Subsidy and Revenue Maximization in Developing Countries

Mbuyu Sumbwanyambe, *Member, IAENG* and A. L. Nel, *Member, IAENG*

***Abstract*—Developing countries have embarked on the promotion of "ICT access for all" through subsidized Information and Communication Technologies (ICTs), especially in underserviced areas of such countries. The main aim of the "ICT access for all" is to extend the communication services to the large areas of underserviced regions through subsidized communication services. In some instances, subsidies may lead to high ICT penetration and high resource utilization while in some instances unsubsidized services may lead to low utilization of resources and low ICT penetration, which may eventually lead to market failure and destroy market efficiency. With explicitly defined objectives, regarding subsidy policy, however, developing countries always fall short on the implementation of such subsidy policy due to economic reasons and unrealistic subsidy driven pricing models. In this paper we investigate the impact of subsidy driven pricing model on resource utilization and revenue maximization in a developing country. In this paper we try to find a middle ground between promoting "ICT access for all" (given a subsidy and diverse income variations between the groups) and resource utilization in a network.**

***Index Terms*—Pricing, Developing country, Subsidy, ICTs**

I. INTRODUCTION

IN developing countries the amount and characteristics of consumers of ICT services affect revenue, quality of service (QoS) and have substantial financial and business model implications for providers and consumers alike [4]. On the whole, competitive environments with numerous service providers are more likely to offer lower costs, high QoS, and are more inclined to create and adjust to a dynamic communications environment. On the other hand, monopolistic service providers who are common in developing countries are inclined to serve large urban areas, and are reluctant to enter smaller and rural markets where there is little or no possibility to derive a profit. To that effect, governments of developing countries have required and persuaded such monopolistic suppliers to serve rural areas through a combination of service requirements and subsidies [3] [5] [8], but have had limited success in creating competitive rural markets. In much of the developing world, therefore, rural access remains limited in scope, regularity, and quality. New entrants typically give precedence to the profitable urban markets, and may enter rural areas and underserviced regions with infrastructure that is insufficient for effective service provision. This observation has led to the exclusion of underserviced areas or rural communities from the urban areas in terms of ICT access.

The exclusion of rural communities and underserviced areas from ICT resources in developing countries perpetuates the problems associated with socio-economic deprivation (specifically poverty). Thus, most rural and underserviced areas in developing countries lack access to basic ICT services, creating a gap between the information "haves" and "have nots"[4] [7] [8].

The gap between the information "haves" and "have nots" creates a group of customers whose preferences and socio-economic status are very diverse. This heterogeneity can arise from economical imbalances due to poverty (income disparity) or from different geographic locations or both [12]. To effectively rectify the existing inequalities in access to ICT services, governments of developing countries use different economic mechanisms, commonly funded by taxes.

The current practice in such countries has been to promote information access by means of tiered subsidized pricing scheme. However, in countries where technical expertise is still in its embryonic stages, subsidized pricing schemes have proved to be very difficult to implement, and, in certain cases, it is unclear whether the user's threshold values and sensitivities towards price are taken into consideration [10]. Subsidy driven pricing mechanisms have, often times, resulted in controversies and may on certain occasion led to resource tragedies in such countries

In a heterogeneous society (especially in developing countries), subsidized resource use can lead to the tragedy of the (anti) commons [12]. If too large a subsidy is given, consumers will demand more than is available creating the tragedy of the commons [6]. In order to prevent the tragedy of the commons, service providers and government policy makers have to cut the subsidy level which will eventually reduce the number of consumers. The decrease in the number of consumers (especially rural consumers) may then lead to the tragedy of the anti-commons [1], leaving a gap in government policy and objectives of promoting social and economic growth. In such an eventuality, the government will have to lower the prices yet again, via a subsidy, in order fulfill their objectives, creating a *see-saw effect*.

In order to prevent this tragedy of the (anti) commons in the telecom markets, a correct subsidy enhanced price structure must be developed for such a market. The use of the correct subsidy level to promote ICT access in underserviced areas can be seen as a balanced two tier-pricing scheme (taking into consideration customer sensitivities and price thresholds). Furthermore, it can be seen as a means of preventing the

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tragedy of the (anti) commons and at the same time promoting social and economic growth. By setting up the right subsidy (an equilibrium subsidy) mechanism, to induce a desirable number of users, the service providers and government can achieve the optimal utility. In this paper, we study the impact of subsidies on the network resource usage by the information "have nots" and propose a subsidy driven pricing model that will mitigate the tragedy of the (anti) commons. We study all the possible outcomes of the subsidy on the information "have nots" and find a fair solution that will prevent the underusage or overusage of network resources by the subsidized group. This paper is organized as follows: In section II we propose our pricing model describing all the necessary mathematical concepts and formulas, and in section III we evaluate how subsidies affect user usage in a heterogeneous society, in section IV we develop our revenue maximizing equation and propose a new class of mechanisms for addressing the tragedy of the (anti)commons problem that arises in the usage of subsidy so as to promote social and economic growth in underserviced areas. Section VI concludes our study.

II. PRICING MODEL

We consider a case of a developing country with heterogeneous consumers and an observable income disparity or geographical disadvantage; the information "haves" (N_2) and "have-nots" (N_1). In heterogeneous societies, such as South Africa and Zambia, consumers have different service requirements and sensitivities towards price. In such developing countries, service providers can offer different charging standards for each consumer and can choose a suitable price based on consumers' price acceptance, sensitivities and threshold values towards a set price and adjust their price accordingly so as to obtain an optimal price at which a both types of consumers will be more willing to pay for the service. Such characteristics can be learned through a market price adaptive research which estimates the threshold values of users and sensitivities in such an environment. In fact, the process of learning price parameters in such a market is a dynamic process with the aim of adjusting the quoted price in line with the consumers' behavior towards price dynamics.

Within this current perspective, we assume that the N_1 consumers are offered a subsidy so as to promote social and economic growth in underserviced areas, while the N_2 users pay the price that is set by the services provider. We take p_1 and p_2 as the price that is payable to the service provider by the heterogeneous consumers N_1 and N_2, respectively, where $p_1 = \beta p_2$, where β is a subsidy factor and is in the range of $0 < \beta < 1$ and $p_2 \in (0, p_\infty)$ while c and λ is the cost of providing network services. The following definition is required.

Definition 1 *The threshold or reserve price, p_{thi} for $i \in (1, 2)$ is the price at which a customer from group* i *is indifferent between subscribing to the current network or opting out of the current network.* □

Assuming that p_{thi} for $i \in (1, 2)$ is the reservation price, the consumer's utility function can thus be written as $u_{(pthi,pi)}$ for $i \in (1, 2)$. If $p_{thi} - p_i > 0$ the consumers has positive utility, if $p_{thi} - p_i < 0$ then the consumer has a negative utility. For notational simplicity we assume that consumers attempt to maximize their utility subject to the reservation price of p_{th1} for N_1 and p_{th2} for N_2, where $p_{th1} < p_{th2} < p_\infty$.

III. TRAGEDIES AND RESOURCE UTILIZATION IN HETEROGENEOUS COMMUNITIES

Depending on the subsidy level, the total number of information "have nots" will increase or decrease leading to the tragedy of the commons or tragedy of the anti-commons respectively. We define the following:

Definition 2 *The tragedy of the commons is a dilemma arising from the situation in which consumers, acting independently and rationally consulting their own self-interest, will ultimately deplete a shared limited resource, even when it is clear that it is not in anyone's long-term interest for this to happen [6].* □

Definition 3 *The tragedy of the anti-commons, describes a coordinated breakdown in a game where the existence of high prices for a public good or numerous holders of certain rights hinder the achievement of more socially desirable outcome [1].* □

In developing countries, it is obvious that pricing structures that are currently at use affect network usage and are not helping in promoting social and economical growth. To that effect, subsidy driven pricing may be a valuable tool in promoting social and economic growth in underserviced regions of developing countries. From the users' point of view, price affects their behavior in terms of network usage, which is correlated to their perception about the price for such a service. Therefore, choosing the right price and subsidy by the service provider and government respectively, is of great importance in maximizing revenue and preventing resource tragedies. It is therefore imperative that government policy makers and service providers find a middle ground between "ICT access for all" and resource overuse or underuse given a subsidy β and users' reservation price i.e. p_{th1} and p_{th2}.

In underserviced regions of some developing countries the reservation price is associated with a number of factors. Primarily, any heterogeneous user is assumed to maximize his or her utility subject to a predetermined level of his or her income or reservation price [2]. Some of the probable factors associated with users' reservation price in developing countries are what is perceived as positive or negative utility experienced by the consumer. For example, for people who live on less than USD 2 a day [4], pricing the internet services beyond such an amount, i.e. USD 2, will constitute what is known as a negative utility towards the customer or consumer. To remove such a negative utility, a subsidy has to be paid by the government to the ISP. However, such a subsidy may have an adverse effect on resource usage, number of users and revenue maximization in a network.

To understand the impact of subsidies on user behavior and revenue maximization in developing countries, especially on the information "have nots", one has to understand how subsidies affect the number of users subscribing to an ISP. The number of users in this case determines the revenue and the number of packets sent in any network. For example, Figure 1 and Figure 2 shows a relationship between packets and price given a government subsidy factor $\beta = 0.1$, $\beta = 1$, sensitivity $\alpha = 0.1$, $\lambda = 100$, $p_{th1} = 40$ and $p_{th2} = 100$. Analysis of the graphs shows that the number of packets seem to fluctuate with increasing or decreasing subsidy factor β. For instance, for a subsidy factor of 0.1, as in Figure 1, the number of packets increases significantly due to an increase in the number of users, while the opposite is true for no subsidy ($\beta = 1$). Obviously, there is an elastic relationship between price and the number of users that will enable us to make some generalized observations about the entire information "have nots" population. We list these observations as below:

- There exists a finite price $p_1 > p_{th1}$ where a customer will not subscribe to any ISP.
- Information "have nots" are rational, they will prefer a subsidized price or a lower price to a higher price.
- The number of information "have nots" who will accept a given price in any heterogeneous communities of developing countries does not increase with price.

Given the above motivation, we can therefore define the number of packets for the N_1 and N_2 in a network as a function of price p_1 and p_2 and the reservation price p_{th1} and p_{th2} respectively. Jagannathan and Almeroth [13] and Li et al [14] investigate user behavior upon price change using a demand function model. We adopt this model with some modifications to reflect our price setting and the number of users. When the price tends to zero the number of users increases significantly and when the price is very high the number of users decreases significantly. The following equations are given for the number of N_1 and N_2 users and number of packets in a network accordingly:

$$N_1 = \left(\frac{\beta p_2}{p_{th1}}\right)^{-\alpha} - 1 \tag{1}$$

$$N_2 = \left(\frac{p_2}{p_{th2}}\right)^{-\alpha} - 1 \tag{2}$$

Combining equations (1) and (2) and multiplying with λ will give us the total number of packets in a network:

$$\text{Total}_{packets} = \lambda\left(\frac{\beta p_2}{p_{th1}}\right)^{-\alpha} - 1 + \lambda\left(\frac{p_2}{p_{th2}}\right)^{-\alpha} - 1 \tag{3}$$

Where $0 < \alpha \leq 1$ is the probabilistic response of consumers towards price, and it increases with an increasing value of α. Actually, α can be described as the perceived utility of consumers given a certain price. From equation (3) we can define the unused capacity for the network by subtracting number of packets from the total network capacity.

IV. SUBSIDIES, TRAGEDIES AND REVENUE MAXIMIZATION IN HETEROGENEOUS SOCIETIES

When a consumer uses shared resources in a network, that consumer should pay a corresponding price. The dynamics of correct subsidy implementation in a heterogeneous community could affect the individual user's (in this case the information "have nots") decision in such a way that tragedy of the commons and tragedy of the anti-commons can be avoided. Given the preferences of consumers for a given price p_2 and a cost c, the profit function of the service provider can be written and defined as follows:

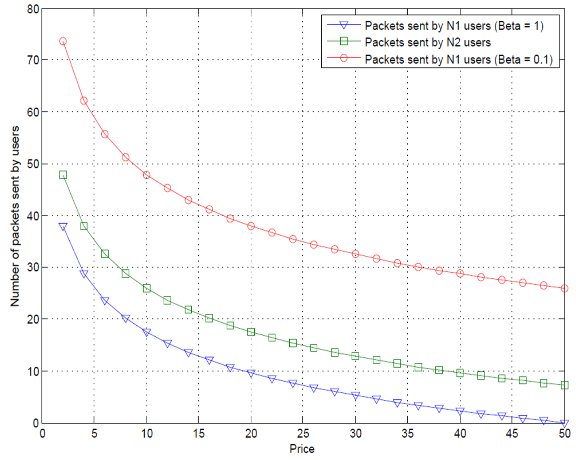

Fig. 1. Number of packets versus the price at different subsidy levels

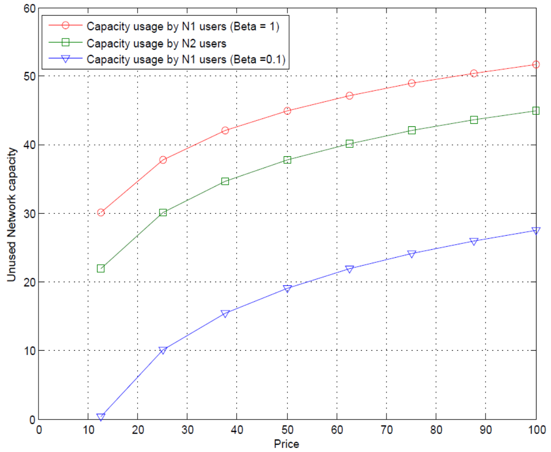

Fig. 2. Unused network capacity versus the price at different subsidy levels (Network capacity = 45 for N_1 and N_2)

$$\Pi = \lambda\left(\left(\frac{\beta p_2}{p_{th1}}\right)^{-\alpha} - 1\right)(p_2 - c)$$
$$+\lambda\left(\left(\frac{p_2}{p_{th2}}\right)^{-\alpha} - 1\right)(p_2 - c) \tag{4}$$

Taking the partial derivatives of 4, we obtain an expression of Π as follows:

$$\frac{\partial \Pi}{\partial p_2} = \frac{\lambda}{(\frac{\beta p_2}{p_{th1}})^{\alpha} - 1} + \frac{\lambda}{(\frac{p_2}{p_{th2}})^{\alpha} - 1} +$$
$$\frac{\lambda\alpha(c - p_2)}{p_{th2}(\frac{p_2}{p_{th2}})^{\alpha+1}} + \frac{\lambda\alpha\beta(c - p_2)}{p_{th1}(\frac{\beta p_2}{p_{th1}})^{\alpha+1}} \tag{5}$$

Further differentiation of equation (5) provides us with:

$$\frac{\partial^2\Pi}{\partial p_2^2} = -\left(\frac{2\lambda\alpha}{(p_{th2}\frac{p_2}{p_{th2}})^{\alpha}+1}\right) - \left(\frac{2\lambda\alpha\beta}{(p_{th1}\frac{\beta p_2}{p_{th1}})^{\alpha}+1}\right)$$
$$-\left(\frac{\lambda\alpha(c-p_2)(\alpha+1)}{(p_{th2}^2(\frac{p_2}{p_{th2}})^{\alpha+2}}\right) - \left(\frac{\lambda\alpha\beta^2(c-p_2)(\alpha+1)}{p_{th1}^2(\frac{\beta p_2}{p_{th1}})^{\alpha+2}}\right) \quad (6)$$

A close analysis of equation (4), (5) and (6) shows that there is an optimal solution (price) to equation (4) that provides the service provider with the maximum revenue or profit. Thus, from equation (4), (5) and (6) the following theorem can be proposed:

Theorem 1 *Suppose the profit function* Π *is twice differentiable and strictly concave in the region* $0 < p_2 < p_{th2}$*, then there exists an optimal price* p_2^* *that maximizes the service provider's revenue due to unsubsidized users.* □

Proof: From equation (4), (5) and (6) we see that $\frac{\partial\Pi}{\partial p_2} = 0$ has a maximum point since $\frac{\partial^2\Pi}{\partial p_2^2} < 0$, meaning that within the interval of $0 < p_2 < p_{th2}$, the profit function is positive for all the values of p_2 in $c_i < p_i < p_{thi}$. Therefore, it is clear that the profit function, Π, is strictly concave in the region of $c_i < p_i < p_{thi}$ for $i \in 1, 2$. Moreover, it can be inferred from Equation (4) that if $\frac{\partial^2\Pi}{\partial p_2^2} < 0$, Π contains at least one maximization point or fixed point (see Kakutani fixed point theorem), at which $\frac{\partial\Pi}{\partial p_2} = 0$ within the range of $c < p_2 < p_{th2}$. Thus the optimal price can be found by equating the first derivative to zero i.e. $\frac{\partial\Pi}{\partial p_2} = 0$ and solving for p_2^*. ■

Depending on the price set by the ISP, and within the range of $0 \le p_1 \le p_{th1}$, the number of information "have-nots" in such a network can be determined by the value of the subsidy β set by the government. Actually, β also determines the maximum profit that the ISP can receive under no network capacity constraints i.e. the assumption of infinite available bandwidth. From theorem (1), we provide the following lemma to support our claim:

Lemma 1 *In any developing country the maximum number of information "have nots"* (N_1) *subscribing to an ISP is determined by the subsidy factor* β*, and is a maximum when* $\beta \approx 0$ *i.e.* $p_1 \approx 0$ □

Proof: Consider equation 3 as follows:

$$N_{total} = \left(\frac{\beta p_2}{p_{th1}}\right)^{-\alpha} - 1 + \left(\frac{p_2}{p_{th2}}\right)^{-\alpha} - 1,$$

by differentiating equation 3 with respect to β we obtain:

$$\frac{\partial N}{\partial \beta} = \frac{\alpha p_2(c-p_2)}{p_{th1}\frac{\beta p_2}{p_{th1}}^{(\alpha+1)}}. \quad (7)$$

From $\frac{\partial N}{\partial \beta}$, we can solve for β^* by equating 7 to zero:

$$\beta = \left(\frac{\alpha p_2(c-p_2)}{p_{th1}p_2^{(\alpha+1)}}\right)^{\frac{1}{(\alpha+1)}} \quad (8)$$

■

It is clear from the lemma that if $\beta^* \approx 0$ then the tragedy of the commons is the most likely outcome. Increasing the value of β such that $\beta^* = 1$, will reduce the number of information "have nots" subscribing to the ISP. Obviously, a solution to equation (8) exists that maximizes the revenue of a service provider without creating the tragedy of the (anti) commons within the range $0 < \beta < 1$. Since the solution to the equation above is mathematically intractable, we use Matlab to approximate the value of β which is the solution to equation (8). Note that this solution is dependent on the value of the threshold values for the two groups and it varies with varying threshold and sensitivity (α) towards price.

V. SIMULATION RESULTS

In this section we provide the simulation results on the consequences of subsidy on the number of users and revenue maximization. We use Matlab to approximate our solutions.

A. *Effect of subsidy on revenue and price*

In Figure 3, 4 and 5, we evaluate the effect of subsidy on revenue maximization, given any price p_2 and the threshold price of consumers i.e. p_{th1} and p_{th2} at a constant sensitivity α. Figure 3, for example, shows that for a subsidy factor of $\beta = 1$ (in this case we assume that the government does not subsidize the information "have nots"), the revenue generated from the information "have nots", is, actually, dependant on the maximum price p_{th1} that they are willing to pay. This phenomenon, especially in underserviced areas of developing countries, may sometimes result in the tragedy of anti-commons since very few information "have nots" will be willing to subscribe to the ISP. For all intents and purposes, it is worthwhile to note that the information "haves nots" are contributing very little towards the ISP's overall revenue since no subsidy is being given by the government towards the ISP (see Figure 3). In order to obtain maximum revenue from the users, it will be meaningful for the ISP to charge the information "have nots" and "haves" an optimal price of 30 units so as to maximize revenue.

Consider again that the government subsidizes the information "have nots" as shown in figure 4. Figure 4 shows that too much subsidy has a drastic consequence on revenue maximization and the number of information "have nots" subscribing to an ISP. Since decreasing the subsidy factor β, i.e. $\beta = 0.1$,increases the number of information "have nots", it is obvious that the maximum revenue obtained by the ISP will be highest when the government pays for all the "information have nots"($\beta \approx 0$) as shown in Figure 4.

In actual fact, Figure 4 represents an outcome that will objectively fulfill government policies of universal access for all, as it allows for more information "have nots" to subscribe to the ISP at a subsidized price. However, such a situation would definitely promote the tragedy of the commons and may lead to market failure in once stable markets. Thus, Figure 4 highlights the implications of heavy subsidy towards revenue maximization in a heterogeneous society, and shows that under heavy government subsidy there will be a skewed information

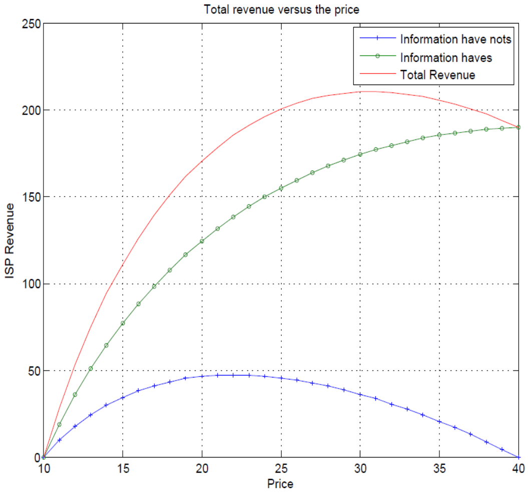

Fig. 3. Revenue versus price at $alpha = 0.3$ $p_{th1} = 40$ $p_{th2} = 100$ $\beta = 1$ $c = 10$

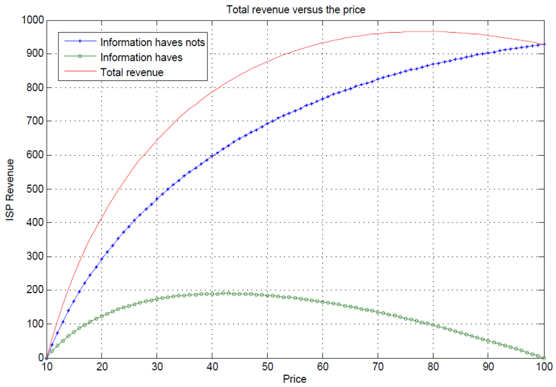

Fig. 4. Revenue versus price at $alpha = 0.3$ $p_{th1} = 40$ $p_{th2} = 100$ $\beta = 0.1$ $c = 10$

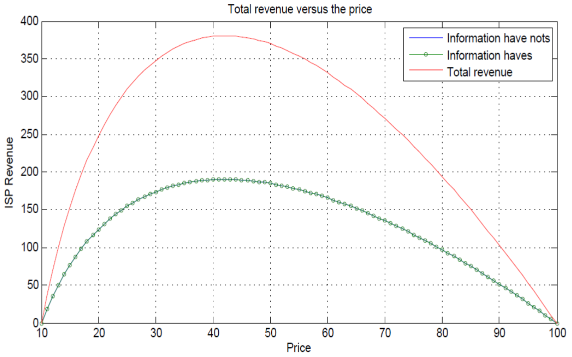

Fig. 5. Revenue versus price at $alpha = 0.3$ $p_{th1} = 40$ $p_{th2} = 100$ $\beta = 0.4$ $c = 10$

access between the information “haves” and the information “have nots” due to unequal subsidy distribution.

By equating (consider equation 4) the revenue generated by the information “have nots” to the “information haves” we can solve for equilibrated value of β. Figure 5, actually, represents an equilibrated game between the information “haves” and “have nots” as the revenue curve of the N_1 and N_2 follow the same trajectory. Both users (as depicted in Figure 5) will not subscribe to the ISP at the price of 100 units preventing free riding. Contrary to Figure 3 and 4, Figure 5 represents a desirable outcome between the information “haves” and “have nots” as it represents a balanced price versus revenue curve trajectory. Generally, Figure 5, in point of fact, may prevent the tragedy of the commons and tragedy of the anti-commons in heterogeneous society as no member of a group is disadvantage due to skewed pricing.

B. Revenue, subsidy and consumer sensitivity towards price

This section presents results on how β and the sensitivity α affects revenue maximization given the two groups of people. We adopt the following values for our simulation: $p_{th1} = 40$ $\lambda = 2$ $c = 10$ $p_{th2} = 100$.

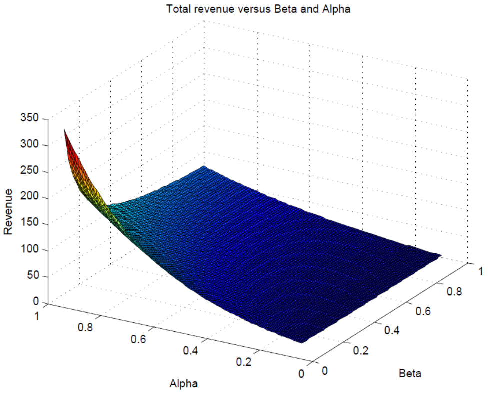

Fig. 6. Revenue versus beta and alpha when $p_2 = 15$

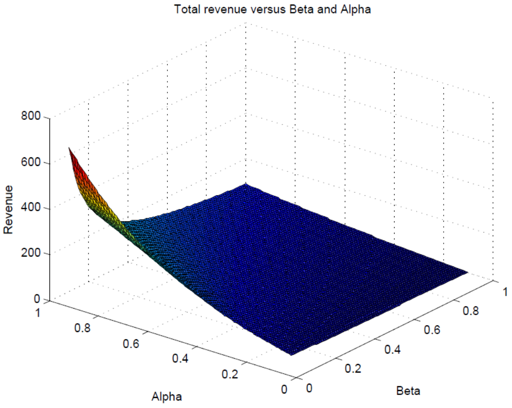

Fig. 7. Revenue versus beta and alpha when $p_2 = 40$

In this subsidy driven pricing model, understanding the properties of subsidy factor β and sensitivity α, is useful to ascertain revenue maximization. The major question to be asked in this pricing model is: how does customer sensitivity α and subsidy factor β affect the revenue obtained by the service provider given the threshold price of the consumer p_{th1} p_{th2}?

As observed from Figure 6, 7 and 8, revenue maximization is affected by the value of the subsidy factor and sensitivity of consumers towards price.

The figures show that by adjusting the value of α and β the government and the ISP can attain a required number of users so as to prevent the tragedy of the commons and the tragedy of anti-commons. As a matter of analysis, Figure 6, 7, 8 expresses the revenue as a function of β and α that determines and optimizes net revenue over the ISP's services horizon.

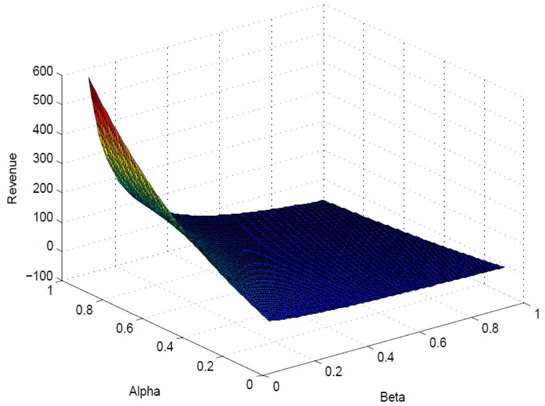

Fig. 8. Revenue versus beta and alpha when $p_2 = 90$

VI. CONCLUSION

We have analyzed the effect of subsidy on the increase and decrease of users in a network which has a direct effect on the underuse and overuse of resources in heterogeneous communities. We have shown that if implemented properly, subsidies can prevent the tragedy of the (anti) commons and could be the solution to promoting social and economical growth in developing countries. In a matter of fact, developing countries must consider the trade off between increasing or decreasing subsidy and its effect on the underuse or overuse of resources. Our subsidy driven pricing model addresses the importance of a balanced subsidy pricing scheme with a view of achieving social and economical growth while enhancing the usage of network resources efficiently. Using this framework, we have shown that a government subsidy ina developing country can result in the increase or decrease of information "have nots" leading to the tragedy of (anti) commons. We have also shown that a correct subsidy, given customer sensitivities, can promote desired revenue and that the revenue is a concave function of price.

We have derived the optimal revenue given the price, subsidy factor β and α as shown in the Figures of 6, 7 and 8. In summary we have shown that an over-subsidized or under-subsidized group of users in a heterogeneous society can lead to either the tragedy of commons or anti-commons accordingly.

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