

UDC 621.646.42: 621.05: 621.016

*S. A. SHEVCHENKO, V. I. KONOKH, YU. A. MITIKOV, A. L. GRIGORIEV***UNIVERSAL ORTHOMETRIC REAL GAS EQUATION OF STATE  
FOR MODELING PROCESSES IN ROCKET-AND-SPACE TECHNOLOGY UNITS**

A domain of the working parameters of the gas unified for all units of pneumatic automation, limited by isochore, adiabatic and two isotherms is formed. The requirements on the real gas equation suitable for dynamic calculations of these units are formulated. The known equations of state for real gas are analyzed and the reasons of their being not convenient to use are indicated. To obtain a new equation, it is proposed to use the theory of orthometric states of gas and the Boyle scales to measure its parameters. The form of the target equation is chosen and by using the standard tabular data the values of its three tuning coefficients are obtained. It turns out that nitrogen, argon, krypton, xenon, methane, oxygen, and air have the same two coefficients of the model, which result boosts the development of the theory of real gas. Similar equations are obtained for helium, xenon, krypton, and methane. It is shown that in the domain of gas working parameters all thermodynamic properties and calorific characteristics follow from the equation of state, the formulas for calculating these functions are given. The calculated characteristics are compared with standard tabular data; the maximum of the compressibility factor error is 0.05 ... 0.1%. An improved compressibility equation for real gas in a cavity is deduced. Methods for solving algebraic equations for the density and temperature arising at the phases of formation of initial conditions and at integration of the equations of compressibility and continuity are described. Directions of further development of the model and its practical applications are indicated.

**Key words:** pneumatic control unit, LRE of multiple ignitions, gas-dynamic analysis, real gas, equation of state, compressibility factor.

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УНІВЕРСАЛЬНЕ ОРТОМЕТРИЧНЕ РІВНЯННЯ СТАНУ РЕАЛЬНОГО ГАЗУ  
ДЛЯ МОДЕЛЮВАННЯ ПРОЦЕСІВ У АГРЕГАТАХ РАКЕТНО-КОСМІЧНОЇ ТЕХНІКИ**

Для агрегатів пневмоавтоматики ракетної техніки визначена область зміни параметрів робочого тіла й сформульовані вимоги до рівняння стану реального газу, придатному для динамічних розрахунків цих агрегатів. Обрана ортометрична форма шуканого рівняння й отримані значення його настроювальних коефіцієнтів. Наведено формули для розрахунків термодинамічних і калоричних функцій, визначена похибка моделі. Отримано уточнене рівняння стисливості реального газу в порожнині. Зазначено напрямки подальшого розвитку моделі та її практичного застосування.

**Ключові слова:** агрегат пневмоавтоматики, РРД багаторазового включення, газодинамічний розрахунок, реальний газ, рівняння стану, фактор стисливості.

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УНІВЕРСАЛЬНОЕ ОРТОМЕТРИЧЕСКОЕ УРАВНЕНИЕ СОСТОЯНИЯ РЕАЛЬНОГО ГАЗА  
ДЛЯ МОДЕЛИРОВАНИЯ ПРОЦЕССОВ В АГРЕГАТАХ РАКЕТНО-КОСМИЧЕСКОЙ ТЕХНИКИ**

Сформирована унифицированная для всех агрегатов пневмоавтоматики область изменения рабочих параметров газа, ограниченная изохорой, адиабатой и двумя изотермами. Сформулированы требования к уравнению состояния, пригодному для газодинамических расчетов. Выполнен анализ известных уравнений реального газа и указаны причины, по которым они неудобны для использования. Предложено для нового уравнения использовать теорию ортометрических состояний газа и бойлевые масштабы измерения его параметров. Выбрана форма уравнения и, при использовании табличных данных, получены значения трех настроечных коэффициентов. Оказалось, что у азота, аргона, кислорода и воздуха два коэффициента модели одинаковы, и этот результат дает импульс для развития теории реального газа. Аналогичные уравнения получены для гелия, ксенона, криптона и метана. Показано, что в области рабочих параметров газа из уравнения состояния следуют все термодинамические свойства и калорические характеристики, приведены формулы для расчета этих функций. Выполнено сопоставление расчетных характеристик со стандартными табличными данными; максимальная погрешность фактора сжимаемости составила 0.05 ... 0.1%. Получено уточненное уравнение сжимаемости реального газа в полости. Описаны методы решения алгебраических уравнений для плотности и температуры, которые возникают при формировании начальных условий и интегрировании уравнений сжимаемости и неразрывности. Указаны направления дальнейшего развития модели и её практических приложений.

**Ключевые слова:** агрегат пневмоавтоматики, ЖРД многократного включения, газодинамический расчет, реальный газ, уравнение состояния, фактор сжимаемости.

**Introduction and formulation of the problem.** When developing complex gas-dynamic systems used in liquid-propellant rocket engines (LRE) as well as pneumatic units mathematical modeling of their dynamic characteristics is performed. The modern level of computerization of scientific researches makes it possible to significantly refine the mathematical models of these processes and improve the quality and the reliability of the obtained results. To this end it is necessary to revise and upgrade some of the theoretical principles (for example, the principles describing the properties of a gaseous working fluid), which were set as a basis for calculations more than 50 years ago and with rare exceptions [1, 2], are still being used. First of all it concerns the change of the equation for describing the state of the gas, from which its refined thermodynamic properties taken into account in the calculations will follow.

The domain of working parameters of the gas used in the automation units of rocket technology is shown in Fig. 1; it is bounded by isochore and adiabat from above simulating slow cooling and rapid release of gas from gas bottles, and at the junction point the gas pressure reaches 34 MPa.

Experts are aware of the fact that at such pressures the compressibility factor of the gas specifies the density up to 10–20 %, therefore the further ignoring this factor by the dynamic calculations (possibly acceptable in first-approximation models) a priori causes a distrust towards their results.

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The equation of state of real gas that solves the indicated problem should be of high accuracy and low complexity (at the level of the well-known *Beatty – Bridgman* [3] or *Novikov – Vukalovich equations* [4]). The error of calculating pressure in the domain of operating parameters should not exceed 0.1 %, which is higher than the capabilities of the models [3, 4]. Moreover the equation of state must hold for all gases listed in Table 1.

Helium is mainly used as a working fluid to control pneumatic valves in rocket technology, since it has the high speed of sound (about 1000 m/s) and provides the minimal duration of transient processes after turning on or off the electromagnetic valves [5]. In Fig. 2, for example, a diagram of a multiple launch system of the LRE installed on the upper stages of launch rocket vehicles is shown. Gas bottle 11 of the pneumatic control unit is filled with helium, and helium or nitrogen pumped into gas bottle 1 can be used for starting-up the turbopump.

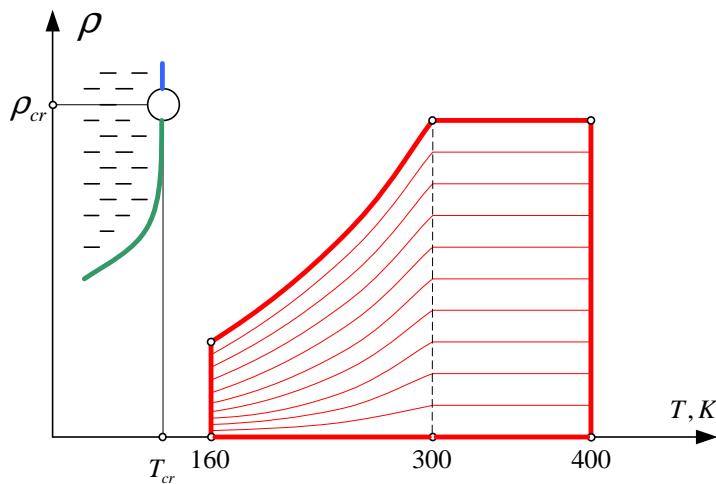


Fig. 1 – Unified domain of gas working parameters for automation units of LRE and inert fluid for spacecraft orientation engines (here and below  $\rho$  – densities;  $T$  – temperatures; to the left from the critical point there is the two-phase region).

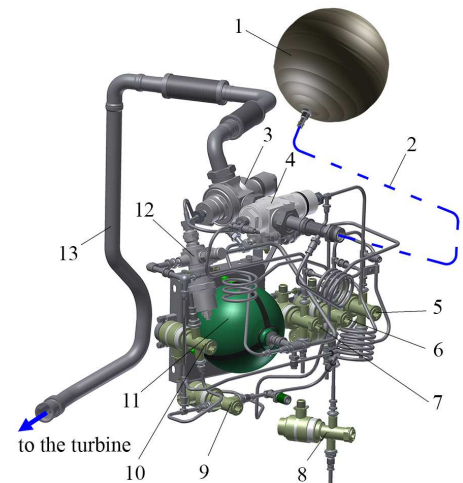


Fig. 2 – Pneumatic starting system of LRE RD861K, Yuzhnoye SDO, UA:  
1, 11 – gas bottles; 2, 13 – inlet and outlet pipelines; 3, 12 – pressure regulators; 4 – pneumatic valve; 5 – 10 – electro-pneumatic valves.

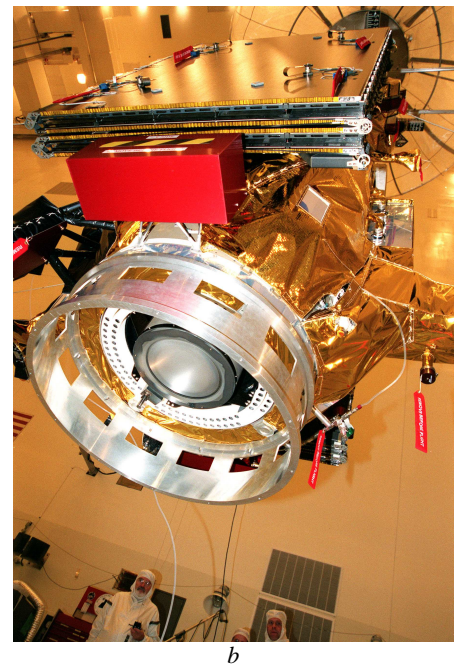


Fig. 3 – Spacecraft Deep Space 1 (NASA, 1998) ion engines using xenon:  
a – Deep Space 1 spacecraft against the background of Borelli’s comet;  
b – preparation for installation in the launch vehicle DELTA II, McDonnell Douglas, USA.



Fig. 4 – The Hall-engine-based Electric Jet Propulsion System (EJPS) is designed for spacecraft orbit correction, stabilization, and interplanetary flights, Yuzhnoye SDO, UA.

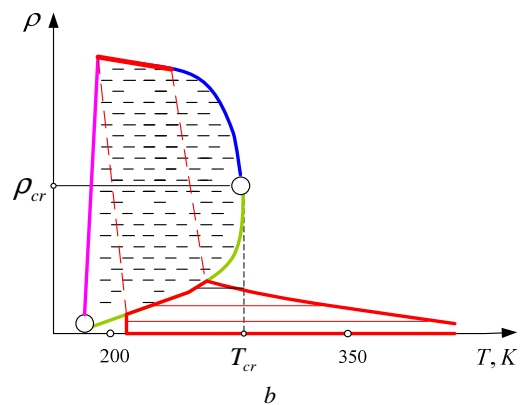
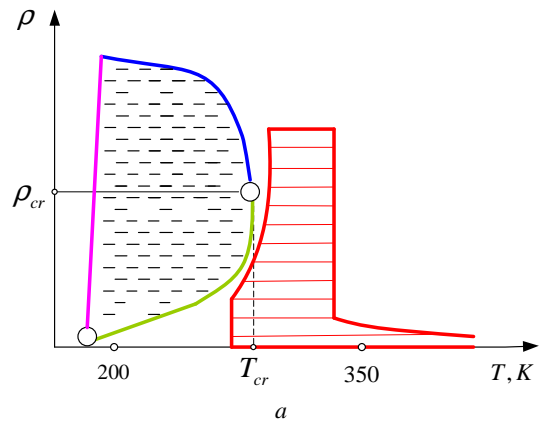


Fig. 5 – Domain of working parameters of xenon for ion (plasma) engine:  
 a – gas stored at supercritical pressure [7];  
 b – kept cooled and liquefied at low pressure [8].

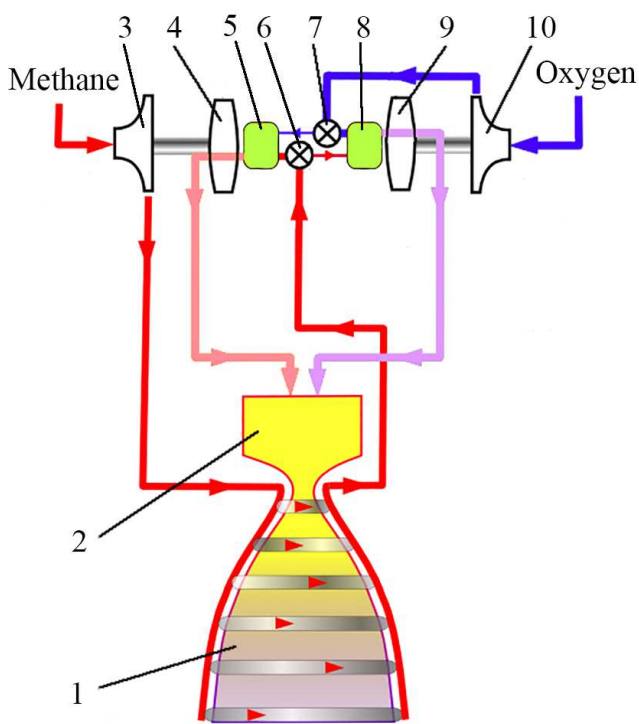


Fig. 6 – Setup of RAPTOR-type LRE:  
 1 – nozzle; 2 – combustion chamber; 3 – fuel pump;  
 4, 9 – turbines; 5, 8 – gas generators;  
 6, 7 – control valves; 10 – oxidizer pump.

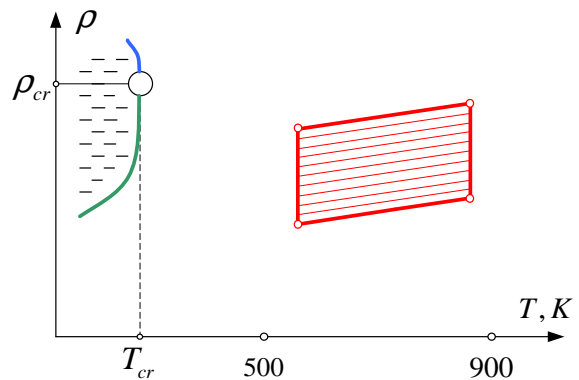


Fig. 7 – Calculated domain of working parameters of methane and oxygen for gasified fuel components of RAPTOR-type LRE.

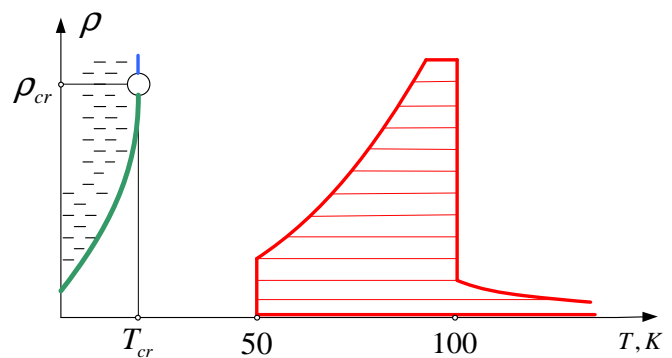


Fig. 8 – Calculated domain of working parameters of helium for system [2] above cold pressurization of tanks of launch vehicle ZENIT, Yuzhnoye SDO, UA.

Table 1 – Basic physical gas constants and critical point parameters

Gas	$k$	$R, J/(kg \cdot K)$	$\rho_{cr}, kg/m^3$	$T_{cr}, K$	$p_{cr}, MPa$	$z_{cr}$
Nitrogen	1.4	296.8	313.1	126.2	3.40	0.2899
Argon	1.667	208.1	535.7	150.7	4.80	0.2857
Helium	1.667	2078	69.64	5.19	0.227	0.3030
Oxygen	1.4	259.8	436.2	154.6	5.04	0.2878
Krypton	1.667	99.22	912.7	209.4	5.49	0.2895
Xenon	1.667	63.33	1109.9	289.7	5.83	0.2863
Methane	1.333	518.3	163.5	190.8	4.63	0.2862
Air	1.4025	287.1	316.5	132.5	3.77	0.3128

In low thrust engines of gas-jet stabilization and orientation systems of spacecrafts compressed argon is used for creating impulse and/or angular momentum. The ignition systems of some LRE operate on an injection of compressed gaseous oxygen, which is stored in their gas bottles the same way as argon. The main physical parameters of the bodies used, such as gas constants  $R$ , adiabatic indices  $k$ , critical temperatures  $T_{cr}$ , densities  $\rho_{cr}$ , pressure  $p_{cr}$ , and compressibility factor  $z_{cr}$ , are listed in Table 1; the critical temperature is below the operating cycle temperatures, which is required for normal operation of the pneumatic system. Due to this and other reasons other gases as well as gas mixtures are not used.

In the laboratory researches of the developed units compressed air is used, that is why its parameters are also listed in Table 1. In addition the same table includes the parameters of krypton and xenon used as working fluids of the electric jet engine ([6], Fig. 3 – 5), and methane used as fuel for the methane-oxygen full-flow staged combustion cycle rocket engine (RAPTOR-type, Fig. 6, 7). Besides the control systems, helium is used to pressurize tanks with liquid fuel components (Fig. 8).

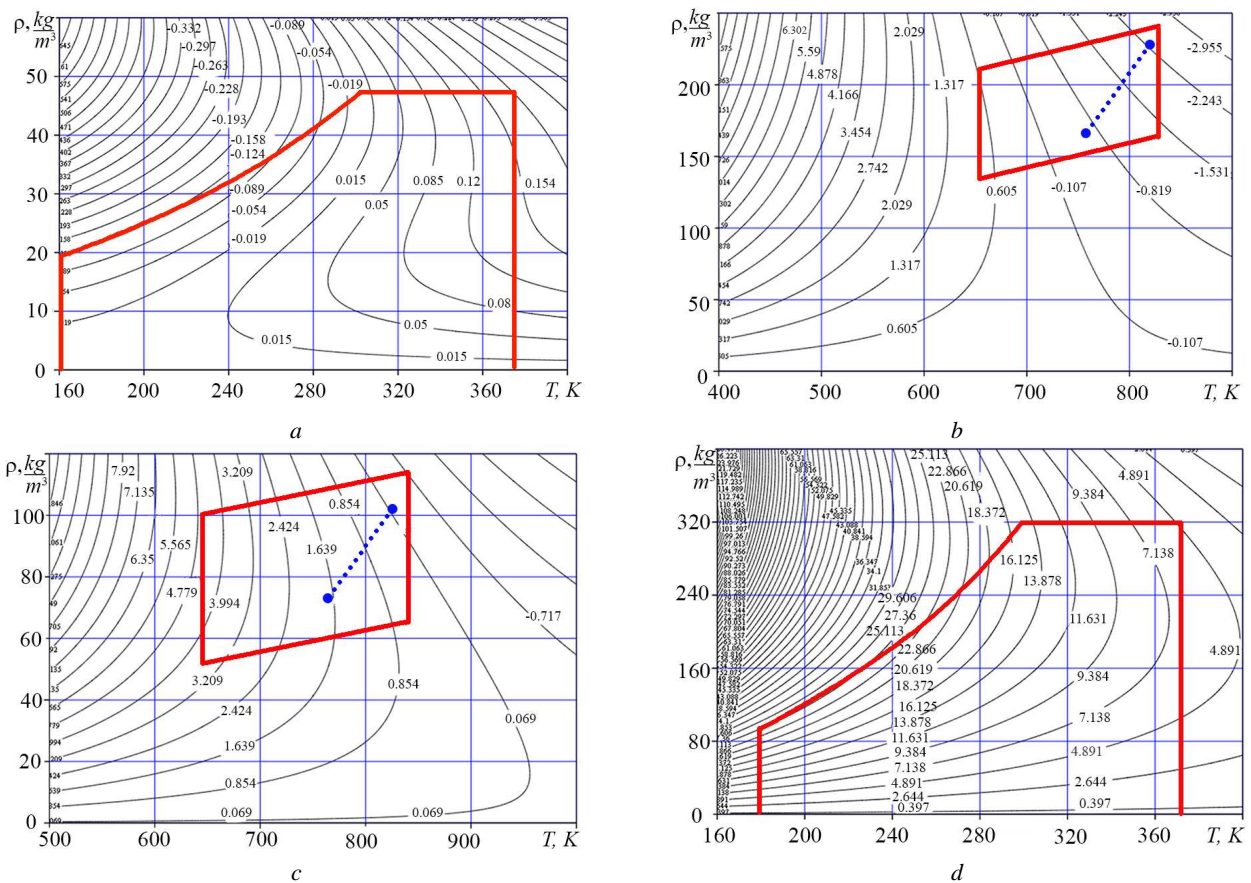


Fig. 9 – Change Abel model of the relative error of calculation  $z$ , % :  $a$  – for helium;  $b$  – for oxygen;  $c$  – for methane;  $d$  – for nitrogen.

**Analysis of the known equations of gas state.** If the gas has low density  $\rho \ll \rho_k$ , then it is in the so-called “ideal gas state” and its thermodynamic properties are described by the *Clapeyron equation*:

$$p = \rho RT ,$$

binding pressure  $p$ , density  $\rho$ , and temperature  $T$ , as well as Mayer’s formulas for specific heat capacities:

$$c_v = R / (k - 1); \quad c_p = k \cdot R / (k - 1) ,$$

and the simplest equation for enthalpy:

$$h = c_p \cdot T.$$

The individual features of the gas are transferred to the model by the values of the first two constants from Table 1 and its equations remain unchanged, which allows using a unified approach to modeling any gaseous fluid.

In work [9] a closed discrete-continual model of the system shown in Fig. 2 is presented. The oscillations of gas parameters in 8 pipelines are simulated using the equations of one-dimensional gas dynamics and changes in gas pressure and temperature in 20 cavities. The parameters of compressed helium used as the working fluid of the pneumatic system are described by the equation of state of ideal gas.

In work [10] this model was refined to take into account the helium compressibility factor  $z$  used in the equation:

$$p = z \rho R T.$$

It is shown that in the range of operating modes of this system the condition  $\rho \ll \rho_k$  is not fulfilled and in some cavities (for example, in gas bottles) the maximum density reaches the point  $0.7\rho_k$ . As a result, the factor  $z$  changes here within the range 1 ... 1.2, which affects the dynamic characteristics.

For helium the condition  $T \gg T_k$  holds in all cavities of the system, which means that the gas is in the high-temperature region. That is why to simulate its properties the simplest model of real gas (the *Abel model*, Fig. 9, a) is used, in which

$$z = 1 / (1 - c_2 \rho) = 1 + b_2 p / T, \quad h = c_p T + c_2 p, \quad b_2 = c_2 / R, \quad c_2 = 0.002862 \text{ m}^3 / \text{kg},$$

and the gas state equation has the form

$$p / \rho = RT + b_2 R p.$$

In work [10], simple formulas were obtained for calculating the speed of sound  $a$  in a pipeline and the gas flow rate  $\dot{m}_{i,j}$  through throttles taking into account the change in the compressibility factor. In the gas compressibility equation for the  $i$ -th cavity of volume  $V_i$  the modulus of elasticity and the formula for the energy flow during mass exchange are specified:

$$\frac{V_i \dot{p}_i}{k z_i p_i} = -\dot{V}_i - R \sum_j \left[ \frac{b_2}{k} + \frac{\sigma_{i,j}}{p_i} \right] \cdot \dot{m}_{i,j} - \frac{(k-1)}{k p_i} \cdot \dot{Q}_i; \quad \sigma_{i,j} = \begin{cases} T_i + b_2(1-1/k) p_i, & \dot{m}_{i,j} > 0; \\ T_j + b_2(1-1/k) p_j, & \dot{m}_{i,j} \leq 0, \end{cases} \quad (1)$$

where  $\dot{Q}_i$  is the heat flow extending into the cavity walls. In the continuity equation:

$$\dot{m}_i = -\sum_j \dot{m}_{i,j}, \quad (2)$$

the mass flow  $\dot{m}_{i,j}$  is calculated taking into account the amendments to the formulas of Saint-Venant. The temperature is from the closure equation:

$$zT = T + b_2 p = p / (R\rho), \quad (3)$$

which for this model is linear.

Note that the temperature can be obtained in other way by integrating the energy conservation equation and from (3) the pressure can be found. But this approach is not used for the following reasons:

- it complicates the use of the method of invariants for calculation of pipelines;
- for pipelines of small cross section the equation for energy is replaced by isotherm formulas;
- the compressibility equation is considered to be more trustworthy and its solution is directly controlled in the experiment.

Unfortunately, for other gases from Table 1 the condition  $T \gg T_k$  is not fulfilled and the Abel model has an inadmissibly high error (Fig. 9, b - d). At the same time the maximum relative density  $\rho / \rho_k$  here is greater than that for helium, and reaches 1 (Fig. 5, a, Fig. 8).

For these gases the *Van der Waals model* is more accurate [11], where the dependence of the compressibility factor on density and temperature is the following:

$$z = 1 / (1 - c_2 \rho) - a_2 \rho / T.$$

The constants  $c_2$  and  $a_2$  are expressed in terms of the parameters of the critical point (Table 1); that is why using the Van der Waals equations would be a good solution to the problem. However, even for this model at tight densities  $\rho \approx \rho_k$  the noticeable deviations of the results from the table values given in monographs [12 - 17] are observed; in addition to this fact the Van der Waals gas has the heat capacity  $c_v$  that does not depend on pressure, which is not the case with the real gases from Table 1.

In [12 - 17] alongside the tables describing the thermodynamic properties, the so-called virial equations of real gas are given in the following form:

$$z = 1 + \sum_{i=1,2,\dots} B_i(T) \rho^i = 1 + \sum_{i=1,2,\dots} \sum_{j=0,1,\dots} b_{i,j} \rho^i T^{-j}. \quad (4)$$

These equations approximate the experimental data in a wide range of changing parameters including the area in

Fig. 1. But the sums in (4) contain about 50 terms and it is impossible to reduce this number without a serious loss of accuracy. The closure equation for the temperature  $T$  is no longer linear as (3) but of high degree (from 5th for argon to 7th for nitrogen and air) and its solution is laborious. That is why using virial equations for calculating the transient processes is considered irrational.

Analysis of these and many other models of real gas reveals a general pattern – they were compiled for domains rectangular in the coordinates  $p-T$  or  $\rho-T$ . When moving along the upper boundary of such a rectangle the decrease in temperature results in the drop of entropy, and the gas parameters approach the critical values. Then to ensure the high accuracy it is necessary to complicate the equation of state.

In the process of gas consumption from gas bottles the working fluid of the automation units preserves (and with noticeable heat exchange with the walls increases) the initial entropy  $s_0$ , which for all gases is higher than the entropy of the critical point. As the result, the adiabat of the upper boundary of the area in Fig. 1 passes further significantly below the critical point.

As a result of the above mentioned facts it is inexpedient to involve well-known models for solving the problem under consideration and it is necessary to develop a new model of real gas which in the best way is going to take into account these features. The authors called this new model *the orthometric high-entropic gaseous fluid* and its equation of state (abbreviated) – the

**OGF-equation.**

If it turns out that among the thousands of previously proposed equations of state of real gas its complete namesake is found then it will have to be remained but this will not affect the practical significance of the proposed solution.

**Orthometric equation of state of real gas.** The term orthometric state of gas was introduced into thermodynamics by *A. I. Bachinsky* [18] in the beginning of the 20th century. By orthometric, he understood such combinations of temperature and density for which the compressibility factor  $z$  equaled 1.

On the thermodynamic surface of the graph of the function  $z = f(\rho, T)$  the condition  $z = 1$  is obviously satisfied by the straight line  $\rho = 0$ , which corresponds to the ideal gas state. And also, what is not obvious but is a firmly established scientific fact, the condition is satisfied by the straight line (Fig. 10)

$$\rho / \rho_b + T / T_b = 1, \tag{5}$$

where  $T_b$  is the *Boyle temperature* and the density  $\rho_b$  is called the *Boyle density*. Their values are given in Table 2; the values of the associated pressure  $p_b = R \rho_b T_b$  and temperature of the inversion  $T_{in}$  are also shown there.

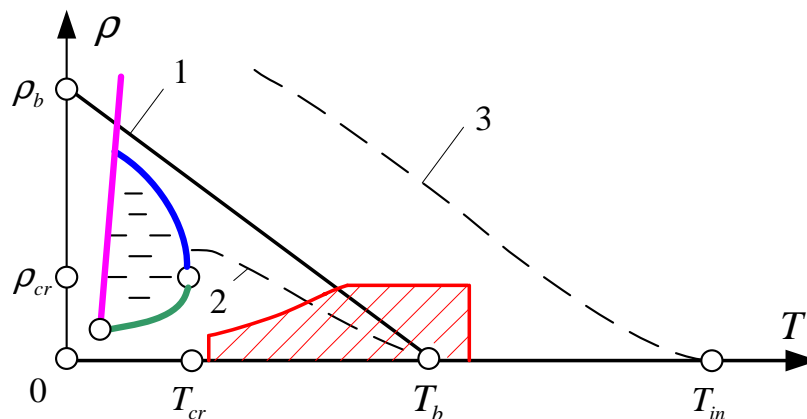


Fig. 10 – “Ideal curves” of a thermodynamic surface: 1 – ideal gas ( $z = 1$ ), 2 – Boyle ( $z'_p = 0$ ), 3 – Joule – Thomson ( $h'_p = 0$ ).

Bachinsky called the gas satisfying condition (5) an *orthometer* and attributed its equation of state to the *Van der Waals type* since Van der Waals gas turned out to be the simplest orthometric model. All gases from Table 1 are orthometers. The compressibility factor of any gas-orthometer is described by the formula:

$$z = 1 + \psi(\rho, T) \cdot \rho \cdot (\rho / \rho_b + T / T_b - 1), \tag{6}$$

where the function  $\psi(\rho, T)$  takes into account the individual characteristics of the gas and can be determined by processing the results of the experiment.

Acting in a similar way Bachinsky obtained the equations of state for isopentane and ethyloxide. The type of the functions  $\psi(\rho, T)$  turned out to be different and was not used by the authors of this work. Bachinsky could not get a one-type equation of state for any class of gases because during his life period the necessary experimental materials were unavailable.

Table 2 – Orthometric scales and inversion temperature

Real gas	$T_b$ , K	$\rho_b$ , kg/m <sup>3</sup>	$p_b$ , MPa	$T_i$ , K	$T_b \div T_k$	$\rho_b \div \rho_k$	$T_i \div T_b$
Nitrogen	325.2	1130	109.1	611.1	2.5769	3.6088	1.8792
Argon	408.2	1870	158.9	746.1	2.7089	3.4908	1.8278
Helium	24.21	211.8	10.65	48.4	4.6647	3.0419	1.9356
Oxygen	407.0	1549	163.8	786.6	2.6329	3.5517	1.9327
Krypton	566.6	3232	181.7	1036.4	2.7059	3.5414	1.8292
Xenon	791.0	3892	195.0	1446.9	2.7301	3.5064	1.8292
Methane	507.5	579.1	152.3	1010.0	2.6603	3.5416	1.9771
Air	342.1	1215	119.3	642.8	2.5819	3.8382	1.8790
Van der Waals model	–	–	–	–	2.6667	3.0000	$\infty$

The straight line of orthometric states is now called the *ideal gas curve*, by analogy with the Boyle, Joule – Thomson, etc. curves (Fig. 10), but its deviations from the straight line have not been detected in experiments. The authors [16] believe that its straightness is a consequence of physical law which is yet to be discovered.

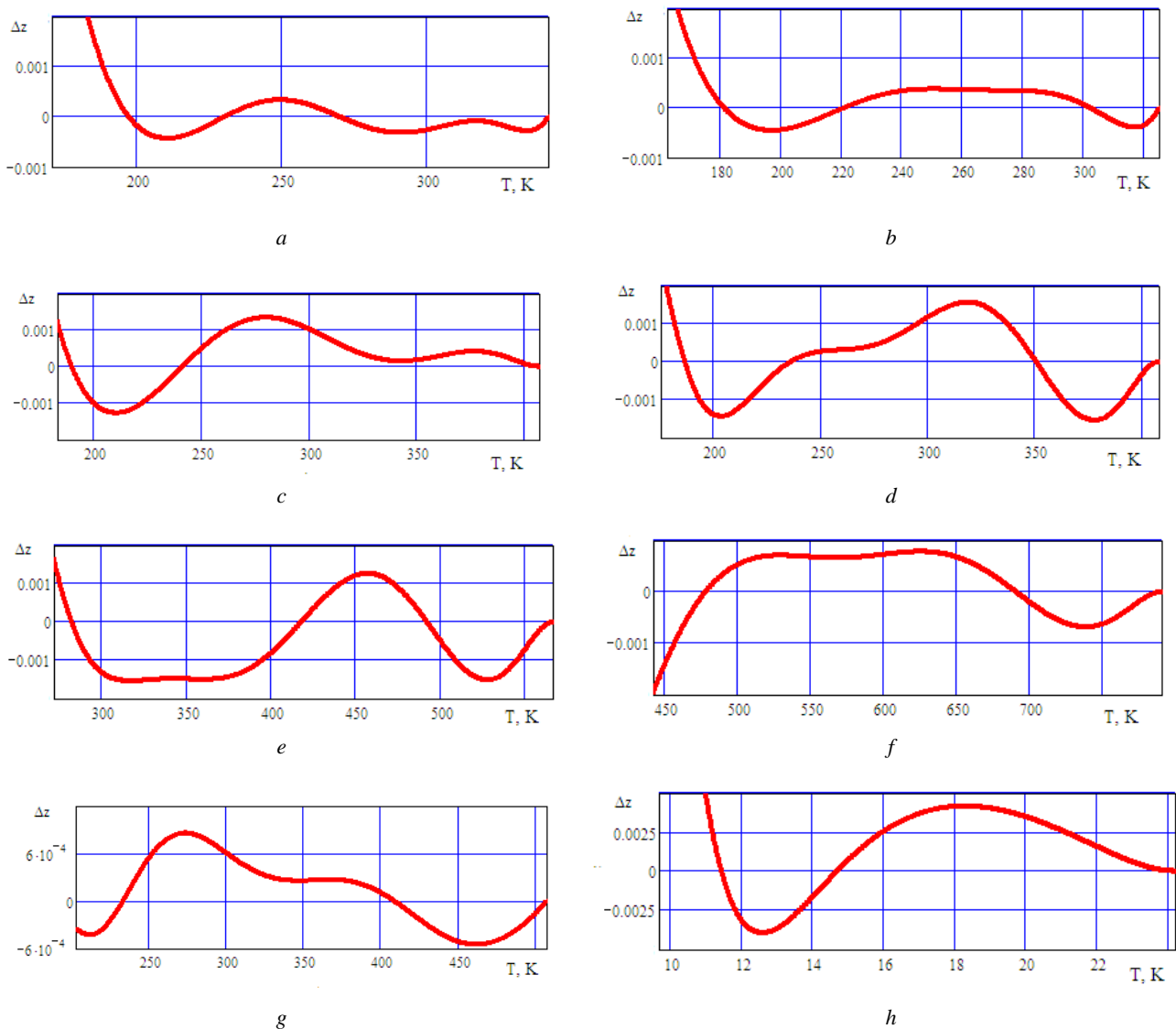


Fig. 11 – Approximation error of the ideal gas curve by virial equations: *a* – for air [13]; *b* – for nitrogen [15]; *c* – for oxygen [14]; *d* – for argon [16]; *e* – for krypton [16]; *f* – for xenon [16]; *g* – for methane [17]; *h* – for helium [12].

As for virial equations (4) from [12 – 17], this condition holds approximately as illustrated in Fig. 11. Moreover, for nitrogen, oxygen, and air, virial coefficients  $B_2(T)$  are negative at temperature  $T = T_b$ . This contradicts the requirements of [18] resulting from the physical meaning of these coefficients and complicates the analysis.

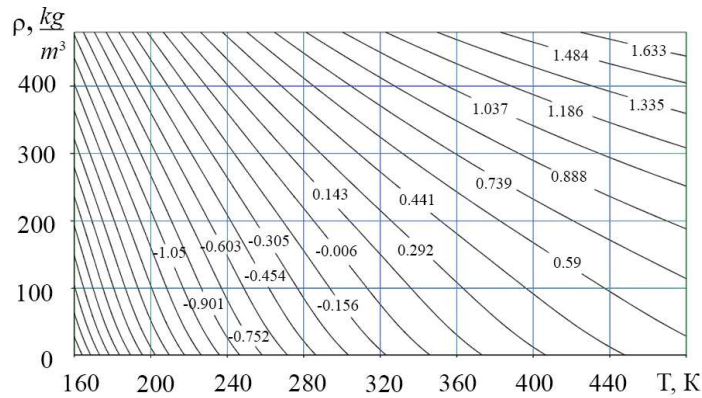


Fig. 12 – Thermodynamic surface level lines  $\psi(\rho, T)$  for the virial air equation [13].

In [16], a differential relation between virial coefficients in the vicinity of the Boyle temperature was noted:

$$B_j(T) \approx B_1^{(j-1)}(T) \cdot (T_b / \rho_b)^{j-1} / (j-1)!, \quad j = 2, 3, \dots \tag{7}$$

The consistent implementation of this idea leads to an approximate replacement of the thermodynamic surface

$$\psi(\rho, T) = (z(\rho, T) - 1) / \rho$$

by the ruled surface and results in the equality

$$z = 1 + \rho B_1(T + T_b \cdot \rho / \rho_b). \tag{8}$$

Note that formula (8) is a special case of (6). We believed that with reliable values of the virial coefficient  $B_1(T)$  it opened the way to creating the desired equation of state of gas. However, verification of equalities (7) and (8) using the data from works [12 – 17] dispelled these hopes. The accuracy of equality (7) is poor even for  $j = 2$  and formula (8) provides a good approximation of the table data only in a small vicinity of the ideal gas curve. Moreover, the best one was the ruled approximation of the surface for argon which is theoretically justified in [16].

The cause of failure lies in the errors of the empirical virial equations obtained in [12 – 17] by the least squares method as well as in the change in the slope of the level lines of the function  $\psi(\rho, T)$  (Fig. 12) which is not taken into account in equation (8).

In monograph [16] and in a number of other works, it is hypothesized that if we choose the end points of the ideal gas curve as scale values and change to dimensionless quantities

$$\omega = \rho / \rho_b, \quad \theta = T_b / T, \quad \pi_b = p / p_b$$

for density, temperature, and pressure, the equations of state of many gases will be the same. But this assumption is exactly fulfilled only for Van der Waals gases, which equation in the new (Boyle) coordinates becomes:

$$z = 1 / (1 - \omega) - \omega \theta;$$

or in orthometric form:

$$z = 1 + \frac{1}{1 - \omega} \cdot \omega (1 - \theta \cdot (1 - \omega)). \tag{9}$$

Fig. 13 shows the graphs of virial coefficients  $B_1(\theta)$  describing the cross section  $\omega = 0$  of the surface of the function

$$\psi(\omega, \theta) = (z(\omega, \theta) - 1) / \omega,$$

determining the deviation of gas from the ideal state. And Fig. 14 shows the graphs of the functions

$$\Omega_1 = (z(\omega, 1) - 1) / \omega^2,$$

which correspond to the cross section  $T = T_b$  of the thermodynamic surface for nitrogen, argon, oxygen, and air (the curves coincide for argon, krypton, and xenon).

As one can see there is no complete match, but for the gap  $\omega \in (0.07; 0.3)$  in Fig. 14 the differences are leveled by choosing an individual constant  $d$  that determines the vertical displacement of the graphs, and for the values  $\omega < 0.07$  in calculating the compressibility factor they are insignificant since here  $\Omega_1 \cdot \omega^2 \approx 0$  and  $z(\omega, 1) \approx 1$ . After aligning the curves they are approximated by the dependence

$$y = c / (1 - c \cdot \omega),$$

which generalizes the function

$$y = 1 / (1 - \omega),$$

used in (9), and for  $\theta \rightarrow 0$  provides the same asymptotics as the *Abel model*.



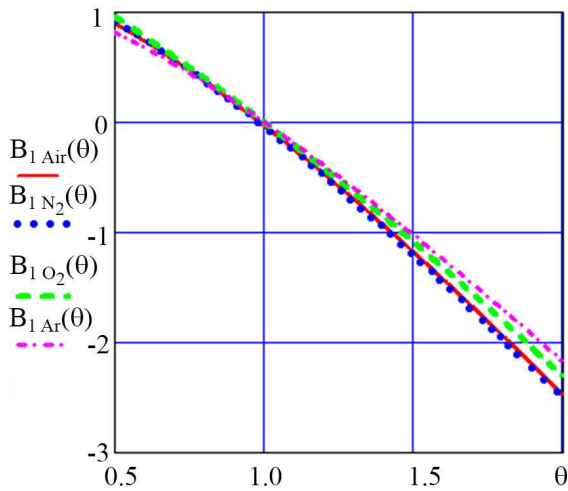


Fig. 13 – Graphs of virial coefficient  $B_1(\theta)$  for air, nitrogen, oxygen, and argon.

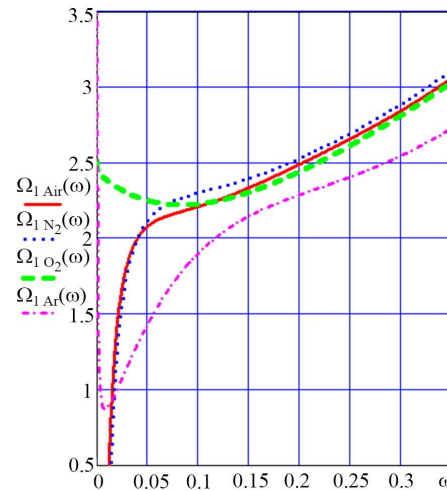


Fig. 14 – Sections  $T = T_b$  of thermodynamic surface for air, nitrogen, oxygen, and argon.

The values of the constants  $d$  and  $c$  are given in Table. 3; it turned out that for the 7 gases the constant  $c = 1.278$ . The value  $c'$  corresponding to the recalculation  $c$  on the critical density is also given. For air  $c' = 1/3$ , which is fully consistent with the theory of the Van der Waals gas, but for the other gases  $c' \neq 1/3$ . Let us change in (6) to the Boyle coordinates:

$$z = 1 + \psi_b(\omega, \theta) \omega(1 - \theta \cdot (1 - \omega)),$$

where, as follows from the previous argument,

$$\psi_b(\omega, 1) = c / (1 - c \omega) + d.$$

Based on this result and the known inversion temperatures  $T_i$  corresponding to the root of the equation

$$\theta B_1'(\theta) + B_1(\theta) = 0,$$

for all graphs shown in Fig. 6 a one-type approximation is derived:

$$B_1(\theta) \approx (c + d \cdot \theta^\gamma) \cdot (1 - \theta), \quad \theta \in (0.5; 2),$$

where  $\gamma = 0.45$ .

Table 3 – Constants of orthometric model

Gas	$c$	$c'$	$d$	$\gamma$
Nitrogen	1.278	0.354	0.800	0.45
Argon	1.278	0.366	0.500	0.45
Krypton	1.278	0.361	0.600	0.45
Xenon	1.278	0.365	0.600	0.45
Oxygen	1.278	0.360	0.670	0.45
Oxygen-HT	1.278	0.360	0.765	0.45
Methane	1.278	0.361	0.670	0.45
Methane-HT	1.278	0.361	0.716	0.45
Air	1.278	0.333	0.750	0.45
Helium-LT	0.578	0.190	0.400	0
Helium-HT	0.578	0.190	1.000	1.00

This means that the initial orthometric equation of state of gas has the form:

$$z = 1 + \left[ \frac{c}{1 - c \omega} + d \theta^\gamma \right] \omega(1 - \theta(1 - \omega)). \tag{10}$$

To simplify the solution of the closure equation an approximate replacement is performed in (10):

$$\theta^\gamma \approx \gamma \cdot \theta + 1 - \gamma. \tag{11}$$

Equation (10) was further tested using amendment (11); it is also recommended for practical use of the model.

The constants for the helium-LT equation (for  $T < 50$  K) are given in Table. 3 to compare results and generalizations; in automatic units such temperatures are not realized. A similar equation was obtained for helium-HT in the high-temperature range ( $T > 100$  K); its constants are also given in Table. 3

Comparing the results one notes that the values of the approximating coefficients  $d, \gamma$  that simulate the effects of

the association of atoms (formation and decay of short-lived dimers [19]) for helium undergo restructuring in the temperature range of 50 ... 100 K. The coefficient  $c$  determining the size of the force field of an individual atom does not change.

A similar restructuring of the model occurs with other gases from Table. 1 at  $T > 600 \dots 700$  K, that is why the results obtained cannot be extended to their high-temperature range.

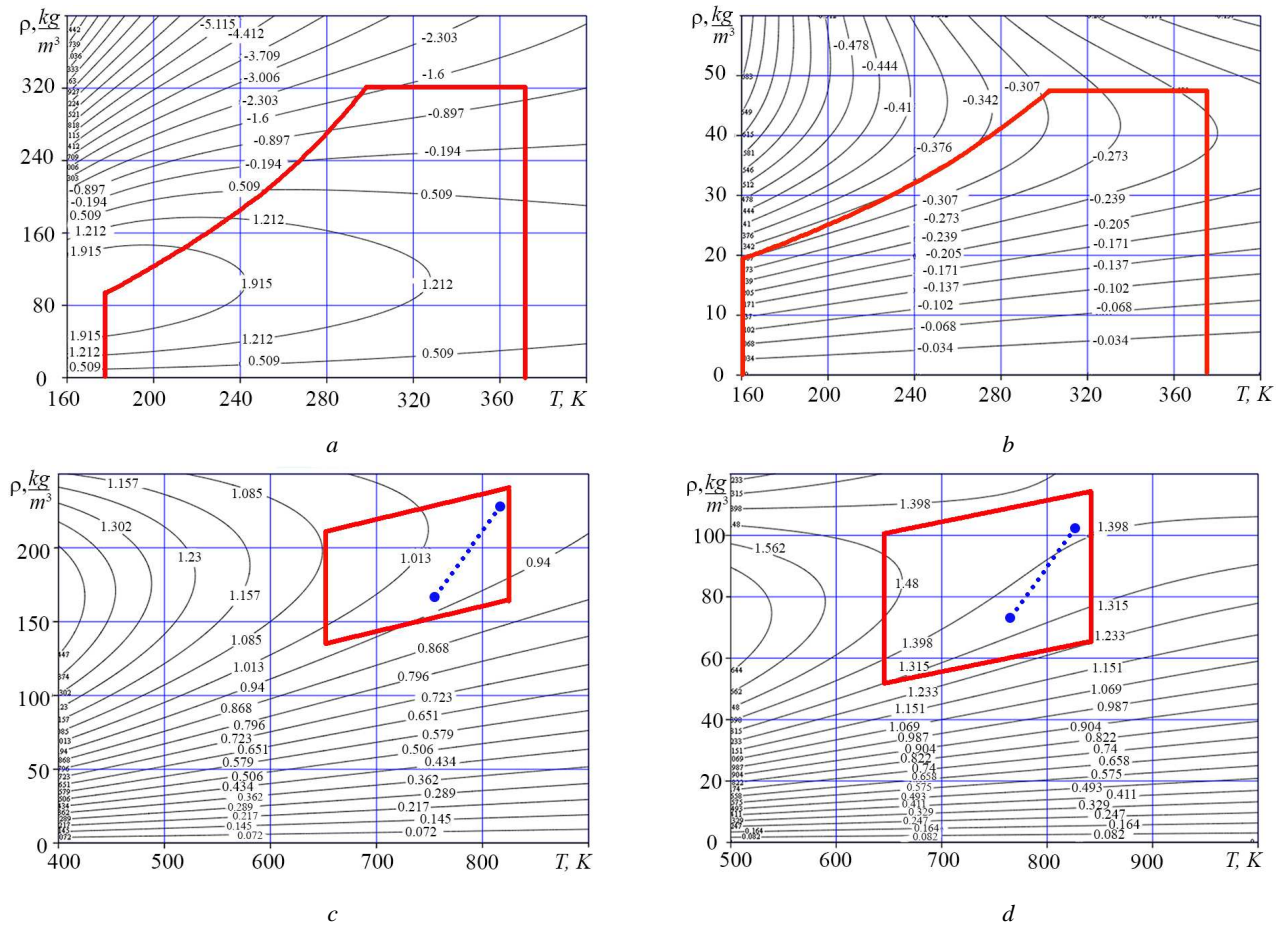


Fig. 15 – Deviations  $\Delta k_T$  of the thermal adiabatic index of real gas from the standard value of the indicator  $k_T$ , % :  
 a – for nitrogen; b – for helium; c – for oxygen; d – for methane.

**Thermodynamic functions of gas.** For helium and argon the heat capacities  $c_{v0}, c_{p0}$  in the ideal gas state do not depend on temperature and for nitrogen, oxygen and air in the range  $T = 160 \dots 360$  K this dependence is weak and can be ignored. As a result, the known formulas [20] for the calculation of specific heats  $c_v, c_p$  and enthalpy  $h$  are simplified and take the form:

$$c_v = R \cdot \left[ \frac{1}{k-1} + \int_0^\omega \theta^2 \cdot \frac{\partial^2 z}{\partial \theta^2} \cdot \frac{d\omega}{\omega} \right]; \quad c_p = c_v + R \cdot \left[ z\theta^2 / z_\omega \right]; \quad h = h_0^0 + c_{p0}T + (z-1)RT + RT_b \int_0^\omega \frac{\partial z}{\partial \theta} \cdot \frac{d\omega}{\omega},$$

where  $z_\theta = z - \theta \cdot \partial z / \partial \theta$ ;  $z_\omega = z + \omega \cdot \partial z / \partial \omega$ ;  $h_0^0$  – sublimation heat;  $c_{p0} = k \cdot R / (k-1)$ .

For convenience of integration and differentiation equality (10) is reduced to the form:

$$z = \frac{1 - \frac{c-1}{c} \cdot \theta}{1 - c \cdot \omega} + \frac{c-1}{c} \cdot \theta - \omega \cdot \theta + d \cdot \omega \cdot (\gamma \cdot \theta + 1 - \gamma)(1 - \theta + \omega \cdot \theta),$$

which implies

$$z_\theta = \frac{1}{1 - c\omega} + d\omega \cdot [\gamma\theta^2(1 - \omega) + 1 - \gamma]; \quad z_\omega = 1 - \frac{(c-1) \cdot (2 - c\omega)}{(1 - c\omega)^2} - 2\theta\omega - d\omega(\gamma\theta + 1 - \gamma) \cdot (2\omega - 2 - 3\theta\omega);$$

$$\int_0^\omega \theta^2 \cdot \frac{\partial^2 z}{\partial \theta^2} \cdot \frac{d\omega}{\omega} = 2\gamma d \theta^2 \omega \cdot (1 - \omega/2); \quad \int_0^\omega \frac{\partial z}{\partial \theta} \cdot \frac{d\omega}{\omega} = \frac{c-1}{c} \cdot \ln \frac{1}{1 - c\omega} - \omega - d\omega \cdot [(1 - \omega/2) \cdot (2\gamma\theta + 1 - \gamma) - \gamma].$$

Note that in the helium-HT model the coefficients  $d, \gamma=1$  and the results are significantly simplified:

$$z_\theta = \frac{1}{1-c\omega} + \theta^2 \cdot \omega \cdot (1-\omega); \quad z_\omega = 1 - \frac{(c-1) \cdot (2-c\omega)}{(1-c\omega)^2} - 2\theta\omega - \omega \cdot \theta \cdot (2\omega - 2 - 3\theta\omega);$$

$$\int_0^\omega \theta^2 \cdot \frac{\partial^2 z}{\partial \theta^2} \cdot \frac{d\omega}{\omega} = 2\theta^2 \omega \cdot (1-\omega/2); \quad \int_0^\omega \frac{\partial z}{\partial \theta} \cdot \frac{d\omega}{\omega} = \frac{c-1}{c} \cdot \ln \frac{1}{1-c\omega} - \omega - \omega \cdot [(2-\omega) \cdot \theta - 1].$$

From here the working formulas for calculating the speed of sound  $a$  are obtained:

$$a = \sqrt{z_\omega (c_p / c_v) RT};$$

and two adiabatic indices of real gas [21] are deduced:

$$k_\rho = \frac{c_p}{c_v} \cdot \frac{z_\omega}{z}; \quad k_T = \frac{(c_p / c_v) z_\theta}{z + (c_p / c_v)(z_\theta - z)}.$$

For ideal gas these indices are the same and equal  $k$ , and for real gas they are different and satisfy two differential adiabatic equations of different form:

$$\frac{d\rho}{\rho} = \frac{1}{k_\rho} \cdot \frac{dp}{p}; \quad \frac{dT}{T} = \frac{k_T - 1}{k_T} \cdot \frac{dp}{p}.$$

For the range of gas working parameters the indicator  $k_\rho$  varies widely and its change must be taken into account. Whereas the indicator  $k_T$  deviates from the value  $k$  no more than by 2% (for helium – by 0.4%) which can be neglected (Fig. 15). For this reason to calculate the gas flow through the throttle at critical pressure drops one can use the theory described in [10] and keep common with [1] recommendation to include a correction factor of the type  $1/\sqrt{z}$  in the *Saint-Venant formula*. The same correction can be applied in the case of non-critical pressure drops, taking into account the known errors of the flow coefficients.

**The errors of the model.** To test the OGF-equation, its thermodynamic functions were compared with tabular data from [12 – 17], and from [22] for argon, which defined the standards of the USSR and the USA. The mean (in absolute value) relative deviations of the compressibility factor  $\delta z$ , specific enthalpy  $\delta h$ , and speed of sound  $\delta a$  were calculated for 200 points from the domain of working parameters located on 10 polygonal chains (Fig. 1). The results are shown in Table. 4.

Table 4 – Average deviations from standard values, %

Gas	$\delta z$	$\delta h$	$\delta a$
Nitrogen	0.068	0.054	0.125
Argon	0.156	0.233	0.157
Oxygen	0.091	0.059	0.233
Air	0.067	0.040	0.082
Helium	0.032	0.041	0.057

In addition to the mean values the maximum deviations were controlled which turned out to be as small as the mean ones. The nature of the change of  $z$  error for nitrogen, methane, oxygen, and helium is shown in Fig. 16. The boundary of the domain of working parameters is superimposed on the level lines. Within this domain the deviations do not exceed  $\pm 0.1\%$  but at low temperatures and high densities (where the gas entropy is significantly lower than the value  $s_i$ ) they are large. Here the shape of the line of the error level is close to the adiabatic graph, which is the line of the entropy level. That is why it should be recognized that in order to maintain the high accuracy of the model it is the decrease in entropy that requires the complication of the equation of state of real gas.

The lowest error values for the OGF-equation were shown for helium; compared with the model of Abel, they decreased by an order of magnitude.

Note that the tabular data have their own errors as shown for example in Fig. 15. The greatest errors are observed in argon, which is explained by the smaller amount of experimental material. But even here the situation is improving and if the virial equations [16, 1975] and the US standard [22, 1999] are compared, the discrepancy (we are sure that the refinement) of the results averaged for the parameter range under research is:

$$\delta z = 0.064\%, \quad \delta h = 0.051\%, \quad \delta a = 1.22\%.$$

It should be noted that the OGF-equation has a calculated critical point at which the pressure is overestimated by about 20%. This point corresponds to the critical compressibility factor  $z_{cr} \approx 0.35$  which is 15% higher than for real gas but lower than the value  $z_{cr} = 0.375$  for Van der Waals gas. In the vicinity of the critical point the developed model

cannot be used but, as a rule, pneumatic automation units are not designed to work with a two-phase working fluid.

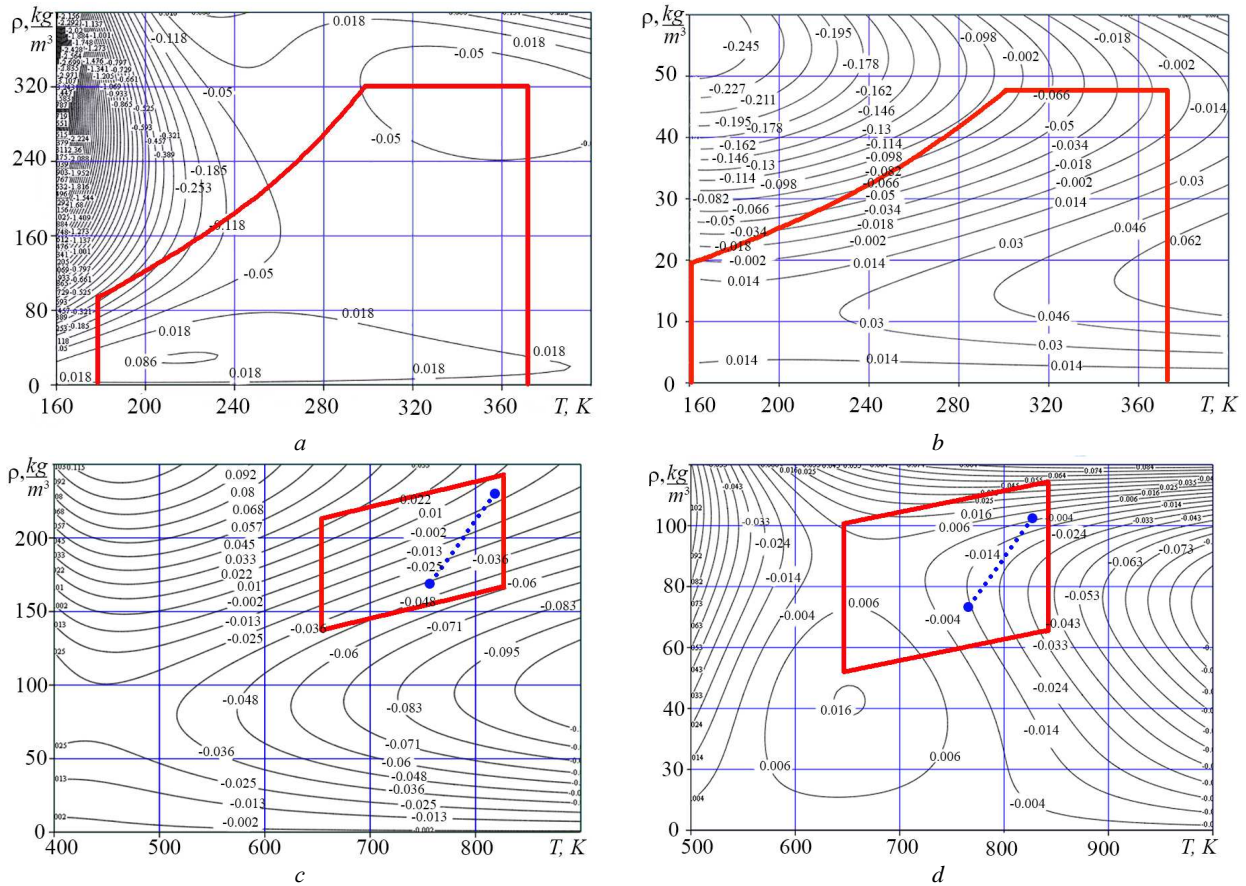


Fig. 16 – Change of the relative error of calculation  $z, \%$ :  $a$  – for nitrogen;  $b$  – for helium-HT;  $c$  – for oxygen-HT;  $d$  – for methane-HT

(the dotted line indicates the change of gas parameters in the turbopump assembly of the engine RAPTOR, SpaceX, USA).

**The compressibility equation of real gas.** Its formal derivation uses the equation of state and the first law of thermodynamics; we show its main points. The equation of state in differentials is:

$$\frac{dp}{p} = \frac{z_\omega}{z} \cdot \frac{d\rho}{\rho} + \frac{z_\theta}{z} \cdot \frac{dT}{T},$$

where  $z_\theta/z$  is the change in the thermal pressure coefficient and the ratio  $z_\theta/z_\omega$  describes the change in the coefficient of volumetric expansion.

On a short time interval any gas-dynamic process can be considered as the result of superposition of the adiabatic and isochoric processes. For adiabat and isochore we get

$$\frac{dp_{ad}}{k_\rho p} = \frac{d\rho}{\rho}, \quad \frac{dp_{is}}{k_\rho p} = \frac{z_\theta}{k_\rho z T} \cdot \frac{\Delta h \cdot dm - dQ}{c_v m},$$

where  $\Delta h$  is the difference of specific enthalpies between the gas in the cavity and the gas of small mass  $dm$  participating in the exchange;  $dQ$  is a small portion of heat going into the wall;  $m$  is the mass of gas in the cavity.

We add the left and right sides of the equations:

$$\frac{dp}{k_\rho p} = \frac{d\rho}{\rho} + \frac{z_\theta}{k_\rho c_v} \cdot \frac{\Delta h \cdot dm - dQ}{z m T}.$$

Analyzing the coefficients of the resulting equation we note that

$$z m T = \frac{pV}{R}, \quad \frac{d\rho}{\rho} = \frac{dm}{m} - \frac{dV}{V}, \quad \frac{z_\theta}{c_v k_\rho} = \frac{k_T - 1}{R k_T},$$

besides it is permissible to assume that  $k_T = k$ . As a result, for  $i$ -th cavity we get the equality:

$$\frac{V_i \cdot \dot{p}_i}{k_\rho \cdot p_i} = -\dot{V}_i + \frac{\dot{m}_i}{\rho_i} - \frac{k-1}{k \cdot p_i} \left( \sum_j \Delta h_{i,j} \cdot \dot{m}_{i,j} + \dot{Q}_i \right), \tag{12}$$

which is a generalization of equation (1).

When using equation (12), the heat of sublimation  $h_0^0$  in the formula for calculating the specific enthalpy is not taken into account because it does not affect the value of the enthalpy difference  $\Delta h_{i,j}$  between  $i$ -th and  $j$ -th cavities.

**Solutions of closure equations.** In Boyle coordinates pressure, density, and temperature are related by the equation

$$\frac{\pi_b \theta}{\omega} = 1 + \left[ \frac{c}{1-c\omega} + \gamma d \theta + (1-\gamma)d \right] \omega (1-\theta(1-\omega)). \quad (13)$$

Algebraic form (13) of the gas state equation is used in the method of dynamic calculation of the system for forming the initial conditions and numerical integrating the equations.

The initial conditions for each cavity are pressure  $p$  and temperature  $T$ , thus  $\pi_b = p/p_b$  and  $\theta = T_b/T$  and the density  $\rho = \omega \rho_b$  is determined from equation (13). After elementary transformations we get the equation:

$$A\omega^4 + B\omega^3 + C\omega^2 + D\omega + E = 0 \quad (14)$$

with known coefficients  $A, B, C, D, E$ .

Algebraic equation (14) is solved by either using the poliroots procedure of the Mathcad package or in the iterative cycle:

$$\omega_0 = 0; \omega_{j+1} = -\frac{E + A\omega_j^4 + B\omega_j^3 + C\omega_j^2}{D}, j = 0, 1, 2 \dots$$

To achieve the required accuracy it is necessary to have 10 iterations. Since the initial conditions are determined here, this task does not affect the overall complexity of the calculation method of the pneumatic system.

After integrating the differential equations in the Boyle values of pressure  $\pi_b$  and density  $\omega$ , from equation (13) the temperature is found. Thanks to approximation (11) this equation for  $\theta$  turns out to be quadratic, the positive root corresponds to the initial temperature. The simple solution of the problem, which is repeated millions of times for each cavity during the simulation of the pneumatic system, allows to escape the complexity of the calculation algorithm.

**Perspectives of further research.** The OGF-equation and its thermodynamic functions are used by the Yuzhnoye State Design Office for developing the new automation units for rocket technology; results valuable for science and practice will be published.

To simulate the conditions of super cold supercharging [2] of the fuel tanks of the Zenit-type launch vehicle the orthometric equation of state is refined to expand its field of use. Similar works concern the refinement of xenon orthometric equations which are relevant for the storage of this working fluid in a cooled liquefied state.

The third perspective direction is the development of similar equations for gas mixtures arising from gasification of oxygen and methane in the gas generators of the Raptor engine.

Besides, the orthometric equations of methane and its mixtures can be used in calculating the equipment used in the transportation of natural gas via high-pressure pipelines. Newly developed space scientific technologies need to be useful in solving problems on the Earth.

**Conclusions.** The domain of variation of the working fluid parameters for pneumatic automation units is formed and the requirements for the equation of real gas suitable for dynamic calculations are formulated.

The analysis of the known equations of state of real gas is performed and the reasons for which they cannot be used to solve the indicated problem are shown.

It is proposed to use the theory of A. I. Bachinsky of orthometric states of gas and the Boyle scaling of its parameters for obtaining a new equation.

The form of the initial equation is chosen and by using tabular data the values of its coefficients are obtained. It turns out that nitrogen, argon, krypton, xenon, oxygen, methane, and air have two same coefficients of the model (out of three) and this new result gives impetus to the development of the theory of real gas.

It is shown that in the domain of variation of the working fluid parameters all thermodynamic properties including caloric characteristics follow from its equation of state and simple formulas for calculating these functions are given.

The calculated characteristics of gas are compared to the tabular data; the average and maximum error do not exceed 0.1 %.

A refined equation of the compressibility of real gas in a cavity is obtained and the methods for solving the closure equations for density and temperature arising at the stages of the formation of initial conditions and integration of the compressibility and continuity equations are described.

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Received (надійшла) 08.09.2018

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УДК 539.3

**Е. Г. ЯНИУТИН, Г. А. ГНАТЕНКО, П. А. ЕГОРОВ**

**НЕСТАЦИОНАРНОЕ ДЕФОРМИРОВАНИЕ ПОДКРЕПЛЕННЫХ ЦИЛИНДРИЧЕСКИХ ОБОЛОЧЕК**

Предложен метод идентификации нестационарной нагрузки, воздействующей на шарнирно-опертые подкрепленные цилиндрические оболочки. Рассмотрены два случая: подкрепление деформирующегося объекта по всей длине охватывающей оболочкой, подкрепление ребрами жесткости, ширина которых мала по сравнению с длиной оболочки. В качестве вспомогательного этапа решения основной задачи приводится решение прямой задачи по исследованию деформированного состояния системы, достоверность которого подтверждается путем сопоставления с МКЭ. Достоверность решения задачи идентификации подтверждена путем сопоставления с исходными данными соответствующей прямой задачи. Интегральные уравнения Вольтерра, получаемые при решении задач, анализируются численно. Некорректность поставленных задач преодолевается с использованием метода регуляризации А.Н. Тихонова.

**Ключевые слова:** оболочка, ребро жесткости, условие контакта, ряд Фурье, преобразование Лапласа, метод регуляризации.

**Є. Г. ЯНИУТИН, Г. О. ГНАТЕНКО, П. А. ЕГОРОВ**

**НЕСТАЦІОНАРНЕ ДЕФОРМУВАННЯ ПІДКРІПЛЕНИХ ЦИЛІНДРИЧНИХ ОБОЛОНОК**

Запропоновано метод ідентифікації нестационарного навантаження, яке діє на шарнірно-обперті підкріплені циліндричні оболонки. Розглянуто два випадки: підкріплення об'єкту, що деформується, за всією довжиною оболонкою, що охоплює дану, підкріплення ребрами жорсткості, ширина котрих є малою у порівнянні з довжиною оболонки. У якості допоміжного етапу розв'язку основної задачі наводиться розв'язок прямої задачі з дослідження деформованого стану системи, достовірність якого підтверджується шляхом зіставлення з МСЕ. Достовірність розв'язку задачі ідентифікації підтверджена шляхом зіставлення з вихідними даними відповідної прямої задачі. Інтегральні рівняння Вольтерра, які отримані при розв'язку задач, аналізуються чисельно. Некоректність поставлених задач долається за допомогою методу регуляризації А. Н. Тихонова.

**Ключові слова:** оболонка, ребро жорсткості, умова контакту, ряд Фур'є, перетворення Лапласа, метод регуляризації.

**YE. G. YANYUTIN, G. A. GNATENKO, P. A. YEGOROV**

**NONSTATIONARY DEFORMATION OF REINFORCED CYLINDRICAL SHELLS**

The article suggests a method of identifying nonstationary load acting on hinged reinforced cylindrical shells. Two cases are considered: reinforcement of a deformable object along the entire length with enclosing shell, reinforcement with stiffeners whose width is small compared to the length of the shell. As a substep of solving the main problem, the solution of the direct problem of studying the deformed state of the system is given. The reliability of solving the direct problem is confirmed by comparison with FEM. The reliability of the identification problem solution is confirmed by comparison with the initial data for the corresponding direct problem. Volterra integral equations obtained in the process of solving the problems are numerically analyzed. The ill-posedness of the problem is succeeded with using Tikhonov's regularization method.

**Key words:** shell, stiffener, contact condition, Fourier series, Laplace transformation, regularization method.

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