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# Rotationally adiabatic pair interactions of para- and ortho-hydrogen with the halogen molecules $\mathrm{F}_{2}, \mathrm{Cl}_{2}$, and $\mathrm{Br}_{2}$ 

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#### Abstract

The present work is concerned with the weak interactions between hydrogen and halogen molecules, i.e., the interactions of pairs $\mathrm{H}_{2}-\mathrm{X}_{2}$ with $\mathrm{X}=\mathrm{F}, \mathrm{Cl}, \mathrm{Br}$, which are dominated by dispersion and quadrupole-quadrupole forces. The global minimum of the four-dimensional (4D) coupled cluster with singles and doubles and perturbative triples $(\operatorname{CCSD}(\mathrm{T}))$ pair potentials is always a T shaped structure where $\mathrm{H}_{2}$ acts as the hat of the T, with well depths $\left(D_{e}\right)$ of $1.3,2.4$, and $3.1 \mathrm{~kJ} / \mathrm{mol}$ for $\mathrm{F}_{2}, \mathrm{Cl}_{2}$, and $\mathrm{Br}_{2}$, respectively. MP2/AVQZ results, in reasonable agreement with $\operatorname{CCSD}(\mathrm{T})$ results extrapolated to the basis set limit, are used for detailed scans of the potentials. Due to the large difference in the rotational constants of the monomers, in the adiabatic approximation, one can solve the rotational Schrödinger equation for $\mathrm{H}_{2}$ in the potential of the $\mathrm{X}_{2}$ molecule. This yields effective two-dimensional rotationally adiabatic potential energy surfaces where $\mathrm{pH}_{2}$ and $o \mathrm{H}_{2}$ are point-like particles. These potentials for the $\mathrm{H}_{2}-\mathrm{X}_{2}$ complexes have global and local minima for effective linear and T-shaped complexes, respectively, which are separated by $0.4-1.0 \mathrm{~kJ} / \mathrm{mol}$, where $o \mathrm{H}_{2}$ binds stronger than $\mathrm{pH}_{2}$ to $\mathrm{X}_{2}$, due to higher alignment to minima structures of the 4D-pair potential. Further, we provide fits of an analytical function to the rotationally adiabatic potentials. © 2014 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4892599]


## I. INTRODUCTION

Hydrogen is the lightest molecule, with a large rotational constant, and thus quantum effects govern the dynamics of the single molecule, its clusters and macroscopic phases. ${ }^{1,2}$ The hydrogen molecule forms two nuclear spin isomers referred to as para- $\left(p \mathrm{H}_{2}\right)$ and ortho-hydrogen $\left(o \mathrm{H}_{2}\right)$. For hydrogen molecules in the electronic ground state, the antisymmetry principle states that $p \mathrm{H}_{2}$ has rotational states with even main rotational quantum numbers, while for $\mathrm{oH}_{2}$ only the odd numbered states are allowed. The resulting differences in the rotational energies lead in turn to differences in the macroscopic properties allowing, for example, the separation of para- from ortho-hydrogen. Due to the low mass, also the translation and vibration of $\mathrm{H}_{2}$ molecules are subject to quantum effects. Theoretical investigations suggest that small clusters of parahydrogen $\left(\left(p \mathrm{H}_{2}\right)_{N}\right)$ are liquid below 14 K , the melting point of bulk $p \mathrm{H}_{2}$, and exhibit superfluid behaviour at temperatures below $1 \mathrm{~K}^{3-6}$ The relative cluster stability is predicted to increase for clusters with $N$ equal to icosahedral cluster magic numbers. ${ }^{7,8}$ Para-hydrogen crystals are translational quantum solids, where, due to the zero point energy, the hcp- is preferred over the fcc-lattice at low temperatures. ${ }^{2,9-11}$

These quantum effects also have to be considered for $\mathrm{H}_{2}$ clusters and solids doped with other molecules. ${ }^{12-14}$ For example, small $p \mathrm{H}_{2}$ clusters show superfluid responses to ro-

[^0]tations of embedded OCS, ${ }^{15-17} \mathrm{CO}_{2},{ }^{18}$ and $\mathrm{CO}^{19}$ molecules. Theoretical investigations rely on the respective pair potentials for the interaction of the dopant with $\mathrm{H}_{2}$. This poses challenges for $a b$ initio electronic structure calculations, as the interaction of $\mathrm{H}_{2}$ with other closed shell molecules is in the order of only a few $\mathrm{kJ} / \mathrm{mol} .{ }^{20-22}$ Therefore, accurate results are currently accessible only for systems with a small number of electrons. Similarly weak interactions are also known for rare-gas halogen systems, which have been studied extensively. ${ }^{23-27}$ In contrast to rare-gas matrices, halogens trapped in solid $p \mathrm{H}_{2}$ can react with the matrix itself. Concerning the hydrogen halogen $\mathrm{H}_{2} \mathrm{X}_{2}$ systems, with $\mathrm{X}=\mathrm{F}$, Cl , and Br , the ab initio interactions of the $(\mathrm{HX})_{2}$ dimers $^{28-32}$ and of the atom-molecule pairs X- $\mathrm{H}_{2}{ }^{33-36}$ have been studied in view of hydrogen bonding and the hydrogen halogen reaction. However, with the exceptions of $\mathrm{H}_{2}-\mathrm{I}_{2}{ }^{37,38}$ and $\mathrm{H}_{2}-$ $\mathrm{F}_{2}$, ${ }^{39}$ no reports about pair potentials for the $\mathrm{H}_{2}-\mathrm{X}_{2}$ dimers could be found in the literature. These potentials are important to understand the properties of hydrogen halogen mixtures prior to reaction. An example are the experiments by Kettwich et al. ${ }^{40}$ who performed flash photolysis of $\mathrm{Cl}_{2}$ diluted in $p \mathrm{H}_{2}$ at cryogenic temperatures. The simultaneous irradiation with IR light, exciting $p \mathrm{H}_{2}$ to the first vibrational state, lead to a significant increase in the yield of HCl , as was proposed by Korolkov et al. ${ }^{41}$ To model the quantum reaction dynamics of the photolysis of $\mathrm{X}_{2}$ in a $p \mathrm{H}_{2}$ matrix, an assumption about the orientation of $\mathrm{X}_{2}$ and translation of $\mathrm{H}_{2}$ within the crystal lattice has to be made. ${ }^{41}$ Here, the $\mathrm{H}_{2}-\mathrm{X}_{2}$ potentials are needed to construct the many body potentials for which the librational ground state wave functions answer these questions. ${ }^{42,43}$

One can in a good approximation adiabatize the pair potentials with respect to, not only the electronic and vibrational degrees of freedom, but also to the rotation of $\mathrm{H}_{2}$, as the rotational constant of $\mathrm{H}_{2}$ is much larger than for the heavier halogen molecules. This reduces the four-dimensional (4D) pair potential to effective two-dimensional (2D) potentials for $p \mathrm{H}_{2}$ and $o \mathrm{H}_{2}$, which facilitates studies of ensembles. ${ }^{44}$ In particular, model functions developed for the quantum dynamics of diatomic molecules trapped in rare gas clusters ${ }^{45-52}$ can be readily applied. Developed for rare gas pair potentials, analytic Hartree-Fock-dispersion (HFD) models ${ }^{53}$ are also suited to parametrize $p / o \mathrm{H}_{2}$ interactions. ${ }^{21,54}$ Accurate intermolecular potentials for simple van der Waals complexes can also help in the development of on top dispersion corrections and atom-atom potentials used in molecular dynamics.

In view of the similarities of $p \mathrm{H}_{2}$ to the rare gas atoms, it is interesting to compare their respective interaction potentials with halogen molecules. The rare gas atom- $\mathrm{X}_{2}$ pair potentials are weakly anisotropic. Similar binding energies for linear and T-shaped complexes have been predicted by high level $a b$ initio calculations ${ }^{23,25,26}$ and have been measured using laserinduced fluorescence and two-color action spectroscopy, ${ }^{55-57}$ see Ref. 58 for a review. We further compare the $p / o \mathrm{H}_{2}-\mathrm{X}_{2}$ interactions with those of $p \mathrm{H}_{2}$ with other small molecules.

## II. AB INITIO PAIR POTENTIALS

## A. Methods

We assume the molecular wave function of the $\mathrm{H}_{2}-\mathrm{X}_{2}$ dimers to be approximately adiabatic with respect to the electronic ground state of the dimer, as well as to the vibrational ground state of the free monomers, i.e., we apply the Born-Oppenheimer and fixed monomer approximation. In the latter, the bond lengths of the monomers are kept constant at the following values $r_{\mathrm{H}_{2}}=75.7 \mathrm{pm}, r_{\mathrm{F}_{2}}=141.2 \mathrm{pm}, r_{\mathrm{Cl}_{2}}$ $=203.3 \mathrm{pm}$, and $r_{\mathrm{Br}_{2}}=228.1 \mathrm{pm}$. The interaction energy then depends on the Jacobi coordinates of the systems, see Fig. 1, hence the distance $R$ between the centers of mass of $\mathrm{H}_{2}$ and $\mathrm{X}_{2}$, the polar $\theta_{\mathrm{H}_{2}}, \theta_{\mathrm{X}_{2}}$, and dihedral angles $\left(\phi_{\mathrm{H}_{2}}-\phi_{\mathrm{X}_{2}}\right)$.

The interaction between non-polar molecules is governed by dispersion and electrostatic quadrupole-quadrupole (QQ) interactions. The former ones are roughly proportional to the polarizabilities of the molecules. ${ }^{59}$ For $\mathrm{H}_{2}-\mathrm{X}_{2}$, they are in the few $\mathrm{kJ} / \mathrm{mol}$ regime, calling for highly accurate wave function based correlation methods, such as full configuration interaction (FCI), coupled cluster with singles and doubles and perturbative triples $(\operatorname{CCSD}(\mathrm{T}))$, and symmetry adapted perturbation theory (SAPT), in conjunction with large one electron basis sets. ${ }^{60,61}$

Here, the interaction energies $\Delta E\left(R, \theta_{\mathrm{X}_{2}}, \theta_{\mathrm{H}_{2}}, \phi_{\mathrm{H}_{2}}\right.$ $-\phi_{\mathrm{X}_{2}}$ ), defined by

$$
\begin{equation*}
\Delta E=E\left(\mathrm{H}_{2} \mathrm{X}_{2}\right)-\left(E\left(\mathrm{X}_{2}\right)+E\left(\mathrm{H}_{2}\right)\right), \tag{1}
\end{equation*}
$$

were calculated at the Møller-Plesset perturbation theory of second order (MP2) and $\operatorname{CCSD}(\mathrm{T})$ level of theories as implemented in the MOLPRO package of programs. ${ }^{6-64}$ For H, F, and Cl all electron aug-cc-pVnZ basis sets ${ }^{65}(\mathrm{AVnZ})$ up to


FIG. 1. Laboratory frame and internal coordinates. The halogen molecule $\left(\mathrm{X}_{2}\right)$ and the hydrogen molecule $\left(\mathrm{H}_{2}\right)$ are depicted by yellow and blue pairs of circles, respectively.
$\mathrm{n}=6(\mathrm{H}, \mathrm{F})$ and $\mathrm{n}=5(\mathrm{Cl})$ were used. For Br the scalarrelativistic effective core potential ECP10MDF ${ }^{66}$ was used in conjunction with the adapted aug-cc-pVnZ basis sets ${ }^{66}$ up to $\mathrm{n}=5$. All interaction energies are corrected for the basis set superposition error via the counterpoise scheme. ${ }^{67}$ Results referring to the complete basis set (CBS) limit are extrapolated from the results of the largest and second largest basis. ${ }^{60}$ In the determination of the correlation energy virtual excitations were restricted to the valence electrons. Total energies are converged to a threshold of $10^{-8}$ Hartree.

## B. $\operatorname{CCSD}(\mathrm{T})$ results

For the six structures of high symmetry, depicted in Fig. 2, $\Delta E(R)$ was calculated up to the $\operatorname{CCSD}(\mathrm{T}) / \mathrm{CBS}$ level of theory. The orientations are abbreviated by (a) letters $T$, $H$, and $X$ forming similar shapes as the complexes and (b) by letters $L$ and $S$ which abbreviate the descriptions, "linear" and "slipped-parallel," of the structures. We further discern the structures $T$ for the $\mathrm{H}_{2}-\mathrm{X}_{2}$ pairs, where $\mathrm{H}_{2}$ forms either the stem (T1) or the hat (T2). The minima of the $\operatorname{CCSD}(\mathrm{T})$ curves, obtained by spline interpolation, as well as curves from MP2/AVQZ are given in Fig. 3. Tabulated values are given in the supplementary material. ${ }^{68}$ Trends within the $\operatorname{CCSD}(\mathrm{T}) / \mathrm{CBS}$ interaction energies are similar for all hydrogen-halogen pairs. The most strongly bound orientation is always $T 2$, with well depths $D_{e}$ increasing by about $1 \mathrm{~kJ} / \mathrm{mol}$ in the order $\mathrm{F}_{2}<\mathrm{Cl}_{2}<\mathrm{Br}_{2}$, and range from 1.3 over 2.4 to $3.2 \mathrm{~kJ} / \mathrm{mol}$. The least bound orientation of the six is always the linear one $(L)$ with minimal interaction energies of $-0.25,-0.5$, and $-0.6 \mathrm{~kJ} / \mathrm{mol}$ for $\mathrm{F}_{2}, \mathrm{Cl}_{2}$, and $\mathrm{Br}_{2}$, respectively, where the minima occur at the largest monomer separation. Intermediate interaction energies are found for the


FIG. 2. Selected symmetric structures of $\mathrm{H}_{2}-\mathrm{X}_{2}$ pairs. The $H_{-}$, T1-, T2-, and $X$-configurations take their names from their geometrical arrangements, whereas $L$ and $S$ stands for "linear" and "slipped parallel," respectively. The corresponding geometrical factors $G$ in Eq. (3) for the quadrupolequadrupole interaction are given in brackets.
$T 1, X, H$, and $S$ structures. For all three systems, the minima of $T 1, X, H$ are found at the smallest monomer separations and are similar in energy. Within this group, the absolute interaction energy decreases from $T 1$ over $H$ to $X$. While for $\mathrm{Cl}_{2}$ and $\mathrm{Br}_{2}$ the interaction energy of $H$ is closer to that of $T 1$ than the one of $X$, for $\mathrm{F}_{2}$ the interaction energies of $H$ and $X$ are closest instead. The minimum of the $\Delta E(R)$ curves of the $S$ orientation is found at distances between the minima of the $T 2$ and $L$ curves. For $\mathrm{Cl}_{2}$ and $\mathrm{Br}_{2}$, the respective absolute interaction energy is always below that of the $T 1, H, X$ group, while for $\mathrm{F}_{2}$, it is in between the close values for $H$ and $X$.

To emphasize the anisotropy of the hydrogen-halogen molecular interaction, we included the hydrogen-hydrogen $\operatorname{CCSD}(\mathrm{T}) / \mathrm{AV} 5 \mathrm{Z}$ molecular pair potentials at equivalent orientations in Fig. 3. For this system, the orientations $T 1$ and $T 2$ degenerate to $T$, with a well depth of $0.43 \mathrm{~kJ} / \mathrm{mol}$. Thus, the interaction of $\mathrm{H}_{2}$ with $\mathrm{H}_{2}$ is $3,5.6$, and 7.5 times weaker than with $\mathrm{F}_{2}, \mathrm{Cl}_{2}$, and $\mathrm{Br}_{2}$, respectively. In contrast to the interaction with halogen molecules, structure $S$ and $T$ are very close in energy, differing by only $0.03 \mathrm{~kJ} / \mathrm{mol}$. Note also the similarity of the interaction energies for $X$ and $H$ to the $L$ orientation. Hence, the $\mathrm{H}_{2}-\mathrm{H}_{2}$ pair interaction is much more isotropic and weaker, in comparison to the hydrogen halogen pair interactions.

## C. Electrostatic contributions

In this subsection, we want to illustrate in how far the interaction energy can be associated to electrostatic forces. While a rigorous approach to quantify all separate types of interactions would involve SAPT theory, ${ }^{61}$ we restrict ourselves to calculate only the electrostatic QQ interaction to obtain at least a qualitative measure for their importance. Because the quadrupole moments are the lowest non-vanishing multipole moments for linear neutral homonuclear molecules. Hence, their interaction is the leading term of the multipole expansion for the electrostatic interaction, which is given by ${ }^{59}$

$$
\begin{equation*}
U_{Q_{X_{2}} Q_{\mathrm{H}_{2}}}(R)=\frac{Q_{\mathrm{X}_{2}} Q_{\mathrm{H}_{2}}}{4 \pi \epsilon_{0} R^{5}} \times G\left(\theta_{\mathrm{X}_{2}}, \theta_{\mathrm{H}_{2}}, \phi_{\mathrm{H}_{2}}-\phi_{\mathrm{X}_{2}}\right) . \tag{2}
\end{equation*}
$$

The components of the quadrupole tensors along the bond axis are $Q_{\mathrm{H}_{2}}=0.4252 e a_{0}^{2}, \quad Q_{\mathrm{F}_{2}}=0.7635 e a_{0}^{2}$,


FIG. 3. Interaction energies $\Delta E$ between $\mathrm{H}_{2}$ and $\mathrm{H}_{2}, \mathrm{~F}_{2}, \mathrm{Cl}_{2}$, and $\mathrm{Br}_{2}$ for the static orientations shown in Fig. 2. Ab initio results (full lines) for $\mathrm{H}_{2}-$ $\mathrm{H}_{2}$ are calculated at the $\operatorname{CCSD}(\mathrm{T}) / \mathrm{AV} 5 \mathrm{Z}$ level of theory. The interactions of $\mathrm{H}_{2}$ with the halogens, denoted by full lines, are of $\mathrm{MP} 2 /(\mathrm{H}, \mathrm{F}, \mathrm{Cl}: \mathrm{AVQZ}$, Br: ECP10MDF AVQZ) quality. Triangles at the maximal $\operatorname{CCSD}(\mathrm{T}) / \mathrm{CBS}$ binding interactions are included as reference points to judge the accuracy of the MP2 results. Dashed lines show hard sphere quadrupole-quadrupole model interactions.
$Q_{\mathrm{Cl}_{2}}=2.5427 e a_{0}^{2}, Q_{\mathrm{Br}_{2}}=0.7635 e a_{0}^{2}$, calculated at the CCSD(T)/AV6Z, MP2/AV6Z, MP2/AV6Z, and MP2/AV5Z levels of theory, respectively. The geometrical factor $G$ is given by

$$
\begin{align*}
& G\left(\theta_{\mathrm{X}_{2}}, \theta_{\mathrm{H}_{2}}, \phi_{\mathrm{H}_{2}}-\phi_{\mathrm{X}_{2}}\right) \\
& =\frac{3}{4}\left\{1-5 \cos ^{2} \theta_{\mathrm{X}_{2}}-5 \cos ^{2} \theta_{\mathrm{H}_{2}}-15 \cos ^{2} \theta_{\mathrm{X}_{2}} \cos ^{2} \theta_{\mathrm{H}_{2}}\right. \\
& \left.\quad+2\left[4 \cos \theta_{\mathrm{X}_{2}} \cos \theta_{\mathrm{H}_{2}}-\sin \theta_{\mathrm{X}_{2}} \sin \theta_{\mathrm{H}_{2}} \cos \left(\phi_{\mathrm{H}_{2}}-\phi_{\mathrm{X}_{2}}\right)\right]^{2}\right\} \tag{3}
\end{align*}
$$

The values of $G$ for the six orientations discussed above are also given in Fig. 2. We truncate $U_{Q_{X_{2}}} Q_{\mathrm{H}_{2}}(R)$ at the contact distances given by a very simple "hard-sphere" (HS) model, where the atoms are represented by hard spheres with van der Waals radii of $110 \mathrm{pm}, 147 \mathrm{pm}, 175 \mathrm{pm}$, and 185 pm for the $\mathrm{H}, \mathrm{F}, \mathrm{Cl}$, and Br atoms, respectively. ${ }^{69}$

The corresponding $\mathrm{QQ}+\mathrm{HS}$ interactions $U_{\mathrm{Q}_{\mathrm{X}_{2}} \mathrm{Q}_{\mathrm{H}_{2}}}$, are depicted in Fig. 3 by dashed lines. For the $\mathrm{H}_{2}-\mathrm{H}_{2}$ complex, this model predicts, in accordance with the results from the electronic structure calculations, structure $T$ as the most stable followed by structure $S$. The QQ+HS interactions for $X, H$, $L$ are repulsive, thus these complexes are bound only due to dispersion forces. The latter also holds for the $\mathrm{H}_{2}-\mathrm{X}_{2}$ pairs. Here, bonding electrostatic contributions are present for $T 1$, $T 2$, and $S$. In the QQ+HS model, the only difference between $T 1$ and T2 is the HS contact distance. The T1 interaction energy has the highest QQ contribution followed by $S$ and $T 2$. From this we deduce, that $\mathrm{H}_{2}-\mathrm{X}_{2}$ structures $T 2$ are highly stabilized by dispersion interactions, while for $T 1$ structures the QQ and dispersion interactions are approximately equal in magnitude. Thus, it is unsurprising that, when comparing $\mathrm{F}_{2}$, $\mathrm{Cl}_{2}$, and $\mathrm{Br}_{2}$, the changes in the QQ interactions are too small to account for the increase in the $T 2$ well depth in this series. On the other hand, for $T 1$ the $\mathrm{QQ}+\mathrm{HS}$ interactions very well describe the actual changes in the interaction energy.

## D. MP2 results

A large number of $a b$ initio calculations are needed to get a fine angular and spatial resolution for the potential energy hypersurfaces $\Delta E\left(R, \theta_{\mathrm{X}_{2}}, \theta_{\mathrm{H}_{2}}, \phi_{\mathrm{H}_{2}}-\phi_{\mathrm{X}_{2}}\right)$. Hence, due to economical reasons we have to resort to the MP2 method in their evaluation, unless symmetry allows otherwise. In the following part, we therefore compare the $\operatorname{CCSD}(\mathrm{T}) / \mathrm{CBS}$ results to those obtained at the MP2 level of theory. Fig. 3 shows full curves for the $R$ dependence of the MP2/AVQZ interaction energy of dimers in fixed orientations and for comparison the location of $\operatorname{CCSD}(\mathrm{T}) / \mathrm{CBS}$ minima. Tables holding the respective $R_{e}$ and $D_{e}$ are provided in the supplementary material. ${ }^{68}$ The CCSD(T)/CBS binding distances $R_{e}$ are well recovered by MP2/AVQZ. Averaging the absolute deviations for $D_{e}$ from MP2/AVQZ calculations, with respect to the $\operatorname{CCSD}(\mathrm{T}) / \mathrm{CBS}$ values, for the six test structures of each complex, yields $0.11 \mathrm{~kJ} / \mathrm{mol}$ for $\mathrm{H}_{2}-\mathrm{F}_{2}, 0.07 \mathrm{~kJ} / \mathrm{mol}$ for $\mathrm{H}_{2}-\mathrm{Cl}_{2}$, and $0.09 \mathrm{~kJ} / \mathrm{mol}$ for $\mathrm{H}_{2}-\mathrm{Cl}_{2}$. In terms of absolute relative deviations, the mean values are $15 \%, 6 \%$, and $7 \%$ for the complexes with $\mathrm{F}_{2}, \mathrm{Cl}_{2}$, and $\mathrm{Br}_{2}$, respectively. Thus, on average MP2/AVQZ performs better for $\mathrm{H}_{2}-\mathrm{Cl}_{2}$ and $\mathrm{H}_{2}-$ $\mathrm{Br}_{2}$ than for $\mathrm{H}_{2}-\mathrm{F}_{2}$. We also tested larger basis sets of $\mathrm{n}=$ 5, 6 quality with the result, see Fig. 4, that the mean absolute relative deviations do not change. Basis sets with $\mathrm{n}=3$, however, increase the deviations considerably, as tested for $\mathrm{F}_{2}$ and $\mathrm{Br}_{2}$.

Next, we compare the performance of MP2/AVQZ for each of the orientations individually. The global minima, i.e., well depths of $T 2$, are in excellent agreement with the $\operatorname{CCSD}(\mathrm{T}) / \mathrm{CBS}$ results, for all three complexes, as $\Delta D_{e}(T 2)$ ranges from $-0.03 \mathrm{~kJ} / \mathrm{mol}\left(\mathrm{H}_{2}-\mathrm{F}_{2}\right)$ to $-0.07 \mathrm{~kJ} / \mathrm{mol}\left(\mathrm{H}_{2}-\right.$


FIG. 4. Basis set dependence of relative deviations of MP2 well depths $D_{e}$ with respect to $\operatorname{CCSD}(\mathrm{T}) / \mathrm{CBS}$ values for the structures shown in Fig. 2.
$\mathrm{Br}_{2}$ ). The MP2/AVQZ well depths for the $S, L$, and $T 1$ structures of the $\mathrm{H}_{2}$ complexes with $\mathrm{Cl}_{2}, \mathrm{Br}_{2}$ agree similarly well $\left(\left|\Delta D_{e}\right|<0.1 \mathrm{~kJ} / \mathrm{mol}\right)$ with the $\operatorname{CCSD}(\mathrm{T}) / \mathrm{CBS}$ reference data. For $\mathrm{H}_{2}-\mathrm{F}_{2}$, the well depths of $L$ and $S$ are also within $0.1 \mathrm{~kJ} / \mathrm{mol}$ of the $\operatorname{CCSD}(\mathrm{T}) / \mathrm{CBS}$ values. However, the well depth of the $T 1$ structure of $\mathrm{H}_{2}-\mathrm{F}_{2}$ is underestimated by $0.16 \mathrm{~kJ} / \mathrm{mol}$, which is a considerable deviation from the MP2/AVQZ errors $\Delta D_{e}(T 1)$ of only $0.002 \mathrm{~kJ} / \mathrm{mol}$ and $0.003 \mathrm{~kJ} / \mathrm{mol}$ for the $\mathrm{H}_{2}-\mathrm{Cl}_{2}$ and $\mathrm{H}_{2}-\mathrm{Br}_{2}$ complexes. The least agreement is found for the $X$ and $H$ structures of all complexes, where MP2/AVQZ systematically underestimates well depths by $0.13 \mathrm{~kJ} / \mathrm{mol}-0.22 \mathrm{~kJ} / \mathrm{mol}$. However, the energetic ordering remains the same as with $\operatorname{CCSD}(\mathrm{T}) / \mathrm{CBS}$. This also holds for all tested structures with the exception of the $H, X$, and $S$ structures of $\mathrm{H}_{2}-\mathrm{F}_{2}$. Here, the $\operatorname{CCSD}(\mathrm{T}) / \mathrm{CBS}$ well depths are within a range of just $0.04 \mathrm{~kJ} / \mathrm{mol}$ and are ordered $H>S>X$, whereas with MP2/AVQZ the ordering changes to $S>H>X$. Overall, the MP2/AVQZ method performs very well for $\mathrm{H}_{2}-\mathrm{Cl}_{2}$ and $\mathrm{H}_{2}-\mathrm{Br}_{2}$, while anisotropies with respect to the $H$ and $X$ structures are slightly overestimated. The pair potential of $\mathrm{H}_{2}-\mathrm{F}_{2}$ is less anisotropic in comparison, thus the MP2 errors, although similar in magnitudes, lead to a less accurate description of the potential. In particular, anisotropies of the $\mathrm{H}_{2}-\mathrm{F}_{2}$ pair potential with respect to the $H, L$, and T1 structures are overestimated by MP2. In summary, the MP2/AVQZ method presents an economic way to describe the interaction energy reasonably well for the six structures for all investigated complexes within the discussed limits.

## III. ROTATIONALLY ADIABATIC (RA) POTENTIAL ENERGY SURFACE

## A. Method

Because the rotational constant of $\mathrm{H}_{2}$ is 62, 244, and 724 times larger than the rotational constants of ${ }^{19} \mathrm{~F}_{2},{ }^{35} \mathrm{Cl}_{2}$, and ${ }^{79} \mathrm{Br}_{2}$, it is a justified approximation, to adiabatize the pair potentials with respect to the rotational degrees of freedom of $\mathrm{H}_{2}$. This yields two-dimensional potentials, depending on the orientation of the halogen molecule and the distance $R$, which is equivalent to an effective point-like $\mathrm{H}_{2}$ molecule representing a certain rotational state. In most of the previous work found in the literature, ${ }^{70-74}$ effects of $\mathrm{H}_{2}$ rotation are eliminated by considering the free rotor ground state only, i.e., by isotropic averaging with respect to the rotational degrees of freedom. In this approximation, perturbations of the free rotor states due to the presence of an intermolecular potential are neglected. Li et al. have compared isotropic averaging with the "adiabatic-hindered-rotor" treatment, for $p \mathrm{H}_{2}-\mathrm{CO}, p \mathrm{H}_{2}-$ $\mathrm{CO}_{2}$, and $p \mathrm{H}_{2}-p \mathrm{H}_{2}$. They concluded, that the latter method results in more accurate binding energies and spectra over the former. ${ }^{44}$ Considering that the binding energies of the $\mathrm{H}_{2}-\mathrm{X}_{2}$ complexes, see Fig. 3, are up to 2, 3.5, and 4.7 times larger than the rotational constant of $\mathrm{H}_{2}$, we have to expect a hindered rotation around the global minima. Consequently, we employ the "adiabatic-hindered-rotor" method, i.e., we calculate RA pair potentials, by solving the rotational Schrödinger equations (SE)

$$
\begin{align*}
& \hat{\mathbf{H}}_{\mathrm{rot}}\left(\theta_{\mathrm{H}_{2}}, \phi_{\mathrm{H}_{2}} ; R, \theta_{\mathrm{X}_{2}}, \phi_{\mathrm{X}_{2}}\right) \Psi_{n}\left(\theta_{\mathrm{H}_{2}}, \phi_{\mathrm{H}_{2}} ; R, \theta_{\mathrm{X}_{2}}, \phi_{\mathrm{X}_{2}}\right) \\
& \quad=W_{n}\left(R, \theta_{\mathrm{X}_{2}}, \phi_{\mathrm{X}_{2}}\right) \Psi_{n}\left(\theta_{\mathrm{H}_{2}}, \phi_{\mathrm{H}_{2}} ; R, \theta_{\mathrm{X}_{2}}, \phi_{\mathrm{X}_{2}}\right), \tag{4}
\end{align*}
$$

in the variables $\theta_{\mathrm{H}_{2}}$ and $\phi_{\mathrm{H}_{2}}$, which are separated from the parameters $R, \theta_{\mathrm{X}_{2}}$, and $\phi_{\mathrm{X}_{2}}$ by the semicolon. The Hamiltonians for the rotation of rigid $\mathrm{H}_{2}$ in the $\mathrm{H}_{2}-\mathrm{X}_{2}$ complex are

$$
\begin{align*}
& \hat{\mathbf{H}}_{\mathrm{rot}}\left(\theta_{\mathrm{H}_{2}}, \phi_{\mathrm{H}_{2}} ; R, \theta_{\mathrm{X}_{2}}, \phi_{\mathrm{X}_{2}}\right) \\
& \quad=B_{0} \hat{J}_{\mathrm{H}_{2}}^{2}+\Delta E\left(R, \theta_{\mathrm{X}_{2}}, \theta_{\mathrm{H}_{2}}, \phi_{\mathrm{H}_{2}}-\phi_{\mathrm{X}_{2}}\right) \tag{5}
\end{align*}
$$

where $B_{0}=0.6812 \mathrm{~kJ} / \mathrm{mol}$ is the rotational constant of the hydrogen molecule in the vibrational ground state, and $\hat{J}_{\mathrm{H}_{2}}$ is the corresponding angular momentum operator. The second term $\Delta E$ denotes the respective $\mathrm{H}_{2}-\mathrm{X}_{2}$ pair potential from quantum chemistry MP2/AVQZ or $\operatorname{CCSD}(\mathrm{T}) / \mathrm{AVQZ}$ calculations, see Sec. II. Equation (4) is solved using a representation in a basis of spherical harmonics $Y_{J M}\left(\theta_{\mathrm{H}_{2}}, \phi_{\mathrm{H}_{2}}\right)$, where the potential energy matrix elements

$$
\begin{align*}
& \Delta \mathbf{E}_{J^{\prime} M^{\prime} J M}\left(R, \theta_{\mathrm{X}_{2}}, \phi_{\mathrm{X}_{2}}\right) \\
& = \\
& =\frac{1}{4 \pi} \int_{1}^{-1} d \cos \theta_{\mathrm{H}_{2}} \int_{0}^{2 \pi} d \phi_{\mathrm{H}_{2}} \\
& \quad \times \Delta E\left(R, \theta_{\mathrm{X}_{2}}, \theta_{\mathrm{H}_{2}}, \phi_{\mathrm{H}_{2}}-\phi_{\mathrm{X}_{2}}\right)  \tag{6}\\
& \quad \times Y_{J^{\prime} M^{\prime}}^{*}\left(\theta_{\mathrm{H}_{2}}, \phi_{\mathrm{H}_{2}}\right) Y_{J M}\left(\theta_{\mathrm{H}_{2}}, \phi_{\mathrm{H}_{2}}\right)
\end{align*}
$$

$$
\begin{align*}
\approx & \frac{1}{4 \pi} \sum_{k=1}^{8} w_{k} \sum_{j=1}^{16} w_{j} \\
& \times \Delta E\left(R, \theta_{\mathrm{X}_{2}}, \theta_{\mathrm{H}_{2}, k}, \phi_{\mathrm{H}_{2}, j}-\phi_{\mathrm{X}_{2}}\right) \\
& \times Y_{J^{\prime} M^{\prime}}^{*}\left(\theta_{\mathrm{H}_{2}, k}, \phi_{\mathrm{H}_{2}, j}\right) Y_{J M}\left(\theta_{\mathrm{H}_{2}, k}, \phi_{\mathrm{H}_{2}, j}\right), \tag{7}
\end{align*}
$$

are approximately computed by means of a Gaussian quadrature, involving the weights $w_{k}$ and $w_{j}$, which implies the $a b$ initio PESs, $\Delta E$, to be evaluated at some definite GaussLegendre grid-points $\theta_{\mathrm{H}_{2}, k}$ and $\phi_{\mathrm{H}_{2}, j}$, with $\phi_{\mathrm{X}_{2}}$ held constant. The pair potentials $\Delta E$ subject to the rotational adiabatization are calculated using the following fixed bond lengths, $r_{\mathrm{H}_{2}}=76.7 \mathrm{pm}, r_{\mathrm{F}_{2}}=141.7 \mathrm{pm}, r_{\mathrm{Cl}_{2}}=203.3 \mathrm{pm}$, and $r_{\mathrm{Br}_{2}}$ $=228.1 \mathrm{pm}$, which correspond to vibrational ground state averages. For each point $\left(R, \theta_{\mathrm{X}_{2}}\right)$ the pair potential $\Delta E$ represented by a $16 \times 8$ Gauss-Legendre grid over $\theta_{\mathrm{H}_{2}, k}$ and $\phi_{\mathrm{H}_{2}, j}$ was calculated at the MP2/AVQZ level of theory, with $\phi_{\mathrm{X}_{2}}=45^{\circ}$. This $16 \times 8$ grid assures the first four eigenvectors to be well converged, for details see Ref. 8 . We performed linear scans over $\theta_{\mathrm{X}_{2}}$, ranging from $0^{\circ}$ to $90^{\circ}$, with an increment of $5^{\circ}$. For each value of $\theta_{\mathrm{X}_{2}}$, a varying number of monomer separations $R$, were evaluated. The smallest increment used in the $R$ direction was 10 pm . Collectively, the $\Delta E$ contain 50176,87552 , and 41600 individual structures for $\mathrm{H}_{2}-\mathrm{F}_{2}, \mathrm{H}_{2}-\mathrm{Cl}_{2}$, and $\mathrm{H}_{2}-\mathrm{Br}_{2}$, respectively. For $\theta_{\mathrm{X}_{2}}=0^{\circ}, \Delta E$ is of cylindrical symmetry, thus independent of $\phi_{\mathrm{H}_{2}}$, which reduces the number of non-equivalent Gauss-Legendre points from 128 to 8 . When $\theta_{\mathrm{X}_{2}}=90^{\circ}$ the potential has $C_{2 v}$ symmetry, hence it is sufficient to evaluate one quarter of the sphere, which means 32 points in our case. Exploiting these symmetries, we also performed $\operatorname{CCSD}(\mathrm{T}) / A V Q Z$ level of theory scans along $R$ for $\theta_{\mathrm{X}_{2}}=0^{\circ}$ and $\theta_{\mathrm{X}_{2}}=90^{\circ}$. Files containing all calculated points are provided in the supplementary material. ${ }^{68}$

The first eigenvector of Eq. (4), $\Psi_{0}$ is asymptotically $(R \rightarrow \infty)$ correlated with the free rotor state with $J=0$, thus it is the rotational ground state wave function of $p \mathrm{H}_{2}$ in the $\mathrm{H}_{2}-\mathrm{X}_{2}$ complex, to which belongs the RA potential $W_{0}\left(R, \theta_{\mathrm{X}_{2}}\right)$. Likewise, the next three states $\Psi_{1}, \Psi_{2}$, and $\Psi_{3}$ are asymptotically correlated with the free rotor states of $o \mathrm{H}_{2}$, with quantum numbers $J=1,(M=-1,0,1)$. Their corresponding eigenvalues $W_{n}\left(R, \theta_{\mathrm{X}_{2}}\right)$ define the RA potentials for $o \mathrm{H}_{2}$. For the linear complex $\left(\theta_{\mathrm{X}_{2}}=0^{\circ}\right)$, the first two $o \mathrm{H}_{2}$ states are degenerate and separated from the third state, hence $W_{1}\left(R, 0^{\circ}\right)=W_{2}\left(R, 0^{\circ}\right)<W_{3}\left(R, 0^{\circ}\right)$.

At large separations $R, W_{0}\left(R, \theta_{\mathrm{X}_{2}}\right)$ converges to zero for the free $p \mathrm{H}_{2}$, while $W_{1-3}\left(R, \theta_{\mathrm{X}_{2}}\right)$ converge to the rotational energy $2 B_{0}$ of free $o \mathrm{H}_{2}$. The interaction energies $\Delta W$ for the complexes are given by $\Delta W_{0}\left(R, \theta_{\mathrm{X}_{2}}\right)=W_{0}\left(R, \theta_{\mathrm{X}_{2}}\right)$ for $p \mathrm{H}_{2}$ and $\Delta W_{1-3}\left(R, \theta_{\mathrm{X}_{2}}\right)=W_{1-3}\left(R, \theta_{\mathrm{X}_{2}}\right)-2 B_{0}$ for $o \mathrm{H}_{2}$.

## B. Nuclear spin effect

Before we discuss the features of the RA pair potentials for the interaction energy $\Delta W_{0}$ and $\Delta W_{1}$ for $p \mathrm{H}_{2}-\mathrm{X}_{2}$ and $o \mathrm{H}_{2}-\mathrm{X}_{2}$ in detail, see Sec. III D, we compare the probability


FIG. 5. Potentials for $\mathrm{H}_{2}-\mathrm{Br}_{2}$ at the MP2/AVQZ level of theory (1st row, in $\mathrm{kJ} / \mathrm{mol}$ ) and the rotational densities for $\mathrm{pH}_{2}$ (2nd row) and $o \mathrm{H}_{2}$ (3rd-5th row), in the linear minimum (left panel, $\theta_{\mathrm{Br}_{2}}=0^{\circ}, R=420 \mathrm{pm}$ ) and the Tshape minimum (right panel, $\theta_{\mathrm{Br}_{2}}=90^{\circ}, R=354 \mathrm{pm}$ ) of the RA potential of $\mathrm{H}_{2}-\mathrm{Br}_{2}$. The position of high symmetry structures is indicated by $T 1, T 2, \ldots$, see Fig. 2.
distributions $\left|\Psi_{n}\right|^{2}$, effective interactions $\Delta W_{n}$, and the perturbing potentials $\Delta E$ for the first four rotational eigenstates of $\mathrm{H}_{2}$ exemplarily for the $\mathrm{H}_{2}-\mathrm{Br}_{2}$ system. We do so for two points ( $R, \theta_{\mathrm{Br}_{2}}$ ) close to the global and local minima of the RA potentials $\Delta W_{n}$. We choose $\mathrm{H}_{2}-\mathrm{Br}_{2}$ because the respective pair interaction is stronger than for $\mathrm{F}_{2}$ and $\mathrm{Cl}_{2}$, and thus perturbations to the free rotor states of $\mathrm{H}_{2}$ are more pronounced.

The coordinates $R=420 \mathrm{pm}, \theta_{\mathrm{Br}_{2}}=0^{\circ}$ are close to the global minima, i.e., the linear case, of the $p \mathrm{H}_{2}-\mathrm{Br}_{2}$ RA potential, see Sec. III D. Here, the part of the 4D pair potential $\Delta E(\mathcal{L})=\Delta E\left(420 \mathrm{pm}, 0^{\circ}, \theta_{\mathrm{H}_{2}}, \phi_{\mathrm{H}_{2}}-45^{\circ}\right)$, given in the top left panel of Fig. 5, depends only on $\theta_{\mathrm{H}_{2}}$. The minimum of
the static $T 2$ structure at $\theta_{\mathrm{H}_{2}}=90^{\circ}$ dominates the interaction, which becomes weaker approaching the unfavourable limit of the static L structure at $\theta_{\mathrm{H}_{2}}=0^{\circ}$ and $\theta_{\mathrm{H}_{2}}=180^{\circ}$. The distribution of $\left|\Psi_{0}\right|^{2}$ is notably perturbed by the potential, i.e., the ground state of $p \mathrm{H}_{2}$ is not spherical. Instead, it is more aligned to the static T2 structure and thereby less to the $L$ structure. Hence, the isotropic averaging approximation would underestimate the binding interaction of $p \mathrm{H}_{2}-\mathrm{Br}_{2}$. The free $o \mathrm{H}_{2}$ molecule has a triply degenerate rotational ground state. In the effective linear complex, this degeneracy is partly lifted. The lowest two $o \mathrm{H}_{2}$ eigenstates $\Psi_{1}, \Psi_{2}$ are degenerate and aligned to the static $T 2$ structure, with two nodes along $\phi\left(\mathrm{H}_{2}\right)$. As this alignment to the strongly bound $T 2$ structure is more pronounced for $o \mathrm{H}_{2}$, it also interacts stronger with $\mathrm{Br}_{2}$ than $p \mathrm{H}_{2}$. Note that this comes at the cost of an increase in kinetic energy by $2 B_{0}$ for $o \mathrm{H}_{2}$ over $p \mathrm{H}_{2}$. The third $o \mathrm{H}_{2}$ eigenstate $\Psi_{3}$ is aligned to the static $L$ structure and thus lies well above the first two states.

The energetic ordering of the rotational states at the T-shape local minimum, see right panel of Fig. 5, can be explained by the potential $\Delta E(\mathcal{T})=\Delta E(354 \mathrm{pm}$, $90^{\circ}, \theta_{\mathrm{H}_{2}}, \phi_{\mathrm{H}_{2}}-45^{\circ}$. First, it is energetically more isotropic than $\Delta E(\mathcal{L})$ as the interaction energies of the static $T 1, H$, and $X$ structures are similar at this intermolecular separation. Hence, interaction energies of the rotational states are closer together as well. Consequently, the para/ortho splitting is expected to be smaller. Second, the interaction is weaker than for the linear-shaped case. Thus, the perturbation with respect to the free rotor states is lower, as can be seen in the very isotropic distribution $\left|\Psi_{0}\right|^{2}$. Third, the potential depends on both rotational degrees of freedom, $\phi_{\mathrm{H}_{2}}$ and $\theta_{\mathrm{H}_{2}}$, of $\mathrm{H}_{2}$, which lifts the degeneracy of the free $o \mathrm{H}_{2}$ ground state. The $p \mathrm{H}_{2}$ density stays approximately spherically symmetric, since its alignment to the $T 1$ structure would imply a node in the wave function, a mixing of higher angular momentum states, that would require kinetic energies in the order of $6 B_{0}$. For $o \mathrm{H}_{2}$, we find that $\left|\Psi_{1}\right|^{2}$ essentially aligns to the $T 1,\left|\Psi_{2}\right|^{2}$ with the $H$, and $\left|\Psi_{3}\right|^{2}$ with the $X$ structure, which are energetically most, second, and least favoured. In summary, for the T-shaped complexes the alignment of $o \mathrm{H}_{2}$ to the $T 1$ structure leads to a larger binding interaction in comparison to $p \mathrm{H}_{2}$.

## C. Fit to analytic function

Analytic expressions of potentials facilitate the future use in simulations. The rotationally adiabatic MP2 pair potentials $\Delta W_{n}$ were fitted to a modified HFD function ${ }^{53}$

$$
\begin{align*}
\Delta W_{n}\left(R, \theta_{\mathrm{X}_{2}}\right)= & a\left(\theta_{\mathrm{X}_{2}}\right) \exp \left[-b\left(\theta_{\mathrm{X}_{2}}\right) R\right] \\
& -S(R)\left(\frac{C_{6}\left(\theta_{\mathrm{X}_{2}}\right)}{R^{6}}+\frac{C_{8}\left(\theta_{\mathrm{X}_{2}}\right)}{R^{8}}\right), \tag{8}
\end{align*}
$$

where the first term represents the Pauli repulsion interaction and the second term the dispersion interaction. The dispersion


FIG. 6. Rotationally adiabatic MP2/AVQZ potentials for the interaction $\Delta W$ of para- and ortho-hydrogen, in their rotational ground states, with fluorine, chlorine, and bromine molecules. Contour levels for $\Delta W$, ranging from -3 to $0 \mathrm{~kJ} / \mathrm{mol}$, are shown at $0.1 \mathrm{~kJ} / \mathrm{mol}$ increments.
decays algebraically, hence a switching function
$S(R)= \begin{cases}\exp \left[-\left(1.28 R_{e}\left(0^{\circ}\right) / R-1\right)^{2}\right], & \text { if } R \leq 1.28 R_{e}\left(0^{\circ}\right), \\ 1, & \text { if } R \geq 1.28 R_{e}\left(0^{\circ}\right),\end{cases}$
which truncates the dispersion energy toward short internuclear distances, becomes necessary. We have simplified the original HFD function, by omitting the $R^{2}$ dependence in the repulsion exponent and the $R^{-10}$ dependence of the dispersion, since 1 D cuts of the potentials for fixed $\theta_{\mathrm{X}_{2}}$, could be reproduced to good accuracy without them. We further decided to omit the angular dependence of the switching function $S$, by making it only dependent on the position $R_{e}\left(0^{\circ}\right)$ of the global minimum of the $p \mathrm{H}_{2}-\mathrm{X}_{2}$ potential. The angular dependence of the other fitting parameters $a, b, C_{6}$, and $C_{8}$ is expanded in terms of the first six even-ordered Legendre polynomials $P_{2 k}$,

$$
\begin{equation*}
X\left(\theta_{\mathrm{X}_{2}}\right)=\sum_{k=0}^{5} X_{2 k} P_{2 k}\left(\cos \theta_{\mathrm{X}_{2}}\right), \tag{10}
\end{equation*}
$$

where $X_{0}$ to $X_{10}$ are the corresponding expansion coefficients. Our 2D HFD fits reproduce the global minima with maximal deviations of $2 \%$. Tabulated fit results and original data points are provided in the supplementary material. ${ }^{68}$

## D. Potential energy surfaces for the interaction of $\mathrm{pH}_{2}$ and $\mathrm{oH}_{2}$ with $\mathrm{X}_{2}$

The rotationally adiabatic pair potentials for the interaction energy, $\Delta W$, of the $p / o \mathrm{H}_{2}-\mathrm{X}_{2}$ dimers for the rotational ground states of $p \mathrm{H}_{2}$ and $o \mathrm{H}_{2}$ have two minima. The respective contour plots, representing the results obtained from the MP2/AVQZ calculations, are given in Fig. 6, for minimum energy paths we refer to Fig. 7. Equilibrium distances and well depths for both minima are provided in Table I. These have been obtained for cuts of the potentials along $\theta_{\mathrm{X}_{2}}=0^{\circ}$


FIG. 7. Minimum MP2/AVQZ interaction energy angular paths for para- and ortho-hydrogen, in their rotational ground states, with fluorine, chlorine, and bromine molecules.

TABLE I. Ab initio well depths $D_{\mathrm{e}}[\mathrm{kJ} / \mathrm{mol}]$ and equilibrium distances $R_{\mathrm{e}}[\mathrm{pm}]$ for the $p / o \mathrm{H}_{2}-\mathrm{X}_{2}$ and $\mathrm{Rg}-\mathrm{X}_{2}$ complexes with $\mathrm{X}=\mathrm{F}, \mathrm{Cl}, \mathrm{Br} ; \mathrm{Rg}=\mathrm{He}, \mathrm{Ne}, \mathrm{Ar}$ as well as small molecules from the literature.

| Complex | Ref. | Method | $\text { Linear }\left(\theta_{\mathrm{X}_{2}}=0^{\circ}\right)$ |  | T-shape ( $\theta_{\mathrm{X}_{2}}=90^{\circ}$ ) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $D_{\text {e }}$ | $R_{\mathrm{e}}$ | $D_{\text {e }}$ | $R_{\text {e }}$ |
| $p \mathrm{H}_{2}-\mathrm{F}_{2}$ | present work | CCSD(T) |  |  | 0.678 | 322 |
|  |  | MP2 | 0.932 | 357 | 0.552 | 328 |
| $o \mathrm{H}_{2}-\mathrm{F}_{2}$ | present work | $\operatorname{CCSD}(\mathrm{T})$ | 0.986 | 356 | 0.725 | 325 |
|  |  | MP2 | 1.081 | 353 | 0.606 | 330 |
| $\mathrm{He}-\mathrm{F}_{2}$ | 25 | $\operatorname{CCSD}(\mathrm{T})$ | 0.429 | 347 | 0.382 | 300 |
| $\mathrm{Ne}-\mathrm{F}_{2}$ | 25 | $\operatorname{CCSD}(\mathrm{T})$ | 0.736 | 359 | 0.735 | 308 |
| Ar-F ${ }_{2}$ | 25 | $\operatorname{CCSD}(\mathrm{T})$ | 1.469 | 388 | 1.316 | 344 |
| $p \mathrm{H}_{2}-\mathrm{Cl}_{2}$ | present work | $\operatorname{CCSD}(\mathrm{T})$ | 1.686 | 412 | 1.390 | 343 |
|  |  | MP2 | 1.882 | 408 | 1.419 | 342 |
| $o \mathrm{H}_{2}-\mathrm{Cl}_{2}$ | present work | $\operatorname{CCSD}(\mathrm{T})$ | 1.938 | 409 | 1.459 | 345 |
|  |  | MP2 | 2.144 | 404 | 1.518 | 343 |
| $\mathrm{He}-\mathrm{Cl}_{2}$ | 23 | MP4 | 0.481 | 420 | 0.449 | 345 |
| $\mathrm{Ne}-\mathrm{Cl}_{2}$ | 23 | MP4 | 0.973 | 427 | 0.925 | 350 |
| $\mathrm{Ar}-\mathrm{Cl}_{2}$ | 23 | $\operatorname{CCSD}(\mathrm{T})$ | 2.572 | 448 | 2.524 | 374 |
| $\mathrm{pH}_{2}-\mathrm{Br}_{2}$ | present work | $\operatorname{CCSD}(\mathrm{T})$ | 2.111 | 428 | 1.539 | 353 |
|  |  | MP2 | 2.330 | 424 | 1.567 | 352 |
| $o \mathrm{H}_{2}-\mathrm{Br}_{2}$ | present work | $\operatorname{CCSD}(\mathrm{T})$ | 2.410 | 425 | 1.623 | 356 |
|  |  | MP2 | 2.638 | 421 | 1.682 | 354 |
| $\mathrm{He}-\mathrm{Br}_{2}$ | 26 | $\operatorname{CCSD}(\mathrm{T})$ | 0.584 | 442 | 0.482 | 358 |
| $\mathrm{Ne}-\mathrm{Br}_{2}$ | 26 | $\operatorname{CCSD}(\mathrm{T})$ | 1.120 | 449 | 0.805 | 360 |
| $\mathrm{Ar}-\mathrm{Br}_{2}$ | 26 | $\operatorname{CCSD}(\mathrm{T})$ | 3.143 | 463 | 2.708 | 380 |


| Complex | Ref. | Method | Linear |  |  |  | T-shape |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $D_{\text {e }}$ | $R_{\text {e }}$ | $D_{\text {e }}$ | $R_{\text {e }}$ | $D_{\text {e }}$ | $R_{\text {e }}$ |
| $\frac{p \mathrm{H}_{2}-p \mathrm{H}_{2}}{p \mathrm{H}_{3}-\mathrm{CO}}$ | 44, 75 | $\operatorname{CCSD}(\mathrm{T})$ | 0.295 | 346 |  |  |  |  |
|  | 44, 76 | $\operatorname{CCSD}(\mathrm{T})$ | $\theta_{1}=0^{\circ}(\mathrm{C})$ |  | $\theta_{1}=180^{\circ}(\mathrm{O})$ |  | $\theta_{1}=85.8^{\circ}$ |  |
|  |  |  | 0.443 | 440 | 0.514 | 400 | 0.610 | 360 |
| $\overline{p \mathrm{H}_{2}-\mathrm{CO}_{2}}$ | 44, 78 | $\operatorname{CCSD}(\mathrm{T})$ |  |  | $\theta_{1}=180^{\circ}$ |  | $\theta_{1}=90.0^{\circ}$ |  |
|  |  |  |  |  | 0.667 | 444 | 1.340 | 319 |
| $\overline{p \mathrm{H}_{2}-\mathrm{N}_{2} \mathrm{O}}$ | 79 | $\operatorname{CCSD}(\mathrm{T})$ | $\theta_{1}=0^{\circ}(\mathrm{N})$ |  | $\theta_{1}=180^{\circ}(\mathrm{O})$ |  | $\theta_{1}=92.58^{\circ}$ |  |
|  |  |  | 0.60 | 460 | 0.821 | 429 | 1.691 | 308 |
| $o \mathrm{H}_{2}-\mathrm{N}_{2} \mathrm{O}$ | 79 |  | $\theta_{1}=0^{\circ}(\mathrm{N})$ |  | $\theta_{1}=180^{\circ}$ |  | $\theta_{1}=92.88^{\circ}$ |  |
|  |  |  | 0.86 | 460 | 1.02 | 430 | 2.09 | 302 |
| $\overline{p \mathrm{H}_{2}-\mathrm{HCN}}$ | 80, 81 | $\begin{aligned} & \hline \operatorname{CCSD}(\mathrm{T})-\mathrm{F} 12 \mathrm{a} \\ & \text { spherical average } \end{aligned}$ | $\theta=0^{\circ}(\mathrm{H})$ |  | $\theta=180^{\circ}(\mathrm{N})$ |  | $\theta \approx 60^{\circ}$ |  |
|  |  |  | $\approx 0.66$ | 410 | 0.948 | 430 | 0.750 | 407 |
| $\overline{p \mathrm{H}_{2}-\mathrm{OCS}}$ | 15,77 | MP4spherical average | $\theta=0^{\circ}(\mathbf{S})$ |  | $\theta=180^{\circ}(\mathrm{O})$ |  | $\theta=105^{\circ}$ |  |
|  |  |  | 1.093 | 452 | 0.828 | 492 | 1.729 | 335 |
| $\overline{p \mathrm{H}_{2}-\mathrm{H}_{2} \mathrm{O}}$ | 22, 82 | $\begin{gathered} \operatorname{CCSD}(\mathrm{T}), \\ \operatorname{CCSD}(\mathrm{T})-\mathrm{R} 12 \\ \hline \end{gathered}$ |  |  | $\theta=110.00^{\circ}, \chi=0^{\circ}$ |  |  |  |
|  |  |  |  |  | 1.167 | 336 |  |  |

and $\theta_{\mathrm{X}_{2}}=90^{\circ}$, respectively. The global minima of $\Delta W$ correspond to linear $\left(\theta_{\mathrm{X}_{2}}=0^{\circ}\right) p / o \mathrm{H}_{2}-\mathrm{X}_{2}$ complexes, which is anticipated, since the underlying static 4D pair potential for $\Delta E$ for $\theta_{\mathrm{X}_{2}}=0^{\circ}$, see Secs. II and III B, involves the static global minima $T 2$ structures. With $\Delta E$ calculated at the MP2/AVQZ level of theory, the minimum interaction energies $\Delta W$ for the linear complexes range from $-0.93 \mathrm{~kJ} / \mathrm{mol}$ for $p \mathrm{H}_{2}-\mathrm{F}_{2}$ to $-2.64 \mathrm{~kJ} / \mathrm{mol}$ for $o \mathrm{H}_{2}-\mathrm{Br}_{2}$. The local minimum of each re-
spective potential $\Delta W$, corresponds to T-shaped ( $\theta_{\mathrm{X}_{2}}=90^{\circ}$ ) $p / o \mathrm{H}_{2}-\mathrm{X}_{2}$ complexes, where the averaging over $\Delta E$ includes mainly the static $T 1, H$, and $X$ structures. The interaction energies for the T-shaped complexes at the MP2/AVQZ level of theory range from $-0.55 \mathrm{~kJ} / \mathrm{mol}$ for $p \mathrm{H}_{2}-\mathrm{F}_{2}$ to $-1.68 \mathrm{~kJ} / \mathrm{mol}$ for $o \mathrm{H}_{2}-\mathrm{Br}_{2}$. The anisotropy in the energy, i.e., the difference between global and local minima, ranges from $0.38 \mathrm{~kJ} / \mathrm{mol}$ for $p \mathrm{H}_{2}-\mathrm{F}_{2}$ to $0.96 \mathrm{~kJ} / \mathrm{mol}$ for $o \mathrm{H}_{2}-\mathrm{Br}_{2}$. The energy values at the
saddle points, which are located in the vicinity of $\theta_{\mathrm{X}_{2}}=40^{\circ}$, range from $-0.42 \mathrm{~kJ} / \mathrm{mol}\left(p \mathrm{H}_{2}-\mathrm{F}_{2}\right)$ over $-0.73 \mathrm{~kJ} / \mathrm{mol}\left(p \mathrm{H}_{2}-\right.$ $\left.\mathrm{Cl}_{2}\right)$ to $-0.95 \mathrm{~kJ} / \mathrm{mol}\left(o \mathrm{H}_{2}-\mathrm{Br}_{2}\right)$ with small para/ortho splittings of $0.07 \mathrm{~kJ} / \mathrm{mol}, 0.10 \mathrm{~kJ} / \mathrm{mol}$, and $0.13 \mathrm{~kJ} / \mathrm{mol}$, for $\mathrm{F}_{2}, \mathrm{Cl}_{2}$, and $\mathrm{Br}_{2}$, respectively. For any given $\theta_{\mathrm{X}_{2}}$, the complexes with $o \mathrm{H}_{2}$ are bound more strongly than those with $p \mathrm{H}_{2}$, which can be explained in terms of a higher degree of alignment to more stable static structures for $\mathrm{oH}_{2}$, see also Sec. III B. The rotationally adiabatic pair potentials with $\mathrm{Cl}_{2}$ and $\mathrm{Br}_{2}$ are more similar in comparison to $\Delta W$ with $\mathrm{F}_{2}$, as the potentials for $p / o \mathrm{H}_{2}-\mathrm{F}_{2}$ are significantly less anisotropic in terms of both energies and distances.

For both minima of the $\Delta W$ potentials, the respective $\operatorname{CCSD}(\mathrm{T}) / \mathrm{AVQZ}$ results, given in Table I, lead essentially to the same conclusions, as drawn from the MP2/AVQZ results. However, we find that the MP2 method overestimates the strength of the van der Waals bonds for the linear $p / o \mathrm{H}_{2}-$ $\mathrm{X}_{2}$ complexes by $10 \%$, in comparison to $\operatorname{CCSD}(\mathrm{T})$ results. For the T-shaped complexes of $p / o \mathrm{H}_{2}$ with $\mathrm{Cl}_{2}$ and $\mathrm{Br}_{2}$, MP2 overestimates the bonding interaction by $2 \%-5 \%$ and underestimates it for the T-shaped $\mathrm{F}_{2}$ complexes by up to $19 \%$. Thus, the $p / o \mathrm{H}_{2}-\mathrm{F}_{2}$ potentials are even less anisotropic than suggested by the MP2 results. While relative deviations up to $19 \%$ seem to be quite high, the corresponding absolute deviations are on the order of only hundredths to tenths of $\mathrm{kJ} / \mathrm{mol}$, due to the overall weakness of the interactions.

## IV. CONCLUSIONS

We calculated the pair interaction of the hydrogen molecule with a fluorine, chlorine, and bromine molecule up to the $\operatorname{CCSD}(\mathrm{T})$ level of theory in the complete basis set limit. The highest binding interaction is present, for structure T2, where the molecular axis of the halogen molecule points perpendicular to the molecular axis of the hydrogen molecule. The second highest binding interaction belongs to structure T1, where the molecular axis of the hydrogen molecules points perpendicular to the molecular axis of the halogen molecule. A simple hard sphere model with quadrupolequadrupole interactions underestimates the binding interaction globally and predicts the highest binding interaction for structure T1. As this model lacks any dispersion forces, the comparison with the quantum chemistry results confirms, that they are essential to describe the pair interaction of the $\mathrm{H}_{2}-\mathrm{X}_{2}$.

To investigate the interaction energy of $p \mathrm{H}_{2}$ and $o \mathrm{H}_{2}$ molecules with the halogen molecules, the pair potentials have been adiabatized with respect to the rotational degrees of freedom of $\mathrm{H}_{2}$. The whole effective pair potentials have been calculated at the MP2/AVQZ level of theory, while for linear and T-shaped complexes additional CCSD(T)/AVQZ calculations could be performed. The global minimum structure is always a linear complex. Due to higher alignment to static low energy structures, the $o \mathrm{H}_{2}-\mathrm{X}_{2}$ complexes are more strongly bound than the respective $p \mathrm{H}_{2}-\mathrm{X}_{2}$ complexes. For bromine and chlorine, the MP2/AVQZ level of theory overestimates the well depths of the linear complex by up to $10 \%$, and up to $5 \%$ for the T-shaped complex. For fluorine, the well depth of the T-shaped complex is underestimated by up to $19 \%$ at
the MP2/AVQZ level of theory. For future utilization of the effective potentials, we provide analytic fits to HDF model functions.

We are now in the position to compare the interactions of $p \mathrm{H}_{2}$ and $o \mathrm{H}_{2}$ with $\mathrm{X}_{2}$ to the respective rare gas atom ( $\mathrm{Rg}=\mathrm{He}, \mathrm{Ne}$, and Ar ) dihalogen interactions from the literature. ${ }^{23,25,26}$ Each of the $\mathrm{Rg}-\mathrm{X}_{2}$ 2D pair potentials also has a global minimum for linear and a local minimum for T shaped structures. However, the potentials are known for their low anisotropy, with similar energies for both structures, see also Table I, whereas our RA potentials with $p \mathrm{H}_{2}$ and $o \mathrm{H}_{2}$ are much more anisotropic. The well depths are ordered $\mathrm{He}<\mathrm{Ne}$ $<\mathrm{pH}_{2}<o \mathrm{H}_{2}<\mathrm{Ar}$, irrespective of the choice of the halogen molecule and for both minima. Equilibrium distances for the $p \mathrm{H}_{2}-\mathrm{X}_{2}$ and $o \mathrm{H}_{2}-\mathrm{X}_{2}$ complexes are similar to those of the respective He or Ne complexes.

Our findings are in line with the results by Darr et al. for the $\mathrm{H}_{2}-\mathrm{I}_{2}$ complex. ${ }^{38}$ They experimentally identified a linearshaped as the most stable and a T-shaped as the second most stable conformer. The experimental binding energy $D_{0}$ of the linear $o \mathrm{H}_{2}-\mathrm{I}_{2}$ complex is $1.42 \mathrm{~kJ} / \mathrm{mol}$ and thus higher than the $1.22 \mathrm{~kJ} / \mathrm{mol}$ found for $p \mathrm{H}_{2}-\mathrm{I}_{2}$. They also reported MP2/AVTZ well depths $\left(D_{e}\right)$ for symmetric $\mathrm{H}_{2}-\mathrm{I}_{2}$ complexes and concluded that they lie between the values for the respective Ne and Ar complexes. Further, they analysed differences in the charge densities indicating a larger quadrupole-quadrupole interaction for the static $T 2$ structure compared to the $L$ orientation.

In his thesis, ${ }^{39}$ Tat Pham Van reported $\operatorname{CCSD}(\mathrm{T}) /$ $\mathrm{AV}(\mathrm{DT}) \mathrm{Z}$ scans of the $\mathrm{H}_{2}-\mathrm{F}_{2}$ potential $\Delta E$, fitted the results to a 5 -site potential and calculated virial coefficients. Our calculations confirm the features of the $\mathrm{H}_{2}-\mathrm{F}_{2}$ potential and extend the angular and spatial resolution.

In addition, we have also collected effective RA pair potentials describing the interaction of $\mathrm{pH}_{2}$ with other small molecules from the literature in the lower part of Table I. The one dimensional $p \mathrm{H}_{2}-p \mathrm{H}_{2}$ potential has a very low well depth of $0.296 \mathrm{~kJ} / \mathrm{mol}$ and an equilibrium distance of 346 $\mathrm{pm} .{ }^{44,75}$ The potential of $p \mathrm{H}_{2}-\mathrm{CO}$ has three minima. ${ }^{44,76}$ In contrast to $\mathrm{pH}_{2}-\mathrm{X}_{2}$, the global minimum corresponds to a slightly disturbed T-shaped complex with a well depth of $0.610 \mathrm{~kJ} / \mathrm{mol}$. Concerning the linear minima with well depths of $0.443 \mathrm{~kJ} / \mathrm{mol}$ and $0.514 \mathrm{~kJ} / \mathrm{mol}, p \mathrm{H}_{2}$ at the O side of CO is favoured. ${ }^{44}$ Compared to the halogen interactions the potential is energetically less anisotropic. T-shaped global minimum structures were also found for the $\mathrm{pH}_{2}-$ OCS, ${ }^{15,77} p \mathrm{H}_{2}-\mathrm{CO}_{2},{ }^{44,78} p \mathrm{H}_{2}-\mathrm{N}_{2} \mathrm{O}$, and $o \mathrm{H}_{2}-\mathrm{N}_{2} \mathrm{O}^{79}$ complexes. The anisotropy of these potentials, however is larger than found for the $p \mathrm{H}_{2}-\mathrm{X}_{2}$ potentials. An example for a triatomic molecule where a linear complex is stronger bound is $\mathrm{HCN} .{ }^{80,81}$ In this complex, $p \mathrm{H}_{2}$ binds stronger to the N -side and a distorted T-shaped structure is bound more weakly, although we have to note that this result is based on spherical averaging. For the $p \mathrm{H}_{2}-\mathrm{H}_{2} \mathrm{O}$ complex, the rotational adiabatization leaves only one minimum on the effective potential energy surface, where $p \mathrm{H}_{2}$ is near a hydrogen nucleus of the water molecule. ${ }^{22,82} \mathrm{We}$ conclude that $p \mathrm{H}_{2}$ interacts less with CO than with $\mathrm{F}_{2}$. The strength of interaction with $p \mathrm{H}_{2}$ is similar for $\mathrm{F}_{2}, \mathrm{H}_{2} \mathrm{O}$, and HCN . Concerning the values for $D_{e}, p \mathrm{H}_{2}$
interacts stronger with $\mathrm{CO}_{2}$ than with $\mathrm{F}_{2}$ and even more with $\mathrm{N}_{2} \mathrm{O}, \mathrm{OCS}, \mathrm{Cl}_{2}$, and $\mathrm{Br}_{2}$.

We have presented the fundamental RA pair potentials for the interaction of para- and ortho-hydrogen with fluorine, chlorine, and bromine, based on MP2 and $\operatorname{CCSD}(\mathrm{T})$ data. Hopefully, the present results will serve as a basis for further theoretical and experimental studies. From the RA potentials for $p \mathrm{H}_{2}-\mathrm{X}_{2}$ and $p \mathrm{H}_{2}-p \mathrm{H}_{2}{ }^{44,75}$ one can, in a pairwise fashion, approximate the many body potential of $\mathrm{X}_{2}\left(p \mathrm{H}_{2}\right)_{n}$ clusters with small $n$, where the $\mathrm{H}_{2}$ rotational states are still mostly determined by the $p \mathrm{H}_{2}-\mathrm{X}_{2}$ interaction. However, when studying $\mathrm{X}_{2}\left(p \mathrm{H}_{2}\right)_{n}$ clusters with large $n$, the RA potentials for $\mathrm{H}_{2}-\mathrm{X}_{2}$ are of limited use, because the $\mathrm{H}_{2}$ rotational dynamics may be dominated by the $p \mathrm{H}_{2}-p \mathrm{H}_{2}$ interaction. In such cases, it may be more convenient to use the tabulated 4 D potentials provided in the supplementary material. ${ }^{68}$

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