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Bethe Ansatz and exact form factors of the $O(N)$ Gross Neveu-model

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Abstract: We apply previous results on the $O(N)$ Bethe Ansatz [1–3] to construct a general form factor formula for the $O(N)$ Gross-Neveu model. We examine this formula for several operators, such as the energy momentum, the spin-field and the current. We also compare these results with the $1/N$ expansion of this model and obtain full agreement. We discuss bound state form factors, in particular for the three particle form factor of the field. In addition for the two particle case we prove a recursion relation for the K-functions of the higher level Bethe Ansatz.

Keywords: Field Theories in Lower Dimensions, Bethe Ansatz, Integrable Field Theories

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Contents

1 Introduction

The $O(N)$ σ- and Gross-Neveu (GN) models are integrable and asymptotically free quantum field theories in 1+1 dimension. The S-matrices of these two models correspond to two solutions of the Yang-Baxter equation [4, 5]. In previous articles we constructed the $O(N)$ nested off-shell Bethe Ansatz [1, 2] and applied this technique to construct the exact form factors for the $O(N)$ σ model [3]. Here we extend this work and construct the form factors for the $O(N)$ Gross-Neveu model for arbitrary number of fundamental particles (for the two-particle case see [6]). The model exhibits a very rich bound state structure and kinks (see e.g. [7]), turning this study even more challenging.

Before we recall the S-matrix and all other details of this model we should mention that the integrable structure present in 1+1 dimension is now becoming relevant and actual in higher dimensional gauge theories under specific circumstances. Remarkably, in the articles [8–11] (see also references therein) a non-perturbative formulation of planar scattering in the $N = 4$ Supersymmetric Yang-Mills theory (SYM) with the so called polygonal Wilson loops was proposed and a new decomposition of the Wilson loops in terms of the fundamental building blocks-Pentagon transitions was introduced. These transitions are directly related to the dynamics of the Gubser-Klebanov-Polyakov flux-tube [12], which can be computed exactly by exploring the integrability. In addition, three axioms about the transitions that single particles must satisfy were postulated and, interestingly, it is possible to verify that these axioms correspond to some deformations of the form factor equations in $1 + 1$- dimensional integrable quantum field theories. Such exact and constructive developments in the $N = 4$ SYM theory opens, indeed, large perspectives in the view of using the exact integrability and the full machinery of the form factor program to get physical insights, specially in the case of non-trivial symmetry groups, such as SU(N) and $O(N)$.

In this article we consider the $O(N)$-Gross-Neveu model for N = even. We do not use any Lagrangian to construct the model, nevertheless, we give the following motivation. The $O(N)$-Gross-Neveu model describes the interaction of $N/2$ Dirac (or N Majorana) fermions defined by the Lagrangian[1] [13]

$$\mathcal{L}^{\mathrm{GN}} = \sum_{\alpha=1}^{N/2} \bar{\psi}_\alpha i\gamma\partial\psi_\alpha + \frac{1}{2}\, g^2 \left(\sum_{i=1}^{N/2} \bar{\psi}_\alpha \psi_\alpha \right)^2 . \tag{1.1}$$

It is known from semi-classical calculations [14] that there are bound states of two fundamental fermions in the scalar and the anti-symmetric tensor channel. Furthermore there are kinks such that the fundamental fermions are kink-kink bound states. The bootstrap program does not use the Lagrangian, but we are looking for an factorizing S-matrix of an $O(N)$-isovector N-plett of self conjugate fundamental fermions. However, now we assume bound states in the scalar and anti-symmetric tensor channel of two of them.

In this article we use the techniques of [1, 3] to construct the form factors of the $O(N)$-Gross-Neveu model. We apply the general results to compute exact form factors for the energy-momentum, the spin-field and the current. The exact results are compared with the ones obtained in perturbation theory using the $1/N$ expansion. The final aim of the form factor program is to obtain explicit results for the correlation functions or Wightman functions in the framework of 2-dimensional integrable QFTs. In [6, 15] the concept of generalized form factors was introduced and developed further by Smirnov [16]. We call the matrix elements of fields with many particle states: "generalized form factors".

[1] The Lagrangian (1.1) is invariant under $O(N)$ transformations of the vector of N Majorana fermi fields $\psi_\alpha^{(i)}$ $(\alpha = 1, \ldots, N/2)$, $i = 1, 2$, where $\psi_\alpha = \psi_\alpha^{(1)} + i\psi_\alpha^{(2)}$ [5].

Matrix difference equations (the generalized Watson's equations) are solved by using the "off-shell Bethe Ansatz" [1, 17–19], which was introduced in [20] to solve the Knizhnik-Zamolodchikov equations. Other approaches to form factors in integrable quantum field theories can be found in [21–29]. For articles considering the form factor program for Bethe Ansatz solvable models with nesting see also [30–33].

The general form factor formula in terms of an integral representation is the main result of this paper. It solves the form factors equations. The matrix element of a local operator $\mathcal{O}(x)$ for a state of n particles of kind α_i with rapidities θ_i

$$\langle\, 0 \,|\, \mathcal{O}(x) \,|\, \theta_1, \ldots, \theta_n \,\rangle^{\rm in}_{\underline{\alpha}} = e^{-ix(p_1+\cdots+p_n)} F^{\mathcal{O}}_{\underline{\alpha}}(\underline{\theta}) \tag{1.2}$$

defines the generalized form factor $F^{\mathcal{O}}_{\underline{\alpha}}(\underline{\theta})$. Here we restrict α to the fundamental particles of the model, which form an isovector N-plett of $O(N)$. Following [6] we write

$$F^{\mathcal{O}}_{\underline{\alpha}}(\underline{\theta}) = K^{\mathcal{O}}_{\underline{\alpha}}(\underline{\theta}) \prod_{1\le i<j\le n} F(\theta_{ij}) \tag{1.3}$$

where $F(\theta)$ is the minimal form factor function.

For the K-function we propose the same Ansatz as for the σ-model in [3] in terms of a nested 'off-shell' Bethe Ansatz

$$K^{\mathcal{O}}_{\underline{\alpha}}(\underline{\theta}) = N^{\mathcal{O}}_n \int_{\mathcal{C}^{(1)}_{\underline{\theta}}} dz_1 \cdots \int_{\mathcal{C}^{(m)}_{\underline{\theta}}} dz_m \, \tilde{h}(\underline{\theta}, \underline{z}) \, p^{\mathcal{O}}(\underline{\theta}, \underline{z}) \, \tilde{\Psi}_{\underline{\alpha}}(\underline{\theta}, \underline{z}) \,. \tag{1.4}$$

Here $\tilde{h}(\underline{\theta}, \underline{z})$ is a scalar function which depends only on the S-matrix. The scalar p-function $p^{\mathcal{O}}(\underline{\theta}, \underline{z})$ which is in general a simple function of e^{θ_i} and e^{z_j} depends on the specific operator $\mathcal{O}(x)$. This Ansatz transforms the complicated form factor matrix equations (see (3.1)–(3.3) below) to simple equations for the scalar function $p^{\mathcal{O}}(\underline{\theta}, \underline{z})$ (see also [19]). The integration contour $\mathcal{C}_{\underline{\theta}}$ will be specified in section 4. The state $\tilde{\Psi}_{\underline{\alpha}}$ in (1.4) is a linear combination of the basic Bethe Ansatz co-vectors (see [3] and (4.1))

$$\tilde{\Psi}_{\underline{\alpha}}(\underline{\theta}, \underline{z}) = L_{\mathring{\underline{\beta}}}(\underline{z}) \, \tilde{\Phi}^{\mathring{\underline{\beta}}}_{\underline{\alpha}}(\underline{\theta}, \underline{z}) \,. \tag{1.5}$$

The nested off-shell Bethe Ansatz is obtained by making for $L_{\mathring{\underline{\beta}}}(\underline{z})$ an Ansatz like (1.4) and iterating this procedure. In the present paper we mainly consider the case where α correspond to the fundamental fermions of the $O(N)$-Gross-Neveu model Lagrangian (1.1). In forthcoming publications we will consider the kinks [34] and we will discuss, in particular, the $O(6)$-Gross-Neveu model in more detail [35].

The 'off-shell' Bethe Ansatz states are highest weight states if they satisfy certain matrix difference equations (see for instance [1]). For n particle states the $O(N)$ weights are

$$(w_1, \ldots, w_{N/2}) = \big(n - n_1, \ldots, n_{N/2-2} - n_- - n_+, n_- - n_+\big)$$

where $n_1 = m, n_2, \ldots$ are the numbers of integrations in (1.4) and the higher levels of the nesting. In particular $n_\pm$ are the numbers of positive/negative chirality spinors. The various levels of the nested Bethe Ansatz correspond to the nodes of the Dynkin diagram of the corresponding Lie algebra (see for instance [36–38] and references therein). Here we have $D_{N/2}$ for N = even (see figure 2). In [3] we used for the $O(N)$ σ-model the group isomorphy $O(4) \simeq \mathrm{SU}(2) \otimes \mathrm{SU}(2)$ to start the nesting procedure with form factors of the SU(2) chiral Gross-Neveu model [39]. For the $O(N)$ Gross-Neveu model it is also possible to use the group isomorphy $O(6) \simeq \mathrm{SU}(4)$ to start the nesting with form factors of the SU(4) chiral Gross-Neveu model [39]. This will be performed in detail in a separate paper [35]. For the on-shell Bethe Ansatz for N even see also [40].

Section 2 provides some known results and the notation for the $O(N)$ Gross-Neveu S-matrix, the bound states, etc. In section 3 we recall the general form factor equations and obtain the minimal form factor function. In section 4 we present the general exact form factors formula for the $O(N)$-Gross-Neveu model and discuss the higher levels of the nested off-shell Bethe Ansatz. In section 5 the general results are applied to some examples. The more complicated proofs and calculations are delegated to the appendices.

2 General settings

2.1 The $O(N)$-Gross-Neveu S-matrix

We consider the fundamental particles of the Lagrangian (1.1) which are fermions and transform as the vector representation of $O(N)$. The structure of the S-matrix is the same as that of the nonlinear σ-model [3] however, here we are looking for a solution of the $O(N)$ -Yang-Baxter equations with a bound state pole in the physical strip $0 < \operatorname{Im}\theta < \pi$. Therefore, here "minimality" implies that the S-matrix for the scattering of two fundamental particles is of the form

$$S(\theta, N) = \frac{\sinh\theta + i\sin\pi\nu}{\sinh\theta - i\sin\pi\nu} S^{\min}(\theta), \quad \text{with } \nu = \frac{2}{N-2}. \tag{2.1}$$

This S-matrix was given by Zamolodchikov-Zamolodchikov [5]. The first factor in (2.1) is the sine-Gordon breather-breather [41] amplitude and $S^{\min}$ is the minimal $O(N)$ S-matrix which is the one of the nonlinear σ-model (see e.g. [3]). The position of the pole is dictated by the condition [42] that the pole has to be cancelled by a zero in the amplitude $S_+^{\min}$. This condition[2] fixes the pole and therefore the bound state mass spectrum

$$m_k = 2m \sin\frac{1}{2}k\pi\nu \quad (k = 1, 2, \ldots, N/2 - 2)\,. \tag{2.2}$$

For each "principal" quantum number k there exist particles $b_k^{(r)}$ which are anti-symmetric tensors of rank $r = k,\, k-2, \dots \geq 0$, i.e. they transform according to the r-th fundamental representation of $O(N)$. These particles are bosons/fermions for k even/odd. In addition there exist "kinks" of mass m which transform as the two spinor representations of $O(N)$ (with positive or negative isotopic chirality).

Note the intimate connection between the spectrum of the GN-model, figure 1, and the Dynkin diagram figure 2. There exist exclusively such one-particle states which transform according to one of the fundamental (or trivial) representations of $O(N)$.

[2] An additional pole in S_+^{GN} would contradict positivity in the Hilbert space (for details see [42]).

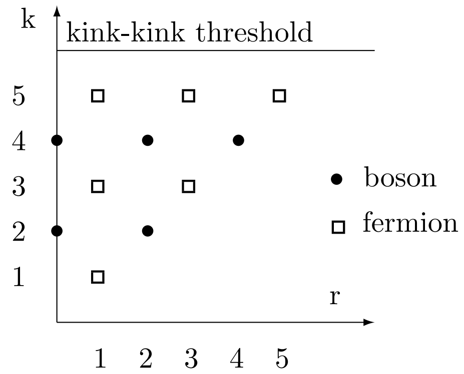


Figure 1. Particle spectrum of the $O(N)$-Gross-Neveu model for $N = 14$.

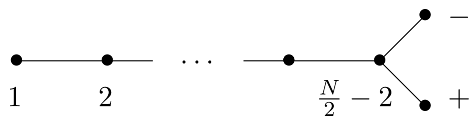


Figure 2. Dynkin diagram for $O(N)$.

For the Bethe Ansatz it is convenient as in [3] to use instead of the real basis $|\alpha\rangle_r$, $(\alpha = 1, 2, \ldots, N)$ the complex basis $|1\rangle, |2\rangle, \ldots, |\bar{2}\rangle, |\bar{1}\rangle$

$$\left.\begin{array}{rcl} |\alpha\rangle &=& \frac{1}{\sqrt{2}} \left(|2\alpha - 1\rangle_r + i|2\alpha\rangle_r\right) \\ |\bar{\alpha}\rangle &=& \frac{1}{\sqrt{2}} \left(|2\alpha - 1\rangle_r - i|2\alpha\rangle_r\right) \end{array}\right\} , \quad \alpha = 1, 2, \ldots, N/2 \,.$$

Then the S-matrix writes in terms of the components as

$$S^{\delta\gamma}_{\alpha\beta}(\theta) = b(\theta)\delta^\gamma_\alpha \delta^\delta_\beta + c(\theta)\delta^\delta_\alpha \delta^\gamma_\beta + d(\theta)\mathbf{C}^{\delta\gamma}\mathbf{C}_{\alpha\beta} \tag{2.3}$$

with the rapidity difference θ of the particles and the "charge conjugation matrices"

$$\mathbf{C}_{\alpha\beta} = \delta_{\alpha\bar{\beta}} \text{ and } \mathbf{C}^{\alpha\beta} = \delta^{\alpha\bar{\beta}}. \tag{2.4}$$

The Yang-Baxter-, crossing- and unitarity-relation write as in [3]. The highest weight amplitude is $a(\theta) = S_+(\theta) = b(\theta) + c(\theta)$

$$\begin{aligned} a(\theta) &= \exp\left(2\int_0^\infty \frac{dt}{t}\left(\frac{e^{-t(1-\nu)} - e^{-t}}{1 + e^{-t}}\right)\sinh t\frac{\theta}{i\pi}\right) \\ &= \frac{\Gamma\left(1 - \frac{1}{2\pi i}\theta\right)\Gamma\left(\frac{1}{2} + \frac{1}{2\pi i}\theta\right)}{\Gamma\left(1 + \frac{1}{2\pi i}\theta\right)\Gamma\left(\frac{1}{2} - \frac{1}{2\pi i}\theta\right)} \frac{\Gamma\left(1 - \frac{1}{2}\nu + \frac{1}{2\pi i}\theta\right)\Gamma\left(\frac{1}{2} - \frac{1}{2}\nu - \frac{1}{2\pi i}\theta\right)}{\Gamma\left(1 - \frac{1}{2}\nu - \frac{1}{2\pi i}\theta\right)\Gamma\left(\frac{1}{2} - \frac{1}{2}\nu + \frac{1}{2\pi i}\theta\right)} \end{aligned} \tag{2.5}$$

with $\nu = 2/(N - 2)$. For later convenience we introduce

$$\tilde{S}^{\delta\gamma}_{\alpha\beta}(\theta) = S^{\delta\gamma}_{\alpha\beta}(\theta)/S_+(\theta) = \tilde{b}(\theta)\delta^\gamma_\alpha \delta^\delta_\beta + \tilde{c}(\theta)\delta^\delta_\alpha \delta^\gamma_\beta + \tilde{d}(\theta)\mathbf{C}^{\delta\gamma}\mathbf{C}_{\alpha\beta} \tag{2.6}$$

with

$$\tilde{b}(\theta) = \frac{\theta}{\theta - i\pi\nu}, \quad \tilde{c}(\theta) = -\frac{i\pi\nu}{\theta - i\pi\nu}, \quad \tilde{d}(\theta) = -\frac{\theta}{\theta - i\pi\nu}\frac{i\pi\nu}{i\pi - \theta}\,. \tag{2.7}$$

We will also need $\mathring{S}(z)$ the S-matrix for $O(N-2)$

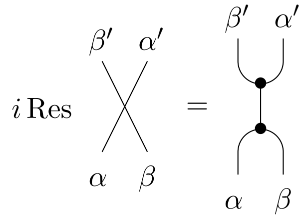
$$\tilde{\mathring{S}}_{\alpha\beta}^{\delta\gamma}(\theta)=\mathring{S}_{\alpha\beta}^{\delta\gamma}(\theta)/\mathring{S}_{+}(\theta)=\tilde{\mathring{b}}(\theta)\delta_{\alpha}^{\gamma}\delta_{\beta}^{\delta}+\tilde{\mathring{c}}(\theta)\delta_{\alpha}^{\delta}\delta_{\beta}^{\gamma}+\tilde{\mathring{d}}(\theta)\mathbf{C}^{\delta\gamma}\mathbf{C}_{\alpha\beta} \tag{2.8}$$

where ν is replaced by $\mathring{\nu}=2/(N-4)$.

2.1.1 Bound states

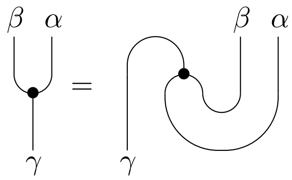
Following [7, 42, 43] we write for the fundamental fermions $\alpha,\beta,\beta',\alpha'$

$$i\operatorname*{Res}_{\theta=i\eta}(\sigma S)_{\alpha\beta}^{\beta'\alpha'}(\theta)=\sum_{\gamma}\Gamma_{\gamma}^{\beta'\alpha'}\Gamma_{\alpha\beta}^{\gamma}: \tag{2.9}$$



where $\sigma=-1$ is the statistics factor. The intertwiner $\Gamma_{\alpha\beta}^{\gamma}$ and the dual one $\Gamma_{\gamma}^{\beta\alpha}$ satisfy the crossing relation

$$\Gamma_{\gamma}^{\beta\alpha}=\mathbf{C}_{\gamma\gamma'}\Gamma_{\alpha'\beta'}^{\gamma'}\mathbf{C}^{\beta'\beta}\mathbf{C}^{\alpha'\alpha}: \tag{2.10}$$



with the charge conjugation matrix $\mathbf{C}$ (2.4). Here we have for $\eta=\pi\nu$

$$\begin{aligned}i\operatorname*{Res}_{\theta=i\pi\nu}(\sigma S)_{\alpha\beta}^{\beta'\alpha'}(\theta)&=-i\operatorname*{Res}_{\theta=i\pi\nu}\frac{\sinh\theta+i\sin\pi\nu}{\sinh\theta-i\sin\pi\nu}\left(S^{\min}\right)_{\alpha\beta}^{\beta'\alpha'}(\theta)\\&=2\tan\pi\nu\,\left(S^{\min}\right)_{\alpha\beta}^{\beta'\alpha'}(i\pi\nu)\,.\end{aligned}$$

3 Generalized form factors

Form factor equations. The form factor equations for the $O(N)$ Gross-Neveu-model are similar to the ones of the $O(N)$ σ-model in [3]. However, here there are additional statistics factors. The $F_{\underline{\alpha}}^{\mathcal{O}}(\underline{\theta})$ defined by (1.2) are considered as the components of a co-vector valued function $F_{1\dots n}^{\mathcal{O}}(\underline{\theta})$ which satisfies:

(i) Watson's equation

$$F_{\dots ij\dots}^{\mathcal{O}}(\dots,\theta_i,\theta_j,\dots)=F_{\dots ji\dots}^{\mathcal{O}}(\dots,\theta_j,\theta_i,\dots)\;(\sigma S)_{ij}(\theta_{ij}) \tag{3.1}$$

with $\theta_{ij}=\theta_i-\theta_j$ and $\sigma_{ij}=-1$ for fermions.

(ii) Crossing equation

$$\begin{aligned}&{}^{\text{out},\bar{1}}\langle\,\theta_1\,|\,\mathcal{O}(0)\,|\,\theta_2,\dots,\theta_n\,\rangle_{2\dots n}^{\text{in,conn.}}\\&\quad=F_{1\dots n}^{\mathcal{O}}(\theta_1+i\pi,\theta_2,\dots,\theta_n)\sigma_1^{\mathcal{O}}\mathbf{C}^{\bar{1}1}=F_{2\dots n1}^{\mathcal{O}}(\theta_2,\dots,\theta_n,\theta_1-i\pi)\mathbf{C}^{1\bar{1}}\end{aligned} \tag{3.2}$$

with the charge conjugation matrix $\mathbf{C}^{\bar{1}1}$ and the statistics factor $\sigma_1^{\mathcal{O}}$ of the operator $\mathcal{O}$ with respect to the particle 1.

JHEP02(2016)042

(iii) Recursion equation

$$\operatorname*{Res}_{\theta_{12}=i\pi} F^{\mathcal{O}}_{1\dots n}(\theta_1,\dots,\theta_n) = 2i\,\mathbf{C}_{12}\,F^{\mathcal{O}}_{3\dots n}(\theta_3,\dots,\theta_n)\left(\mathbf{1} - \sigma^{\mathcal{O}}_2\,(\sigma S)_{2n}\dots(\sigma S)_{23}\right), \quad (3.3)$$

(iv) Because there are bound states in the model the function $F^{\mathcal{O}}_{\underline{\alpha}}(\underline{\theta})$ has additional poles. If for instance the particles 1 and 2 form a bound state (12), there is a pole at $\theta_{12} = i\eta$, $(0 < \eta < \pi)$ such that

$$\operatorname*{Res}_{\theta_{12}=i\eta} F^{\mathcal{O}}_{12\dots n}(\theta_1,\theta_2,\dots,\theta_n) = F^{\mathcal{O}}_{(12)\dots n}(\theta_{(12)},\dots,\theta_n)\,\sqrt{2}\Gamma^{(12)}_{12} \quad (3.4)$$

where the bound state intertwiner $\Gamma^{(12)}_{12}$ of (2.9) and the values of θ_1, θ_2, $\theta_{(12)}$ and η are given in [7, 42, 43].

(v) Lorentz covariance

$$F^{\mathcal{O}}_{1\dots n}(\theta_1+\mu,\dots,\theta_n+\mu) = e^{s\mu}\,F^{\mathcal{O}}_{1\dots n}(\theta_1,\dots,\theta_n) \quad (3.5)$$

if the local operator transforms under Lorentz transformations as $\mathcal{O} \to e^{s\mu}\mathcal{O}$ where s is the "spin" of $\mathcal{O}$.

The statistics factors in (ii) and (iii) are not arbitrary, but consistency and crossing implies that both are the same and that the for anti-particle $\sigma^{\mathcal{O}}_1\sigma^{\mathcal{O}}_{\bar{1}} = 1$ holds (see also [19]). In [18, 43] was shown that the form factor equations follow from general LSZ assumptions and "maximal analyticity".

Minimal form factors. The solutions of Watson's and the crossing equations (i) and (ii) for two particles

$$\begin{aligned} F(\theta) &= S(\theta)\,F(-\theta) \\ F(i\pi-\theta) &= F(i\pi+\theta) \end{aligned} \quad (3.6)$$

with no poles in the physical strip $0 \le \operatorname{Im}\theta \le \pi$ and at most a simple zero at $\theta = 0$ are the minimal form factors [6]

$$F^{\min}_{+}(\theta) = \exp\int_0^\infty \frac{dt}{t\sinh t}\,\frac{e^{-t(1-\nu)}-e^{-t}}{1+e^{-t}}\left(1-\cosh t\left(1-\frac{\theta}{i\pi}\right)\right) \quad (3.7)$$

$$F^{\min}_{-}(\theta) = \frac{\cosh\frac12(i\pi-\theta)\,\Gamma^2\left(\frac12+\frac12\nu\right)}{\Gamma\left(1+\frac12\nu-\frac{1}{2\pi i}\theta\right)\Gamma\left(\frac12\nu+\frac{1}{2\pi i}\theta\right)}F^{\min}_{+}(\theta) \quad (3.8)$$

$$F^{\min}_{0}(\theta) = \frac{2\tanh\frac12(i\pi-\theta)}{i\pi-\theta}F^{\min}_{-}(\theta)\,. \quad (3.9)$$

They belong to the S-matrix eigenvalues $S_\pm = b \pm c$ and $S_0 = b + c + Nd$ (see (2.3)). For the construction of the off-shell Bethe Ansatz the minimal solution of the form factor equation (3.1) for the highest weight eigenvalue of the $O(N)$ S-matrix

$$F(\theta) = \sigma S_+(\theta)\,F(-\theta) = -a(\theta)F(-\theta) \quad (3.10)$$

JHEP02(2016)042

is essential. We take the solution[3]

$$
\begin{aligned}
F(\theta) &= c\cosh\frac{1}{2}(i\pi-\theta)\ F_{+}^{\min}(\theta) \\
&= c\exp\left(\int_0^\infty \frac{dt}{t\sinh t}\frac{1+e^{-t(1-\nu)}}{1+e^{-t}}\left(1-\cosh t\left(1-\frac{\theta}{i\pi}\right)\right)\right)
\end{aligned}
\tag{3.11}
$$

or[4]

$$
F(\theta) = \frac{G\left(\frac{1}{2}\frac{\theta}{i\pi}\right)G\left(1-\frac{1}{2}\frac{\theta}{i\pi}\right)}{G\left(\frac{1}{2}+\frac{1}{2}\frac{\theta}{i\pi}\right)G\left(\frac{3}{2}-\frac{1}{2}\frac{\theta}{i\pi}\right)}\frac{G\left(\frac{1}{2}-\frac{1}{2}\nu+\frac{1}{2}\frac{\theta}{i\pi}\right)G\left(\frac{3}{2}-\frac{1}{2}\nu-\frac{1}{2}\frac{\theta}{i\pi}\right)}{G\left(1-\frac{1}{2}\nu+\frac{1}{2}\frac{\theta}{i\pi}\right)G\left(2-\frac{1}{2}\nu-\frac{1}{2}\frac{\theta}{i\pi}\right)}
$$

where $G(z)$ is Barnes G-function, which satisfies (see e.g. [44])

$$
G(1+z) = \Gamma(z)\,G(z)\ .
$$

For convenience we have introduced the constant c (see (4.9))

$$
c = G^2\left(\frac{1}{2}\right)G^2\left(1-\frac{1}{2}\nu\right)G^{-2}\left(\frac{3}{2}-\frac{1}{2}\nu\right)\ . \tag{3.12}
$$

The full 2-particle form factors are

$$
F_{+,-,0}(\theta) = \frac{-\cos^2\frac{1}{2}\pi\nu}{\sinh\frac{1}{2}(\theta-i\pi\nu)\sinh\frac{1}{2}(\theta+i\pi\nu)}F_{+,-,0}^{\min}(\theta)\ . \tag{3.13}
$$

They are non-minimal solutions of (3.6) containing the bound state pole at $\theta = i\pi\nu$ (see (2.16) of [6]).

4 $O(N)$ form factors and Bethe Ansatz

4.1 The fundamental theorem

Following [6] we write the general form factor $F^{\mathcal{O}}_{1\dots n}(\underline{\theta})$ for n-fundamental particles as (1.3) where $F(\theta)$ is the minimal form factor function (3.11). The K-function $K^{\mathcal{O}}_{1\dots n}(\underline{\theta})$ is determined by the form factor equations (i)–(v). We propose the K-function in terms of a nested 'off-shell' Bethe Ansatz (1.4) as a multiple contour integral.

The basic Bethe Ansatz co-vectors in (1.5) are defined as (for more details see [1, 3])

$$
\tilde{\Phi}^{\mathring{\underline{\beta}}}_{\underline{\alpha}}(\underline{\theta},\underline{z}) = \left(\Pi^{\mathring{\underline{\beta}}}_{\underline{\beta}}(\underline{z})\Omega\tilde{T}_1^{\beta_m}(\underline{\theta},z_m)\dots\tilde{T}_1^{\beta_1}(\underline{\theta},z_1)\right)_{\underline{\alpha}}\ . \tag{4.1}
$$

The matrix $\Pi^{\mathring{\underline{\beta}}}_{\underline{\beta}}(\underline{z})$ intertwines[5] between the S-matrices S of $O(N)$ and $\mathring{S}$ of $O(N-2)$

$$
\tilde{\mathring{S}}_{ij}(z_{ij}\mathring{\nu}/\nu)\Pi_{\dots ij\dots}(\underline{z}) = \Pi_{\dots ji\dots}(\underline{z})\tilde{S}_{ij}(z_{ij})\,. \tag{4.2}
$$

[3]The minus sign in (3.10) and the factor $\cosh\frac{1}{2}(i\pi-\theta)$ is due to fermionic statistics of the fundamental particles (see also eq. 4.14 of [18]).

[4]Private communication: Karol K. Kozlowski pointed out to one of the authors (M.K.), that the minimal form factors may be expressed in terms of Barnes G-function.

[5]This matrix Π is trivial for the SU(N) Bethe Ansatz because the SU(N) S-matrix does not depend on N for a suitable normalization and parametrization.

The Bethe Ansatz co-vectors (4.1) are generalizations of vectors introduced by Tarasov [45] for the Korepin-Izergin model. Below we will use the following relations for special components of Π (for more details see [1–3])

$$\Pi_{\underline{\beta}}^{\underline{\mathring{\beta}}} = \begin{cases} 0 & \text{for } \beta_1 = 1, \text{ or } \beta_m = \bar{1} \\ \delta_{\beta_1}^{\mathring{\beta}_1} \Pi_{\beta_2 \dots \beta_m}^{\mathring{\beta}_2 \dots \mathring{\beta}_m} & \text{for } \beta_1 \neq \bar{1} \\ \Pi_{\beta_1 \dots \beta_{m-1}}^{\mathring{\beta}_1 \dots \mathring{\beta}_{m-1}} \delta_{\beta_m}^{\mathring{\beta}_m} & \text{for } \beta_m \neq 1\,. \end{cases} \tag{4.3}$$

The scalar function $\tilde{h}(\underline{\theta}, \underline{z})$ in (1.4) depends only on the S-matrix and not on the specific operator $\mathcal{O}(x)$

$$\tilde{h}(\underline{\theta}, \underline{z}) = \prod_{i=1}^{n} \prod_{j=1}^{m} \tilde{\phi}(\theta_i - z_j) \prod_{1 \le i < j \le m} \tau(z_i - z_j)\,. \tag{4.4}$$

The functions $\tilde{\phi}$ and τ satisfy the shift equations

$$\tilde{\phi}(\theta - 2\pi i) = -\tilde{b}(\theta)\tilde{\phi}(\theta) \tag{4.5}$$

$$\tau(z - 2\pi i)/\tilde{b}(2\pi i - z) = \tau(z)/\tilde{b}(z) \tag{4.6}$$

which are related to the form factor equation (ii) or (3.2) [1–3]. Here for the $O(N)$ Gross-Neveu model

$$\tau(z) = \frac{1}{\tilde{\phi}(-z)\tilde{\phi}(z)} \tag{4.7}$$

where $\tilde{\phi}(\theta)$ is[6]

$$\tilde{\phi}(\theta) = \Gamma\left(1 - \frac{1}{2}\nu + \frac{1}{2\pi i}\theta\right)\Gamma\left(-\frac{1}{2\pi i}\theta\right). \tag{4.8}$$

The form factor equation (iii) or (3.3) (as will be discussed in appendix A) requires that

$$F(\theta)F(\theta + i\pi)\tilde{\phi}(-\theta - i\pi + i\pi\nu)\tilde{\phi}(-\theta) = 1. \tag{4.9}$$

The function (4.8) satisfies this relation. Notice that the equations (4.8) and (4.9) also determine the normalization constant c in (3.11) and (3.12).

Similar as in [3] the integration contours $\mathcal{C}_{\underline{\theta}}^{(j)}$ in (1.4) depend on whether j is even or odd, they are depicted in figure 3. The Ansatz (1.3) and (1.4) transforms the matrix equations (i)–(v) (see (3.2)–(3.5)) into much simpler scalar equations for the scalar p-function $p^{\mathcal{O}}(\underline{\theta}, \underline{z})$. This function depends on the specific operator $\mathcal{O}(x)$ and is in general a simple function of e^{θ_i} and e^{z_j}.

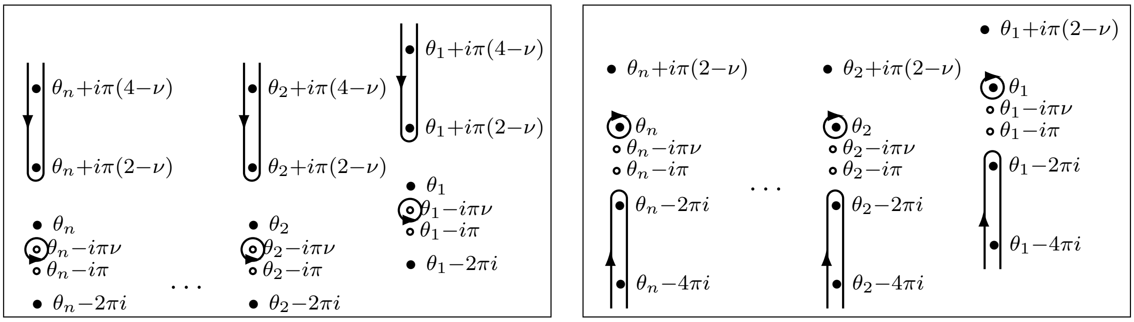
Theorem 1 *Assume that:*

1. *The p-function $p^{\mathcal{O}}(\underline{\theta}, \underline{z})$ satisfies the equations*

$$\left.\begin{array}{ll} (\mathrm{i}') & p^{\mathcal{O}}(\underline{\theta}, \underline{z}) \text{ is symmetric under } \theta_i \leftrightarrow \theta_j \\ (\mathrm{ii}'_1) & p^{\mathcal{O}}(\underline{\theta}, \underline{z}) = \sigma^{\mathcal{O}}(-1)^m p^{\mathcal{O}}(\theta_1 + 2\pi i, \theta_2, \dots, \underline{z}) \\ (\mathrm{ii}'_2) & p^{\mathcal{O}}(\underline{\theta}, \underline{z}) = (-1)^n p^{\mathcal{O}}(\underline{\theta}, z_1 + 2\pi i, z_2, \dots) \\ (\mathrm{iii}') & p^{\mathcal{O}}(\underline{\theta}, \underline{z}) = p^{\mathcal{O}}(\underline{\check{\theta}}, \underline{\check{z}}) \end{array}\right\} \tag{4.10}$$

where in (iii′) $\theta_{12} = i\pi$, $z_1 = \theta_1 - i\pi\nu$ *and* $z_2 = \theta_2$. *The short notations* $\underline{\check{\theta}} = (\theta_3, \dots, \theta_n)$ *and* $\underline{\check{z}} = (z_3, \dots, z_m)$ *are used.*

[6] This is in contrast to the σ-model case where the $\tilde{\phi}$-functions depend on whether j in (4.4) is even or odd.



(odd) (even)

Figure 3. The integration contours $\mathcal{C}^{(o)}_{\underline{\theta}}$ and $\mathcal{C}^{(e)}_{\underline{\theta}}$. The bullets refer to poles of the integrand resulting from $\tilde{\phi}(\theta_i - z_j)$ and the small open circles refer to poles originating from $\tilde{S}(\theta_i - z_j)$.

2. *The higher level function* $L_{\underline{\beta}}(\underline{z})$ *in (1.5) satisfies* $(\mathrm{i})^{(k)}$–$(\mathrm{iii})^{(k)}$ *of (4.13)–(4.15) for* $k = 1$.

3. *A suitable choice of the normalization constants in (1.4).*

Then the co-vector valued function $F_{\underline{\alpha}}(\underline{\theta})$ *given by the Ansatz (1.3) and the integral representation (1.4) satisfies the form factor equations* (i)–(v) *of (3.1)–(3.5).*

The proof of this theorem can be found in appendix A.

4.2 Higher level off-shell Bethe Ansatz

For this discussion it is convenient to introduce the variables u, v defined by $\theta = i\pi\nu_k u,\ z = i\pi\nu_k v$ and $\nu_k = 2/(N-2k-2)$. For the $O(N-2k)$ S-matrix $S^{(k)}(u)$ we write as in (2.6)

$$\tilde{S}^{(k)}(u) = S^{(k)}/S^{(k)}_{+} = \tilde{b}(u)\mathbf{1} + \tilde{c}(u)\mathbf{P} + \tilde{d}_k(u)\mathbf{K} \qquad (4.11)$$
$$\tilde{b}(u) = \frac{u}{u-1},\quad \tilde{c}(u) = \frac{-1}{u-1},\quad \tilde{d}_k(u) = \frac{u}{u-1}\frac{1}{u-1/\nu_k}\,.$$

and define

$$K^{(k)}_{\underline{\alpha}}(\underline{u}) = \tilde{N}^{(k)}_{m_k}\int_{\mathcal{C}^{(1)}_{\underline{u}}} dv_1 \cdots \int_{\mathcal{C}^{(m_k)}_{\underline{u}}} dv_{m_k}\, \tilde{h}(\underline{u},\underline{v}) p^{(k)}(\underline{u},\underline{v})\ \tilde{\Psi}^{(k)}_{\underline{\alpha}}(\underline{u},\underline{v}) \qquad (4.12)$$
$$\tilde{\Psi}^{(k)}_{\underline{\alpha}}(\underline{u},\underline{v}) = L^{(k)}_{\underline{\mathring{\beta}}}(\underline{v})\ \left(\tilde{\Phi}^{(k)}\right)^{\underline{\mathring{\beta}}}_{\underline{\alpha}}(\underline{u},\underline{v}),\qquad L^{(k)}_{\underline{\mathring{\beta}}}(\underline{v}) = K^{(k+1)}_{\underline{\mathring{\beta}}}(\underline{v})$$

with $\underline{u} = u_1, \ldots, u_{n_k}$, $\underline{v} = v_1, \ldots, v_{m_k}$ and $m_k = n_{k+1}$.

The equations $(\mathrm{i})^{(k)}$–$(\mathrm{iii})^{(k)}$ for $k>0$ are in terms of these variables similar as in [3]

$(\mathrm{i})^{(k)}$

$$K^{(k)}_{\dots ij\dots}(\dots,u_i,u_j,\dots)=K^{(k)}_{\dots ji\dots}(\dots,u_j,u_i,\dots)\,\tilde S^{(k)}_{ij}(u_{ij}) \tag{4.13}$$

$(\mathrm{ii})^{(k)}$

$$K^{(k)}_{1\dots n_k}(u_1+2/\nu,u_2,\dots,u_{n_k})\mathbf{C}^{\bar 11}=K^{(k)}_{2\dots n_k 1}(u_2,\dots,u_{n_k},u_1)\mathbf{C}^{1\bar 1} \tag{4.14}$$

$(\mathrm{iii})^{(k)}$

$$\operatorname*{Res}_{u_{12}=1/\nu_k} K^{(k)}_{1\dots n_k}(u_1,\dots,u_{n_k})=\prod_{i=3}^{n_k}\tilde\phi(u_{i1}+1)\tilde\phi(u_{i2})\mathbf{C}_{12}K^{(k)}_{3\dots n_k}(u_3,\dots,u_{n_k})\,. \tag{4.15}$$

in addition we have here the bound state relation

$(\mathrm{iv})^{(k)}$

$$\operatorname*{Res}_{u_{12}=1} F^{(k)}_{12\dots n}(u_1,u_2,\dots,u_n)\;=F^{(k)}_{(12)\dots n}(u_{(12)},\dots,u_n)\,\sqrt{2}\Gamma^{(12)}_{12}\,. \tag{4.16}$$

The form factor equations (i)–(iv) of (3.1)–(3.4) for $O(N-2k)$ are similar to these higher level equations. There are, however, two differences: 1) The shift in $(\mathrm{ii})^{(k)}$ is the one of $O(N)$ but not that of $O(N-2k)$. 2) There is only one term on the right hand side in $(\mathrm{iii})^{(k)}$.

We assume that the p-function $p^{(k)}(\underline u,\underline v)$ satisfies the equations

$$\begin{aligned}&(\mathrm{i}')\quad p^{(k)}(\underline u,\underline v)\text{ is symmetric under } u_i\leftrightarrow u_j,\; v_i\leftrightarrow v_j\\ &(\mathrm{ii}')\quad p^{(k)}(\underline u,\underline v)=(-1)^{m_k}p^{(k)}(u_1+2/\nu,u_2,\dots,\underline v)=(-1)^{n_k}p^{(k)}(\underline u,v_1+2/\nu,v_2,\dots)\\ &(\mathrm{iii}')\; p^{(k)}(\underline u,\underline z)=p^{(k)}(\check{\underline u},\check{\underline v})\text{ for } u_{12}=1/\nu_k,\; v_1=u_1-1\text{ and } v_2=u_2\end{aligned} \tag{4.17}$$

where we use the short notations $\check{\underline u}=(u_3,\dots,u_{n_k})$ and $\check{\underline v}=(v_3,\dots,v_{m_k})$.

Lemma 2 *For $0<k<\frac12(N-4)$ the functions $K^{(k)}_{\underline\alpha}(\underline u)$ of (4.12) satisfy the equations $(\mathrm{i})^{(k)}$, $(\mathrm{ii})^{(k)}$ and $(\mathrm{iii})^{(k)}$, if the corresponding relations are satisfied for $K^{(k+1)}_{\underline{\mathring\beta}}(\underline v)$ and if suitable choice of the normalization constants in (4.12) is assumed. The weights of the operator $\mathcal O$*

$$w^{\mathcal O}=(w_1,\dots,w_{N/2})=\big(n_0-n_1,\dots,n_{N/2-2}-n_--n_+,n_--n_+\big) \tag{4.18}$$

determine the numbers $m_k=n_{k+1}$ for a given number of particles $n=n_0$

The proof of this lemma can be found in appendix C.1.

5 Examples

In this section, to illustrate our general results we present some simple examples.

5.1 Current

The $O(N)$ Noether current[7]

$$J_\mu^{\alpha\beta} = \bar{\psi}^\alpha \gamma_\mu \psi^\beta$$

transforms as the antisymmetric tensor representation of $O(N)$. This operator has therefore the weights $w^J = (w_1, \dots, w_{N/2}) = (1, 1, 0, \dots, 0)$ (see [1, 2]), which implies with (4.18) that

$$n - 2 = n_1 - 1 = n_2 = \dots = n_{N/2-2} = n_- + n_+, n_- = n_+ \,.$$

where n_i are the numbers of integrations in the various levels of the off-shell Bethe Ansatz. The existence of a pseudo-potential $J^{\alpha\beta}(x)$ follows from the conservation law $\partial^\mu J_\mu^{\alpha\beta} = 0$

$$J_\mu^{\alpha\beta}(x) = \epsilon_{\mu\nu} \partial^\nu J^{\alpha\beta}(x).$$

For the form factors of both operators we have the relation

$$F_{\underline{\alpha}}^{J_\mu}(\underline{\theta}) = -i\epsilon_{\mu\nu} \left(\sum p_i^\nu \right) F_{\underline{\alpha}}^J(\underline{\theta}) \,. \tag{5.1}$$

Because the Bethe Ansatz yields highest weight states we obtain the matrix elements of the highest weight component of $J^{\alpha\beta}$ which means in the complex basis $J(x) = J^{12}(x)$.

We propose the form factors of the operator $J(x)$ (for $n = m+1 = n_1+1 = n_2+2$ even)

$$\begin{aligned} \langle 0|J(0)|\underline{\theta}\rangle_{\underline{\alpha}} = F_{\underline{\alpha}}^J(\underline{\theta}) &= \prod_{i<j} F(\theta_{ij}) K_{\underline{\alpha}}^J(\underline{\theta}) \\ K_{\underline{\alpha}}^J(\underline{\theta}) &= N_n^J \int_{\mathcal{C}_{\underline{\theta}}^{(1)}} dz_1 \dots \int_{\mathcal{C}_{\underline{\theta}}^{(m)}} dz_m \, \tilde{h}(\underline{\theta}, \underline{z}) p^J(\underline{\theta}, \underline{\underline{z}}) \, \tilde{\Psi}_{\underline{\alpha}}(\underline{\theta}, \underline{z}) \,. \end{aligned}$$

Expressing $\tilde{\Psi}_{\underline{\alpha}}(\underline{\theta}, \underline{z})$ in terms of all higher level Bethe Ansatzes there appears the product of all level p-functions $p^J(\underline{\theta}, \underline{\underline{z}})$. For the example of the current it depends on the θ_i and the second level $z_j^{(2)}$

$$p^J(\underline{\theta}, \underline{\underline{z}}) = e^{\frac{1}{2}\left(\sum_{i=1}^n \theta_i - \sum_{j=1}^{n_2} z_j^{(2)} - \frac{1}{2} n_2 i\pi\mathring{\nu} \right)} \Big/ \sum_{i=1}^n e^{\theta_i} + e^{-\frac{1}{2}\left(\sum_{i=1}^n \theta_i - \sum_{j=1}^{n_2} z_j^{(2)} - \frac{1}{2} n_2 i\pi\mathring{\nu} \right)} \Big/ \sum_{i=1}^n e^{-\theta_i} \tag{5.2}$$

which satisfies (4.10) with

charge	$Q^J = 0$
weight vector	$w^J = (1, 1, 0, \dots, 0)$
statistics factor	$\sigma^J = 1$
spin	$s^J = 0, \; s^{J_\mu} = 1 \,.$

[7] In the real basis.

For example for 2-particle form factor we obtain (see appendix B)

$$F_{\alpha_1\alpha_2}^{J^{\alpha\beta}}(\theta_1,\theta_2) = im\left(\delta^{\alpha}_{\alpha_1}\delta^{\beta}_{\alpha_2} - \delta^{\beta}_{\alpha_1}\delta^{\alpha}_{\alpha_2}\right)\frac{1}{\cosh\frac{1}{2}\theta_{12}}F_-(\theta) \tag{5.3}$$

$$F_{\alpha_1\alpha_2}^{J_\mu^{\alpha\beta}}(\theta_1,\theta_2) = i\left(\delta^{\alpha}_{\alpha_1}\delta^{\beta}_{\alpha_2} - \delta^{\beta}_{\alpha_1}\delta^{\alpha}_{\alpha_2}\right)\bar{v}(\theta_1)\gamma_\mu u(\theta_2)F_-(\theta) \tag{5.4}$$

with $F_-(\theta)$ of (3.13) and (3.8), $\bar{v}(\theta_1)\gamma^{\pm}u(\theta_2) = \pm i2me^{\pm\frac{1}{2}(\theta_1+\theta_2)}$ and (5.1). This result agrees with [6].

5.2 Field

For the fundamental field $\psi^\alpha(x)$ in (1.1) the numbers n_i of integrations in the various levels of the off-shell Bethe Ansatz satisfy

$$n-1 = n_1 = n_2 = \cdots = n_{N/2-2} = n_- + n_+, n_- = n_+$$

because ψ^α transforms as the vector representation of $O(N)$ (see [1, 2] and (4.18)). We restrict to component $\psi = \psi^1$ because as usual the Bethe Ansatz yields highest weight states. For convenience we multiply the field with the Dirac operator and take

$$\chi(x) = i(-i\gamma\partial + m)\psi(x)\,. \tag{5.5}$$

We propose for the n-particle form factors ($n = m+1$ odd) for the spinor components $\chi^{(\pm)}$

$$\langle 0|\chi^{(\pm)}(0)|\underline{\theta}\rangle_{\underline{\alpha}} = F_{\underline{\alpha}}^{\chi^{(\pm)}}(\underline{\theta}) = \prod_{i<j}F(\theta_{ij})K_{\underline{\alpha}}^{\chi^{(\pm)}}(\underline{\theta}) \tag{5.6}$$

$$K_{\underline{\alpha}}^{\chi^{(\pm)}}(\underline{\theta}) = N_n^{\chi}\int_{\mathcal{C}_{\underline{\theta}}^{(1)}}dz_1\ldots\int_{\mathcal{C}_{\underline{\theta}}^{(m)}}dz_m\,\tilde{h}(\underline{\theta},\underline{z})p^{\chi^{(\pm)}}(\underline{\theta},\underline{z})\,\tilde{\Psi}_{\underline{\alpha}}(\underline{\theta},\underline{z}) \tag{5.7}$$

with the p-function (for $n = m+1 = \text{odd} > 1$)

$$p^{\chi^{(\pm)}}(\underline{\theta},\underline{z}) = \exp\left(\mp\frac{1}{2}\left(\sum_{j=1}^{n}\theta_j - \sum_{j=1}^{m}z_j - \frac{1}{2}mi\pi\nu\right)\right) \tag{5.8}$$

which solves (4.10) with

charge	$Q^\psi = 1$
weight vector	$w^\psi = (1,\ldots,0)$
statistics factor	$\sigma^\psi = -1$
spin	$s^\psi = \frac{1}{2}$

The one particle form factor is trivial

$$\langle 0|\psi(0)|\theta\rangle_\alpha = F_\alpha^\psi(\theta) = \delta_\alpha^1\,u(\theta)\,.$$

For the three particle form factor ($n=3,\ m=2$) the equations (5.6) and (5.7) write as

$$\begin{aligned}\langle 0|\chi(0)|\underline{\theta}\rangle_{\underline{\alpha}} &= F^{\chi}_{\underline{\alpha}}(\underline{\theta}) = F(\theta_{12})F(\theta_{13})F(\theta_{23})K^{\chi}_{\underline{\alpha}}(\underline{\theta})\\ K^{\chi}_{\underline{\alpha}}(\underline{\theta}) &= N^{\chi}_{3}\int_{\mathcal{C}^{(o)}_{\underline{\theta}}} dz_1 \int_{\mathcal{C}^{(e)}_{\underline{\theta}}} dz_2\, \tilde{h}(\underline{\theta},\underline{z})p^{\chi}(\underline{\theta},\underline{z})\,\tilde{\Psi}_{\underline{\alpha}}(\underline{\theta},\underline{z})\end{aligned} \tag{5.9}$$

with

$$\begin{aligned}\tilde{h}(\underline{\theta},\underline{z}) &= \prod_{i=1}^{3}\tilde{\phi}(\theta_i-z_1)\tilde{\phi}(\theta_i-z_2)\frac{1}{\tilde{\phi}(z_{12})\tilde{\phi}(-z_{12})}\\ p^{\chi^{\pm}}(\underline{\theta},\underline{z}) &= e^{\mp\frac{1}{2}(\theta_1+\theta_2+\theta_3-z_1-z_2-i\pi\nu)}\\ \tilde{\Psi}_{\underline{\alpha}}(\underline{\theta},\underline{z}) &= L_{\underline{\mathring{\beta}}}(\underline{z})\left(\Pi^{\underline{\mathring{\beta}}}_{\underline{\beta}}(\underline{z})\Omega\tilde{T}^{\beta_2}_{1}(\underline{\theta},z_2)\tilde{T}^{\beta_1}_{1}(\underline{\theta},z_1)\right)_{\underline{\alpha}}.\end{aligned}$$

Lemma 2 for the $O(N-2)$ weights $w=(0,\dots,0)$ yields for the higher level function $L_{\mathring{\beta}_1\mathring{\beta}_2}(\underline{z}) = \mathring{\mathbf{C}}_{\mathring{\beta}_1\mathring{\beta}_2}L(z_{12})$ with

$$L(z) = \frac{\Gamma\left(1-\frac{1}{2}\nu-\frac{z}{2\pi i}\right)\Gamma\left(-\frac{1}{2}\nu+\frac{z}{2\pi i}\right)}{\Gamma\left(1+\frac{1}{2}(1-\nu)-\frac{z}{2\pi i}\right)\Gamma\left(\frac{1}{2}(1-\nu)+\frac{z}{2\pi i}\right)} \tag{5.10}$$

(see appendix C.2). We could not perform the integrations[8] in (5.9) for general N, but we expand the exact expression in $1/N$-expansion to compare the result with the $1/N$-expansion of the $O(N)$ Gross-Neveu model in terms of Feynman graphs.

1/N expansion. We obtain the 3-particle form factor of $\chi(x)$ up to $O(N^{-2})$ as (see appendix E.1)

$$F^{\chi^{\delta}}_{\alpha\beta\gamma} = \frac{8\pi m}{N}\left(\delta^{\delta}_{\gamma}\mathbf{C}_{\alpha\beta}\frac{\cosh\frac{1}{2}\theta_{12}}{\theta_{12}-i\pi}u(\theta_3) - \delta^{\delta}_{\beta}\mathbf{C}_{\alpha\gamma}\frac{\cosh\frac{1}{2}\theta_{13}}{\theta_{13}-i\pi}u(\theta_2) + \delta^{\delta}_{\alpha}\mathbf{C}_{\beta\gamma}\frac{\cosh\frac{1}{2}\theta_{23}}{\theta_{23}-i\pi}u(\theta_1)\right) \tag{5.11}$$

which agrees with the $1/N$ expansion using Feynman graphs (see appendix E.2).

Bound state form factor of ψ. We discuss the bound state fusion of 2 fundamental fermions $f+f\to b_2$, a boson of mass m_2 (see (2.2)). Writing (5.5) as

$$\psi(x) = (i\gamma\partial+m)\tilde{\chi}(x),\quad \tilde{\chi}(x) = -i\left(\Box+m^2\right)^{-1}\chi(x)\,,$$

we apply the form factor equation (iv), i.e. (3.4)

$$\operatorname*{Res}_{\theta_{12}=i\pi\nu} F^{\mathcal{O}}_{123}(\theta_1,\theta_2,\theta_3) = F^{\mathcal{O}}_{(12)3}(\theta_{(12)},\theta_3)\,\sqrt{2}\Gamma^{(12)}_{12}$$

to the operator $\mathcal{O}=\tilde{\chi}$,[9]

$$\operatorname*{Res}_{\theta_{12}=i\pi\nu} F^{\tilde{\chi}^{(\pm)}}_{1\bar{1}1}(\underline{\theta}) = F^{\tilde{\chi}^{(\pm)}}_{b_2 1}(\theta_0,\theta_3)\sqrt{2}\Gamma^{b_2}_{1\bar{1}}\,.$$

[8]Doing one integral we obtain a generalization of Meijer's G-functions. The second integration does not yield known functions (to our knowledge). One could, of course, apply numerical integration techniques and determine the asymptotic behavior for large θ's which is under investigation [46].

[9]Strictly speaking $F^{\tilde{\chi}}_{1\bar{1}1} \pm F^{\tilde{\chi}}_{\bar{1}11}$ give $F^{\tilde{\chi}}_{b_2^{(0,2)}1}$.

JHEP02(2016)042

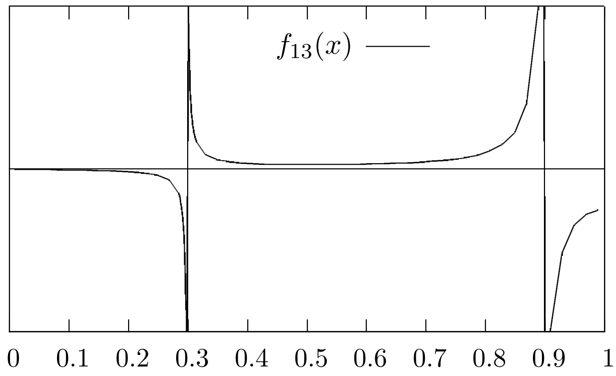


Figure 4. Plot of the bound state form factor function $f_{13}(x)$, $(\theta = i\pi x)$ for $N = 12$ $(\nu = 1/5)$.

The result may be written as

$$\begin{aligned} F_{b_2 1}^{\tilde{\chi}(\pm)}(\theta_0, \theta_3) &= e^{\mp\frac{1}{2}\theta_0}\left(e^{\pm\frac{1}{4}i\pi\nu} f_{13}(\theta_{03}) + e^{\mp\frac{1}{4}i\pi\nu} f_{32}(\theta_{03})\right) \\ &\quad + e^{\mp\frac{1}{2}\theta_3}\left(e^{\pm\frac{1}{2}i\pi\nu} f_{11}(\theta_{03}) + e^{\mp\frac{1}{2}i\pi\nu} f_{22}(\theta_{03})\right). \end{aligned}$$

where the functions f_{ij} may be calculated in terms of hypergeometric functions ${}_3F_2$ (for more details see appendix D). For example f_{13} is plotted for $N = 12$ in figure 4.

The pole at $\theta = \frac{3}{2}i\pi\nu$ (here $x = 0.3$) belongs to the bound state fusion $b_2^{(r)} + f \to b_3^{(r\pm1)}$, a fermion of mass m_3 (see (2.2)). The pole at $\theta = i\pi\left(1 - \frac{1}{2}\nu\right)$ (here $x = 0.9$) belongs to the bound state fusion $b_2^{(r)} + f \to f$, which is again the fundamental fermion. These are examples of the general "bootstrap principal" [43].

5.3 Energy momentum

The energy momentum tensor is in terms of fields is

$$T^{\mu\nu}(x) = \frac{1}{2} i\bar{\psi}\gamma^{\mu}\overleftrightarrow{\partial^{\nu}}\psi - g^{\mu\nu}\mathcal{L}$$

with the trace

$$T^{\mu}_{\ \mu}(x) = m\bar{\psi}\psi\,.$$

Because $T^{\mu\nu}$ is an $O(N)$ iso-scalar we have the weights $w = (w_1, \ldots, w_{N/2}) = (0, \ldots, 0)$ (see [1, 2]) which implies that

$$n = n_1 = \cdots = n_{N/2-2} = n_- + n_+, n_- = n_+\,.$$

We write the energy momentum tensor in terms of an energy momentum potential (see e.g. [3])

$$\begin{aligned} T^{\mu\nu}(x) &= R^{\mu\nu}(i\partial_x)T(x) \\ R^{\mu\nu}(P) &= -P^{\mu}P^{\nu} + g^{\mu\nu}P^2 \\ T^{\mu}_{\ \mu}(x) &= (i\partial_x)^2 T(x)\,. \end{aligned}$$

For $\bar\psi\psi$ we propose the n-particle form factor as

$$\begin{aligned}
\langle 0|\bar\psi\psi(0)|\underline{\theta}\rangle_{\underline{\alpha}} = F^{\bar\psi\psi}_{\underline{\alpha}}(\underline{\theta}) &= N^{\bar\psi\psi}_n \prod_{i<j} F(\theta_{ij}) K^{\bar\psi\psi}_{\underline{\alpha}}(\underline{\theta}) \\
K^{\bar\psi\psi}_{\underline{\alpha}}(\underline{\theta}) &= \int_{\mathcal{C}^{(1)}_{\underline{\theta}}} dz_1 \dots \int_{\mathcal{C}^{(m)}_{\underline{\theta}}} dz_m \, \tilde{h}(\underline{\theta},\underline{z}) p^{\bar\psi\psi}(\underline{\theta},\underline{z}) \, \tilde{\Psi}_{\underline{\alpha}}(\underline{\theta},\underline{z})
\end{aligned} \tag{5.12}$$

with $m = n =$ even and

$$\begin{aligned}
\tilde{h}(\underline{\theta},\underline{z}) &= \prod_{i=1}^{n}\prod_{j=1}^{m} \tilde{\phi}(\theta_i - z_j) \prod_{1\le i<j\le m} \tau(z_{ij}), \\
p^{\bar\psi\psi}(\underline{\theta},\underline{z}) &= 1 \\
\tilde{\Psi}_{\underline{\alpha}}(\underline{\theta},\underline{z}) &= L_{\underline{\mathring{\beta}}}(\underline{z}) \left(\Pi^{\underline{\mathring{\beta}}}_{\underline{\beta}}(\underline{z}) \Omega \tilde{T}^{\beta_m}_1(\underline{\theta},z_m) \dots \tilde{T}^{\beta_1}_1(\underline{\theta},z_1) \right)_{\underline{\alpha}} .
\end{aligned} \tag{5.13}$$

We do not calculate the integrals in (5.12) for general N, but the 2 particle form factor follows from lemma 4 in appendix C.2

$$F^{\bar\psi\psi}_{\alpha_1\alpha_2}(\underline{\theta}) = \langle\, 0 \,|\, \bar\psi\psi(0) \,|\, \theta_1,\theta_2 \,\rangle^{\text{in}}_{\alpha_1\alpha_2} = \mathbf{C}_{\alpha_1\alpha_2} \, \bar{v}(\theta_1) u(\theta_2) \, F_0(\theta_{12}) \tag{5.14}$$

and

$$\begin{aligned}
F^{T^{\mu\nu}}_{\alpha_1\alpha_2}(\underline{\theta}) = \langle\, 0 \,|\, T^{\mu\nu}(0) \,|\, \theta_1,\theta_2 \,\rangle^{\text{in}}_{\alpha_1\alpha_2} &= \mathbf{C}_{\alpha_1\alpha_2} \bar{v}(\theta_1)\gamma^\mu u(\theta_2) \, \frac{1}{2}(p_1^\nu - p_2^\nu) \, F_0(\theta_{12}) \\
&= \mathbf{C}_{\alpha_1\alpha_2} \bar{v}(\theta_1) u(\theta_2) \, m \frac{(p_1-p_2)^\mu \, (p_1^\nu - p_2^\nu)}{(p_1-p_2)^2} \, F_0(\theta_{12}) \\
F^{T}_{\alpha_1\alpha_2}(\underline{\theta}) = \langle\, 0 \,|\, T(0) \,|\, p_1,p_2 \,\rangle^{\text{in}}_{\alpha_1\alpha_2} &= \mathbf{C}_{\alpha_1\alpha_2} \frac{\bar{v}(\theta_1) u(\theta_2)}{4m \cosh^2 \frac{1}{2}\theta_{12}} \, F_0(\theta_{12})
\end{aligned}$$

with $F_0(\theta)$ given by (3.9) and (3.13).

1/N expansion. For $N \to \infty$ we obtain

$$F^{\bar\psi\psi}_{\alpha_1\alpha_2}(\underline{\theta}) = \mathbf{C}_{\alpha_1\alpha_2} \bar{v}(\theta_2) u(\theta_1) \frac{2 \coth \frac{1}{2}\theta_{12}}{\theta_{12} - i\pi} + O(1/N) \, .$$

This result agrees with the one obtained by computing Feynman graphs as was done in [6].

6 Conclusions

In this article we have enlarged our $O(N)$ Bethe Ansatz knowledge of the $O(N)$ Gross-Neveu model, which exhibits a very rich bound state structure and, consequently, creates a rich form factor hierarchy. We have computed the form factors for the fundamental Fermi field, which transforms as a vector representation of $O(N)$. Then we have also constructed the form factors for the Noether current and the energy-momentum tensor. In addition for the two particle case we have proved the recursion relation for the higher level K-functions. Finally we have checked our results against the usual $1/N$ expansion and found full agreement. In a forthcoming paper we will investigate the kink form factors, possibly proving a kink field equation. Moreover, we will perform a detailed analysis of the $O(6)$ Gross-Neveu model, a starting point in the nesting procedure.

JHEP02(2016)042

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A Proof of the main theorem 1

The identity

$$\int_{\mathcal{C}_a} dz\Gamma(a-z)f(z) = 2\pi i \operatorname*{Res}_{z=a} \sum_{l=-\infty}^{\infty} \Gamma(a-z-l)f(z+l) \tag{A.1}$$

where the $\mathcal{C}_a$ encircles the poles of $\Gamma(a-z)$ anti-clockwise may be used to write the K-function $K^{\mathcal{O}}_{\underline{\alpha}}(\underline{\theta})$ defined by the integral representation (1.4) as a sum of "Jackson-type Integrals" as investigated in [1]. These expressions satisfy symmetry properties and a matrix difference equation which are equivalent to the form factor equations (i) and (ii). We have to prove, that due to the assumptions of theorem 1 in addition the residue relations (iii)

$$\operatorname*{Res}_{\theta_{12}=i\pi} F^{\mathcal{O}}_{1\dots n}(\theta_1,\dots,\theta_n) = 2i\,\mathbf{C}_{12}\,F^{\mathcal{O}}_{3\dots n}(\theta_3,\dots,\theta_n)\left(\mathbf{1}-\sigma^{\mathcal{O}}_2\,(\sigma S)_{2n}\dots(\sigma S)_{23}\right)$$

and (iv)

$$\operatorname*{Res}_{\theta_{12}=i\pi\nu} F^{\mathcal{O}}_{12\dots n}(\theta_1,\theta_2,\dots,\theta_n) = F^{\mathcal{O}}_{(12)\dots n}(\theta_{(12)},\dots,\theta_n)\,\sqrt{2}\Gamma^{(12)}_{12}$$

are satisfied.

Proof. We prove that the K-function $K^{\mathcal{O}}_{1\dots n}(\underline{\theta})$ defined by (1.3) and (1.4) satisfies the form factor equations (i)–(iii) which read in terms of $K^{\mathcal{O}}_{1\dots n}(\underline{\theta})$ as

$$K^{\mathcal{O}}_{\dots ij\dots}(\dots,\theta_i,\theta_j,\dots) = K^{\mathcal{O}}_{\dots ji\dots}(\dots,\theta_j,\theta_i,\dots)\,\tilde{S}_{ij}(\theta_{ij}) \tag{A.2}$$

$$K^{\mathcal{O}}_{1\dots n}(\theta_1+2\pi i,\theta_2,\dots,\theta_n)\sigma^{\mathcal{O}}_1\mathbf{C}^{\bar{1}1} = K^{\mathcal{O}}_{2\dots n1}(\theta_2,\dots,\theta_n,\theta_1)\mathbf{C}^{1\bar{1}} \tag{A.3}$$

$$\operatorname*{Res}_{\theta_{12}=i\pi} K_{1\dots n}(\underline{\theta}) = \frac{2i}{F(i\pi)}\mathbf{C}_{12}\prod_{i=3}^{n}\tilde{\phi}(\theta_{i1}+i\pi\nu)\tilde{\phi}(\theta_{i2})K_{3\dots n}(\theta_3,\dots,\theta_n)\left(\mathbf{1}-\sigma^{\mathcal{O}}_2 S_{2n}\dots S_{23}\right) \tag{A.4}$$

where (4.9) has been used.

- (i) follows as in [1],
- (ii) follows as in [1], however, here (4.10) is responsible for the statistics factor $\sigma^{\mathcal{O}}_1$ in (A.3).

- (iii) the residue of

$$K^{\mathcal{O}}_{1\dots n}(\underline{\theta}) = N^{\mathcal{O}}_n \int_{\mathcal{C}^{(1)}_{\underline{\theta}}} dz_1 \cdots \int_{\mathcal{C}^{(m)}_{\underline{\theta}}} dz_m\, \tilde{h}(\underline{\theta}, \underline{z})\, p^{\mathcal{O}}(\underline{\theta}, \underline{z})\; \tilde{\Psi}_{1\dots n}(\underline{\theta}, \underline{z}) \tag{A.5}$$

consists of two terms

$$\operatorname*{Res}_{\theta_{12}=i\pi} K_{1\dots n}(\underline{\theta}) = \left(\overset{(1)}{\operatorname*{Res}_{\theta_{12}=i\pi}} + \overset{(2)}{\operatorname*{Res}_{\theta_{12}=i\pi}} \right) K_{1\dots n}(\underline{\theta})\,.$$

This is because for each z_j integration with j even the contours will be "pinched" at two points (see figure 3):

(1) $z_j = \theta_2 \approx \theta_1 - i\pi$

(2) $z_j = \theta_1 - 2\pi i \approx \theta_2 - i\pi$

We prove in appendix C.1 the residue formulas for general level k of the off-shell Bethe Ansatz. In particular for $k = 0$ the general result implies that the pinching (1) gives

$$\overset{(1)}{\operatorname*{Res}_{\theta_{12}=i\pi}} K_{1\dots n}(\underline{\theta}) = \frac{2i}{F(i\pi)} \mathbf{C}_{12} \prod_{i=3}^{n} \tilde{\phi}(\theta_{i1} + i\pi\nu)\tilde{\phi}(\theta_{i2}) K_{3\dots n}(\theta_3, \dots, \theta_n) \tag{A.6}$$

for a suitable choice of the normalization constants in (A.5). Therefore we have proved

$$\overset{(1)}{\operatorname*{Res}_{\theta_{12}=i\pi}} F_{1\dots n}(\theta_1, \dots, \theta_n) = 2i\, \mathbf{C}_{12}\, F_{3\dots n}(\theta_3, \dots, \theta_n)\,.$$

We use (ii) and (i) to write

$$\begin{aligned} F_{1\dots n}(\underline{\theta})\sigma^{\mathcal{O}}_1 &= \mathbf{C}_{1\bar{1}} F_{2\dots n1}(\theta_2, \dots, \theta_n, \theta_1 - 2\pi i)\mathbf{C}^{1\bar{1}} \\ &= \mathbf{C}_{1\bar{1}} F_{21\dots n}(\theta_2, \theta_1 - 2\pi i, \dots, \theta_n)\mathbf{C}^{1\bar{1}}\, (\sigma S)_{\bar{1}n} \dots (\sigma S)_{\bar{1}3}\,. \end{aligned}$$

Then the result for $\overset{(1)}{\operatorname*{Res}_{\theta_1=\theta_2+i\pi}}$ implies for the contribution of the pinching at $z_j = \theta_1 - 2\pi i \approx \theta_2 - i\pi$

$$\begin{aligned} \overset{(2)}{\operatorname*{Res}_{\theta_1=\theta_2+i\pi}} F_{1\dots n}(\underline{\theta})\sigma^{\mathcal{O}}_1 &= - \overset{(1)}{\operatorname*{Res}_{\theta_2=(\theta_1-2\pi i)+i\pi}} \mathbf{C}_{1\bar{1}} F_{21\dots n}(\theta_2, \theta_1 - 2\pi i, \dots, \theta_n)\mathbf{C}^{1\bar{1}}\, (\sigma S)_{\bar{1}n} \dots (\sigma S)_{\bar{1}3} \\ &= -\mathbf{C}_{1\bar{1}} 2i\, \mathbf{C}_{21}\, F^{\mathcal{O}}_{3\dots n}(\theta_3, \dots, \theta_n)\mathbf{C}^{1\bar{1}}\, (\sigma S)_{\bar{1}n} \dots (\sigma S)_{\bar{1}3} \\ &= -2i\mathbf{C}_{12}\, F^{\mathcal{O}}_{3\dots n}(\theta_3, \dots, \theta_n)\sigma^{\mathcal{O}}_2\, (\sigma S)_{2n} \dots (\sigma S)_{23}\, \sigma^{\mathcal{O}}_1 \end{aligned}$$

using $\sigma^{\mathcal{O}}_1 \sigma^{\mathcal{O}}_{\bar{1}} = 1$.

(iv) Because there are bound states we also have to discuss the form factor equation (iv) (3.4)

$$\operatorname*{Res}_{\theta_{12}=i\pi\nu} F^{\mathcal{O}}_{12\dots n}(\theta_1, \theta_2, \underline{\hat{\theta}}) = F^{\mathcal{O}}_{(12)\dots n}(\theta_{(12)}, \underline{\hat{\theta}})\, \sqrt{2}\Gamma^{(12)}_{12}\,.$$

The bound state form factor $F^{\mathcal{O}}_{(12)\dots n}(\theta_{(12)},\underline{\hat{\theta}})$ is then obtained from the residue

$$\operatorname*{Res}_{\theta_{12}=i\pi\nu} K^{\mathcal{O}}_{12\dots n}(\underline{\theta}) = \operatorname*{Res}_{\theta_{12}=i\pi\nu} N^{\mathcal{O}}_n \int_{\mathcal{C}^{(1)}_{\underline{\theta}}} dz_1 \cdots \int_{\mathcal{C}^{(m)}_{\underline{\theta}}} dz_m\, \tilde{h}(\underline{\theta},\underline{z})\, p^{\mathcal{O}}(\underline{\theta},\underline{z})\ \tilde{\Psi}_{1\dots n}(\underline{\theta},\underline{z})\,.$$

Similar as in the proof of (iii) the residue is obtained from pinching at:
$z_j = \theta_1 - i\pi\nu \approx \theta_2$ for $\mathcal{C}^{(o)}$ and $z_j = \theta_2 \approx \theta_1 - i\pi\nu$ for $\mathcal{C}^{(e)}$.

Here we will not perform the lengthy calculations and write the complicated result, but in appendix D we will calculate the bound state form factors for the examples of section 5.
■

B Two-particle current form factor

Derivation of (5.3) and (5.4):

Proof. The two-particle K-function of the current is

$$K^J_{\underline{\alpha}}(\underline{\theta}) = N^J_2 \int_{\mathcal{C}^{(o)}_{\underline{\theta}}} dz\, \tilde{h}(\underline{\theta},z) p^J(\underline{\theta},z)\, \tilde{\Psi}_{\underline{\alpha}}(\underline{\theta},z)$$

with the p-function (5.2) for $n=2$ and $m=1$

$$p^J(\underline{\theta},\underline{\underline{z}}) = \frac{e^{\frac{1}{2}(\theta_1+\theta_2)}}{e^{\theta_1}+e^{\theta_2}} + \frac{e^{-\frac{1}{2}(\theta_1+\theta_2)}}{e^{-\theta_1}+e^{-\theta_2}} = \frac{1}{\cosh\frac{1}{2}\theta_{12}} \tag{B.1}$$

and the Bethe state

$$\tilde{\Psi}_{\underline{\alpha}}(\underline{\theta},z) = \delta^2_{\alpha_1}\delta^1_{\alpha_2}\tilde{c}(\theta_1-z) + \delta^1_{\alpha_1}\delta^2_{\alpha_2}\tilde{b}(\theta_1-z)\tilde{c}(\theta_2-z)\,.$$

Doing the integral we obtain

$$K^J_{21}(\theta_{12}) = N^J_2 \int_{\mathcal{C}^{(o)}_{\underline{\theta}}} dz\, \tilde{h}(\underline{\theta},z) p^J(\underline{\theta},z)\, \tilde{\Psi}_{21}(\underline{\theta},z) = -N^J_2 8\pi^3 4^\nu \Gamma(1-\nu)\, c\frac{F_-(\theta)}{F(\theta)} \tag{B.2}$$

and $K^J_{12}(\theta) = -K^J_{21}(\theta)$.

We use again the variables $u=\theta/(i\pi\nu)$ and $v=z/(i\pi\nu)$, consider the component $K^J_{21}(\theta)$ and calculate the integral

$$I = \frac{1}{2\pi i}\int_{\mathcal{C}^{(o)}_{\underline{u}}} dv\, I(\underline{u},v)$$
$$I(\underline{u},v) = \tilde{h}(\underline{u},v)\tilde{\Psi}(\underline{u},v),\ \ \tilde{h}(\underline{u},v) = \tilde{\phi}(u_1-v)\tilde{\phi}(u_2-v),\ \ \tilde{\Psi}(\underline{u},v) = \tilde{c}(u_1-v)\,.$$

Writing the integrals in terms of sums over residues we obtain (see figure 3)

$$I = I_1 + I_2 = \sum_{l=0}^{\infty} s_1(u_1,u_2,l) + \sum_{l=0}^{\infty} s_2(u_1,u_2,l) \tag{B.3}$$
$$s_i(u_1,u_2,l) = \operatorname*{Res}_{v=v_o(u_i,l)} I(u_1,u_2,v),\ \ v_o(u,l) = u - 1 + 2l/\nu\,.$$

Using the Gauss formula

$$
{}_2F_1(a,b;c;1) = \sum_{n=0}^{\infty} \frac{\Gamma(a+n)}{\Gamma(a)} \frac{\Gamma(b+n)}{\Gamma(b)} \frac{\Gamma(c)}{\Gamma(c+n)} \frac{\Gamma(1)}{\Gamma(1+n)} = \frac{\Gamma(c)\,\Gamma(c-a-b)}{\Gamma(c-a)\,\Gamma(c-b)} \tag{B.4}
$$

we get

$$
\begin{aligned}
I &= I_1 + I_2 \\
&= 2^\nu \pi\sqrt{\pi} \frac{\Gamma\left(-\frac{1}{2}\nu\right)\Gamma\left(\frac{1}{2}\nu+\frac{1}{2}\right)\cos\frac{1}{2}\pi\nu}{\sin\frac{1}{2}\pi\nu\,(u_{12}+1)\sin\frac{1}{2}\pi\nu\,(u_{12}-1)} \frac{1}{\Gamma\left(\frac{1}{2}\nu+\frac{1}{2}\nu u_{12}\right)\Gamma\left(1+\frac{1}{2}\nu-\frac{1}{2}\nu u_{12}\right)}
\end{aligned}
$$

which agrees with (B.2) taking (B.1) into account. Therefore using (3.11) and (3.13) we finally obtain with the normalization constant

$$
N_2^J = \frac{1}{8} 4^{-\nu} \frac{im}{c\pi^3\Gamma(1-\nu)} \tag{B.5}
$$

for the pseudo-potential $J^{\alpha\beta}(x)$ the two-particle form factors (5.3) and (5.4) for the current. The normalization is chosen such that the form factor agrees for $F_-(\theta) \to F_-(i\pi) = 1$ with the free field expression. ■

C Higher level K-functions

C.1 Proof of lemma 2

Remark 3 *If in (4.15)* Res *is replaced by* $\overset{(1)}{\text{Res}}$ *Lemma 2 also holds for* $k = 0$ *as explained in appendix A.*

For the discussion of the general k-level Bethe Ansatz it is convenient to use the variables u, v defined by $\theta = i\pi\nu_k u$, $z = i\pi\nu_k v$ and $\nu_k = 2/(N-2k-2)$ (for the S-matrix see (4.11)). In the proof we will replace $p^{(k)}(\underline{u}, \underline{v})$ by 1 which will not change the results, if the $p^{(k)}$ satisfy the conditions (4.17).

Proof. As above in the proof of theorem 1 the relations $(\mathrm{i})^{(k)}$ and $(\mathrm{ii})^{(k)}$ follow from the results of [1]. The proof of $(\mathrm{iii})^{(k)}$ is the same as the corresponding one in [3], only the functions $\tilde{\psi}(u)$ and $\tilde{\chi}(u)$ have to replaced by $\tilde{\phi}(u)$ and $\tau_{ij}(v)$ by $\tau(v)$. As in [3] one finally obtains

$$
\begin{aligned}
&\operatorname*{Res}_{u_{12}=1/\nu_k} K^{(k)}_{\underline{\alpha}}(\underline{u}) \\
&\quad = \text{const.} \left(\operatorname*{Res}_{v=1/\nu_{k+1}} \tilde{d}_{k+1}(v)\right)^{-1} \operatorname*{Res}_{u_{12}=1/\nu_k} \oint_{u_1-1} dv_1 \tilde{c}(u_1-v_1) \left(-\oint_{u_2}\right) dv_2 \\
&\qquad \times \tilde{d}_k(u_{12}) \left(\prod_{i=1}^{2}\prod_{j=1}^{2} \tilde{\phi}(u_i - v_j)\right) \tau(v_{12}) \prod_{i=3}^{n_k} \tilde{\phi}(u_{i1}+1)\tilde{\phi}(u_{i2}) \frac{1}{\tilde{N}^{(k)}_{m_k-2}} \mathbf{C}_{\alpha_1\alpha_2} K^{(k)}_{\underline{\check{\alpha}}}(\underline{\check{u}})
\end{aligned}
$$

with $\underline{\check{\alpha}} = \alpha_3 \dots \alpha_{n_k}$. It has been used that for $u_{12} = 1/\nu_k$, $v_{12} = 1/\nu_{k+1}$, $u_2 = v_2$, $u_1 = v_2 + 1/\nu_k = v_1 + 1$

$$
\left(\frac{a_{k+1}(v_{1j})a_{k+1}(v_{2j})}{a_k(u_1-v_j)a_k(u_2-v_j)} \tilde{\phi}(v_{j1}+1)\tilde{\phi}(v_{j2})\right) \left(\tilde{\phi}(u_1-v_j)\tilde{\phi}(u_2-v_j)\tau(v_{1j})\tau(v_{2j})\right) = 1\,.
$$

This can be shown by means of (4.5) and the formulas

$$
\begin{aligned}
a_k(u_1)a_k(u_2) &= \tilde{b}(-u_2)/\tilde{b}(u_1) \\
\tilde{b}(u)\tilde{\phi}(u) &= -\tilde{\phi}(1-u)\,.
\end{aligned}
$$

The final result is that equation (4.15) holds for a suitable choice of the normalization constants in (4.12). ■

C.2 Two-particle higher level K-functions

We need higher level K-functions for the examples of section 5, in particular, in the iso-scalar two-particle channel (with weights $w = (0, \dots, 0)$) the K-function $K^{(k)}_{\alpha_1\alpha_2}(\theta_1, \theta_2)$ (level $k = 0, 1, 2, \dots$) belonging to $O(N-2k)$. It is of the form

$$
K^{(k)}_{\alpha_1\alpha_2}(u_1, u_2) = \mathbf{C}^{(N-2k)}_{\alpha_1\alpha_2} K(u_{12}, k) \tag{C.1}
$$

where $\mathbf{C}^{(N-2k)}_{\alpha_1\alpha_2}$ is the $O(N-2k)$ charge conjugation matrix.[10] From the weight vector

$$
w = (w_1, \dots, w_{N/2}) = (0, \dots, 0) = \left(n - n_1, \dots, n_{N/2-2} - n_- - n_+, n_- - n_+\right)
$$

follows that for all levels $n_k = 2$.

Lemma 4 *The vector valued functions $K^{(k)}_{\alpha_1\alpha_2}(u_1, u_2)$ with*

$$
K(u,k) = \frac{\Gamma\left(1 - \frac{1}{2}\nu - \frac{1}{2}\nu u\right)\Gamma\left(-\frac{1}{2}\nu + \frac{1}{2}\nu u\right)}{\Gamma\left(\frac{3}{2} - \frac{1}{2}k\nu - \frac{1}{2}\nu u\right)\Gamma\left(\frac{1}{2} - \frac{1}{2}k\nu + \frac{1}{2}\nu u\right)} \tag{C.2}
$$

satisfy for $k = 0, 1, 2, \dots < N/2 - 2$ the recursion relation

$$
\begin{aligned}
K^{(k)}_{\underline{\alpha}}(\underline{u}) &= N^{(k)} \int_{\mathcal{C}^{(o)}_{\underline{u}}} dv_1 \int_{\mathcal{C}^{(e)}_{\underline{u}}} dv_2\, \tilde{h}(\underline{u}, \underline{v})\, L^{(k)}_{\underline{\mathring{\beta}}}(\underline{v}, k)\, \tilde{\Phi}^{(k)}{}^{\underline{\mathring{\beta}}}_{\underline{\alpha}}(\underline{u}, \underline{v}) \\
L^{(k)}_{\underline{\mathring{\beta}}}(\underline{v}, k) &= K^{(k+1)}_{\underline{\mathring{\beta}}}(\underline{v}) = \mathbf{C}^{(N-2k-2)}_{\underline{\mathring{\beta}}} K(v_{12}, k+1)
\end{aligned} \tag{C.3}
$$

with

$$
\begin{aligned}
\tilde{h}(\underline{u}, \underline{v}) &= \prod_{i=1}^{2} \left(\tilde{\phi}(u_i - v_1)\tilde{\phi}(u_i - v_2)\right) \frac{1}{\tilde{\phi}(v_{12})\tilde{\phi}(-v_{12})} \\
\tilde{\Phi}^{(k)}{}^{\underline{\mathring{\beta}}}_{\underline{\alpha}}(\underline{u}, \underline{v}) &= \left(\Pi^{\underline{\mathring{\beta}}}_{\underline{\beta}}(\underline{v})\, \Omega\, \tilde{T}_1^{\beta_2}(\underline{u}, v_2)\, \tilde{T}_1^{\beta_1}(\underline{u}, v_1)\right)^{(k)}_{\underline{\alpha}}
\end{aligned}
$$

and the normalization

$$
N^{(k)} = \frac{-2^{-\nu}}{8\pi^2} \frac{\Gamma\left(\frac{1}{2} - \frac{1}{2}k\nu\right)\Gamma\left(1 - \frac{1}{2}k\nu - \frac{1}{2}\nu\right)}{\Gamma\left(1 - \frac{1}{2}k\nu\right)\Gamma\left(\frac{1}{2} - \frac{1}{2}k\nu + \frac{1}{2}\nu\right)\left(\Gamma\left(-\frac{1}{2}\nu\right)\right)^2}\,. \tag{C.4}
$$

[10] In the real basis this would be $\delta_{\alpha_1\alpha_2}$.

JHEP02(2016)042

Proof. The function (C.2) satisfies

$$
\begin{aligned}
\text{(i)}:\qquad K(u,k) &= K(-u,k)\tilde{S}_0^{(k)}(u)\\
\text{(ii)}:\quad K(1/\nu-u,k) &= K(1/\nu+u,k)
\end{aligned}
\tag{C.5}
$$

with the scalar eigenvalue of $\tilde{S}^{(k)}(u)=\tilde{S}^{O(N-2k)}(u)$

$$
\tilde{S}_0^{(k)}(u)=S_0^{(k)}(u)/S_+^{(k)}(u)=\frac{u+1/\nu_k}{u-1/\nu_k}\frac{u+1}{u-1}=\frac{u+(1/\nu-k)}{u-(1/\nu-k)}\frac{u+1}{u-1}\,.
$$

The minimal solution (with no poles in the physical strip $0\leq \operatorname{Re}u\leq 1/\nu$) is

$$
K_m(u,k)=\frac{1}{\Gamma\left(\frac{3}{2}-\frac{1}{2}k\nu-\frac{1}{2}\nu u\right)\Gamma\left(\frac{1}{2}(1-\nu k)+\frac{1}{2}\nu u\right)\Gamma\left(1+\frac{1}{2}\nu-\frac{1}{2}\nu u\right)\Gamma\left(\frac{1}{2}\nu+\frac{1}{2}\nu u\right)}
\tag{C.6}
$$

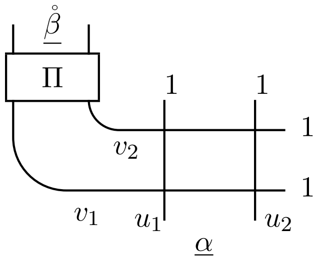
whereas $K(u,k)$ has the bound state pole at $u=1$ ($\theta=i\pi\nu_k$).

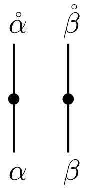
The Bethe state in (C.3) is (see [1])

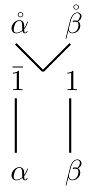
$$
\tilde{\Phi}^{(k)}{}^{\underline{\mathring{\beta}}}_{\underline{\alpha}}(\underline{u},\underline{v})=\left(\Pi^{(k)}\right)^{\underline{\mathring{\beta}}}_{\underline{\beta}}(\underline{v})\left(\Omega\tilde{T}_1^{\beta_2}(\underline{u},v_2)\tilde{T}_1^{\beta_1}(\underline{u},v_1)\right)^{(k)}_{\underline{\alpha}}
$$

$$
\left(\Pi^{(k)}\right)^{\mathring{\beta}_1\mathring{\beta}_2}_{\beta_1\beta_2}(\underline{v})=\delta^{\mathring{\beta}_1}_{\beta_1}\delta^{\mathring{\beta}_2}_{\beta_2}+f_k(v_{12})\mathring{\mathbf{C}}^{\mathring{\beta}_1\mathring{\beta}_2}\delta^{\overline{k+1}}_{\beta_1}\delta^{k+1}_{\beta 2}\,,\quad f_k(v)=\frac{1}{v+1/\nu_k-1}
$$

which may be depicted (for $k=0$) as

$\Phi^{\underline{\mathring{\beta}}}_{\underline{\alpha}}(\underline{u},\underline{w})=$ 

$,\quad \Pi^{\mathring{\alpha}\mathring{\beta}}_{\alpha\beta}(u_1,u_2)=$ 

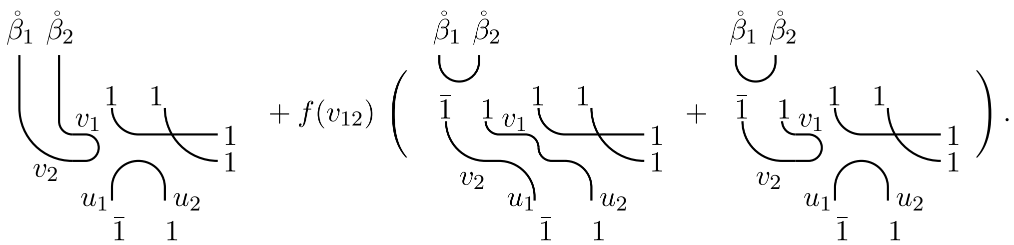
$+f(u_{12})$ 

. (C.7)

Because of (C.1) it is sufficient to consider only one component of (C.3) and for convenience we take $\tilde{\Phi}^{(k)}{}^{\underline{\mathring{\beta}}}_{\underline{\alpha}}$ with $\underline{\alpha}=\overline{k+1},k+1$ and define

$$
\begin{aligned}
&\tilde{\Phi}_k(\underline{u},\underline{v})\\
&=\mathbf{C}^{(N-2k-2)}_{\mathring{\beta}_1\mathring{\beta}_2}\tilde{\Phi}^{(k)}{}^{\mathring{\beta}_1\mathring{\beta}_2}_{\overline{k+1},k+1}(\underline{u},\underline{v})\\
&=\mathbf{C}^{(N-2k-2)}_{\mathring{\beta}_1\mathring{\beta}_2}\mathbf{C}^{\mathring{\beta}_1\mathring{\beta}_2}_{(N-2k-2)}\left(\tilde{c}(u_1-v_2)\tilde{d}_k(u_1-v_1)+f_k(v_1-v_2)\left(\tilde{c}(u_1-v_1)+\tilde{d}_k(u_1-v_1)\right)\right)\\
&=(N-2k-2)\frac{-(v_1-v_2-1)}{(u_1-v_1-1)(u_1-v_2-1)(v_1-v_2+1/\nu_k-1)}
\end{aligned}
$$

where (for $k=0$) $\tilde{\Phi}^{\mathring{\beta}_1\mathring{\beta}_2}_{\bar{1},1}(\underline{u},\underline{v})$ may be depicted as



Then because $L^{(k)}_{\underline{\mathring{\beta}}}(\underline{v})\tilde{\Phi}^{(k)}{}^{\underline{\mathring{\beta}}}_{k+1,k+1}(\underline{u},\underline{v}) = K(v_{12}, k+1)\tilde{\Phi}_k(\underline{u},\underline{v})$

$$\begin{aligned} K^{(k)}_{k+1,k+1}(\underline{u}) &= N^{(k)} \int_{\mathcal{C}^{(o)}_{\underline{u}}} dv_1 \int_{\mathcal{C}^{(e)}_{\underline{u}}} dv_2\, \tilde{h}(\underline{u},\underline{v})\, K(v_{12}, k+1)\tilde{\Phi}_k(\underline{u},\underline{v}) \\ &= N^{(k)}(2\pi i)^2(N-2k-2)J(u_{12}) \end{aligned} \tag{C.8}$$

where

$$\begin{aligned} J(u_{12}) &= \frac{1}{(2\pi i)^2} \int_{\mathcal{C}^{(o)}_{\underline{u}}} dv_1 \int_{\mathcal{C}^{(e)}_{\underline{u}}} dv_2\, J(\underline{u},\underline{v}) \\ J(\underline{u},\underline{v}) &= \tilde{\phi}(u_1-v_1)\tilde{c}(u_1-v_1)\tilde{\phi}(u_1-v_2)\tilde{c}(u_1-v_2)\tilde{\phi}(u_2-v_1)\tilde{\phi}(u_2-v_2)\varphi(v_{12}) \\ \varphi(v) &= \frac{(1-v)\, K(v, k+1)}{\tilde{\phi}(v)\tilde{\phi}(-v)\,(v+1/\nu-k-1)}. \end{aligned} \tag{C.9}$$

Because the function $J(u)$ satisfies (C.5) it is proportional to $K(u,k)$ (as was shown in general in [6]) if there are no zeroes and exactly one pole[11] at $u=1$ in $0 \le \mathrm{Re}\, u \le 1/\nu$. Finally we obtain

$$J(u) = \frac{2^{\nu-1}\nu\left(\Gamma\left(-\frac{1}{2}\nu\right)\right)^2 \Gamma\left(\frac{1}{2}-\frac{1}{2}k\nu+\frac{1}{2}\nu\right)\Gamma\left(1-\frac{1}{2}k\nu\right)}{\Gamma\left(-\frac{1}{2}\nu+1-\frac{1}{2}k\nu\right)\Gamma\left(\frac{3}{2}-\frac{1}{2}k\nu\right)} K(u,k) \tag{C.10}$$

where the constant is calculated by taking the residue at $u=1$ on both sides of (C.10). Finally we turn to (C.3). By (C.8) and (C.1) we have

$$K(u,k) = K^{(k)}_{k+1,k+1}(\underline{u}) = N^{(k)}(2\pi i)^2(N-2k-2)J(u_{12}) \tag{C.11}$$

which provides the normalization (C.4). ■

In particular for $k=0$ and $k=1$

$$\begin{aligned} K(u) = K(u,0) &= \frac{-2}{\pi}\frac{\cos\frac{1}{2}\pi\nu u}{\nu u-1}\Gamma\left(-\frac{1}{2}\nu+\frac{1}{2}\nu u\right)\Gamma\left(1-\frac{1}{2}\nu-\frac{1}{2}\nu u\right) \\ L(u) = K(u,1) &= \frac{\Gamma\left(1-\frac{1}{2}\nu-\frac{1}{2}\nu u\right)\Gamma\left(-\frac{1}{2}\nu+\frac{1}{2}\nu u\right)}{\Gamma\left(1+\frac{1}{2}(1-\nu)-\frac{1}{2}\nu u\right)\Gamma\left(\frac{1}{2}(1-\nu)+\frac{1}{2}\nu u\right)}. \end{aligned}$$

The function $L(u)$ is that of (5.10) and it is used to calculate the 2-particle form factor on the energy momentum (5.14) and also to calculate the 3-particle form factor of the field 5.9. In particular the 2-particle K-function of the scalar operator $\bar{\psi}\psi$ is up to a constant equal to $K(u)$. With the normalization in (5.12)

$$N_2^{\bar{\psi}\psi} = 2m/\left(\pi^2\nu^2 c\Gamma^2\left(\frac{1}{2}(1-\nu)\right)\right)$$

we obtain (5.14)

$$F^{\bar{\psi}\psi}_{\alpha_1\alpha_2}(\underline{\theta}) = \mathbf{C}_{\alpha_1\alpha_2}\,\bar{v}(\theta_1)u(\theta_2)\,F_0(\theta_{12})$$

which agrees with the result of [6]. The normalization is chosen such that the form factor agrees for $\theta \to i\pi$ with the free field expression.

[11] This is suggested by numerical calculations using mathematica.

JHEP02(2016)042

D Bound state form factors

We discuss the form factor equation (iv)

$$\operatorname*{Res}_{\theta_{12}=i\eta} F^{\mathcal{O}}_{12\dots n}(\theta_1,\theta_2,\dots,\theta_n) = F^{\mathcal{O}}_{(12)\dots n}(\theta_{(12)},\dots,\theta_n)\,\sqrt{2}\Gamma^{(12)}_{12}$$

for the examples of section (5). Of course, one may easily calculate the residues for two-particle form factors for the pseudo-potential $J^{\alpha\beta}(x)$ (5.3) and $\bar\psi\psi(x)$ (5.14) directly, however we will check here whether the general pinching procedure of appendix A will give the same result. In addition we obtain the bound state form factor of the three-particle form factor for the field.

Two-particle current form factor. By the form factor equation (iv) (3.4) the two-particle bound state form factor for the pseudo-potential $J^{\alpha\beta}(x)$ is

$$\operatorname*{Res}_{\theta_{12}=i\pi\nu} F^{J^{\alpha\beta}}_{12}(\theta_1,\theta_2) = F^{J^{\alpha\beta}}_{(12)}(\theta_{(12)})\,\sqrt{2}\Gamma^{(12)}_{12}\,,\quad \theta_{(12)}=\frac{1}{2}(\theta_1+\theta_2)$$

where the bound state intertwiner $\Gamma^{(12)}_{12}$ is given by (2.9) and (2.10).

In appendix B we calculated the two-particle form factor for the pseudo-potential $J^{\alpha\beta}(x)$ in terms of the integral

$$\begin{aligned}
I(u_{12}) &= \frac{1}{2\pi i}\int_{\mathcal{C}^{(o)}_{\underline{u}}} dv I(\underline{u},\underline{v})\\
I(\underline{u},v) &= \tilde h(\underline{u},v)\tilde\Psi(\underline{u},v),\ \ \tilde h(\underline{u},v)=\tilde\phi(u_1-v)\tilde\phi(u_2-v),\ \ \tilde\Psi(\underline{u},v)=\tilde c(u_1-v)
\end{aligned}$$

with the result

$$I(u) = \frac{2^\nu\pi\sqrt{\pi}\Gamma\left(-\frac12\nu\right)\Gamma\left(\frac12\nu+\frac12\right)\cos\frac12\pi\nu}{\sin\frac12\pi\nu\,(u+1)\sin\frac12\pi\nu\,(u-1)\,\Gamma\left(\frac12\nu+\frac12\nu u\right)\Gamma\left(1+\frac12\nu-\frac12\nu u\right)}$$

and the residue

$$\operatorname*{Res}_{u=1} I(u) = -\left(\Gamma\left(-\frac{1}{2}\nu\right)\right)^2.$$

In appendix A we remarked that the residue is obtained from pinching at: $z=\theta_1-i\pi\nu\approx\theta_2$ for $\mathcal{C}^{(o)}$

$$\begin{aligned}
\operatorname*{Res}_{u_{12}=1} I(u_{12}) &= \operatorname*{Res}_{u_{12}=1}\frac{1}{2\pi i}\oint_{u_1-1} dv\,\tilde\phi(u_1-v)\tilde\phi(u_2-v)\tilde c(u_1-v)\\
&= \operatorname*{Res}_{u_{12}=1}\tilde\phi(1)\tilde\phi(-u_{12}+1) = -\left(\Gamma\left(-\frac12\nu\right)\right)^2
\end{aligned}$$

which means that the pinching procedure gives the same result as the direct calculation.

Two-particle form factor of $\bar\psi\psi$. By the form factor equation (iv) (3.4) the two-particle bound state form factor for $\bar\psi\psi$ is

$$\operatorname*{Res}_{\theta_{12}=i\pi\nu} F^{\bar\psi\psi}_{12}(\theta_1,\theta_2) = F^{\bar\psi\psi}_{(12)}(\theta_{(12)})\,\sqrt{2}\Gamma^{(12)}_{12}\,.$$

In appendix C.2 we calculated the two-particle form factor for $\bar{\psi}\psi(x)$ in terms of the integral

$$
\begin{aligned}
J(u_{12}) &= \frac{1}{(2\pi i)^2}\int_{\mathcal{C}^{(o)}_{\underline{u}}} dv_1 \int_{\mathcal{C}^{(e)}_{\underline{u}}} dv_2\, J(\underline{u},\underline{v}) \\
J(\underline{u},\underline{v}) &= \tilde{\phi}(u_1-v_1)\tilde{c}(u_1-v_1)\tilde{\phi}(u_1-v_2)\tilde{c}(u_1-v_2)\tilde{\phi}(u_2-v_1)\tilde{\phi}(u_2-v_2)\varphi(v_{12}) \\
\varphi(v) &= \frac{(1-v)\,K(v,k+1)}{\tilde{\phi}(v)\tilde{\phi}(-v)\,(v+1/\nu-k-1)}
\end{aligned}
$$

with the result

$$
J = \frac{c_2}{\cos\pi\nu}K(u,0) = \frac{c_2}{\cos\pi\nu}\frac{\Gamma\left(1-\frac{1}{2}\nu-\frac{1}{2}\nu u\right)\Gamma\left(-\frac{1}{2}\nu+\frac{1}{2}\nu u\right)}{\Gamma\left(1+\frac{1}{2}-\frac{1}{2}\nu u\right)\Gamma\left(\frac{1}{2}+\frac{1}{2}\nu u\right)}
$$

and the residue is

$$
\operatorname*{Res}_{u=1} J(u) = \frac{c_2}{\cos\pi\nu}\operatorname*{Res}_{u=1} K(u,0) = \frac{4}{(1-\nu)\,\pi}\left(\Gamma\left(-\frac{1}{2}\nu\right)\right)^2 .
$$

In appendix A we remarked that the residue is obtained from pinching at: $z=\theta_1-i\pi\nu\approx\theta_2$ for $\mathcal{C}^{(o)}$ and $z_j=\theta_2\approx\theta_1-i\pi\nu$ for $\mathcal{C}^{(e)}$, therefore (see (C.9))

$$
\begin{aligned}
\operatorname*{Res}_{u_{12}=1} J &= \operatorname*{Res}_{u_{12}=1}\frac{1}{(2\pi i)^2}\left(\oint_{u_1-1} dv_1\int_{\mathcal{C}^{(e)}_{\underline{u}}} dv_2 - \int_{\mathcal{C}^{(o)}_{\underline{u}}} dv_1 \oint_{u_2} dv_2\right) J(\underline{u},\underline{v}) = R_1+R_2 \\
R_1 &= -\operatorname*{Res}_{u_{12}=1}\sum_{l_2=0}^{\infty}\left(s_{11}(u_1,u_2,0,l_2)+s_{12}(u_1,u_2,0,l_2)\right) .
\end{aligned}
$$

with

$$
\begin{aligned}
s_{ij}(u_1,u_2,l_1,l_2) &= \operatorname*{Res}_{v_1=v_o(u_i,l_1)}\operatorname*{Res}_{v_2=v_{e(u_j,l_2)}} J(u_1,u_2;v_1,v_2) \\
v_o(u,l) &= u-1+2l/\nu, \quad v_e(u,l)=u-2l/\nu
\end{aligned}
$$

It turns out that s_{12} gives no contribution and

$$
R_1 = -\operatorname*{Res}_{u_{12}=1}\sum_{l_2=0}^{\infty} s_{11}(u_1,u_2,0,l_2) = 2\frac{\left(\Gamma\left(-\frac{1}{2}\nu\right)\right)^2}{\pi\,(1-\nu)}
$$

such that again

$$
\operatorname*{Res}_{u_{12}=1} J = 4\frac{\left(\Gamma\left(-\frac{1}{2}\nu\right)\right)^2}{\pi\,(1-\nu)}
$$

which means that the pinching procedure gives the same result as the direct calculation.

3-particle form factor of ψ. We discuss the bound state fusion of 2 fundamental fermions $f+f\to b_2$. We write (5.5) as

$$
\psi(x) = (i\gamma\partial+m)\tilde{\chi}(x), \quad \tilde{\chi}(x) = -i\left(\Box+m^2\right)^{-1}\chi(x)
$$

and apply the form factor equation (3.4) to $\tilde{\chi}$,[12]

$$\operatorname*{Res}_{\theta_{12}=i\pi\nu} F^{\tilde{\chi}}_{1\bar{1}1}(\underline{\theta}) = F^{\tilde{\chi}(\pm)}_{b_2 1}(\theta_0,\theta_3)\sqrt{2}\Gamma^{b_2}_{1\bar{1}}\,.$$

The component $K^{\tilde{\chi}}_{1\bar{1}1}$ of the K-function (similar as for $\bar{\psi}\psi$ in appendix C.2) can be written in terms of

$$\begin{aligned}
J^{\chi}(\underline{u}) &= \frac{1}{(2\pi i)^2}\int_{\mathcal{C}^{(o)}_{\underline{u}}} dv_1 \int_{\mathcal{C}^{(e)}_{\underline{u}}} dv_2\, J^{\chi}(\underline{u},\underline{v}) p^{\chi}(\underline{u},\underline{v}) \\
J^{\chi}(\underline{u},\underline{v}) &= \left(\prod_{i=1}^{3}\prod_{j=1}^{2}\tilde{\phi}(u_i-v_j)\right)\tilde{b}(u_1-v_1)\tilde{b}(u_1-v_2)\tilde{c}(u_2-v_1)\tilde{c}(u_2-v_2)\varphi(v_{12}) \\
\varphi(v) &= \frac{(1-v)\,K(v,1)}{\tilde{\phi}(v)\tilde{\phi}(-v)\,(v+1/\nu-1)}\,.
\end{aligned}$$

In appendix A we remarked that the residue is obtained from pinching at:
$z_1=\theta_1-i\pi\nu\approx\theta_2$ $(v_1=u_1-1\approx u_2)$ for $\mathcal{C}^{(o)}$ and $z_2=\theta_2\approx\theta_1-i\pi\nu$ $(v_2=u_2\approx u_1-1)$ for $\mathcal{C}^{(e)}$. Therefore the bound state form factor is obtained from

$$\operatorname*{Res}_{u_{12}=1} J^{\chi}(\underline{u}) = \operatorname*{Res}_{u_{12}=1}\frac{1}{(2\pi i)^2}\left(\oint_{u_1-1}\int_{\mathcal{C}^{(e)}_{\underline{u}}} - \int_{\mathcal{C}^{(o)}_{\underline{u}}}\oint_{u_2}\right) dv_1\,dv_2 J^{\chi}(\underline{u},\underline{v})p^{\chi}(\underline{u},\underline{v})\,.$$

The integrals may be calculated in terms of hypergeometric functions ${}_3F_2$. We obtain

$$\begin{aligned}
F^{\tilde{\chi}(\pm)}_{b_2 1}(\theta_0,\theta_3) = e^{\mp\frac{1}{2}\theta_0}&\left(e^{\pm\frac{1}{4}i\pi\nu}f_{13}(\theta_{03})+e^{\mp\frac{1}{4}i\pi\nu}f_{32}(\theta_{03})\right)\\
&+e^{\mp\frac{1}{2}\theta_3}\left(e^{\pm\frac{1}{2}i\pi\nu}f_{11}(\theta_{03})+e^{\mp\frac{1}{2}i\pi\nu}f_{22}(\theta_{03})\right)
\end{aligned}$$

where $f_{1i}(\theta_{03})$ and $f_{i2}(\theta_{03})$ are the results from the integrations

$$\oint_{u_1-1} dv_1 \int_{\mathcal{C}_{u_i}} dv_2\ldots \text{ and } \oint_{\mathcal{C}_{u_i}} dv_1\int_{u_2} dv_2\ldots \text{ respectively.}$$

For example up to a constant (see figure 4)

$$\begin{aligned}
f_{13}(u) = &\frac{\left(\Gamma\left(1-\frac{1}{4}\nu-\frac{1}{2}\nu u\right)\right)^2\Gamma\left(-\frac{3}{4}\nu+\frac{1}{2}\nu u\right)\Gamma\left(-\frac{1}{4}\nu+\frac{1}{2}\nu u\right)}{\Gamma\left(\frac{3}{2}-\frac{3}{4}\nu+\frac{1}{2}\nu u\right)\Gamma\left(\frac{3}{2}-\frac{1}{4}\nu-\frac{1}{2}\nu u\right)\cot\frac{1}{2}\pi\nu\left(u-\frac{1}{2}\right)\cot\frac{1}{2}\pi\nu\left(u+\frac{1}{2}\right)}\\
&\times {}_3F_2\left(-\frac{1}{2}\nu+1,-\frac{3}{4}\nu+\frac{1}{2}\nu u,-\frac{1}{2}+\frac{1}{4}\nu+\frac{1}{2}\nu u;\frac{1}{4}\nu+\frac{1}{2}\nu u,\frac{3}{2}-\frac{3}{4}\nu+\frac{1}{2}\nu u;1\right)F_b(u)
\end{aligned}$$

where $F_b(u)$ is the minimal highest weight form factor function in the $b_2^{(r)}+f$ sector

$$F_b(\theta) = \text{const.}\left(\sinh\frac{1}{2}\theta\right)\frac{F^{\min}_{+}(\theta+\frac{1}{2}i\pi\nu)F^{\min}_{+}(\theta-\frac{1}{2}i\pi\nu)}{\Gamma(1+\frac{1}{4}\nu-\frac{\theta}{2i\pi})\Gamma(\frac{1}{4}\nu+\frac{\theta}{2i\pi})}$$

or explicitly in terms of $G(z)$ Barnes G-function

$$F_b(u) = \frac{\left(\sinh\frac{1}{2}\theta\right)G\left(\frac{1}{4}\nu+\frac{1}{2}\nu u\right)G\left(\frac{3}{2}-\frac{3}{4}\nu-\frac{1}{2}\nu u\right)G\left(1+\frac{1}{4}\nu-\frac{1}{2}\nu u\right)G\left(\frac{1}{2}-\frac{3}{4}\nu+\frac{1}{2}\nu u\right)}{G\left(\frac{1}{2}+\frac{1}{4}\nu+\frac{1}{2}\nu u\right)G\left(2-\frac{3}{4}\nu-\frac{1}{2}\nu u\right)G\left(\frac{3}{2}+\frac{1}{4}\nu-\frac{1}{2}\nu u\right)G\left(1-\frac{3}{4}\nu+\frac{1}{2}\nu u\right)}$$

[12] Strictly speaking $F^{\tilde{\chi}}_{1\bar{1}1}\pm F^{\tilde{\chi}}_{\bar{1}11}$ give $F^{\tilde{\chi}}_{b_2^{(0,2)}1}$.

with $u = \theta/(i\pi\nu)$. It satisfies Watson's equation

$$\frac{F_b(\theta)}{F_b(-\theta)} = a\left(\theta + \frac{1}{2}i\pi\nu\right) a\left(\theta - \frac{1}{2}i\pi\nu\right) \frac{\theta + \frac{1}{2}i\pi\nu}{\theta - \frac{1}{2}i\pi\nu} = a_b(\theta)$$

where $a_b(\theta)$ is the highest weight scattering amplitude in the $b_2^{(r)} + f$ sector.

E 1/N expansion

E.1 1/N expansion of the exact 3-particle field form factor

For $\chi^\delta(x) = i(-i\gamma\partial + m)\psi^\delta(x)$ we derive for the highest weight component $\chi(x) = \chi^1(x)$

$$F^{\chi}_{\bar{1}11}(\underline{\theta}) = \frac{8\pi m}{N}\left(\frac{\cosh\frac{1}{2}\theta_{12}}{\theta_{12} - i\pi} u(\theta_3) - \frac{\cosh\frac{1}{2}\theta_{13}}{\theta_{13} - i\pi} u(\theta_2)\right) + O(N^{-2}) \tag{E.1}$$

which is equivalent to (5.11).

Proof. The p-function of $\chi(x)$ for three particles and $\nu = 0$ is

$$p^{\chi(\pm)} = \exp\left(\mp\frac{1}{2}\left(\theta_1 + \theta_2 + \theta_3 - z_1 - z_2\right)\right).$$

We have to consider (up to const.)

$$K^{\chi(+)}_{\bar{1}11}(\underline{\theta}) = \int_{\mathcal{C}_{\underline{\theta}}} dz_1 \int_{\mathcal{C}_{\underline{\theta}}} dz_2 \prod_{i=1}^{3}\left(\tilde{\phi}(\theta_i - z_1)\tilde{\phi}(\theta_i - z_2)\right) \frac{1}{\tilde{\phi}(z_{12})\tilde{\phi}(-z_{12})} p^{\chi(+)}(\underline{z})\,\tilde{\Psi}_{\bar{1}11}(\underline{\theta}, \underline{z}).$$

This formula is similar as (C.3) for $k = 0$ (which correspond to the operator $\bar{\psi}\psi$), only we have here to add the factor $\left(\tilde{\phi}(\theta_3 - z_1)\tilde{\phi}(\theta_3 - z_2)p^{\chi(\pm)}(\underline{\theta}, \underline{z})\right)$. Therefore we get using (C.11) for small ν (up to constants)

$$\begin{aligned}
K^{\chi(\pm)}_{\bar{1}11}(\underline{\theta}) &= K(\theta_{12}, 0)\frac{\exp\left(\mp\frac{1}{2}\theta_3\right)}{\sinh\frac{1}{2}\theta_{13}\sinh\frac{1}{2}\theta_{23}} + (2 \leftrightarrow 3) \\
&= \frac{\cosh\frac{1}{2}\theta_{12}}{(\theta_{12} - i\pi)\sinh\frac{1}{2}\theta_{12}} \frac{\exp\left(\mp\frac{1}{2}\theta_3\right)}{\sinh\frac{1}{2}\theta_{13}\sinh\frac{1}{2}\theta_{23}} + (2 \leftrightarrow 3) + O(\nu) \\
&= \frac{1}{\theta_{12} - i\pi}\coth\frac{1}{2}\theta_{12}\frac{\exp\left(\mp\frac{1}{2}\theta_3\right)}{\sinh\frac{1}{2}\theta_{13}\sinh\frac{1}{2}\theta_{23}} + (2 \leftrightarrow 3) + O(\nu) \\
F^{\chi}_{\bar{1}11}(\underline{\theta}) &= \frac{\cosh\frac{1}{2}\theta_{12}}{\theta_{12} - i\pi} u(\theta_3) - \frac{\cosh\frac{1}{2}\theta_{13}}{\theta_{13} - i\pi} u(\theta_2) + O(\nu)
\end{aligned}$$

which is (E.1) up to a constant. The normalization is obtained by the form factor equation (iii)

$$\begin{aligned}
\operatorname*{Res}_{\theta_{12}=i\pi} F^{\psi}_{\bar{1}11}(\underline{\theta}) &= 2i\left(1 - a(\theta_{23})\right) F^{\psi}_{1}(\theta_3) \\
&= \frac{4\pi}{N}\left(\frac{1}{\sinh\theta_{23}} - \frac{1}{\theta_{23}}\right) u(\theta_3) + O(N^{-2})
\end{aligned}$$

where

$$F^{\psi}_{\alpha\beta\gamma}(\underline{\theta}) = \frac{i(\gamma\,(p_1 + p_2 + p_3) + m)}{8m^2\cosh\frac{1}{2}\theta_{12}\cosh\frac{1}{2}\theta_{13}\cosh\frac{1}{2}\theta_{23}} F^{\psi\chi}_{\alpha\beta\gamma}(\underline{\theta})\,.$$

It has been used that

$$\begin{aligned}
K(\theta,0) &= -2i\pi\frac{\cosh\frac{1}{2}z}{(z-i\pi)\sinh\frac{1}{2}z}+O(\nu)\\
\tilde{\phi}(\theta) &= \frac{-i\pi}{\sinh\frac{1}{2}\theta}+O(\nu)\\
F(\theta) &= -i\sinh\frac{1}{2}\theta+O(\nu)\\
a(\theta) &= 1+\nu i\pi\left(\frac{1}{\sinh\theta}-\frac{1}{\theta}\right)+O\left(\nu^2\right).
\end{aligned}$$

■

E.2 1/N perturbation theory

Introducing the auxiliary field $\sigma(x)$ the Lagrangian (1.1) may be written as

$$\mathcal{L}^{\mathrm{GN}}=\bar{\psi}(i\gamma\partial-\sigma)\psi-\frac{1}{2g^2}\sigma^2$$

and the Green's functions in $1/N$ expansions are obtained from the expansion of

$$\begin{aligned}
Z(\xi,\bar{\xi}) &= \int d\sigma\,\exp\left(i\mathcal{A}_{\mathrm{eff}}(\sigma)-\bar{\xi}S\xi\right)\\
\mathcal{A}_{\mathrm{eff}}(\sigma) &= -i\frac{1}{2}N\ \mathrm{tr}\ \ln(i\gamma\partial-\sigma)-\int d^2x\,\frac{1}{2g^2}\sigma^2
\end{aligned}$$

with the σ propagator [5, 6]

$$\tilde{\Delta}_\sigma(k)=\left(\frac{1}{2}N\int\frac{d^2p}{(2\pi)^2}\,\mathrm{tr}\left(\frac{1}{\gamma p-m}\left(\frac{1}{\gamma(p+k)-m}-\frac{1}{m}\right)\right)\right)^{-1}=-\frac{4\pi i}{N}\,\frac{\tanh\frac{1}{2}\phi}{\phi}$$

where $k^2=-4m^2\sinh^2\frac{1}{2}\phi$. This propagator together with the simple vertex of figure 5 yield the Feynman rules which allow to calculate general vertex functions in the $1/N$-expansion. For example the four point vertex function is

$$\tilde{\Gamma}^{(4)DC\delta\gamma}_{AB\alpha\beta}(-p_3,-p_4,p_1,p_2)=\delta^\delta_\alpha\delta^\gamma_\beta\,G^{DC}_{AB}(p_2-p_3)-\delta^\gamma_\alpha\delta^\delta_\beta\,G^{CD}_{AB}(p_3-p_1) \tag{E.2}$$

where A,B,C,D are spinor indices, $\alpha\beta\gamma\delta$ are isospin indices and G is given by the Feynman graph of figure 6. Taking into account the contributions from the propagator we obtain

$$G(k)=-1\otimes1\,\tilde{\Delta}_\sigma(k)=\frac{4\pi i}{N}1\otimes1\,\frac{\tanh\frac{1}{2}\phi}{\phi}\,. \tag{E.3}$$

where the tensor product structure of the spinor matrices is obvious from figure 6.

3-particle form factor of the fundamental fermi field. We now calculate the three particle form factor of the fundamental fermi field in $1/N$-expansion in lowest nontrivial order. For convenience we multiply the field with the Dirac operator

$$\chi^{\delta D}(x)=i(-i\gamma\partial+m)^D_{D'}\psi^{\delta D'}(x)$$

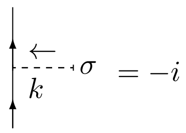


Figure 5. The elementary vertex for the $O(N)$ Gross-Neveu model. With respect to isospin the vertex is proportional to the unit matrix.

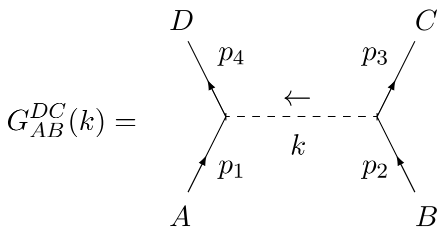


Figure 6. The four point vertex.

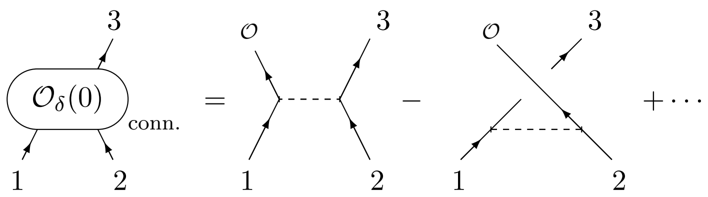


Figure 7. The connected part of the three particle form factor of the fundamental fermi field in $1/N$-expansion.

and define

$$ {}^{\gamma}_{\text{out}}\langle\, p_3 \,|\, \chi^{\delta D}(0) \,|\, \theta_1, \theta_2 \,\rangle^{\text{in}}_{\alpha\beta} = F^{\eta^{\delta D}}{}^{\gamma}_{\alpha\beta}(\theta_3; \theta_1, \theta_2)\,. $$

By means of LSZ-techniques one can express the connected part in terms of the 4-point vertex function (E.2) in lowest order given by the Feynman graphs of figure 7

$$ F^{\chi^{\delta D}\,\gamma}_{\text{conn}\alpha\beta}(\theta_3; \theta_1, \theta_2) = \bar{u}_C(p_3) \left\{ \delta_{\alpha\delta}\delta_{\beta\gamma}\, G^{DC}_{AB}(p_2 - p_3) - \delta_{\alpha\gamma}\delta_{\beta\delta}\, G^{CD}_{AB}(p_3 - p_1) \right\} u^A(p_1) u^B(p_2) \tag{E.4} $$

where G is given by figure 6 and eq. (E.3) and the spinors by $u_\pm(p) = \sqrt{m}e^{\mp\theta/2}$. It turns out that for p_1, p_2 and p_3 on-shell several terms vanish or cancel and we obtain up to order $1/N$ using $\bar{u}(\theta_1)u(\theta_2) = 2m \cosh \frac{1}{2}\theta_{12}$

$$ F^{\chi^{\delta D}\;\gamma}_{\text{conn.}\alpha\beta} = \frac{i\pi}{N}\, 8m \left\{ \delta^{\delta}_{\alpha}\delta^{\gamma}_{\beta} \frac{\sinh \frac{1}{2}\theta_{23}}{\theta_{23}}\, u^D(p_1) - \delta^{\gamma}_{\alpha}\delta^{\delta}_{\beta} \frac{\sinh \frac{1}{2}\theta_{13}}{\theta_{13}}\, u^D(p_2) \right\}. \tag{E.5} $$

By crossing ($\theta_3 \to \theta_3 + i\pi$) this gives $F^{\chi^\delta}_{\alpha\beta\bar{\gamma}}$ and agrees with the $1/N$ expansion of the exact form factor (5.11).

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JHEP02(2016)042