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O.P. Arsenyeva, O.V. Ved, L.V. Solovey, A.P. Yuzbashian

“COMPUTER SCIENCE AND PROGRAMMING”
(Engineering calculations in Microsoft Excel)

Teaching and practical guide
for the students of chemical specialization of all education forms
(in English)

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Reviewers:

П.О. Капустенко, канд. техн. наук, проф. каф. ІТПА, НТУ «ХПІ»;

М.Л. Угрюмов, д-р техн. наук, проф. кафедри інформатики Національного аерокосмічного університету ім. М. Є. Жуковського «Харківський авіаційний інститут»

С74 «Обчислювальна математика та програмування» (Інженерні розрахунки в середовищі Excel) : навч.-метод. посіб. / О. П. Арсеньєва, О. В. Ведь, Л. В. Соловей, А. П. Юзбашьян. – Харків: НТУ «ХПІ», 2017. – 104 с. – Англ. мовою.

Навчально-методичний посібник присвячений вивченню середовища Excel та практичному застосуванню цього програмного забезпечення для інженерних розрахунків. Наведено велику кількість прикладів вирішення розрахункових завдань різної складності. До прикладів надаються пояснення. Усі приклади забезпечені результатами виконання. По кожній темі представлені практичні завдання для виконання лабораторних робіт.

Призначено для студентів хімічних спеціальностей, які бажають застосовувати середовище Excel для інженерних розрахунків.

The teaching and practical guide is devoted to study how to make the engineering calculations using the Microsoft Excel software. Different examples of engineering tasks with the varied complexity are provided with step-by-step explanation with corresponding illustrations. The obtained results are provided. Each subject includes the explained case studies and the tasks for work in class and individually.

The teaching and practical guide is aimed for the students studying in chemical engineering, which want to use Microsoft Excel software for engineering calculations.

Іл.: 86 Табл.: 26 Бібліогр.: 4

УДК 519.6

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Introduction

Excel is a program in the Microsoft Office system that allows easy entry and manipulation of figures, equations, and text. Excel can be used to create and format workbooks (a collection of spreadsheets) in order to analyze data and make more informed business decisions. Specifically, Excel helps

- to track data,
- build models for analyzing data,
- write formulas to perform calculations on that data,
- pivot the data in numerous ways, and
- present data in a variety of professional looking charts.

Common scenarios for using Excel include: accounting; budgeting; billing and sales; reporting; planning; tracking; using calendars.

The Excel spreadsheets are generally used by many researchers for engineering calculations, as they can enter initial data in the table format and then evaluate them using arithmetical and build-in statistical functions. The Solver add-in in Excel provides the possibility to solve the problems of non-linear equations and optimizations. Results from the spreadsheets are readily plotted and procedural language (Visual Basic for Applications) can be used for more complex tasks.

The present teaching and practical guide observes 9 subjects connected to engineering calculations in Excel for chemical engineering. Subject 1 and Subject 2 describe the ways of basic arithmetic calculations in Excel using built-in functions. The logical functions and their usage are given in Subject 3. The 4^{th} subject examines the possibility of plotting the data. The main approaches for working with tables are listed in Subject 5. Subjects 6 – 9 are devoted to more complex mathematical problems, such as solving the systems of equations, finding the local minimum and maximum of functions and approximation of the experimental data. Each section contains the example with the detailed step-by-step explanation of the required action to solve the problem and corresponding illustrations. The number of tasks for individual work is provided for each subject.

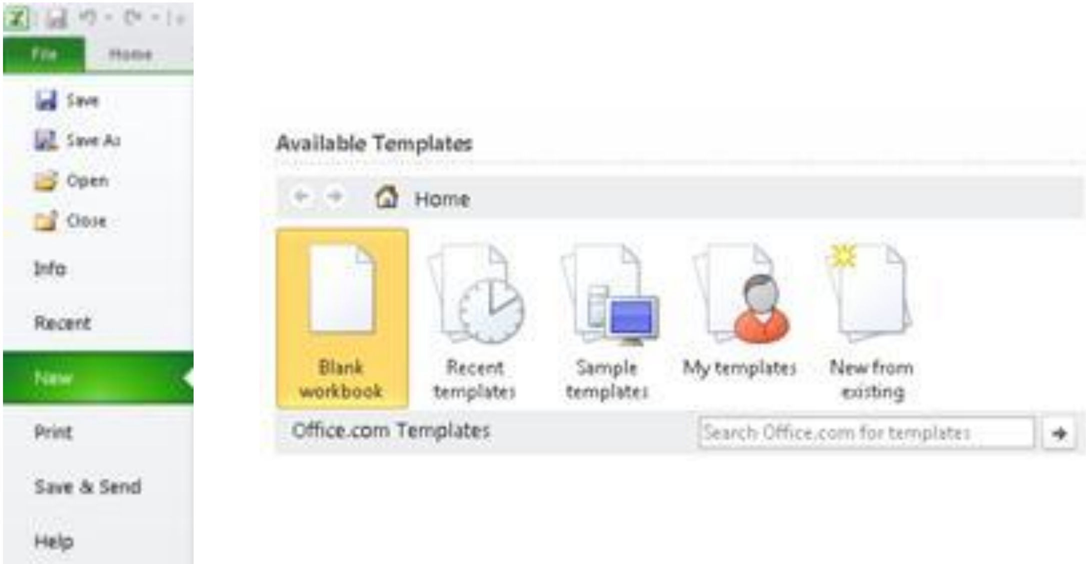
Subject 1. GETTING STARTED WITH EXCEL

1.1. Create a new workbook

1. Click the File tab and then click New (Figure 1.1*a*).

2. Under Available Templates, click Blank Workbook (Figure 1.1*b*).

3. Click Create. The new Excel document presented in Figure 1.2 will appear.



a *b*

Figure 1.1 – Creating new document

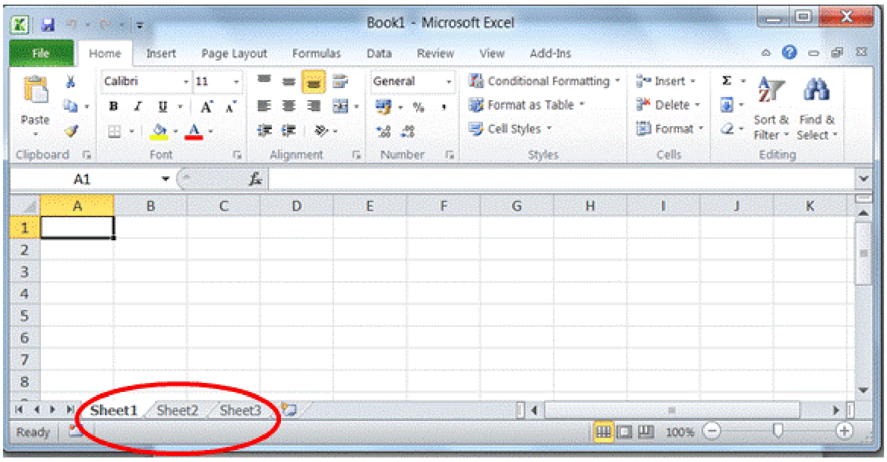


Figure 1.2 – The new Excel document

1.2. Working with worksheets

To insert a new worksheet do the following:

1. To quickly insert a new worksheet at the end of the existing worksheets, click the ***Insert Worksheet*** tab at the bottom of the screen (Figure 1.3).

2. To insert a new worksheet in front of an existing worksheet, select that worksheet and then, on the ***Home*** tab, in the ***Cells*** group, click ***Insert***, and then click ***Insert Sheet***.

Figure 1.3 – The Insert Worksheet tab

To rename a worksheet:

1. On the ***Sheet*** tab bar, right-click the sheet tab that you want to rename, and then click ***Rename Sheet*** (Figure 1.4).

2. Select the current name, and then type the new name.

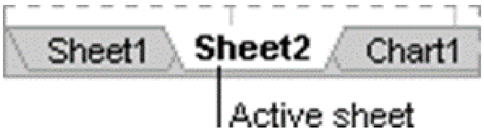


Figure 1.4 – The renaming of the Worksheet Sheet2

Entering data in a worksheet:

You have several options when you want to enter data manually in Excel.

You can enter data in one cell;

– in several cells at the same time, or
– on more than one worksheet at the same time.

The data that you enter can be

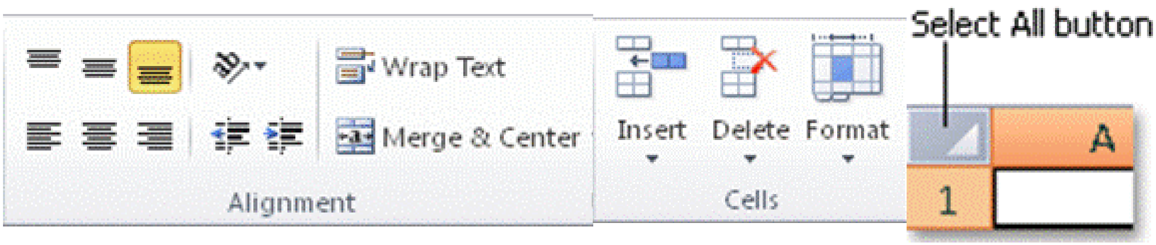
– numbers;
– text;
– dates;
– time.

1.3. Adjust settings

1. To wrap text in a cell, select the cells that you want to format, and then on the **Home** tab, in the **Alignment** group, click **Wrap Text** (Figure 1.5*a*).

2. To adjust column width and row height to automatically fit the contents of a cell, select the columns or rows that you want to change, and then on the **Home** tab, in the **Cells** group, click **Format**(Figure 1.5*b*).

3. To quickly autofit all columns or rows in the worksheet, click the **Select All** button (Figure 1.5*c*), and then double-click any boundary between two column or row headings.



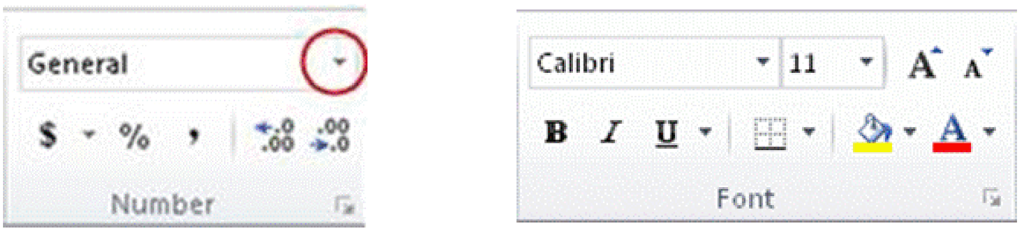
a *b* *c*

Figure 1.5 – Adjust settings

1.4. Format the data

1. To apply number formatting, click the cell that contains the numbers that you want to format, and then on the **Home** tab, in the **Number** group, click the arrow next to **General** (Figure 1.6*a*), and then click the format that you want.

2. To change the font, select the cells that contain the data that you want to format, and then on the **Home** tab, in the **Font** group (Figure 1.6*b*), click the format that you want.



a *b*

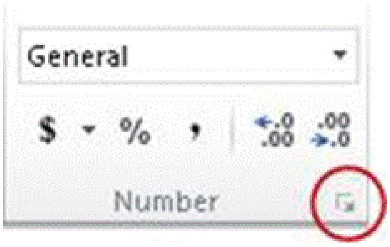
Figure 1.6 – Format data

1.5. Format numbers

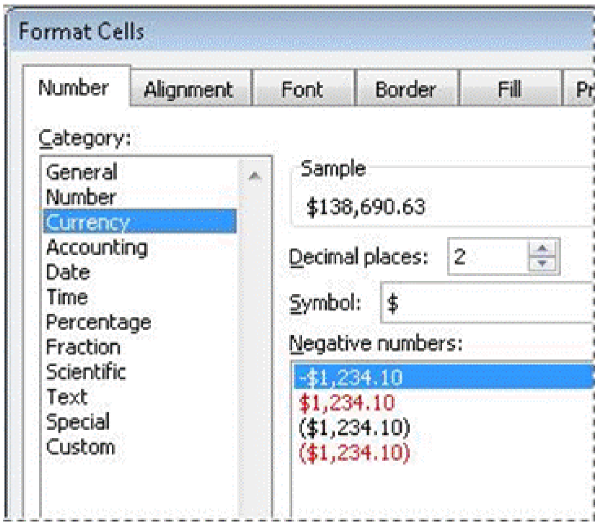
1. Select the cells that you want to format.

2. One the **Home** tab, in the **Number** group, click the **Dialog Box Launcher** next to **Number** (Figure 1.7*a*).

3. In the **Category** list, click the format that you want to use, and then adjust settings, if necessary (Figure 1.7*b*).



a



b

Figure 1.7 – Format numbers

Available number formats

By applying different number formats (Table 1.1), you can change the appearance of a number without changing the number itself. A number format does not affect the actual cell value that Excel uses to perform calculations. The actual value is displayed in the formula bar.

Table 1.1 – The list of available number formats

Format	Description
General	The default number format that Excel applies when you type a number. For the most part, numbers that are formatted with the **General** format are displayed just the way you type them
Number	Used for the general display of numbers. You can specify the number of decimal places that you want to use, whether you want to use a thousands separator, and how you want to display negative numbers.
Currency	Used for general monetary values and displays the default currency symbol with numbers. You can specify the number of decimal places that you want to use, whether you want to use a thousands separator, and how you want to display negative numbers.

The end of the table 1.1

Format	Description
Accounting	Also used for monetary values, but it aligns the currency symbols and decimal points of numbers in a column.
Date	Displays date and time serial numbers as date values, according to the type and locale (location) that you specify.
Time	Displays date and time serial numbers as time values, according to the type and locale (location) that you specify.
Percentage	Multiplies the cell value by 100 and displays the result with a percent (**%**) symbol. You can specify the number of decimal places that you want to use.
Fraction	Displays a number as a fraction, according to the type of fraction that you specify.
Scientific	Displays a number in exponential notation, replacing part of the number with *E*+*n*, where *E* (which stands for Exponent) multiplies the preceding number by 10 to the *n*th power. For example, a 2-decimal **Scientific** format displays 12345678901 as 1.23*E*+10, which is 1.23 times 10 to the 10th power.
Text	Treats the content of a cell as text and displays the content exactly as you type it, even when you type numbers.
Special	Displays a number as a postal code (ZIP Code), phone number, or Social Security number.
Custom	Allows you to modify a copy of an existing number format code.

1.6. Apply cell borders

1. Select the cell or range of cells that you want to add a border to.

2. On the **Home** tab, in the **Font** group, click the arrow next to Borders (Figure 1.8), and then click the border style that you want.

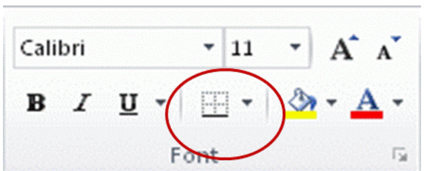


Figure 1.8 – Applying cell borders

1.7. Absolute, relative and mixed references

1. *Relative references* (Figure 1.9*a*)

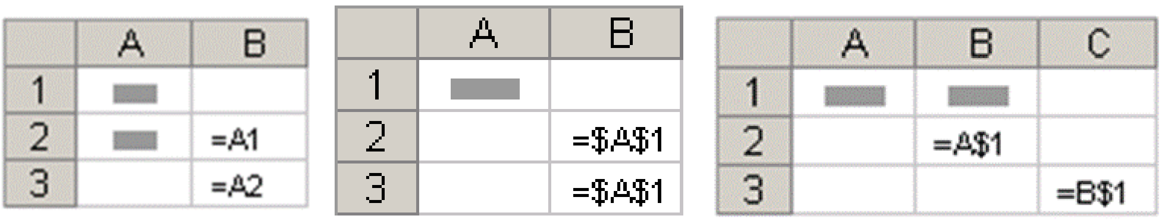
A *relative* cell reference in a formula, such as **A1**, is based on the *relative position* of the cell that contains the formula and the cell the reference refers to. If ***the position of the cell*** that contains the formula ***changes, the reference is changed***. If you copy or fill the formula across rows or down columns, the reference automatically adjusts. Bydefault, new formulas use relative references. For example, if you copy or fill a relative reference in cell B2 to cell B3, it automatically adjusts from =A1 to =A2.

2. *Absolute references* (Figure 1.9*b*)

An *absolute* cell reference in a formula, such as **$A$1**, always refer to a cell in a *specific location*. If ***the position of the cell*** that contains the formula ***changes***, the absolute ***reference remains the same***. If you copy or fill the formula across rows or down columns, the absolute reference does not adjust. By default, new formulas use relative references, so you may need to switch them to absolute references. For example, if you copy or fill an absolute reference in cell B2 to cell B3, it stays the same in both cells: =$A$1.

3. *Mixed references* (Figure 1.9*c*)

A*mixed* reference has either an absolute column and relative row, or absolute row and relative column. An absolute column reference takes the form **$A1**, **$B1**, and so on. An absolute row reference takes the form A$1, B$1, and so on. If the position of the cell that contains the formula changes, the relative reference is changed, and the absolute reference does not change. If you copy or fill the formula across rows or down columns, the relative reference automatically adjusts, and the absolute reference does not adjust. For example, if you copy or fill a mixed reference from cell A2 to B3, it adjusts from =A$1 to =B$1.



a *b* *c*

Figure 1.9 – Absolute, relative and mixed references

Subject 2. CALCULATIONS IN EXCEL

2.1. Calculation operators and precedence

Operators specify the type of calculation that you want to perform on the elements of a formula. There is a default order in which calculations occur, but you can change this order by using parentheses.

There are four different types of calculation operators:

- arithmetic,
- comparison,
- text concatenation, and
- reference.

Arithmetic operators

To perform basic mathematical operations, such as addition, subtraction, multiplication, or division; combine numbers; and produce numeric results, use the arithmetic operators presented in Table 2.1.

Table 2.1 – The list of available arithmetic operators

Arithmetic operator	Meaning	Example
+ (plus sign)	Addition	3+3
– (minus sign)	Subtraction Negation	3–1 –1
* (asterisk)	Multiplication	3*3
/ (forward slash)	Division	3/3
% (percent sign)	Percent	20%
^ (caret)	Exponentiation	3^2

Comparison operators

It is possible to compare two values with the operators presented in Table 2.2. When two values are compared by using these operators, the result is a logical value – either **TRUE** or **FALSE**.

Table 2.2 – The list of available comparison operators

Comparison operator	Meaning	Example
= (equal sign)	Equal to	A1=B1
> (greater than sign)	Greater than	A1>B1
< (less than sign)	Less than	A1<B1
>= (greater than or equal to sign)	Greater than or equal to	A1>=B1
<= (less than or equal to sign)	Less than or equal to	A1<=B1
<> (not equal to sign)	Not equal to	A1<>B1

Text concatenation operator

To produce a single piece of text the operators such as *ampersand* (&) to **concatenate** (join) one or more text strings are used (Table 2.3).

Table 2.3. – The text concatenation operator

Textoperator	Meaning	Example
& (ampersand)	Connects, or concatenates, two values to produce one continuous text value	"North"&"wind" results in "Northwind"

Reference operators

Reference operators combine ranges of cells for calculations with the operators presented in Table 2.4.

Table 2.4 – The list of available reference operators

Reference operator	Meaning	Example
: (colon)	Range operator, which produces one reference to all the cells between two references, including the two references.	B5:B15
, (comma)	Union operator, which combines multiple references into one reference	SUM(B5:B15,D5:D15)
(space)	Intersection operator, which produces one reference to cells common to the two references	B7:D7 C6:C8

Calculation order

Formulas calculate values in a specific order.

1. A formula in Excel always begins with an equal sign (=). Excel interprets the characters that follow the equal sign as a formula.
2. Following the equal sign are the elements to be calculated (the operands), such as constants or cell references.
3. These are separated by calculation operators.
4. Excel calculates the formula from left to right, according to a specific order for each operator in the formula.

Operator precedence

If you combine several operators in a single formula, Excel performs the operations in the order shown in the Table 2.5.

Table 2.5 – Operator precedence

Operator	Description
: (colon) (single space) , (comma)	Reference operators
–	Negation (as in –1)
%	Percent
^	Exponentiation
* and /	Multiplication and division
+ and –	Addition and subtraction
&	Connects two strings of text (concatenation)
= <> <= >= <>	Comparison

If a formula contains operators with the same precedence Excel evaluates the operators from left to right.

Use of parentheses

To change the order of evaluation, enclose in parentheses the part of the formula to be calculated first.

Table 2.5 – Operator precedence

Problem	**Solution**	
To change the order of evaluation, enclose in parentheses the part of the formula to be calculated first.	***=5+2*3***	***Will be 11***
In contrast, if you use parentheses to change the syntax:	***=(5+2)*3***	***Will be 21***
In the following example, the parentheses that enclose the first part of the formula force Excel to calculate B4+25 first and then divide the result by the sum of the values in cells D5, E5, and F5.	***=(B4+25)/SUM(D5:F5)***	

Formulas in Excel

Formulas are equations that perform calculations on values in your worksheet. A ***formula always starts with an equal sign*** (=).

Depending on the type of formula that you create, a formula can contain any or all of the following parts (Figure 2.1):

1. **Functions**: The PI() function returns the value of pi: 3.142...
2. **References**: A2 returns the value in cell A2.
3. **Constants**: Numbers or text values entered directly into a formula, such as 2.
4. **Operators**: The ^ (caret) operator raises a number to a power, and the * (asterisk) operator multiplies numbers.

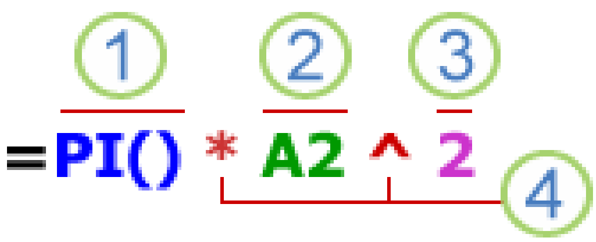


Figure 2.1 – Parts of standard formula

Using constants in formulas

A ***constant*** is a value that is not calculated; it *always stays the same*. For example, the date 10/9/2008, the number 210, and the text "Quarterly Earnings" are all constants. An expression or a value resulting from an expression is not a constant.

Using calculation operators in formulas

Operators specify the type of calculation that you want to perform on the elements of a formula. There is a default order in which calculations occur (this follows general mathematical rules), but you can change this order by using parentheses.

Using the A1 reference style in formulas

A reference identifies a cell or a range of cells on a worksheet, and tells Excel where to look for the values or data you want to use in a formula. It is possible to settle references to use data contained in different parts of a worksheet in one formula or use the value from one cell in several formulas. You can also refer to cells on other sheets in the same workbook, and to other workbooks. References to cells in other workbooks are called links or external references. The reference formats are listed in Table 2.6.

Table 2.6 – Reference style in Excel

To refer to	**Use**
The cell in column A and row 10	A10
The range of cells in column A and rows 10 through 20	A10:A20
The range of cells in row 15 and columns B through E	B15:E15
All cells in row 5	5:5
All cells in rows 5 through 10	5:10
All cells in column H	H:H
All cells in columns H through J	H:J
The range of cells in columns A through E and rows 10 through 20	A10:E20

Using functions and nested functions in formulas

Functions are predefined formulas that perform calculations by using specific values, called arguments, in a particular order, or structure. Functions can be used to perform simple or complex calculations. The example of nested function is given in Figure 2.2 and explained in Table 2.7.

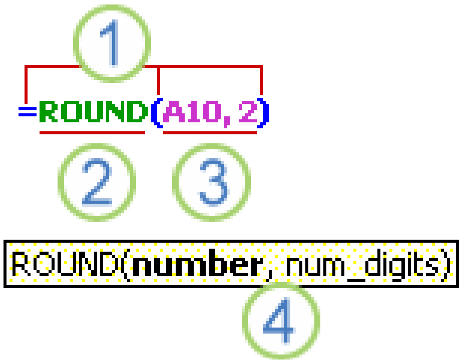


Figure 2.2 – Nested functions in formula

Table 2.7 – The parts of nested functions in formula

1	Structure. The structure of a function begins with an equal sign (=), followed by the function name, an opening parenthesis, the arguments for the function separated by commas, and a closing parenthesis.
2	Function name. For a list of available functions, click a cell and press SHIFT+F3.
3	Arguments. Arguments can be numbers, text, logical values such as TRUE or FALSE, arrays, error values such as #N/A, or cell references. The argument you designate must produce a valid value for that argument. Arguments can also be constants, formulas, or other functions.
4	Argument tooltip. A tooltip with the syntax and arguments appears as you type the function. For example, type =ROUND(and the tooltip appears. Tooltips appear only for built-in functions.

Entering functions

If to create a formula that contains a function, it is possible to use the *Insert Function* dialog box to enter worksheet functions. When entering a function into the formula, the *Insert Function* dialog box displays the following parameters:

– the name of the function,
– each of its arguments,
– a description of the function and each argument,
– the current result of the function, and
– the current result of the entire formula.

To make it easier to create and edit formulas and minimize typing and syntax errors, use Formula AutoComplete. After you type an = (equal sign) and beginning letters or a display trigger, Excel displays, below the cell, a dynamic drop-down list of valid functions, arguments, and names that match the letters or trigger. You can then insert an item from the drop-down list into the formula.

The list of some math, trigonometry and array functions available in Excel with their explanation is given in Table 2.8. The list of logical functions is presented in Table 2.9.

Table 2.8 – Math, trigonometry and array functions in Excel

Function	**Description**
ABS	Returns the absolute value of a number
ACOS	Returns the arccosine of a number
ASIN	Returns the arcsine of a number
ATAN	Returns the arctangent of a number
COS	Returns the cosine of a number
DEGREES	Converts radians to degrees
EVEN	Rounds a number up to the nearest even integer
EXP	Returns *e* raised to the power of a given number
FACT	Returns the factorial of a number
FLOOR	Rounds a number down, toward zero
GCD	Returns the greatest common divisor
INT	Rounds a number down to the nearest integer
LCM	Returns the least common multiple
LN	Returns the natural logarithm of a number
LOG	Returns the logarithm of a number to a specified base
LOG10	Returns the base-10 logarithm of a number
MDETERM	Returns the matrix determinant of an array
MINVERSE	Returns the matrix inverse of an array
MMULT	Returns the matrix product of two arrays
MOD	Returns the remainder from division
MROUND	Returns a number rounded to the desired multiple
MULTINOMIAL	Returns the multinomial of a set of numbers
ODD	Rounds a number up to the nearest odd integer

The end of the table 2.8

Function	Description
PI	Returns the value of pi
POWER	Returns the result of a number raised to a power
PRODUCT	Multiplies its arguments
QUOTIENT	Returns the integer portion of a division
RADIANS	Converts degrees to radians
RAND	Returns a random number between 0 and 1
ROMAN	Converts an arabic numeral to roman, as text
ROUND	Rounds a number to a specified number of digits
ROUNDDOWN	Rounds a number down, toward zero
ROUNDUP	Rounds a number up, away from zero
SIGN	Returns the sign of a number
SIN	Returns the sine of the given angle
SQRT	Returns a positive square root
SUM	Adds its arguments
SUMIF	Adds the cells specified by a given criteria
TAN	Returns the tangent of a number
TANH	Returns the hyperbolic tangent of a number
TRUNC	Truncates a number to an integer

Table 2.9 – Logical functions in Excel

Function	Description
AND	Returns TRUE if all of its arguments are TRUE
FALSE	Returns the logical value FALSE
IF	Specifies a logical test to perform
IFERROR	Returns a value you specify if a formula evaluates to an error; otherwise, returns the result of the formula
NOT	Reverses the logic of its argument
OR	Returns TRUE if any argument is TRUE
TRUE	Returns the logical value TRUE

2.2. Calculation examples in Excel

Task 2.1. Calculate the value of the expression

$$z = \frac{\sin x}{\sqrt{1 + m^2 \sin^2 x}}$$

With the given values: $x = 1.5;\ m = 5.74$.

Solution(Figure 2.3):

1. In cells A1 and A2 input the description text "*x*=" and "*m*=".
2. In cell A3 input the description text "*z*=".
3. Enter the values of *x* and *m* to cells B1 and B2 correspondingly. B1 cell should contain the number 1,5 and B2 – 5,74. Format the data applying number format.
4. In the cell B3 enter the formula:

= SIN(B1) / SQRT(1+B2^2*SIN(B1)^2)

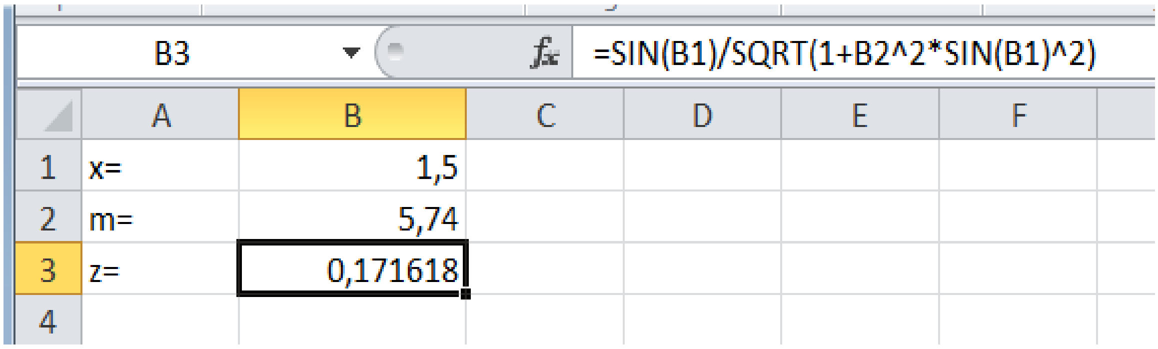


Figure 2.3 – The solution of Task 2.1 in Excel

Task 2.2. Tasks for laboratory work.

Table 2.10 – Variants for calculation

№	Relations for calculation	Initial data
1	2	3
1	$a = \frac{2\cos(x - \pi/6)}{1/2 + \sin^2 y}$ $b = 1 + \frac{z^2}{3 + z^2/5}$	$x = 1.426$ $y = -1.22$ $z = 3.5$

The end of the table 2.10

1	2	3
2	$\gamma = \left\| x^{y/x} - \sqrt[3]{y/x} \right\|$ $\psi = (y-x)\dfrac{y - z/(y-x)}{1+(y-x)^2}$	$x = 1.825$ $y = 18.225$ $z = -3.298$
3	$y = e^{-bt}\sin(at+b) - \sqrt{\|bt+a\|}$ $s = b\sin(at^2\cos 2t) - 1$	$a = -0.5$ $b = 1.7$ $t = 0.44$
4	$\omega = \sqrt{x^2+b} - b^2\sin^3(x+a)/x$ $y = \cos^2 x^3 - x\big/\sqrt{a^2+b^2}$	$a = 1.5$; $b = 15.5$ $x = -2.9$
5	$s = x^3\mathrm{tg}^2(x+b)^2 + a\big/\sqrt{x+b}$ $Q = \dfrac{bx^2 - a}{e^{ax} - 1}$	$a = 16.5$ $b = 3.4$ $x = 0.61$
6	$R = x^2(x+1)/b - \sin^2(x+a)$ $S = \sqrt{xb/a} + \cos^2(x+b)^3$	$a = 0.7$ $b = 0.05$ $x = 0.5$
7	$y = \sin^3(x^2+a)^2 - \sqrt{x/b}$ $z = \dfrac{x^2}{a} + \cos(x+b)^3$	$a = 1.1$ $b = 0.004$ $x = 0.2$
8	$f = \sqrt[3]{m\,\mathrm{tg}\,t + \|c\sin t\|}$ $z = m\cos(bt\sin t) + c$	$m = 2$; $c = -1$ $t = 1.2$; $b = 0.7$
9	$y = \ln^3(1+x^2)$ $F = \sin x^2 \cos\dfrac{7x-2}{3{,}75\pi}$	$x = 1.45$
10	$f = \ln(a+x^2) + \sin^2(x/b)$ $z = e^{-cx}\dfrac{x+\sqrt{x+a}}{x-\sqrt{\|x-b\|}}$	$a = 10.2$; $b = 9.2$ $x = 2.2$; $c = 0.5$

Task 2.3.Tasks for individual work

Table 2.11 – Variants for calculation

№	Relations for calculation	Initial data
1	2	3
1	$z=\sqrt{ax\sin 2x+e^{-2x}(x+b)}$ $\omega=\cos^2 x^3 - x\big/\sqrt{a^2+b^2}$	$a=0.5$ $b=3.1$ $x=1.4$
2	$U=\dfrac{a^2x+e^{-x}\cos bx}{bx-e^{-x}\sin bx+1}$ $f=e^{2x}\ln(a+x)-b^{3x}\ln(b-x)$	$a=0.5$ $b=2.9$ $x=0.3$
3	$z=\dfrac{\sin x}{\sqrt{1+m^2\sin^2 x}}-c\,m\ln mx$ $s=e^{-ax}\sqrt{x+1}+e^{-bx}\sqrt{x+1.5}$	$m=0.7;\ c=2.1$ $x=1.7;\quad a=0.5$ $b=1.08$
4	$\omega=\sqrt{x^2+b}-b^2\sin^3(x+a)/x$ $y=\cos^2 x^3 - x\big/\sqrt{a^2+b^2}$	$a=1.5$ $b=15.5$ $x=-2.9$
5	$y=-2c\big/(b-b^{0.5}-4ac)$ $F=9.2\cos x^2-\left\|\sin\dfrac{x}{1.2-x}\right\|$	$a=2;\quad b=3$ $c=4;\quad x=2.34$
6	$y=2^{-x}\cdot\sqrt{x+\sqrt[4]{x-c}}$ $F=12.4\sin\left\|\dfrac{x}{2\pi}\right\|-8.3\cos\|x/2\pi\|$	$x=4.741$ $c=0.5$
7	$y=\sqrt[3]{e^x-\sin x+c}$ $F=\left\|\cos\dfrac{x}{2.7\pi}\right\|-9.1\sin 2x$	$x=2.312$ $c=1.5$

Continuation of the table 2.11

1	2	3
8	$y = b\,\mathrm{tg}^2 x - \dfrac{a}{\sin^2(x/a)}$ $d = ae^{-\sqrt{a}}\cos(bx/a)$	$a = 3.2$ $b = 17.5$ $x = -4.8$
9	$t = \sqrt{2h/g}\,,\quad v = gt^2/2$ $F = \dfrac{\cos\lvert 2x\rvert}{2\pi - x} - \sin(3x + 2.1)$	$g = 9.81;\quad h = 350$ $x = 3.65$
10	$\gamma = \left\lvert x^{y/x} - \sqrt[3]{y/x}\right\rvert$ $\psi = (y - x)\dfrac{y - z/(y - x)}{1 + (y - x)^2}$	$x = 1.825$ $y = 18.225$ $z = -3.298$
11	$y = \sin^3(x^2 + a)^2 - \sqrt{x/b}$ $z = \dfrac{x^2}{a} + \cos(x + b)^3$	$a = 1.1$ $b = 0.004$ $x = 0.2$
12	$y = b\,\mathrm{tg}^2 x - \dfrac{a}{\sin^2(x/a)}$ $d = ae^{-\sqrt{a}}\cos(bx/a)$	$a = 3.2$ $b = 17.5$ $x = -4.8$
13	$y = \dfrac{a^{2x} + b^{-x}\cos(a + b)x}{x + 1}$ $R = \sqrt{x^2 + b} - b^2\sin^3(x + a)/x$	$a = 0.3$ $b = 0.9$ $x = 0.61$
14	$y = e^{-bt}\sin(at + b) - \sqrt{\lvert bt + a\rvert}$ $s = b\sin(at^2\cos 2t) - 1$	$a = -0.5$ $b = 1.7$ $t = 0.44$
15	$R = x^2(x + 1)/b - \sin^2(x + a)$ $S = \sqrt{xb/a} + \cos^2(x + b)^3$	$a = 0.7$ $b = 0.05$ $x = 0.5$

The end of the table 2.11

1	2	3
16	$c=(a^2-b^2)/a-(a^3-b^3)/(a^2-b^2)$ $F=\sin x\cos x^2-\sin^2\dfrac{x-3.1}{2\pi}$	$a=2.71;\quad b=1.5$ $x=1.78$
17	$c=(1+(a+x)^{-1})/(1-(a+x)^{-1})$ $F=\left\|tg\dfrac{2\pi-x}{3.1\pi}\right\|$	$a=1.25$ $x=0.25$
18	$a=(2+\sqrt{d}),\ \ b=(2-\sqrt{d})$ $c=(a+1)^{-1}+(b+1)^{-1}$ $F=\cos x^2\sin\dfrac{2x}{1.15\pi}+5.1$	$d=3$ $x=1.7$
19	$c=(x\cdot y)^z$ $F=\dfrac{\sin(2x-3)}{\cos(2\pi+1.2x)+2.56}$	$x=4.65$ $y=2.87$ $z=3$
20	$s=x^3\mathrm{tg}^2(x+b)^2+a/\sqrt{x+b}$ $Q=\dfrac{bx^2-a}{e^{ax}-1}$	$a=16.5$ $b=3.4$ $x=0.61$
21	$y=e^{-bt}\sin(at+b)-\sqrt{\|bt+a\|}$ $s=b\sin(at^2\cos 2t)-1$	$a=-0.5$ $b=1.7$ $t=0.44$
22	$f=\ln(a+x^2)+\sin^2(x/b)$ $z=e^{-cx}\dfrac{x+\sqrt{x+a}}{x-\sqrt{\|x-b\|}}$	$a=10.2;\quad b=9.2$ $x=2.2;\quad c=0.5$

Subject 3. LOGICAL FUNCTIONS IN EXCEL

3.1. Decision Structure

The decision structure takes place when one of two possible actions is taken, depending on a condition.

In flowchart notation a diamond symbol indicates a yes/no question (Figure 3.1):

1. If the answer to the question is yes, the flow follows one path.
2. If the answer is no, the flow follows another path

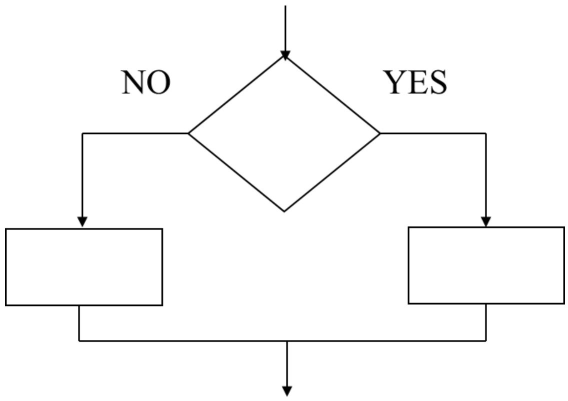


Figure 3.1 – Flowchart with diamond symbol of decision structure

In the flowchart segment below, the question "***is x < y?***" is asked. If the answer is no, then ***process A*** is performed. If the answer is yes, then ***process B*** is performed (Figure 3.2).

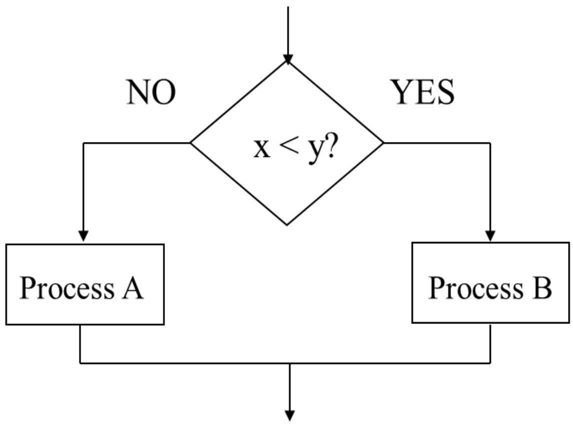


Figure 3.2 – Flowchart with selection if *x* less than *y*

The decision structure in Excel (**IF function)** is shown on flowchart segment at Figure 3.3. In case, when $x < y$ the resulting value is $x + y$, in other case it is $2 \cdot x$. Mathematically it can be expressed by following relation:

$$m = \begin{cases} x + y, & \text{if} \quad x < y \\ 2 \cdot x, & \text{if} \quad x \geq y \end{cases}$$

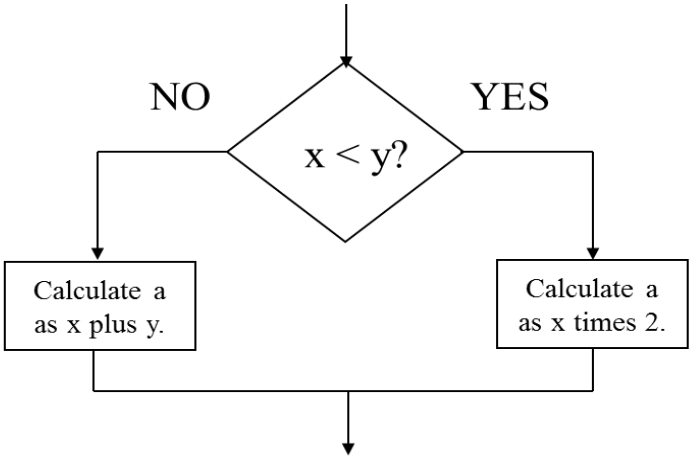


Figure 3.2 – Flowchart with selection if *x* less than *y*

Let's consider, that x value is in A1 cell and y value is in B2 cell. In this case the decision structure in Excel will be expressed by following **IF function**:

$$= \boldsymbol{IF}\ (\boldsymbol{A1} < B\boldsymbol{2}\ ,\ \boldsymbol{A1} * \boldsymbol{2},\ \ \boldsymbol{A1} + \boldsymbol{A2}\ \ )$$

3.2. IF … THEN function

Some functions do not calculate values but instead do logical tests using logical comparison operators. When two values are compared by using these operators, the result is a *logical value* — either **TRUE** or **FALSE**. The list of comparison operators used in Excel is listed in Table 3.1. Such test allows to do one thing when the comparison is true and something different when it is false.

The **IF function** has three arguments inside parentheses, which are separated by commas:

- The comparison statement;
- The cell value to use when the comparison is true;
- The cell value to use when the comparison is false.

Table 3.1 – Comparison operators

Comparisonoperator	Meaning	Example
= (equal sign)	Equal to	A1=B1
> (greater than sign)	Greater than	A1>B1
< (less than sign)	Less than	A1<B1
>= (greater than or equal to sign)	Greater than or equal to	A1>=B1
<= (less than or equal to sign)	Less than or equal to	A1<=B1
<> (notequaltosign)	Not equal to	A1<>B1

The general form of an IF function is:

$$= IF\ (\ \boldsymbol{logical\ comparison},\ \ value\ if\ \boldsymbol{TRUE},\ value\ if\ \boldsymbol{FALSE})$$

A resulted value can be a number, text within double quotes, a cell reference, a formula, or another logical test.

3.3. Example of IF function

In the example the list of people in the group with their test results are listed in Excel spreadsheet (Figure 3.3). The person passes the examination, if his percent is more than 70. At the third column it is needed to provide the corresponding information: pass or failed.

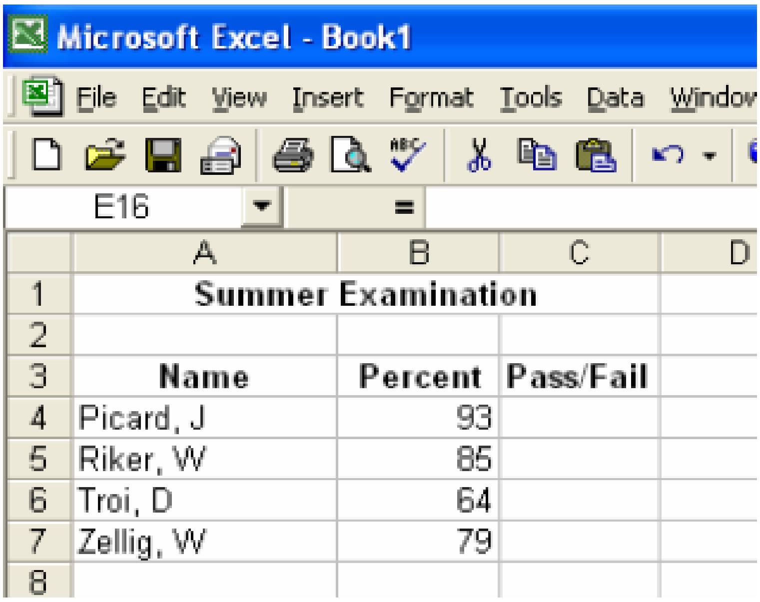
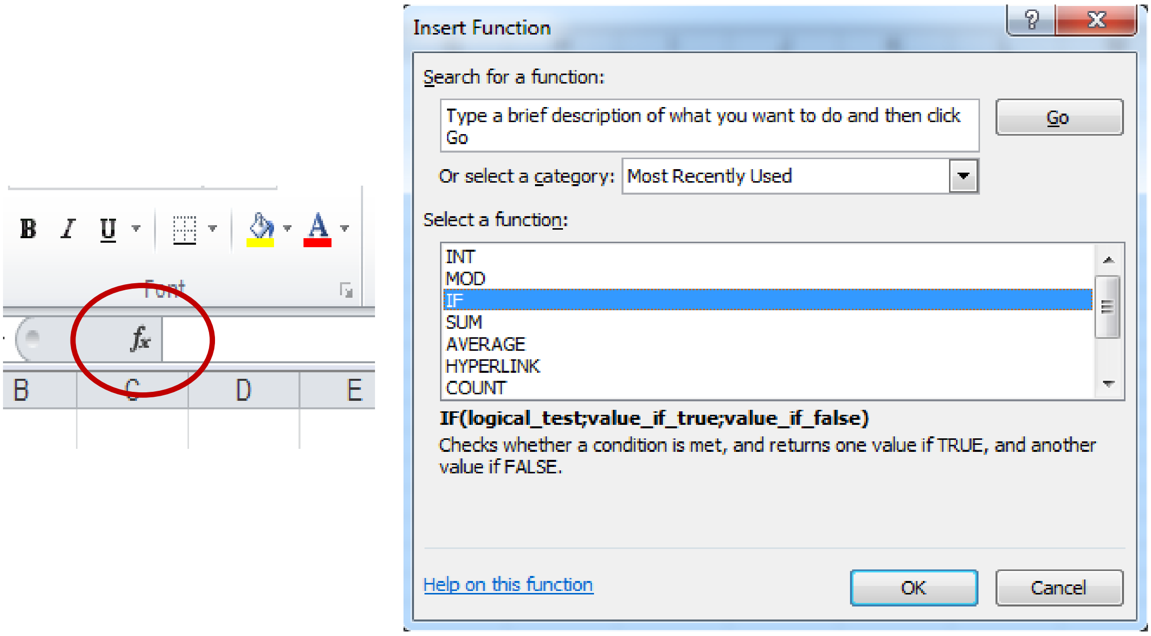


Figure 3.3 – Summer examination example

Solution:

1. Enter the information provided in Figure 3.3.
2. Move the cursor to cell C4.
3. Use the function wizard: click on f_x button (Figure 3.4*a*). The dialog box of function selection will appear (Figure 3.4*b*).Select IF function.



a *b*

Figure 3.4 – Insert function dialog box

4. The Function Arguments box will appear (Figure 3.5).

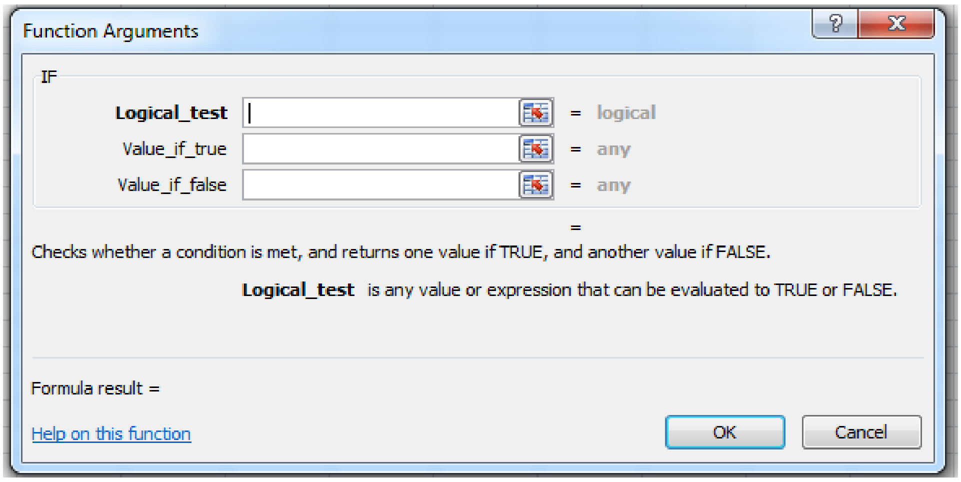


Figure 3.5 – IF Function Arguments box

5. The first box "*logical test*" is for estimating the condition. It gives the result of logical test: TRUE or FALSE. In this example, it is needed to know if the percentage for the examination is more than 70. The result of Picard, J examination is in **B3** cell. It is needed to test if the value in B3 cell is greater than 70. Type $\boldsymbol{B3} > 70$ in the box with *logical test* (Figure 3.6).

6. When the logical test is in place, it is needed to tell what we want to display if the condition is met (**TRUE**) and what we want to display if this condition is not met (**FALSE**). In this example when the value in percent column is more than 70 (TRUE), we need to display Pass. Type "*Pass*" in the "Value_if_true" box (Figure 3.6).

7. In opposite case (when FALSE) we need to display Fail. Type "*Fail*" in the "Value_if_false"box (Figure 3.6).

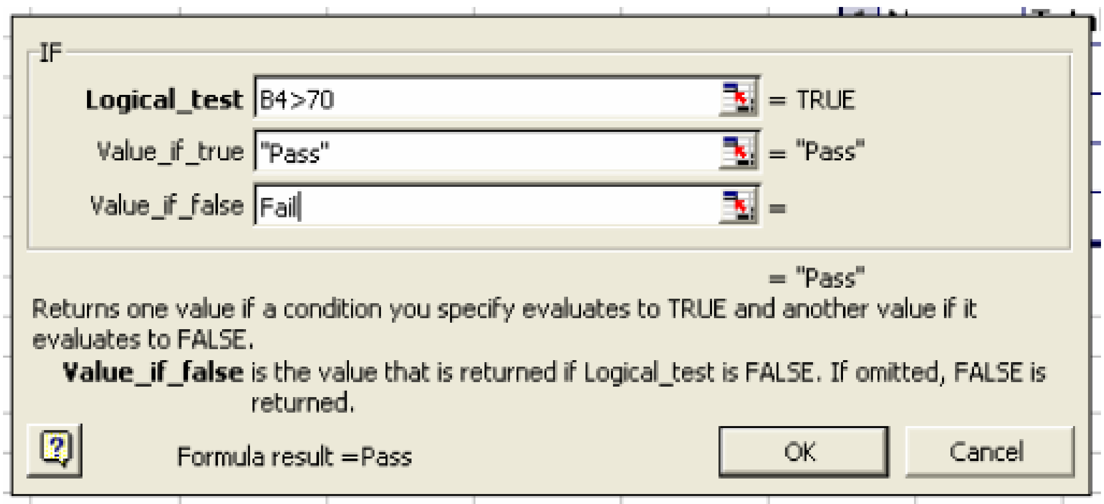


Figure 3.6 – The complete parameters in IF Function Arguments box

8. Press OK button. In C4 cell the information "Pass" will appear.

9. Copy the C4 cell with the function to the other cells (C5:C7). It should provide the proper results.

Task 3.4

Calculate the value of the following function:

$$m = \begin{cases} t^2 + p, & \text{if } \ t \le 0 \\ t - p \cdot c, & \text{if } \ t > 0 \end{cases}$$

With the following parameters: p = 0,5; c = 1,2; t = –2 and 3.

Solution

1. In cells **A1**, **A2** and **A3** input the description text “*p*=“, “*c*=“ and “*t*=“. In cell **A4** enter “*m*=“.
2. Enter the values of x and m to cells **B1**, **B2** and the first value of t(-2) in **B3** cell correspondingly.
3. In the cell **B4** start to enter the formula “**IF**” (Figure 3.7):
4. Choose **IF function** and click button “f_x”.

The Function Argument dialog box for function **IF** appears. Enter the proper arguments in the fields and press OK button (Figure 3.8).

5. Enter the next value of t in **B3** ($t = 3$).

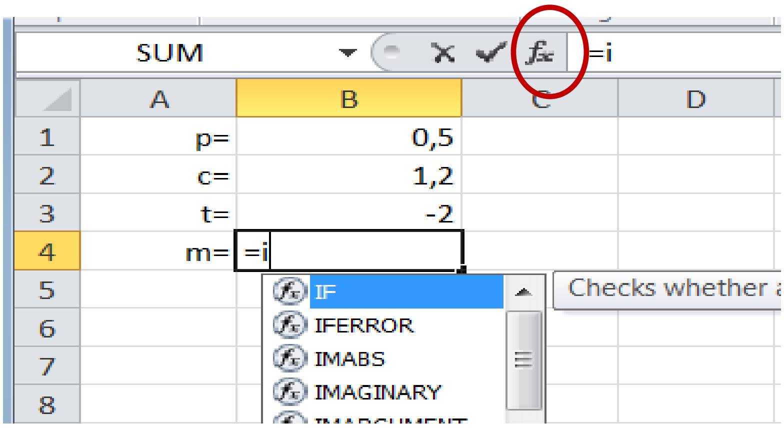


Figure 3.7 – The entering of function

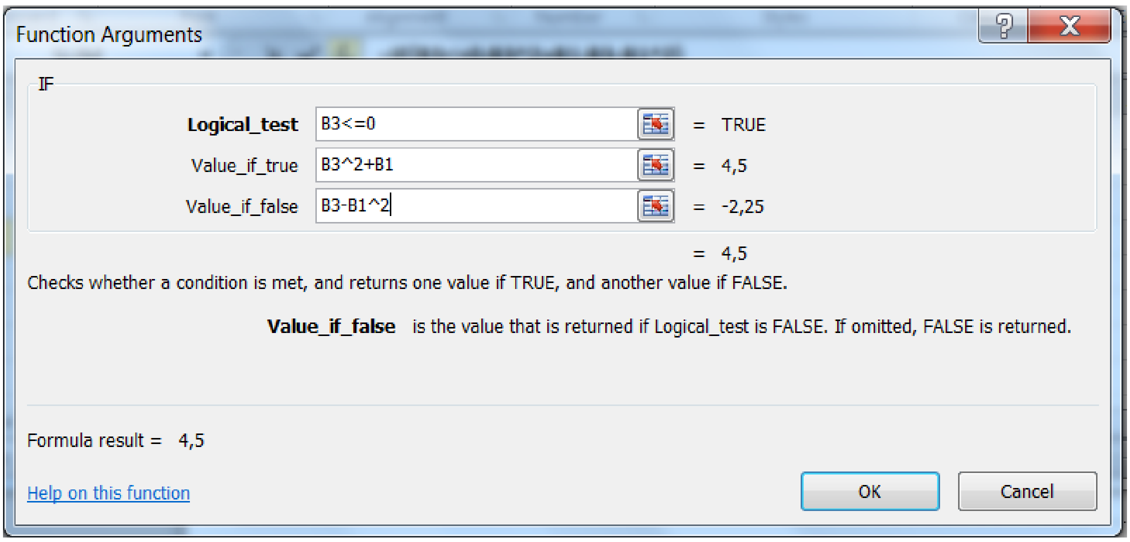
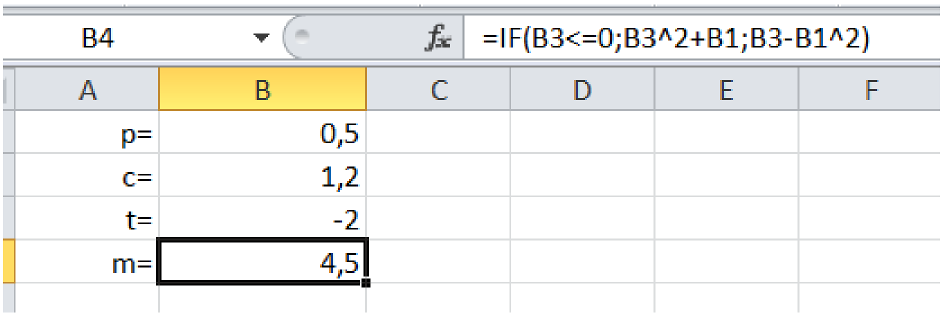


Figure 3.8 – The completed parameters in IF Function Arguments box for Task 3.4

The results of Task 3.4 for different t values are given in Figure 3.9.

1. $t = -2$



2. $t = 3$

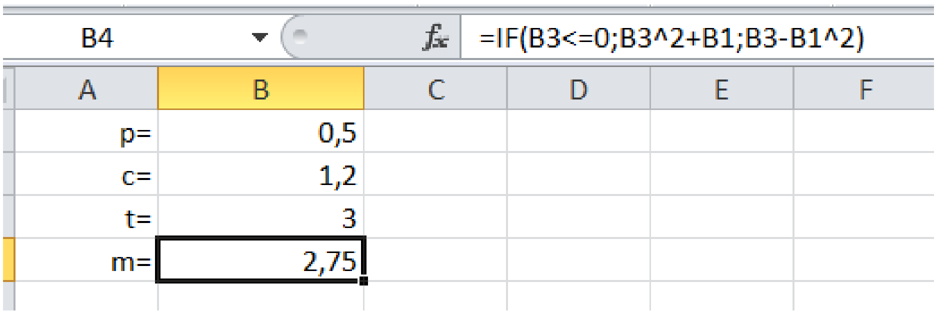


Figure 3.9 – The calculation results of Task 3.4

Task 3.5. Tasks for laboratory work

Calculate the expressions listed in Table 3.2 for two cases: $x = 10$ and $x = -10$ using Excel IF function.

Table 3.2 - Variants for calculation

№	Relations for calculation	№	Relations for calculation
1	2	3	4
1	$\begin{cases} \sqrt{1+2\cdot x^2-\sin(x)^2}, & x \le 0 \\ \dfrac{2+x}{\sqrt[3]{2+e^{-0.1\cdot x}}}, & x > 0 \end{cases}$	2	$\begin{cases} \dfrac{\sqrt{1+\lvert x\rvert}}{2+\cos^2(2x)} & x \ge 0 \\ \dfrac{-2\cdot x+\sqrt{\lvert x^5\rvert}}{1+\lvert x^3\rvert} & x < 0 \end{cases}$

The end of the table 3.2

1	2	3	4
3	$\begin{cases} \dfrac{2+x}{\sqrt{2\cdot e^{-0.1x^3}}}, & x \le 2 \\ \dfrac{\sqrt{1+\sqrt[5]{\lvert 2\cdot\sin\ 5x^2 \rvert}}}{\lvert x\rvert+3}, & x > 2 \end{cases}$	4	$\begin{cases} \dfrac{\sqrt{1+\lvert x\rvert}}{2+\lvert x\rvert}, & x \le -1 \\ \dfrac{1+x}{2+\cos(x)^3}, & x > -1 \end{cases}$
5	$\begin{cases} -2\cdot x+\sqrt{\lvert x\rvert^3}, & x \le 2 \\ \dfrac{x}{\sqrt[4]{e^{2\cdot x+\sqrt{x^3}}}}, & x > 2 \end{cases}$	6	$\begin{cases} \lvert x\rvert^{\frac{1}{3}}, & x \le 3 \\ \dfrac{2+x}{\sqrt{2\cdot e^{-0.1x^3}}}, & x > 3 \end{cases}$
7	$\begin{cases} \sqrt{1+\lvert x\rvert-\sin(x)}, & x \le 0 \\ \dfrac{x}{\sqrt[4]{e^{-0.1x}}}, & x > 0 \end{cases}$	8	$\begin{cases} \dfrac{1+\sin(x)}{1+(2\cdot\cos(x))^2}, & x \le 0 \\ \sqrt{1+\lvert x\rvert}, & x > 0 \end{cases}$
9	$\begin{cases} \dfrac{\lvert x\rvert}{1+x^2}\cdot e^{-2x}, & x < 0 \\ \sqrt{1+x^2}, & x \ge 0 \end{cases}$	10	$\begin{cases} \lvert x\rvert\cdot e^{-2x}, & x < 1 \\ \dfrac{1}{\sqrt{1+x^2}}, & x \ge 1 \end{cases}$
11	$\begin{cases} \sqrt{1+2\cdot x^2}, & x \le 0 \\ \dfrac{1+x}{1+\sqrt[3]{1+e^x}}, & x > 0 \end{cases}$	12	$\begin{cases} 2\cdot\sqrt{1+x^2}, & x \le 0 \\ \dfrac{1}{1+\sqrt[3]{e^x}}, & x > 0 \end{cases}$

Task 3.6

Calculate the value of the following function:

$$m = \begin{cases} (t^2+1)bc, & t < 0 \\ 1/(t+1), & 0 \le t \le 4 \\ \sin^2(t+1), & t > 4 \end{cases}$$

With the following parameters: b = 0.5; c = 1.2; t = –2; 3;5.

Solution

1. In cells **A1**, **A2** and **A3** input the description text “*b*=“, “*c*=“ and “*t*=“ correspondingly. In cell **A4** enter “*m*=“.
2. Enter the values of x and m to cells **B1**, **B2** and the first value of t in **B3** cell correspondingly.
3. In the cell **B4** start to enter the formula “**IF**” (Figure 3.10).

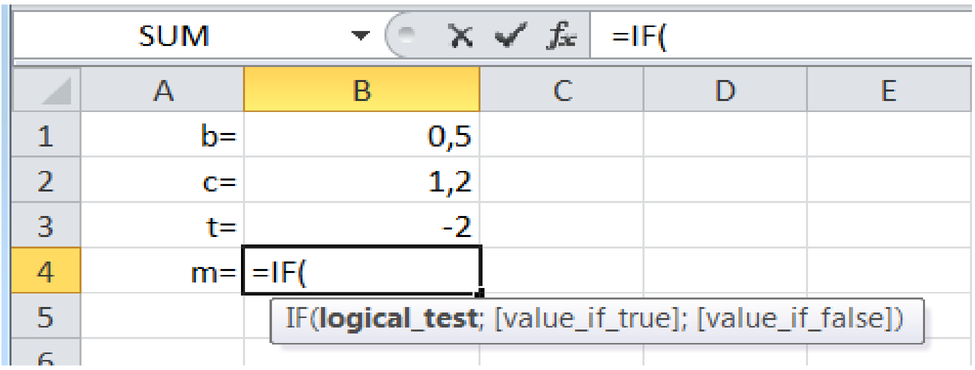


Figure 3.10 – The entering of function

4. Choose **IF function** and click button “f_x”
5. Start to enter arguments of **IF function** in Function Argument dialog box. The entered values for “*Logical_test*” - B3 <0 and “*Value_if_true*” - B3^2+1)*B1*B2 are presented in Figure 3.11. Then close it by clicking OK.

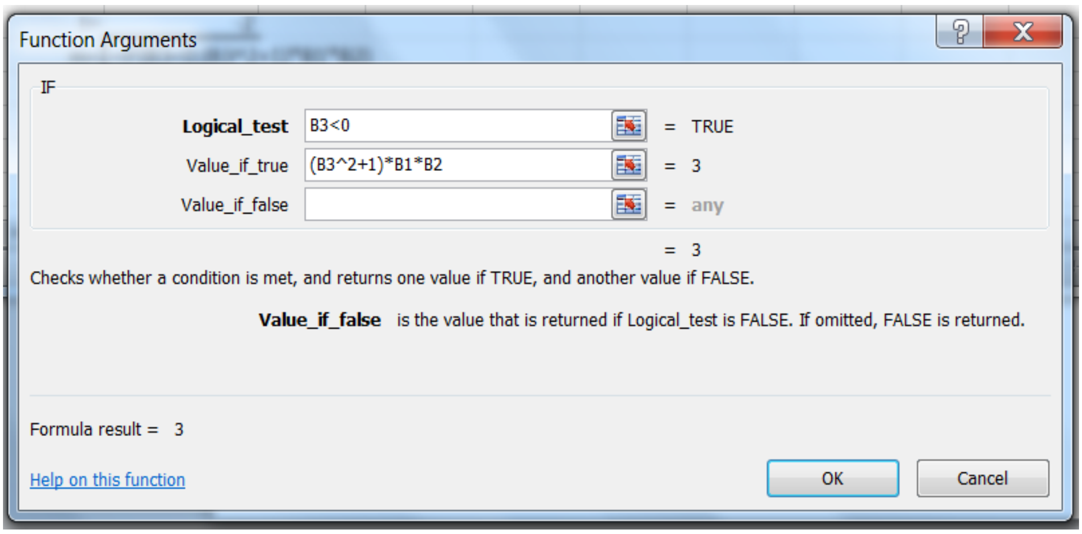


Figure 3.11 – The entering of IF function parameters

6. As “*Value_if_false*” argument in the formula box start enter **IF function** and then press button “f_x” (Figure 3.12).

7. The Function Argument dialog box for the nested function IF appears. Enter the remained arguments in the proper fields and press OK button (Figure 3.13).

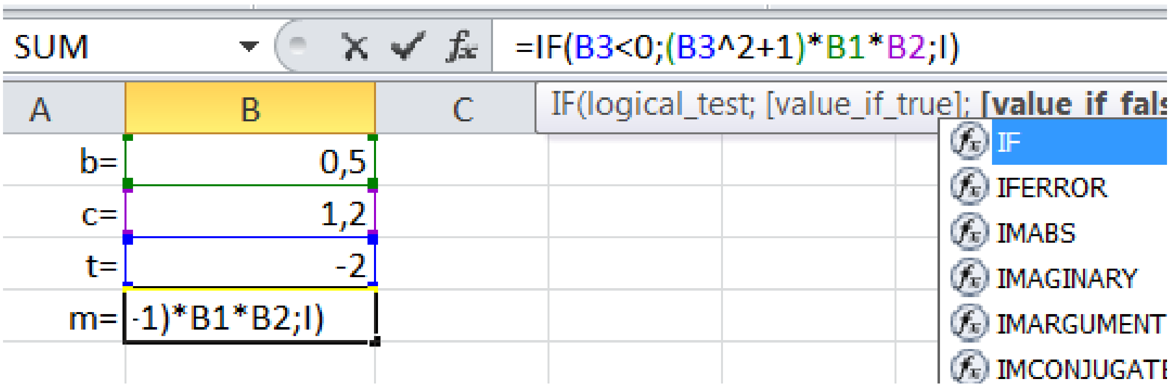


Figure 3.12 – The entering of nested IF function

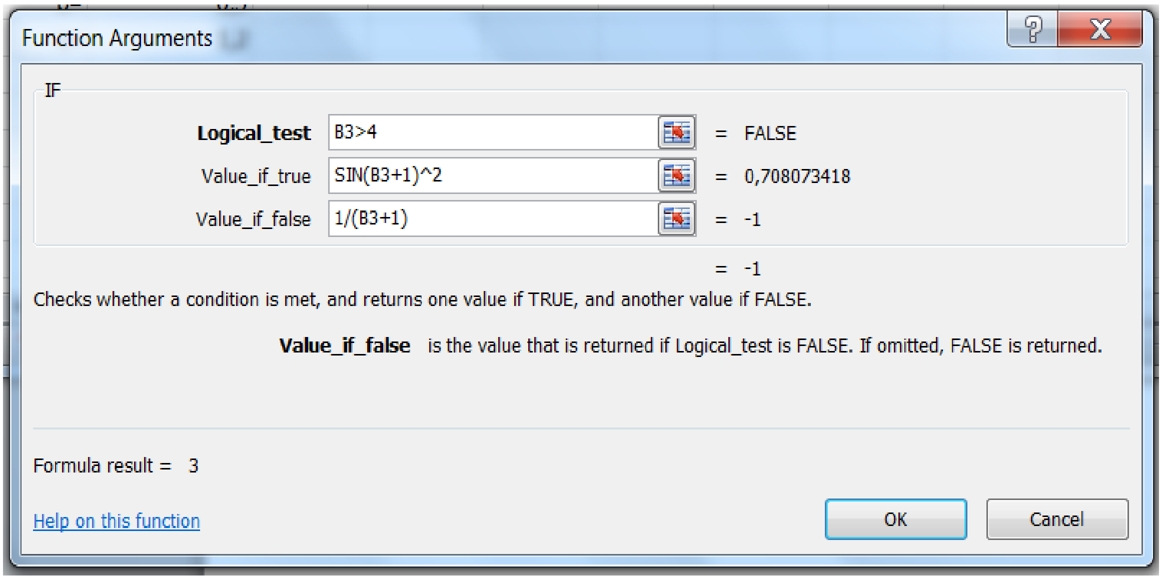
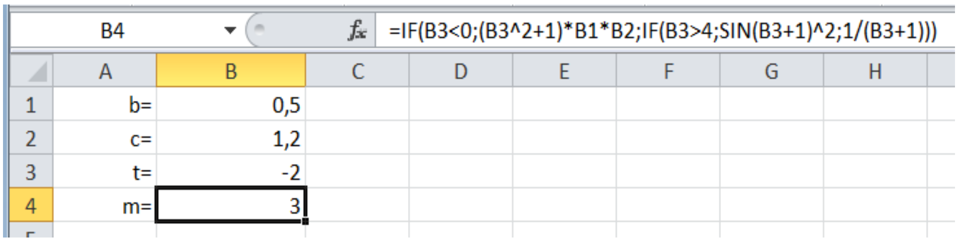


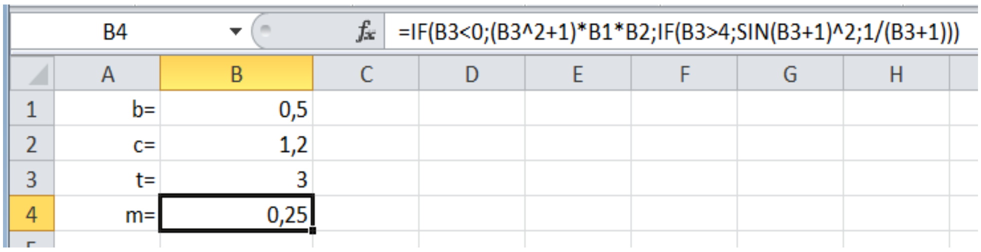
Figure 3.13 – The entering of nested IF function parameters

The results of Task 3.4 for different *t* values are given in Figure 3.14.

1. $t = -2$



2. $t = 3$



3. $t = 5$

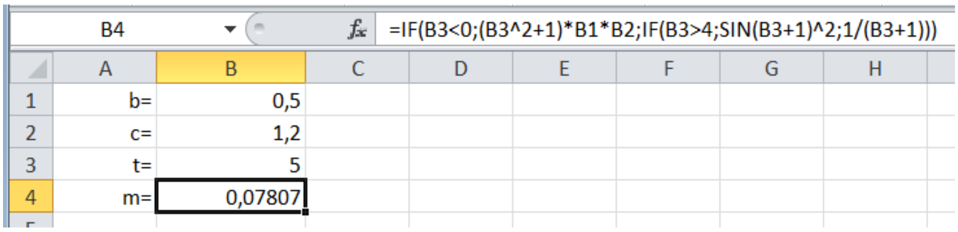


Figure 3.14 – The calculation results of Task 3.4

Task 3.7. Tasks for laboratory work

Table 3.3 – Variants for calculation

№	Relations for calculation	Test condition	Initial data
1	2	3	4
1	$\omega = \begin{cases} x\sqrt[3]{x-a} \\ x\sin ax \\ e^{-ax}\cos ax \end{cases}$	$x > a$ $x = a$ $x < a$	$a = 2.5$ $x = 3;\ 2.5;\ 1$
2	$x = \begin{cases} \lg(ay+1) \\ (ay+b)/2 \\ a+1 \end{cases}$	$y < 10$ $y = 10$ $y > 10$	$a = 0.2;\ b = 0.01;$ $y = 2;\ 10;\ 12.5$

The end of the table 3.3

1	2	3	4
3	$s=\begin{cases}\cos(x)\\ a+b \;/\; x+1\\ e^{x}+\sin x\end{cases}$	$x<2.8$ $2.8\le x<6$ $x\ge 6$	$a=2.6$ $b=-0.39$ $x=2;\ 3;\ 6.2$
4	$f=\begin{cases}(a-b)/2+x\\ x\lg(a+b)\\ ((a+b)x/a\end{cases}$	$x>1$ $x=1$ $x<1$	$a=0.7;\ b=0.31$ $x=2.4;\ 1;\ 0.5$
5	$s=\begin{cases}1-x,\\ (1+xb)/a\\ (1+x)/0.5\end{cases}$	$x=0$ $x<0$ $x>0$	$a=7.2;\ \ b=0.001$ $x=0;-3;\ 5.4$
6	$x=\begin{cases}(m-1.5y)/2\\ mb-y\\ y+mb\end{cases}$	$y>2$ $y<2$ $y=2$	$b=\ 0.2;\ \ m\ =\ 5;$ $y=4;\ 1.5;\ 2$
7	$s=\begin{cases}3.5x-15.2y\\ 8.1x+\sin(y-1)\\ (1.2x+5.1y)/a\end{cases}$	$y>5$ $y=5$ $y<5$	$a=\ 0.1;\ \ x=3$ $y=7;\ 5;\ 2.67$
8	$f=\begin{cases}y-1\\ y+b\\ ((y+1)b)/(a-b)\end{cases}$	$y>1$ $y=1$ $y<1$	$a=0.2;\ \ \ b=0.3$ $y=2;\ \ 1;-4.8$
9	$f=\begin{cases}1-\cos x-y\\ (ay+b)/2\\ a+1\end{cases}$	$y<0$ $y=0$ $y>0$	$a=0.2;\ \ b=0.001$ $x=2$ $y=-2;\ 0;\ 5.8$
10	$m=\begin{cases}1-a\cos x\\ ax+b\\ x+(b/a)\end{cases}$	$x<1$ $x=1$ $x>1$	$a=0.001;\ \ b=5.1$ $x=-3;\ 1;\ 8.56$

Task 3.8. Tasks for individual work

Table 3.4 – Variants for calculation

№	Relations for calculation	Test condition	Initial data
1	2	3	4
1	$y=\begin{cases}\sin^2 x\\ \ln^3 \quad 1+x^2\\ x+c\end{cases}$	$x>0$ $x<0$ $x=0$	$c=1.57$ $x=3;\ -2.5;\ 0$
2	$y=\begin{cases}a/\sin(x+c)\\ (a^2+c)x\\ a+c\end{cases}$	$x>0$ $x<0$ $x=0$	$a = 1.5;\quad c = -0.45$ $x=2;-3.5;\ 0$
3	$Q=\begin{cases}\pi x^2-7/x^2\\ ax^3+7\sqrt{x}\\ \ln(x+7)\cdot\sqrt{\lvert x+a\rvert}\end{cases}$	$x<1.4$ $x=1.4$ $x>1.4$	$a=1.65$ $x=1;\ 1.4;\ 5$
4	$g=\begin{cases}y^3-0.3\\ 0\\ y^2+y\end{cases}$	$y<0$ $0\le y\le 1$ $y>1$	$y = z + 2;$ $z = -3; -1.5;\ 0$
5	$w=\begin{cases}(x^2-1)+b\\ (x+1)/b\\ b\end{cases}$	$x<0$ $x>0$ $x=0$	$b = 1.5$ $x=-2;\ 5;\ 0$
6	$c=\begin{cases}x/(1+x^2/2)\\ x-(x/(1+x^2/2))\\ e^x-(x/(1+x^2/2))\end{cases}$	$x<1$ $1\le x\le 2$ $x>2$	$x=\ 0.5;\ 1.85;\ 3$
7	$y=\begin{cases}x/12\\ x^2+15\\ x^3-4\end{cases}$	$x<3$ $3\le x\le 10$ $x>10$	$x=0.5;\ 7;\ 14$

Continuation of the table 3.4

1	2	3	4
8	$\omega=\begin{cases} ax^2+bx+c \\ a/x+\sqrt{x^2+1} \\ (a+bx)/\sqrt{x^2+1} \end{cases}$	$x<1.2$ $x=1.2$ $x>1.2$	$a=2.8;\ b=-0.3;$ $c=4$ $x=\ 1;\ 1.2;\ 3$
9	$m=\begin{cases} (t^2+1)bc \\ 1/(t+1) \\ \sin(t+1)^2 \end{cases}$	$t<0$ $0\le t\le 4$ $t\ge 4$	$b\ =\ 0.5;\quad c\ =\ 1.2;$ $t\ =-2;\ 3;\ 5.67$
10	$\omega=\begin{cases} x\sqrt[3]{x-a} \\ x\sin ax \\ e^{-ax}\cos ax \end{cases}$	$x>a$ $x=a$ $x<a$	$a=2.5$ $x=3;\ 2.5;\ 1$
11	$x=\begin{cases} \lg(ay+1) \\ (ay+b)/2 \\ a+1 \end{cases}$	$y<10$ $y=10$ $y>10$	$a=0.2;\ b=0.01;$ $y=2;\ 10;\ 12.5$
12	$z=\begin{cases} (\ln^3 x+x^2)/\sqrt{x+t} \\ \sqrt{x+t}+1/t \\ \cos x+t\sin^2 x \end{cases}$	$x<0.5$ $x=0.5$ $x>0.5$	$t=2.2$ $x=0.3;\ 0.5;\ 1$
13	$s=\begin{cases} a+b\ /(e^x+\cos x) \\ a+b\ /\ x+1 \\ e^x+\sin x \end{cases}$	$x<2.8$ $2.8\le x<6$ $x\ge 6$	$a=2.6$ $b=-0.39$ $x=2;\ 3;\ 6.2$
14	$\omega=\begin{cases} \frac{a}{i}+bi^2+c \\ i \\ ai+bi^3 \end{cases}$	$i<4$ $4\le i\le 6$ $i>6$	$a=2.1;\ b=1.8$ $c=-20.5$ $i=\ 3;\ 5;\ 6.7$

The end of the table 3.4

1	2	3	4
15	$\omega = \begin{cases} \sqrt{at^2 + b\sin t + 1} \\ at + b \\ \sqrt{at^2 + b\cos t + 1} \end{cases}$	$t < 0.1$ $t = 0.1$ $t > 0.1$	$a = 2.5$ $b = 0.4$ $t =\ 0.05;\ 0.1;\ 2$
16	$f = \begin{cases} (a-b)/2 + x \\ x\lg(a+b) \\ (a+b)x/a \end{cases}$	$x > 1$ $x = 1$ $x < 1$	$a = 0.7;\ b = 0.31$ $x = 2.4;\ 1;\ 0.5$

Task 3.9. Working with flowchart

Using **IF function**, do the calculations according to the flow chart presented in Figure 3.15 with the following *x* values: $x = -10,\ 2,\ 5,\ 15$.

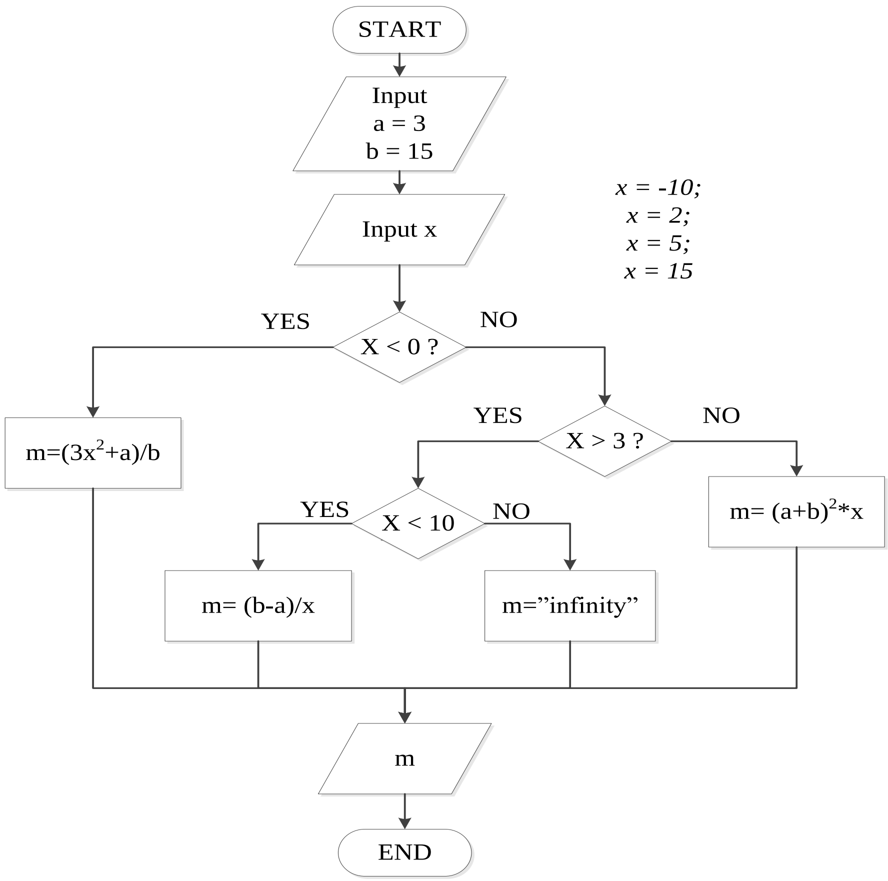


Figure 3.15 – The flowchart for Task 3.9

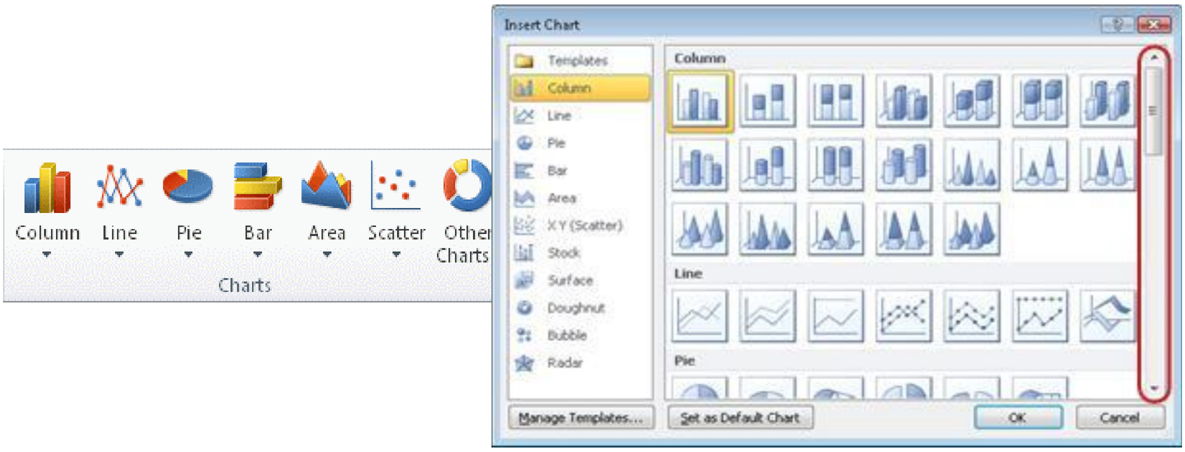
Subject 4. PLOTTING CHARTS

4.1. Chart the data

Charts are used to display series of numeric data in a graphical format to make it easier to understand large quantities of data and the relationship between different series of data.

To create a *chart* in Excel, it is needed to enter the numeric data for the chart on a worksheet. Then it is possible to plot that data into a chart by selecting the chart type on the Insert tab, in the Charts group. The procedure needs to do the following steps:

1. Select the data to chart.
2. On the *Insert* tab, in the *Charts* group, select the chart type (Figure 4.1*a*), and then a chart subtype (Figure 4.1*b*).



a *b*

Figure 4.1 – Charts group on Insert tab

3. Use the *Chart* Tools on *Design*, *Layout* and *Format* tab to add chart elements such as titles and data labels, to change the design, layout, or format of your chart.

Excel provides the following chart types:

– Column charts
– Line charts
– Pie charts
– Bar charts
– Area charts
– XY (scatter) charts
– Stock charts
– Surface charts
– Doughnut charts
– Bubble charts
– Radar charts

The elements of a chart

The chart area in Excel consists of the basic elements presented in Figure 4.2 and listed in Table 4.1.

Figure 4.2 – Chart elements

Table 4.1 – The description of chart elements

№	Description
1	The chart area of the chart.
2	The plot area of the chart.
3	The data points of the data series that are plotted in the chart.
4	The horizontal (category) and vertical (value) axis along which the data is plotted in the chart.
5	The legend of the chart.
6	A chart and axis title that you can use in the chart.
7	A data label that you can use to identify the details of a data point in a data series

Task 4.2

Plot the graph of function

$$f(t) = 65.5 \cdot e^{-0.15 \cdot t}$$

In the range $0 \leq t \leq 15$; with the step $h = 1$.

Solution

1. Input the names of columns ***t*** and ***f(t)*** in cells **C2** and **D2** correspondingly (Figure 4.3).

2. Input the first and the second value of ***t*** in cells **C3** and **C4**$(\boldsymbol{t_1 = 0; t_2 = 1})$ and enter the formula for function ***f(t)*** in **D3** cell $(\boldsymbol{= 65,5 * EXP(-0,15 * C3)})$ (Figure 4.3).

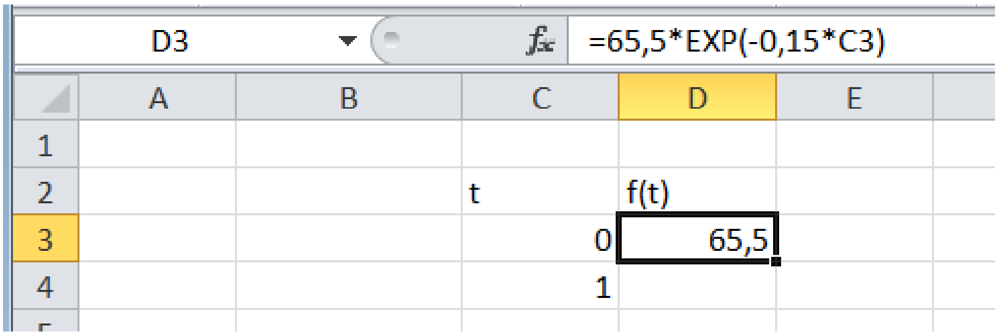


Figure 4.3 – Task 4.2 solution, step 1 and 2

3. By selecting two cells **C3** and **C4** and then dragging them, increase the ***t*** values till it will be equal 15 (Figure 4.4).

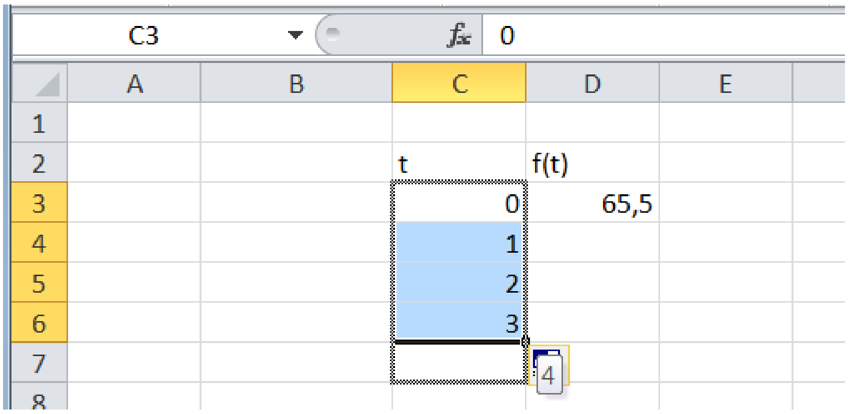
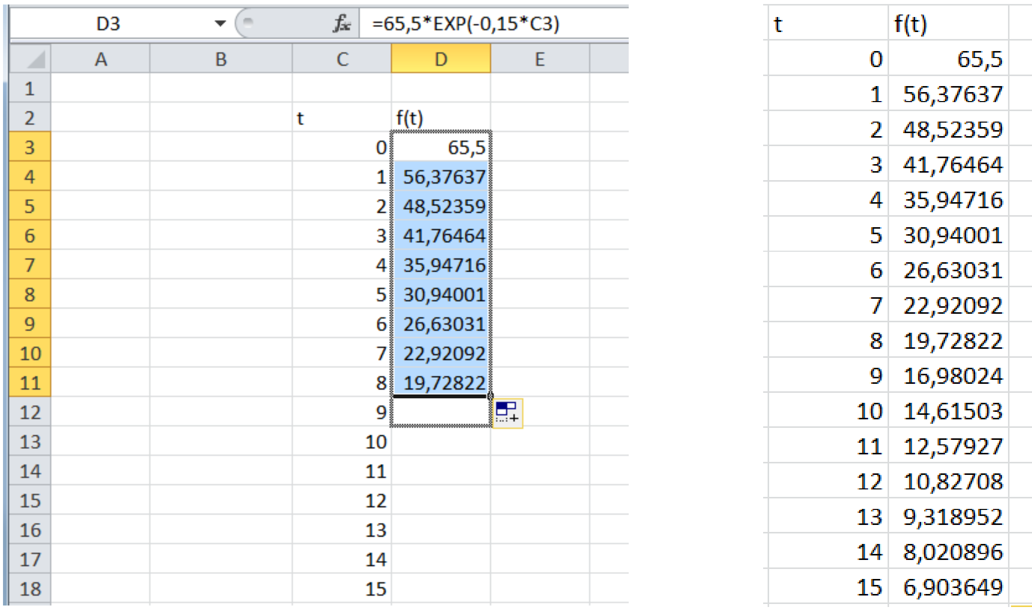


Figure 4.4 – Creating the column with ***t*** values

4. And copy the formula by dragging the cell **D3** (Figure 4.5*a*).The two columns presented in Figure 4.5*b* one with the argument ***t*** values and other with function ***f(t)***values should be created.



a *b*

Figure 4.5 – Creating the columns with argument and function values

5. Select the cell **F7** where the graph will be plotted and go to ***Insert tab***, ***Charts group***, select ***Scatter with smooth lines and markers*** (Figure 4.6).

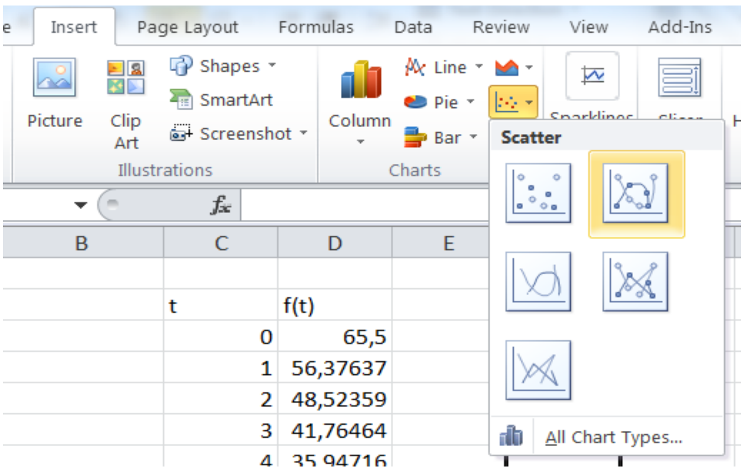


Figure 4.6 – Selecting ***Scatter with smooth lines and markers*** to plot the graph by points $(t\,;f(t))$

6. Press ***Select Data*** button(Figure 4.7).

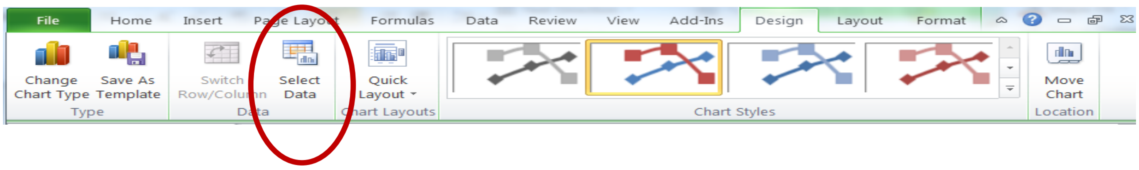


Figure 4.7 – Selecting data for the chart

7. Input the chart data range by selecting cells ***C3:D19*** on the worksheet (Figure 4.8).

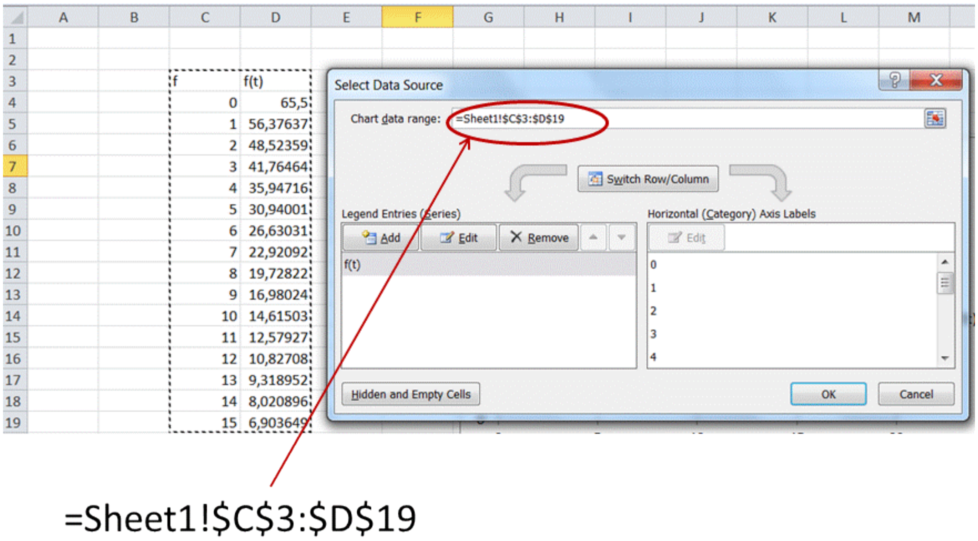


Figure 4.8 – Selecting data for the chart

8. Press ***Edit button*** and check values for t and $f(t)$(Figure 4.9).

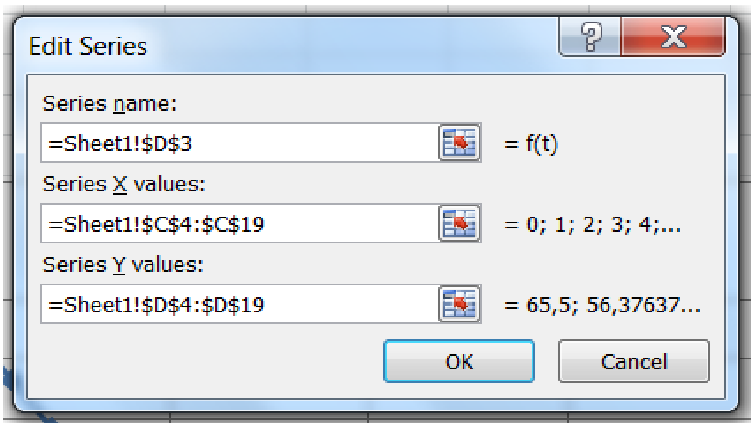


Figure 4.9 – Edit data for the chart

9. Use the ***Layout tab*** to modify the view of your chart (Figure 4.10).

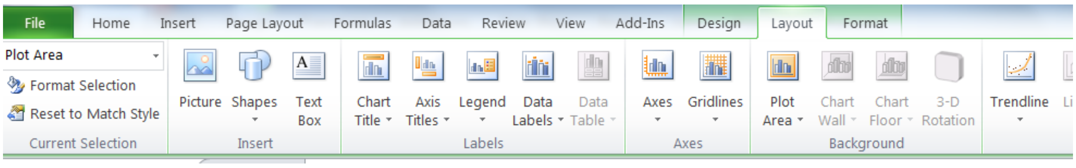


Figure 4.10 – Layout tab to modify the view of the chart

10. Press OK button. The graph of function presented in Figure 4.11 will appear.

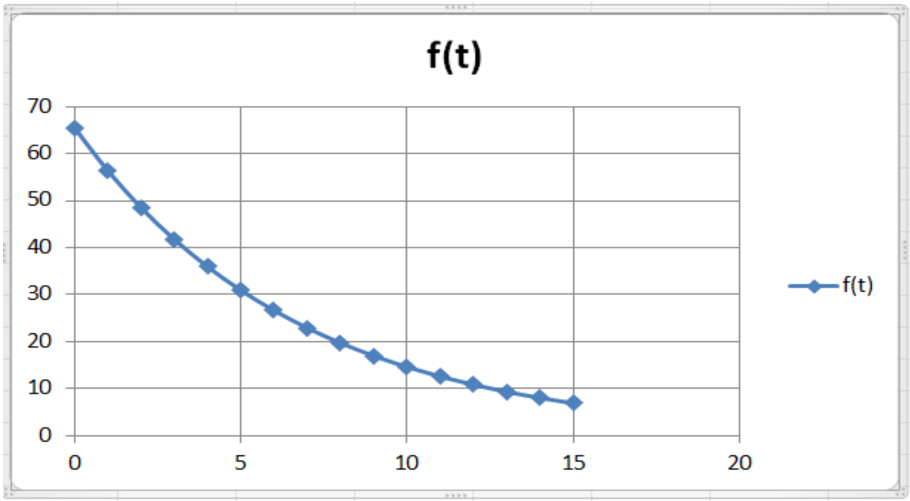


Figure 4.11 – The graph of function $f(t) = 65.5 \cdot e^{-0.15 \cdot t}$ in the range $0 \le t \le 15$, step $h = 1$

Task 4.3. Tasks for laboratory work

Plot the graph of function presented in Table 4.2 on the given range. Take the appropriate increment for building the graph of function.

Table 4.2 – Variants for calculation

№	**Function**	**The range of variables**
1	2	3
1	$Y = 2 \cdot x + cos(x) - 0.5$	$-3.14 < x < 3.14$

The end of the table 4.2

1	2	3
2	$Y = x + ln(x) - 0.8$	$1 < x < 10$
3	$Y = 2 \cdot x - lg(x) - 7$	$2 < x < 10$
4	$Y = (x + 1)^{0.5} - 1/x$	$2 \leq x < 5$
5	$Y = x^2 + 4 \cdot sin(x)$	$-3.14 < x < 3.14$
6	$Y = x + cos(x) - 1$	$1.7 < x < 4.5$
7	$Y = x^3 - sin(x)$	$-3.14 < x < 3.14$
8	$Y = x \cdot (x + 1)^2 - 1$	$0 < x < 10$
9	$Y = 2 \cdot x + lg(x) + 0.5$	$2 < x < 6$
10	$Y = sin(x) + cos(x)$	$-3.14 < x < 3.14$

Task 4.4

Do the Tasks listed in Table 4.3.

Table 4.3 – Tasks for calculation

№	**Task description**
1	2
1	Plot the graph of the reaction velocity according to the relation: $K = 78.56 \cdot e^{-0.1037\tau}; \quad 1 \leq \tau \leq 25; \quad \Delta\tau = 1$
2	Plot the graph of the humidity *x* (in percent of dry substance) depending from time *t*, min according to the relation: $x = 10^{1.4568 - 0.00938t}; \quad 0 \leq t \leq 100; \quad \Delta t = 5$
3	Plot the graph of relation between the mixture amount y (in %), which is taken from the evaporation unit, and its output *x* (in kg/h) according to the equation: $y = 5.789x^{2.071} \cdot 10^{-7} + 0.048; \quad 250 \leq x \leq 2000; \quad \Delta x = 100$

The end of the table 4.3

1	2
4	Plot the graph of relation between the atmospheric pressure *p* (in mm Hg) and barometric height *h* (in km), according to the equation: $p = 760 \cdot 10^{-0.0531h}; \quad 0 \le h \le 6; \quad \Delta h = 0.2$
5	Plot the graph of relation between the atmospheric pressure *p* (in mm Hg) and barometric height *h* (in km), according to the equation: $p = 760 \cdot 10^{-0.0531h}; \quad 0 \le h \le 6; \quad \Delta h = 0.2$
6	Plot the graph of the humidity *x* (in percent of dry substance) depending from time *t*, min according to the relation: $x = 10^{1.4568-0.00938t}; \quad 0 \le t \le 100; \quad \Delta t = 5$
7	Plot the graph of relation between the air temperature inside the dryer *T* (in K) during the day, where τ are the hours, according to the equation $T = 80 + 14.99\cos(15\tau - 37) + 18\cos(30\tau - 353)$, $0 \le \tau \le 24$
8	Plot the graph of the relation between the solubility of the sodium salt of hypochlorous acid in water *x* (in g/100 g of water) from its temperature *t* (in °C) according to the equation: $x = 7.894e^{0.1939t} - 4.82; \quad 0 \le t \le 60; \quad \Delta t = 2$
9	Plot the graph of the relation between the friction factor in tubes τ depending from the Reynolds number Re: $\tau = \frac{0.398}{\mathrm{Re}^{0.254}}, \quad 3000 \le \mathrm{Re} \le 16000; \quad \Delta \mathrm{Re} = 1000$
10	Plot the graph of the relation between the air volume content in water ν (in ml, taken at given temperature *t*, °C and 760 mm Hg) and temperature *t*, °C: $\nu = (t-5)/(-1.386 - 0.0375t) + 25.68; \quad 0 \le t \le 25; \quad \Delta t = 1$

Task 4.5.Tasks for individual work

Plot the graph of function presented in Table 4.4 on the given range with the specified increment.

Table 4.4 – Variants for calculation

№	Function	The range of variables	Increment
1	$y = 2 \cdot x + \cos(x) - 0.5$	$-3.14 \le x \le 3.14$	$h = 0.1$
2	$y = x + \ln(x) - 0.8$	$1 \le x \le 7$	$h = 0.2$
3	$y = 2 \cdot x + \ln(x) - 3$	$0.1 \le x \le 3$	$h = 0.2$
4	$y = (x+1)^{0.5} - 1/x$	$0.2 \le x \le 5$	$h = 0.2$
5	$y = x^2 + 4 \cdot \sin(x)$	$-\pi \le x \le \pi$	$h = 0.1$
6	$y = x + \cos(x) - 1$	$-1.7 \le x \le 4.5$	$h = 0.5$
7	$y = x^3 - \sin(x)$	$-\pi \le x \le \pi$	$h = 0.1$
8	$y = x \cdot (x+1)^2 - 1$	$0 \le x \le 10$	$h = 1$
9	$y = 5 \cdot x^2 + \lg(x) - 5$	$1 \le x \le 8$	$h = 0.5$
10	$y = \sin(x) + \cos(x)$	$-\pi \le x \le \pi$	$h = 0.1$
11	$y = x \cdot \lg(x+1) - 4$	$0{,}1 \le x \le 8$	$h = 0.1$
12	$y = 2 \cdot \ln(x) - x/2 + 1$	$1 \le x \le 50$	$h = 5$
13	$y = arctg(x) - x^3$	$0 \le x \le 25$	$h = 5$
14	$y = x^{\sin(x) - x}$	$0.1 \le x \le 4$	$h = 0.1$
15	$y = 5 \cdot \sin(x) - x$	$-10 \le x \le 10$	$h = 1$
16	$y = x^3 - \sin(x) + 5$	$-\pi \le x \le \pi$	$h = 0.1$
17	$y = x \cdot (x+1)^2 - 90$	$0 \le x \le 10$	$h = 1$
18	$y = 3 \cdot x^2 - \lg(x) - 56$	$0.2 \le x \le 5$	$h = 0.2$
19	$y = \sin(x) + \cos(x)$	$-\pi \le x \le \pi$	$h = 0.1$
20	$y = x^2 + 4 \cdot \sin(x)$	$-\pi \le x \le \pi$	$h = 0.1$

Subject 5. WORKING WITH TABLES IN EXCEL

Task 5.1

The typical engineering problems can be solved in Excel. For, example to find the content of working mass, what is the standard problem in heat engineering.

It is needed to find the **lowest combustion heat** ($Q_L, kJ/kg$), and **highest combustion heat** ($Q_H, kJ/kg$) of the fuel with the following content of ***combusting mass***: C^c= 78.5%; H^c = 5.6%; S^c= 0.4%; N^c= 2.5%; O^c = 13.0%. The ash content of working mass is A^w = 13.2%.The wetness of the ***working mass*** is W^w = 12.0%.

The formulas for the highest and lowest combustion heat of the fuel (Q_L and Q_H) are:

$$Q_L = 338\mathrm{C}^w + 1025\mathrm{H}^w - 108.5\ \mathrm{O}^w - \mathrm{S}^w\ - 25W^w, \qquad (5.1)$$

$$Q_H = Q_L + 225\mathrm{H}^w + 25W^w. \qquad (5.2)$$

The equation to determine the content working (w) mass:

$$\mathrm{X}^w = X^c \cdot \frac{100 - \ A^w + W^w}{100}, \qquad (5.3)$$

where X is the one of the fuel component: C – carbon; H – hydrogen; S – sulfur; N – nitrogen; O – oxygen.

Solution

1. Create the table with initial data:
   - C^c value, which is equal to 78.5% is in **B3** cell;
   - H^c value equal to 5.6% is in **B4** cell;
   - S^c value equal to 0.4% is in **B5** cell;
   - N^c value equal to 2.5% is in **B6** cell;
   - O^c value equal to 13.0%is in **B7** cell;
   - A^w value equal to 13.2%is in **C8** cell;
   - W^w value equal to 12.0% is in **C9** cell.

Draw the table borders and format the alignment of the data inside the cells. The resulting table with initial data is presented in Figure 5.1.

	A	B	C	D
1				
2	Fuel content	Combusting mass, %	Working mass, %	
3	C	78,5		
4	H	5,6		
5	S	0,4		
6	N	2,5		
7	O	13		
8	A	0	13,2	
9	W	0	12	
10	Summa			
11	Lowest combustion heat, KJ/kg			
12	Highest combustion heat, KJ/kg			
13				

Figure 5.1 – The table with initial data for Task 5.1

2. Complete the table with the values, which should be calculated.

In cell C3 enter the following formula according to Equation (5.3) with the reference to the corresponding cells:

$$= B3 * ((100 - (C8 + C9))/100)$$

3. Use the absolute reference to the cellsC8 and C9 with ash content of working mass value (A^w) and the wetness of the working mass value (W^w) by putting "$" sign before the column letter and row number. The changed formula cell C3 should look like:

$$= B3 * ((100 - (\$C\$8 + \$C\$9))/100)$$

4. Copy the content of C3 cell to cells C4:C7.
5. Calculate the sums automatically. It should be equal to 100 % for column with combusting mass (*B*) and for column with working mass (C).

Calculate the **lowest combustion heat** ($Q_L\ , \frac{kJ}{kg}$) of the fuel according to Equation (5.1), with reference to the cells with corresponding values

$$= 338 * C3 + 1025 * C4 - 108.5 * (C7 - C5) - 25 * C9;$$

and **highest combustion heat** $(Q_H\, , kJ/kg)$ of the fuel according to Equation (5.2)

$$= B11 + 225 * C4 + 25 * C9;$$

The resulting table is presented in Figure 5.2.

	A	B	C	D
1				
2	Fuel content	Combusting mass, %	Working mass, %	
3	C	78,5	58,718	
4	H	5,6	4,1888	
5	S	0,4	0,2992	
6	N	2,5	1,87	
7	O	13	9,724	
8	A	0	13,2	
9	W	0	12	
10	Summa	100	100	
11	Lowest combustion heat, KJ/kg	22817,61		
12	Highest combustion heat, KJ/kg	24060,09		
13				

Figure 5.2 – The resulting table for Task 5.1

Task 5.2

Create the table presented in Figure 5.3 keeping the information in proper cells of the Excel worksheet. The task is to complete the table, doing the questions, listed below. You should make a judgement about which type of if statement is needed before you attempt to work out the formula. You can use the following functions:

- **IF… then**
- **IF(AND…)**
- **IF(OR…)**
- **Nested IF statements**

THEATRE COMPANY

Number of Seats Available	500

Date	Day	Time of Performance	Number of Seats Sold	Performance? Matinee/Evening	Ticket Price	Sold Out?	Reduce Ticket Price Due To Undersold Matinee	Increase Remaining Ticket Price Due To On Target Sales or Saturday	Final Sales Price For Remaining Tickets
2/16/2014	Thursday	14	465						
2/16/2014	Thursday	19	410						
2/17/2014	Friday	14	132						
2/17/2014	Friday	19	500						
2/18/2014	Saturday	14	435						
2/18/2014	Saturday	19	500						
2/19/2014	Sunday	14	477						
2/19/2014	Sunday	19	365						
2/20/2014	Monday	14	475						
2/20/2014	Monday	19	471						
2/21/2014	Tuesday	14	245						
2/21/2014	Tuesday	19	147						
2/22/2014	Wednesday	14	456						
2/22/2014	Wednesday	19	357						

Figure 5.3 – The table with initial data for Task 5.2

Question 1

In column F, a formula needs to be written to identify whether a performance is in the evening or if it is a matinee performance.

1. Write a formula in cell F9, so if a performance is on at 14.00 hours then the word "Matinee" should be displayed, otherwise "Evening" should be displayed.

2. Replicate this formula for all performances.

Question 2

In column G, a formula needs to be written to display the cost of a performance. The cost depends on whether a performance is a matinee or evening performance.

1. Write a formula in cell G9, so if a performance is a matinee, the cost displayed is 80.00 UAH, otherwise it is 100.00 UAH.

2. Replicate this formula for all performances.

Question 3

In column H, a formula needs to be written to display whether a performance is sold out. All performances have the same number of seats available.

1. Write a formula in cell H9, so if the number of seats sold is equal to 500, "Sold Out" should be displayed, if the number of seats sold is greater than or equal to 400, "On Target" should be displayed, otherwise "Undersold" should be displayed.

2. Replicate this formula for all performances.

Question 4

In column I, a formula needs to be written to decrease the cost of remaining tickets for a performance. The new ticket price is calculated by checking if a performance is a matinee and is undersold.

1. Write a formula in cell I9, so that if a performance is a matinee AND it is undersold, the price displayed should be the original ticket price – 10 UAH, otherwise the price should remain the same.

2. Replicate this formula for all performances.

Question 5

In column J, a formula needs to be written to increase the cost of remaining tickets for a performance. The new ticket price is calculated by checking if sales are on target or the performance is on a Saturday.

1. Write a formula in cell J9, so that if a performance is on a Saturday OR is currently on target with sales, the price displayed should be the original ticket price + 10 UAH, otherwise the price should remain the same.

2. Replicate this formula for all performances.

Question 6

In column K, a formula needs to be written to display the final sales price for any remaining tickets. The highest price should be the selling price.

1. Write a formula in cell K9, so that if the ticket price in I9 is greater than J9, display the ticket price in I9, otherwise display the price in J9.

2. Replicate this formula for all performances.

The correct formulas:

Q1 =IF(D9=14, "Matinee", "Evening")

Q2 =IF(F9="Matinee", 80, 100)

Q3 =IF(E9=$E$6,"Sold Out",IF(E9>=400,"On Target","Undersold"))

Q4 =IF(AND(F9="Matinee", H9="Undersold"), G9-10, G9)

Q5 =IF(OR(H9="On Target", C9="Saturday"), G9+10, G9)

Q6 =IF(I9>J9,I9,J9)

Tasks for laboratory work

Task 5.3

Create the Table 5.1.

Calculate the following values:

– **Total World** – total amount for the presented countries,

– **Average** – the average value for the world,

– **Min** – minimum value of the presented numbers,

– **Max**– maximum value of the presented numbers,

– **Count** – the number of listed countries;

– **% of Global** – the percent producing by the country in total world amount

Table 5.1 – Initial data for Task 5.3.

Natural Gas			Oil		
Country	Trillion Cubic Feet	% of Global	Country	Trillion Cubic Feet	% of Global
Russia	21.1		Saudi Arabia	4.1	
USA	18.2		Russia	3.5	
Canada	6.5		USA	2.5	
Algeria	3.1		Iran	1.5	
UK	3.07		Mexico	1.4	
Iran	55.1		China	1.3	
Total World			Total World		
Average			Average		
Min			Min		
Max			Max		
Count			Count		

Task 5.4

Create the table presented in Figure 5.4.

	A	B	C	D	E	F	G
1	***Test Results***				Pass mark	6	
2							
3	**Name**	**Test 1**	**Pass/Fail**	**Test 2**	**Pass/Fail**	**improvement**	**Estimation**
4	Sophie	4		9			
5	James	6		10			
6	Alice	8		6			
7	Laura	2		5			
8	Tom	9		6			

Figure 5.4 – The table for Task 5.4

Complete the table according to the following questions:

1. In cell **C4** write a formula that will display “pass” if the result in **B4** is greater or equal to 5 and “fail” if it is not.

Replicate this formula to cells **C5** to **C8.**

2. In cell **E4** write a formula that will display “pass” if the result in **B4** is greater or equal to the value in cell **F1**, and fail if it is not.

Replicate this formula to cells **E5** to **E8**

3. In cell **F4** write a formula that will display “improvement” if the result in **D4** is greater than the result in **B4**, if not, display “worst”

Replicate this formula to cells **F5** to **F8**

4. In cell **G4** write a formula that will display “good” if score 1 and score 2 are greater than or equal to 8 and the total is**17** or more. If it isn't, display “poor”.

Replicate this formula to cells **G5** to **G8**.

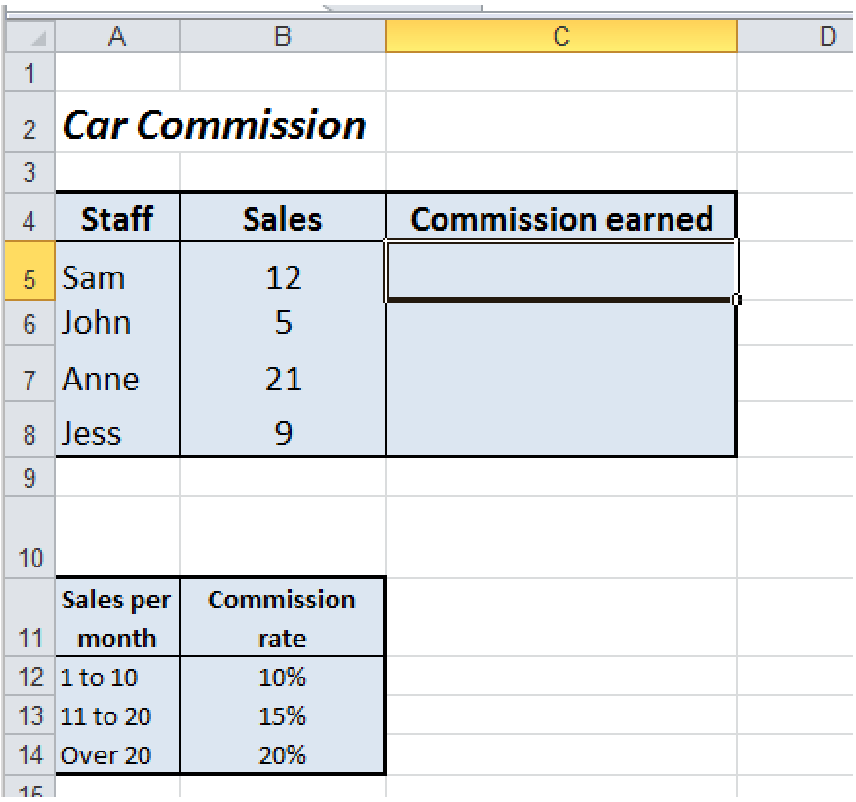
Hint, this is an IF(AND.. formula:

If(AND(first condition; second condition; third condition);"what to display if true"; "what to display if false").

Make sure that you have the same number of closing brackets as opening brackets.

Task 5.5

Create the table presented in Figure 5.5.



Car Commission

Staff	Sales	Commission earned
Sam	12	
John	5	
Anne	21	
Jess	9	

Sales per month	Commission rate
1 to 10	10%
11 to 20	15%
Over 20	20%

Figure 5.5 – The table for Task 5.5

Complete the table according to the following questions:

1. In cell **C5** write a formula that will display the commission rate of the staff according to the second table.

Replicate this formula to cells **C6** to **C8.**

Task 5.6

Create the table presented in Figure 5.6.

	A	B	C	D	E	F	G	H	I
1	**Concerts**								
2									
3–4	**VENUE**	**HALL COST**	**ARTIST COST**	**ADMIN COST**	**TOTAL COST**	**TICKET PRICE**	**SIZE OF HALL**	**MAXIMUM INCOME**	**PROFIT/ LOSS**
5	CARDIFF	1,200.00	2,500.00	1,200.00	4,900.00	10.00	1500	$ 15,000.00	$ 10,100.00
6	BRISTOL	1,600.00	2,000.00	1,050.00	4,650.00	15.00	800	$ 12,000.00	$ 7,350.00
7	EASTBOURNE	1,550.00	2,500.00	450.00	4,500.00	8.00	600	$ 4,800.00	$ 300.00
8	YORK	750.00	2,000.00	890.00	3,640.00	7.50	850	$ 6,375.00	$ 2,735.00
9	LANCASTER	875.00	2,000.00	1,200.00	4,075.00	9.50	1200	$ 11,400.00	$ 7,325.00
10	DUBLIN	950.00	1,500.00	1,600.00	4,050.00	10.00	1375	$ 13,750.00	$ 9,700.00
11									
12									
13									
14–15	Commission rate								

Figure 5.6 – The table for Task 5.6

Complete the table according to the following questions:

1. Insert a **new column** after ticket price. Give it the title, "*Agency Commission"*

2. In cell **B14**, enter 5%

3. In cell **G5**, calculate the commission by multiplying the *'ticket price'* by the *'commission rate'*. (This is an absolute reference. Replicate the formula to the other venues.)

4. Change the *'commission rate'* to 7%. What other columns does this affect?

5. In cell **K3** enter the title *'Book Venue'*
In cell **K5**, write a formula that will display *'book'* if the profit is greater than 5,000. If not, then it should display *'cancel'*

Task 5.7. Tasks for individual work

Do the Tasks listed in Table 5.2.

Table 5.2 – Tasks for creating and calculating tables in Excel

№	Task description
1	2
1	The user enters a person's height and weight. The RELATION column varies depending of the following: If the weight is less than 70, then issue a rating of "NORMAL" If more, depending of te growth. If growth is more than 180 then type "NORMAL" otherwise type "Inconsistency between weight and height"
2	The user enters the PC devices estimations (from 1 to 5) Further, taking these estimations, the whole PC performance is determined according to the following rules: The total score greater and equal to 15, and no estimate of "1" then the final PC performance is "GOOD PC" The total score less than 15, but more then 10 then the final PC performance is "MEDIUM PC" The total sum is less than 10, then the final PC performance is "LOW POWER PC"

Height	Weight	Relation
170	75	
181	75	

Device	**Device estimations 1**	**Device estimations 2**	**Device estimations 3**
Processor	3	3	1
RAM (Random Access Memory)	3	3	1
HDD (Hard Disk Drive)	3	3	2
Video	2	2	2
System bus	2	4	2
Sum			
Total score			

Continuation of the table 5.2

<table>
<tr><td>1</td><td>2</td></tr>
<tr><td>3</td><td>
The user enters two numbers (Number1 and Number2)
Then user enters the assumed average number of these two (Average)
If the input Average is correct, computer wrights "TRUE"
If the average number is more, then "NEED LESS"
If the average number is bigger, then "NEED MORE"
<table>
<tr><th></th><th>Number1</th><th>Number2</th><th>Assumed AVERAGE</th><th>Evaluation of PC</th></tr>
<tr><td>1</td><td>6</td><td>8</td><td>10</td><td>Need more</td></tr>
<tr><td>2</td><td>6</td><td>8</td><td>8</td><td>Need less</td></tr>
<tr><td>3</td><td>6</td><td>8</td><td>7</td><td>True</td></tr>
</table>
</td></tr>
<tr><td>4</td><td>
The car's fuel economy rating is given in the Table.
Provide the proper FUEL ECONOMY depending on the rating.
Calculate the cost of the fuel economy tax based on the list price of the car.
<table>
<tr><th>Title</th><th>List Price</th><th>Code</th><th>Fuel Economy</th><th>Cost of fuel economy tax</th></tr>
<tr><td>Nissan Micra</td><td>6000</td><td>A</td><td></td><td></td></tr>
<tr><td>Ford Fusion</td><td>9000</td><td>B</td><td></td><td></td></tr>
<tr><td>Ford Fiesta</td><td>6000</td><td>A</td><td></td><td></td></tr>
<tr><td>Toyota Lexus</td><td>40000</td><td>C</td><td></td><td></td></tr>
</table>
<table>
<tr><th>Rating</th><th>Fuel Economy</th><th>Tax Rate</th></tr>
<tr><td>A</td><td>Economical</td><td>0,05</td></tr>
<tr><td>B</td><td>Medium</td><td>0,075</td></tr>
<tr><td>C</td><td>Expensive</td><td>0,1</td></tr>
</table>
</td></tr>
<tr><td>5</td><td>
The sales per day are given for each person.
Estimate the Commission earned depending on the Comission rate
<table>
<tr><th>Employee</th><th>Sales per day</th><th>Commission earned</th></tr>
<tr><td>Sam</td><td>30</td><td></td></tr>
<tr><td>John</td><td>12</td><td></td></tr>
<tr><td>Anne</td><td>23</td><td></td></tr>
<tr><td>Jess</td><td>9</td><td></td></tr>
</table>
<table>
<tr><th>Hours per day</th><th>The Comission rate</th></tr>
<tr><td>1 to 10</td><td>0,1</td></tr>
<tr><td>11 to 20</td><td>0,15</td></tr>
<tr><td>Over 20</td><td>0,2</td></tr>
</table>
</td></tr>
</table>

The end of the table 5.2

1	2
6	The working hours per 5-days week week are given for the employees. Estimate the salary coefficient depending on the Coefficient rate

Employee	Working hours per week	Salary coefficient
Sam	40	
John	30	
Anne	32	
Jess	16	

Hours per day	The salary rate
1 to 4	0,5
5 to 7	0,75
Over 7	1

1	2
7	Provide the estimations of the student average scores

If s = 5 then "excellent"
If 4<= s <5 then "good"
If 3 <= s < 4 then "satisfactory"
If 2 <= s < 3 then "poor"
If s > 5 and s < 2 then "Invalid data"

Student	Score for subject				Average score (s)	Estimations
	Math	Russian	Informatics	Physics		
Semenov	5	5	5	5		
Makarov	2	3	3	2		
Ivanov	3	3	3	3		
Petrov	4	4	4	4		
Sidorov	1	1	1	1		

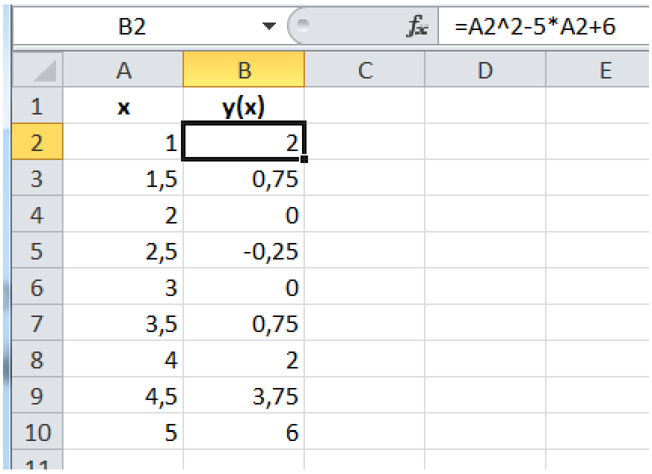
Subject 6. SOLVING THE EQUATIONS USING THE SOLVER ADD-IN APPLICATION

Task 6.1. Solve the quadratic equation:

$$x^2 - 5x + 6 = 0$$

Solution.

1. To estimate the number and approximate values of the equation roots, it is needed to build the graph of function $y(x) = x^2 - 5x + 6$ on the range $1 \le x \le 5$. The data points for $(x, y(x))$values are presented in Figure 6.1.


B2 | f_x | =A2^2-5*A2+6

	A	B	C	D	E
1	x	y(x)			
2	1	2			
3	1,5	0,75			
4	2	0			
5	2,5	-0,25			
6	3	0			
7	3,5	0,75			
8	4	2			
9	4,5	3,75			
10	5	6			

Figure 6.1 – The data points for plotting the graph (Task 6.1)

2. Approximately estimate the values, where $y(x) = 0$. For this function the points are $x_1 = 2$; $x_2 = 3$ (Figure 6.2).

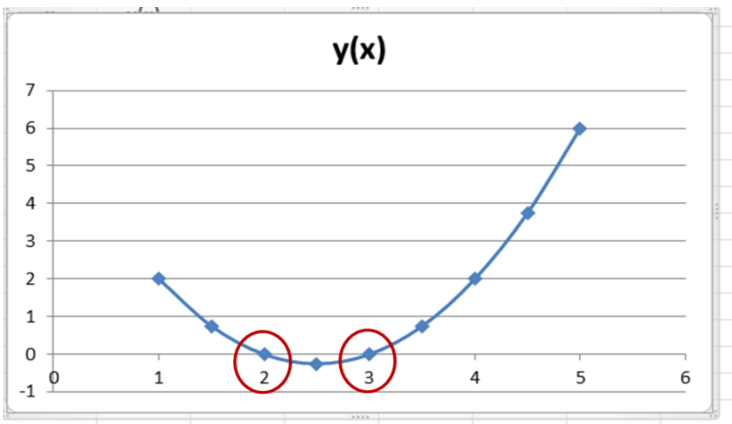


Figure 6.2 – The initial points of the roots

3. Excel will check if these two points are the equation's roots or it will find the exact values. For the automatic solving firstly it is needed to prepare two cells **A21** and **B21**:

– **A21** is the *decision variable* cell;

here we type in the initial guess value (= 1) of the equation's root.

– **B21** is *objective* cell;

type the formula $= A21^2 - 5 * A21 + 6$

The number in **B21** is the value of this function at $x = 1$ (the value in cell **A21**).

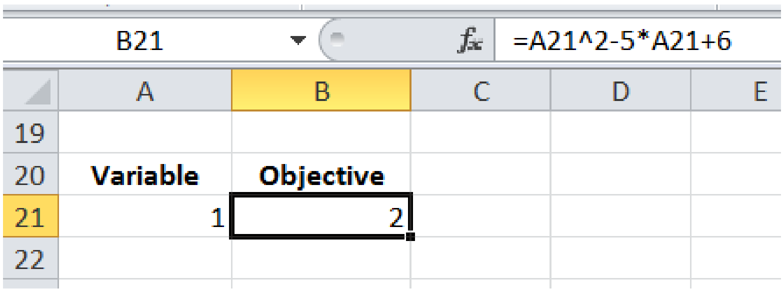


Figure 6.3 – Preparing the cells with initial variable and objective values

4. Now it is needed to determine the value of A21 cell under which the function in cell B21 will be equal to 0 using Solver. On the **Data tab**, in the **Analysis group**, click **Solver** (Figure 6.4).

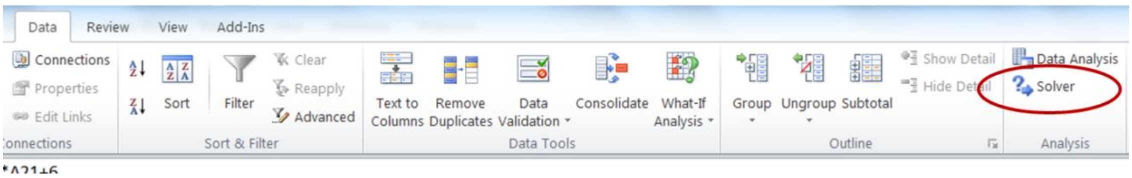


Figure 6.4 – Preparing the cells with initial variable and objective values

The **Solver Parameters** dialog box will appear. Set the following parameters (Figure 6.5):

– The set objective – the absolute reference to the objective cell **B21**;

– By changing variable cell – the absolute reference to the decision variable cell **A21**;

– To – choose the value of equal to **zero**.

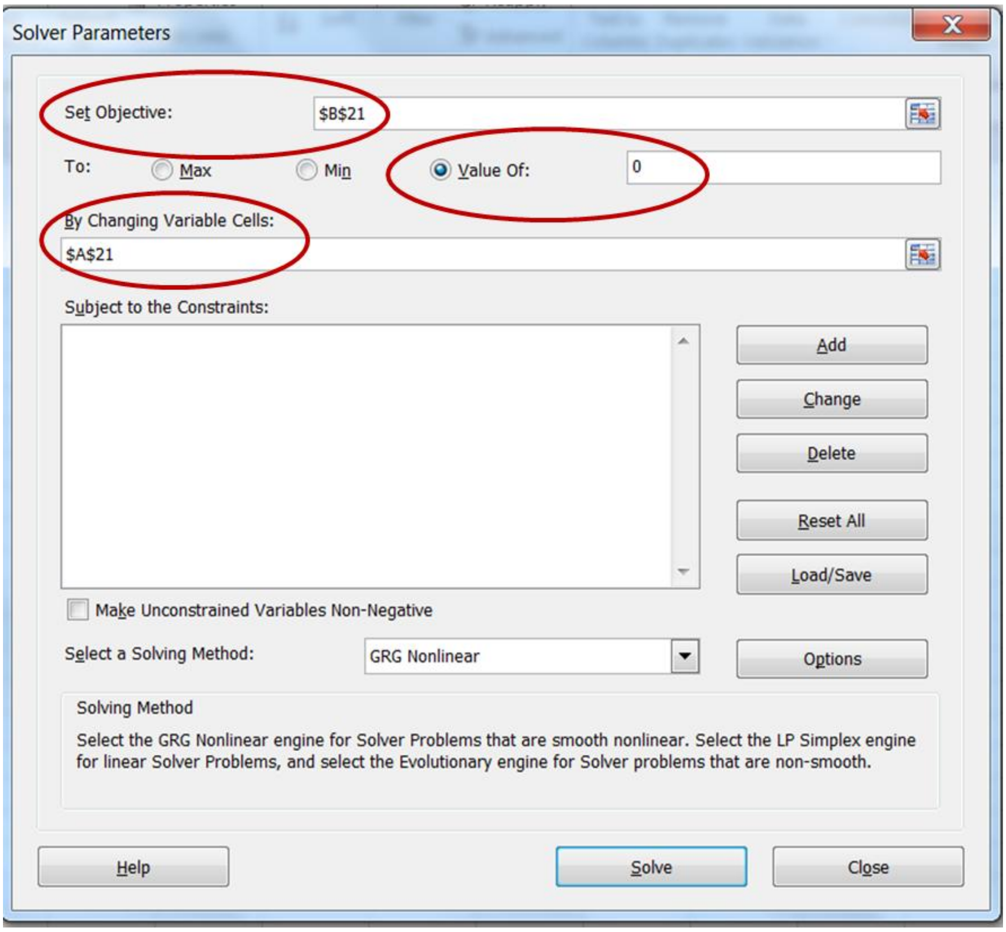


Figure 6.5 – The Solver Parameters dialog box

– Select a solving method – choose GRG Nonlinear Solving Method as we have nonlinear function and press Options button. Change the values in the Options dialog box. And press OK button. The more detailed description of Options dialog box is given in **Chapter 6.3**.

– You will return to the **Solver Parameters** dialog box. Press **Solve** button. The solver results window will appear (Figure 6.6c). Choose **Keep solver Solution** and press OK.

– The results of the solving are presented in Figure 6.6. The Solver add-in found one root of the quadratic equation$x^2 - 5x + 6 = 0$, which is in **A21** cell and equal $x_1 = 2.00000059$.

5. The obtained result is only one of the equation's roots. Do the same actions (step 4) for the cells **A22** (typing (= 4) as initial value) as decision variable cell and for the objective cell – cell **B22**. After solving you will get the result presented in Figure 6.6. The other root of the

quadratic equation $x^2 - 5x + 6 = 0$, which is in **A22** cell and equal $x_2 = 2.99999917$.

6. *The answer:*

The quadratic equation $x^2 - 5x + 6 = 0$ has two roots: $x_1 = 2.00000059$ and $x_2 = 2.99999917$.

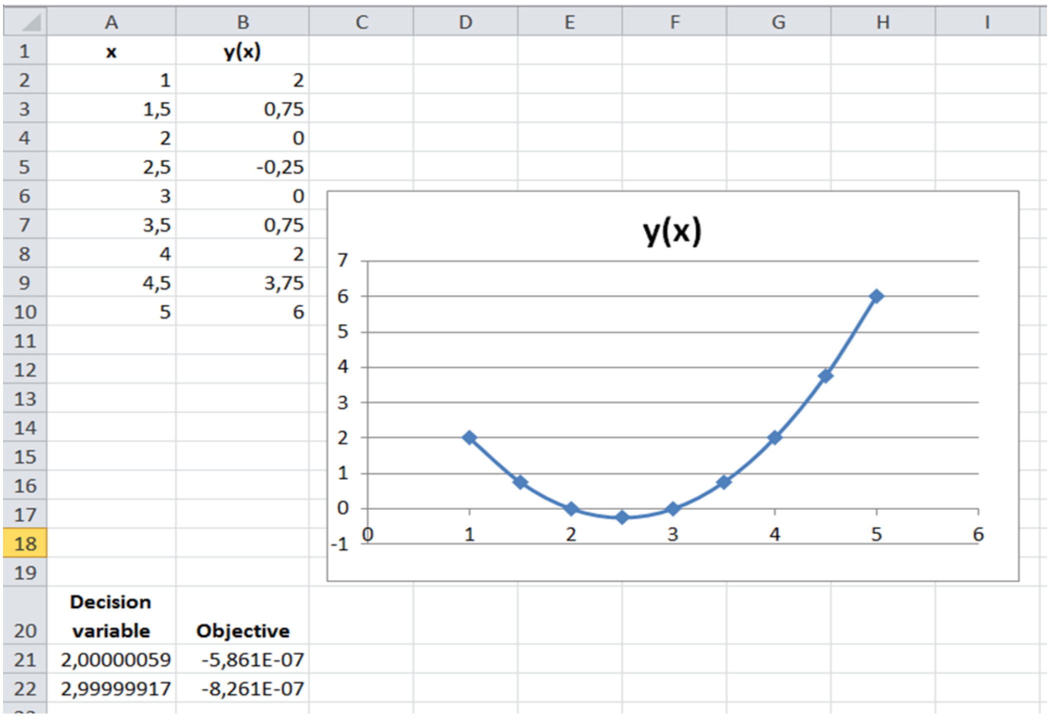


Figure 6.6 – The found values of equation roots

Task 6.2. Tasks for laboratory and individual work

Find the roots of the equations listed in Table 6.1.

Table 6.1 – Variants for calculation

№	Equation	The range of variables
1	2	3
1	$x^3 - 12x - 4 = 0$	$-5 \le x \le 7$
2	$x^3 - 24x + 11 = 0$	$-4 \le x \le 7$
3	$x^3 + 2x - 7 = 0$	$-5 \le x \le 5$
4	$x^3 - 21x + 7 = 0$	$-10 \le x \le 10$

The end of the table 6.1

1	2	3
5	$x^3 - 51x + 1 = 0$	$-10 \le x \le 10$
6	$x^3 + 3x^2 - 14x - 1 = 0$	$-7 \le x \le 7$
7	$x^3 - 9x^2 + 20x - 11 = 0$	$-3 \le x \le 4$
8	$x^3 - 12x + 5 = 0$	$-3 \le x \le 4$
9	$x^3 - 27x - 17 = 0$	$-3 \le x \le 10$
10	$x^3 + 6x^2 + 6x - 7 = 0$	$-5 \le x \le 5$
11	$x^3 - 9x^2 + 5x + 30 = 0$	$-5 \le x \le 10$
12	$x^3 - 3x^2 - x + 2 = 0$	$0 \le x \le 5$
13	$x^3 - 10x^2 + 29 = 0$	$-10 < x < 10$
14	$x^3 + 3x^2 - 3 = 0$	$-3 \le x \le 3$
15	$x^3 + 2x^2 - 15x + 8 = 0$	$-3 \le x \le 3$
16	$x^3 - 3x^2 - 4x + 1 = 0$	$-3 \le x \le 3$
17	$x^3 + 32x^2 - 7x - 45 = 0$	$-5 \le x \le 5$
18	$x^3 + 5x^2 + 2x - 7 = 0$	$-5 \le x \le 5$
19	$x^3 - 4x^2 - 3x + 7 = 0$	$-5 \le x \le 5$
20	$x^3 - 2x^2 - 6x + 3 = 0$	$-5 \le x \le 5$
21	$x \cdot \sin(x) - 1 = 0$	$-5 \le x \le 5$
22	$8\cos(x) - x = 6$	$-5 \le x \le 5$

6.3. Changing the Solver parameters

To change Solver options that can affect the nature and accuracy of the solution, or the time Solver will spend seeking a solution, click **Options** in the **Solver Parameters** dialog. To clear all cell selections and reset all Solver options to their default values, click *Reset All*.

Change **Options** for all Solving Methods (Figure 6.7):

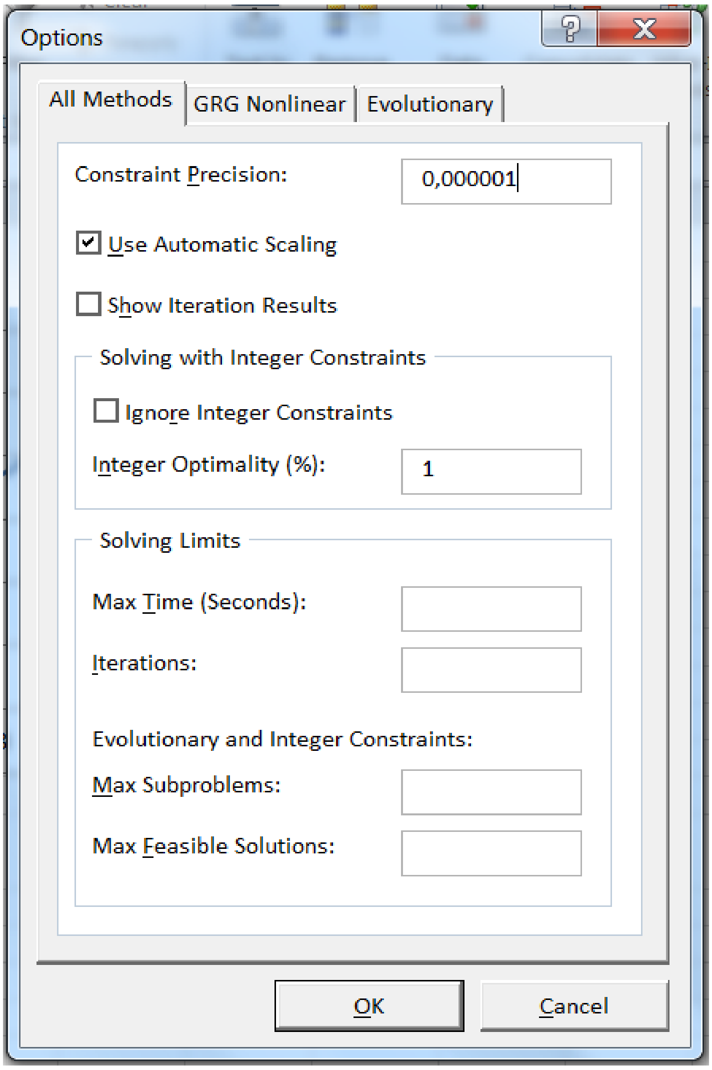


Figure 6.7 – The All Methods tab in Solver Options dialog box

1. **Constraint precision** /In the *Constraint Precision* box, type the degree of precision that you want. For a constraint to be considered satisfied, the relationship between the Cell Reference and the Constraint value cannot be violated by more than this amount. The smaller the number, the higher the precision.

2. **Use Automatic Scaling** /Select the *Use Automatic Scaling* check box to specify that Solver should internally rescale the values of variables, constraints and the objective to similar magnitudes, to reduce the impact of extremely large or small values on the accuracy of the solution process.

3. **Show Iteration Results** /Select the *Show Iteration Results* check box to see the values of each trial solution.

4. **Solving with Integer Constraints** /Select the Ignore Integer Constraints check box to cause all integer, binary and all different constraints to be ignored when you next click Solve. /In the Integer Optimality % box, type the maximum percentage difference Solver should accept between the objective value of the best integer solution found and the best known bound on the true optimal objective value before stopping.

5. **Solving Limits**.

– In the **Max Time** (Seconds) box, type the number of seconds that you want to allow Solver to run.

– In the **Iterations** box, type the maximum number of iterations that you want to allow Solver to perform.

The following limits apply only to problems that include integer restrictions on variables, or problems that use the **Evolutionary Solving Method**:

– In the *Max Subproblems* box, type the maximum number of subproblems that you want to allow.

– In the *Max Feasible Solutions* box, type the maximum number of feasible solutions that you want to allow. For problems with integer restrictions, this is the maximum number of integer feasible solutions.

To change Options for **GRG Nonlinear Solving Method** (Figure 6.8):

1. **Convergence**. In the Convergence box, type the amount of relative change that you want to allow in the last five iterations before Solver stops with the message "Solver converged to the current solution." Smaller values here usually mean that Solver will take more time, but will stop at a point closer to the optimal solution.

2. **Derivatives**. In the Derivatives group box, select Forward to estimate derivatives through forward differencing, or select Central to estimate derivatives through central differencing. Forward is the default choice. Central differencing yields more accurate derivatives, but requires twice as many calculations of the worksheet at each new trial solution.

3. **MultiStart Options** for Global Optimization:

a) Select the **Use Multistart** check box to use the multistart method for global optimization. If this box is selected when you click Solve, the GRG Nonlinear method will be run repeatedly, starting from different (automatically chosen) starting values for the decision variables. This process may find a better solution, but it will take more computing time than a single run of the GRG Nonlinear method.

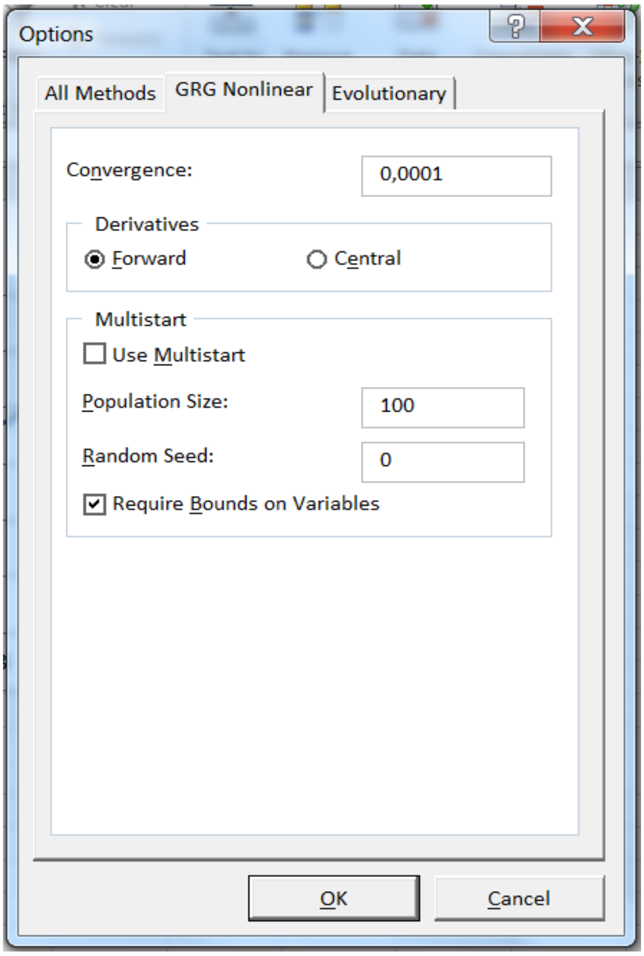


Figure 6.8 – GRG Nonlinear Solving Method in Solver Options dialog box

b) In the **Population Size** box, type the number of different starting points (values for the decision variables) you want the multistart method to consider. The minimum population size is 10; if you supply a value less than 10 in this box, or leave it blank, the multistart method uses a population size of 10 times the number of decision variables, but no more than 200.

c) In the **Random Seed** box, type a positive integer number to be used as the (fixed) seed for the random number generator used to generate candidate starting points for the GRG Nonlinear method. If you enter a number here, the multistart method will use the same starting points each time you click Solve. If you leave this box blank, the random number generator will use a different seed each time you click Solve, which may yield a different (better or worse) final solution.

d) Select the **Require Bounds** on **Variables** check box to specify that the multistart method should run only if you have defined lower and upper bounds on all decision variables in the Constraints list box. The multistart method is far more effective if you define bounds on all variables; the tighter the bounds on the variables that you can specify, the better the multistart method is likely to perform.

Change Options for **Evolutionary Solving Method**(Figure 6.9):

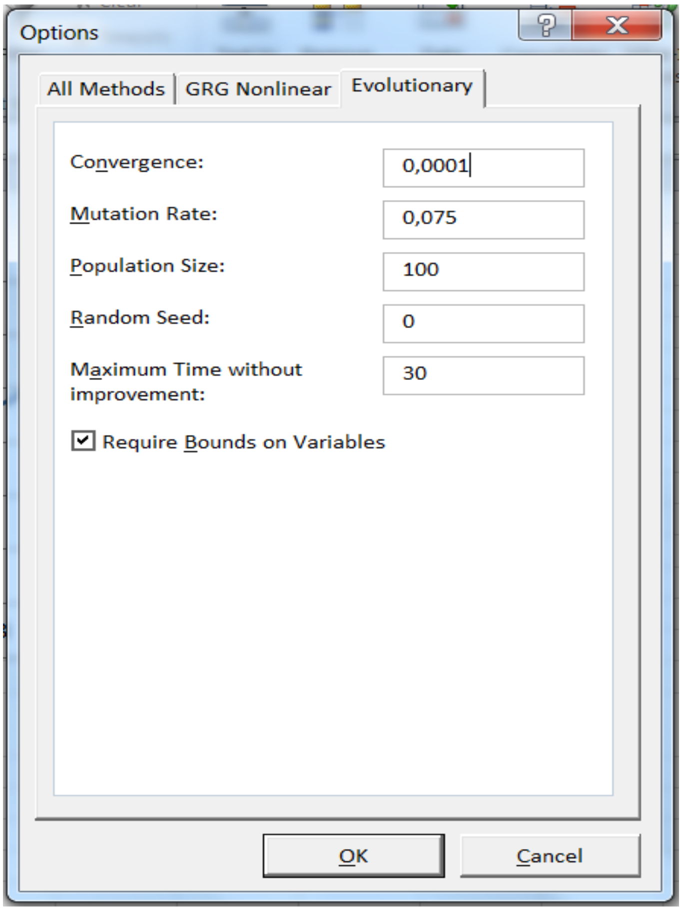


Figure 6.9 – Change Options for Evolutionary Solving Method

1. **Convergence**. In the **Convergence** box, type the maximum percentage difference in objective values for the top 99% of the population that Solver should allow in order to stop with the message "Solver converged to the current solution." Smaller values here normally mean that

Solver will take more time, but will stop at a point closer to the optimal solution.

2. **Mutation Rate**. In the **Mutation Rate** box, type a number between 0 and 1, the relative frequency with which some member of the population will be altered or "mutated" to create a new trial solution, during each "generation" or subproblem considered by the **Evolutionary method**. A higher **Mutation Rate** increases the diversity of the population and the chance that a new, better solution will be found; but this may increase total solution time.

3. **Population Size**. In the **Population Size** box, type the number of different points (values for the decision variables) you want the Evolutionary method to maintain at any given time in its population of candidate solutions. The minimum population size is 10 members; if you supply a value less than 10 in this box, or leave it blank, the Evolutionary Solver uses a population size of 10 times the number of decision variables in the problem, but no more than 200.

4. **Random Seed**. In the **Random Seed** box, type a positive integer number to be used as the (fixed) seed for the random number generator used for a variety of random choices in the Evolutionary method. If you enter a number here, the Evolutionary method will use the same choices each time you click Solve. If you leave this box blank, the random number generator will use a different seed each time you click Solve, which may yield a different (better or worse) final solution.

5. **Maximum Time without Improvement**. In the Maximum Time without Improvement box, type the maximum number of seconds you want the Evolutionary method to continue without a meaningful improvement in the objective value of the best solution in the population, before it stops with the message "Solver cannot improve the current solution."

6. **Require Bounds on Variables**. Select the Require Bounds on Variables check box to specify that the Evolutionary method should run only if you have defined lower and upper bounds on all decision variables in the Constraints list box. The Evolutionary method is far more effective if you define bounds on all variables; the tighter the bounds on the variables that you can specify, the better the Evolutionary method is likely to perform.

Subject 7. FINDING THE EXTREMES OF FUNCTION USING EXCEL SOLVER ADD-IN APPLICATION

Task 7.1

Find the minimum and maximum values of the following function

$$y = \frac{1 - x + x^2}{1 + x - x^2},$$

In the range[0 ... 1].

Solution.

1. Build the graph of unction$y(x) = (1 - x + x^2)/(1 + x - x^2)$.

In cells **A2:A12** enter x values from 0to 1, take the increment $\Delta = 0.1$. In cells **B2:B12** enter the function $y(x) = (1 - x + x^2)/(1 + x - x^2)$, referencing to the cell with corresponding x value. Select the cells with values, then on the **Insert** tab, in **Chart** group select *Scatter with Smooth Lines and Markers*. The graph, presented in Figure 7.1 will appear.

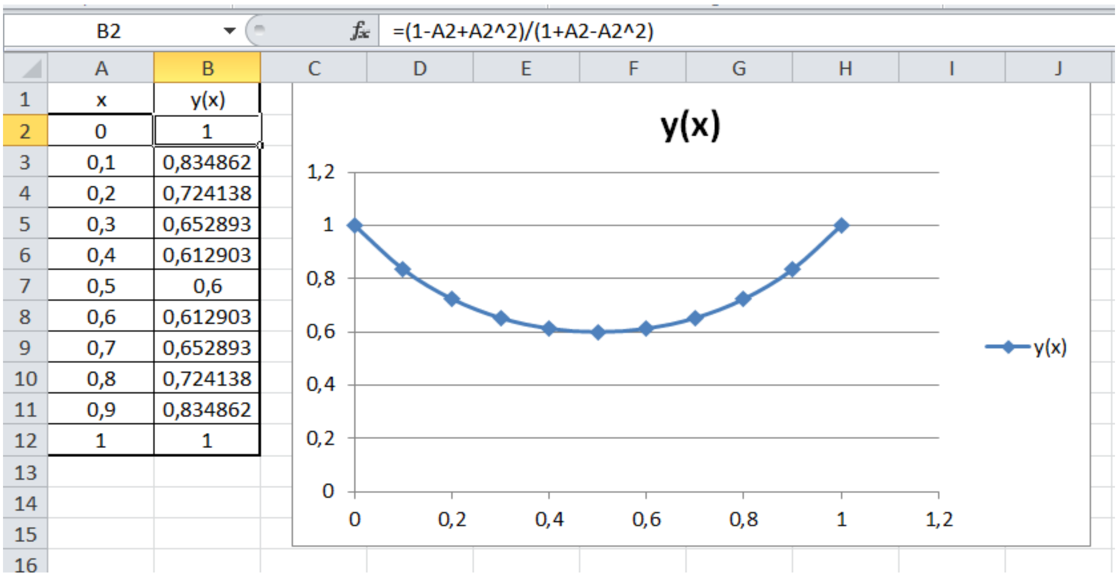


Figure 7.1 – The graph for Task 7.1

2. Make the analysis of the graph.

The obtained function on the investigated range has global minimum approximately at point $= 0.4$. It will be the initial value for finding the minimum value of the function.

To estimate the proper minimum value in point (x, y) the Solver add-in application can be used.

3. Preparation of the cells for using Solver application (Figure 7.2).

The cell **A18**will be the *variable* cell. Type the initial approximation in it " = 0.4"

The cell **B18** will be the *objective* cell. Type here the equation with the observed function, with the reference to x value in **A18** cell:

$" = (1 - A18 + A18\text{^}2)/(1 + A18 - A18\text{^}2)".$

As the value in**A18** is equal to 0.4, the value in **B18** cell will be equal to 0.6129.

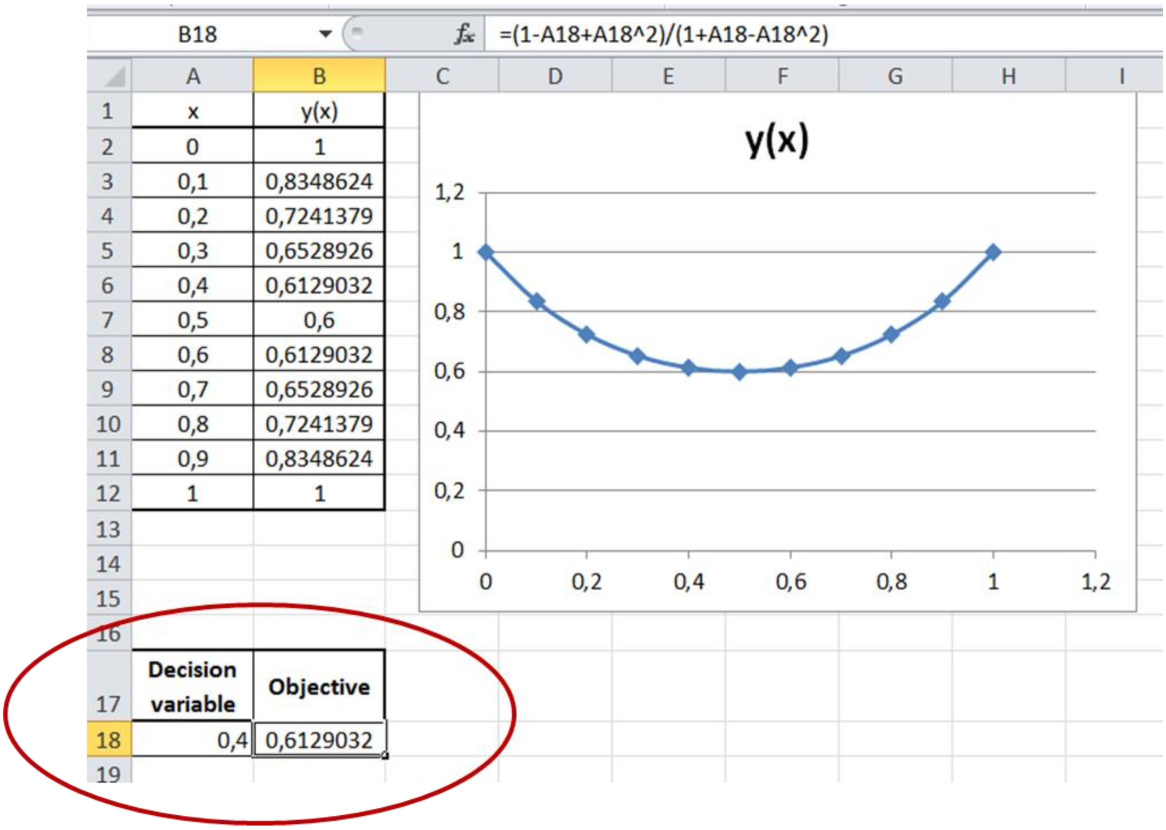


Figure 7.2 – Initial values for finding minimum with Solver add-in application

4. Using **Solver** for estimation of function’s minimal value.

At this step it is needed to determine the value of **A18** cell under which the function in cell **B18** will take the minimal value. Use **Solver** add-in application.

On the **Data** tab, in the **Analysis** group, click **Solver**. (Figure 7.3).

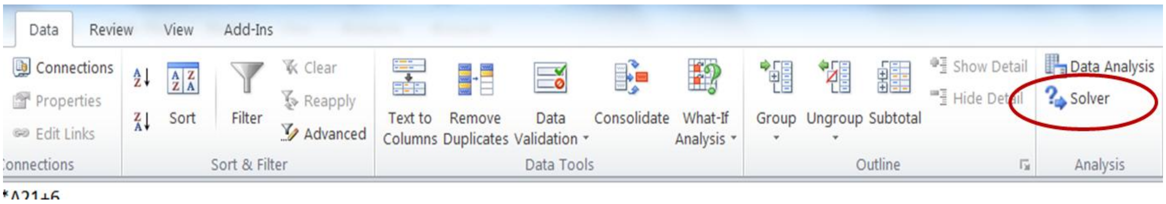


Figure 7.3 – Solver add-in application

5. Configure the parameters of **Solver**.

After selecting Solver, the *Solver Parameters* dialog box will appear. Set the following parameters (Figure 7.4):

1) The set objective – the absolute reference to the objective cell **B18**;
2) By changing variable cell – the absolute reference to the decision variable cell **A18**;
3) To – choose the **Min** value.
4) Add Subject to the Constraints.

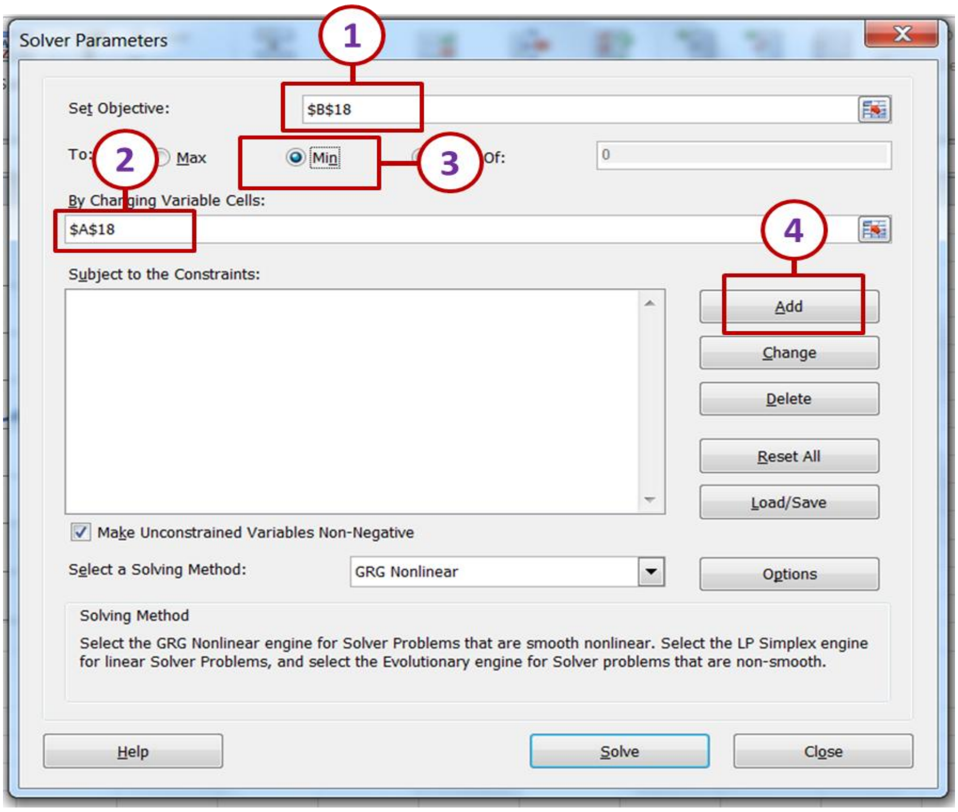


Figure 7.4 – The Solver Parameters dialog box

6. Add the constraints (Figure 7.5).

In Task 7.1 the variable is changing on the range from 0 to 1, and the argument of the function should be limited by this range. So, we need to specify, that the value in variable cell should be greater or equal to 0 and less or equal to 1.

To add the constrains press **Add** button in *Solver Parameters* dialog box. The *Add Constrains* dialog box will appear. Enter the following:

1) Add the constraint, that$x \geq 0$(Figure 7.5*a*).

In cell reference point the cell with x value. It is **A18** cell.

Select the proper sign“>=”. In Constraint cell enter the value “0”.

Then push Add button for adding the constraint.

2) Add the constraint, that$x \leq 1$(Figure 7.5*b*).

In cell reference point the cell with x value. It is **A18** cell.

Select the proper sign“<=”. In Constraint cell enter the value “1”.

Then push Add button for adding the constraint.

a

b

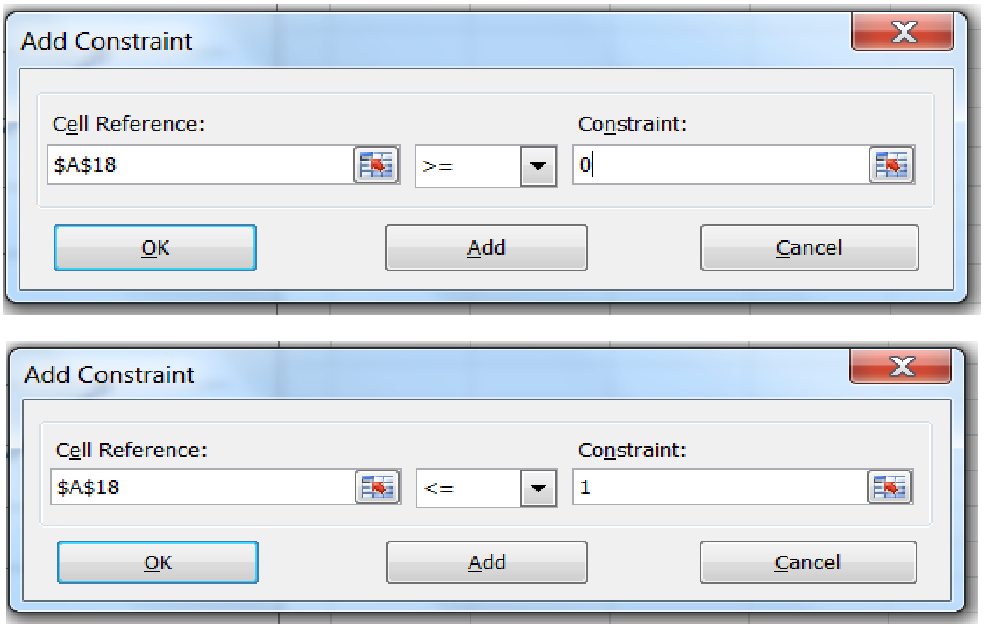


Figure 7.5 – The Add Constrains dialog box

7. The *Solver Parameter* dialog box will appear with the settled constrains, which are indicated in the box “*Subject of the Constraints*” (Figure 7.6).

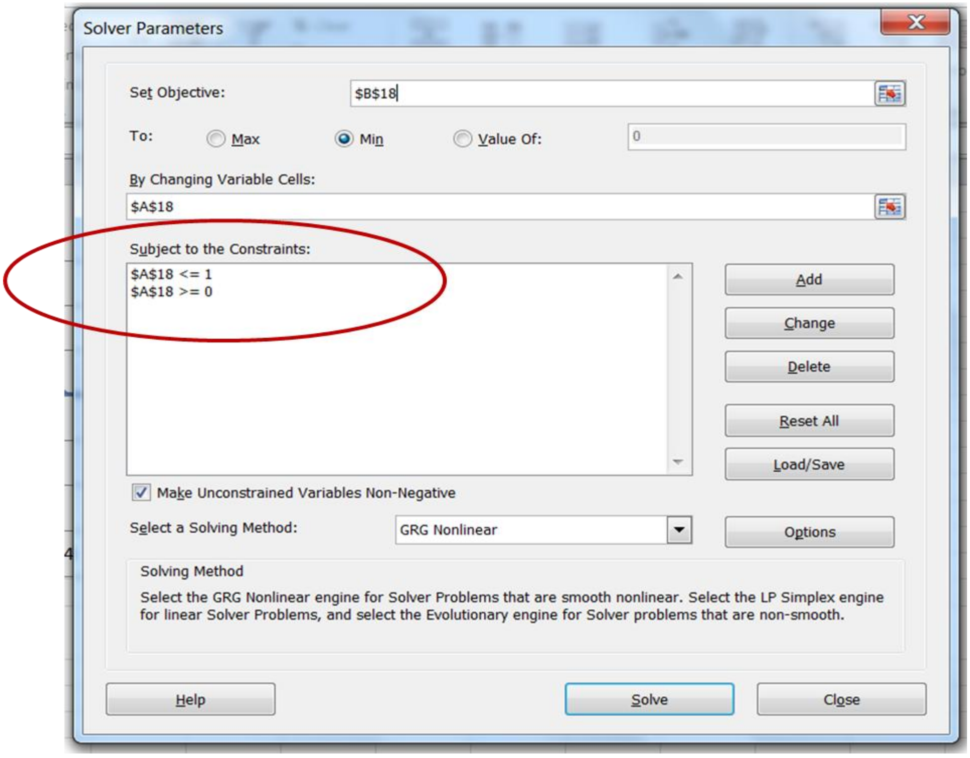


Figure 7.6 – The Solver Parameters dialog box with Subject of Constraints

8. Select the solving method and specify its options. The *Options* dialog box will appear (Figure 7.7).After specifying the options press **OK**. The *Solver Parameters* dialog box will appear. Press **Solve** button.

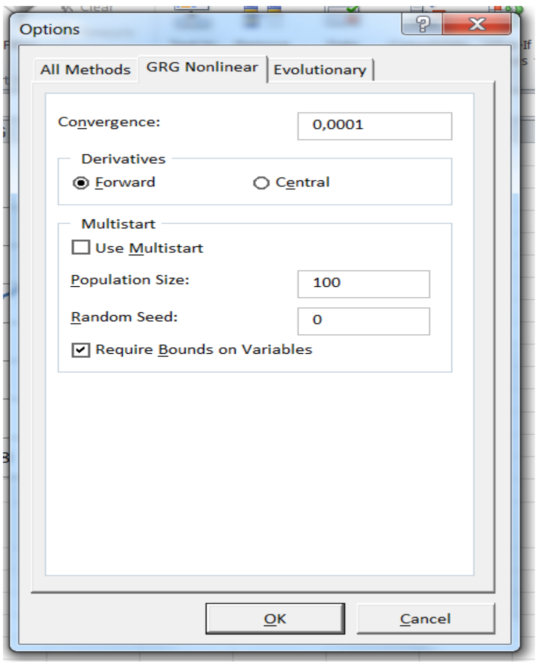


Figure 7.6 – The Solver Parameters dialog box with Subject of Constraints

9. The *Solver Result* dialog box will be on the screen. Choose "*Keep solver solution*" and press **OK** button.

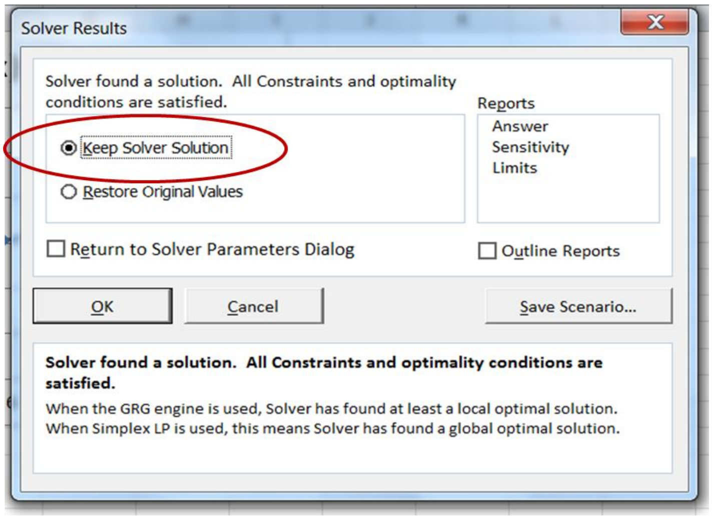


Figure 7.7 – The Solver Results dialog box

10. The solution will appear in the variable and objective cells (Figure 7.8).

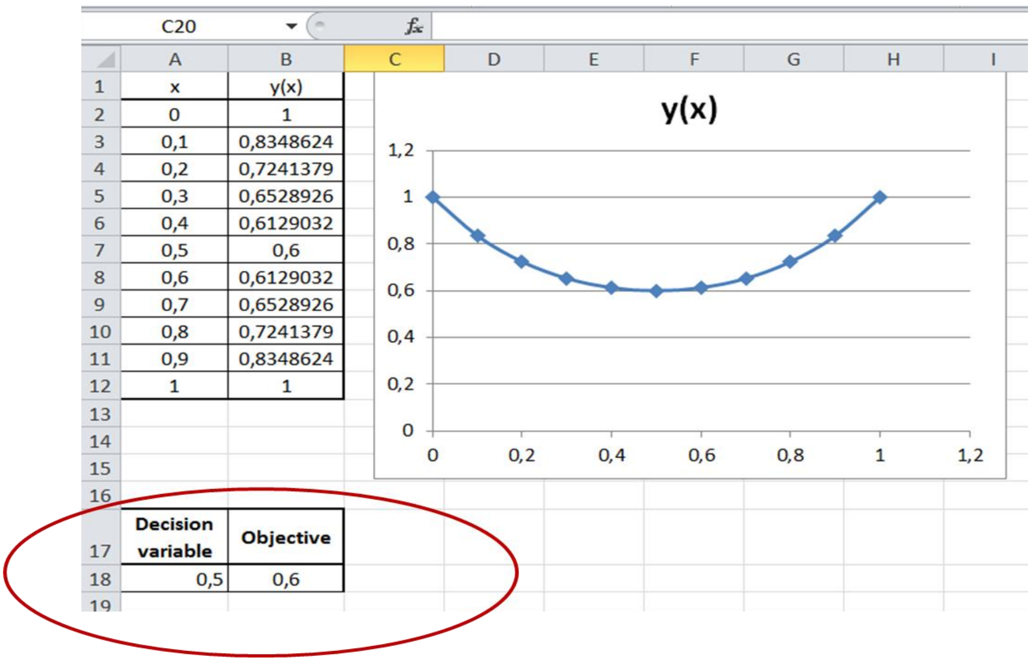


Figure 7.8 – The result of finding the minimum of function using Solver

Answer: the function $y(x) = (1 - x + x^2)/(1 + x - x^2)$ on the range $[0 \ldots 1]$ has minimum in point $(0.5; 0.6)$.

Task 7.2

Find the minimum and maximum values of the following function

$$y = -3x^4 + 6x^2,$$

In the range$[-2 \dots 2]$.

Solution.

1. Build the graph of function$y(x) = -3 \cdot x^4 + 6 \cdot x^2$ on the range $[-2 \dots 2]$ (Figure 7.9). The increment was taken $\Delta = 0.4$.

B2 — f_x =-3*A2^4+6*A2^2

	A	B
1	x	y(x)
2	-2	-24
3	-1,6	-4,3008
4	-1,2	2,4192
5	-0,8	2,6112
6	-0,4	0,8832
7	0	0
8	0,4	0,8832
9	0,8	2,6112
10	1,2	2,4192
11	1,6	-4,3008
12	2	-24

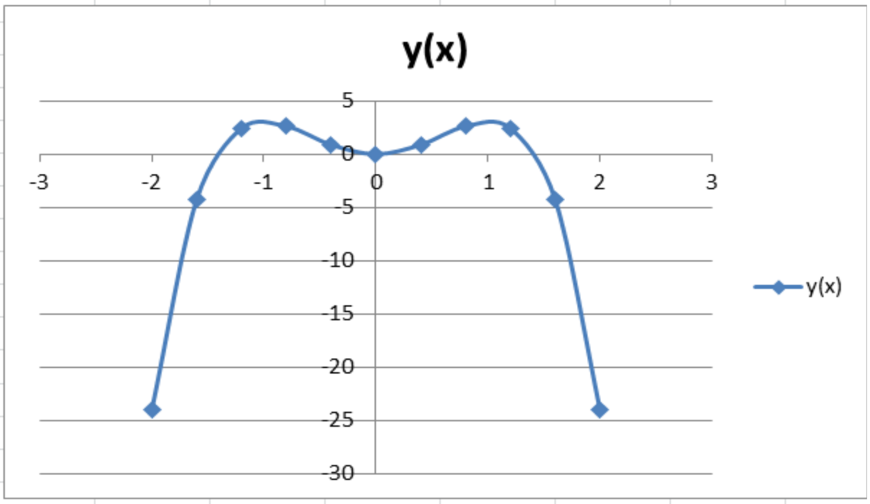


Figure 7.9 – The graph for Task 7.2

2. Make the analysis of the graph.

The function on the given range has two local maximums and one local minimum. The approximate points are:

1st maximum at $x = 0.5$

2nd maximum at $x = -0.5$

minimum at $x = 0$

It will be the initial value for finding the proper value.

To estimate the proper minimum values, three points:(x_1, y_1)-1st maximum; (x_2, y_2) – 2nd maximum; (x_3, y_3) – minimum should be found using the Solver add-in application.

Firstly, let's find the 1st maximum at $x = 0.5$.

3. Preparation of the cells for using Solver application (Figure 7.10).

The cell **A19**will be the *variable* cell. Type the initial approximation in it " = 0.5"

The cell **B19** will be the *objective* cell. Type here the equation with the observed function, with the reference to x value in **A19** cell:

$" = -3 * A2\text{^}4 + 6 * A2\text{^}2"$.

As the value in**A19** is equal to 0.5, the value in **B19** cell will be equal to 1.312.

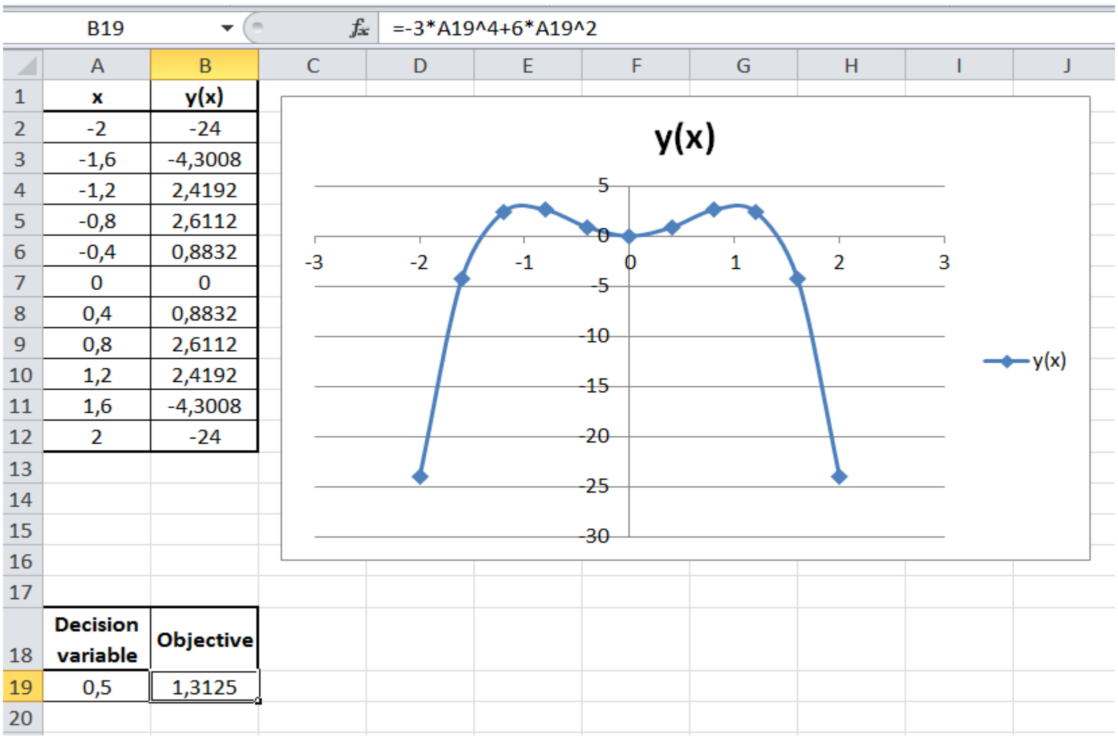


Figure 7.10 – Initial values for finding maximum with Solver add-in application

4. Use **Solver** add-in application to estimate the exact value of the argument and function in maximum point.

At this step it is needed to determine the value of **A19** cell under which the function in cell **B19** will take the maximal value. Use **Solver** add-in application.

On the **Data** tab, in the **Analysis** group, click **Solver**. (Figure 7.3).

5. Configure the parameters of **Solver**.

After selecting Solver, the *Solver Parameters* dialog box will appear. Set the following parameters (Figure 7.11):

1) The set objective – the absolute reference to the objective cell **B19**;
2) By changing variable cell – the absolute reference to the decision variable cell **A19**;
3) To – choose the **Max** value.
4) Add Subject to the Constraints.

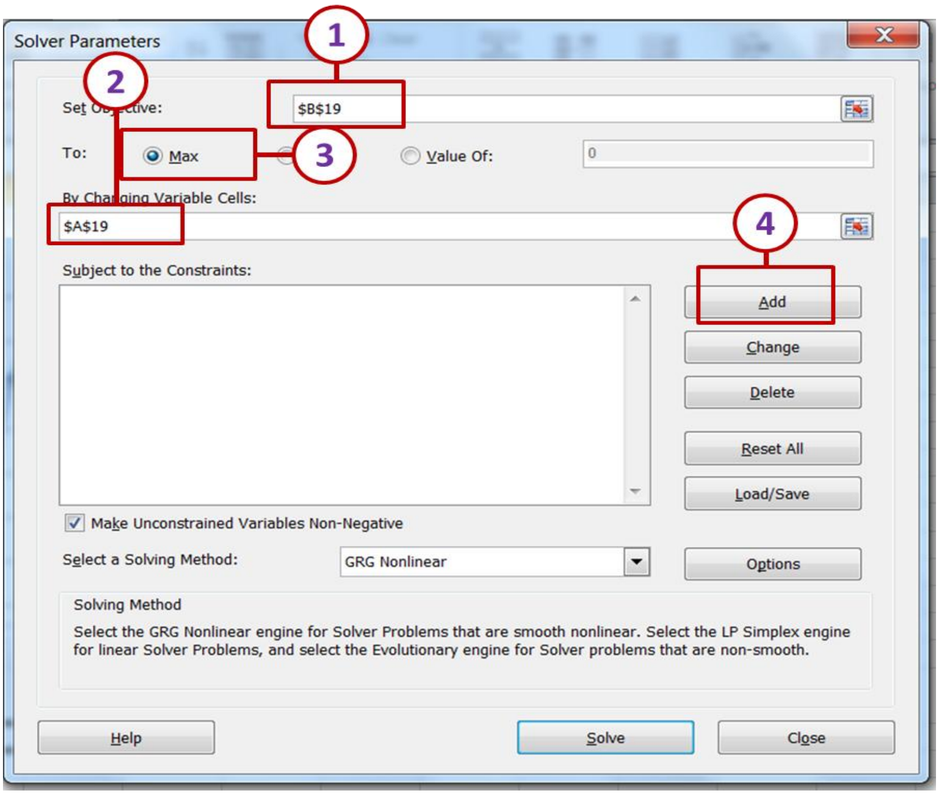


Figure 7.11 – The Solver Parameters dialog box

6. Add the constraints (Figure 7.12).

In Task 7.2 the variable is changing on the range $[-2 \dots 2]$, and the argument of the function should be limited by this range. So, we need to specify, that the value in variable cell should be greater or equal to -2 and less or equal to 2.

To add the constrains press **Add** button in *Solver Parameters* dialog box. The *Add Constrains* dialog box will appear. Enter the following:

1) Add the constraint, that$x \geq -2$(Figure 7.12*a*).

In cell reference point the cell with x value. It is **A19** cell.

Select the proper sign“>=”. In Constraint cell enter the value “-2”.

Then push Add button for adding the constraint.

2) Add the constraint, that $x \leq 2$ (Figure 7.12*b*).
In cell reference point the cell with x value. It is **A19** cell.
Select the proper sign "<=". In Constraint cell enter the value "2".
Then push Add button for adding the constraint.

a

b

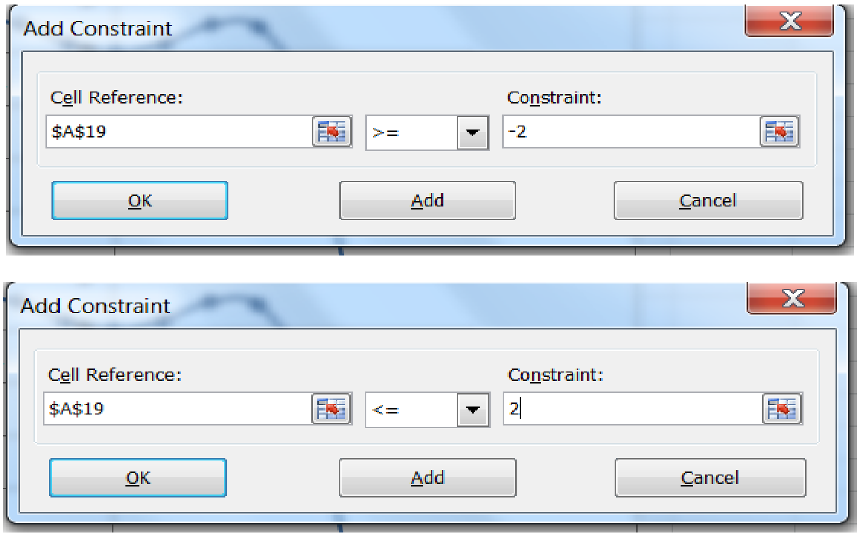


Figure 7.12 – The Add Constrains dialog box

7. The *Solver Parameter* dialog box will appear with the settled constrains, which are indicated in the box "*Subject of the Constraints*" (Figure 7.13).

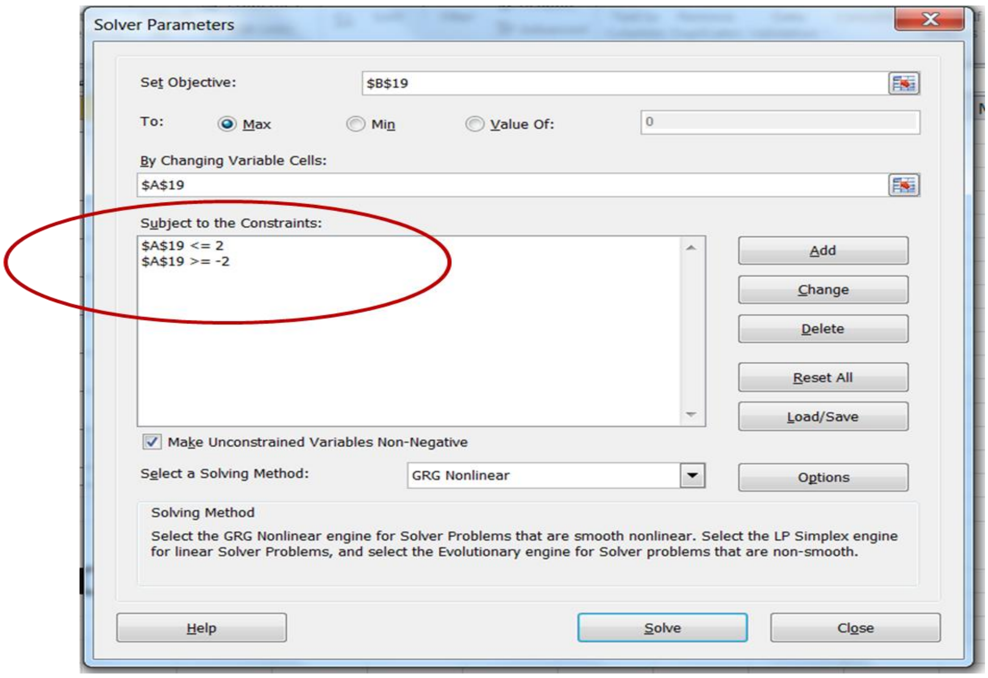


Figure 7.13 – The Solver Parameters dialog box with constraints

8. Select the solving method and specify its options. The *Options* dialog box will appear (Figure 7.7). After specifying the options press **OK**. The *Solver Parameters* dialog box will appear. Press **Solve** button.

9. The *Solver Result* dialog box will be on the screen. Choose "*Keep solver solution*" and press **OK** button.

10. The exact value of function local maximum point will appear in the variable and objective cells (Figure 7.14).

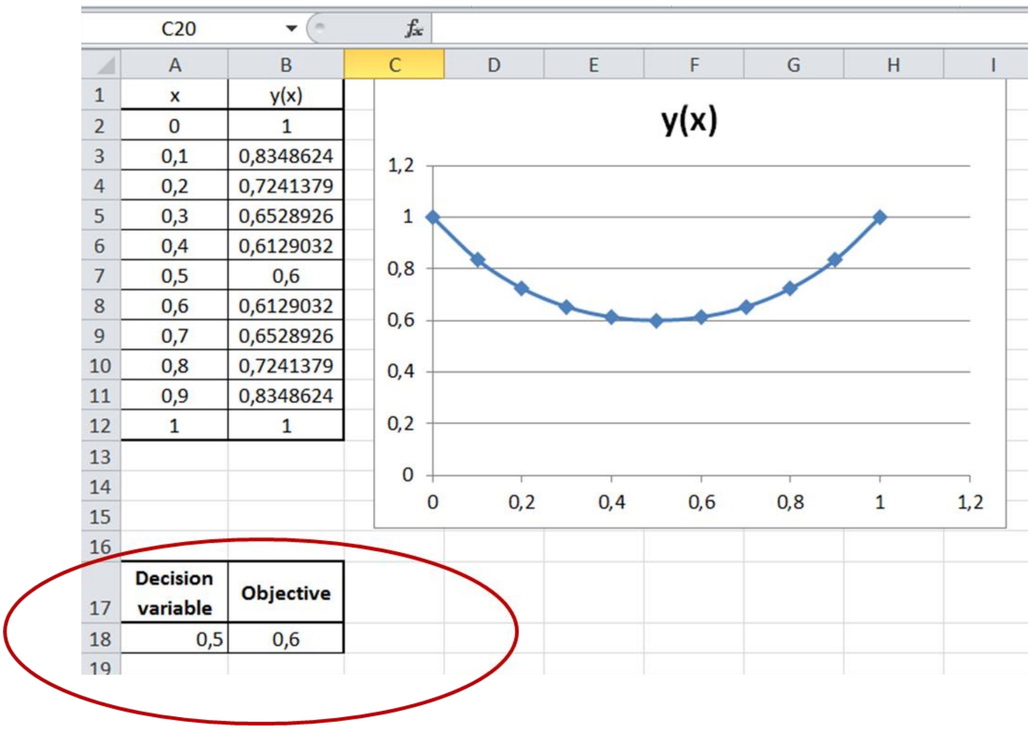


Figure 7.14 – The 1st maximum point obtained using Solver

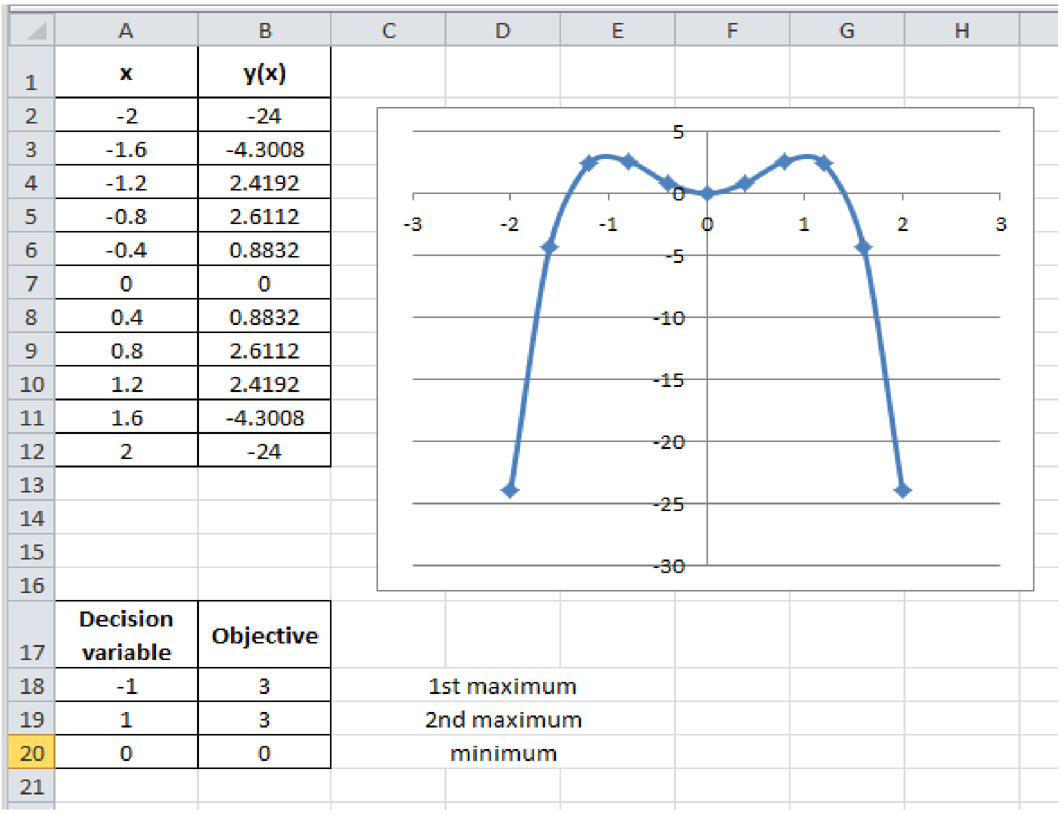
11. Find the 2nd maximum at $x = -0.5$ by repeating the steps 3-10.

The cell **A20** use as the variable cell; the cell **B20** use as the objective cell. The function and constraints are the same.

12. Find the minimum at $x = 0$ by repeating the steps 3-10.

The cell **A21** use as the variable cell; the cell **B21** use as the objective cell. The function and constraints are the same.

13. The solution is presented in Figure 7.15.



	A	B	C	D
1	x	y(x)		
2	-2	-24		
3	-1.6	-4.3008		
4	-1.2	2.4192		
5	-0.8	2.6112		
6	-0.4	0.8832		
7	0	0		
8	0.4	0.8832		
9	0.8	2.6112		
10	1.2	2.4192		
11	1.6	-4.3008		
12	2	-24		
13				
14				
15				
16				
17	Decision variable	Objective		
18	-1	3		1st maximum
19	1	3		2nd maximum
20	0	0		minimum
21				

Figure 7.15– The solution of Task 7.2

Answer: the function$y(x) = -3 \cdot x^4 + 6 \cdot x^2$ on the range $[-2 \ldots 2]$ has two local maximums in points $(-1; 3)$ and $(1; 3)$ and one local minimum in point $(0; 0)$.

Task 7.3. Tasks for laboratory and individual work

Find local minimum and maximums of the functions listed in Table 7.1, column (2), on the given range, column (3). For drawing the graph of function use the increment presented in column (4).

Table 7.1 - Variants for calculation

№	**Function**	**The range of variables**	**Increment**
1	2	3	4
1	$F(x) = 5^{-x} - x^{\sin(x)} + 7$	$1 \le x \le 15$	$h = 1$
2	$P(x) = 2^{-x} - 0.5x^{2\cos(x)} + 5$	$0 \le x \le 10$	$h = 0.5$

The end of the table 7.1

1	2	3	4
3	$W(x) = x + 2 \cdot \mathrm{e}^{x \cdot \cos(x)}$	$-5 \le x \le 5$	$h = 0.5$
4	$Q(x) = x + 2 \cdot e^{x \cdot \sin(x)}$	$-5 \le x \le 5$	$h = 0.5$
5	$Y(x) = 2 - \ln(x)^2 + 3\cos(3x)$	$1 \le x \le 5$	$h = 0.2$
6	$G(x) = (\cos(x)^3 + 0.5) / (x + \cos(x))$	$1 \le x \le 7$	$h = 0.5$
7	$M(x) = (4x + 7)^{1/3} - 3\cos(x) - 4$	$1 \le x \le 10$	$h = 0.5$
8	$Q(x) = 1 - \lg(x)^{\sin(x)}$	$1 \le x \le 10$	$h = 0.5$
9	$C(x) = 1 - \ln(x)^2 \cdot \cos(x)^2$	$1 \le x \le 10$	$h = 0.5$
10	$y = x^2 + 45 \cdot \sin(x)$	$-10 \le x \le 10$	$h = 1$
11	$R(x) = 1 - arctg(x)^2 \cdot \cos(x)$	$1 \le x \le 10$	$h = 1$
12	$K(x) = 2 - \ln(x)^2 \cdot \sin(x)$	$1 \le x \le 10$	$h = 1$
13	$L(x) = 1 - \lg(x)^{\cos(x)}$	$1 \le x \le 10$	$h = 1$
14	$D(x) = \ln(x)^3 \cdot \sin(x)^2 + x - 10$	$1 \le x \le 10$	$h = 1$
15	$W(x) = (x - 1)^{\lg(x)} - 5e^{\sin(x)}$	$2 \le x \le 15$	$h = 1$
16	$P(x) = x^{\lg(x)} - 0{,}5e^4 \cdot \cos(x)$	$1 \le x \le 15$	$h = 1$
17	$Y(x) = \ln(x)^3 \cdot \sin(x)^2 - \ln(x)^2$	$1 \le x \le 7$	$h = 0.5$
18	$T(x) = \sin(x)^3 / (x - 2\cos(x)^3)$	$1 \le x \le 9$	$h = 0.5$
19	$Y(x) = \lg(x)^3 \cdot \sin(x)^2 - \lg(x)$	$1 \le x \le 10$	$h = 0.5$
20	$B(x) = \lg(x)^{\cos(x)^2} - \sin(x)$	$1 \le x \le 10$	$h = 0.5$
21	$Y(x) = \ln(x)^5 \cdot \sin(x)^2 + x - 20$	$1 \le x \le 10$	$h = 0.5$
22	$Z(x) = \sin(x)^3 + 0.3 \Big/ x^{\cos(x)} + 5$	$1 \le x \le 8$	$h = 0.5$

Subject 8. SOLVING THE SET OF LINEAR EQUATIONS USING MICROSOFT EXCEL

A system of linear equations is a collection of two or more linear equations involving the same set of variables. A general system of m linear equations with n unknowns can be written as

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\ \vdots \qquad \vdots \qquad\qquad \vdots \quad & \quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &= b_m \end{aligned} \tag{8.1}$$

Or it can be represented as:

$$\sum_{j=1}^{n} a_{ij}x_j = b_i \tag{8.2}$$

where $i = 1, 2, \ldots, m$ is the equation number, totally n;

$j = 1, 2, \ldots, n$ is the number of variables in equations set, totally m;

a_{ij} is the matrix of coefficients in the left part of equations (the coefficients of the system);

b_i is the vector of right parts (the constant terms);

x_j are unknowns.

8.1. Matrix method

Representing the system of linear equations in matrix form, we will get:

$$AX = B, \tag{8.3}$$

where

– matrix A containing a_{ij} elements, which is the matrix of coefficients of the system;

– matrix B containing b_i elements, which is the vector of the constant terms;

– matrix X containing x_j elements, which are unknowns.

The system in matrix form is $AX = B$. Matrix solution stands, that if the equation system is expressed in the matrix form, the entire solution set can also be expressed in matrix form. If the matrix *A* is square (has *m* rows and *n*=*m* columns) and has full rank (all *m* rows are independent), then the system has a unique solution given by

$$X = A^{-1}B. \qquad (8.4)$$

where A^{-1} is the inverse of A.

Task 8.1

Find values x, y, z, which are the solution of the following system of linear equations:

$$\begin{cases} x - 2y + 3z = 6 \\ 2x + 3y - 4z = 16 \\ 3x - 2y - 5z = 12 \end{cases}$$

Solution

1. Analysis of the set of linear equations to obtain matrix *A* containing a_{ij} elements, which is the matrix of coefficients of the system and matrix *B* containing b_i elements, which is the vector of the constant terms.

The matrices *A* and *B* for Task 8.1 are presented in Table 8.1.

Table 8.1 – The matrices *A* and *B* for Task 8.1

The matrix of equation set			**The right parts**
x	y	z	
1	−2	3	6
2	3	−4	16
3	−2	−5	12

2. Create the matrices with initial data in Excel

Enter the values of A matrix of equation set (a_{ij}) to the cells **A4:C6** (Figure 8.1)

Enter B vector of right parts (b_i) to cells **G4:G6** (Figure 8.1)

	A	B	C	D	E	F	G
1	The solving of equations' set						
2							
3	The matrix of the equations' set				The vector of right parts		
4	1	-2	3				6
5	2	3	-4				16
6	3	-2	-5				12
7							

Figure 8.1– The matrices A and B for Task 8.1 in Excel

3. Create the inverse matrix A^{-1} from Eq.(8.4) in Excel.

To create the inverse matrix of initial data matrix, containing in cells **A4:C6** do the following:

1) Select the range **A10:C12** then press f_x button.
2) Find the function **MINVERSE** and press OK button (Figure 8.2).

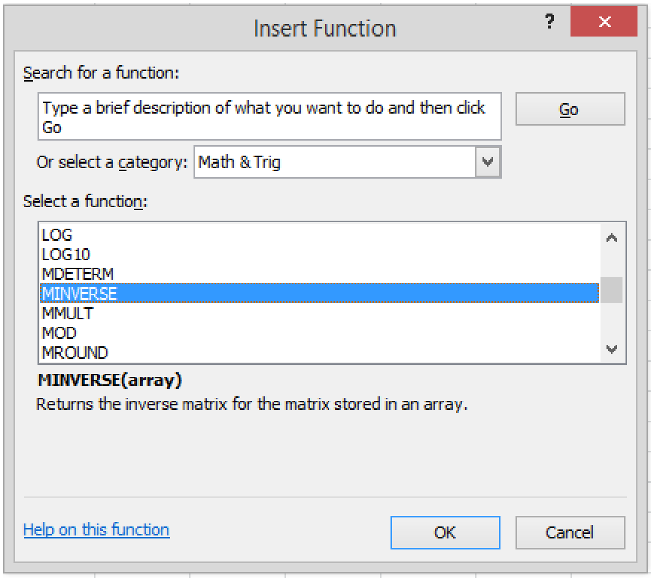


Figure 8.2 – Insert Function dialog box with **MINVERSE** function

3) The **MINVERSE** function arguments dialog box will open (Figure 8.3).

4) Select **A4:C6** cells as **ARRAY**(Figure 8.3).

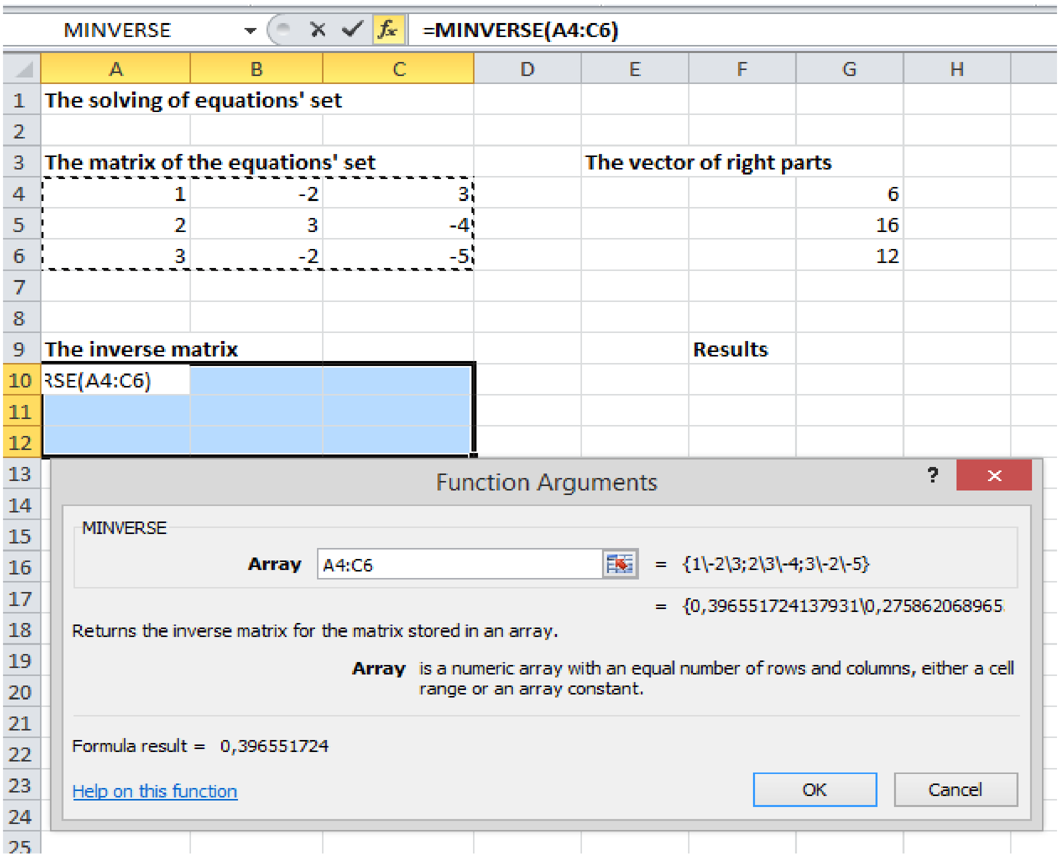


Figure 8.3 – Insert Function dialog box with **MINVERSE** function

5) Press $\underline{<Ctrl>+<Shift>+<Enter>}$instead OK button.

! Don't press OK button !

6) The inverse matrix $A^{-1}$is obtained in **A10:C12** cells (Figure 8.4).

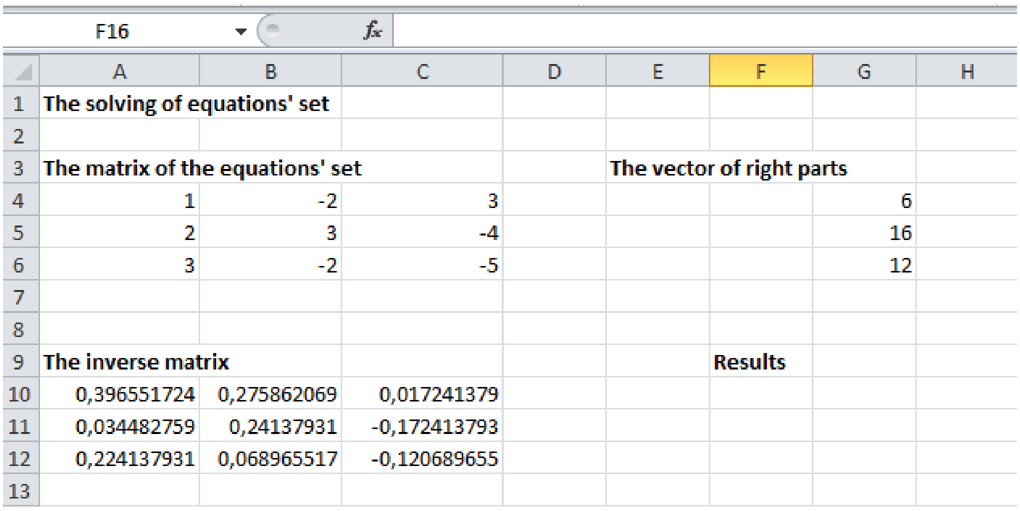


Figure 8.4 – The inverse matrix in cells **A10:C12**

4. Get the solution vector X containing x_j elements, which are unknowns.

To obtain the vector with unknowns in Excel do the following:

1) Select the range **F10:F12**, where the results will be placed, then press f_x button.

2) Find the function **MMULT** and press OK button.

3) The **MMULT** function arguments dialog box will open (Figure 8.5).

4) Select **A10:C12** cells as **Array1** and **G4:G6** cells as **Array2** (Figure 8.5).

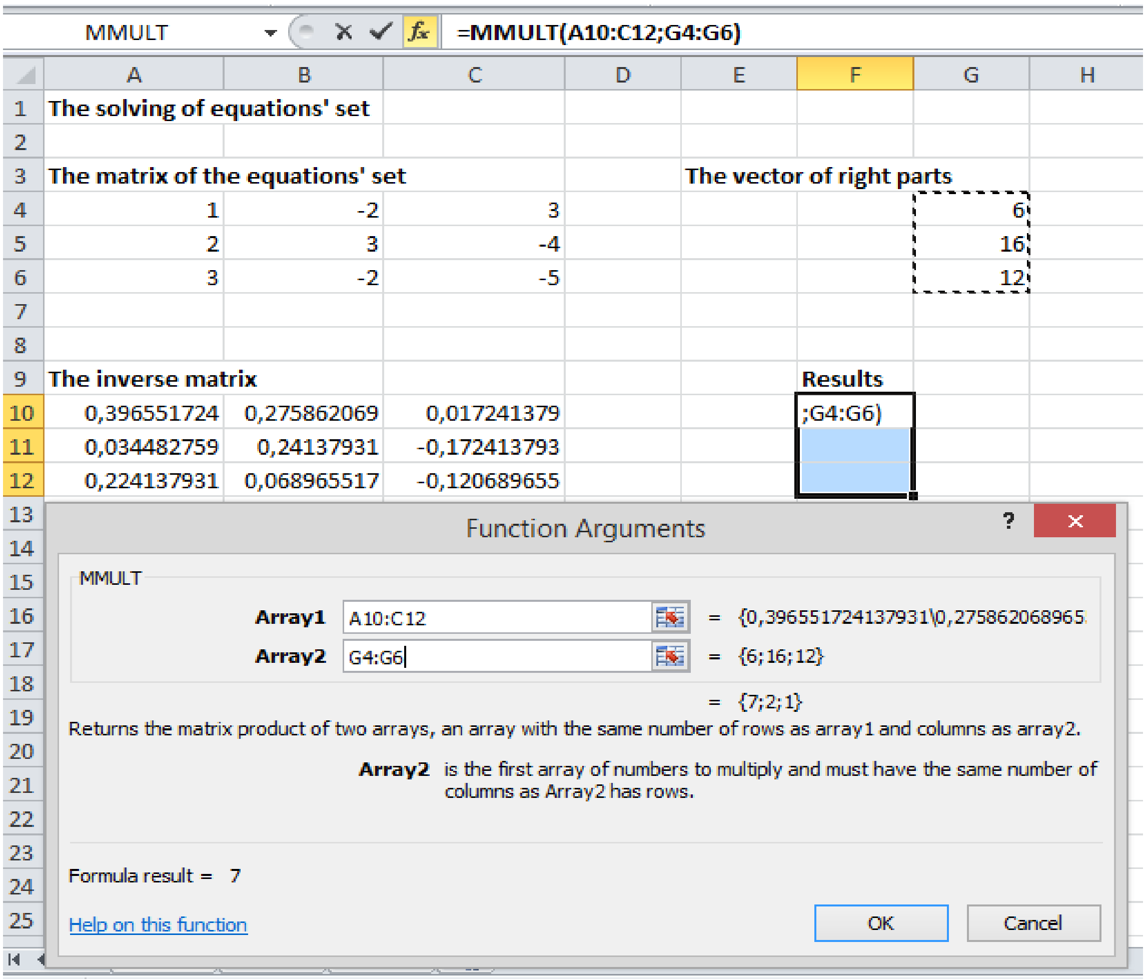


Figure 8.5 – MMULT function arguments dialog box

5) Press $< Ctrl > + < Shift > + < Enter >$ instead OK button.

! Don't press OK button !

6) The solution vector X is obtained in **F10:F12**cells (Figure 8.6).

	A	B	C	D	E	F	G	H
1	The solving of equations' set							
2								
3	The matrix of the equations' set				The vector of right parts			
4	1	-2	3				6	
5	2	3	-4				16	
6	3	-2	-5				12	
7								
8								
9	The inverse matrix					Results		
10	0,396551724	0,275862069	0,017241379			7		
11	0,034482759	0,24137931	-0,172413793			2		
12	0,224137931	0,068965517	-0,120689655			1		

Figure 8.6 – The obtained solution for Task 8.1

Answer: the solution of equation set from Task 8.1 is: $x = 7$; $y = 2$; $z = 1$.

8.2. Solving the system of linear equations according to Cramer's rule

Consider a system of n linear equations for n unknowns, represented in matrix multiplication form as follows:

$$Ax = b \tag{8.5}$$

where the $n \times n$ matrix A has a nonzero determinant; the vector $x = x_1, x_2, \ldots, x_n{}^T$ is the column vector of the variables.

Then the theorem states that in this case the system has a unique solution, whose individual values for the unknowns are given by:

$$x_i = \frac{\det A_i}{\det A}, \quad i = 1, 2, \ldots, n \tag{8.6}$$

Where A_i is the matrix formed by replacing the i-th column of A by the column vector b.

Task 8.2

Find values x, y, z, which are the solution of the following system of linear equations:

$$\begin{cases} x - 2y + 3z = 6 \\ 2x + 3y - 4z = 16 \\ 3x - 2y - 5z = 12 \end{cases}$$

Solution

1. Analysis of the set of linear equations to obtain matrix *A* containing a_{ij} elements, which is the matrix of coefficients of the system and matrix *B* containing b_i elements, which is the vector of the constant terms.

The matrices *A* and *B* for Task 8.2 are the same as for Task 8.1 and are presented in Table 8.1.

2. Create the matrices with initial data in Excel

Enter the values of *A* matrix of equation set (a_{ij}) to the cells **A4:C6** (Figure 8.1)

Enter *B* vector of right parts (b_i) to cells **G4:G6** (Figure 8.1)

3. Estimate the main determinant $\det A$.

To estimate the main determinant of the equation set, it is needed to create the matrix *A* with the values from the matrix, containing a_{ij} elements, which is the matrix of coefficients of the system. Then find its determinant doing the following:

1) Copy the cells from the range **A4:C6** to the range **A10:C12** then select **C13** cell and press f_x button.
2) Find the function **MDETERM** and press OK button.
3) The **MDETERM** function arguments dialog box will open.
4) Select **A10:C12** cells as **ARRAY** (Figure 8.7*a*).
5) Press OK button.
6) In cell **C13** the result presented in Figure 8.7*b* is obtained.

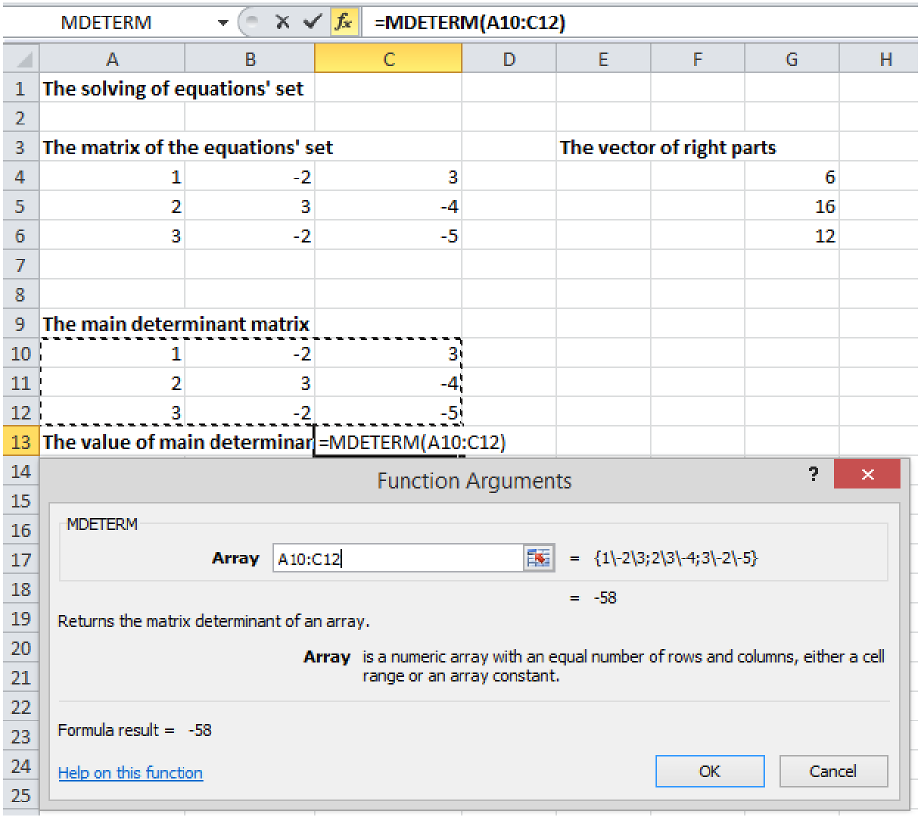
4. The estimation of additional determinants for *X, Y, Z*

1) Enter the matrix to estimate the additional determinant $\det(A_i)$, $i = 1$ in range **A16:C18** (Figure 8.8).Using the function **MDETERM** find the value of $\det(A_i)$, $i = 1$ in **C19** cell (Figure 8.8).

2) Enter the matrix for additional determinant $\det(A_i)$, $i = 2$ estimation in range **E16:G18**. Using the function **MDETERM** find the value of $\det(A_i)$, $i = 2$ in **G19** cell (Figure 8.8).

3) Enter the matrix for additional determinant $\det(A_i)$, $i = 3$ estimation in range **J16:L18**. Using the function **MDETERM** find the value of $\det(A_i)$, $i = 3$ in **L19** cell (Figure 8.8).

a



b

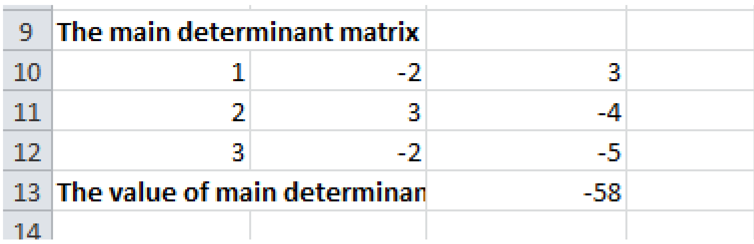


Figure 8.7 – The estimation of main determinant

	A	B	C	D	E	F	G	H	I	J	K	L
1	The solving of equations' set											
2												
3	The matrix of the equations' set				The vector of right parts							
4	1	-2	3				6					
5	2	3	-4				16					
6	3	-2	-5				12					
7												
8												
9	The main determinant matrix											
10	1	-2	3									
11	2	3	-4									
12	3	-2	-5									
13	The value of main determinan		-58									
14												
15	The additional determinant matrix for X				The additional determinant matrix for Y					The additional determinant matrix for Z		
16	6	-2	3		1	6	3			1	-2	6
17	16	3	-4		2	16	-4			2	3	16
18	12	-2	-5		3	12	-5			3	-2	12
19	The value of X determinant		-406		The value of X determinant		-116			The value of X determinant		-58
20												

Figure 8.8 – The estimation of additional determinant

5. Get the solution vector X containing x_j elements, which are unknowns according to Eq.(8.6).

To obtain the vector with unknowns in Excel do the following:

1) In cell **B22** find the x value ($x = \det A_1 / \det A$), for this in **B22** enter

"= C19/C13".

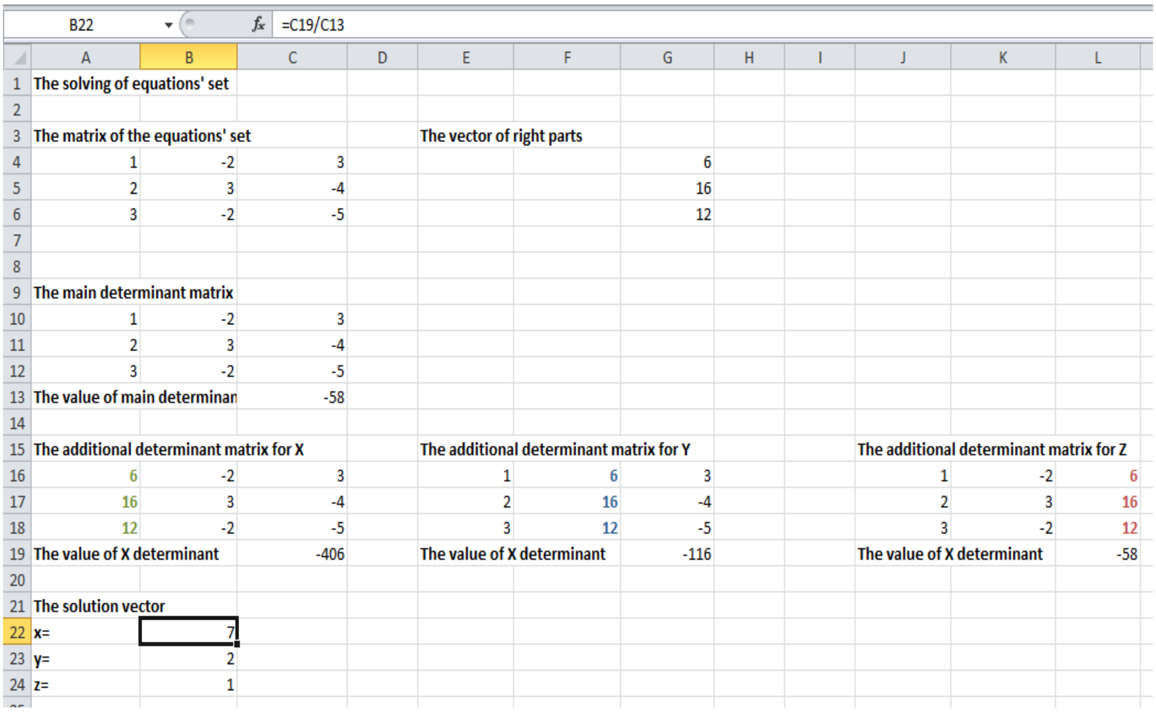
2) In cell **B23** find the y value($y = \det A_2 / \det A$), for this in **B23** enter "

= G19/C13".

3) In cell **B24** find the z value($z = \det A_3 / \det A$), for this in **B24** enter

"= L19/C13".

4) The solution vector X is obtained in **B22:B24**cells (Figure 8.9).



B22 | f_x =C19/C13

	A	B	C	D	E	F	G	H	I	J	K	L
1	The solving of equations' set											
2												
3	The matrix of the equations' set				The vector of right parts							
4	1	-2	3				6					
5	2	3	-4				16					
6	3	-2	-5				12					
7												
8												
9	The main determinant matrix											
10	1	-2	3									
11	2	3	-4									
12	3	-2	-5									
13	The value of main determinan		-58									
14												
15	The additional determinant matrix for X				The additional determinant matrix for Y					The additional determinant matrix for Z		
16	6	-2	3		1	6	3			1	-2	6
17	16	3	-4		2	16	-4			2	3	16
18	12	-2	-5		3	12	-5			3	-2	12
19	The value of X determinant		-406		The value of X determinant		-116			The value of X determinant		-58
20												
21	The solution vector											
22	x=	7										
23	y=	2										
24	z=	1										

Figure 8.9 – The obtained solution for Task 8.2

Answer: the solution of equation set from Task 8.2 is: $x = 7$; $y = 2$; $z = 1$.

8.3. Solving the system of equations set using SOLVER add-in application

To find the solution of linear equation set in form given in Eq.(8.1) using SOLVER add-in application it is needed to get the full function of equations in the following form:

$$\begin{aligned} &F(x_1, x_2, \ldots, x_n) = \\ &= \left(x_1 \cdot a_{11} + x_2 \cdot a_{12} + \ldots + x_n \cdot a_{1m} - b_1\right)^2 + \\ &+ \left(x_2 \cdot a_{21} + x_2 \cdot a_{22} + \ldots + x_n \cdot a_{2m} - b_2\right)^2 + \\ &+ \ldots + \left(x_n \cdot a_{n1} + x_n \cdot a_{n2} + \ldots + x_n \cdot a_{nm} - b_n\right)^2 \end{aligned} \quad (8.7)$$

Then using SOLVER add-in application it is possible to find the solution of Eq.(8.7) with $x_1, x_2, \ldots, x_n$ unknowns.

Task 8.3

Find values x, y, z, which are the solution of the following system of linear equations:

$$\begin{cases} x-2y+3z=6 \\ 2x+3y-4z=16 \\ 3x-2y-5z=12 \end{cases}$$

Solution

1. Analysis of the set of linear equations to represent the linear equation set in the form of Eq.(8.7).

The equation set from Task 8.3 can be represented as following function:

$$F(x,y,z) = \left(x-2y+3z-6\right)^2 + \left(2x+3y-4z-16\right)^2 + \left(3x-2y-5z-12\right)^2$$

2. Prepare the cells for **SOLVER**.

1) Enter the first guessed values of variables $x,\ y,\ z$.

For Task 8.3 the initial guessed values can be taken as $x = 1,\ y = 1$, $z = 1$).

Enter 1 in cell **A5**, **B5** and **C5**.

2) Enter the function $F(x, y, z)$to cell **E5**, selecting the references to the cells**A5**, **B5**, **C5** with values:

$$= (A5 - 2 * B5 + 3 * C5 - 6)^2 + (2 * A5 + 3 * B5 - 4 * C5 - 16)^2 + (3 * A5 - 2 * B5 - 5 * C5 - 12)\text{^}2$$

3. In **Data** ribbon find **SOLVER** add-in (Figure 8.10).

1) The set objective – the absolute reference to the objective cell **E5** cell ($E$5);
2) By changing variable cell – the absolute reference to the decision variable cells **A5:C5**;
3) To – select "0"to find the root of the function $F(x, y, z)$.

4. In cells **A5**, **B5** and **C5**will appear the solution presented in Figure 8.11.

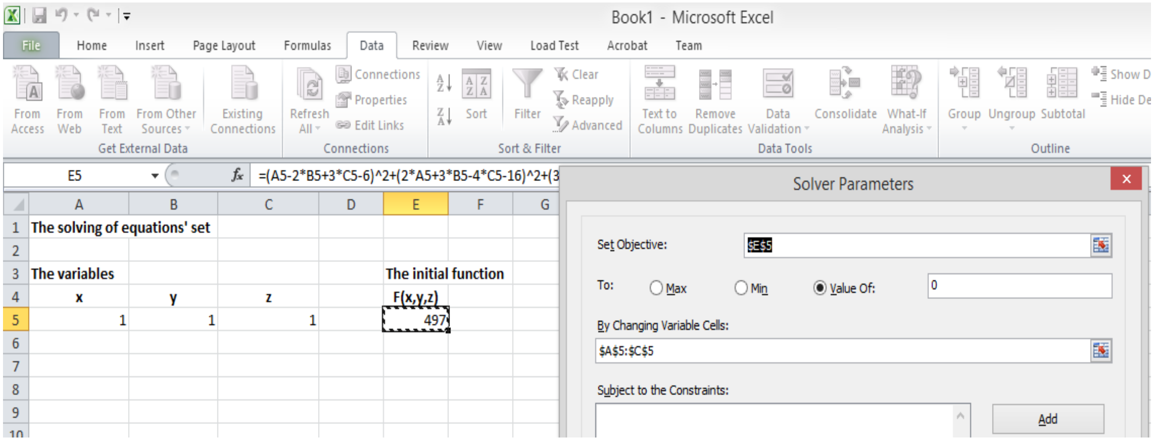


Figure 8.10 – The Solver Parameters dialog box for Task 8.3

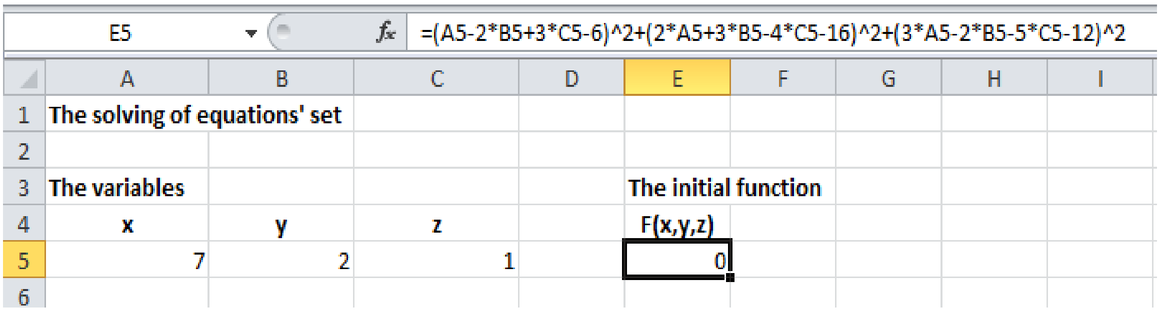


Figure 8.11 – The obtained solution for Task 8.3

Answer: the solution of equation set from Task 8.3 is: $x = 7$; $y = 2$; $z = 1$.

Task 8.4. Tasks for laboratory and individual work

Solve the sets of equations listed in Table 8.2 using the approaches observed above.

Table 8.2 – Tasks for calculation

№	Set of Equations
1	2
1	$\begin{cases} -0.85x + 0.05y - 0.08z + 0.14t = 0.48 \\ 0.32x - 1.43y + 0.12z + 0.11t = -1.24 \\ 0.17x + 0.06y - 1.08z + 0.12t = -1.15 \\ 0.21x - 0.16y + 0.36z - t = 0.88 \end{cases}$

The end of the table 8.2

1	2
2	$\begin{cases} -x+0.28y-0.17z+0.06t=-0.21 \\ 0.52x-y+0.12z+0.17t=1.17 \\ 0.17x-0.18y+0.79z=0.81 \\ 0.11x+0.22y+0.03z-0.95t=-0.72 \end{cases}$
3	$\begin{cases} 2.5x+4y-7z=12.115 \\ x-3y+z=-0.87 \\ 7x+2y-1.5z=35.93 \end{cases}$
4	$\begin{cases} x+7y+20z=91.65 \\ 4x-5y-2z=-14.85 \\ 10x+2y+15z=87.2 \end{cases}$
5	$\begin{cases} 2x+15y-8z=206.45 \\ 12x-7y+3z=39.06 \\ 7x+2y-12z=148.14 \end{cases}$
6	$\begin{cases} 6.05x+0.13y+8.57z=19.6 \\ 15.46x-8y+13.94z=23.8 \\ 7.18x-12.6y+0.07z=-0.04 \end{cases}$
7	$\begin{cases} x+2y-3z+5t=1 \\ x+3y-13z+22t=-1 \\ 3x+5y+z-2t=5 \\ 2x+3y+4z-7t=4 \end{cases}$
8	$\begin{cases} -0.87x_1+0.27x_2-0.22x_3-1.28x_4=-1.21 \\ -0.21x_1-0.3x_2-0.45x_3+0.18x_4=0.33 \\ 0.12x_1+0.13x_2-1.33x_3+0.18x_4=0.48 \\ 0.33x_1-0.05x_2+0.06x_3-1.28x_4=0.17 \end{cases}$
9	$\begin{cases} x_1+2x_2+4x_3=31 \\ 5x_1+x_2+x_3=29 \\ 3x_1-x_2+x_3=10 \end{cases}$

Subject 9. ADDING THE TREND LINES

9.1. Trend lines in Excel

Trend line is a graphic representation of trends in data series, such as a line sloping upward to represent increased sales over a period of months. Trend lines are used for the study of problems of prediction, also called regression analysis.

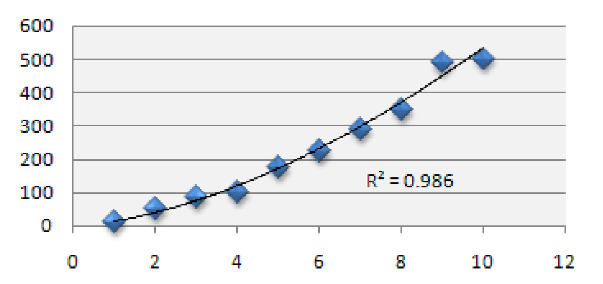
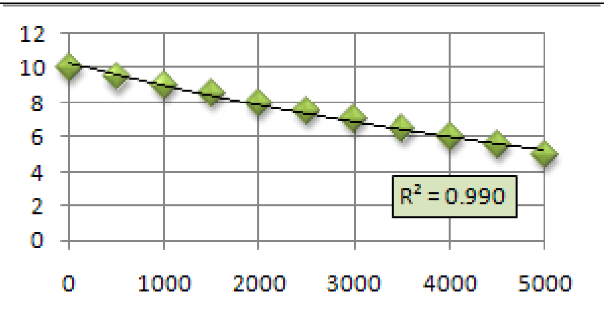
By using regression analysis, it is possible to extend a trend line in a chart beyond the actual data to predict future values.

R-squared value. A number from 0 to 1 that reveals how closely the estimated values for the trend line correspond to the actual data. A trend line is most reliable when its R-squared value is at or near 1. R-squared value is also known as the *coefficient of determination*.

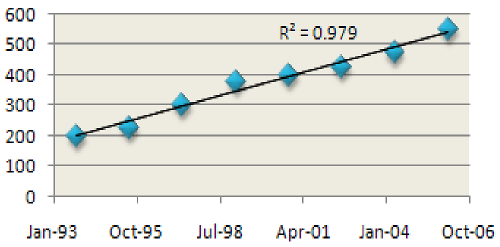
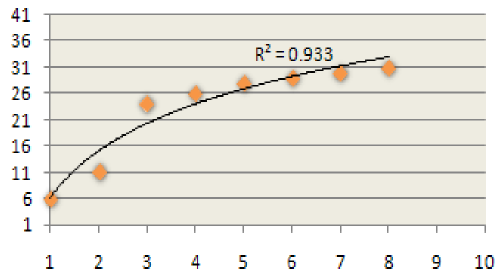
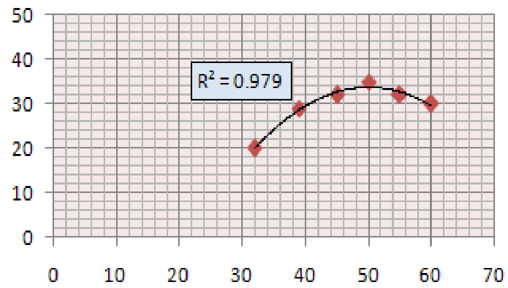
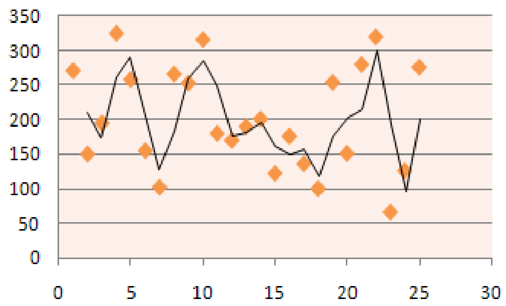
Microsoft Office Excel provides six different trend or regression types: linear trend lines, logarithmic trend lines, polynomial trend lines, power trend lines, exponential trend lines, or moving average trend lines.

The type of data that is needed to determine depends on the type of trend line, which is used. The standard approximation functions are listed in Table 9.1.

Table 9.1 – Standard functions used for approximations

Name	Example of graph	Equation
1	2	3
Power function		$y = c \cdot x^b$
Exponential		$y = c \cdot e^{b \cdot x}$

The end of the table 9.1

<table>
<tr><td>1</td><td>2</td><td>3</td></tr>
<tr><td>Linear</td><td>
</td><td>$y = m \cdot x + b$</td></tr>
<tr><td>Logarithmic</td><td>
</td><td>$y = c \cdot \ln(x) + b$</td></tr>
<tr><td>Polynomial</td><td>
</td><td>$y = b + c_1 \cdot x + c_2 \cdot x^2 + c_n \cdot x^n$</td></tr>
<tr><td>Moving average</td><td>
</td><td>$F_t = \frac{A_t + A_{t-1} + \cdots + A_{t-n+1}}{n}$</td></tr>
</table>

Task 9.1

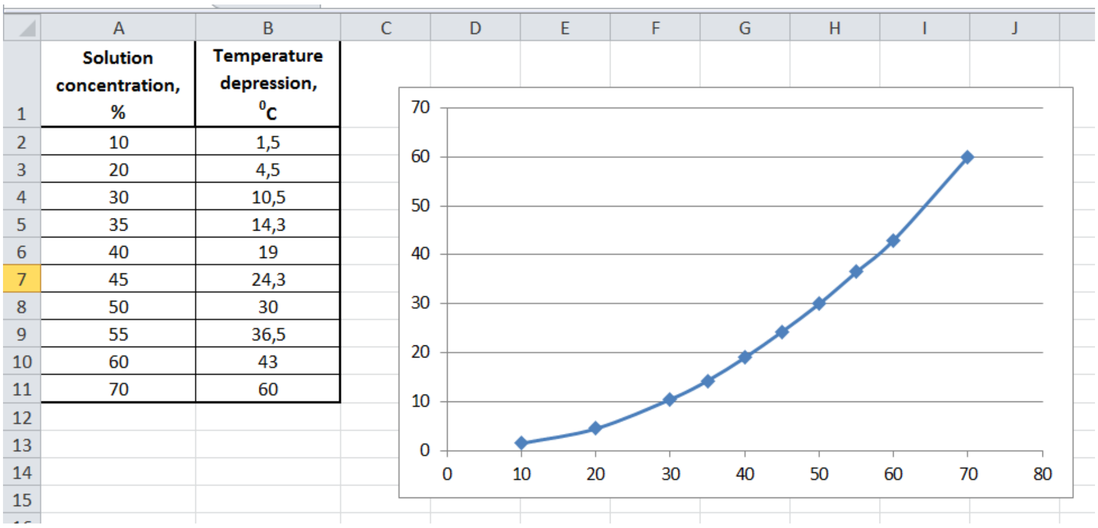
Use the following data for solution concentration C, [%] in paint from the temperature depression, [°C]:

Solution concentration, %	10	20	30	35	40	45	50	55	60	70
Temperature depression, °C	1.5	4.5	10.5	14.3	19	24.3	30	36.5	43	60

It is needed to find the functional dependence of temperature depression from solution concentration, select the curve type and estimate the functional dependence number.

Solution.

1. Enter the table with initial data and plot the corresponding graph (Figure 9.1).



Solution concentration, %	Temperature depression, °C
10	1,5
20	4,5
30	10,5
35	14,3
40	19
45	24,3
50	30
55	36,5
60	43
70	60

Figure 9.1 – The initial data and graph for Task 9.1

2. Add the trend line to the graph.

1) Press the right mouse button and in the appeared menu select “Add Trend line…” (Figure 9.2).

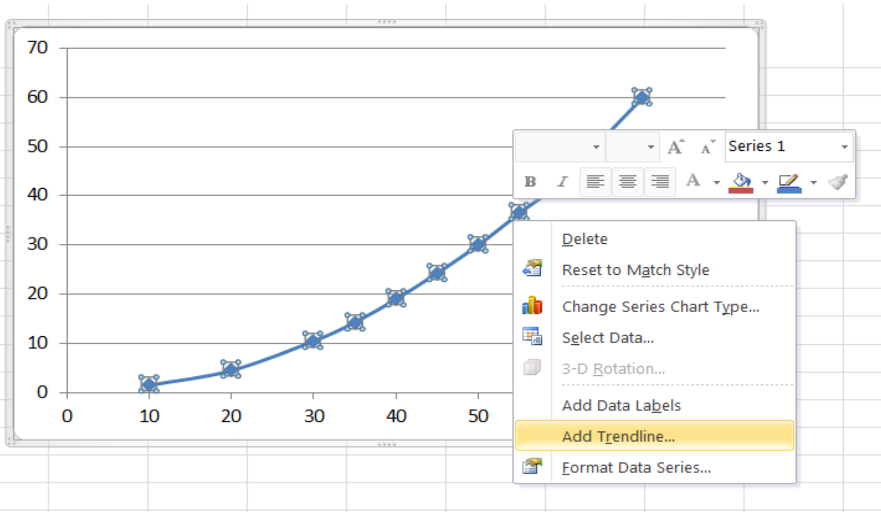


Figure 9.2 – Adding trend line to the graph

2) Select the first regression type in the *Format Trend line* dialog box.

For the data presented in Figure 9.1 the power regression can be in good agreement.

Select the *Power regression* type, mark the "*Display Equation on chart*" and "*Display R-squared value on chart*" (Figure 9.3).

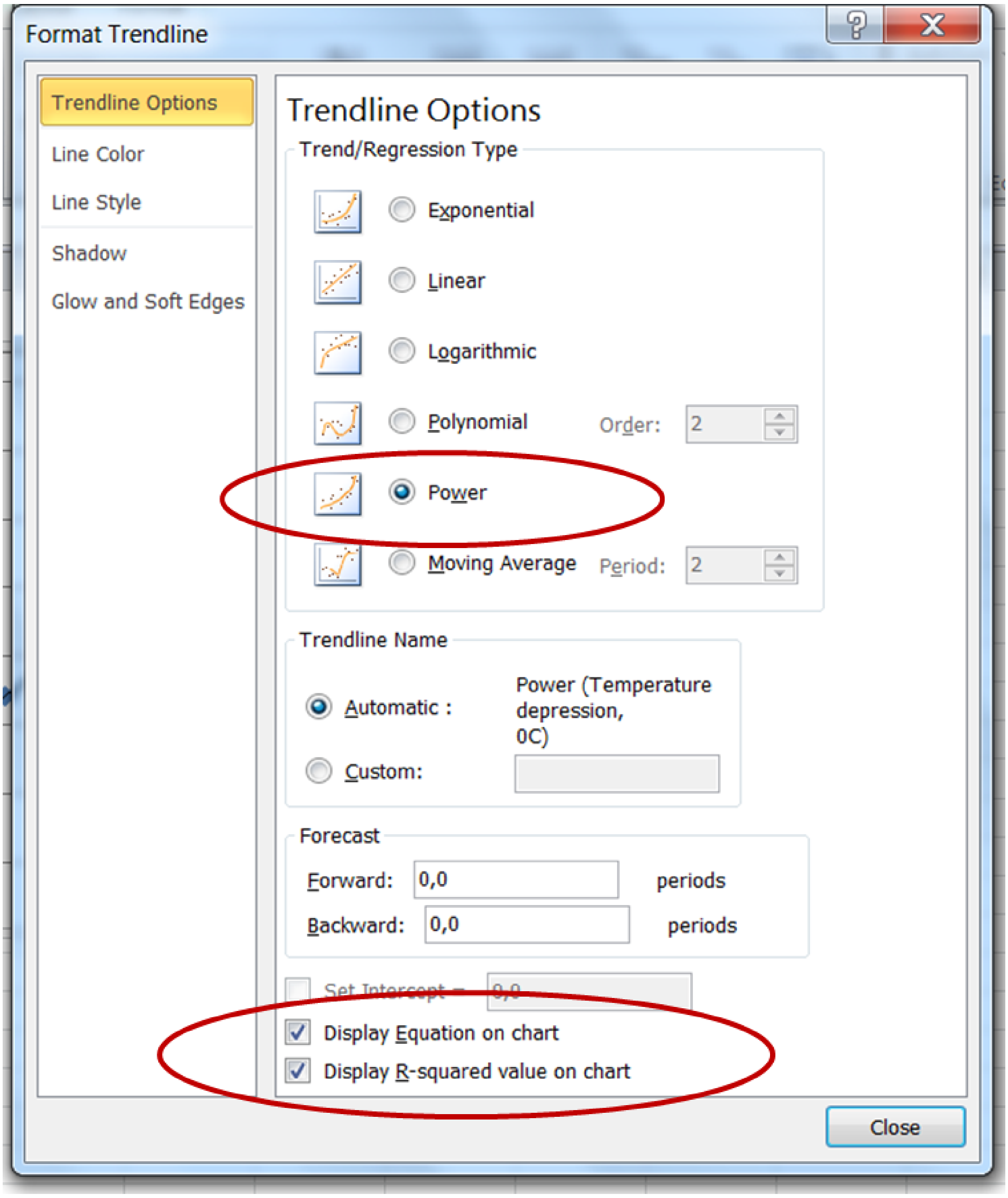


Figure 9.3 – Changing parameters in Format Trend line dialog box

3) The trend line, its function and *R*-squared value will appear on the graph (Figure 9.4).

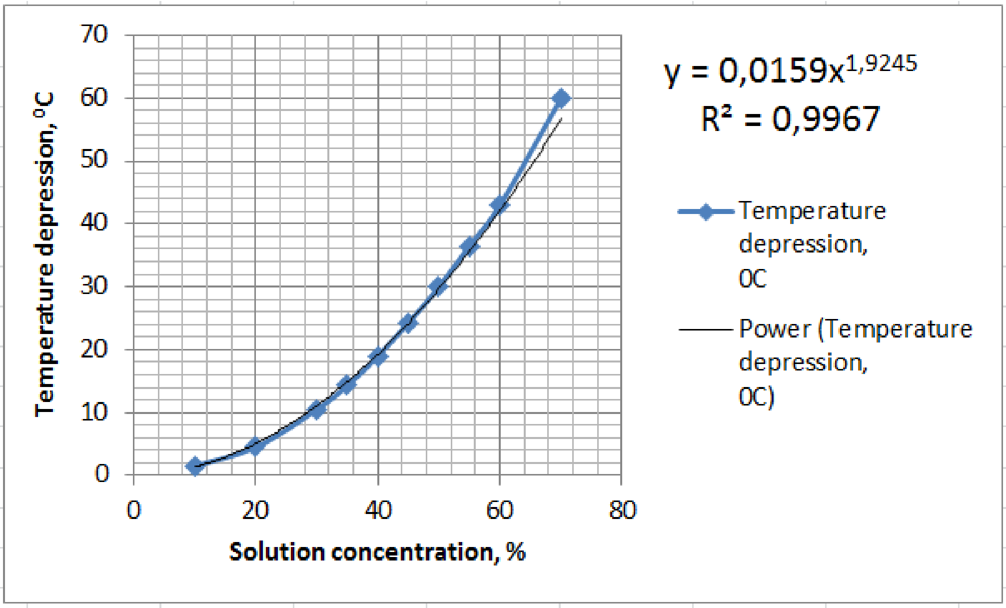


Figure 9.4 – Power regression for Task 9.1

4) Select the other regression type.

Plot the other graph for the data from Task 9.1 and add the trend line with *exponential* type of regression. The obtained trend line, its function and *R*-squared value are presented in Figure 9.5.

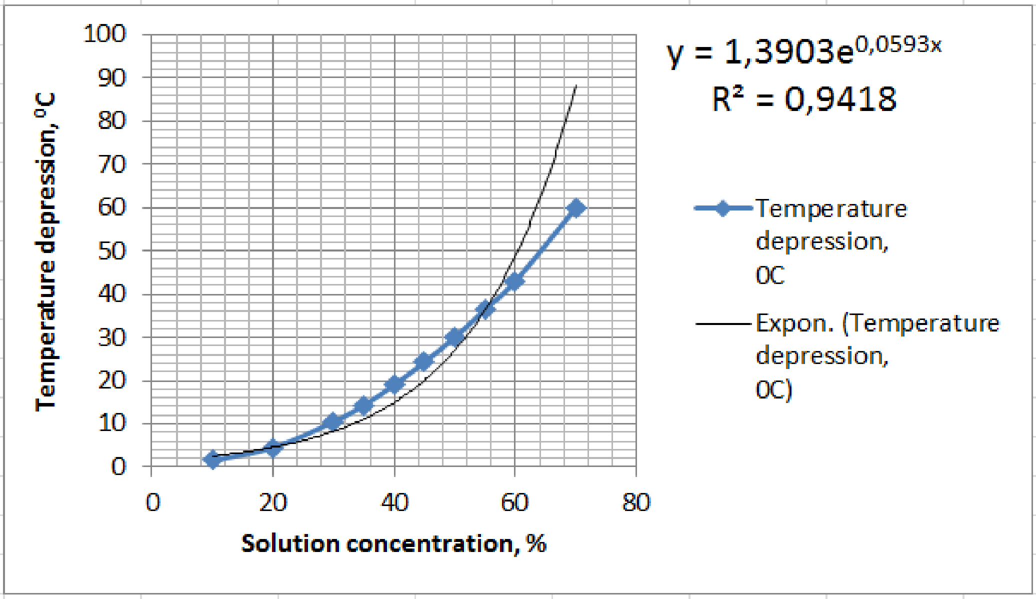


Figure 9.5 – Exponential regression for Task 9.1

3. Select the best approximation function comparing R-squared value.

For power regression the R-squared value is equal to $R^2 = 0.9967$, and for exponential regression $R^2 = 0.9418$. The value for the power regression is closer to 1, and is better.

Answer: the approximation function for data from Task 8.3 is:
$y(x) = 0.1059 \cdot x^{1.9245}$;
The R-squared value is equal to $R^2 = 0.9967$.

Task 9.2

Using Table 9.2 find the functional dependence of specific heat capacity from the temperature. The temperature T, [°C] is given in the table heading. The specific heat capacity data are listed for the substances according to the variant. For example, variant 2 contains the data for acetone.

Table 9.2 – Data foe task 9.2

№	Substance	Specific heat capacity at the temperature, °C (in kJ/(kg·°C))						
		-20	0	20	40	60	80	100
1	Ammonia (liquid)	4.522	4.606	4.731	4.857	5.108	5.443	5.736
2	Acetone	2.052	2.114	2.177	2.24	2.303	2.37	2.445
3	Dichloroethane	0.971	1.057	1.147	1.23	1.327	1.419	1.512
4	$CaCl_2$ (25%)	2.818	2.889	2.939	2.973	3.057	3.098	3.14
5	Methanol	2.382	2.466	2.567	2.667	2.763	2.864	2.964
6	H_2SO_4 (75%)	1.805	1.872	1.939	2.006	2.073	2.145	2.207
7	Toluene	1.52	1.612	1.704	1.796	1.888	1.98	2.068
8	Chlorine benzene	1.193	1.256	1.319	1.382	1.445	1.507	1.574
9	CCl_4	0.812	0.837	0.863	0.892	0.921	0.946	0.975

References

1.Товажнянский Л. Л. Компьютерные технологии в инженерной химии. Информатика.: учеб. пособие / Л. Л. Товажнянский, Б. Д. Зулин, В. А. Коцаренко – Харьков : НТУ «ХПИ», 2004. – 456 с.

2. Долженков В. А. Самоучитель Microsoft Excel 2000 / В. А. Долженков, Ю. В. Колесников – СПб. : BHV – Санкт-Петербург, 2000. – 355 с.

3. Додж М. Эффективная работа с Microsoft Excel 2000. / М. Додж, К. Стинсон. – СПб. : Питер, 2001. – 1056 с.

4. Лабораторный практикум по курсу «Компьютерные технологии» : учеб. пособие / Л. Л. Товажнянский, Т. Г. Бабак, В. А. Коцаренко, Е. Д. Пономаренко, А. В. Сатарин. – 2-е изд., перераб. и доп. – Харьков : НТУ «ХПИ», 2002. – 364 с.

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АРСЕНЬЄВА Ольга Петрівна
ВЕДЬ Олена Валеріївна
СОЛОВЕЙ Людмила Валентинівна
ЮЗБАШЬЯН Анна Петрівна

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для студентів хімічних спеціальностей

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