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Precisely Timing Dissipative Quantum Information Processing

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Dissipative engineering constitutes a framework within which quantum information processing protocols are powered by system-environment interaction rather than by unitary dynamics alone. This framework embraces noise as a resource and, consequently, offers a number of advantages compared to one based on unitary dynamics alone, e.g., that the protocols are typically independent of the initial state of the system. However, the time independent nature of this scheme makes it difficult to imagine precisely timed sequential operations, conditional measurements, or error correction. In this work, we provide a path around these challenges, by introducing basic dissipative gadgets which allow us to precisely initiate, trigger, and time dissipative operations while keeping the system Liouvillian time independent. These gadgets open up novel perspectives for thinking of timed dissipative quantum information processing. As an example, we sketch how measurement-based computation can be simulated in the dissipative setting.

One of the main goals in quantum information science and in quantum technologies in general is to understand the information processing power available within the framework of quantum mechanics. Several quantum computational models have been proposed and analyzed, and each has its strengths and weaknesses. Every such model will suffer severely from the problem of decoherence due to noise from a practically inevitable environment, so that a lot of effort has been invested in developing methods for isolating systems from their destructive environment. In such a picture, quantum noise and dissipation are seen as necessarily being detrimental for coherent quantum state manipulation. Recently, however, a new avenue has been suggested for addressing this issue: actively exploiting the dissipation into the environment. In a number of recent studies, it has been shown both experimentally [1,2] and theoretically [3–11] that several of the basic tasks in quantum information science (quantum computation [3], state preparation [3–5,7,8], distillation [9], storage [10], and cooling [11]) can be performed by engineering the system environment coupling instead of isolating a subsystem and performing coherent controlled unitary dynamics on it.

The most counterintuitive application of such a paradigm might be dissipative quantum computing (DQC) [3], a model of quantum computation in which one assumes a system-environment interaction described by a Markovian master equation, where the computation is encoded in the Lindblad operators and the outcome of the computation is encoded in the unique stationary state of the open system. The main benefit with this approach (that under appropriate conditions the unique stationary state is reached rapidly starting from any initial state) appears to imply a potentially severe disadvantage: that operations cannot be performed sequentially in time. Steps of a procedure cannot be conditioned on previous steps, it is not clear how to "stop preparing" a state, and it is far from clear how to incorporate error correction into any such scheme.

In this work, we open up new perspectives for dissipative quantum information processing by introducing and analyzing a number of dissipative "gadgets." They can be combined to act as time triggers for various dissipative operations; i.e., they allow a certain dissipative process to start (or stop) acting after a very specific point in time: They serve as convenient "clocks" in a framework with time independent Liouvillians. The functioning of these tools is based on a counterintuitive property of classical Markov chains called the "cutoff phenomenon" [12], which can be lifted up into the quantum setting [13]. We also show how such gadgets can be used in order to "translate" any scheme involving unitary dynamics, measurements, and conditional dynamics into a time independent dissipative setting. We then outline how these time triggers can be used to perform dissipative measurement-based quantum computation [14]. The timer gadgets can be used to show that DQC is universal also for geometrically local interactions (6-local if embedded in a 3D lattice). In the original proof of DQC [3], the interactions were made 5-local but in a way which cannot be embedded in a lattice geometry. We show that timed dissipation-driven quantum information processing is indeed possible.

The Letter is set up as follows. We first introduce an initialization gadget which allows us to prepare a given state for a finite amount of time. Next, we combine the initialization gadget with a special behavior from Markov chain mixing to construct a rudimentary timer gadget, which allows us to trigger a dissipative operation at a specific point in time. Rigorous error estimates for these gadgets are given in Supplemental Material [15]. We go on

to outline an application of these gadgets: a dissipative one-way quantum computation (D1WQC) scheme.

Basic building blocks.—The basic conceptual building blocks for the dissipative gadgets considered here are an idea for initialization and a particular mixing behavior called the "cutoff phenomenon." Throughout, we will assume that the open system dynamics are properly modeled by a Markovian master equation with local terms in the Liouvillian $\mathcal{L}$, so that

$$\frac{d}{dt}\rho = \mathcal{L}(\rho) = \sum_j \left(L_j \rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho\} \right),$$

where each Lindblad operator L_j has a local support which is independent of the system size. This amounts to local interactions and local heat baths. From this point on, we will not invoke the heat baths explicitly but assume that all of the Lindblad operators are derived from the interaction with the environment.

The initialization gadget.—For initialization to be successful, (i) the qubit must be prepared in the desired state in a time which scales at most polynomially in the gadget size M while the error decreases exponentially with M, and (ii) the state preparation must stop acting after a time which is specified by the desired dissipative process. We show how to initialize a single qubit in $|0\rangle$, with a preparation time logarithmic in the gadget size. For this, consider a star-shaped graph as in Fig. 1(a), and label the central qubit as c and the auxiliary qubits as $j = 1, \ldots, M$ [16]. The central qubit—the one we want to initialize—is surrounded by M auxiliary qubits. The Liouvillian of the gadget can be written $\mathcal{L}^{\text{ini}} = \mathcal{L}^{\text{ad}} + \mathcal{L}^{\text{cp}}$, where $\mathcal{L}^{\text{ad}}$ represents a collection of amplitude damping channels acting independently on the M auxiliary qubits with rate ω:
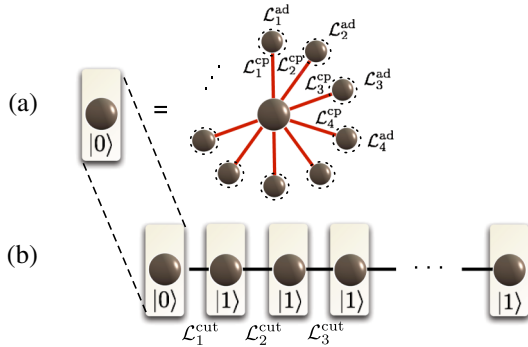

FIG. 1 (color online). (a) Depiction of an initialization gadget. The central circle depicts the target qubit, the outer circles the auxiliary qubits. The full red lines connecting the timer qubit to the auxiliary ones illustrate the conditional state preparation local Liouvillians $\mathcal{L}_j^{\text{cp}}$, and the dotted circles around the auxiliary qubits illustrate unconditional amplitude damping Liouvillians $\mathcal{L}_j^{\text{ad}}$; i.e., $\mathcal{L}^{\text{ini}} = \mathcal{L}^{\text{cp}} + \mathcal{L}^{\text{ad}}$. (b) Depiction of the linear timer gadget. The circles represent the timer qubits, the lines connecting the timer qubits are the local Liouvillians $\mathcal{L}_j^{\text{cut}}$, and the boxes illustrate the initialization devices from (a).

$$L_k^{\text{ad}} = \sqrt{\omega}|0_k\rangle\langle 1_k|, \tag{1}$$

for $k = 1, \ldots, M$, and $\mathcal{L}^{\text{cp}}$ represents the preparation of the central qubit in state $|0_c\rangle\langle 0_c|$, at a rate Γ, conditioned on the auxiliary qubits being in the state $|1_k\rangle\langle 1_k|$:

$$L_k^{\text{cp}} = \sqrt{\Gamma}|0_c\rangle\langle 1_c| \otimes |1_k\rangle\langle 1_k|. \tag{2}$$

Intuitively, after a time $t_M = \mathcal{O}(\log M/\omega)$, the M auxiliary qubits will all have relaxed to $|0\rangle$ to within an exponentially small error, so that the state preparation essentially stops acting. We set out to show that, for all but an exponentially small fraction of input states, the central qubit will be in the desired state $|0_c\rangle$ after the time t_M with an error that is exponentially small in M. Importantly, note that the dynamics of the initializer gadget acts exclusively on the diagonal elements of the system's density matrix. This implies that no off-diagonal element contributes to the mixing time analysis.

Theorem 1 (initialization).—Let ρ be an arbitrary input state, and let $\mathcal{L}^{\text{ini}}$ be the Liouvillian for the initialization gadget. If there exist $\delta, c > 0$ and a subset $S \subset \{1, \ldots, M\}$, with $|S| = cM$ such that $\langle 1_j|\rho_j|1_j\rangle > \delta$ for all $j \in S$, then for any $\varepsilon > 0$, there exists a $\tau = \mathcal{O}(\log(M))$ such that, for all $t > \tau$,

$$\langle 1_c|\rho_c(t)|1_c\rangle \leq M e^{-\mu M} + \varepsilon,$$

where ρ_j is the restriction of ρ to subsystem j, $\rho_c(t) = \text{tr}_{\text{aux}} e^{t\mathcal{L}^{\text{ini}}}(\rho)$ is the partial trace over the auxiliary qubits, and μ is some positive constant which depends on $\{c, \delta, \omega, \Gamma\}$.

The precise form of μ is given in the Supplemental Material [15]. The theorem states that if a sufficient number of auxiliary bits have a nonzero overlap with the $|1\rangle$ state, then the stationary state of the central qubit will be exponentially close to $|0_c\rangle$. It also states that the system equilibrates in a time $\mathcal{O}(\log(M))$. It becomes clear in the proof that the assumptions on the initial state can be further weakened, without affecting the result too severely.

The timer gadget.—We will now show how to trigger a dissipative operation at a particular point in time and, by concatenation, how to perform sequential dissipative operations. Intuitively, one might be tempted to introduce time scales by simply imposing different decay rates $\gamma_1 \ll \gamma_2 \ll \cdots \ll \gamma_N$ before each desired dissipative operation $\mathcal{L}_j$. But it becomes evident that either the convergence time or the error of such a process will scale poorly in N. A way around this problem is to further exploit the cutoff phenomenon [12]. A dissipative process is said to exhibit cutoff-type behavior [13] if there exists an initial state which does not converge to stationarity for a long time and converges exponentially fast (in the number of qubits) after a specific time.

We provide perhaps the simplest example of a timer gadget: a line of N qubits such that the last one exhibits a cutoff from $|1_N\rangle$ to $|0_N\rangle$ in a time $\mathcal{O}(N)$. Each qubit of the

timer must first be initialized in the state which will lead the chain to exhibit a cutoff. As depicted in Fig. 1(a), the timer gadget consists of a main line of N "timer" qubits, connected by $(N - 1)$ local Liouvillians $\mathcal{L}^{\mathrm{cut}}$ with Lindblad operators: $L_j^{\mathrm{cut}} = \sqrt{\gamma}|0_j\rangle\langle 0_j| \otimes |0_{j+1}\rangle\langle 1_{j+1}|$, for $j = 1, \ldots, N - 1$. The local Liouvillian $\mathcal{L}_j^{\mathrm{cut}}$ acts as an amplitude damping channel with rate γ on qubit $j + 1$ if qubit j is in $|0_j\rangle$ and acts trivially otherwise. The initialization gadget guarantees that we start in the desired state vector $|\phi_0\rangle := |0\rangle \otimes |1\rangle^{\otimes(N-1)}$; i.e., the first qubit is in $|0\rangle$ and all of the other qubits are in $|1\rangle$. The qubit which will act as a trigger is the last one, so we want to estimate how long it will take before the Nth qubit is in $|0\rangle$, assuming proper initialization. This can be calculated explicitly. Let tr_{N-1} be the partial trace over the first $N - 1$ qubits, and then the following proposition proves cutoff behavior of the timer gadget in arguably the strongest possible sense.

Theorem 2 (timing).—Let N be the number of timer qubits, and then for $t_N(x) = (N + x\sqrt{N})/\gamma$,

$$\frac{1}{2}||\mathrm{tr}_{N-1}[e^{t_N(x)\mathcal{L}^{\mathrm{cut}}}(\phi_0)] - |0_N\rangle\langle 0_N|\,||_1$$
$$= 1 - \Phi(x) + \theta(1/\sqrt{N}),$$

where $\Phi(x) = \int_{-\infty}^{x} e^{-t^2/2}dt/\sqrt{2\pi}$ is the cumulative normal distribution function.

For any real $c > 0$, it follows from Theorem 2 that

$$\lim_{N\to\infty} \langle 0_N|\mathrm{tr}_{N-1}[e^{ct_N(0)\mathcal{L}^{\mathrm{cut}}}(\phi_0)]|0_N\rangle = 0, \quad \text{for } c < 1,$$
$$\lim_{N\to\infty} \langle 0_N|\mathrm{tr}_{N-1}[e^{ct_N(0)\mathcal{L}^{\mathrm{cut}}}(\phi_0)]|0_N\rangle = 1, \quad \text{for } c > 1,$$

which is the definition of a cutoff given in Ref. [13]. However, Theorem 2 makes a stronger statement yet. It gives us a cutoff time ($t_N = N/\gamma$), as well as the accurate cutoff window ($\Delta t_N = \sqrt{N}$), and the deviation away from the cutoff time in the form of the cumulative distribution function. This last piece of information is especially useful, as it allows us to estimate the inaccuracy of the timer and analyze the error propagation for multiple uses of the gadget.

It follows that if the state of the lattice is properly initialized to $|\phi_0\rangle$, then an operation conditioned on the last qubit will start taking place only after a time of order $t_N = N/\gamma$, while it is exponentially suppressed before that. The trigger will operate on a time scale of order $\sqrt{N}$, and the error in the triggering will be given by the tail of a normal distribution. In practice, this means that a polynomial number of dissipative operations can be triggered each in linear time, with a linear time interval between them, while accurately triggering each operation with exponential precision. Rigorous error analysis is given in the Supplemental Material [15]. We note that the above linear timer gadget displays three dissipative time scales, given by γ, the rate decay of the timer Lindblad operators, Γ, the decay rate of the initialization Lindblad operators,

and ω, the decay rate of the amplitude damping operators on the auxiliary qubits of the initialization gadget. All of these time scales are constant in the size and in the number of the gadgets, so we can choose them in such a way that they clearly separate active dissipative processes. For optimal functionality we then want $\gamma \ll \omega \ll \Gamma$, so that the timer Lindblad operators effectively do not start acting before $|\phi_0\rangle$ is initialized. We now turn to applications.

Dissipative one-way quantum computation.—We sketch how to perform D1WQC by using the timer and initialization gadgets. Before we start, let us emphasize that one does not need to have any coherent unitary control over the quantum system but instead merely makes use of dissipative dynamics exhibiting quantum and classical features. Still, it is the very point of our schemes that, at the end of the day, the statistics of the measurement outcomes are exactly as if one had performed this unitary transformation. We need to be able to perform the following three main steps: (i) initialize a two-dimensional lattice in a cluster state [14], (ii) perform conditional projective measurements in a time-ordered fashion on the cluster state which simulate a universal gate set, and (iii) propagate the classical boolean phase information from one measurement to another. All three of these steps have to be performed purely dissipatively, in such a way that the stationary state is essentially unique and that it is reached in a time which scales polynomially in the system size. It turns out that the obvious bottlenecks in the program—initialization of the cluster state and timing of operations—are solved by the timer and initialization gadgets introduced above. It can also be seen that, by construction, every Lindblad operator is geometrically local.

Indeed, a graph state [17] of N qubits can be prepared dissipatively in a time which scales at most as $\mathcal{O}(\log N)$ [13], using 5-local Liouvillians in case of a 2D lattice. Moreover, the state can be initialized by conditionally controlling each Lindblad operator of the graph state preparation with an initialization gadget. In particular, we can use the star geometry above, as the graph state can be prepared in time $\mathcal{O}(\log N)$.

Projective measurements can also be very naturally cast into a dissipative setting. Let $\{|\xi_k\rangle\}$ be the orthogonal basis associated with a projective measurement on subsystem s, and let r denote the classical registry storing the measurement; then the master equation describing the measurement is

$$\mathcal{L}^{\mathrm{meas}}(\rho) = \sum_k \langle\xi_k|\rho_s|\xi_k\rangle\mathbb{1}_s \otimes |\xi_k\rangle\langle\xi_k| - \rho.$$

Note that measurement is on the same footing as logical gates in this model, and, again, all resources are treated equally.

In D1WQC it is also necessary to perform measurements conditional on the previous measurement outcomes. This can be performed by keeping track of the pseudomeasurements on a classical information bus which is updated by

conditioning. This bus only needs to keep track of two bits per measurement and can be fed into future measurements by conditioning. This can be performed locally on a three-rail bus. Here, each step in the D1WQC is triggered by a global clock (timer gadget). A logical CNOT gate can be performed by simultaneous measurements on nine qubits with two input and two output qubits [14]. Arbitrary one-qubit measurements can be performed on a line of five qubits, but the measurements have to be performed sequentially with feedback [14].

Rather than performing a full analysis of the dissipative versions of these schemes, which is beyond the scope of this Letter, we will give a detailed analysis of a subroutine, which already contains essentially all of the main ingredients of this model: qubit state transfer along a line. Consider a 1D chain of odd-n spins, where we want to transfer the state of the first spin to the last spin by directed measurements. We assume that the initial state is already encoded in the cluster state at the input. In the standard setting, this is done by first preparing the chain in the state $|\varphi_{\text{in}}\rangle \otimes |+\rangle^{\otimes(n-1)}$, where $|\pm\rangle := (|0\rangle \pm |1\rangle)/\sqrt{2}$, and then applying an entangling unitary operation $S_j = |0_j\rangle\langle 0_j| + |1_j\rangle\langle 1_j| \otimes \sigma_{j+1}^z$ onto the state. The cluster state with $|\varphi_{\text{in}}\rangle$ encoded in the input will be denoted $|\varphi_{\text{in}}\rangle_{\mathcal{C}} := S|\varphi_{\text{in}}\rangle$.

Now, in the standard state transfer by measurement, one would measure qubits 1 through $n-1$ in the σ_x basis and obtain measurement outcomes $\{s_1, \ldots, s_{n-1}\}$ where $s_j \in \{0, 1\}$, and $s_j = 0$ corresponds to the $|+_j\rangle$ projection and $s_j = 1$ corresponds to the $|-_j\rangle$ projection. The final state vector after $n-1$ measurements is $|\phi_{\text{out}}\rangle = |s_1\rangle \otimes \cdots \otimes |s_{n-1}\rangle \otimes |\varphi_{\text{out}}\rangle$. Indeed, $|\varphi_{\text{out}}\rangle = U_\Sigma^{\text{st}}|\varphi_{\text{in}}\rangle$, where $U_\Sigma^{\text{st}} = \sigma_x^{s_1+s_3+\cdots+s_{n-2}}\sigma_z^{s_2+\cdots+s_{n-1}}$. In order to recover the original state, one needs to perform the inverse unitary $(U_\Sigma^{\text{st}})^\dagger$ onto $|\varphi_{\text{out}}\rangle$. In order to illustrate the analogous dissipative construction, we show here how to perform the transfer of a state along a three qubit chain (for the n qubit state transfer, see the Supplemental Material [15]).

Consider a chain of three qubits, labeled $\{1, 2, 3\}$, together with two auxiliary qubits, labeled $\{4, 5\}$. As in the standard measurement-based state transfer, we prepare the cluster state of three qubits with $|\varphi_{\text{in}}\rangle$ encoded in the first: i.e., $|\varphi_{\text{in}}\rangle_{\mathcal{C}}$. We will then act on the five qubits of the system with two maps sequentially: i.e., act with $e^{t\mathcal{L}^A}$ until equilibrium is reached, and then act with $e^{t\mathcal{L}^B}$. Both of these semigroups can be seen to mix rapidly, as they are conditional depolarizing semigroups with few conditioners. The sequential application of the maps can be performed by attaching each to a timer qubit which prescribes a specific starting time to each map. The Liouvillians $\mathcal{L}^A$ and $\mathcal{L}^B$ are defined by $L_{1,j}^A = \sqrt{\omega}|+_j\rangle \times \langle +_j| \otimes |0_{j+3}\rangle\langle 1_{j+3}|$ and $L_{2,j}^A = \sqrt{\omega}|-_j\rangle\langle -_j| \otimes |1_{j+3}\rangle \times \langle 0_{j+3}|$, respectively, where $j = \{1, 2\}$. The second Liouvillian has Lindblad operators $L_1^B = \sqrt{\omega}\sigma_3^z \otimes |0, 0\rangle \times \langle 0, 1|$, $L_2^B = \sqrt{\omega}\sigma_3^x \otimes |0, 0\rangle\langle 1, 0|$, and $L_3^B = \sqrt{\omega}\sigma_3^z\sigma_3^x \otimes$

$|0, 0\rangle\langle 1, 1|$, where the bras and kets with zeros and ones always refer to the auxiliary qubits. These maps will do the following: $e^{t\mathcal{L}^A}$ takes $|\varphi_{\text{in}}\rangle_{\mathcal{C}}$ to an even superposition of

$$|+, +\rangle \otimes |\varphi_{\text{in}}\rangle \otimes |0, 0\rangle, \quad |+, -\rangle \otimes (\sigma_z|\varphi_{\text{in}}\rangle) \otimes |0, 1\rangle,$$

$$|-, +\rangle \otimes (\sigma_x|\varphi_{\text{in}}\rangle) \otimes |1, 0\rangle, \quad |-, -\rangle \otimes (\sigma_x\sigma_z|\varphi_{\text{in}}\rangle) \otimes |1, 1\rangle.$$

In other words, it performs a measurement on qubits one and two but keeps a record of each outcome (on the classical registry $\{4, 5\}$). The second map $e^{t\mathcal{L}^B}$ acts as the unitary correction U_Σ^{st} by rotating the superposition of states above onto the vector $|\phi_{\text{out}}\rangle\langle\phi_{\text{out}}| = \mathbb{1}_{1,2} \otimes |\varphi_{\text{in}}\rangle \times \langle\varphi_{\text{in}}| \otimes |0, 0\rangle\langle 0, 0|$, from which $|\varphi_{\text{in}}\rangle$ can be read off directly. The CNOT and single qubit rotation are performed in much the same way with a slightly more involved classical bus for measurement and feedback. Likewise, concatenation of elementary gates can be handled efficiently, as the classical bus only ever has to keep two bits of information per updated qubit.

Summary and outlook.—In this work, we have introduced a basic toolbox for timing dissipative operations with a time independent Liouvillian. We have shown that dissipative operations can be performed sequentially with only a polynomial overhead in qubits or in Lindblad operators. We also show how to initialize (i.e., switch on and off) dissipative operations, which then leads to full dissipative state initialization. We have provided the necessary error analysis to show that these gadgets can be applied sequentially while remaining efficient. We outlined how these gadgets can be used to perform a dissipative version of one-way computation, holding the promise of geometrically local dissipative computation. The gadgets proposed here give rise to a versatile toolbox for constructing novel schemes of dissipative quantum information processing, opening up avenues for dissipative engineering. Finally, we mention a very promising application for our gadgets: dissipative error correction, where the interplay between active and passive operations become critical. A fully passive error correction scheme remains elusive, but clues suggest that dissipative engineering might be the right framework (see Supplemental Material [15]).

[1] J. T. Barreiro, M. Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller, and R. Blatt, Nature (London) **470**, 486 (2011).
[2] H. Krauter, K. Jensen, W. Wasilewski, J. Petersen, J. Cirac, and E. Polzik, Phys. Rev. Lett. **107**, 080503 (2011).
[3] F. Verstraete, M. M. Wolf, and J. I. Cirac, Nat. Phys. **5**, 633 (2009).

[4] S. Diehl, A. Micheli, A. Kantian, B. Kraus, H. P. Büchler, and P. Zoller, Nat. Phys. **4**, 878 (2008).

[5] B. Kraus, H. Büchler, S. Diehl, A. Kantian, A. Micheli, and P. Zoller, Phys. Rev. A **78**, 042307 (2008).

[6] J. Eisert and T. Prosen, arXiv:1012.5013.

[7] M. J. Kastoryano, F. Reiter, and A. S. Sørensen, Phys. Rev. Lett. **106**, 090502 (2011).

[8] C. A. Muschik, E. S. Polzik, and J. I. Cirac, Phys. Rev. A **83**, 052312 (2011).

[9] K. G. H. Vollbrecht, C. A. Muschik, and J. I. Cirac, Phys. Rev. Lett. **107**, 120502 (2011).

[10] F. Pastawski, L. Clemente, and J. I. Cirac, Phys. Rev. A **83**, 012304 (2011).

[11] A. Mari and J. Eisert, Phys. Rev. Lett. **108**, 120602 (2012).

[12] P. Diaconis, Proc. Natl. Acad. Sci. U.S.A. **93**, 1659 (1996).

[13] M. J. Kastoryano, D. Reeb, and M. M. Wolf, J. Phys. A **45**, 075307 (2012).

[14] R. Raussendorf and H. J. Briegel, Phys. Rev. Lett. **86**, 5188 (2001); Quantum Inf. Comput. **6**, 433 (2002).

[15] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.110.110501 for a rigorous proofs of Theorems 1 and 2, and for a discussion of dissipative error correction.

[16] Many physical systems such as quantum dots in the condensed-matter context naturally have a star structure, so that, ultimately, the type of locality desired for the initialization gadget will depend on the physical setup. More generally, all the Liouvillians discussed here can be physically implemented by adding an auxiliary mode that is undergoing a suitable continuous measurement process. In particular, in the quantum optics setting, such schemes are natural.

[17] M. Hein, J. Eisert, and H. J. Briegel, Phys. Rev. A **69**, 062311 (2004).