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Euclidian Geometry: Proposed Lesson Plans to Teach Throughout a One Semester Course

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Proposed Lesson Plans to teach throughout a one semester course

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Overview

We provide several engaging lesson plans that would aid in the teaching of geometry, specifically targeting Euclidian Geometry, towards students of high school age. The audience of this piece would be high school or college students who have not yet had an introduction to geometry, but have completed the standard mathematical courses leading up to this point (i.e. algebra, elementary math, etc.). This being the case the lessons and concepts realized in Chapter 1 target a basic understanding of what Euclidian Geometry is and the subsequent chapters aim specifically at underlying properties of a geometry. The main source of reference for these lessons and this document is the book **Foundations of Geometry Second Edition** by Gerard A. Venema.

These lessons are laid out as individual lessons that could be taught at any given point of a class that was dealing with the topic of the lesson at the time. These lessons are snapshots of what would be happening in a classroom and the idea is that lessons and teaching happen in-between each of the individual lessons and ideas presented here. Each chapter will begin with a summary of the main concepts and big ideas to be addressed in the chapter. I then offer the general structure of the lesson and how it could be taught. This includes what the teacher would say in the lesson and student misconceptions and questions. My hope is that this document would act as a teaching resource for teachers looking for individual lesson plans to be implemented in their own classroom during moments that they feel are appropriate. A lesson in this paper should take one class period to teach, which I have timed out at an hour. Being that most class periods are about 45 to 50 minutes this can be shortened or it could be spread out over several days as needed and appropriate. I appreciate your reading of this document and wish you a lovely day,

Joseph Willert

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Lesson 1: The False Proposition Problem

Title of the Lesson: A False Proposition of Triangle Side Congruency

Technology Lesson: Yes

Length of Lesson: 30 minutes to 1 hour (This length may be extended or shortened as needed depending on the level of the class and the pace at which they work)

Targeted Audience: High School/ Intro Level College Geometry

Grade Level: 10^{th} - Undergrad

Lesson Source: Foundations of Geometry 2^{nd} Edition

Student Learning Outcomes:

- Be able to construct geometric objects according to writing.
- Be able to analyze and interpret a mathematical proof.
- Ability to locate areas of falsity or confusion in mathematical proofs.
- Be able to follow a mathematical proof from its assumptions to its conclusion.

Concept Statement:

The aim of this lesson is to help with an understanding of mathematical proof, as well as show how a reliance on diagrams or assumptions can lead us to false conclusions. We start with a statement that we should be able to immediately recognize as false, and then give a mathematical proof to show to the statement is true. Since we know the statement must be false something must be amiss within our proof. By following a proof and analyzing the steps students will gain a better understanding of the underlying concepts of geometry and an increased ability of problem solving.

Standards Addressed (Math: Common Core)

CCSS.MATH.CONTENT.HSG.CO.C.10

Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

CCSS.MATH.CONTENT.HSG.CO.D.12

Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle;*

constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

Handouts: A False Proposition Worksheet

False Proposition Worksheet

We know that the below proposition is false as we can construct scalene triangles such that no two sides are congruent. Knowing that this is false can you go through the proof and find the error that is present? What kind of error do you believe it is? Try solving this problem first using only a pencil and piece of paper. If you get no luck here trying using a compass and straightedge. If still no luck you may log onto the computer and open up geogebra to see if it helps in finding the error. Once you have identified the error let your teacher know so that they may review your work and how you went about doing it.

Problem

False Proposition. *If ΔABC is any triangle, then side $\overline{AB}$ is congruent to side $\overline{AC}$.*

Proof. Let l be the line that bisects the angle $\angle BAC$ and let G be the point at which l intersects $\overline{BC}$. We know that either l is perpendicular to $\overline{BC}$ or it is not.

If l is perpendicular to $\overline{BC}$ then we know that $\Delta AGB \cong \Delta AGC$ from our ASA (angle-side-angle) congruence property of triangles. Since these two triangles are congruent then we know that $\overline{AB} \cong \overline{AC}$ because of CPCTC (corresponding parts of congruent triangles are congruent).

If l is not perpendicular to $\overline{BC}$ then we can let m be our perpendicular bisecting line of $\overline{BC}$ and M be our midpoint of $\overline{BC}$. Since m is now perpendicular, and l is not; we know that m is not equal or parallel to l. Therefore l and m must intersect at some point (due to all lines intersecting in at least one point if not parallel) which we can label D. Now we can draw perpendicular lines from D to the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ (The full lines extending off from the sides of our triangle). We can call the points of intersection of these perpendicular lines with $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ E and F, respectively.

There are now only three cases for the location of D: it is either on our triangle, inside our triangle, or on the outside of our triangle. We can look at each of these cases separately.

If D is on our triangle, then we know that $\Delta ADE \cong \Delta ADF$ by AAS (angle-angle-side). We then get that $\overline{AE} \cong \overline{AF}$ and $\overline{DE} \cong \overline{DF}$ from CPCTC. We also know that $\overline{BD} \cong \overline{CD}$ since D = M and M is the midpoint of $\overline{BC}$. We then obtain from the Hypotenuse-Leg Theorem (if we have two right triangles that have a congruent hypotenuse and a congruent leg, then the triangles are congruent) that $\Delta BDE \cong \Delta CDF$ so $\overline{BE} \cong \overline{CF}$ by CPCTC. We then get that $\overline{AE} + \overline{EB} \cong \overline{AF} + \overline{FC}$ so $\overline{AB} \cong \overline{AC}$.

If D is inside of our triangle then $\Delta ADE \cong \Delta ADF$ by AAS just like before. Again we get that $\overline{AE} \cong \overline{AF}$ and $\overline{DE} \cong \overline{DF}$. Also $\Delta BMD \cong \Delta CMD$ by SAS (side-angle-side) so $\overline{BD} \cong \overline{CD}$. We also get $\Delta BDE \cong \Delta CDF$ using the Hypotenuse-Leg Theorem like before, so again $\overline{BE} \cong \overline{CF}$. Therefore; $\overline{AE} + \overline{EB} \cong \overline{AF} + \overline{FC}$ so $\overline{AB} \cong \overline{AC}$.

Our last case is if D is outside of our triangle. We still have $\Delta ADE \cong \Delta ADF$ by AAS and $\overline{AE} \cong \overline{AF}$ and $\overline{DE} \cong \overline{DF}$. Also like before $\Delta BMD \cong \Delta CMD$ by SAS and $\overline{BD} \cong \overline{CD}$. Using Hypotenuse-Leg Theorem gives us $\Delta BDE \cong \Delta CDF$ and again $\overline{BE} \cong \overline{CF}$. Now we can see that $\overline{AE} - \overline{BE} \cong \overline{AF} - \overline{CF}$ so $\overline{AB} \cong \overline{AC}$.

Materials List: Worksheet, Pencil, Paper, Markers, Whiteboard, Compass, Straightedge, access to a computer and the program geogebra.

Geogebra Construct that shows why the proof is flawed: https://ggbm.at/bngmxnqz

Advanced Preparations:

This lesson would be best taught to a group of students whom were already familiar with the concept of a mathematical proof. While this wouldn't necessarily be critical it would help immensely and if there was no familiarity with proof this lesson should be presented slower and step by step as a whole class activity. Students will also need a basic familiarity with the standard geometric symbols presented in the proof and the meaning of each. They should also know how to create geometric constructs such as an angle bisector or perpendicular using a straight edge and compass. Finally, they should have at least some familiarity with geogebra before using it for their constructs, a small lesson or in class help here could be used to obtain this familiarity.

ENGAGEMENT		**Time: 10 minutes**
What the Teacher Will Do	**Probing Questions & Answers**	**What the Student Will Do**
Explain the Big Idea behind the lesson today.	Today we are going to be exploring mathematical proofs and a geometric construct. Mathematicians use proofs to help show that theorems and ideas work and these theorems are used to improve our world. By proving something in math we know that it will work every time and not just for a specific shape or set of numbers we have.	• Ask any questions they have about the lesson. • Begin thinking about mathematical proof and why it might be important. • Do we need proofs? Can everything be proved?
Ask if anyone has had experience writing proofs or if they can think of an example of where they may have seen proofs before.	Has anyone in class seen a proof before or written one themselves? Can you think of places or things in the world where we use the word proof?	• Generate ideas about where they may have seen proofs or used them. • Law Enforcement has to prove things, math book have proofs, Your ID proves who you are.

EXPLORATION	Time: 30 minutes	
What the Teacher Will Do	**Probing Questions & Answers**	**What the Student Will Do**
Hand out the "False Proposition" Worksheet and have students break up into groups of 3-4.	I am now handing out our worksheet for today. It's titled a false proposition; does anyone know what that means? Why is our proposition false? If our proposition is false can our proof for it be true? If our proposition is false, then that means something in our proof must be false because we cannot prove something that isn't true. Today's lesson is to find out if you can figure out where the error is in the proof I handed out. You will work in groups of 3 to 4 and be able to generate ideas on what in the proof is incorrect.	• Ask any questions they have on the worksheet. • Break up into groups of 3 or 4. • Answer questions about propositions and proofs. • "A proposition is like a statement" • "I don't know what that is"- Talk about what it is • "Our proposition is false because we can have triangles without equal sides" • "False because all sides of a triangle are equal" –Discuss why this isn't true • "It's not false" – Discuss why this isn't true. • "We can't prove something if it isn't true" • "Yes our proof can be true" – Go over why we can't have a true proof if proposition is false.
Introduce the idea of going about the problem 3 ways.	For today's lesson I want everyone to try and approach this problem 3 different ways. First I want you all in a group to figure out if you can see what is wrong using only	• Ask questions about what they are doing. • "Can we use geogebra right away?" • "How do we use a compass?"

	pencil and paper. After about 10 minutes you can begin using a compass and straight edge if you'd like. After another 10 minutes as a group you can try constructing shapes in geogebra to see if that helps you find an issue in the proof.	• "Do we get to choose our own groups?"- this is up to the teacher to decide depending on the class.
Have extra work ideas for students who finish early.	If there are students that finish early you could ask them how they would change the proof to make it more accurate. Does this proof work for isosceles triangles? Why or why not?	• Think about ways to change the proof or correct it. • "We could take out the parts about the points falling inside or outside." • "It works for isosceles because they have equal sides which is what we are proving." • "It doesn't work because it's a false proof and we can't use it." – Go over how even something false can have valuable information.

EXPLANATION/ELABORATION		**Time: 10 minutes**
What the Teacher Will Do	**Probing Questions & Answers**	**What the Student Will Do**
Go over different ways that students went about solving the problem.	I saw some really good different attempts at solving this problem. Several groups tried making the triangles on geogebra or with their rulers to see if something was awry. I'd like to have the groups present what they did in front of the class and share with us what they learned.	• Listen to different ideas that students tried.

Have students present their findings in groups in front of the class with different things they tried and found.	Would any group like to volunteer to come up first? – If no volunteers can start calling on groups to present. Ask questions about what they tried and what they found out. Do any students have any questions for the groups?	• Talk about what they did as a group and what they found out. • “We tried making the triangles and drawing the parts of them.” • “We thought that the proof was worded badly so that’s why it was false.” • “We tried making different sizes and shapes of triangles.” • “We used geogebra and the triangles wouldn’t work.”
Show the students a geogebra construct and explain why the proof is false. i.e. that the diagrams cannot be made in such a manner and they are where we get our error.	Present a pre-made construct of the proof. Show that the intersection points don’t rest entirely on the inside or outside of the triangle but on both. Explain how this shows the diagram is misleading.	• Ask questions they have about the diagram and how it was made. • “How did you make that?” • “Doesn’t it sometimes work?”

EVALUATION		**Time: 10 Minutes**
What the Teacher Will Do	**Probing Questions & Answers**	**What the Student Will Do**
Ask the students what they believe went well and what could have gone better.	What do you all think we did well today as a class? How could we improve for next time?	• Talk about what they thought went well and what didn’t. • “I liked that we got to work in groups.” • “I didn’t understand how the proof worked.” • “I think we should have had more help at the beginning.”

		• "I feel like I understand proofs a little better now." • "Next time you should bring in treats for the class."
Start a conversation with the students on anything they learned about formulating or reading proofs.	Could someone share with me something that they learned about proofs today? What about constructing things in geometry?	• Share ideas about what they learned in proofs and constructions. • "Constructions are hard to make without tools." • "Proofs take a lot of work to create." • "Proofs can look true but still be false." • "Geometry is a tough subject." • "I learned how to make triangles in geogebra."
Leave with the idea that we shouldn't always believe everything we're told and we should use our abilities to question things as much as we can.	I hope that everyone had fun with today's lesson and we can learn something valuable from it. We can't always believe everything we see and we should try out best to question things that don't seem to make sense of us. Just like in math in life we can also try to prove things to ourselves but shouldn't be persuaded just by people showing us something.	• Students can ask any final questions or state any final thoughts they had about the lesson.

Lesson 2: A Truncated Pyramid

Title of the Lesson: Finding the Volume of a Truncated Pyramid

Technology Lesson: No

Length of Lesson: 30 minutes to 1 hour (This length may be extended or shortened as needed depending on the level of the class and the pace at which they work)

Targeted Audience: High School/ Intro Level College Geometry

Grade Level: 10th - Undergrad

Lesson Source: Foundations of Geometry 2nd Edition

Student Learning Outcomes:

- **Gain experience with the concept of volume and how it relates to the dimensions of an object**
- **Understand how to apply volume formulas to objects and where those formulas come from**
- **Gain confidence in testing out different ideas and problem solving.**

Concept Statement:

This lesson is about figuring out how to apply information that the students already know (volume of a pyramid with a square base) to gather new information about the world around them (volume of a truncated pyramid). To successfully do this, students have to be able to break more complex objects down into ones that they are familiar with, and manipulate the properties of these objects to best suit their needs. This lesson also helps in gaining familiarity with algebraic manipulation and using variables to represent information.

Standards Addressed (Math: Common Core)

CCSS.MATH.CONTENT.HSG.GMD.A.1
Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments*.

CCSS.MATH.CONTENT.HSG.GMD.A.3
Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

CCSS.MATH.CONTENT.HSG.MG.A.1
Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

Handouts: Math Worksheet Lesson 2

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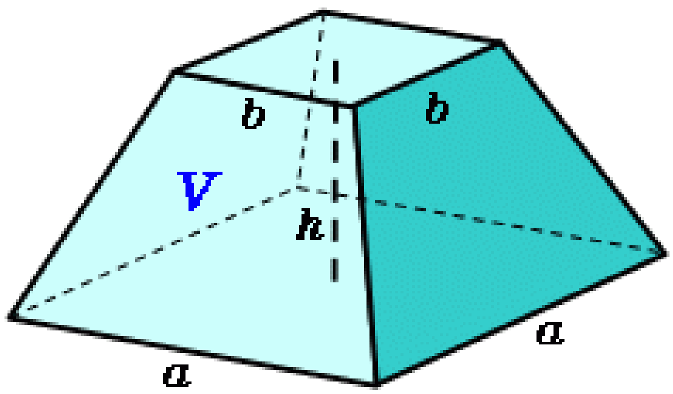
Volume of a Truncated Pyramid

Ancient Egyptians really enjoyed their pyramids. According to the *Moscow papyrus* the Egyptians developed a formula for finding the volume of a truncated pyramid with square a base. This formula is give below:

$$V = \frac{h}{3}(a^2 + ab + b^2)$$



In this formula, the base of the pyramid is an $a\ x\ a$ square, the top is a $b\ x\ b$ square and the height of the truncated pyramid is h. All of these measurements can be seen in the image above. You have previously learned that the volume of a pyramid is one-third the area of the base times height ($v = \frac{1}{3}a^2h$). Use this fact along with geometric properties you know to verify that the Egyptian formula is correct. When might this formula have been useful to the Egyptians?

Materials List: Worksheet, Pencil, Paper, Markers, Whiteboard, possible solution

Possible Solution: A possible solution to this problem would be to take the volume of the total square pyramid and subtract the smaller pyramid that is missing. The idea would be that the total pyramid would have some height h+x where x is the missing length. We can then set up a similarity relation which is that x/b = (h+x)/a. Solving this for x we obtain x = (bh)/(a-b). We can then use the formula V=(1/3)a^2*total height to get V = (1/3)a^2*(h+x)= (1/3)a^2*(h+bh/(a-b)) which we can simplify down to the proper formula through algebra.

Advanced Preparations:

This lesson is a really great lesson to introduce to students who are just starting to find out about volume and how it relates to the dimensions of an object. They should already be able to find the volume of several shapes (sphere, cylinder, box, pyramid, etc.) and this lesson should be thought of as an extension on what they already know. This lesson may be too easy for certain students and could potentially not last very long. If this is the case have other activities that address volume on standby.

ENGAGEMENT		**Time: 10 Minutes**
What the Teacher Will Do	**Probing Questions & Answers**	**What the Student Will Do**
Introduce the lesson of the day and explain what the students will be doing.	"Today we are going to be tackling a lesson on figuring out the volume of a truncated pyramid. To do this we are going to want to use what we already know about the volume of a pyramid and apply this into a pyramid with its top cut off, called a truncated pyramid."	• Ask any questions they have about what they are doing that day. • "What is a truncated pyramid?" • "How do we find volume of a normal pyramid?"
Get the students involved by thinking about volume and how it relates to objects around us.	"We know that the volume of an object is how much space that object takes up in the world. Math allows us to	• Answer questions about volume and connections to real world problems.

	calculate this space and use it in different ways. Why might we want to know the volumes of different objects? How would this information be useful to us?”	• “Volumes help us know where to place objects.” • “They can help with how much space we have.” • “They can help tell us how much we need to fill an object or build an object.”

EXPLORATION		**Time: 20 Minutes**
What the Teacher Will Do	**Probing Questions & Answers**	**What the Student Will Do**
Pass out the worksheet for finding the truncated volume of a pyramid. Also, pass out whiteboards and markers for the students to work with.	“Here are all of the materials that we are going to be using today. You are also welcome to use geogebra and if you have an idea that you would like to test out just let me know about that as well.”	• Begin working on their worksheets and trying to solve for the volume of the pyramid. • Ask for materials if they need them.
Walk around the room and make sure that students are staying on task. Answer any questions they may have and help students that appear to be struggling.	“Raise your hand if you’d like me to come by and answer a question or look at what you are doing.” “What kind of shape is our truncated pyramid? How is it that we are truncating it? What does this tell us about our new pyramid?”	• Ask questions they have about the assignment or their progress. • “How can I find the volume of this?” • “Can I break up the pyramid into smaller shapes?” • “How do I find the length of this part of the pyramid?”

EXPLANATION/ELABORATION		**Time: 10 Minutes**
What the Teacher Will Do	**Probing Questions & Answers**	**What the Student Will Do**

Gather the students and go through how to find the volume of the truncated pyramid systematically.	"You all did a good job today trying to find the volume of a shape that you may not have been familiar with. Now as a class we are going to go through and see how to solve for the volume together."	• Be prepared to answer questions about finding the volume.
Have students volunteer and come up to the board in turns to show how to go about finding the volume.	"Who would like to come to the board and help with the first step of finding the volume of this shape? What is the volume of a normal pyramid? What do we do to our normal pyramid to get our truncated pyramid? How does this affect the volume?"	• Take turns coming up to the board to answer the questions. • "Volume is 1/3 base times height." • "We cut our pyramid to get truncated." • "It makes the volume smaller."

EVALUATION		**Time: 5 Minutes**
What the Teacher Will Do	**Probing Questions & Answers**	**What the Student Will Do**
Check in with the students and see what they have learned about volumes of shapes and how they can be found.	"Would someone be willing to share with me what they learned about volume today? How can we apply what we've learned about volumes to real world situations? How does thinking through a problem in this way help us outside of mathematics?"	• Answer questions about volumes and their use in real world situations. • "I learned that volume is how much space something occupies." • "In the real world volume is seen in things like gas tanks, boats, cars, milk, etc." • "Can't apply what we know outside of math." • "Thinking like this helps us solve problems we run into in our lives."
Offer additional shapes that students can try and find the	"I've prepared some additional objects that you can	Students will take the homework and ask any final

volume of when they go home as either homework or an extra credit activity.	take with you and try to find the volume of. The first 5 are homework and the last 2 are extra credit."	questions they have over the lesson or the work.

Lesson 3: Proving Congruence Properties of Triangles

Title of the Lesson: Proving AAS and ASA triangle congruence properties.

Technology Lesson: Yes

Length of Lesson: 30 minutes to 1 hour (This length may be extended or shortened as needed depending on the level of the class and the pace at which they work)

Targeted Audience: High School/ Intro Level College Geometry

Grade Level: 10th - Undergrad

Lesson Source: Foundations of Geometry 2nd Edition

Student Learning Outcomes:

- **Students will be able to explain why 2 angles and 1 side determine a triangle.**
- **Students will be able to determine if 2 triangles are congruent given two angles and a side.**
- **Students will be able to write a mathematical proof that addresses geometric principles and constructs.**
- **Students will know the congruence properties of triangles.**

Concept Statement:

The goal of this lesson is to help familiarize students with triangle congruency and have them be able to find out for themselves why congruence properties of triangles work. Many students memorize properties such as these and try to apply them where they can but never really understand why they work or how they come about. By constructing their own proofs and trying to figure out how two angles and a side can determine congruency of a triangle, students will be building their own mathematical understanding of geometry and congruency. This lesson will also help them get used to reading and writing mathematical proof statements and trying to construct one that is both understandable as well as neatly presented.

Standards Addressed (Math: Common Core)

CCSS.MATH.CONTENT.HSG.SRT.B.4

Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*

CCSS.MATH.CONTENT.HSG.SRT.B.5
Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures

CCSS.MATH.CONTENT.HSG.CO.B.8
Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Handouts: Math Worksheet 3

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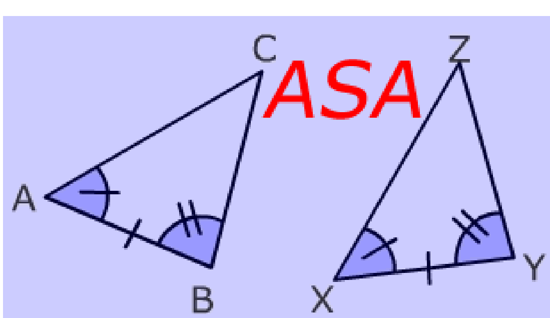
Proving Congruence Properties of Triangles

By now you may have heard about several congruence properties of triangles. These are properties that we can use to prove that one triangle is congruent to another, that is to say they occupy the same area in space. Today's lesson is to try and prove the Angle-Side-Angle (ASA) and Angle-Angle-Side (AAS) congruence properties of triangles. The idea is to see if you can show that two triangles must be congruent given two of their angles and one of their sides. These two properties are given below. To prove this try starting out with what you know and writing what you want to show. How do these triangles relate to each other? And how can we use this to convince ourselves they are congruent? You may use Geogebra, White Boards, and Paper to try out some ideas and see if you can prove these two theories below. Good Luck!

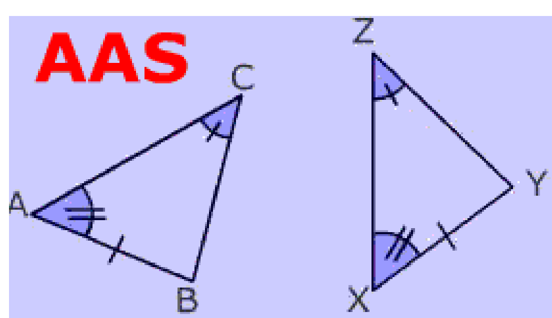
ASA: If two angles and the side between them of a triangle are congruent to the corresponding parts of a second triangle, then these two triangles are congruent.

AAS: If two angles and the side following these two angles of a triangle are congruent to the corresponding parts of a second triangle, then these triangles are congruent.

ASA Congruence Property



AAS Congruence Property



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Materials List: Math Worksheet 3, Pencil, Paper, Markers, Whiteboard, Compass, Straightedge, access to a computer and the program geogebra.

Possible Solution: One way of proving the ASA congruence properties of triangles is to overlay the triangles one on top of the other. By superimposing 2 triangles together, and allowing for one angle, side, and angle to be congruent, we can then go about assuming that they are not congruent triangles. We could have statements about the properties and relationships the two triangles must have if they are not congruent and reach a point of contradiction. Here we would have our proof that the triangles must be congruent. Proof by contradiction would also work for AAS as well.

Advanced Preparations: This lesson should be taught to students after they've already been exposed to similarity and congruence properties of triangles. The idea of this lesson is to further solidify why these properties prove triangle congruency, as well as help with mathematical proof writing. This lesson could be taught before students had encountered these properties but if this was the case extra attention should be given to what these properties are and how they are applied in triangles. Assistance should also be offered for writing proofs and helping the students through their proof writing and writing their own ideas in mathematical terms. Offer challenges and questions as to what the student knows and how they can show this through their words.

ENGAGEMENT		**Time: 10 Minutes**
What the Teacher Will Do	**Probing Questions & Answers**	**What the Student Will Do**
Introduce the lesson on Triangle Congruency and what the students will be working on that day.	"Today we are going to be trying to prove some of the triangle congruency properties we've been working with. Now that we know how to apply ASA and AAS we can see if we can figure out why we are allowed to do that in a mathematical way."	• Ask any questions they have about the lesson of the day. • Ask questions on ASA or AAS if they are confused with those properties.
Talk to the students about proof writing and why it is applicable to mathematics.	"Proof writing is important to mathematics as it allows mathematicians to show that	• Come up with ideas about where proofs might be needed in

Ask if other types of fields have similar writing structures.	something is true. In math, we cannot accept a property or theorem as true until we have proved it mathematically. Are there other fields out there where they have to prove things? What other types of jobs might you need to come up with a proof for?”	other jobs or areas of life. • “Lawyers use proofs to show if someone is guilty or innocent.” • “Editors of books and newspapers have to ‘proof’ read something.” • “Scientists have to have proofs to show their conclusion is correct.”

EXPLORATION		**Time: 20 Minutes**
What the Teacher Will Do	**Probing Questions & Answers**	**What the Student Will Do**
Pass out the worksheet for proving congruence properties.	“This worksheet will explain the 2 different congruence properties you are going to work on proving today. I’ll be breaking everyone up into groups and walking around so if you have any questions just let me know.”	• Collect the worksheet for the day. • Ask any questions about the worksheet or the lesson plan.
Split the students up into groups so that they may work in groups on the assignment.	“Ask each other questions and help each other out as you go through this. Don’t forget about what we need in order to prove that something is true. How can we know if we’ve proved something or not?”	• Get into groups and begin the worksheet. • Answer questions the teacher asks. • “We know if something is proved once our conclusion is true and everyone agrees with it.” • “We know we’ve proved something if it’s correct.”
Walk around the room answering questions and monitoring progress.	“What should you do after this step? Have you proved this property yet? How do you	• Students will ask questions they have

	know this about these 2 triangles?”	about the assignment or proofs. • “How do I do this?” • “Should I try to prove that these sides are congruent?” • “I don’t know what I’m doing here.” • “Oh I need to prove this next.” • “I know that these two sides are the same.”

EXPLANATION/ELABORATION Time: 10 Minutes		
What the Teacher Will Do	**Probing Questions & Answers**	**What the Student Will Do**
Bring the students back into a full class conversation about proof writing.	“How is writing a mathematical proof different from other types of writing? What do we have to do when we write a proof that we normally wouldn’t have to do with other types of writing? How does the structure of a proof help us understand what is going on?” In math our proofs are founded on axioms, and we have to show how we got from one step to another in a logical order.	• Talk about proof writing and how it is successfully done. • “Mathematical proofs have to have more behind them.” • “Math proofs are harder.” • “We have to try and explain how we know things.” • “We have to have a specific structure.” • “The structure tells us where we got our information from.”
Ask the students what they learned in their groups.	“Could I get a few volunteers to tell me something they learned about congruence properties or proofs in their groups today. How was this	• Students will answer questions about what they learned in groups. • “I learned why SAS is a property that proves

	activity valuable to your understanding of triangles?”	triangles are the same.” • “I didn’t learn anything.” • “I learned how to write a proof.”
Walk through how to prove the ASA congruence property as a class.	“Let’s now try to prove the ASA property as a class. Could I get a volunteer to start us out with this proof?” “Great where do we go from there? What would our next step be?”	• Students will take turns working through the proof with the teacher. • “I’ll volunteer.” • “We start by stating what we know about the triangles.” • “We want to try and show that this side is the same.” • “We can see that if we put this triangle on top of this other one we get two triangles.” • “I don’t know what to do next.”

EVALUATION		**Time: 5 Minutes**
What the Teacher Will Do	**Probing Questions & Answers**	**What the Student Will Do**
Ask the students if they made any discoveries about triangles or proofs from the assignment that day.	“Did anyone learn anything new about triangles or proofs today? Could someone share something that they learned with me today? Why might knowing how to prove something be a valuable skill for us?”	• Students will answer questions about what they learned. • “I learned how to write a proof.” • “I learned that making proofs is hard.” • “I learned why we can use ASA.” • “Proving things is valuable in life because sometimes we have to show

		something is true and a proof is a good way to do it." • "This doesn't apply to real life."
Either offer additional proof writing as homework, or transition into using those proofs to solve problems and proving that triangles are congruent.	"I have some homework for everyone to work on that involves using triangle congruence properties to show that triangles are congruent. Have a great day and great job." One possible type of homework here would be to have students write different combinations of angles and sides, and see which ones give actual congruence rules. By trying all different configurations they can get a better sense of why certain rules are congruency properties and others are not.	Students will take the homework and leave the classroom feeling ready to work on it and more comfortable with proof writing and triangle congruency properties.