Towards Holography via Quantum Source-Channel Codes

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While originally motivated by quantum computation, quantum error correction (QEC) is currently providing valuable insights into many-body quantum physics, such as topological phases of matter. Furthermore, mounting evidence originating from holography research (AdS/CFT) indicates that QEC should also be pertinent for conformal field theories. With this motivation in mind, we introduce quantum source-channel codes, which combine features of lossy compression and approximate quantum error correction, both of which are predicted in holography. Through a recent construction for approximate recovery maps, we derive guarantees on its erasure decoding performance from calculations of an entropic quantity called conditional mutual information. As an example, we consider Gibbs states of the transverse field Ising model at criticality and provide evidence that they exhibit nontrivial protection from local erasure. This gives rise to the first concrete interpretation of a bona fide conformal field theory as a quantum error correcting code. We argue that quantum source-channel codes are of independent interest beyond holography.

Driven by the prospects of building a large scale quantum computer, the field of quantum information [1] has flourished and within it, quantum error correction (QEC). QEC is the framework for quantum channel coding and allows us to understand how noisy degrees of freedom (DoFs) may be used to implement a smaller number of reliable DoFs [2]. In contrast, compression (source coding) describes how statistical redundancies may be exploited to reduce the naive number of required resources. Seeking separation of concerns, these two techniques are conventionally dealt with separately, particularly in the classical regime where vast volumes of data justify the use of Shannon's source-channel separation theorem [3].

In this Letter, we depart from the customary approach of separating source and channel coding. We justify this from two perspectives. From a coding theory perspective, joint source-channel coding provides improved performance when the source is not an asymptotic i.i.d. distribution [4]. Our second motivation comes from fundamental physics, more specifically, from quantum gravity.

In the holographic approach [5–8], quantum gravity on a $d + 1$-dimensional asymptotically anti–de Sitter (AdS) background is understood to be isomorphic to certain conformal field theories (CFTs) in d dimensions. For example, the vacuum state in the CFT corresponds to (is dual to) the AdS vacuum, while thermal states in the CFT correspond to a static black hole in the center of AdS [9,10]. It has been argued that this isomorphism, dubbed AdS/CFT correspondence, shows both features of quantum error correction [11] and data compression [12].

It is difficult to reconcile how error correction features can show up in a single state of a CFT, such as the vacuum or a full rank thermal state, from the perspective of traditional quantum error correction codes (QECCs). The main difficulty lies

in the absence of a proper subspace of full Hilbert space, where logical information can be deposited. We require a distinct framework and introduce the name source-channel codes to emphasize that the task being achieved is different from that of traditional QECCs and closely follows earlier work by Barnum and Knill [13] instead. Through an example, we show that error-correcting properties of CFTs can be made explicit in the framework of quantum source-channel codes. Our analysis confirms predictions from holography, even beyond its expected regime of validity.

Our presentation is structured as follows. We first review some basics about source coding and channel coding from a purely information theoretic perspective. We then discuss source-channel coding and show, using recent results from quantum information theory, how recovery guarantees can be given by calculating entropies. Finally, guided by the QECC interpretation of AdS/CFT [11,14–17], we consider thermal states of the transverse field Ising CFT as an example of quantum source-channel coding. This is supported by numerical evidence of nontrivial error correction guarantees with respect to local erasures.

Source and channel coding.—In traditional source coding, statistical properties of an input source distribution X are identified and exploited to reduce the message size. The standard procedure is to discard atypical inputs. Shannon's source coding theorem [3] considers the asymptotic i.i.d. setting (i.e., the source consists of a large number n of messages sampled over the same distribution X). It states that $\lceil nH(X) \rceil$ bits are sufficient to support most of this product distribution (compression fails with vanishing probability), where $H(X)$ denotes the Shannon entropy of X; Schumacher compression [18] achieves this in the quantum setting. However, if the source space is larger than the output space, compression will necessarily be lossy. In

particular, compression will be either trivial or lossy for any finite n.

QECCs are the paragon example of (quantum) channel coding. For concreteness, we focus on qubit codes parametrized by $[[n, k, d]]$. An encoding is an isometric map from k qubits to $n \geq k$ qubits. In particular, the code subspace is a 2^k dimensional subspace of the 2^n dimensional physical Hilbert space. For us, it will be important that such an isometry may be viewed as a unitary from k logical qubits and $n - k$ ancillas onto the physical Hilbert space (see Fig. 1, left). In certain settings, such as when the noise channel affects no more than $d - 1$ identified qubits or $(d - 1)/2$ unspecified qubits, the code space allows for perfect recovery. In general, however, if one aims for approximate recovery from the outset, it is possible to design codes which deal with a much broader set of noise channels; this is the approach taken by approximate quantum error-correcting codes [19].

Source-channel separation theorems state that it is possible to separate the problem of source coding and channel coding without any sacrifice in the achievable communication rate [3]. While these results generalize to the quantum setting (see Ref. [21] and references therein), they hold only in an asymptotic i.i.d. limit, assuming both the source and channel are memoryless. In contrast, for the single shot setting, which is relevant for practical communication problems with correlated data, joint source-channel coding has been found to be advantageous [4]. In particular, we consider the source Hilbert space to be of equal dimension as the target Hilbert space such that the combined source-channel encoding is a unitary U and lossless in the case of no erasure. This choice is also motivated by the holographic duality, which we seek to interpret as a unitary isomorphism.
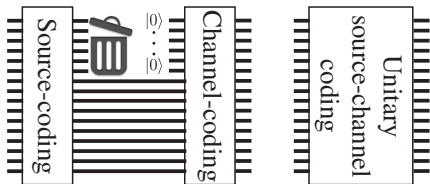

FIG. 1. For a density matrix ρ, Schumacher compression may be performed by a unitary that concentrates the information of ρ^n on the last $k \approx S(\rho)n$ qubits. As a result of the compression unitary, we may think of the remaining $n - k$ qubits as being set to zero, with high probability, and discarding them leads to minimal loss. Indeed, the compression can also be interpreted as an algorithmic cooling unitary [20] in which the discarded qubits approximate a known pure state. Precisely how small a loss will depend on ρ and the choice of k. Conversely, traditional channel coding for a $[[n, k, d]]$ QECC may be performed by a unitary that takes as input k data qubits and $n - k$ ancillary qubits, which are initially set to $|0\rangle$. We may view unitary source-channel coding as the result of matching parameters and skipping the intermediate reinitialization of ancillas.

We interpret a decomposition of a density matrix $\sigma = \int_\phi |\phi\rangle\langle\phi|\mu(\phi)d\phi$ into a convex combination of pure states as specifying a source distribution. We will refer to the probability measure μ as a resolution of σ into an ensemble of pure states, where the von Neumann entropy $S(\sigma)$ upper bounds the amount of classical information conveyed [22]. We defer the difficult problem of identifying an adequate source-channel encoding unitary U for a given source distribution and noise model to future work. Our analysis focuses on the erasure recovery of an encoded distribution ρ, related to σ by $\rho = U\sigma U^\dagger$.

In our approach, choosing a maximally mixed density matrix ρ within a given code subspace will correspond to considering average fidelity in traditional subspace encoding QECCs (we elaborate on this in Supplemental Material A [23], which builds on Refs. [24,25]). However, a density matrix ρ with generic spectrum (possibly supported over the full Hilbert space) will correspond to a nontrivial prior σ on the distribution of input states. In this sense, our approach combines source coding with channel coding.

Fidelities.—We now review fidelity measures and their relations used to benchmark the performance of source-channel codes. Given a noise channel $\mathcal{N}$ and the recovery channel $\mathcal{R}$, one may calculate the fidelity with which a single pure state $|\psi\rangle \in \mathcal{H}_{BC}$ is recovered as

$$F^2(|\psi\rangle, \mathcal{E}) := \langle\psi|\mathcal{E}(|\psi\rangle\langle\psi|)|\psi\rangle, \tag{1}$$

where $\mathcal{E} := \mathcal{R}\circ\mathcal{N}$ combines noise and recovery. Given a distribution μ of pure states, the weighted average fidelity

$$\bar{F}(\mu, \mathcal{E}) := \int_\psi \mu(\psi)\langle\psi|\mathcal{E}(|\psi\rangle\langle\psi|)|\psi\rangle d\psi \tag{2}$$

quantifies the corresponding weighted average. To obtain bounds on the average fidelity, we make use of the Schumacher entanglement fidelity, proposed in Ref. [26]. For any density matrix $\rho = \int_\psi |\psi\rangle\langle\psi|\mu(\psi)d\psi$, it is defined as

$$F_e(\rho, \mathcal{E}) := F^2(|\Psi_\rho\rangle, \mathrm{id}_A \otimes \mathcal{E}), \tag{3}$$

where $|\Psi_\rho\rangle$ is an arbitrary purification of ρ, i.e., a pure state on an enlarged Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_{BC}$ such that $\mathrm{Tr}_A(|\Psi_\rho\rangle\langle\Psi_\rho|) = \rho$. If ρ is a thermal state of a Hamiltonian with energies E_j, the thermofield double state [9]

$$|\Psi_{\mathrm{TFD}}\rangle := \frac{1}{\sqrt{\mathcal{Z}}}\sum_j e^{-\beta E_j/2}|j\rangle_A|j\rangle_{BC}, \tag{4}$$

with $\mathcal{Z} := \sum_j e^{-\beta E_j}$, provides an explicit choice for such a purification. However, definition (3) is independent of the purification used on the rhs.

Schumacher [26] proved that the average fidelity $\bar{F}$ with respect to any resolution μ of ρ is lower bounded by the entanglement fidelity

$$F_e(\rho, \mathcal{E}) \leq \bar{F}(\mu, \mathcal{E}). \qquad (5)$$

It is thus sufficient to focus on the entanglement fidelity $F_e(\rho, \mathcal{E})$ of a density matrix ρ in order to provide a lower bound on the weighted average fidelity (2) for any ensemble μ of states generating the statistics of ρ. Our next task will be to guarantee the existence of a recovery map $\mathcal{R}$ such that $\mathcal{E} := \mathcal{R} \circ \mathcal{N}$ realizes a high entanglement fidelity for a given noise map $\mathcal{N}$.

Markov condition and recovery.—If we know on which subsystem C the noise channel $\mathcal{N}$ acts nontrivially, we can invoke powerful prescriptions [27–29] to specify a recovery map $\mathcal{R}$. Given a tensor product Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$, and a noise channel $\mathcal{N}$ acting only on subsystem C, there exists a recovery map $\mathcal{R}$ such that

$$I(A:C|B) \geq -2 \log F(\rho_{ABC}, \mathrm{id}_A \otimes \mathcal{R}), \qquad (6)$$

where $I(A:C|B) := S_{AB} + S_{BC} - S_{ABC} - S_B$ is the conditional mutual information (CMI) of $A:C$ conditioned on B. Moreover, the recovery map $\mathcal{R}$ will act exclusively on BC, attempting to reconstruct C from B without access to A. Intuitively, the recovery map assumes the worst possible scenario for subsystem C and ignores its content altogether (i.e., $\mathcal{R} = \mathcal{R} \circ \mathcal{N} \equiv \mathcal{E}$). For $I(A:C|B) \ll 1$, we can use Eq. (6) to derive

$$1 - I(A:C|B) \leq F^2(\rho_{ABC}, \mathrm{id}_A \otimes \mathcal{R}). \qquad (7)$$

Furthermore, for pure ρ_{ABC}, the CMI simplifies to

$$I(A:C|B) = S_C + S_{BC} - S_B, \qquad (8)$$

where we have chosen an expression without reference to A. Interpreting $\mathcal{H} = \mathcal{H}_B \otimes \mathcal{H}_C$ as the physical Hilbert space and $\mathcal{H}_A$ as a fictitious ancilla space purifying ρ_{BC}, this corresponds to the problem of global recovery in which B includes all physical information not affected by noise $\mathcal{N}$. Assuming $\rho_{ABC} := |\Psi_\rho\rangle\langle\Psi_\rho|$ to be a pure state, we recognize the entanglement fidelity (3) in the rhs of Eq. (7). We then have our final result

$$1 - S_C + S_{BC} - S_B \leq F_e(\rho, \mathcal{E}) \leq \bar{F}(\mu, \mathcal{E}). \qquad (9)$$

While the general prescription [29] for $\mathcal{R}$ depends on ρ_{ABC}, given ρ_{BC}, it is independent of its purification on ABC. To summarize, given a state ρ_{BC} with a resolution μ and a noise map $\mathcal{N}$ acting exclusively on subsystem C, it is possible to specify a fixed recovery map $\mathcal{R}$ which recovers the state with entanglement (weighted average) fidelity above $1 - \epsilon$, provided $S_C + S_{BC} - S_B \leq \epsilon$.

Numerical evaluation on a CFT.—We observe that the two "natural" separations of CFT thermal distributions into degrees of freedom are far from i.i.d. A real space blocking of CFT degrees of freedom could lead to identical blocks, which are, however, not independent; CFTs present correlations at all scales. Traditionally, i.i.d. source coding would not take advantage of the correlations, leading to

a significant overestimate in the total source entropy, unless the block length is much larger than the CFT thermal length. A momentum space separation of CFT degrees of freedom would lead to independent degrees of freedom for a free CFT, which are, however, far from being identical to one another. This suggests that CFT thermal distributions might be good candidates for the application of source-channel coding.

We now discuss a concrete CFT model and the corresponding numerical results and later comment on the geometric interpretation suggested by holography. The critical transverse field Ising model on a ring

$$H_{\mathrm{TF}} = \sum_{j=1}^{n} (-\sigma_j^x \sigma_{j+1}^x - \sigma_j^z) \qquad (10)$$

is arguably the simplest spin Hamiltonian approximating a CFT. The parity operator $P := \bigotimes_{j=1}^{n} \sigma_j^z$ is a conserved quantity for H_{TF}, and $P_{\mathrm{even}} := (\mathbb{1} + P)/2$ and $P_{\mathrm{odd}} := (\mathbb{1} - P)/2$ are the corresponding projectors. Within each parity sector, H_{TF} is equivalent to a free fermion Hamiltonian

$$H_{\mathrm{maj}}^{(\mathrm{odd,even})} := \sum_{j=1}^{2n-1} i w_j w_{j+1} \pm i w_{2n} w_1, \qquad (11)$$

(also restricted to a specific fermionic parity sector [30]) where we have introduced Majorana fermion operators w_j, satisfying the commutation relations $\{\omega_i, \omega_j\} = \delta_{i,j}\mathbb{1}$. We will consider the thermal state at inverse temperature $\beta > 0$, restricted to the even parity sector

$$\rho_{BC} = \rho_{\mathrm{even}}^{(\beta)} := \frac{P_{\mathrm{even}} e^{-\beta H_{\mathrm{TF}}}}{\mathrm{tr}(P_{\mathrm{even}} e^{-\beta H_{\mathrm{TF}}})}. \qquad (12)$$

Since the original thermal state is block diagonal with respect to P, we are simply normalizing the even block.

We justify the parity projection of Eq. (12) from two perspectives. From a physical viewpoint, P distinguishes fermionic parity superselection sectors in the fermionic interpretation (11) of H_{TF}. A manifestation of the fermionic Hilbert space not being of product form is that the Fermionic parity cannot be changed by a physical noise process. However, the qubit erasure channel does not enforce this. The global parity projection P_{even} of (12) compensates for this at the level of the state in a relatively simple way. As we will see, the parity projection is also more favorable in terms of its coding properties (particularly, when we take $\beta \propto n$, as we will). While the parity projected state $\rho_{\mathrm{even}}^{(\beta)}$ seems to lead to a quadratic scaling $O(n^{-2})$ of the CMI of Eq. (8), the full thermal state results in a "trivial" $O(1/n)$ scaling [31].

The price we pay for including the parity projection is that, contrary to the full thermal state of (11), $\rho_{\mathrm{even}}^{(\beta)}$ is not a fermionic Gaussian state (see Supplemental Material B

[23], for a comparison to Gaussian states). This forces us to use exact diagonalization for the numerical evaluation of the CMI (8), which has a computational cost scaling exponentially in n instead of polynomially. In turn, this hinders us from considering $n > 13$ without dedicating numerical resources beyond the scope of this Letter.

The Hamiltonian H_{TF} (respectively, H_{maj}) has a linear dispersion relation and a gap scaling as $1/n$. In order to maintain the global entropy $S_{BC} = S(\rho_{even}^{(\beta)})$ roughly independent of the system size n, it will also be necessary to scale $\beta \propto n$ (i.e., we scale the temperature $1/\beta$ linearly with the Hamiltonian gap). Note that in the interpretation as quantum source-channel codes n, the entropy S_{BC} and $|C|$, respectively, play roles analog to n, k, and d in traditional $[[n, k, d]]$ QECCs, where to each d, we may associate a guaranteed performance in terms of average fidelity (or lack thereof). In Supplemental Material C [23], we discuss the possibility of encoding more logical information by scaling β sublinearly with n.

Take C to be a contiguous subregion of the lattice ($C \subset [1, n]$). While starting from Ref. [32], there are numerous analytical calculations of entanglement entropies in CFTs [33–36], we are unaware of one allowing for the evaluation of Eq. (8) in terms of the joint scaling $\beta \propto n \to \infty$ and for a specific parity sector to a precision which goes beyond constant terms (whereas, cancellation is only confirmed for divergent terms). We, therefore, numerically calculate the CMI in Eq. (8) for $\rho_{even}^{(\beta)}$ and confirm that there is a nontrivial increase in protection of the encoded information as n increases, see Fig. 2.

The holographic interpretation.—We now turn to connecting insights from quantum source-channel coding with holography. Recently, a connection between the holographic approach to quantum gravity [5–8,37] and quantum information notions such as entanglement [38], compression [12], and QECCs [11,14–17,39] has emerged. Toy models discussed so far [14,16] build upon notions of subspace coding, where the code space corresponds to the low energy subspace. In contrast, source-channel codes allow interpreting the full holographic duality as a unitary isomorphism between bulk and boundary Hilbert spaces, while preserving error correcting properties to a certain extent.

In the holographic dictionary, CFT thermal states are interpreted as being dual to static black holes, where the black hole area is given by the thermal entropy in Planck units [9,10]; the constant black hole area corresponds to constant thermal entropy. It has furthermore been argued that certain purifications of the thermal state, such as the thermofield double $|\Psi_{TFD}\rangle$, admit a geometric interpretation beyond the black hole horizon as wormholes [40].

Recently, a code theoretic interpretation for general holographic geometries has been put forth [17]. In this interpretation, black hole horizons are interpreted as logical degrees of freedom carrying an amount of information equal to their horizon area. Corresponding CFT degrees of
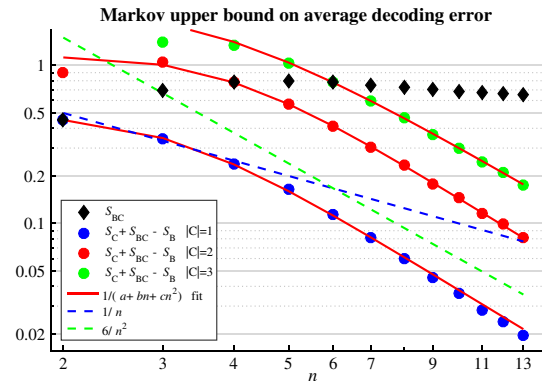

FIG. 2. We consider $\rho_{BC} = \rho_n^{(\beta)}$ from Eq. (12). The number of physical qubits is given by $n = |BC|$ and take $\beta = 0.2n$ to maintain the global entropy S_{BC} (black diamonds) approximately constant. We plot CMI of Eq. (8), associated to a small subsystem C, using blue, red, and green dots, corresponding to $|C| = 1, 2, 3$. These provide convincing numerical evidence that the CMI converges quadratically for large n. We include $1/n$ (dashed blue) and $6/n^2$ (dashed green) lines as visual guides to the eye as well as Lorenzian fits to the data (solid red); we omit small $n < 2|C|$ from fitting.

freedom are interpreted as physical degrees of freedom. In this setting, the degrees of freedom associated to a black hole are recoverable from a region B of the boundary CFT if and only if the minimal surface χ_B separating B from its boundary complement includes the black hole horizon. Using the Ryu-Takayanagi prescription [38] of identifying the area of minimal bulk surfaces with the entropy of corresponding boundary regions (see Fig. 3), the geometric
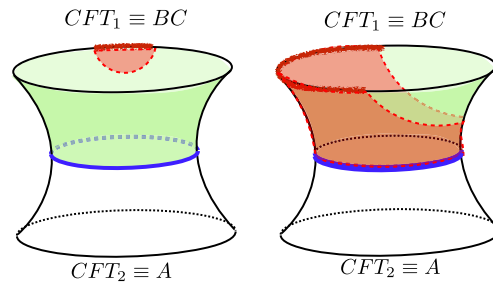

FIG. 3. Illustration of the wormhole geometry, which corresponds to thermofield double state $|\Psi_{TFD}\rangle$. In our interpretation, only $CFT_1 \equiv BC$ is physical, whereas $CFT_2 \equiv A$ is simply included for illustrative purpose. The bulk regions in green illustrate the entanglement wedge of B [42,43]; this is the region contained between B and the minimal bulk surface χ_B, separating B from its boundary complement AC. Left: a small CFT subregion C is lost (red boundary), and the entanglement wedge of B (green) reaches all the way up to the wormhole throat or black hole horizon (blue). This corresponds to accurate reconstruction on B of thermally active degrees of freedom. Right: the CFT region C that has been lost is large (roughly half), and the entanglement wedge of B (green) no longer touches the wormhole throat. In this scenario, it is impossible to recover any of the information associated to the thermally active degrees of freedom.

condition can be identified as the CMI condition (8). The critical Ising CFT is not expected to be dual to a classical geometry since the AdS radius is the Planck scale [$N = O(1)$], and the central charge $c = 1/2$ of the model are of order one [41]; nevertheless, our numerical findings are in qualitative agreement with an interpretation of a Schwartzschild-AdS geometry (see Fig. 3).

Conclusion.—We have shown that the thermal state of a CFT can be interpreted as a quantum source-channel code, which is approximately protected from small local errors. While we provided strong numerical evidence that the relevant CMI goes to zero as n^{-2}, a rigorous analytic treatment of the CMI convergence is still lacking. One obstruction for such an analytic treatment is the crucial projection onto a superselection sector, which spoils the noninteracting character of the model. However, it would be interesting to see if the recent analytical results of Ref. [44] can be adapted to the interpretation of CFTs as source-channel codes.

It is clear that as $|C|/n$ increases, our lower bound on the recovery fidelity suffers. Future work should elucidate whether the precise scaling predicted by AdS/CFT entanglement wedge hypothesis [42] for the transition between recoverability and scrambling is reproduced by simple CFT models in terms of n, S_{BC}, and $|C|$. While we have focused on global recovery due to the small accessible system size, it is natural to considering local recovery, wherein, the recovery map is restricted to act on a small patch of the physical system (i.e., C plus some constant buffer region around it) as is done in recent work on approximate quantum error correction [45–47]. In this case, geometric considerations [17,46] indicate that the necessary buffer region around C will have a width proportional to the size of C instead of constant. Our example suggests that restricting to a specific topological sector of the CFT might be necessary in order to allow nontrivial reconstruction of the thermally active degrees of freedom—indicating that gauge invariance may play a role in the QECC features, as suggested by Refs. [48,49]. It is the hope that the present Letter can significantly stimulate such further endeavors identifying the precise connection between holography and quantum error correction.

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