The allocation of authority under limited liability

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Abstract

Authority is modelled as the right to undertake a non-contractible decision in a joint project. We show that the allocation of authority depends on bargaining power and differences in both parties cost functions. The decision-maker is assumed to exert an externality on the other parties. Overall surplus is shared according to generalized Nash bargaining. Under limited liability, the agent with the larger cost parameter receives authority if the agents' cost parameters are very different. If the agents have similar cost parameters, bargaining power determines the allocation of authority. Possible applications include the introduction of a new product.

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1   Introduction

This paper analyzes the allocation of authority in a joint project. Authority is viewed as the right to undertake a project-oriented decision. This decision affects not only the decision-maker but also some other parties involved. If the decision itself is non-contractible, limited liability creates distortion because of a trade off between surplus maximization and rent extraction. How this trade off is solved depends on two factors. First, the differences between the agents' cost functions are decisive. Second, the relationship between an agent's costs and her bargaining power is also influential. Our setup allows to analyze the impact of limited liability, which is the only source of distortion in our model.

In detail, the project-oriented decision determines all parties' costs as well as the project's success probability and therefore its expected outcome. The success of the project is assumed to be observable so that the contract between the parties can specify transfer payments conditional on success and failure. If the project-oriented decision is contractible, the first-best outcome which maximizes overall surplus is implemented. In case of a non-contractible decision, the right to undertake the decision is contractually assigned to the decision-maker. The decision-maker is considered to have full authority. As long as transfers are unrestricted, the first-best is reached even though the decision itself is non-contractible. Transfer payments are used to incentivize the decision-maker who can be compensated for a first-best efficient decision.

Since payments condition on project outcome, they are carried out ex post after the project is done. As long as the transfers simply lead to a sharing of the realized project output, this is unproblematic. But in some settings, a net payment ex post might not be enforceable. For example, wealth constraints might prevent a party from carrying out such payments. Since this problem is anticipated by all parties, it seems reasonable to restrict the set of possible transfers to those which simply implement a sharing of the realized output. That is, limited liability is assumed. Due to limited liability, the decision-maker can no longer be compensated for a surplus-maximizing decision. She chooses opportunistically a project which is in her own interest and cannot be incentivized to choose the first-best project. In general, a project different from first-best is implemented. If authority is allocated exogenously to the agent with the larger costs, the surplus is larger compared to the alternative allocation of authority. But if the agents bargain over authority, they do not necessarily end up with this allocation. We show that bargaining instead allocates authority to the one with the smaller costs if and only if the parties

are not too different concerning their cost functions and the party with the smaller costs has a large bargaining power. It turns out that this party's decision leads to a smaller overall surplus compared to the alternative allocation, but authority enables her to extract a larger share of the rent. This is shown to be beneficial to her if the parties are not too different in costs (ensuring that the decrease in surplus is not too large) and her bargaining power is large (ensuring that the increase in rent share is large). Due to her large bargaining power, this party is able to receive the desired authority. If the cost structures of the parties are quite different, bargaining power does not influence the allocation of authority and the one with the larger costs decides.

Whenever several persons or institutions undertake a joint project, decisions which might influence all involved parties have to be made. Who should and who will make such a decision in a world of incomplete contracts? This question arises in very different applications. For example, consider two firms or two departments within one firm working on a new product. While one firm resp. department designs the product, the other one is working on a marketing strategy. Decisions about the quality of the product or the included features have an impact on both parties. A high-quality or complex product is harder to develop and more difficult to explain to the customers. On the other hand, such a quality-decision also influences the expected sales. As another example, take two firms forming a research joint venture or two departments of a firm working on a joint research project. The two parties have to decide what kind of scientific experiment to carry out. If they conduct a complex experiment, their workload and therefore their costs increase while the expected outcome is influenced as well. They are able to find more relevant results than within a simpler framework. Similar ideas apply to co-authorship. All these examples have in common that a certain project-oriented decision influences the costs of all parties as well as the expected outcome independently of who makes the decision.

If the project-oriented decision is contractible, the involved parties may specify it in a contract. But in many situations, contracts are necessarily incomplete. A variable might be non-contractible because it is unobservable to a third party or unverifiable in court. Even if all contractual partners observe this variable, they cannot enforce the contract in a trial. In addition, it might be too costly to specify everything in detail. Instead of specifying the project in a contract, one can determine who has the right to make the remaining decisions. This concept of the allocation of authority is used in Grossman and Hart (1986), Aghion and Bolton (1992), or Bester (2005).

The property rights approach according to Grossman and Hart (1986) usually assumes a decision that is not describable at the contracting stage but verifiable at the bargaining stage, while the action in Aghion and Bolton (1992) and Bester (2005) remains unverifiable ex post. We follow the latter approach and assume that the project-oriented decision remains unverifiable ex post. Only project output can be contracted upon. The complexity of a scientific experiment or the detailed quality properties of a new product are too hard to identify for a third party, especially a court. Aghion and Rey (2003) analyze contracts which simply allocate authority and show that their optimality is often robust to the introduction of message games under ex post non-verifiability, which justifies the restriction to this class of models in our paper.

The enforcement of an allocation of authority is not specified in our model explicitly. Grossman and Hart (1986) as well as Hart and Moore (1990) use asset ownership to enforce authority. The owner of an asset has residual rights of control, that is the right to undertake any decision not determined in the contract. Rajan and Zingales (1998) use access to assets instead of ownership to allocate authority. A specific privileged access to a resource is more restrictive than residual rights of control. Our model fits into the framework of residual control rights as well as privileged access to assets. Since there is only one decision to be undertaken, it makes no difference if the decision-maker has residual rights of control or just access to an asset which enables her to determine this single decision. Baker, Gibbons, and Murphy (1999) constitute informal authority via repeated interaction, which suits our model as well. Many partnerships continue over several periods. For example, two departments of a firm working together probably expect that there will be further joint projects in the future.

All parties incur costs from undertaking the project. These costs might be disutility from work, caused from effort to spend. But the decision analyzed here is not an effort choice in the usual sense. An effort choice as modeled in Aghion and Tirole (1997) and other papers influences the decision-makers costs only. In the applications considered here, the decision determines the costs of all parties involved. It can be considered as an externality the decision-maker exerts on the other parties. This is a key feature of our model which distinguishes it from the literature on effort choice: It is not only the decision-maker whose costs are affected by the decision. Bester (2005) also analyzes a model of externalities and the allocation of authority in a firm, but deals with asymmetric information. Aghion and Tirole (1997) describe

how authority influences the agent's incentive to acquire information and therefore focus on the impact of delegation on the information structure. In contrast, Dessein (2002) and Bester (2005) take the information structure as given and analyze the revelation of the given information, which might be viewed as communication. Bester (2005) supports truthful revelation of the agent's private information through the possibility of trading authority. In Dessein (2002), the principal chooses either to delegate the decision rights to the better informed agent or to keep authority and communicate with the agent about his private information. While delegation causes a loss of control, keeping authority causes a loss of information because communication is noisy. In our model, the information structure is given as well, but in contrast to Dessein (2002) and Bester (2005), information is completely symmetric.

Most of the mentioned papers model the allocation of authority to be the job of a principal who makes a take-it-or-leave-it offer to an agent. Instead of a principal-agent model, we consider the broader approach of two agents who share the expected overall surplus according to generalized Nash bargaining. In many applications, it is reasonable to assume some differences in bargaining power whereas a principal-agent model seems to constitute too much asymmetry between the parties. In, for example, Grossman and Hart (1986) as well as Aghion and Rey (2003), bargaining takes place ex post so that the contract, esp. the allocation of authority, determines the starting point of the bargaining procedure. In difference, our model uses ex ante bargaining at the contracting stage. Therefore, the terms of contract and especially the allocation of authority are not the starting point, but the result of the bargaining.

If the parties involved are wealth-constraint or a party cannot be forced contractually to pay something ex post, it seems reasonable to assume limited liability. Imposing limited liability constraints creates a trade-off between rent extraction and surplus maximization which leads to an overall surplus lower than first-best in our model. It is no longer possible to compensate the decision-maker for a surplus-maximizing decision. While Aghion and Bolton (1992) and Aghion and Rey (2003) analyze parties with different wealth, our model assumes both parties' liability to be limited completely. This is a reasonable assumption if it is impossible to enforce ex post payments. In Aghion and Rey (2003), ex post efficiency is increased if (under wealth constraints) authority is allocated to the party with the lower bargaining power. But in our model, it does not depend on bargaining power which

allocation creates a larger surplus. It always results in a larger surplus to let the party with the larger cost parameter decide. But bargaining power might influence the allocation of authority because the allocation is a bargaining outcome. If the agents are very different in cost parameters, bargaining power does not influence the allocation of authority. Both agents benefit from the larger surplus created by the decision of the agent with the larger cost parameter. Surplus maximization is favored over rent extraction. But if the agents are similar in cost parameters, the allocation of authority hardly influences overall surplus and rent extraction becomes more important. The allocation of authority now depends on bargaining power.

The rest of the paper is structured as follows: Section 2 describes a formal model of the allocation of authority in a joint project. The benchmark case of contractible project choice is analyzed in section 3. Section 4 studies non-contractible project choice, while in section 5 the allocation of authority under limited liability is examined. A brief summary of the results and conclusions as well as some open research questions are given in section 6.

2   The Model

This section describes a formal model of the allocation of authority under limited liability. The timing of the contract game is as follows: In the initial stage, the agents bargain and sign the contract. If project choice is contractible, it is determined in the contract. In case of non-contractible project choice, the contract specifies the decision-maker, who chooses one of the possible projects. The project is undertaken, private costs occur and project output is realized. The payment scheme is executed. The details are given in the remaining section. To illustrate the assumptions, we consider the example of two firms working together on a new product.

Two agents $i = 1, 2$ jointly undertake a project. They negotiate a contract in order to specify the terms of their partnership. The agents could be, for example, a scientist developing a new product and an advertising director creating the marketing strategy. We elaborate an example below.

There is a set of possible projects $\mathcal{D} = [0, 1]$. Every project can either succeed or fail. The project output is a random variable which can take two values: In case of success, the project generates a positive output $X > 0$. Instead, if the project fails, no output is generated and the random variable takes the value zero. Once the project has been undertaken, the realized

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output is observable and therefore contractible. The success probability is given by the project characteristic $d \in \mathcal{D}$ which could describe, for example, the size of the project or the level of complexity. Depending on the interpretation of d, the meaning of this structure is that a large project is more likely to succeed than a small one or that a project is more likely to succeed if it is undertaken with a high intensity level. For example, a new product is more likely to succeed on the market if it has a high quality.

If project d is undertaken, each agent incurs private costs. Agent i's cost function is $c_i d^2$ with $c_i > 0$. Without loss of generality agent 1 is assumed to have a smaller cost parameter than agent 2, that is $c_1 \leq c_2$. To avoid rather uninteresting corner solutions, we assume c_1 and c_2 to fulfill some further restrictions. A sufficient condition for all relevant projects to be in $\mathcal{D} = [0, 1]$ is $2c_1 + c_2 > X$. This condition can be relaxed in several situations. The costs may be non-monetary costs like disutility from effort to spent on the project. A complex or large project might force both agents to work harder and increase their private costs. Even though the costs might describe disutility from work, the action d should not be interpreted as an effort choice[1] or a task assignment[2], since in our model the choice of d influences the costs of *all* agents, not only the decision-maker's costs. For example, when d represents the quality of a new product, a higher d implies more difficult and therefore more costly work for the product designer as well as for the marketing specialist - the latter has to explain a more complex product to the customers. A complex software is harder to develop and its features are harder to communicate to the customers. The level of complexity influences the private costs of both the software developer and the marketing specialist.

The contract between the agents specifies a payment scheme (w_h, w_l) carried out conditional on the verifiable project output. If the project succeeds, agent 1 receives the output X and compensates agent 2 with a transfer payment w_h (which might exceed the output). In case of project failure, there is a transfer w_l from agent 1 to agent 2. If a transfer is negative, it is in fact a payment from agent 2 to agent 1. We assume a budget balance condition in the sense that the whole output is given to the agents and there is no possibility to threaten to throw away part of the output instead. Nevertheless,

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[1]An effort choice typically influences only the decision-maker's costs but not the other party's costs (e.g. Aghion and Tirole (1997)).

[2]The assignment of tasks as in Holmstrom and Milgrom (1991) differs from our allocation of authority since only the agent responsible for a task incurs the costs of this task.

we do not model a team problem since the agents do not coordinate their actions.

The agents are risk-neutral and their payoffs are composed of their expected benefits and their private costs. With probability d, the project succeeds and agent 1 receives $X - w_h$, while with probability $1 - d$, she gets $-w_l$. Subtracting the private costs $c_1 d^2$ results in the payoff function U_1. Analogously, agent 2's payoff U_2 is derived. Rearranging the terms gives rise to a slightly different interpretation which considers w_l as a transfer independent of success while $w_h - w_l$ is in addition carried out in case of success only. The payoff functions turn out to be

$$
\begin{aligned}
U_1(w_h, w_l, d) &= d(X - w_h) + (1 - d)(-w_l) - c_1 d^2 \\
&= d(X - w_h + w_l) - w_l - c_1 d^2 \\
U_2(w_h, w_l, d) &= dw_h + (1 - d)w_l - c_2 d^2 \\
&= d(w_h - w_l) + w_l - c_2 d^2 \quad .
\end{aligned}
\tag{1}
$$

Each agent's outside option gives a zero payoff. The overall expected surplus $U_1 + U_2$ is independent of the payment scheme and a function of the project d only. A project d is called (first-best) efficient if and only if it maximizes $U_1 + U_2$.

Project d is not necessarily contractible. In our example, the quality of the new product might be determined by too many details to specify it in a contract. Note that non-contractible project choice implies non-contractible private costs since there is an one-to-one relationship between costs and project characteristic. While the project d itself is non-contractible, the right to choose a project is assumed to be contractible. This decision right is called authority. Instead of specifying a certain project, the contract specifies the allocation of authority denoted by r. If $r = 1$, agent 1 receives authority over the project choice, $r = 2$ gives the decision right to agent 2. The agent who receives authority is called the decision-maker. After the contract has been signed, the decision-maker chooses the project to be undertaken, i.e. the project characteristic d.

The agents share the available expected surplus $U_1 + U_2$ according to the generalized Nash bargaining solution.[3] Let $\alpha \in [0, 1]$ indicate agent 1's ex-

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[3]Nash (1950) introduced this concept for the case of equal bargaining power. It can easily be generalized by dropping this symmetry assumption in order to allow for agents with different bargaining power.

ogenously given bargaining power and $(1-\alpha)$ the bargaining power of agent 2. The agents sign a contract which maximizes

$$B(w_h, w_l, d) = U_1^\alpha(w_h, w_l, d)\, U_2^{1-\alpha}(w_h, w_l, d), \qquad (2)$$

subject to $d \in \mathcal{D}$ and the individual rationality constraints (participation constraints) $U_1, U_2 \geq 0$. Such a contract is called optimal. In case of $\alpha = 0$ (resp. $\alpha = 1$), the generalized Nash bargaining results in a principal-agent-model with agent 2 (resp. agent 1) as a principal who makes a take-it-or-leave-it offer.

If project d is contractible, a contract specifies a payment scheme and a project so that the maximization of the bargaining function B is over (w_h, w_l, d). If d is non-contractible, a contract consists of a payment scheme and the allocation of authority r so that maximization is over (w_h, w_l, r). The decision-maker will choose a project which maximizes her own payoff. Both agents anticipate this behavior during the negotiation. Thus, the optimization of B is restricted by the incentive constraint $d \in \mathrm{argmax}_{d' \in \mathcal{D}}\, U_r(w_h, w_l, d')$. If an optimal contract specifies an allocation of authority, this allocation is called optimal.

As long as the set of possible payment schemes is unrestricted, the following situation might arise: If $w_l \neq 0$ or $w_h \notin [0, X]$, one agent might have to pay more than she receives after the realization of output. This is not possible if the agent is wealth-constraint. Even with wealthy agents, such ex post payments might not be enforceable. Since both agents anticipate this situation, they restrict the possible payment schemes by the limited liability constraints $w_l = 0$ and $0 \leq w_h \leq X$. If these constraints are fulfilled, there is no incentive to walk away after the project is done.[4] Both agents stick to the contract in order to receive their predetermined share of project output. Whenever it is impossible or too costly to enforce a contract which requires an agent to accept a loss after the realization of output, it seems reasonable to impose the limited liability constraints (e.g. a non-slavery condition).

The following sections compare contractible versus non-contractible project choice as well as limited versus unlimited liability. While the other three cases serve as benchmarks, the main focus of the paper is on non-contractible project choice under limited liability.

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[4]We assume implicitly here that agent 1, if walking away, cannot take the output X with her. This simply means that a stipulated sharing of output can always be enforced, even if additional payments cannot.

3 Contractible Project Choice

This section studies the benchmark case of contractible project choice. A contract consists of a payment scheme (w_h, w_l) and a project $d \in \mathcal{D}$. Lemma 1 characterizes the first-best efficient solution which maximizes overall expected surplus $U_1 + U_2$. As Proposition 1 shows, the first-best efficient project is implemented even if limited liability constraints are required.

Lemma 1 (First-Best Project) *When the project characteristic d is contractible, the overall surplus $U_1 + U_2$ is maximized if and only if the project*

$$d_e := \frac{X}{2(c_1 + c_2)} \in \mathcal{D} \tag{3}$$

is implemented. The resulting surplus is

$$U_1 + U_2 = \frac{X^2}{4(c_1 + c_2)} > 0 \quad . \tag{4}$$

Proof: see Appendix A.

For any given project d, the overall surplus is independent of the payment scheme. Accordingly, the surplus maximizing project d_e is also independent of the payment scheme. The larger the possible project output X and the smaller the cost parameters c_1 and c_2, the larger is the first-best efficient project as well as the maximum reachable surplus. The trade-off between maximizing success probability and minimizing costs is solved in favor of a high success probability if success pays a lot and costs do not increase too fast.

As described in section 2, a contract is optimal (that is, a possible bargaining outcome) if and only if it solves

$$\max_{w_h, w_l, d} U_1^\alpha(w_h, w_l, d) \, U_2^{1-\alpha}(w_h, w_l, d) \quad , \tag{5}$$

subject to $d \in \mathcal{D}$ and the participation constraints $U_1, U_2 \geq 0$. Under limited liability, the additional constraints $w_l = 0$ and $0 \leq w_h \leq X$ are imposed. Proposition 1 states the results of the maximization.

Proposition 1 (Contractible Project Choice) *Suppose project choice is contractible. Then each optimal contract implements the first-best efficient project d_e. The agents share the resulting surplus according to their bargaining power, that is $U_1 = \alpha(U_1 + U_2)$ and $U_2 = (1-\alpha)(U_1 + U_2)$. These results still hold true if limited liability constraints are imposed.*
Proof: see Appendix A.

An explicit construction of the optimal contracts can be found in Appendix A. If a project choice is given, there are infinitely many payment schemes which implement a desired distribution of overall surplus. An optimal contract implements project d_e which maximizes the surplus and results in a distribution according to bargaining power. This distribution can be implemented via every payment scheme fulfilling $(1-\alpha)(U_1 + U_2)(d_e, w_h, w_l) = U_2(d_e, w_h, w_l)$, that is fulfilling the linear equation

$$d_e w_h + (1 - d_e)w_l - c_2 d_e^2 = (1 - \alpha)\frac{X^2}{4(c_1 + c_2)} \quad . \tag{6}$$

The payment scheme does not influence the expected overall surplus, but determines its distribution.

Intuitively, the contract has the three instruments w_h, w_l and d to control only two problems, surplus and distribution. One of the variables can be chosen freely, as can be seen from (6). Therefore, we have infinitely many optimal contracts (that is, infinitely many bargaining solutions) under unlimited liability. Under limited liability, w_l is fixed to be zero so that the number of effective instruments is reduced to two. Both optimizations can still be done independently so that the first-best solution is still implemented. But there is only one payment scheme fulfilling (6) and the limited liability constraints simultaneously, leading to a unique bargaining outcome. This unique optimal contract implements a payment scheme with $w_l = 0$ and

$$w_h = X/2 \left(\frac{c_2}{c_1 + c_2} + 1 - \alpha \right) \quad . \tag{7}$$

If the project succeeds, the agents share half of the project output X according to bargaining power, the other half according to cost share. E.g. agent 2 who has bargaining power $1 - \alpha$ and cost share $c_2/(c_1 + c_2)$ receives w_h given in (7). Overall expected surplus is shared according to bargaining power.

In summary, if project choice is contractible, there are no distortions even if limited liability is required. There are at least as many instruments available as problems to solve. But under limited liability, the multiplicity of optimal contracts boils down to a unique solution.

4 Non-Contractible Project Choice

In what follows, the project choice is non-contractible and only the allocation of authority can be contracted upon. Throughout this section, assume unlimited liability so that $w_l, w_h \in \mathbb{R}$. We show that the first-best efficient project is still implemented and the allocation of authority, even if exogenously fixed, does not have any impact.

A contract consists of a payment scheme (w_h, w_l) and an allocation of authority $r \in \{1, 2\}$. Agent r is the decision-maker who receives the right to choose the project $d \in \mathcal{D}$ once the contract is signed. At the contracting stage, both agents anticipate that the decision-maker will choose a project which maximizes her own payoff. This adds an incentive constraint to the maximization problem. An optimal contract solves

$$\max_{w_h, w_l, r} U_1^\alpha(w_h, w_l, d)\, U_2^{1-\alpha}(w_h, w_l, d) \tag{8}$$

subject to $U_1, U_2 \geq 0$ and

$$d \in \operatorname{argmax}_{d' \in \mathcal{D}} U_r(w_h, w_l, d') \quad . \tag{9}$$

The future decision-making of the decision-maker is reflected by (9). Proposition 2 shows the results.

Proposition 2 (Non-Contractible Project Choice) *Suppose   project choice is non-contractible. There are exactly two optimal contracts, one gives the authority to agent 1 while the other one gives it to agent 2. Both optimal contracts implement the first-best efficient project d_e and the agents share the resulting surplus according to their bargaining power, that is $U_1 = \alpha(U_1 + U_2)$ and $U_2 = (1 - \alpha)(U_1 + U_2)$.*
Proof: see Appendix A.

If agent 1 is the decision-maker, her incentive constraint is

$$d = \frac{X - w_h + w_l}{2c_1} \quad , \tag{10}$$

while it is

$$d = \frac{w_h - w_l}{2c_2} \tag{11}$$

if agent 2 has the decision right. Therefore, the project choice is determined by the decision-maker's cost parameter together with $w_h - w_l$. Given an allocation of authority and $w_h - w_l$, the size of overall expected surplus is fixed. The distribution of overall expected surplus still varies with w_l. While the appropriate $w_h - w_l$ ensures the implementation of the first-best efficient project, the appropriate w_l enforces a distribution of overall surplus according to bargaining power. A contract has the two instruments w_h, w_l to determine surplus and distribution. Therefore, surplus and distribution may be optimized independently and the first-best solution is reached. The first-best payment scheme is independent of the allocation of authority since maximizing one agent's payoff and overall surplus implies maximizing the other agent's payoff. Since efficiency is attained, the optimal payment scheme is necessarily one of the optimal payment schemes in case of contractible project choice given in (6). Even if the decision right is exogenously fixed, the first-best efficient project is implemented and payoffs are independent of the allocation of authority.

To compare these results with the case of limited liability in section 5, recall that we might consider w_l as a transfer independent of success and $w_h - w_l$ as an additional transfer carried out in case of success only. Therefore, $w_h - w_l$ and $X - w_h + w_l$ can be viewed as a bonus received if the project succeeds. In this case, each agent receives a share of the project output according to her share of overall costs: agent 1 earns $X - w_h + w_l = c_1/(c_1 + c_2) \, X$ and agent 2 gets $w_h - w_l = c_2/(c_1 + c_2) \, X$. This bonus is independent of bargaining power because it is the instrument to optimize overall surplus, that is, to implement the first-best efficient project. This project is independent of bargaining power as well.
On the other hand, the transfer

$$
\begin{aligned}
w_l &= \frac{X^2}{4(c_1 + c_2)} \left(\frac{c_1}{c_1 + c_2} - \alpha \right) \\
&= \frac{X^2}{4(c_1 + c_2)} \left(1 - \alpha - \frac{c_2}{c_1 + c_2} \right)
\end{aligned} \tag{12}
$$

depends crucially on the relationship between bargaining power and cost share. It is the instrument to implement the optimal distribution which is determined by bargaining power.

The agent who has a bargaining power larger than the cost share receives a payment. Such an agent might be called "powerful" since her bargaining power is large compared to her cost share. If $\alpha < c_1/(c_1 + c_2)$, it is $w_l > 0$ and agent 1 pays a transfer to agent 2 who is the powerful one in this situation. In case of $\alpha > c_1/(c_1 + c_2)$, agent 1 is the powerful agent and receives a payment since $w_l < 0$. The optimal payment scheme is one of the optimal payment schemes in case of contractible project choice under unlimited liability, but it is different from the optimal payment scheme in case of contractible project choice under limited liability. If project choice is non-contractible, the optimal contract fulfills the limited liability constraints if and only if $\alpha = c_1/(c_1 + c_2)$, that is, if bargaining power equals cost share.

In case of non-contractible project choice, there are no inefficiencies and the allocation of authority does not have any impact. There are as many instruments available as problems so solve. Payments (w_h, w_l) determine the surplus and its distribution. Bargaining power determines the optimal distribution of surplus, but does not play any further role. But for any bargaining power $\alpha \neq c_1/(c_1 + c_2)$, limited liability is not fulfilled and imposing limited liability is expected to create some distortions.

5 Non-Contractible Project Choice under Limited Liability

We now consider the case of non-contractible project choice under the limited liability constraints $w_l = 0$ and $0 \leq w_h \leq X$. The agents simply share the project output and do not perform any additional transfer payments. Imposing limited liability constraints usually creates a trade-off between surplus maximization and rent extraction. Under limited liability, it is no longer possible to maximize overall surplus and extract a rent share according to bargaining power at the same time. Subsection 5.1 analyzes the case of an exogenously given allocation of authority. We show that the first-best efficient project is no longer implemented as soon as bargaining power does not reflect the relative cost share. The results are used in subsection 5.2 to analyze the impact of bargaining power and cost structure on the allocation of authority. It turns out that large differences in cost parameters make the bargaining power less important since both agents prefer the allocation of authority which leads to a larger surplus compared to the alternative allocation. If the agents are similar in cost parameters, it is mainly the bargaining power which determines the allocation of authority.

5.1 Exogenous Allocation of Authority

With $w_l = 0$ the payoff functions become

$$
\begin{aligned}
U_1 &= d(X - w_h) - c_1 d^2 \quad , \\
U_2 &= d w_h - c_2 d^2 \quad .
\end{aligned}
\tag{13}
$$

If the allocation of authority is exogenously given to be $r = \bar{r}$, an optimal contract solves

$$
\max_{w_h} U_1^{\alpha}(w_h, d) \, U_2^{1-\alpha}(w_h, d)
\tag{14}
$$

subject to $U_1, U_2 \geq 0$, $0 \leq w_h \leq X$ and

$$
d \in \operatorname{argmax}_{d' \in \mathcal{D}} U_{\bar{r}}(w_h, d') \quad .
\tag{15}
$$

Lemma 2 (Agent 1 decides) *Consider the case of non-contractible project choice under limited liability. Let agent 1 be the decision-maker so that $r = 1$. The optimal contract given this allocation of authority implements the project*

$$
d_1 := \frac{(1+\alpha)X}{2(2c_1 + c_2)}
\tag{16}
$$

which is the first-best efficient project d_e if and only if $\alpha = c_1/(c_1 + c_2)$. It is

$$
d_1 \gtreqless d_e \iff \alpha \gtreqless c_1/(c_1 + c_2) \quad .
\tag{17}
$$

The payment in case of success is

$$
w_h = \frac{c_2 + (1-\alpha)c_1}{2c_1 + c_2} X \quad .
\tag{18}
$$

Proof: see Appendix A.

Solving $d_1 = d_e$ for α shows that in case of $r = 1$, the first-best efficient project d_e is implemented if and only if each agent's bargaining power equals her cost share so that $\alpha = c_1/(c_1 + c_2)$. If the decision-maker agent 1 is too powerful in terms of bargaining power, that is $\alpha > c_1/(c_1 + c_2)$, she implements an inefficiently large project $d_1 > d_e$. Since $X - w_h$ is increasing in α, a powerful agent 1 can extract a large share of the output in case of success. She benefits from a large project choice. Instead, if $\alpha < c_1/(c_1 + c_2)$, it does not pay to choose a large and, therefore, costly project since the extra rent

from success is small. A project smaller than first-best is implemented. The following Lemma states the respective results for agent 2 being the decision-maker, which is simply a mirror of Lemma 2 - replace $w_h \leftrightarrow X - w_h, \alpha \leftrightarrow 1 - \alpha, c_1 \leftrightarrow c_2$ and note that the agents changed their roles.

Lemma 3 (Agent 2 decides) *Consider the case of non-contractible project choice under limited liability. Let agent 2 be the decision-maker, that is $r = 2$. The optimal contract given this allocation of authority implements the project*

$$d_2 := \frac{(2 - \alpha)X}{2(c_1 + 2c_2)} \tag{19}$$

which is the first-best efficient project d_e if and only if $\alpha = c_1/(c_1 + c_2)$.
Proof: see Appendix A.

Exactly as in Lemma 2, the first-best efficient project d_e is implemented if and only if $\alpha = c_1/(c_1 + c_2)$. Again, a decision-maker who is too powerful, that is $1 - \alpha > c_2/(c_1 + c_2)$ resp. $\alpha < c_1/(c_1 + c_2)$, implements a project larger than first-best since she can extract a large share of output in case of success. If instead $\alpha > c_1/(c_1 + c_2)$, the decision-maker agent 2 implements a project smaller than first-best. As soon as $\alpha \neq c_1/(c_1 + c_2)$, the first-best efficient project is no longer implemented. Giving the decision right to the agent with bargaining power larger than cost share will lead to a project "too large" while giving it to the other agent will result in a project "too small" compared to the first-best efficient one.

Under unlimited liability, we have seen in section 4 that $w_h - w_l$ determines the project characteristic while w_l determines (given $w_h - w_l$) the distribution of surplus. Both the surplus and its distribution are optimized independently. But under limited liability, $w_l = 0$ is fixed and a contract has only one instrument to determine surplus and distribution. The wage w_h determines both the project choice (which fixes the surplus) and the distribution of surplus. Those cannot be optimized simultaneously since there is only one instrument to solve two problems. A trade off between surplus maximization and rent extraction (which is distribution) occurs. A w_h which compensates the decision-maker for a surplus-oriented decision-making - that is, a w_h leading to the implementation of the first-best project - would result in a distribution of surplus far from reflecting bargaining power. On the other hand, a w_h which implements a distribution according to bargaining power would result in a surplus much smaller than the first-best surplus.

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5.2 Allocation of Authority via Bargaining

If the allocation of authority is a result of the bargaining process, the trade off between rent extraction and surplus maximization concerns not only the payment scheme but also the allocation of authority. We start this subsection with the observation that the cost parameters determine differences in overall surplus for the two allocations. Proposition 3 analyzes the allocation of authority subject to bargaining power and cost parameters. It turns out that if the agents are similar in cost structure, bargaining power determines the allocation and rent extraction is favored over surplus maximization. But in case of very different cost parameters, the surplus is decisive and the allocation of authority is mainly influenced by cost structure. In this case even a powerful agent does not receive authority if this would lead to a decrease of overall surplus. As a benchmark, Lemma 4 describes the impact of authority on the overall surplus.

Lemma 4 (Allocation of Authority and Overall Surplus) *In case of non-contractible project choice under limited liability, overall surplus depends on the allocation of authority if and only if $\alpha \neq c_1/(c_1 + c_2)$. The distortion relative to first-best for the two allocations can be compared by*

$$
\begin{aligned}
|d_1 - d_e| &= d_e \frac{|\alpha(c_1 + c_2) - c_1|}{2c_1 + c_2} \\
|d_2 - d_e| &= d_e \frac{|\alpha(c_1 + c_2) - c_1|}{c_1 + 2c_2}
\end{aligned}
\tag{20}
$$

If the agent with the larger cost share receives the decision right, overall surplus is larger compared to the alternative allocation of authority. Since $c_1 \leq c_2$ by assumption, the surplus is larger if authority is allocated to agent 2.
Proof: see Appendix A.

Since overall expected surplus $U_1 + U_2$ is a symmetric function with its unique maximum in d_e, the distance $|d - d_e|$ is an appropriate measure for the loss of surplus (compared to first-best) occurring if project d is implemented. As already mentioned, the decision-maker's project choice is distorted away from the first-best efficient project because bargaining power does not reflect cost share. This is measured by $|\alpha(c_1 + c_2) - c_1|$ which is independent of the allocation of authority. If any other source of distortion is eliminated by assuming agents who differ only in bargaining power, that is assuming

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$c_1 = c_2$, both possible allocations of authority result in the same amount of distortion $|d_1 - d_e| = |d_2 - d_e|$. What one agent chooses "too much" is exactly what the other one chooses "too little" compared to first-best. Still, the size of the distortion depends on bargaining power.

If $c_1 \neq c_2$, there is an additional source of distortion measured by the terms $2c_1 + c_2$ resp. $c_1 + 2c_2$. When choosing the project, the decision-maker cares about her own costs, but does not fully internalize the externalities exerted on the other agent. That is, she puts too little weight to the other agent's costs. This is reflected by the denominators in (20), showing that the distortion is less drastic, if authority is given to the agent with the larger cost share. Therefore, the overall surplus is larger if agent 2 (who has the larger cost parameter) is the decision-maker.

But the bargaining might as well result in agent 1 being the decision-maker. Now consider the bargaining outcome. An optimal contract solves

$$\max_{w_h, r} U_1^\alpha(w_h, d) \, U_2^{1-\alpha}(w_h, d) \tag{21}$$

subject to $U_1, U_2 \geq 0$, $0 \leq w_h \leq X$ and

$$d \in \mathrm{argmax}_{d' \in \mathcal{D}} \, U_r(w_h, d') \quad . \tag{22}$$

To state the following Proposition, we define

$$
\begin{aligned}
\phi(k, \alpha) \quad := \quad & (2k+1)^{1+\alpha} \, (2-\alpha)^{2-\alpha} \, \alpha^\alpha \\
& - \quad (k+2)^{2-\alpha} k^\alpha \, (1+\alpha)^{1+\alpha} \, (1-\alpha)^{1-\alpha}
\end{aligned}
\tag{23}
$$

with $k := c_1/c_2$.

Proposition 3 (Allocation of Authority) *Consider the case of non-contractible project choice under limited liability. Generalized Nash bargaining results in the unique optimal contract if and only if $\phi(k, \alpha) \neq 0$. If $\phi(k, \alpha) < 0$, agent 1 receives authority, while in case of $\phi(k, \alpha) > 0$ it is allocated to agent 2. If $\phi(k, \alpha) = 0$, there are two optimal contracts which differ in the allocation of authority.*
Proof: see Appendix A.

Figure 1 shows a contour plot of ϕ with k on the horizontal axes and α on the vertical axes. The plot approximates the sets of (k, α) with $\phi(k, \alpha) > 0$ resp. $\phi(k, \alpha) < 0$. If the agent's cost parameters are similar so that k is large,
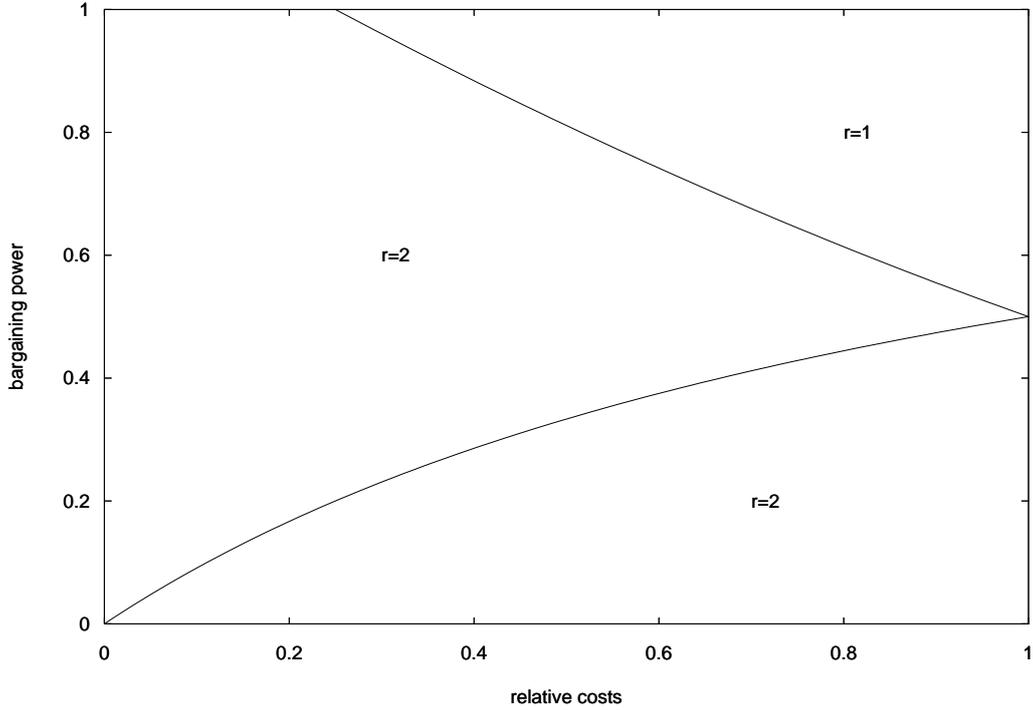
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Figure 1: Contour plot of $\phi(k, \alpha)$ describing the allocation of authority r

authority is allocated to agent 2 if her bargaining power $1 - \alpha$ is large. If agent 1's bargaining power α increases sufficiently, the allocation of authority switches towards agent 1. Instead, for small k the allocation is independent of the bargaining power and agent 2 decides. On the other hand, given a small α, the decision-maker is agent 2 independent of k. Given a large α and a small k, authority is given to agent 2. But a sufficient increase in k changes the allocation so that agent 1 receives the decision right. The upper right corner which is characterized by a large k and a large α describes the parameter constellations which result in agent 1 being the decision-maker. If agent 1's bargaining power is large and simultaneously her cost share nearly equals agent 2's cost share, she receives authority. The allocation of authority is not unique along the curve separating this area from the remaining parameter constellations. This curve describes a switch from $\phi(k, \alpha) > 0$ to $\phi(k, \alpha) < 0$, which is a switch in the allocation of authority. In addition, the allocation is not unique along the curve $\alpha = k/(k+1)$ which in fact is $\alpha = c_1/(c_1 + c_2)$. This curve does not describe a switch in the allocation, it is $\phi(k, \alpha) > 0$ above as well as below this line. Below this curve, the decision-maker agent 2 is powerful (having bargaining power larger than cost share) and chooses a project larger than the first-best project, while above this curve she is

still the decision-maker, but chooses a project smaller than first-best. On the curve $\alpha = c_1/(c_1 + c_2)$, the allocation is not unique but the first-best project is implemented. If α and k are large enough so that agent 1 decides, the decision-maker is again powerful and a project larger than first-best is implemented.

Under unlimited liability, each agent receives a fixed share of the surplus determined by her bargaining power. The agents' common goal is to maximize the surplus. Under limited liability, there is a trade off between surplus maximization and rent extraction. If the agents have similar cost shares, the allocation of authority does not have much influence on the surplus. Rent extraction becomes important. In the extreme $c_1 = c_2$, the overall expected surplus is even independent of the allocation of authority. For each agent, getting the decision right increases the payoff. The agent with the larger bargaining power receives authority. To understand the shape of the parameter constellation allocating authority to agent 1, start with $k = 1$ and consider a small decrease in k. Now the allocation of authority has two impacts: If agent 1 is the decision-maker, overall surplus is smaller compared to agent 2 being the decision-maker. But on the other hand, agent 1 can extract a larger share of overall surplus. If she receives authority, agent 1 ends up with a larger share of a smaller surplus. An increase in α increases the impact of the allocation on the share of the rent.[5] If agent 1 is the decision-maker, an increase in α decreases overall surplus but increases her share of the rent. The larger α, the more agent 1 benefits from having the decision-right. If α is above a certain threshold, agent 1 is better off if she receives authority instead of agent 2 being the decision-maker. In addition, the large α enables her to get the desired authority. If α and k are large enough, agent 1 receives authority. If k is quite small instead, even a very powerful agent 1 does not receive authority, but benefits from the large surplus realized by the decision of agent 2. Therefore, the cost structure plays a decisive role in allocating decision rights which - in some parameter constellations - even outweighs the influence of bargaining power.

6   Conclusion

In this paper, we have developed a simple model of the allocation of authority in a joint project. Generalized Nash bargaining allocates authority to the agent with the larger cost share as long as the other agent has a significantly

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[5]This can be seen from direct calculation of rent share.

smaller cost share or is not too powerful. This allocation results in a larger surplus compared to the alternative allocation. But a very powerful agent can receive the decision right even though she has the smaller cost share so that her decision-making results in a smaller surplus. This happens if the agents differ not too much in cost shares. In this case, the decision-maker benefits from authority by receiving a larger share of a smaller surplus compared to the alternative allocation of authority. If on the other hand the agents are very different in cost shares, even a very powerful agent does not receive authority. Both agents benefit from the larger surplus reached by allocating authority to the one with the larger cost share.

In our model, a decision undertaken by one of the agents has an impact on all agents involved so that an externality is exerted. As long as the decision is contractible, first-best efficiency is reached even if limited liability constraints are imposed. If the decision is non-contractible, the decision right is contractually assigned to one of the agents. Under unlimited liability, the first-best efficient project is still implemented. The allocation of authority is undetermined - both possible allocations are first-best efficient bargaining solutions. This is in line with the findings in Aghion and Bolton (1992) and Aghion and Rey (2003) for wealthy agents. Imposing limited liability constraints now creates a trade off between rent extraction and surplus maximization. The decision-maker cannot be compensated for choosing the first-best efficient project. The resulting loss compared to the first-best surplus is smaller if authority is allocated to the the agent with the larger cost share, independent of the agents' bargaining power. This result is different from Aghion and Rey (2003) whose model requires to allocate authority to the party with the lower bargaining power to increase efficiency. Their model considers ex post renegotiation while our model, in contrast, incorporates ex ante bargaining.

Bargaining power influences the surplus in two ways. First, for an exogenously given allocation of authority, bargaining power influences decision-making. The loss compared to the first-best surplus depends on how the agents' bargaining power reflects the cost structure. The closer the decision-maker's bargaining power is to her cost share, the smaller is the loss. Second, the distortion of the decision-maker's choice also depends on the relationship between the two agent's costs. The allocation of authority influences overall surplus. Because the allocation is a bargaining outcome, it clearly depends on bargaining power. Therefore, bargaining power influences who receives authority and how this party executes authority.

While ex ante participation constraints create inefficiencies in Aghion and Bolton (1992) as well as Aghion and Rey (2003), they play a minor role in our model, because participation constraints do not bind in any of the cases except for agents without any bargaining power.

Our results for the case of non-contractible project choice under limited liability base upon the fact that there is only one instrument to solve two problems, surplus maximization and distribution of surplus. Adding another instrument should solve the arising inefficiencies. Therefore, one might argue that our results rely on the binary character of the project outcome. To see that this is not the case, assume the project has three instead of two possible outcomes so that the payment scheme contains an additional payment carried out if the third outcome occurs. If the probabilities are further on assumed to be linear, the situation with three possible project outcomes can be reduced to a situation with only two possible outcomes[6], but possibly with a strict positive project outcome in case of failure. Allowing for a positive outcome in case of failure does not change the results substantially. Our model turns out to be robust to the number of possible project outcomes.

The approach has a wide variety of applications, ranging from two-person projects to institutional relationships. Research joint ventures or other cooperations inside or between firms might be modeled. The project, for example, could be the introduction of a new product. The product characteristic influences the costs of several departments or several firms involved as well as the probability of a successful launch. Possible extensions of the model include the division of tasks in multi-task projects, the introduction of effort incentives for the involved agents and third-party involvement with collusion. These are left for future research.

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[6]To see this, you simply have to rearrange the payoff functions.

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Appendix

A Proofs

Proof of Lemma 1:
The surplus $U_1 + U_2$ is a function of d independent of the payment scheme. Maximizing $U_1 + U_2$ by solving the first order condition yields the project d_e and the surplus $U_1 + U_2 = (U_1 + U_2)(d_e) = X^2/[4(c_1 + c_2)]$. Since $0 < X < c_1 + 2c_2 < 2(c_1 + c_2)$, we have $d_e \in [0,1] = \mathcal{D}$ and $U_1 + U_2 > 0$. ∎

Proof of Proposition 1:
Each optimal contract solves $\max_{w_h, w_l, d} B(w_h, w_l, d)$, that is

$$\max_{w_h, w_l, d} U_1^\alpha(w_h, w_l, d)\, U_2^{1-\alpha}(w_h, w_l, d) \quad , \tag{24}$$

subject to $d \in \mathcal{D}$ and the participation constraints $U_1, U_2 \geq 0$. The first order conditions (suppressing the arguments for notational purposes) are given by

$$\alpha U_1^{\alpha-1} U_2^{1-\alpha} \frac{\partial U_1}{\partial d} + (1-\alpha) U_1^\alpha U_2^{-\alpha} \frac{\partial U_2}{\partial d} = 0$$

$$\alpha U_1^{\alpha-1} U_2^{1-\alpha} \frac{\partial U_1}{\partial w_l} + (1-\alpha) U_1^\alpha U_2^{-\alpha} \frac{\partial U_2}{\partial w_l} = 0$$

$$\alpha U_1^{\alpha-1} U_2^{1-\alpha} \frac{\partial U_1}{\partial w_h} + (1-\alpha) U_1^\alpha U_2^{-\alpha} \frac{\partial U_2}{\partial w_h} = 0 \quad . \tag{25}$$

For the moment, assume there is at least one contract with $B > 0$ fulfilling the constraints so that each optimal contract satisfies $B > 0$ as well. One of the following three cases must apply: It is either $U_1, U_2 > 0$ or $U_1 > 0, U_2 = 0, \alpha = 1$ or $U_1 = 0, U_2 > 0, \alpha = 0$. The last two cases imply $B = U_1 + U_2$ and Lemma 1 gives $d = d_e \in \mathcal{D}$. Furthermore, it follows that $\alpha U_2 = (1-\alpha)U_1$ and therefore $U_1 = \alpha(U_1 + U_2)$ and $U_2 = (1-\alpha)(U_1 + U_2)$.

If $U_1, U_2 > 0$ holds, then $U_1^{\alpha-1}U_2^{-\alpha} > 0$ holds as well. The system (25) can therefore be rewritten as

$$
\begin{aligned}
\alpha U_2 \frac{\partial U_1}{\partial d} + (1-\alpha)U_1\frac{\partial U_2}{\partial d} &= 0 \\
\alpha U_2 \frac{\partial U_1}{\partial w_l} + (1-\alpha)U_1\frac{\partial U_2}{\partial w_l} &= 0 \\
\alpha U_2 \frac{\partial U_1}{\partial w_h} + (1-\alpha)U_1\frac{\partial U_2}{\partial w_h} &= 0 \quad .
\end{aligned}
\tag{26}
$$

Since $\partial U_1/\partial w_l = -\partial U_2/\partial w_l = -(1-d)$ and $\partial U_1/\partial w_h = -\partial U_2/\partial w_h = -d$, and the two expressions cannot be zero at the same time, the second and third equation are fulfilled if and only if $\alpha U_2 = (1-\alpha)U_1$. This yields the sharing rule $U_1 = \alpha(U_1 + U_2)$ resp. $U_2 = (1-\alpha)(U_1 + U_2)$ as stated in the Proposition.

From $U_1, U_2 > 0$ it follows that $\alpha U_2 = (1-\alpha)U_1 > 0$ and $\alpha \in (0,1)$. Plugging $\alpha U_2 = (1-\alpha)U_1$ into the first equation gives $\partial U_1/\partial d + \partial U_2/\partial d = 0$ which is the first order condition for maximizing $U_1 + U_2$ in d. Lemma 1 again implies $d = d_e \in \mathcal{D}$.

Given $d = d_e$, $\alpha U_2 = (1-\alpha)U_1$ results in

$$
w_h = X/2\left(\frac{c_2}{c_1 + c_2} + 1 - \alpha\right) + w_l\left(1 - \frac{2(c_1 + c_2)}{X}\right) \quad .
\tag{27}
$$

Since $\alpha U_2 = (1-\alpha)U_1$ leads to $U_1 = \alpha(U_1 + U_2) \geq 0$ with equality if and only if $\alpha = 0$ and $U_2 = (1-\alpha)(U_1 + U_2) \geq 0$ with equality if and only if $\alpha = 1$, the ad-hoc-assumption of $B > 0$ is justified. A contract is optimal if and only if $d = d_e$ and (27) are satisfied.

Now impose the limited liability constraints $w_l = 0$ and $0 \leq w_h \leq X$. The contract with $d = d_e$, $w_l = 0$ and

$$
w_h = X/2\left(\frac{c_2}{c_1 + c_2} + 1 - \alpha\right)
\tag{28}
$$

meets the limited liability constraints and is optimal (even without requiring limited liability) since the payment scheme fulfills (27). To see the uniqueness of the optimal contract under limited liability, note that any other contract either hurts the limited liability constraints or equation (27). In the latter case, the contract results in a smaller B and is not optimal. $\blacksquare$

Proof of Proposition 2:
A contract is optimal if and only if it solves

$$\max_{w_h, w_l, r} U_1^\alpha(w_h, w_l, d) \, U_2^{1-\alpha}(w_h, w_l, d) \tag{29}$$

subject to $U_1, U_2 \geq 0$ and

$$d \in \text{argmax}_{d' \in \mathcal{D}} \, U_r(w_h, w_l, d') \quad . \tag{30}$$

To find the optimal contract(s), calculate the optimal contract(s) given $r = 1$ as well as the optimal contract(s) given $r = 2$. Among these contracts, find the optimal one(s) by evaluating the bargaining function.

Let $r = 1$ so that agent 1 is the decision-maker. Once the contract is signed, she will choose project d^* maximizing U_1 over $\mathcal{D}$ given w_h, w_l, that is

$$d^* = \frac{X - w_h + w_l}{2c_1} \tag{31}$$

if this expression is an element of $\mathcal{D}$. Assume for the moment that $d^* \in \mathcal{D}$ and note that d^* is the unique maximum of $U_1(w_h, w_l, d)$. Both agents anticipate the later implementation of d^* during the bargaining process. Plugging in d^* gives

$$
\begin{aligned}
U_1 &= \frac{(X - w_h + w_l)^2}{4c_1} - w_l \quad , \\
U_2 &= \frac{X - w_h + w_l}{2c_1}(w_h - w_l) + w_l - \frac{c_2(X - w_h + w_l)^2}{4c_1^2} \quad .
\end{aligned}
\tag{32}
$$

The first order conditions for maximizing B are

$$
\begin{aligned}
\alpha U_1^{\alpha-1} U_2^{1-\alpha} \frac{\partial U_1}{\partial w_l} + (1-\alpha) U_1^\alpha U_2^{-\alpha} \frac{\partial U_2}{\partial w_l} &= 0 \\
\alpha U_1^{\alpha-1} U_2^{1-\alpha} \frac{\partial U_1}{\partial w_h} + (1-\alpha) U_1^\alpha U_2^{-\alpha} \frac{\partial U_2}{\partial w_h} &= 0 \quad .
\end{aligned}
\tag{33}
$$

From now on, the procedure is similar to the proof of Proposition 1.
Making the ad-hoc assumption of $B > 0$ for the optimal contract(s), there are again three possible cases: $U_1, U_2 > 0$ or $U_1 > 0, U_2 = 0, \alpha = 1$ or $U_1 = 0, U_2 > 0, \alpha = 0$. The latter two cases imply $B = U_1 + U_2$. Some straight forward calculations show that the maximum of $B = U_1 + U_2$ is reached in $w_h - w_l = c_2/(c_1 + c_2) \, X$. Using (32), it follows $\alpha U_2 = (1 - \alpha)U_1$ so that $U_1 = \alpha(U_1 + U_2)$ and $U_2 = (1 - \alpha)(U_1 + U_2)$.

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If $U_1, U_2 > 0$, then $U_1^{\alpha-1} U_2^{-\alpha} > 0$. Dividing the first order conditions through $U_1^{\alpha-1} U_2^{-\alpha}$ results in

$$
\begin{aligned}
\alpha U_2 \frac{\partial U_1}{\partial w_l} + (1-\alpha) U_1 \frac{\partial U_2}{\partial w_l} &= 0 \quad, \\
\alpha U_2 \frac{\partial U_1}{\partial w_h} + (1-\alpha) U_1 \frac{\partial U_2}{\partial w_h} &= 0 \quad.
\end{aligned}
\tag{34}
$$

Since $\partial U_1/\partial w_l = -1 - \partial U_1/\partial w_h$ and $\partial U_2/\partial w_l = 1 - \partial U_2/\partial w_h$, the conditions in case of $U_1, U_2 > 0$ are equivalent to

$$
\begin{aligned}
\alpha U_2 &= (1-\alpha) U_1 \quad, \\
0 &= \frac{\partial U_1}{\partial w_l} + \frac{\partial U_2}{\partial w_1} \quad.
\end{aligned}
\tag{35}
$$

The first equation implies $\alpha \in (0,1)$. Rearranging the second equation again leads to

$$
w_h - w_l = \frac{c_2}{c_1 + c_2} X
\tag{36}
$$

which now holds for all three cases. Using (31) gives

$$
d^* = d_e \quad.
\tag{37}
$$

The assumption $d^* \in \mathcal{D}$ is justified. According to Lemma 1, the implementation of the project results in a surplus $U_1 + U_2 = X^2/[4(c_1 + c_2)]$. Combined with $\alpha U_2 = (1-\alpha) U_1$, it follows that $U_1 \geq 0$ with equality if and only if $\alpha = 0$ and $U_2 \geq 0$ with equality if and only if $\alpha = 1$. The ad-hoc assumption $B > 0$ is justified. Some straightforward calculations using these results together with (32) and (36) give the unique payment scheme

$$
\begin{aligned}
w_l &= \frac{X^2}{4(c_1 + c_2)^2} \left[-\alpha c_2 + (1-\alpha) c_1 \right] \\
w_h &= \frac{X^2}{4(c_1 + c_2)^2} \left[-\alpha c_2 + (1-\alpha) c_1 \right] + \frac{c_2}{c_1 + c_2} X \quad.
\end{aligned}
\tag{38}
$$

To summarize, the unique optimal contract given $r = 1$ is described by (38).

Now assume $r = 2$ and proceed analogue to the case $r = 1$. Note that the two problems are symmetric by $\alpha \leftrightarrow (1-\alpha), c_1 \leftrightarrow c_2, w_l \leftrightarrow -w_l$ and $w_h \leftrightarrow X - w_h$. The decision-maker agent 2 implements

$$
d^{**} = \frac{w_h - w_l}{2c_2}
\tag{39}
$$

once the contract is signed. Maximizing the bargaining function B results in exactly the same payment scheme and project as if $r = 1$. The payment scheme is determined by (38) and the project d_e is implemented.

To summarize, there are two contracts which are candidates for an optimal contract, one with $r = 1$ and one with $r = 2$. Both contracts implement project d_e and payment scheme (38). Hence the payoffs U_1 and U_2 as well as the bargaining function B take the same values in both cases. The two candidates turn out to be the optimal contracts. ∎

Proof of Lemma 2:

An optimal contract solves

$$\max_{w_h} U_1^\alpha(w_h, d) U_2^{1-\alpha}(w_h, d) \tag{40}$$

subject to $0 \leq w_h \leq X$, $U_1, U_2 \geq 0$, and $d \in \text{argmax}_{d' \in \mathcal{D}} U_1(w_h, d')$. Once the contract is signed, the decision-maker agent 1 chooses a project in order to maximize U_1. This is the project

$$d_1 := \frac{X - w_h}{2c_1} \tag{41}$$

as long as it is an element of $\mathcal{D}$. For the moment, assume $d_1 \in \mathcal{D}$. Plugging d_1 in yields to

$$
\begin{aligned}
U_1 &= \frac{(X - w_h)^2}{4c_1} \quad, \\
U_2 &= \frac{(X - w_h)w_h}{2c_1} - \frac{c_2(X - w_h)^2}{4c_1^2}
\end{aligned} \tag{42}
$$

and

$$
\begin{aligned}
\frac{\partial U_1}{\partial w_h} &= -\frac{X - w_h}{2c_1} \quad, \\
\frac{\partial U_2}{\partial w_h} &= \frac{c_1 + c_2}{2c_1^2}(X - w_h) - \frac{w_h}{2c_1} \quad.
\end{aligned} \tag{43}
$$

The remaining proof is similar to the proof of Proposition 2. Assume that there is at least one contract fulfilling $B > 0$ and the required constraints so that each optimal contract satisfies $B > 0$ as well. Since $U_1 = 0$ implies $w_h = X$ which in turn implies $U_2 = 0$ and finally $B = 0$, it is necessarily $U_1 > 0$ and $w_h < X$. There are only two possible cases, namely $U_1, U_2 > 0$ or $U_1 > 0, U_2 = 0, \alpha = 1$.

Consider $U_1, U_2 > 0$. Dividing $\partial B / \partial w_h = 0$ through $U_1^{\alpha-1} U_2^{-\alpha} > 0$ leads to the first order condition

$$\alpha U_2 \frac{\partial U_1}{\partial w_h} + (1 - \alpha) U_1 \frac{\partial U_2}{\partial w_h} = 0 \tag{44}$$

which by (42) and (43) is

$$\alpha \left[-\frac{(X - w_h)^2 w_h}{4c_1^2} + \frac{c_2(X - w_h)^3}{8c_1^3} \right] +$$

$$(1 - \alpha) \left[\frac{(c_1 + c_2)(X - w_h)^3}{8c_1^3} - \frac{(X - w_h)^2 w_h}{8c_1^2} \right] = 0 \quad . \tag{45}$$

Since $w_h = X$ is already ruled out, the unique solution of (45) is

$$w_h = \frac{c_2 + (1 - \alpha)c_1}{2c_1 + c_2} X \quad . \tag{46}$$

Solving $U_2 = 0$ for w_h gives exactly the same payment for the case $U_1 > 0, U_2 = 0, \alpha = 1$.

This payment fulfills the limited liability constraints and leads to

$$d_1 = \frac{(1 + \alpha)X}{2(2c_1 + c_2)} \tag{47}$$

which is in $\mathcal{D}$ since $1 + \alpha \leq 2$ and $2c_1 + c_2 > X$ by assumption. Using (42), (46) and (47) give

$$U_1 = \frac{(1 + \alpha)^2 X^2 c_1}{4(2c_1 + c_2)^2} \quad > \quad 0 \tag{48}$$

and

$$U_2 = \frac{(1 - \alpha)(1 + \alpha)X^2}{4(2c_1 + c_2)} \quad \geq \quad 0 \tag{49}$$

with equality if and only if $\alpha = 1$. The ad-hoc assumption $B > 0$ is justified.

∎

Proof of Lemma 3:

The proof is completely analogue to Lemma 2, simply replace $w_h \leftrightarrow X - w_h, \alpha \leftrightarrow 1 - \alpha, c_1 \leftrightarrow c_2$ and note that the agents changed their roles. The decision-maker agent 2 chooses the project $d_2 = w_h/(2c_2)$ which turns out to be

$$d_2 = \frac{(2 - \alpha)X}{2(c_1 + 2c_2)} \tag{50}$$

in the end. It is $d_2 \in \mathcal{D}$ since $2 - \alpha \leq 2$ and $c_1 + 2c_2 \geq 2c_1 + c_2 > X$ by assumption. The payoffs are

$$U_1 = \frac{\alpha(2-\alpha)X^2}{4(c_1 + 2c_2)} \quad \geq \quad 0 \tag{51}$$

$$U_2 = \frac{(2-\alpha)^2 X^2 c_2}{4(c_1 + 2c_2)^2} \quad > \quad 0 \tag{52}$$

with equality if and only if $\alpha = 0$ so that $B > 0$. ∎

Proof of Lemma 4:
Note that $U_1 + U_2 = dX - (c_1 + c_2)d^2$ is a parabola open below with its maximum in d_e. To put it differently, $U_1 + U_2$ is strictly decreasing in $|d - d_e|$. It is

$$
\begin{aligned}
|d_1 - d_e| &= \left| \frac{(1+\alpha)X}{2(2c_1 + c_2)} - \frac{X}{2(c_1 + c_2)} \right| \\
&= \left| \frac{X[\alpha c_2 - (1-\alpha)c_1]}{2(2c_1 + c_2)(c_1 + c_2)} \right| \\
&= d_e \frac{|\alpha(c_1 + c_2) - c_1|}{2c_1 + c_2}
\end{aligned}
\tag{53}
$$

and

$$
\begin{aligned}
|d_2 - d_e| &= \left| \frac{(2-\alpha)X}{2(c_1 + 2c_2)} - \frac{X}{2(c_1 + c_2)} \right| \\
&= \left| \frac{X[\alpha c_2 - (1-\alpha)c_1]}{2(c_1 + 2c_2)(c_1 + c_2)} \right| \\
&= d_e \frac{|\alpha(c_1 + c_2) - c_1|}{c_1 + 2c_2} \quad .
\end{aligned}
\tag{54}
$$

It is $|d_1 - d_e| = |d_2 - d_e| = 0$ if and only if $\alpha = c_1/(c_1 + c_2)$. The allocation of authority does not influence overall surplus in this case. If $\alpha \neq c_1/(c_1 + c_2)$, we have

$$|d_1 - d_e| \geq |d_2 - d_e| \iff \frac{1}{2c_1 + c_2} \geq \frac{1}{c_1 + 2c_2} \iff c_1 \leq c_2 \quad . \tag{55}$$

The surplus is increased by allocating authority to the agent with the larger cost share who is agent 2 by assumption $c_1 \leq c_2$. ∎

Proof of Proposition 3:

Consider $r = 1$. Using U_1 and U_2 from the proof of Lemma 2, the value of the bargaining function is calculated to be

$$B = \frac{X^2}{4} \left(\frac{1}{2c_1 + c_2} \right)^{1+\alpha} c_1^\alpha (1+\alpha)^{1+\alpha} (1-\alpha)^{1-\alpha} =: B_1 \quad . \tag{56}$$

If $r = 2$, the proof of Lemma 3 shows

$$B = \frac{X^2}{4} \left(\frac{1}{c_1 + 2c_2} \right)^{2-\alpha} c_2^{1-\alpha} (2-\alpha)^{2-\alpha} \alpha^\alpha =: B_2 \quad . \tag{57}$$

If and only if $B_1 = B_2$, there are two optimal contracts which differ in the allocation of authority. If and only if $B_1 \neq B_2$, the optimal contract (and therefore the allocation of authority) is unique. Agent 1 is the decision-maker in case of $B_1 > B_2$, while agent 2 receives authority if $B_2 > B_1$.

Define $k := c_1/c_2$ so that

$$B_1 = \frac{X^2}{4} (2k+1)^{-1-\alpha} k^\alpha c_2^{-1} (1+\alpha)^{1+\alpha} (1-\alpha)^{1-\alpha} \tag{58}$$

and

$$B_2 = \frac{X^2}{4} (k+2)^{-2+\alpha} c_2^{-1} (2-\alpha)^{2-\alpha} \alpha^\alpha \quad . \tag{59}$$

It follows

$$B_2 \gtreqqless B_1 \iff \phi(k,\alpha) \gtreqqless 0 \tag{60}$$

with

$$
\begin{aligned}
\phi(k,\alpha) \quad := \quad & (2k+1)^{1+\alpha} (2-\alpha)^{2-\alpha} \alpha^\alpha \\
- \quad & (k+2)^{2-\alpha} k^\alpha (1+\alpha)^{1+\alpha} (1-\alpha)^{1-\alpha} \quad .
\end{aligned}
\tag{61}
$$

∎

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