Essays on Family Economics in Developing Countries

Author: Jacob Penglase

Boston College Electronic Thesis or Dissertation, 2018

ESSAYS ON FAMILY ECONOMICS IN DEVELOPING COUNTRIES

Jacob Penglase

A dissertation
submitted to the Faculty of
the department of Economics
in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy

Boston College
Morrissey College of Arts and Sciences
Graduate School

April 2018

ESSAYS ON FAMILY ECONOMICS IN DEVELOPING COUNTRIES

Jacob Penglase

Advisors: Prof. S. Anurkriti, Prof. Arthur Lewbel, Prof. Donald Cox

In this dissertation, I attempt to better understand the inner workings of the household: Do parents favor certain types of children? When do parents decide to have their children work? How can we identify inequality within the household? These issues are fundamental to economic development and closely related to individual welfare. However, studying these questions is difficult since the household is in many ways a blackbox to economists; consumption data is typically collected at the household level, and concepts like bargaining power are not observable. My research examines these questions in a variety of different contexts in the developing world. In Chapter 1, I test for consumption inequality between foster and non-foster children in Sub-Saharan Africa. In Chapter 2, I examine the relationship between child labor and fertility in Nigerian households. Lastly, I study the identification of intrahousehold inequality in collective households in Chapter 3.

Chapter 1: In *"Consumption Inequality Among Children: Evidence from Child Fostering in Malawi"*, I study how resources are allocated among foster and non-foster children in Malawi. Child fostering is widespread in parts of Africa and the wellbeing of these children, who may be particularly vulnerable to impoverishment, is not well known. However, identifying individual-level consumption is difficult, since goods are shared and consumption is measured at the household level. Recent work on intrahousehold resource allocation has inferred child consumption from household-level spending on child-specific goods (e.g., child clothing).[1] This literature is often dependent on the existence of goods in the data that are consumed exclusively by a particular type of person in the household. These studies are therefore limited by the level of assignability of goods within the consumption survey. Stated differently, to identify inequality between foster and non-foster children using existing techniques, I would need to observe expenditure on a good that is consumed separately by foster and non-foster children. Because I do not, I develop a new methodology using the

[1] See Dunbar, Lewbel, and Pendakur (2013), for example.

collective framework to measure consumption inequality between foster and non-foster children. I find little evidence of inequality between foster and non-foster children. I then divide foster children by whether the child is orphaned, and I find that orphan-foster child consumption is 23 percent less than non-orphan foster child consumption. The results of this paper suggest that policymakers should design programs to improve the relative standing of orphan-foster children in the household. The methodological contribution of this paper is applicable to other contexts as intrahousehold inequality among children is widespread.

Chapter 2: In *"Child Labor Laws and Household Fertility Decision: Evidence from Nigeria"* I study the Child Rights Act of Nigeria (CRA). In 2003, the Nigerian National Assembly implemented this law, which codified existing child labor standards and dramatically increased the penalties for employing children. I exploit the Child Rights Act to both understand the employment consequences of a child labor legislation, and to analyze the effect of lowering the economic value of children on fertility rates. Identification comes from variation in the timing of when each Nigerian state adopted the law, and from variation in the law's age restrictions. Consistent with recent theoretical and empirical evidence, I find the Child Rights Act increased child employment at both the intensive and extensive margins. I then model household fertility decisions to demonstrate that the demand for children is increasing in child wages and therefore influenced by changes in the child labor market. I empirically test the model implications by examining the effect of the Child Rights Act on fertility rates, but find little to no effect.

Chapter 3: In *"Identification of Resource Shares with Multiple Assignable Goods"* (with Caitlin Brown and Rossella Calvi), we study intrahousehold inequality. We develop a new methodology using the collective framework to identify resource shares, defined as the fraction of household resources consumed by each household member. We build upon recent work by Dunbar, Lewbel, and Pendakur (2013) (DLP) who identify resource shares by observing how expenditure on a single private assignable good varies with household income and size.[2] They achieve identification by mak-

[2]A private good is a good that is not shared. An assignable good is a good that is consumed

ing semi-parametric restrictions on preferences across either household members or household sizes. Because our data contain *multiple* private assignable goods, we are able to employ this additional data to weaken the DLP preference restrictions using a different approach, which we call "Differenced-Similar Across People" (D-SAP). Under D-SAP, preferences for the assignable goods are allowed to differ entirely across both household members. However, we introduce a weaker restriction that requires that preferences differ across people in a similar way across goods.

exclusively by an individual known to the econometrician.

ACKNOWLEDGEMENTS

I would like to thank my advisors S Anukriti, Arthur Lewbel and Donald Cox for their invaluable guidance through the dissertation process, and teaching me almost all of what I know about doing research. I appreciate their patience in allowing my to make (many) mistakes over the past couple of years, and their knowledge that has contributed immensely to this dissertation. I would also like to thank Andrew Beauchamp and Scott Fulford who's classes were tremendously helpful.

There are many friends and classmates who have contributed in a variety of ways to this dissertation, however I would be remiss not to mention Rossella Calvi, Nicholas Diebel, Mehmet Ezer, Sylvia Hristakeva, Deeksha Kale, John Lindner, Ethan Struby, Lauren Hoehn-Velasco, and Solvejg Wewel. I spent too many hours in the economics department, but it was tolerable entirely because of the people I was working with.

Finally, I would like to thank my parents, Richard and Essa Penglase. They have assisted me in all parts of my life and I owe any success I've had to them. I feel tremendously fortunate to have gone through graduate school with my parents so close by. I would not have graduated without their support.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

CONSUMPTION INEQUALITY AMONG CHILDREN: EVIDENCE FROM CHILD FOSTERING IN MALAWI

1 Introduction

Do parents favor certain types of children? Dating back to Becker (1960), economists have recognized that parents can to some degree choose the "quality" of their children through schooling decisions, health investments, and consumption allocations. While many parents treat their children equally, some parents may have a preferred type of child. Gender, birth order, prenatal endowments, and degree of kinship are all child characteristics that may impact parental treatment.

In this paper, I study intrahousehold consumption inequality. Do parents allocate a larger share of the household budget to certain types of children? This question is difficult to answer as consumption data is collected at the household level and goods are shared among family members. Existing work has used reduced-form methods to identify the existence of discrimination, but not it's extent. For example, Deaton (1989) tests for gender discrimination by examining how expenditure on adult goods varies with the number of boys and girls in the household. This approach is similar to the Rothbarth method (Rothbarth (1943)) which also relies upon strong preference stability assumptions across household compositions. In this paper, I develop a new methodology using a structural model of intrahousehold resource allocation to identify the existence *and extent* of consumption inequality among children. I rely only on standard household-level survey data and am able to identify the share of total household resources allocated to each type of child within the household. I apply this method to child fostering in Malawi, where many children live in households away from both of their biological parents.

Following Chiappori (1988, 1992), I model households as a collection of individuals, each with their own utility function. I obtain a measure of individual-level consumption by identifying *resource shares*, defined as the share of the total household budget allocated to each household member. Dunbar et al. (2013) (DLP henceforth) demonstrate that resource shares can be identified by observing how expenditure on *assignable* goods vary with household income and size, where a good is assignable if it is consumed exclusively by a particular type of person in the household (e.g., men's clothing). DLP obtain identification by inverting Engel curves for the assignable goods within the framework of a structural model. While the DLP identification results and related studies (Bargain and Donni (2012)) have allowed economists to identify inequality between men, women, and children within the household, these existing methods are often unable to uncover inequality *among* children within the household. This limitation is due to the nature of consumption surveys, which include expenditures on goods that can be assigned to children (clothing, shoes, toys), but not goods that can be assigned to individual children.[1]

In this study, I overcome this common data limitation. I develop a new framework to identify inequality among children using Engel curves for *partially assignable* goods. A good is partially assignable if the researcher can, to a limited extent, determine which individuals in the household consume it. For example, children's clothing expenditures are partially assignable to boys and girls, or foster and non-foster children. Identification proceeds as follows: First, I note that children's clothing expenditures can be assigned exclusively to a specific type of child if the household only contains that type of child, that is, children's clothing expenditures are assignable to boys if the household only has boys. It follows that in these households, I can use the DLP methodology to separately identify resource shares for each child type. I next move to households with both types of children (boys and girls, foster and non-foster children, etc.), where children's clothing expenditures are not assignable. The key

[1]There are a limited number of surveys that include individual-level consumption data, such as the Bangladesh Integrated Household Survey and the China Health and Nutrition Survey.

assumption is to impose a modest similarity restriction between the clothing Engel curves in households with one type of child, which have already been identified, and households with both types of children. With these similarity restrictions, resource shares can now be identified.

In this framework, I maintain the key identifying assumptions of DLP: I assume that resource shares are independent of household expenditure,[2] and I impose one of two semi-parametric restrictions on individual preferences for clothing. As in DLP, the model parameters are identified by comparing the slopes of clothing Engel curves across individuals or household sizes.

With this methodological contribution, I add to the growing literature that examines intrahousehold resource allocation using the collective household framework. This strand of research, beginning with work by Chiappori (1988, 1992), Apps and Rees (1988), and Browning et al. (1994) models households as a collection of individuals, each with their own distinct preferences. Within this field, my paper relates mostly to work on the identification of the level of resource shares, such as Lewbel and Pendakur (2008), Browning et al. (2013) (BCL), and DLP.[3] I differ from this literature in several ways. Unlike Lewbel and Pendakur (2008) and BCL, I am able to identify resource shares for children, and unlike DLP, my identification method is not dependent on the existence of assignable goods within the data. The identification results of this paper can therefore be used to quantify inequality in variety of contexts where assignable goods often do not exist, such as inequality between boys and girls, first-born children and children of lower birth order, or inequality among children with different prenatal endowments.[4]

In the empirical application, I study foster children in Sub-Saharan Africa (SSA). Foster children have become a population of increasing interest as economists have come to recognise the variety of household structures that exist in SSA. Child fostering is practiced across all of SSA and varies from 8 percent in Burkina Faso to

[2] I discuss the validity of this assumption in Section 3.

[3] A different approach places bounds on resource shares using revealed preference inequalities. Cherchye et al. (2011) and Cherchye et al. (2015) are examples of these studies.

[4] See Almond and Mazumder (2013) for a review of the literature on the relationship between prenatal endowments and parental investments in children.

as high as 25 percent in Zimbabwe.[5] In Malawi, 12 percent of children are fostered and 17 percent of households have a foster child. While some of these children are orphans, the majority are children who are voluntarily sent away by their parents to live with close relatives.[6] Children are fostered for a variety of reasons including child labour, education, or to share risk across households.[7] Because foster children live away from their parents, they may be particularly vulnerable to unequal treatment within the household. Existing work on foster child welfare has focused on education (Case et al. (2004), Fafchamps and Wahba (2006), Ainsworth and Filmer (2006), Evans and Miguel (2007)), but much less is known about consumption, which I study in this paper. A notable exception is Case et al. (2000) who study how *household* food expenditures vary by the fostering status of the household's children. I build upon Case et al. (2000) by using a structural model to estimate resource shares, which allows for a clearer picture of the extent of inequality within the household.

I estimate the model using detailed household-level consumption and expenditure data from Malawi. The resulting structural estimates allow me to quantify resource shares separately for foster and non-foster children, and I find no evidence of inequality. I then divide foster children into two categories based on whether or not they are orphaned. I find that orphaned foster children are particularly disadvantaged with consumption that is 23 percent less than the consumption of non-orphaned foster children. The results also suggest that foster children living in matrilineal villages consume a larger share of the household budget relative to foster children living in patrilineal villages. Gender does not appear to be a determinant of foster child treatment. The results highlight the importance of orphanhood and kinship networks in the wellbeing of foster children, which motivates future work investigating the mechanisms underlying this paper's results.

I use the predicted resource shares to estimate foster and non-foster child poverty

[5] These figures are taken from Grant and Yeatman (2012) who use Demographic and Health Survey data to compute foster rates for 14 countries in Sub-Saharan Africa.

[6] I use "orphan" to describe a child who has lost at least one parent. This is consistent with the UNICEF and UNAIDS definition. In Malawi, 34 percent of foster children are orphans.

[7] Ainsworth (1995), Akresh (2009), and Serra (2009) examine the economic reasons children are fostered.

rates. Traditional measures of poverty implicitly assume an equal distribution of resources across household members. I move away from the traditional approach by using the predicted resource shares to determine each household member's individual consumption. I show that using household-level poverty rates dramatically understates child poverty rates, which is in line with DLP and Brown et al. (2016). Furthermore, I also find that orphaned foster child poverty is being miscalculated at an even higher rate. This result is important for several reasons. First, coverage of government programs is rarely universal, and policymakers must find ways to determine who is poor. Different methods that are used to identify the poor, such as proxy-means testing, use household-level measures. I demonstrate that these methods are unsatisfactory, since poor individuals do not necessarily live in poor households. My results suggest that anti-poverty programs that specifically target orphans, such as the Kenya Cash Transfer for Orphans and Vulnerable Children program, would be more effective. Furthermore, programs that improve the relative standing of children in the household, such as cash transfer programs that are conditional on children being enrolled in school, would also be beneficial.

The remainder of the paper is organised as follows. Section 2 presents the collective household model. Section 3 discusses the identification results. I then apply the identification method to child fostering in Malawi in Section 4. Section 5 examines the robustness of my results. I conclude in Section 6.

2 Collective Model of the Household

This section presents a structural model of Malawian households using the collective framework of Browning et al. (2013). The household consists of four types of individuals denoted by t: adult men (m), adult women (w), foster children (a), and non-foster children (b). Person types a and b could refer to boys and girls, or young and old children and everything that follows would be exactly the same. I index household types by the number of foster and non-foster children within the household, denoted by the subscript s.[8] Consistent with the standard characterization of collective households, I make no assumptions about the bargaining process which determines how resources are allocated across household members, only that the ultimate allocation is Pareto efficient.[9] I account for economies of scale in consumption using a Gorman (1976) linear technology function.[10] Individuals have caring preferences, in the sense that they are allowed to get utility from the utility of other household members, though not the consumption of specific goods by the other household members.

Households consume K types of goods at market prices $p = (p^1, ..., p^K)'$. Let $z_s = (z_s^1, ..., z_s^K)$ be the K - vectors of observed quantities purchased by the household. The vector of unobserved quantities consumed by individuals within the household is denoted by $x_t = (x_t^1, ..., x_t^K)$. The household-level quantities are converted into private good equivalents x_t using a linear consumption technology as follows: $z_s = A(s_f x_f + s_m x_m + s_a x_a + s_b x_b)$ where A is a $K \times K$ matrix which accounts for economies of scale in consumption,[11] and s_t denotes the number of each person type within the household. If good x^k is not shared, then what the household purchases is equal to the sum of what individuals consume, and the element in the k'th row in the k'th column of matrix A takes a value of one with all off-diagonal elements in that row and column equal to zero. Goods that are shared have values along the diagonal of matrix A that are less than one, as the sum of what individuals consume is greater than what the household purchases.

Each individual member has a monotonically increasing, continuously twice differentiable strictly quasi-concave utility function over a bundle of goods. Let $U_t(x_t)$

[8] I occasionally denote household type by s_{ab} to explicitly indicate the number of foster and non-foster children within the household. For example, s_{21} denotes a household with two foster children, and one non-foster child.

[9] Pareto efficiency in household consumption allocations has been analysed in many different contexts and usually cannot be rejected. Notable papers that analyse this assumption include Browning and Chiappori (1998), Bobonis (2009), and Attanasio and Lechene (2014). Pareto efficiency has at times been rejected in the context of household agricultural production decisions, especially in West Africa. See Udry (1996) for example.

[10] See Browning et al. (2013) for a detailed explanation of accounting for economies of scale and sharing in collective households.

[11] The use of private good equivalents was introduced in Browning et al. (2013). This approach differs from the Chiappori (1988, 1992) version of the collective model where goods are either purely public or purely private; here goods can be purely public, purely private, or partially shared, and is therefore a more general framework.

be the utility of an individual of type t who consumes x_t goods while living in the household. This utility function is assumed to be separable from leisure, savings, or any other goods not included in the commodity bundle. Individuals of the same type are assumed to have the same utility function. For the empirical results, the utility function for each person type is allowed to differ over observable characteristics such as age and education.

Each household maximises the Bergson-Samuelson social welfare function, $\tilde{U}$ where each individual's utility function is discounted by the Pareto weights $\mu_t(p/y)$ where y is total household expenditure:[12]

$$\tilde{U}(U_m, U_f, U_a, U_b, p/y) = \sum_{t \epsilon \{m,f,a,b\}} \mu_t(p/y) U_t \tag{1.1}$$

The household then solves the following maximisation problem:

$$\begin{aligned} \max_{x_m, x_f, x_a, x_b} \tilde{U}(U_m, U_f, U_a, U_b, p/y) \quad & \text{such that} \\ & z_s = A(s_f x_f + s_m x_m + s_a x_a + s_b x_b) \\ & y = z_s' p \end{aligned} \tag{1.2}$$

Solving this system results in bundles of private good equivalents. If these goods are priced at within household prices $A'p$,[13] I obtain the resource share η_s^t, which is defined as the fraction of total expenditure that is allocated to each individual of type t.[14] By definition, resource shares for men, women, foster, and non-foster children sum to one. I will ultimately compare resource shares of foster and non-foster children to test for intrahousehold inequality.

With Pareto efficiency, I can reformulate the household's problem as a two step process using the second welfare theorem; In the first stage, resources are optimally

[12]The Pareto weights may also be a function of distribution factors. These are variables that may affect bargaining power, but not preferences. Because distribution factors are not necessary for identification, they are omitted.

[13]The within household price vector is different than the market prices faced by the household since some goods are jointly consumed.

[14]Resource shares have a one-to-one correspondence with the Pareto weights, where the Pareto weights are the marginal response of $\tilde{U}$ to U_t.

allocated across household members. In the second stage, each individual chooses x_t to maximise their own personal utility function U_t subject to the shadow budget constraint $\sum_k A_k p^k x_t^k = \eta_s^t y$.

Identification of resource shares relies on observing Engel curves, where an Engel curve is defined as a functional relationship between budget shares and total household expenditure holding prices constant. I write the household-level demand for a certain subset of goods whose properties substantially reduce the data requirements necessary for identification. Define these goods as *private assignable goods*, which are goods that are not shared across household member types (private), and that are consumed by a person of known type t (assignable). Examples of private goods include food and clothing; if the father drinks a glass of milk, the mother cannot consume that same glass of milk. Food however is not assignable; my data provides information on the total amount of food consumed, but not who in the household consumed it. On the contrary, clothing is both private and assignable, in the sense that men's clothing is observable in the data, and can safely be assumed to be consumed only by men.[15]

The motivation for relying on private assignable goods is that the household-level demand functions for these goods are substantially simpler than the demand functions for non-private goods. Intuitively, household-level demand for men's clothing will behave fairly similarly to men's demand for men's clothing. On the other hand, the household's demand for non-private goods, such as gasoline, depends on the degree to which gasoline is shared within the household, and also on each individual's preferences for gasoline.

Let $W_s^t(y, p)$ be the share of household expenditure y spent on person type t's private assignable good in a household of type s. DLP derive the household demand functions for the private assignable goods, which can be written as follows:[16]

$$W_s^t(y, p) = \ s_t \ \eta_s^t \ w_s^t(A'p, \eta_s^t y) \tag{1.3}$$

[15] This is true for men and women, but children's clothing is only partially assignable for foster and non-foster children.

[16] See Section 1.B in the appendix for the details of the DLP derivation.

where w_s^t is the amount of the private assignable good that a person of type t living in a household of type s would hypothetically demand had they lived alone with income $\eta_s^t y$ facing price vector $A'p$. Note that the resource shares and the individual demand functions are unobservable, and hence the system is not identified without more assumptions (for each equation there are two unknowns).[17] In what follows I discuss how to identify the parameters of interest.

3 Identification

DLP demonstrate how resource shares can be identified by observing how budget shares for assignable clothing vary with household expenditure and size. The key data requirement for their identification strategy is household-level expenditure on a private assignable good for each person type within the household. In this context, that would mean separately observing expenditure on foster child clothing and non-foster child clothing, neither of which are available in the data. Thus, a direct application of the DLP methodology is infeasible. I work around this data limitation by making use of expenditure on partially assignable goods, children's clothing in particular, which is partially assignable to both foster and non-foster children.[18]

I demonstrate two different sets of assumptions to identify resource shares in this context. I begin in Section 3.1 by summarizing how DLP use private assignable goods to identify resource shares. I then present two new approaches that identify resource shares using expenditure on private partially assignable goods in Sections 3.2.1 and 3.2.2, respectively. Throughout this discussion I emphasize where and why I differ.

3.1 Identification with Private Assignable Goods

If foster and non-foster child clothing expenditures are observed separately, the DLP method would identify resource shares using four separate Engel curves for assignable

[17]BCL achieve identification by assuming w_s^t is "observed" using data from households that have only men, or only women. In households with only single men, or only single women, the household and individual demand functions are the same. This clearly does not work in a context where children are present, as children do not live alone.

[18]Children's clothing expenditure would also be partially assignable to boys and girls, for example.

clothing. From Equation (1.3), this system can be written as follows:

$$\begin{aligned} W_s^m(y) &= \eta_s^m \; w_s^m(\eta_s^m y) \\ W_s^f(y) &= \eta_s^f \; w_s^f(\eta_s^f y) \\ W_s^a(y) &= s_a \; \eta_s^a \; w_s^a(\eta_s^a y) \\ W_s^b(y) &= s_b \; \eta_s^b \; w_s^b(\eta_s^b y) \end{aligned} \tag{1.4}$$

The number of foster and non-foster children in the household is given by s_a and s_b, and this determines the household type given by the subscript s. To simplify notation, the household is assumed to have only one man ($s_m = 1$) and one woman ($s_f = 1$). To achieve identification, resource shares are assumed to be independent of household expenditure; this is the key identifying assumption.[19] Resource shares can however depend on variables highly correlated with expenditure, such as household member wages, remittances, or wealth.

In the empirical application I assume individuals have preferences over clothing given by Muelbauer's PIGLOG indirect utility function, and this assumption facilitates a discussion of identification so it is used henceforth.[20] The PIGLOG indirect utility function takes the following functional form:

$$V_t(p, y) = b_t(p)(\ln y - a_t(p)) \tag{1.5}$$

Using Roy's identity, the budget share functions are written as follows:

$$w_t(p, y) = \delta_t(p) + \beta_t(p) \ln y \tag{1.6}$$

where $\delta_t(p)$ is a function of $a_t(p)$ and $b_t(p)$, and $\beta_t(p)$ is minus the price elasticity of

[19] Menon et al. (2012) show this assumption to be quite reasonable. They rely on a household survey question that asked Italian parents what percentage of household expenditures they allocated to children. Their answers did not vary considerably across expenditure levels. Cherchye et al. (2015) use a revealed preference approach to place bounds on resource shares and also find that they do not vary with household expenditure. Lastly, resource shares need to be independent of household expenditure only at low levels of household expenditure.

[20] A more general functional form is used in the proof in the appendix. No preference restriction is made on the other goods.

$b_t(p)$ with respect to the price of person t's assignable good. Substituting Equation (1.6) into Equation (1.4) results in the system of Engel curves given below:

$$\begin{aligned} W_s^m &= \eta_s^m\,[\delta_s^m + \beta_s^m \ln(\eta_s^m)] + \eta_s^m \beta_s^m \ln y \\ W_s^f &= \eta_s^f\,[\delta_s^f + \beta_s^f \ln(\eta_s^f)] + \eta_s^f \beta_s^f \ln y \\ W_s^a &= s_a \eta_s^a\,[\delta_s^a + \beta_s^a \ln(\eta_s^a)] + s_a \eta_s^a \beta_s^a \ln y \\ W_s^b &= s_b \eta_s^b\,[\delta_s^b + \beta_s^b \ln(\eta_s^b)] + s_b \eta_s^b \beta_s^b \ln y \end{aligned} \tag{1.7}$$

where W_s^t are budget shares for the private assignable good for person type t in household s. I drop prices from Equation (1.7), as Engel curves describe the relationship between budget shares and total expenditure holding prices fixed. DLP demonstrate one of two additional assumptions are necessary for identification: (1) Preferences for the assignable good are similar across household types (SAT), so $\beta_s^t = \beta^t$; or (2) Preferences for the assignable good are similar across people (SAP), so $\beta_s^t = \beta_s$.[21]

The SAT restriction was first used in Lewbel and Pendakur (2008) and is equivalent to assuming price differences across household types can be absorbed into an income deflator. Under this restriction, identification is achieved by comparing Engel curves across households of different sizes for a given individual type. To better understand what this restriction entails, consider the demand for a purely public good such as housing. As the household size increases, the shadow price of rent decreases. This change in the price of housing may have an effect on each person's demand for clothing. However, under SAT, this price change can only affect the demand for clothing through a person-specific income deflator.

The SAP restriction is a more commonly used preference restriction in the demand literature, and is a weaker version of shape-invariance (Pendakur (1999), Lewbel (2010)). Under this restriction, identification is achieved by comparing Engel curves across individuals for a given household type.

[21] A sufficient restriction on the indirect utility function for SAT to hold is that $b(p) = \bar{b}_t(p_t, \bar{p})$, where p_t is the price of the assignable good, and $\bar{p}$ is the price of the private non-assignable goods. In effect, $\bar{b}_t(\cdot)$ is assumed not to vary with the prices of the shared goods, and thus independent of household size. A sufficient restriction on the indirect utility function for SAP to hold is that $b_t(p) = b(p)$, and therefore does not vary across people.

Assuming resource shares sum to one, the model parameters can then be identified with either preference restriction by inverting the Engel curves. It is important to note that the relative size of the budget shares for foster and non-foster child clothing does not necessarily determine which child type has higher resource shares. It is entirely possible for $\eta_s^b > \eta_s^a$ with $W_s^a > W_s^b$, since preferences are allowed to be different across individuals.

The key complication in both identification methods for my purposes is the absence of a separate private assignable good for foster and non-foster children in the data; I do not observe the budget shares for foster and non-foster child clothing, W_s^a and W_s^b, but rather their sum $W_s^c = W_s^a + W_s^b$, where W_s^c is the budget share for *child* clothing. This is a widespread data problem that is present in a variety of settings where inequality among children is of interest; consumption surveys rarely contain data on goods that are assignable to specific types of children. To work around the lack of sufficient data, I now develop a new methodology to identify resource shares in the absence of private assignable goods using private partially assignable goods.

3.2 Identification with Private Partially Assignable Goods

3.2.1 Approach 1: An Extension of SAT Identification

Without private assignable goods for foster and non-foster children, I rewrite the Engel curves for foster and non-foster child clothing in system (1.7) as a single Engel curve for children's clothing, and I begin by using the SAT restriction (i.e., $\beta_s^t = \beta^t$):

$$
\begin{aligned}
W_s^m &= \eta_s^m \; [\delta_s^m + \beta^m \ln(\eta_s^m)] \; + \; \eta_s^m \beta^m \ln y \\
W_s^f &= \eta_s^f \; [\delta_s^f + \beta^f \ln(\eta_s^f)] \; + \; \eta_s^f \beta^f \ln y \\
W_s^c &= s_a \eta_s^a \; [\delta_s^a + \beta^a \ln(\eta_s^a)] \; + \; s_b \eta_s^b \; [\delta_s^b + \beta^b \ln(\eta_s^b)] \\
&\quad + \; \ln y \; (s_a \eta_s^a \beta^a + s_b \eta_s^b \beta^b)
\end{aligned} \tag{1.8}
$$

Here, the Engel curve for children's clothing is given as the sum of the Engel curves for foster and non-foster child clothing. I have simply taken the bottom two equations from system (1.7) and summed them together.[22] As before, I allow preferences for clothing to vary considerably by person type through both the intercept parameter δ_s^t and the slope parameter β^t.

The identification proof proceeds in two steps. First, I demonstrate that resource shares are identified in *one-child-type* households, that is, households with only foster children, or only non-foster children. This follows directly from DLP as children's clothing expenditures are fully assignable in these households. I then move to the *composite* households, or households with both foster and non-foster children, where children's clothing expenditures are not assignable. The key new assumption is to impose some similarity between the one-child-type households and the composite households.

The identification proof starts with identifying resource shares in the one-child-type households. Suppose there are four one-child-type households $s \,\epsilon\, \{s_{10}, s_{20}, s_{01}, s_{02}\}$ where, for example, s_{10} denotes a household with one foster child and no foster children.[23] I can use a simple counting exercise to show that the order condition is satisfied. With three Engel curves for each household type, and four household types, there are twelve Engel curves. Moreover, for each of the four household types resource shares must sum to one. This results in a system of sixteen equations in total. In terms of the number of unknowns, each Engel curve has one resource share η_s^t that needs to be identified (twelve total), and there are four shape parameters β^t that need to be identified.[24] This leads to sixteen unknowns, and with sixteen equations, the order condition for identification is satisfied. A formal proof that the rank condition holds for the one-child-type households is provided in the appendix.

I next move to the composite households, which is where the main contribution of this paper lies. With SAT, preferences for clothing are similar across household sizes. I modify this restriction by assuming that preferences are both similar across

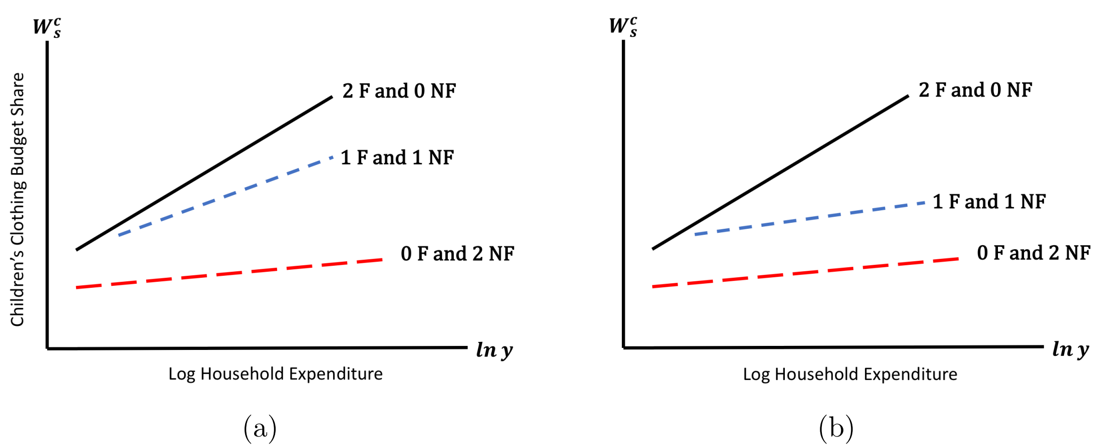
[22]Implicit in summing the two Engel curves is the assumption that foster and non-foster children do not share clothing. The validity of this assumption is analysed in Section 5.

[23]In the empirical application the sample includes households with as many as five children.

[24]It is necessary to identify the slope preference parameter β^t to identify the resource shares, however it is not necessary to identify the intercept preference parameter δ_s^t, and it is therefore ignored.

households sizes *and across household compositions*; that is, preferences for clothing are similar across one-child-type and composite households. In words, the foster child's marginal propensity to consume clothing, as their expenditure increases, is independent of the number of non-foster children present in the household, and vice versa. I take β^t from the one-child-type households, and assume it is the same in the composite households. It follows that the resource shares for men and women can be immediately recovered since the slope coefficients for their Engel curves ($\beta^m \eta_s^m$ and $\beta^f \eta_s^f$) are identified by a simple OLS-type regression of the budget shares on log expenditure. Furthermore, the slope coefficient on the Engel curve for children's clothing ($\beta^a \eta_s^a + \beta^b \eta_s^b$) is identified. This coefficient contains two unknowns. I can then use that resource shares sum to one to identify the resource shares for foster and non-foster children. A formal proof for composite households is provided in the appendix in Section 1.D.1.

Figure 1.1: Children's Clothing Engel Curves by Household Composition



Notes: In Figure 1.1a, the slope of the children's Engel curve in the composite household (1 F 1 NF) is more similar to the foster only household (2 F 0 NF) which suggests that in the composite household, the foster child is allocated more of the budget. The opposite is true in Figure 1.1b.

The intuition for this method can be understood using the graphical example in Figure 1.1, which presents children's clothing Engel curves for three household types, each with two children: (1) a foster only household "2 F 0 NF", (2) a non-foster only household "0 F 2 NF", (3) a composite household "1 F 1 NF". The slope of the children's clothing Engel curve in the composite household ($\beta^a \eta_{11}^a + \beta^b \eta_{11}^b$) is identified and the parameters β^a and β^b are known from the one-child-type households. If this slope is more similar to the slope of the children's clothing Engel curve in the foster only household ($\beta^a\eta^a_{20}$) as opposed to the one in the non-foster household ($\beta^b\eta^b_{02}$), then that would suggest the parents are placing more weight on the foster child's preferences for clothing. Placing more weight on a specific child's preferences for clothing implies that that child is given a larger share of the budget. This case is demonstrated in Figure 1.1a. If instead the children's clothing Engel curve was more similar to the children's clothing Engel curve in the non-foster only household as in Figure 1.1b, then the non-foster child is allocated more of the budget. This exercise is possible since I assume preferences for clothing are similar across the household compositions.

3.2.2 Approach 2: An Extension of SAP Identification

Without private assignable goods for foster and non-foster children, I again rewrite the Engel curves for foster and non-foster child clothing in system (1.7) as a single Engel curve for children's clothing, and assume SAP (i.e., $\beta^t_s = \beta_s$):

$$\begin{aligned} W^m_s &= \eta^m_s \; [\delta^m_s + \beta_s \ln(\eta^m_s)] \; + \; \eta^m_s \beta_s \ln y \\ W^f_s &= \eta^f_s \; [\delta^f_s + \beta_s \ln(\eta^f_s)] \; + \; \eta^f_s \beta_s \ln y \\ W^c_s &= s_a \eta^a_s \; [\delta^a_s + \beta_s \ln(\eta^a_s)] \; + \; s_b \eta^b_s \; [\delta^b_s + \beta_s \ln(\eta^b_s)] \\ &\quad + \; \ln y \; (s_a \eta^a_s \beta_s + s_b \eta^b_s \beta_s) \end{aligned} \tag{1.9}$$

This system of equations is identical to system (1.8) except now the shape parameter β is allowed to vary with the household type s, but not the person type t. Resource shares are still identified in the one-child-type households. To see how the order condition is satisfied, note that for each household type there are three resource shares (η^m_s, η^f_s, and either η^a_s or η^b_s) and a single preference parameter β_s that need to be identified. Moreover, there are four equations: three Engel curves, and the restriction that resource shares sum to one. With four equations and four unknowns, resource shares are identified for each one-child-type household.

Moving to the composite households, it is easy to see how identification fails. For each household type, there are five unknowns; four resource shares (both η_s^a and η_s^b are now nonzero) and again a single preference parameter β_s. However, the number of equations is still four, so the order condition is no longer satisfied. It is important understand why the SAP restriction fails here, but the SAT restriction does not. With the SAT restriction, as the number of household types increases, the number of preference parameters β^t does not change. However, with the SAP restriction, there is a different β_s for each household type, and therefore as the number of household types increases, so too does the number of preference parameters that need to be identified.

The SAP restriction is easier to estimate than the SAT restriction so I now introduce several new model assumptions to make the SAP restriction employable. To do this, I add structure to the model by introducing additional restrictions which limit how foster and non-foster child resource shares vary by household type. Restriction 1 is given below:

$$\frac{\eta^a_{s_{a0}}}{\eta^a_{s_{a+1,0}}} = \frac{\eta^a_{s_{ab}}}{\eta^a_{s_{a+1,b}}} \text{ and } \frac{\eta^b_{s_{0b}}}{\eta^b_{s_{0,b+1}}} = \frac{\eta^b_{s_{ab}}}{\eta^b_{s_{a,b+1}}} \tag{1.10}$$

where the household type is now given as s_{ab} to explicitly indicate the number of foster and non-foster children present. In words, this restriction requires that (1) the ratio of foster child resource shares in households with s_a and s_{a+1} foster children is independent of the number of non-foster children present; and (2) the ratio of non-foster child resource shares in households with s_b and s_{b+1} non-foster children is independent of the number of foster children present. For both equations, the left-hand-side is identified from the one-child-type households, which are used to identify the composite households on the right-hand-side.

I do not restrict the levels of foster child resource shares to be a specific value, only that the ratio of foster child resource shares in two different household types be independent of the number of non-foster children present. This ratio is assumed to be the same whether or not there are zero, one, or two, non-foster children are present, which greatly reduces the number of parameters that need to be identified.

Next, I make an additional assumption, Restriction 2, relating to composite households with one of each child type:

$$\frac{\eta^a_{s10}}{\eta^a_{s11}} = \frac{\eta^b_{s01}}{\eta^b_{s11}} \tag{1.11}$$

This restriction states that the degree of unequal treatment *within* a household with one of each child type is proportional to the degree of unequal treatment *across* households with one foster child or one non-foster child. With both restrictions, I identify how resource shares vary across household sizes in the one-child-type households, and assume resource shares behave in a similar way in the composite households. I comment on the validity of these restrictions in Section 5.

With these additional model restrictions, resource shares are now identified in the composite households. I limit my attention to the following household types: $s \epsilon \{s_{11}, s_{21}, s_{12}, s_{22}\}$. To see that the order condition is satisfied, note that with three Engel curves for each household type and four household types, there are twelve Engel curves in total. And again, for each household type resource shares sum to one. This results in four additional equations. Finally, Restriction 1 generates four additional equations and Restriction 2 leads to one additional equation, resulting in a system of twenty-one equations in total. In terms of unknowns, with four household types, and four resource shares for each household type, there are sixteen resource shares that need to be identified. For each household type, there is a preference parameter β_s that needs to be identified (four total). This results in twenty unknowns, so the order condition is satisfied.[25] A proof of the rank condition is provided in the appendix in Section 1.D.2.

In summary, both identification approaches use the one-child-type households to help identify the model parameters in the composite households. In Approach 1, preferences for clothing in the composite households are assumed to be similar to

[25]For the household type s_{22}, I restrict both $\eta^a_{22} = \frac{\eta^a_{20} \times \eta^a_{12}}{\eta^a_{10}}$ and $\eta^b_{22} = \frac{\eta^b_{02} \times \eta^b_{21}}{\eta^b_{01}}$, however only one of these two ratios needs to be assumed for identification.

preferences in the one-child-type households, but resource shares are allowed to vary considerably across household types. On the other hand, in Approach 2, preferences for clothing are allowed to vary flexibly across household types, but the way in which resource shares vary across composite household types is restricted using what can be identified from the one-child-type households.

4 Application: Child Fostering in Malawi

4.1 Background

Child fostering, or kinship care, is the practice of sending one's biological children to live with close relatives. I use a broader definition of foster children to include all individuals age 14 and under who are living in households away from both of their parents. This definition includes children in kinship care, but also orphans. Child fostering rates vary by country and are highest in West African societies (Grant and Yeatman (2012)). In Malawi, fostering is also quite common with 17 percent of households having a foster child.[26] Figure 1.2a presents the percentage of children fostered by age in Malawi (the green solid line). Overall, 12.5 percent of children are fostered (Malawi Integrated Panel Survey 2013), and fostering rates are increasing with age. The red and blue lines show the number of children living away from their father and mother, respectively.[27] Figure 1.2b displays orphan rates by age. I use the UNICEF definition of "orphan", defined as any child who has lost at least one parent. A double orphan is a child who has lost both parents, and a maternal or paternal orphan is a child who has lost either their mother or father. By definition, double orphans are foster children. Comparing Figure 1.2a with Figure 1.2b demonstrates that the majority of foster children are not double orphans, suggesting orphanhood is not the primary cause of fostering.

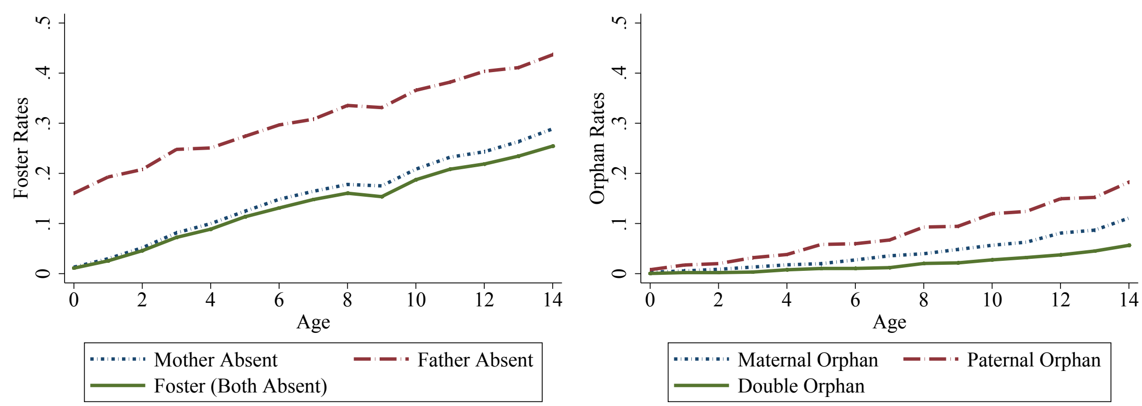
The literature divides foster children into two categories: those who are fostered for voluntary reasons, and those who are not (Serra (2009)). Non-voluntary, or crisis

[26] Grant and Yeatman (2012) use DHS data to examine the prevalence of fostering and orphanhood across sub-Saharan African countries.

[27] Fathers are more likely than mothers to live away from their children, potentially due to migration for work, or the AIDS epidemic.

fostering occurs when the child is orphaned, or has parents who are ill and unable to care for their child. Non-voluntary fostering has become substantially more common as a result of the AIDS epidemic. Voluntary, or purposive child fostering occurs when the child's parents voluntarily send the child to another household. There are a myriad of reasons parents may choose to do this: to provide educational access for the child, to strengthen kinship networks, to increase fertility, to reallocate child labour across households, or due to agricultural shocks.[28] Children are also often fostered as a result of their parents divorce and subsequent remarriage (Grant and Yeatman (2014)). This is especially prevalent in Malawi as almost half of all marriages end in divorce, with remarriage rates being equally high (Reniers (2003)).

Figure 1.2: Foster and Orphan Status by Age



(a) (b)

Notes: Malawi Integrated Panel Survey 2013. The sample includes all children age 14 and under. Foster children are individuals living in households away from both of their biological parents. Figure 1.2a presents the mean number of children fostered by age. Figure 1.2b presents the mean number of children orphaned by age.

Data limitations prevent me from examining in detail the reasons households foster children, as I only observe the receiving household. With additional data, I would be able to analyse both how foster children are treated within the household, *and* whether the reason for fostering affects foster child treatment. Without additional data, I am unable to determine whether, for example, children fostered due to nega-

[28]See Ainsworth (1995), Akresh (2009), and Serra (2009) for a detailed analysis of why households foster children.

tive agricultural shocks are treated differently than children fostered for child labour related reasons. I can however differentiate between children who are involuntarily fostered due to orphanhood and those who are not, and I find this distinction matters for foster child treatment.

There are several reasons why foster children may be treated worse than non-foster children. First, parents are likely to be more altruistic towards their own biological children. This theory, known as Hamilton's Rule (Hamilton (1964)), hypothesizes that altruism is increasing in relatedness; parents care more for their children relative to their nephews and nieces and they care more about their nephews and nieces than their neighbor's children. This theory has a basis in evolutionary biology and is sometimes referred to as "inclusive fitness". Hamilton's Rule has direct implications in the context of child fostering since children who are more closely related to their caregivers should experience better access to education, lower levels of child labour, and a higher share of household consumption. I test the implications of Hamilton's Rule in this study. The second reason for unequal treatment is related to the parent's expectation of old age care. Specifically, parents may invest more in children that they believe will care for them in old age (Becker (1992)). If adult children primarily support their biological parents, then parents may be inclined to favor non-foster children. Unfortunately, I am unable to test this hypothesis given the available data.

4.2 Data

I use the Malawi Integrated Households Survey (IHS3) and the Malawi Integrated Panel Survey. The IHS3 consists of 12,288 households surveyed in 2010, of which, 4,000 were resurveyed in 2013. Both are nationally representative household surveys and contain detailed information on individual education, employment, migration, health, and other demographic characteristics as well as household-level expenditure data. I rely primarily on the expenditure module in the estimation of the structural model.

From the survey, I can determine whether or not each child's parents are present in the household, and if not, whether their parents are living or dead. This allows me to identify both foster children and orphans.

Identifying resource shares requires detailed household expenditure data, and in particular, expenditure on men's, women's, and children's clothing. In both surveys, households are asked their expenditure on different categories (shirts, shoes, pants, etc.) of men's, women's, children's clothing, which I use to construct the corresponding budget shares. Because the model is estimated using Engel curves, price data is not necessary. I account for heterogeneity across households using data on the age, orphan status, education, health status, and gender of the households men, women, foster, and non-foster children. Other household-level variables include an indicator for whether the household is located in an urban or rural area, an indicator for residence in a matrilineal village, and region indicators.

From the data, I select a sample of 10,763 households. For ease of estimation, I exclude households that have less than one or more than four men and women, or less than one or more than five children. I also exclude households that are in the top or bottom percentile of expenditure to eliminate outliers. Households are dropped if they are missing information on any of the covariates listed in Table 1.1. Sample sizes for each household type are provided in the appendix in Table 1.10.

Table 1.1 reports descriptive statistics for the estimation sample. Households have on average 5.27 individuals. The average age of foster children (9.26) is significantly higher than that of non-foster children (5.80). This is consistent with child labour and education being reasons households foster children. Roughly 37 percent of foster children have lost at least one parent, indicating a majority of foster children are voluntarily sent away by their biological parents. Households in Malawi are very poor, with the average real annual per capita household expenditure equal to 126,580 MWK (approximately 1,147 US$).[29] Lastly, households spend a large fraction of their income on food (62 percent), which consistent with the high level of poverty in Malawi.

[29]The median per capita household expenditure is considerably lower at 871 US$.

Table 1.1: Descriptive Statistics

	Mean	Std. Dev.	Min	Max	Sample Size
Household Characteristics					
Household Size	5.273	1.712	3	13	10,763
Men	1.394	0.713	1	4	10,763
Women	1.344	0.646	1	4	10,763
Children	2.580	1.254	1	5	10,763
Non-Foster	2.305	1.370	0	5	10,763
Foster	0.275	0.696	0	5	10,763
Per Capita Total Expenditures (1000s MWK)	126.58	106.58	145.63	1,266.38	10,763
Men's Clothing Budget Shares	0.006	0.014	0	0.142	10,763
Women's Clothing Budget Shares	0.009	0.016	0	0.139	10,763
Child's Clothing Budget Shares	0.009	0.017	0	0.149	10,763
Food Budget Shares	0.625	0.137	0.077	0.963	10,763
Preference Factors					
Year=2010	0.744	0.436	0	1	10,763
Foster Child Age	9.255	3.343	0	14	1,933
non-Foster Child Age	5.803	3.397	0	14	9,864
Proportion Orphaned of Foster Children	0.376	0.467	0	1	1,933
Proportion One Parent Absent of Non-Foster Children	0.139	0.330	0	1	9,864
Proportion Female of non-Foster	0.502	0.365	0	1	9,864
Proportion Female of Foster	0.554	0.447	0	1	1,933
Average Age Women	30.560	10.831	15	99	10,763
Average Age Difference (Men-Women)	1.96	12.658	-77	60	10,763
Education Men	6.666	3.719	0	14	10,763
Education Women	5.180	3.646	0	14	10,763
Share Women Age 15-18	0.077	0.183	0	1	10,763
Share Men Age 15-18	0.114	0.247	0	1	10,763
Rural	0.804	0.397	0	1	10,763
Matrilineal Village	0.608	0.488	0	1	10,763
North	0.200	0.400	0	1	10,763
Central	0.369	0.483	0	1	10,763
South	0.431	0.495	0	1	10,763

Notes: Households with 1-4 men and women, and 1-5 children. Children are age 14 or younger. Malawi Third Integrated Household Survey and Integrated Household Panel Survey.

4.3 Estimation

To estimate the model, I add an error term to the clothing Engel curves for men, women, and children. Since the error terms of the Engel curves are likely to be correlated across equations, the system is estimated using Non-linear Seemingly Unrelated Regression. To match the data used in the empirical analysis, I now explicitly account for households with multiple men and women with s_f and s_m denoting the number of women and men, respectively.

$$W_s^m = s_m \eta_s^m [\delta_s^m + \beta_s^m \ln(\eta_s^m)] + s_m \eta_s^m \beta_s^m \ln y + \epsilon_m$$

$$
\begin{aligned}
W_s^f =& s_f \eta_s^f [\delta_s^f + \beta_s^f \ln(\eta_s^f)] + s_f \eta_s^f \beta_s^f \ln y + \epsilon_f \qquad (1.12) \\
W_s^c =& s_a \eta_s^a [\delta_s^a + \beta_s^a \ln(\eta_s^a)] + s_b \eta_s^b [\delta_s^b + \beta_s^b \ln(\eta_s^b)] \\
&+ (s_a \eta_s^a \beta_s^a + s_b \eta_s^b \beta_s^b) \ln y + \epsilon_c
\end{aligned}
$$

The objects of interest are the resource shares for foster and non-foster children, given by η_s^a and η_s^b, respectively. The estimation allows for considerable heterogeneity as each parameter is a linear function of the household characteristics provided in Table 1.1. To estimate how resource shares differ by household composition, I include indicator variables for household types in the parameterization of the foster and non-foster child resource share functions. I therefore omit constant terms, as those are already captured by the household type indicators. Resource shares for foster children are then parameterized as follows:

$$
\eta_{s_{ab}}^a = \underbrace{\Big(\sum_{i=0}^{5} \sum_{j=0}^{5} \eta_{s_{ij}}^a I\{s_{ab} = s_{ij}\} \Big)}_{\text{Household type indicators}} + \mathbf{X}'\gamma, \quad 1 \leq i + j \leq 5 \qquad (1.13)
$$

where the first set of terms are the indicators for household types. The vector of household characteristics is given by X. Resource shares for non-foster children are parameterized similarly. For men and women, I assume that their resource shares increase linearly in the number of men, women, foster, and non-foster children in the household.[30]

To identify resource shares, I must impose the parameter restrictions discussed in either Approach 1 or 2. With Approach 1, the slope preference parameter is assumed to be the same across household types ($\beta_s^t = \beta^t$). With Approach 2, the slope preference parameter is assumed to be similar across people ($\beta_s^t = \beta_s$), and Restrictions

[30] This assumption is for computational reasons. Determining household types by the number of men and women in the household, in addition to the number of foster and non-foster children, would result in a significant increase in the number of parameters needed to be estimated. Calvi (2016) parameterizes resource shares for men, women, and children this way. For robustness, I include indicators for the number of men and women in the parametrization of men's and women's resource shares and the results are unaffected.

1 and 2 are substituted directly into the resource share functions. The parameter restrictions for foster child resource shares are provided below:

Restriction 1: $\eta^a_{41} = \frac{\eta^a_{40} \times \eta^a_{11}}{\eta^a_{10}}$, $\eta^a_{31} = \frac{\eta^a_{30} \times \eta^a_{11}}{\eta^a_{10}}$, $\eta^a_{21} = \frac{\eta^a_{20} \times \eta^a_{11}}{\eta^a_{10}}$, $\eta^a_{22} = \frac{\eta^a_{20} \times \eta^a_{12}}{\eta^a_{10}}$

$\eta^a_{32} = \frac{\eta^a_{30} \times \eta^a_{12}}{\eta^a_{10}}$, $\eta^a_{23} = \frac{\eta^a_{20} \times \eta^a_{13}}{\eta^a_{10}}$

Restriction 2: $\eta^a_{11} = \frac{\eta^b_{11} \times \eta^a_{10}}{\eta^b_{01}}$

Restriction 1 is also imposed for non-foster children. While the above estimation strategies are viable, both Approaches 1 and 2 have weaknesses in terms of precisely identifying resource shares. In regards to the SAT restriction used in Approach 1, DLP note that this preference restriction appears to perform considerably worse than the SAP restriction because identification depends heavily on the assumption that resource shares sum to one, as opposed to something like a log sum. In this context, where I am further weakening the data requirements, the effectiveness of this approach is unsatisfactory.[31] However, using the second approach may also perform poorly as there are many household types, and identifying a different preference parameter β_s for each household type is difficult. I therefore take an intermediate approach. I impose SAT, while simultaneously restricting how resource shares vary across household types using Restrictions 1 and 2.[32] This combined approach has the benefit of being empirically tractable, while not overly restrictive. This is the preferred specification and the one used in the results that follow unless noted otherwise. For robustness, I estimate other specifications that assume both SAT and SAP ($\beta^t_s = \beta$), while also imposing Restrictions 1 and 2 to strengthen identification.

Lastly, I would ideally like to estimate resource shares separately for orphaned and non-orphaned foster children. However, given the small number of orphans in the sample, this is infeasible. Instead, I include the proportion of foster children who are orphaned as a covariate of the resource share functions. Moreover, in some

[31]See DLP for a more detailed discussion of the weaknesses of identification using the SAT restriction. See Tommasi and Wolf (2016) for a more general discussion of potential estimation complications with the DLP identification method.

[32]To further improve precision I restrict $\beta^f = \beta^m$. I fail to reject the hypothesis that these parameters are equal.

specifications I interact the proportion of foster children who are orphaned with other covariates, such as gender and an indicator for rural residence. This allows foster child resource shares to vary somewhat flexibly with the share of foster children who are orphaned.

4.4 Results

Figure 1.3 presents estimates for the predicted resource shares for foster and non-foster children ($\hat{\eta}_s^a$ and $\hat{\eta}_s^b$). The resource shares are *per child*. The solid bars denote foster child resource shares, and the line-patterned bars denote non-foster child resource shares.[33] Each quadrant corresponds to a different household size, defined by the number of children in the household. Within each quadrant, predicted resource shares for foster and non-foster children are given by household composition, which is determined by the number foster and non-foster children present, where for example, "1 NF 0 F" indicates a household with 1 non-foster child and 0 foster children. The motivation for this grouping of the results is that, if all children are treated equally, then foster and non-foster child resource shares should not vary for a given household size. The predictions are made for a reference household, which I define as a household with one man, one woman, and all other covariates set to their median value.[34][35] The brackets are the 95 percent confidence intervals of the predicted values.

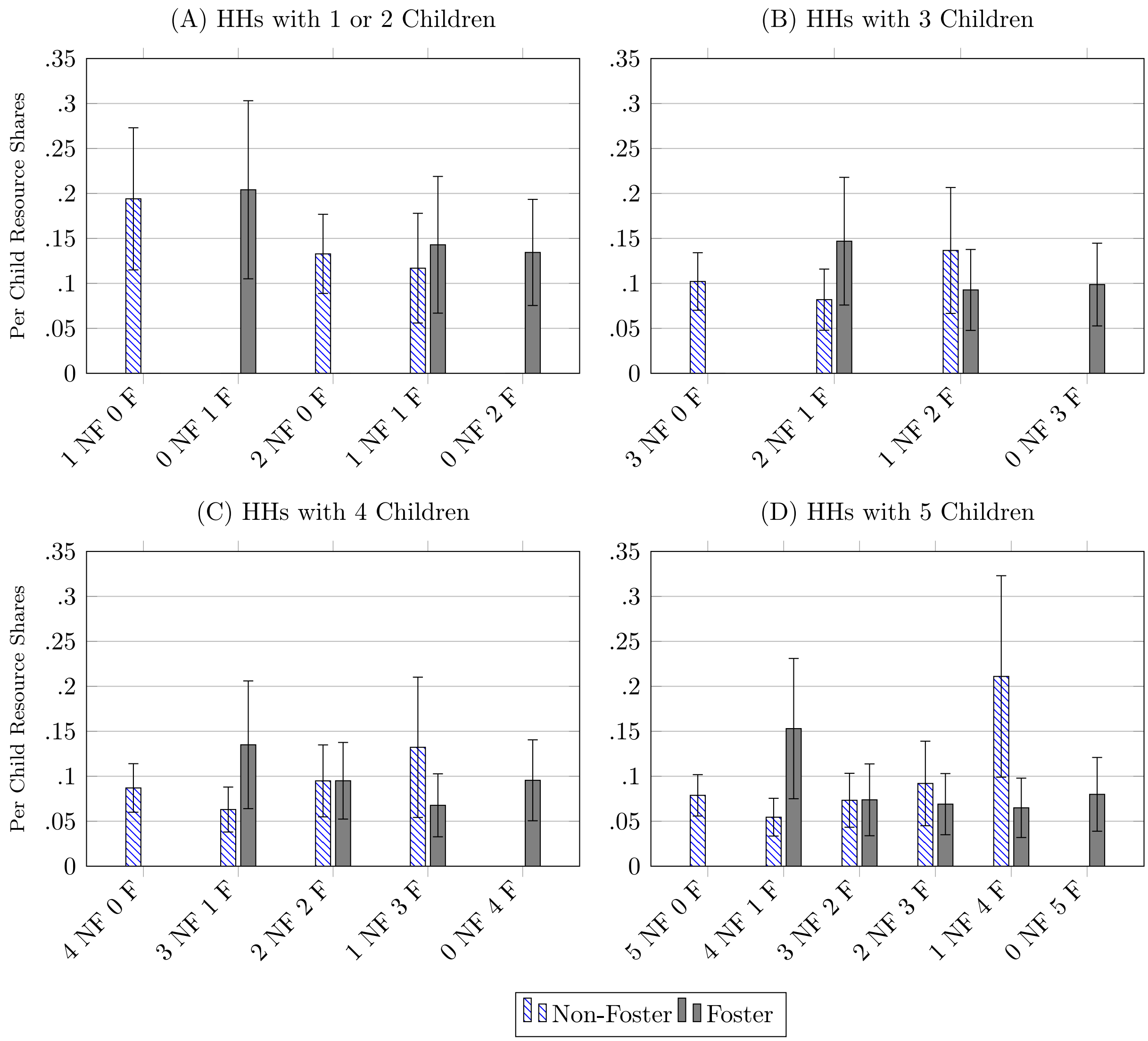
Panel A of Figure 1.3 provides the predicted resource shares for reference households with one or two children. For households with one non-foster child, and zero foster children ("1 NF 0 F"), the non-foster child consumes 19.4 percent of the household budget. Similarly, for households with one foster child, and zero non-foster children ("0 NF 1 F"), the foster child is allocated roughly 20.4 percent of the household budget. This provides little evidence of discrimination. Panels B, C, and D present the results for households with three, four, and five total children respectively, and again, the

[33]Resource shares for men and women are also estimated, but are omitted to facilitate the presentation of the more relevant results. See Figure 1.6 in the appendix for the results for men and women.

[34]Instead of using the median value for foster and non-foster child age, I set both to seven to make the predicted resource shares more comparable.

[35]Using mean values for the predictions instead of median values does not meaningfully affect the results.

Figure 1.3: Predicted Resource Shares: Reference Household



Note: Malawi Third Integrated Household Survey and Integrated Household Panel Survey. Robust standard errors. The brackets are the 95 percent confidence intervals. Each quadrant presents non-foster and foster child resource shares for a different household size defined by the number of children. Within each quadrant, foster, and non-foster child resource shares are presented by household type which is defined by the number of foster and non-foster children, respectively. A reference household is a household with 1 man, 1 woman, and all other covariates at their median value, excluding foster and non-foster child age, which are both set to 7.

results do not demonstrate a systematic pattern of discrimination against foster children. It should be emphasized that this lack of discrimination is for households with all covariates at their median value, and that the median foster child is non-orphaned. I therefore examine how heterogeneity in these covariates, such as orphanhood, relate to foster child treatment.

Table 1.2 presents the parameter estimates of the determinants of resource shares for foster and non-foster children. I omit the coefficients on the household composition variables as those are displayed in Figure 1.3.[36] The results provide some evidence that child resource shares are increasing in age, at least for foster children. Living in a matrilineal village is beneficial to foster children, with foster child resource shares being 3.2 percentage points higher if the majority of households in the village follow matrilineal customs. Greater involvement of female matrilineal relatives in the child's upbringing could explain this finding. Lastly, the results provide no evidence of gender discrimination among foster or non-foster children. This result is counter to what DLP find.

Orphanhood does seem to matter considerably for child welfare, as the results suggest that orphaned foster children are treated significantly worse than non-orphaned foster children.[37] Specifically, if all foster children are orphaned, foster child resource shares are 4.2 percentage points lower than they would be if all foster children were non-orphaned. This translates into orphaned children consuming roughly 76.9 percent of what non-orphaned foster children consume.

To better illustrate the importance of orphanhood in foster child treatment, Figure 1.4 presents the predicted resource shares for households where the foster children are orphaned. The earlier predicted resource shares in Figure 1.3 were for households with non-orphaned foster children, as the median foster child is non-orphaned. To facilitate a comparison between non-orphaned and orphaned foster children, I reproduce the results from Figure 1.3 for households with four children in panel (A) while

[36] The actual parameter estimates of the household type indicators are provided in the appendix in Table 1.11. Because most of the covariates are demeaned, the indicators for the household type variables are largely similar to the predicted values found in Figure 1.3.

[37] An orphaned foster child is a foster child who has lost at least one parent.

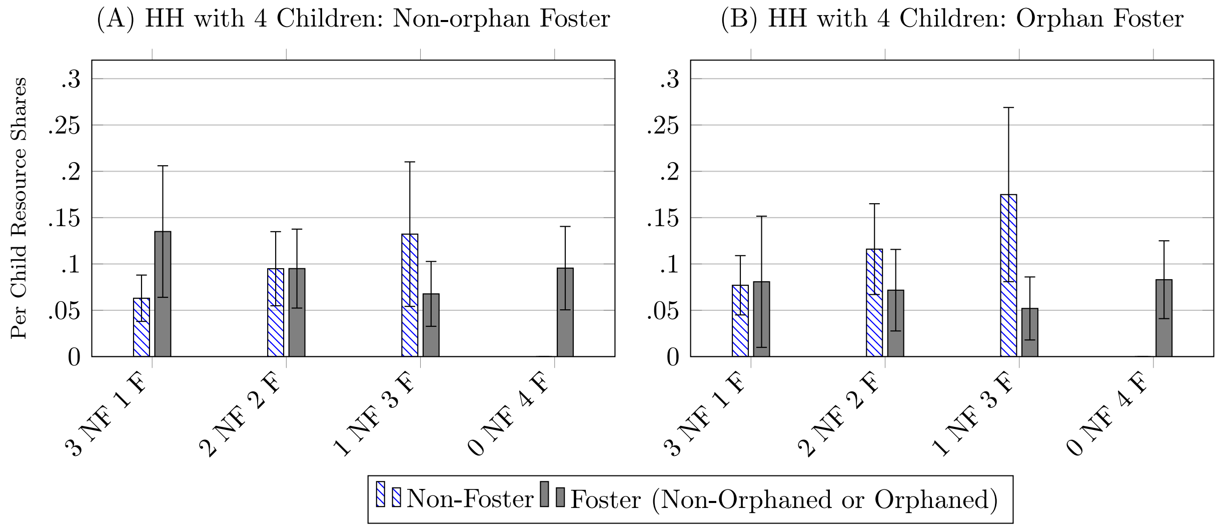
Table 1.2: Determinants of Resource Shares

	non-Foster Children (1) NLSUR	Foster Children (2) NLSUR
North	0.0207	0.0347
	(0.0212)	(0.0226)
Central	-0.00619	-0.00295
	(0.0146)	(0.0156)
Year=2010	-0.0242*	-0.0166
	(0.0143)	(0.0137)
Average Age non-Foster	0.625	-0.0422
	(0.774)	(0.893)
Average Age non-Foster^2	0.00965	0.0266
	(0.0568)	(0.0652)
Average Age Foster	-1.333	2.618**
	(1.686)	(1.225)
Average Age Foster^2	0.0864	-0.125*
	(0.0947)	(0.0759)
Proportion of Fostered Orphaned	0.0430	-0.0427**
	(0.0278)	(0.0206)
Proportion of Non-Fostered One Parent Absent	0.00529	-0.000158
	(0.0233)	(0.0208)
Fraction Female non-Foster	-0.0292	0.0167
	(0.0192)	(0.0186)
Fraction Female Foster	-0.0105	0.0270
	(0.0260)	(0.0280)
Average Age Women	0.488	0.0699
	(0.331)	(0.349)
Average Age Women^2	-0.00725*	-0.00300
	(0.00431)	(0.00388)
(Average Age Men - Average Age Women)	0.121	0.0360
	(0.0759)	(0.0810)
(Average Age Men - Average Age Women)^2	0.000616	0.000789
	(0.00234)	(0.00164)
Average Education Men	0.00464**	0.00254
	(0.00225)	(0.00327)
Average Education Women	-0.00472*	-0.00310
	(0.00253)	(0.00264)
Rural	-0.00775	-0.00306
	(0.0172)	(0.0149)
Share of Adult Women Age 15-18	0.0587	-0.0227
	(0.0514)	(0.0426)
Share of Adult Men Age 15-18	0.000428	0.00950
	(0.0310)	(0.0283)
Matrilineal Village	0.0151	0.0323*
	(0.0163)	(0.0174)
N	10,763	
Log Likelihood	92,338	

Notes: Malawi Third Integrated Household Survey and Integrated Household Panel Survey. The sample includes all households with 1-4 men and women, and 1-5 children. Robust standard errors in parentheses. Age variables are divided by 100 to ease computation. The education and age variables are demeaned. South Malawi is the omitted region. Coefficients on the household composition indicators are omitted for conciseness. * p<0.1, ** p<0.05, *** p<0.01

in panel (B), I present the predicted resource shares for non-foster and orphaned foster children. The results illustrate a clear pattern of unequal treatment of orphaned

Figure 1.4: Predicted Resource Shares by Presence of Orphans



Notes: Malawi Third Integrated Household Survey and Integrated Household Panel Survey. Robust standard errors. The brackets are the 95 percent confidence intervals. Panels A and B present predicted foster and non-foster child resource shares for households with four children. In Panel A, all covariates are set to the median value of households with no orphaned foster children present. Panel B sets all covariates to the median value of households with orphaned foster children present. Predicted values are computed assuming households have one man and one woman. Comparing panel A with panel B demonstrates differences in treatment for foster children by orphan status.

foster children relative to non-fostered children. For example, focusing on households with two non-foster children and two foster children (“2 NF 2 F"), when the foster children are non-orphaned, the predicted per child resource shares for non-foster and foster children are 9.4 and 9.5 percent respectively. However, when the foster children are orphaned, the predicted per child resource shares are now 11.6 and 7.7 percent for non-foster and foster children. Similar differences are found across the different household types.

The above results are estimated assuming preferences for assignable clothing are similar across households types (Approach 1), and that the way in which resource shares for foster children vary across household types is independent of the number of non-foster children present, and vice vera (Restrictions 1 and 2). To examine the robustness of these results, I estimate the model using several alternative identification assumptions. Table 1.3 presents the results to each different specification. In the interest of conciseness, I limit the displayed parameter estimates to several key household characteristics and household type indicators. The four different combinations of assumptions are as follows: (1) SAT; (2) SAT and Restrictions 1 and 2 (this specification is what is used in the main analysis); (3) SAP and Restrictions 1 and 2; (4) SAT, SAP, and Restrictions 1 and 2. Columns (1a) - (4a) present the results for non-foster children, and columns (1b) - (4b) do the same for foster children.

The results are reassuringly similar across specifications. As expected, estimating the model assuming only SAT (Approach 1) leads to large standard errors. Moreover, using this approach requires dropping several household types in order for the system to converge. Across specifications, none of the parameter estimates on the household type indicators are statistically different, and overall are quite similar to each other. Looking at the household characteristics, the results are again for the most part consistent. Orphanhood is only statistically significant when the SAP restriction is not imposed, though the magnitude is again similar across all specifications.

Neither SAT or SAP on their own are rejected by the data. However, the preferred results are presented in columns (2a) and (2b) of Table 1.3, in which I impose SAT and Restrictions 1 and 2, but not SAP. This combination of assumptions has the advantage being relatively flexible (preferences are allowed to be different across people), while simultaneously having standard errors that are significantly more precise than the results presented in columns (1a) and (1b) where Restrictions 1 and 2 are not imposed.

Table 1.3: Determinants of Resource Shares: Estimates by Identification Assumptions

	Non-Foster Children				Foster Children			
Preference Restriction Restrictions 1 and 2	SAT No	SAT Yes	SAP Yes	SAP+SAT Yes	SAT No	SAT Yes	SAP Yes	SAP+SAT Yes
	(1a)	(2a)	(3a)	(4a)	(1b)	(2b)	(3b)	(4b)
Household Type Indicators								
3 Non-Foster 0 Foster	0.299***	0.336***	0.324***	0.325***				
	(0.0581)	(0.0567)	(0.0401)	(0.0411)				
2 Non-Foster 1 Foster	0.134**	0.194***	0.181***	0.180***	0.160	0.121***	0.160***	0.152***
	(0.0564)	(0.0425)	(0.0352)	(0.0359)	(0.100)	(0.0343)	(0.0374)	(0.0366)
1 Non-Foster 2 Foster	0.139	0.167***	0.152***	0.151***	0.161***	0.159***	0.198***	0.189***
	(0.0931)	(0.0417)	(0.0368)	(0.0379)	(0.0588)	(0.0429)	(0.0418)	(0.041)
0 Non-Foster 3 Foster					0.255***	0.270***	0.342***	0.329***
					(0.0893)	(0.0697)	(0.0686)	(0.0696)
Covariates								
Average Age non-Foster	1.681**	0.625	2.358***	2.373***	-0.0803	-0.0422	-0.265	-0.191
	(0.659)	(0.774)	(0.836)	(0.866)	(0.952)	(0.893)	(0.958)	(0.914)
Average Age non-Foster^2	-0.0540	0.00965	-0.120*	-0.120*	0.0235	0.0266	0.0282	0.0241
	(0.0475)	(0.0568)	(0.0638)	(0.0660)	(0.0648)	(0.0652)	(0.0698)	(0.0671)
Average Age Foster	-1.715	-1.333	-0.856	-0.622	3.291**	2.618**	1.820	2.247
	(1.868)	(1.686)	(2.187)	(2.315)	(1.457)	(1.225)	(1.939)	(1.755)
Average Age Foster^2	0.0952	0.0864	0.0622	0.0508	-0.163*	-0.125*	-0.0951	-0.122
	(0.0990)	(0.0947)	(0.123)	(0.129)	(0.0849)	(0.0759)	(0.114)	(0.103)
Matrilineal Village	0.00299	0.0151	0.00309	0.00257	0.0257*	0.0323*	0.0238	0.0256*
	(0.0122)	(0.0163)	(0.0159)	(0.0161)	(0.0151)	(0.0174)	(0.0154)	(0.0156)
Proportion of	0.0349	0.0430	0.0371	0.0373	-0.0438*	-0.0427**	-0.0398	-0.0388
Fostered Orphaned	(0.0222)	(0.0278)	(0.0348)	(0.0350)	(0.0240)	(0.0206)	(0.0321)	(0.0287)
N	10,433	10,763	10,763	10,763	10,433	10,763	10,763	10,763
Log Likelihood	86,945	92,338	92,259	92,245	86,945	92,338	92,259	92,245

Notes: Malawi Third Integrated Household Survey and Integrated Household Panel Survey. The sample includes all households with 1-4 men and women, and 1-5 children. Robust standard errors in parentheses. Coefficients on the household type are not per child. Age variables are divided by 100 to ease computation. Coefficients on the household composition indicators are omitted for conciseness. Estimates for certain household types and preferences factors are omitted for conciseness. Columns (1a-4a) and (1b-4b) differ by identification assumptions. In columns 1a and 1b, I impose SAT. In columns 2a and 2b, I impose SAT and Restrictions 1 and 2. In columns 3a and 3b, I impose SAP and Restrictions 1 and 2. In columns 4a and 4b, I impose SAT, SAP, and Restrictions 1 and 2. * p<0.1, ** p<0.05, *** p<0.01

I conduct several other robustness checks. First, I estimate a more flexible model that allows for the relationship between orphanhood and foster child treatment to vary with both gender and rural residence. These results are provided in Table 1.12. For conciseness, I again limit the parameter estimates displayed in the table to several preference factors and household type indicators. Columns (2a) and (2b) present the results for non-foster and foster children respectively, whereas columns (1a) and (1b) reproduce the main results as a point of comparison. The number of interactions make a clean interpretation of the relationship between orphanhood and foster child treatment difficult, however, the results still suggest the extent of unequal treatment between foster and non-foster children increases with the presence of orphaned foster children. I next limit the sample to only nuclear households with one man and one woman who are married.[38] Columns (3a) and (3b) or Table 1.12 display these results. While several parameter estimate differ somewhat in magnitude, none are statistically different.

4.5 Poverty Analysis

Resource shares are a desirable object to identify in part because they allow for the estimation of individual-level consumption. I can therefore use the predicted resource shares to estimate foster and non-foster child poverty rates that account for the unequal distribution of goods within the household. Importantly, everyone in the household may not be poor; it is possible for the adults to be living above the poverty line, but the children below it. Moreover, not all children need to be poor; non-foster children may be above the poverty line with the foster children below it, and vice versa. This analysis therefore differs from the more traditional approach to estimating poverty which relies on household-level measures that ignore intrahousehold inequality. In a setting where intrahousehold inequality is likely, accounting for an unequal distribution of resources is essential, and highly relevant for accurately targeting poverty programs.

[38]Only 54.7 percent of the estimation sample households consist of a single married couple with no other adult men or women present. Because of the much smaller sample size I limit the number of preference factors and household types, but the results are quantitatively similar.

I classify adults as poor using a US \$1.90 a day poverty line.[39] For children, I use several different poverty lines based on the average age of foster or non-foster children in the household. Setting a single poverty line for children abstracts from potential inequality as older children require more resources than younger children to maintain the same standard of living, and foster children tend to be significantly older than non-foster children. To determine these age-specific poverty lines, I assume that the child poverty line is proportional to the calorie requirements for children of that age relative to adults.[40] So if a six-year-old child requires half as many calories as an adult, their poverty line would be half of the adult poverty line, or US \$0.95 a day. The choice of poverty line is arbitrary, however the results are still somewhat comparable to DLP as the "average" child poverty line across all ages is roughly 60 percent of the adult poverty line, which is the child poverty line used by DLP for children of all ages.

As a point of comparison, I calculate household-level poverty rates where I assume an equal distribution of resources within the household. The household-level poverty measures use the OECD adult equivalent scale, where the number of adult equivalents in the household is given by $1 + 0.5 \times N_c + 0.7 \times (N_a - 1)$, where N_c is the number of children and N_a is the number of adults. A household is poor if per adult equivalent consumption is less than US \$1.90 a day. Since the OECD equivalence scale is somewhat arbitrary, the main focus of the poverty analysis is to examine relative levels of poverty across individuals, rather than levels of poverty.[41]

Table 1.4 presents poverty rates for individuals by household size, defined by the number of children in the household. Columns (1) - (5) provide individual poverty rates computed using the predicted resource shares. Column (6) presents the household-

[39] This is the World Bank 2011 extreme poverty line.

[40] I use the United States Department of Health and Human Services estimated daily calorie needs by age. I abstract from gender differences for children and assume adults require 2400 calories per day.

[41] Adult equivalence scales are used to account for economies of scale in household consumption. Without estimating the consumption technology function (the A - Matrix in Section 2), the individual type-specific poverty estimates cannot account for economies of scale. While the consumption technology function can in principle be identified, as in BCL, I lack sufficient price data to estimate it. As a result the household and individual levels of poverty are not directly comparable.

level poverty rates. Comparing column (6) to the individual-level poverty rates clearly illustrates that traditional household-level measures fail to identify individuals who are poor, particularly women and children. This result is consistent with DLP and recent work on using health measures to analyse the ability of household-level measures to capture individual-level poverty (Brown et al. (2016)). Moving to the individual poverty rates, I separate orphaned from non-orphaned foster children to emphasize differences in foster child treatment by orphan status. This choice is motivated by the results in Section 4.4, and as expected, orphaned foster child poverty rates are greater than non-orphaned foster child poverty. For example, comparing households with either one non-orphaned foster child, or one orphaned foster child, I find that 29.7 percent of non-orphaned foster children are poor, whereas 49.4 percent of orphaned foster children are poor.

Table 1.4: Estimated Poverty Rates by Household Size

Number of Children	Sample Size: # Households	Individual Poverty Rates					Assuming Equal Distribution
		Foster		Non-Foster	Men	Women	
		Non-Orphaned (1)	Orphaned (2)	(3)	(4)	(5)	(6)
1	2,639	0.297	0.494	0.190	0.160	0.257	0.090
2	2,840	0.404	0.494	0.326	0.153	0.305	0.119
3	2,595	0.405	0.591	0.465	0.178	0.338	0.167
4	1,777	0.384	0.692	0.563	0.240	0.378	0.225
5	912	0.424	0.731	0.632	0.240	0.437	0.261
All Households	10,673	0.385	0.597	0.467	0.182	0.327	0.167

Notes: Malawi Third Integrated Household Survey and Integrated Household Panel Survey. A household is poor if per adult equivalent expenditures are less than $1.90 a day. Individual poverty rates measure consumption as the product of predicted resource shares and total expenditure. The child poverty line is less than the adult poverty line and is determined based on the average age of foster or non-foster children in the household. The exact child poverty line is proportional to the calorie requirements for children of a given age relative to adults.

Comparing columns (1) and (3) of Table 1.4, the estimated poverty rates seem to suggest that non-orphaned foster child poverty is mostly below non-foster child poverty. However, these estimates on their own do not suggest foster children are treated better than non-foster children, since households with non-orphaned foster children tend to be wealthier. It is common for parents to foster out their children due to negative income shocks, and send them to households with the financial means to take care of additional children (Akresh (2009)). Selection into wealthier households likely occurs more frequently for non-orphaned children as they are *purposively* fostered. On the other hand, orphans are more likely to be involuntarily fostered into a household that may not have the means to care for them. This could partially explain why intrahousehold inequality is more prevalent among orphaned foster children.

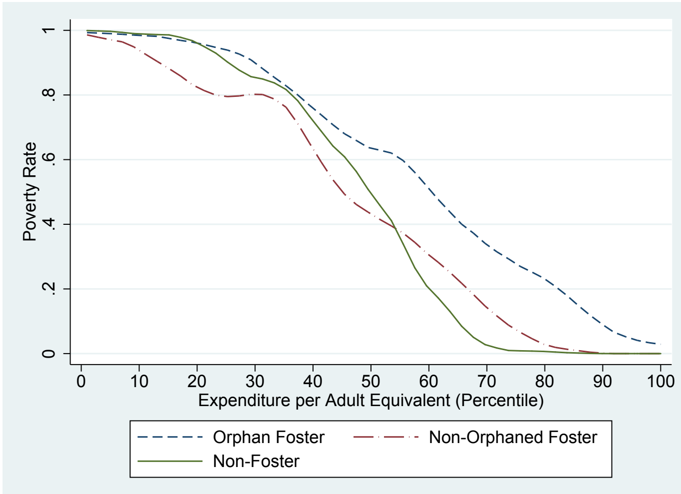
Because household-level expenditure is correlated with both individual poverty rates and the presence of foster children in the household, I present the results in a different way. I plot individual poverty rates for non-foster, orphaned foster, and non-orphaned foster children by percentiles of the per adult equivalent household expenditure distribution. These results are displayed in Figure **??**.[42] As expected, individual poverty rates decline as household expenditure increases. However, for certain levels of household expenditure, orphaned foster child poverty (the blue dashed line) is significantly higher than non-foster child poverty (the green solid line). This result suggests orphaned foster children often live below the poverty line, despite living in households that are not considered poor. In effect, household-level measures of poverty are likely to misclassify orphaned foster children as non-poor more frequently than both non-foster and non-orphaned foster children. Lastly, non-foster and non-orphaned foster poverty rates are no longer as starkly different as the results in Table 1.4 would suggest.

It is important to note that I am not making welfare statements about child fostering as an institution. Even if foster children sometimes receive a smaller share of household resources relative to other household members, the counterfactual of staying with their biological parents may result in a higher resource share, but lower total resources due to a smaller household budget.

Existing work has demonstrated that women and children are often misclassified as non-poor using household-level measures. I highlight a new population, orphaned-foster children, who are even more frequently misclassified as non-poor. These results demonstrate the importance of accounting for intrahousehold inequality when design-

[42]Figure **??** is analogous to Figure 2 in Brown et al. (2016) who plot different measures of undernutrition against percentiles of household wealth.

Figure 1.5: Individual Poverty Rates by Household Expenditure Percentile



Notes: The graph shows the proportion of different child types who are poor at each per adult equivalent household expenditure percentile. A lowess regression is used to fit the line.

ing policy. To efficiently target poverty programs, it is essential to accurately identify poor individuals, not just poor households.

4.6 Why are Orphans Treated Worse?

The previous results suggest that non-orphaned foster children are treated equally to non-foster children. However this lack of discrimination does not hold if the foster child is orphaned. I next examine one potential explanation for this unequal treatment; non-orphaned foster children have a better outside option. More specifically, since both of their biological parents are alive, non-orphaned foster children can potentially return to live with their parents. Foster children who have lost at least one parent, on the other hand, do not have that same advantage. From the perspective of Nash bargaining, non-orphaned foster children have a higher threat point. To determine the plausibility of this hypothesis, I take advantage of the panel structure of the data and compute the probability of a foster child in 2010 being in the same household in 2013. If orphaned foster children are more likely to still be present, that suggests their outside option is worse, as they are forced to remain in their current household. I assign children into four mutually exclusive groups: non-orphaned non-foster (g=1); orphaned non-foster (g=2); non-orphaned foster (g=3); orphaned foster (g=4). I then estimate the following probit regression:

$$Y_{ihsg} = \alpha + \gamma_1 O_i + \gamma_2 F_i + \gamma_3 (O_i \times F_i) + \psi_s + X_{ih}\delta + \epsilon_{ihsg} \tag{1.14}$$

where Y_{ihsg} is an indicator for whether child i in household h in region s in orphan-foster group g was present in the same household in 2013 as they were in 2010. F_i and O_i are indicators for foster and orphan status respectively. I include a vector of individual and household characteristics X_{ih} that includes child age, age squared, gender, household expenditure, residence in a rural area, and the number of men, women, male, and female siblings.

Table 1.5: Probability of Staying in Same HH by Foster and Orphan Status

	Foster Child Sample		Full Sample	
	(1)	(2)	(3)	(4)
Orphaned Foster	0.0189	0.0244***	0.0370**	0.0453***
	(0.020)	(0.009)	(0.012)	(0.017)
Non-Orphaned Non-Foster			0.0708***	0.0713***
			(0.014)	(0.017)
Orphaned Non-Foster			0.0237	0.0285
			(0.016)	(0.283)
Mean Dependent Variable	0.871	0.871	0.915	0.915
Sample Size	746	746	6,076	6,076
Region Fixed Effects	Yes	Yes	Yes	Yes
Individual Controls	Yes	Yes	Yes	Yes
Household Controls		Yes		Yes

Notes: The sample includes all children age 0-11 in 2010. The dependent variable is an indicator for whether or not the child in the 2010 sample was still in the same household in 2013. The omitted category are non-orphaned foster children. Standard errors are clustered at the region level. Individual controls include age age^2, and gender. Household controls include the number of male and female siblings age 0-6 and 7-14, the number of adult men and women, log household expenditure, and whether or not the household is in a rural area. * p<0.1, ** p<0.05, *** p<0.01

Table 1.5 presents the marginal effects of a probit regression of Equation (1.14). The omitted category in each specification is group 3: non-orphaned foster children.[43] In columns (1) and (2) the sample is restricted to only foster children, while columns (3) and (4) include both foster and non-foster children. If orphaned foster children

[43]Equation (1.14) is modified so that this is the case.

are more likely than non-orphaned foster children to remain in the same household throughout the sample period, then γ_3 should be positive. I limit the sample to children age eleven and under in 2010.[44] Column (1) shows no difference in the probability of remaining in the same household, however once variation in household characteristics is accounted for, the results suggest orphaned foster children are 2.44 percentage points more likely than non-orphaned foster children to stay. Including non-foster children in the analysis in columns (3) and (4) yields qualitatively similar estimates. Table 1.15 in the appendix further divides orphaned foster children by whether they are single orphans (maternal or paternal) or double orphans. As expected, the probability of remaining in the same household is significantly higher in magnitude for double orphans. This finding suggests that double orphans have the worst outside option of all children, which may weaken their standing withiin the household.

5 Robustness

5.1 Is Clothing a Private Good?

A key assumption of the model is that clothing is not shared across person types. This assumption means that foster children cannot share clothes with non-foster children, and vice versa.[45] While this assumption may at first seem worrisome, there are several reasons it is not too great of a concern. First, clothing includes both shoes and school uniforms, both of which can be reasonably assumed to not be shared. Secondly, foster children are typically different ages than the non-foster children within the household. Fostering is often used to balance the demographic structure of the household, both in terms of child age and gender, in order to maximise household production (Akresh (2009)). As a result, it is somewhat rare to have a foster and non-foster child of the same age and gender in a given household.

[44] The consumption analysis defines children as anyone fourteen and under. Therefore I restrict the sample to children age fourteen and under in 2013, or eleven and under in 2010. Using different age thresholds does not meaningfully affect the results.

[45] Hand-me-down clothing is not considered shared clothing. I define children's clothing expenditures to be the amount the household spends on children's clothing within the past year. Hand-me-down clothing is therefore not considered in the analysis and does not factor in to whether clothing is shared or not. It is assumed to be separable from purchased clothing.

To examine the merit of this assumption, I conduct two tests. First, I include a covariate for the age difference between foster and non-foster children in the resource share equation. If this parameter is positive, then that suggests that clothing is shared, since differently aged children would need more resources. This parameter proves to not be statistically different from zero. To be even more cautious, I drop all households with both foster and non-foster children within the following age groups from the sample: 0-3, 4-7, 8-11, and 12-14. Since foster and non-foster children in different age groups are unlikely to share clothing, I can confidently assume clothing is private in this restricted sample. The results from the restricted sample are qualitatively similar to the main results, which suggests that the privateness of clothing does not interfere with the findings of this paper. Table 1.13 in the appendix presents these results.

5.2 Are the Restrictions Valid?

In Section 3.2.2, I take the "Similar Across People" assumption on individual preferences from DLP, and demonstrate that if we restrict the way in which resource shares vary across household types, resource shares can be recovered using expenditure on private partially assignable goods. In particular, I first restrict the way in which foster child resource shares vary across household types to be independent of the number of non-foster children present, and vice versa (Restriction 1). Secondly, I assume the extent of discrimination in a composite household with one child of each type to be the same as the extent of the discrimination across two one-child-type households, each with one foster or non-foster child (Restriction 2). Whether or not these two assumptions hold is important to the reliability of this identification method.

To test the validity of these restrictions, I estimate the model assuming preferences are *similar across household types* (SAT), and test whether or not the estimated resource shares are consistent with the ratios implied by Restriction's 1 and 2. As discussed in Section 3.2.1, if I make the SAT restriction, I do not need to restrict how resource shares vary across household types. This allows for a direct test of the second approach to identification. Specifically, I test the following null hypotheses which are assumed to hold by Restriction 1: $\eta^a_{21} = \frac{\eta^a_{11}\eta^a_{20}}{\eta^a_{10}}$, $\eta^a_{31} = \frac{\eta^a_{11}\eta^a_{30}}{\eta^a_{10}}$, $\eta^b_{12} = \frac{\eta^b_{11}\eta^b_{02}}{\eta^b_{01}}$; and

Restriction 2: $\eta^b_{11} = \frac{\eta^a_{11}\eta^b_{01}}{\eta^a_{10}}$. I omit several household types because they have too few observations.[46] Overall, I consistently fail to reject the hypothesis that the restrictions hold. While the resource shares are not estimated that precisely and therefore the hypotheses are difficult to reject, the restrictions are still largely consistent with the estimated resource shares. The estimation results are presented in the appendix in Table 1.14.[47]

I next examine these restrictions in a more indirect way. Since Restriction 1 requires that resource shares vary across household types independently of the the number of foster and non-foster children present, I next ask, are *household characteristics* independent of the number of foster and non-foster children present? Stated differently, are one-child-type and composite households similar? To answer this question, I compute sample means of different household characteristics for one-child-type and composite households. If households with only foster (or non-foster) children differ from composite households over observable characteristics, that may suggest they differ in unobservable ways, which would limit the validity of the restrictions. Table 1.16 presents sample means for several household characteristics by the different household compositions.

The results are mixed; encouragingly, foster and non-foster child characteristics, such as age and gender, do not seem to vary much between one-child-type and composite households. Unfortunately, adult characteristics, such as age and education, differ substantially across one-child-type foster households and the composite households. The underlying reason for this is that households that have only foster children tend to be households where the foster children are cared for by grandparents, while in composite households foster children are typically cared for by their aunt and uncle, who have their own non-foster (biological) children. Table 1.18 presents the percentage of foster children cared for by different relatives in households with only foster children, and in households with both foster and non-foster children.

Since one-child-type and composite households do seem to differ across the entire

[46] This is one reason why identification with the SAT assumption without Restrictions 1 and 2 (Approach 1) is less than satisfactory in this context.

[47] Other results from this estimation were presented previously in Table 1.3, columns (1) and (5).

sample, I next examine if there is at least some amount of overlap among subsamples of the different household types. To do this, I select two subsamples of one-child-type households (foster only and non-foster only) that are most similar to the composite households using a propensity score matching procedure.[48] The results are presented in Table 1.17. Columns (1) and (2) compare households with only non-foster children to households with both non-foster and foster children. I do the same for foster one-child-type households in columns (3) and (4). None of the estimated means are statistically different across the matched subsamples. Then since the model does allow for observable heterogeneity in the resource share parameters, concerns regarding potential violations due to differences in composite and one-child-type households are likely minimal.

Each of the above tests of the restrictions is meant to examine the extent to which resource shares are well-behaved; do they vary across household types in a predictable way? The above results suggest that resource shares mostly do behave in such a way. Lastly, it is useful to note that in principle, these restrictions are testable with additional data. If I observed assignable goods for foster and non-foster children, I could precisely estimate the model without Restrictions 1 and 2 and compare those results to the findings recovered in this paper without having to make the additional restrictions. I leave that for future work.

5.3 Is There Selection Bias?

Foster and non-foster children are not randomly assigned into households. The decision to foster one's children, and the decision to receive a foster child is a complicated process. Furthermore, households that decide to accept a foster child may be different from households without foster children in unobservable ways that are correlated with the treatment of foster and non-foster children. For example, a household with non-foster children that refuses to take in a foster child may do so because they prefer

[48] I use nearest neighbor propensity score matching, where households are selected based on the covariates listed in Table 1.1. In comparing non-foster one-child-type households with composite households, I drop one-child-type households and match them with the full sample of composite households. When I compare foster one-child-type households with composite households, I select a subsample of similar one-child-type foster households and composite households.

to devote more resources to their own biological children.

In this paper, I do not model the fostering decision as others have done (Ainsworth (1995), Akresh (2009), Serra (2009)), but instead analyse child welfare conditional on being in a given household. In other words, I do not analyse the causal effect of living in a foster household on child treatment. Nevertheless, I briefly examine whether or not selection of children into different household types affects foster and non-foster child treatment. The primary concern is that there is a subset of one-child-type, non-foster households who are driving the results, and that these households are different in unobservable ways from the composite households. If this were true, I should not be imposing any similarity between these different household types. To determine the severity of this concern, I attempt to drop these "problem" households. I conduct a matching exercise to select a subsample of one-child-type, non-foster households that are most similar to the composite households using nearest neighbor propensity score matching.[49] I estimate the model on the subsample of one-child-type households and compare these results to the main results from Section 4.4 in Table (1.19). Columns (1) and (2) display the predicted per non-foster child resource shares for a reference household.[50] Column (1) presents the results for the full sample, while column (2) does the same for the restricted sample. Overall, there are no statistical differences between the results, suggesting that for non-foster children, selection bias is not too great of a concern.[51]

6 Conclusion

The household is in many ways a black box to economists. Understanding the inner workings of households is difficult and measuring the treatment of children within the household is far from straightforward. I build upon recent work by DLP to demonstrate how resource shares can be identified using expenditure on partially assignable

[49]See Table 1.17 columns (1) and (2).

[50]As before, I define a reference household to be a household with all covariates at their median value.

[51]I lack a sufficient number of households to proceed with a similar analysis of one-child-type foster households.

clothing. Like DLP, I rely on observing how clothing budget shares vary with household expenditure to identify resource shares. I differ in that I weaken the data requirements necessary for identification. Future work can use this methodology in other contexts where intrahousehold inequality is of interest, but assignable goods are not present in the data.

I use this new approach to measure inequality among children. While the unequal treatment of children is present in a variety of contexts, I focus on foster children in Malawi who live in situations that may leave them particularly susceptible to impoverishment. The findings of this paper demonstrate that for the most part, foster children are treated the same as other children and that extended family members are capable caretakers. However, the results suggest *orphaned* foster children are disadvantaged. I find orphaned foster child poverty is being substantially understated by poverty measures that rely on household-level measures of consumption. This result emphasizes the importance of designing government programs that target not just poor households, but also orphaned children, regardless of the poverty level of the household. Future work should connect the findings of this paper with past research on *why* children are fostered (Ainsworth (1995), Akresh (2009)). Bridging these two areas of study will help determine the underlying mechanisms that influence foster children treatment, and ultimately allow for better policy design.

The weaknesses of the unitary household framework are well known (Attanasio and Lechene (2002), Duflo (2003), Bobonis (2009)). This study adds to the growing literature that stresses the importance of thinking about individuals within the household, as opposed to the household as a single economic agent. This distinction is even more relevant where intrahousehold inequality may be present, as the results of this paper demonstrate in regards to child fostering. This project identifies a second, less emphasized, limitation of household-level studies, in that they typically ignore kinship networks. Individuals within a kinship network interact along many dimensions, with child fostering being a central component. The finding that non-orphaned foster children are treated better than orphaned foster children suggests kinship networks play a role in child welfare; having living parents in another household influences how foster children are treated. Recognising the role of extended families in child welfare is therefore critical to designing policies that help children. Future work should analyse in more depth the relationship between different types of kinship systems (matrilineal vs. patrilineal), as well as the role of different types of relatives (grandparents vs. aunts and uncles) in foster and non-foster child treatment.

APPENDIX

1.A School Enrollment and Child labor

To provide context to the consumption results, I examine intrahousehold inequality among foster and non-foster children along two other dimensions of welfare: education and child labor. As discussed in Section 4.1, education and child labor are centrally linked to why parents foster their children. In terms of education, if the household does not live close to a school, or if the nearby school is low quality, parents may send their children to live with a relative who lives in a village with better educational access. Moreover, households may be more amenable to accepting foster children if the foster children work. For example, a household with a newborn child benefits from fostering in a young teenage girl who can care for the newborn. Alternatively, if a household has a stronger than normal harvest, they may foster in children to help with the farm work. This suggests child labor may be higher among foster children.

1.A.1 Empirical Strategy

Unlike consumption, both school enrollment and work hours are observable at the individual level using standard household-level survey data. This facilitates a direct comparison of enrollment rates and child labor between foster and non-foster children. I begin by assigning children to two mutually exclusive groups: both biological parents absent (g=1); at least one parent present (g=2). Children in group 1 are foster children, while children in group 2 are non-foster children. I am therefore ignoring orphan status for now.

For a child i age 6-14 living in household h, living in region r in year t, I estimate the following regression,

$$Y_{ihst} = \alpha + \gamma F_i + \pi_h + \psi_{st} + X_i\delta + \epsilon_{ihst} \tag{1.15}$$

where Y_{ihst} is an indicator for school enrollment or some measure of child labor, and F_i is an indicator variable equal to one if the child is fostered. The parameter of interest is γ, which captures the effect of the absence of a child's parents on the various outcomes of interest. The omitted category is children with at least one biological parent present. In some specifications I include household fixed effects to control for any unobserved heterogeneity that does not vary over time. Household fixed effects allow for the direct examination of unequal treatment between foster and non-foster children, as I am relying only on within household variation. Lastly, I include region-year fixed effects to account for any region specific year effects that are common across foster status and households. There are four years of data and three regions so I cluster standard errors at the region-year level.

The consumption results suggest orphanhood is an important factor in how children are treated. I modify the above estimation to account for orphan status in order to examine whether a similar pattern emerges here. I now assign children into four mutually exclusive groups consistent with the consumption analysis: non-orphaned non-foster (g=1); orphaned non-foster (g=2); non-orphaned foster (g=3); orphaned foster (g=4). I estimate the following specification:

$$Y_{ihstg} = \alpha + \gamma_1 O_i + \gamma_2 F_i + \gamma_3 (O_i \times F_i) + \pi_h + \psi_{st} + X_i \delta + \epsilon_{ihstg} \tag{1.16}$$

where F_i and O_i are indicators for foster and orphan status respectively. The parameters of interest are now γ_g, which captures the differential effects of the child's foster and orphan status on school enrollment or child labor. The omitted category is non-orphaned children with at least one biological parent present. I again use the Malawi Integrated Households Survey (IHS3) 2010 and the Malawi Integrated Panel Survey 2013. Descriptive statistics are presented in Table 1.20 in the appendix.

1.A.2 Results

I begin by analyzing the difference in school enrollment rates between foster and non-foster children. I estimate Equation (3.21) and present the results in Table 1.6. The coefficient of interest γ describes the difference in treatment for foster and non-foster children. Column 1 provides an estimate of differences in means by foster status, controlling for child age and gender. This specification ignores any household characteristics that may be associated with both school enrollment rates and the types of households that foster in children. Columns 2 and 3 attempt to uncover evidence of intrahousehold discrimination of foster children. In column 2, I account for observable household characteristics, including the education, age, and gender of the household head, household composition measures, and log per capita household expenditure. In column 3, I include household fixed effects, which accounts for any unobservable household characteristics that do not vary across time. The results provide no evidence of discrimination based on foster status. This is largely consistent with the consumption analysis.

Table 1.6: School Enrollment by Foster Status

	OLS			Probit
	(1)	(2)	(3)	(4)
Foster Child	-0.029 (0.013)	-0.020 (0.019)	-0.047 (0.036)	-0.023 (0.019)
Sample Size	20,371	20,371	20,371	20,371
Region-Year Fixed Effects	Yes	Yes	Yes	Yes
Individual Controls	Yes	Yes	Yes	Yes
Household Controls		Yes	Yes	Yes
Household Fixed Effects			Yes	

Notes: The sample includes all children age 6-14. The omitted fostering category are children with at least one biological parent present. Standard errors are clustered at the region-year level. Individual controls include age age^2, and gender. Household controls include the number of male and female siblings age 0-6 and 7-14, the number of adult men and women, log household expenditure, and demographic characteristics of the household head. * p<0.1, ** p<0.05, *** p<0.01

The consumption results imply orphans are particularly mistreated within the household. To examine whether this pattern holds for education, I estimate Equation (1.16) with four foster categories that account for orphanhood. The results are presented in Table 1.7. The results provide evidence orphanhood matters greatly for foster child treatment; Each specification demonstrates that foster children who are orphans have enrollment rates that are statistically lower than children whose biological parents are present in the household. Column (1), which reports differences in means between orphan-foster groups controlling for child age and gender shows that on average, orphaned foster children have school enrollment rates that are 4 percentage points lower than non-orphaned, non-foster children. The results in column (2) are lower in magnitude than the results in column (1) at 2.7 percentage points, suggesting the lower school enrollment rates are partially due to differences in observable household characteristics. However the difference in school enrollment rates is still statistically significant. Once I account for household fixed effects in column (3), the estimated coefficient is negative and significant, suggesting that orphaned foster children are subject to intrahousehold discrimination.

Table 1.7: School Enrollment by Foster Status (Detailed Categories)

	OLS			Probit
	(1)	(2)	(3)	(4)
Fostering Categories				
Non-Orphaned Foster	-0.025	-0.019	-0.053	-0.024
	(0.022)	(0.028)	(0.054)	(0.031)
Orphaned Foster	-0.040***	-0.027**	-0.033***	-0.031***
	(0.003)	(0.003)	(0.001)	(0.005)
Orphaned Non-Foster	-0.022	-0.013	0.016	-0.014
	(0.013)	(0.012)	(0.029)	(0.010)
Sample Size	20,371	20,371	20,371	20,371
Region-Year Fixed Effects	Yes	Yes	Yes	Yes
Individual Controls	Yes	Yes	Yes	Yes
Household Controls		Yes	Yes	Yes
Household Fixed Effects			Yes	

Notes: The sample includes all children age 6-14. The omitted fostering category are non-orphaned children with at least one biological parent present. Standard errors are clustered at the region-year level. Individual controls include age age^2, and gender. Household controls include the number of male and female siblings age 0-6 and 7-14, the number of adult men and women, log household expenditure, and demographic characteristics of the household head. * p<0.1, ** p<0.05, *** p<0.01

Table 1.8 provides the child labor results. In columns 1 to 3, I examine the relationship between foster status and hours worked doing chores,[52] while columns 4 to 6 focus on hours worked for a household farm, household enterprise, or wage work outside the household. I add controls moving from left to right. The results again provide little evidence that work around the house differs substantially between foster and non-foster children, which is contrary to what the literature suggests (Serra (2009)). This lack of any effect is partially due to the limited definition of chores (only fetching wood and water), and possible measurement error in the data, as parents may be unwilling to reveal that their children work. The same lack of an association is apparent in examining the relationship foster status on work hours in columns 4 to 6. Table 1.9 accounts for orphanhood when examining the effect of foster status on child labor. The results demonstrate little difference by foster or orphan status in terms of child labor, which again is likely due to data issues.

[52]Chores include fetching wood and fetching water.

Table 1.8: Weekly Hours Worked by Fostering Status

	Chores			Work Outside HH		
	(1)	(2)	(3)	(4)	(5)	(6)
Fostering Categories						
Foster Child	0.077	0.007	-0.215	0.134	-0.163	-0.070
	(0.085)	(0.095)	(0.095)	(0.141)	(0.087)	(0.201)
Sample Size	20,371	20,371	20,371	20,371	20,371	20,371
Region-Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Individual Controls	Yes	Yes	Yes	Yes	Yes	Yes
Household Controls		Yes	Yes		Yes	Yes
Household Fixed Effects			Yes			Yes

Notes: The sample includes all children age 6-14. The omitted fostering category are children with both biological parents present. Standard errors are clustered at the region-year level. Individual controls include age age^2, and gender. Household controls include the number of male and female siblings age 0-6 and 7-14, the number of adult men and women, log household expenditure, and demographic characteristics of the household head. * p<0.1, ** p<0.05, *** p<0.01

Table 1.9: Weekly Hours Worked by Fostering Status (Detailed Categories)

	Chores			Work Outside HH		
	(1)	(2)	(3)	(4)	(5)	(6)
Fostering Categories						
Non-Orphaned Foster	-0.041	-0.140	-0.334*	0.034	-0.298	-0.068
	(0.099)	(0.100)	(0.114)	(0.187)	(0.137)	(0.294)
Orphaned Foster	0.253	0.211	-0.089	0.325*	0.018	-0.068
	(0.093)	(0.115)	(0.134)	(0.094)	(0.058)	(0.227)
Orphaned Non-Foster	-0.033	-0.032	-0.100	0.113	-0.097	-0.028
	(0.084)	(0.051)	(0.068)	(0.057)	(0.119)	(0.430)
Sample Size	20,371	20,371	20,371	20,371	20,371	20,371
Region-Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Individual Controls	Yes	Yes	Yes	Yes	Yes	Yes
Household Controls		Yes	Yes		Yes	Yes
Household Fixed Effects			Yes			Yes

Notes: The sample includes all children age 6-14. The omitted fostering category are non-orphaned children with at least one parent present. Standard errors are clustered at the region-year level. Individual controls include age age^2, and gender. Household controls include the number of male and female siblings age 0-6 and 7-14, the number of adult men and women, log household expenditure, and demographic characteristics of the household head. * p<0.1, ** p<0.05, *** p<0.01

1.B Fully Specified Model

The estimation in this study identifies resource shares from Engel curves for assignable clothing. In this section, I follow DLP and write a fully specified household demand model consistent with the restrictions contained in the clothing Engel curves. In particular, Engel curves for clothing are required to be linear in log expenditure, and resource shares must be independent of household expenditure.

Let y be household expenditure, and $\tilde{p}$ be the price vector of all goods aside from men's, women's, and children's clothing, which is denoted by p. While more general formulations are possible, I start with assuming individuals have subutility over clothing given by Muellbauer's Price Independent Generalized Logarithmic (PIGLOG) functional form.

$$\ln V_t(p, y) = \ln[\ln(\frac{y}{G^t(p_t, \tilde{p})})] + p_t e^{-a' \ln \tilde{p}} \tag{1.17}$$

where G^t is some function that is nonzero, differentiable, and homogeneous of degree one, and some constant vector a with elements a_k summing to one. Each member of the same type is assumed to have the same utility function. This assumption could be dropped with a data set that has goods that are assignable at a more detailed level.

The household weights individual utilities using the following Bergson-Samuelson social welfare function:

$$\tilde{U}_s(U_f, U_m, U_a U_b, p/y) = \sum_{t \epsilon \{m,f,a,b\}} \omega_t(p)[U_t + \rho_t(p)] \tag{1.18}$$

where $\omega_t(p)$ are the Pareto weight functions and $\rho_t(p)$ are the externality functions. Individuals are allowed to receive utility from another person's utility, but not from another person's consumption of a specific good. This can be considered a form of restricted altruism.

The household's problem is to maximize the social welfare function subject to a budget constraint, and a consumption technology constraint.

$$\max_{x_m, x_f, x_a, x_b, z_s} \omega(p) + \sum_{t \epsilon \{m,f,a,b\}} \omega_t(p) U_t$$

$$\text{s.t} \quad y = z_s' p \ \text{ and}$$

$$z_s^k = A_s^k (x_m^k + x_f^k + s_a x_a^k + s_b x_b^k) \ \text{ for each good } \ k$$

where the household type is given by s, or the number of foster and non-foster children present in the household and $\omega(p) = \sum_{t\epsilon\{m,f,a,b\}} \omega_t(p)\rho_t(p)$. Matrix A_s is the consumption technology function. It is a $k \times k$ diagonal matrix and determines the relative publicness or privateness of good k. If good k is private, then the k,k'th element is equal to one, and what the household purchases is exactly equal to individual consumption.

By Pareto efficiency, the household maximisation can be decomposed into two step process; In the first stage, resource shares are optimally allocated, and in the second stage, each individual maximizes their individual utility subject to the budget constraint $A_s^k p^k x_t^k = \eta_s^t y$. Resource shares can then be defined as $\eta_s^t = x^t A_s p/y = \sum_k A_s^k p^k x_t^k/y$ evaluated at the optimized level of expenditures x_t. The optimal utility level is given by the individual's indirect utility function V^t evaluated at Lindahl prices, $V_t(A_s'p, \eta_s^t, y)$.

Using the functional form assumptions regarding individual indirect utility functions, the household problem can again be rewritten:

$$\max_{\eta_s^m,\eta_s^f,\eta_s^a,\eta_s^b} \quad \omega(p) + \sum_{t\epsilon\{m,f,a,b\}} \tilde{\omega}_s^t(p)\ln(\frac{\eta_s^t y}{G^t(A_s'p)})$$
$$\text{s.t} \quad \eta_s^m + \eta_s^f + s_a\eta_s^a + s_b\eta_s^b = 1 \tag{1.19}$$

where $\tilde{\omega}(p) = \omega_t \exp(A_t p_t e^{-a'(\ln\tilde{p}+\ln\tilde{A}_s)})$

The first order conditions from this maximisation problem are as follows:

$$\frac{\tilde{\omega}_s^m(p)}{\eta_s^m} = \frac{\tilde{\omega}_s^f(p)}{\eta_s^f} = \frac{\tilde{\omega}_s^a(p)}{s_a\eta_s^a} = \frac{\tilde{\omega}_s^b(p)}{s_b\eta_s^b}, \text{ and } \sum_{t\epsilon\{m,f,a,b\}} s_t\eta_s^t = 1 \tag{1.20}$$

Solving for person specific resource shares gives the following equations:

$$\eta_s^t(p) = \frac{\tilde{\omega}_s^t(p)}{\tilde{\omega}_s^m + \tilde{\omega}_s^f + \tilde{\omega}_s^a + \tilde{\omega}_s^b} \text{for } t\,\epsilon\,\{m,f\} \tag{1.21}$$

$$\eta_s^t(p) = \frac{\tilde{\omega}_s^t(p)/s_t}{\tilde{\omega}_s^m + \tilde{\omega}_s^f + \tilde{\omega}_s^a + \tilde{\omega}_s^b} \text{for } t\,\epsilon\,\{a,b\} \tag{1.22}$$

With each person now allocated their share of household resources, each person can then maximize there own utility, subject to their own personal budget constraint. In particular, individuals choose x_t to maximize $U_t(x_t)$ subject to $\eta_s^t y = \sum_k A_s^k p_k x_t^k$. Individual demand functions can be derived using Roy's Identify on the indirect utility functions given in Equation (1.33), where individual income is used $\eta_s^t y$ and individuals face the Lindahl price vector $A_s p$.

$$h_t^k(\eta_s^t y, A_s p) = \frac{\eta_s^t y}{G^t}\frac{\partial G^t}{\partial A_s p^k} - \frac{\partial(Ap^k e^{-a' \ln \tilde{p}})}{\partial Ap^k}[\ln \eta_s^t y - \ln G^t]\eta_s^t y \tag{1.23}$$

for any good k for person of type t. This can be written more concisely:

$$h_t^k(\eta_s^t y, A_s' p) = \tilde{\delta}_t^k(A_s' p)\eta_s^t y - \psi_t^k(A_s' p)\eta_s^t y \ln(\eta_s^t y) \tag{1.24}$$

Using the individual demand functions, household demand for good k is written in general terms as follows accounting for the consumption technology function:

$$z_s^k = A_s \sum_{t \epsilon \{m,f,a,b\}} h_t^k(A_s' p, \eta_s^t(p)y) \tag{1.25}$$

Dividing the individual demand functions by income produces the budget share equations:

$$\frac{h_t^k(\eta_s^t y, A_s' p)}{y} = \tilde{\delta}_t^k(A_s' p)\eta_s^t - \psi_t^k(A_s' p)\eta_s^t \ln(\eta_s^t y) \tag{1.26}$$

The analysis in this paper uses Engel curves for private goods, which simplifies the above equation even further. First, Engel curves demonstrate how budget shares vary with income holding prices constant. Thus prices can be dropped from the above equation. Secondly, the consumption technology drops out for private goods, as the element in the A matrix takes a value of 1 for private goods. The Engel curves are then written as follows:

$$W_s^t(y) = \frac{h_s^t(y)}{y} = \eta_s^t \delta_s^t + \eta_s^t \beta_s^t (\ln y + \ln \eta_s^t) \tag{1.27}$$

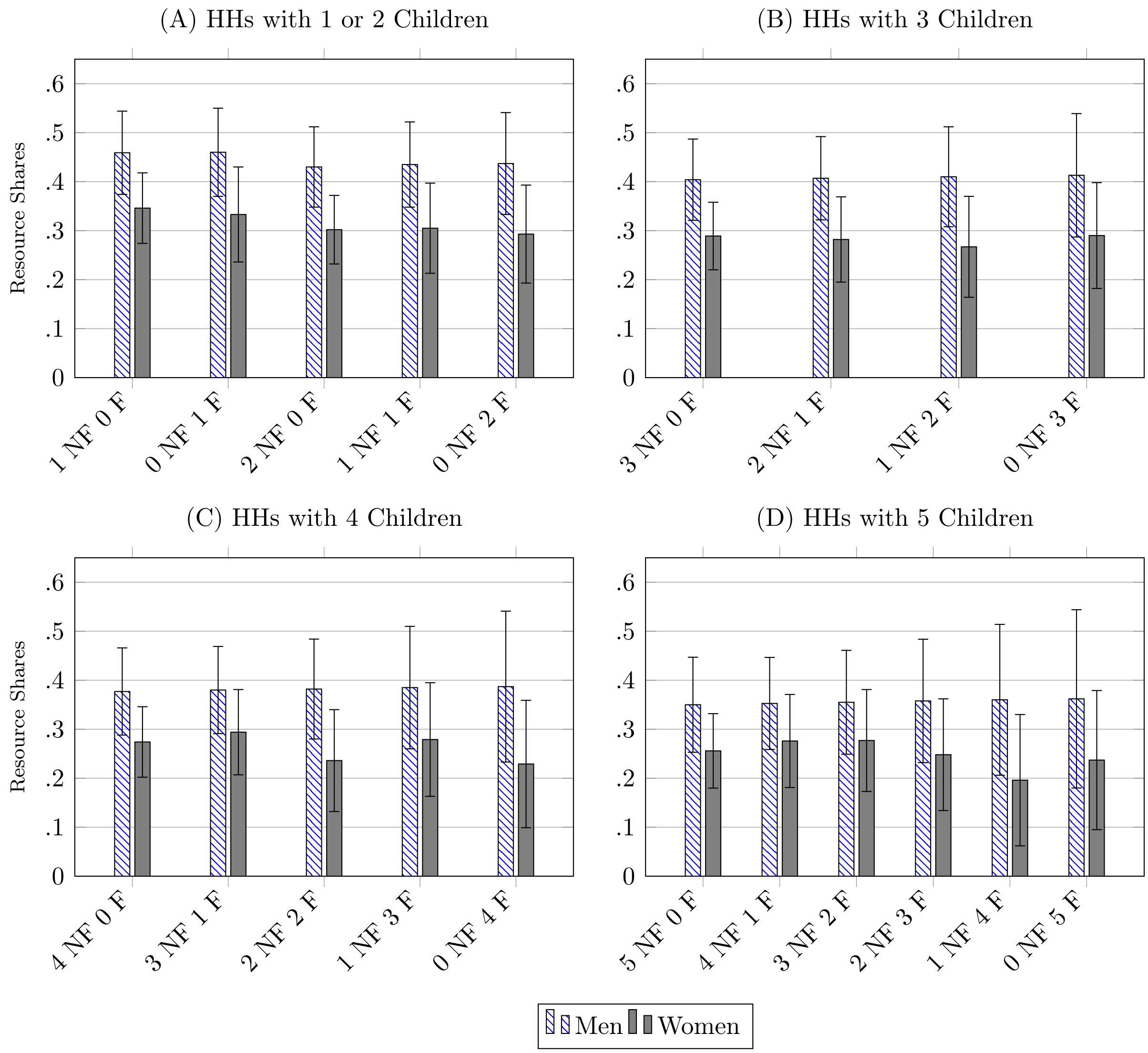
1.B.1 Additional Tables

Table 1.10: Household Structure

		# Foster					
		0	1	2	3	4	5
# Non-Foster	0	0	480	234	107	55	23
	1	2,159	283	79	22	18	0
	2	2,323	242	57	23	0	0
	3	2,167	170	41	0	0	0
	4	1,473	99	0	0	0	0
	5	708	0	0	0	0	0

Notes: Malawi Third Integrated Household Survey and Integrated Household Panel Survey. Households with 1-4 men and women, and 1-5 children.

Figure 1.6: Predicted Men's and Women's Resource Shares: Reference Household



Note: Malawi Third Integrated Household Survey and Integrated Household Panel Survey. Robust standard errors. The brackets are the 95 percent confidence intervals. Each quadrant presents men's and women's resource shares for a different household size defined by the number of children. Within each quadrant, men's and women's resource shares are presented by household type which is defined by the number of foster and non-foster children, respectively. A reference household is a household with 1 man, 1 woman, and all other covariates at their median value, excluding foster and non-foster child age, which are both set to 7.

Table 1.11: Determinants of Resource Shares: Household Type Indicators

	non-Foster Children (1) NLSUR	Foster Children (2) NLSUR
1 non-Foster 0 Foster	0.224***	
	(0.0469)	
2 non-Foster 0 Foster	0.296***	
	(0.0522)	
3 non-Foster 0 Foster	0.336***	
	(0.0567)	
4 non-Foster 0 Foster	0.379***	
	(0.0622)	
5 non-Foster 0 Foster	0.424***	
	(0.0685)	
0 non-Foster 1 Foster		0.178***
		(0.0489)
1 non-Foster 1 Foster	0.147***	0.117***
	(0.0366)	(0.0349)
2 non-Foster 1 Foster	0.194***	0.121***
	(0.0425)	(0.0343)
3 non-Foster 1 Foster	0.221***	0.109***
	(0.0469)	(0.0365)
4 non-Foster 1 Foster	0.248***	0.127***
	(0.0519)	(0.0410)
0 non-Foster 2 Foster		0.242***
		(0.0584)
1 non-Foster 2 Foster	0.167***	0.159***
	(0.0417)	(0.0429)
2 non-Foster 2 Foster	0.220***	0.164***
	(0.0489)	(0.0430)
3 non-Foster 2 Foster	0.250***	0.121***
	(0.0539)	(0.0418)
0 non-Foster 3 Foster		0.270***
		(0.0697)
1 non-Foster 3 Foster	0.162***	0.177***
	(0.0447)	(0.0516)
2 non-Foster 3 Foster	0.214***	0.183***
	(0.0538)	(0.0520)
0 non-Foster 4 Foster		0.356***
		(0.0897)
1 non-Foster 4 Foster	0.241***	0.233***
	(0.0600)	(0.0667)
0 non-Foster 5 Foster		0.373***
		(0.106)
No. Men	0.00475	-0.00369
	(0.00926)	(0.00935)
No. Women	-0.0183	0.00255
	(0.0112)	(0.0118)
N	10,763	
Log Likelihood	92294.57811	

Notes: Malawi Third Integrated Household Survey and Integrated Household Panel Survey. The sample includes all households with 1-4 men and women, and 1-5 children. Robust standard errors in parentheses. Age variables are divided by 100 to ease computation. The education and age variables are demeaned. Coefficients on the covariates (age, education, etc.) are omitted for conciseness. The parameter estimates are not per child, but rather the total resources allocated to foster or non-foster children. * p<0.1, ** p<0.05, *** p<0.01

Table 1.12: Determinants of Resource Shares: Orphan Interactions and Nuclear Households

	Non-Foster Children			Foster Children		
	Main Results (1a)	Orphan Interactions (2a)	Nuclear Households (3a)	Main Results (1b)	Orphan Interactions (2b)	Nuclear Households (3b)
Household Type Indicators						
3 Non-Foster 0 Foster	0.336***	0.332***	0.255***			
	(0.0567)	(0.0609)	(0.0522)			
2 Non-Foster 1 Foster	0.194***	0.199***	0.158***	0.121***	0.130***	0.0690*
	(0.0425)	(0.0454)	(0.0403)	(0.0343)	(0.0399)	(0.0404)
1 Non-Foster 2 Foster	0.167***	0.176***	0.0923***	0.159***	0.186***	0.157***
	(0.0417)	(0.0454)	(0.0350)	(0.0429)	(0.0502)	(0.0588)
0 Non-Foster 3 Foster				0.270***	0.306***	0.281***
				(0.0697)	(0.0825)	(0.106)
Covariates						
Average Age non-Foster	0.625	0.517	2.230***	-0.0422	0.0727	1.462
	(0.774)	(0.771)	(0.760)	(0.893)	(0.936)	(1.104)
Average Age non-Foster^2	0.00965	0.0251	-0.135**	0.0266	0.0131	-0.0624
	(0.0568)	(0.0541)	(0.0527)	(0.0652)	(0.0674)	(0.0822)
Average Age Foster	-1.333	-1.522	-1.501	2.618**	2.858	1.478
	(1.686)	(1.855)	(2.142)	(1.225)	(1.769)	(1.852)
Average Age Foster^2	0.0864	0.0989	0.0765	-0.125*	-0.143	-0.0619
	(0.0947)	(0.103)	(0.117)	(0.0759)	(0.102)	(0.105)
Proportion Non-Foster Female	-0.0292	-0.0294	0.0254	0.0167	0.0244	-0.0224
	(0.0192)	(0.0183)	(0.0180)	(0.0186)	(0.0208)	(0.0250)
Proportion Foster Female	-0.0105	0.00522	0.00863	0.0270	-0.0178	0.0156
	(0.0260)	(0.0342)	(0.0300)	(0.0280)	(0.0411)	(0.0331)
Rural	-0.00775	-0.0123	-0.0388*	-0.00306	-0.0123	0.00627
	(0.0172)	(0.0181)	(0.0229)	(0.0149)	(0.0181)	(0.0187)
Matrilineal Village	0.0151	0.0134	0.0119	0.0323*	0.0306*	0.0235
	(0.0163)	(0.0158)	(0.0171)	(0.0174)	(0.0180)	(0.0200)
Proportion of Fostered Orphaned	0.0430	0.0401	0.0276	-0.0427**	-0.0567	-0.0601*
	(0.0278)	(0.0600)	(0.0349)	(0.0206)	(0.0441)	(0.0341)
Proportion of Fostered Orphaned × Proportion Female Foster		0.000358			0.0462	
		(0.0621)			(0.0484)	
Proportion of Fostered Orphaned × Rural		-0.0178			0.00520	
		(0.0459)			(0.0349)	
N	10,763	10,763	5,850	10,763	10,763	5,850
Log Likelihood	92,338	92,378	49,077	92,338	92,378	49,077

Notes: Malawi Third Integrated Household Survey and Integrated Household Panel Survey. The sample includes all households with 1-4 men and women, and 1-5 children. Robust standard errors in parentheses. Age variables are divided by 100 to ease computation. Estimates for certain household types and preferences factors are omitted for conciseness. The parameters on the household type indicators are not per child, but the total allocation to all foster or non-foster children within the household. * p<0.1, ** p<0.05, *** p<0.01

Table 1.13: Determinants of Resource Shares: Age-Restricted Sample

	Non-Foster Children		Foster Children	
	Estimation Sample (1a)	Age-Restricted Sample (2a)	Estimation Sample (1b)	Age-Restricted Sample (2b)
Household Type Indicators				
3 Non-Foster 0 Foster	0.336***	0.350***		
	(0.0567)	(0.0802)		
2 Non-Foster 1 Foster	0.194***	0.202***	0.121***	0.110**
	(0.0425)	(0.0583)	(0.0343)	(0.0522)
1 Non-Foster 2 Foster	0.167***	0.174***	0.159***	0.174***
	(0.0417)	(0.0587)	(0.0429)	(0.0667)
0 Non-Foster 3 Foster			0.270***	0.301***
			(0.0697)	(0.111)
Covariates				
Average Age non-Foster	0.625	-0.0347	-0.0422	0.570
	(0.774)	(0.866)	(0.893)	(1.016)
Average Age non-Foster^2	0.00965	0.0410	0.0266	-0.0191
	(0.0568)	(0.0615)	(0.0652)	(0.0841)
Average Age Foster	-1.333	-2.373	2.618**	3.886
	(1.686)	(2.901)	(1.225)	(3.185)
Average Age Foster^2	0.0864	0.157	-0.125*	-0.204
	(0.0947)	(0.154)	(0.0759)	(0.193)
Matrilineal Village	0.0151	0.0103	0.0323*	0.0284
	(0.0163)	(0.0179)	(0.0174)	(0.0226)
Proportion of Fostered Orphaned	0.0430	0.0274	-0.0427**	-0.0484
	(0.0278)	(0.0299)	(0.0206)	(0.0372)
N	10,763	10,368	10,763	10,368
LL	92,338	88,701	92,338	88,701

Notes: Malawi Third Integrated Household Survey and Integrated Household Panel Survey. The sample includes all households with 1-4 men and women, and 1-5 children. Robust standard errors in parentheses. Age variables are divided by 100 to ease computation. Estimates for certain household types and preferences factors are omitted for conciseness. The parameters on the household type indicators are not per child, but the total allocation to all foster or non-foster children within the household. The Age-Restricted Sample drops households with both foster and non-foster children in any of the following age groups: 0-3, 4-7, 8-11, and 12-14. * p<0.1, ** p<0.05, *** p<0.01

Table 1.14: Determinants of Resource Shares: Estimation with SAT Restriction

	non-Foster Children (1) NLSUR	Foster Children (2) NLSUR
1 non-Foster 0 Foster	0.188***	
	(0.0465)	
2 non-Foster 0 Foster	0.260***	
	(0.0532)	
3 non-Foster 0 Foster	0.299***	
	(0.0581)	
4 non-Foster 0 Foster	0.335***	
	(0.0638)	
5 non-Foster 0 Foster	0.376***	
	(0.0705)	
0 non-Foster 1 Foster		0.122**
		(0.0537)
1 non-Foster 1 Foster	0.142**	0.0722
	(0.0571)	(0.0559)
2 non-Foster 1 Foster	0.134**	0.160
	(0.0564)	(0.100)
3 non-Foster 1 Foster	0.138*	0.193**
	(0.0714)	(0.0802)
0 non-Foster 2 Foster		0.205***
		(0.0715)
1 non-Foster 2 Foster	0.139	0.161***
	(0.0931)	(0.0588)
0 non-Foster 3 Foster		0.255***
		(0.0893)
No. Men	0.000230	0.00480
	(0.00861)	(0.00940)
No. Women	-0.00609	0.00820
	(0.0128)	(0.0148)
N	10,443	
No. Parameters	211	
Log Likelihood	92294.57811	

Notes: Malawi Third Integrated Household Survey and Integrated Household Panel Survey. The sample includes all households with 1-4 men and women, and 1-5 children. Robust standard errors in parentheses. Age variables are divided by 100 to ease computation. Coefficients on the covariates (age, education, etc.) are omitted for conciseness. Several household types are dropped from the sample due to too few observations. The parameter estimates are not per child, but rather the total resources allocated to foster or non-foster children. Restrictions 1 and 2 are NOT imposed in the estimation. * p<0.1, ** p<0.05, *** p<0.01

Table 1.15: Probability of Staying in Same HH by Foster and Orphan Status (Double vs. Single)

	Foster Child Sample		Full Sample	
	(1)	(2)	(3)	(4)
Double-Orphaned Foster	0.0307*	0.0309***	0.0573***	0.0622***
	(0.0167)	(0.009)	(0.006)	(0.016)
Single-Orphaned Foster	0.0118	0.0201	0.0254	0.0355**
	(0.025)	(0.015)	(0.016)	(0.016)
Non-Orphaned Non-Foster			0.0708***	0.0713***
			(0.014)	(0.017)
Orphaned Non-Foster			0.0237	0.0285
			(0.016)	(0.283)
Mean Dependent Variable	0.871	0.871	0.915	0.915
Sample Size	746	746	6,076	6,076
Region Fixed Effects	Yes	Yes	Yes	Yes
Individual Controls	Yes	Yes	Yes	Yes
Household Controls		Yes		Yes

Notes: The sample includes all children age 0-11 in 2010. The dependent variable is an indicator for whether or not the child in the 2010 sample was still in the same household in 2013. The omitted category are non-orphaned foster children. Standard errors are clustered at the region level. Individual controls include age age^2, and gender. Household controls include the number of male and female siblings age 0-6 and 7-14, the number of adult men and women, log household expenditure, and whether or not the household is in a rural area. * $p<0.1$, ** $p<0.05$, *** $p<0.01$

Table 1.16: Sample Means by Household Composition

	One-Child-Type		Composite
	Only Non-Foster	Only Foster	
	(1)	(2)	(3)
Men	1.353	1.527	1.621
Women	1.303	1.474	1.578
Non-Foster	2.575		2.007
Foster		1.784	1.310
Log Expenditure per Person	11.490	11.607	11.572
North	0.194	0.192	0.255
Central	0.375	0.326	0.362
Year=2010	0.753	0.720	0.691
Average Age Non-Foster	5.721		6.506
Average Age Foster		9.228	9.282
Proportion Female Non-Foster	0.504		0.487
Proportion Female Foster		0.552	0.555
Proportion Orphaned Foster		0.339	0.409
Average Age Women	28.789	47.126	31.327
Average Age Men	31.744	40.582	32.070
Average Education Women	5.216	3.749	6.094
Average Education Men	6.642	5.939	7.491
Rural	0.813	0.806	0.730
Proportion Female Adults Age 15-18	0.071	0.102	0.107
Proportion Male Adults Age 15-18	0.098	0.209	0.163
Matrilineal Village	0.608	0.640	0.574
Sample Size	8,830	899	1,034

Notes: Out of all households with 1-4 men and women, and 1-5 children. Malawi Third Integrated Household Survey and Integrated Household Panel Survey. One-child-type households contain either only non-foster children or only foster children. Composite households contain both foster and non-foster children.

Table 1.17: Sample Means by Household Composition

	Matched Sample		Matched Sample	
	Non-Foster Only	Composite	Foster Only	Composite
	(1)	(2)	(3)	(4)
Men	1.625	1.621	1.648	1.634
Women	1.572	1.578	1.661	1.691
Non-Foster	1.979	2.007		1.763
Foster		1.310	1.528	1.544
Log Expenditure per Person	11.590	11.572	11.631	11.706
North	0.248	0.255	0.219	0.252
Central	0.349	0.362	0.323	0.325
Year=2010	0.693	0.691	0.706	0.685
Average Age Non-Foster	6.518	6.506		7.919
Average Age Foster		9.282	8.929	8.920
Proportion Female Non-Foster	0.495	0.487		0.483
Proportion Female Foster		0.555	0.532	0.546
Proportion Orphaned Foster		0.409	0.334	0.324
Average Age Women	31.031	31.327	37.741	37.756
Average Age Men	31.963	32.070	35.822	35.786
Average Education Women	6.181	6.094	5.131	5.246
Average Education Men	7.614	7.491	6.933	6.915
Rural	0.727	0.730	0.744	0.726
Proportion Female Adults Age 15-18	0.116	0.107	0.138	0.143
Proportion Male Adults Age 15-18	0.157	0.163	0.198	0.195
Matrilineal Village	0.571	0.574	0.607	0.595
Sample Size	1,034	1,034	489	489

Notes: Malawi Third Integrated Household Survey and Integrated Household Panel Survey. One-child-type households contain either only non-foster children or only foster children. Composite households contain both foster and non-foster children. Matched samples are selected using propensity score matching. In total, there are 1,034 composite households that are matched with a corresponding one-child-type non-foster household. There are 899 households with only foster children, and out of those households I select 489 to match with the most similar composite households. None of the variables are statistically different at the 5% level across one-child-type and composite households.

Table 1.18: Distribution of Foster Caretakers by Household Composition

	All Foster Households	Households With Both Foster and Non-Foster Children	Households With Only Foster Children
	(1)	(2)	(3)
Foster Caretaker			
Grandparent(s) and Uncle/Aunt	34.27	37.71	31.12
Uncle/Aunt Only	14.16	21.38	7.55
Grandparent(s) Only	25.26	5.72	43.14
Adopted	14.00	16.50	11.71
Other*	12.31	18.69	6.47
Observations	1,243	594	649

Notes: Malawi Integrated Household Panel Survey 2013. The sample includes all foster children. *Other includes children living with an older sibling, other relatives, or other non-relatives.

Table 1.19: Predicted Resource Shares: Households with Only Non-Foster Children

Household Type	Full Sample (1)	Restricted Sample (2)
1 non-Foster 0 Foster	0.194***	0.204***
	(0.0403)	(0.0892)
2 non-Foster 0 Foster	0.133***	0.132***
	(0.0224)	(0.0510)
3 non-Foster 0 Foster	0.102***	0.104***
	(0.0163)	(0.0383)
4 non-Foster 0 Foster	0.087***	0.0933***
	(0.0136)	(0.0327)
5 non-Foster 0 Foster	0.079***	0.0902***
	(0.0121)	(0.0311)
Sample Size	10,763	1,034
Log Likelihood	92,338	8,933

Notes: Malawi Third Integrated Household Survey and Integrated Household Panel Survey. The full sample includes all households with 1-4 men and women, and 1-5 children. The restricted sample is selected using nearest neighbor propensity score matching. In total, there are 1,034 composite households which are matched with one-child-type non-foster households. These matched households comprise the restricted sample Robust standard errors in parentheses. The predicted resource shares are per-child. * p<0.1, ** p<0.05, *** p<0.01

Table 1.20: Descriptive Statistics: Education and Child Labour

	Mean	SD	Min	Max	Sample Size
Foster Status					
Both Parents Present	0.622	0.485	0	1	20371
Father Present Mother Absent and Alive	0.013	0.115	0	1	20371
Father Present Maternal Orphan	0.009	0.092	0	1	20371
Mother Present Father Absent and Alive	0.116	0.320	0	1	20371
Mother Present Paternal Orphan	0.058	0.234	0	1	20371
Both Absent and Alive	0.107	0.309	0	1	20371
Double Orphan	0.027	0.163	0	1	20371
Both Absent Paternal Orphan	0.026	0.158	0	1	20371
Both Absent Maternal Orphan	0.022	0.146	0	1	20371
Individual and Household Characteristics					
Enrolled in School	0.880	0.325	0	1	20371
Hours Worked in Chores Past Week	1.825	5.674	0	96	20371
Hours Worked (Excluding Chores) Past Week	2.166	4.021	0	49	20371
Expenditure per Capita (1000s MWK)	115.992	136.494	6.896	2,976.659	20371
Remmitances Per Capita (1000s MWK)	1.943	15.409	0	1,751.6	20371
North	0.207	0.405	0	1	20371
Central	0.362	0.481	0	1	20371
South	0.432	0.495	0	1	20371
Year = 2010	0.739	0.439	0	1	20371
Male Sibling Age 0-6	0.613	0.762	0	5	20371
Female Siblings Age 0-6	0.621	0.759	0	4	20371
Male Siblings Age 7-14	0.656	0.784	0	6	20371
Female Siblings Age 7-14	0.662	0.788	0	6	20371
Men in HH	1.370	0.958	0	9	20371
Women in HH	1.485	0.810	0	7	20371
Age	9.694	2.596	6	14	20371
Female	0.508	0.500	0	1	20371
Rural	0.825	0.380	0	1	20371
Age Household Head	44.032	12.999	16	104	20371
Female Household Head	0.174	0.379	0	1	20371
Education of Household Head	5.530	4.123	0	14	20371

Notes: Malawi Third Integrated Household Survey and Integrated Household Panel Survey.
All children age 6-14.

1.C Identification Theorems

What follows are extended versions of the identification theorems in DLP. Theorem 1 demonstrates how resource shares can be identified using the SAT restriction, while Theorem 2 does the same using the SAP restriction. Parts of both theorems and their respective proofs are similar to what is found in DLP, and I will therefore point out the parts where I differ.

Let $h_t^k(p, y)$ be the Marshallian demand function for good k and let the utility function of person t be defined as $U_t(x_t)$. Individual t chooses x_t to maximize $U_t(x_t)$ under the budget constraint $p'x_t = y$ with $x_t = h_t(p, y)$ for all goods k. Define the indirect utility function $V_t(p, y) = U_t(h_t(p, y))$ where $h_t(p, y)$ is the vector of demand functions for all goods k.

The household solves the following maximisation problem where each individual person type has their own utility function:[53]

$$\max_{x_m, x_f, x_a, x_b} \tilde{U}_{s_{ab}}[U_m(x_m), U_f(x_f), U_a(x_a), U_b(x_b), p/y] \text{ such that}$$
$$z_{s_{ab}} = A_{s_{ab}}[x_m + x_f + s_a x_a + s_b x_b] \text{ and} \tag{1.28}$$
$$y = z'p$$

The household demand functions are given by $H^k_{s_{ab}}(p, y)$. Let $A^k_{s_{ab}}$ be the row vector given by the k'th row of the linear technology function $A_{s_{ab}}$. Each individual faces the shadow budget constraint defined by the Lindahl price vector $A'_{s_{ab}}p$ and individual income $\eta^t_{s_{ab}}y$. Then household demand can be written as follows:

$$z^k_{s_{ab}} = H^k_{s_{ab}}(p, y) = A^k_{s_{ab}}\Big[\sum_{t\epsilon\{m,f,a,b\}} s_t h_t(A'_{s_{ab}}p, \eta^t_{s_{ab}}y)\Big] \tag{1.29}$$

where $\eta^t_{s_{ab}}$ are the resource shares of person t in a household with s_a foster children and s_b non-foster children. Resource shares by construction must sum to one.

$$\eta^m_{s_{ab}} + \eta^f_{s_{ab}} + s_a\eta^a_{s_{ab}} + s_b\eta^b_{s_{ab}} = 1 \tag{1.30}$$

ASSUMPTION A1: Equations (1.28), (1.29), and (1.30) hold with resource shares $\eta^t_{s_{ab}}$ that do not depend on y.

Resource shares being independent of household expenditure is the key identify-

[53] For simplicity, I have assumed there are one man and one woman in each household.

ing assumption. Resource shares can still depend on other variables correlated with household expenditure such as the individual wages for men and women.

DEFINITION: A good k is a *private* good if, for any household size s_{ab}, the matrix $A_{s_{ab}}$, has a one in position k, k and has all other elements in row k and column k equal to zero.

DEFINITION: A good k is an *assignable* good if it only appears in one of the utility functions U_m, U_f, U_a, and U_b.

Men's and women's clothing expenditures are examples of private assignable goods. These goods are central to identification in DLP and they are here as well. What makes private assignable goods unique and especially useful for identification is that by definition, the quantities that the household purchases are equivalent to what individuals in the household consume. In other words, there are no economies of scale or sharing for these goods making household-level consumption in some sense equivalent to individual-level consumption. However, because I lack a private assignable good for foster and non-foster children, I must make use of partially assignable goods.

DEFINITION: A good k is a *partially assignable* good if it only appears in two of the utility functions U_m, U_f, U_a, and U_b.

An example of a partially assignable good is children's clothing expenditures, which are partially assignable to foster and non-foster children. Specifically, children's clothing only appears in the utility functions for foster and non-foster children, U_a and U_b. In other contexts, children's clothing expenditures can be classified as partially assignable to boys and girls, or potentially to young and old children. Other examples of partially assignable goods commonly found in household survey data include alcohol and tobacco, which are assignable to adults, but only partially assignable to adult men and women.

The distinction between assignable and partially assignable goods is in some ways determined by the question the researcher is interested in answering. For example, DLP are interested in estimating intrahousehold inequality between men, women, and children within the household, and are therefore less interested in understanding inequality *among* children within the household, as I am in this context. They assume all children have the same utility function, U_c, or that $U_a = U_b$.[54] As a result, chil-

[54] All utility functions are allowed to vary by observable household characteristics, such as age,

dren's clothing expenditures are assignable, as they only appear in U_c. In my context, where I allow foster and non-foster children to have different utility functions and ultimately different resource shares, children's clothing expenditures now appear in both U_a and U_b and are therefore no longer assignable.

ASSUMPTION A2: Assume that the demand functions include a private assignable good for men and women, denoted as goods m and f. Assume that the demand functions include a private partially assignable good for foster and non-foster children, denoted as good c.

The household demand functions for the private assignable goods for men and women can be written as follows:

$$z^k_{s_{ab}} = H^k_{s_{ab}} = h^k(A'_{s_{ab}}p, \eta^k_{s_{ab}}(p)y) \text{ for } k \in \{m, f\} \tag{1.31}$$

For the foster and non-foster children, household demand functions for the private partially assignable good can be written as follows:

$$z^c_{s_{ab}} = H^c_{s_{ab}} = s_a h^a(A'_{s_{ab}}p, \eta^a_{s_{ab}}(p)y) + s_b h^b(A'_{s_{ab}}p, \eta^b_{s_{ab}}(p)y) \tag{1.32}$$

In practice, I take the household demand functions for foster child clothing, and non-foster child clothing, and sum them together. Taking this action is possible since the goods are private. In the empirical application, this means that I assume clothing is not shared across child types.

Define p_m and p_f to be the prices of the private assignable goods and define p_c to be the price of the private partially assignable good. Define $\bar{p}$ to be the vector of prices for all private goods excluding p_m, p_f, and p_c. Assume $\bar{p}$ is nonempty.

ASSUMPTION A3: Each person $t \in \{m, f, a, b\}$ has the following indirect utility function:[55]

$$V_t(p, y) = \psi_t\Big[u_t\Big(\frac{y}{G^t(\tilde{p})}, \frac{\bar{p}}{p_t}\Big), \tilde{p}\Big] \tag{1.33}$$

where G^t is some function that is nonzero, differentiable, and homogeneous of degree

education, and gender.

[55] As discussed in DLP, the indirect utility function only has to take this form for low levels of expenditure. For simplicity, I assume the indirect utility function is the same across all expenditure levels.

one, ψ_t and u_t are strictly positive, differentiable, and strictly monotonically increasing in their first arguments, and differentiable and homogenous of degree zero in their remaining elements.[56]

By Roy's identity, the demand functions for the private assignable goods $k \in \{m, f, a, b\}$ can be written as follows:

$$h^k(y,p) = \frac{\partial u_k\Big(\frac{y}{G^k(\tilde{p})}, \frac{\bar{p}}{p_k}\Big)'}{\partial(\bar{p}/p_k)} \frac{\bar{p}}{p_k^2} \frac{G^k(\tilde{p})}{u_k'\Big(\frac{y}{G^k(\tilde{p})}, \frac{\bar{p}}{p_k}\Big)} = \tilde{f}_k\Big(\frac{y}{G^k(\tilde{p})}, p_k, \bar{p}\Big)y$$

Since p_k and $\bar{p}$ do not change when replaced by $A'_{s_{ab}}p$, substituting the above equation into Equation (1.31) gives the household demand functions for the assignable goods:

$$H^k_{s_{ab}}(y,p) = \tilde{f}_k\Big(\frac{\eta^k_{s_{ab}}(p)y}{G^k(\tilde{A'}_{s_{ab}}\tilde{p})}, p_k, \bar{p}\Big)\eta^k_{s_{ab}}(p)y$$

The Engel curve by definition holds price constant, and can then be written as:

$$H^k_{s_{ab}}(y) = \tilde{f}_k\Big(\frac{\eta^k_{s_{ab}}y}{G^k_{s_{ab}}}\Big)\eta^k_{s_{ab}}y \tag{1.34}$$

However, because there are no private assignable goods for foster and non-foster children, I write the Engel curve for the private partially assignable good for children in place of $H^a_{s_{ab}}$ and $H^b_{s_{ab}}$ as follows:

$$H^c_{s_{ab}}(y) = \tilde{f}_a\Big(\frac{\eta^a_{s_{ab}}y}{G^a_{s_{ab}}}\Big)s_a\eta^a_{s_{ab}}y + \tilde{f}_b\Big(\frac{\eta^b_{s_{ab}}y}{G^b_{s_{ab}}}\Big)s_b\eta^b_{s_{ab}}y \tag{1.35}$$

Define the matrix Ω' by

$$\Omega' = \begin{bmatrix} \frac{\eta^m_{10}}{\eta^m_{20}} & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\eta^f_{10}}{\eta^f_{20}} & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\eta^m_{01}}{\eta^m_{02}} & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\eta^f_{01}}{\eta^f_{02}} & -1 \\ 0 & -1 & 0 & 0 & 0 & \frac{\eta^m_{10}}{\eta^m_{01}} & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & \frac{\eta^f_{10}}{\eta^f_{01}} \\ \frac{\eta^m_{10}}{\eta^m_{20}} - \frac{\eta^a_{10}}{\eta^a_{20}} & 0 & \frac{\eta^f_{10}}{\eta^f_{20}} - \frac{\eta^a_{10}}{\eta^a_{20}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\eta^m_{01}}{\eta^m_{02}} - \frac{\eta^b_{01}}{\eta^b_{02}} & 0 & \frac{\eta^f_{01}}{\eta^f_{02}} - \frac{\eta^b_{01}}{\eta^b_{02}} & 0 \end{bmatrix}$$

ASSUMPTION A4: The matrix Ω' is finite and nonsingular. $f^k(0) \neq 0$ for $k \in$

[56] Asssumption A3 is a modified version of Assumption B3 in DLP.

$\{m, f, a, b\}$.

Finiteness of Ω' requires that resource shares are never zero. The matrix is non-singular provided resource shares are not equal across household sizes. An example of a potential violation would be if parents in households with one fostered child have the exact same resource shares as parents in households with two fostered children, which is unlikely.

The condition that $f^k(0) \neq 0$ requires that the Engel curves for the private assignable and partially assignable goods are continuous and bounded away from zero.

DEFINITION: A *composite* household is a household that contains at least one foster and one non-foster child, or more concisely ($s_a > 0$ and $s_b > 0$).

DEFINITION: A *one-child-type* household is a household that has children, but is not a composite household, or more concisely ($s_a > 0$ and $s_b = 0$) or ($s_a = 0$ and $s_b > 0$) .

ASSUMPTION A5: Assume households with either only foster children, or only non-foster children are observed. With four different person types, there must be at least four different one-child-type households in the data.

For Assumption A5 to hold in this context, it is necessary to observe both one-child-type households with one or two foster children (s_{10} and s_{20}), and also one-child-type households with one or two non-foster children (s_{01} and s_{02}). This requirement is easily met but may be more difficult in other contexts. For example, if one was interested in analyzing intrahousehold inequality between widows and non-widow adult women, it is rare to have multiple widows in the same household. In this case, identification could be achieved by observing a one-child-type household with only a widow present, and three different household types with only non-widowed adult women present.

Using one-child-type and composite households in some sense mirrors the central identification assumption of Browning et al. (2013). They use households with single men or single women ("one-person-type households") to identify preferences in households with married couples ("composite households"). Similarly, I use the one-child-type households to impose structure on the composite households. I would however argue that my use of one-child-type households is much weaker than their use of single person households as married men and women likely have different preferences than single men and women, while it is not obvious why foster and non-foster child preferences should differ significantly across one-child-type and composite households.

ASSUMPTION A6: Preferences for clothing for foster and non-foster children are not identical. That is, $f^a(0) \neq f^b(0)$.

Resource shares will be identified by determining whether preferences for children's clothing in the composite households look more like the foster only households, or the non-foster only households. If those preferences are identical, then this method will not work.

Theorem 1. *Let Assumptions A1, A2, A3, A4, A5 and A6 hold for all household sizes s_{ab} in some set S, with one-child-type households $s_{ab} \in \{s_{01}, s_{10}, s_{02}, s_{20}\}$, and composite households s_{ab}.*[57] *Assume the household's Engel curves for the private, assignable good $H^t_{s_{ab}}(y)$ for $t \in \{m, f\}$ and $s_{ab} \in S$ are identified. Assume the household's Engel curve for the private, partially assignable good $H^c_{s_{ab}}$ for $s_{ab} \in S$ is identified. Then resource shares $\eta^t_{s_{ab}}$ for all household members $t \in \{m, f, a, b\}$ in household sizes $s_{ab} \in S$ are identified.*

The above theorem is a generalization of the DLP identification strategy using the SAT restriction. I next show how resource shares can be recovered using the SAP restriction. This theorem is an extension of Theorem 1 in DLP.

Define p_m and p_f to be the prices of the private assignable goods. Define p_c to be the price of the private partially assignable goods. The price of all other goods is given by $\tilde{p}$. As in DLP, define the square matrix $\tilde{A}_{s_{ab}}$ such that the set of prices given by $A'_{s_{ab}}$ includes the private and partially assignable good prices, p_m, p_f, and p_c, as well as all other prices, given by $A'_{s_{ab}}$.

ASSUMPTION B3: Assume each person $t \in \{m, f, a, b\}$ faces the budget constraint defined by (y, p) and has preferences over the private assignable and partially assignable goods, $k \in \{m, f, c\}$ given by the following indirect utility function:

$$V_t(p, y) = \psi_t\Big[\nu(\frac{y}{G^t(p)}) + F^t(p), \tilde{p}\Big] \tag{1.36}$$

[57] Resource shares are identified for any composite household provided there is a sufficient number of one-child-type households. In the empirical application, there are ten such households.

for some some functions ψ_t, F, and G^t where G^t is nonzero, differentiable, and homogenous of degree one, ν is differentiable and strictly monotonically increasing, $F^t(p)$ is differentiable, homogenous of degree zero, and is such that $\partial F^t(p)/\partial p_t = \phi(p) \neq 0$. Lastly, ψ_t is differentiable and strictly monotonically increasing in its arguments, and differentiable and homogenous of degree zero in the remaining arguments.

ASSUMPTION B4: For foster and non-foster children, the person-specific expenditure deflators are equal. That is, $G^a = G^b = G^c$, where G^c denotes the expenditure deflator for children.

By Roy's identity the demand functions for private assignable goods are as follows:

$$\begin{aligned} h^k(y,p) =& \frac{v'\left(\frac{y}{G^k(p)}\right)\frac{y}{G^{k2}(p)}\frac{\partial G^k(p)}{\partial p_k} + \frac{\partial F^k(p)}{\partial p_k}}{v'\left(\frac{y}{G^k(p)}\right)\frac{1}{G^k(p)}} \\ =& \frac{y}{G^k(p)}\frac{\partial G^k(p)}{\partial p_k} + \frac{\phi(p)}{v'\left(\frac{y}{G^k(p)}\right)}\frac{y}{y/G^k(p)} = \delta^k(p)y + g\Big(\frac{y}{G^k(p)}, p\Big)y \end{aligned}$$

Adding the demand functions for foster and non-foster child assignable goods results in the following equation:

$$h^a(y,p) + h^b(y,p) = \Big(\delta^a(p) + \delta^b(p)\Big)y + g\Big(\frac{y}{G^c(p)}, p\Big)y$$

For the private assignable goods for adults, I derive the following household-level demand function.

$$H^k(y,p) = \delta^k(A'_{s_{ab}}p)\eta^k_{s_{ab}}(p)y + g\Big(\frac{\eta^k_{s_{ab}}(p)y}{G^k(A'_{s_{ab}}p)}, p\Big)\eta^k_{s_{ab}}(p)y$$

Let $\eta^c = s_a\eta^a_{s_{ab}} + s_b\eta^b_{s_{ab}}$. Then the household-level demand functions for children's clothing is given by:

$$H^c(y,p) = \Big(\delta^a(A'_{s_{ab}}p) + \delta^b(A'_{s_{ab}}p)\Big)\eta^c_{s_{ab}}(p)y + g\Big(\frac{\eta^c_{s_{ab}}(p)y/(s_a + s_b)}{G^c(A'_{s_{ab}}p)}\Big)\eta^c_{s_{ab}}(p)y$$

The Engel curves for adults ($k \in \{m, f\}$) and children are then as follows:

$$H^k_{s_{ab}}(y) = \delta^k_{s_{ab}}\eta^k_{s_{ab}}y + g_{s_{ab}}\Big(\frac{\eta^k_{s_{ab}}y}{G^k_{s_{ab}}}\Big)\eta^k_{s_{ab}}y \tag{1.37}$$

and

$$H^c_{s_{ab}}(y) = \Big(\delta^a_{s_{ab}} + \delta^b_{s_{ab}}\Big)\eta^c_{s_{ab}} y + g\Big(\frac{\eta^c_{s_{ab}} y/(s_a + s_b)}{G^c_{s_{ab}}}\Big)\eta^c_{s_{ab}} y \tag{1.38}$$

ASSUMPTION B5:[58] The function $g_{s_{ab}}$ is twice differentiable. Let $g'_{s_{ab}}(y)$ and $g''_{s_{ab}}(y)$ be the first and second derivatives of $g_{s_{ab}}$. Assume either that $\lambda_{s_{ab}} = \lim_{y \to 0}[y^{\zeta} g''_{s_{ab}}(y)/g'_{s_{ab}}]^{\frac{1}{1-\zeta}}$ is finite and nonzero for some constant $\zeta \neq 1$ or that $g_{s_{ab}}$ is a polynomial in $\ln y$.

Assumption B5 requires that there be some nonlinearity in the demand function so that g'' is not zero.

ASSUMPTION B6: The ratio of foster and non-foster child resource shares in households with s_a and $s_{a'}$, and s_b and $s_{b'}$ foster and non-foster children is constant across household sizes.

$$\frac{\eta^a_{s_{a0}}}{\eta^a_{s_{a+1,0}}} = \frac{\eta^a_{s_{ab}}}{\eta^a_{s_{a+1,b}}} \text{ and } \frac{\eta^b_{s_{0b}}}{\eta^b_{s_{0,b+1}}} = \frac{\eta^b_{s_{ab}}}{\eta^b_{s_{a,b+1}}} \tag{1.39}$$

for s_a and $s_b \ \epsilon \ \{1, 2\}$.

This assumption restricts the way in which resource shares vary across household types. In effect, it imposes that resource shares for foster and non-foster children in one-child-type and composite households behave in a similar fashion. Stated differently, this is an independence assumption: the ratio of foster child resource shares in a households with s_a and s_{a+1} foster children is independent of the number of *non-foster* children present in those households, and vice versa.

Other studies using the DLP identification strategy have imposed similar restrictions to improve precision in the estimation, but not for identification reasons. For example, Calvi (2016) parametrizes resource shares in such a way that per person resource shares decrease linearly in the number of household members. In the notation of this study, that would mean assuming $\eta^a_{s_{a,0}} - \eta^a_{s_{a+1,0}} = \eta^a_{s_{ab}} - \eta^a_{s_{a+1,b}}$. On the contrary, I impose that the percent decline is constant, as opposed to the absolute decline. In several specifications, DLP make a similar restriction that per child resource shares decrease linearly in the number of children.

ASSUMPTION B7: The degree of unequal treatment *within* a household with one of each child type is proportional to the degree of unequal treatment *across* households

[58] This is Assumption A4 from DLP.

with one foster child or one non-foster child.

$$\frac{\eta^a_{s_{10}}}{\eta^b_{s_{01}}} = \frac{\eta^a_{s_{11}}}{\eta^b_{s_{11}}} \tag{1.40}$$

Similar to Assumption B6, this restriction assumes households with only foster on non-foster children are similar to households with both types of children. To better understand this restriction, consider the following example given below:

Household	s_a	s_b	$\eta^a_{s_{ab}}$	$\eta^b_{s_{ab}}$	Assumption B6
A	1	0	24	0	
B	0	1	0	21	
C	1	1	16	14	$\frac{24}{21} = \frac{\eta^a_{10}}{\eta^b_{01}} = \frac{\eta^a_{11}}{\eta^b_{11}} = \frac{16}{14}$

Here, Household's A and B are one-child-type, whereas Household C is a composite household. Assumption B6 requires that foster and non-foster child resource shares in Household C, $\eta^a_{11}$and η^b_{11}, are proportional to foster and non-foster child resource shares in Household's A and B. In particular, if $\eta^a_{10} = 24$, and $\eta^b_{01} = 21$, then $\frac{\eta^a_{11}}{\eta^b_{11}} = \frac{24}{21}$. Importantly, this restriction directly applies to only a single composite household type.

Define the matrix Ω'' by

$$\Omega'''' = \begin{bmatrix}
\frac{\eta^m_{11}}{\eta^a_{11}} + \frac{\eta^f_{11}}{\eta^a_{11}} + 1 & \frac{\eta^m_{11}}{\eta^b_{11}} + \frac{\eta^f_{11}}{\eta^b_{11}} + 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2(\frac{\eta^m_{21}}{\eta^a_{21}} + \frac{\eta^f_{21}}{\eta^a_{21}} + 1) & \frac{\eta^m_{21}}{\eta^b_{21}} + \frac{\eta^f_{21}}{\eta^b_{21}} + 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\eta^m_{12}}{\eta^a_{12}} + \frac{\eta^f_{12}}{\eta^a_{12}} + 1 & 2(\frac{\eta^m_{12}}{\eta^b_{12}} + \frac{\eta^f_{12}}{\eta^b_{12}} + 1) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\eta^m_{22}}{\eta^a_{22}} + \frac{\eta^f_{22}}{\eta^a_{22}} + 1 & \frac{\eta^m_{22}}{\eta^b_{22}} + \frac{\eta^f_{22}}{\eta^b_{22}} + 1 \\
-1 & 0 & \frac{\eta^a_{10}}{\eta^a_{20}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & \frac{\eta^a_{10}}{\eta^a_{20}} & 0 \\
0 & -1 & 0 & 0 & 0 & \frac{\eta^b_{01}}{\eta^b_{02}} & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & \frac{\eta^b_{01}}{\eta^b_{02}} \\
\frac{1}{\eta^a_{10}} & \frac{-1}{\eta^b_{01}} & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

ASSUMPTION B8: The matrix Ω'' is finite and nonsinuglar.

This is true as long as resource shares are nonzero.

Theorem 2. *Let Assumptions A1, A2, A5, B3, B4, B5, B6, B7, and B8 hold for all household sizes* s_{ab} *in some set S, with* $s_{ab} \in \{s_{01}, s_{10}, s_{02}, s_{20}, s_{11}, s_{12}, s_{21}, s_{22}\}$*. Assume the household's Engel curves for the private, assignable good* $H^k_{s_{ab}}(y)$ *for* $k \in$

$\{m, f\}$ for $s_{ab} \in S$ are identified. Assume the household's Engel curve for the private, partially assignable good $H^c_{s_{ab}}$ for $s_{ab} \in S$ is identified. Then resource shares $\eta^t_{s_{ab}}$ for all household members $t \in \{m, f, a, b\}$ in household sizes $s_{ab} \in S$ are identified.

1.D Identification Proofs

1.D.1 Proof of Theorem 1

This proof follows the proof of Theorem 2 in DLP, and extends it to identify resource shares in the absence of assignable goods for each person type. The proof proceeds in two steps. In the first step, I demonstrate resource shares are identified in the one-child-type households; this follows directly from DLP. In the second step, I extend DLP to demonstrate how resource shares can be identified in the absence of private assignable goods.

By Assumption A3, the Engel curve functions for the assignable and partially assignable goods are given by Equations (1.34) and (1.35). Let $s_{ab} \in \{s_{10}, s_{20}, s_{01}, s_{02}\}$ be the different one-child-type households. Then since the functions H^k and H^c are identified, ζ^k_{20}, ζ^k_{02}, and ζ^k_{01} defined as $\zeta^k_{20} = \lim_{y \to 0} H^k_{10}(y)/H^k_{20}(y)$, $\zeta^k_{02} = \lim_{y \to 0} H^k_{10}(y)/H^k_{02}(y)$, and $\zeta^k_{01} = \lim_{y \to 0} H^k_{10}(y)/H^k_{01}(y)$ can all be identified for $k \in \{m, f\}$. Moreover, $\zeta^a_{20} = \lim_{y \to 0} H^a_{10}(y)/H^a_{20}(y)$ and $\zeta^b_{02} = \lim_{y \to 0} H^b_{01}(y)/H^b_{02}(y)$ can be identified for foster and non-foster children, respectively.

Then for $k \in \{m, f\}$:

$$\zeta^k_{20} = \frac{f^k(0)\eta^k_{10}}{f^k(0)\eta^k_{20}} = \frac{\eta^k_{10}}{\eta^k_{20}} \quad \text{and} \quad \zeta^k_{02} = \frac{f^k(0)\eta^k_{01}}{f^k(0)\eta^k_{02}} = \frac{\eta^k_{01}}{\eta^k_{02}} \quad \text{and} \quad \zeta^k_{01} = \frac{f^k(0)\eta^k_{10}}{f^k(0)\eta^k_{01}} = \frac{\eta^k_{10}}{\eta^k_{01}}$$

The same ratio for foster and non-foster children in households with only one child type can be identified:

$$\zeta^a_{20} = \frac{(f^a(0)\eta^a_{10} + 0 \times f^b(0)\eta^b_{10})}{(2f^a(0)\eta^a_{20} + 0 \times f^b(0)\eta^b_{20})} = \frac{\eta^a_{10}}{2\eta^a_{20}} \quad \text{and} \quad \zeta^b_{02} = \frac{(0 \times f^a(0)\eta^a_{01} + f^b(0)\eta^b_{01})}{(0 \times f^a(0)\eta^a_{02} + 2f^b(0)\eta^b_{02})} = \frac{\eta^b_{01}}{2\eta^b_{02}}$$

Using that resource shares must sum to one, the following equations can be written, first for households with only non-foster children:

$$\begin{aligned}
\zeta^m_{s_{20}}\eta^m_{s_{20}} + \zeta^f_{s_{20}}\eta^f_{s_{20}} + \zeta^a_{s_{20}} s_a \eta^a_{s_{20}} &= \eta^m_{10} + \eta^f_{10} + \eta^a_{10} = 1 \\
\zeta^m_{s_{20}}\eta^m_{s_{20}} + \zeta^f_{s_{20}}\eta^f_{s_{20}} + \zeta^a_{s_{20}}(1 - \eta^m_{s_{20}} - \eta^f_{s_{20}}) &= 1 \\
(\zeta^m_{s_{20}} - \zeta^a_{s_{20}})\eta^m_{s_{20}} + (\zeta^f_{s_{20}} - \zeta^a_{s_{20}})\eta^f_{s_{20}} &= 1 - \zeta^a_{s_{20}}
\end{aligned}$$

and then for households with only foster children:

$$
\begin{aligned}
&\zeta_{s02}^m \eta_{s02}^m + \zeta_{s02}^f \eta_{s02}^f + \zeta_{s02}^b s_b \eta_{s02}^b = \eta_{01}^m + \eta_{01}^f + \eta_{01}^b = 1 \\
&\zeta_{s02}^m \eta_{s02}^m + \zeta_{s02}^f \eta_{s02}^f + \zeta_{s02}^b (1 - \eta_{s02}^m - \eta_{s02}^f) = 1 \\
&(\zeta_{s02}^m - \zeta_{s02}^b)\eta_{s02}^m + (\zeta_{s02}^f - \zeta_{s02}^b)\eta_{s02}^f = 1 - \zeta_{s02}^b
\end{aligned}
$$

These above equations for $t \in \{m, f\}$, give the matrix equation

$$
\begin{bmatrix}
\zeta_{20}^m & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \zeta_{20}^f & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \zeta_{02}^m & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \zeta_{02}^f & -1 \\
0 & -1 & 0 & 0 & 0 & \zeta_{01}^m & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & \zeta_{01}^f \\
\zeta_{20}^m - \zeta_{20}^a & 0 & \zeta_{20}^f - \zeta_{20}^a & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \zeta_{02}^m - \zeta_{02}^b & 0 & \zeta_{02}^f - \zeta_{02}^b & 0
\end{bmatrix}
\times
\begin{bmatrix}
\eta_{20}^m \\ \eta_{10}^m \\ \eta_{20}^f \\ \eta_{10}^f \\ \eta_{02}^m \\ \eta_{01}^m \\ \eta_{02}^f \\ \eta_{01}^f
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 - \zeta_{20}^a \\ 1 - \zeta_{02}^b
\end{bmatrix}
$$

The 8×8 matrix in this equation equals the previously defined matrix Ω' which was assumed to be nonsingular. Therefore the system can be solved for η_{sa0}^m, η_{s0b}^m, η_{sa0}^f, and η_{s0b}^f. Non-foster child resource shares and foster child resource shares can then be identified for one-child-type only households by $\eta_{sa0}^a = (1 - \eta_{sa0}^m - \eta_{sa0}^f)/s_a$ and $\eta_{s0b}^b = (1 - \eta_{s0b}^m - \eta_{s0b}^f)/s_b$.

I now show resource shares are identified in any given composite household. Recall that the functions H^k are identified for $k \in \{m, f\}$. It follows that for any household type s_{ab}, $\zeta_{s_{ab}}^k$ defined as $\zeta_{s_{ab}}^k = \lim_{y \to 0} H_{10}^k(y)/H_{s_{ab}}^k(y)$ can be identified.

Then for $k \in \{m, f\}$:

$$
\zeta_{s_{ab}}^k = \frac{f^k(0)\eta_{10}^k}{f^k(0)\eta_{s_{ab}}^k} = \frac{\eta_{10}^k}{\eta_{s_{ab}}^k}
$$

With η_{10}^k already identified, resource shares for men and women in the composite household types can be recovered. This is a simple extension of DLP where there are more household types than individual types.

I now aim to separately identify non-foster and foster child resource shares in households with both types of children. Define $\zeta_{s_{ab}}^a$ as follows: $\zeta_{s_{ab}}^a = \lim_{y \to 0} H_{s_{ab}}^c(y)/H_{10}^c(y)$. Moreover, define $\zeta_{01}^b = \lim_{y \to 0} H_{01}^c(y)/H_{10}^c(y)$. Then we can write:

$$
\zeta_{s_{ab}}^a = \frac{f^a(0)\eta_{s_{ab}}^a + f^b(0)\eta_{s_{ab}}^b}{f^a(0)\eta_{10}^a} = \frac{\eta_{s_{ab}}^a}{\eta_{10}^a} + \frac{f^b(0)\eta_{s_{ab}}^b}{f^a(0)\eta_{10}^a} \tag{1.41}
$$

Furthermore,

$$
\zeta_{01}^b = \frac{f^b(0)\eta_{01}^b}{f^a(0)\eta_{10}^a} \rightarrow \frac{f^b(0)}{f^a(0)} = \frac{\zeta_{01}^b \eta_{10}^a}{\eta_{01}^b} = \kappa
$$

where η^a_{10} and η^b_{01} have already been identified. Thus, the ratio $f^b(0)/f^a(0) = \kappa$ is identified. Substituting κ into equation (1.41) results in the following expression:

$$\zeta^a_{s_{ab}} = \frac{\eta^a_{s_{ab}}}{\eta^a_{10}} + \kappa \frac{\eta^b_{s_{ab}}}{\eta^a_{10}} \tag{1.42}$$

where only $\eta^a_{s_{ab}}$ and $\eta^b_{s_{ab}}$ are unknown. Then since resource shares for men and women have already been identified for households of type s_{ab}, and because resource shares sum to one, we can solve for $\eta^a_{s_{ab}}$ and $\eta^b_{s_{ab}}$. This has a unique solution following Assumption A6.

1.D.2 Proof of Theorem 2

The proof proceeds in two steps. In the first step, I demonstrate resource shares are identified in the one-child-type households; this follows directly from DLP. In the second step, I extend DLP to demonstrate how resource shares can be identified in the absence of private assignable goods.

By Assumption B3, Engel curves for the private assignable goods for men and women are given by Equation (1.37) and by Assumptions B3 and B4, the Engel curve for the private partially assignable good is given by Equation (1.38). Define $\tilde{h}^k_{s_{ab}}(y) = \partial[H^k_{s_{ab}}(y)/y]\partial y$ and $\lambda_{s_{ab}} = \lim_{y\to 0}[y^\zeta g''_{s_{ab}}(y)/g'_{s_{ab}}]^{\frac{1}{1-\zeta}}$, where $\zeta \neq 1$ (the log polynomial case, where $\zeta = 1$ is considered in the second case).

Case 1: $g_{s_{ab}}$ is not a polynomial in logarithms.

Let $s_c = s_a + s_b$ be the total number of children. Then since $H^k_{s_{ab}}(y)$ are identified for $k \in \{m, f, c\}$, we can identify $\kappa^k_{s_{ab}}$ for men and women defined as follows:

$$\begin{aligned}
\kappa^k_{s_{ab}} &= \left(y^\zeta \frac{\partial \tilde{h}^k_{s_{ab}}(y)/\partial y}{\tilde{h}^k_{s_{ab}}(y)} \right)^{\frac{1}{1-\zeta}} \\
&= \left(\left(\frac{\eta^k_{s_{ab}}}{G^k_{s_{ab}}}\right)^{-\zeta} \left(\frac{\eta^k_{s_{ab}} y}{G^k_{s_{ab}}}\right)^{\zeta} \left[g''_{s_{ab}}\left(\frac{\eta^k_{s_{ab}} y}{G^k_{s_{ab}}}\right) \frac{\eta^{k^3}_{s_{ab}}}{G^{k^2}_{s_{ab}}} \right] / \left[g'_{s_{ab}}\left(\frac{\eta^k_{s_{ab}} y}{G^k_{s_{ab}}}\right) \frac{\eta^{k^2}_{s_{ab}}}{G^k_{s_{ab}}} \right] \right)^{\frac{1}{1-\zeta}} \\
&= \frac{\eta^k_{s_{ab}}}{G^k_{s_{ab}}} \left(y^\zeta_{k,s_{ab}} \frac{g''_{s_{ab}}(y_{k,s_{ab}})}{g'_{s_{ab}}(y_{k,s_{ab}})} \right)^{\frac{1}{1-\zeta}}
\end{aligned}$$

and for children:

$$\kappa^c_{s_{ab}} = \left(y^{\zeta} \frac{\partial \tilde{h}^c_{s_{ab}}(y)/\partial y}{\tilde{h}^c_{s_{ab}}(y)} \right)^{\frac{1}{1-\zeta}} =$$

$$\left(\Big(\frac{\eta^c_{s_{ab}}}{G^c_{s_{ab}} s_c}\Big)^{-\zeta} \Big(\frac{\eta^c_{s_{ab}} y}{G^c_{s_{ab}} s_c}\Big)^{\zeta} \Big[g^{''}_{s_{ab}} \Big(\frac{\eta^c_{s_{ab}} y}{G^c_{s_{ab}} s_c}\Big) \frac{\eta^{c^3}_{s_{ab}}}{G^{c^2}_{s_{ab}} s_c^2} \Big] / \Big[g^{'}_{s_{ab}} \Big(\frac{\eta^c_{s_{ab}} y}{G^c_{s_{ab}} s_c}\Big) \frac{\eta^{c^2}_{s_{ab}}}{G^c_{s_{ab}} s_c} \Big] \right)^{\frac{1}{1-\zeta}}$$

$$= \frac{\eta^c_{s_{ab}}}{G^c_{s_{ab}} s_c} \left(y^{\zeta}_{c,s_{ab}} \frac{g^{''}_{s_{ab}}(y_{c,s_{ab}})}{g^{'}_{s_{ab}}(y_{c,s_{ab}})} \right)^{\frac{1}{1-\zeta}}$$

Then for $k \, \epsilon \, \{m, f\}$, $\kappa^k_{s_{ab}}(0) = \frac{\eta^k_{s_{ab}}}{G^k_{s_{ab}}} \lambda_{s_{ab}}$, and we can identify $\rho^k_{s_{ab}}(y)$ defined as:

$$\rho^k_{s_{ab}}(y) = \frac{\tilde{h}^k_{s_{ab}}(y/\kappa^k_{s_{ab}}(0))}{\kappa^k_{s_{ab}}(0)} = g^{'}_{s_{ab}} \Big(\frac{y}{\lambda_{s_{ab}}}\Big) \frac{\eta^k_{s_{ab}}}{\lambda_{s_{ab}}}$$

and for $k = c$, $\kappa^c_{s_{ab}}(0) = \frac{\eta^c_{s_{ab}}}{G^c_{s_{ab}} s_c} \lambda_{s_{ab}}$, and we can identify $\rho^c_{s_{ab}}(y)$ defined as:

$$\rho^c_{s_{ab}}(y) = \frac{\tilde{h}^c_{s_{ab}}\Big(y/\kappa^c_{s_{ab}}(0)\Big)}{\kappa^c_{s_{ab}}(0)} = g^{'}_{s_{ab}} \Big(\frac{y}{\lambda_{s_{ab}}}\Big) \frac{\eta^c_{s_{ab}}}{\lambda_{s_{ab}}}$$

and we can write $\gamma^k_{s_{ab}}$ for $k \, \epsilon \, \{m, f\}$:

$$\gamma^k_{s_{ab}} = \frac{\tilde{\rho}^k_{s_{ab}}}{\tilde{\rho}^c_{s_{ab}}} = \left(g^{'}_{s_{ab}} \Big(\frac{y}{\lambda_{s_{ab}}}\Big) \frac{\eta^k_{s_{ab}}}{\lambda_{s_{ab}}} \right) / \left(g^{'}_{s_{ab}} \Big(\frac{y}{\lambda_{s_{ab}}}\Big) \frac{\eta^c_{s_{ab}}}{\lambda_{s_{ab}}} \right) = \frac{\eta^k_{s_{ab}}}{(s_a \eta^a_{s_{ab}} + s_b \eta^b_{s_{ab}})} \tag{1.43}$$

Case 2: Before proceeding with the proof, I examine the case where $g_{s_{ab}}$ is a polynomial in logarithms (the end result will be Equation (1.43) and I will proceed with both cases simultaneously afterwards). Suppose $g_{s_{ab}}$ is a polynomial of degree λ in logarithms. Then

$$g_{s_{ab}} \Big(\frac{\eta^k_{s_{ab}} y}{G^k}\Big) = \sum_{l=0}^{\lambda} \left(\ln \Big(\frac{\eta^k_{s_{ab}}}{G^k_{s_{ab}}}\Big) + \ln(y) \right)^l c_{s_{ab},l}$$

Then for $k \, \epsilon \, \{m, f\}$:

$$\gamma^k_{s_{ab}} = \Big(\frac{\partial^{\lambda}[H^k_{s_{ab}}(y)/y]}{\partial(\ln y)^{\lambda}}\Big) / \Big(\frac{\partial^{\lambda}[H^c_{s_{ab}}(y)/y]}{\partial(\ln y)^{\lambda}}\Big) =$$

$$\frac{c_{s_{ab},\lambda} \eta^k_{s_{ab}}}{c_{s_{ab},\lambda}(s_a \eta^a_{s_{ab}} + s_b \eta^b_{s_{ab}})} = \frac{\eta^k_{s_{ab}}}{(s_a \eta^a_{s_{ab}} + s_b \eta^b_{s_{ab}})} \tag{1.44}$$

which is the same as Equation (1.43). Then since resource shares must sum to one:

$$\begin{aligned}
&\gamma^m_{s_{ab}}(s_a\eta^a_{s_{ab}} + s_b\eta^b_{s_{ab}}) + \gamma^f_{s_{ab}}(s_a\eta^a_{s_{ab}} + s_b\eta^b_{s_{ab}}) + s_a\eta^a_{s_{ab}} + s_b\eta^b_{s_{ab}} = \\
&\eta^m_{s_{ab}} + \eta^f_{s_{ab}} + s_a\eta^a_{s_{ab}} + s_b\eta^b_{s_{ab}} = 1 \\
&s_a\eta^a_{s_{ab}}(\gamma^m_{s_{ab}} + \gamma^f_{s_{ab}} + 1) + s_b\eta^b_{s_{ab}}(\gamma^m_{s_{ab}} + \gamma^f_{s_{ab}} + 1) = 1 \qquad (1.45)
\end{aligned}$$

For one-child-type households, s_a or s_b equals zero, and Equation (1.45) simplifies significantly. For households that only have foster children, Equation (1.45) can be written as follows:

$$s_a\eta^a_{s_{ab}}(\gamma^m_{s_{ab}} + \gamma^f_{s_{ab}} + 1) = 1$$

which can be solved for $\eta^a_{s_{ab}} = \frac{1}{s_a(\gamma^m_{s_{ab}}+\gamma^f_{s_{ab}}+1)}$. Similarly, $\eta^b_{s_{ab}} = \frac{1}{s_b(\gamma^m_{s_{ab}}+\gamma^f_{s_{ab}}+1)}$.

With resource shares for foster and non-foster children identified, resource shares for men and women in the one-child-type households can then be solved for since $\eta^t_{s_{ab}} = \gamma^t_{s_{ab}}(s_a\eta^a_{s_{ab}} + s_b\eta^b_{s_{ab}})$ for $t \in \{m, f\}$.

I next move to the composite households $s_{ab} \in \{s_{11}, s_{21}, s_{12}, s_{22}\}$. Note that now, for each household type, resource shares for both foster and non-foster children need to be identified (η^a and η^b). For the one-child-type households, one of those two parameters was zero. From Equation (1.45) I can write the following four equations:

$$\begin{bmatrix}
1+\gamma^m_{11}+\gamma^f_{11} & 1+\gamma^m_{11}+\gamma^f_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2(1+\gamma^m_{21}+\gamma^f_{21}) & 1+\gamma^m_{21}+\gamma^f_{21} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1+\gamma^m_{12}+\gamma^f_{12} & 2(1+\gamma^m_{12}+\gamma^f_{12}) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1+\gamma^m_{22}+\gamma^f_{22} & 1+\gamma^m_{22}+\gamma^f_{22}
\end{bmatrix} \times \begin{bmatrix} \eta^a_{11} \\ \eta^b_{11} \\ \eta^a_{21} \\ \eta^b_{21} \\ \eta^a_{12} \\ \eta^b_{12} \\ \eta^a_{22} \\ \eta^b_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \frac{1}{2} \end{bmatrix}$$

Clearly the above system is underidentified as there are eight unknowns and only four equations. I now impose Assumptions B6 and B7, which add an additional five equations to the system. Note that the resource shares for the one-child-type households have already been identified (i.e. η^a_{10} is known at this point). This results in the following system of nine equations

$$
\begin{bmatrix}
1+\gamma_{11}^{m}+\gamma_{11}^{f} & 1+\gamma_{11}^{m}+\gamma_{11}^{f} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2(1+\gamma_{21}^{m}+\gamma_{21}^{f}) & 1+\gamma_{21}^{m}+\gamma_{21}^{f} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1+\gamma_{12}^{m}+\gamma_{12}^{f} & 2(1+\gamma_{12}^{m}+\gamma_{12}^{f}) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1+\gamma_{22}^{m}+\gamma_{22}^{f} & 1+\gamma_{22}^{m}+\gamma_{22}^{f} \\
-1 & 0 & \frac{\eta_{10}^{a}}{\eta_{20}^{a}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & \frac{\eta_{10}^{a}}{\eta_{20}^{a}} & 0 \\
0 & -1 & 0 & 0 & 0 & \frac{\eta_{01}^{b}}{\eta_{02}^{b}} & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & \frac{\eta_{01}^{b}}{\eta_{02}^{b}} \\
\frac{1}{\eta_{10}^{a}} & \frac{-1}{\eta_{01}^{b}} & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix}
\eta_{11}^{a} \\ \eta_{11}^{b} \\ \eta_{21}^{a} \\ \eta_{21}^{b} \\ \eta_{12}^{a} \\ \eta_{12}^{b} \\ \eta_{22}^{a} \\ \eta_{22}^{b}
\end{bmatrix}
=
\begin{bmatrix}
1 \\ 1 \\ 1 \\ \frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{bmatrix}
$$

This eight by nine matrix is equal to the matrix Ω'' defined earlier with $\gamma_{s_{ab}}^{t} = \frac{\eta_{s_{ab}}^{m}}{s_a\eta_{s_{ab}}^{a}+s_b\eta_{s_{ab}}^{b}}$, which is nonsingular by Assumption B8. The system can therefore be solved for $\eta_{s_{ab}}^{a}$ and $\eta_{s_{ab}}^{b}$. Resource shares for men and women can then be solved for since $\eta_{s_{ab}}^{t} = \gamma_{s_{ab}}^{t}(s_a\eta_{s_{ab}}^{a} + s_b\eta_{s_{ab}}^{b})$ for $t \; \epsilon \; \{m, f\}$.

CHAPTER 2
CHILD LABOR LAWS AND HOUSEHOLD FERTILITY DECISIONS: EVIDENCE FROM NIGERIA

1 Introduction

What is the effect of child labor laws on child employment? Do child labor laws affect household fertility decisions? Despite the prevalence of these laws, there is limited empirical evidence on their effectiveness in reducing child labor. Moreover, little is known about how a child labor law, through its effect on the economic value of children, affects fertility rates. In this study, I analyze both questions in the context of Nigeria by examining the effect of the Nigerian Child Rights Act of 2003 (CRA) on child employment and household fertility decisions.

Nigeria provides an ideal setting for this analysis. First, child labor is widespread, with UNICEF reporting as many as one third of all Nigerian children currently working (MICS 2011). Furthermore, fertility rates in Nigeria are some of the highest in the world with the average woman having more than 5 children over the course of her lifetime (DHS 2013).[1] Consequently, Nigeria's population is expected to triple by 2050 and is expected become the third largest country in the world.[2] Reducing child labor and fertility rates are thus both goals of the current Nigerian government and whether or not the CRA succeeded in achieving these goals is what I aim to determine.

I begin my analysis by examining the child employment effects of the CRA at both the intensive and extensive margins of employment. To my knowledge, no pre-

[1] As a point of comparison the total fertility rate in India is 2.3 (SRS 2013).

[2] Pew Center Research Report http://www.pewglobal.org/2014/01/30/attitudes-about-aging-a-global-perspective.

vious study has examined the employment effects of child labor laws at the intensive margin using hourly employment data. For identification, I rely on yearly variation in when each Nigerian state adopted the law, and from the age restrictions that determined to whom the child labor law applied. The results suggest that the CRA increased child employment levels at both the intensive and extensive margins. In particular, work outside the household by children age 6-11 increased by an average of 0.337 hours per week. This result is consistent with recent empirical evidence on the employment effects of the Child Labor (Prohibition and Regulation) Act of 1986 in India (Bharadwaj et al. (2013)).

The theoretical foundation for this counterintuitive result can be understood as follows: The CRA, in practice, acts as a tax on firms for employing children, which results in lower child labor demand. Moreover, children have been shown to have negatively sloped labor supply curves (Bhalotra (2007)); parents do not want their children to work, but may be forced to in order to reach some subsistence level of consumption. If child wages decrease due to lower demand, then households are poorer, and farther below the subsistence threshold. To make up for the lost income, children need to work more hours.[3]

Lower child wages due to the CRA are likely to not only affect child employment, but also household fertility decisions as well. In particular, if household demand for children is a positive function of child wages, and the CRA lowered child wages, then fertility rates may have decreased as a result. To make the relationship more concrete, I present a simple model of household fertility decisions based on Rosenzweig and Evenson (1977). I then empirically analyze this relationship by examining how the CRA affected fertility rates. In order to determine how this law impacted fertility rates, I would ideally study how household fertility decisions are impacted by child wages, as that is the hypothesized mechanism. However, given that I do not observe child wages, I attempt a more indirect approach.[4] I first take my previous results that show that child employment levels increased, and I interpret these changes in

[3]See Basu and Van (1998) and Bharadwaj et al. (2013) for a more detailed discussion.

[4]Children often work in the informal sector and are typically paid in kind or not at all. As a result they rarely receive an easily measurable wage.

employment as suggestive evidence that child wages decreased. After establishing the CRA lowered child wages, I next measure the effect of the CRA on fertility and find that it had little to no impact.

This paper relates to two strands of literature; first to studies on the employment effects of child labor laws, and secondly, to research in the growth literature that seeks to understand the causes of the demographic transition. Key empirical studies on the effects of child labor laws on child employment include Moehling (1999) and Bharadwaj et al. (2013). Moehling (1999) examines the effect of U.S. state child labor laws and finds that the laws resulted in a minimal, if any, change in child employment rates. Bharadwaj et al. (2013) analyze the 1986 Child Labor (Prohibition and Regulation) Act in India, and finds that the law increased child employment rates and decreased child wages. The authors extend the child labor model of Basu and Van (1998) to a two sector labor market to account for the fact that the Indian child labor law did not apply to agricultural workers. A similar partial ban was implemented in Nigeria and as in Bharadwaj et al. (2013), labor market frictions play a central role in how the child labor ban impacted sector specific employment levels.

More recently, Piza and Souza (2016) analyze a child labor ban in Brazil and find it reduced child employment rates for boys, with the effects mostly concentrated in the informal sector. Piza and Souza (2016) use a difference-in-difference strategy that relies on variation in the minimum age of child work across time. Their results could be driven by age-group specific time trends that would violate the parallel trends assumption. Specifically, if employment for children below the age threshold in Brazil was trending downward at a faster rate than it was for children above the age threshold, then differences-in-differences would not accurately measure the effect of the child labor law. My study avoids this potential source of bias by using variation in not just the minimum age of child work across time, but also variation in the timing of adoption of the law across Nigerian states.

Overall, there is little empirical evidence that child labor laws reduce child labor, and some (Bharadwaj et al. (2013)) that it increases it. My study contributes to the literature on the effects of child labor laws by using hourly employment data.

While previous work has focused on the extensive margin, the hourly employment data allows me to examine both the extensive and intensive margins and therefore I can more clearly understand changes in employment as a result of the CRA.

The second strand of literature I contribute to is research on the causes of the demographic transition. The economic growth literature has discussed in great detail the relationship between the demographic transition, the demand for human capital, and the quantity-quality tradeoff (Galor and Weil (2000), Hazan and Berdugo (2002), Doepke and Zilibotti (2005)). Moreover, growth economists have also recognized that in the past, declines in fertility rates have typically coincided with falling child employment rates. As a result, they have suggested that child labor laws can theoretically speed the demographic transition. For example, Doepke (2004) calibrates a model of economic growth and fertility decline focusing specifically on the effects of child labor and compulsory schooling laws and finds that countries that require children to attend school and ban child labor go through the demographic transition sooner than countries that do not enact these policies. Doepke (2004) achieves these results by incorporating the quantity-quality tradeoff into a model of economic growth. I build off his results by empirically analyzing the relationship between child labor laws and fertility using household-level data, which to the best of my knowledge, has not been done. This paper is therefore the first to provide a causal analysis of a relationship that is potentially relevant to both population and child welfare policy.

The paper is structured as follows: In Section 2, I describe in detail the CRA. In Section 3, I then provide a brief motivating model that illustrates the relationship between the economic value of children, child labor laws, and household fertility decisions. Section 4 summarizes the data. I then discuss my empirical strategy in Section 5, before moving to the results in Section 6. Finally, I conclude with a discussion of my results in Section 7.

2 The 2003 Child Rights Act

In 2003, the Nigerian legislature passed the CRA. The law was the culmination of years of effort to bring Nigeria in line with international standards on child rights.

While Nigeria had previously adopted the United Nation's Convention on the Rights of the Child (CRC) in 1991 and the African Union Charter on the Rights and Welfare of the Child (CRCW) in 2001, the enforcement of these treaties was nonexistent.[5] This lack of enforcement generated political pressure to enact a law designed to implement the principles and laws established in these international treaties, and in response, the Nigerian National Assembly passed the CRA in July, 2003.

The CRA covers many topics related to child welfare, including child trafficking, adoption, conscription, corporal punishment, and child marriage. For the purposes of this study, the key provisions of the CRA are the sections regarding child labor. While Nigeria had previously enacted the Labour Act of 1990 which set the minimum age of work to 12, the monetary penalty for violating the law was only 100 Naira (∼20 USD) and there was no criminal penalty. The CRA increased the fine to as much as 50,000 Naira (∼466 USD) and a prison term of up to five years.[6] Overall, the CRA left much of the Labour Act in place, but dramatically increased the penalties for employing children. The relevant clauses of Sections 28.2-28.3 of the CRA are given below:

> No child shall be...employed to work in any capacity except where he is employed by a member of a family on light work of an agricultural, horticultural or domestic character...Any person who [violates the child labor regulations] is liable on conviction to a fine not exceeding fifty thousand Naira or imprisonment for a term of five years or to both such a fine and imprisonment. (pg. 6)

Furthermore, the CRA specifies that the "provisions relating to young persons in Sections 58, 59, 60, 61, 62 and 63 of the Labour Act shall apply to children under this Act." These sections define different age thresholds for different types of works; Children age 14 and under are prohibited from being employed unless they work on a "day-to-day basis" with a "daily wage" and "return each night to the place of residence of his parents or guardian" at a reasonable time. Children age 15 and under

[5] See Nwapi (2011).

[6] In 1990 prices, the CRA fine is roughly 2,500 Naira.

are prohibited from doing any "industrial undertaking."[7]

The CRA is written is such a way that there is some ambiguity over the minimum age of work. While the Labour Act defines a child to be anyone under age 12, the CRA classifies children as individuals under the age of 18. However, this does not mean the minimum age of work is 18; The CRA explicitly states that certain clauses from the Labour Act that specify various age thresholds below age 18 apply to the CRA. Discussions with various policymakers familiar with the CRA confirm that the age thresholds are determined by the Labour Act. The reason children are defined as individuals below age 18 is related to other provisions of the CRA, especially those governing conscription and child marriage.

Nigeria has a federal system of government similar to the United States. For legislation impacting child rights, the federal government passes laws, but it is the responsibility of the 36 Nigerian States to adopt and enforce them.[8] As of 2013, only 23 states had adopted the law.[9] Table 2.1 shows which states of adopted the CRA and provides the timing of the adoption. The variation in when each state has adopted the law is central to this study's identification strategy. Importantly, the decision of each state to adopt the CRA was primarily related to popular views of child marriage and therefore likely independent of the state's view of child labor.[10] Non-adopting CRA states also opposed sections dealing with child adoption and family courts. These issues are discussed extensively in the Nigerian legal literature.[11] To assuage any potential concern that states decided not to adopt the CRA due to their views on child labor, I use several specifications where the analysis is conducted solely using a sample of states that did adopt the law, and rely on variation in the timing of adoption to identify its effect.

As in other developing countries, enforcement of employment laws is far from per-

[7]What constitutes an industrial undertaking is left undefined.

[8]The capital territory, FCT-Abuja, automatically adopts any law passed by the federal government.

[9]In 2014 Jigawa, which had previously adopted the CRA, put the law "under review".

[10]The predominantly Muslim states in the north follow sharia law in family and sometimes criminal legal matters. According to Sharia law, girls are allowed to be married at age 12, which drastically conflicts with the minimum age of marriage set in the CRA of 18.

[11]See Akinwumi (2010), Ogunniran (2010), Kawu and Abdur-Rahman (2014) for example.

fect. Government statistics on the number of child labor violations and convictions are difficult to find. This is especially hard when evaluating the CRA because enforcement is at the state-level and thus require state-level statistics. What is clear is that enforcement of the CRA is limited as children still work at high rates. There is, however, circumstantial evidence that the law is at least being partially enforced. This is supported by the significant effort NGOs in Nigeria, and in particular UNICEF, have made in lobbying all states to adopt the CRA. For the purposes of this study, the law only has to be partially enforced to have some effect on child labor and fertility.

3 Model

This section presents a model to formalize the potential relationship between child labor laws and household fertility decisions. I describe a simple household that makes a static decision regarding child labor, child education, and fertility that closely follows Rosenzweig and Evenson (1977). The goal is to show how a child labor law, through its effect on child wages, can decrease the demand for children.

The household is modeled using the household production framework of Becker (1965). Parents are altruistic and derive utility from the number of children they have, the level of education and leisure of their children, and some composite commodity representing the household's standard of living. Children are therefore considered normal goods as in Becker and Lewis (1974). The household's utility is given below:

$$U = U(Z_n, Z_e, Z_l, Z_s)$$

where Z_n is the number of children, Z_e is the level of schooling, Z_l is child leisure, and Z_s is a composite commodity representing the household's standard of living. Each good is produced using a linear homogenous production function.

Consider a nuclear household with a husband, a wife, and Z_n children. The husband and wife work T_{m_h} and T_{m_w} units of time in market work at wages W_h and W_w respectively. The wife can also spend time producing children T_{n_w} or the composite commodity T_{s_w}. Each child works T_{m_c} units of time in market work at wage W_c and also spends time producing their own education T_{e_c} and leisure T_{l_c}.[12]

The production functions of each good are given below:

$$Z_e = \gamma_e(X_e, T_{e_c}), \;\; Z_l = \gamma_l(X_l, T_{l_c})$$

$$Z_n = \gamma_n(X_n, T_{n_w}), \;\; Z_s = \gamma_s(X_s, T_{s_w})$$

Here, aggregated bundles of goods are given by X_i and are purchased at market price P_i, $i = n, e, l, s$.

Each child has T_c units of total time to allocate between market work, education, and leisure. For simplicity the husband is assumed to spend all his time T_h in market work. The wife has T_w units of time to allocate between market work, producing children, and producing the composite household commodity. The household budget constraint can then be written as follows:

$$\begin{aligned} W_cT_cZ_n + W_wT_w + W_hT_h =& Z_n(p_nx_n + t_{n_a}W_w) + Z_nZ_e(p_ex_e + t_{e_c}W_c) + \\ & Z_s(p_sx_s + t_{s_a}W_w) + Z_nZ_l(p_lx_l + t_{l_c}W_c) \end{aligned}$$

where x_j and t_{j_i} are the marginal price and input coefficients.

Maximizing the utility function subject to the budget and time constraints gives the following shadow prices:

$$\begin{aligned} \pi_n =& p_nx_n + t_{n_a}W_w - t_{m_c}W_c + Z_ep_ex_e + Z_lp_lx_l \\ \pi_e =& Z_n(p_ex_e + t_{e_c}W_c) \\ \pi_l =& Z_n(p_lx_l + t_{l_c}W_c) \\ \pi_s =& p_sx_s + t_{s_a}W_w \end{aligned}$$

The shadow price of children, π_n, is therefore a negative function of child wages, which is the key mechanism through which the CRA affects fertility rates.

[12]Including adult leisure would not change the model implications and are therefore ignored for simplicity.

Rosenzweig and Evenson (1977) show that the uncompensated substitution elasticity of the number of children with respect to child wages is given as follows:

$$\eta^N_{W_c} = \tilde{\eta}^N_{W_c} + \frac{W_c Z_N T_{m_c}}{F}\epsilon_N \tag{2.1}$$

where $\tilde{\eta}^N_{W_c}$ is the compensated substitution elasticity of the number of children with respect to child wages, F is full income, and ϵ_N is the pure income elasticity of N. Then $\eta^N_{W_c}$ is unambiguously postive since an increase in child wages would decrease the relative price of child quantity and increase household income. Ultimately I will qualify this implication by noting that female wages also enter the shadow price of children, and that changes in the female wage as a result of the CRA can potentially also impact fertility rates. In the following section I describe in more detail how a child labor law affects fertility rates.

3.1 Adding a Child Labor Law

Following Basu (2005), I interpret a child labor law as a tax on child labor. More specifically, with some probability p, each firm is fined F dollars for employing children. pF can be interpreted as the expected fine a firm faces when employing children. The expected cost of employing children is $W_c + pF$. An increase in the expected fine will then result in an increase in the marginal cost of employing children. Labor demand for children will then decrease in response to an increase in pF. With labor demand shifting down, equilibrium child wages will also decrease. Equation (2.1) implies that this decrease in child wages results in households demanding fewer children.

The model therefore suggests the CRA should lead to a reduction in fertility rates. A potential complication to this reasoning is that the CRA could impact the adult labor market in addition to the child labor market. The effect on the adult labor market can be understood as follows: children are now working more as a result of lower wages,[13] giving firms the ability to substitute child labor for adult labor, therefore decreasing the demand for adult workers.[14] With decreased adult

[13]Recall children have backwards bending labor supply curves.

[14]This assumes adults and children are substitutes in the production process, which is consistent

labor demand, equilibrium adult wages are now lower. The shadow price of producing children is an increasing function of female wages, and therefore a decrease in adult wages lowers the opportunity cost for women to have children. Thus, changes in adult wages as a result of the CRA may offset the fertility effect of changes in child wages.

4 Data

To examine the effect of the CRA on child labor, I use the third and fourth rounds of the Nigeria Multiple Indicator Cluster Survey (MICS) which took place in 2007 and 2011, respectively.[15] I take advantage of the detailed information on child employment; parents are asked whether their children work for anyone outside the household, and if so, for how many hours. Similar questions are asked about work for household enterprises or household farms, and household chores.[16] With this data, I can examine the impact of the CRA on employment at both the intensive and extensive margin. The data also includes demographic and socioeconomic information on the household including, parent's education, religion, household size, urban or rural status, the household composition, and a computed wealth index.

Table 2.2 provides descriptive statistics from the third and fourth waves of the MICS. Among children age 6-11, 14 percent do some work outside the household and 23 percent of children work on a household farm or household enterprise. The amount of time per week these children work varies considerably. The majority however work less than 7 hours a week which is consistent with children working and attending school at the same time.

To examine the effect of the CRA on fertility, I use the first four rounds of the Nigeria Demographic and Healthy Survey[17] (DHS) which took place in 1990,[18] 2003, 2008, and 2013. The DHS is a national sample survey that provides detailed infor-

with recent empirical evidence (Doran (2013)). This assumption is central to the Basu and Van (1998) and Bharadwaj et al. (2013) models of child labor.

[15]The first and second waves do not include sufficiently detailed child labor data.

[16]Chores include fetching wood, fetching water, childcare, cooking, and cleaning.

[17]While many DHS surveys include information on child labor, the Nigeria DHS unfortunately does not.

[18]18 of Nigeria's 36 states were created after 1990. I therefore use DHS GPS data to convert each woman's 1990 state of residence to their 2013 state of residence.

mation on female wellbeing, including topics ranging from fertility and marriage to child health and domestic violence. To study the effects of the CRA on fertility, I exploit the birth history information in the DHS. Specifically, I know in any given year whether a woman gave birth. This allows me to construct a woman-year panel of each woman's birth history. I focus on married women age 19-49 to alleviate any concerns about changes in minimum age of marriage laws affecting the analysis.

Table 2.3 demonstrates the high birth rates in Nigeria. In 2013 for example, 18 percent of women age 15-49 had given birth in the prior year. The total fertility rate has not fallen considerably since 1990. Lastly, most of the observations come from the 2008 and 2013 waves which is useful given that that is the period with the most variation in state adoption of the CRA.

Figures 1 and 2 use an event study framework to illustrate graphically how trends in birth rates have changed as a result of the CRA. I regress a measure of fertility on time relative to adoption of the CRA, while controlling for age, age squared, wealth, education, year, and age of first marriage. In Figure 1, I plot the coefficients on time relative to adoption, where I use an indicator for whether the woman gave birth as the dependent variable. Similarly, Figure 2 uses the same framework, except with the number of living children as the dependent variable. In both figures, there is little change consistent with what is found in the empirical results. The rise towards the end of the figure is due to the composition of states that are in the treatment group changing over time.

5 Empirical Strategy

5.1 Employment Effects of CRA

To estimate the effect of the CRA on child employment, I use a difference-in-difference-in-differences (DDD) empirical strategy. I compare hours worked for children under 12 to children 12 and older in states that have adopted the CRA to states that have not, before and after the CRA was enacted. I rely on variation in which states adopted the law, the timing of the adoption, and the age group to which the law applied. As stated earlier, the CRA penalizes firms for employing children only if the child is younger than 12 years old. I am therefore following the same technique employed by Moehling (1999).

For this estimation strategy, the traditional DD empirical strategy using state and time differences is modified to include a separate treatment and control group within each treated state. The assumptions for estimation of DDD are weaker than for DD. For DD, the key identifying assumption is that in the absence of the adoption of the CRA, the differences in employment rates for children under 12 in treatment and control states would be the same in period t as in period $t+1$, or what is known in the literature as the parallel trends assumption. This is likely to be violated given that Nigerian states differ in their industry structure and there is a strong geographic relationship between which states have and have not adopted the CRA.

With DDD instead, any time trend specific to the treated states is differenced out. Here, the key identifying assumption is that any time trend specific to the treated states is the same for children under 12 as it is for children 12 and over. Possible violations of this assumption are significantly more difficult to find, and thus I use DDD to identify the parameters of interest.

The following regression is then used:

$$\begin{aligned} Y_{istg} &= \beta_{DDD}(Treat_{st} \times YoungAge_g) \\ &+ Post_t \times \phi_s + \psi_{sg} + \gamma_{gt} + X_i\delta + \epsilon_{istg} \end{aligned} \tag{2.2}$$

where Y_{istg} is an indicator for whether child i in state s in year t of age group g is employed. In other specifications it is a measure of hours worked. $Treat_{st}$ is an indicator equal to 1 if the state had passed the law prior to year t. I allow the year effects to vary by state with $Post_t \times \phi_s$, where $Post_t$ is an indicator equal to 1 if the year is 2011. This variable accounts for any state specific year effects that are common across age groups. I also include state-age group and age group-year fixed effects, ψ_{sg}, and γ_{gt}, which account for any age group specific state effects common across time, and any age group specific time effects that are common across states. $YoungAge_g$ is an indicator for whether the child is in the treated age group (i.e. under age 12). In other specifications, I interact $Treat_{st}$ with age dummies. I include a vector of covariates X_i which consists of age, religion, household wealth, parental characteristics, and gender. Standard errors are clustered at the state level with 37 total clusters.[19] The parameter of interest is β_{DDD} which represents the average treatment effect on the treated.

One modification to the above estimation strategy that is likely to lead to more robust results is to exclude states that have never adopted the CRA. These states are almost exclusively located in Nigeria's northern region, and are poorer and significantly more Muslim than the mostly Christian south. The difference in religion is key, as the mainly Muslim states implemented Sharia law in 2000 and are under a different legal system. Therefore, omitting these states and using only states that have adopted the CRA by 2007 as a control group could lessen concerns about different time trends unique to certain age groups in the northern states.[20]

5.2 Fertility Effects of CRA

To examine the effect of the CRA on fertility I again exploit the quasi-experimental nature of the law's implementation. I begin by constructing a woman-year panel of birth histories using four waves of the DHS (1990, 2003, 2008, 2013). Using this data, I know for each woman in any given year both her number of living children and whether or not she gave birth in the previous year. I use variation in which states adopted the CRA, and in the timing of the adoption, to estimate a difference-in-differences model. This allows me to identify the effect of the CRA on fertility rates provided treatment and control states would follow the same time trend in the absence of the treatment. While making this assumption is not ideal, I am able to weaken this assumption by including state specific time trends.

The following specification is used to identify the effect of the CRA on total fertility:

$$Y_{ist} = \alpha + \gamma_t + \lambda_s + \phi_s \times t + \beta_{DD} Treat_{s,t-1} + W_i \delta + \epsilon_{ist} \tag{2.3}$$

[19] There are 36 Nigerian states and the Federal Capital Territory.

[20] The data is from 2007 and 2011, and 14 states adopted the CRA between 2003 and 2007, while 10 did so between 2007 and 2011.

where Y_{ist} is the number of living children for woman i in state s in year t. $Treat_{s,t-1}$ is an indicator variable equal to 1 if the woman is living in a state that adopted the CRA in the previous year. This accounts for the fact that childbirth takes nine months. Lastly, I include a vector of covariates W_i which includes age, age squared, age at first marriage, years of education, household wealth, religion, and whether the woman lives in an urban area.

As noted earlier, the CRA also raised the minimum age of marriage to 18. This change could also affect fertility rates. To account for this potential bias, I exclude women younger than 19 from the analysis, and control for age of first marriage.

To determine if the length of exposure to the CRA impacts fertility rates, I interact $Treat_{s,t-1}$ with the number of years exposed to the law. In principle, women who have lived in a CRA state for longer should have fewer children relative to those who have done so for a shorter amount of time. I therefore modify equation (3.12) as follows:

$$Y_{ist} = \alpha + \gamma_t + \lambda_s + \phi_s \times t + \beta_{DD}(Treat_{s,t-1} \times Exposure_{s,t-1}) + W_i\delta + \epsilon_{ist} \quad (2.4)$$

where $Exposure_{s,t-1}$ is the number of years since state s adopted the CRA in year $t-1$.

For robustness, I modify the estimation strategy following Güneş (2016) by restricting the sample to women above age 25 at the time of the survey, and using the number of children born by age 25 as the dependent variable. This is due to the fact that the fertility history of women in the DHS is censored in the sense that I do not observe completed fertility for the majority of the women in the sample. The treatment variable for this specification is therefore the number of years woman i was exposed to the CRA prior to the woman's 25th birthday.[21]

[21] Using alternative age thresholds yielded qualitatively similar results.

6 Results

6.1 Child Employment

To examine the effect of the CRA on child labor, I study changes in employment at both the extensive and intensive margins, and look at three different types of work; (1) household chores, (2) work for a household farm or enterprise, or (3) work outside the household. As discussed in Section 2, the CRA penalized firms and individuals for employing children outside their household.

Table 2.4 reports the DDD coefficients from specification (3.11). Here, any time trend specific to the treatment states is differenced out. The key identifying assumption is that any treatment state specific trend is the same for children under 12, and children 12 and older. The results show the effect of the CRA on weekly hours in work outside the household, chores, and for a household enterprise or farm. Odd numbered columns include separate state and year fixed effects, while in even numbered columns, state-year fixed effects are included as well. The sample includes all children age 6-15. The object of interest is found in the first row, which corresponds to β_{DDD} in equation (3.11). The results here suggest work outside the household increased in response to the CRA. More specifically, the results in column (2) suggest that on average, children age 6-11 are working 0.343 hours more per week outside the household, which is equivalent to a 24 percent increase above the pretreatment control group mean. This seemingly counterintuitive result is consistent with the Basu and Van (1998) model of child labor. A plausible explanation for this result is that the CRA lowered child wages, resulting in families falling further below some subsistence level of consumption. To compensate for the lost income, households responded by sending their children to work for longer hours.

Furthermore, columns (3) and (4) show that child hours worked in chores increased roughly 2 hours per week. What explains this large increase? A ban on work outside the household lowers wages in that sector, and therefore impacts the benefits of working outside the household relative to participating in work in chores, or for household enterprises or farms. Children should then reallocate their labor to chores and work in household enterprises or farms until wages equalize in all sectors.[22] Secondly, the CRA only applied to work outside the household, making this transition in sectors more likely. The lack of any impact of the CRA on work in household enterprises or farms, and the fact that work outside the household increased, suggests that there are likely labor market frictions preventing this reallocation.

Table 2.5 presents a similar specification to what is found in Table 2.4, except now the treatment variable is interacted with age dummies, as opposed to an indicator for whether the child is below age 12. The reference category is the interaction between treatment and children age 12. The results demonstrate that most of the effect of the law is concentrated among the youngest children. In particular, the results suggest that on average, children age 6 are working 0.571 more hours outside the household per week after the implementation of the CRA after differencing out time effects and changes in hours worked among 12 year olds in treated states. On the other hand, the CRA seems to have had little effect on the difference in hours worked between 11 and 12 year olds in treated and untreated states. This could be because labor hours for the younger children were so low to begin with that they had more room to grow. Alternatively, this could simply be because children age 11 look similar to 12 year old children, and therefore the law did not affect differences in treatment between them.

Table 2.6, like Table 2.4, presents results from specification (3.11). However, the sample is now limited to states that have at any point adopted the CRA, and the results rely on variation in the timing of adoption. The treated states are defined as any state that adopted the law between 2007 and 2010, while control states consist of any state that adopted the CRA prior to 2007. The motivation behind this subsample analysis is to eliminate any unobserved differences between the treated and never treated states specific to different age groups. The results here are mostly consistent with the results presented in Table 2.4; on average, children age 6-11 are working 0.369 hours more per week outside the household, which is nearly identical to the corresponding estimate in Table 4. The magnitude of the effect on chores is slightly smaller, with children age 6-11 working 1.723 hours more per week as a result of the

[22] For a formal model and analyses of these mechanisms see Bharadwaj et al. (2013)

CRA.

Moving to the extensive margin of employment, Table 2.7 presents probit estimates of equation (3.11) where the dependent variables are indicators for whether the child did any of the three types of work during the previous week. The given coefficients represent the marginal effects on the binary employment outcomes. The results again suggest that the CRA increased work outside the household. In particular, employment outside of the household is increased by 4.1 percentage points for children under age 12 who were exposed to the CRA. Why might children who were not previously working now begin working after a decrease in child wages? Imagine a 9 year old child with an employed 11 year old sibling. Now that the older sibling has lower wages, the household decides to both increase the 11 year olds hours, *and* send the 9 year old to work as well. Studies without hourly employment data have relied on this mechanism to analyze the employment effects of child labor laws. For example, Bharadwaj et al. (2013) examines the 1986 Child Labor (Prohibition and Regulation) Act in India using National Sample Survey employment data, which only contains information on whether or not the child works (in addition to child wages). Their model suggests that child employment will change as a result of changes in sibling wages. Thus, they categorize a child as treated if the child has a sibling under the minimum age of work, and see how this law affects child employment at the extensive margin. My results are consistent with their findings.

6.2 Fertility

The results from the previous subsection suggest that the CRA resulted in a small increase in employment levels. In effect, the CRA functions as a negative shift in the demand for child labor, which together with a negatively sloped child labor supply curve results in increased child employment. These results therefore provide circumstantial evidence that child wages decreased as well following the passage of the law. Given that the model discussed in Section (3) suggests child wages are an input in household fertility decisions, I expect there to potentially be a corresponding effect of the CRA on fertility.

Table 2.8 presents results from specifications (3.12) and (3.13). Here the dependent variable is the number of children woman i has given birth to by year t. I am therefore examining the effect of the CRA on total fertility. Columns (1) and (2) restrict the effect of the CRA to be independent of the length of exposure to the law, while columns (3) and (4) allow the treatment effect to vary by the duration of exposure. The object of interest is found in the first row, which corresponds to β_{DD} in equations (3) and (4). In column (1), the results suggest that the CRA caused a reduction of 0.298 in the number of children born to a woman in a treated state. However, after accounting for state specific time trends, this effect disappears. Columns (3) and (4) follow a similar pattern.

For robustness, I restrict the sample to women age 25 and over and use total children born by age 25 as the dependent variable. This accounts for any issues arising due to total fertility being censored for most women in the sample. The results for this specification are presented in Table 2.9. Again there is negative effect on total fertility when state specific time trends are unaccounted for, but no effect when they are included. Overall, the results suggest that the CRA had little impact on fertility rates.

7 Discussion

The CRA seems to have resulted in increased child employment, clearly contrary to the intended purpose of the law. This suggests that the CRA did have some effect on child wages, which should have an impact on household fertility. However, the data provide no statistical evidence of any effect of the CRA on fertility rates. There are a myriad of possible reasons for this. First, the change in child wages may have been fairly small, and therefore any corresponding effect on fertility would also be low in magnitude. Secondly, the effect of the law on fertility rates may not be immediate. Implementation of the CRA is a slow process and the infrastructure needed to enforce such a wide ranging law takes time to establish. For this reason, meaningful fertility effects may not become evident until several years after the passing of the law. Unfortunately, it is not easy to identify long term effects of the CRA at this time given the data available.

Another reason fertility may not have declined as a result of the CRA, and discussed in Section 3, involves the adult labor market. With the quantity of child labor increasing, firms now demand fewer adult workers, which lowers adult wages. The opportunity cost for women of having children is now lower, and therefore they have more children.

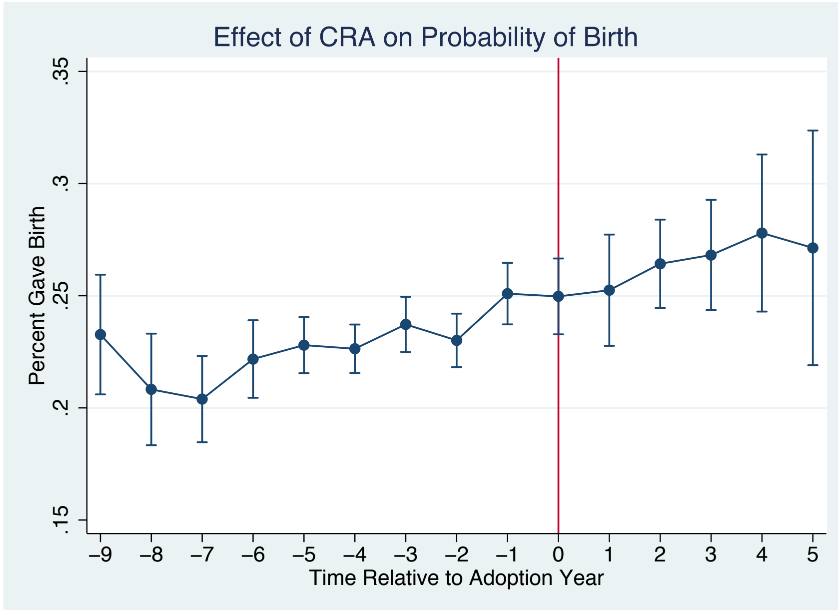
Lastly, the CRA may not have impacted fertility rates is that women simple lack the means of easily reducing their fertility. As of 2013, only 26 percent of married women of child bearing age who did not want children within the next two years used any contraceptive method (DHS 2013).

8 Conclusion

This study examines the impact of the Nigerian Child Rights Act of 2003 on both child labor and fertility rates. The CRA was a landmark piece of legislation that significantly increased the penalty for employing children under age 12. The theoretical model presented in Section 3 suggests that such a child labor law should impact fertility rates as a result of changes in the economic value of children. The results are not conclusive. I find the CRA likely resulted in increased child employment outside the household consistent with the Basu and Van (1998) and Bharadwaj et al. (2013) models of child labor. However, the empirical evidence does not suggest there is a corresponding effect on fertility rates. This could be due to offsetting effects that I am unable to disentangle.

This paper contributes to the existing empirical literature on the employment effects of child labor laws. Moreover, it is the first to analyze child employment at the intensive margin. This study also focuses on Nigeria, an understudied country of growing global importance. Secondly, this is the first paper to empirically study the relationship between child labor laws and fertility rates using household level data. While the results provide no clear answer to the relationship between child labor and fertility rates, it is an important first step in understanding a relationship that is central to models of economic growth and the demographic transition.

Figure 2.1: Effect of CRA on Probability of Giving Birth



Notes: This figure plots the β_k coefficients from the following regression, with the year of adoption of the CRA as the omitted year: $Y_{ist} = \sum_{k=-9}^{5} \beta_k * 1[Year_t = k] + W_i\delta + \gamma_t + \epsilon_{ist}$. Y_{ist} is an indicator for giving birth for woman i in state s in year t. 95% confidence intervals. The above figures are for treated states only.

Table 2.1: Timing of CRA Adoption

CRA States		Non-CRA States
State	Year of Adoption	State
FCT-Abuja	2003	Adamawa
Anambra	2004	Bauchi
Ebonyi	2004	Borno
Imo	2004	Enugu
Ogun	2004	Gombe
Abia	2007	Kaduna
Bayelsa	2007	Kano
Jigawa	2007	Katsina
Kwara	2007	Kebbi
Nasarawa	2007	Sokoto
Ondo	2007	Yobe
Plateau	2007	Zamfara
Rivers	2007	
Taraba	2007	
Akwa Ibom	2008	
Edo	2008	
Lagos	2008	
Oyo	2008	
Benue	2010	
Cross River	2010	
Delta	2010	
Kogi	2010	
Niger	2010	
Osun	2010	

Notes: No single source contained all years of adoption. The timing of adoption was collected from various ILO, UNICEF, and World Bank documents. FCT-Abuja, the capital city, automatically adopted the CRA following the National Assembly's passing of the CRA in July 2003. Jigawa is no longer abiding by the CRA as the state legislature put the CRA "under review" in 2013.

Table 2.2: Descriptive Statistics (MICS)

	Mean	**Std. Dev.**	**Min.**	**Max.**
Employment				
Outside Household	0.14	0.35	0	1
Chores	0.72	0.45	0	1
Household Enterprise or Farm	0.23	0.42	0	1
Weekly Hours Conditional on Working				
Outside Household	6.22	7.56	1	76
Chores	8.01	7.81	1	80
Household Enterprise or Farm	7.17	7.40	1	77
Enrolled in School	0.71	0.45	0	1
Household Characteristics				
Muslim	0.54	0.50	0	1
Father's Education	0.89	0.88	0	2
Mother's Education	0.73	0.84	0	2
Urban	0.29	0.46	0	1
Female	0.49	0.50	0	1

Notes: Children age 6-11. MICS 2007 and 2011

Table 2.3: Descriptive Statistics (DHS)

Survey Year	1990	2003	2008	2013
Gave Birth$_{t-1}$	18.63	17.44	17.73	18.07
Living Children	2.62	2.37	2.47	2.54
Marital Status				
Single	20.61	28.81	25.38	26.68
Monogamously Married	49.64	47.38	50.35	50.58
Polygamously Married	29.75	23.82	24.28	22.74
Religion				
Muslim	48.62	47.26	46.28	47.70
Education Level				
No Education	51.70	39.44	39.66	35.28
Primary	24.05	21.86	19.74	18.24
Secondary	21.76	32.31	32.66	36.99
Higher	2.48	6.39	7.93	9.49
Residence				
Rural	59.80	59.88	68.58	60.09
Sample Size	8,781	7,620	33,385	38,948

Notes: All values are percents. Sample includes all women age 15-49. DHS 1990, 2003, 2008, and 2013

Table 2.4: Effect of CRA on Hours Worked

Dependent Variable	Outside HH		Chores		HH Enterprise or HH Farm	
	(1)	(2)	(3)	(4)	(5)	(6)
$Treat_{st} \times YoungAge_g = 1$	0.393***	0.343***	1.873**	2.139***	0.385	0.346
	(0.095)	(0.088)	(0.557)	(0.618)	(0.221)	(0.210)
$Post_t \times YoungAge_g = 1$	-0.332***	-0.281***	-1.578***	-1.631***	0.156	0.116
	(0.084)	(0.079)	(0.255)	(0.251)	(0.139)	(0.144)
$Treat_{st}$	-0.103		-5.422***		-2.063***	
	(0.245)		(1.232)		(0.413)	
Sample Size	56,628	56,628	56,628	56,628	56,628	56,628
Pretreatment Control Group Mean	1.439	1.439	3.685	3.685	1.613	1.613
State-Age Group Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
State Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
State-Year Fixed Effects		Yes		Yes		Yes

* p<0.05, ** p<0.01, *** p<0.001. Notes: $Treat_{st}$ is an indicator equal to 1 if the year=2007 and the state was treated prior to 2007 or if the year=2011 and the state was ever treated. $YoungAge_g$ is an indicator equal to one if the child is age 6-11, 0 if the child is age 12-15. Standard errors clustered at the state-year level. Covariates include gender, age, mother's education, religion, wealth, household composition controls, and whether the respondent lives in an urban area.

Table 2.5: Effect of CRA on Hours Worked by Age

Dependent Variable	Outside HH		Chores		HH Enterprise or HH Farm	
	(1)	(2)	(3)	(4)	(5)	(6)
$Treat_{st} \times Age = 6$	0.642***	0.642***	2.900**	2.900**	0.482	0.482
	(0.161)	(0.161)	(0.957)	(0.957)	(0.544)	(0.544)
$Treat_{st} \times Age = 7$	0.430**	0.430**	2.111**	2.111**	0.236	0.236
	(0.152)	(0.152)	(0.696)	(0.696)	(0.518)	(0.518)
$Treat_{st} \times Age = 8$	0.343*	0.343*	1.170	1.170	-0.302	-0.302
	(0.137)	(0.137)	(0.624)	(0.624)	(0.469)	(0.469)
$Treat_{st} \times Age = 9$	0.160	0.160	1.952**	1.952**	-0.765	-0.765
	(0.190)	(0.190)	(0.678)	(0.678)	(0.583)	(0.583)
$Treat_{st} \times Age = 10$	0.150	0.150	0.379	0.379	-0.206	-0.206
	(0.146)	(0.146)	(0.499)	(0.499)	(0.530)	(0.530)
$Treat_{st} \times Age = 11$	-0.081	-0.081	1.022	1.022	0.196	0.196
	(0.306)	(0.306)	(0.849)	(0.849)	(0.600)	(0.600)
$Treat_{st} \times Age = 13$	-0.079	-0.079	-0.225	-0.225	-1.333	-1.333
	(0.241)	(0.241)	(0.435)	(0.435)	(0.735)	(0.735)
$Treat_{st} \times Age = 14$	0.082	0.082	-0.253	-0.253	-0.342	-0.342
	(0.179)	(0.179)	(0.654)	(0.654)	(0.720)	(0.720)
$Treat_{st} \times Age = 15$	-0.370	-0.370	-0.174	-0.174	-0.556	-0.556
	(0.251)	(0.251)	(0.716)	(0.716)	(0.568)	(0.568)
Sample Size	56,628	56,628	56,628	56,628	56,628	56,628
Pretreatment Control Group Mean	1.439	1.439	3.685	3.685	1.613	1.613
State-Age Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
State Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
State-Year Fixed Effects		Yes		Yes		Yes

* p<0.05, ** p<0.01, *** p<0.001. Notes: $Treat_{st}$ is an indicator equal to 1 if the year=2007 and the state was treated prior to 2007 or if the year=2011 and the state was ever treated. The omitted age group is children age 12. Coefficients represent marginal effects. Standard errors clustered at the state-year level. Covariates include gender, age, mother's education, religion, wealth, household composition controls, and whether the respondent lives in an urban area.

Table 2.6: Effect of CRA on Hours Worked Treated State Subsample

Dependent Variable	Outside HH		Chores		HH Enterprise or HH Farm	
	(1)	(2)	(3)	(4)	(5)	(6)
$Treat_{st} \times YoungAge_g = 1$	0.434**	0.369**	1.491*	1.723*	0.316	0.283
	(0.132)	(0.115)	(0.690)	(0.749)	(0.309)	(0.308)
$Post_t \times YoungAge_g = 1$	-0.375**	-0.312**	-1.173*	-1.187*	0.229	0.188
	(0.122)	(0.107)	(0.477)	(0.486)	(0.254)	(0.266)
$Treat_{st}$	-0.645**		-4.221**		-1.761***	
	(0.208)		(1.372)		(0.439)	
Sample Size	28,590	28,590	28,590	28,590	28,590	28,590
Pretreatment Control Group Mean	0.723	0.723	5.182	5.182	1.842	1.842
State-Age Group Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
State Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
State-Year Fixed Effects		Yes		Yes		Yes

* p<0.05, ** p<0.01, *** p<0.001. Notes: $Treat_{st}$ is an indicator equal to 1 if the year=2007 and the state was treated prior to 2007 or if the year=2011 and the state was ever treated. $YoungAge_g$ is an indicator equal to one if the child is age 6-11, 0 if the child is age 12-15. Standard errors clustered at the state-year level. Covariates include gender, age, mother's education, religion, wealth, household composition controls, household composition controls, and whether the respondent lives in an urban area. I exclude all states that have never passed the CRA.

Table 2.7: Effect of CRA on Employment (Probit)

Dependent Variable	Outside HH		Chores		HH Enterprise or HH Farm	
	(1)	(2)	(3)	(4)	(5)	(6)
$Treat_{st} \times YoungAge_g = 1$	0.054**	0.041**	0.001	-0.006	-0.028	-0.018
	(0.017)	(0.016)	(0.018)	(0.0178)	(0.020)	(0.019)
$Post_t \times YoungAge_g = 1$	-0.022*	-0.019	-0.071***	-0.059***	0.006	0.001
	(0.009)	(0.010)	(0.008)	(0.008)	(0.012)	(0.011)
$Treat_{st}$	0.025		0.011		-0.035	
	(0.033)		(0.034)		(0.039)	
Sample Size	56,587	56,587	56,587	56,587	56,587	56,587
Pretreatment Control Group Mean	0.148	0.148	0.668	0.668	0.246	0.246
State-Age Group Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
State Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
State-Year Fixed Effects		Yes		Yes		Yes

* p<0.05, ** p<0.01, *** p<0.001. Notes: $Treat_{st}$ is an indicator equal to 1 if the year=2007 and the state was treated prior to 2007 or if the year=2011 and the state was ever treated. $YoungAge_g$ is an indicator equal to one if the child is age 6-11, 0 if the child is age 12-15. Standard errors clustered at the state-year level. Covariates include gender, age, mother's education, religion, wealth, household composition controls, and whether the respondent lives in an urban area.

Table 2.8: Effect of CRA on Number of Children

	(1)	(2)	(3)	(4)
$Treat_{st}$	-0.298**	0.054		
	(0.084)	(0.055)		
$Treat_{st} \times Exposure_{st}$			-0.067**	-0.003
			(0.019)	(0.015)
Sample Size	61,170	61,170	61,170	61,170
Year Fixed Effects	Yes	Yes	Yes	Yes
State Fixed Effects	Yes	Yes	Yes	Yes
State Specific Time Trends		Yes		Yes

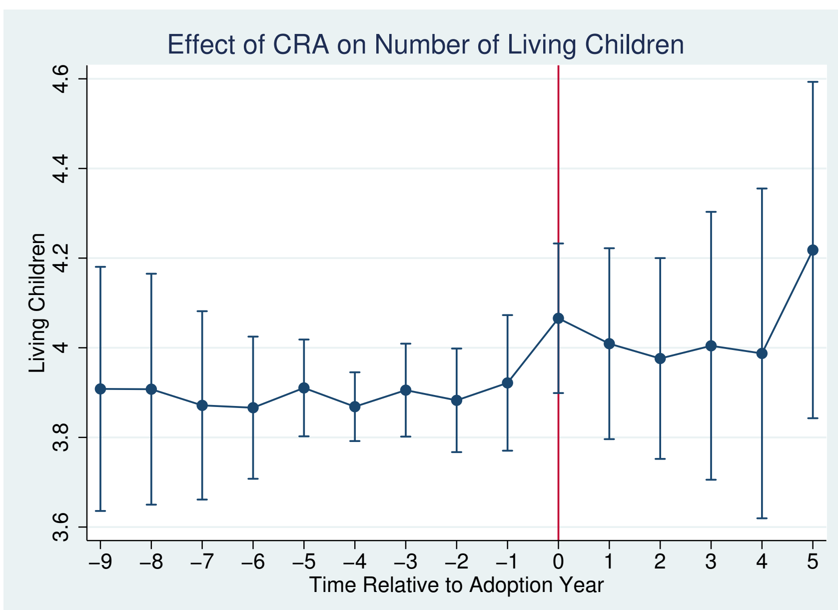
* p<0.05, ** p<0.01, *** p<0.001. Notes: The dependent variable of children born. Columns (1) and (2) correspond to specification (3.12) while Columns (3) and (4) correspond to specification (3.13). Standard errors clustered at the state level. The sample includes all married women age 19-49. Covariates include age, age^2, age of first marriage, education, household wealth, religion, and whether the respondent lives in an urban area.

Table 2.9: Effect of CRA on Number of Children at Age 25

	(1)	(2)	(3)	(4)
$Treat_{st}$	-0.164*	-0.072		
	(0.070)	(0.053)		
$Treat_{st} \times Exposure_{st}$			-0.040*	-0.017
			(0.017)	(0.014)
Sample Size	46,315	46,315	46,315	46,315
Year Fixed Effects	Yes	Yes	Yes	Yes
State Fixed Effects	Yes	Yes	Yes	Yes
State Specific Time Trends		Yes		Yes

* p<0.05, ** p<0.01, *** p<0.001. Notes: The dependent variable of children born by age 25. Columns (1) and (2) correspond to specification (3.12) while Columns (3) and (4) correspond to specification (3.13). Standard errors clustered at the state level. The sample includes all married women age 25-49. Covariates include age, age^2, age of first marriage, education, household wealth, religion, and whether the respondent lives in an urban area.

Figure 2.2: Effect of CRA on Number of Living Children



Notes: This figure plots the β_k coefficients from the following regression, with the year of adoption of the CRA as the omitted year: $Y_{ist} = \sum_{k=-9}^{5} \beta_k * 1[Year_t = k] + W_i\delta + \gamma_t + \epsilon_{ist}$. Y_{ist} is the number of living children for woman i in state s in year t. 95% confidence intervals. The above figures are for treated states only.

CHAPTER 3

IDENTIFYING RESOURCE SHARES USING MULTIPLE PRIVATE ASSIGNABLE GOODS (WITH CAITLIN BROWN AND ROSSELLA CALVI)

1 Introduction

A major focus for government and international development organizations is measuring poverty. This task is complicated for a variety of reasons, but especially in developing countries due to difficulties in measuring income and consumption. One challenge that has received more attention recently are the consequences of measuring household-level poverty as opposed to individual-level poverty (e.g., Brown et al. (2016)). Because surveys are typically conducted at the household level, the resulting poverty rates are by necessity also at the household level. The underlying assumption in this action is that either everyone in the household is poor, or everyone is not poor. While this may seem like an unavoidable part of measuring poverty, there are potential consequences to using household-level measures in the presence of substantial *intra-household* inequality. Mainly, poverty rates for specific groups that may have less power within the household (e.g., women and children) are likely underestimated.

Fundamentally, there is an identification problem. Standard consumption data is at the household level, but we are interested in consumption differences across individuals within the household. Because of these data limitations our analysis requires a structural model. In this paper, we develop a new identification method using a structural model of intra-household resource allocation to identify the share of household resources allocated to each household member.

The starting point of our analysis the collective household model of Chiappori (1988). The central assumption of this model is that the household reaches a Pareto efficient allocation of goods. While this is a fairly weak assumption, it is still not sufficient to identify how resources are allocated within the household. A large body of research within this field has demonstrated this non-identification result (Browning et al. (1994), Browning and Chiappori (1998), Vermeulen (2002), and Chiappori and Ekeland (2009))).

In response, a growing literature has sought to solve this identification problem by adding more structure to the model. There are several approaches. Browning et al. (2013) demonstrate that if we assume some preference similarity across household compositions (single men, single women, and married couples) we can identify the sharing rule, and also economies of scale in consumption. Other studies using this type of identification restriction include Lewbel and Pendakur (2008), Bargain and Donni (2012), and Lise and Seitz (2011). Because these preference stability assumptions are somewhat unattractive, other studies have tried to place bounds on the sharing rule, as opposed to point identifying it. Cherchye et al. (2011), and Cherchye et al. (2015) place bounds on the sharing rule using revealed preference inequalities.

A different strand of research that closely relates to our approach uses an Engel curve framework. The key assumption in this literature is that resource shares are independent of total household expenditure. This restriction is quite powerful, and requires only modest additional assumptions to identify resource shares. Dunbar et al. (2013) use this assumption along with semi-parametric restrictions on individual preferences for a single assignable good to identify resource shares. No price variation is required and the only data requirement is an assignable good for each person type within the household. Because of its simplicity and small data requirements it has become one of the more popular approaches.[1] Dunbar et al. (2017) slightly modify this approach and show that the preference restrictions of DLP are no longer necessary if there are a sufficient number of distribution factors in the data.[2]

Our approach extends this recent literature. Like DLP and DLP2, we require that resource shares be independent of household expenditure. We differ in several

[1]See Calvi (2017), Calvi et al. (2017), and Penglase (2018).

[2]In some ways, a distribution factor can be thought of as a preference restriction, in that these variables are required to not affect preferences.

ways. First, we impose significantly weaker preference restrictions than DLP in that preferences for the assignable goods are allowed to differ quite flexibly across people and household sizes. Second, unlike DLP2, we do not require distribution factors, which are often difficult to find in the data. Third, we require multiple assignable goods, which are commonly found in data sets with assignable goods. For example, data with assignable clothing often also includes assignable shoes.

2 Model

Let households consist of J types of individuals, indexed by j. Denote the number of each person type within the household by $\sigma_j \epsilon \{\sigma_1, ..., \sigma_J\}$. Let household composition be given by the subscript s, which is determined by the number of each person type within the household. The household purchases a vector of goods $z = (z^1, z^2, ..., z^K)$ at market prices $p = (p^1, p^2, ..., p^K)$. The vector of quantities consumed by individual j within the household is given by $x_j = (x_j^1, x_j^2, ..., x_j^K)$, where $z = A(\sum_{j=1}^{J} \sigma_j x_j)$. The matrix A accounts for economies of scale of consumption, which converts what the household purchases into "private good equivalents". The private good equivalents are then divided among household members. The sum of the private good equivalents is weakly larger than what the household purchases due to the sharing of goods.

Each person type has there own utility function $U_j(x_j)$. Following BCL and standard characterizations of the collective model, we assume the household reaches a Pareto efficient allocation of goods. Using standard results in the collective household literature, we can write the household's problem as follows:

$$\begin{gathered} \max_{x_1,...,x_J} \tilde{U}_s[\ U_1(x_1),\ \ ....\ ,\ \ U_J(x_J),\ \ p/y\ ] \\ \text{such that} \\ y = z_s^{'}p \quad \text{and} \quad z_s = A_s[\ \sum_{j=1}^{J} \sigma_j x_j\ ] \end{gathered} \tag{3.1}$$

Solving this problem results in bundles of private good equivalents. If we price these goods at within household prices $A_s^{'}$p, we can calculate resource shares η_{js}.

We can derive household-level demand functions for each good the household purchases. The key insight of DLP is that we can focus on a subset of these goods that have a variety of simplifying properties, that is, private assignable goods. A good is private if it is not shared, and it is assignable if from the survey we can determine who consumed it. Because we observe individual-level food consumption, we have several assignable goods to choose from.

DLP derive the following household-level demand functions for the private assignable good k .

$$W_{js}^{k}(y,p) = \eta_{js}(y,p)\ \omega_{js}^{k}(\eta_{js}(y,p)y,\ A_{s}^{'}p)$$

where W_{js}^{k} is the budget share for good k and ω is the individual-level demand function. Our identification approach relies on Engel curves, so we rewrite the household-level demand function as follows:

$$W_{js}^{k}(y) = \eta_{js}(y)\ \omega_{js}^{k}(\eta_{js}(y)y) \qquad (3.2)$$

The central identification problem is that both resource shares and individual-level preferences are unknown.

3 Identification

The goal of the model is to identify resource shares. We follow the methodology of DLP who identify resource shares by comparing Engel curves for private assignable goods across either people, or household sizes. DLP make two key assumptions for identification. First, they assume that resource shares are independent of household expenditure and secondly, they impose one of two semi-parametric restrictions on individual preferences for the assignable good. While DLP require only a single private assignable good, we make use of our detailed data which contain multiple private assignable goods. This additional data allows us to extend DLP by weakening the model assumptions.

Let $p = [p_j,\ \bar{p},\ \tilde{p}]$ for $j \in \{1, ..., J\}$ where p_j are the prices of the private assignable goods for each person type j. For our identification strategy, we require at least two such goods ($k = 1, 2$) for each person type, with prices denoted by p_j^1, and p_j^2, respectively. We define $\bar{p}$ to correspond to the subvector of private non-assignable good prices, and $\tilde{p}$ to correspond to the subvector of shared good prices.

We assume individuals have PIGLOG preferences over the private assignable goods in the empirical section and this functional form facilitates a discussion of identification so we use it henceforth. We show identification with a more general functional form in the appendix. The standard PIGLOG indirect utility function takes the following form:

$$V_j(p, y) = d_j(p)\Big(\ln y - \ln a_j(p)\Big)$$

By Roy's Identity, the budget share functions are written as follows:

$$w_j(y, p) = \alpha_j(p) + \gamma_j(p) \ln y$$

where the budget share functions are linear in $\ln y$.

DLP obtain identification by making one of two semi-parametric restrictions on the shape parameter $\gamma_j(p)$: either preferences are similar across people (SAP), so $\gamma_j(p) = \gamma(p)$,[3] or preferences are similar across household types (SAT), so $\gamma_j(p) = \bar{\gamma}_j(p_j, \bar{p})$.[4] With SAP, the shape preference parameter does not vary across people since $\gamma(p)$ lacks a j subscript, and with SAT, the shape preference parameter does not vary across household types since $\bar{\gamma}_j(p_j, \bar{p})$ is not a function of the prices of shared goods $\tilde{p}$, and therefore does not vary with household size.[5]

[3]The indirect utility function for SAP takes the following form: $V_j(p, y) = d(p)\Big(\ln y - \ln a_j(p)\Big)$. This is a weaker form of shape invariance. See Pendakur (1999) for details.

[4]The indirect utility function for SAT takes the following form: $V_j(p, y) = \bar{d}_j(p_j, \bar{p})\Big(\ln y - \ln a_j(p)\Big)$. SAT is a restriction on how the prices of shared goods enters the utility function. In effect, it restricts changes in the prices of shared goods to have a pure income effect on the demand for the private, assignable goods.

[5]A second way to identify resource shares within this framework is to use distribution factors d in place of semi-parametric restrictions on the assignable goods, as in DLP2. Identification comes from observing that resource shares must some to one for different values of the distribution factor. This results in additional equations in the model and allows DLP2 to obtain identification without

Under the SAT restriction, the household-level Engel curves for person j's assignable good is given by:

$$W_{js} = \eta_{js}[\alpha_{js} + \gamma_j \ln \eta_{js}] + \gamma_j \eta_{js} \ln y \tag{3.3}$$

and with SAP:

$$W_{js} = \eta_{js}[\alpha_{js} + \gamma_j \ln \eta_{js}] + \gamma_s \eta_{js} \ln y \tag{3.4}$$

With these restrictions, DLP show that resource shares can be identified with a *single* private assignable good for each person type. Since we observe *multiple* private assignable goods for each person type, we develop two new approaches that employ this additional data.

3.1 D-SAT

In the first approach, we demonstrate that the SAT restriction of DLP can be substantially weakened by using multiple private assignable goods. Unlike DLP, we do not assume that preferences for the assignable goods are similar across household sizes, but rather, we allow preferences to differ considerably across household sizes, but require them to do so in the same way across two different private assignable goods.[6] Because of this, we call our approach "Differenced Similar Across Types", or "D-SAT".

We illustrate this method using a PIGLOG indirect utility function $V_j(p, y) = e^{F_j(p)}(\ln y - \ln a_j(p))$. Our assumption requires that

$$\frac{\partial F_j(p)}{\partial p_j^1} - \frac{\partial F_j(p)}{\partial p_j^2} = \theta_j(p_j^1, p_j^2, \bar{p}) \tag{3.5}$$

where $\theta_j(p_j^1, p_j^2, \bar{p})$ does not vary across household sizes.[7]

D-SAT holds if $F_j(p)$ takes the following form: $F_j(p) = b_j(p_j^1 + p_j^2,\ \bar{p},\ \tilde{p}) + r_j(p_j^1,\ p_j^2,\ \bar{p})$, where $r_j(\cdot)$ does not depend on the prices of shared goods, and therefore

restricting the preference parameter $\gamma_j(p)$.

[6]Having a third assignable good would not meaningfully reduce the assumptions necessary for identification.

[7]DLP impose a stronger version of this with $\partial F_j(p)/\partial p_j^1 = \tilde{\theta}_j(p_j^1, \bar{p})$.

does not vary by household size. Moreover, p_j^1 and p_j^2 are additively separable in $b_j(\cdot)$ which results in preferences that differ across households sizes in the same way across goods.

We can use Roy's Identity to derive the budget share functions for goods $k \epsilon \{1, 2\}$:

$$\frac{h_j^k(p, y)}{y} = \Big(\frac{\partial b_j(p_j^1 + p_j^2,\ \bar{p},\ \tilde{p})}{\partial p_j^k} + \frac{\partial r_j(p_j^1\ , p_j^2,\ \bar{p})}{\partial p_j^k}\Big) \ln y + \alpha_j^k(p) \tag{3.6}$$

The household-level Engel curves for person j's two assignable goods can then be written as follows:

$$\begin{aligned} W_{js}^1 =& \eta_{js}[\alpha_{js}^1 + \gamma_j^1 \ln \eta_{js}] + (\beta_{js} + \gamma_j^1)\eta_{js} \ln y \\ W_{js}^2 =& \eta_{js}[\alpha_{js}^2 + \gamma_j^2 \ln \eta_{js}] + (\beta_{js} + \gamma_j^2)\eta_{js} \ln y \end{aligned} \tag{3.7}$$

If we compare equations (3.3) and (3.7), we can see how we weaken the SAT restriction. As in DLP, preferences for the assignable goods are allowed to differ across people, both in α_{js}^k and in γ_j. Unlike DLP, we also allow preferences to differ across household sizes in the slope parameter β_{js}.[8] However, we restrict preferences to differ across household sizes in the same way across goods, that is, β_{js} is the same for both goods. SAT is therefore a special case of D-SAT with $\beta_{js} = 0$.

To better understand our assumptions, consider the following example. Suppose we observe assignable cereals and proteins (meat, dairy, and fish) for men, women, and children in a sample of nuclear households with one to three children. The SAT restriction would require that the man's marginal propensity to consume cereals be the same regardless of the number of children in the household. With D-SAT, we allow his marginal propensity to consume cereals to differ considerably across household sizes. However, we require that the difference in the man's preferences for cereals across household sizes be similar to the difference in his preferences for proteins across household sizes. The same must be true for women and children.

Our identification assumption can be understood a different way by rewriting

[8]DLP do not require preferences for the assignable good to be identical across household size, as the intercept parameter α_{js} does vary with household size.

equation (3.7); let $\psi^1_{js} = \beta_{js} + \gamma^1_j$ and $\psi^2_{js} = \beta_{js} + \gamma^2_j$ be the shape preference parameters for goods 1 and 2, respectively. With the SAT restriction, DLP implicitly assume that $\psi^1_{js} - \psi^1_{j,s+1} = 0$. Our alternative restriction allows this quantity to be nonzero, however, it has to be the same for both goods. Stated differently: $\psi^1_{js} - \psi^1_{j,s+1} = \psi^2_{js} - \psi^2_{j,s+1}$. Preferences for these goods should differ in the same way across household sizes.

To show that resource shares are identified, first let $\lambda_{js} = \beta_{js} + \gamma^1_j$ and $\kappa_j = \gamma^2_j - \gamma^1_j$. Then we can rewrite system (3.7) as follows for $j \in \{1, ..., J\}$:

$$W^1_{js} = \ldots \; + \; \eta_{js} \; \lambda_{js} \ln y$$
$$W^2_{js} = \ldots \; + \; \eta_{js} \; (\lambda_{js} + \kappa_j) \ln y$$

If we then subtract person j's budget share function for good 2 from their budget share function for good 1, we are left with a set of equations that are identical to the SAT system of equations from DLP with $j \in \{1, ..., J\}$:

$$W^1_{js} - W^2_{js} = \ldots \; + \; \eta_{js} \; \kappa_j \ln y$$

An OLS-type regression of the observable budget shares on log expenditure identifies the slope coefficient for each person type j. Comparing the slopes of the Engel curves across household sizes, and assuming resource shares sum to one allows us to recover the resource share parameters.

The order condition is satisfied with J household types. To see this, first note that there are J Engel curves for each of the J household types, resulting in J^2 equations. Moreover, for each household type resource shares must sum to one. This results in $J(J+1)$ equations in total. In terms of unknowns, there are J^2 resource shares, and J preference parameters (κ_j), or $J(J+1)$ unknowns in total. A proof of the rank condition can be found in the appendix.

3.2 D-SAP

In the second approach, we demonstrate that the SAP restriction of DLP can also be substantially weakened by using multiple private assignable goods. Unlike DLP, we do not assume that preferences for the assignable goods are similar across people, but rather, we allow preferences to differ considerably across people, but require them to do so in the same way across two different private assignable goods. Here, we call our assumption "Differenced Similar Across People", or "D-SAP".

Again, we demonstrate identification using a PIGLOG indirect utility function $V_j(p, y) = e^{F_j(p)}(\ln y - \ln a_j(p))$. Our second assumption requires that

$$\frac{\partial F_j(p)}{\partial p_j^1} - \frac{\partial F_j(p)}{\partial p_j^2} = \theta(p) \tag{3.8}$$

where $\theta(p)$ does not vary across people.[9]

Our assumption holds if $F_j(p)$ takes the following form: $F_j(p) = b_j(p_j^1 + p_j^2,\ \bar{p},\ \tilde{p}) + r(p)$, where $r(p)$ does not vary across people. Moreover, p_j^1 and p_j^2 are again additively separable in $b_j(\cdot)$ which results in preferences that differ across people in the same way across goods.

We again use Roy's Identity to derive the budget share function for goods $k \ \epsilon \ \{1, 2\}$:

$$\frac{h_j^k(p, y)}{y} = \Big(\frac{\partial b_j(p_j^1 + p_j^2,\ \bar{p},\ \tilde{p})}{\partial p_j^k} + \frac{\partial r(p)}{\partial p_j^k}\Big) \ln y + \alpha_j^k(p) \tag{3.9}$$

The household-level Engel curves for person j's two assignable goods can then be written as follows:

$$\begin{aligned} W_{js}^1 =& \eta_{js}[\alpha_{js}^1 + \gamma_s^1 \ln \eta_{js}] + (\beta_{js} + \gamma_s^1)\eta_{js} \ln y \\ W_{js}^2 =& \eta_{js}[\alpha_{js}^2 + \gamma_s^2 \ln \eta_{js}] + (\beta_{js} + \gamma_s^2)\eta_{js} \ln y \end{aligned} \tag{3.10}$$

If we compare equations (3.4) and (3.10), we can see how we weaken the SAP restriction. As in DLP, preferences for the assignable goods are allowed to differ

[9] DLP impose a stronger version of this with $\partial F_j(p)/\partial p_j^1 = \tilde{\theta}(p)$.

entirely across household sizes, both in α^k_{js} and in γ_s. Unlike DLP, we also allow preferences to differ across people in the slope parameter β_{js}.[10] However, we restrict preferences to differ across people in the same way across goods, that is, β_{js} is the same for both goods. SAP is therefore a special case of our set of assumptions with $\beta_{js} = 0$.

We can again use an example to illustrate the differences between DLP and our method. Suppose we observe assignable cereals and proteins (meat, dairy, and fish) for men, women, and children in a sample of nuclear households with one to three children. The SAP restriction would require that the man's marginal propensity to consume cereals be the same as the woman's.[11] With our assumption, we allow his marginal propensity to consume cereals to differ considerably from hers. However, we require that this difference in the man's and woman's preferences for cereals be similar to the difference in their preferences for proteins.

Once again, our identification assumption can be understood a different way using the above system of equations; let $\psi^1_{js} = \beta_{js} + \gamma^1_s$ and $\psi^2_{js} = \beta_{js} + \gamma^2_s$ be the shape preference parameters for goods 1 and 2, respectively. With the SAP restriction, DLP implicitly assume that $\psi^1_{js} - \psi^1_{j',s} = 0$. Our alternative restriction allows this quantity to be nonzero, however, it has to the the same for both goods. Stated differently: $\psi^1_{js} - \psi^1_{j',s} = \psi^2_{js} - \psi^2_{j',s}$.

To show that resource shares are identified, first let $\lambda_{js} = \beta_{js} + \gamma^1_s$ and $\kappa_s = \gamma^2_s - \gamma^1_s$. Then we can rewrite system (3.10) as follows for $j \in \{1, ..., J\}$:

$$\begin{aligned} W^1_{js} &= \ldots \ + \ \eta_{js}\ \lambda_{js} \ln y \\ W^2_{js} &= \ldots \ + \ \eta_{js}\ (\lambda_{js} + \kappa_s) \ln y \end{aligned}$$

If we then subtract person j's budget share function for good 2 from their budget share function for good 1, we are left with a set of equations that are identical to the

[10]DLP do not require preferences for the assignable good to be identical across people, as the intercept parameter α_{js} does across people.

[11]In DLP, the SAP restriction is imposed on the function $F_j(p)$ with $\partial F_j(p)/\partial p_j = \theta(p)$. Instead, we assume $\partial F_j(p)/\partial p^1_j - \partial F_j(p)/\partial p^2_j = \tilde{\theta}(p)$.

SAP system of equations for $j \epsilon \{1, ..., J\}$:

$$W^1_{js} - W^2_{js} = \dots \; + \eta_{js} \; \kappa_s \ln y$$

We can easily demonstrate resource shares are identified. An OLS-type regression of the observable budget shares on log expenditure identifies the slope coefficients $c_{js} = \eta_{js}\kappa_s$. Then since resource shares sum to one, $\sum_{j=1}^{J} c_{js} = \sum_{j=1}^{J} \eta_{js}\kappa_s = \kappa_s$ is identified. It follows that $\eta_{js} = c_{js}/\kappa_s$.

3.3 Graphical Intuition for D-SAP

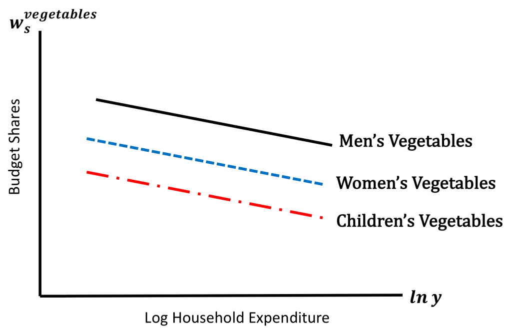
To understand the results graphically, we first plot hypothetical *individual-level* Engel curves for two assignable goods (e.g., vegetables and proteins). Under SAP, DLP assume that preferences for the assignable good are similar across person types. With PIGLOG preferences, that results in individual-level Engel curves with the same slopes as seen in Figure (3.1a) and (3.1b).[12]

We differ in that we allow allow preferences for the assignable goods to differ completely across individuals. Figures (3.1c) and (3.1d) illustrate this point as the slopes are no longer identical across people. However, we restrict preferences to differ across people in the same way across goods. Intuitively, this means that if women have a higher marginal propensity to consume vegetables than men, then they also have a higher marginal propensity to consume proteins than men. Moreover, this difference in preferences between person types is the same across goods.

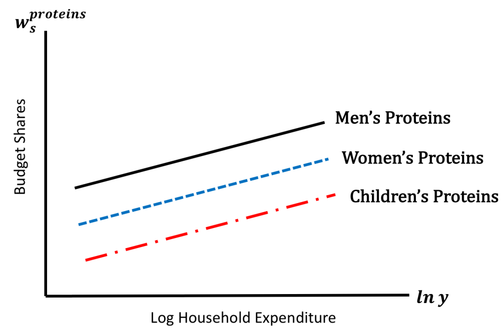
It is important to note that DLP also implicitly impose some similarity across goods. Relating to our example, DLP impose that men and women have the same marginal propensity to consume vegetables *and* men and women have the same marginal propensity to consume proteins. In that sense, the difference in marginal propensities to consume vegetables across men and women is the same as it is for proteins, in that it does not differ.

[12] The following individual-level Engel curves satisfies SAP: $w_j(y, p) = \delta_j(p) + \beta(p) \ln y$ since $\beta(p)$ does not vary across people.

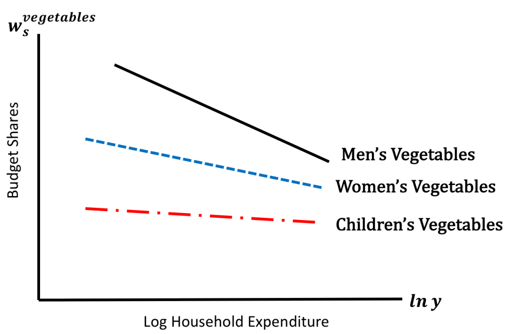
Figure 3.1: Individual-level Engel curves for assignable clothing and shoes. Figures (3.1a) and (3.1b) illustrate Engel curves under the SAP restriction. Figures (3.1c) and (3.1d) illustrate Engel curves under the D-SAP restriction. The Engel curves in Figures (1c) and (1d) do not exhibit shape invariance, however, the difference in slopes across men, women, and children differ in the same way across goods.



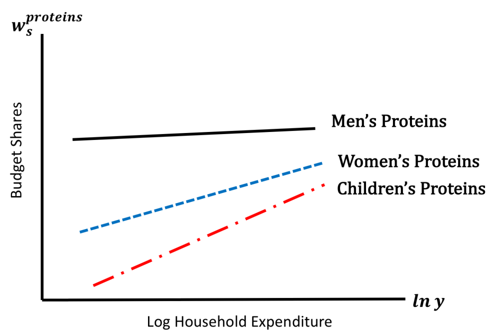
(a) SAP-Vegetables



(b) SAP-Proteins



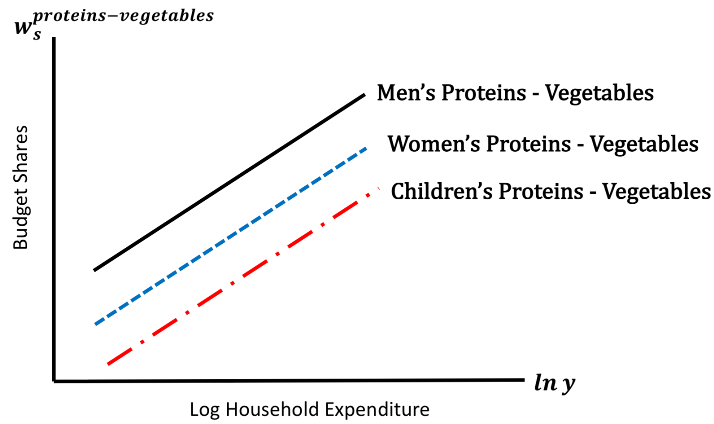
(c) D-SAP-Vegetables



(d) D-SAP-Proteins

With this assumption, if we difference the Engel curves we end up with Figure (3.2). Here, the differenced *individual-level* Engel curves are parallel, similar to SAP. Essentially the differenced Engel curves are shape invariant. We can therefore use the DLP identification results to recover resource shares.

Figure 3.2: Differences in individual-level Engel curves across assignable clothing and shoes. The Engel curves are derived by taking the difference of Figures (1d) and (1c). By assumption, the difference across Engel curves will have the same slope. Any difference in the slopes of the *household-level* differenced Engel curves can then be attributed to differences in resource shares, as in SAP.



4 Conclusion

In this paper, we develop a new method to identify inequality within the household. Our method weakens the necessary assumptions to identify resource shares by employing multiple private assignable goods. This methodology can be applied to a variety of projects where inequality within the household is of interest, and the data contains multiple assignable goods. The authors of this paper are currently working on a project that applies this method to inequality within Bangladeshi households using individual-level food consumption expenditures. Using budget shares for different food groups, such as cereals and vegetables, we document inequality among boys and girls, young and old children, and adults.

APPENDIX

3.A Theorems

The section provides the two main theorems of the paper. Both are extensions of Theorems 1 and 2 in DLP, and therefore share much of the same content. The main differences are in the data requirements (we need more) and the assumptions (we need fewer). The key differences can be found in Assumptions A2$'$, A3$'$, B3$'$. Otherwise, we follow DLP.

3.A.1 Theorem 1

Let j denote individual person types with $j \epsilon \{1, ..., J\}$. The Marshallian demand function for a person type j and good k is given by $h_j^k(p, y)$. Each individual chooses x_j to maximize their own utility function $U_j(x_j)$ subject to the budget constraint $p^{'}x_j = y$, where p is vector of prices and y is total expenditure. Denote the vector of demand functions as $h_j(p, y)$ for all goods k. Let the indirect utility function be given by $V_j(p, y) = U_j(h_j(p, y)$.

Let z_s denote the vector of goods purchased by a household of composition s, where the subscript s indexes the household types. Let σ_j denote the number of individuals of type j in the household. From the BCL, we write the household's problem as follows:

$$\max_{x_1...,x_J,z_s} = \tilde{U}[U_1(x_1), ..., U_J(x_J), p/y] \quad (3.11)$$

$$\text{such that } z_s = A_s\Big[\sum_{j=1}^{J} \sigma_j x_j\Big] \text{ and } y = z_s^{'}p$$

where A_s is a matrix that accounts for the sharing of goods within the household. From the household's problem we can derive household-level demand functions $H_s^k(p, y)$ for good k in a household of size s:

$$z_s^k = H_s^k(p, y) = A_s^k \Big[\sum_{j=1}^{J} h_j(A_s^{'} p, \eta_{js} y) \Big] \tag{3.12}$$

where A_s^k denotes the row vector given by the k'th row of matrix A_s, and η_{js} is the resource share for a person of type j in a household of size s. Lastly, resource shares sum to one:

$$\sum_{j=1}^{J} \sigma_j \eta_{js} = 1 \tag{3.13}$$

ASSUMPTION A1: Equations (3.11), (3.12), and (3.13) hold, and resource shares are independent of household expenditure at low levels of household expenditure.

Definition: A good k is a private good if the Matrix A_s takes the value one in position k, k and has all other elements in row and column k equal to zero.

Definition: A good k is assignable if it only appears in one of the utility functions U_j.

ASSUMPTION A2$^{'}$: Assume that the demand functions include at least 2 private, assignable goods, denoted as goods j^1 and j^2 for each person type.

DLP require a single assignable good for each person j. We differ in that we require at least 2 different goods for each person.

Let $\tilde{p}$ be the price of the goods that are not both private and assignable. Let p_j^k be the prices of the private assignable goods, with $k \in \{1, 2\}$.

ASSUMPTION A3$'$: For $j \epsilon$ {1,...,J} let

$$V_j(p,y) = I(y \leq y^*(p))\psi_j\Big[\nu(\frac{y}{G_j(p)}) + F_j(p), \tilde{p}\Big] + I(y > y^*(p))\Psi(y,p) \tag{3.14}$$

where $F_j(p) = b_j(p_j^1 + p_j^2, \bar{p}, \tilde{p}) + e(p)$, and y^*, ψ_j, Ψ, ν, b_j e, and G_j are functions with y^* is strictly positive, G_j is nonzero, differentiable, and homogenous of degree one. The function ν is differentiable and strictly monotonically increasing. The functions b_j and e are homogenous of degree 0. Lastly, Ψ and ψ are differentiable and strictly increasing in their first arguments, differentiable, and homogenous of degree zero in their remaining arguments.

This assumption differs from Assumption A3 in DLP in the function $F_j(p)$. DLP restrict $F_j(p)$ to not vary across people with $\partial F_j(p)/\partial p_j = \phi(p)$. Here, we allow $F_j(p)$ to vary across people in the function $b_j(\cdot)$. However, the way $F_j(p)$ varies across people is restricted to be the same across goods 1 and 2: $\partial b_j(\cdot)/\partial p_j^1 = \partial b_j(\cdot)/\partial p_j^2$. This holds since the prices for goods 1 and 2 enter $b_j(\cdot)$ in an additively separable way. The function $e(p)$ does not vary across people.

Use Roy's Identity to derive individual-level demand functions for goods $k \, \epsilon \, \{1, 2\}$:

- For $I(y > y^*)$

$$h_j^k(y,p) = -\Big[\partial\Psi_j(y,p)/\partial p_j^k\Big] / \Big[\partial\Psi_j(y,p)/\partial y\Big]$$

- For $I(y \leq y^*)$

$$\begin{aligned}
h_j^k(p,y) &= -\frac{\frac{\partial V_j(p,y)}{\partial p_j^k}}{\frac{\partial V_j(p,y)}{\partial y}} \\
&= \frac{y}{G_j(p)}\frac{\partial G_j(p)}{\partial p_j^k} + \Big(\frac{\partial b_j(p_j^1 + p_j^2, \bar{p}, \tilde{p})}{\partial p_j^k} + \frac{\partial e(p)}{\partial p_j^k}\Big)\frac{1}{\nu'(\frac{y}{G_j(p)})}G_j(p) \\
&= \frac{y}{G_j(p)}\frac{\partial G_j(p)}{\partial p_j^k} + \Big(\frac{\partial b_j(p_j^1 + p_j^2, \bar{p}, \tilde{p})}{\partial p_j^k} + \frac{\partial e(p)}{\partial p_j^k})\Big)\frac{1}{\nu'(\frac{y}{G_j(p)})}\frac{y}{y/G_j(p)}
\end{aligned}$$

$$=a_j^k(p)y + \Big(\frac{\partial b_j(p_j^1+p_j^2,\bar{p},\tilde{p})}{\partial p_j^k} + \frac{\partial e(p)}{\partial p_j^k}\Big)g(\frac{y}{G_j(p)})y$$

For $I(y \leq y^*)$, we can then write the household-level Engel curves for the private, assignable goods for $j \in \{1, ..., J\}$ in a given price regime p:

$$H_{js}^k(y) = a_{js}^k s_j \eta_{js} y + \Big(\tilde{b}_{js} + \tilde{e}_s^k\Big) g_s(\frac{\eta_{js} y}{G_{js}}) s_j \eta_{js} y \tag{3.15}$$

ASSUMPTION A4: The function $g_s(y)$ is twice differentiable. Let $g_s^{'}(y)$ and $g_s^{''}(y)$ denote the first and second derivatives of $g_s(y)$. Either $\lim_{y\to 0} y^{\zeta} g_s^{''}(y)/g_s^{'}(y)$ is finite and nonzero for some constant $\zeta \neq 1$ or $g_s(y)$ is a polynomial in $\ln y$.

Theorem 1: *Let Assumptions A1, A2, A3, and A4 hold. Assume the household-level Engel curves for the private assignable goods H_{js}^1 and H_{js}^2 are identified for $j \in \{1, ..., J\}$. Then the resource shares η_{js} are identified for $j \in \{1, ..., J\}$.*

3.A.2 Theorem 2

Let $\tilde{p}$ be the price of the goods that are not both private and assignable. Let p_j^k be the prices of the private assignable goods, with $k \in \{1, 2\}$ and $j \in \{1, ..., J\}$. Let $\bar{p}$ be the price of the private goods that are not assignable.

ASSUMPTION B3$'$: For $j \in$ {1,...,J} let

$$V_j(p,y) = I(y \leq y^*(p))\psi_j\Big[u_j\Big(\frac{y}{G_j(p)}\Big) + b_j(p_j^1+p_j^2,\bar{p},\tilde{p}) + e_j(p_j^1,p_j^2,\bar{p}),\tilde{p}),\tilde{p}\Big] + I(y > y^*(p))\Psi(y,p) \tag{3.16}$$

where y^*, ψ_j, Ψ, u_j, b_j e, and G_j are functions with y^* is strictly positive, G_j is nonzero, differentiable, and homogenous of degree one. The function ν is differentiable and strictly monotonically increasing. The functions b_j and e are homogenous of degree 0. Lastly, Ψ and ψ are differentiable and strictly increasing in their first arguments, differentiable, and homogenous of degree zero in their remaining arguments.

This assumption differs from Assumption B3 in DLP as follows: We replace $u_j(\frac{y}{G(\tilde{p})}, \frac{\bar{p}}{p_j})$ with $u_j(\frac{y}{G_j(p)}) + b_j(p_j^1 + p_j^2, \bar{p}, \tilde{p}) + e_j(p_j^1, p_j^2, \bar{p})$. The function $u_j(\cdot)$ is still restricted to not depend on the prices of shared goods, however, we have included the function $b_j(\cdot)$ which is allowed to depend on the prices of shared goods, and therefore varies across household size. However, the way in which $b_j(\cdot)$ varies across household size is restricted to be the same across goods 1 and 2: $\partial b_j(\cdot)/\partial p_j^1 = \partial b_j(\cdot)/\partial p_j^2$. This holds since the prices for goods 1 and 2 enter $b_j(\cdot)$ in an additively separable way.

Use Roy's Identity to derive individual-level demand functions for goods $k \epsilon \{1, 2\}$:

- For $I(y > y^*)$

$$h_j^k(y,p) = -\Big[\partial \Psi_j(y,p)/\partial p_j^k\Big] / \Big[\partial \Psi_j(y,p)/\partial y\Big]$$

- For $I(y \leq y^*)$

$$\begin{aligned}
h_j^k(p,y) =& -\frac{\frac{\partial V_j(p,y)}{\partial p_j^k}}{\frac{\partial V_j(p,y)}{\partial y}} \\
=& \frac{u_j'(\frac{y}{G_j(p)})\frac{y}{G_j(p)^2}\frac{\partial G_j(p)}{\partial p_j^k} + (\frac{\partial b_j(p_j^1+p_j^2,\bar{p},\tilde{p})}{\partial p_j^k} + \frac{\partial e_j(p_j^1+p_j^2,\bar{p})}{\partial p^k)j})}{u_j'(\frac{y}{G_j(p)})\frac{1}{G_j(\tilde{p})}} \\
=& \frac{y}{G_j(p)}\frac{\partial G_j(p)}{\partial p_j^k} + \Big(\frac{\partial b_j(p_j^1 + p_j^2, \bar{p}, \tilde{p})}{\partial p_j^k} + \frac{\partial e(p_j^1, p_j^2, \bar{p})}{\partial p_j^k})\Big)\frac{1}{u_j'(\frac{y}{G_j(p)})}\frac{y}{y/G_j(p)} \\
=& a_j^k(p)y + \Big(\frac{\partial b_j(p_j^1 + p_j^2, \bar{p}, \tilde{p})}{\partial p_j^k} + \frac{\partial e(p_j^1, p_j^2, \bar{p})}{\partial p_j^k}\Big) f_j(\frac{y}{G_j(p)})y
\end{aligned}$$

For $I(y \leq y^*)$, we can then write the household-level Engel curves for the private, assignable goods for $j \ \epsilon \ \{1, ..., J\}$ in a given price regime p:

$$H_{js}^k(y) = a_{js}^k s_j \eta_{js} y + \Big(\tilde{b}_{js} + \tilde{e}_j^k\Big) f_j(\frac{\eta_{js} y}{G_{js}}) s_j \eta_{js} y \tag{3.17}$$

We take the ratio of resource shares for person j across two different household types, which results in the following equation:

$$\frac{\eta_{j1}}{\eta_{js}} = \zeta_{js} \tag{3.18}$$

for $j \in \{1, ..., J-1\}$ and $s \in \{2, ..., S\}$. In total, this results in $(S-1)(J-1)$ equations. Moreover, in the proof we will use that resource shares sum to one to write the following system of equations:

$$\sum_{j=1}^{J-1} (\zeta_{js} - \zeta_{Js})\eta_{js} = 1 - \zeta_{Js} \tag{3.19}$$

for $s \in \{2, ..., S\}$. Equation (3.19) results in $S - 1$ equations.

We can stack the system of equations given by Equations (3.18) and (3.19). This results in a system of $J(S-1)$ equations. In matrix form, let E be a $J(S-1) \times 1$ vector of η_{js} for $j \in \{1, ..., J-1\}$ and $s \in \{1, ..., S\}$ such that $\Omega \times E = B$, where Ω is a $J(S-1) \times J(S-1)$ matrix, and B is a $J(S-1) \times 1$ vector.

ASSUMPTION B4: The matrix Ω is finite and nonsingular, and $f_j(0) \neq 0$ for $j \in \{1, ..., J\}$.

Theorem 2: *Let Assumptions A1, A2, B3, and B4 hold. Assume there are $S \geq J$ household types. Assume the household-level Engel curves for the private assignable goods H^1_{js} and H^2_{js} are identified for $j \in \{1, ..., J\}$. Then the resource shares η_{js} are identified for $j \in \{1, ..., J\}$.*

3.B Proofs

3.B.1 Proof of Theorem 1

The proof will consist of two cases. In the first case, we assume g_s is not a polynomial of degree λ in logarithms. In the second case we assume that it is. Define

$$\tilde{h}_{js}^k(y) = \partial[H_{js}^k(y)/y]/\partial y = \Big(\tilde{b}_{js} + \tilde{e}_s^k\Big) g_s^{'}(\frac{\eta_{js} y}{G_{js}})\frac{\eta_{js}^2}{G_{js}}$$
$$\lambda_s = \lim_{y\to 0}[y^\zeta g_s^{''}(y)/g_s^{'}(y)]^{\frac{1}{1-\zeta}}$$

Case 1: $\zeta \neq 1$

Then since $H_{js}^k(y)$ are identified, we can identify $\kappa_{js}^k(y)$ for $y \leq y^*$:

$$\begin{aligned}\kappa_{js}^k(y) =&\Big(y^\zeta \frac{\partial \tilde{h}_{js}^k(y)/\partial y}{\tilde{h}_{js}^k(y)}\Big)^{\frac{1}{1-\zeta}}\\ =&\Big((\frac{\eta_{js}}{G_{js}})^{-\zeta}(\frac{\eta_{js}y}{G_{js}})^\zeta\Big[(\tilde{b}_{js}+\tilde{e}_s^k)g_s^{''}(\frac{\eta_{js}y}{G_{js}})\frac{\eta_{js}^3}{G_{js}^2}\Big]/\Big[(\tilde{b}_{js}+\tilde{e}_s^k)g_s^{'}(\frac{\eta_{js}y}{G_{js}})\frac{\eta_{js}^2}{G_{js}}\Big]\Big)^{\frac{1}{1-\zeta}}\\ =&\frac{\eta_{js}}{G_{js}}\Big(y_{js}^\zeta \frac{g^{''}(y)}{g^{'}(y)}\Big)^{\frac{1}{1-\zeta}}\end{aligned}$$

Then we can define $\rho_{js}^1(y)$ and $\rho_{js}^2(y)$ by

$$\rho_{js}^1(y) = \frac{\tilde{h}_{js}^1(y/\kappa_{js}^1(0))}{\kappa_{js}^1(0)} = (\tilde{b}_{js}+\tilde{e}_s^1)g_s^{'}(\frac{y}{\lambda_s})\frac{\eta_{js}}{\lambda_s}$$
$$\rho_{js}^2(y) = \frac{\tilde{h}_{js}^2(y/\kappa_{js}^2(0))}{\kappa_{js}^2(0)} = (\tilde{b}_{js}+\tilde{e}_s^2)g_s^{'}(\frac{y}{\lambda_s})\frac{\eta_{js}}{\lambda_s}$$

Taking the difference of the above two equations, we derive the following expression similar to DLP

$$\rho_{js}^2(y) - \rho_{js}^1(y) = \hat{\rho}_{js}(y) = (\tilde{e}_s^2 - \tilde{e}_s^1)g_s^{'}(\frac{y}{\lambda_s})\frac{\eta_{js}}{\lambda_s} = \phi_s\eta_{js}$$

Then since resource shares sum to one, we can identify resource shares as follows:

$$\eta_{js} = \frac{\hat{\rho}_{js}}{\sum_{j=1}^{J} \hat{\rho}_{js}}$$

Case 2: g_s is a polynomial of degree λ in logarithms

$$g_s(\frac{\eta_{js}y}{G_{js}}) = \sum_{l=0}^{\lambda} \Big(\ln \Big(\frac{\eta_{js}}{G_{js}} \Big) + \ln y \Big)^l c_{sl}$$

for some constants c_{sl}. We can then identify

$$\tilde{\rho^1}_{js} = \frac{\partial^{\lambda}[H_s^1(y)/y]}{\partial(\ln y)^{\lambda}} = (\tilde{b}_{js} + \tilde{e}_s^1)d_{s\lambda}^1\eta_{js}$$
$$\tilde{\rho^2}_{js} = \frac{\partial^{\lambda}[H_s^2(y)/y]}{\partial(\ln y)^{\lambda}} = (\tilde{b}_{js} + \tilde{e}_s^2)d_{s\lambda}^2\eta_{js}$$

Taking the difference of the above two equations, we derive the following expression similar to DLP

$$\tilde{\rho}_{js}^2(y) - \tilde{\rho}_{js}^1(y) = \hat{\rho}_{js}(y) = (\tilde{e}_s^2 d_{s\lambda}^2 - \tilde{e}_s^1 d_{s\lambda}^1)\eta_{js} = \phi_s\eta_{js}$$

Then since resource shares sum to one, we can identify resource shares as follows:

$$\eta_{js} = \frac{\hat{\rho}_{js}}{\sum_{j=1}^{J} \hat{\rho}_{js}}$$

3.B.2 Proof of Theorem 2

The household-level Engel curves for person $j \ \epsilon \ \{1, ..., J\}$ and good k:

$$H_{js}^k(y) = a_{js}^k\eta_{js}y + \Big(\tilde{b}_{js} + \tilde{e}_j^k\Big) f_j(\frac{\eta_{js}y}{G_{js}})\eta_{js}y$$

For each $j \epsilon \{1, ..., J\}$ take the difference of the Engel curves for private, assignable goods $k = 1$ and $k = 2$.

$$\tilde{H}_{js}(y) = H^2_{js}(y) - H^1_{js}(y) = \tilde{a}_{js}\eta_{js} + \tilde{e}_j\tilde{f}_j(\frac{\eta_{js}y}{G_{js}})\eta_{js}y$$

Let s and 1 be elements of S. Since the Engel curves are identified, we can identify ζ_{js} defined by $\zeta_{js} = \lim_{y\to 0}\tilde{H}_{j1}(y)/\tilde{H}_{js}(y)$ as follows for $j \epsilon \{1, ..., J\}$ and $s \epsilon \{2, ..., S\}$

$$\zeta_{js} = \frac{\tilde{e}_j\tilde{f}_j(0)\eta_{j1}y}{\tilde{e}_j\tilde{f}_j(0)\eta_{js}y} = \frac{\eta_{j1}}{\eta_{js}} \tag{3.20}$$

Then since resource shares sum to one,

$$\sum_{j=1}^{J}\zeta_{js}\eta_{js} = \sum_{j=1}^{J}\eta_{j1} = 1$$
$$\sum_{j=1}^{J-1}\zeta_{js}\eta_{js} + \zeta_{Js}\Big(1 - \sum_{j=1}^{J-1}\eta_{js}\Big) = 1$$
$$\sum_{j=1}^{J-1}(\zeta_{js} - \zeta_{Js})\eta_{js} = 1 - \zeta_{Js} \tag{3.21}$$

for $s \epsilon \{2, ..., S\}$.

We then stack Equation (3.20) for $j \epsilon \{1, ..., J-1\}$ and $s \epsilon \{2, ..., S\}$ and Equation (3.21) for $s \epsilon \{2, ..., S\}$. This results in a system of $J(S-1)$ equations. In matrix form, this can be written as the previously defined system of equations $\Omega \times E = B$, where E is a $J(S-1) \times 1$ vector of η_{js} for $j \epsilon \{1, ..., J-1\}$ and $s \epsilon \{1, ..., S\}$, Ω is a $J(S-1) \times J(S-1)$ matrix, and B is a $J(S-1) \times 1$ vector. By Assumption B4, Ω is nonsingular. It follows that for any given household type s, we can solve for $J-1$ of the η's. Then since resource shares sum to one, we can solve for η_{Js}.

Bibliography

Ainsworth, M. (1995). Economic aspects of child fostering in cote d'ivoire. *Research in Population Economics*, 8:25–62.

Ainsworth, M. and Filmer, D. (2006). Inequalities in children's schooling: Aids, orphanhood, poverty, and gender. *World Development*, 34(6):1099–1128.

Akinwumi, O. S. (2010). Legal impediments on the practical implementation of the child right act 2003. *International Journal of Legal Information*, 37(3):10.

Akresh, R. (2009). Flexibility of household structure: Child fostering decisions in burkina faso. *Journal of Human Resources*, 44(4):976–997.

Almond, D. and Mazumder, B. (2013). Fetal origins and parental responses. *Annual Review of Economics*, 5(1):37–56.

Apps, P. F. and Rees, R. (1988). Taxation and the household. *Journal of Public Economics*, 35(3):355–369.

Attanasio, O. P. and Lechene, V. (2002). Tests of income pooling in household decisions. *Review of Economic Dynamics*, 5(4):720–748.

Attanasio, O. P. and Lechene, V. (2014). Efficient responses to targeted cash transfers. *Journal of Political Economy*, 122(1):178–222.

Bargain, O. and Donni, O. (2012). Expenditure on children: A rothbarth-type method consistent with scale economies and parents' bargaining. *European Economic Review*, 56(4):792–813.

Basu, K. (2005). Child labor and the law: Notes on possible pathologies. *Economics Letters*, 87(2):169–174.

Basu, K. and Van, P. H. (1998). The economics of child labor. *American Economic Review*, pages 412–427.

Becker, G. S. (1965). A theory of the allocation of time. *The Economic Journal*, pages 493–517.

Becker, G. S. (1992). Fertility and the economy. *Journal of Population Economics*, 5(3):185–201.

Becker, G. S. and Lewis, H. G. (1974). Interaction between quantity and quality of children. In *Economics of the Family: Marriage, Children, and Human Capital*, pages 81–90. University of Chicago Press.

Bhalotra, S. (2007). Is child work necessary? *Oxford Bulletin of Economics and Statistics*, 69(1):29–55.

Bharadwaj, P., Lakdawala, L. K., and Li, N. (2013). Perverse consequences of well intentioned regulation: Evidence from india's child labor ban. Technical report, National Bureau of Economic Research.

Bobonis, G. J. (2009). Is the allocation of resources within the household efficient? new evidence from a randomized experiment. *Journal of Political Economy*, 117(3):453–503.

Brown, C., Ravallion, M., and van de Walle, D. (2016). How well do household poverty data identify africa's nutritionally vulnerable women and children.

Browning, M., Bourguignon, F., Chiappori, P.-A., and Lechene, V. (1994). Income and outcomes: A structural model of intrahousehold allocation. *Journal of Political Economy*, pages 1067–1096.

Browning, M. and Chiappori, P.-A. (1998). Efficient intra-household allocations: A general characterization and empirical tests. *Econometrica*, pages 1241–1278.

Browning, M., Chiappori, P.-A., and Lewbel, A. (2013). Estimating consumption economies of scale, adult equivalence scales, and household bargaining power. *The Review of Economic Studies*, page rdt019.

Calvi, R. (2016). Why are older women missing in india? the age profile of bargaining power and poverty. *Unpublished Manuscript*, 45:58–67.

Calvi, R. (2017). Why are older women missing in india? the age profile of bargaining power and poverty.

Calvi, R., Lewbel, A., and Tommasi, D. (2017). LATE with Mismeasured or Misspecified Treatment: An Application to Women's Empowerment in India. (ECARES 2017-27).

Case, A., Lin, I.-F., and McLanahan, S. (2000). How hungry is the selfish gene? *The Economic Journal*, 110(466):781–804.

Case, A., Paxson, C., and Ableidinger, J. (2004). Orphans in africa: Parental death, poverty, and school enrollment. *Demography*, 41(3):483–508.

Cherchye, L., De Rock, B., Lewbel, A., and Vermeulen, F. (2015). Sharing rule identification for general collective consumption models. *Econometrica*, 83(5):2001–2041.

Cherchye, L., De Rock, B., and Vermeulen, F. (2011). The revealed preference approach to collective consumption behaviour: Testing and sharing rule recovery. *The Review of Economic Studies*, 78(1):176–198.

Chiappori, P.-A. (1988). Rational household labor supply. *Econometrica*, pages 63–90.

Chiappori, P.-A. (1992). Collective labor supply and welfare. *Journal of Political Economy*, pages 437–467.

Chiappori, P.-A. and Ekeland, I. (2009). The microeconomics of efficient group behavior: Identification. *Econometrica*, 77(3):763–799.

Deaton, A. (1989). Looking for boy-girl discrimination in household expenditure data. *The World Bank Economic Review*, 3(1):1–15.

Doepke, M. (2004). Accounting for fertility decline during the transition to growth. *Journal of Economic growth*, 9(3):347–383.

Doepke, M. and Zilibotti, F. (2005). The macroeconomics of child labor regulation. *The American Economic Review*, 95(5):1492–1524.

Doran, K. B. (2013). How does child labor affect the demand for adult labor? evidence from rural mexico. *Journal of Human Resources*, 48(3):702–735.

Duflo, E. (2003). Grandmothers and granddaughters: Old-age pensions and intra-household allocation in south africa. *The World Bank Economic Review*, 17(1):1–25.

Dunbar, G. R., Lewbel, A., and Pendakur, K. (2013). Children's resources in collective households: Identification, estimation, and an application to child poverty in malawi. *The American Economic Review*, 103(1):438–471.

Dunbar, G. R., Lewbel, A., Pendakur, K., et al. (2017). *Identification of Random Resource Shares in Collective Households Without Preference Similarity Restrictions*. Bank of Canada.

Evans, D. K. and Miguel, E. (2007). Orphans and schooling in africa: A longitudinal analysis. *Demography*, 44(1):35–57.

Fafchamps, M. and Wahba, J. (2006). Child labor, urban proximity, and household composition. *Journal of Development Economics*, 79(2):374–397.

Galor, O. and Weil, D. N. (2000). Population, technology, and growth: From malthusian stagnation to the demographic transition and beyond. *The American Economic Review*, 90(4):806–828.

Gorman, W. M. (1976). Tricks with utility functions. *Essays in Economic Analysis*, pages 211–243.

Grant, M. J. and Yeatman, S. (2012). The relationship between orphanhood and child fostering in sub-saharan africa, 1990s–2000s. *Population Studies*, 66(3):279–295.

Grant, M. J. and Yeatman, S. (2014). The impact of family transitions on child fostering in rural malawi. *Demography*, 51(1):205–228.

Güneş, P. M. (2016). The impact of female education on teenage fertility: Evidence from turkey. *The BE Journal of Economic Analysis & Policy*, 16(1):259–288.

Hamilton, W. (1964). *The Genetical Evolution of Social Behaviour. I.*

Hazan, M. and Berdugo, B. (2002). Child labour, fertility, and economic growth. *The Economic Journal*, 112(482):810–828.

Kawu, H. A. and Abdur-Rahman, A. S. (2014). Emerging nigerian muslim fear over the implementation of child rights act. *International Journal of Education and Literacy Studies*, 1(2):27–33.

Lewbel, A. (2010). Shape-invariant demand functions. *The Review of Economics and Statistics*, 92(3):549–556.

Lewbel, A. and Pendakur, K. (2008). Estimation of collective household models with engel curves. *Journal of Econometrics*, 147(2):350–358.

Lise, J. and Seitz, S. (2011). Consumption inequality and intra-household allocations. *The Review of Economic Studies*, 78(1):328–355.

Menon, M., Pendakur, K., and Perali, F. (2012). On the expenditure-dependence of children's resource shares. *Economics Letters*, 117(3):739–742.

Moehling, C. M. (1999). State child labor laws and the decline of child labor. *Explorations in Economic History*, 36(1):72–106.

Nwapi, C. (2011). International treaties in nigerian and canadian courts. *African Journal of International and Comparative Law*, 19(2011):38–65.

Ogunniran, I. (2010). The child rights act versus sharia law in nigeria: Issues, challenges and a way forward. *Child. Legal Rts. J.*, 30:62.

Pendakur, K. (1999). Semiparametric estimates and tests of base-independent equivalence scales. *Journal of Econometrics*, 88(1):1–40.

Piza, C. and Souza, A. P. (2016). The causal impacts of child labor law in brazil: Some preliminary findings. *The World Bank Economic Review*, page lhw024.

Reniers, G. (2003). Divorce and remarriage in rural malawi.

Rosenzweig, M. R. and Evenson, R. (1977). Fertility, schooling, and the economic contribution of children of rural india: An econometric analysis. *Econometrica: Journal of the Econometric Society*, pages 1065–1079.

Rothbarth, E. (1943). Note on a method of determining equivalent income for families of different composition. *Appendix IV in War-Time Pattern of Saving and Expenditure by Charles Madge, University Press, Cambridge.*

Serra, R. (2009). Child fostering in africa: When labor and schooling motives may coexist. *Journal of Development Economics*, 88(1):157–170.

Tommasi, D. and Wolf, A. (2016). Overcoming weak identification in the estimation of household resource shares.

Udry, C. (1996). Gender, agricultural production, and the theory of the household. *Journal of Political Economy*, 104(5):1010–1046.

Vermeulen, F. (2002). Collective household models: Principles and main results. *Journal of Economic Surveys*, 16(4):533–564.