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“COMPUTER SCIENCE AND PROGRAMMING”
(Engineering calculations in Mathcad)

Teaching and practical guide
for the students of chemical specialization of all education forms
(in English)

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С74 «Обчислювальна математика та програмування» (Інженерні розрахунки в середовищі Mathcad) : навч.-метод. посіб. / О. П. Арсеньєва, О. В. Ведь, Л. В. Соловей, А. П. Юзбашьян. – Харків : НТУ «ХПІ», 2017. – 100 с. – Англ. мовою.

Навчально-методичний посібник присвячений вивченню середовища Mathcad та практичному застосуванню цього програмного забезпечення для інженерних розрахунків. Наведено велику кількість прикладів вирішення розрахункових завдань різної складності. До прикладів надаються пояснення. Усі приклади забезпечені результатами виконання. По кожній темі представлені практичні завдання для виконання лабораторних робіт.

Призначено для студентів хімічних спеціальностей, які бажають застосовувати середовище Mathcad для інженерних розрахунків.

The teaching and practical guide is devoted to study how to make the engineering calculations using the Mathcad software. The different examples of engineering tasks with the varied complexity are provided with step-by-step explanation with corresponding illustrations. The obtained results are provided. Each subject includes the explained case studies and the tasks for work in class and individually.

The teaching and practical guide is aimed for the students studying in chemical engineering, which want to use Mathcad for engineering calculations.

Іл.: 11 Табл.: 22 Бібліогр.: 5

УДК 519.6

Contents

Introduction

Mathcad is used to perform, document, and share the calculations and design work. Mathcad's interface is seen in standard mathematical notation, text, and graphs collected in a single worksheet, what makes Mathcad perfect for engineering calculations and further collaboration.

In the most general sense, Mathcad combines

- engineering-oriented mathematics notation and functionality,
- a numeric and symbolic computational engine,
- flexible, full-featured word processing and visualization tools.

Mathcad is used for engineering problem-solving productivity and presentation of solutions. It allows engineers to work with powerful mathematics tools. Mathcad delivers all the solving capabilities, functionality, and robustness needed for calculation, data manipulation, and engineering design work. Its interface makes commonly used features accessible and natural. By allowing text, math, and graphics to be combined in a single worksheet environment, solutions are easy to visualize, illustrate, verify, and annotate.

The present teaching and practical guide observes 12 subjects connected to engineering calculations in Mathcad in the field of chemical engineering. Subjects 1 – 6 give the basic approaches for calculation in Mathcad, such as defining the variables, performing basic arithmetic calculations, working with functions and matrixes. The solution of more complex mathematical problems, such as finding the local minimum and maximum of functions, solving the equations in Mathcad and the system of differential equations are presented in Subjects 7 – 10. Subject 11gives the examples of experimental data approximation using different approaches. The standard programming operators in Mathcad and their application for solving the engineering problems containing logical test and loops are given. Each section contains the example with the detailed step-by-step explanation of the required action to solve the problem and corresponding illustrations. The number of tasks for individual work is provided for each subject.

Subject 1. GETTING STARTED WITH MATHCAD

Mathcad 15 is the standard for Engineering Calculation and Communication. Mathcad is designed for engineering problem-solving productivity and presentation of solutions. Mathcad allows engineers to work with the most natural, powerful mathematics tools available while enabling managers to easily access, track, and reuse work done in their departments. Mathcad delivers all the solving capabilities, functionality, and robustness needed for calculation, data manipulation, and engineering design work. Its interface makes commonly used features accessible and natural. By allowing text, math, and graphics to be combined in a single worksheet environment, solutions are easy to visualize, illustrate, verify, and annotate.

Mathcad is the only enterprise-wide solution for managing engineering design and attributes. The produced files are publishable into a variety of formats: XML, HTML, PDF, and RTF, and with the use of the Mathcad Calculation Server they can published live math on the Web using existing Mathcad worksheets. Mathcad's open application architecture combined with its support of .NET and its native XML format make it easy to integrate Mathcad into other engineering applications.

In the most general sense, Mathcad combines

- engineering-oriented mathematics notation and functionality,
- a powerful numeric and symbolic computational engine,
- flexible, full-featured word processing and visualization tools.

1.1. Mathcad Toolbars

The main Worksheet window view is presented in Fig. 1.1. You are currently in the Resources window, so make sure you are looking at the window with the toolbar pictured below. If you can't see it, try moving this window to the right, so both windows line up next to each other.

The Worksheet window consists of the ***Resources*** or E-book window and the ***Worksheet*** window:

– The ***Resources*** window has content that Mathsoft has written. You can add your own comments or annotations. You cannot delete or move any of the content that appears in the Resources window. You can, however, drag content or copy content into the Worksheet window and modify it there.

– The ***Worksheet*** window in Mathcad is your own workspace. Each worksheet can be saved as a new file or you can open existing files in the ***Worksheet*** window. Some of the activities in these tutorials require you to work in the ***Worksheet*** window.

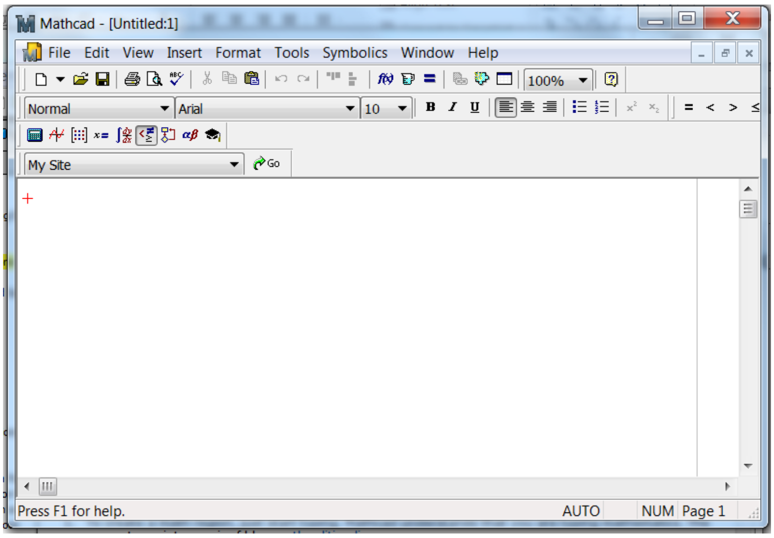


Figure 1.1 – The main view of Worksheet window

Mathcad toolbars provide access to a number of following features, functions, and math operators:

– The ***Menu*** bar.
– The ***Math*** toolbar.
– The ***Standard*** toolbar.
– The ***Formatting*** toolbar.

1.2. The Menu bar

Mathcad ***Menu*** Commands consist of following items: File, Edit, View, Insert, Format, Tools, Symbolics, Window, Help.

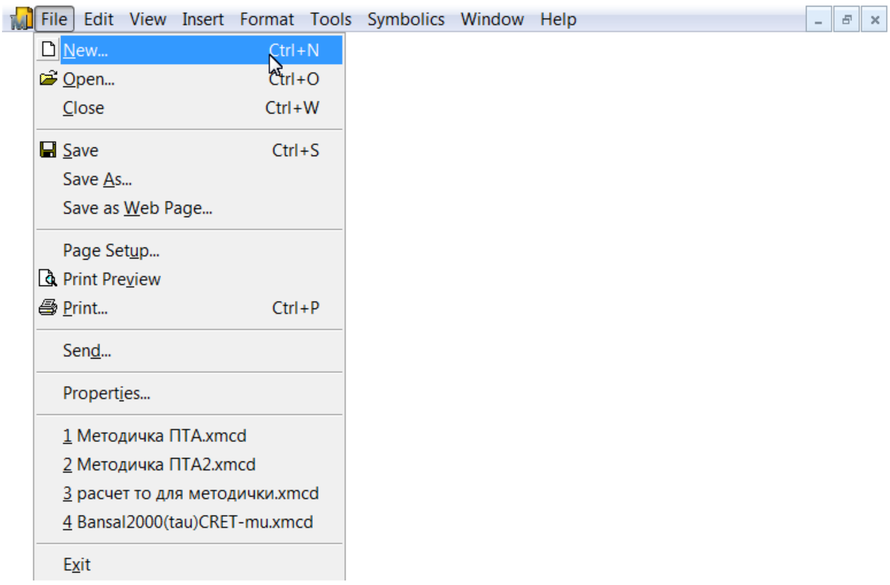


Figure 1.2 – The view of the ***File menu*** bar

The File menu contains the following items (Fig. 1.2):

– New

Opens a dialog displaying a list of templates to choose from. Choose "Normal" to create a new blank worksheet.

– Open

Lists worksheets in the current directory or allows user to browse by changing the selection in the Look in drop-down menu or by typing the full path to the file in the "File Name" box.

– Close

Closes the current worksheet.

– Close & Return

Closes the worksheet and returns to the OLE container application. This command is only available when you are editing an embedded worksheet in a separate window, as opposed to in-place.

– Update

Updates the worksheet in an OLE container application. This command is only available when you are editing an embedded worksheet in a separate window, rather than in-place.

– Save

Saves the worksheet in the current format. If the worksheet is new, Mathcad prompts you to specify the name and format. You can save a file as *.XMCD (Mathcad's native XML format), or *.XMCDZ (compressed XML format, which is a GZIP (RFC1952) format).

– Save As

Saves the worksheet as a new file, allowing you to save the worksheet with a new name or in a different format.

– Save as Web Page

Saves the worksheet as an HTML file for posting on a Web site.

– Save Copy As

Saves a copy of the embedded worksheet as a stand-alone file. This option is available only when Mathcad is used as an OLE server and is not in-place activated.

– Page Setup

Controls the page layout options for the active worksheet, including margins, paper size, source, and orientation, and printing text beyond the right margin.

– Print Preview

Displays the worksheet as it will be sent to the printer, according to the current page setup instructions.

– Print

Prints the active worksheet. Using the Print dialog, you can specify options such as specific pages and number of copies to print.

– Send

Attaches your worksheet to a new email message, if you have a MAPI-compliant email application.

– Properties

Opens the File Properties dialog, allowing you to create and modify metadata for your worksheet.

– Recent Files

Lists recently opened files; click on a name to open the worksheet. The number of files listed is controlled from the Tools > Preferences dialog.

– Exit

Closes Mathcad. If you edited an open worksheet since it was last saved, you are prompted to save it before quitting.

The view of other menu bars is presented in Fig. 1.3. The ***Window*** Menu provides the possibility to arrange the open windows and ***Help*** Menu contains the references to the Help topics and tutorials.

1.3. The Math toolbar

The view of the Math toolbar is presented in Fig. 1.4.

Each button in the ***Math*** toolbar opens another toolbar of operators or symbols. To see which toolbar each button brings up, move the cursor over the button until a tooltip (small popup window) will show the button title, and a description of the button appears on the message line of the ***Status*** Bar.

The ***Math*** toolbar contains following buttons: Calculator; Graphing; Matrix; Evaluation; Calculus; Boolean; Programming; Greek; Symbolic. The content of each toolbar is presented in Fig. 1.5.

1.4. The Standard toolbar

The Standard toolbar buttons are shortcuts to many of the commands in the File and Insert menus (Fig. 1.6). To learn what a button does, hover the cursor over the button until a tooltip shows the title, and a description appears on the message line of the Status Bar. To customize the Standard toolbar, right-click on it and choose Customize.

1.5. The Formatting toolbar

The ***Formatting*** toolbar contains scrolling lists and buttons to specify font characteristics in equations and text (Fig. 1.7). To learn what a button does, hover the cursor over the button until a tooltip shows the title and a description appears on the message line of the ***Status*** Bar.

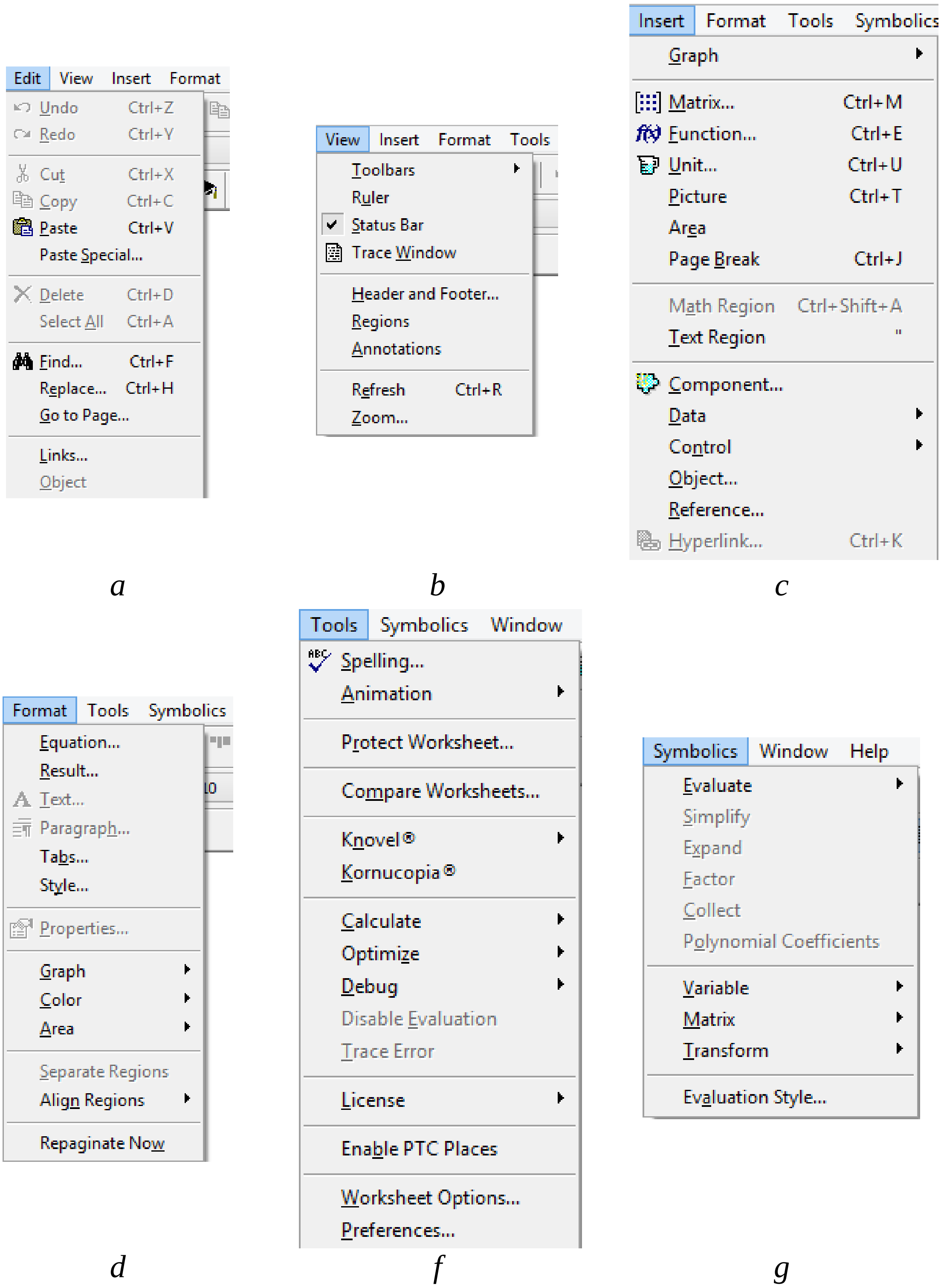
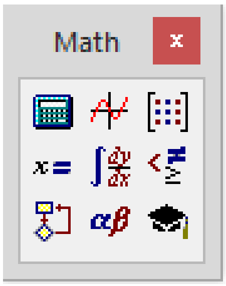
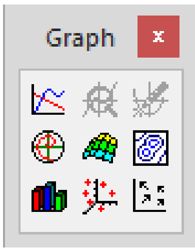


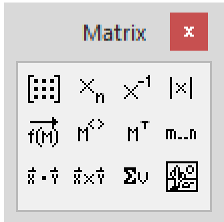
Figure 1.3 – The view of the following ***Menu*** bars:
a – Edit; b –View; c – Insert; d – Format; f – Tools; g – Symbolics



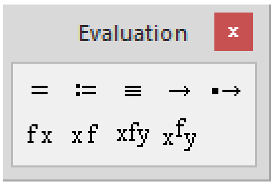
Calculator toolbar
Graph toolbar
Vector and Matrix toolbar
Evaluation toolbar
Calculus toolbar
Boolean toolbar
Programming toolbar
Greek Symbol toolbar
Symbolic Keyword toolbar

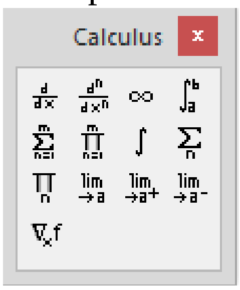
Figure 1.4 – The view of ***Math*** toolbar

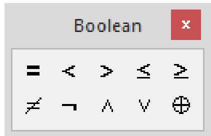




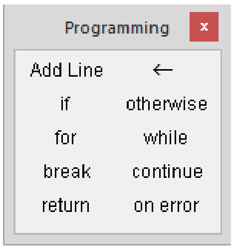
a – Calculator Toolbar; *b* – Graph Toolbar; *c* – Matrix Toolbar;

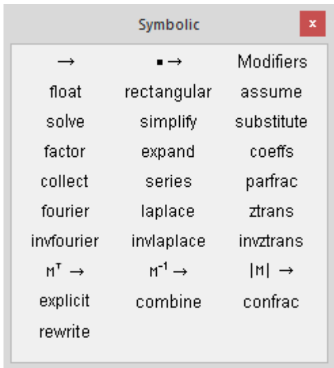






d – Evaluation Toolbar; *f* – Calculus Toolbar; *g* – Boolean Toolbar;





j – Programming Toolbar; *k* – Greek Toolbar; *l* – Symbolic Toolbar

Figure 1.5 – The view of the ***Math*** toolbars

Figure 1.6 – The view of ***Standard*** toolbar

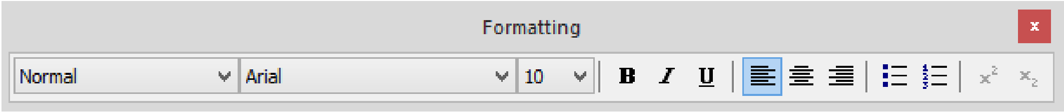


Figure 1.7 – The view of ***Formatting*** toolbar

1.6. Creating regions

All worksheets are comprised of regions - text regions, math regions, graphs, and images. Math regions often depend on other math regions to calculate correctly.

A definition must appear above or to the left of where it is being used, a procedure referred to as ***top/down***, ***left/right*** protocol. With the exception of global definitions, math expressions in a worksheet are evaluated from left to right and top to bottom. Before using a variable or function, you must define it above and to the left of the expression in which you wish to use it. As you evaluate new expressions, you will see results flow down the page.

In Mathcad the following expressions exist:

– Numeric expressions are the expressions involving numbers and operators;

– Symbolic expressions are expressions involving numbers, unknowns (variables), and operators;

– Result format is the type of result, including: decimal, fractional, scientific, etc.

Besides math regions, you can also add text and images next to the math in your worksheets. To create a text region:

– Click to start the region.

– Choose ***Text Region*** from the ***Insert*** menu, or press the double-quote ["] key. You can also just start typing text and once you type a space, Mathcad begins a text region. A text box with a red text cursor appears.

– As you type, the text box grows. The text is formatted according to the "Normal" text style.

– When you finish typing, click outside the text region. Do not press [Enter]. Doing so simply creates a new line inside the text region.

Subject 2. CALCULATION OF MATHEMATICAL EXPRESSIONS IN MATHCAD. THE BUILT-IN FUNCTIONS

2.1. Mathcad built-in constants

Mathcad sometimes returns special symbolic constants that cannot be evaluated directly using the numerical equal sign. The constants predefined in Mathcad are presented in Table. 2.1 and Table 2.2. Mathcad can evaluate these constants numerically using the symbolic equal sign with the keyword "float." Alternatively, it is possible to define new constants using the output of the symbolic equal sign and then evaluate these constants with the numerical equal sign.

Table 2.1 – Math constants

Name	Keystroke	Default Value
∞	[Ctrl] [Shift] Z	10^{307}
e	e	Value of *e* to 17 digits (with "Show trailing zeros" enabled): 2.71828182845904500
π	[Ctrl] [Shift] p or p [Ctrl] g	Value of *π* to 17 digits (with "Show trailing zeros" enabled): 3.14159265358979300
γ	g [Ctrl] g	Value of *γ* (Euler's constant) to 17 digits (with "Show trailing zeros" enabled): 0.57721566490153290
i or j	1i or 1j	The imaginary unit.
%	%	0.01; multiplying by % gives you the appropriate conversion. You can type [expression] % for inferred multiplication or use it in the unit placeholder.
NaN	NaN	Not a number.

Table 2.2 – Some system constants

Name	Default Value	Use
TOL	.001	Controls iterations on some numerical methods.
CTOL	.001	Controls convergence tolerance in Solve Blocks.
ORIGIN	0	Controls array indexing.
FRAME	0	Controls animations.
ERR	NA	Size of the sum of squares error for the approximate solution to a Solve Block.
1L, 1M, 1T, 1Q, 1K, 1C, 1S	assigned by the selected unit system	Define custom unit definitions for the base dimensions.

2.2. Mathcad operators

Operators are symbols like "+" and "−" that link variables and numbers together to form expressions. The variables and numbers linked together are called **operands**.

For example, in “ a^{x+y} “ the operands for the "+" are *x* and *y*. The operands for the exponent operator are *a* and the expression *x* + *y*.

Insert arithmetic operators in Mathcad using standard keystrokes, like **[*]** and **[+]**. Alternatively, all of Mathcad's operators can be inserted from the Math toolbars. Choose **Toolbars** from the **View** menu to see the Math toolbars with operators. The keystrokes for each operator (Table 2.3) depend on the localized keyboard and operating system. The tooltips are for the US keyboards only.

Table 2.3 – Mathcad keystrokes for math operators

Display	**Keystroke**	**Operator**
1	2	3
:=	[:]	***definition*** – defines a value as a variable to use in subsequent calculations
=	[=]	***evaluation*** – returns the numerical evaluation for the left side as a result on the right-hand side
≡	[~] (tilde)	***global definition*** – returns a globally defined variable
+	[+]	***addition*** – returns the sum
+ ...	[Ctrl] [Enter]	***addition with linebreak*** – returns the sum with terms on different lines
−	[−]	***negation*** – returns the negative or ***subtraction*** – subtracts one number from another
·	[*]	***multiplication*** – returns the product
$\frac{y}{x}$	[/]	***division*** – return the result of one number divided by another
÷	[Ctrl] [/]	***in-line division***
..	[;]	Precedes last number in ***range***
$\sqrt{\blacksquare}$	[\]	***square root***
$\sqrt[n]{\blacksquare}$	[Ctrl] [\]	***n-th root***
∫	[&]	***integration***

The end of the table 2.3

1	2	3
x^y	[^]	***exponentiation*** or ***matrix inverse***
$\frac{d}{dx}\blacksquare$	[?]	***derivative***
$\frac{d^n}{dx^n}\blacksquare$	[Ctrl] [Shift] [/]	***n-th derivative***
v_i	[v[i	***array subscript***
$M^{\langle n\rangle}$	[Ctrl] 6	***column selection***
×	[Ctrl] 8	***cross product*** – returns the vector cross product of
$\vec{v}$	[Ctrl] [−]	***vectorize***
$\|x\|$	[|]	***magnitude*** or ***determinant*** – returns the magnitude (norm) of a vector, the absolute value of a scalar, or the determinant of a matrix
M^T	[Ctrl] 1	***transpose*** the matrix *M*
$\sum_{i=m}^{n} X$	[Ctrl] [Shift] 4	***summation*** – returns the sum of the expression *X* as *i* goes from *m* to *n*
$\sum_{i} X$	[$]	***range sum*** – returns the sum over *i* of the expression *X*
Σv	[Ctrl] 4	***sum of elements in vector*** – returns the sum of all the elements in the vector *v*.
$\prod_{i=m}^{n} X$	[Ctrl] [Shift] 3	***product*** – returns the product as *i* goes from m to *n* of the expression *X*
$\prod_{i} X$	[#]	***range product*** – returns the product over *i* of the expression *X*
n!	[!]	***factorial*** – returns $n(n-1)(n-2)\ldots1$
=	[Ctrl] [=]	***equal to*** – returns 1 if $x = y$, 0 otherwise
≠	[Ctrl] 3	***not equal*** – returns 1 if $x \neq y$, 0 otherwise
<	[<]	***less than*** – returns 1 if $x < y$, 0 otherwise
>	[>]	***greater than*** – returns 1 if $x > y$, 0 otherwise
≤	[Ctrl] 9	***less than or equal*** – returns 1 if $x \leq y$, 0 otherwise
≥	[Ctrl] 0	***greater than or equal*** – returns 1 if $x \geq y$, 0 otherwise

2.3. Built-in Functions

It can be inserted any built-in function by choosing **Function** from the **Insert** menu to open the **Insert Function** dialog box. It is possible also to type the names of the built-in functions directly into Mathcad. Spelling and capitalization are important. The list of built-in function is presented in Table 2.4.

Many common operations are represented by the appropriate mathematical notation, and so are considered operators, described separately. If you to install optional Extension Packs, you will have additional built-in functions. These also are available from the Insert > Function menu under the name of the Extension Pack.

Table 2.4 – Built-in functions

Mathcad function	**Description**
Log and Exponential Functions	
exp(z)	Returns the number e raised to the power *z* (e^z)
log(z, [b])	Returns the base *b* logarithm of *z*. If b is omitted, returns base 10 log of *z*
ln(z)	Returns the natural logarithm (base *e*) of *z*
Trigonometric Functions	
sin(z)	Return the trigonometric functions sine, cosine, tangent, secant, cosecant, and cotangent of *z*, respectively
cos(z)	
tan(z)	
sec(z)	
csc(z)	
cot(z)	
Inverse Trigonometric Functions	
asin(z)	Return the value in radians of the arcsine, arccosine, arctangent, arcsecant, arccosecant, and arccotangent, respectively, that is, the angle whose sin, cos, and so on, is *z*
acos(z)	
atan(z)	
asec(z)	
acsc(z)	
acot(z)	

Insert Function Dialog Box contains the following items:

– Function Category

Lists categories of built-in functions. Choose "All" to see all the functions.

– Function Name

Displays a list of built-in functions in the selected category.

– Syntax

Shows the quantity and type of arguments required for the function.

– Description

Displays a description of the function.

– Insert and OK buttons

Inserts the function (together with placeholders for its arguments) into your worksheet.

To apply a function to an expression you have already entered, surround the expression with the editing lines and then click OK button.

2.4. Mathematical expressions

Math in a Mathcad worksheet appears in familiar math notation – division with a fraction bar, exponents in a raised position, and built-up fractions just as in a book. Entering math expressions is very straightforward. The standard elements of the mathematical expression are presented in Fig. 2.1.

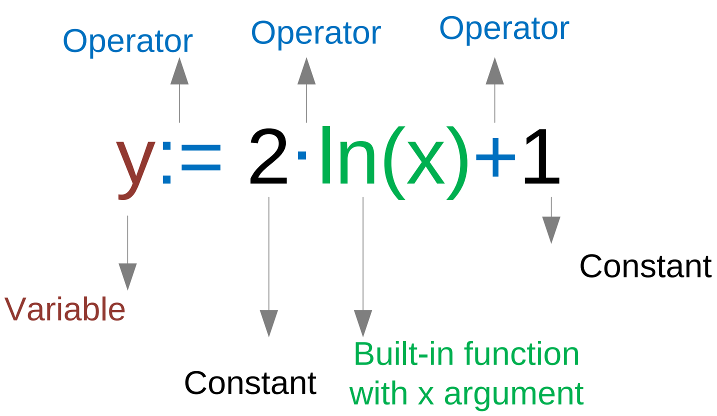


Figure 2.1 – Parts of the mathematical expression

Task 2.1

Calculate the value of the expression $(x+5)^2$ at $x = 1$.

Mathcad text:

1. Definition of the variable x

$x := 1$

2. Check the variable x value

$x = 1$

3. Calculation of the expression

$(x+5)^2 = 36$

To calculate the value of the expression, which contains a variable, it is needed firstly to define the variable and its value, and then to apply all the operators, as presented in (3), Mathcad text of Task 2.1.

Task 2.2

Calculate the value of the expression

$$y = \frac{a^{2x} + b^{-x}\cos(a+b)x}{x+1}$$

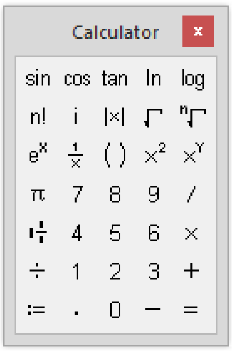
$$R = \sqrt{x^2+b} - b^2\sin^3(x+a)/x$$

where $a = 0.3$; $b = 0.9$; $x = 0.61$

Mathcad text:

1. Definition of the variables a, b, x

$a := 1$ $\quad b := 0.9$ $\quad x := 0.61$



2. The expressions

$$y := \frac{a^{2\cdot x} + b^{-x}\cdot\cos(a+b)\cdot x}{x+1}$$

$$R := \sqrt{x^2+b} - b^2\cdot\frac{\sin(x+a)^3}{x}$$

3. Output the results

$y = 0.49$ $\quad R = -0.197$

Task 2.3

Using Table 2.5 calculate the value of the expression in column (2) with the given initial data of variables in column (3).

Table 2.5 – Data for calculation

Variant	**Expression**	**The values of variables**
1	2	3
1	$a=\dfrac{2\cos(x-\pi/6)}{1/2+\sin^2 y}$ $b=1+\dfrac{z^2}{3+z^2/5}$	$x=1.426;$ $y=-1.22;$ $z=3.5$
2	$\gamma=\left\lvert x^{y/x}-\sqrt[3]{y/x}\right\rvert$ $\psi=(y-x)\dfrac{y-z/(y-x)}{1+(y-x)^2}$	$x=1.825;$ $y=18.225;$ $z=-3.298$
3	$y=e^{-bt}\sin(at+b)-\sqrt{\lvert bt+a\rvert}$ $s=b\sin(at^2\cos 2t)-1$	$a=-0.5;$ $b=1.7;$ $t=0.44$
4	$\omega=\sqrt{x^2+b}-b^2\sin^3(x+a)/x$ $y=\cos^2 x^3-x\big/\sqrt{a^2+b^2}$	$a=1.5;$ $b=15.5;$ $x=-2.9$
5	$s=x^3tg^2(x+b)+a\big/\sqrt{x+b}$ $Q=\dfrac{bx^2-a}{e^{ax}-1}$	$a=16.5;$ $b=3.4;$ $x=0.61$
6	$R=x^2(x+1)\big/b-\sin^2(x+a)$ $S=\sqrt{xb/a}+\cos^2(x+b)^3$	$a=0.7;$ $b=0.05;$ $x=0.5$
7	$y=\sin^3(x^2+a)^2-\sqrt{x/b}$ $z=\dfrac{x^2}{a}+\cos(x+b)^3$	$a=1.1;$ $b=0.004;$ $x=0.2$

The end of the table 2.5

1	2	3
8	$f=\sqrt[3]{mtgt+\lvert c\sin t\rvert}$ $z=m\cos(bt\sin t)+c$	$m=2;\quad c=-1;$ $t=1.2;\quad b=0.7$
9	$y=btg^2x-\dfrac{a}{\sin^2(x/a)}$ $d=ae^{-\sqrt{a}}\cos(bx/a)$	$a=3.2;$ $b=17.5;$ $x=-4.8$
10	$f=\ln(a+x^2)+\sin^2(x/b)$ $z=e^{-cx}\dfrac{x+\sqrt{x+a}}{x-\sqrt{\lvert x-b\rvert}}$	$a=10.2;\quad b=9.2;$ $x=2.2;\quad c=0.5$
11	$a=1.2c$ $b=3c/5$ $k=(a^{3/2}+b^{3/2})/(a^2-ab)^{3/2}$ $F=\cos(x^2+1.43\pi)+\dfrac{x}{2}$	$c=2.15;$ $x=2.5$
12	$z=(a+b-c)/(a+b+c)$ $y=(a^2-b^2-c^2+2ab)$ $F=\sin\dfrac{2x}{\pi-4.1}+\cos^3 2x$	$a=8.6;\quad b=3^{1/2};$ $c=3.3;\quad x=3.65$

Subject 3. DEFINING RANGE VARIABLES AND FUNCTIONS IN MATHCAD. PLOT A GRAPH OF FUNCTION

3.1. Defining functions in Mathcad

A function is a mathematical relationship between a set of arguments and a numerical result. The arguments are the variables that appear between the parentheses you use to define or evaluate a function.

To define a function:

1. Type the function name followed by a left parenthesis. A placeholder appears.

   f(▪

2. Type a list of arguments separated by commas followed by a right parenthesis.

   f(x,y)

3. Type **[:]** to see the definition symbol, **:=**. Another placeholder appears.

   f(x,y) := ▪

4. Type anexpression or a string in the placeholder.

   f(x,y) := x + sin(y)

Make sure any variables used in the right-hand expression are either:

- previously defined, or
- part of the argument list.

If a variable in the right-hand expression doesn't satisfy one of these conditions, it appears in red as undefined.

Task 3.1

Define the function $f = x^2 - 3 \cdot x - 2$ in points: $x = 0$ and $x = 10$.

Solution in Mathcad:

$f(x) := x^2 - 3x - 2$

$f(0) = -2$ $\quad$ $f(10) = 68$

Task 3.2

Define the function $L = S(x^2 - 2x)^3 - 4S^2\ x + 1$, where $S(f) = f + \sin^3 f \ /0,5$ in point: $x = 2$.

Solution in Mathcad:

$$x := 2$$

$$S(f) := f + \frac{\sin(f)^3}{0.5}$$

$$L := S\left(x^2 - 2x\right)^3 - 4\,S(x)^2 + 1$$

$$L = -48.102$$

3.2. Defining a range variables

The range variable is the variable, for which some range of its variation is determined.

To define a range variable:

1.Type the name of the variable followed by a colon **[:]**, to create a definition.

2.In the placeholder, type the first number in the sequence of values.

3.Type a comma, or, if you're defining a sequence in increments of one, go to step 5.

4.Type the second number in the sequence.

5.Type a semicolon **[;]**. Note that it appears as two dots ".."

6.In the remaining placeholder, type the last number in the sequence.

In Table 3.1. the examples of range variables are presented. And Table 3.2 contains the example how to define the range variables in Mathcad.

Table 3.1 – Range variables examples

Displayed in Mathcad	Explanation
1..20	Defines the range of numbers 1, 2, 3 ... 20. The increment is equal to 1.
20..1	Defines the range of numbers 20, 19, 18 ... 1. The increment is equal to –1.
1,1.1..5	Defines the range of numbers 1, 1.1, 1.2 ...5. The increment is equal to 0.1.
5,3..-7	Defines the range of numbers 5, 3, 1, –1 ...–7. The increment is equal to –2.

Table 3.2 – Range variables definition in Mathcad

No	Type:	See:
1.	**b: 1 [Tab] 10**	$b := 1..10$
	You get a range of numbers between 1 and 10 at whole number increments.	
2.	**k:3.5;12.5**	$k := 3.5..12.5$
	You get a range of numbers between 3.5 and 12.5 at whole number increments.	
3.	**d:3.5,4.0;12.5**	$d := 3.5, 4..12.5$
	You get a range of numbers between 3.5 and 12.5 in increments of 0.5.	
4.	**q:1/4 [Spacebar] ,1/2 [Spacebar] ;7/4**	$q := \frac{1}{4}, \frac{1}{2}..\frac{7}{4}$
	You get a range of numbers between $\frac{1}{4}$ and $\frac{7}{4}$ in increments of $\frac{1}{4}$.	
5.	**n:-6,-5.9;6**	$n := -6, -5.9..6$
	You get a range of numbers between –6 and 6 in increments of 0.1.	

3.3. Graphing Functions

Graphs are easy to create and modify in Mathcad. You can create 2D and 3D plots in Mathcad or graphs from functions or data sets. A 2D graph can contain up to sixteen plots, all formatted differently. Three-dimensional plots can be created from a matrix, a function, or a set of vectors. In the case of matrices, the row and column numbers correspond to the *x* and *y* axes, respectively. The values of the matrix elements represent the heights above the *xy* plane. Similarly, a function uses ranges of *x* and *y* to generate values for height *z* (or a range over a single parameter for all axes). For a set of vectors or matrices, specify the *x*, *y*, and *z* coordinates independently.

Two-dimensional plots in Mathcad:

– *X–Y* Plot

Inserts the *X-Y* plot operator containing placeholders for *x* and *y* values as well as endpoints of the axes. The independent range variable or vector should be defined, or Mathcad will choose a range using a QuickPlot.

– Polar Plot

Inserts the polar plot operator containing placeholders for an angle variable and a radial expression. The independent range variable or vector should be defined, or Mathcad will choose a range using a QuickPlot.

Three-dimensional plots in Mathcad:

– Surface Plot

Creates a surface plot or type [Ctrl] 2 or choose the Surface Plot button on the Graph toolbar.

– Contour Plot

Creates a contour plot or type [Ctrl] 5 or choose the Contour Plot button on the Graph toolbar.

– 3D Scatter Plot

Creates a scatter plot for which you can specify color and symbol properties or choose the Scatter Plot button on the Graph toolbar. No surface is drawn to connect the points.

– 3D Bar Chart

Creates a 3D bar chart or choose the Bar Plot button on the Graph toolbar.

– Vector Field Plot

Creates a 2D vector field plot of a matrix or choose the Vector Field Plot button on the Graph toolbar.

The terms definitions:

– **plot** is a single line, or trace, in a graph region;
– **graph** is a set of axes with a scale into which plots may be placed;
– **argument** is a variable or expression used in a function or graph to define the region.

3.3.1. Graphing two-dimensional plots

To create an *X–Y* plot:

1. Define a range variable for the range of values over which to plot the expression, for example

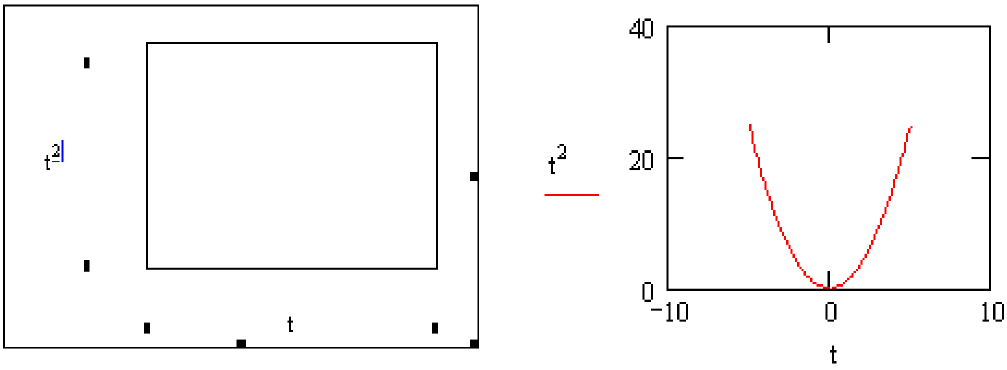
$t := -5, -4.8 .. 5$

2. Choose **Graph > *X–Y* Plot** from the **Insert** menu, or type **@**, to create an *x–y* plot operator. The Math toolbar (Fig. 1.4) and Graph toolbar (Fig. 1.5b) should be opened.

3. In the middle placeholder underneath the *x*-axis (horizontal axis), enter the range variable.

4. In the middle placeholder beside the *y*-axis (vertical axis), enter the expression you want to plot (Fig. 3.1a).

5. Click away from the plot or press **[Enter]**. One point is plotted for each value of the range variable and, unless you specify otherwise, they are connected with straight lines (Fig. 3.1b).



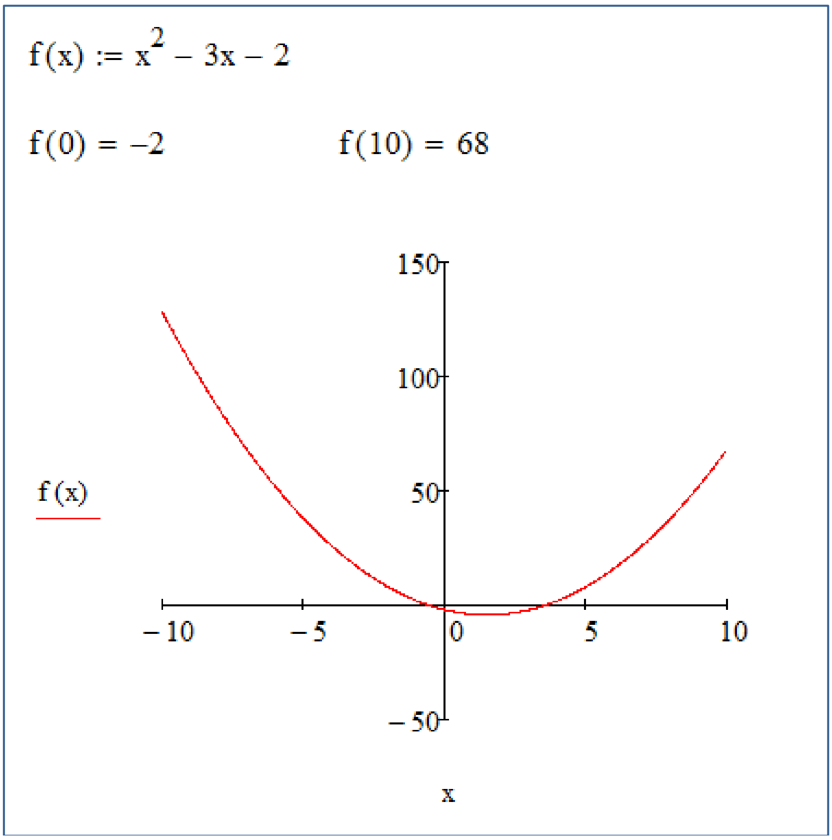
a *b*

Figure 3.1 – Plotting *x*–*y* graph

Task 3.3

Define the function $= x^2 - 3 \cdot x - 2$, calculate its values in points: $x = 0$ and $x = 10$. Plot the graph of function.

Solution in Mathcad:



Note: If the variable range is not defined, Mathcad will plot the graph within default values –10 to 10.

Task 3.4

Plot the graphs of two functions: $y = \sin(x)$ and $y = \cos(x)$ in one graph. The variable x varies from 1 to 10, the increment is equal to 0.1.

Solution in Mathcad:

1. Define the functions

$y(x) := \sin(x)$

$z(x) := \cos(x)$

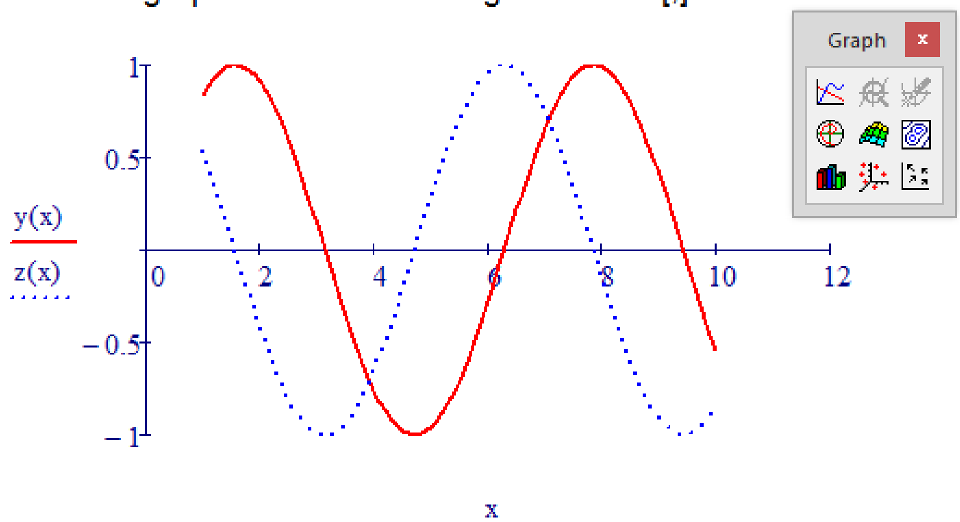
2. Define the range of variable x

$x := 1, 1.1 .. 10$

3. Obtained values of argument x and functions y(x) and z(x) are

x =	y(x) =	z(x) =
1	0.841	0.54
1.1	0.891	0.454
1.2	0.932	0.362
1.3	0.964	0.267
1.4	0.985	0.17
1.5	0.997	0.071
1.6	1	-0.029
1.7	0.992	-0.129
1.8	0.974	-0.227
1.9	0.946	-0.323
2	0.909	-0.416
...	...	...

4. Plot the graph of functions listing them with [,] comma



Task 3.5

Using Table 3.3 plot the graph of the function from column (2) with the range of variable given in column (3) and increment provided in column (4).

Example of Task 3.5 solution

Plot the graph of function $y = 2x + lg(x) + 0.5$, in the range $2 \le x \le 5$, taking increment $h = 0.1$.

Solution in Mathcad:

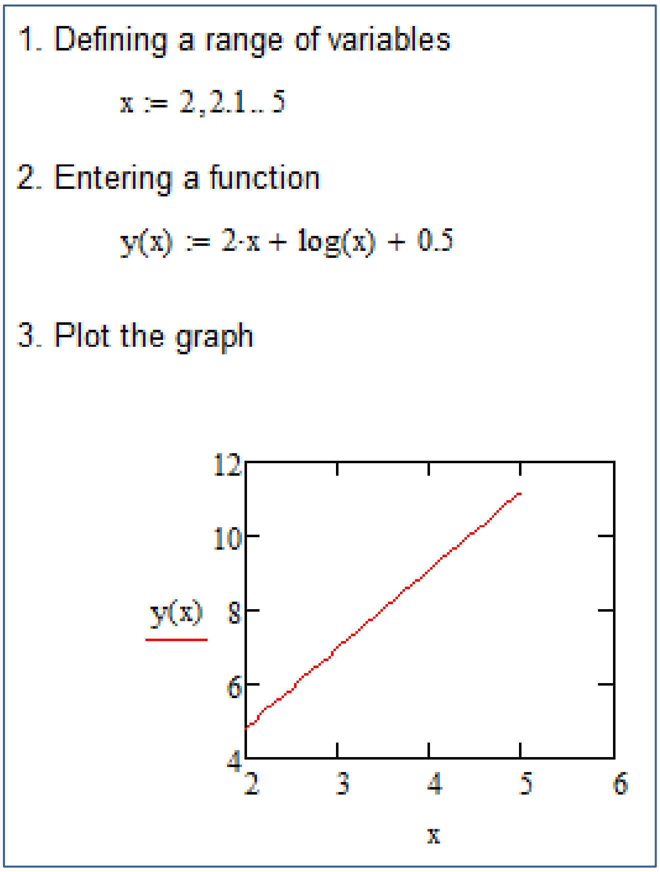


Table 3.3 – Data for calculation

No	Function	The range of variables	Increment
1	2	3	4
1	$y = \sin(x) + \cos(x)$	$-\pi \le x \le \pi$	$h = 0.1$
2	$y = x^2 + 4 \cdot \sin(x)$	$-\pi \le x \le \pi$	$h = 0.1$
3	$y = x + \cos(x) - 1$	$-7 \le x \le 4{,}5$	$h = 0.1$
4	$y = 2 \cdot x + \cos(x) - 7$	$-\pi \le x \le \pi$	$h = 0.1$

The end of the table 3.3

1	2	3	4
5	$y = x \cdot \lg(x+1) - 3$	$0 \le x \le 15$	h = 0.1
6	$y = 2 \cdot \ln(x) - x/2 + 1$	$1 \le x \le 50$	h = 0.1
7	$y = x + \cos(x) - 2$	$-1{,}7 \le x \le 4{,}5$	h = 0.1
8	$y = x^2 + 4 \cdot \sin(x)$	$-\pi \le x \le \pi$	h = 0.1
9	$y = x \cdot (x-4)^2 - 9$	$0{,}5 \le x \le 7$	h = 0.1
10	$y = x^4 - x^3 - 2 \cdot x^2 + 3 \cdot x - 52$	$0 \le x \le 6$	h = 0.1
11	$y = arctg(x) - x^3$	$-5 \le x \le 10$	h = 0.1
12	$y = x + \sin(3x) - 1$	$0 \le x \le 6$	h = 0.1

Task 3.6

Do the Tasks listed in Table 3.4.

Table 3.4 – Tasks for calculation

No	Task description
1	2
1	Plot the graph of the reaction velocity according to the relation: $K = 78.56 \cdot e^{-0.1037\tau}$; $1 \le \tau \le 25$; $\Delta\tau = 1$
2	Plot the graph of the humidity x (in percent of dry substance) depending from time t, min according to the relation: $x = 10^{1.4568-0.00938t}$; $0 \le t \le 100$; $\Delta t = 5$
3	Plot the graph of relation between the mixture amount y (in %), which is taken from the evaporation unit, and its output x (in kg/h) according to the equation: $y = 5.789x^{2.071} \cdot 10^{-7} + 0.048$; $250 \le x \le 2000$; $\Delta x = 100$
4	Plot the graph of relation between the atmospheric pressure p (in mm Hg) and barometric height h (in km), according to the equation: $p = 760 \cdot 10^{-0.0531h}$; $0 \le h \le 6$; $\Delta h = 0.2$
5	Plot the graph of relation between the atmospheric pressure p (in mm Hg) and barometric height h (in km), according to the equation: $p = 760 \cdot 10^{-0.0531h}$; $0 \le h \le 6$; $\Delta h = 0.2$

The end of the table 3.4

1	2
6	Plot the graph of the humidity *x* (in percent of dry substance) depending from time *t*, min according to the relation: $x = 10^{1.4568-0.00938t}; \quad 0 \le t \le 100; \quad \Delta t = 5$
7	Plot the graph of relation between the air temperature inside the dryer *T* (in K) during the day, where τ are the hours, according to the equation $T = 80 + 14.99\cos(15\tau - 37) + 18\cos(30\tau - 353), \quad 0 \le \tau \le 24$
8	Plot the graph of the relation between the solubility of the sodium salt of hypochlorous acid in water *x* (in g/100 g of water) from its temperature *t* (in °C) according to the equation: $x = 7.894e^{0.1939t} - 4.82; \quad 0 \le t \le 60; \quad \Delta t = 2$
9	Plot the graph of the relation between the friction factor in tubes τ depending from the Reynolds number Re: $\tau = \frac{0.398}{\mathrm{Re}^{0.254}}, \quad 3000 \le \mathrm{Re} \le 16000; \quad \Delta\mathrm{Re} = 1000$
10	Plot the graph of the relation between the air volume content in water *ν* (in ml, taken at given temperature *t*, °C and 760 mm Hg) and temperature *t*, °C: $\nu = (t-5)/(-1.386 - 0.0375t) + 25.68; \quad 0 \le t \le 25; \quad \Delta t = 1$

3.3.2. Graphing three-dimensional plots

Three-dimensional scatter plots allow to visualize *x*, *y*, and *z* coordinates in space. They are useful for drawing parametric curves or for observing clusters of data in 3D space. To create a scatter plot, it is needed to define a function, a matrix of *z*-coordinates, or three vectors or matrices specifying the *x*, *y*, and *z* coordinates. Then,

1. Choose **Insert > Graph > Scatter Plot** to create a scatter plot region. Functions for scatter plots can be parametric.

2. Place the name of the matrix, function, or set of functions in the placeholder at the bottom left of the plot. If you enter a set of function names, defining a parametric surface, enclose them in parentheses (Fig. 3.2).

3. Click outside the plot or press [Enter].

The collection of points is rendered in three-dimensional space. Mathcad draws the points with black dots, unless you specify otherwise. Double-click on the plot to get formatting options.

A scatter plot of a function is created using default ranges and grids for the independent variables. To change them, double-click on the graph and use the options in the QuickPlot Data tab of the 3D Plot Format dialog box.

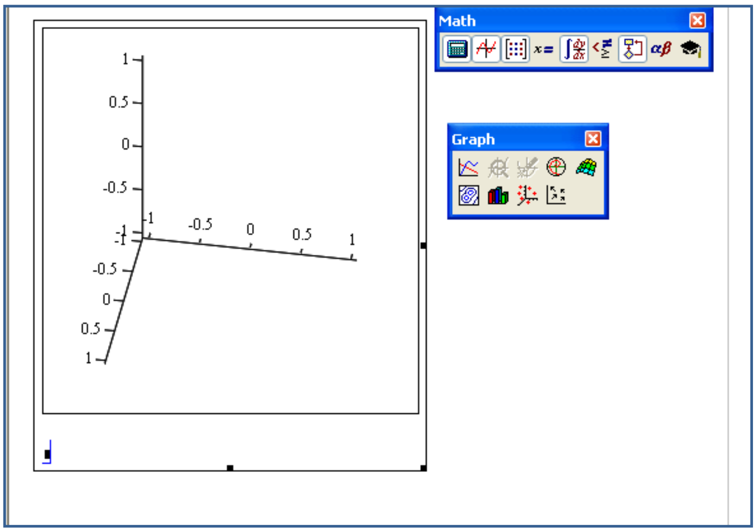
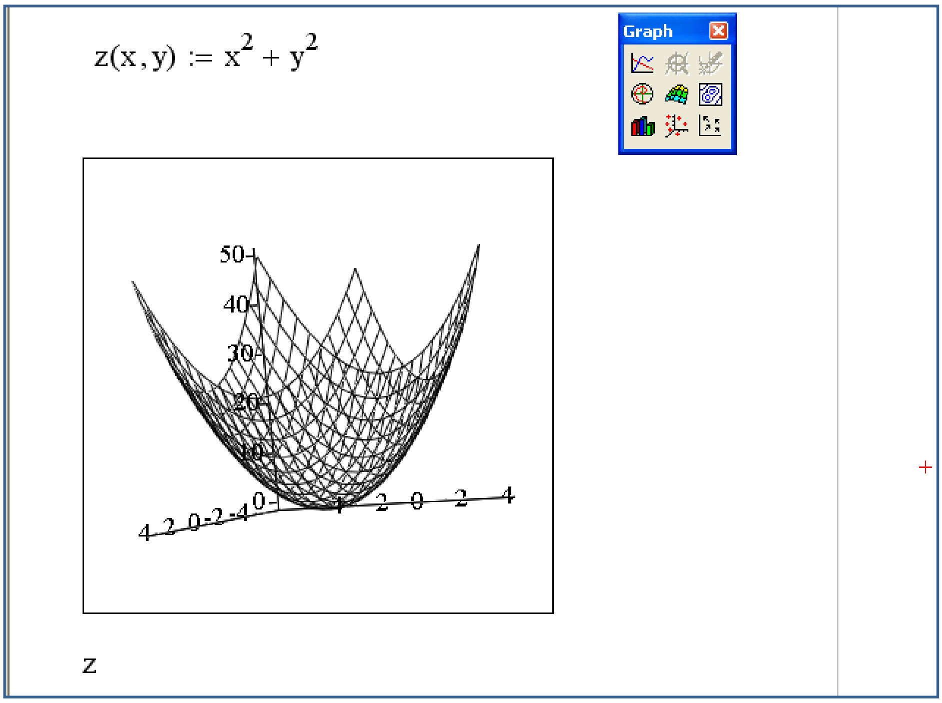


Figure 3.2 – Plotting 3D scatter

Task 3.7
Plot the 3D graph of the function $z(x, y) = x^2 + y^2$.
Solution in Mathcad:



Task 3.8

Plot the 3D graph of the matrix S,

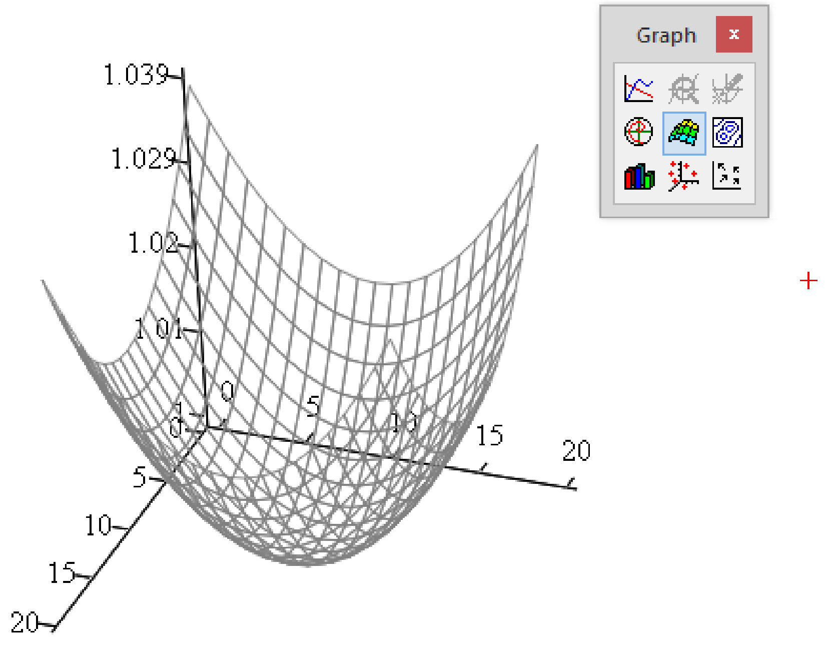
$$S_{i,j} = \sqrt{1 + \left(\frac{1}{5} - \frac{i}{50}\right)^2 + \left(\frac{1}{5} - \frac{j}{50}\right)^2}$$

where $i = 0 \ldots 20$, $j = 0 \ldots 20$.

Solution in Mathcad:

$i := 0..20 \quad j := 0..20$

$$S_{i,j} := \sqrt{1 + \left(\frac{1}{5} + \frac{-i}{50}\right)^2 + \left(\frac{1}{5} + \frac{-j}{50}\right)^2}$$



Task 3.9

Plot the 3D graph of the matrix $M_{i,j} = f(x_i, y_j)$, where function $f(x, y) = \sin(x^2 + y^2)$, x and y are matrixes with the values calculated according to the following relations:

$x_i = -1.5 + 0.15 \cdot i; \quad y_j = -1.5 + 0.15 \cdot j,$

$i = 0 \ldots 20$, $j = 0 \ldots 20$.

Solution in Mathcad:

1. The dimension of x and y matrixes

$N := 20 \qquad i := 0..N \qquad j := 0..N$

2. Calculation of the elements in matrix x and y

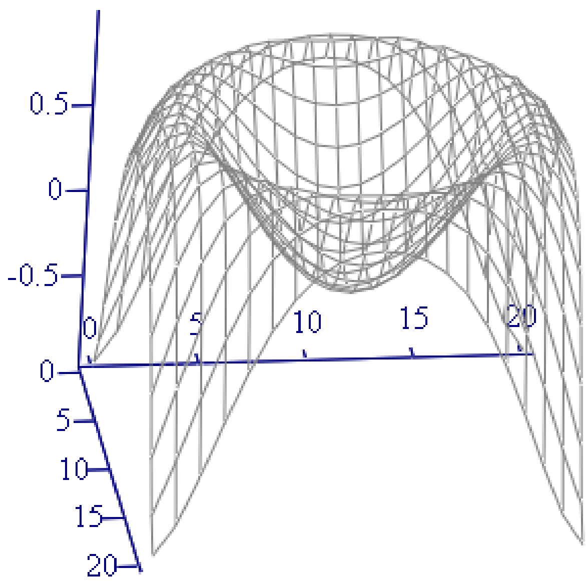
$x_i := -1.5 + 0.15 \cdot i \qquad y_j := -1.5 + 0.15 \cdot j$

3. Defining the function f and corresponding matrix M

$f(x,y) := \sin\left(x^2 + y^2\right)$

$M_{i,j} := f\left(x_i, y_j\right)$

4. Plot the graf of matrix M using Surface Plot



M

Subject 4. CALCULATION OPERATORS. NUMERICAL ESTIMATION OF DERIVATIVES AND INTEGRALS. SYMBOLIC CALCULATIONS

4.1. Evaluating expressions numerically or analytically

You can use calculus operators to evaluate expressions numerically or analytically. Click on the ***Calculus*** icon button in the ***Math*** toolbar to bring up the ***Calculus*** toolbar (Fig. 4.1), or choose ***Toolbars*** > ***Calculus*** from the ***View*** menu.

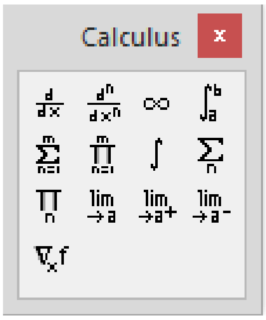


Figure 4.1 – ***Calculus*** toolbar

The Calculus toolbar contains the following operators: derivative, n-th derivative, infinity, definite integral, indefinite integral, summation, summation with range variables, iterated product, iterated product with range variables, two-sided limit right-hand limit, left-hand limit, gradient.

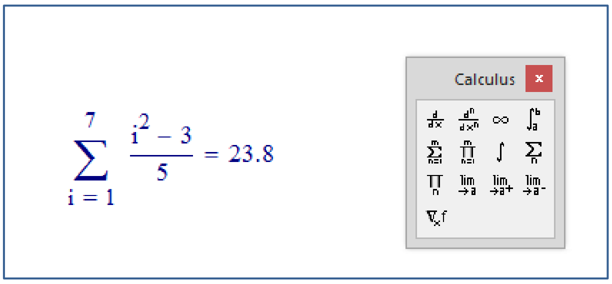
For example, to perform the summation using the summation operator, taken over all values of an index, it is needed either cast the elements to be summed or multiplied as vectors, or implicitly specify the elements in the range of the operator itself.

Task 4.1

Calculate the value of the expression

$$\sum_{i=1}^{7} \frac{i^2 - 3}{5}$$

Solution in Mathcad:



Task 4.2

For the given function $y(x) = (x^2 + 1) \cdot \sin(x - 0.5)$ on the specified range [–1, 1]. Calculate:

1. The value of differential in point, which divide this range with the step 0.5;
2. The integral of this function on this range.

Solution in Mathcad:

1. Defining a range within the points

$x := -1, -0.5 .. 1$

2. Definition of function y

$y(x) := (x^2 + 1) \cdot \sin(x - 0.5)$

3. Calculating the value of the differential in points(given by the defined range x)

$\frac{d}{dx} y(x) =$

2.136
1.517
0.878
1.25
2.714

4. Calculating the integral on the range from -1 to 1

$\int_{-1}^{1} y(x)\, dx = -1.036$

Task 4.3

Using Table 4.1 take the function from column (2) on the range specified in column (3) and calculate:

1. The value of differential in point, which divide this range with the step 0.5;
2. The integral of this function on this range.

Table 4.1 – Data for calculation

Variant	**Function**	**The range of variables**
1	2	3
1	$0.37e^{\sin(x)}$	[0, 2]

The end of the table 4.1

1	2	3
2	$0.5x + x\lg(x)$	[1, 3]
3	$\frac{1}{x \cdot \ln(x+2)}$	[1, 3]
4	$(x+1.9)\cdot\sin(x/3)$	[0, 3]
5	$\frac{3\cdot\cos(x)}{2x}$	[1, 3]
6	$\frac{2x}{\cos(x/2)}$	[–2, 0]
7	$2.6x^2\cdot\ln(x)$	[1, 3]
8	$x^2\cos(x/4)$	[2, 4]
9	$3x+\ln(x)$	[1, 4]
10	$x^2 tg(x/2)$	[1, 3]
11	$1.3x^2\cdot\lg(x)$	[1, 3]
12	$(x+4)\cdot\sin(x-0.1)$	[1, 3]

4.2. Symbolic Calculations in Mathcad

Besides doing numerical calculations, Mathcad can also perform operations on symbolic expressions – expressions that contain variables or mathematical symbols – and return the results in symbolic form.

These expressions are evaluated using the ***symbolic equal sign*** →, rather than the ***numerical equal sign*** =. To insert the symbolic equal sign, press **[Ctrl] [.]**. It is possible to evaluate all of the standard Mathcad operators and many built-in functions symbolically.

The features of evaluation of the expression symbolically:

- Unlike the numerical equal sign, the symbolic equal sign doesn't require to assign values to the variables in an expression before evaluating it.
- Symbolic results can reveal relationships among variables that might not be apparent from numerical results.
- Symbolic calculations are immune to the round-off errors that are inherent in numerical calculations.

There are two ways to perform symbolic operations on an expression:

1. Evaluate it using the symbolic equal sign. This is the recommended because the results are "live": if you later make a change to the worksheet anywhere above the expression or to the left of it in the same line, Mathcad updates the result automatically. To perform specific symbolic operations, such as factoring or expanding the expression, you can insert keywords before the symbolic equal sign.

2. Use the ***Symbolics*** menu commands. The results are static, one-time calculations that ignore all previously defined variables and functions. These results do not display the symbolic equals sign or keywords.

Task 4.4

Calculate the value of the integral of function $y(x) = \cos(x)/(5 + 4\cos(x))$.

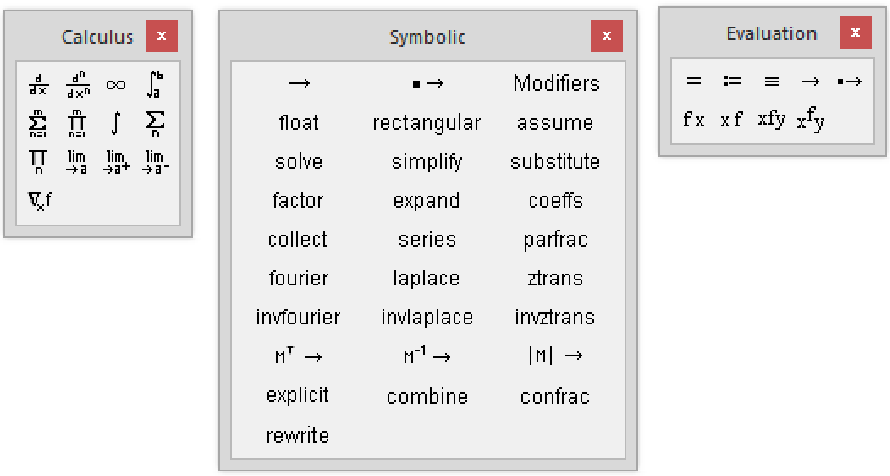
Solution in Mathcad:

1. Definition of function y

$$y(x) := \frac{\cos(x)}{5 + 4\cos(x)}$$

2. Symbolic calculation of integral from y(x) function

$$\int y(x)\,dx \rightarrow \frac{x}{4} - \frac{5\cdot\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{3}\right)}{6}$$



Subject 5. CREATING VECTORS AND MATRICES

5.1. Defining array

In two dimensions there are just two types of arrays: ***vectors*** and ***matrices***.

A ***vector*** is an array of just one row or column, while a ***matrix*** has multiple rows and columns. Mathcad functions that expect a vector argument generally require a column vector.

There are several ways to define an array:

- Create an array of empty placeholders with the Insert Matrix dialog box (Fig. 1.5c). Then click on each placeholder and enter a value or use **[Tab]** to move through the placeholders.
- Use the subscript operator to define individual array elements, for example

v[0 := 4i ; v[1 := 1+3i

- Create an array variable using the subscript operator and a formula relating the value of an array element to its indices with range variables, for example

i := 0 ..9
j := 0 ..9
X[i,j := i^2+j/2

- Type numbers as a comma-separated list into a vector defined with a range variable, for example

v[i := 1.345, 2.567, 3.678, 4.39, ...

- Use one of the functions that generate special arrays or use a random number generator to create a vector of random numbers.
- Import data from a file.

Once an array is defined in terms of a variable name, you can use that variable in any calculation, where you would like to represent the array.

Arrays can contain numbers, expressions, or strings. Arrays can also contain nested arrays. The simplest way to define an array element that is itself an array is to assign the element using the subscript operator.

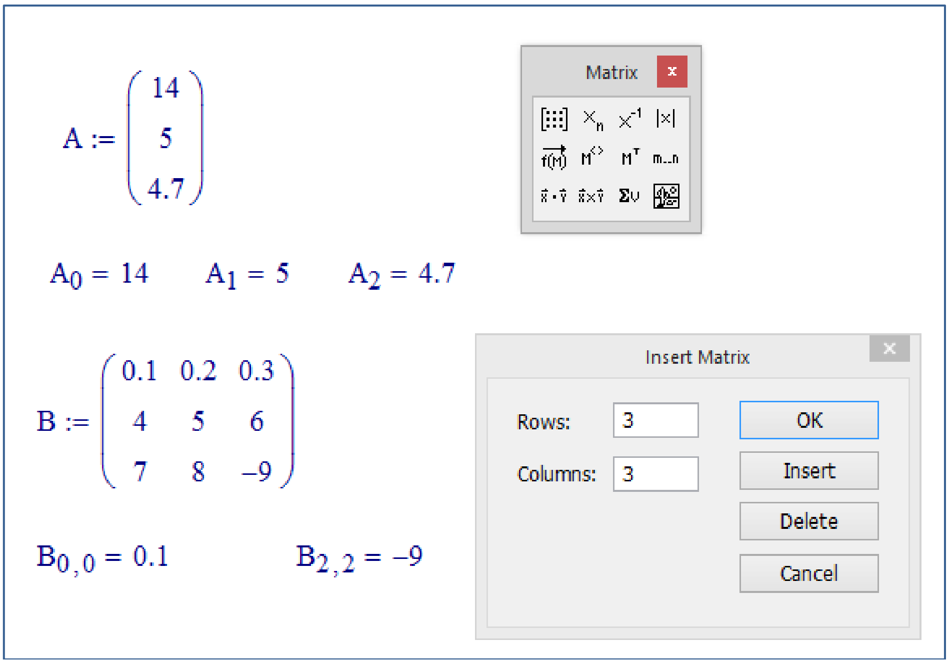
Task 5.1

Create the vector $A = \{14,\ 5,\ 4.7\}$ and matrix *B*,

$$B = \begin{matrix} 0.1 & 0.2 & 0.3 \\ 4 & 5 & 6 \\ 7 & 8 & -9 \end{matrix}$$

Display all elements of vector A and the first and the last elements of matrix B.

Solution in Mathcad:



5.2. Indexing an array

Arrays begin with index 0 by default. This can be changed by adjusting the built-in variable **ORIGIN** in the worksheet. Beware of not including all elements in an array in a calculation by creating a range variable which begins after the **ORIGIN**, or ends before the last element of the matrix or vector. For **ORIGIN = 0**, the last index of a vector from n elements is $(n-1)$ index.

To change the **ORIGIN** for the whole worksheet, choose **Worksheet Options** from the **Tools** menu and go to the ***Built-in Variables*** tab, then enter a new value in the dialog box.

It is possible also to reset the **ORIGIN** locally by redefining it in your worksheet, for example

ORIGIN := 1

sets the array index **ORIGIN** to **1** for all arrays below this definition.

Changing the Task 5.1 indexing to starting with the first element index equal to 1 is listed below:

ORIGIN := 1

$$A := \begin{pmatrix} 14 \\ 5 \\ 4.7 \end{pmatrix}$$

$A_1 = 14$ $A_2 = 5$ $A_3 = 4.7$

$$B := \begin{pmatrix} 0.1 & 0.2 & 0.3 \\ 4 & 5 & 6 \\ 7 & 8 & -9 \end{pmatrix}$$

$B_{1,1} = 0.1$ $B_{3,3} = -9$

Task 5.2

Plot a graph by the points x and y. Points x are vectors, calculated according to the following relations: $x_i = i \cdot 0.5$. Point y are the function $y = \sin(x)$. Take 16 initial points. Display the transposed vector x.

Solution in Mathcad:

1. Definition of elements' indexes

$i := 0..15$

2. Definition of elements of vector x

$x_i := i \cdot 0.5$

3. Creating the y vector by calculation the function y(x) for each point of vector x

$y_i := \sin(x_i)$

4. Displaying the obtained x and y vectors

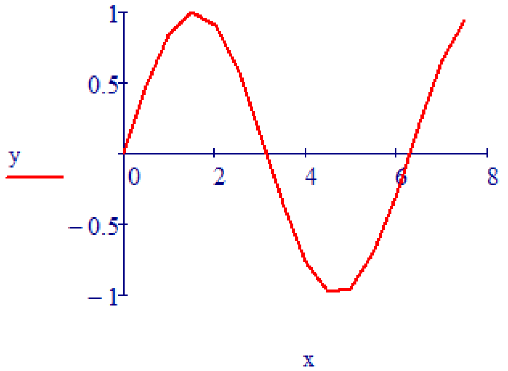
x =

	0
0	0
1	0.5
2	1
3	1.5
4	2
5	2.5
6	3
7	3.5
8	...

y =

	0
0	0
1	0.479
2	0.841
3	0.997
4	0.909
5	0.598
6	0.141
7	-0.351
8	...

5. Plot the graph y(x)



6. The transposed x vector

$x^T =$

	0	1	2	3	4	5	6	7	8	9
0	0	0.5	1	1.5	2	2.5	3	3.5	4	...

Task 5.3

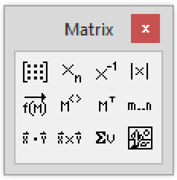
Define t vector, which is the second row in matrix M.

$$M = \begin{matrix} 2 & 1 & 6 & 8 \\ -8 & 34 & 5 & 1.4 \\ 3.6 & 6 & 3 & 5.8 \end{matrix}$$

Solution in Mathcad:

1. Array indexing from 1

ORIGIN := 1



2. Input matrix M with dimension 3 x 4

$$M := \begin{pmatrix} 2 & 1 & 6 & 8 \\ -8 & 34 & 5 & 1.4 \\ 3.6 & 6 & 3 & 5.8 \end{pmatrix}$$

3. Define t vector, which is the second column of matrix M

$$t := M^{\langle 2 \rangle}$$

4. Display the resulted t vector

$$t = \begin{pmatrix} 1 \\ 34 \\ 6 \end{pmatrix}$$

Task 5.4

1) Create the one-dimensional array $A = \{a_1, a_2, \dots, a_{25}\}$. The array elements are calculated according to the relation: $a_i = 10 \cdot i/\sin(i)$, where $i = 1 \dots 25$. Display the array elements.

2) Calculate the value of the following expression:

$$R = \sum_{i=5}^{10} a_i - \prod_{i=7}^{18} a_i$$

Solution in Mathcad:

1. Array indexing from 1

$ORIGIN := 1$

2. Define the array's indexing numbers

$i := 1 .. 25$

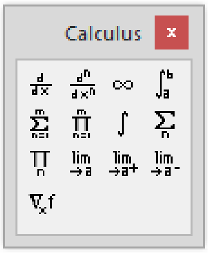
3. Calculate the elements a from array

$$a_i := 10 \cdot \frac{i}{\sin(i)}$$

4. Display the resulted array

a =

	1
1	11.884
2	21.995
3	212.585
4	-52.854
5	-52.142
6	-214.734
7	106.547
8	80.86
9	218.384
10	-183.816
11	...



5. Calculate the expression

$$R := \sum_{i=5}^{10} a_i + \left(-\prod_{i=7}^{18} a_i\right) = -2.021 \times 10^{27}$$

Task 5.5

Using Table 5.1 do the following:

1) Create the one-dimensional array $A = \{a_1, a_2, \dots, a_N\}$, where N is the number of elements in array listed in column (3). The array elements are calculated according to the relation presented in column (2), where $i = 1 \dots N$. Display the array elements.

2) Calculate the value of the following expression:

$$R = \sum_{i=n}^{k} a_i - \prod_{i=m}^{l} a_i$$

where $n,\ k,\ m,\ l$ are the numbers listed in column (4)

Table 5.1 – Data for calculation Task 5.5

No	**Relation for array elements**	**N value**	**Values $n,\ k,\ m,\ l$**
1	$a_i = 1/2^i + 1/3^i$	$N = 20$	$n = 3;\ k = 11;\ m = 2;\ l = 18$
2	$a_i = (2 \cdot i - 1) \cdot 2^i$	$N = 20$	$n = 2;\ k = 10;\ m = 1;\ l = 8$
3	$a_i = 1/(3 \cdot i - 2) \cdot (3 \cdot i + 1)$	$N = 25$	$n = 5;\ k = 13;\ m = 3;\ l = 15$
4	$a_i = 10^i / i$	$N = 18$	$n = 4;\ k = 12;\ m = 3;\ l = 14$
5	$a_i = 10 / i^i$	$N = 25$	$n = 9;\ k = 19;\ m = 3;\ l = 9$
6	$a_i = \ln(i) / i^2$	$N = 25$	$n = 5;\ k = 20;\ m = 4;\ l = 12$
7	$a_i = \mathrm{i}^{\ln i} / (\ln(5 \cdot i))^i$	$N = 30$	$n = 7;\ k = 25;\ m = 1;\ l = 9$
8	$a_i = 10 / i$	$N = 19$	$n = 4;\ k = 16;\ m = 5;\ l = 15$
9	$a_i = (e^i) / i$	$N = 18$	$n = 2;\ k = 15;\ m = 4;\ l = 14$
10	$a_i = i^2 \cdot e^i$	$N = 20$	$n = 1;\ k = 18;\ m = 2;\ l = 17$
11	$a_i = i^2 + 2 \cdot e^i$	$N = 15$	$n = 3;\ k = 9;\ m = 2;\ l = 12$
12	$a_i = \dfrac{10 + i}{\sin(i)}$	$N = 25$	$n = 5;\ k = 10;\ m = 7;\ l = 18$

Task 5.6

Calculate the value of the following expression:

$$d=\sum_{i=1}^{8} a_i + b_i \;^{1/3},$$

where a_i are the elements of one-dimension array $A=(a_1,a_2,\ldots a_8)$;

b_i are the elements of one-dimension array $B=(b_1,b_2,\ldots b_8)$.

The components of arrays are optional numbers.

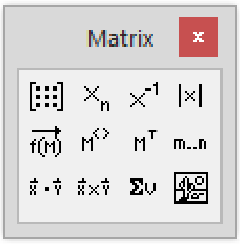
Solution in Mathcad:

1. Array indexing from 1

ORIGIN := 1

2. Input optional elements for array A and B

$$a := \begin{pmatrix} 3 \\ 4.5 \\ 7 \\ 12 \\ 1 \\ 14 \\ 8 \\ 9 \end{pmatrix} \qquad b := \begin{pmatrix} 1 \\ 4 \\ 7 \\ 8 \\ 2.3 \\ 12 \\ 14 \\ 17 \end{pmatrix}$$



3. Calculate the expression

$$d := \sum_{i=1}^{8} \left(a_i + b_i\right)^{\frac{1}{3}} = 18.969$$

Task 5.7
Do the Tasks listed in Table 5.2.

Table 5.2 – Tasks for calculation

No	Task description
1	2
1	Calculate the value of the following expression: $$y = \sum_{i=1}^{20} a_i b_i ,$$ where $i = 1,2,\ldots,20$; a_i are the elements of one-dimension array $A = (a_1, a_2, \ldots a_{20})$; b_i are the elements of one-dimension array $B = (b_1, b_2, \ldots b_{20})$. The components of arrays are optional numbers.
2	Calculate the value of the following expression: $$F = \sum_{i=1}^{10} b_i + k \cdot \sum_{i=1}^{10} c_i ,$$ where $i = 1,2,\ldots,10$; $k = 2.55$; b_i are the elements of one-dimension array $B = (b_1, b_2, \ldots b_{10})$; c_i are the elements of one-dimension array $C = (c_1, c_2, \ldots c_{10})$. The components of arrays are optional numbers.
3	Calculate the value of the following expression: $$Z = \left\{ \sum_{j=1}^{20} (y_j + x_j) \right\}^{1/2} ,$$ where $j = 1,2,\ldots,20$; x_j are the elements of one-dimension array $X = (x_1, x_2, \ldots x_{20})$; y_j are the elements of one-dimension array $Y = (y_1, y_2, \ldots y_{20})$. The components of arrays are optional numbers.

Continuation of the table 5.2

1	2
4	Calculate the elements of array z_j according to the following expression: $$z_j = (x_j + y_j) / p,$$ where $j = 1,2,\dots,10$; $p = 23$; x_j are the elements of one-dimension array $X = (x_1, x_2, \dots x_{10})$; y_j are the elements of one-dimension array $Y = (y_1, y_2, \dots y_{10})$; z_j are the elements of one-dimension array $Z = (z_1, z_2, \dots z_{10})$. The components of arrays $X = (x_1, x_2, \dots x_{10})$ and $Y = (y_1, y_2, \dots y_{10})$ are optional numbers.
5	Calculate the value of the following expression: $$P = \sum_{i=1}^{20} x_i + 5.6 y_i \quad ,$$ where $i = 1,2,\dots,20$; x_i are the elements of one-dimension array $X = (x_1, x_2, \dots x_{20})$; y_i are the elements of one-dimension array $Y = (y_1, y_2, \dots y_{20})$. The components of arrays are optional numbers.
6	Calculate the value of the following expression: $$R = \sum_{i=5}^{20} \cos x_i \cdot \sin y_i \quad ,$$ where $i = 1,2,\dots,20$; x_i are the elements of one-dimension array $X = (x_1, x_2, \dots x_{20})$; y_i are the elements of one-dimension array $Y = (y_1, y_2, \dots y_{20})$. The components of arrays are optional numbers.
7	Calculate the value of the following expression: $$Z1 = x^2 \cdot \sum_{i=3}^{20} (m_i + 1),$$ where $i = 1,2,\dots,20$; $x = 2.76$; m_i are the elements of one-dimension array $M = (m_1, m_2, \dots m_{20})$ The components of array are optional numbers.

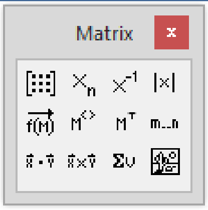
The end of the table 5.2

1	2
8	Calculate the value of the following expression: $$X = \sqrt[3]{\sum_{i=2}^{15}(z_i + a)},$$ where $i = 1,2,...,15$; $a = 4.65$; z_i are the elements of one-dimension array $Z = (z_1, z_2, ... z_{15})$. The components of array are optional numbers.
9	Calculate the value of the following expression: $$T = (x-1)\cdot\sqrt[3]{\sum_{i=1}^{20} a_i} + (x+1)\ ,$$ where $i = 1,2,...,20$; $x = 6.45$; a_i are the elements of one-dimension array $A = (a_1, a_2, ... a_{20})$. The components of array are optional numbers.
10	Calculate the value of the following expression: $$X = \sqrt[3]{\sum_{i=1}^{15}(2z_i + a)},$$ where $i = 1,2,...,15$; $a = 5.85$; z_i are the elements of one-dimension array $Z = (z_1, z_2, ... z_{15})$. The components of array are optional numbers.
11	Calculate the value of the following expression: $$F1 = c^2 \prod_{i=2}^{8}(y_i \cdot x)^2,$$ where $i = 1,2,...,8$; $c = 12$; $x = 1.5$; y_i are the elements of one-dimension array $Y = (y_1, y_2, ... y_8)$. The components of arrays are optional numbers.
12	Calculate the elements of array z_j according to the following expression: $$q_i = e^x(a_i - (k+1)^{1/3}) + b_i\ ,$$ where $i = 1,2,...,10$; $x = 2$; $k = 3.5$; a_i, b_i, q_i, are the elements of one-dimension arrays $A = (a_1, a_2, ... a_{10})$; $B = (b_1, b_2, ... b_{10})$ and $Q = (q_1, q_2, ... q_{10})$ correspondingly. The components of arrays $A = (a_1, a_2, ... a_{10})$ and $B = (b_1, b_2, ... b_{10})$ are optional numbers.

Subject 6
VECTOR AND MATRIX OPERATORS AND FUNCTIONS

6.1. Vector and matrix operators

The vector and matrix operators used in Mathcad are presented in Table 6.1. The examples of these operators are listed below.



Array indexing from 1

ORIGIN := 1

Definiton of M matrix and v and w vectors

$$M := \begin{pmatrix} 0 & 1 & 2 \\ 3 & 0 & 2 \\ 5 & 3 & 1 \end{pmatrix} \qquad v := \begin{pmatrix} 13 \\ -3 \\ 50 \end{pmatrix} \qquad w := 2 \cdot v \qquad w = \begin{pmatrix} 26 \\ -6 \\ 100 \end{pmatrix}$$

Summation of vector v elements

$$\sum v = 60$$

Determinant of M matrix

$$|M| = 25$$

Scalar (dot) product

$$v \cdot w = 5.356 \times 10^3$$

Inverse matrix

$$N := M^{-1} \qquad N = \begin{pmatrix} -0.24 & 0.2 & 0.08 \\ 0.28 & -0.4 & 0.24 \\ 0.36 & 0.2 & -0.12 \end{pmatrix}$$

Transposed matrix

$$w^T = \begin{pmatrix} 26 & -6 & 100 \end{pmatrix}$$

Dot product

$$M \cdot N = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Joining the matrices

$$Mv := \text{augment}(M, v) \qquad Mv = \begin{pmatrix} 0 & 1 & 2 & 13 \\ 3 & 0 & 2 & -3 \\ 5 & 3 & 1 & 50 \end{pmatrix}$$

$$vM := \text{augment}(v, M) \qquad vM = \begin{pmatrix} 13 & 0 & 1 & 2 \\ -3 & 3 & 0 & 2 \\ 50 & 5 & 3 & 1 \end{pmatrix}$$

Table 6.1 – Vector and matrix operators in Mathcad

Operator	**Displayed**	**Keystroke**	**Description**
1	2	3	4
Multiplication and Dot Product Operator	$x \cdot y$	**[*]**	If *x* and *y* are scalars, returns the product of *x* and *y*. If *x* and *y* are vectors, returns the dot product (inner product) of *x* and *y*, a scalar formed by multiplying element-wise the entries of the first vector with the complex conjugate of the entries of the second vector, and summing the results. If *x* and *y* are matrices, returns the matrix product of *x* and *y*. If one operand is a scalar and the other is a matrix, each element in the matrix is multiplied by the scalar.
Division	$\frac{x}{y}$	**[/]** **[Ctrl] [/]**	Return *x* divided by *y*. If either *x*, *y*, or both, are vectors, division is performed element-wise. If *y* is a square matrix, then the result is $x \cdot y^{-1}$, that is, *x* times the inverse of *y*.
Addition	$x+y$	**[+]**	Returns the sum of *x* and *y*. If *x* and *y* are two matrices, they should have equal number of rows and columns. Then it summarizes the corresponding elements. If *x* is a matrix and *y* is scalar, it adds *y* to every element in *x*.
Negation	$-x$	**[−]**	Negation for arrays negates each element in the array.
Subtraction	$x-y$	**[−]**	Subtracts *y* from *x* (*x* and *y* are scalars or arrays having the same dimensions). If both *x* and *y* are arrays, they must have the same number of rows and columns. If *x* is an array and *y* is a scalar, then $x - y$ is an array formed by subtracting *y* from every element in *x*.

The end of the table 6.1

1	2	3	4
Powers of matrix	y^x	**[^]**	Returns *y* raised to the *x* power. If *y* is a scalar and *x* is a vector, the result is a vector of the same size as *x*, whose i-th entry is *y* raised to the power x_i. If *y* is a square matrix, the result depends on *x*. – $x = 0$ returns the identity matrix whose dimensions match those of *y*. – $x = -1$ returns inverse of *y*, if it exists. – When *x* is a positive integer, *y* is multiplied by itself *x* times. – When *x* is a negative integer, the inverse of *y* is multiplied by itself *x* times.
Determinant or Magnitude	| **v** | or $\|M\|$	**[|]**	Returns the magnitude (norm) of a vector, the absolute value of a scalar, or the determinant of a matrix.
Transpose	M^T	**[Ctrl] 1**	It returns the $n \times m$ array formed by interchanging the rows and columns of an $m \times n$ array.
Vector Cross Product	**u × v**	**[Ctrl] 8**	Returns the vector cross product of *u* and *v*.
Vectorize	$\overrightarrow{v}$	**[Ctrl] [−]**	Applies operators and functions element-wise to an array, allowing you to perform iterative calculations without using a range variable.
Column	$M^{<n>}$	**[Ctrl] 6**	Returns the nth column of the array *M*.
Array Subscript	v_n or $M_{m,n}$	**v[n** **M[m,n**	Returns the nth element of v. Returns the element in row *m* and column *n* of matrix *M*.

6.2. Vector and matrix functions

The vector and matrix functions used in Mathcad are presented in Table 6.2. The examples of these functions are listed below.

Array indexing from 1

$ORIGIN := 1$

Definiton of M matrix

$$M := \begin{pmatrix} 0 & 1 \\ 5 & 3 \\ 6 & -2 \end{pmatrix}$$

The number of columns in M matrix

$cols(M) = 2$

The number of rows in M matrix

$rows(M) = 3$

Maxima and Minima of an M matrix

$min(M) = -2$

$max(M) = 6$

Definiton of v vector

$$v := \begin{pmatrix} 13 \\ 3 \\ 5 \end{pmatrix}$$

The number of elements in v

$length(v) = 3$

The index of last element

$last(v) = 3$

Table 6.2 – Vector and matrix operators in Mathcad

Function	**Description**
rows(A)	Returns the number of rows in *A*.
cols(A)	Returns the number of columns in *A*.
max(A, B, C, ...)	Returns the largest value from *A*, *B*, *C*, ...
min(A, B, C, ...)	Returns the smallest value from *A*, *B*, *C*, ...
length(v)	Returns the number of elements in the vector *v*.
last(v)	Returns the index of the last element in the vector *v*.

Subject 7. SOLVING THE SET OF EQUATIONS

A system of linear equations is a collection of two or more linear equations involving the same set of variables. A general system of m linear equations with n unknowns can be written as

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\ \vdots \qquad \vdots \qquad\quad & \quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &= b_m \end{aligned}$$

Or it can be represented as:

$$\sum_{j=1}^{n} a_{ij}x_j = b_i$$

where $i = 1, 2, \ldots, m$ is the equation number, totally *n*;

$j = 1, 2, \ldots, n$ is the number of variables in equations set, totally *m*;

a_{ij} is the matrix of coefficients in the left part of equations (the coefficients of the system);

b_i is the vector of right parts (the constant terms);

x_j are unknowns.

7.1. Matrix method

Representing the system of linear equations in matrix form, we will get:

$$AX = B,$$

where

– matrix *A* containing a_{ij} elements, which is the matrix of coefficients of the system;

– matrix *B* containing b_i elements, which is the vector of the constant terms;

– matrix *X* containing x_j elements, which are unknowns.

The system in matrix form is *AX* = *B*. Matrix solution stands, that if the equation system is expressed in the matrix form, the entire solution set can also be expressed in matrix form. If the matrix *A* is square (has *m* rows and n=m columns) and has full rank (all m rows are independent), then the system has a unique solution given by

$$X = A^{-1}B.$$

where A^{-1} is the inverse of A.

Task 7.1

Find values x, y, z, which are the solution of the following system of linear equations:

$$\begin{cases} x - 2y + 3z = 6 \\ 2x + 3y - 4z = 16 \\ 3x - 2y - 5z = 12 \end{cases}$$

Solution in Mathcad:

1. Create matrix A with 3 rows and 3 collumns.

$$A := \begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -4 \\ 3 & -2 & -5 \end{pmatrix}$$

2. Create matrix B of the costant terms.

$$B := \begin{pmatrix} 6 \\ 16 \\ 12 \end{pmatrix}$$

3. Use the inverse matrixes to solve the system of linear equations:

$$A^{-1} \cdot B = \begin{pmatrix} 7 \\ 2 \\ 1 \end{pmatrix}$$

7.2. Using Mathcad function lsolve( ) or solve block

Mathcad contains the special function for solving linear systems – lsolve(*A*, *b*).

It returns the solution x for the linear system of equations $A \cdot x = b$, using LU decomposition.

Arguments of lsolve(*A*, *b*) function:

– *A* is a real or complex matrix. If the matrix is square, it must be non-singular;

– *b* is a real or complex vector or matrix having the same number of rows as *A*.

Task 7.2

Solve the following linear system:

$$\begin{cases} -0.87x_1 + 0.27x_2 - 0.22x_3 - 0.18x_4 = -1.21 \\ -0.21x_1 - x_2 - 0.45x_3 + 0.18x_4 = 0.33 \\ 0.12x_1 + 0.13x_2 - 1.33x_3 + 0.18x_4 = 0.48 \\ 0.33x_1 - 0.05x_2 + 0.06x_3 - 1.28x_4 = 0.17 \end{cases}$$

Solution in Mathcad:

1. Array indexing from 1

$$ORIGIN := 1$$

2. Input matrix A of coefficients of the system

$$A := \begin{pmatrix} -0.87 & 0.27 & -0.22 & -0.18 \\ -0.21 & -1 & -0.45 & 0.18 \\ 0.12 & 0.13 & -1.33 & 0.18 \\ 0.33 & -0.05 & 0.06 & -1.28 \end{pmatrix}$$

2. Input matrix B of constant terms

$$B := \begin{pmatrix} -1.21 \\ 0.33 \\ 0.48 \\ 0.17 \end{pmatrix}$$

3. Define X matrix, which is the matrix of unknowns

$$X := \mathrm{lsolve}(A, B)$$

4. Display the obtained solution

$$X = \begin{pmatrix} 1.277 \\ -0.444 \\ -0.262 \\ 0.202 \end{pmatrix}$$

4. Check the obtained results applying 1st equation

$$-0.87 \cdot X_1 + 0.27 \cdot X_2 - 0.22 \cdot X_3 - 0.18 \cdot X_4 = -1.21$$

The other way to solve the system of linear equations in Mathcad is to use solve block.

A "***Solve Block***" refers to a group of Mathcad commands used to solve a system of linear, differential or partial differential equations, an optimization problem, or a linear programming problem. The steps for these kinds of solving differ slightly, but each ***Solve Block*** involves the keyword **Given**, a set of constraints, a set of equations, and a solving function.

Parts of a solve block:

1. The keyword **Given**, typed as a math region.

2. A set of equations and/or inequalities representing the constraints. The following operators (from Boolean toolbar, Fig. 1.5g) are used to create these constraints:

– boolean equals,

– greater than,

– greater than or equal to,

– less than, or

– less than or equal to.

3. An equation or system of equations to solve, along with initial conditions in the case of differential equations. Equality is indicated using the Boolean equals, **[Ctrl] [=].**

4. A solving function:

• The functions **Find** or **Minerr** are used to solve a system of equations.

Function **Find(var1, var2, ...)** returns the values of ***var1, var2...*** that satisfy the equations and inequalities in a Solve Block. If you are solving for *n* variables, the solve block must have *n* equations. Matrix notation is allowed, as is solving for matrix variables. The Find function chooses an appropriate method from a group of available methods, depending on whether the problem is linear or nonlinear, and other attributes.

Output of the **Find** function may be assigned to a single variable, a vector of explicit variable names, or a function of other variable names within the solve block (including guess variables), parameterizing the solve block.

Task 7.3

Solve the following linear system:

$$\begin{cases} -0.87x_1 + 0.27x_2 - 0.22x_3 - 0.18x_4 = -1.21 \\ -0.21x_1 - x_2 - 0.45x_3 + 0.18x_4 = 0.33 \\ 0.12x_1 + 0.13x_2 - 1.33x_3 + 0.18x_4 = 0.48 \\ 0.33x_1 - 0.05x_2 + 0.06x_3 - 1.28x_4 = 0.17 \end{cases}$$

Solution in Mathcad:

1. Array indexing from 1

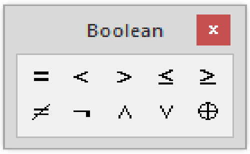
ORIGIN := 1

2. Provide the initial guessed values for the Solve block

x1 := 1 x2 := 1 x3 := 1 x4 := 1

3. Start the Solve block from word "Given"

Given



4. Provide the system of equations

$$-0.87\cdot x1 + 0.27\cdot x2 - 0.22\cdot x3 - 0.18\cdot x4 = -1.21$$

$$-0.21\cdot x1 - x2 - 0.45\cdot x3 + 0.18\cdot x4 = 0.33$$

$$0.12\cdot x1 + 0.13\cdot x2 - 1.33\cdot x3 + 0.18\cdot x4 = 0.48$$

$$0.33\cdot x1 - 0.05\cdot x2 + 0.06\cdot x3 - 1.28\cdot x4 = 0.17$$

5. Use find function to obtain the X vector with unknowns

X := Find(x1 ,x2 ,x3 ,x4)

6. Display the solution

$$X = \begin{pmatrix} 1.277 \\ -0.444 \\ -0.262 \\ 0.202 \end{pmatrix}$$

7. Check the obtained results applying 1st equation

$$-0.87\cdot X_1 + 0.27\cdot X_2 - 0.22\cdot X_3 - 0.18\cdot X_4 = -1.21$$

A "***Solve Block***" with function **Find(var1, var2, ...)** is suitable to solve the systems of nonlinear equations.

Task 7.4

Solve the following system of nonlinear equations:

$$\begin{cases} 7x + 5y + \sin(z) = 9 \\ \sin^2(x) - 3y + 2z = 2 \\ 2x + 5\cos(y) - 4z = 3 \end{cases}$$

Solution in Mathcad:

1. Array indexing from 1

   ORIGIN := 1

2. Provide the initial guessed values for the Solve block

   x := 1   y := 1   z := 1

3. Start the Solve block from word "Given"

   Given

4. Provide the system of equations

   $7 \cdot x + 5 \cdot y + \sin(z) = 9$

   $\sin(x)^2 - 3 \cdot y + 2 \cdot z = 2$

   $2 \cdot x + 5 \cdot \cos(y) - 4 \cdot z = 3$

5. Use find function to obtain the X vector with unknowns

   X := Find(x,y,z)

6. Display the solution

   $$X = \begin{pmatrix} 1.009 \\ 0.222 \\ 0.974 \end{pmatrix}$$

7. Check the obtained results applying 1st equation

   $7 \cdot X_1 + 5 \cdot X_2 + \sin(X_3) = 9$

Task 7.5

Do the Tasks listed in Table 7.1.

Table 7.1 – Tasks for calculation

№	Set of Equations
1	$\begin{cases} -0.85x+0.05y-0.08z+0.14t=0.48 \\ 0.32x-1.43y+0.12z+0.11t=-1.24 \\ 0.17x+0.06y-1.08z+0.12t=-1.15 \\ 0.21x-0.16y+0.36z-t=0.88 \end{cases}$
2	$\begin{cases} -x+0.28y-0.17z+0.06t=-0.21 \\ 0.52x-y+0.12z+0.17t=1.17 \\ 0.17x-0.18y+0.79z=0.81 \\ 0.11x+0.22y+0.03z-0.95t=-0.72 \end{cases}$
3	$\begin{cases} 2.5x+4y-7z=12.115 \\ x-3y+z=-0.87 \\ 7x+2y-1.5z=35.93 \end{cases}$
4	$\begin{cases} x+7y+20z=91.65 \\ 4x-5y-2z=-14.85 \\ 10x+2y+15z=87.2 \end{cases}$
5	$\begin{cases} 2x+15y-8z=206.45 \\ 12x-7y+3z=39.06 \\ 7x+2y-12z=148.14 \end{cases}$
6	$\begin{cases} 6.05x+0.13y+8.57z=19.6 \\ 15.46x-8y+13.94z=23.8 \\ 7.18x-12.6y+0.07z=-0.04 \end{cases}$
7	$\begin{cases} x+2y-3z+5t=1 \\ x+3y-13z+22t=-1 \\ 3x+5y+z-2t=5 \\ 2x+3y+4z-7t=4 \end{cases}$
8	$\begin{cases} -0.87x_1+0.27x_2-0.22x_3-1.28x_4=-1.21 \\ -0.21x_1-0.3x_2-0.45x_3+0.18x_4=0.33 \\ 0.12x_1+0.13x_2-1.33x_3+0.18x_4=0.48 \\ 0.33x_1-0.05x_2+0.06x_3-1.28x_4=0.17 \end{cases}$
9	$\begin{cases} x_1+2x_2+4x_3=31 \\ 5x_1+x_2+x_3=29 \\ 3x_1-x_2+x_3=10 \end{cases}$

Subject 8. FINDING THE ROOTS OF EQUATIONS $\boldsymbol{f(x) = 0}$

In mathematics, an equation is an equality containing one or more variables. Solving the equation consists of determining which values of the variables make the equality true. In this situation, variables are also known as unknowns and the values which satisfy the equality are known as solutions.

Algebra studies two main families of equations: ***polynomial*** equations and, among them, ***linear*** equations. Polynomial equations have the form $P(x) = 0$, where P is a polynomial. Linear equations have the form $a(x) + b = 0$, where a is a linear function and b is a vector.

8.1. Finding the roots of equations in Mathcad

Mathcad provides the possibility to solve the equation with one unknown by the graphing method. It requires the presentation of the equation in the following form $f(x) = 0$, when all its members are moved to the left part. The next step is to plot the graph of function $y = f(x)$. The points where the graph crosses the x-axis, are the solutions of the equation. If the graph doesn't cross the x-axis, the equation doesn't have the solution.

The Mathcad function, which finds the roots of the equation is

root(f(var1, var2, ...), var1, [a, b]).

It returns the value of ***var1*** to make the function f equal to zero.

If ***a*** and ***b*** are specified, root finds ***var1*** on the interval **[*a*,*b*]**. Otherwise, ***var1*** must be defined with a guess value before root is called. When a guess value is used, root uses the Secant or Mueller method; in the case where root bracketing is used, root uses the Ridder or Brent method.

Arguments of root() function are as follows:

- ***f*** is a scalar-valued function of any number of variables.
- ***var1*** is a scalar variable found in *f*, the variable with respect to which the root is found.
- ***a, b*** (optional) are real numbers, $a < b$, such that $f(a)$ and $f(b)$ have opposite signs. **root** searches for a root on the interval $a \le x \le b$.

Task 8.1

Find the roots of the equation:

$$\sin(x) + \cos(x) = 0$$

In the range $-\pi \le x \le \pi$.

Solution in Mathcad:

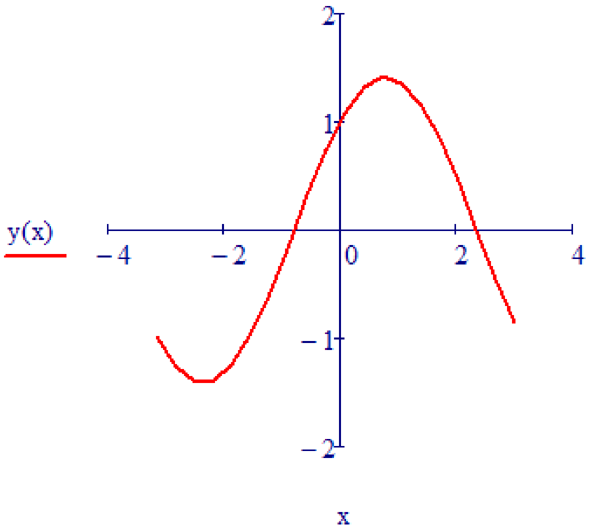
1. Define the function

y(x) := sin(x) + cos(x)

2. Define the range for x taking increment 0.314

x := −3.15 ,−3.14 + 0.314 .. 3.14

3. Plot the graph of function



4. Provide the initial guessed values of equation roots

x1 := −1 x2 := 2.2

5. Find the roots using root() function and guessed values

root(y(x1) ,x1) = −0.785

root(y(x2) ,x2) = 2.356

Answer:
the equation $\sin(x)+\cos(x)=0$ on the range $-\pi \le x \le \pi$ has two roots: $x_1 = -0.785$; $x_2 = 2.356$.

8.2. Finding the roots of polynomial equations

In mathematics, a polynomial is an expression consisting of variables and coefficients which only employs the operations of addition, subtraction, multiplication, and non-negative integer exponents.

A polynomial in a single indeterminate x can always be written (or rewritten) in the form:

$$v_n x^n + \ldots v_2 x^2 + v_1 x + v_0$$

where $v_0, \ldots, v_n$ are the constants, and x is the indeterminate.

A polynomial equation, also called algebraic equation, is an equation of the form:

$$v_n x^n + \ldots v_2 x^2 + v_1 x + v_0 = 0$$

In case of a univariate polynomial equation, the variable is considered an unknown, and one seeks to find the possible values for which both members of the equation evaluate to the same value (in general more than one solution may exist).

The Mathcad function, which finds the roots of polynomial equation is ***polyroots(v)***.

It returns a vector containing the roots of the polynomial whose coefficients are in ***v***. By default, polyroots uses a LaGuerre method. LaGuerre's method is iterative and searches for solutions in the complex plane.

Argument of polyroots() function:

- ***v*** is a vector containing coefficients of a polynomial in which the first element is the constant term, and $2 \le \text{length}(\mathbf{v}) \le 99$.

Task 8.2

Find the roots of the equation:

$$2x^3 - 11x + 5 = 0.$$

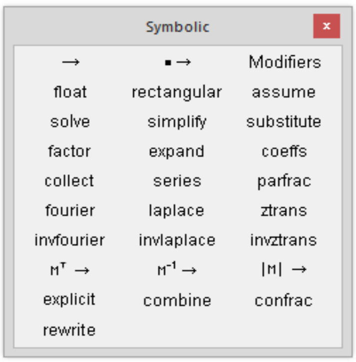
Solution in Mathcad:

1. Define the function

$$p(x) := 2 \cdot x^3 - 11 \cdot x + 5$$

2. Estimate the vector v of the polinomial symbolically

$$v := p(x) \text{ coeffs } \rightarrow \begin{pmatrix} 5 \\ -11 \\ 0 \\ 2 \end{pmatrix}$$



Mathcad returns a vector containing the coefficients of the polynomial. In the column of v, the coefficients are listed in order of increasing powers of x.

3. Apply the function polyroots to the vector v:

$$x := \text{polyroots}(v) = \begin{pmatrix} -2.546 \\ 0.474 \\ 2.072 \end{pmatrix}$$

4. Display the obtained solution

$$x^T = (-2.546 \quad 0.474 \quad 2.072)$$

Task 8.3

Using Table 8.1 find the roots of the equation from column (2) on the range given in column (3).

Table 8.1 – Data for calculation

No	Equation	Range of x variable
1	$x^3 - 4x^2 - 3x + 7 = 0$	$-5 \le x \le 5$
2	$x^3 - 2x^2 - 6x + 3 = 0$	$-5 \le x \le 5$
3	$x \cdot \sin(x) - 1 = 0$	$-5 \le x \le 5$
4	$8\cos(x) - x = 6$	$-5 \le x \le 5$
5	$2 \cdot \ln(x+7) - 5 \cdot \sin(x) = 0$	$-5 \le x \le 5$
6	$4 \cdot \cos(x) + 0.3x = 0$	$-6 \le x \le 5$
7	$2x^2 - 5 = 2^x$	$-5 \le x \le 5$
8	$5^x + 45x^2 - 170 = 0$	$-5 \le x \le 5$
9	$5^x + 45x + 70 = 0$	$-5 \le x \le 5$
10	$3^{x-1} - 4 - x = 0$	$-5 \le x \le 5$
11	$2^x - 3x - 2 = 0$	$-5 \le x \le 5$
12	$4^{x-1} - 9x = 0$	$-2 \le x \le 5$

Subject 9. FINDING MINIMUM AND MAXIMUM VALUES OF A FUNCTION

9.1. Finding extrema according to Fermat's theorem

In mathematical analysis, the maxima and minima of a function, known collectively as extrema, are the largest and smallest value of the function, either within a given range (the local or relative extrema) or on the entire domain of a function (the global or absolute extrema). Local extrema of differentiable functions can be found by Fermat's theorem, which states that they must occur at critical points.

Fermat's theorem (also known as Interior extremum theorem) is a method to find local maxima and minima of differentiable functions on open sets by showing that every local extremum of the function is a stationary point (the function derivative is zero in that point).

Fermat's theorem statement:

– If a function has a local extremum at some point, then the function's derivative at that point must be zero.

So, to find the extrema of the function, it is needed to find the points, where the derivative of the function is equal to zero.

Task 9.1

Find the extrema (local minima and maxima points) of the following function:

$$T(x) = \frac{3\sin(x)^3 + 2.3}{x - 2\cos(x)^3},$$

at the range $3 \le x \le 7$, taking the increment $h = 0.1$.

Solution in Mathcad:

1. Define the function

$$T(x) := \frac{3 \cdot \sin(x)^3 + 2.3}{x - 2 \cdot \cos(x)^3}$$

2. Estimate the derivative function T1(x) using symbolic evaluation

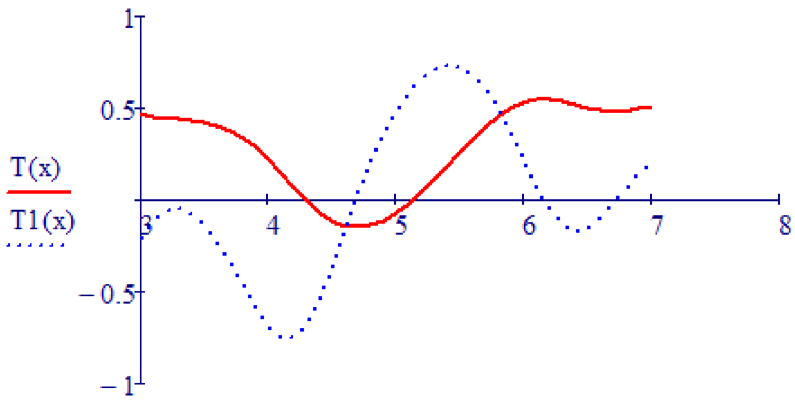
$$T1(x) := \frac{d}{dx} T(x)$$

$$T1(x) \rightarrow \frac{9 \cdot \cos(x) \cdot \sin(x)^2}{x - 2 \cdot \cos(x)^3} - \frac{\left(6 \cdot \sin(x) \cdot \cos(x)^2 + 1\right) \cdot \left(3 \cdot \sin(x)^3 + 2.3\right)}{\left(x - 2 \cdot \cos(x)^3\right)^2}$$

3. Provide the range of x variable to plot the graphs of functions T(x) and its derivative T1(x)

x := 3 ,3.1 .. 7

4. Plot the graphs for extrema location estimation



5. From the graph approximately estimate the points, where T1(x)=0

x1 := 4.6   x2 := 6.2   x3 := 6.7

6. Find the points, where 1(x)=0 using root() function

x1 := root(T1(x1) ,x1) = 4.696

x2 := root(T1(x2) ,x2) = 6.157

x3 := root(T1(x3) ,x3) = 6.727

7. Calculate the value of T(x) function in the obtained points to find the extrema

T(x1) = −0.149

T(x2) = 0.546

T(x3) = 0.483

<u>Answer</u>:

the function $T(x)=\dfrac{3\sin(x)^3+2.3}{x-2\cos(x)^3}$ has the following local extrema on the range $3 \le x \le 7$:

1) minimum at point $x_1 = 4.696$; $T(x_1) = -0.149$;
2) minimum at point $x_2 = 6.727$; $T(x_2) = 0.483$;
3) maximum at point $x_3 = 6.157$; $T(x_3) = 0.546$.

9.2. Finding extrema using Mathcad functions

The functions to find minimum and maximum values of the function in Mathcad are

Minimize(f, var1, var2, ...)
Maximize(f, var1, var2, ...)

This function returns the values of ***var1, var2...*** that satisfy the constraints in a ***Solve Block***, and make the function $f\ (var1, var2, \ldots)$ take on its smallest or largest value, respectively.

Minimize and **Maximize** differ from **Find** and **Minerr** in that they can refer to functions defined outside of the ***Solve Block***, rather than defined in the body of the ***Block***. Functions are used as objective functions, rather than as constraints, as they are with **Find** and **Minerr**. If you are solving for *n* variables, the solve block must have *n* equations. The functions choose an appropriate method from a group of available methods, depending on whether the problem is linear or nonlinear, and other attributes.

Arguments of **Minimize / Maximize** function**:**

- ***var1, var2,...*** are the scalar or array variables that you want to optimize. Guess values for each variable must be defined above the **Given** keyword, or within the body of the ***Solve Block***. If solutions are expected to be complex, complex guess values must be used.
- ***f*** is a function defined above the ***Solve Block***. The function is supplied without its arguments to **Maximize** and **Minimize**.

Task 9.2

Find the extrema (local minima and maxima points) of the following function:

$$f(x) = x^4 - 3x^3 + 2x^2 + x - 0.5,$$

on the range $-0.7 \le x \le 1.8$.

Solution in Mathcad:

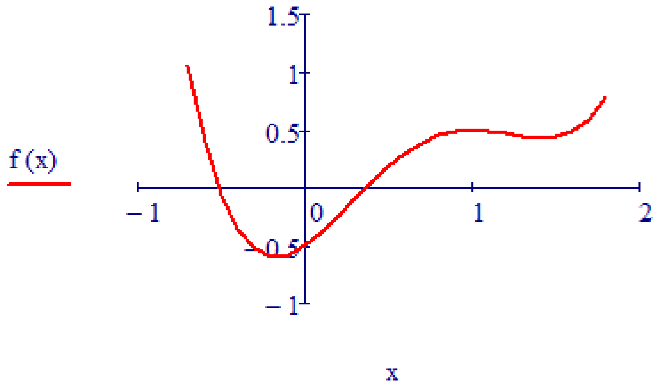
1. Define the function

$$f(x) := x^4 - 3\cdot x^3 + 2\cdot x^2 + x - 0.5$$

2. Provide the range of x variable to plot the graph of function f(x)

$$x := -0.7, -0.6 .. 1.8$$

3. Plot the graph of function f(x)



4. Find clearly seen extrema (firstly provide the guessed value, then use function Minimize() to obtaine x point and then calculate the value of function in this point

x1 := −0.2   x1 := Minimize(f ,x1) = −0.175

f(x1) = −0.597

x2 := 0.8   x2 := Maximize(f ,x2) = 1

f(x2) = 0.5

5. Find the implicit minimum

Provide guessed value

x3 := 1.4

Use Solve Block starting from Given

Given

Provide the range where to find the extremum using boolean toolbar

1.2 < x3 < 2

Use Minimize() function to find minimum

x3 := Minimize(f ,x3) = 1.425

f(x3) = 0.429

Answer:

the function $f(x) = x^4 - 3x^3 + 2x^2 + x - 0.5$ has the following local extrema on the range $-0.7 \le x \le 1.8$:

1) minimum at point $x_1 = -0.175$; $f(x_1) = -0.597$;
2) maximum at point $x_2 = 1$; $f(x_2) = 0.5$;
3) minimum at point $x_3 = 1.425$; $f(x_3) = 0.429$.

Task 9.3

Using Table 9.1 find the extrema of the function from column (2) on the range given in column (3) using the increment h.

Table 9.1 – Data for calculation

No	Equation	Range of x variable
1	$Y(x) = \lg(x)^3 \cdot \sin(x)^2 - \lg(x)$	$1 \le x \le 10 \quad h = 0.2$
2	$B(x) = \lg(x)^{\cos(x)^2} - \sin(x)$	$1 \le x \le 10 \quad h = 0.1$
3	$Y(x) = \ln(x)^5 \cdot \sin(x)^2 + x - 20$	$1 \le x \le 10 \quad h = 0.2$
4	$Z(x) = \sin(x)^3 + 0.3 \Big/ x^{\cos(x)} + 5$	$1 \le x \le 8 \quad h = 0.1$
5	$T(x) = \sin(x)^3 - 0.6 \Big/ x^2$	$1 \le x \le 10 \quad h = 0.1$
6	$F(x) = 5^{\cos(x)} + 5\sin(x)^2 - 3$	$0 \le x \le 10 \quad h = 0.25$
7	$C(x) = (\sin(x)^3 - 0.7)/x$	$1 \le x \le 10 \quad h = 0.1$
8	$S(x) = (\cos(x)^3 - 0.2)/x$	$1 \le x \le 10 \quad h = 0.2$
9	$Z(x) = (\cos(x)^3 - 0.5x)/(x^{\sin(x)} + 7)$	$1 \le x \le 7 \quad h = 0.1$
10	$T(x) = \dfrac{3\sin(x)^3 + 2.3}{x - 2\cos(x)^3}$	$3 \le x \le 9 \quad h = 0.1$
11	$F(x) = (\sin(x)^3 - 0.5)/(x^{\sin(x)} + 7)$	$1 \le x \le 10 \quad h = 0.2$
12	$G(x) = (\cos(x)^3 + 0.5)/(x + \cos(x))$	$1 \le x \le 10 \quad h = 0.1$

Subject 10. DIFFERENTIAL EQUATION SOLVING. THE SYSTEMS OF DIFFERENTIAL EQUATIONS

10.1. Ordinary differential equations

Ordinary differential of a function *f*(*x*) of a variable *x* is a measure of the rate at which the value of the function changes with respect to the change of the variable. It is called the *derivative* of *f* with respect to *x*. If *x* and *y* are real numbers, and if the graph of *f* is plotted against *x*, the derivative is the slope of this graph at each point.

Ordinary differential equation (ODE) solvers in Mathcad solve an equation or system of equations for an unknown functions of one variable. To solve an ODE directly without creating a ***Solve Block***, one of the ODE solvers can be used, which solve systems of ODEs of the form

$$\frac{d}{dx}y = \begin{bmatrix} f_1(x,y) \\ f_2(x,y) \\ \cdots \\ f_n(x,y) \end{bmatrix},$$

where *y* is vector of unknown functions of the independent variable *x*.

To solve ODE in Mathcad use function

rkfixed(init, x1, x2, intvls, D)

It uses the fourth-order Runge-Kutta fixed-step method.

The arguments used in rkfixed() function:

- **init** is either a vector of n real initial values, where *n* is the number of unknowns, or a single scalar initial value, in the case of a single ODE.
- ***x1*** and ***x2*** are real, scalar endpoints of the interval over which the solution to the ODE(s) is evaluated. Initial values in **init** are the values of the ODE function(s) evaluated at ***x1***.
- **intvls** is the integer number of discretization intervals used to interpolate the solution function. The number of solution points is the number of intervals + 1.
- **D** is a vector function of the form ***D(x,y)*** specifying the right-hand side of the system:

$$\frac{d}{dx}y = D(x,y) = \begin{bmatrix} f_1(x,y) \\ f_2(x,y) \\ \cdots \\ f_n(x,y) \end{bmatrix}$$

Task 10.1
Solve the following ordinary differential equation:

$$y' = \sin xy\,,$$

at the range $[0,\ \pi]$, with initial condition $y(0) = 1$.
Plot the graph of the solution.

Solution in Mathcad:

1. Array indexing from 1

ORIGIN := 1

2. Provide the initial value of the first y value (1 is the index)

$y_1 := 1$

3. Provide the ODE function

$D(x,y) := \sin(x \cdot y_1)$

4. To obtain the solution points provide the start point of the range x1, the last point of the range x2 and the total number of points N

x1 := 0 x2 := π N := 20

5. Use rfixed() function to find the solution.

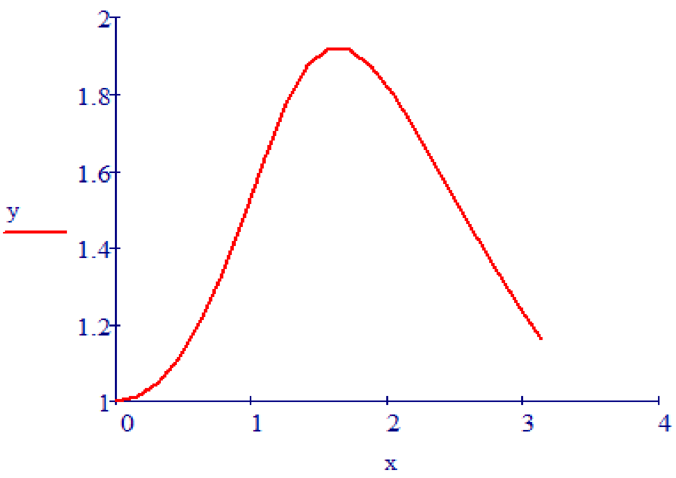
Z := rkfixed(y ,x1 ,x2 ,N ,D) Z =

	1	2
1	0	1
2	0.157	1.012
3	0.314	1.05
4	0.471	1.115
5	0.628	1.208
6	0.785	1.331
7	0.942	1.477
8	1.1	1.633
9	1.257	1.774
10	1.414	...

6. Get the vectors of points x and y(x)

$x := Z^{\langle 1 \rangle}$ $y := Z^{\langle 2 \rangle}$

7. Plot the graph



Task 10.2

Solve the following ordinary differential equation:

$$y' = x^2 y^3 \sin^3(x + y),$$

at the range $[0, 3]$, with initial condition $y(0) = 1$, taking increment $h = 0.15$.

Plot the graph of the solution.

Solution in Mathcad:

1. Array indexing from 1

$$ORIGIN := 1$$

2. Provide the initial value of the first y value (1 is the index)

$$y_1 := 1$$

3. Provide the ODE function

$$D(x, y) := x^2 \cdot (y_1)^3 \cdot \sin(x + y_1)^3$$

4. To obtain the solution points provide the start point of the range x1, the last point of the range x2 and the total number of points N

$x1 := 0$ $\quad$ $x2 := 3$ $\quad$ $N := \dfrac{x2 - x1}{0.15} = 20$

5. Use rfixed() function to find the solution.

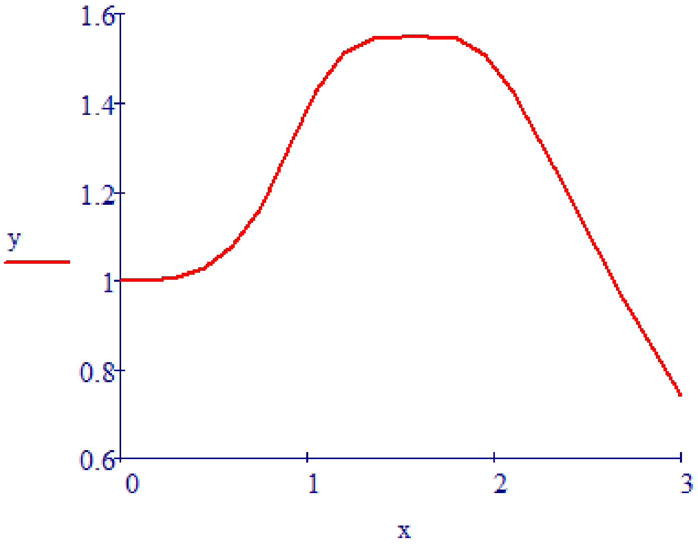
$Z := \mathrm{rkfixed}(y, x1, x2, N, D)$ $Z =$

	1	2
1	0	1
2	0.15	1.001
3	0.3	1.008
4	0.45	1.029
5	0.6	1.077
6	0.75	1.166
7	0.9	1.296
8	1.05	1.427
9	1.2	1.511
10	1.35	...

6. Get the vectors of points x and y(x)

$x := Z^{\langle 1 \rangle}$ $y := Z^{\langle 2 \rangle}$

7. Plot the graph



10.2. System of ordinary differential equations

Using the rkfixed(init, *x1*, *x2*, intvls, *D*) function, it is possible to solve the system of ordinary differential equations. The difference with ODE solution is in provided arguments for ODE system:

- **init** is either a vector of *n* real initial values for each ODE function.
- ***D*** is a vector function of the form ***D(x,y)*** specifying the right-hand side of the system and providing the corresponding functions.

Task 10.3

Solve the following system of ordinary differential equations:

$$\begin{cases} y_1' = y_2 \\ y_2' = e^{-x \cdot y_1} \end{cases},$$

at the range [0, 3], with initial conditions $y_1(0) = 0;\quad y_2(0) = 0$, taking increment h = 0.1. Plot the graph of the solution.

Solution in Mathcad:

1. Array indexing from 1

ORIGIN := 1

2. Provide the initial values of functions

$$y := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

3. Provide the matrix D with ODE functions of the system

$$D(x,y) := \begin{pmatrix} y_2 \\ \exp(-x \cdot y_1) \end{pmatrix}$$

4. To obtain the solution points provide the start point of the range x1, the last point of the range x2 and the total number of points N

x1 := 0   x2 := 3 $N := \dfrac{x2 - x1}{0.1} = 30$

5. Use rfixed() function to find the solution.

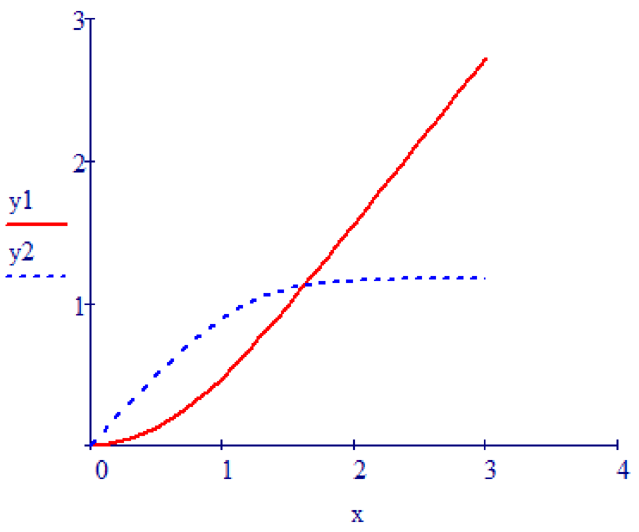
Z := rkfixed(y ,x1 ,x2 ,N ,D)   Z =

	1	2	3
1	0	0	0
2	0.1	$5 \cdot 10^{-3}$	0.1
3	0.2	0.02	0.2
4	0.3	0.045	0.299
5	0.4	0.08	0.397
6	0.5	0.124	0.492
7	0.6	0.178	0.584
8	0.7	0.241	0.672
9	0.8	0.312	0.753
10	0.9	0.391	...

6. Get the vectors of points x and y(x)

$x := Z^{\langle 1 \rangle}$ $y1 := Z^{\langle 2 \rangle}$ $y2 := Z^{\langle 3 \rangle}$

7. Plot the graph



Task 10.4

Using Table 10.1 do the presented tasks.

Table 10.1 – Data for Task 10.4

No	**Task description**
1	2
1	Solve the ordinary differential equation: $y'+\frac{3}{x}\cdot y=\frac{2}{x^3}$; on the range $1\le x\le 4$ with initial condition $y\ \ 1\ \ =1$. Plot the graph of function $y = f(x)$.
2	Solve the ordinary differential equation: $y'+y\cdot tgx=\cos^2 x$; on the range $\frac{\pi}{2}\le x\le \pi$ with initial condition $y\left(\frac{\pi}{2}\right)=\frac{1}{2}$. Plot the graph of function $y = f(x)$.
3	Solve the ordinary differential equation: $y'-\frac{1}{x+2}\cdot y=x^2+4x+5$; on the range $-1\le x\le 3$ with initial condition $y\ \ -1\ \ =\frac{3}{2}$. Plot the graph of function $y = f(x)$.

The end of the table 10.1

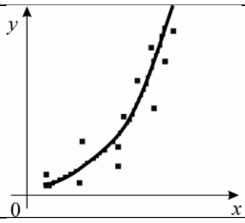
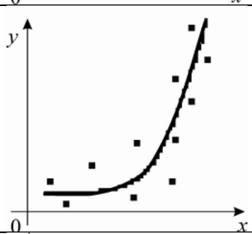
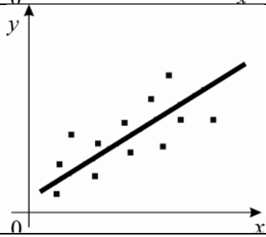
1	2
4	Solve the ordinary differential equation: $x^2(y^3+5)dx-(x^3+5)y^2dy=0$; on the range $0\le x\le 4$ with initial condition $y(0)=2$. Plot the graph of function $y=f(x)$.
5	Solve the ordinary differential equation: $\frac{dy}{dx}+3y=0$; on the range $0\le x\le 4$ with initial condition $y(0)=4$. Plot the graph of function $y=f(x)$.
6	Solve the ordinary differential equation: $tg\,y\,dx-x\ln x dy=0$; on the range $\frac{\pi}{8}\le x\le \frac{\pi}{2}$ with initial condition $y\left(\frac{\pi}{8}\right)=e$. Plot the graph of function $y=f(x)$.
7	Solve the ordinary differential equation: $y'=4x+\frac{y}{x}+\left(\frac{y}{x}\right)^2$; on the range $1\le x\le 4$ with initial condition $y(1)=2$. Plot the graph of function $y=f(x)$.
8	Solve the ordinary differential equation: $(x^4+6x^2y^2+y^4)dx-4x^2y(x^2+y^2)dy=0$; on the range $1\le x\le 4$ with initial condition $y(1)=2$. Plot the graph of function $y=f(x)$.
9	Solve the ordinary differential equation: $y'=5\sqrt{y+1}+\frac{x^2}{3y^2}$; on the range $1\le x\le 3$ with initial condition $y(1)=2$. Plot the graph of function $y=f(x)$.
10	Solve the ordinary differential equation: $xy'=y\left(1+\ln\frac{y}{x}\right)$; on the range $2\le x\le 7$ with initial condition $y(1)=\frac{1}{\sqrt{e}}$. Plot the graph of function $y=f(x)$.

Subject 11. APPROXIMATION OF EXPERIMENTAL DATA

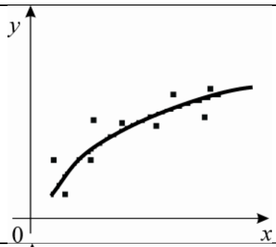
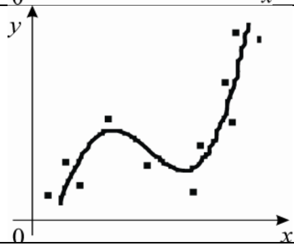
In statistical modeling, regression analysis is a statistical process for estimating the relationships among variables. It includes many techniques for modeling and analyzing several variables, when the focus is on the relationship between a dependent variable and one or more independent variables (or 'predictors'). More specifically, regression analysis helps one understand how the typical value of the dependent variable (or 'criterion variable') changes when any one of the independent variables is varied, while the other independent variables are held fixed. Most commonly, regression analysis estimates the conditional expectation of the dependent variable given the independent variables – that is, the average value of the dependent variable when the independent variables are fixed. In regression analysis, it is also of interest to characterize the variation of the dependent variable around the regression function which can be described by a probability distribution.

The selection of the proper approximation model depends on the behavior of the initial (experimental) data, as the approximation function $F(x)$ determines the mathematical object or phenomenon, what is under consideration. The standard approximation functions are listed in Table 11.1.

Table 11.1 – Standard functions used for approximations

No	**Name**	**Graph**	**Equation**
1	2	3	4
1	Power function		$y = a \cdot x^b$
2	Exponential		$y = a \cdot e^{b \cdot x}$
3	Linear		$y = b \cdot x + a$

The end of the table 11.1

1	2	3	4
4	Logarithmic		$y = b \cdot \ln(x) + a$
5	Polynomial		$y = b_0 + b_1 \cdot x + b_2 \cdot x^2 + b_n \cdot x^n$

The approximation of data points supposes the estimation of the proper function form and estimation of constants from the equations: a and b.

To calculate the deviation between the initial data and obtained approximating function, the following Mathcad function is used: **corr(x, y)**

It returns the Pearson's r correlation coefficient of the elements in ***X*** and ***Y***:

$$r = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \overline{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \overline{y})^2}}$$

where $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ is the sample mean, and analogously for $\overline{y}$.

Arguments:

- ***X*** and ***Y*** are real or complex arrays of the same size.

11.1. Linear regression of linear function

The linear functions in math are expressed by the relation:

$$y(x) = a + b \cdot x$$

where x is the function argument, and b, a are the constants, which should be specified for the approximation.

The build-in functions in Mathcad used for the linear approximation are:

– **line(*vx*, *vy*)**

Returns a vector containing the coefficients a and b for a line of the form $a + b \cdot x$ that best approximates the data in vectors ***vx*** and ***vy*** in the least-squares sense. If you want the coefficients independently, use:

– **slope(*vx*, *vy*)**

Returns the slope of line that best fits data in ***vx*** and ***vy***.

– **intercept(*vx*, *vy*)**

Returns the intercept of line that best fits data in ***vx*** and ***vy***.

– **medfit(*vx*, *vy*)**

Returns a vector containing the coefficients a and b for a line of the form $a + b \cdot x$ that best approximates the data in vectors ***vx*** and ***vy*** using median-median regression.

Arguments:

– ***vx*** is a vector of real numbers representing the *x* values.

– ***vy*** is a vector of real numbers representing the *y* values. ***vy*** must be the same length as ***vx***.

Task 11.1

Find the functional dependence of wet content *W*, [%] in paint from the time *t*, [h] in the drying machine during the 16 hours drying process:

t, (°C)	0	2	4	6	8	10	12	14	16
W, (%)	51	46	39	30	24	18	12	7.5	4.5

Solution in Mathcad:

1. Enter the matrix of the initial data

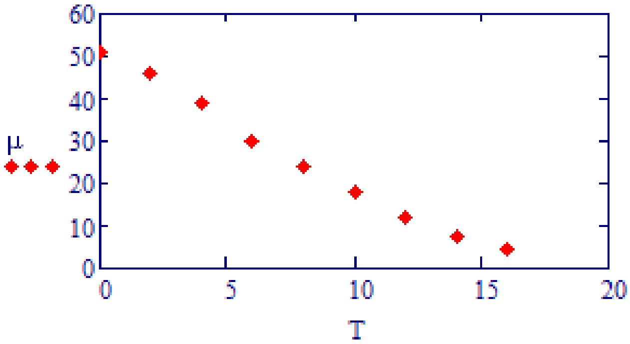
$$d := \begin{pmatrix} 0 & 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 \\ 51 & 46 & 39 & 30 & 24 & 18 & 12 & 7.5 & 4.5 \end{pmatrix}$$

2. Create two vectors: for x values and for y values. You need to transpose matrix d and take the corresponding column: 0 for t values and 1 for W values (by default ORIGIN=0).

$$T := \left(d^T\right)^{\langle 0 \rangle} \qquad \mu := \left(d^T\right)^{\langle 1 \rangle}$$

$$T = \begin{pmatrix} 0 \\ 2 \\ 4 \\ 6 \\ 8 \\ 10 \\ 12 \\ 14 \\ 16 \end{pmatrix} \qquad \mu = \begin{pmatrix} 51 \\ 46 \\ 39 \\ 30 \\ 24 \\ 18 \\ 12 \\ 7.5 \\ 4.5 \end{pmatrix}$$

3. Build the points of initial data



4. Find the values a and b of linear function y=a+b*x using intercept() and slope() functions

$a := \text{intercept}(T, \mu)$ $\qquad a = 50.278$

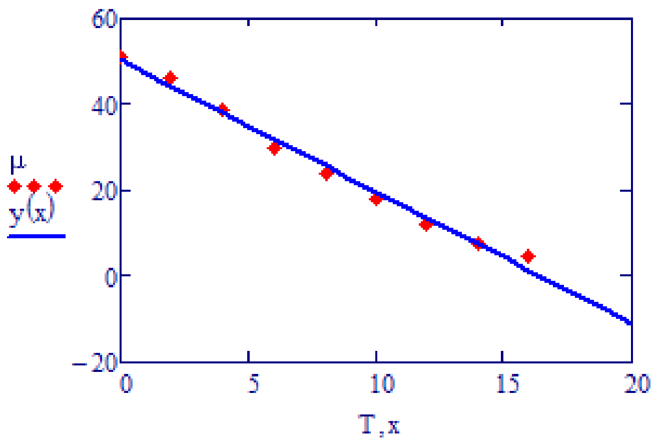
$b := \text{slope}(T, \mu)$ $\qquad b = -3.063$

5. The aproimating linear function is

$y(x) := a + b \cdot x$

6. Plot the results of calculations

$x := 0..20$



7. Find the deviation between the initial data and points of approximating function

$\text{corr}\left(\mu, \overrightarrow{y(T)}\right) = 0.994$

Answer: The function for data approximation is

$$W(t) = 50.278 - 3.063 \cdot t.$$

The correlation between initial data and the obtained function is equal to 0.994.

11.2. Linear regression of non-linear function

In case, when data behavior differs from linear, it is also possible to apply the linear approximation. But the a and b values from the linear function $y(x) = a + b \cdot x$ will be functions. For example, if experimental data are likely to be the exponential function: $y(x) = a \cdot e^{b \cdot x}$, it is possible to use ***slope(vx, vy)*** and ***intercept(vx, vy)*** Mathcad functions, where *vx* is the vector od initial x points; as $y(x) = a \cdot e^{b \cdot x} \Rightarrow b$ function takes the form inverse to exponential and is $\ln(y(x))$. Here Mathcad functions will be ***slope(vx, ln(vy))*** and ***intercept(vx, ln(vy))***.

Task 11.2

Find the functional dependence of wet content *W*, [%] in paint from the time *t*, [h] in the drying machine during the 16 hours drying process:

t, (°C)	0	2	4	6	8	10	12	14	16
W, (%)	51	46	39	30	24	18	12	7.5	4.5

Solution in Mathcad:

1. Enter the matrix of the initial data

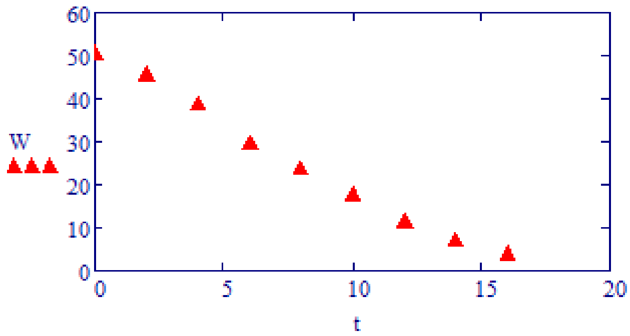
$$d := \begin{pmatrix} 0 & 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 \\ 51 & 46 & 39 & 30 & 24 & 18 & 12 & 7.5 & 4.5 \end{pmatrix}$$

2. Create two vectors of initial ponts t and W, where W(t) (by default ORIGIN=0).

$$t := \left(d^T\right)^{\langle 0 \rangle} \qquad W := \left(d^T\right)^{\langle 1 \rangle}$$

$$t = \begin{pmatrix} 0 \\ 2 \\ 4 \\ 6 \\ 8 \\ 10 \\ 12 \\ 14 \\ 16 \end{pmatrix} \qquad W = \begin{pmatrix} 51 \\ 46 \\ 39 \\ 30 \\ 24 \\ 18 \\ 12 \\ 7.5 \\ 4.5 \end{pmatrix}$$

3. Plot the initial data points



4. The plotted graph W(t) is a non-linear function likele to be the exponential dependence of form y(x)=a*exp(b*x). Using slope() and intercept() functions, find constants a and b

$a := \exp\left(\text{intercept}\left(t, \overrightarrow{\ln(W)}\right)\right)$ $\qquad a = 66.021$

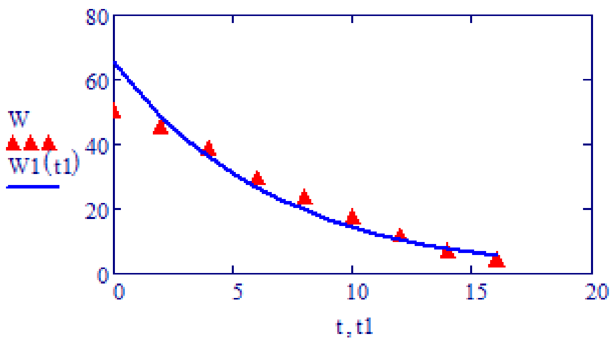
$b := \text{slope}\left(t, \overrightarrow{\ln(W)}\right)$ $\qquad b = -0.15$

5. The approximation function W1(t1) is

$W1(t1) := a \cdot \exp(b \cdot t1)$

6. Plot the graph of obtained function and initial points

$t1 := 0 .. 16$



7. Find the deviation between the initial data points and approximating function

$\text{corr}\left(W, \overrightarrow{W1(t)}\right) = 0.968$

Answer:

The function for data approximation is

$$W(t) = 66.021 \cdot e^{-0.15 \cdot t}.$$

The correlation between initial data and the obtained function is equal to 0.968.

11.3. Linear regression by the least squares method

The method of least squares is a standard approach in regression analysis to the approximate solution of overdetermined systems, i.e., sets of equations in which there are more equations than unknowns. "Least squares" means that the overall solution minimizes the sum of the squares of the errors made in the results of every single equation. Least squares problems fall into two categories: linear or ordinary least squares and non-linear least squares, depending on whether or not the residuals are linear in all unknowns.

The using of least square method defines the determination of b, a coefficients from the equation $y(x) = a + b \cdot x$ from the condition of minimization of summation of square distances from the initial values and mathematically can be presented as follows:

$$F = \sum_{i=1}^{N} \left(y_i - \hat{y}_i \right)^2 \Rightarrow \min$$

or ,

$$F = \sum_{i=1}^{N} \left(y_i - \left(b_0 + b_1 x_i \right) \right)^2 \Rightarrow \min$$

where x_i are the initial *x* points;

y_i are the initial *y* points (depending points);

$\hat{y}_i$ are the values, calculated according to the approximation function $\hat{y}_i = b_0 + b_1 x_i$;

N is the number of experimental points.

According to the least square method for linear regression the b, a coefficients are specified according to the following relations:

$$b = \frac{N\sum_{i=1}^{N}(y_i x_i) - \sum_{i=1}^{N} x_i \cdot \sum_{i=1}^{N} y_i}{N\sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i \right)^2}; \qquad a = \frac{\sum_{i=1}^{N} y_i}{N} - b\frac{\sum_{i=1}^{N} x_i}{N};$$

Task 11.3

Find the functional dependence of air viscosity *μ*, [kPa·s] from temperature *T*, [°C]:

T, (°C)	0	100	200	300	400	500	600	700	800
μ, (kPa·s)	0.0171	0.0218	0.0259	0.0294	0.0320	0.0357	0.0384	0.0411	0.0437

Solution in Mathcad:

1. Enter the matrix of the initial data and set the ORIGIN to 1

ORIGIN := 1

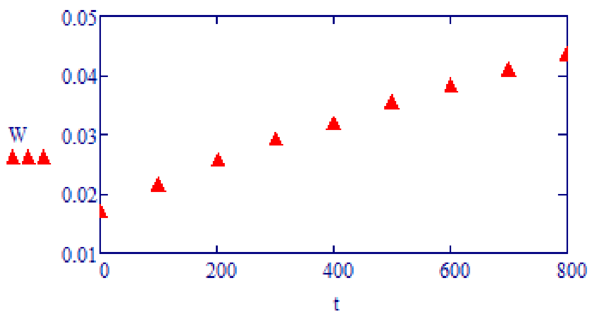
$$d := \begin{pmatrix} 0 & 100 & 200 & 300 & 400 & 500 & 600 & 700 & 800 \\ 0.0171 & 0.0218 & 0.0259 & 0.0294 & 0.0320 & 0.0357 & 0.0384 & 0.0411 & 0.0437 \end{pmatrix}$$

2. Create two vectors: for x values and for y values.

$$t := \left(d^T\right)^{\langle 1\rangle} \qquad W := \left(d^T\right)^{\langle 2\rangle}$$

$$t = \begin{pmatrix} 0 \\ 100 \\ 200 \\ 300 \\ 400 \\ 500 \\ 600 \\ 700 \\ 800 \end{pmatrix} \qquad W = \begin{pmatrix} 0.017 \\ 0.022 \\ 0.026 \\ 0.029 \\ 0.032 \\ 0.036 \\ 0.038 \\ 0.041 \\ 0.044 \end{pmatrix}$$

3. Plot initial data points



4. Set the number of experimental points N=9. And rename the matrices: let x be t and y be W

$$N := \text{length}(t) = 9 \qquad X := t \qquad Y := W$$

5. Using the least square method find all the needed summations:

$$\sum_{i=1}^{N}(y_i \cdot x_i);\quad \sum_{i=1}^{N} y_i;\quad \sum_{i=1}^{N} x_i;\quad \sum_{i=1}^{N} x_i^2$$

$$SUMxy := \sum_{i=1}^{N} \left(X_i \cdot Y_i\right) = 133.6$$

$$SUMx := \sum_{i=1}^{N} X_i = 3.6 \times 10^3$$

$$SUMy := \sum_{i=1}^{N} Y_i = 0.285$$

$$SUMx2 := \sum_{i=1}^{N} (X_i)^2 = 2.04 \times 10^6$$

6. Find a and b coefficients

$$b := \frac{N \cdot SUMxy - SUMx \cdot SUMy}{N \cdot SUMx2 - SUMx^2} \qquad b = 3.26 \times 10^{-5}$$

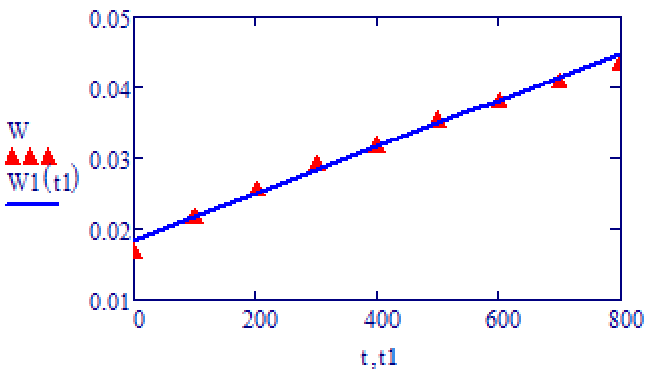
$$a := \frac{SUMy}{N} + \left(-b \cdot \frac{SUMx}{N}\right) \qquad a = 0.019$$

7. The obtained function is

$$W1(t1) := a + b \cdot t1$$

8. Plot the resulting function and initial points

$$t1 := 0, 50 .. 800$$



9. Find the deviation between the initial data and approximated

$$corr\left(W, \overrightarrow{W1(t)}\right) = 0.995$$

Answer:

The function for data approximation is

$$\mu(T) = 0.019 + 3.26 \cdot 10^{-5} \cdot T .$$

The correlation between initial data and the obtained function is equal to 0.995.

11.4. Polynomial regression

The polynomial functions in math are expressed by the relation:

$$Y(x) = b_0 + b_1 \cdot x + b_2 \cdot x^2 + ... + b_k \cdot x^k$$

where x is the function argument, and $b_0, b_1, ..., b_k$ are the constants, which should be specified for the approximation; k is the polynomial level.

When $k = 1$ the polynomial becomes a line;

$k = 2$ it is parabola.

The build-in functions in Mathcad used for the polynomial approximation:

– **loess(*vx*, *vy*, span)** or **loess(*Mx*, *vy*, span)**

Returns a vector which **interp** uses to find a set of second-order polynomials that best fit the neighborhood of *x* and *y* data values in ***vx*** and ***vy*** in the least-squares sense. The size of the neighborhood is controlled by span. Can also be used for multivariate regression, where a matrix ***Mxy*** of *k* independent variables and a vector of dependent values, ***vy***, are used to fit second-order polynomial surfaces in *k* dimensions.

– **interp(*vs*, *vx*, *vy*, *x*)** or **interp(*vs*, *Mx*, *vy*, *X*)**

Returns the interpolated *y*-value corresponding to *x* using the output vector ***vs*** from **loess**. If **loess** has been used to fit a multidimensional surface, ***X*** is a vector of independent variables at which to calculate the interpolated y-value.

Arguments:

- ***vx*** and ***vy*** are vectors of real data values and are of the same length;
- ***Mx*** is a matrix of real data values. There is one column for each independent variable ($k \leq 4$ columns). In this case, ***vy*** has the same number of rows as ***Mx*;**
- ***vs*** is a vector generated by **loess**;
- span is a positive real number specifying the size of the data neighborhood. Use larger values of span when the data behaves very differently over different ranges of *x*. A good default value is span = 0.75;
- ***x*** is the value of the independent variable at which you want to evaluate the regression curve;
- ***X*** is the vector of values of the independent variables at which you want to evaluate the regression surface.

Task 11.4

Find the functional dependence (3rd level polynomial) of wet content *W*, [%] in paint from the time *t*, [h] in the drying machine during the 16 hours drying process:

t, (h)	0	2	4	6	8	10	12	14	16
W, (%)	51	46	39	30	24	18	12	7.5	4.5

Solution in Mathcad:

1. Enter the matrix of the initial data.

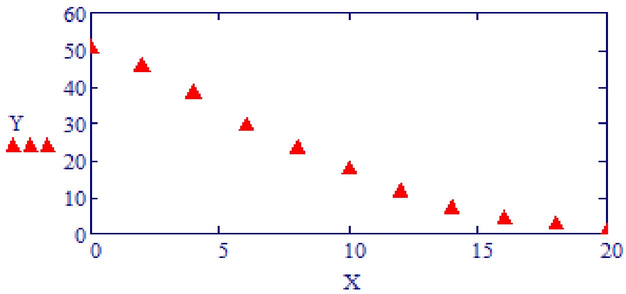
$$d := \begin{pmatrix} 0 & 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 & 18 & 20 \\ 51 & 46 & 39 & 30 & 24 & 18 & 12 & 7.5 & 4.7 & 3 & 1.5 \end{pmatrix}$$

2. Create two vectors: for x values and for y values (by default ORIGIN=0).

$$X := \left(d^T\right)^{\langle 0 \rangle} \qquad Y := \left(d^T\right)^{\langle 1 \rangle}$$

$$X = \begin{pmatrix} 0 \\ 2 \\ 4 \\ 6 \\ 8 \\ 10 \\ 12 \\ 14 \\ 16 \\ 18 \\ 20 \end{pmatrix} \qquad Y = \begin{pmatrix} 51 \\ 46 \\ 39 \\ 30 \\ 24 \\ 18 \\ 12 \\ 7.5 \\ 4.7 \\ 3 \\ 1.5 \end{pmatrix}$$

3. Plot initial data points



4. Set the level of the polynomial (KN=3) and get the range of summarized members (k):

$KN := 3 \qquad k := 0..KN$

5. Find the number of approximated points N:

The number of elements in vector X can be found using build-in Mathcad function LENGTH(X)

$N := \text{length}(X) \qquad N = 11$

6. Get the matrix of polynomial elements:

$$X1^{\langle k \rangle} := \overrightarrow{X^k}$$

7. Find the $b_0, b_1, \ldots, b_k$ coefficients

Set the range of x variable. The number of experimental points is 11. The ORIGIN=0, so we need to count the 0 element, and i will vary from 0 to 10 (N-1)

$i := 0..N-1$

Determine the $b_0, b_1, \ldots, b_k$ coefficients (matrix B):

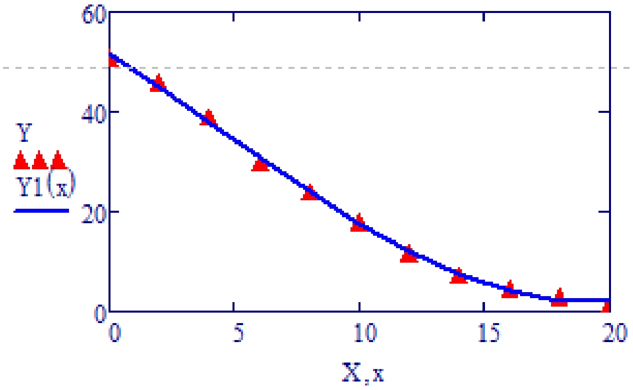
$$B := \left(X1^T \cdot X1\right)^{-1} \cdot \left(X1^T \cdot Y\right) \qquad B = \begin{pmatrix} 51.763 \\ -3.249 \\ -0.065 \\ 5.174 \times 10^{-3} \end{pmatrix}$$

The obtained approximation function is

$$Y1(X) := B_0 + B_1 \cdot X + B_2 \cdot X^2 + B_3 \cdot X^3$$

8. Plot the results of calculations

$x := 0, 2..20$



9. Find the deviation between the initial data and approximated

$$\mathrm{corr}\left(Y, \overrightarrow{Y1(X)}\right) = 0.999$$

Answer:
The function for data approximation is

$$W(t) = 51.763 - 3.249 \cdot t - 0.065 \cdot t^2 + 5.174 \cdot t^3.$$

The correlation between initial data and the obtained function is equal to 0.999.

11.5. Linear combination of functions

The linear combination of functions in math is expressed by the polynomial:

$$y(x)=a_0\cdot f_0(x)+a_1\cdot f_1(x)+a_2\cdot f_2(x)+\ldots$$

where $f_i(x)$ is the i-th function of argument x, and it should be specified for the approximation.

The build-in functions in Mathcad used for the approximation by linear combination of functions:

– **linfit(*vx*,*vy*,*F*)**

Returns a vector containing the parameters used to create a linear combination of the functions in vector ***F*** which best approximates the data in ***vx*** and ***vy*** in the least-squares sense.

Arguments of **linfit()** function:

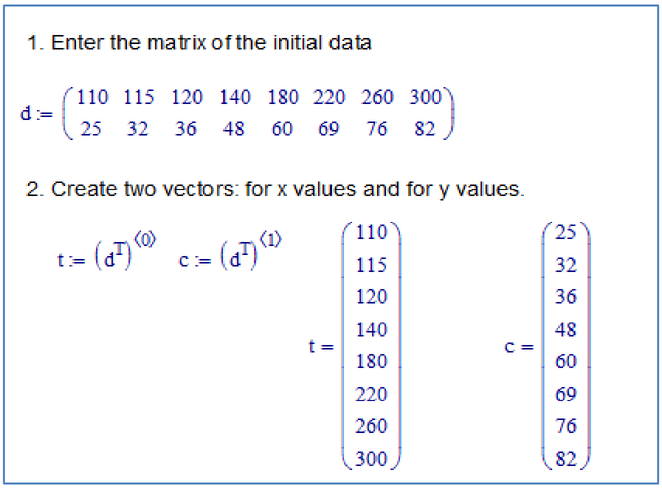
- ***vx*** and ***vy*** are vectors of real data values of the same length, corresponding to the *x* and *y*-values in the data set. There must be at least as many data points as there are terms in ***F***.
- ***F(x)*** is a vector of functions; each element is one linear functional term in the fit function. In the case of a single linear function, ***F*** is a scalar, ***F*** $=\{f_0(x), f_1(x), f_2(x),\ldots\}$..

Task 11.5

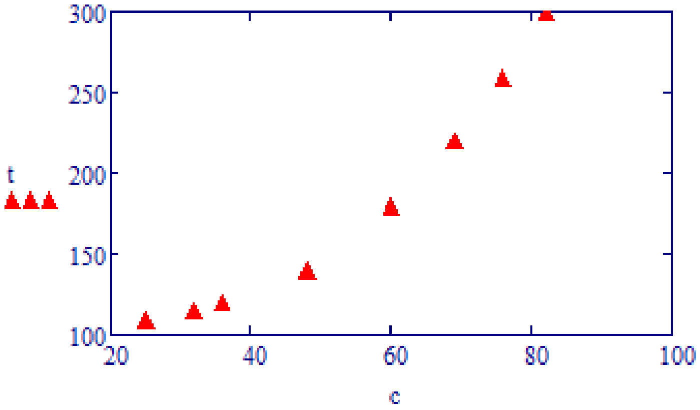
Find the functional dependence of mixture boiling temperature *T*, [°C] from its concentration *C*, [% of weight] from the:

T, (°C)	110	115	120	140	180	220	260	300
C, (% of weight)	25	32	36	48	60	69	76	82

Solution in Mathcad:



3. Plot initial data points



4. Analyzing the points dependence

The form of function is likely to be the linear combination of functions, which in Mathcad can be expressed in the following relation:

$$y(x) = a \cdot x^2 + b \cdot \left(\frac{x}{1+x^2} \right) + d$$

It is needed to find the coefficients a,b and d of this function
The linear functions are

$f_0(x)$ is x^2, $f_0(x)=x^2$

$f_1(x)$ is $x/(1+x^2)$, $f_1(x)=x/(1+x^2)$

$f_2(x)$ is 1, $f_2(x)=1$

5. Create a vector of the approximation function

$$F(c) := \begin{pmatrix} c \\ \dfrac{c}{1+c^2} \\ 1 \end{pmatrix}$$

6. Use function LINFIT(c,t,F) to find the values the approximation numbers.

c is the depending variable C (% of weight) in matrix form
t are the points of T (temperature) given in matrix form
F is the vector of functions

$S := \text{linfit}(c, t, F)$ $\qquad$ $S = \begin{pmatrix} 5.82 \\ 5.716 \times 10^3 \\ -256.665 \end{pmatrix}$

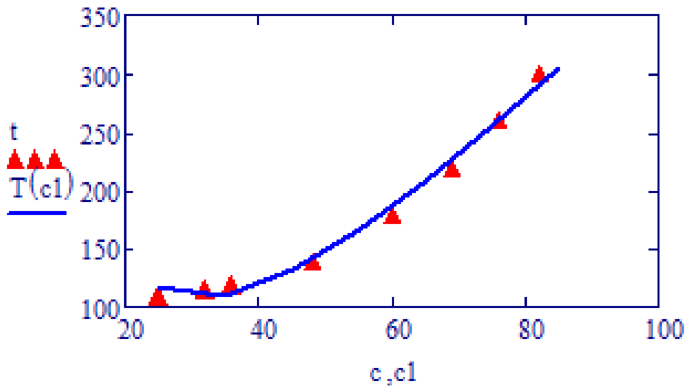
7. Plot the results of calculations

$c1 := 25, 35 .. 85$

Define the function T(C), with will take the calculated approximation function and points of the range c1, so t(c1).

$T(c1) := F(c1) \cdot S$

Plot the graphs



8. Find the deviation between the initial data and approximated

It is needed to estimate c1 points as initial range

$c1 := c$

$corr\left(t, \overrightarrow{T(c1)}\right) = 0.995$

<u>Answer</u>:

The function for data approximation is

$$t(c) = 0.039 \cdot c^2 + 2021 \cdot \left(\frac{c}{1 + c^2}\right) + 8.686.$$

The correlation between initial data and the obtained function is equal to 0.995.

Task 11.6

Using Table 11.2 find the functional dependence of specific heat capacity from the temperature. The temperature *T*, [°C] is given in the table heading. The specific heat capacity data are listed for the substances according to the variant. For example, variant 2 contains the data for acetone.

Table 11.2 – Data for Task 11.6

Variant	Substance	Specific heat capacity at the temperature, ^{o}C (in $kJ/(kg \cdot {}^{o}C)$)							
		–20	0	20	40	60	80	100	120
1	Ammonia (liquid)	4.522	4.606	4.731	4.857	5.108	5.443	5.736	6.197
2	Acetone	2.052	2.114	2.177	2.24	2.303	2.37	2.445	2.49
3	Dichloroethane	0.971	1.057	1.147	1.23	1.327	1.419	1.512	1.599
4	$CaCl_2$ (25%)	2.818	2.889	2.939	2.973	3.057	3.098	3.14	3.182
5	Methanol	2.382	2.466	2.567	2.667	2.763	2.864	2.964	3.065
6	H_2SO_4 (75%)	1.805	1.872	1.939	2.006	2.073	2.145	2.207	2.274
7	Toluene	1.52	1.612	1.704	1.796	1.888	1.98	2.068	2.119
8	Chlorine benzene	1.193	1.256	1.319	1.382	1.445	1.507	1.574	1.637
9	CCl_4	0.812	0.837	0.863	0.892	0.921	0.946	0.976	1.005
10	Ethyl acetate	1.775	1.846	1.918	1.989	2.064	2.135	2.207	2.278

Subject 12. PROGRAMMING IN MATHCAD

12.1. Programming operators in Mathcad

In Mathcad, a program is entered in the programming operator, a multi-step container for Mathcad program-control operators. Specific programming operators can be used to specify local assignments to variables or functions, loop over calculations, conditionally evaluate branches, add breakpoints, trap errors, and return values.

Mathcad evaluates the sequence of statements in a program in the order specified by the programming operators then returns the result of the last step. Here is a simple example of a program.

The operators comprise the Mathcad programming "language" are listed in Table 12.1.

Table 12.1 – Programming operators in Mathcad

Operator	**Keystroke**	**Description**
1	2	3
Add line	]	Creates a new, empty line within a program, or creates the first line in a program if used in a blank region of the worksheet. Also called the ***Add Line*** operator.
if	**[}]**	*x* if *y* Evaluates *x* if *y* is nonzero (true). The enclosing program proceeds to the next line regardless of whether *x* is evaluated or not. Conditional statements allow Mathcad to execute or skip certain calculations. Use a conditional statement whenever you want to direct program execution along a particular branch.
for	**[Ctrl] ["]**	for $x \in y$ z Evaluates *z* for each value of *x* over the range *y*. Typically, at least one expression in the loop body, *z*, uses the value of *x* to change the calculation for each evaluation. Use a **for loop** when you know exactly how many times you want the body of the loop to execute.
return	**[Ctrl] [Shift] [\]**	The ***return*** operator halts the program and returns *x*.

The end of the table 12.1

1	2	3
break	**[Ctrl] [Shift] [**	The ***break*** operator halts execution of the current loop and returns the last value calculated. The ***break*** operator is used in conjunction with a conditional statement to halt execution of a loop and return control to the first statement after the loop. This operator takes no arguments.
Local assignment	**[{]**	$x \leftarrow y$ Evaluates *y* numerically and assigns its contents to *x*. Variables and functions defined with this operator are only defined locally within the current definition, for example, within a program. Returns the value of the left-hand-side.
otherwise	**[Ctrl] [Shift]]**	*x* otherwise Evaluates *x* if the **if** statement immediately preceding *x* is 0 (false). The **otherwise** operator only works with the **if** operator.
while	**[Ctrl]]**	while x y Evaluates *y* while *x* is nonzero (true). The condition expression is evaluated at the beginning of the **while** loop, so it is possible that the loop never executes. The loop stops iterating as soon as the condition is false and returns the last value calculated in its body on the previous iteration.
continue	**[Ctrl] [**	The ***continue*** operator skips the remainder of the current iteration and returns to the first loop statement. In a for loop, the iteration variable is incremented. The ***continue*** operator is used in conjunction with a conditional statement to skip the current iteration and proceed to the next one. It takes no arguments.
on error	**[Ctrl] [']**	*x* on error y Evaluates *y*. If y produces an error, evaluates and returns *x*. Otherwise returns the results of *y*. The **on error** operator is a distinct type of conditional operator, guiding execution only in the event of an error during calculation.

Conditional (if and otherwise) Operators

x if y

Evaluates *x* if *y* is nonzero (true). The enclosing program proceeds to the next line regardless of whether *x* is evaluated or not. Conditional statements allow Mathcad to execute or skip certain calculations. Use a conditional statement whenever you want to direct program execution along a particular branch.

x otherwise

Evaluates *x* if the **if** statement immediately preceding *x* is 0 (false). The **otherwise** operator only works with the **if** operator. In the following example,

$$f(x) := \begin{vmatrix} 0 \quad \text{if} \quad |x| > 2 \\ \sqrt{4 - x^2} \quad \text{otherwise} \end{vmatrix}$$

the function returns 0 if *x* is greater than 2 or less than −2. When *x* is between −2 and 2, the function returns the square root of $4 - x^2$.

Operands:

- *x* is any valid Mathcad expression.
- *y* is any valid Mathcad expression that can evaluate to 0 in some cases. Only the return value of *y* is considered; it can be a boolean expression or any other Mathcad expression. For example, a local assignment or a sequence of programming steps is allowed.

Program Loops

A loop is a block of code that causes one or more statements (the body of the loop) to iterate until a termination condition occurs. There are two kinds of loops:

- For loops are used when you know exactly how many times the body of the loop should execute.
- While loops are used when you want to stop execution upon the occurrence of a condition but you don't know exactly when that condition will occur.

For Loops

$\text{for } x \in y$

$\quad z$

Evaluates *z* for each value of *x* over the range *y*. Typically, at least one expression in the loop body, *z*, uses the value of *x* to change the calculation for each evaluation. Use a **for loop** when you know exactly how many times you want the body of the loop to execute.

Operands:

- *x* is any valid Mathcad variable name.
- *y* is a value or sequence of values. Most frequently, this is a range value, but you can also use a vector, or a comma-separated list of scalars or vectors, each of which results in a series of scalar values to be assumed by the iteration variable *x*. *y* can also be a series of matrices, separated by commas; *x* assumes the value of each matrix in turn. This is a convenient way to apply the same calculation to a set of matrices.
- *z* is any valid Mathcad expression or sequence of expressions. For example, a local assignment, or a sequence of programming steps is allowed here. Use the Add Line operator to insert placeholders for additional statements.

Example of ***for*** operator:

While Loops

while x

y

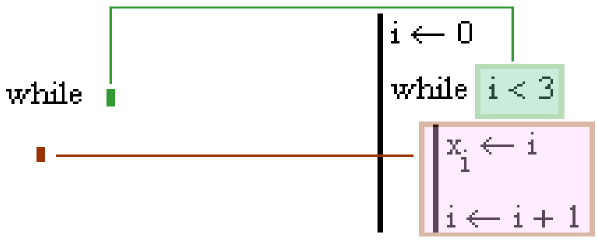
Evaluates *y* while *x* is nonzero (true). The condition expression is evaluated at the beginning of the **while** loop, so it is possible that the loop never executes. The loop stops iterating as soon as the condition is false and returns the last value calculated in its body on the previous iteration.

Operands:

- *x* is any valid Mathcad expression or sequence of expressions that can evaluate to 0, frequently a boolean expression.
- *y* is any valid Mathcad expression or sequence of expressions. At least one expression in the loop body must change the condition

expression, *x*, so that it becomes false; otherwise, the loop executes indefinitely and you need to stop it by pressing **[Esc]**. Use the Add Line operator to insert placeholders for additional statements.

Example of ***while*** operator:



Task 12.1

Calculate the value of the expression

$$y = \begin{cases} (a \cdot x - b \cdot c)/2 & \text{at } x>1 \\ a \cdot x \cdot \cos(x) & \text{at } x=1 \\ a \cdot x + b & \text{at } x<1 \end{cases}$$

where $a = 0.5$; $b = 1.2$; $c = 15.1$

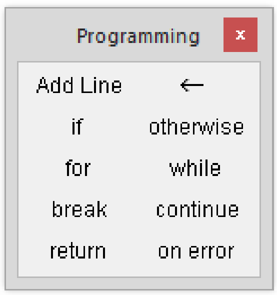
Mathcad text:

1. Defining a, b, c values

$a := 0.5 \qquad b := 1.2 \qquad c := 15.1$

2. Definition of function y using programming operators from Programming toolbar and <,> operators from Boolean toolbar

$$y(x) := \begin{vmatrix} \dfrac{(a\cdot x - b\cdot c)}{2} & \text{if } x > 1 \\ a\cdot x\cdot \cos(x) & \text{if } x = 1 \\ a\cdot x + b & \text{if } x < 1 \end{vmatrix}$$

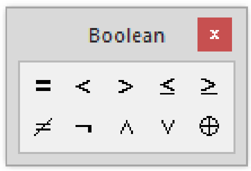


3. Calculating the value of function in different points

$y(12) = -6.06$

$y(1) = 0.27$

$y(-5) = -1.3$



Task 12.2

Using Table 12.2 calculate the conditional expression from column (2) using the coefficients given in column (3).

Table 12.2 – Data for calculation

No	**Expressions for calculation**	**Initial data**
1	2	3
1	$m = \begin{cases} x-1, & x<1 \\ a \cdot x + b, & x=1 \\ 1+b/a, & x>1 \end{cases}$	$a = 12.4;\ b = 56.1$
2	$y = \begin{cases} \sin^2 x, & x>0 \\ \ln^3(1+x^2), & x<0 \\ x+c, & x=0 \end{cases}$	$c = 1.57$
3	$g = \begin{cases} (1-x)/2, & x>3 \\ \sin(x)-2, & x=3 \\ (1-x)/(a-b), & x<3 \end{cases}$	$a = 5.5;\ b = 1.2$
4	$m = \begin{cases} 1-a \cdot \cos(x), & x<1 \\ a \cdot x + b, & x=1 \\ x+b/a, & x>1 \end{cases}$	$a = 0.001;\ b = 5.1$
5	$w = \begin{cases} b-y \cdot x, & y>1 \\ \sin(b-y), & y=1 \\ (x-y)/2, & y<1 \end{cases}$	$b = 0.15;\ x = 2$
6	$f = \begin{cases} 1-\cos(x)-y, & y<0 \\ (a \cdot y + b)/2, & y=0 \\ a+1, & y>0 \end{cases}$	$a = -0.2;\ b = 0.01$
7	$\omega = \begin{cases} x\sqrt[3]{x-a}, & x>0 \\ x\sin ax, & x=0 \\ e^{-ax}\cos ax, & x<0 \end{cases}$	$a = 2.5$

The end of the table 12.2

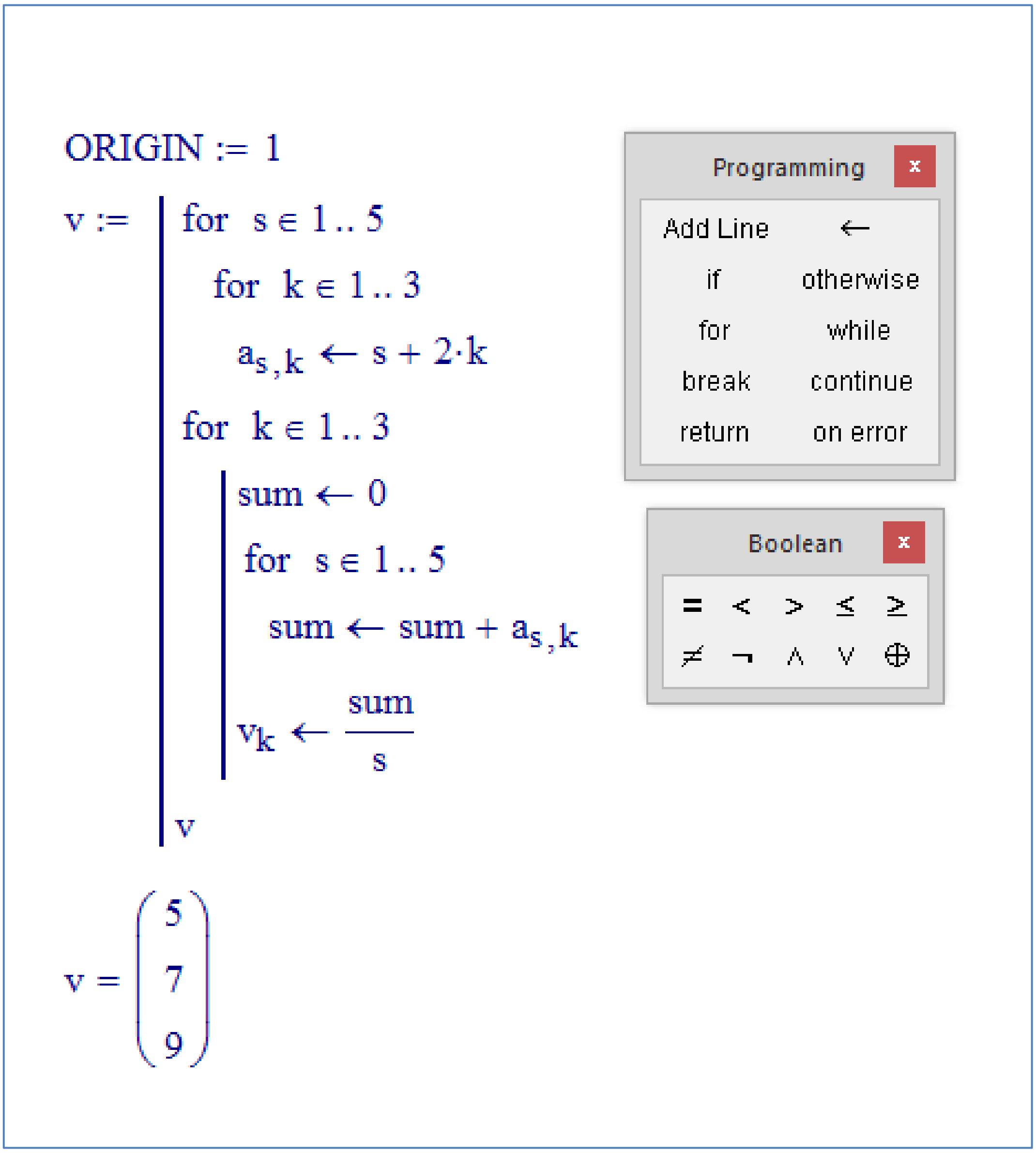
1	2	3
8	$y=\begin{cases}\pi x^2-7/x^2, & x<1.3\\ ax^3+7\sqrt{x}, & x=1.3\\ \lg(x)+7\sqrt{x}, & x>1.3\end{cases}$	$a=1.5$
9	$w=\begin{cases}\lg(a\cdot y+1), & y<2\\ (a\cdot y+b)/2, & y=2\\ a+1, & y>2\end{cases}$	$a=-0.2;\ b=0.01$
10	$y=\begin{cases}at^2\ln t, & t=2\\ 1, & t<2\\ e^{at}\cos bt, & t>2\end{cases}$	$a=-0.5;\ b=2$
11	$f=\begin{cases}1-\cos(x)-y, & y<0\\ (a\cdot y+b)/2, & y=0\\ a+1, & y>0\end{cases}$	$a=-0.2;\ b=0.01$
12	$y=\begin{cases}\sin^2 x, & x>0\\ \ln^3(1+x^2), & x<0\\ x+c, & x=0\end{cases}$	$c=1.57$

Task 12.3

Create the program, which will calculate the elements of matrix A according to relationship $A_{s,k}=s+2\cdot k$ and receive the vector with the following elements:

1) The mean value of the sum of the martix' column elements – vector V;
2) The mean value of the sum of the martix' row elements – vector $V1$:
3) The number of matrix rows $s=5$; columns $k=3$.

Solution in Mathcad:



Task 12.4

Using data from Table 12.3 create the program, which will calculate the elements of matrix A according to relationship presented in column (2) and receive the vector with the following elements:

1) The mean value of the sum of the martix' column elements – vector V;

2) The mean value of the sum of the martix' row elements – vector $V1$:

3) The number of matrix rows and columns are given in column (3).

Table 12.3 – Data for calculation

No	**Relations for calculation the matrix elements**	**Values of *s* and *k***
1	$a_{s,k} = 1/(s+k)$	s = 3; k = 4
2	$a_{s,k} = 10 \cdot s + 1/k$	s = 4; k = 5
3	$a_{s,k} = 2 \cdot s + 5 \cdot (k+1)$	s = 4; k = 3
4	$a_{s,k} = 10 \cdot s + 2 \cdot k$	s = 3; k = 5
5	$a_{s,k} = (s+2)/(0.5+k)$	s = 3; k = 5
6	$a_{s,k} = 2 \cdot s + 10 \cdot k$	s = 5; k = 4
7	$a_{s,k} = (1+k)/2 \cdot s$	s = 3; k = 4
8	$a_{s,k} = 0.5 \cdot s + 0.1 \cdot k$	s = 4; k = 5
9	$a_{s,k} = (s+k) \cdot 0.5$	s = 3; k = 5
10	$a_{s,k} = 3 \cdot s + 2 \cdot k$	s = 4; k = 3
11	$a_{s,k} = (0.1 \cdot s + k) + 1$	s = 3; k = 4
12	$a_{s,k} = 2 \cdot s + 5 \cdot (1+k)$	s = 4; k = 3

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(Інженерні розрахунки в середовищі Mathcad)

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для студентів хімічних спеціальностей

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