ} = MDF(Q; \gamma)$ is the marginal damage function and γ is a damage parameter. Pollution is assumed to be uniformly mixed. I assume that $U_i' = \frac{dU_i}{dq_i} > 0$, $U_i'' \leq 0$, and $DF' = \frac{dDF}{dQ} > 0$, $DF'' \geq 0$.

With complete information about the costs and benefits of emission reductions, the ultimate problem is to maximize the social welfare with respect to the pollution vector $\mathbf{q}$:

$$\max_{\mathbf{q}} W(\mathbf{q}) = \sum_{i=1}^{n} U_i(q_i; \theta_i) - DF\left(\sum_{i=1}^{n} q_i; \gamma\right). \tag{3.1}$$

[5]Consider again the example of climate change. The consequences of climate change are very uncertain. It has implications for economic growth and for local climate and weather conditions, among other things. Even if it is difficult to determine the causal relations between different factors, it is reasonable to argue that firms' investment costs and possible damage caused by climate change have some common factors, and thus there may be a statistical dependence between them.

[6]There could also be only one firm, but I assume for now that there are a number of firms.

The first-best (interior) solution to this problem is denoted by a vector $\mathbf{q}^\star = (q_1^\star, \ldots, q_n^\star)$, where $0 < q_i^\star < q_i^{bau}$ for all i and $Q^\star = \sum_{i=1}^{n} q_i^\star$. In the first-best, the pollution of each firm is reduced to a level where the marginal value of pollution equals the first-best price, i.e. $u_i(q_i^\star; \theta_i) = p^\star$. In addition, the first-best price equals the value of the marginal damage function, i.e. $p^\star = MDF(Q^\star; \gamma)$.

The first-best solution is unknown to the regulator and firms, while they do not have exact information about the true damage function nor about the true emission reduction costs. However, even if the cost parameters θ_i are unknown to firms and to the regulator when the regulation is implemented, firms are privately better informed than the regulator at the outset and learn their true abatement costs when they start to put the new abatement technologies into operation. Hence, I assume that each firm receives a noisy estimate, i.e. signal s_i, of its cost parameter before the regulation is implemented. I denote by $\mathbf{s} = (s_1, \ldots, s_n)$ the signal vector and by $\mathbf{s}_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n)$ the signal vector of every other firm but firm i.

Suppose that the regulator implements a regulation denoted by r. Let $\pi_{i,r}(q_i; \theta_i)$ denote the profits of firm i given the regulation r. Thus the profit-maximizing solution of firm i after the revelation of θ_i is denoted by $q_{i,r} = \arg\max \pi_{i,r}(q_i; \theta_i)$. Respectively, let $\mathbf{q}_r = (q_{1,r}, \ldots, q_{n,r})$ denote the vector of profit-maximizing pollution levels. $Q_r = \sum_{i=1}^{n} q_{i,r}$ is the total level of pollution. The regulator chooses a regulation that maximizes the expected welfare given the reactions of firms to the regulation. An equivalent problem is to minimize the expected deadweight loss:

$$\begin{aligned} \min_r E[DWL_r] &= E[W(\mathbf{q}^\star) - W(\mathbf{q}_r)] \qquad (3.2) \\ &= E\left[\sum_{i=1}^{n} U_i(q_i^\star; \theta_i) - \sum_{i=1}^{n} U_i(q_{i,r}; \theta_i) - DF(Q^\star; \gamma) + DF(Q_r; \gamma)\right] \\ s.t. & \\ q_{i,r} &= \arg\max \pi_{i,r}(q_i; \theta_i). \end{aligned}$$

The regulation I consider in this paper is implemented in two stages. The regulation takes into account the imperfections of the permit market and the information structure of the regulatory environment. In order to derive tractable results I introduce an affine linear model. These are described next.

3.2.1 Two-stage regulation with an imperfect permit market

To begin with, I introduce the timing of the regulation. The two-stage regulation is conducted by the following steps:

- t_0: All firms and the regulator learn the distribution functions of uncertain parameters and the functional forms of firms' abatement costs. Each firm receives its own signal s_i.
- t_1: The first stage - the information stage. The regulator conducts an auction, in which emission permits are initially allocated to firms. The regulator announces both the rules of the auction mechanism and rules of the regulation during the regulation period. In the auction, each firm i simultaneously submits a demand schedule to the regulator. The regulator sets the clearing price and the total quantity of permits to be allocated. Then it distributes permits to firms and collects the auction payments from firms. The regulation period starts after the auction.
- $t_1 - t_2$: The second stage - the regulation stage. Firms are allowed to trade permits with each other. In addition, they may purchase more permits from the regulator or sell permits back to the regulator according to non-constant permit supply schedules. Firms learn their cost parameters during the regulation period.
- t_2: All firms have learned their cost parameters. The time point t_2 is not known to any firm or the regulator at the outset. The true pollution damage is not revealed.

In the information stage, the regulator conducts a mechanism, the main goal of which is to collect private information from regulated firms. In addition, the first-stage mechanism serves as an initial allocation method for permits. Utilizing a generalized Vickrey auction, the initial allocation is ex-post efficient, if the incentive compatibility conditions are satisfied. The incentive compatibility of this regulation is defined as follows.

Definition 3.1. *The Vickrey auction in the information stage is said to be incentive compatible (IC) if bidding sincerely in the auction is the best response to other bidders' strategies when they bid sincerely.*

Three important issues must be addressed. First, without any regulation, no abatement and thus no learning about the abatement costs will occur. Second, the regulator is not able to conduct auctions continuously one after another. There is always a time period between two consecutive auctions. During this period between two possible auctions, firms take different actions. They make decisions about abatement technologies, they produce products for primary markets, they trade inputs, outputs and, perhaps, emissions permits with each other. Most importantly, firms learn and want to adjust their permit holdings. Due to the learning and firms' actions, the regulator is willing to implement a second-stage regulation in the period following an auction. Hence, at the beginning of the regulation period, the regulator allocates non-constant permit schedules to firms. After the initial allocation, firm i may purchase more permits from the regulator or sell excess permits back to the regulator according to the schedule allocated to it. In addition, firm i is free to make transactions in the permit market with other firms.

Third, at least with the structure of this paper, the regulator does not need the private information of firms in order to implement the second-best regulation if the permit market is perfect and if the regulator uses a non-constant regulation.[7] However, firms' private information is valuable to the regulator if the permit market is not perfect. Optimal permit schedules take into account the frictions of the permit market as well as the information from the first-stage auction.

With an imperfect permit market, it is reasonable to reduce the amount of trading in the permit market and to make the initial allocation of permits as efficient as possible. I model permit market trade frictions as increasing costs of actual trades in the market. The costs of trading affect symmetrically both the seller and the buyer side of the market. I keep the model relatively general and I do not specify the nature or source of trade frictions. Frictions can be a consequence of transaction costs (e.g. Stavins 1995), market power (e.g. Malueg and Yates 2009), asymmetric information between traders (Myerson and Satterthwaite 1983) or any other source of market imperfection. However, to give a name to these costs, I call them trading costs. The trading cost function is equal for all firms and is denoted by $TC(\Delta q_i)$, where $\Delta q_i \geq 0$ is the amount of trading in the permit market of firm i. The more firm i trades in the market, the higher the costs of the transactions are. Hence $TC \geq 0$ and $TC' \geq 0$. The functional form of the trading costs is explained in the following section.

Also, the information about signals and distribution functions related to abatement costs is valuable to the regulator if there is a statistical dependence between the benefits and costs of emissions reductions. The signals are jointly affected by uncertain abatement costs and pollution damage, which should be taken into account when implementing the regulation. Without proper information, the regulator is not able to derive the relevant expected marginal damage function and hence optimal permit schedules. In this paper I assume that the regulator does know the distribution functions of the unknown cost parameters and the functional forms of firms' abatement costs. However, the regulator does not know the firms'

[7] I explain the result of a perfect permit market in Appendix 3.A.

signal realization.[8]

3.2.2 Affine linear model

I follow previous literature (e.g. Weitzman 1974, 1978) and linearize the unknown marginal functions around the first-best. As Weitzman (1978, p. 686) puts it:

> "A linear approximation might be rationalized on one of two grounds. The amount of uncertainty could be small enough to keep the range of output responses sufficiently limited to justify a first-order approximation. Or, it might just happen that total cost and benefit functions are almost quadratic to begin with. At any rate, the possibility of sharply characterizing an optimal solution makes the linear case a natural preliminary to any more general analysis."

In addition, I apply the affine information structure from Vives (2010, 2011). In this model all random variables are normally distributed. Although fairly detailed, this is convenient when calculating the conditional expectations of the payoff-relevant variables. The conditional expectations of the value parameters are linear functions of the agents' information. With linear marginal value functions and with linear strategies it is then possible to construct a Linear Bayesian Demand Function Equilibrium for the model (see Vives 2011).

Definition 3.2. *The affine linear model is defined by equations (3.3) - (3.14), where the distribution functions of the uncertain variables and the functional forms of abatement costs and pollution damage are common knowledge.*

Taking a second-order approximation of $U_i(q_i;\theta_i)$ around the first-best gives the following linear marginal value function (or the marginal abatement cost function):

$$u_i(q_i;\theta_i) = \theta_i - \beta q_i. \tag{3.3}$$

Cost parameter θ_i and slope β are such that $\theta_i \equiv U_i'(q_i^\star;\theta_i) - U_i''(q_i^\star;\theta_i)\, q_i^\star > 0$ and $\beta \equiv -U_i''(q_i^\star;\theta_i) \geq 0$. To simplify the model, the slope parameter is assumed to be constant and common knowledge to all firms and to the regulator. As noted already, the cost parameters are uncertain, but they are drawn from the same prior distribution, $\theta_i \sim N\left(\bar{\theta}, \sigma_\theta^2\right)$. Moreover, the cost parameters are symmetrically correlated between firms with a covariance, $cov\left[\theta_i, \theta_j\right] = \rho\sigma_\theta^2$, where only positive correlations are assumed, i.e. $\rho > 0$. Thus the average

[8] Cases where the distribution functions of the unknown parameters, the performance of the permit market and the functional forms of abatement costs are unknown to the regulator are left for future work.

cost parameter $\theta_m = \frac{1}{n}\sum_{i=1}^{n}\theta_i$ is normally distributed with an expected value $E[\theta_m] = \bar{\theta}$ and a variance $var[\theta_m] = \frac{1}{n}(1+(n-1)\rho)\sigma_\theta^2$.

At the initial time point t_0, each firm receives a noisy signal of its own cost parameter, $s_i = \theta_i + \varepsilon_i$. The noise terms are identically and independently distributed around zero, $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$. The average signal is denoted by $s_m = \frac{1}{n}\sum_{i=1}^{n} s_i$. Its distribution has an expected value $E[s_m] = \bar{\theta}$ and a variance $var[s_m] = \frac{1}{n}(\sigma_\varepsilon^2 + (1+(n-1)\rho)\sigma_\theta^2)$.[9] The expected marginal values are interdependent and I thus assume that $\sigma_\varepsilon^2 > 0$ and $0 < \rho < 1$. [10]

Initially firms can condition their cost parameters only on their own signals. However, given the structure of the model and given that the Vickrey auction is incentive compatible (Definition 3.1), the clearing price of the auction reveals the average signal s_m. In other words, the clearing price p is sufficient statistics for s_m and $E[\theta_i|\, s_m]$ is informationally equivalent to $E[\theta_i|\, p]$. I explain the procedure later in more detail (see also Chapter 2 of this thesis and Vives 2011). Furthermore, due to the symmetric correlation between firms' cost parameters, we may also write $E[\theta_i|\, \mathbf{s}] = E[\theta_i|\, s_m] = E[\theta_i|\, p]$. Thus firms are able to update their beliefs about their own cost parameters conditional on information from the auction. The expected value of θ_i conditional on signal s_i and the clearing price of the Vickrey auction (or the average signal s_m, or the whole signal vector $\mathbf{s}$) writes as[11]

$$E[\theta_i|\, \mathbf{s}] = A\bar{\theta} + Bs_i + Cns_m, \tag{3.4}$$

where

$$\begin{aligned} A &= \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + (1+(n-1)\rho)\sigma_\theta^2} \\ B &= \frac{(1-\rho)\sigma_\theta^2}{\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2} \\ C &= \frac{\rho\sigma_\theta^2\sigma_\varepsilon^2}{(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2)(\sigma_\varepsilon^2 + (1+(n-1)\rho)\sigma_\theta^2)}. \end{aligned}$$

The variance of θ_i conditional on $\mathbf{s}$ is

$$var[\theta_i|\, \mathbf{s}] = (B+C)\sigma_\varepsilon^2. \tag{3.5}$$

[9]Note also that $var[s_m] = cov[s_i, s_m]$.

[10]The model would be one with independent private values if $\rho = 0$. The pure common values case is when cost parameters are perfectly correlated and thus $\rho = 1$.

[11]See Appendix 2.A and e.g. DeGroot (1970). The expected value of θ_i conditional only on signal s_i is $E[\theta_i|\, s_i] = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2+\sigma_\theta^2}\bar{\theta} + \frac{\sigma_\theta^2}{\sigma_\varepsilon^2+\sigma_\theta^2}s_i$. Note also that $A + B + nC = 1$ and thus $1 - A = B + nC$.

From (3.4) the conditional expected value and the variance of the average cost parameter are written as, respectively,

$$E\left[\theta_m|\,\mathbf{s}\right] = \bar{\theta} + (1-A)\left(s_m - \bar{\theta}\right), \tag{3.6}$$

$$var\left[\theta_m|\,\mathbf{s}\right] = A \cdot var\left[\theta_m\right] = (B+nC)\,\frac{1}{n}\sigma_\varepsilon^2. \tag{3.7}$$

I denote the remaining uncertainty of firm i's cost parameter by $\varepsilon_i^s = \theta_i - E\left[\theta_i|\,\mathbf{s}\right]$, which has a normal distribution, i.e. $\varepsilon_i^s \sim N\left(0,(B+C)\,\sigma_\varepsilon^2\right)$. The covariance between ε_i^s and ε_j^s is $cov\left[\varepsilon_i^s, \varepsilon_j^s\right] = C\sigma_\varepsilon^2$. Respectively, the distribution of the remaining aggregate uncertainty, i.e. $n\varepsilon_m^s = \sum_{i=1}^n \varepsilon_i^s$, has parameters $n\varepsilon_m^s \sim N\left(0,(B+nC)\,n\sigma_\varepsilon^2\right)$.

From (3.3) and (3.4) the expected marginal value function conditional on $\mathbf{s}$ is written as

$$\begin{aligned} v_i\left(q_i;\mathbf{s}\right) &\equiv E\left[u_i\left(q_i;\theta_i\right)|\,s_i,\mathbf{s}_{-i}\right]. \\ &= A\bar{\theta} + Bs_i + Cns_m - \beta q_i. \end{aligned} \tag{3.8}$$

Respectively, the damage function is also approximated around the first-best. The first-order linear approximation of the marginal damage function is written as

$$MDF\left(Q;\gamma\right) = \gamma + \delta Q, \tag{3.9}$$

where slope $\delta \equiv DF''\left(Q^\star;\gamma\right) \geq 0$ is common knowledge. The damage parameter $\gamma \equiv DF'\left(Q^\star;\gamma\right) - DF''\left(Q^\star;\gamma\right)Q^\star$ is, however, uncertain. It is a draw from the distribution with parameters $\gamma \sim N(\bar{\gamma},\sigma_\gamma^2)$. Furthermore, I allow γ to be correlated with θ_m and I denote the covariance by $cov\left[\gamma,\theta_m\right] = \sigma_{\gamma\theta}$. Using these, the expected marginal damage function conditional on the sum of all signals $ns_m = \sum\limits_{i=1}^{n} s_i$ is

$$\begin{aligned} y\left(Q;\mathbf{s}\right) &\equiv E\left[MDF\left(Q;\gamma\right)|\,\mathbf{s}\right] \\ &= \bar{\gamma} + nZ\left(s_m - \bar{\theta}\right) + \delta Q, \end{aligned} \tag{3.10}$$

where $E\left[\gamma|\,\mathbf{s}\right] = \bar{\gamma} + nZ\left(s_m - \bar{\theta}\right)$ and

$$Z = \frac{cov\left[\gamma, ns_m\right]}{var\left[ns_m\right]} = \frac{\sigma_{\gamma\theta}}{\sigma_\varepsilon^2 + \left(1+(n-1)\,\rho\right)\sigma_\theta^2}. \tag{3.11}$$

I assume that the true damage parameter remains uncertain in the model. The uncertainty related to the damage parameter conditional on $\mathbf{s}$ is $\varepsilon_\gamma^s = \gamma - \bar{\gamma} - Z\left(ns_m - n\bar{\theta}\right)$. Furthermore, the variance of γ conditional on $\mathbf{s}$ is

$$var\left[\gamma|\,\mathbf{s}\right] = var\left[\varepsilon_\gamma^s\right] = \sigma_\gamma^2 - nZ\sigma_{\gamma\theta}, \tag{3.12}$$

and the conditional covariance between the average cost and damage parameters is simply

$$cov\left[\gamma, \theta_m|\,\mathbf{s}\right] = A\sigma_{\gamma\theta}. \tag{3.13}$$

I also assume that the marginal trading cost function is linear:

$$MTC\left(\Delta q_i\right) = \omega \Delta q_i, \tag{3.14}$$

where $MTC\left(\Delta q_i\right) = TC'\left(\Delta q_i\right)$ and ω denotes the coefficient of market performance. For simplicity, I assume that ω is common knowledge and exogenous to the other parameters of the model. Note that the assumption about the linear and increasing marginal trading costs is just a simplification. For instance, Stavins (1995) discusses transaction costs in tradeable permit markets with either an increasing, decreasing or constant marginal transaction cost function. However, to my understanding, using a trading cost function with decreasing or constant marginal trading costs would not change the results qualitatively.

Now, given this structure, the pollution of firm i in the first-best is given by $q_i^\star = \frac{1}{\beta}\left(\theta_i - p^\star\right)$, where $p^\star$ is the first-best price:

$$p^\star = \frac{\delta\theta_m + \frac{\beta}{n}\gamma}{\frac{\beta}{n} + \delta}. \tag{3.15}$$

Furthermore, the welfare maximizing pollution level may write

$$Q^\star = \frac{\theta_m - \gamma}{\frac{\beta}{n} + \delta}. \tag{3.16}$$

Moreover, the second-best regulation minimizes the following linear approximation of the expected deadweight loss equation:

$$\begin{aligned} \min_r E\left[DWL_r\right] &= E\left[\sum_{i=1}^n \left\{\int_{q_{i,r}}^{q_i^\star} u_i\left(x;\theta_i\right)dx\right\} - \int_{Q_r}^{Q^\star} MDF\left(X;\gamma\right)dX\right] \\ &\approx E\left[\sum_i^n \theta_i\left(q_i^\star - q_{i,r}\right) - \frac{\beta}{2}\sum_i^n\left(q_i^{\star 2} - q_{i,r}^2\right) - \gamma\left(Q^\star - Q_r\right) - \frac{\delta}{2}\left(Q^{\star 2} - Q_r^2\right)\right]. \end{aligned} \tag{3.17}$$

This problem is a linearized version of (3.2).

The main objective of this paper is to derive conditions in which the information mechanism of the two-stage regulation is incentive compatible. If this is the case, the regulator learns the firms' private information and is able to provide an ex-post efficient allocation of permits after the auction. The ex-post efficient allocation, conditional on the revelation of signals, is denoted by $q^e(\mathbf{s}) = (q_1^e(\mathbf{s}), \ldots, q_n^e(\mathbf{s})) \equiv E[q^\star | \mathbf{s}]$. I show in Proposition 3.1 that given the affine linear model, the information mechanism is incentive compatible whenever the correlation between emissions reduction benefits and costs is not too high. More precisely, the IC condition is satisfied if

$$-\frac{n\delta}{\beta} \leq \frac{\sigma_{\gamma\theta}}{var[\theta_m]} \leq 1. \tag{3.18}$$

Particularly, the latter inequality $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} \leq 1$ ensures that the aggregate quantity rule of the first-stage auction is weakly increasing in each bidder's signal. Note that $nZ \leq 1 - A$ is equivalent to $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} \leq 1$. Moreover, the former inequality $-\frac{n\delta}{\beta} \leq \frac{\sigma_{\gamma\theta}}{var[\theta_m]}$ guarantees that the optimal permit supply schedules of the second-stage regulation are weakly increasing in permit purchases. This, on the other hand, is equivalent to $-\frac{n\delta}{\beta}(1 - A) \leq nZ$.

I solve the problem backwards. In the next section, I derive the optimal permit supply schedules given the information the regulator has at the time of implementing the non-constant quantity regulation. I also examine the value of firms' private information to the regulator in different information structures and permit market conditions. In Section 3.4, I then describe the Vickrey payment rule, which gives incentives to firms to reveal their expected values to the regulator in the information stage.

3.3 Regulation stage

In this section, I derive the second-best regulation in the second stage given the information the regulator has. The regulator's information parameter is denoted by I. In particular, $I = 0$ denotes that the regulator knows only the prior information. Then the regulation is implemented in the absence of the private information of firms. In contrast, if $I = \mathbf{s}$, the regulator has complete information about signal vector $\mathbf{s}$ and is able to implement the ex-post efficient allocation of permits at the beginning of the regulation period. Following Roberts and Spence (1976; later RS), Weitzman (1978; later W) and Yates (2012), I assume that the regulator uses the following linear permit schedules in the regulation stage:

$$T_i(q_i; I) = \bar{p}(I) + \tau(q_i - \bar{q}_i(I)). \tag{3.19}$$

The price $\bar{p}(I) = E[p^\star | I]$ denotes the expected first-best price and $\bar{q}_i(I) = E[q_i^\star | I]$ the expected first-best allocation of permits to firm i conditional on information I. The slope of the permit supply schedule is τ. Weitzman (1978) uses general tax functions but the optimal schedules are linear in his model due to the linearized system, but also due to regularity assumptions about the distributions of the random variables. In contrast, Yates (2012) does not make any assumptions about distributions of uncertain parameters, but he restricts his analysis to linear schedules. Due to the linearized system and normal distributions of the random variables, the linear schedules are optimal in this model. With knowledge of prior information only, the expected first-best price is

$$\bar{p}(0) = \frac{n\delta\bar{\theta} + \beta\bar{\gamma}}{\beta + n\delta}. \tag{3.20}$$

Respectively, if $I = \mathbf{s}$, the expected first-best price is

$$\begin{aligned} \bar{p}(\mathbf{s}) &= \frac{n\delta\bar{\theta}_m(\mathbf{s}) + \beta\bar{\gamma}(\mathbf{s})}{\beta + n\delta} \\ &= \bar{p}(0) + \frac{n\delta(1-A) + \beta n Z}{\beta + n\delta}\left(s_m - \bar{\theta}\right). \end{aligned} \tag{3.21}$$

The last line comes from inserting $\bar{\theta}_m(\mathbf{s}) = E[\theta_m | \mathbf{s}]$ from (3.6) and $\bar{\gamma}(\mathbf{s}) = E[\gamma | \mathbf{s}]$ from (3.10). The initial allocation is given by $\bar{q}_i(I) = \frac{1}{\beta}\left(\bar{\theta}_i(I) - \bar{p}(I)\right)$, where $\bar{\theta}_i(I) = E[\theta_i | I]$. Without any information about signals, $\bar{\theta}_i(0) = \bar{\theta}$ for all i and the regulator allocates an equal amount of permits to each firm:

$$\bar{q}_i(0) = \bar{q} = \frac{\bar{\theta} - \bar{\gamma}}{\beta + n\delta}. \tag{3.22}$$

On the other hand, when the regulator knows the signal vector, $\bar{\theta}_i(\mathbf{s}) = A\bar{\theta} + Bs_i + Cns_m$ from (3.4), the regulator can implement an ex-post efficient allocation:

$$\begin{aligned} \bar{q}_i(\mathbf{s}) &= q_i^e(\mathbf{s}) \\ &= \underbrace{\bar{q} + \left(\frac{1 - A - nZ}{\beta + n\delta}\right)\left(s_m - \bar{\theta}\right)}_{\bar{q}_m(\mathbf{s})} + \frac{B}{\beta}(s_i - s_m), \end{aligned} \tag{3.23}$$

where $\bar{q}_m(\mathbf{s}) = \frac{1}{n}\bar{Q}(\mathbf{s})$ is the allocation of the average firm indexed with m and receiving signal s_m. Firm i may purchase (or sell back) permits from (to) the regulator according to the price schedule (3.19). The net purchases of firm i from the regulator are denoted by h_i. This is a sum of the initial allocation $\bar{q}_i(I)$ and transactions with the regulator in the regulation period. In addition, firms are allowed to trade permits with each other. I assume full compliance. At the end of the regulation period each firm holds an amount of permits equal to its emissions in the regulation period. Firms report their emissions honestly to the regulator. Hence, $\Delta q_i = |q_i - h_i| \geq 0$ is firm i's amount of trading in the permit market. Firm i is a buyer of permits if $q_i > h_i$ and a permit seller if $h_i > q_i$.[12]

The problem of the regulator is to find a slope τ that minimizes the expected deadweight loss from (3.17). It has to take into account the reactions of firms. Firm i maximizes its profits with respect to pollution q_i and purchases from the regulator h_i given the supply schedule $T_i(q_i; I)$ and the market performance of the secondary market defined by ω:

$$\begin{aligned} \max_{q_i, h_i} \pi_{i,\tau}(q_i, h_i; \theta_i) &= \int_{\bar{q}_i(I)}^{q_i} u_i(x; \theta_i)\, dx - \int_{\bar{q}_i(I)}^{h_i} T_i(x; I)\, dx \\ &+ \int_0^{\Delta q_i} \{p_\tau - \omega x\}\, dx \mathbf{1}_{\{h_i > q_i\}} - \int_0^{\Delta q_i} \{p_\tau + \omega x\}\, dx \mathbf{1}_{\{q_i > h_i\}}, \end{aligned} \tag{3.24}$$

where p_τ is the equilibrium price of the secondary market and $\mathbf{1}$ is the indicator function with a value of 1 if its argument is true and otherwise 0. The first-order conditions with respect to pollution q_i and permit purchases h_i are given by, respectively,

$$\frac{d\pi_{i,\tau}(q_i, h_i; \theta_i)}{dq_i} = u_i(q_i; \theta_i) - p_\tau + \omega(h_i - q_i) = 0 \tag{3.25}$$

and

$$\frac{d\pi_{i,\tau}(q_i, h_i; \theta_i)}{dh_i} = -T_i(h_i; I) + p_\tau - \omega(h_i - q_i) = 0. \tag{3.26}$$

Let $q_{i,\tau}$ and $h_{i,\tau}$ denote the profit-maximizing outcomes. From the first-order conditions it is easy to see that $u_i(q_{i,\tau}; \theta_i) = T_i(h_{i,\tau}; I)$, which implies

$$\tau h_{i,\tau} + \beta q_{i,\tau} = (\beta + \tau)\bar{q}_i(I) + \left(\theta_i - \bar{\theta}_i(I)\right). \tag{3.27}$$

Summing (3.27) from 1 to n and using the market-clearing condition $\sum_{i=1}^n h_{i,\tau} = \sum_{i=1}^n q_{i,\tau} = Q_\tau$ gives the total level of pollution:

$$Q_\tau = \bar{Q}(I) + \left(\frac{n}{\beta + \tau}\right)\left(\theta_m - \bar{\theta}_m(I)\right). \tag{3.28}$$

[12]Note that if $\tau = 0$ the permit supply schedule reduces to constant tax $T_i = \bar{p}(I)$. On the other hand, if $\tau \to \infty$, the regulation is a constant quantity regulation. Then, in the beginning of the regulation period, the regulator allocates pollution permits to firms according to $\bar{q}(I) = (\bar{q}_1(I), \ldots, \bar{q}_n(I))$ and lets firms trade permits freely in the secondary market. These two constant regulations following the information mechanism are examined in Chapter 2.

The average firm (with cost parameter θ_m) does not trade in the equilibrium:

$$q_{m,\tau} = h_{m,\tau} = \bar{q}_m(I) + \left(\frac{1}{\beta+\tau}\right)\left(\theta_m - \bar{\theta}_m(I)\right). \tag{3.29}$$

Plugging (3.29) into the first-order condition $p_\tau = u_m(q_{m,\tau};\theta_m)$ gives the equilibrium price:

$$p_\tau = \bar{p}(I) + \left(\frac{\tau}{\beta+\tau}\right)\left(\theta_m - \bar{\theta}_m(I)\right). \tag{3.30}$$

Moreover, using (3.25), (3.27) and (3.30) the equilibrium outcomes of firm i are

$$q_{i,\tau} = \bar{q}_i(I) + \left(\frac{1}{\beta\tau+\omega\beta+\omega\tau}\right)\left\{(\tau+\omega)\left(\theta_i - \bar{\theta}_i(I)\right) - \left(\frac{\tau^2}{\beta+\tau}\right)\left(\theta_m - \bar{\theta}_m(I)\right)\right\}, \tag{3.31}$$

and

$$h_{i,\tau} = \bar{q}_i(I) + \left(\frac{1}{\beta\tau+\beta\omega+\tau\omega}\right)\left\{\omega\left(\theta_i - \bar{\theta}_i(I)\right) + \left(\frac{\beta\tau}{\beta+\tau}\right)\left(\theta_m - \bar{\theta}_m(I)\right)\right\}. \tag{3.32}$$

Together these imply

$$q_{i,\tau} - h_{i,\tau} = \left(\frac{\tau}{\beta\tau+\omega\beta+\omega\tau}\right)\left\{(\theta_i - \theta_m) - \left(\bar{\theta}_i(I) - \bar{\theta}_m(I)\right)\right\}. \tag{3.33}$$

Let us assume that $\tau \geq 0$. Firm i is then on the demand side of the permit market ($q_{i,\tau} > h_{i,\tau}$), if the positive deviation of firm i's cost parameter from the average value, i.e. $\theta_i - \theta_m$, is greater than the difference between the expected cost parameter of firm i and the expected average cost parameter, i.e. $\bar{\theta}_i(I) - \bar{\theta}_m(I)$. Hence, even if the cost parameter of firm i decreases from the expected value, firm i may be a demander of permits in the secondary market if, on average, the cost parameters decrease even more.

Figure 3.1 describes the equilibrium outcomes in a three-firm market (firms j, m and k) in different market conditions, where $\theta_j < \theta_m < \theta_k$ with $\theta_m = \frac{1}{2}(\theta_j + \theta_k)$. Figure 3.1a presents the extreme cases of a perfect ($\omega = 0$) and collapsed ($\omega \to \infty$) permit market, whereas Figure 3.1b presents the case where $0 < \omega < \infty$. Suppose, for simplicity, that the regulator has no information about firms' signals and thus $I = 0$.[13] When the regulator has only prior information available, it implements the same permit supply schedule for each firm, i.e. $T_i(q_i) = \bar{p} + \tau(q_i - \bar{q}_i)$ for all $i = j, m, k$. In Figure 3.1, the slope of the permit supply schedule τ is fixed in different cases of ω. Respectively, from the regulator's point of

[13] To simplify notation, I have omitted the argument of the information parameter $I = 0$ in Figure 3.1 and in what follows. The market performance $\omega \in (0, \infty)$ is denoted with a superscript.

view, each firm has the same expected marginal value function $v_i(q_i)$. I have also assumed that $\theta_m > \bar{\theta}$ and, hence, the equilibrium price is greater than the expected first-best price, $p_\tau > \bar{p}$. Due to symmetry, market frictions do not affect the equilibrium price, but they reduce trading in the market.

Suppose first that the permit market is perfect ($\omega = 0$, Figure 3.1a). Then all firms purchase an equal amount of permits from the regulator $h^0_{i,\tau}$. Furthermore, in the absence of permit market trade frictions, the market mechanism provides a cost-efficient solution where $p_\tau = u_i\left(q^0_{i,\tau}\right) = T_i\left(h^0_{i,\tau}\right)$ for all i. The average firm m does not trade in the permit market, whereas firm k buys $\Delta q_k = q^0_{k,\tau} - h^0_{i,\tau} = h^0_{i,\tau} - q^0_{j,\tau} = \Delta q_j$ permits from firm j. In equilibrium, purchasing one more unit from the regulator or reducing one more unit of pollution is more costly for every firm than the benefits it receives by selling one more permit in the market. The problem of the regulator is then only to care about the aggregate efficiency, because the permit market trading equalizes the marginal abatement costs of firms.

With positive trading costs ($\omega > 0$, Figure 3.1b), each firm's benefits of purchasing permits from the regulator and selling them to other firms is reduced. Hence, the seller of permits in the secondary market, i.e. firm j, reduces its purchases from the regulator to $h^\omega_{j,\tau} < h^0_{i,\tau}$. Respectively, firm k buys less permits from the market and more units from the regulator $h^\omega_{k,\tau} > h^0_{i,\tau}$. Hence, trading between firms reduces as ω increases. In equilibrium, the marginal abatement cost function is equal to the marginal costs of purchasing permits from the regulator, i.e. $u_i\left(q^\omega_{i,\tau}\right) = T_i\left(h^\omega_{i,\tau}\right)$. Moreover, for the permit buyer (firm k), these must be equal to the marginal cost of purchasing permits from the market, i.e. $p_\tau + \omega \Delta q_k$. For the permit seller (firm j), on the other hand, $u_j\left(q^\omega_{j,\tau}\right) = T_j\left(h^\omega_{j,\tau}\right) = p_\tau - \omega \Delta q_k$. Thus the equilibrium is not cost-efficient, while $u_j\left(q^\omega_{j,\tau}\right) < u_m\left(q_{m,\tau}\right) < u_k\left(q^\omega_{k,\tau}\right)$.

In another extreme case, the permit market is totally collapsed ($\omega \to \infty$, Figure 3.1a) and every firm purchases all the permits it needs from the regulator $h^\infty_{i,\tau} = q^\infty_{i,\tau}$. This increases the deviation of equilibrium pollution from the cost-efficient solution.

Hence, the regulator has to take into account the expected deviations when choosing the permit schedule functions. This is done by lowering the slope of the permit schedules when ω increases. However, at the same time the aggregate pollution becomes more price-sensitive and the risks of high environmental damage increase. Hence there is a trade-off between cost-efficiency and aggregate efficiency when the permit market is imperfect.

Using $q_{i,\tau}$ from (3.31) and Q_τ from (3.28) and plugging these into (3.17) gives the expected deadweight loss of the non-constant regulation defined in equation (3.19). I derive the ex-

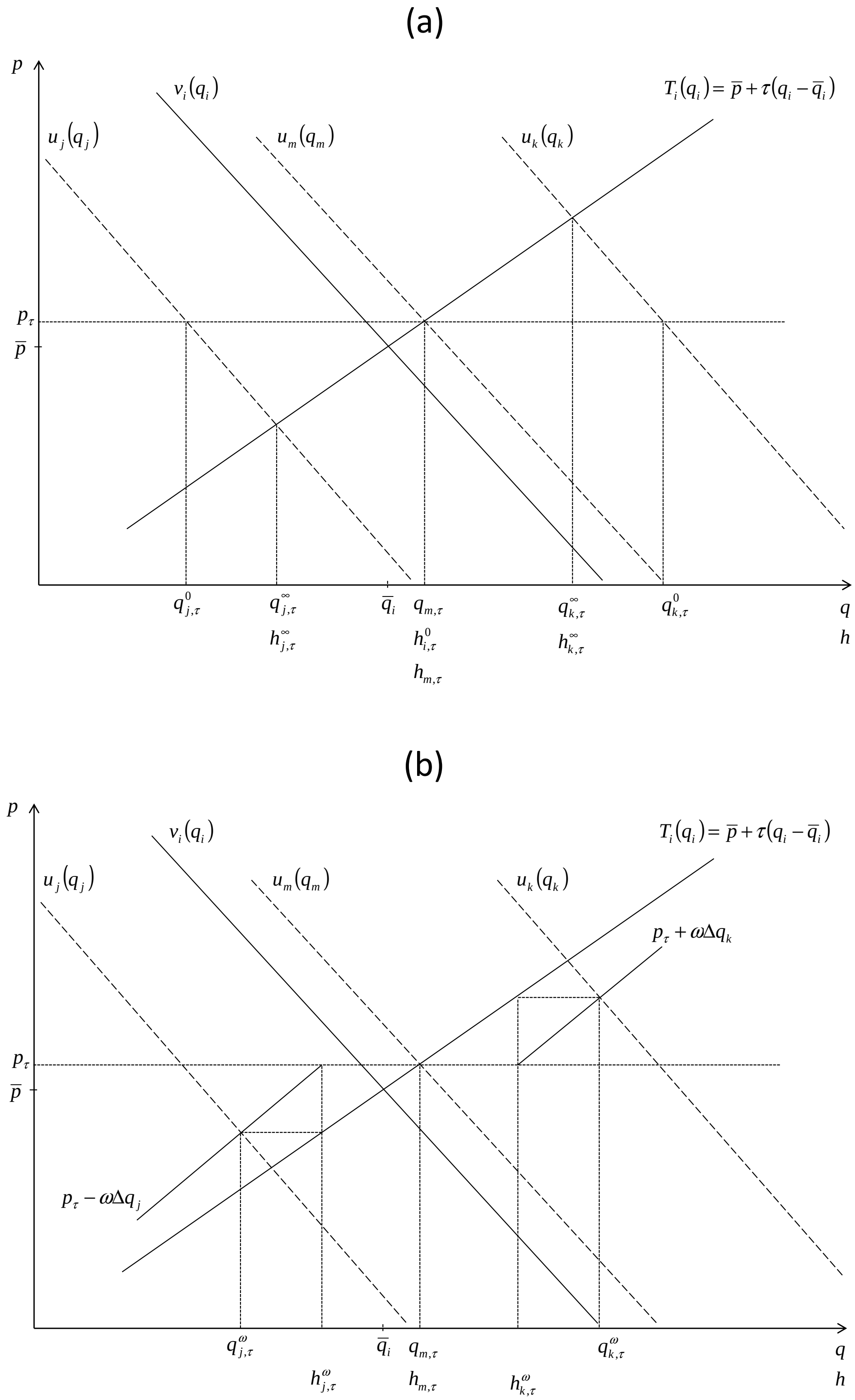

Figure 3.1: Equilibrium of the regulation stage.

pected deadweight loss in Appendix 3.B. It is written as

$$E\left[DWL_{\tau}\left(I,\omega\right)\right] = \frac{1}{2}\left(\frac{1}{\frac{\beta}{n}+\delta}\right)\left\{\left(\frac{n\delta-\tau}{\beta+\tau}\right)^{2} var\left[\theta_{m}\right|I] + var\left[\gamma\right|I]\right\} + \left(\frac{1}{\frac{\beta}{n}+\delta}\right)\left(\frac{n\delta-\tau}{\beta+\tau}\right) cov\left[\gamma,\theta_{m}\right|I] + \frac{n}{2\beta}\left(\frac{\tau}{\frac{\beta\tau}{\omega}+\beta+\tau}\right)^{2}\left\{var\left[\theta_{i}\right|I] - var\left[\theta_{m}\right|I]\right\}. \quad (3.34)$$

The optimal τ, given information I and market performance ω, solves $\frac{d(E[DWL_{\tau}(I,\omega)])}{d\tau} = 0$ or, respectively, the following equation:

$$f_{\tau}\left(\tau\right|I,\omega) \equiv \left(\frac{n\delta-\tau}{\beta+\tau}\right) var\left[\theta_{m}\right|I] + cov\left[\gamma,\theta_{m}\right|I] - \left(\frac{\tau\left(\beta+\tau\right)^{2}}{\left(\frac{\beta\tau}{\omega}+\beta+\tau\right)^{3}}\right)\left(var\left[\theta_{i}\right|I] - var\left[\theta_{m}\right|I]\right) = 0. \quad (3.35)$$

The optimal slope is denoted by $\tau\left(I,\omega\right) \equiv \left(\tau : f_{\tau}\left(\tau\right|I,\omega) = 0\right)$. Note that, when $I = \mathbf{s}$, equation (3.35) yields

$$f_{\tau,s}\left(\tau\right) \equiv f_{\tau}\left(\tau\right|I=\mathbf{s},\omega) = \left(\frac{n\delta-\tau}{\beta+\tau}\right)\left(1-A\right) + nZ - \left(\frac{\tau\left(\beta+\tau\right)^{2}}{\left(\frac{\beta\tau}{\omega}+\beta+\tau\right)^{3}}\right)\left(n-1\right)B = 0. \quad (3.36)$$

I use equation (3.36) when I later derive the main results of the paper. However, the main results of this section are provided in the following four lemmas.

Lemma 3.1. *Given the affine linear model, the slopes of the permit supply schedules in the Weitzman (1978) and Roberts and Spence (1976) models, respectively τ^{W} and τ^{RS}, are the lower and upper bounds of the permit supply schedule slope $\tau\left(I,\omega\right)$, when ω moves from 0 to ∞. Given that $-\frac{n\delta}{\beta} \leq \frac{\sigma_{\gamma\theta}}{var[\theta_m]} \leq 1$, Weitzman's non-constant tax schedule provides the lower bound of the slope, $0 \leq \tau^{W}\left(I\right) \leq \tau\left(I,\omega\right) \leq \tau^{RS}\left(I\right)$. If $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} < -\frac{n\delta}{\beta}$, Weitzman's non-constant tax schedule provides the upper bound of the slope, $\tau^{RS}\left(I\right) \leq \tau\left(I,\omega\right) \leq \tau^{W}\left(I\right) < 0$.*

Proof. Suppose first that the resale market performs perfectly and thus $\omega = 0$. Then the regulator chooses $\tau(I,0) \equiv \tau^{RS}(I)$ such that

$$\begin{aligned} f_\tau(\tau| I, \omega = 0) &= \left(\frac{n\delta - \tau}{\beta + \tau}\right) var[\theta_m| I] + cov[\gamma, \theta_m| I] = 0 \\ &\Rightarrow \tau^{RS}(I) = \frac{\beta \cdot cov[\gamma, \theta_m| I] + n\delta \cdot var[\theta_m| I]}{var[\theta_m| I] - cov[\gamma, \theta_m| I]}. \end{aligned}$$

This is the optimal slope of the RS non-constant permit supply model when pollution damage and abatement costs are correlated. When $\sigma_{\gamma\theta} = 0$, the slope is $\tau^{RS} = n\delta$. On the other hand, when the resale market is collapsed or absent ($\omega \to \infty$), we get for $\tau(I,\infty) \equiv \tau^W(I)$:

$$\begin{aligned} f_\tau(\tau| I, \omega \to \infty) &= \left(\frac{n\delta - \tau}{\beta + \tau}\right) var[\theta_m| I] + cov[\gamma, \theta_m| I] \\ &\quad - \left(\frac{\tau}{\beta + \tau}\right)(var[\theta_i| I] - var[\theta_m| I]) = 0 \\ &\Rightarrow \tau^W(I) = \frac{\beta \cdot cov[\gamma, \theta_m| I] + n\delta \cdot var[\theta_m| I]}{var[\theta_i| I] - cov[\gamma, \theta_m| I]}. \end{aligned}$$

This is equal to the slope of Weitzman's (1978) non-constant tax regulation. Note that $var[\theta_m| \mathbf{s}] = A \cdot var[\theta_m]$ and $cov[\gamma, \theta_m| \mathbf{s}] = A \cdot cov[\gamma, \theta_m| 0]$. While $var[\theta_i| I] \geq var[\theta_m| I]$, it always holds that $\left|\tau^W(I)\right| \leq \left|\tau^{RS}(I)\right|$, when $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} \leq 1$. These prove the latter part of the Lemma. ■

The cases where $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} > 1$ are not considered in Lemma 3.1. However, when $var[\theta_m| I] < cov[\gamma, \theta_m| I] < var[\theta_i| I]$ the non-constant taxes are increasing ($\tau^W(I) > 0$) and the permit supply schedules of the RS model are decreasing ($\tau^{RS}(I) < 0$). When $cov[\gamma, \theta_m| I] > var[\theta_i| I]$ all these functions are decreasing such that $\tau^W(I) < \tau^{RS}(I) < -\beta$.

Lemma 3.2. *Given the affine linear model and a perfect permit market, the regulator does not need the private information of firms in order to implement the second-best regulation.*

Proof. Using again $var[\theta_m| \mathbf{s}] = A \cdot var[\theta_m]$ and $cov[\gamma, \theta_m| \mathbf{s}] = A \cdot cov[\gamma, \theta_m| 0]$ gives

$$\tau^{RS}(\mathbf{s}) = \frac{\beta A \sigma_{\gamma\theta} + n\delta A \cdot var[\theta_m]}{A \cdot var[\theta_m] - A\sigma_{\gamma\theta}} = \frac{\beta \sigma_{\gamma\theta} + n\delta \cdot var[\theta_m]}{var[\theta_m] - \sigma_{\gamma\theta}} = \tau^{RS}(0). \tag{3.37}$$

Denoting $\tau^{RS}(I) = \tau^{RS}$ we get

$$\frac{n\delta - \tau^{RS}}{\beta + \tau^{RS}} = \frac{-n\sigma_{\gamma\theta}}{(1 + (n-1)\rho)\sigma_\theta^2} = -\frac{nZ}{1-A}.$$

Furthermore, given a perfect secondary market ($\omega = 0$), the deadweight loss depends on the price difference $p_\tau - p^\star$ (see equation 3.61 in Appendix 3.B). Hence, if this price difference is independent of information I, the regulator is able to implement the second-best regulation without the information $I = \mathbf{s}$. Thus, in order to show that $DWL_\tau(\mathbf{s}, 0) = DWL_\tau(0, 0)$ when using $\tau = \tau^{RS}$, it remains to show that $p_\tau(\mathbf{s}) = p_\tau(0)$. From (3.30) we get

$$
\begin{aligned}
p_\tau(\mathbf{s}) &= \bar{p}(\mathbf{s}) + \left(\frac{\tau^{RS}}{\beta + \tau^{RS}}\right)\left(\theta_m - \bar{\theta}_m(\mathbf{s})\right) \\
&= \underbrace{\bar{p}(0) + \left(\frac{\tau^{RS}}{\beta + \tau^{RS}}\right)\left(\theta_m - \bar{\theta}\right)}_{p_\tau(0)} \\
&\quad + \left(\frac{n\delta(1-A) + \beta n Z}{\beta + n\delta} - \left(\frac{\tau^{RS}}{\beta + \tau^{RS}}\right)(1-A)\right)\left(s_m - \bar{\theta}\right) \\
&= p_\tau(0) + \left(\frac{1}{1 + \frac{n\delta}{\beta}}\right)\underbrace{\left\{\left(\frac{n\delta - \tau^{RS}}{\beta + \tau^{RS}}\right)(1-A) + nZ\right\}}_{=0}\left(s_m - \bar{\theta}\right). \blacksquare
\end{aligned}
$$

Lemma 3.3. *Given the affine linear model, the expected deadweight loss is decreasing in the slopes of the marginal damage function δ and the aggregate abatement cost function $\frac{\beta}{n}$, and increasing in the conditional variance of the damage parameter $var[\gamma|\theta_m]$, when the permit market is perfect and the regulator uses permit schedules from (3.19) with $\tau = \tau^{RS}$.*

Proof. Plugging $\frac{n\delta - \tau^{RS}}{\beta + \tau^{RS}} = -\frac{nZ}{1-A}$ into (3.34) when $\omega = 0$ and, for instance, $I = 0$ the expected deadweight loss reduces to

$$
\begin{aligned}
E\left[DWL_\tau(0,0)\right] &= \frac{1}{2}\left(\frac{1}{\frac{\beta}{n} + \delta}\right)\left\{\left(\frac{n\delta - \tau^{RS}}{\beta + \tau^{RS}}\right)^2 var\left[\theta_m|\, 0\right] + var\left[\gamma|\, 0\right]\right\} \\
&\quad + \left(\frac{1}{\frac{\beta}{n} + \delta}\right)\left(\frac{n\delta - \tau^{RS}}{\beta + \tau^{RS}}\right) cov\left[\gamma, \theta_m|\, 0\right] \\
&= \frac{1}{2}\left(\frac{1}{\frac{\beta}{n} + \delta}\right)\left\{\sigma_\gamma^2 - \frac{\left(\sigma_{\gamma\theta}\right)^2}{\frac{1}{n}\left(1 + (n-1)\rho\right)\sigma_\theta^2}\right\} \\
&= \frac{1}{2}\left(\frac{1}{\frac{\beta}{n} + \delta}\right) var\left[\gamma|\, \theta_m\right]. \blacksquare
\end{aligned}
$$

Lemma 3.3 implies that the correlation between emission reduction costs and benefits, whether negative or positive, improves welfare when the permit market performs perfectly. The regulator is able to use the private information of firms, which reveals valuable information about pollution damage and thus improves the outcome of the regulation.

Lemma 3.4. *Given the affine linear model, the slopes of the optimal permit supply schedules are not independent of information in the absence of a permit market.*

Proof. With non-constant taxes from the Weitzman (1978) model, we have

$$\tau^W(0) = \frac{\beta\sigma_{\gamma\theta} + n\delta \cdot var[\theta_m]}{\sigma_\theta^2 - \sigma_{\gamma\theta}}. \tag{3.38}$$

Note that $\frac{var[\theta_i|\mathbf{s}]}{A} = var[\theta_m] + (n-1)B \cdot var[s_m]$, and thus when $-\frac{n\delta}{\beta} < \frac{\sigma_{\gamma\theta}}{var[\theta_m]} < 1$,

$$\begin{aligned} \tau^W(\mathbf{s}) &= \frac{\beta\sigma_{\gamma\theta} + n\delta \cdot var[\theta_m]}{(n-1)B \cdot var[s_m] + var[\theta_m] - \sigma_{\gamma\theta}} \\ &= \frac{\beta\sigma_{\gamma\theta} + n\delta \cdot var[\theta_m]}{(1+(n-1)\rho B)\sigma_\theta^2 - \sigma_{\gamma\theta}} \\ &< \tau^W(0). \blacksquare \end{aligned} \tag{3.39}$$

Lemma 3.4 implies that $E[DWL_\tau(\mathbf{s},\infty)] < E[DWL_\tau(0,\infty)]$. More generally, the slopes of the optimal permit supply schedules are not independent of information when $\omega > 0$, which further implies $E[DWL_\tau(\mathbf{s},\omega)] < E[DWL_\tau(0,\omega)]$ whenever $\rho, \sigma_\theta^2 > 0$. See Appendix 3.D for the numerical simulations. Finally, if there was only one firm ($n = 1$), the slope of the Weitzman non-constant tax regulation would coincide with the slope of the Roberts and Spence non-constant quantity regulation.

3.4 Information stage

Now I turn to the auction mechanism of the information stage.[14] I consider a generalized VCG mechanism (or Vickrey auction). With interdependent values, finding a mechanism that is able to implement efficient allocation may not be possible (Jehiel and Moldovanu 2001). However, Ausubel and Cramton (2004) prove in their Theorem 1 that for any value functions satisfying continuity, value monotonicity and the single-crossing property, a Vickrey auction with reserve pricing has truthful bidding as an ex-post equilibrium. This holds for any monotonic aggregate quantity rule $\bar{Q}(\mathbf{s})$ and associated monotonic efficient assignment rule $q_i^e(\mathbf{s})$.

In addition, even if an equilibrium in an auction without resale is typically not an equilibrium in a auction followed by a resale market, the resale market does not distort the equilibrium of a Vickrey auction. In particular, Ausubel and Cramton state in their Theorem 2 that if the Vickrey auction with reserve pricing is followed by any resale process that is coalitionally-rational against individual bidders, truthful bidding remains an ex-post equilibrium.

[14] In Chapter 2 I use the same information structure and auction mechanism and explain the Vickrey auction in more detail.

In this section, I will derive conditions in which it is optimal for a firm to bid sincerely in a Vickrey auction, when every other bidder is bidding sincerely and when the auction is followed by the regulation stage.

3.4.1 Vickrey auction

I model the auction using the indirect interpretation of the generalized VCG mechanism introduced by Montero (2008). Montero applied the auction in a pure private values environment, whereas the values are interdependent in this model. Nevertheless, the rules of the auction mechanism are similar. Let us assume for a moment that every firm bids sincerely. Hence, each firm is a price-taker in the auction. Given this assumption I characterize the auction equilibrium strategies of firms. Then I examine whether an individual firm has incentives to deviate from the strategy of sincere bidding, given that every other firm is bidding sincerely. This gives the conditions of incentive compatibility.

Before describing the auction mechanism, first note that the expected marginal abatement cost function $v_i(q_i;\mathbf{s})$ from (3.8) satisfies continuity, value monotonicity and the single-crossing property. Secondly, in this model the aggregate quantity rule may be written as

$$\bar{Q}(\mathbf{s}) = \begin{cases} y^{-1}\left(v_{-i}\left(q^e_{-i}(\mathbf{s});\mathbf{s}\right);\mathbf{s}\right), & \text{if } q^e_i(\mathbf{s}) = 0 \\ y^{-1}\left(v_i\left(q^e_i(\mathbf{s});\mathbf{s}\right);\mathbf{s}\right), & \text{if } q^e_i(\mathbf{s}) > 0. \end{cases} \tag{3.40}$$

According to the aggregate quantity rule (3.40), the price-elastic supply of permits is thus

$$\begin{aligned} Q_S(p;\mathbf{s}) &= y^{-1}(p;\mathbf{s}) \\ &= \frac{1}{\delta}\left(p - \bar{\gamma} - nZ\left(s_m - \bar{\theta}\right)\right). \end{aligned} \tag{3.41}$$

Given that the Vickrey auction is ex-post efficient, it must hold that the clearing price is given by $p_v(\mathbf{s}) = \bar{p}(\mathbf{s}) = \bar{p}(0) + \left(\frac{\delta(1-A)+\beta Z}{\frac{\beta}{n}+\delta}\right)\left(s_m - \bar{\theta}\right)$. From this it is easy to see that the aggregate quantity rule $\bar{Q}(\mathbf{s})$ is weakly increasing in each bidder's signal if

$$\frac{dp_v(\mathbf{s})}{ds_i} \geq Z \;\Rightarrow\; var\left[\theta_m\right] \geq \sigma_{\gamma\theta}.$$

This gives the second inequality of (3.18).

In the auction, firms report continuous and decreasing bid functions $D_i(p;s_i)$ to the regulator. The inverse bid function is written $P_i(q_i;s_i) \equiv D_i^{-1}(q_i;s_i)$. The regulator clears the auction and determines the clearing price p_v at which total demand equals supply. Winning bids are all bids equal or above the clearing price. Total demand in the auction is $D(p;\mathbf{s}) = D_i(p;s_i) + D_{-i}(p;\mathbf{s}_{-i})$ where $D_{-i}(p;\mathbf{s}_{-i}) = \sum_{j\neq i} D_j(p;s_j)$ is the aggregate demand of every other bidder but bidder i.

The final price firms have to pay for the units received is not the clearing price. In a private values case, the marginal payment is the opportunity cost of each particular unit won by bidder i. I explain later how of the Vickrey price in the private values model differs from the model of interdependent values. Montero (2008) derives the Vickrey pricing rule as follows. In addition to the clearing price and the allocation of permits, the regulator determines paybacks for each firm. The share of the paybacks is defined by

$$\alpha_i = 1 - \frac{\int_0^{q_i} RS_i^{-1}(x;\mathbf{s}_{-i})\,dx}{RS_i^{-1}(q_i;\mathbf{s}_{-i})\,q_i}, \tag{3.42}$$

where $RS_i(p;\mathbf{s}_{-i}) = Q_S(p;\mathbf{s}) - D_{-i}(p;\mathbf{s}_{-i})$ is the residual supply for bidder i. While $p_v(\mathbf{s}) = RS_i^{-1}(q_i;\mathbf{s}_{-i})$ in the equilibrium, the total payment of bidder i in the auction writes as

$$\begin{aligned} R_{i,v} &= (1-\alpha_i)\,p_v(\mathbf{s})\,q_i \\ &= \int_0^{q_i} RS_i^{-1}(x;\mathbf{s}_{-i})\,dx. \end{aligned} \tag{3.43}$$

The payback mechanism induces bidders to bid with their expected marginal value functions, conditional on the aggregate information. The payback function of firm i is determined by the strategies of other bidders and thus the Vickrey price is equal to the residual supply function. The clearing price of the auction contains the information of other bidders' signals. Given the linear affine model, firms utilize linear strategies defined by

$$D_i(p;s_i) = a + bs_i - cp, \tag{3.44}$$

where a, b and c are some positive constants. Thus the aggregate demand in the auction is

$$D(p;\mathbf{s}) = na + nbs_m - ncp. \tag{3.45}$$

Furthermore, using (3.41) and (3.45) yields

$$ns_m = \left(\frac{1}{b+\frac{Z}{\delta}}\right)\left(\frac{1}{\delta}\left(p - \bar{\gamma} + Zn\bar{\theta}\right) - na + ncp\right). \tag{3.46}$$

With the affine linear model, the clearing price is sufficient statistics for ns_m and hence $E[\theta_i|\mathbf{s}]$ is informationally equivalent to $E[\theta_i|s_i,p]$ (Vives, 2011). The first-order condition in the price-taking auction equilibrium is

$$E\left[\theta_i|s_i,p\right]-\beta q_i-p=0. \tag{3.47}$$

Furthermore, plugging equation (3.4) into the first-order equation (3.47) and solving for q_i yields the equilibrium allocation. This must be equal to $D_i(p;s_i)=a+bs_i-cp$ and we hence get

$$a+bs_i-cp=\frac{1}{\beta}\left\{A\bar{\theta}+Bs_i+Cns_m-p\right\}.$$

Plugging in ns_m from (3.46) and solving this three equation system[15], gives the linear Bayesian demand function equilibrium strategy $D_i(p;s_i)=a+bs_i-cp$, where

$$\begin{aligned}
a &= \frac{1}{\beta}\left(\frac{1}{B+nC+\frac{\beta}{\delta}Z}\right)\left\{\left(AB+\frac{\beta}{\delta}\left(A+nC\right)Z\right)\bar{\theta}-\frac{\beta}{\delta}C\bar{\gamma}\right\} &(3.48)\\
b &= \frac{1}{\beta}B &(3.49)\\
c &= \frac{1}{\beta}\left(\frac{B-\frac{\beta}{\delta}C+\frac{\beta}{\delta}Z}{B+nC+\frac{\beta}{\delta}Z}\right). &(3.50)
\end{aligned}$$

Using (3.41) and (3.45) the clearing price is

$$p_v\left(\mathbf{s}\right)=\frac{n\delta a+\bar{\gamma}+n\delta b\bar{\theta}+n\left(\delta b+Z\right)\left(s_m-\bar{\theta}\right)}{n\delta c+1}. \tag{3.51}$$

Plugging (3.48) - (3.50) into (3.51), it is easy to show that $p_v\left(\mathbf{s}\right)=\bar{p}\left(\mathbf{s}\right)$. Also, $D_i(p_v\left(\mathbf{s}\right);s_i)=\bar{q}_i(\mathbf{s})=q_i^e(\mathbf{s})$ where $\bar{p}\left(\mathbf{s}\right)$ and $q_i^e(\mathbf{s})$ are given by (3.21) and (3.23).

Note that with interdependent values the total payment of firm i in the Vickrey auction is not the full externality cost, in contrast to the pure private values case. With pure private values the total payment is the sum of the pollution externality of increased pollution and the pecuniary externality, i.e. the value to other bidders of those units which are not assigned to them due to firm i's participation (Montero 2008). With interdependent values the payment does not include the informational externality of signal s_i to other bidders' values and to pollution damage (see Chapter 2, Figure 2.1).

[15] See the derivation in Chapter 2.

3.4.2 Incentive compatibility of the two-stage regulation

Next I explore the question of the incentive compatibility of a Vickrey auction followed by the regulation stage. First note that the auction payment $R_{i,v}(\mathbf{s}) = \int_0^{\bar{q}_i(\mathbf{s})} RS_i^{-1}(x; \mathbf{s}_{-i})\,dx$ depends only on s_i through its end point $\bar{q}_i(\mathbf{s})$. Let $\tilde{s}_{-i} = \frac{1}{n-1}\sum_{j\neq i} s_j$ denote the average signal of every other firm but firm i. Consider for a moment that firm i receives a signal $\hat{s}_i$. Given that every other bidder is bidding sincerely, it is then easy to derive the clearing price p_v as a function of $\hat{\mathbf{s}} = (\hat{s}_i, \mathbf{s}_{-i})$ from (3.51):

$$p_v(\hat{\mathbf{s}}) = \frac{n\delta a + \bar{\gamma} - nZ\bar{\theta} + (n-1)(\delta b + Z)\tilde{s}_{-i}}{n\delta c + 1} + \frac{\delta b + Z}{n\delta c + 1}\hat{s}_i, \tag{3.52}$$

where $\frac{\delta b+Z}{n\delta c+1} = \frac{\delta(1-A)+\beta Z}{\beta+n\delta}$. In the auction equilibrium

$$\bar{q}_i(\hat{s}_i, \mathbf{s}_{-i}) = Q_S(p_v(\hat{\mathbf{s}}); \hat{s}_i, \mathbf{s}_{-i}) - D_{-i}(p_v(\hat{\mathbf{s}}); \mathbf{s}_{-i}),$$

and we may further write

$$\begin{aligned} \hat{s}_i(\mathbf{s}_{-i}, q_i) &= \left(\frac{nc\delta + 1}{(nc\delta+1)b - c(b\delta + Z)}\right) \\ &\quad \times \left\{\frac{-a + c\bar{\gamma} - ncZ\bar{\theta} + c(n-1)(b\delta + Z)\tilde{s}_{-i}}{nc\delta + 1} + q_i\right\}. \end{aligned} \tag{3.53}$$

Hence, while the inverse residual supply function goes through the equilibrium point $(p_v(\mathbf{s}), q_i^e(\mathbf{s}))$, it can be written as

$$RS_i^{-1}(q_i; \mathbf{s}_{-i}) = p_v(\mathbf{s}) + \tau_v(q_i - q_i^e(\mathbf{s})). \tag{3.54}$$

Thus, plugging (3.53) into (3.52) gives the slope:

$$\begin{aligned} \tau_v &= \frac{\delta b + Z}{(nc\delta+1)b - c(\delta b + Z)} \\ &= \frac{\beta\sigma_{\gamma\theta} + n\delta\cdot var[\theta_m]}{\left(1 + \frac{n\delta}{\beta}\right)(n-1)B\cdot var[s_m] + var[\theta_m] - \sigma_{\gamma\theta}}. \end{aligned} \tag{3.55}$$

Note that in the absence of the second stage, sincere bidding in the Vickrey auction would be incentive compatible if $\tau_v \geq -\beta$. This holds whenever

$$\left(1 + \frac{n\delta}{\beta}\right)(n-1)B\cdot var[s_m] + var[\theta_m] - \sigma_{\gamma\theta} \geq 0. \tag{3.56}$$

Note also that if there was only one firm ($n = 1$), equation (3.56) reduces to $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} \leq 1$. In that case the slope of the inverse (residual) supply function is equal to the slope of the Roberts and Spence non-constant quantity regulation and, in fact, the information mechanism is not needed. In that case the information mechanism coincides with the second-best regulation in the regulation stage.

Now, suppose that firm i bids according to signal s_i' when its true signal is s_i, and, thus, firm i uses a deviation strategy $D_i(p; s_i', s_i)$.[16]. The initial allocation of permits to firm i, given $\mathbf{s}' = (s_i', \mathbf{s}_{-i})$, is

$$\begin{aligned} \bar{q}_i(\mathbf{s}') &= a + bs_i' - cp_v(\mathbf{s}') \qquad (3.57) \\ &= q_i^e(\mathbf{s}) - \left(b - \frac{\delta b + Z}{n\delta + \frac{1}{c}}\right)(s_i - s_i') \\ &= q_i^e(\mathbf{s}) - \left(\frac{\left(1 + \frac{n\delta}{\beta}\right)(n-1)\,B \cdot var\,[s_m] + var\,[\theta_m] - \sigma_{\gamma\theta}}{(\beta + n\delta)\,var\,[s_m]}\right)\frac{1}{n}(s_i - s_i'). \end{aligned}$$

Hence from (3.56), sincere bidding in a Vickrey auction without the regulation stage is incentive compatible if the equilibrium allocation $\bar{q}_i(\mathbf{s})$ is increasing in s_i. However, the condition $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} \leq 1$ gives a lowest upper bound for $\sigma_{\gamma\theta}$ that guarantees the incentive compatibility of the Vickrey auction also when the auction mechanism is followed by the secondary market.

Let $\pi_{i,v}(s_i'; s_i, \mathbf{s}_{-i})$ denote the profit of firm i in the Vickrey auction when using the deviation strategy $D_i(p; s_i', s_i)$. The loss in the Vickrey auction may thus be written as

$$\begin{aligned} L_i(s_i'; s_i, \mathbf{s}_{-i}) &= \pi_{i,v}(s_i; s_i, \mathbf{s}_{-i}) - \pi_{i,v}(s_i'; s_i, \mathbf{s}_{-i}) \qquad (3.58) \\ &= \int_{\bar{q}_i(\mathbf{s}')}^{q_i^e(\mathbf{s})} \left\{v_i(x; \mathbf{s}) - RS_i^{-1}(x; \mathbf{s}_{-i})\right\} dx. \end{aligned}$$

From (3.24) the expected profit in the secondary market is

$$\begin{aligned} \pi_{i,\tau}(s_i'; s_i, \mathbf{s}_{-i}) = &\int_{\bar{q}_i(\mathbf{s}')}^{q_{i,\tau}(\mathbf{s}')} v_i(x; \mathbf{s})\,dx - \int_{\bar{q}_i(\mathbf{s}')}^{h_{i,\tau}(\mathbf{s}')} T_i(x; \mathbf{s}')\,dx \\ &+ \int_0^{\Delta q_i(\mathbf{s}')} \{p_\tau(\mathbf{s}') - \omega x\}\,dx\mathbf{1}_{\{h_{i,\tau}(\mathbf{s}') > q_{i,\tau}(\mathbf{s}')\}} \\ &- \int_0^{\Delta q_i(\mathbf{s}')} \{p_\tau(\mathbf{s}') + \omega x\}\,dx\mathbf{1}_{\{q_{i,\tau}(\mathbf{s}') > h_{i,\tau}(\mathbf{s}')\}}, \end{aligned}$$

where $p_\tau(\mathbf{s}')$ is the expected equilibrium price of the permit market given the initial allocation according to $\mathbf{s}'$. Respectively, $q_{i,\tau}(\mathbf{s}')$, $h_{i,\tau}(\mathbf{s}')$ and $\Delta q_i(\mathbf{s}') = |q_{i,\tau}(\mathbf{s}') - h_{i,\tau}(\mathbf{s}')|$ are the

[16] Note that firm i may use any bid function that goes through the point $(p_v(\mathbf{s}'), \bar{q}_i(\mathbf{s}'))$.

corresponding expected equilibrium outcomes of the secondary market.

When all bidders bid sincerely in the Vickrey auction the allocation is ex-post efficienct and the expected profits of firm i in the secondary market are zero, $\pi_{i,\tau}(s_i; s_i, \mathbf{s}_{-i}) = 0$. Hence the Vickrey auction in the first stage is incentive compatible if the loss from (3.58) is greater than the expected profits in the secondary market:

$$\begin{aligned}\Delta_{IC} &= \pi_{i,v}(s_i; s_i, \mathbf{s}_{-i}) + \pi_{i,\tau}(s_i; s_i, \mathbf{s}_{-i}) - \pi_{i,v}(s_i'; s_i, \mathbf{s}_{-i}) - \pi_{i,\tau}(s_i'; s_i, \mathbf{s}_{-i}) \\ &= L_i(s_i'; s_i, \mathbf{s}_{-i}) - \pi_{i,\tau}(s_i'; s_i, \mathbf{s}_{-i}) \\ &\geq 0.\end{aligned} \tag{3.59}$$

Proposition 3.1. *Given the affine linear model, the information mechanism of the two-stage regulation is incentive compatible whenever the aggregate quantity rule of the first stage auction is weakly increasing in each bidder's signal and the optimal permit supply schedules of the second-stage regulation are weakly increasing in permit purchases, i.e. whenever* $-\frac{n\delta}{\beta} \leq \frac{\sigma_{\gamma\theta}}{var[\theta_m]} \leq 1$.

Proof. See Appendix 3.C.

Suppose that $-\frac{n\delta}{\beta} \leq \frac{\sigma_{\gamma\theta}}{var[\theta_m]} \leq 1$ and firm i uses a deviation strategy according to $s_i' < s_i$ when every other firm is bidding sincerely in the Vickrey auction. Then $q_{i,\tau}(\mathbf{s}') > h_{i,\tau}(\mathbf{s}')$ and the IC condition writes as

$$\begin{aligned}\Delta_{IC} = &\underbrace{\int_{\bar{q}_i(\mathbf{s}')}^{h_{i,\tau}(\mathbf{s}')} \left\{T_i(x; \mathbf{s}') - RS_i^{-1}(x; \mathbf{s}_{-i})\right\} dx}_{\Delta_{IC}^{I}} \\ &+ \underbrace{\int_{h_{i,\tau}(\mathbf{s}')}^{q_{i,\tau}(\mathbf{s}')} \left\{p_\tau(\mathbf{s}') + \omega(x - h_{i,\tau}(\mathbf{s}')) - RS_i^{-1}(x; \mathbf{s}_{-i})\right\} dx}_{\Delta_{IC}^{II}} \\ &+ \underbrace{\int_{q_{i,\tau}(\mathbf{s}')}^{q_i^e(\mathbf{s})} \left\{v_i(x; \mathbf{s}) - RS_i^{-1}(x; \mathbf{s}_{-i})\right\} dx.}_{\Delta_{IC}^{III}}\end{aligned} \tag{3.60}$$

These terms are described in Figure 3.2. Δ_{IC}^{I} denotes the excess payments for permits which firm i purchases from the regulator due to the deviation strategy. Δ_{IC}^{I} is non-negative for all $s_i' < s_i$, because the permit supply schedule $T_i(q_i; \mathbf{s}')$ crosses the inverse residual supply $RS_i^{-1}(q_i; \mathbf{s}_{-i})$ from below at $\bar{q}_i(\mathbf{s}')$, when these functions are increasing and thus when $-\frac{n\delta}{\beta} \leq \frac{\sigma_{\gamma\theta}}{var[\theta_m]} \leq 1$.

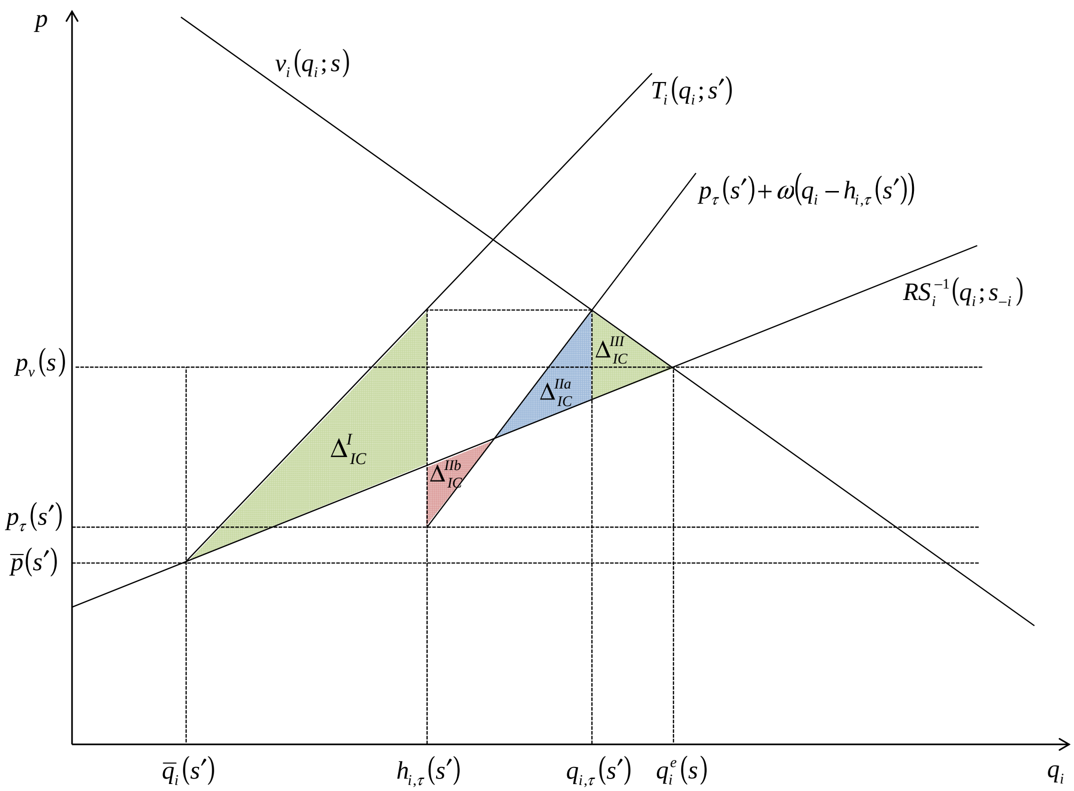

Figure 3.2: Incentive compatibility of the Vickrey auction.

Δ_{IC}^{III} denotes the net loss of Vickrey payoffs due to the deviation strategy. When $s_i' < s_i$, the expected equilibrium pollution of firm i after using the deviation strategy in the auction $q_{i,\tau}(\mathbf{s}')$ is lower than the ex-post efficient level of pollution $q_i^e(\mathbf{s})$. Δ_{IC}^{III} is non-negative for all $s_i' < s_i$, when $-\frac{n\delta}{\beta} \leq \frac{\sigma_{\gamma\theta}}{var[\theta_m]} \leq 1$.

Finally, (the negative of) Δ_{IC}^{II} denotes the net gains from trading due to the deviation strategy. However, the sign of Δ_{IC}^{II} is not clear. When the permit market is perfect ($\omega = 0$), it is non-negative. When $\omega \to \infty$, the limit value of Δ_{IC}^{II} is zero. Hence, $\Delta_{IC} \geq 0$ in both of these extremes. When ω increases from zero, $h_{i,\tau}(\mathbf{s}')$ increases but τ, $p_\tau(\mathbf{s}')$, and $q_{i,\tau}(\mathbf{s}')$ decreases. This implies that Δ_{IC}^{III} increases but Δ_{IC}^{I} and Δ_{IC}^{II} may either increase or decrease. From Figure 3.2, the incentive constraint is non-negative if $\Delta_{IC}^{I} + \Delta_{IC}^{III} + \Delta_{IC}^{IIa} \geq \left|\Delta_{IC}^{IIb}\right|$. Unfortunately, I am not able to derive an analytical proof that $\Delta_{IC} \geq 0$ for all $\omega \geq 0$. However, applying some numerical simulations I show in Appendix 3.C that $\Delta_{IC} \geq 0$ for all $\omega \geq 0$ when the regulator uses a two-stage regulation with a Vickrey auction in the information stage and permit supply schedules $T_i(q_i;\mathbf{s}) = \bar{p}(\mathbf{s}) + \tau(q_i - \bar{q}_i(\mathbf{s}))$ in the regulation stage and whenever $-\frac{n\delta}{\beta} \leq \frac{\sigma_{\gamma\theta}}{var[\theta_m]} \leq 1$ holds.

Suppose next that $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} < -\frac{n\delta}{\beta}$. Then the regulator cannot be sure that a Vickrey auction in the information stage followed by the regulation stage is incentive compatible. In this case both $T_i(q_i; \mathbf{s}')$ and $RS_i(q_i; \mathbf{s}_{-i})$ are decreasing functions and $T_i(q_i; \mathbf{s}')$ has a steeper slope. We thus have $-\beta \leq \tau \leq \tau_v \leq 0$. In both extremes of market performance, i.e. when $\omega = 0$ and when $\omega \to \infty$, the IC condition is negative (see Appendix 3.C). However, in Chapter 2 I show that if the regulator implements a constant price regulation in the second stage, and thus sets $\tau = 0$, the Vickrey auction is incentive compatible whenever $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} < -\frac{n\delta}{\beta}$. Moreover, the regulator could also set $\tau = \tau_v$, which guarantees incentive compatibility and would improve the results of the constant price regulation.

Also when $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} > 1$, the Vickrey auction is not incentive compatible even without a secondary market. However, at least when $\sigma_{\gamma\theta} \approx var[\theta_m]$ and when the market performance is not too bad ($\omega \approx 0$), the firms' private information is not valuable to the regulator, because the aggregate supply in the Vickrey auction is inelastic in s_m. In that case the regulator should apply a constant quantity regulation without any auction.

3.5 Conclusions

This paper considers a commons problem, which addresses two important issues. Hence, in addition to the pollution externality, two other market failures exist. Firstly, trading in pollution permits suffers from frictions. Secondly, the information with regards to emission abatement costs is incomplete and distributed asymmetrically. To tackle these issues, I have proposed a two-stage regulation of tradeable pollution permits. In the first stage the regulator conducts an information mechanism. Applying a generalized Vickrey-Clarke-Groves (VCG) mechanism, the regulator collects private information from regulated firms and allocates emission permits to the firms. In the second stage, the regulator implements a quantity regulation, where the non-constant permit supply schedules take into account the frictions of the permit market and the private information of firms. I have applied a linearized model with an affine information structure. Given this structure, the marginal abatement cost functions and the marginal damage function are linear functions, the conditional expectations are linear in information, and the slopes and distribution functions are common knowledge. Also, pollution is assumed to be uniformly mixed and uncertain benefits and costs of emissions reductions may be correlated. Given the affine linear model, I have shown that in the information mechanism followed by the second-best regulation, sincere bidding satisfies incentive compatibility conditions whenever the correlation between emissions reduction benefits and costs is not too high. Moreover, depending on the performance of the permit market, the slopes of the permit supply functions in the second stage are between the slopes of the Weitzman (1978) non-constant tax schedules and the Roberts and Spence (1976) non-constant quantity regulation permit supply schedules.

The model builds on the symmetric linear structure. A natural next step is to ask how much the assumptions of the linear affine model could be relaxed in order for the results of this paper to continue to hold? In what conditions is the regulator independent of the private information of firms when the permit market is perfect? What are the general incentive compatibility conditions of the Vickrey auction followed by the regulation stage? Might there be another, optimal mechanism in the environment I consider? How would the two-stage regulation change if the distribution functions of the unknown parameters, the performance of the permit market and the functional forms of the abatement costs were unknown to the regulator? These issues are left for future research.

Bibliography

[1] Ausubel, L., and Cramton, P., 2004. Vickrey Auctions with Reserve Pricing. Economic Theory 23, 493-505.

[2] Clarke, E., 1971. Multipart Pricing of Public Goods. Public Choice 2, 19-33.

[3] Dasgupta, P., Hammond, P., and Maskin, E., 1980. On Imperfect Information and Optimal Pollution Control. Review of Economic Studies 47(5), 857-860.

[4] Dasgupta, P., and Maskin, E., 2000. Efficient Auctions. Quarterly Journal of Economics 115(2), 341-388.

[5] DeGroot, M.H., 1970. Optimal Statistical Decisions. New York: McGraw-Hill.

[6] Groves, T., 1973. Incentives in Teams. Econometrica 41, 617-631.

[7] Jehiel, P., and Moldovanu, B., 2001. Efficient Design with Interdependent Valuations. Econometrica 69(5), 1237-1259.

[8] Kennedy, P., Laplante, B., and Whittington, D., 2010. Simple Pricing Schemes for Pollution Control under Asymmetric Information. The B.E. Journal of Economic Analysis & Policy 10(1), Article 12.

[9] Malueg, D.A., and Yates, A.J., 2009. Bilateral Oligopoly, Private Information, and Pollution Permit Markets. Environmental and Resource Economics 43(4), 553-572.

[10] Montero, J.-P., 2008. A Simple Auction Mechanism for the Optimal Allocation of the Commons. American Economic Review 98(1), 496-518.

[11] Myerson, R.B, and Satterthwaite, M.A., 1983. Efficient Mechanisms for Bilateral Trading. Journal of Economic Theory 28, 265-281.

[12] Roberts, M.J., and Spence, M., 1976. Effluent Charges and Licenses under Uncertainty. Journal of Public Economics 5(3-4), 193-208.

[13] Stavins, R.N., 1995. Transaction Costs and Tradeable Permits. Journal of Environmental Economics and Management 29, 133-148.

[14] Stavins, R.N., 1996. Correlated Uncertainty and Policy Instrument Choice. Journal of Environmental Economics and Management 30, 218-232.

[15] Vickrey, W., 1961. Counterspeculation, Auctions and Competitive Sealed Tenders. Journal of Finance 16, 8-37.

[16] Vives, X., 2010. Asset Auctions, Information, and Liquidity. Journal of the European Economic Association 8(2-3), 467-477.

[17] Vives, X., 2011. Strategic Supply Function Competition with Private Information. Econometrica 79(6), 1919-1966.

[18] Weitzman, M.L., 1974. Prices vs. Quantities. Review of Economic Studies 41(4), 477-491.

[19] Weitzman, M.L., 1978. Optimal Rewards for Economic Regulation. American Economic Review 68, 683-691.

[20] Yates, A.J., 2012. On a Fundamental Advantage of Permits Over Taxes for the Control of Pollution. Environmental and Resource Economics 51, 583-598.

Appendices

3.A Second-best regulation with a perfect permit market

Consider the affine linear model. Suppose, for simplicity, that the abatement costs and the pollution damage are not correlated, i.e. $\sigma_{\gamma\theta} = 0$. Hence the regulator and all firms have the same information about the expected marginal damage function:

$$y\left(Q; \mathbf{s}\right) = y\left(Q\right) = \bar{\gamma} + \delta Q.$$

Then, given that the permit market is perfect, the solution to the regulator's problem (3.17) is to i) offer a non-constant aggregate permit supply that equals the expected marginal damage function, $T\left(Q\right) = y\left(Q\right)$, and ii) let firms trade permits freely with each other in the regulation period (e.g. Roberts and Spence 1976, Yates 2012). When the permit market is perfect, it is irrelevant in terms of efficiency how the regulator distributes permits initially between firms. The permit market allocates permits cost-efficiently among polluting firms. For instance, if the firms are identical ex-ante, the regulator may implement an equal individual permit supply function for each firm such that

$$T_i\left(q_i\right) = y\left(nq_i\right).$$

Firms may thus purchase permits from the regulator according to this schedule. Let h_i denote firm i's permit purchases. In the equilibrium, each firm buys an equal amount of permits from the regulator, i.e. $h_i = h_m = \frac{1}{n}Q^{\star\star}$, where $Q^{\star\star}$ denotes the second-best aggregate pollution. The equilibrium is thus defined by

$$T_i\left(h_m\right) = u_i\left(q_i; \theta_i\right) = p^{\star\star}$$

where $p^{\star\star}$ is equilibrium (second-best) price in the permit market. Hence firms that have low abatement costs and thus $u_j\left(h_m; \theta_j\right) < T_j\left(h_m\right)$ sell $\Delta q_j = h_m - q_j$ permits in the secondary market. Respectively, high-cost firms, i.e. firms with $u_k\left(h_m; \theta_k\right) > T_k\left(h_m\right)$, buy $\Delta q_k = q_k - h_m$ from the permit market. The average firm (indexed with m and receiving signal θ_m) does not buy or sell any permits in the permit market and it thus has $T_m\left(h_m\right) = u_m\left(h_m; \theta_m\right)$. In equilibrium, buying one more permit from the regulator or abating one more unit of emissions is more costly than the benefit of selling the permit in the market. The solution is second-best, while the marginal abatement costs are equal across firms and the aggregate abatement cost is equal to the expected marginal damage. Most importantly, the regulator does not need the private information of firms in order to implement the second-best regulation. This feature remains true even if the aggregate abatement costs and the pollution damage are correlated. However, the regulator needs to be able to derive the expected marginal damage function conditional on the average abatement costs (θ_m).

3.B Derivation of the expected deadweight loss

In order to derive the expected deadweight loss and the optimal slope τ, it is easy to see using $q_i^\star = \frac{1}{\beta}(\theta_i - p^\star)$, and from (3.3), (3.31) and (3.33) that

$$q_i^\star - q_{i,\tau} = \frac{1}{\beta}(p_\tau - p^\star) + O_{i,\tau},$$

where $O_{i,\tau} = \frac{1}{\beta}\left(\frac{\omega\tau}{\beta\tau+\omega\beta+\omega\tau}\right)\left(\left(\theta_i - \bar{\theta}_i(I)\right) - \left(\theta_m - \bar{\theta}_m(I)\right)\right)$. Furthermore,

$$q_i^{\star 2} - q_{i,\tau}^2 = \frac{1}{\beta}(p_\tau - p^\star)\left(2q_i^\star - \frac{1}{\beta}(p_\tau - p^\star)\right) + 2O_{i,\tau}\frac{1}{\beta}(\theta_i - p_\tau) - O_{i,\tau}^2.$$

For the market as a whole we get

$$\begin{aligned} Q^\star - Q_\tau &= \frac{n}{\beta}(p_\tau - p^\star), \\ Q^{\star 2} - Q_\tau^2 &= \frac{n}{\beta}(p_\tau - p^\star)\left(2Q^\star - \frac{n}{\beta}(p_\tau - p^\star)\right). \end{aligned}$$

Plugging these into the deadweight loss equation (3.17) results in

$$DWL_\tau(I,\omega) = \underbrace{\frac{1}{2}\left(1 + \frac{n\delta}{\beta}\right)\frac{n}{\beta}(p_\tau - p^\star)^2}_{DWL_\tau^0} + \frac{\beta}{2}\sum_i^n O_{i,\tau}^2. \tag{3.61}$$

We may first examine the term DWL_τ^0. From (3.15) and (3.30), the price difference is written as

$$p_\tau - p^\star = -\frac{\beta}{(\beta + n\delta)}\left(\left(\frac{n\delta - \tau}{\beta + \tau}\right)\left(\theta_m - \bar{\theta}_m(I)\right) + (\gamma - \bar{\gamma}(I))\right),$$

and we hence get

$$DWL_{\tau}^{0} = \frac{1}{2}\left(\frac{1}{\frac{\beta}{n}+\delta}\right)\left\{\left(\frac{n\delta-\tau}{\beta+\tau}\right)^{2}\left(\theta_{m}-\bar{\theta}_{m}(I)\right)^{2}+\left(\gamma-\bar{\gamma}(I)\right)^{2}\right\} \quad (3.62)$$
$$+\left(\frac{1}{\frac{\beta}{n}+\delta}\right)\left(\frac{n\delta-\tau}{\beta+\tau}\right)\left(\theta_{m}-\bar{\theta}_{m}(I)\right)\left(\gamma-\bar{\gamma}(I)\right).$$

Taking the expectation from (3.62) yields

$$E\left[DWL_{\tau}^{0}\right] = \left\{\left(\frac{n\delta-\tau}{\beta+\tau}\right)^{2} var\left[\theta_{m}|\, I\right]+var\left[\gamma|\, I\right]\right\} \quad (3.63)$$
$$+\left(\frac{1}{\frac{\beta}{n}+\delta}\right)\left(\frac{n\delta-\tau}{\beta+\tau}\right) cov\left[\gamma,\theta_{m}|\, I\right].$$

However, the expected value of the second term in (3.61) is not zero if $\omega > 0$. It may write as

$$\frac{\beta}{2}E\left[\sum_{i}^{n}O_{i,\tau}^{2}\right] = \frac{n}{2\beta}\left(\frac{\tau}{\frac{\beta\tau}{\omega}+\beta+\tau}\right)^{2}\left\{var\left[\theta_{m}|\, I\right]-2cov\left[\theta_{m},\theta_{i}|\, I\right]+var\left[\theta_{i}|\, I\right]\right\}$$
$$= \frac{n}{2\beta}\left(\frac{\tau}{\frac{\beta\tau}{\omega}+\beta+\tau}\right)^{2}\left\{var\left[\theta_{i}|\, I\right]-var\left[\theta_{m}|\, I\right]\right\}. \quad (3.64)$$

Thus taking the expected value of (3.61) and plugging in (3.63) and (3.64) results in the following expected deadweight loss formula:

$$E\left[DWL_{\tau}(I,\omega)\right] = \left\{\left(\frac{n\delta-\tau}{\beta+\tau}\right)^{2} var\left[\theta_{m}|\, I\right]+var\left[\gamma|\, I\right]\right\} \quad (3.65)$$
$$+\left(\frac{1}{\frac{\beta}{n}+\delta}\right)\left(\frac{n\delta-\tau}{\beta+\tau}\right) cov\left[\gamma,\theta_{m}|\, I\right]$$
$$+\frac{n}{2\beta}\left(\frac{\tau}{\frac{\beta\tau}{\omega}+\beta+\tau}\right)^{2}\left\{var\left[\theta_{i}|\, I\right]-var\left[\theta_{m}|\, I\right]\right\}.$$

3.C Proof of Proposition 3.1

Suppose that other firms bid sincerely in the information stage and firm i bids according to signal s_i' when its true signal is s_i. Suppose also that $s_i' < s_i$. Before the actual proof, I derive the expected permit market price and the expected equilibrium outcomes of firm i when it uses a deviation strategy $D_i\left(p; s_i', s_i\right)$ in the information stage. Note that

$$\bar{\theta}_i\left(\mathbf{s}\right) - \bar{\theta}_i\left(\mathbf{s}'\right) = \left(B + C\right)\left(s_i - s_i'\right),$$

and

$$\bar{\theta}_m\left(\mathbf{s}\right) - \bar{\theta}_m\left(\mathbf{s}'\right) = \left(\frac{B}{n} + C\right)\left(s_i - s_i'\right).$$

Thus from (3.30):

$$\begin{aligned} p_\tau\left(\mathbf{s}'\right) &= \underbrace{p_v\left(\mathbf{s}\right) - \left(\frac{n\delta\left(1-A\right) + \beta n Z}{\beta + n\delta}\right)\frac{1}{n}\left(s_i - s_i'\right)}_{p_v\left(\mathbf{s}'\right)} \\ &\quad + \left(\frac{\tau}{\beta+\tau}\right)\left(1-A\right)\frac{1}{n}\left(s_i - s_i'\right) \\ &= p_v\left(\mathbf{s}\right) - \left(\frac{\left(\frac{n\delta-\tau}{\beta+\tau}\right)\left(1-A\right) + nZ}{\left(1+\frac{n\delta}{\beta}\right)}\right)\frac{1}{n}\left(s_i - s_i'\right). \end{aligned} \tag{3.66}$$

The expected equilibrium price is lower than the expected first-best price $p_\tau\left(\mathbf{s}'\right) \leq p_v\left(\mathbf{s}\right)$ whenever $s_i' \leq s_i$ and $\tau > 0$. This follows from equation (3.36), which yields

$$\left(\frac{n\delta - \tau}{\beta+\tau}\right)\left(1-A\right) + nZ = \left(\frac{\tau\left(\beta+\tau\right)^2}{\left(\frac{\beta\tau}{\omega} + \beta + \tau\right)^3}\right)\left(n-1\right)B \geq 0.$$

The expected equilibrium pollution $q_{i,\tau}(\mathbf{s}')$ may derive from (3.31) and (3.57):

$$
\begin{aligned}
q_{i,\tau}(\mathbf{s}') &= \bar{q}_i(\mathbf{s}') \qquad (3.67)\\
&+\left\{\left(\frac{\tau+\omega}{\beta\tau+\omega\beta+\omega\tau}\right)\left(\frac{n-1}{n}\right)B+\left(\frac{1}{\beta+\tau}\right)\left(\frac{B}{n}+C\right)\right\}(s_i-s_i')\\
&= q_i^e(\mathbf{s})-\left\{\frac{1}{\beta}(n-1)B+\left(\frac{1}{\beta+n\delta}\right)[1-A-nZ]\right\}\frac{1}{n}(s_i-s_i')\\
&+\left(\frac{\tau+\omega}{\beta\tau+\omega\beta+\omega\tau}\right)(n-1)B\frac{1}{n}(s_i-s_i')\\
&+\left(\frac{\tau+\omega}{\beta\tau+\omega\beta+\omega\tau}\right)(1-A)\frac{1}{n}(s_i-s_i')\\
&-\left(\frac{1}{\beta\tau+\omega\beta+\omega\tau}\right)\left(\frac{\tau^2}{\beta+\tau}\right)(1-A)\frac{1}{n}(s_i-s_i')\\
&= q_i^e(\mathbf{s})-\left\{1+\frac{n\delta}{\beta}-\left(\frac{\beta+\tau}{\frac{\beta\tau}{\omega}+\beta+\tau}\right)^2\right\}\\
&\qquad\times\left(\frac{1}{\beta+n\delta}\right)\left(\frac{1}{1+\frac{\beta}{\tau}+\frac{\beta}{\omega}}\right)\left(\frac{n-1}{n}\right)B(s_i-s_i').
\end{aligned}
$$

Respectively, the expected equilibrium purchases from the regulator may derive from (3.32):

$$
\begin{aligned}
h_{i,\tau}(\mathbf{s}') &= \bar{q}_i(\mathbf{s}') \qquad (3.68)\\
&+\left\{\left(\frac{\omega}{\beta\tau+\omega\beta+\omega\tau}\right)\left(\frac{n-1}{n}\right)B+\left(\frac{1}{\beta+\tau}\right)\left(\frac{B}{n}+C\right)\right\}(s_i-s_i').
\end{aligned}
$$

Hence, a deviating firm is expected to be on the demand side of the permit market and thus $\bar{q}_i(\mathbf{s}')\leq h_{i,\tau}(\mathbf{s}')\leq q_{i,\tau}(\mathbf{s}')\leq q_i^e(\mathbf{s})$, if $\left(1+\frac{\beta}{\tau}+\frac{\beta}{\omega}\right)\geq 0$ and $\left(1+\frac{n\delta}{\beta}\right)\geq\left(\frac{1}{1+\frac{\beta\tau}{\omega(\beta+\tau)}}\right)^2$. These hold for all $\tau\geq 0$.

According to (3.60), we may decompose the incentive condition into three parts, $\Delta_{IC}=\Delta_{IC}^{I}+\Delta_{IC}^{II}+\Delta_{IC}^{III}$. First, Δ_{IC}^{I} is non-negative if the permit schedule $T_i(q_i;\mathbf{s}')$ is above the inverse residual supply function $RS_i^{-1}(q_i;\mathbf{s}_{-i})$ for $q_i\geq\bar{q}_i(\mathbf{s}')$. Note that these are equal at $\bar{q}_i(\mathbf{s}')$, when the bid functions are based on $\mathbf{s}'=(s_i',\mathbf{s}_{-i})$. Then, in order for $T_i(q_i;\mathbf{s}')\geq RS_i(q_i;\mathbf{s}_{-i})$ for $q_i\geq\bar{q}_i(\mathbf{s}')$, it is enough to show that $\tau\geq\tau_v$. Moreover, when $I=\mathbf{s}$ and $\tau\geq 0$, the slope τ is bounded by the optimal slopes of the Roberts and Spence (1976) and Weitzman (1978) models from (3.37) and (3.39). Then $\infty\geq\tau^{RS}\geq\tau\geq\tau^{W}(\mathbf{s})\geq\tau_v\geq 0$ whenever

$-\frac{n\delta}{\beta} \leq \frac{\sigma_{\gamma\theta}}{var[\theta_m]} \leq 1$. The first part of the IC condition is then non-negative:

$$\begin{aligned}
\Delta_{IC}^{I} &= \int_{\bar{q}_i(\mathbf{s}')}^{h_{i,\tau}(\mathbf{s}')} \left\{ T_i\left(x;\mathbf{s}'\right) - RS_i^{-1}\left(x;\mathbf{s}_{-i}\right)\right\} dx \\
&= \frac{1}{2}\left(\tau-\tau_v\right)\left(h_{i,\tau}\left(\mathbf{s}'\right)-\bar{q}_i\left(\mathbf{s}'\right)\right)^2 \\
&\geq 0.
\end{aligned}$$

Second, Δ_{IC}^{III} is always non-negative if $v_i\left(q_i;\mathbf{s}\right) \geq RS_i^{-1}\left(q_i;\mathbf{s}_{-i}\right)$ for all $q_i \leq q_i^e(\mathbf{s})$ and $v_i\left(q_i;\mathbf{s}\right) \leq RS_i^{-1}\left(q_i;\mathbf{s}_{-i}\right)$ for all $q_i \geq q_i^e(\mathbf{s})$. This holds if $\tau_v \geq -\beta$. This, on the other hand, is satisfied whenever $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} \leq 1$, which also guarantees that the aggregate quantity rule is weakly increasing in each bidder's signal. Hence if $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} \leq 1$,

$$\begin{aligned}
\Delta_{IC}^{III} &= \int_{q_{i,\tau}(\mathbf{s}')}^{q_i^e(\mathbf{s})} \left\{ v_i\left(x;\mathbf{s}\right) - RS_i^{-1}\left(x;\mathbf{s}_{-i}\right)\right\} dx \\
&= \frac{1}{2}\left(\beta+\tau_v\right)\left(q_i^e(\mathbf{s})-q_{i,\tau}\left(\mathbf{s}'\right)\right)^2 \\
&\geq 0.
\end{aligned}$$

Third, the second part of the IC condition writes as

$$\begin{aligned}
\Delta_{IC}^{II} &= \int_{h_{i,\tau}(\mathbf{s}')}^{q_{i,\tau}(\mathbf{s}')} \left\{ p_\tau\left(\mathbf{s}'\right) + \omega\left(x-h_{i,\tau}\left(\mathbf{s}'\right)\right) - RS_i^{-1}\left(x;\mathbf{s}_{-i}\right)\right\} dx \\
&= \left(\beta+\tau_v\right)\left(q_i^e(\mathbf{s})-q_{i,\tau}\left(\mathbf{s}'\right)\right)\left(q_{i,\tau}\left(\mathbf{s}'\right)-h_{i,\tau}\left(\mathbf{s}'\right)\right) - \frac{1}{2}\left(\omega-\tau_v\right)\left(q_{i,\tau}\left(\mathbf{s}'\right)-h_{i,\tau}\left(\mathbf{s}'\right)\right)^2.
\end{aligned}$$

The sign of Δ_{IC}^{II} is not clear. However, I show next that when (3.18) holds, the incentive constraint Δ_{IC} is non-negative for all ω.

Firstly, Δ_{IC} is clearly positive when $\omega = 0$, because then $q_{i,\tau}\left(\mathbf{s}'\right) = q_i^e(\mathbf{s})$ which implies $p_\tau\left(\mathbf{s}'\right) = p_v\left(\mathbf{s}\right) \geq RS_i^{-1}\left(q_i;\mathbf{s}_{-i}\right)$ for all $q_i \leq q_{i,\tau}\left(\mathbf{s}'\right)$. Thus,

$$\Delta_{IC}^{II}\Big|_{\omega=0} = \int_{h_{i,\tau}(\mathbf{s}')}^{q_{i,\tau}(\mathbf{s}')} \left\{ p_v\left(\mathbf{s}\right) - RS_i^{-1}\left(x;\mathbf{s}_{-i}\right)\right\} dx \geq 0,$$

and

$$\Delta_{IC}\Big|_{\omega=0} = \Delta_{IC}^{I} + \Delta_{IC}^{II} \geq 0.$$

Respectively, when $\omega \to \infty$ the limit value of the incentive constraint is positive, while then $\lim_{\omega\to\infty}\left\{q_{i,\tau}\left(\mathbf{s}'\right)-h_{i,\tau}\left(\mathbf{s}'\right)\right\} = 0$. This gives $\lim_{\omega\to\infty}\Delta_{IC} = \Delta_{IC}^{I} + \Delta_{IC}^{III} \geq 0$.

Further, when $\tau > 0$, the slope τ is decreasing in ω. From (3.36) we get

$$\begin{aligned}
\frac{d\tau}{d\omega} &= -\frac{\partial f_{\tau,s}}{\partial \omega} / \frac{\partial f_{\tau,s}}{\partial \tau} \\
&= -\left\{ \frac{3\beta\left(\frac{\tau}{\omega}\right)^2 (n-1) B}{\left(\frac{\frac{\beta\tau}{\omega}+\beta+\tau}{\beta+\tau}\right)^4 (\beta+n\delta)(1-A) + \beta\left\{1-2\frac{\beta}{\omega}\left(\frac{\tau}{\beta+\tau}\right)\right\}(n-1)B} \right\}.
\end{aligned}$$

It is easy to show that $\left.\frac{d\tau}{d\omega}\right|_{\omega=0} = 0$ and $\lim_{\omega\to\infty} \frac{d\tau}{d\omega} = 0$. According to the numerical simulations (see Appendix 3.D) and also by $\tau(\mathbf{s},\omega)|_{\omega=0} = \tau^{RS} \geq \tau \geq \tau^{W}(\mathbf{s}) = \lim_{\omega\to\infty} \tau(\mathbf{s},\omega)$ we get $\frac{d\tau}{d\omega} \leq 0$ when $\tau \geq 0$. This furthermore gives

$$\left.\frac{dp_\tau(\mathbf{s}')}{d\omega}\right|_{\omega=0} = \left(\frac{\beta}{(\beta+\tau^{RS})^2}\right)\left(\frac{B}{n}+C\right)(s_i - s_i')\left.\frac{d\tau}{d\omega}\right|_{\omega=0} = 0.$$

Hence taking a derivative of the incentive constraint with respect to ω yields

$$\begin{aligned}
\frac{d\Delta_{IC}}{d\omega} &= -\frac{d\pi_{i,\tau}(s_i'; s_i, \mathbf{s}_{-i})}{d\omega} \\
&= \underbrace{\{p_\tau(\mathbf{s}') + \omega(q_{i,\tau}(\mathbf{s}') - h_{i,\tau}(\mathbf{s}')) - v_i(q_{i,\tau}(\mathbf{s}'); \mathbf{s})\}}_{=0} \frac{dq_{i,\tau}(\mathbf{s}')}{d\omega} \\
&\quad + \underbrace{\{T_i(h_{i,\tau}(\mathbf{s}'); \mathbf{s}') - p_\tau(\mathbf{s}') - \omega(q_{i,\tau}(\mathbf{s}') - h_{i,\tau}(\mathbf{s}'))\}}_{=0} \frac{dh_{i,\tau}(\mathbf{s}')}{d\omega} \\
&\quad + \int_{\bar{q}_i(\mathbf{s}')}^{h_{i,\tau}(\mathbf{s}')} \frac{dT_i(x;\mathbf{s}')}{d\omega} dx + \int_0^{\Delta q_i(\mathbf{s}')} \left\{\frac{dp_\tau(\mathbf{s}')}{d\omega} + x\right\} dx \\
&= \frac{1}{2}(h_{i,\tau}(\mathbf{s}') - \bar{q}_i(\mathbf{s}'))^2 \frac{d\tau}{d\omega} + (q_{i,\tau}(\mathbf{s}') - h_{i,\tau}(\mathbf{s}'))\frac{dp_\tau(\mathbf{s}')}{d\omega} + \frac{1}{2}(q_{i,\tau}(\mathbf{s}') - h_{i,\tau}(\mathbf{s}'))^2.
\end{aligned}$$

Thus Δ_{IC} is positive and increasing in ω when evaluated at $\omega = 0$, while

$$\left.\frac{d\Delta_{IC}}{d\omega}\right|_{\omega=0} = \frac{1}{2}\left(\left(\frac{n-1}{n}\right)\frac{B}{\beta}(s_i - s_i')\right)^2 \geq 0.$$

Note also that $\lim_{\omega\to\infty} \frac{d\Delta_{IC}}{d\omega} = 0$. Moreover, according to the numerical simulations there is only one point where $\frac{d\Delta_{IC}}{d\omega} = 0$ when $\omega \geq 0$. Thus Δ_{IC} first increases and then decreases when ω increases from zero. This implies that $\Delta_{IC} \geq 0$ for all $\omega \geq 0$ when the regulator uses permit schedules $T_i(q_i; \mathbf{s}) = \bar{p}(\mathbf{s}) + \tau(q_i - \bar{q}_i(\mathbf{s}))$ and when (3.18) holds.

Suppose, on the contrary, that the permit supply schedules and inverse residual supply functions are decreasing and $-\beta < \tau < \tau_v < 0$. I thus relax the assumption $-\frac{n\delta}{\beta} \leq \frac{\sigma_{\gamma\theta}}{var[\theta_m]}$. With decreasing inverse residual supply functions and permit schedules, $T_i(q_i; \mathbf{s}') \leq RS_i^{-1}(q_i; \mathbf{s}_{-i})$

for all $q_i \geq \bar{q}_i(\mathbf{s}')$ and $p_v(\mathbf{s}) \leq RS_i^{-1}(q_i;\mathbf{s}_{-i})$ for all $q_i \leq q_i^e(\mathbf{s})$. I next show that $\Delta_{IC} \leq 0$ when the permit market is perfect ($\omega = 0$) and also when it is collapsed ($\omega \to \infty$) if $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} < -\frac{n\delta}{\beta}$. Consider first that $\omega = 0$. This case is presented in Figure 3.3a. From (3.66) and (3.67) we get $p_\tau(\mathbf{s}') = p_v(\mathbf{s})$ and $q_{i,\tau}(\mathbf{s}') = q_i^e(\mathbf{s})$. Furthermore,

$$\begin{aligned} h_{i,\tau}(\mathbf{s}') &= q_i^e(\mathbf{s}) - \frac{1}{\beta}\left(\frac{n-1}{n}\right)B(s_i - s_i') \\ &\leq q_i^e(\mathbf{s}). \end{aligned}$$

Hence the IC condition is non-positive:

$$\Delta_{IC}\big|_{\omega=0} = \int_{\bar{q}_i(\mathbf{s}')}^{h_{i,\tau}(\mathbf{s}')} \left\{T_i(x;\mathbf{s}') - RS_i^{-1}(x;\mathbf{s}_{-i})\right\} dx + \int_{h_{i,\tau}(\mathbf{s}')}^{q_i^e(\mathbf{s})} \left\{p_v(\mathbf{s}) - RS_i^{-1}(x;\mathbf{s}_{-i})\right\} dx \leq 0.$$

Moreover, if $s_i' < s_i$ and $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} < -\frac{n\delta}{\beta}$ the IC condition is strictly negative and sincere bidding is not incentive compatible.

Suppose next that $\omega \to \infty$ (see Figure 3.3b). This gives

$$\begin{aligned} q_{i,\tau}(\mathbf{s}') = h_{i,\tau}(\mathbf{s}') &= q_i^e(\mathbf{s}) + \left(\frac{\frac{n\delta}{\beta}}{(\beta + n\delta)\left(\frac{\beta}{|\tau|} - 1\right)}\right)\left(\frac{n-1}{n}\right)B(s_i - s_i') \\ &\geq q_i^e(\mathbf{s}). \end{aligned}$$

The limit value of the IC condition is then non-positive:

$$\lim_{\omega\to\infty} \Delta_{IC} = \int_{\bar{q}_i(\mathbf{s}')}^{q_i^e(\mathbf{s})} \left\{T_i(x;\mathbf{s}') - RS_i^{-1}(x;\mathbf{s}_{-i})\right\} dx + \int_{q_i^e(\mathbf{s})}^{q_{i,\tau}(\mathbf{s}')} \left\{T_i(x;\mathbf{s}') - v_i(x;\mathbf{s})\right\} dx \leq 0.$$

With $s_i' < s_i$ and $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} < -\frac{n\delta}{\beta}$ we get $\lim_{\omega\to\infty} \Delta_{IC} < 0$. Thus the regulator cannot guarantee that the Vickrey auction in the information stage is incentive compatible if $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} < -\frac{n\delta}{\beta}$. Similar arguments hold when $s_i' > s_i$. ■

3.D Simulations

In order to give some intuition of the results I have run some numerical simulations. I have used nine different information structures, where I have varied the standard deviation of the signal noise (σ_ε) and the correlation coefficient of the aggregate abatement costs and pollution damage ($\rho_{\gamma\theta}$). The other parameters of the affine linear model are kept fixed.

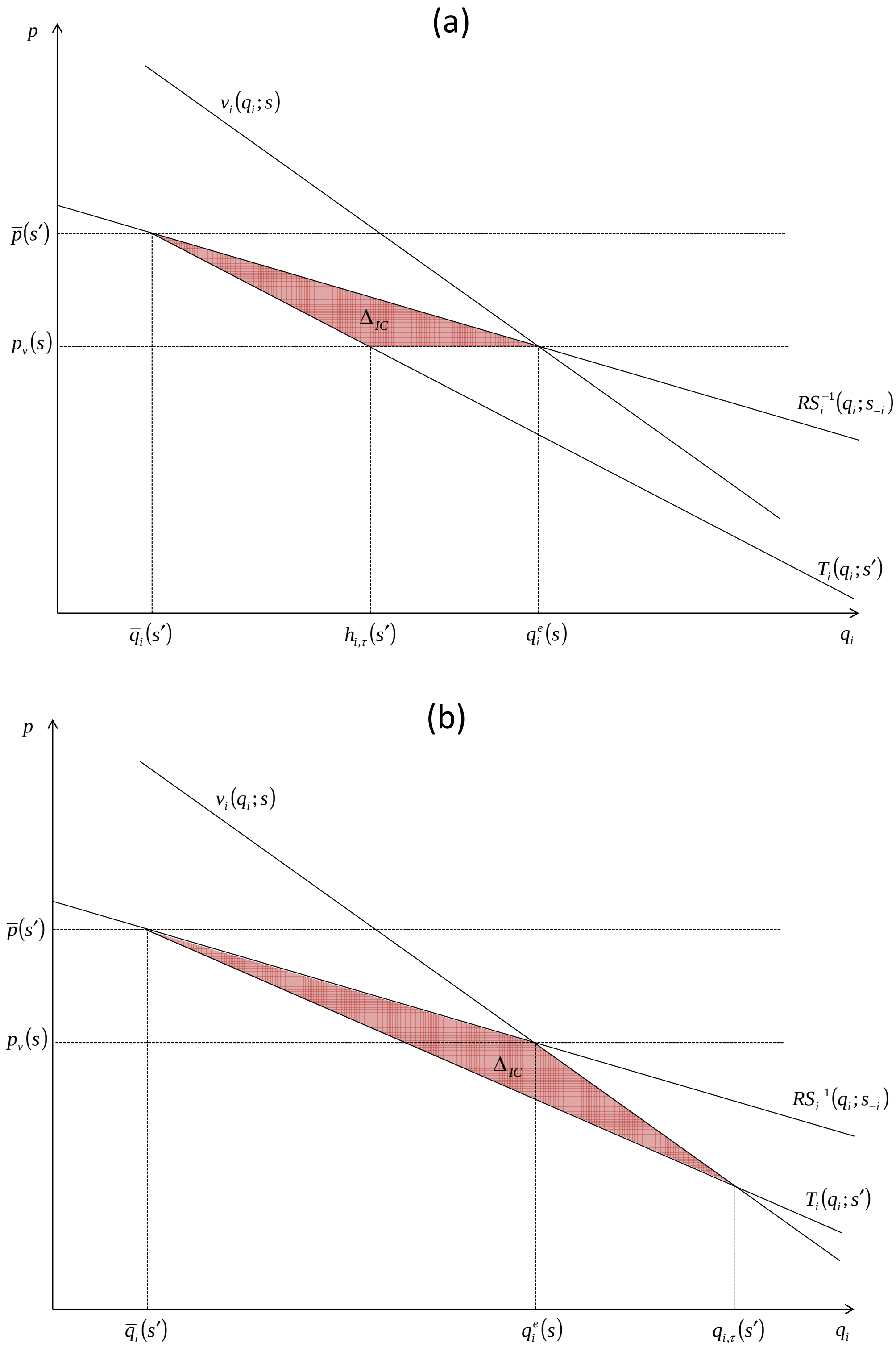

Figure 3.3: Incentive compatibility of the Vickrey auction when $\frac{\sigma_{\gamma\theta}}{var[\theta_m]} < -\frac{n\delta}{\beta}$ and when the permit market is (a) perfect ($\omega = 0$) or (b) collapsed ($\omega \to \infty$).

The fixed parameter values are presented in Table 3.1 and the values of the two variating variables in Table 3.2. For example in Simulation 5, I have used $\sigma_\varepsilon = 10$ and $\rho_{\gamma\theta} = 0$. For each information structure (simulations 1-9) I have let the coefficient of the market performance ω to rise from 0 to 30. The simulation results are presented in Figures 3.4, 3.5 and 3.6. Figures 3.4 and 3.5 present the results from Section 3.3 and thus the expected deadweight losses and slopes of the permit supply schedules of the second-best regulation, Weitzman's (1978) non-constant tax regulation and Roberts and Spence's (1976) non-constant quantity regulation. The results are calculated given that the regulator knows only the prior information ($I = 0$) and when the regulator is aware of the private information of firms ($I = \mathbf{s}$). Figure 3.6 presents the IC conditions of the Vickrey auction followed by the second-best regulation.

Table 3.1: Simulations - fixed parameter values

Variable		Value
n	Number of firms	10
σ_θ	Standard deviation of the abatement cost parameter	10
ρ	Correlation coefficient of the abatement costs	0.5
β/n	Slope of the aggregate abatement costs	1
σ_γ	Standard deviation of the damage parameter	10
δ	Slope of the pollution damage	1

Table 3.2: Simulations - values of variating variables

Variable			σ_ε Standard deviation of the signal noise		
		Value	5	10	20
$\rho_{\gamma\theta}$	Correlation coefficient	-0.2	Simulation 1	Simulation 2	Simulation 3
	of abatement costs	0	Simulation 4	Simulation 5	Simulation 6
	and pollution damage	0.2	Simulation 7	Simulation 8	Simulation 9

In Figure 3.4 the bold curves describe the deadweight losses of the second-best regulation when the regulator knows only the prior information (red curve) and when the regulator knows the private information of firms (blue curves). The solid thin curves are the results of the Roberts and Spence non-constant quantity regulation and the dashed curves the Weitzman non-constant tax regulation. When moving from the left panels to the right panels, and thus when the signals become more noisy, less is gained from the information mechanism and the closer the red and blue curves come to each other when the permit market is not perfect. Also, the more noisy the signals are and the better the permit market performs the greater the difference is between the Weitzman non-constant tax regulation and the second-best regulation. On the other hand, when the correlation between emission abatement costs and pollution damage increases from negative to positive (from the top panels to the bottom panels) the wider the gap between results with and without private information becomes outside the case of a perfect permit market.

Figure 3.5 presents the slopes of the permit supply schedules. According to the numerical simulations, the slope of the second-best regulation ($\tau > 0$) is decreasing in ω and increasing in $\rho_{\gamma\theta}$ and σ_ε (when $I = \mathbf{s}$ and $\omega > 0$). Moreover, the difference between the slopes of the Weitzman, and Roberts and Spence regulations increases in $\rho_{\gamma\theta}$ but decreases in σ_ε (if $I = \mathbf{s}$).

Figure 3.6 presents the IC condition from equation (3.59) in the form of $\frac{\Delta_{IC}}{\left(s_i - s_i'\right)^2}$. Hence it is independent whether s_i' is greater or smaller than the true signal s_i. According to the simulations, $\frac{\Delta_{IC}}{\left(s_i - s_i'\right)^2}$ first increases and then decreases when ω increases if $-\frac{n\delta}{\beta} \leq \frac{\sigma_{\gamma\theta}}{var[\theta_m]} \leq 1$ is satisfied. Thus in all simulations $\frac{\Delta_{IC}}{\left(s_i - s_i'\right)^2} \geq 0$.

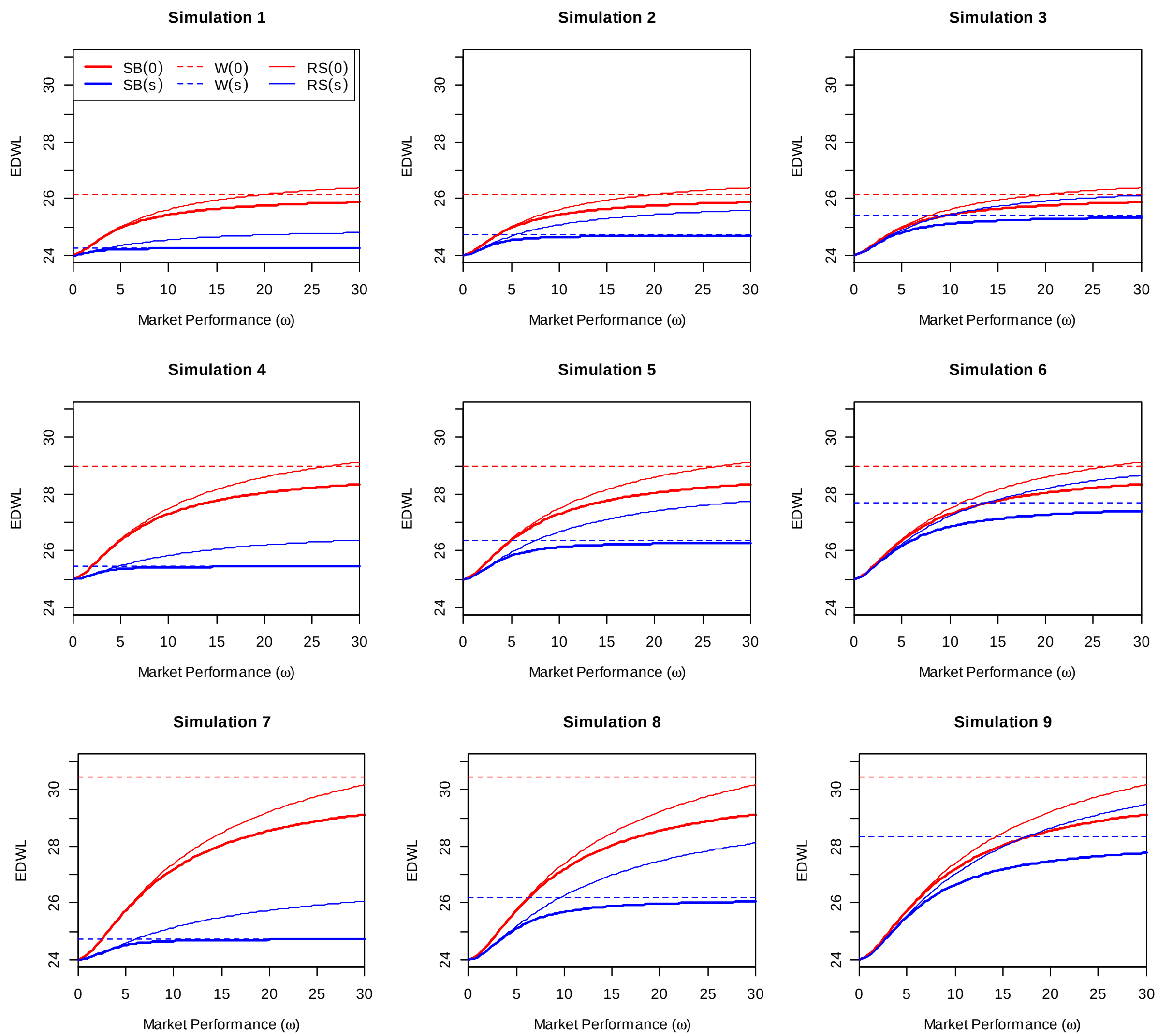

Figure 3.4: Expected deadweight losses of the second-best regulation (SB), the Weitzman (1978) non-constant tax regulation (W), and the Roberts and Spence (1976) non-constant quantity regulation (RS) when the coefficient of market performance ω increases. The red curves describe the cases when the regulator knows only the prior information ($I = 0$) and the blue curves the cases when the regulator knows the private information of firms ($I = \mathbf{s}$). The parameter values of the various simulations are presented in Tables 3.1 and 3.2. In the simulations the signal noise increases from left to right and the correlation between emission reduction costs and benefits from top to bottom.

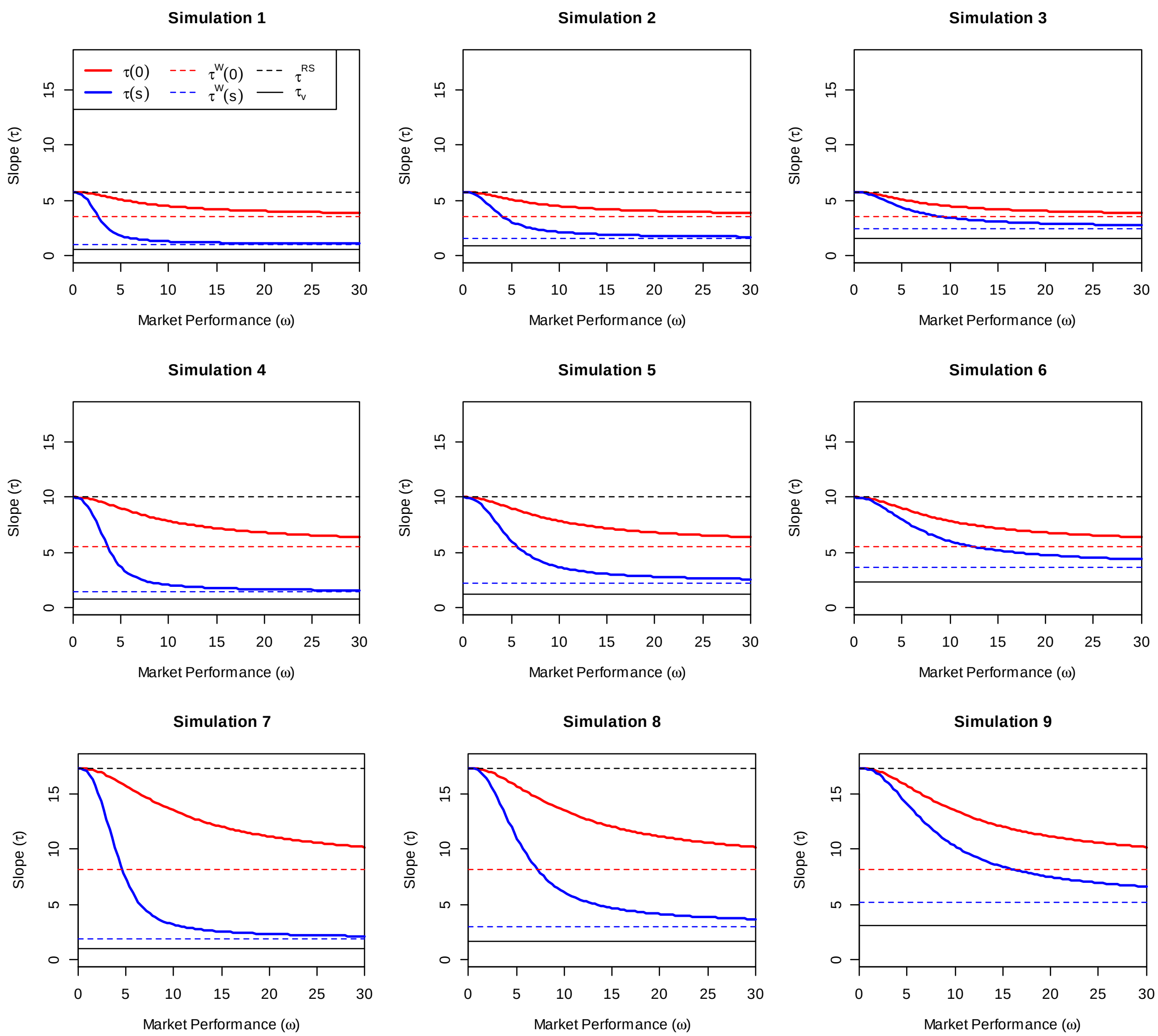

Figure 3.5: Slopes of the permit supply schedules when the coefficient of market performance ω increases. The second-best regulation (τ), the Weitzman (1978) non-constant tax regulation (τ^W), the Roberts and Spence (1976) non-constant quantity regulation (τ^{RS}) and the inverse residual supply of the Vickrey auction (τ_v). The red curves describe the cases when the regulator knows only the prior information ($I = 0$) and the blue curves the cases when the regulator knows the private information of firms ($I = \mathbf{s}$). The parameter values of the various simulations are presented in Tables 3.1 and 3.2. In the simulations the signal noise increases from left to right and the correlation between emission reduction costs and benefits from top to bottom.

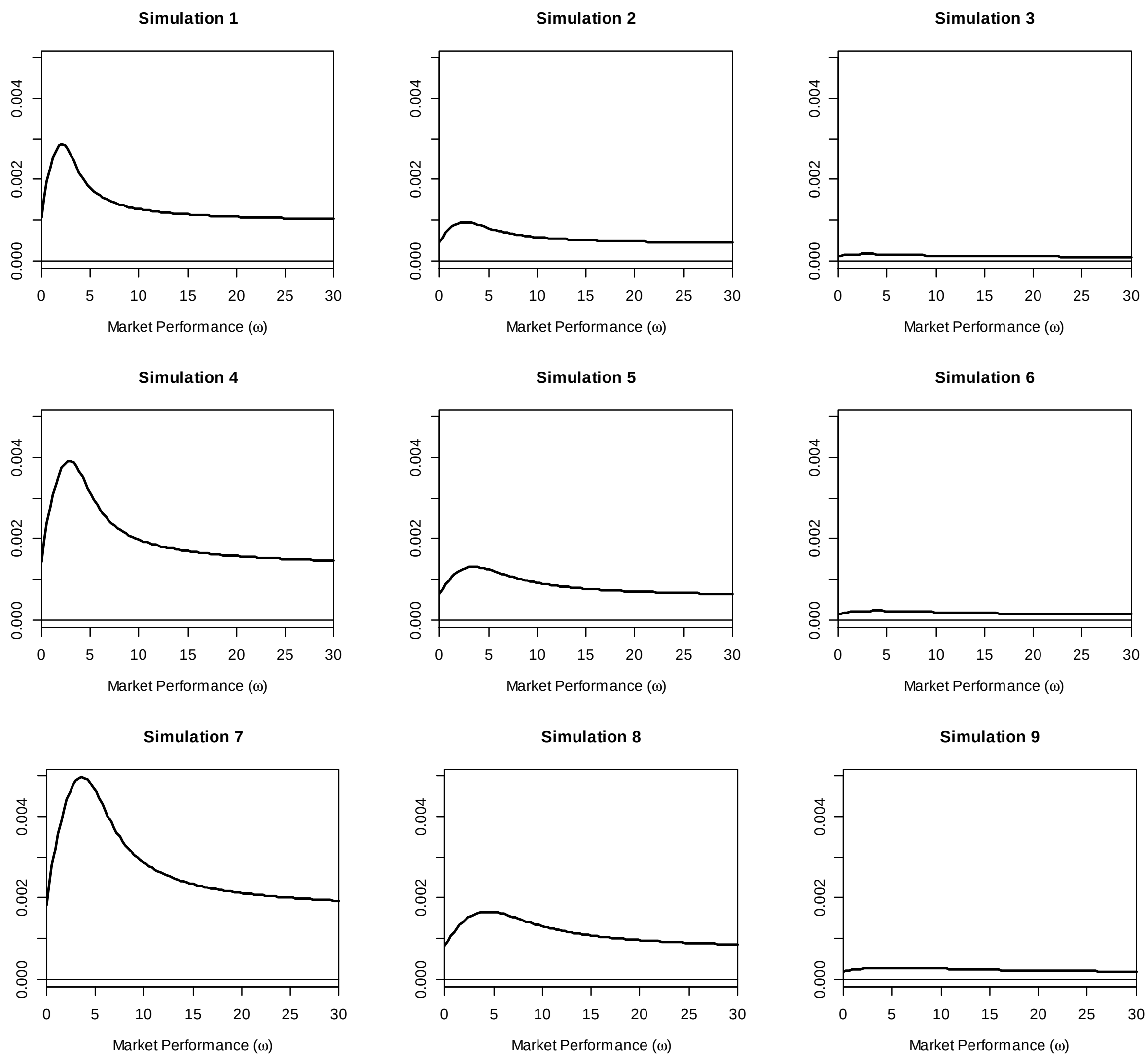

Figure 3.6: Incentive compatibility condition of the Vickrey auction followed by the second-best regulation, $\Delta_{IC}/\left(s_i - s_i'\right)^2$. The parameter values of the various simulations are presented in Tables 3.1 and 3.2. In the simulations the signal noise increases from left to right and the correlation between emission reduction costs and benefits from top to bottom.

Chapter 4

Collusion in emission allowance auctions

Abstract

I examine the Vickrey auction and the uniform price auction in allocations of emission allowances without an allowance resale market. I study oligopolistic competition in an allowance auction and build a demand function equilibrium model with linear strategies and private values. In the model, the market consists of two parts, a competitive fringe and a number of strategic firms. The fringe balances the market and hence I do not have to consider Wilson's (1979) low price equilibria. I link the auction model with a coalition formation game, where any subgroup of firms may coordinate their bids in the auction. I calculate the impact of collusion on the efficiency of the allowance allocation and auction revenues. Montero (2008) shows, with a similar kind of private values model, that the Vickrey auction provides efficient allocation of allowances even if firms collude. However, all strategic firms have strong incentives to form one big coalition in a Vickrey auction, which reduces revenues. The uniform price auction creates a coalition game with positive externalities. The more concentrated the coalition structure is, the better off coalition outsiders are. I examine three examples of coalition formation game in a uniform price auction: a cartel game with either myopic or farsighted firms and an open membership game with multiple coalitions. The stable coalition structure and hence the efficiency and revenues of the uniform price auction depend heavily on the coalition game and the structure of the market.

4.1 Introduction

In many emissions trading programs, auction mechanisms are used to allocate emission allowances to firms. For instance, the European Union Emissions Trading System (EU ETS), California's Cap-and-Trade Program and the U.S.'s Regional Greenhouse Gas Initiative (RGGI) use a uniform price format to auction emission allowances. The primary objective of pollution regulation is the efficient allocation of pollution rights. The total costs of emission reductions will be minimized if the marginal abatement costs, or the marginal values of the right to pollute, are equal across firms. Also, an efficient allocation creates the right price signal for emissions reductions in other sectors of the economy. Contrary to many other auctions, maximizing auction revenues is not the main objective of pollution regulation. However, revenues may be important if revenues can be recycled into the economy in a profitable way. For instance, the revenues from auctions can be used to reduce distortionary taxes such as labour taxes or increase firms' incentives for research and development activities of cleaner and more efficient technologies.

Collusion is one of the concerns of efficient markets and auctions. In this paper, collusion means that firms can communicate prior to an auction, but cannot make binding agreements, because such agreements are illegal. By communicating, strategic firms or subgroups of firms, i.e. coalitions, may coordinate their bidding behavior in the auction. This, on the other hand, has consequences for the efficiency and revenues of the auction. However, while agreements are not binding, it is easy for an individual firm to deviate from the agreement. Thus a stable coalition structure must be self-enforcing and it must benefit all coalition members. The objective of this paper is to examine two different auction designs for allocating pollution rights to firms where firms may collude. I make comparisons between the commonly used design of the uniform price auction and the Vickrey auction, in terms of efficiency and revenues. In addition, I study three examples of coalition formation games: a cartel game with myopic firms, a cartel game with farsighted firms and an open membership game with multiple coalitions.

The contribution of this paper is to link multi-unit auction mechanisms to non-cooperative coalition games. Papers studying collusion in multi-unit auctions are surprisingly scarce. However, Wilson (1979) and Back and Zender (1993), and subsequent papers, have shown that demand reduction or collusive-seeming bidding may result in very low equilibrium prices in uniform price auctions. Furthermore, Fabra (2003) and Dechenaux and Kovenock (2007), for instance, examine tacit collusion in infinitely repeated uniform price auctions[1]. They study how perfect collusion can be sustained among the capacity-constrained firms. However,

[1] Fabra (2003) also studies discriminatory price auctions.

these are equilibria in non-cooperative games between individual firms. On the other hand, Montero (2008) shows that collusion does not distort the efficient solution of the Vickrey auction.

The model of this paper is a static one. I examine the non-cooperative auction game of coalitions, and how this game affects the formation of coalitions by firms. I apply the partition function approach (e.g. Yi 2003). Thus the coalition formation is a two-stage game. In the first stage, strategic firms decide on their participation in coalitions. In the second stage, coalitions play a non-cooperative auction game, which is assumed to have a unique Nash equilibrium outcome for any coalition structure. Hence the whole game can be interpreted as a one-stage game, where firms need to choose their membership strategies, because the payoffs of the second-stage auction can be calculated given the information about the auction design and the coalition structure. Moreover, to simplify the model, I assume that coalition members may transfer emission allowances between themselves inside each coalition, but there is no other resale market after the auction. To my knowledge, coalition formation has not been studied in the context of multi-unit auction models in a similar set-up before.

In uniform price auctions, bidders pay the uniform clearing price for all the units they win. In general, this makes uniform price auctions rather complex to model. Bidders will not bid with their true valuations if they can influence the clearing price and, consequently, their payoffs by their bidding. The uniform pricing creates strong incentives for bid-shading and demand reduction, which may cause inefficient allocation of allowances and decrease the revenues. Bidders shade their bids more, the relatively larger they are. Due to bid-shading, the clearing price is lower than the competitive price and large bidders receive less units than in the efficient outcome. (E.g. Ausubel et al. 2013.)

Under-pricing can be eliminated or reduced e.g. 1) if inelastic supply can be adjusted after the submission of bids (Back and Zender 2001, McAdams 2007); 2) by making the supply elastic (LiCalzi and Pavan 2005); or 3) by forcing the bid functions to be discrete (Kremer and Nyborg 2004). Moreover, strategic bidding models with a uniform price format have often applied the supply function equlibrium (SFE) models developed originally by Klemperer and Meyer (1989). If there are n strategic agents, the solution of SFE is a system of n differential equations. Usually, these models have a continuum of equilibria bounded by the Bertrand and Cournot equilibria. However, the number of equilibria can be reduced by assumptions about the initial conditions or end-conditions of the supply functions. For instance, in models of electric power markets, capacity and price constraints reduce the range of equilibria (e.g. Green and Newbery 1992, Holmberg 2008, 2009). Also, assuming linear model and linear bid schedules simplifies the model and offers tractable solutions (e.g. Green 1996, Baldick et al. 2004, Ausubel et al. 2013). Another important feature of the uniform price auction is that coalitions are beneficial for members of coalitions, but they are even more beneficial for those firms that stay outside. Hence, under uniform pricing coalition formation is a game with positive externalities. In these games, large coalitions are hard to sustain, because firms have strong incentives to deviate.

In addition to the uniform price auction, I study the Vickrey auction, or its multi-unit extension the Vickrey-Clarke-Groves (VCG) mechanism (Vickrey 1961, Clarke 1971, Groves 1973). In particular, I apply the indirect interpretation of the VCG mechanism introduced by Montero (2008). When bidders' values are private, i.e. each bidder knows its valuations[2] and these valuations do not depend on other bidders' valuations, the VCG mechanism implements efficient allocation in dominant strategies. Hence it is the best response for each firm to bid sincerely in a Vickrey auction whatever other bidders do. This is induced by the Vickrey pricing rule. The marginal price bidders pay for each unit they win is the externality the bidder causes to other bidders by participating in the auction and winning the unit. Montero (2008) derives Vickrey prices by a particular payback rule, which determines the share of paybacks to each bidder after the auction. The marginal payment schedule and thus the share of the paybacks is determined by the total supply of allowances and the bid schedules of other bidders, and is thus independent of the bidder's own bidding in the auction. The greater the bidder's effect on the auction equilibrium, the greater the share of the paybacks. Furthermore, Montero shows that collusion does not distort the efficiency of the outcome. However, if colluding firms are able to agree on the distribution of extra profits due to collusion, the Vickrey pricing rule creates incentives for stratgic firms to form as large coalitions as possible, because coalitions do not benefit outsiders but only insiders. Coalition profits are greatest with grand coalition where all strategic firms are members of a single coalition. With an efficient cartel agreement, each strategic firm is better off than if it participated in the auction individually.

To analyze these two auction designs, I derive a linear-quadratic demand function equilibrium with a fixed supply of emission allowances. I assume that the market consists of two parts. In addition to ex-ante symmetric strategic firms, there is a fringe of competitive firms. The fringe firms act as price-takers in the market and thus the residual supply of allowances for strategic firms is an increasing function of price. This reduces the possibilities for underpricing in a uniform price auction. Furthermore, under a uniform price auction, I consider three endogenous coalition formation games. The first two games are cartel games, where only one coalition, i.e. a cartel, is formed. However, the stable structures are different depending on

[2]In the case of emission allowance auctions, the marginal valuations are determined by the marginal abatement costs.

whether the firms are myopic or farsighted. If a firm is myopic, it does not take into account the final result of its decisions but only the immediate reaction of the other players. A farsighted firm, on the other hand, understands that after its actions other firms or coalitions may react and these reactions might result in further reactions and thus affect the final outcome of the game. In addition to these two cartel games, I consider one game with multiple coalitions. Multiple coalition games become relatively complicated when the number of strategic agents increases. The number of possible coalition structures increases rapidly in the number of strategic firms (e.g. Sáiz et al. 2006). Fortunately, the analysis becomes simpler when the strategic firms are symmetric. Another distinction in coalition formation games is whether the coalition formation is a sequential process (e.g. Bloch 1995, 1996) or is it a simultaneous coalition formation game (e.g. Yi 1997). Moreover, what happens to the coalition if one of the firms or any subgroup of firms leaves the coalition? Do the remaining members of the coalition continue as a new coalition or does the coalition break down into singletons? For simplicity, I consider only simultaneous coalition formation games, where deviation strategies do not break down the remaining coalition. Moreover, due to the assumptions of ex-ante symmetric firms and a simultaneous coalition formation game, I do not have to consider transfers among coalition members. Each member of the same coalition gets the same payoff.

Applying a numerical example, I show the existence of stable structures in all three coalition formation games with a linear-quadratic model. Due to efficient allocation, there is no welfare loss in the Vickrey auction. However, the revenue loss increases in the market share of the strategic firms. In a uniform price auction, on the other hand, the stable coalition structure and hence the efficiency and revenues depend heavily on the coalition game as well as the structure of the market. If firms are not able to make binding agreements (a cartel game with myopic firms and an open membership game), the allocation of allowances is almost efficient and the auction revenues are almost as great as in a competitive market, at least for a large enough number of strategic firms and a large enough market share of fringe firms. If firms are farsighted and can agree on the least competitive cartel agreement, the uniform price auction will not offer an efficient allocation and the auction revenues will be relatively low, at least if strategic firms have a large market share.

This chapter is organized as follows. In Section 4.2, I introduce the linear-quadratic model. Section 4.3 explains the payment schedules of the two auction designs and Section 4.4 the three coalition formation games. In Section 4.5, I run a numerical simulation and derive the results of the paper. Section 4.6 concludes.

4.2 Model

I consider an auction model with a large number of firms, which are divided into two types, namely firms in a competitive fringe acting as one market participant indexed with f and a number of $n \geq 3$ identical strategic firms indexed with $i \in N = \{1, \ldots, n\}$. Allowances will be allocated to firms by two alternative auction designs: the Vickrey auction (VA) and the uniform price auction (UPA). The auction and payment rules are described later.

Pollution in the industry is denoted by a vector $\mathbf{q}_I = (q_f, \mathbf{q})$, where $\mathbf{q} = (q_1, \ldots, q_n)$ is a vector of strategic firms' pollution and the total pollution of the fringe is denoted by q_f. The aggregate pollution is thus $Q = q_f + \sum_{i=1}^{n} q_i$. At the outset, the firms' (business-as-usual) pollution is denoted by $Q^0 = q_f^0 + \sum_{i=1}^{n} q_i^0$, where q_i^0 denotes the pollution of strategic firm i and q_f^0 is the business-as-usual pollution of the competitive fringe. Hence, I denote the market share of strategic firms by $\lambda \equiv \frac{\sum_{i=1}^{n} q_i^0}{Q^0}$ and the market share of the fringe is respectively $1 - \lambda = \frac{q_f^0}{Q^0}$.

Reducing emissions is costly to firms. The total costs of reducing pollution below the business-as-usual levels, for firm i, are given by the abatement cost function $AC_i(q_i) = \int_{q_i}^{q_i^0} u_i(x)\, dx$, where $u_i(q_i)$ is the marginal abatement cost function in terms of emissions, i.e. the marginal value of the avoided abatement costs.[3] Thus the gross value of the avoided abatement costs of firm i, i.e. the gross value of pollution, writes $U_i(q_i) = AC_i(0) - AC_i(q_i) = \int_0^{q_i} u_i(x)\, dx$.

4.2.1 Linear-quadratic model and auction rules

For simplicity, I use a linear-quadratic model. I thus assume that $U_i(q_i)$ is given by the equation:

$$U_i(q_i) = \theta_i q_i - \frac{1}{2}\beta_i q_i^2, \tag{4.1}$$

where the intercept of the marginal value function is assumed, for simplicity, to be constant for every firm in the economy and thus $\theta_i = \theta$ for all i and for all fringe firms. This specification defines the linear-quadratic auction model.

Definition 4.1. *The model described in equations (4.1) - (4.7) is called the linear-quadratic model.*

The marginal abatement cost function of the entire industry, or the aggregate inverse demand

[3] Let z_i denote emission reductions and $AC_i^z(z_i)$ is the abatement cost function in emission reductions, i.e. $AC_i(q_i) \equiv AC_i^z(z_i(q_i))$, where $z_i(q_i) = q_i^0 - q_i \geq 0$. The marginal abatement cost function is $MAC_i^z(z_i) = \frac{dAC_i^z(z_i)}{dz_i} \equiv \frac{dAC_i(q_i)}{dq_i}\frac{dq_i}{dz_i} = -u_i(q_i)$. Thus, $\frac{dAC_i^z(z_i)}{dq_i} = \frac{dAC_i^z(z_i)}{dz_i}\frac{dz_i}{dq_i} = u_i(q_i)$.

function for emission allowances, is written as

$$u(Q) = \theta - \beta Q. \tag{4.2}$$

Hence, the true demand function of the industry is $u^{-1}(p) = \frac{\theta}{\beta} - \frac{1}{\beta}p$.

Pollution causes damage. The pollution reduction target is determined by the regulator's expectations about marginal abatement costs and the marginal damage of pollution. I assume that the regulator observes Q^0 but has no exact information about θ or β. Hence the regulator allocates a fixed supply of allowances $L = \delta Q^0$, such that:

$$E[u(L)] = E[MDF(L)], \tag{4.3}$$

where $MDF(Q)$ is the marginal damage function and $0 < (1-\delta) < 1$ is the reduction target as a share of business-as-usual emissions. Note that while MDF is uncertain in the model, we live in a second-best world. Hence $L = Q^\star = q_f^\star + \sum_{i=1}^{n} q_i^\star$, where the second-best solution, i.e. the cost-minimizing solution given the fixed supply of allowances L, is denoted by a vector $\mathbf{q}_I^\star = \left(q_f^\star, \mathbf{q}^\star\right)$. Business-as-usual emissions are found at the point where the marginal value of pollution falls to zero. Hence $Q^0 = \frac{\theta}{\beta}$, which further gives

$$L = \frac{\delta\theta}{\beta}. \tag{4.4}$$

Given the market share of the fringe $1-\lambda$, the aggregate marginal value of pollution of the fringe is written as

$$u_f(q_f) = \theta - \beta_f q_f, \tag{4.5}$$

where $\beta_f = \frac{1}{1-\lambda}\beta$. All the strategic firms are symmetric and each of them thus has a true inverse demand for allowances as follows:

$$u_i(q_i) = \theta - \beta_i q_i. \tag{4.6}$$

Furthermore, while the strategic firms are identical, the slope of an individual strategic firm is $\beta_i = \frac{n}{\lambda}\beta$.

In the auction, either in VA or UPA, each bidder x reports its demand schedule $D_x(p)$ to the regulator. The inverse demand schedule is written $P_x(q_x) \equiv D_x^{-1}(q_x)$. The schedules are limited to be linear and strictly decreasing in p. Furthermore, I assume that the bid functions have an equal constant term θ, which is common knowledge among bidders. Thus the reported demand schedules are written as

$$P_x(q_x) = \theta - b_x q_x, \tag{4.7}$$

where the strategy of bidder x is determined solely by the slope of the inverse bid function b_x. In the Vickrey auction it is a dominant strategy to bid sincerely, and thus $b_x = \beta_x$. In the linear equilibrium of the uniform price auction, on the other hand, bidders do not shade their bids at $q_x = 0$ and bid-shading increases in quantity (Ausubel et al. 2013). Hence, the bid function coincides with the marginal valuation at $q_x = 0$ and is steeper than the marginal valuation for $q_x > 0$. Thus the linear bid function from (4.7) with $b_x \geq \beta_x$ is well justified in both auction designs.

Moreover, the relative size of a single fringe firm is so small that I assume it to bid with its marginal costs. Hence, $P_f(q_f) = u_f(q_f)$ from (4.5). Given the pollution target L, the residual supply of emissions allowances for the oligopolistic market, i.e. for all the strategic firms, is thus

$$\begin{aligned} RS_s(p) &= L - u_f^{-1}(p) \\ &= (\delta + \lambda - 1)\frac{\theta}{\beta} + \left(\frac{1-\lambda}{\beta}\right)p, \end{aligned} \tag{4.8}$$

and the inverse residual supply for the oligopolistic market is $RS_s^{-1}(q_s) = \left(1 - \frac{\delta}{1-\lambda}\right)\theta + \left(\frac{\beta}{1-\lambda}\right)q_s$, where $q_s = \sum_{i=1}^{n} q_i$.[4]

Respectively, given the bid schedules of other bidders, the residual supply function for bidder x is

$$RS_x(p) = L - D_{-x}(p), \tag{4.9}$$

where, $D_{-x}(p)$ is the aggregate demand reported by every other bidder but bidder x.

The regulator computes the total demand from the bid schedules and clears the auction. The auction equilibrium is defined by the clearing rule:

$$p = RS_x^{-1}(l_x) \begin{cases} \geq P_x(l_x), & \text{if } l_x = 0 \\ = P_x(l_x), & \text{if } 0 < l_x < L \\ \leq P_x(l_x), & \text{if } l_x = L \end{cases}, \tag{4.10}$$

[4] A fixed supply of allowances is one special case in Montero (2008). He studies a more general model where the supply of allowances is defined by the marginal damage function of pollution. Note that the model of this paper also generalizes to the case of a non-constant supply of allowances. For example, in the absence of the fringe, we could also write $RS_s^{-1}(q_s) \equiv MDF(Q)$ where $q_s = Q$.

where l_x is the number of emission allowances allocated to bidder x and $RS_x^{-1}(q_x)$ is the inverse residual supply. In a uniform price auction bidders pay the clearing price for all the allowances they win. In a Vickrey auction, bidders receive paybacks after the auction and the marginal price that bidders pay for units is called the Vickrey price. The total costs of a firm are the sum of its abatement costs $AC_x(l_x)$ and its auction payments R_x^A in auction $A = va$, upa. In the Vickrey auction the costs are

$$TC_x^{va}(b_x^\star) = \underbrace{\int_{l_x^\star}^{q_x^0} u_x(z)\,dz}_{AC_x(l_x^\star)} + \underbrace{(1-\alpha_x)\,p^\star l_x^\star}_{R_x^{va}}, \tag{4.11}$$

where $p^\star$ is the clearing price of the Vickrey auction, $l_x^\star$ is the equilibrium allocation of pollution permits and α_x is the share of the paybacks to bidder x after the auction. Due to efficient allocation, the clearing price of the Vickrey auction is the second-best price. In the uniform price auction total costs of bidder x are

$$TC_x^{upa}\left(\hat{b}_x\right) = \underbrace{\int_{\hat{l}_x}^{q_x^0} u_x(z)\,dz}_{AC_x\left(\hat{l}_x\right)} + \underbrace{\hat{p}\hat{l}_x}_{R_x^{upa}}, \tag{4.12}$$

where, respectively, $\hat{p}$ is the clearing price of the uniform price auction and $\hat{l}_x$ is the equilibrium allocation of pollution permits. I explain later how the payment schedules affect the firms' bidding behavior and, respectively, their strategies in coalition formation.

4.2.2 Coalition structure

Strategic firms may co-operate when reporting their schedules to the regulator in the auction. If this is the case, a coalition of co-operating firms reports one joint demand schedule or one of the collusive firms reports the aggregate schedule while the others report null schedules. After the auction, the coalition shares the costs and allowances between its members using some internal mechanism which I assume to be efficient and equal with respect to the member firms' cost functions. Other than transactions inside coalitions, I assume that there is no other resale market. Due to the symmetry, each firm inside a coalition earns equal profits. The joint true inverse demand of coalition c_i, consisting of n_i strategic firms, can be written as

$$u_{c_i}(q_{c_i}) = \theta - \beta_{c_i} q_{c_i}, \tag{4.13}$$

where $\beta_{c_i} = \frac{n}{n_i \lambda}\beta$. The pollution of coalition c_i is q_{c_i} and the pollution of firm j in coalition c_i is denoted by $q_{c_i(j)} = \frac{q_{c_i}}{n_i}$. I assume that firms may form multiple coalitions of different sizes. I assume that the intercept θ and the slopes β_x are common knowledge among polluting firms, given the knowledge related to the coalition structure C.

Definition 4.2. *The coalition structure* $C \equiv \{n_1, n_2, \ldots, n_m\}$ *is a partition of strategic firms, such that coalition* c_i *includes* n_i *strategic firms;* $c_i \cap c_j = \varnothing$, $\forall i \neq j$; $\sum_{i=1}^{m} n_i = n$; *and* $n \geq n_1 \geq n_2 \geq \cdots \geq n_m \geq 1$.

Hence I assume that each strategic firm is a member of one and only one coalition. Also, coalitions are ordered such that the largest coalition is c_1 and the smallest is c_m, where m is the number of coalitions. If $m = 1$, then there is only one coalition and it is called a grand coalition, i.e. $c_1 \equiv c_{gc}$ and $n_1 \equiv n_{gc} = n$. On the other hand, if $m = n$, then all coalitions are singletons, i.e. $n_i = 1$ for all i. To simplify the notation, consider the following structure:

$$\begin{aligned} C &= \left\{ \underbrace{n_1, \ldots, n_{m_1}}_{m_1 \text{ coalitions}}, \underbrace{n_{m_1+1}, \ldots, n_{m_1+m_2}}_{m_2 \text{ coalitions}}, \ldots, \underbrace{n_{(m_1+\ldots+m_{h-1}+1)}, \ldots, n_m}_{m_h \text{ coalitions}} \right\} \\ &\equiv \left\{ \eta_1^{(m_1)}, \eta_2^{(m_2)}, \ldots, \eta_h^{(m_h)} \right\}, \end{aligned} \tag{4.14}$$

where $\eta_1 = n_1 = \cdots = n_{m_1}$ and $\eta_i = n_{\left(1+\sum_{j=1}^{i-1} m_j\right)} = \cdots = n_{\left(\sum_{j=1}^{i} m_j\right)}$ with $\eta_1 > \eta_2 > \cdots > \eta_h$ and $m = \sum_{i=1}^{h} m_i$. Hence each of the first m_1 symmetric coalitions denoted by $\kappa_1 \equiv \eta_1^{(m_1)}$ has exactly η_1 members, the next m_2 coalitions denoted by $\kappa_2 \equiv \eta_2^{(m_2)}$ have η_2 members each and so forth.[5]

I assume that coalitions of equal size apply symmetric strategies in auctions, i.e. $b_{c_j} = b_{\kappa_i}$ for all $n_j = \eta_i$. Thus the inverse residual supply for coalition c_j is

$$RS_{c_j}^{-1}\left(q_{c_j}\right) = \max\left[0, \left(\theta - \left(\frac{1}{\sum_{i\neq j}^{m}\left\{\frac{1}{b_{c_i}}\right\} + \frac{1}{\beta_f}}\right) L + \left(\frac{1}{\sum_{i\neq j}^{m}\left\{\frac{1}{b_{c_i}}\right\} + \frac{1}{\beta_f}}\right) q_{c_j}\right)\right], \tag{4.15}$$

[5]Respectively, coalition c_j can be denoted by $c_j \equiv n_j^{(1)}$. Hence $c_j \equiv \kappa_i$ if $n_j = \eta_i$.

or respectively using notation with κ_i:

$$RS_{\kappa_i}^{-1}\left(q_{\kappa_i}\right) = \max\left[0, \left(\theta - \left(\frac{1}{\frac{m_i-1}{b_{\kappa_i}} + \sum_{j\neq i}^{h}\left\{\frac{m_j}{b_{\kappa_j}}\right\} + \frac{1}{\beta_f}}\right) L \right.\right. \\ \left.\left. + \left(\frac{1}{\frac{m_i-1}{b_{\kappa_i}} + \sum_{j\neq i}^{h}\left\{\frac{m_j}{b_{\kappa_j}}\right\} + \frac{1}{\beta_f}}\right) q_{\kappa_i}\right)\right]. \tag{4.16}$$

Suppose that the total costs $TC_{\kappa_i}^{A}\left(b_{\kappa_i}\right)$ of coalition κ_i, given the coalition structure $C = \left\{\eta_1^{(m_1)}, \ldots, \eta_h^{(m_h)}\right\}$, are determined by the unique Nash equilibrium of the non-cooperative auction game A and thus are derived from (4.11) or (4.12). Hence each coalition structure maps the total costs for every coalition and, consequently, for every individual firm, given the equilibrium of auction A. The per-member costs of coalition κ_i, given the coalition structure C and the auction design $A = va$, upa, are thus given by the following partition function:

$$TC^{A}\left(\eta_i; C\right) \equiv \frac{TC_{\kappa_i}^{A}\left(b_{\kappa_i}\right)}{\eta_i}.$$

I omit the superscript A when it is not relevant. Due to the symmetry, each member of a coalition is assumed to earn equal profits. I consider three different coalition formation games. However, I first explain the payment rules of the second-stage auction game and derive two metrics for comparing auction designs. These are the relative welfare loss and the relative revenue loss. The aggregate abatement costs are compared to the abatement costs by the efficient allocation of allowances (i.e. the second-best allocation) denoted by AC^e. Auction revenues are compared to "competitive revenues", which are the revenues if the regulator received the second-best price $p^\star$ from all the allowances it sells in the auction, i.e. $R^e = p^\star L$.

4.3 Auction

4.3.1 Vickrey auction

In the Vickrey auction (see Montero 2008), in addition to the auction procedure described in the previous section, every bidder gets paybacks after the auction. The share of the paybacks to coalition c_i is defined as

$$\alpha_{c_i} = 1 - \frac{\int\limits_0^{l_{c_i}} RS_{c_i}^{-1}(z)dz}{RS_{c_i}^{-1}(l_{c_i})l_{c_i}}. \tag{4.17}$$

In equilibrium $RS_{c_i}^{-1}(l_{c_i}) = p$. Hence the total costs of coalition c_i in a Vickrey auction are

$$\begin{aligned} TC_{c_i}^{va}\left(b_{c_i}\right) &= \int\limits_{l_{c_i}}^{q_{c_i}^0} u_{c_i}\left(z\right) dz + \left(1 - \alpha_{c_i}\right) p l_{c_i} \qquad (4.18) \\ &= \int\limits_{l_{c_i}}^{q_{c_i}^0} \left\{\theta - \beta_{c_i} z\right\} dz + \int\limits_0^{l_{c_i}} RS_{c_i}^{-1}(x)dx. \end{aligned}$$

The first term is the abatement costs of reducing emissions from $q_{c_i}^0$ to l_{c_i} and the second term is the auction payment. By Proposition 3 in Montero (2008), it is optimal for each bidder to report its true demand curve irrespective of the other bidders' reports due to the paybacks. Thus the mechanism implements efficient allocation in dominant strategies. In equilibrium, all firms face the same clearing price and receive the efficient amount of allowances, but the final (average) prices, i.e. $(1 - \alpha_j)\, p$, differs between bidders, unless they all have identical bid schedules. The marginal price, i.e. the Vickrey price, is determined by the inverse residual supply function $RS_{c_i}^{-1}\left(q_{c_i}\right)$. The final payment is equal to the externality the bidder causes to other bidders. When the total supply of allowances is fixed, it is the external cost due to the increase in allowance price.[6] If the number of strategic firms increases to the limit and if firms do not coordinate their bids, the share of paybacks closes to zero and the Vickrey price for all units is equal to the uniform second-best price $p^{\star}$.

Due to truthful reporting, the Vickrey auction implements efficient allocation of allowances and thus a cost-minimizing solution for pollution control. The second-best allocation holds even if firms coordinate their bids in the auction. Still, the dominant strategy is to report truthfully and the outcome will be the second-best. However, compared to a competitive market, the paybacks to coalitions are greater due to their greater impacts on aggregate demand and the larger residual supply at every price. Under collusion, the auction revenues of the regulator will be lower than in a more competitive market structure.

The paybacks to an individual fringe firm are zero, i.e. $\alpha_f = 0$. The share of paybacks to

[6] If we had an emission damage function in the model, as in Montero (2008), the externality would consist of two factors: an increase in the allowance price and the total damage of the bidder's own pollution.

coalition c_i is (see Appendix 4.A for derivation)[7]

$$\alpha_{c_i} = \begin{cases} \frac{n_i \lambda \delta}{2(n-n_i\lambda)(1-\delta)} & \text{if } D^{va}_{-c_i}(0) \geq L \\ \frac{n_i \lambda \delta}{2(n-n_i\lambda)(1-\delta)} \left[1 - \left(\frac{n-n_i\lambda-n\delta}{n_i\lambda\delta}\right)^2\right], & \text{if } D^{va}_{-c_i}(0) < L \end{cases}. \tag{4.19}$$

Keeping the number of strategic firms n fixed, the share of paybacks to coalition c_i increases in the size of collusive firms n_i, in the market share of strategic firms λ and in the stringency of the environmental policy δ. Thus it is profitable for strategic firms to form a grand coalition c_{gc} with $n_{gc} = n$. The share of paybacks to the grand coalition is

$$\alpha_{gc} = \begin{cases} \frac{\lambda \delta}{2(1-\lambda)(1-\delta)}, & \text{if } 1-\lambda \geq \delta \\ \frac{\lambda \delta}{2(1-\lambda)(1-\delta)} \left[1 - \left(\frac{1-\delta-\lambda}{\lambda\delta}\right)^2\right], & \text{if } 1-\lambda < \delta \end{cases}.$$

The total costs of bidder $x = f, c_{gc}$ are from (4.18) $TC^{va}_x = \int_{l^*_x}^{q^0_x} \{\theta - \beta_x z\}\, dz + (1-\alpha_x)\, p^* l^*_x$, where $p^* = (1-\delta)\,\theta$; $l^*_{gc} = \lambda \frac{\delta\theta}{\beta}$; and $l^*_f = (1-\lambda)\, \frac{\delta\theta}{\beta}$. Due to the efficient allocation, the total abatement costs of the entire industry are minimized:

$$\begin{aligned} AC^{va} = AC^e &= \int_L^{Q^0} \{\theta - \beta z\}\, dz \\ &= \frac{(1-\delta)^2\,\theta^2}{2\beta}. \end{aligned} \tag{4.20}$$

Hence there is no welfare loss due to efficient allocation in the Vickrey auction,

$$\Delta AC^{va} = \frac{AC^{va} - AC^e}{AC^e} = 0.$$

Furthermore, the "competitive revenues" are $R^e = p^\star L = \frac{(1-\delta)\delta\theta^2}{\beta}$. The share of paybacks and thus the relative revenue loss compared to there being competitive revenues is

$$\Delta R^{va} = \frac{R^e - R^{va}}{R^e} = \lambda \alpha_{gc} = \begin{cases} \frac{\lambda^2 \delta}{2(1-\lambda)(1-\delta)}, & \text{if } 1-\lambda \geq \delta \\ \frac{\lambda^2 \delta}{2(1-\lambda)(1-\delta)} \left[1 - \left(\frac{1-\delta-\lambda}{\lambda\delta}\right)^2\right], & \text{if } 1-\lambda < \delta \end{cases}. \tag{4.21}$$

[7] The correction, when $D^{va}_{-c_i}(0) < L$, is due to the fact that the inverse residual supply cannot be negative.

4.3.2 Uniform price auction

In a uniform price auction it is no longer profitable to report demand truthfully as long as firms can influence the auction price. The objective of coalition c_i is to minimize its total costs with respect to the slope of the bid function b_{c_i}:

$$\min_{b_{c_i}} TC^{upa}_{c_i}(b_{c_i}) = \int_{l_{c_i}(b_{c_i})}^{q^0_{c_i}} u_{c_i}(z)dz + l_{c_i}(b_{c_i}) RS^{-1}_{c_i}(l_{c_i}(b_{c_i})). \tag{4.22}$$

However, bidders have to take into account other bidders' actions and how their own actions affect other bidders' actions. Hence coalitions play a non-cooperative game. The first-order condition of (4.22) is written as

$$\begin{aligned} 0 &= -u_{c_i}(l_{c_i})\frac{dl_{c_i}}{db_{c_i}} + RS^{-1}_{c_i}(l_{c_i})\frac{dl_{c_i}}{db_{c_i}} + l_{c_i}\frac{dRS^{-1}_{c_i}(l_{c_i})}{dl_{c_i}}\frac{dl_{c_i}}{db_{c_i}} \qquad (4.23) \\ RS^{-1}_{c_i}(l_{c_i}) &= u_{c_i}(l_{c_i}) - l_{c_i}\frac{dRS^{-1}_{c_i}(l_{c_i})}{dl_{c_i}}. \end{aligned}$$

Consider coalition structure $C = \left\{\eta_1^{(m_1)}, \eta_2^{(m_2)}, \ldots, \eta_h^{(m_h)}\right\}$. By symmetry, coalitions of equal size use symmetric strategies. In equilibrium, $P_{\kappa_i}(l_{\kappa_i}) = RS^{-1}_{\kappa_i}(l_{\kappa_i})$. Assuming $RS^{-1}_{\kappa_i}(l_{\kappa_i}) > 0$ and $l_{\kappa_i} > 0$, this yields from (4.23):

$$b_{\kappa_i} = \beta_{\kappa_i} + \left(\frac{1}{\frac{m_i - 1}{b_{\kappa_i}} + \sum_{j \neq i}^{h}\left\{\frac{m_j}{b_{\kappa_j}}\right\} + \frac{1}{\beta_f}}\right).$$

Solving this for b_{κ_i} gives a quadratic equation:

$$\Gamma_{\kappa_i} b^2_{\kappa_i} + \left((m_i - 2)\beta_f - \Gamma_{\kappa_i}\beta_{\kappa_i}\right) b_{\kappa_i} - (m_i - 1)\beta_f \beta_{\kappa_i} = 0,$$

where $\Gamma_{\kappa_i} = 1 + \beta_f \sum_{j \neq i}^{h}\left\{\frac{m_j}{b_{\kappa_j}}\right\}$. Because $b_{\kappa_i} \geq \beta_{\kappa_i}$ we get the best response strategy for coalition κ_i as

$$\begin{aligned} b_{\kappa_i} &\equiv BR_{\kappa_i}(\mathbf{b}_{-\kappa_i}) \qquad (4.24) \\ &= \frac{1}{2}\left[\beta_{\kappa_i} - \left(\frac{m_i - 2}{\Gamma_{\kappa_i}}\right)\beta_f + \sqrt{\left(\beta_{\kappa_i} + \left(\frac{m_i - 2}{\Gamma_{\kappa_i}}\right)\beta_f\right)^2 + 4\frac{\beta_f \beta_{\kappa_i}}{\Gamma_{\kappa_i}}}\right], \end{aligned}$$

where $\mathbf{b}_{-\kappa_i} = \left(b_{\kappa_1}, \ldots, b_{\kappa_{i-1}}, b_{\kappa_{i+1}}, \ldots, b_{\kappa_h}\right)$ denotes the vector of strategies other than coalitions κ_i.

The slopes of the bid functions are strategic complements. Thus the best response functions are increasing in other bidders' strategies. The less aggressive other bidders play (the steeper their bid functions), the steeper the bid function of bidder κ_i. Without the fringe, the bid functions of strategic firms could have infinite slopes. Wilson (1979) has shown the possibility of extreme low price equilibria in uniform price auctions. With a big enough fringe or with an endogenous supply, this can be avoided.

The $BR_{\kappa_i}(\mathbf{b}_{-\kappa_i})$ function is derived similarly as in Klemperer and Meyer (1989), where they define unique supply function equilibrium using exogenous uncertainty in the industry demand (see also Akgün 2004). The demand in Klemperer and Meyer is analogous to the residual supply for strategic firms $RS_s(p) \equiv L - u_f^{-1}(p)$ in this paper. Without uncertainty, Klemperer and Meyer show with a general model that there is an infinite number of supply functions which satisfy the sufficient and necessary conditions for the optimum. In this model, I restrict the demand functions to be linear and to have a constant intercept parameter $\theta_i = \theta$. This is common knowledge to all bidders. Thus the demand function is fully defined for the whole support by a single parameter, i.e. the slope of the bid function. By this construction, I define the unique demand function equilibrium as $\hat{\mathbf{b}} = \left(\hat{b}_{\kappa_1}, \ldots, \hat{b}_{\kappa_h}\right)$ and the equilibrium price is thus written as

$$\hat{p} = \theta - \frac{1}{\left(\frac{1}{\beta_f} + \sum_{i=1}^{h} \frac{m_i}{\hat{b}_{\kappa_i}}\right)} L. \tag{4.25}$$

The allocation of allowances to bidder $x = f, \kappa_i$ is $\hat{l}_x = \frac{\theta}{b_x} - \frac{1}{b_x}\hat{p}$, where $b_f = \beta_f$. The total costs from (4.12) can be decomposed into the abatement costs $AC_x^{upa} = \int_{\hat{l}_x}^{q_x^0} \{\theta - \beta_x z\}\, dz$ and into the auction payments (revenues) $R_x^{upa} = \hat{p}\hat{l}_x$. The relative welfare loss due to inefficient allocation is thus

$$\Delta AC^{upa} = \frac{\int_{\hat{l}_f}^{q_f^0} \{\theta - \beta_f z\}\, dz + \sum_{i=1}^{h} m_i \int_{\hat{l}_{\kappa_i}}^{q_{\kappa_i}^0} \{\theta - \beta_{\kappa_i} z\}\, dz}{\int_{L}^{Q^0} \{\theta - \beta z\}\, dz} - 1, \tag{4.26}$$

and the relative revenue loss is

$$\Delta R^{upa} = 1 - \frac{\hat{p}}{p^*}. \tag{4.27}$$

4.3.3 Comparison of the Vickrey and uniform price auctions - a numerical example

Figure 4.1 illustrates the difference between the Vickrey auction (the left panel) and the uniform price auction (the right panel) in the case of a grand coalition (a cartel). Initially, the market is equally shared between the fringe and strategic firms. Thus the initial emissions of the cartel and the fringe are equal $q_f^0 = q_{gc}^0$ and they both have identical (true) inverse demand functions $u_{gc}(q_{gc}) = u_f(q_f)$. The pollution target is to halve total emissions from business-as-usual Q^0. Hence, the parameter values of this numerical example are $\lambda = \delta = 0.5$ and the abatement costs are normalized such that $\theta = 100$ and $\beta = 1$.

In the Vickrey auction, both bidders (the cartel and the fringe) bid truthfully and the allowance allocation is cost-efficient. The abatement costs of both the fringe and the cartel are illustrated by AC_{gc}^{va} (the red triangle). In the auction they both pay first the amount of $R_f = p^\star l_f^\star$, but the cartel receives paybacks and the final payment of the cartel is R_{gc}^{va} (the blue triangle).

In the uniform price auction, the cartel reduces its demand and reports schedule $P_{gc}^{upa}(q_{gc})$, which lies below the true demand function $u_{gc}(q_{gc})$ at every positive q_{gc}. Due to demand reduction (or bid-shading), the cartel receives allowances of an amount which is strictly less than in the Vickrey auction. The abatement costs of the cartel, AC_{gc}^{upa}, are thus higher than in the second-best. However, the equilibrium price of allowances is lower than the second-best price and the auction revenues from the cartel are only $R_{gc}^{upa} = \hat{p}\hat{l}_{gc}$. Interestingly, the strategic behavior of the cartel makes fringe firms strictly better off because the allowance price is lower. The abatement costs of the fringe are reduced to the triangle $AC_f^{upa} \equiv \triangle(X\hat{l}_f q_f^0)$ and the regulator collects revenues from the fringe amounting to $R_f^{upa} = \hat{p}\hat{l}_f$. The total costs of the fringe are lower than in the Vickrey auction and the costs are lower than the total costs of the cartel.

Comparing the results of the example drawn in Figure 4.1, we may conclude that the Vickrey auction is strictly a better auction design from the regulator's point of view in the case of a grand coalition, if the objective of the regulator is to achieve efficient allocation of allowances and to maximize the revenues of the auction. The total abatement costs are minimized and the revenues are larger than in a uniform price auction. Given these parameter values, the total abatement costs are $AC^{va} = 1250$ and $AC^{upa} = 1389$, and the revenues are $R^{va} = 1875$ and $R^{upa} = 1667$. However, a grand coalition may not be stable in the case of a uniform price auction, because each member of the cartel may have incentives to deviate from the cartel due to the positive externality the cartel provides for outsiders. The willingness to deviate depends on the coalition formation game. Three examples of these games are described next.

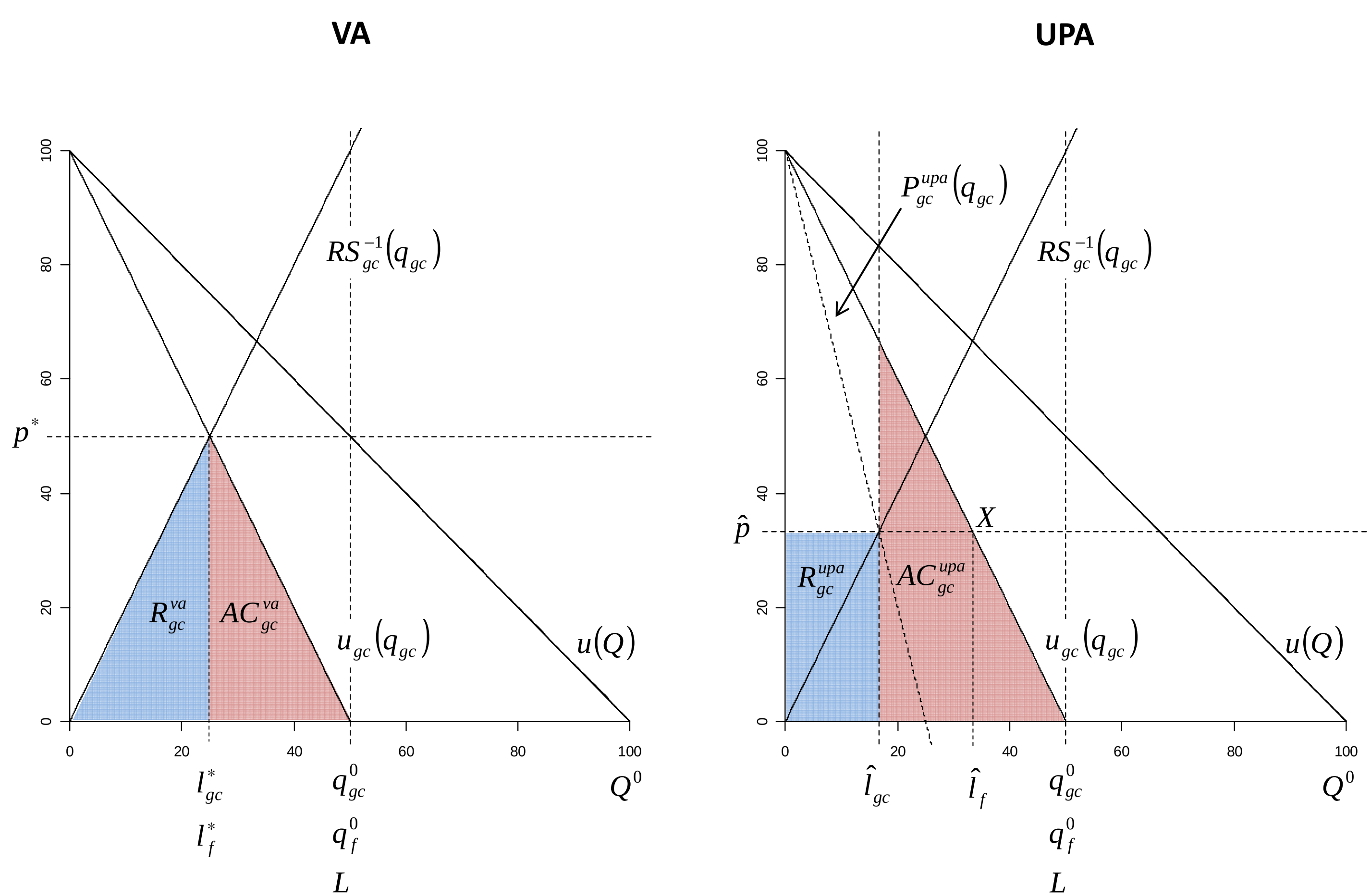

Figure 4.1: The Vickrey auction (VA) and the uniform price auction (UPA) with a grand coalition and a fringe of competitive firms of equal size. Parameter values: $\theta = 100$, $\beta = 1$ and $\lambda = \delta = 0.5$.

4.4 Coalition formation

The coalition formation game is denoted by $G = \left\{N, (\Sigma_i)_{i\in N}, (TC_i)_{i\in N}\right\}$, where $N = \{1, \ldots, n\}$ is the set of strategic firms, Σ_i is the set of membersip strategies of firm i in the game, and $TC_i = TC\left(\eta_{l(i)}, C(\sigma)\right)$ is the payoff of firm i for being a member of coalition η_l given the coalition structure $C(\sigma)$. The coalition structure $C(\sigma)$ is defined by the strategy vector of all firms $\sigma \in \Sigma \equiv \Pi_{i\in N}\Sigma_i$.

Before introducing the three coalition formation games, let me note that there are several useful equilibrium concepts when explaining the stable coalition structures in these games. In Appendix 4.B the reader will find definitions of Nash equilibrium (NE), strong Nash equilibrium (SNE), and coalition-proof Nash equilibrium (CPNE), as well as definitions of stand-alone stability and the concentration of a coalition.

In uniform price auctions, firms inside coalitions are better-off the more concentrated, and thus the less competitive, the coalition structure is. However, in order to sustain large coalitions, problems arise due to the positive externalities large coalitions provide to outsiders. The more concentrated the coalition structure is, the more the outsiders gain. The stable coalition structure should thus be such that no member of any coalition wants to deviate.

4.4.1 Cartel game with myopic firms

The coalition formation game is called a cartel game if there is only a single coalition with $\eta_1 \in [1, n]$ members. The rest of the strategic firms are singletons, i.e. $\eta_2 = 1$. Thus, the set of membership strategies in the cartel game is $\Sigma_i = \{0, 1\}$, where $\sigma_i = 0$ means that firm i is a singleton in the auction game, whereas $\sigma_i = 1$ implies that firm i joins the cartel. Hence the coalition structure of the cartel game is $C = \left\{\eta_1^{(1)}, 1^{(n-\eta_1)}\right\}$, where $\eta_1 = \sum_{i=1}^{n}\sigma_i$. The total costs of each cartel member $TC(\eta_1; C)$ and each individual strategic firm $TC(1; C)$ are defined by the size of the cartel. If strategic firms are myopic in the cartel game, two stability conditions are required. These are internal and external stability (D'Aspremont et al. 1983).

Definition 4.3. *Let $\tilde{\eta}_1$ denote the number of cartel members in the stable coalition structure of a cartel game with myopic firms. In particular, the coalition structure $C^{CGM} = \left\{\tilde{\eta}_1^{(1)}, 1^{(n-\tilde{\eta}_1)}\right\}$ of the cartel game is internally stable if*

$$TC\left(\tilde{\eta}_1; C^{CGM}\right) \leq TC\left(1; C'\right), \tag{4.28}$$

where $C' = \left\{(\tilde{\eta}_1 - 1)^{(1)}, 1^{(n-\tilde{\eta}_1+1)}\right\}$.

Definition 4.4. *Let $\tilde{\eta}_1$ denote the number of cartel members in the stable coalition structure of a cartel game with myopic firms. In particular, the coalition structure $C^{CGM} = \left\{\tilde{\eta}_1^{(1)}, 1^{(n-\tilde{\eta}_1)}\right\}$ of the cartel game is externally stable if*

$$TC\left(1; C^{CGM}\right) \leq TC\left(\tilde{\eta}_1 + 1; C''\right), \tag{4.29}$$

where $C'' = \left\{\left(\tilde{\eta}_1 + 1\right)^{(1)}, 1^{(n-\tilde{\eta}_1-1)}\right\}$.

The first rule (4.28) implies that no individual cartel member wants to deviate from the cartel. It is thus equivalent to the definition of stand-alone stability (see Appendix 4.B). The second rule (4.29) implies that no individual strategic firm wants to join the cartel. Hence the coalition structure $C^{CGM} = \left\{\tilde{\eta}_1^{(1)}, 1^{(n-\tilde{\eta}_1)}\right\}$ is a stable structure in a cartel game with myopic firms if it is both internally and externally stable according to Definitions 4.3 and 4.4.

Table 4.1 provides an example of a uniform price auction with a cartel game of $n = 30$ strategic firms. In the example, the market share of strategic firms is $\lambda = 0.5$, the cost parameters are $\theta = 100$ and $\beta = 1$, and the pollution target is $\delta = 0.5$. The stable structure of the cartel game with myopic firms is only $\tilde{\eta}_1 = 3$ members in the cartel. Deviation from the cartel is not profitable, because from (4.28)

$$TC\left(3; \left\{3^{(1)}, 1^{(27)}\right\}\right) = 62.1270 < 62.1273 = TC\left(1; \left\{2^{(1)}, 1^{(28)}\right\}\right).$$

Also, for an individual firm it is not profitable to join the three-firm cartel, while from (4.29)

$$TC\left(4; \left\{4^{(1)}, 1^{(26)}\right\}\right) = 62.096 < 62.079 = TC\left(1; \left\{3^{(1)}, 1^{(27)}\right\}\right).$$

In fact, the stable structure is independent of the model structure and the number of strategic firms for $n \geq 3$. In every model I consider, the stable structure of the cartel game with myopic firms is $C^{CGM} = \left\{3^{(1)}, 1^{(n-3)}\right\}$ and thus the Nash equilibrium (NE) of the game is all the strategy profiles where exactly three firms have $\sigma_i = 1$, where $\{i\} \subset c_1$ and remaining firms have $\sigma_j = 0$, where $\{j\} \subset N \setminus c_1$. This is also a coalition-proof Nash equilibrium (CPNE), because there are no self-enforcing deviations by any group from $C^{CGM} = \left\{3^{(1)}, 1^{(n-3)}\right\}$. However, no strong Nash equilibria (SNE) can be found for this game. For every coalition structure it is possible to find a profitable deviation strategy for some group of firms (see Thoron 1998).[8]

[8] See the definitions of NE, SNE and CPNE in Appendix 4.B.

Table 4.1: Per-member costs of the cartel game in the uniform price auction. Parameter values: $n = 30$, $\theta = 100$, $\beta = 1$, and $\lambda = \delta = 0.5$.

η_1	$TC(\eta_1; C)$	$TC(1; C)$	η_1	$TC(\eta_1; C)$	$TC(1; C)$	η_1	$TC(\eta_1; C)$	$TC(1; C)$
1	62.151	62.151	11	61.528	60.768	21	59.436	56.088
2	62.145	62.1273	12	61.393	60.476	22	59.120	55.347
3	**62.1270**	**62.079**	13	61.244	60.151	23	58.780	54.539
4	62.096	62.006	14	61.079	59.792	24	58.414	53.656
5	62.054	61.909	15	60.898	59.395	25	58.020	52.693
6	61.998	61.786	16	60.701	58.960	**26**	**57.597**	**51.640**
7	61.931	61.638	17	60.486	58.483	27	57.141	50.488
8	61.850	61.462	18	60.253	57.961	28	56.651	49.225
9	61.756	61.260	19	60.002	57.390	29	56.124	47.839
10	61.649	61.029	20	59.730	56.768	30	55.556	-

4.4.2 Cartel game with farsighted firms

The primitives of a cartel game with farsighted firms are similar to the cartel game with myopic firms. Hence there is only a single coalition, with $\eta_1 \in [1, n]$ members, and remaining strategic firms are singletons ($\eta_2 = 1$). The set of membership strategies in the cartel game is $\Sigma_i = \{0, 1\}$. Firm i is a singleton if $\sigma_i = 0$ and a member of the cartel if $\sigma_i = 1$. The coalition structure of the game is $C = \left\{\eta_1^{(1)}, 1^{(n-\eta_1)}\right\}$, where $\eta_1 = \sum_{i=1}^{n} \sigma_i$.

The cartel game with farsighted firms is similar to Carraro and Moriconi (1997) and Finus and Rundshagen (2001). It is a special case of an equilibrium-binding agreement game (Ray and Vohra 1997). If firms are farsighted, larger cartels can be sustained. I explain the formation of a stable structure again with the example of the uniform price auction described in Table 4.1. Consider, for instance, that the cartel contains $\eta_1 = 5$ members in a uniform price auction. If any of the cartel members was myopic, it would find deviation profitable, because playing as an individual strategic firm against four-firm cartel (and against the fringe, and remaining individual strategic firms) would provide greater profits than being a member of a cartel of five firms. However, a farsighted firm would notice that members of the four-firm cartel would also have incentives to deviate. Thus the outcome that a five-firm cartel member should compare its outcome to is not the outcome of a four-firm cartel game but the outcome of a three-firm cartel game, which is a stable coalition structure even if firms were myopic. Now because

$$TC\left(5; \left\{5^{(1)}, 1^{(25)}\right\}\right) = 62.054 < 62.079 = TC\left(1; \left\{3^{(1)}, 1^{(27)}\right\}\right),$$

a deviation strategy would not be profitable and the structure $C = \left\{5^{(1)}, 1^{(25)}\right\}$ is a stable structure for a cartel game with farsighted firms. Using similar reasoning the structure $C = \left\{8^{(1)}, 1^{(22)}\right\}$ is also a stable structure: 1) a five-firm cartel is stable as we already noticed; 2) if a firm deviated from a cartel of $\eta_1 = 8$ members, there would also be other deviators until the cartel had only five members; 3) the total costs of a member of an eight-firm cartel are lower than the costs of a singleton in a uniform price auction with a five-firm cartel:

$$TC\left(8; \left\{8^{(1)}, 1^{(22)}\right\}\right) = 61.850 < 61.909 = TC\left(1; \left\{5^{(1)}, 1^{(25)}\right\}\right).$$

In the case of our example, the stable structures of the cartel game are those structures for which the number of cartel members belongs to a set $\eta_1^{CGF} \in \{3, 5, 8, 12, 18, 26\}$ (see the rows written in red in Table 4.1). If firms are able to agree on binding cartel agreements and firms are farsighted, the most profitable agreement is a coalition with the largest number of cartel members: $C^{CGF} = \left\{\hat{\eta}_{1,1}^{(1)}, 1^{(n-\hat{\eta}_{1,1})}\right\}$, where $\hat{\eta}_{1,1} = \max\left(\eta_1^{CGF}\right)$. I call this the least competitive stable structure and it is the most concentrated structure of the set of stable structures. The least competitive stable structure with $n = 30$ would then be $C^{CGF} = \left\{26^{(1)}, 1^{(4)}\right\}$. Stable structures depend on the number of strategic firms n, but also on the other parameter values. For example, when $\lambda = 0.7$, the least competitive stable structure with otherwise the same model ($n = 30$, $\theta = 100$, $\beta = 1$, and $\delta = 0.5$) is $C^{CGF} = \left\{24^{(1)}, 1^{(6)}\right\}$. Table 4.2 presents all the stable structures for $n = 3, \ldots, 100$ strategic firms in the example model with different market shares of oligopolistic firms ($\lambda \in (0.3; 0.5; 0.7)$.

Table 4.2: Stable structures (η_1^{CGF}) of the cartel game with farsighted firms. Parameter values: $\theta = 100$, $\beta = 1$, and $\delta = 0.5$.

n	$\eta_1^{CGF} \mid \lambda = 0.3$	$\eta_1^{CGF} \mid \lambda = 0.5$	$\eta_1^{CGF} \mid \lambda = 0.7$
3	3	3	3
4	3	3	3
5	3, 5	3, 5	3, 5
6	3, 5	3, 5	3, 5
7	3, 5	3, 5	3, 5, 7
8	3, 5, 8	3, 5, 8	3, 5, 7
9	3, 5, 8	3, 5, 8	3, 5, 7
10	3, 5, 8	3, 5, 8	3, 5, 8
⋮	⋮	⋮	⋮
20	3, 5, 8, 12, 18	3, 5, 8, 12, 17	3, 5, 8, 12, 17
30	3, 5, 8, 12, 18, 26	3, 5, 8, 12, 18, 26	3, 5, 8, 12, 17, 24
40	3, 5, 8, 12, 18, 26, 37	3, 5, 8, 12, 18, 26, 37	3, 5, 8, 12, 17, 24, 33
50	3, 5, 8, 12, 18, 26, 37	3, 5, 8, 12, 18, 26, 37	3, 5, 8, 12, 18, 26, 36, 48
60	3, 5, 8, 12, 18, 26, 37, 53	3, 5, 8, 12, 18, 26, 37, 52	3, 5, 8, 12, 18, 26, 37, 51
70	3, 5, 8, 12, 18, 26, 37, 53	3, 5, 8, 12, 18, 26, 37, 52	3, 5, 8, 12, 18, 26, 37, 51, 68
80	3, 5, 8, 12, 18, 26, 37, 53, 75	3, 5, 8, 12, 18, 26, 37, 52, 72	3, 5, 8, 12, 18, 26, 37, 52, 70
90	3, 5, 8, 12, 18, 26, 37, 53, 75	3, 5, 8, 12, 18, 26, 37, 53, 74	3, 5, 8, 12, 18, 26, 37, 52, 71
100	3, 5, 8, 12, 18, 26, 37, 53, 75	3, 5, 8, 12, 18, 26, 37, 53, 74	3, 5, 8, 12, 18, 26, 37, 52, 72, 96

Defining the stable structure in CGF formally, we need a new definition for internal stability.

Definition 4.5. *Let $\eta_1^{CGF} \in \{\hat{\eta}_{1,1}, \hat{\eta}_{1,2}, \hat{\eta}_{1,3}, ...\}$ denote the set of numbers of cartel members in all the stable coalition structures of the cartel game with farsighted firms, with $\hat{\eta}_{1,i} > \hat{\eta}_{1,j}$ for all $i > j$. In particular, the coalition structure $C_i^{CGF} = \left\{\hat{\eta}_{1,i}^{(1)}, 1^{(n-\hat{\eta}_{1,i})}\right\}$ of the cartel game is internally stable if*

1. *for all $\hat{\eta}_{1,i} > \min\left(\eta_1^{CGF}\right)$,*

$$TC\left(\hat{\eta}_{1,i}; C_i^{CGF}\right) \leq TC\left(1; C_{i+1}^{CGF}\right), \tag{4.30}$$

   where $C_{i+1}^{CGF} = \left\{\hat{\eta}_{1,i+1}^{(1)}, 1^{(n-\hat{\eta}_{1,i+1})}\right\}$; and

2. *for $\hat{\eta}_{1,i} = \min\left(\eta_1^{CGF}\right)$, the coalition structure $C_i^{CGF} = \left\{\hat{\eta}_{1,i}^{(1)}, 1^{(n-\hat{\eta}_{1,i})}\right\}$ is stand-alone stable.*

Hence the coalition structure $C_i^{CGF} = \left\{\hat{\eta}_{1,i}^{(1)}, 1^{(n-\hat{\eta}_{1,i})}\right\}$ is a stable structure of the cartel game with farsighted firms if it is both internally and externally stable according to Definitions

4.4 and 4.5. However, in Definition 4.4 one needs to change C^{CGM} to C_i^{CGF} and thus $C'' = \left\{ (\hat{\eta}_{1,i} + 1)^{(1)}, 1^{(n - \hat{\eta}_{1,i} - 1)} \right\}$.

In a cartel game with myopic firms, the solution concept is CPNE, whereas the stable structure in a cartel game with farsighted firms is not any of NE, SNE or CPNE, unless $\hat{\eta}_{1,i} = \min \left(\eta_1^{CGF} \right)$. The notions of NE, SNE and CPNE consider only the deviation strategies of deviators, keeping the other agents' strategies fixed. However, in the cartel game with farsighted firms, the stable structure is determined by the deviations of those players outside the group of original deviators. Also, while there are several stable structures in the cartel game with farsighted firms, the cartel agreement of the least competitive stable structure must be more binding than the stable structures of the two other coalition formation games considered in this paper, which both have unique stable structures.

4.4.3 Open membership game

Yi (1997), Yi and Shin (2000) and Finus and Rundshagen (2001), for instance, examine an open membership game with positive externalities. It is a simultaneous move game, where players are allowed to form coalitions freely, as long as no player is exluded from joining a coalition.[9] In the coalition formation process, players announce messages (or addresses). If two or more players have announced the same message, they form a coalition. Formally, player $i \in N$ can choose any message from the strategy set $\Sigma_j = \{0, \sigma^1, \ldots, \sigma^n\} = \{0, 1, \ldots, n\}$. The strategy $\sigma_j = 0$ means that firm j is a singleton and $\sigma_j = \sigma^i$ that firm j joins coalition c_i. Let a_j be firm j's action. If firms j and l have chosen the same message, they belong to the same coalition, i.e. if $a_j = a_l = \sigma^i$, they both are members of coalition c_i. For instance,

[9] If players can be exluded, the game is an exclusive membership game (e.g. Hart and Kurz 1983, Yi and Shin 2000, Finus and Rundshagen 2001). These games may have multiple stable structures in the setup of this paper, some of them more concentrated than the unique stable structure of the open membersip game.

a strategy profile

$$\sigma = \Bigg\{ \underbrace{1,\ldots,1}_{\#1=\eta_1}, \underbrace{2,\ldots,2}_{\#2=\eta_1}, \ldots, \underbrace{m_1,\ldots,m_1}_{\#m_1=\eta_1},$$

$$\underbrace{(m_1+1),\ldots,(m_1+1)}_{\#(m_1+1)=\eta_2}, \ldots, \underbrace{\left(\sum_{i=1}^{2} m_i\right),\ldots,\left(\sum_{i=1}^{2} m_i\right)}_{\#\left(\sum_{i=1}^{2} m_i\right)=\eta_2}$$

$$,\ldots,$$

$$\underbrace{\left(\sum_{i=1}^{h-1} m_i+1\right),\ldots,\left(\sum_{i=1}^{h-1} m_i+1\right)}_{\#\left(\sum_{i=1}^{h-1} m_1+1\right)=\eta_h}, \ldots, \underbrace{\left(\sum_{i=1}^{h} m_i\right),\ldots,\left(\sum_{i=1}^{h} m_i\right)}_{\#\left(\sum_{i=1}^{h} m_i\right)=\eta_h} \Bigg\}$$

constitutes a coalition structure $C = \left\{\eta_1^{(m_1)}, \ldots, \eta_h^{(m_h)}\right\}$.

Yi and Shin (2000) prove that if the following four conditions of the positive externality game hold, there is a unique coalition-proof Nash equilibriumin in the open membership game.[10]

Condition 4.1. *$TC(n_i; C') < TC(n_i; C)$, where C' is more concentrated than C; $n_i \in C'$; and $n_i \in C$. (**C.4.1.**)*

Condition 4.2. *$TC(n_i; C) < TC(n_j; C)$, where $n_j > n_i$, for any $C = \{n_1, \ldots, n_m\}$. (**C.4.2.**)*

Condition 4.3. *$TC(n_j; C) > TC(n_j - 1; C')$, where $C = \{n_1, \ldots, n_m\}$; $C' = C \setminus \{n_i, n_j\} \cup \{n_i + 1, n_j - 1\}$; and $n_i \geq n_j \geq 2$. (**C.4.3.**)*

Condition 4.4. *Suppose that $C = \{n_1, \ldots, n_m\}$ is stand-alone stable. If $n_1 \geq n_m + 2$, then there exists n_j, $n_1 \geq n_j + 2$, such that $TC(n_1; C) > TC(n_j + 1; C')$, where $C' = C \setminus \{n_1, n_j\} \cup \{n_1 - 1, n_j + 1\}$. (**C.4.4.**)*

The first condition (C.4.1) states that if the coalition structure becomes more concentrated and coalition c_i is not part of the concentration process, then members of c_i are better-off. According to (C.4.2), a member of a small coalition is better-off than a member of a large coalition in any coalition structure. Condition (C.4.3) states that, if a member of a coalition c_j leaves its coalition and joins a larger or equal-size coalition c_i, the remaining members of coalition c_j are better-off. Finally, by (C.4.4), if the largest coalition in a stand-alone stable coalition structure exceeds the size of the smallest coalition by 2 or more, a member of the largest coalition becomes better-off by joining a coalition, which is smaller by 2 or more than

[10] Again, see the definitions of stand-alone stability and the concentration of a coalition in Appendix 4.B.

the largest coalition. Hence, given the conditions (C.4.1) - (C.4.4), the stable structure must be symmetric with only one or two types of coalitions. Moreover, if there are two types of coalitions, the size of these types must be such that $\eta_1 = \eta_2 + 1$.

Conditions (C.4.1) - (C.4.3) hold trivially in the uniform price auction game. I do not prove analytically that condition (C.4.4) also holds. However, I apply a numerical simulation and check that the unique coalition-proof Nash equilibrium introduced by Yi and Shin (2000) is a stable structure in the open membership game with a uniform price auction. The stable coalition structure can be characterized by the following proposition.

Proposition 4.1. *(Yi and Shin 2000, Proposition 5.) Let $I(n/m)$ denote the next higher integer to n/m including n/m. Furthermore, suppose that $C^\star = \left\{k^{\star(m^\star - r^\star)}, (k^\star - 1)^{(r^\star)}\right\}$ is stand-alone stable, where $k^\star = I(n/m^\star)$ and $r^\star = m^\star k^\star - n\,(\geq 0)$. Suppose also that $C' = \left\{k'^{(m'-r')}, (k'-1)^{r'}\right\}$ is not stand-alone stable, where $k' = I(n/m')$ and $r' = m'k' - n$, for all $m' = 1, \ldots, m^\star - 1$. Then, in the open memebership game:*

1. *under (C.4.4), $C^\star$ is the most concentrated Nash equilibrium coalition structure; and*
2. *under (C.4.1) - (C.4.4), $C^\star$ is the unique coalition-proof Nash equilibrium coalition structure.*

Proof. Yi and Shin (2000, Appendix A). ■

Yi and Shin (2000) examine the formation of research coalitions with positive spillovers. Finus and Rundshagen (2001) model coalition formation in a problem of global pollution control. Both models use a linear-quadratic Cournout structure and are thus close to the model of this paper. In Yi and Shin (2000) and in Finus and Rundshagen (2001) the stable structure of the open membership game is given by $k^\star = 3$ (see Proposition 4.1). More formally, the stable structure of the open membership game with a uniform price auction can be characterized by the following corollary.

Corollary 4.1. *Let $m^3 \equiv (m : I(n/m) = 3)$, where $I(n/m)$ denotes the next higher integer to n/m including n/m, and $r^3 \equiv 3m^3 - n$. Then, for $n \geq 3$, the stable structure of the open membership game with the linear-quadratic model and the uniform price auction is*

$$C^{OMG} = \left\{\bar{\eta}_1^{(\bar{m}_1)}, \bar{\eta}_2^{(\bar{m}_2)}\right\} = \left\{3^{\left(m^3 - r^3\right)}, 2^{\left(r^3\right)}\right\}.$$

The proof of Corollary 4.1. is omitted, but it is similar to the proof of Proposition 5 in Yi and Shin (2000) or Proposition 11 in Finus and Rundshagen (2001). However, I show in Appendix

4.C, applying a numerical simulation that $C^{OMG} = \left\{ \bar{\eta}_1^{(\bar{m}_1)}, \bar{\eta}_2^{(\bar{m}_2)} \right\}$ is the unique stable structure of all the symmetric coalition structures $C^k = \left\{ k^{\left(m^k - r^k\right)}, (k-1)^{r^k} \right\}$, $m^k = 1, \ldots, n$, where $k = I(n/m^k)$ and $r^k = m^k k - n$. Ignoring integer constraints, these structures may also be written as: $\{n\}$, $\left\{\frac{n}{2}, \frac{n}{2}\right\}$, $\left\{\frac{n}{3}, \frac{n}{3}, \frac{n}{3}\right\}$, $\left\{\frac{n}{4}, \frac{n}{4}, \frac{n}{4}, \frac{n}{4}\right\}$,..., $\{2, 2, \ldots, 2\}$, $\{2, 2, \ldots, 2, 1, 1\}$,..., $\{2, 1, 1, \ldots, 1\}$, and$\{1, 1, \ldots, 1\}$ (see Yi and Shin 2000). Even though there is a unique coalition-proof stable structure, there is a number of strategy profiles which constitute this coalition structure. Hence the coalition-proof Nash equilibrium in the coalition formation stage is any strategy profile which has a form equal to the strategy profile

$$\sigma^{\star} = \{1, 1, 1, 2, 2, 2, \ldots, \bar{m}_1, \bar{m}_1, \bar{m}_1, (\bar{m}_1 + 1), (\bar{m}_1 + 1), \ldots, (\bar{m}_1 + \bar{m}_2), (\bar{m}_1 + \bar{m}_2)\}.$$

4.5 Results

In this section I run numerical simulations of the coalition game with the two alternative auction designs. In the simulations I use fixed values for the parameters $\theta = 100$, $\beta = 1$ and $\delta = 0.5$. However, I let the number of strategic firms n run from 3 to 100 and give different values for the share of the oligopolistic market: $\lambda \in (0.3; 0.5; 0.7)$. I calculate the welfare and auction revenue effects from (4.21), (4.26) and (4.27) given the share of the oligopolistic market, the number of strategic firms and the stable structures of different coalition formation games. With the Vickrey auction, strategic firms form a grand coaltion in every coalition formation game. With the uniform price auction, the stable structure and thus the results are different depending on the coalition formation game. When comparing the results, the benchmark case is the second-best allocation of allowances, where the equilibrium price is $p^* = (1 - \delta)\theta$, the total abatement costs are $AC^e = \frac{(1-\delta)^2 \theta^2}{2\beta}$ and the total revenues collected are the competitive revenues $R^e = \frac{(1-\delta)\delta\theta^2}{\beta}$.

Figure 4.2 shows the excess abatement costs relative to the benchmark. For instance, if the number of strategic firms is 5, the aggregate abatement costs after UPA with the open memebership game (UPA-OMG, the red line) are 0.6%, 2.7% and 7.8% higher than in the second-best as the share of the oligopolistic market λ is assigned the value 0.3, 0.5 and 0.7, respectively. However, the excess abatement costs decrease quite rapidly as the number of strategic firms increases. Already for $n = 10$, the relative welfare loss is 0.1%, 0.6% and 1.3%, respectively.

The welfare loss is smaller and decreases even more rapidly with the cartel game with myopic firms (UPA-CGM, the blue dashed line), but if firms are farsighted and form the least competitive cartel agreement (UPA-CGF, the blue solid line), the welfare loss varies between 0.8

- 2.3% for $\lambda = 0.3$, between 3.8 - 11.1% for $\lambda = 0.5$, and between 12.8 - 39.6% for $\lambda = 0.7$. The welfare loss depends heavily on the number of members in the least competitive stable structure. The Vickrey auction (VA, the black line) provides the second-best allocation and thus no welfare loss.

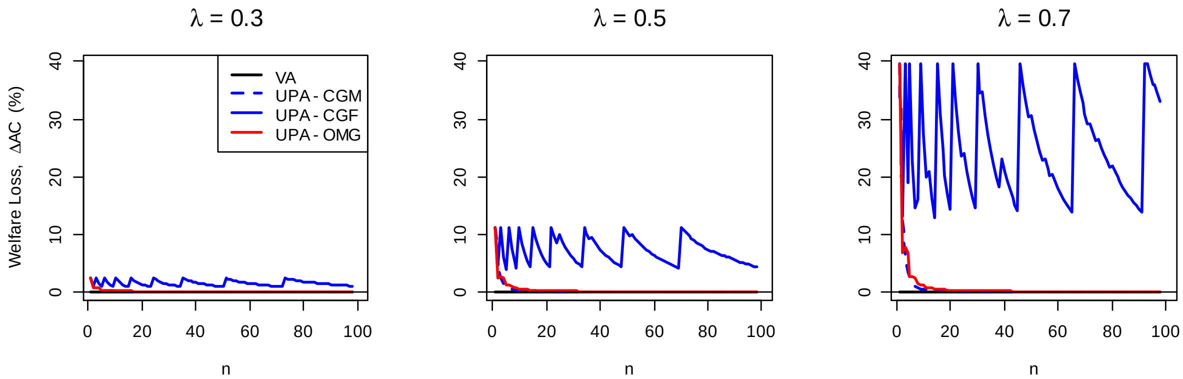

Figure 4.2: Welfare loss. Parameter values: $\theta = 100$, $\beta = 1$, and $\delta = 0.5$.

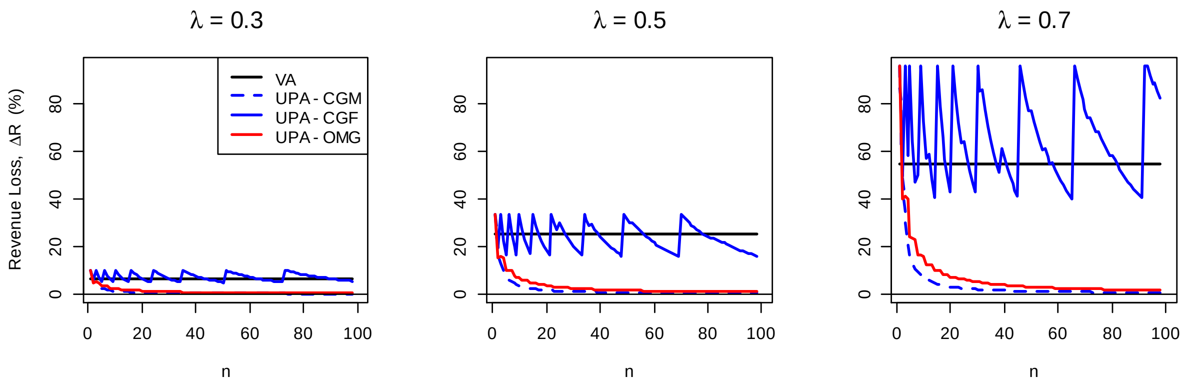

Figure 4.3: Revenue loss. Parameter values: $\theta = 100$, $\beta = 1$, and $\delta = 0.5$.

Figure 4.3 shows the loss of revenues relative to the competitive revenues, i.e. if the auctioneer received the second-best price for all the allowances auctioned. Where the Vickrey auction did not produce any welfare loss due to the efficient allocation of allowances, the revenues will be much lower than in the competitive auction at least when the market share of strategic firms is high. This is due to collusion. The revenue losses of the Vickrey auction are 6.4%, 25% and 55% for λ with values of 0.3, 0.5 and 0.7, respectively. The number of strategic firms has no effect on revenue losses, because in every model structure strategic firms act as if there were only one single monopolistic firm.

Again, in the uniform price auction the revenue losses depend highly on the coalition game considered. With the myopic cartel game (UPA-CGM) and the open membersip game (UPA-OMG), the revenue losses are fairly modest already for $n = 20$ strategic firms, irrespective of the market share of strategic firms. Again, the revenue losses are much higher if firms are farsighted (UPA-CGF). In that case the revenue losses fluctuate around the revenue loss of the Vickrey auction and are highly dependent on the number of strategic firms and the number of members in the least competitive stable coalition.

We can conclude that if cartel agreements are weak and there are not too few strategic firms, the uniform price auction may be approximately equal to the competitive market. A weak cartel agreement means (in uniform price auctions) that strategic firms cannot make a binding agreement on collusive bidding. Instead, some firms, individually or jointly, find it profitable to deviate from the cartel due to the fact that cartel makes outsiders better off. Thus, in equilibrium, the allocation of allowances is almost second-best (for large n) and the auction revenues are almost as great as in the competitive market, at least for a large enough market share of the fringe. If, on the other hand, farsighted firms can agree on the least competitive cartel agreement, the uniform price auction will not offer an efficient allocation and the auction revenues will be relatively low, at least for a large market share of strategic firms. Using the Vickrey auction, the regulator can always guarantee efficient allocation but the auction revenues will decrease in increasing collusive behavior, which, on the other hand, is profitable for strategic firms.

4.6 Conclusions

I have compared two auction mechanisms to allocate emission allowances when firms are able to collude: the Vickrey auction and the uniform price auction. Firms may form multiple coalitions and the two-stage game reduces to a one-stage coalition formation game using the partition function approach (Yi 2003). Firms may trade emission allowances inside but not between coalitions after the auction. Hence I have assumed that there are no resale markets. This is of course a very simplifying assumption and does not hold in existing emissions markets. However, the model gives some insights into the incentives the various auction mechanisms provide for participating firms.

With private values, the Vickrey auction provides an efficient allocation of pollution rights but at the same time strong incentives for firms to coordinate their bids in an auction. This reduces the revenues. Modeling the uniform price auction is a more complex task even in the relatively simple framework of this paper. I have applied a linear-quadratic model and a fringe of competitive firms, which balances the market. By these simplifying assumptions

I have been able to derive a unique Nash equilibrium of the auction. In addition, I have considered three coalition formation games.

Depending on the coalition game, the results of the uniform price auction vary substantially. It should be noted that I have considered only three examples of coalition formation games. However, the interpretation of the results of the uniform price auction is the following. The regulator should understand the market structure in order to estimate the risk of collusion and low price equilibria. First, if the market share of strategic firms is relatively small, the welfare loss and the revenue loss are naturally relatively low. Second, even if the market share of strategic firms is high, but not too high, and if coalition formation is free, the market behaves almost competitively, unless there are only a few strategic firms. However, if (farsighted) firms can coordinate coalition formation and form only a single cartel, then the collusion might be a threat to the regulation.

Bibliography

[1] Akgün, U., 2004. Mergers with Supply Functions. The Journal of Industrial Economics 52(4), 535-546.

[2] Ausubel, L.A., Cramton, P., Pycia, M., Rostek, M., and Weretka, M., 2013. Demand Reduction and Inefficiency in Multi-unit Auctions. Mimeo.

[3] Back, K., and Zender, J.F., 1993. Auctions of Divisible Goods: On the Rationale for the Treasury Experiment. Review of Financial Studies 6, 733-764.

[4] Back, K., and Zender, J.F., 2001. Auctions of Divisible Goods with Endogenous Supply. Economics Letters 73, 29-34.

[5] Baldick, R., Grant, R., and Kahn, E., 2004. Theory and Application of Linear Supply Function Equilibrium in Electricity Markets. Journal of Regulatory Economics 25(2), 143-167.

[6] Bloch, F., 1995. Endogenous Structures of Associations in Oligopolies. Rand Journal of Economics 26(3), 537-556.

[7] Bloch, F., 1996. Sequential Formation of Coalitions in Games with Externalities and Fixed Payoff Division. Games and Economic Behavior 14, 90-123.

[8] Carraro, C., and Moriconi, F., 1997. International Games on Climate Change Control. Fondazione Eni Enrico Mattei Working Paper No. 22.98.

[9] Clarke, E., 1971. Multipart Pricing of Public Goods. Public Choice 2, 19-33.

[10] D'Aspremont, C., Jacquemin, A., Gabszewicz, J.J., and Weymark, J.A., 1983. On the Stability of Collusive Price Leadership. Canadian Journal of Economics 16(1), 17-25.

[11] Dechenaux, E., and Kovenock, D., 2007. Tacit Collusion and Capacity Withholding in Repeated Uniform Price Auctions. RAND Journal of Economics 38(4), 1044-1069.

[12] Fabra, N., 2003. Tacit Collusion in Repeated Auctions: Uniform versus Discriminatory. Journal of Industrial Economics 51, 271-293.

[13] Finus, M., and Rundshagen, B., 2001. Endogenous Coalition Formation in Global Pollution Control. Fondazione Eni Enrico Mattei Working Paper No. 43.2001.

[14] Green, R.J., 1996. Increasing Competition in the British Electricity Spot Market. Journal of Industrial Economics 44(2), 205-216.

[15] Green, R.J., and Newbery, D.M., 1992. Competition in the British Electricity Spot Market. Journal of Political Economy 100, 929-953.

[16] Groves, T., 1973. Incentives in Teams. Econometrica 41, 617-631.

[17] Hart, S., and Kurz, M., 1983. Endogenous Formation of Coalitions. Econometrica 51, 1047-1064.

[18] Holmberg, P., 2008. Unique Supply Function Equilibrium with Capacity Constraints. Energy Economics 30(1), 148-172.

[19] Holmberg, P., 2009. Numerical Calculation of an Asymmetric Supply Function Equilibrium with Capacity Constraints. European Journal of Operational Research 199, 285-295.

[20] Klemperer, P.D., and Meyer, M.A., 1989. Supply Function Equilibria in Oligopoly under Uncertainty. Econometrica 57(6), 1243-1277.

[21] Konishi, H., Breton, M.L., and Weber, S., 1999. On Coalition-proof Nash Equilibria in Common Agency Games. Journal of Economic Theory 85, 122-139.

[22] Kremer, I., and Nyborg, K.G., 2004. Underpricing and Market Power in Uniform Price Auctions. The Review of Financial Studies 17(3), 849-877.

[23] LiCalzi, M., and Pavan, A., 2005. Tilting the Supply Schedule to Enhance Competition in Uniform-price Auctions. European Economic Review 49, 227-250.

[24] McAdams, D., 2007. Adjustable Supply in Uniform Price Auctions: Non-commitment as a Strategic Tool. Economics Letters 95, 48-53.

[25] Montero, J.-P., 2008. A Simple Auction Mechanism for the Optimal Allocation of the Commons. American Economic Review 98(1), 496-518.

[26] Ray, D., and Vohra, R., 1997. Equilibrium Binding Agreements. Journal of Economic Theory 73, 30-78.

[27] Sáiz, E.M., Hendrix, E.M.T., and Olieman, N.J., 2006. On the Computation of Stability in Multiple Coalition Formation Games. Computational Economics 28, 251-275.

[28] Thoron, S., 1998. Formation of Coalition-proof Stable Cartels. Canadian Journal of Economics 31, 63-76.

[29] Vickrey, W., 1961. Counterspeculation, Auctions and Competitive Sealed Tenders. Journal of Finance 16, 8-37.

[30] Wilson, R., 1979. Auctions of Shares. Quarterly Journal of Economics 94, 675-689.

[31] Yi, S., 1997. Stable Coalition Structures with Externalities. Games and Economic Behavior 20, 201-237.

[32] Yi, S., 2003. Endogenous Formation of Economic Coalitions: A Survey of the Partition Function Approach. In Carraro, C. (ed.) The Endogenous Formation of Economic Coalitions. Edward Elgar, Cheltenham, UK, 80-127.

[33] Yi, S., and Shin, H., 2000. Endogenous Formation of Research Coalitions with Spillovers. International Journal of Industrial Organization 18, 229-256.

Appendices

4.A Vickrey auction payback functions.

Given that truthful bidding is a dominant strategy in the Vickrey auction, the residual supply function for coalition c_i can be simplified as:

$$\begin{aligned}
RS_{c_i}^{-1}(q_{c_i}) &= \max\left[0, \left(\theta - \left(\frac{1}{\sum\limits_{j\neq i}^{m}\frac{1}{b_{c_j}}+\frac{1}{\beta_f}}\right)L + \left(\frac{1}{\sum\limits_{j\neq i}^{m}\frac{1}{b_{c_j}}+\frac{1}{\beta_f}}\right)q_{c_i}\right)\right] \\
&= \max\left[0, \left(\underbrace{\theta - \left(\frac{n\beta}{n-n_i\lambda}\right)L}_{\Omega_{c_i}} + \underbrace{\left(\frac{n\beta}{n-n_i\lambda}\right)}_{\tau_{c_i}}q_{c_i}\right)\right] \\
&= \max\left[0, (\Omega_{c_i} + \tau_{c_i}q_{c_i})\right],
\end{aligned}$$

where I have used $b_{c_j} = \beta_{c_j} = \frac{n}{n_j\lambda}\beta$ and $\beta_f = \frac{1}{1-\lambda}\beta$. Equalizing this with the true inverse demand function (4.13) gives the second-best solution:

$$l_{c_i} = \frac{n_i\lambda}{n}L.$$

Using $RS_{c_i}^{-1}(q_{c_i}) = \Omega_{c_i}+\tau_{c_i}q_{c_i}$ from above, the paybacks of coalition c_i in the VCG mechanism are derived using

$$\begin{aligned}
\alpha_{c_i} &= 1 - \frac{\int_{\hat{q}_{c_i}}^{l_{c_i}}(\Omega_{c_i}+\tau_{c_i}x)\,dx}{(\Omega_{c_i}+\tau_{c_i}l_{c_i})\,l_{c_i}} \\
&= \frac{1}{2}\left(\frac{1}{1+\frac{\Omega_{c_i}}{\tau_{c_i}l_{c_i}}}\right) + \frac{\Omega_{c_i}\hat{q}_{c_i}+\frac{1}{2}\tau_{c_i}\hat{q}_{c_i}^2}{\Omega_{c_i}l_{c_i}+\tau_{c_i}l_{c_i}^2},
\end{aligned}$$

where $\hat{q}_{c_i} = \max\left[0, -\frac{\Omega_{c_i}}{\tau_{c_i}}\right]$. Now, provided that $D_{-c_i}^{va}(0) \geq L$, and thus $\hat{q}_{c_i} = 0$, the share of the paybacks is

$$\alpha_{c_i} = \frac{1}{2}\left(\frac{n_i\lambda}{n-n_i\lambda}\right)\left(\frac{\delta}{1-\delta}\right),$$

where I have used Ω_{c_i},τ_{c_i}, and $L = \delta Q^0 = \delta\frac{\theta}{\beta}$. If, on the other hand, $D_{-c_i}^{va}(0) < L$, then

$\hat{q}_{c_i} = -\frac{\Omega_{c_i}}{\tau_{c_i}} = \frac{\theta\left(\delta - 1 + \frac{n_i\lambda}{n}\right)}{\beta} > 0$, and we thus get

$$\begin{aligned}
\alpha_{c_i} &= \frac{1}{2}\left(\frac{n_i\lambda}{n - n_i\lambda}\right)\left(\frac{\delta}{1-\delta}\right) - \frac{\hat{q}_{c_i}^2}{2l_{c_i}\left(l_{c_i} - \hat{q}_{c_i}\right)} \\
&= \frac{1}{2}\left(\frac{n_i\lambda}{n - n_i\lambda}\right)\left(\frac{\delta}{1-\delta}\right) - \frac{\left(1 - \delta - \frac{n_i\lambda}{n}\right)^2}{2\delta\left(\frac{n_i\lambda}{n}\right)(1-\delta)\left(1 - \frac{n_i\lambda}{n}\right)} \\
&= \frac{n_i\lambda\delta}{2\left(n - n_i\lambda\right)(1-\delta)}\left[1 - \left(\frac{n - n_i\lambda - n\delta}{n_i\lambda\delta}\right)^2\right].
\end{aligned}$$

For example, in the case of a grand coalition $c_1 \equiv c_{gc}$ (and thus $n_1 = n$), we get $l_{gc} = \lambda\frac{\delta\theta}{\beta}$; $\Omega_{gc} = \left(1 - \frac{\delta}{1-\lambda}\right)\theta$; and $\tau_{gc} = \frac{\beta}{1-\lambda}$. Furthermore, if $1 - \lambda \geq \delta$,

$$\alpha_{gc} = \frac{\lambda\delta}{2(1-\lambda)(1-\delta)},$$

and if $1 - \lambda < \delta$ the share of the grand coalition paybacks is

$$\alpha_{gc} = \frac{\lambda\delta}{2(1-\lambda)(1-\delta)}\left[1 - \left(\frac{1-\delta-\lambda}{\lambda\delta}\right)^2\right].$$

4.B Equilibrium concepts of coalition formation games

Consider a coalition formation game $G = \left\{N, (\Sigma_i)_{i\in N}, (TC_i)_{i\in N}\right\}$. Moreover, consider any group of firms $S \subset N$ and let $\sigma_{N\setminus S} = (\sigma_j)_{j\in N\setminus S}$ denote a profile of strategies of all firms not included in coalition S. For every coalition $S \subset N$, the restriction of game G to firms in S is defined as

$$\bar{G}_S = \left\{S, (\Sigma_i)_{i\in S}, \left(\overline{TC}_i\right)_{i\in S}\right\},$$

where the strategies of other firms outside S, i.e. $(\bar{\sigma}_j)_{j\in N\setminus S}$, are held fixed and, for every firm $i \in S$ and every strategy $\sigma_i \in \Sigma_i$,

$$\overline{TC}_i = TC\left(\eta_{l(i)}, C\left((\sigma_i)_{i\in S}, (\bar{\sigma}_j)_{j\in N\setminus S}\right)\right).$$

Definition 4.6. *The profile of strategies $\sigma^\star = (\sigma_1^\star, \ldots, \sigma_n^\star)$ is a Nash equilibrium (NE) of game G if for all $i \in N$ and every $\sigma_i \in \Sigma_i$*

$$TC\left(\eta_{l(i)}, C\left(\sigma^\star\right)\right) \leq TC\left(\eta_{l(i)}, C\left(\sigma_i, \sigma_{N\setminus\{i\}}^\star\right)\right).$$

Definition 4.7. *The profile of strategies $\sigma^\star = (\sigma_1^\star, \ldots, \sigma_n^\star)$ is a (strictly) strong Nash equilibrium (SNE) of game G if there exists no $S \subset N$ and $\sigma_S \in \Pi_{i\in S}\Sigma_i$, such that*

$$TC\left(\eta_{l(i)}, C\left(\sigma^\star\right)\right) \geq TC\left(\eta_{l(i)}, C\left(\sigma_S, \sigma^\star_{N\setminus S}\right)\right),$$

for all $i \in S$, and

$$TC\left(\eta_{l(i)}, C\left(\sigma^\star\right)\right) > TC\left(\eta_{l(i)}, C\left(\sigma_S, \sigma^\star_{N\setminus S}\right)\right),$$

for at least one $i \in S$.

Definition 4.8. *(i) Suppose $n = 1$. Then strategy $\sigma_1^\star$ is a (strictly) coalition-proof Nash equilibrium (CPNE) of game G if and only if it is a Nash equilibrium.*

(ii) Suppose $n > 1$ in game G. Then the profile of strategies $\sigma^\star = (\sigma_1^\star, \ldots, \sigma_n^\star)$ is a (strictly) self-enforcing profile of strategies if for all $S \subset N$, $(\sigma_i^\star)_{i\in S}$ is a (strictly) coalition-proof Nash equilibrium (CPNE) of game $\bar{G}_S$, which is a restriction of game G to firms in S.

(iii) The profile of strategies $\sigma^\star = (\sigma_1^\star, \ldots, \sigma_n^\star)$ is a (strictly) coalition-proof Nash equilibrium of game G if it is (strictly) self-enforcing and there is no other (strictly) self-enforcing profile σ' such that

$$TC\left(\eta_{l(i)}, C\left(\sigma^\star\right)\right) \geq TC\left(\eta_{l(i)}, C\left(\sigma'\right)\right),$$

for all $i \in N$, and

$$TC\left(\eta_{l(i)}, C\left(\sigma^\star\right)\right) > TC\left(\eta_{l(i)}, C\left(\sigma'\right)\right),$$

for at least one $i \in N$.

In NE no single firm finds it profitable to deviate from the given coalition structure, whereas SNE requires that deviations of any subgroup of firms are not profitable. Hence a SNE is also NE. In many games the requirement of SNE is too strict, because SNE may not exist. CPNE requires that deviations of any subgroup, which are self-enforcing, are not profitable. Hence, SNE must also be CPNE and while any singleton $\{i\}$ is also a subgroup $S \subset N$, we may write SNE $\subset$ CPNE $\subset$ NE.[11] Also, the following two definitions are useful (see Yi 1997).

Definition 4.9. *(Yi 1997, Definition 5.1.) Coalition structure $C = \{n_1, \ldots, n_m\}$ is stand-alone stable if and only if $TC(n_i; C) \leq TC(1; C')$ where $C' = C \setminus \{n_i\} \cup \{n_i - 1, 1\}$ for all $i = 1, \ldots m$.*

[11] Note that the deviation strategies used in definitions 4.7 and 4.8 are weak deviations. Using strict deviations instead would mean that a group of firms deviates only if all of its members are better-off, which would give definitions of a weakly strong Nash equilibrium and a weakly coalition-proof Nash equilibrium (CPNE) (see e.g. Konishi et al. 1999).

Definition 4.10. *(Yi 1997, Definition 2.2.) Coalition structure* $C = \{n_1, \ldots, n_m\}$ *is a concentration of* $C' = \{n'_1, \ldots, n'_{m'}\}$, $m' \geq m$, *if and only if there exists a sequence of coalition structures* $C^1 = \left\{n^1_1, \ldots, n^1_{m(1)}\right\}$, $C^2 = \left\{n^2_1, \ldots, n^2_{m(2)}\right\}$, ..., $C^R = \left\{n^R_1, \ldots, n^R_{m(R)}\right\}$ *such that*

1. $C = C^1$ and $C = C^R$; and
2. $C^{r-1} = C^r \setminus \left\{n^r_{i(r)}, n^r_{j(r)}\right\} \cup \left\{n^r_{i(r)} + 1, n^r_{j(r)} - 1\right\}$, $n^r_{i(r)} \geq n^r_{j(r)}$, for some $i\,(r)$, $j\,(r) = 1, \ldots, m\,(r)$ and for all $r = 2, \ldots, R$.

4.C Stable structure in the open membership game with a uniform price auction.

Let $C^{OMG} = \left\{\bar{\eta}_1^{(\bar{m}_1)}, \bar{\eta}_2^{(\bar{m}_2)}\right\}$ denote the stable structure of the open membership game with a uniform price auction introduced in Corollary 4.1. First, to give an alternative, and perhaps a more clear, definition of this stable structure, Finus and Rundshagen (2001, Proposition 11) defines the structure C^{OMG} as follows (for $n \geq 3$):

$$C^{OMG} = \left\{\bar{\eta}_1^{(\bar{m}_1)}, \bar{\eta}_2^{(\bar{m}_2)}\right\} = \begin{cases} \left\{3^{(m^\star)}, 2^{(0)}\right\}, & \text{if } R = 0 \\ \left\{3^{(m^\star - 1)}, 2^{(2)}\right\}, & \text{if } R = 1 \\ \left\{3^{(m^\star)}, 2^{(1)}\right\}, & \text{if } R = 2 \end{cases},$$

where $m^\star = \hat{I}(n/3)$, with $\hat{I}(n/k)$ denoting the closest integer lower or equal to n/k , and $R = n - 3m^\star \in (0, 1, 2)$. Note that $\hat{I}(n/k)$ is not equal to $I(n/m)$ in Corollary 4.1.

Consider next a symmetric coalition structure $C^k = \left\{\eta_1^{\left(m_1^k\right)}, \eta_2^{\left(m_2^k\right)}\right\} = \left\{k^{\left(m^k - r^k\right)}, (k-1)^{r^k}\right\}$, where $k = I(n/m^k)$ and $r^k = m^k k - n$. Thus $C^3 = C^{OMG}$. Consider also the following neighboring structures:

$$C^k_{\eta_1, \eta_1} = \left\{(\eta_1 + 1)^{(1)}, \eta_1^{\left(m_1^k - 2\right)}, \eta_2^{\left(m_2^k + 1\right)}\right\}, \quad \text{if } m_1^k > 1,$$
$$C^k_{\eta_1, 0} = \left\{\eta_1^{\left(m_1^k - 1\right)}, \eta_2^{\left(m_2^k + 1\right)}, 1^{(1)}\right\}, \quad \text{if } m_1^k \geq 1,$$

and

$$C^k_{\eta_2,\eta_1} = \left\{(\eta_1+1)^{(1)}, \eta_1^{\left(m_1^k-1\right)}, \eta_2^{\left(m_2^k-1\right)}, (\eta_2-1)^{(1)}\right\}, \quad \text{if } m_1^k, m_2^k \geq 1,$$
$$C^k_{\eta_2,0} = \left\{\eta_1^{\left(m_1^k\right)}, \eta_2^{\left(m_2^k-1\right)}, (\eta_2-1)^{(1)}, 1^{(1)}\right\}, \quad \text{if } m_2^k \geq 1,$$
$$C^k_{\eta_2,\eta_2} = \left\{\eta_1^{\left(m_1^k+1\right)}, \eta_2^{\left(m_2^k-2\right)}, (\eta_2-1)^{(1)}\right\}, \quad \text{if } m_2^k > 1.$$

The first two neighboring structures describe the deviation structures for which a member of a larger coalition κ_1 deviates. Hence in $C^k_{\eta_1,\eta_1}$ a member of coalition κ_1 joins another coalition of the same size. In $C^k_{\eta_1,0}$ a member of coalition κ_1 deviates and becomes a singleton. I do not consider the neighboring structure $C^k_{\eta_1,\eta_2}$, i.e. if a member of coalition κ_1 joins coalition κ_2, because the coalition structure would remain the same and it would not change the payoff of the deviating firm. The last three neighboring structures describe the structures where a member of a smaller coalition κ_2 deviates. In $C^k_{\eta_2,\eta_1}$ the deviating firm joins a larger coalition κ_1, in $C^k_{\eta_2,0}$ it becomes a singleton and in $C^k_{\eta_2,\eta_2}$ it joins coalition κ_2.

The structure C^k is not stable if any of the deviating strategy provides lower total costs for a single deviating firm:

$$TC\left(k; C^k\right) > TC\left(k+1; C^k_{\eta_1,\eta_1}\right),$$
$$TC\left(k; C^k\right) > TC\left(1; C^k_{\eta_1,0}\right),$$

and

$$TC\left(k-1; C^k\right) > TC\left(k+1; C^k_{\eta_2,\eta_1}\right),$$
$$TC\left(k-1; C^k\right) > TC\left(1; C^k_{\eta_2,0}\right),$$
$$TC\left(k-1; C^k\right) > TC\left(k; C^k_{\eta_2,\eta_2}\right).$$

Table 4.3 presents the per-member total costs of coalitions for the stable structure of the model $C^{OMG} = C^3 = \left\{3^{\left(m^3-r^3\right)}, 2^{\left(r^3\right)}\right\}$ and for the neighboring structures described above, with parameter values $n = 3, \ldots, 100$; $\theta = 100$; $\beta = 1$; and $\delta = \lambda = 0.5$. The per-member costs are normalized such that the costs are compareable for different n. The normalized costs are thus denoted by $TC_n(\eta_i; C) = \frac{n}{100} TC(\eta_i; C)$. Moreover, Table 4.4 presents the same calculations for coalition structure $C^2 = \left\{2^{\left(m^2-r^2\right)}, 1^{\left(r^2\right)}\right\}$ and Table 4.5 for coalition structure $C^4 = \left\{4^{\left(m^4-r^4\right)}, 3^{\left(r^4\right)}\right\}$.

In all tables 4.3 - 4.5, the per-member costs of the deviating firm are written in either blue or red. If the per-member costs of the deviator are written in blue, the deviation strategy is not profitable for the deviator. On the other hand, the red color indicates that the deviator is better-off if it deviates. Note that I have omitted coalition structures $C^2_{\eta_2,0}$ and $C^2_{\eta_2,\eta_2}$ from Table 4.4, because $C^2_{\eta_2,\eta_2}$ is not feasible, and $C^2_{\eta_2,0}$ is equal to C^2 and thus $TC\left(1; C^2\right) = TC\left(1; C^2_{\eta_2,0}\right)$ by assumption.

The structure $C^{OMG} = C^3$ is indeed the unique stable structure, while there are no profitable deviation strategies for a single firm. Also, deviations of any larger group of firms are not profitable. This is intuitively clear while (according to Condition 4.4) any profitable deviation from $C^{OMG} = C^3$ must be such that the size of a new coalition of deviating firms is smaller than the original coalition.[12] Hence, if no single firm finds deviation profitable, the deviation of any subgroup of firms must not be profitable either.

Moreover, given the structure C^4 (see Table 4.5), there is at least one profitable single-firm deviating strategy for every n and thus C^4 is not a stable structure. This is also true for all other C^k, where $k > 4$, due to the breaking-down of the stand-alone stable condition.

With C^2 (see Table 4.4) it is always profitable for a singleton to join a larger coalition, while $TC\left(1; C^2\right) > TC\left(3; C^2_{\eta_2,\eta_1}\right)$, if $m^2_2 = 1$. However, the neighboring structures will not provide profitable deviations if $m^2_2 = 0$. Even though if there is no profitable strategy for a single deviator from coalition κ_1 in C^2, the deviation may affect other players' incentives to deviate. Thus there are profitable deviation strategies for some subgroup of firms. This can be described with the following example. Consider the case $n = 6$ and thus the structure $C^2 = \left\{2^{(3)}\right\}$ in Table 4.4. Suppose that one of the firms deviates and joins another coalition. This gives a structure $C^2_{\eta_1,\eta_1} = \left\{3^{(1)}, 2^{(1)}, 1^{(1)}\right\}$. Note that this is exactly the same structure as $C^3_{\eta_1,0}$ in Table 4.3 with $n = 6$. According to Table 4.3, the singleton of this structure will find it profitable to join coalition $\{2\}$, because

$$TC_n\left(1; C^3_{\eta_1,0}\right) = 17.306 > 17.265 = TC_n\left(3; C^3\right).$$

Hence, even if the original deviation does not seem to be profitable, while (see Table 4.4)

$$TC_n\left(2; C^2\right) = 17.712 < 17.763 = TC_n\left(3; C^2_{\eta_1,\eta_1}\right),$$

the resulting structure is not $C^2_{\eta_1,\eta_1} = C^3_{\eta_1,0}$, but the stable structure C^3. Thus, at the end of the day, the deviation is profitable, because (see tables 4.3 and 4.4)

$$TC_n\left(2; C^2\right) = 17.712 > 17.265 = TC_n\left(3; C^3\right).$$

A similar story can be told for every n for which $m^2_2 = 0$. Thus, C^2 is not a stable structure either. Hence, C^3 is the unique stable structure of all symmetric coalition structures $C^k = \left\{k^{\left(m^k - r^k\right)}, (k-1)^{r^k}\right\}$, $m^k = 1, \ldots, n$, where $k = I(n/m^k)$ and $r^k = m^k k - n$.

[12]Suppose that one or more firms deviate to larger coalitions. If those deviations are profitable, the resulting coalition structures violate, presumably, the stand-alone stability and are thus not CPNE. Calculations are however omitted.

Table 4.3: Stable structures $C^3 = \left\{3^{\left(m_1^3\right)}, 2^{\left(m_2^3\right)}\right\}$ and neighboring deviation strategies of the open membership game. Parameter values: $\theta = 100$, $\beta = 1$, and $\delta = \lambda = 0.5$. Per-member costs are normalized such that $TC_n(\eta_i; C) = \frac{n}{100} TC(\eta_i; C)$.

n	m_1^3	m_2^3	$TC_n(3; C^3)$	$TC_n(2; C^3)$	$TC_n(4; C^3_{\eta_1,\eta_1})$	$TC_n(1; C^3_{\eta_1,0})$	$TC_n(4; C^3_{\eta_2,\eta_1})$	$TC_n(1; C^3_{\eta_2,0})$	$TC_n(3; C^3_{\eta_2,\eta_2})$
3	1	0	16.667			**16.732**			
4	0	2		17.265				**17.448**	**17.408**
5	1	1	17.412	16.993		**17.453**	**17.343**	**17.131**	
6	2	0	17.265		**17.444**	**17.306**			
7	1	2	17.801	17.600		**17.819**	**17.757**	**17.657**	**17.635**
8	2	1	17.730	17.571	**17.821**	**17.746**	**17.690**	**17.616**	
9	3	0	17.712		**17.784**	**17.726**			
10	2	2	17.977	17.881	**18.032**	**17.985**	**17.953**	**17.906**	**17.897**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
20	6	1	18.295	18.272	**18.308**	**18.296**	**18.289**	**18.278**	
30	10	0	18.435		**18.440**	**18.435**			
40	12	2	18.522	18.517	**18.525**	**18.523**	**18.521**	**18.518**	**18.518**
50	16	1	18.564	18.561	**18.566**	**18.564**	**18.563**	**18.561**	
60	20	0	18.593		**18.594**	**18.593**			
70	22	2	18.618	18.616	**18.619**	**18.618**	**18.618**	**18.617**	**18.617**
80	26	1	18.633	18.632	**18.634**	**18.633**	**18.633**	**18.632**	
90	30	0	18.645		**18.646**	**18.645**			
100	32	2	18.657	18.656	**18.658**	**18.657**	**18.657**	**18.657**	**18.657**

Table 4.4: Coalition structures $C^2 = \left\{2^{(m_1^2)}, 1^{(m_2^2)}\right\}$ and neighboring deviation strategies of the open membership game. Parameter values: $\theta = 100$, $\beta = 1$, and $\delta = \lambda = 0.5$. Per-member costs are normalized such that $TC_n(\eta_i; C) = \frac{n}{100} TC(\eta_i; C)$.

n	m_1^2	m_2^2	$TC_n(2; C^2)$	$TC_n(1; C^2)$	$TC_n(3; C^2_{\eta_1,\eta_1})$	$TC_n(1; C^2_{\eta_1,0})$	$TC_n(3; C^2_{\eta_2,\eta_1})$
3	1	1	17.444	16.732		**17.712**	**16.667**
4	2	0	17.265		**17.408**	**17.448**	
5	2	1	17.703	17.453	**17.782**	**17.805**	**17.412**
6	3	0	17.712		**17.763**	**17.785**	
7	3	1	17.940	17.819	**17.976**	**17.988**	**17.801**
8	4	0	17.963		**17.990**	**18.000**	
9	4	1	18.098	18.028	**18.119**	**18.126**	**18.020**
10	5	0	18.119		**18.135**	**18.141**	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
20	10	0	18.435		**18.439**	**18.439**	
30	15	0	18.540		**18.542**	**18.542**	
40	20	0	18.593		**18.594**	**18.594**	
50	25	0	18.624		**18.625**	**18.625**	
60	30	0	18.645		**18.646**	**18.646**	
70	35	0	18.660		**18.661**	**18.661**	
80	40	0	18.672		**18.672**	**18.672**	
90	45	0	18.680		**18.681**	**18.681**	
100	50	0	18.687		**18.688**	**18.688**	

Table 4.5: Coalition structures $C^4 = \left\{4^{\left(m_1^4\right)}, 3^{\left(m_2^4\right)}\right\}$ and neighboring deviation strategies of the open membership game. Parameter values: $\theta = 100$, $\beta = 1$, and $\delta = \lambda = 0.5$. Per-member costs are normalized such that $TC_n(\eta_i; C) = \frac{n}{100} TC(\eta_i; C)$.

n	m_1^4	m_2^4	$TC_n(4; C^4)$	$TC_n(3; C^4)$	$TC_n(5; C^4_{\eta_1,\eta_1})$	$TC_n(1; C^4_{\eta_1,0})$	$TC_n(5; C^4_{\eta_2,\eta_1})$	$TC_n(1; C^4_{\eta_2,0})$	$TC_n(4; C^4_{\eta_2,\eta_2})$
7	1	1	17.382	17.084		**17.285**	**15.259**	**17.122**	
8	2	0	17.265		**17.430**	**17.194**			
9	0	3		17.712				**17.726**	**15.151**
10	1	2	17.782	17.644		**17.736**	**15.474**	**17.657**	**15.368**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
20	5	0	18.119		**18.141**	**18.107**			
30	6	2	18.353	18.340	**18.363**	**18.348**	**16.302**	**18.340**	**16.292**
40	10	0	18.435		**18.440**	**18.432**			
50	11	2	18.506	18.502	**18.510**	**18.504**	**16.510**	**18.502**	**16.507**
60	15	0	18.540		**18.543**	**18.539**			
70	16	2	18.574	18.572	**18.576**	**18.573**	**16.602**	**18.572**	**16.600**
80	20	0	18.593		**18.594**	**18.592**			
90	21	2	18.613	18.611	**18.614**	**18.612**	**16.653**	**18.611**	**16.652**
100	25	0	18.624		**18.625**	**18.624**			