Essays on Financial Markets

Dawid Brychcy
Supervisor: Gabriel Pérez Quirós

Submitted to
Departament d'Economia i d'Història Econòmica
Universitat Autònoma de Barcelona
in partial fulfilment of the requirements for the degree of
Doctor in Economics

Author:	Supervisor:	Tutor:
Dawid Brychcy	Gabriel Pérez Quirós	Inés Macho Stadler

June 2013

Acknowledgments

There are many people who have contributed to this thesis and many require special acknowledgement. Above all, I would like to thank my wife – Eli, for her continuous support and enormous patience at all time. Without her support and encouragement this thesis would not be possible. I also would like to mention my little daughter Julia that appeared in the very last part of this thesis. Thank you both once again. I also want to thank my family and friends for all the support.

I want to thank my supervisor, Gabriel Pérez Quirós for his comments and ideas. His criticism of my work and support contributed a lot to this thesis. I am always impressed by his brilliant remarks and suggestions. Many thanks to Rebeca Jiménez-Rodríguez for help at the very early stage of this thesis and valuable critic of Maximo Camacho shortly before finishing this work.

I am very grateful to the members of the Departament d'Economia i d'Història Econòmica at Universitat Autònoma de Barcelona. In particular I would like to thank Hugo Rodríguez, Evi Pappa and Luca Gambetti for the time they spent helping me. I would like also to mention Inés Macho Stadler for her help during the program and Jordi Massó, Jordí Caballé, Carlos Velasco and Hugo for their classes during the program.

I want to thank my fellow colleagues and friends in Barcelona. Especially, I would like to thank Fernanda for her friendship and encouragement and Magda, Ricardo, Francisco, Conchi, Eduardo, Isabel, Brindusa, Sergio and Agnes with whom I have really enjoyed my stay in Barcelona.

I gratefully acknowledge the financial support from the FPU doctoral fellowship (Ref. number AP2003-2252) financed by the Spanish Ministry of Education, Culture and Sport.

Contents

Chapter 1

Introduction

This thesis consists of three self-contained essays, all of them analyze financial markets and each one concentrates on one particular issue. This thesis contributes to the empirical analysis of the changes in financial markets due to some external factor like oil prices (the case of the first essay), the changes in correlations between financial markets and their consequences for investors (second essay) and finally to analysis of changes in the volatility of financial markets (third essay).

All three essays are motivated by real life importance of these issues. The first essay was motivated by the concerns about the stock market performance during the oil prices increases $2003-2007$ and the repeatedly appearing comments in the financial press about the negative impact of these oil price peaks on the daily stock market results. The second essay discusses the changes in correlations between Central and Eastern European stock markets and Western European stock markets when these Central and Eastern European countries joined the European Union in 2004 and the possible changes in the prospects of these countries as the investment opportunity. The third essay analyzes the volatility of the Spanish stock index (IBEX 35) and the Dow Jones Industrial Average over the last 20 years concentrating heavily on the last financial crisis and high levels of volatility not seen before.

Chapter 1, *Impact of oil prices on international financial markets*, analyzes the relation between oil price returns, volatility of oil price return and returns of stock indices. We consider daily prices of WTI crude and daily quotations of five main world stock indices - DJIA, S&P500, FTSE100, DAX and NIKKEI225. We investigate first the linear relationship between oil price returns and stock market returns taking the oil variables as the explanatory variable in the mean equation. In the second step we consider the non-linear transformations of oil prices. Hamilton (1996) and Lee *et al.* (1995) redefine the measure of the oil price changes and propose the non-linear transformation of the oil price returns. Further we investigate the threshold effect in the relationship between oil returns, volatility of oil returns and stock market returns. Finally, we analyze the links between the volatilities of the returns of oil prices and stock market returns in the dynamic setting using the bivariate multivariate GARCH model.

The results of the paper deliver new insight into the relationship between changes in oil prices and changes in stock markets. The changes in oil prices affect DJIA, S&P500 and DAX – this impact is negative, as expected with the economic theory. The increase in oil prices (positive returns) lowers the return on the stock index. The estimated coefficient of oil price returns even if small in magnitude accounts for about one third of the daily average return of the stock index. Lagged oil returns have no influence on the stock market, neither there is any asymmetry in this relationship. The returns of DJIA and S&P 500 react to high volatility and falling oil prices. This impact is positive since the falling oil prices could be seen as a positive cost factor for companies. DAX reacts positively to low volatility of oil returns and negatively to high volatility of oil returns combined with increasing prices - the consequences of geopolitical events. In the same way NIKKEI also negatively reacts to the high oil return volatility.

There is no relationship between the non-linear transformations of oil prices and the stock market returns. We also do not detect any threshold effect in the relationship between oil returns and stock market returns or oil return volatility and stock market returns.

During the periods of very high oil return volatility we observe negative and statistically significant contemporaneous correlations between shocks to oil returns and shocks to stock index returns for all the stock markets we consider. The positive shocks to oil price returns (thus increases in prices) are transmitted immediately to stock markets in form of negative shocks to stock market returns. During the period of low oil volatility shocks to oil prices will results in the increase in the stock index volatility next day. There is also a feedback reaction of stock index volatility to the level of volatility of oil prices - the low level of oil volatility diminishes the volatility of stock market indexes next day.

Chapter 2, *Changes in correlations between CEE stock markets and European stock markets*, analyses the changes in correlations among Western European and CEE stock markets before and after the EU entrance in 2004. This analysis allows investigating the benefits from EU integration. Contrary to the previous works based on the analysis of the long-run relationship between stock markets (cointegration analysis) we concentrate on short term co-movements and analyze changes in correlations among CEE stock markets and three European stock market indexes. These short term relations between stock markets are of a great importance when setting the optimal investment strategies and constructing the optimal investment portfolio. One of the convenient ways of analyzing the changes in correlation is the Dynamic Conditional Correlation GARCH Model proposed by Engle and Sheppard (2002) and the modification of this model to allow for asymmetric behavior of the conditional correlations and also allowing for structural break in the conditional correlation. In our analysis we consider the daily prices over the period 1994 – 2006 for the following stock market indexes - WIG20 from Poland, PX50 from Czech Republic, BUX from Hungary, SAX from Slovakia and SBI from Slovenia, DAX30 from Frankfurt, FTSE100 from London and WBI from Vienna.

Using the Asymmetric Dynamic Correlation GARCH Model we estimate conditional correlation series between CEE stock markets and three important European stock indexes. In the next step we detect a structural break in the conditional correlation series some months before May 2004 - the date of the entrance of all the countries to the EU. The successive increase in correlation of the Central and Eastern European financial markets and Western Europe is the sign of the higher integration of these markets with European stock markets.

Chapter 3, *Structural changes in the volatility of IBEX35*, analyzes the changes in the volatility of the Spanish blue chip index – IBEX 35 over all his history from 1992 – 2011. The changes in volatility levels of stock market indexes are important in many aspects of business and financial life. Using Quadratic GARCH model of Sentana (1995) and modified ICSS algorithm (see Sansó et al. (2004)) to detect the structural changes in the volatility of the index, we detect several structural breaks in the volatility of IBEX 35. The subsamples defined by the breaks differ in the persistence and the asymmetry of the impact of shocks on volatility. The last two months of observations are left for out-of-sample forecasting. We observe a better forecasting performance of the model with breaks than the benchmark, which is the QGARCH model estimated over the entire period of interest. Our analysis includes the period of the current financial crisis when the volatility of the financial markets increased to the levels not seen before. Comparing to the other stock market index – the mostly followed U.S. Dow Jones Industrial Average index we observe that the Spanish as of 2011 was still in the regime of the rather high volatility whereas the DJIA after this period of turmoil in the financial markets already returned to the regime of lower volatility.

Bibliography

[1] Engle, R.F. and K. Sheppard, "Theoretical and Empirical Properties of Dynamic Conditional Correlation Multivariate GARCH", NBER, Working Paper 8554, 2001.

[2] Hamilton, J. (1996), "This is what happened to the oil price–macroeconomy relationship?", *Journal of Monetary Economy,* 38, 215-220.

[3] Lee, K., S. Ni and R.S. Ratti (1995), "Oil shocks and the macroeconomy: The role of price variability.", *Energy Journal,* 16, 39-56.

[4] Sentana, E., "Quadratic ARCH Models", *Review of Economic Studies*, 62 (1995), 639-661.

[5] Sansó, A., Arragó, V. and J. Carrion-Silvestre, "Testing for change in the unconditional variance of financial time series", *Revista de Economía Financiera*, 4 (2004), 32-53.

Chapter 2

Impact of oil prices on international financial markets

2.1 Introduction

Oil is one of the important resources in the economy and plays the crucial role in setting the economic policies. The relation between oil price changes, economic activity and employment is an issue that has been studied during long time. In a pioneer work Hamilton (1983) shows that oil price increases are responsible for almost every post World War II US recession, except the one in 1960. Mork *et al.*(1994) survey the extensive literature on relationship between oil prices and macroeconomy and evidence a clear negative correlation between oil prices and measures of output or employment.

The oil prices affect economy through many channels. The initial impact of changes in oil prices is through the transfer of income from consumers to producers, and on the international level from oil-importing countries to oil-exporting countries. Higher oil prices increase production costs in almost all industries, particularly in such energy-intensive sectors like transport, and are likely to lead to an increase in inflation, which in turn will depend on the extent to which companies pass the higher oil prices on their final product, on the consequences for wages and on the effectiveness of the anti-inflationary policies. A tightening of macroeconomic policies in response to higher oil prices and increasing inflation would have an impact on global financial markets. This impact of higher oil prices on disponsible income, business profits and inflation lowers the value of financial assets.

Stock prices can be regarded as the discounted values of expected future cash flows the company will generate. Oil prices can affect both the expected cash flows and discount rates. The increasing oil prices rise the cost of production and lower the benefits of the companies. The expected discount rate is the sum of the expected inflation rate and expected real interest rate, both of which may in turn depend on oil prices. Rising oil prices are often indicative of inflationary pressures which central banks can control by raising interest rates. Higher interest rates make bonds look more attractive than stocks leading to a fall in stock prices. The overall impact of rising oil prices on stock prices depends of course on whether a company is a consumer or producer of oil and oil related products.

Although a bulk of economic research has studied the relation between oil price changes and economic activity, there is little research on the relationship between oil price changes and financial markets.

In the related literature most of the authors (Jones and Kaul (1996), Huang *et al.*(1996), Sadorsky (1999)) focuses on the linear relationship between oil price returns and stock returns. Huang *et al.* (1996) conclude that oil futures returns do lead only individual oil companies and the petroleum index sector but do not have impact on S&P500 stock index or other sector indices; Sadorsky (1999) shows that oil prices and the volatility of oil prices do affect real stock returns and that the oil price increases have a greater impact on economic activities than oil price decreases. Nandha and Faff (2008) analyze monthly returns of 35 global industry indices and conclude that oil price rises have a negative impact on equity returns for all sectors except mining, and oil and gas industries and provide little evidence of any asymmetry in the oil price - stock market indices relationship.

Although the academic literature is rather scarce and gives no clear answer to the question if and how the oil prices affect stock markets, the financial press assumes that oil prices influence the stock markets and that daily moves of many stock markets can be explained by the changes in oil prices. Whereas many academic papers are based on the monthly data, our work will shed light on this relationship since we consider daily data in our analysis.

Understanding of the relationship between stock markets and oil prices is of high interest of stock market investors, especially in the period when the oil prices are more and more volatile and the levels of oil prices changes in the shorter period of time. Detection of impact of oil price returns on the stock market returns and spill-over effect from volatility of oil prices to volatility of stock markets will allow setting the best investment strategies.

In this work we use daily data for the period 1984 - 2005 to analyze and assess the relation between oil price returns, volatility of oil price return and returns of stock indices. We will consider the prices of WTI crude and five main world stock indices - DJIA, S&P500, FTSE100, DAX and NIKKEI225.

We investigate first the linear relationship between oil price returns and stock market returns taking the oil variables as the explanatory variable in the mean equation. In the second step we consider the non-linear transformations of oil prices. In the mid 1980s the economist observed the change in the oil prices – macroeconomy relationship that became non-linear. Hamilton (1996) and Lee *et al.* (1995) redefine the measure of the oil price changes and propose the non-linear transformation of the oil price returns. Further we investigate the threshold effect in the relationship between oil returns, volatility of oil returns and stock market returns. Finally, we analyze the links between the volatilities of the returns of oil prices and stock market returns in the dynamic setting using the bivariate multivariate GARCH model.

We treat the oil prices as exogenous variable in our analysis and allow for the impact of oil prices on financial markets and not for the impact of stock markets on oil prices.

The latest stream in the research is treating the oil prices as endogenous variable arguing that oil prices respond to factors that also affect stock prices. Kilian and Park (2009) show that the response of aggregate U.S. real stock returns may differ greatly depending on whether the increase in the oil price is driven by the demand or supply shocks in the crude oil market. Oil market specific demand shocks such as increases in the precautionary demand for oil that reflect concerns about future oil supply shortfalls confirm the traditional view that higher oil prices cause lower stock markets. Yet, the positive shocks to the global demand for industrial commodities (expectations about global economic expansion) cause both higher real oil prices and higher stock prices. Oil supply shocks have no significant effects on stock returns. Apergis and Miller (2009) extend the analysis and add seven developed countries. They find that international stock market returns do not respond in a large way to oil market shocks. That is, the significant effects that exist prove to be small in magnitude.

In this paper we do not decompose the oil price shocks since this task gets complicated when using daily data. This will be left for further research.

The changes in oil prices affect DJIA, S&P500 and DAX. The remaining stock markets are not affected by the daily changes in oil prices. The impact of the oil returns on DJIA, S&P and DAX is similar in both the nature and magnitude. This impact is negative, as expected with the economic theory. The increase in oil prices (positive returns) lowers the return on the stock index. The estimated coefficient of oil price returns even if small in magnitude accounts for about one third of the daily average return of the stock index.

Lagged oil returns have no influence on the stock market neither there is any asymmetry in this relationship.

The returns of DJIA and S&P 500 react to high volatility and falling oil prices. This impact is positive since the falling oil prices could be seen as a positive cost factor for companies. DAX reacts positively to low volatility of oil returns and negatively to high volatility of oil returns combined with increasing prices and geopolitical events. In the same way NIKKEI also negatively reacts to the high oil return volatility.

There is no relationship between the non-linear transformations of oil prices and the stock market returns. We also do not detect any threshold effect in the relationship between oil returns and stock market returns or oil return volatility and stock market returns.

During the periods of very high oil return volatility we observe negative and statistically significant contemporaneous correlations between shocks to oil returns and shocks to stock index returns for all the stock markets we consider. The positive shocks to oil price returns (thus increases in prices) are transmitted immediately to stock markets in form of negative shocks to stock market returns (so decreases in the level of stock index).

During the period of low oil volatility shocks to oil prices will results in the increase in the stock index volatility next day. There is also a feedback reaction of stock index volatility to the level of volatility of oil prices - the low level of oil volatility diminishes the volatility of stock market indexes next day.

Our results allow constructing different investment strategies that should benefit from the resulting oil price changes or changes in the oil price volatility. To benefit from the negative impact of oil prices on DJIA, S&P and DAX we could go long futures on these indexes when the oil prices are going down or short futures when the oil prices are going up.

To cash-out the environment of high volatility and falling oil prices we should go long futures on DJIA and S&P 500 that will gain as the indexes will rise in such an environment.

Being in the regime characterized by stable oil prices and low volatility the profitable strategy is to go long futures on DAX. The opposite strategy should be used in the case of the regime defined by increasing oil prices and geopolitical unrest in the oil producing countries - the best strategy is to short futures on DAX or short futures on NIKKEI and enjoy the falling index quotations and falling futures prices.

During the period of low oil price volatility to benefit from shocks to oil prices and the resulting increase in stock index volatility we can go long straddle which is the combination of long call and long put option with the same strike and maturity. There is no exposure to the underlying index in such strategy yet this combination is very sensitive to the changes in the volatility of the underlying. To benefit from the low level of the oil price volatility and the resulting decrease in the stock index volatility we should construct the opposite strategy – short straddle, a combination of short call and short put on the stock index with the same strike and the same maturity which will profit from the decrease in the oil price volatility.

The paper is organized as follows. Section 2 discusses the specification of the models we use in this paper. Section 3 presents the data used in this study. In Section 4 we discuss the empirical results. Section 5 concludes and sketches further research possibilities. In the appendix we present the specifications tested in this paper, discuss the tests used and present the figures and detailed results of the estimation.

2.2 Methodology

This section presents the specification of the models estimated in the empirical part. As mentioned in the introduction we analyze the impact of oil prices on the stock markets on two levels - on the level of returns and the level of volatility. In the first part we investigate, using both the linear and non-linear specification, the impact of changes in oil price returns on stock market returns. In the second part of the analysis we concentrate on the links between the volatilities and transmission of shocks between oil prices and stock markets.

The starting point is to determine the GARCH models for each series of returns. We define the best specification of the conditional mean by considering the Schwarz Information Criterion (BIC), that takes the lowest value for the best model.

To check the presence of GARCH effects in the conditional volatility equation we use the ARCH-LM test proposed by Engle (1982) and to detect the leverage effects in conditional volatility (asymmetry) we consider the Sign Bias, Negative and Positive Size Bias tests proposed by Engle and Ng (1993). All the tests are discussed in the appendix.

For the conditional variance for each of the time series we consider the linear GARCH (see Bollerslev (1986)) and non-linear GARCH models. To account for observed asymmetry in the volatility of stock markets we consider the GJRGARCH model of Glosten *et al.* (1993).

The simplest representation of these models are the linear $GARCH(1,1)$ in which the conditional volatility evolves as

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$$

and the asymmetric $GJRGARCH(1,1)$

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \gamma S_{t-1}^{-}\varepsilon_{t-1}^2 + \beta h_{t-1}$$

where ε_t are the residuals from the mean equation and S_{t-i}^{-} is the dummy variable that takes the value of 1 when the $\varepsilon_{t-i} < 0$ and 0 otherwise. The leverage effect is captured by the use of γ - the positive news have an impact of α, while the negative of $\alpha + \gamma$.

2.2.1 Linear specification

The first analysis concentrates on the impact of oil returns on each stock market.

Specification 1 incorporates the oil price returns as the explanatory variable in the mean equation. This specification tests if there is impact of oil prices on each of the stock markets. We also take the lagged returns of oil prices as the explanatory variable (Specification 2) to investigate if the changes of oil prices in the past influence the stock markets contemporaneously. We check up to five-day lag of oil returns.

Further we construct the dummy variable that accounts for the sign of the returns on oil prices to see if there is an asymmetry in this relationship (Specification 3). This specification coincides with the one proposed by Mork (1989). We consider both the negative dummy variable which accounts for the decreases in oil prices and positive dummy variable which accounts for price increases. We check one-day lag in these dummies as well.

Increased volatility in energy and oil prices can affect the present value of the discounted stream of dividend payments (stock price), through increasing uncertainty about product demand and by increasing uncertainty about the future return on investment. Specification 4 takes squared returns of oil prices (proxy for the volatility of oil prices) as the explanatory variable in the mean equation of the stock market returns. We also add the oil returns to the mean equation in order to correctly isolate the impact of oil price volatility on the stock market return.

2.2.2 Non-linear specification

The first approach in investigating the impact of oil prices of the macroeconomic variables was the linear specification. By the mid-1980s, this estimated linear relationship between oil prices and macroeconomic variables began to lose significance. The declines in oil prices that occurred over the second part of the 1980s were found to have smaller positive effect on economic activity than the predictions made by the linear models. This motivated researchers to propose the non-linear transformations of the oil price variables.

In this paper we use two of them - $NOPI$ (*net oil price increases*) proposed by Hamilton (1996) and $SOPI$ (*scaled oil price increases*) proposed by Lee *et al.* (1995).

Hamilton (1996) claims that it seems more appropriate to compare the prevailing price of oil with what it was during the previous year, rather than during the previous quarter. He therefore defines a new measure, the NOPI - net oil price increase. In our setting we define the $NOPI_t$ as the amount by which the return on oil prices on day t, r_oil_t, exceeds the maximum value over the previous n days; and 0 otherwise. We will consider $n = 5, 6, ..., 10$ to account for the maximum in the period of one to two weeks.

We define the $NOPI_t$ variable as

$$NOPI_t = \max\{0, r\_oil_t - \max\{r\_oil_{t-1}, r\_oil_{t-2}, ..., r\_oil_{t-n}\}\}$$

The specification proposed by Lee *et al.* (1995) $SOPI_t$ - scaled oil price increases, focuses on volatility of returns on oil prices and argues that the oil price increases after a period of price stability have stronger macroeconomic consequences than those that are corrections to the greater oil price decreases. Lee *et al.* (1995) propose to use the GARCH model with the appropriate mean specification and define $SOPI_t$ as the positive standardized residuals

$$SOPI_t = \max\left(0, \widehat{\varepsilon}_t / \sqrt{\widehat{h}_t}\right)$$

where $\widehat{\varepsilon}$ are the estimated residuals from the mean equation and $\widehat{h}_t$ is the estimated conditional variance of returns on oil prices.

Finally, we will use the Hansen (2000) procedure to test for the threshold effect based on a threshold regression model where observations fall into classes or regimes that depend on the unknown value of the observed variable.

We check both if the oil price returns and oil price return volatility show a threshold level in their impact on stock markets. Huang *et al.* (2005) apply the multivariate threshold model to investigate the impacts of an oil price change and its volatility on economic activities (changes in industrial production and real stock returns) for Canada, USA and Japan using monthly data. They detect the threshold levels for both the oil returns and oil volatility but conclude that oil price change seems to have better explanatory power on economic activities than oil price volatility.

In this setting y is the dependent variable, x is the explanatory variable for which we want to test the presence of the threshold effect, z is the set of exogenous explanatory variables and $I(\cdot)$ is the indicator function. To test for the threshold effect we estimate the following regression

$$y_{it} = \beta_0 + \beta_{a1} x_{it} I(x_{it} \leq \gamma) + \beta_{a2} x_{it} I(x_{it} > \gamma) + \beta_z z_{it} + u_{it}$$

Hansen (2000) recommends obtaining the least square estimate $\widehat{\gamma}$ as the value that minimizes the sum of squared errors $S_I(\gamma)$. We test the significance of the detected threshold using the following hypothesis

$$\begin{aligned} H_0 &: \quad \beta_{a1} = \beta_{a2} \\ H_1 &: \quad \beta_{a1} \neq \beta_{a2} \end{aligned}$$

in which H_0 states that the linear model is appropriate whereas H_1 is in favour of the threshold model.

One complication is that γ is not identified under the null so that the classical tests do not have standard distribution and critical values cannot be read off from the standard distribution tables. Hansen (1996) proposes the likelihood ratio test statistic and the bootstrapping method for finding the $p - value$. We present the details of the test in the appendix.

2.2.3 Volatility linkages

Following the success of the ARCH and GARCH models in describing the time-varying variances of economic data in the univariate case the extension to the multivariate case has been developed immediately. Bauwens *et al.* (2006) discuss the most important developments in multivariate ARCH-type modelling. Several applications of multivariate GARCH models can be found in the financial literature: Bollerslev (1990), Karolyi (1995),Tse and Tsui (2000), among others. The multivariate GARCH models offer a suitable framework to investigate the nature of the transmission of shocks among financial time series.

The extension from a univariate GARCH model to the N - variate model requires allowing the conditional variance-covariance matrix of the N dimensional zero mean random variables ε_t (errors from the mean equation) to depend on the elements of the information set. Let $\{z_t\}$ be a sequence of $(N\ x\ 1)$ i.i.d vector such that

$$z_t \sim F(0, I_N)$$

with F continuous density function. Let $\{\varepsilon_t\}$ be a sequence $(N\ x\ 1)$ random vectors defined as

$$\varepsilon_t = H_t^{1/2} z_t$$

where

$$E_{t-1}(\varepsilon_t) = 0, \quad E_{t-1}(\varepsilon_t \varepsilon_t^{'}) = H_t$$

where H_t is a matrix (NxN) positive definite.

In our paper we estimate the Extended Constant Conditional Correlation GARCH model (ECCC-GARCH thereafter) which is the extension proposed by Jeantheau (1998) of the Constant Conditional Correlation GARCH model (CCC-GARCH) (see Bollerslev (1990)). This model allows the interactions among volatilities of time series.

Engle and Sheppard (2002) propose a test for constant versus dynamic correlation structure. We apply this test for the bivariate structure (stock market returns and oil returns). The test rejects the dynamic nature of the conditional correlation between these series therefore ECCC-GARCH best suit the nature of the constant correlation.

ECCCC-MVGARCH model

Bollerslev (1990) introduces the Constant Conditional Correlation GARCH model. In this model, the conditional correlation matrix is time invariant. The assumption of constant correlation makes estimating a large model feasible and ensures that the estimator is positive definite, simply by requiring each univariate conditional variance to be non-zero and the correlation matrix to be full rank.

In this model the matrix of variances-covariances H_t is proposed to be

$$\{H_t\}_{ii} = h_{it}$$

$$\{H_t\}_{ij} = \sqrt{h_{ijt}} = \rho_{ij}\sqrt{h_{it}}\sqrt{h_{jt}} \qquad i \neq j$$

We can partition the matrix H_t as

$$H_t = D_t R D_t$$

where D_t is the (NxN) diagonal matrix with the conditional standard deviations along the diagonal, $\{D_t\}_{ii} = \sqrt{h_{iit}}$ and R denote the matrix of conditional correlations with $(i,j)^{th}$ element being ρ_{ij} and $\rho_{ii} = 1$. So it follows that the $(i,j)^{th}$ element of H_t is given as

$$h_{ijt} = \rho_{ij}\sqrt{h_{iit}h_{jjt}}$$

H_t will be positive definite for all t if and only if each element of the N conditional variances are well defined and R is positive definite.

The diagonal structure implies that each variance behaves like a univariate GARCH model. The only interaction between volatilities is through contemporaneous constant correlation. The main drawback of this diagonal specification is that it rules out the possible interactions between volatilities.

For the bivariate cases we consider in this paper (stock market returns and oil returns) the CCC-GARCH model is defined as

$$\begin{bmatrix} h_{1t} \\ h_{2t} \end{bmatrix} = \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} \alpha_{11} & 0 \\ 0 & \alpha_{22} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{1t-1} \\ \varepsilon^2_{2t-1} \end{bmatrix} + \begin{bmatrix} \gamma \\ 0 \end{bmatrix} \begin{bmatrix} S^-_{t-1}\varepsilon^2_{1t-1} \\ \varepsilon^2_{2t-1} \end{bmatrix} + \begin{bmatrix} \beta_{11} & 0 \\ 0 & \beta_{22} \end{bmatrix} \begin{bmatrix} h_{1t-1} \\ h_{2t-1} \end{bmatrix}$$

since we consider GJRGARCH for stock market returns and GARCH for oil price returns.

The positivity of each conditional variance in the CCC-GARCH model can simply be achieved by assuming that the parameters of each equation satisfy the conditions derived in Nelson and Cao (1992) and Glosten *et al.* (1993).

To account for the possible interactions between contemporaneous and past volatilities Jeantheau (1998) proposes the Extended Constant Conditional Correlation GARCH model (ECCC-GARCH) which relaxes the assumption about the diagonal matrixes and allows the past squared returns and variances of all series to enter the individual conditional variance equation. This in turn allows to account for possible volatility spillovers. Wong *et al.* (2000) apply the ECCC-GARCH for modelling the interactions between S&P500 index and the Sydney All Ordinaries One, and among three major exchange rates.

Using the ECCC-GARCH model we investigate the dynamic links between volatilities. In the bivariate setting we model each volatility of the series using the univariate GARCH model (GJRGARCH for the series of stock returns (h_{1t}) and linear GARCH for the oil prices (h_{2t})). As mentioned in the introduction we treat the oil prices as exogenous variable in our model and do not consider the possibility of impact of stock markets on oil prices. To be consistent with this assumption we need to impose $\alpha_{21} = 0$ and $\beta_{21} = 0$.

Our bivariate ECCC-GARCH model is specified as

$$\begin{bmatrix} h_{1t} \\ h_{2t} \end{bmatrix} = \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ 0 & \alpha_{22} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{1t-1} \\ \varepsilon^2_{2t-1} \end{bmatrix} + \begin{bmatrix} \gamma \\ 0 \end{bmatrix} \begin{bmatrix} S^-_{t-1}\varepsilon^2_{1t-1} \\ \varepsilon^2_{2t-1} \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} \\ 0 & \beta_{22} \end{bmatrix} \begin{bmatrix} h_{1t-1} \\ h_{2t-1} \end{bmatrix}$$

If β_{12} is statistically significant we have an impact of the past volatility of the oil prices on the current volatility of stock markets, if α_{12} is statistically significant we will also observe the impact of oil shocks on the volatility of oil prices.

This model that allows for volatility feedback of either the positive or negative sign requires the reformulation of the positivity constraints for the conditional volatility. In the CCC-GARCH model negative spillovers were ruled out by the assumption that all the parameters of the model are nonnegative. Conrad and Karanasos (2010) discuss the conditions to guarantee a positive definite variance-covariance matrix even if some parameters are negative. They show that the positive definiteness of the conditional covariance matrix can be guaranteed even if some of the parameters are negative.

2.3 Data

In this paper we analyze the links between oil prices and main stock markets. We consider the DJIA and S&P500 as the most important stock market indices in the United States. The FTSE100 and DAX30 are the two main European stock market indices from the UK and Germany respectively. Finally we include in the analysis NIKKEI225 as the main index on the Tokyo Stock Exchange. The appendix shows the plots of the stock market indexes versus the prices of oil crude. For the crude oil prices we use one of the two mostly watched spot prices - the price of the West Texas Intermediate (WTI) Cushing Crude Oil.

All the data we obtain from Bloomberg and are the closing prices. Using the historical exchange rates we convert the values of the stock market indices from local currency into dollar terms.

We remove all the non-trading days and obtain seven time series of 4949 observations. The data spans from 01/01/1984 (initiation of FTSE 100) to 30/06/2005. Finally to have stationary series we consider continuously compounded returns on the stock market indices and oil prices.

Figure 1 shows the evolution of stock market indexes and WTI over the period of interest.

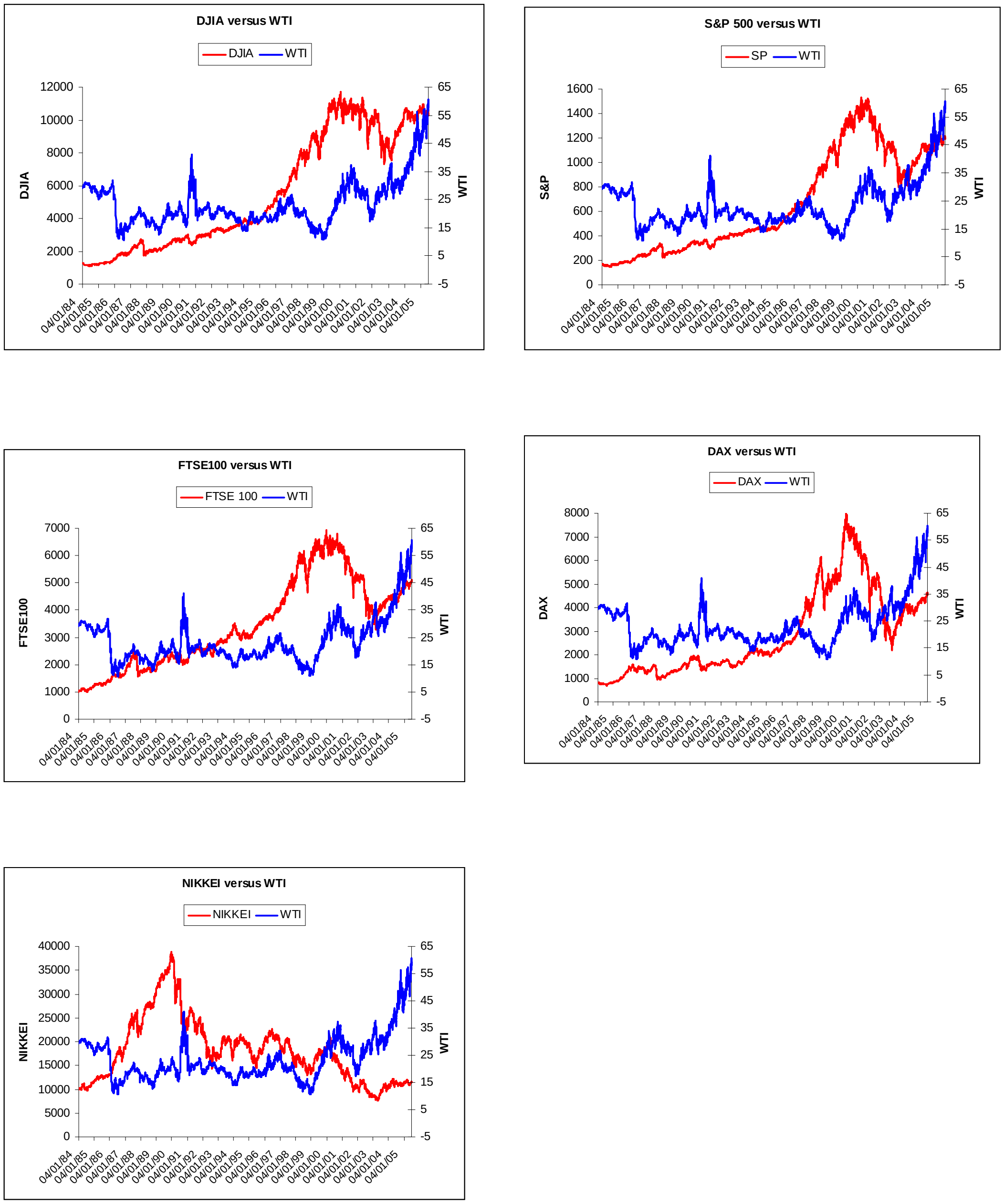


Figure 1. The evolution of stock market indexes and WTI over the period 1984 - 2005.

Looking at the Figure 1 we observe periods when stock markets and oil prices are moving in the same direction and periods when they were moving in the opposite direction.

2.3.1 Oil prices

In this paper we consider the price of the West Texas Intermediate (WTI) Cushing Crude Oil as the oil variable. Figure 2 shows the evolution of oil prices over 1984 - 2005.

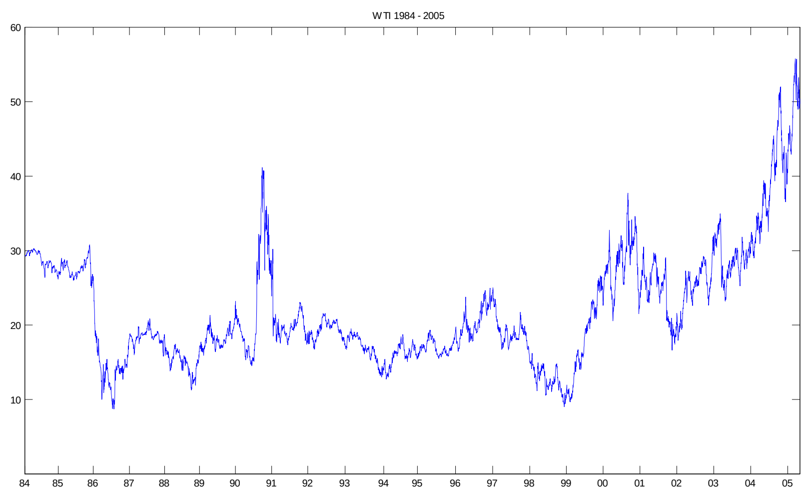


Figure2. WTI prices 1984 - 2005.

In 1985 the OPEC production falls to the 20-year low of 13.7 m bl/d (barrels per day)[1]. Saudi Arabia stopped playing a marker role and fought to increase market share. In 1986 OPEC decided to support oil price of around $18/bl and set this price as the reference level.

Between 1990 and 1997, the oil price averaged $18/bl, although there was a sharp spike during the Gulf war period. In 1998, the combination of rising production and the Asian economic crisis saw the oil price fall to under $10/bl. OPEC and major non-OPEC producers, Mexico and Norway, cut the production three times and the price recovered to over $25/bl by the end of 1999.

Following the recovery in oil price, OPEC introduces a $22-28/bl target price band in March 2000. Between 2000 - 2003 the oil price stayed at the level of $26/bl. It did fall sharply following the 11 September terrorist attack, dropping to $16/bl. However, since 2004 the lack of spare OPEC capacity has resulted in sharp increase in prices, with the price rising to over $70/bl.

[1] This is based on "A brief history of the oil price", Global Equity Research, Lehman Brothers, August 2007.

2.3.2 Volatility of oil prices

Looking at Figure 2 we can observe that the oil prices were changing differently over the period 1984 - 2005. We expect that there are periods of low and high oil price volatility. We expect that also the stock markets will react differently to changes in oil prices during the periods of low and high volatility.

In the first step we detect the structural breaks in the unconditional volatility of oil prices.

Inclan and Tiao (1994) are the first to provide a method of detecting structural breaks in volatility. They propose the Iterative Cumulative Sums of Squares (ICSS) algorithm to detect multiple changes in variance. The ICSS algorithm uses cumulative sums of squares and searches for change points in unconditional volatility systematically at different moments of time. The most serious drawback of the test proposed by Inclan and Tiao (1994) is that its asymptotic distribution is critically dependent on the assumption about the i.i.d. $N(0, \sigma_r^2)$ distribution of the returns. In fact, Sansó *et al*. (2004) show that IT statistic can be oversized for processes that follow different distribution, among them GARCH processes which depend on the past values.

To address this problem and allow the r_t to follow a variety of dependent processes, among them GARCH processes, a nonparametric adjustment based on the Bartlett Kernel is applied to the original test statistic.

We apply the algorithm proposed by Sansó *et al*. (2004) to oil price returns and detect six periods of different levels of volatility. Figure 3 shows the squared oil returns (proxy for volatility) and the six regimes we detect.

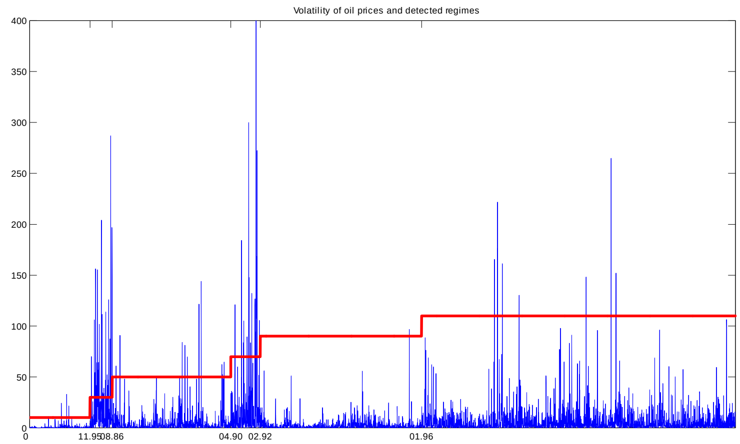


Figure 3. Volatility of oil prices and detected structural breaks in the conditional volatility

The first regime - January 1984 - November 1985 is a period of low volatility (the sample standard deviation is 0.9204). The oil price was oscillating between \$24 - \$28/bl.

The second regime - December 1985 - July 1986 is characterized by high volatility (sample standard deviation 5.145) - OPEC decided to fight for market share. This high volatility was combined with falling prices - the oil price fell from around \$30/bl in December 1985 to \$9/bl in July 1986.

Next regime - August 1986 - April 1990 is a period of lower volatility (sample standard deviation 2.2192) In December 1986 OPEC set the reference price of \$18/bl and during the next three years the oil price was fluctuating in the range \$14 - \$18/bl.

The fourth regime - May 1990 - February 1992 is again a short period of very high volatility (sample standard deviation 5.5631). In this rather short period of time some geopolitical events influenced the level and the volatility of oil prices - in August 1990 Iraq invaded Kuwait, in November 1990 an earthquake hit Iran's oil-producing region, in January 1991 the Gulf war started, as the US launched air attacks against Iraq. Shortly after that the leaving Iraqi soldiers set the Kuwaiti oil fields on fire, finally in December 1991 the Soviet Union collapsed and the Soviet Union suspended oil exports due to the growing fuel shortages.

March 1992 - January 1996 - the fifth regime is the return to tranquil period. The volatility of oil prices decreased to 1.5922 as the OPEC managed to maintain stable production level and the oil prices were moving between \$15/bl and \$20/bl.

The last regime that started in February 1996 is again return to rising prices and higher level of volatility of oil prices (sample standard deviation 2.6009). Between 1996 and 1999 the oversupply in the market reduces the oil prices to below \$10/bl but the consecutive production cuts lifted the oil prices to \$25/bl at the end of 1999. Following this recovery in oil prices OPEC introduced a \$22-28/bl target price band in March 2000. Until 2003 the oil prices were moving in this range with the sudden drop to \$16/bl after the 11 September terrorist attack but since June 2004 the lack of spare OPEC capacity and increase in demand resulted in sharp increase in prices, reaching the level of around \$55/bl in mid 2005.

When analyzing the impact of oil volatility on the stock market returns and the links between volatility of oil prices and volatility of stock markets we will take these detected regimes into account and will check if the before mentioned relationships change depending on the oil volatility level.

2.3.3 Daily returns of stock markets

The series of interest are the continuously compounded returns. Given the daily quotations of the index (P_t) we define continuously compounded returns as $r_t = 100 \ln(P_t/P_{t-1})$.

Table 1 displays the summary statistics of the data.

	$Mean$	$St.Dev.$	$Skewness$	$Kurtosis$	Augmented Dickey-Fuller test
$DJIA$	0.0425	1.1301	-2.54	61.36	-51.855 (0.0000)
$S\&P$	0.0399	1.1044	-1.97	44.67	-69.128 (0.0000)
$FTSE$	0.0326	1.0858	-0.78	12.81	-69.364 (0.0000)
DAX	0.0464	1.5159	-0.25	7.06	-70.586 (0.0000)
$NIKKEI$	0.0181	1.6170	-0.01	9.10	-70.458 (0.0000)
WTI	0.0134	2.5653	-1.09	20.75	-52.746 (0.0000)

Table 1. The main statistics of the data. Critical value for the Augmented Dickey-Fuller test at 5% level of significance -2.8619.

The returns on oil prices show the highest standard deviation - the highest volatility among all the series. The value of the skewness in all the cases is negative showing the left-skewed series and the kurtosis indicates fat tails in the distribution. These are the common stylized facts observed in the series of returns on stock markets.

All the series of returns, as tested using the Augmented Dickey-Fuller test are stationary.

We take into account the differences in opening and closing time of the stock markets since the stock exchanges are located in the different time zones. When analyzing the impact of returns on oil prices on the European and Japanese markets we will take the first lag of the returns on oil prices since the data we consider (WTI) are from the New York Stock Exchange. Marten and Poon (2001) show that using non-synchronous data results in significant downward bias in correlation, as compared to *pseudo*-closed, which means simply constructed by sampling the data at the same time.

The European and Japanese stock markets are closed when the American markets open - at day t the FTSE, DAX and NIKKEI react to $t-1$ returns of oil prices (WTI is quoted in New York).

Table 2 shows the correlations between the series on returns and the corresponding p-values for the statistical significance.

i	$DJIA$	$S\&P$	$FTSE$	DAX	$NIKKEI$	WTI
$DJIA$	1.0000 (−)	−	−	−	−	−
$S\&P$	0.9544* (0.0000)	1.0000 (−)	−	−	−	−
$FTSE$	0.3122* (0.0000)	0.3344* (0.0000)	1.0000 (−)	−	−	−
DAX	0.2376* (0.0000)	0.2457* (0.0000)	0.5351* (0.0000)	1.0000 (−)	−	−
$NIKKEI$	0.2651* (0.0000)	0.2800* (0.0000)	0.2997* (0.0000)	0.2994* (0.0000)	1.0000 (−)	−
WTI	−0.0470* (0.0009)	−0.0380* (0.0075)	0.0157 (0.2690)	−0.0366* (0.0100)	−0.0282* (0.0476)	1.0000 (−)

Table 2. The correlations of the series and the corresponding p-values (* - statistically significant at 5%)

We see that the correlation between American stock markets, DAX and NIKKEI and oil prices is negative, small and statistically significant. In the case of FTSE and WTI the correlation is positive but not statistically significant.

The correlation between stock markets and oil prices is a dynamic process. In the appendix we present the plots of the correlations between stock market returns and oil price returns computed in the 3-month-windows. We observe that the correlation was changing over time - the American markets follow very similar pattern - high negative spikes at the beginning of 1990s, positive one around 1992 and significant changes around 1995 - 1996. There are similar, but lower, spikes in the case of European and Japanese market.

To show the dynamic behavior of the correlation we compute the average monthly correlations across markets in a very similar manner as Campbell *et al.* (2001). First, we have calculated monthly non-overlapping correlation coefficients for each pair of the stock returns and oil price returns. We then average the correlations between returns to compute a synthetic equally weighted index of the average correlation.

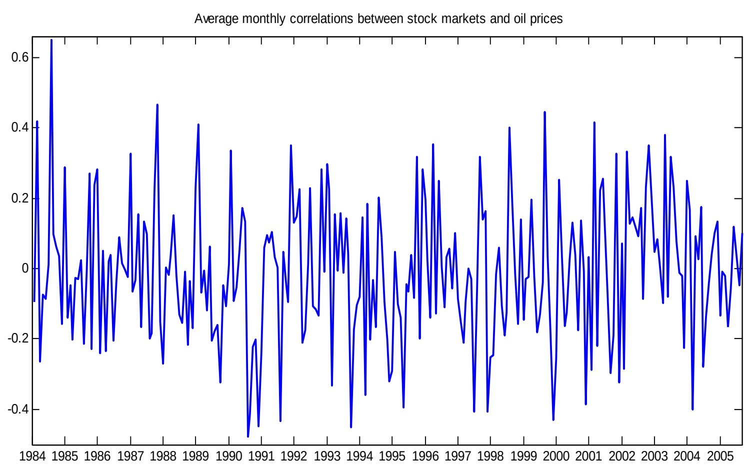


Figure 4. Average monthly correlations.

Figure 4 shows average monthly correlation between stock market returns and oil price returns. This correlation is changing over time around the level of zero, but there are periods of both positive and negative correlations between both series.

2.4 Empirical evidence

This section discusses the empirical results of the estimation of both univariate and multivariate GARCH models. In the appendix we present the detailed results of the estimations.

2.4.1 Linear specification

In this section we present the results of the estimation of the linear specification. We start by discussing the models for the series of returns followed by analyzing the results of the linear specification.

Univariate models for oil price returns and stock market returns

We start by investigating the model for oil price returns.

First we determine the conditional mean equation defined as the mixture of the autoregressive part and lagged innovations. The lowest value of the BIC criterion we obtain for ARMA(1,2).

Engle (1982) develops a test for conditional heteroscedasticity in the context of ARCH models based on the Lagrange Multiplier principle. We present the details of the test in the appendix.

We apply the ARCH-LM test to residuals ε_t from the mean equation and compute the ARCH-LM test statistics for the values of $q = 1, 5, 10$. Following we investigate the asymmetry in the conditional volatility. This idea was motivated by the empirical observation that the volatility of stock markets reacts differently to positive and negative shocks. We use Sign Bias, Negative Size Bias and Positive Size Bias tests proposed by Engle and Ng (1993), discussed in the appendix. For the Sign Bias we calculate the t-statistic for the parameter γ_1 and compute the statistics for Negative Size Bias and Positive Size Bias test.

Table 3 presents the results of these tests.

$ARCH(1)$	$ARCH(5)$	$ARCH(10)$
58.30(0.00)	145.07(0.00)	285.21(0.00)
$Sign\_Bias$	$Negative\_Size$	$Positive\_Size$
0.0367(0.00)	-6.399(0.00)	5.883(0.00)

Table 3. ARCH-LM and Sign Bias, Positive and Negative Size Bias tests for the oil price returns - the value of the test statistics and p-values.

The ARCH-LM test confirms the presence of ARCH effects, so we model the conditional volatility as the GARCH model. The results of the Sign Bias, Positive and Negative Size tests show the evidence of asymmetric ARCH effects.

Following we estimate the models for the returns on oil prices - ARMA(1,2) and consider the volatility specification as GARCH(1,1) and GJRGARCH(1,1) with normally distributed errors. Although the test proposed by Engle and Ng (1993) gives evidence of the asymmetric conditional volatility the parameter that governs this asymmetry is not significant in GJRGARCH. The model we propose for oil price returns is therefore $ARMA(1,2) - GARCH(1,1)$.

We follow similar steps with the series of stock market returns. We define first the conditional mean equation, check the presence of volatility and its nature. As the asymmetric models for volatility we consider GJRGARCH. The best model we choose are - for DJIA, NIKKEI and DAX - $ARMA(0,0)-GJRGARCH(1,1)$, for S&P500 and FTSE - $ARMA(1,0)-GJRGARCH(1,1)$.

The advantage of using the $GJRGARCH$ model for the conditional volatility is the straightforward understanding of the model that governs the dynamics of the conditional volatility. The parameter γ in the conditional volatility stands for the dummy variable that takes the value of 1 when the previous day shocks are negative. This parameter is expected to be positive to confirm the empirical fact that the negative shocks to the series increase the volatility stronger than the positive ones.

We check the adequacy of the variance model by examining the series $\{\widehat{z}_t\}$, the series of standardized residuals defined as $\widehat{\varepsilon}_t/\sqrt{\widehat{h}_t}$, where $\widehat{\varepsilon}_t$ are the estimated residuals from the mean equation and $\sqrt{\widehat{h}_t}$ is the estimated conditional volatility. The Ljung-Box test statistics of $\widehat{z}_t$ are used to check the adequacy of the mean equation and those of the $\widehat{z}_t^2$ of the volatility equation. If the model for the series of returns is correctly specified we expect not to have any autocorrelation in the series of standardized and standardized squared residuals. We compute the Ljung-Box test statistics for 5 and 10 lags.

Table 4 shows the results for each of the series.

	$DJIA$	$S\&P$	$FTSE$	DAX	$NIKKEI$	WTI
c	0.0403* (3.11)	0.0320* (2.33)	0.0341* (2.73)	0.0429* (2.22)	0.0365* (2.01)	0.0018 (0.37)
ϕ_1		0.0379* (2.26)	0.0231** (1.69)			0.7350* (9.12)
θ_1						0.7191* (8.87)
θ_2						0.0589* (3.76)
ω	0.0298* (15.02)	0.0237* (2.32)	0.0326** (1.88)	0.0617* (2.00)	0.0604* (3.57)	0.0339* (8.33)
α	0.0176* (2.89)	0.0104** (1.86)	0.0567* (2.88)	0.0473* (3.18)	0.0442* (3.17)	0.0969* (9.55)
γ	0.1155* (4.21)	0.1237* (4.32)	0.0804* (1.94)	0.0728* (2.28)	0.1050* (5.12)	
β	0.8997* (65.03)	0.9069* (47.81)	0.8736* (19.72)	0.8891* (25.66)	0.8843* (47.51)	0.9028* (88.93)
$Q(5)$	9.72 (0.08)	8.60 (0.12)	5.34 (0.37)	2.02 (0.84)	2.86 (0.72)	7.34 (0.19)
$Q(10)$	13.97 (0.17)	16.66 (0.08)	14.27 (0.16)	7.89 (0.63)	15.51 (0.11)	13.09 (0.21)
$Q(5)^2$	1.72 (0.88)	2.11 (0.83)	4.90 (0.42)	7.82 (0.16)	0.57 (0.98)	1.96 (0.85)
$Q(10)^2$	5.87 (0.82)	4.22 (0.93)	6.06 (0.80)	10.63 (0.38)	1.29 (0.99)	11.60 (0.31)

Table 4. Empirical estimation of the series of returns. We present the conditional mean equation and conditional volatility equation defined either as linear GARCH model or GJRGARCH model. The model is defined as $r_t = c + \phi_1 r_{t-1} + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \varepsilon_t$, $\varepsilon_t = \sqrt{h_t} z_t$, $h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$ (GARCH) or $h_t = \omega + \alpha\varepsilon_{t-1}^2 + \gamma S_{t-1}^{-}\varepsilon_{t-1}^2 + \beta h_{t-1}$ (GJRGARCH). In parenthesis we report the t-statistics for the parameters and p-values for the Ljung-Box test statistics (Q(5) and Q(10) for standardized residuals and $Q(5)^2$ and $Q(10)^2$ for squared standardized residuals. * - statistically significant at 5% level, ** - at 10% level.

All the parameters (except for constant in the case of WTI) are statistically significant at 5% or 10% level of significance.

The Ljung-Box test statistics for both standardized and squared standardized residuals do not show any remaining autocorrelation in standardized and squared standardized residuals so that the mean and volatility equations are correctly specified.

The figures below show the estimated conditional volatilities for DJIA (other stock markets show a very similar figures) and WTI (all the plots are presented in the appendix).

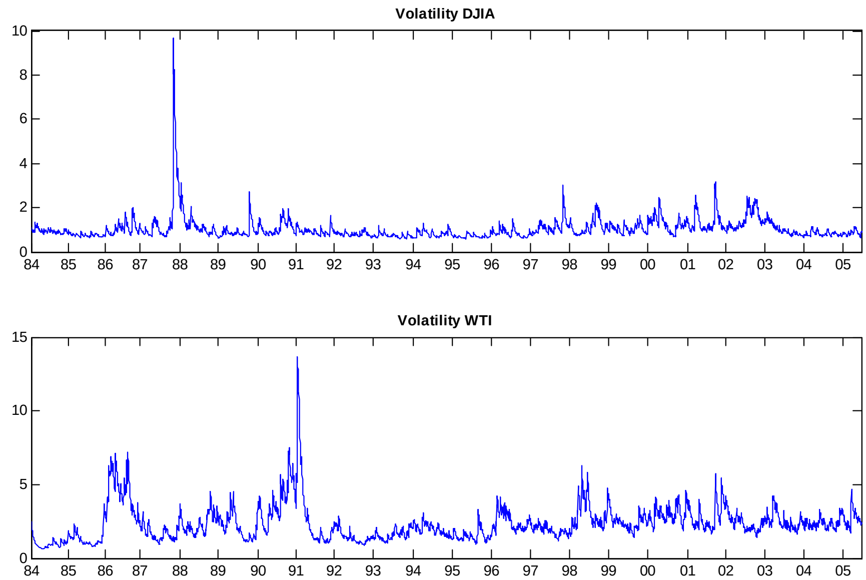


Figure 5. Estimated conditional volatility of DJIA and WTI

Figure 5 shows the evolution of conditional volatility over the period of interest. In case of DJIA we observe a high peak around the end of 1987, which reflects the stock market turbulences in October 1987 when DJIA lost during the single day more than 20%, high volatile periods at the beginning of 1990 (Gulf war), Asian and Russian financial crises (1997-1998), dot com bubble (2000-2001).

The volatility of oil prices shows much higher levels of volatility and periods of turbulences are more frequent. Until 1986 Saudi Arabia acted as the swing producer cutting its production to stop the fall in prices. By early 1986 they linked their oil price to the spot market for crude and increased their production from 2 m bl/s (million barrels per day) to 5 m bl/d Crude oil prices plummeted below $10 per barrel by mid-1986.

The price of oil increased significantly in 1990 with the cuts in the production caused by the Iraqi invasion on Kuwait (August 1990) and the following Gulf war. In 1998 due to the financial crises the Asian Pacific oil consumption declined for the first time since 1982, higher OPEC production sent the prices into the downward spiral. In the fears of the economic downturn after the terrorist attack in September 2001 the price of WTI was down by 35 percent by the middle of November. In March 2003 the US military action commenced in Iraq.

The univariate GARCH models with the oil price returns as the explanatory variable - Specification 1, 2 and 3

Specification 1 and 2 test if the stock market returns react to changes in the oil prices. In Specification 1 (see results in Table 5) we use the oil price returns as the explanatory variable in the mean equation (see appendix), in Specification 2 (see results in Table 6) we use one-day lag of the oil price return as the explanatory variable.

Specification 1					
	$DJIA$	$S\&P$	$FTSE$	DAX	$NIKKEI$
r_oil	−0.0131* (−3.12)	−0.0098* (−2.27)	0.0053 (0.90)	−0.0135** (−1.70)	−0.0132 (−1.55)

Table 5. The results of the estimation of Specification 1 models. We only report the value of the coefficient of the oil price return as the explanatory variable (rest of the results are presented in the appendix). In parenthesis we report the t-statistics for the parameters and p-values, * - statistically significant at 5% level, ** - at 10% level.

Specification 2					
	$DJIA$	$S\&P$	$FTSE$	DAX	$NIKKEI$
r_oil	0.0047 (0.90)	0.0046 (0.90)	0.0024 (0.43)	0.0052 (0.65)	−0.0039 (−0.40)

Table 6. The results of the estimation of Specification 2 models. We only report the value of the coefficient of the lagged oil price return as the explanatory variable (rest of the results are presented in the appendix). In parenthesis we report the t-statistics for the parameters and p-values, * - statistically significant at 5% level, ** - at 10% level.

The results show that the changes in oil prices affect the returns of DJIA, S&P500 and DAX. The remaining stock markets are not affected by the daily changes in oil prices.

The impact of the oil prices on DJIA, S&P and DAX is similar in both the nature and magnitude. This impact is negative, as expected with the economic theory. The increase in oil prices (positive returns) lowers the return on the stock index. The estimated coefficient of oil price returns even if small in magnitude accounts for about one third of the daily average return.

U.S. economy is the world´s first consumer of oil and accounts for 22% of the world consumption[2] and net importer of oil. For the oil importing country the effect of increase in oil

[2] www.eia.gov

prices should have a negative impact on the stock markets and this is what we observe for DJIA and S&P500.

The impact of oil prices on DAX is also negative - Germany is the biggest European oil importer but also the German index includes the leading world automobile companies whose shares are negatively affected by the rising oil prices.

FTSE 100 is positively affected by the oil prices but this relationship is not statistically significant. The UK for the long period of time was the oil net exporter, so would benefit from increases in oil prices. Moreover the FTSE 100 includes the leading oil producing companies and many commodity producing companies. Nandha and Hammamoudeh (2007) show natural resources prices are positively correlated with oil prices, hence mining corporations also perform better during rising oil prices.

Interestingly there no statistically significant impact of oil prices on the Japanese stock market even though that Japan the world´s third oil importer.

To benefit from this result we go long futures on the DJIA, S&P500 and DAX when the oil prices are going down (so since the coefficient of the oil return is negative the index should go up and the futures as well) or short futures on these indexes when oil prices are going up (the index will go down and futures as well). Futures are no-cost strategy and such liquid markets like futures on DJIA, S&P500 and DAX allow closing the long or short position quickly.

We check if the lagged returns on the oil prices have any influence on the stock market returns and we do not detect any such relationship in any of the markets (Specification 2).

Finally, we investigate the possible asymmetry in the relationship between oil prices and stock markets by computing a dummy variable for negative and positive oil returns and we consider this variable as the new explanatory variable in the mean equation (Specification 3). In the next step we also analyze one-day lag of such dummies. In none of the cases we obtain statistically significant results and conclude that there is no asymmetry in this relationship. This lack of asymmetry in the response of the daily stock returns to positive and negative oil price returns coincides with the results of Park and Ratti (2008) who using the monthly data for US and 13 European countries and VAR models do not detect any asymmetry in the reaction of stock markets to changes in oil prices. The same results obtain Kilian and Vigfusson (2009) as well.

The univariate GARCH models with the volatility of returns on oil prices as the explanatory variable - Specification 4

The aim of this analysis is to investigate the impact of volatility of oil price returns on stock market returns.

We consider the squared oil price returns (proxy for oil volatility), as the explanatory variable in the mean equation for the returns on oil prices. We also add the oil price return as the explanatory variable to be able to isolate the impact of oil return volatility on stock index returns.

In Table 7 we present the estimated coefficients and the t-statistic for the proxy of oil volatility for each of the stock markets and each regime.

	$DJIA$	$S\&P$	$FTSE$	DAX	$NIKKEI$
Regime 1	0.0053 (0.47)	0.0176 (1.06)	0.0129 (0.38)	0.0295* (2.46)	0.0136 (1.06)
Regime 2	0.0044* (3.04)	0.0043* (4.09)	−0.0017 (−0.80)	−0.00004 (−0.001)	0.0021 (1.27)
Regime 3	0.0014 (0.47)	−0.0001 (−0.04)	0.0035 (1.23)	0.0025 (0.71)	0.0006 (0.27)
Regime 4	0.0011 (1.59)	0.0007 (1.34)	0.0003 (0.80)	−0.0015* (−2.14)	−0.0020* (−1.85)
Regime 5	0.0002 (0.06)	−0.0026 (−0.87)	−0.0063 (−0.97)	−0.0014 (−0.15)	0.0037 (0.44)
Regime 6	−0.0016 (−1.15)	−0.0013 (−0.92)	−0.0003 (−0.23)	−0.00001 (0.007)	−0.0012 (−0.58)

Table 7. The impact of oil return volatility on stock market returns. We present the estimated coefficient with the t-statistics in parenthesis, * - statistically significant at 5% level, ** - at 10% level.

The DJIA and S&P 500 react to high volatility and falling oil prices (Regime 2). This impact is positive since the falling oil prices could be seen as a positive cost factor for companies. DAX reacts positively to low volatility of oil prices (Regime 1) and negatively to high volatility of oil prices combined with increasing prices and geopolitical events (Regime 4). In the same way NIKKEI also negatively reacts to the high volatility present during the Regime 4.

Interestingly, from March 1992 (Regime 5 and 6) we do not observe any impact of oil price volatility on stock market returns.

To cash-out these results we go long futures on DJIA and S&P 500 when we observe an environment like in Regime 2 - high volatility and falling oil prices. The positive reaction of the

American markets will be reflected in the stock index increase and the futures on these indexes will rise as well. Being in the regime characterized by stable oil prices and low volatility the profitable strategy is to go long futures on DAX since these market conditions boost the index and the futures as well. The opposite strategy should be used in the case of the regime defined by increasing oil prices and geopolitical unrest in the oil producing countries - the best strategy is to short futures on DAX or short futures on NIKKEI and enjoy the falling index quotations and falling futures prices.

2.4.2 Non-linear specification

We analyze the results of using $SOPI_t$ variable as the explanatory variable in the mean equation. We construct the series of $SOPI_t$ variable in the way the Lee, Ni and Ratti (1995) discuss. In our case the model for the oil price returns is $ARMA(1,2) - GARCH(1,1)$. The variable of interest is defined as

$$SOPI_t = \max\left(0, \widehat{\varepsilon}/\sqrt{\widehat{h}_t}\right)$$

where $\widehat{\varepsilon}$ are the estimated residuals from the mean equation and $\widehat{h}_t$ is the estimated conditional variance of returns on oil prices.

Table 8 presents the value of the estimated parameters and in parenthesis the t-statistic (we present results in appendix).

	$DJIA$	$S\&P$	$FTSE$	DAX	$NIKKEI$
$SOPI_t$	-0.0047 (-0.26)	-0.0063 (-0.35)	0.0190 (0.81)	-0.0367 (-0.97)	0.0369 (0.99)

Table 8. The results of the estimation of the models with SOPI_t as the explanatory variable in the mean equation. In parenthesis the values of the t-statistic.

The results of the estimation show that for each of the stock market indices the explanatory variable $SOPI_t$ - proxy for positive shocks of returns on oil prices is not statistically significant at 5% level of significance. The positive shocks of oil prices do not directly affect the returns on stock market indices.

In the second part of the analysis we consider another nonlinear transformation of oil price variable $NOPI_t$ - the net oil price increases as discussed before. We take into account different length of the series starting from $n = 5$ (a week) to $n = 10$ (two weeks). This variable will account for "significant" oil price increases during the period of n days.

We present the estimated coefficients of the $NOPI_t$ variable for $n = 5$ (a week) to $n = 10$ (two weeks) in the appendix for all the markets. We do not detect any impact of the $NOPI_t$ variable on returns of any stock market.

In the last part of the analysis we want to discuss the results of the Hansen test for the threshold effects in the relationship between oil price returns and stock market returns and between volatility of oil price return and stock market returns. Table 9 and Table 10 present the result of the test

	$DJIA$	$S\&P$	$FTSE$	DAX	$NIKKEI$
$Threshold$	-1.67	0.44	0.44	-1.76	-1.94
$statistic$	3.34	2.45	6.82	2.02	4.54
$p-value$	0.45	0.62	0.11	0.72	0.29

Table 9. The results of the Hansen (1996,2000) test for the threshold effect in the relationship between oil price returns and stock market returns.

	$DJIA$	$S\&P$	$FTSE$	DAX	$NIKKEI$
$Threshold$	1.33	1.37	1.37	1.92	1.29
$statistic$	2.09	1.68	1.18	1.53	3.15
$p-value$	0.78	0.87	0.94	0.87	0.53

Table 10. The results of the Hansen (1996,2000) test for the threshold effect in the relationship between volatility of oil price return and returns on stock market indices.

Table 9 and Table 10 present the results of the Hansen threshold test for threshold variables as oil price returns and oil price volatility. For any of the markets we cannot reject the null hypothesis that there is no threshold effect in the relationship between oil returns and stock market returns or oil volatility and stock market returns.

2.4.3 Volatility linkages - ECCC-GARCH

In this section we discuss the links between volatilities of the stock market returns and oil price returns.

We consider the ECCC-GARCH model of Bollerslev (1990) as indicated by the Engle and Sheppard (2002) test for the constant versus dynamic correlation structure test. We work in the bivariate framework - stock market returns and oil price returns.

First we filter the series by removing the deterministic component for each of the series to obtain pure stochastic errors from the model.

Engle and Sheppard (2002) propose a test to determine the nature of the conditional correlation among time series. They point out that testing models for constant correlation has proven to be a difficult problem, as testing for dynamic correlation with data that has time-varying volatilities can result in misleading conclusions and rejection of constant correlation when it is true due to the misspecified volatility model. They propose a test that only requires consistent estimate of the constant conditional correlation, and can be implemented using a vector autoregression. We discuss the details of the test in the appendix.

The table below shows the results of the Engle and Sheppard test for constant versus dynamic correlation structure in the bivariate framework - returns on given stock market and returns on oil prices.

We present the results for the bivariate models for lags from 1 to 5 with corresponding p-value in parenthesis.

lag	$DJIA$	$S\&P$	$FTSE$	DAX	$NIKKEI$
1	1.26 (0.53)	1.92 (0.38)	0.49 (0.77)	0.86 (0.64)	1.18 (0.553)
2	4.70 (0.19)	6.60 (0.08)	3.16 (0.36)	2.59 (0.45)	6.56 (0.087)
3	5.69 (0.22)	6.71 (0.15)	4.69 (0.31)	2.61 (0.62)	9.07 (0.059)
4	7.21 (0.20)	8.17 (0.14)	4.73 (0.44)	3.58 (0.61)	9.076 (0.106)
5	9.07 (0.16)	8.48 (0.20)	4.98 (0.54)	4.19 (0.64)	10.05 (0.122)

Table 11. Results of the test for constant correlation structure.

For each bivariate model we accept the hypothesis about the constant correlation structure at 5% level of significance. Following the results of the test we consider the ECCC-GARCH model for the bivariate case.

To check the links between volatilities of oil price returns and stock market returns we take into account again different regimes of the oil return volatility we have previously discussed. For each regime we estimate the ECCC-GARCH model with imposed conditions for positiveness and stationarity. For each stock market and each regime we only present the values of the parameters of interest - α_{12} which measures the impact of lagged oil shocks on the stock return volatility, β_{12} which measures the impact of lagged oil return volatility on stock return volatility and estimated ρ which measures the correlation between shocks to volatilities.

Table 12 presents the values of the estimated parameters and the values of the asymmptotic t-statistics.

	$DJIA$	$S\&P$	$FTSE$	DAX	$NIKKEI$
Regime 1					
α_{12}	0.0509* (2.08)	0.0367* (2.52)	0.0165 (0.89)	0.0655 (0.97)	0.1598* (3.63)
β_{12}	−0.0608* (−2.27)	−0.0456* (−2.85)	−0.0272** (−1.65)	−0.0140 (−0.18)	−0.1051* (4.911)
ρ	0.082** (1.81)	0.089** (1.82)	−0.0060 (−0.11)	0.1532* (2.92)	0.1206* (2.43)
Regime 2					
α_{12}	0.000 (0.00)	0.000 (0.00)	0.000 (0.00)	0.000 (0.00)	0.000 (0.00)
β_{12}	0.0024 (0.52)	0.0005 (0.18)	−0.0016 (−0.91)	−0.0445 (1.752)	−0.0116 (−1.09)
ρ	−0.1808* (−2.46)	−0.1423** (−1.71)	0.0564 (0.70)	0.0111 (0.15)	−0.0422 (−0.50)
Regime 3					
α_{12}	0.0000 (0.00)	0.0000 (0.00)	0.0109* (2.43)	0.000 (0.00)	0.0036 (0.73)
β_{12}	−0.0047* (−3.06)	−0.0028* (−2.10)	−0.0198* (−4.47)	−0.0007 (−1.57)	−0.0036 (−0.32)
ρ	0.0372 (1.01)	0.0228 (0.61)	0.1059* (2.64)	0.0434 (1.33)	0.0542** (1.76)
Regime 4					
α_{12}	0.0000 (0.00)	0.0000 (0.00)	0.0008 (0.48)	0.0051** (1.70)	0.0000 (0.00)
β_{12}	−0.0004 (−0.48)	−0.0005 (−0.76)	−0.0018 (−1.30)	−0.0052 (−0.606)	−0.0043 (−1.21)
ρ	−0.3007* (−4.75)	−0.2937* (−5.06)	−0.2423* (−3.12)	−0.3506* (−5.52)	−0.2532* (−3.72)
Regime 5					
α_{12}	0.0117* (3.31)	0.0092* (4.97)	0.0166* (3.42)	0.0127 (0.91)	0.0330* (2.54)
β_{12}	−0.0266* (−2.92)	0.0218* (−6.72)	−0.0369* (−2.79)	−0.0182 (−0.48)	−0.0871* (−3.40)
ρ	−0.0372 (−1.29)	−0.0304 (−1.11)	−0.0528 (−1.54)	−0.0403 (1.33)	0.0233 (0.80)

Regime 6					
α_{12}	0.0016 (1.05)	0.0018 (1.41)	0.0021* (1.89)	0.0045* (1.89)	0.0060 (1.53)
β_{12}	−0.0049 (−1.18)	.−0.0050 (−1.40)	−0.0093* (−2.41)	−0.0172* (−2.38)	−0.0281* (−2.30)
ρ	−0.0145 (−0.71)	−0.0063 (−0.30)	0.0120 (0.57)	0.0037 (−0.18)	−0.0172 (−0.79)

Table 12. Estimation of ECCC-GARCH model for each stock market and each oil price volatility regime. * - statistically significant at 5% level, ** - at 10% level.

The results of estimation of ECCC-GARCH model for each stock market return and oil returns in each regime of oil volatility reveal interesting results.

During the periods of very high oil price volatility (Regime 4 and Regime 2) we observe negative and statistically significant contemporaneous correlations between shocks to oil price returns and shocks to stock index returns for all the stock markets we consider. The positive shocks to oil price returns (thus increases in prices) are transmitted immediately to stock markets in form of negative shocks to stock market returns (so decreases in the level of stock index).

During the period of low oil price volatility (Regime 1,3,5,6 - we consider these regimes jointly as low oil volatility regimes although as already discussed the standard deviation of oil returns for these regimes is between 0.9204 and 2.6009, however the results are similar) shocks to oil prices will results in the increase in the stock index volatility next day (α_{12} is statistically significant and positive for all the stock markets of interest). There is also a feedback reaction of stock index volatility to the level of volatility of oil prices - the low levels of oil volatility diminishes the volatility of stock market indexes next day (β_{12} appear negative and statistically significant for all the stock markets we consider).

To profit from the predicted changes in the volatility of oil price returns we can use different option strategies. Contrary to the futures positions, these are not the cost-free strategies (going long an option an investor incurs a cost in form of premium paid). In particular, during the period of low oil price volatility to benefit from shocks to oil prices and the resulting increase in stock index volatility we can go long straddle which is the combination of long call and long put option with the same strike and maturity. To enter a straddle we pay premiums for both a call and a put. The combination of these two options results in no exposure to the underlying (the stock market index) yet it is extremely sensitive to the volatility of the index. This V-shaped strategy will be profitable if the volatility increases – will benefit from large moves in the underlying in either direction. Of course the gain on the options should cover at least the cost of the strategy.

To benefit from the low level of the oil price volatility and the resulting decrease in the stock index volatility we should construct the opposite strategy we have just discussed. We should go short straddle which is the combination of short call and short put on the stock index with the same strike and the same maturity – in this case we receive premiums for both put and call (since we sell the options) and again there is no exposure to the underlying index yet this position is sensitive to the changes in volatility of the index and will benefit from the decreases in the index volatility.

2.5 Conclusions

In this work we analyze and assess the relation between oil prices and oil price volatility and main stock market indices. We consider the prices of WTI crude and five main world stock indexes - DJIA, S&P500, DAX, FTSE100 and NIKKEI225.

The results show different channels the oil prices impact stock markets.

The changes in oil prices affect DJIA, S&P500 and DAX. The remaining stock markets are not affected by the daily changes in oil prices. The impact of the oil returns on DJIA, S&P and DAX is similar in both the nature and magnitude. This impact is negative, as expected with the economic theory. The increase in oil prices (positive returns) lowers the return on the stock index. The estimated coefficient of oil price returns even if small in magnitude accounts for about one third of the daily average return of the stock index. Lagged oil returns have no influence on the stock market neither there is any asymmetry in this relationship.

The returns of DJIA and S&P 500 react to high volatility and falling oil prices. This impact is positive since the falling oil prices could be seen as a positive cost factor for companies. DAX reacts positively to low volatility of oil returns and negatively to high volatility of oil prices combined with increasing prices and geopolitical events. In the same way NIKKEI also negatively reacts to the high oil return volatility.

There is no relationship between the non-linear transformations of oil prices and the stock market returns. We also do not detect any threshold effect in the relationship between oil returns and stock market returns or oil return volatility and stock market returns.

During the periods of very high oil return volatility we observe negative and statistically significant contemporaneous correlations between shocks to oil returns and shocks to stock index returns for all the stock markets we consider. The positive shocks to oil price returns (thus increases in prices) are transmitted immediately to stock markets in form of negative shocks to stock market returns (so decreases in the level of stock index).

During the period of low oil volatility shocks to oil prices will results in the increase in the stock index volatility next day. There is also a feedback reaction of stock index volatility to the level of volatility of oil prices - the low level of oil volatility diminishes the volatility of stock market indexes next day.

The straightforward extension of this analysis is the sector analysis. Analyzing the sector indices (e.g. transportation, energy, banks) we could detect the reaction of different groups of companies on the changes in the oil prices and this could be a good tool when optimizing the portfolio composition since we could give hints which portfolio management strategies to consider when there are changes in oil prices.

Bibliography

[1] Apergis, N. and S.M. Miller (2009), "Do Structural Oil-Market Shocks Affect Stock Prices", *Energy Economics*, 31, 569-575.

[2] Bauwens, L., S. Laurent and J.V.K. Rombouts (2006), "Multivariate GARCH Models: A Survey", *Journal of Applied Econometrics*, 21, 79-110.

[3] Bollerslev, T. (1986), "Generalized Autoregressive Conditional Heteroscedasticity", *Journal of Econometrics,* 31, 307-327.

[4] Bollerslev, T. (1990), "Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Model", *Review of Economics and Statistics,* 72, 498-505.

[5] Bollerslev, T. and J. Wooldridge (1992), "Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time Varying Covariance", *Econometric Reviews*, 11, 143-172.

[6] Campbell, J.Y., M. Lettau, B.G. Malkiel and Y. Xu (2001), "Have Individual Stock Become More Volatile? An Empirical Exploration of the Idiosyncratic Risk", *Journal of Finance*, 56, 1-43.

[7] Cappiello, L., R.F. Engle and K. Sheppard (2006), "Asymmetric Dynamics in the Correlations of Global Equity and Bond Returns", *Journal of Financial Econometrics*, 4, 537-572.

[8] Conrad, Ch. and M. Karanasos (2010), "Negative Volatility Spillovers in the Unrestricted ECCC-GARCH Model", *Econometric Theory,* 26, 838-862.

[9] Engle, R. F. (1982), "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation", *Econometrica,* 50, 987-1008.

[10] Engle, R.F. and V.K. Ng (1993), "Measuring and Testing the Impact of News on Volatility", *Journal of Finance,* 48, 1749-1778.

[11] Engle, R.F. and K. Sheppard (2002), "Theoretical and Empirical Properties of Dynamic Conditional Correlation Multivariate GARCH", *NBER Working Paper* 8554.

[12] Glosten, L., R. Jagannathan and D. Runkle (1993), "On the Relation between Expected Return on Stocks", *Journal of Finance*, 48, 1779-1801.

[13] Hamilton, J. (1983), "Oil and Macroeconomy since World War II" , *Journal of Political Economy,* 91, 228-248.

[14] Hamilton, J. (1996), "This is what happened to the oil price–macroeconomy relationship?", *Journal of Monetary Economy,* 38, 215-220.

[15] Hansen, B. (1996), "Inference when the Nuisance Parameter is not Identified under the Null Hypothesis", *Econometrica,* 64, 413-430.

[16] Hansen, B. (2000), "Sample Splitting and Threshold Estimation", *Econometrica*, 68, 575-603.

[17] Huang, R.D., R.W. Masulis and H.R. Stoll (1996), "Energy Shocks and Financial Markets", *Journal of Futures Markets*, 16, 1-27.

[18] Huang, B.-N., Hwang M.J. and H.P. Peng (2005), "The Asymmetry of the Impact of Oil Price Shocks on Economic Activities: An Application of the Multivariate Threshold Model", *Energy Economics*, 27, 455-476.

[19] Inclan, C. and G.C. Tiao, “Use of Cumulative Sums of Squares for Retrospective Detection of Changes of Variance”, *Journal of American Statistical Association*, 89 (1994), 913-923.

[20] Jeantheau, T. (1998), "Strong Consistency of Estimators for multivariate ARCH models", *Econometrics Theory*, 14, 70-86.

[21] Jones, C.M. and G. Kaul (1996), "Oil and Stock Markets", *Journal of Finance*, 51, 463-491.

[22] Karolyi, K.R. (1995), "A Multivariate GARCH Model of International Transmissions of Stock Returns and Volatility: The Case of United States and Canada", *Journal of Business and Economic Statistics,* 13, 11-25.

[23] Kilian, L. and C. Park (2009), "The Impact of Oil Price Shocks on the U.S. Stock Market", *International Economic Review*, 50, 1267-1287.

[24] Kilian, L. and R. Vigfusson (2009), "Pitfalls in Estimating Asymmetric Effects of Energy Price Shocks", *CEPR Discussion Paper* No. DP7284.

[25] Lee, K., S. Ni and R.S. Ratti (1995), "Oil shocks and the macroeconomy: The role of price variability.", *Energy Journal,* 16, 39-56.

[26] Lundberg, S. and T. Teräasvirta, (2002), "Evaluating GARCH model", *Journal of Econometrics,* 110, 417-435.

[27] Marten, M. and S.H., Poon (2001), "Return Synchronization and Daily Correlation Dynamics between International Stock Markets", *Journal of Banking and Finance*, 25, 1805-27.

[28] Mork, K. (1989), "Oil and the Macroeconomy when Prices Go Up and Down: An extension of Hamilton's results.", *Journal of Political Economy,* 97, 740-744.

[29] Mork, K., O.Olsen and H. Mysen (1994), "Macroeconomic responses to oil price increases and decreases in seven OECD countries", *Energy Journal,* 15, 19-35.

[30] Nakatani, T. and T. Teräsvirta (2008), "Positivity Constraints on the Conditional Variances in the Family of Conditional Correlation GARCH Models", Working Paper No. 675, Stockholm School of Economics.

[31] Nandha, M. and R. Faff (2008), "Does Oil Move Equity Prices? A Global View", *Energy Economics*, 30, 986-997.

[32] Nelson, D.B. and C.Q. Cao (1992), "Inequality constraints in the Univariate GARCH Model", *Journal of Business and Economic Statistics*, 10, 229-235.

[33] Park, J.W. and R.A. Ratti (2008), "Oil Price Shocks and Stock Markets in the U.S. and 13 European Countries", *Energy Economics*, 30, 2587-2608.

[34] Sadorsky, P. (1999), "Oil Price Shocks and Stock Market Activity", *Energy Economics*, 2, 449-469.

[35] Sansó, A., Arragó, V. and J. Carrion-Silvestre, "Testing for change in the unconditional variance of financial time series", *Revista de Economía Financiera*, 4 (2004), 32-53.

[36] Tse, Y.K., and A.K.C. Tsui (2002), "A Multivariate GARCH Model with Time-Varying Correlations, *Journal of Business and Economic Statistics,* 20, 351-362.

[37] Wong, H., W. K. Li, and S. Ling (2000), “A Cointegrated Conditional Heteroscedastic Model with Financial Applications,” No. 2000-5, Department of Applied Mathematics, The Hong Kong Polytechnic University, Hong Kong.

2.6 Appendix 1. Specification of the models

2.6.1 Specifications

In this section we want to present the specification

Let us denote r_stock_t as the return on given stock market at time t, $ARMA$ - autoregressive - moving average specification of the conditional mean equation, specific for every stock markets. The $ARMA$ specification is given as

$$r\_stock_t = c + \alpha_1 r\_stock_{t-1} + ... + \alpha_n r\_stock_{t-n} + \varepsilon_t + \beta_1 \varepsilon_{t-1} + ... + \beta_m \varepsilon_{t-m}$$

where m, n are specific for every stock markets.

r_oil_t is the return on oil prices at time t, vol_t a series of conditional volatility of stock prices.

We will test different specifications for the mean equation and taking an appropriate GARCH model as the model for conditional volatility.

Specification 1 tests if there is any impact of the oil prices on each of the stock markets. The relevant equation is given as

$$r\_stock_t = ARMA\ + r\_oil_t + \varepsilon_t$$

where the dynamics of the shocks is modelled by appropriate (symmetric or asymmetric GARCH model).

We also analyze the lagged returns of oil prices as the explanatory variable (Specification 2) to investigate if the changes of oil prices in the past influence the stock markets contemporaneously. This specification takes into account the lagged oil prices and is given by

$$r\_stock_t = ARMA\ + r\_oil_{t-1} + \varepsilon_t$$

Further to analyze the way the oil prices influence stock market we construct the dummy variable that accounts for the sign of the returns on oil prices to investigate if there is any asymmetry in the relationship between returns on stock markets and returns on oil prices (Specification 3). Let us define the dummy variable d_t that takes the value of 1 when $r\_oil_t < 0$ and zero otherwise.

$$r\_stock_t = ARMA \;\; + d_t + \varepsilon_t$$

Another specification we will test takes the estimated conditional volatility of returns on oil prices as the explanatory variable in the mean equation of the returns on stock markets Specification 3), here we can analyze if the returns of the stock markets depend on the volatility of oil prices.

$$r\_stock_t = ARMA \;\; + vol_t + \varepsilon_t$$

.

2.7 Appendix 2: Tests

2.7.1 ARCH - LM test - Engle (1982)

Engle(1982) developed a test for conditional heteroscedasticity in the context of ARCH models based on the Lagrange Multiplier principle. The LM test can be computed as nR^2, where n is the sample size and R^2 is obtained from a regression of the squared residuals on the constant and q of its lags. The LM test statistic has an asymptotic $\chi^2(q)$ distribution.

2.7.2 Sign Bias, Positive Size Bias and Negative Size Bias tests - Engle and Ng (1993)

Engle and Ng (1993) propose tests to check whether positive and negative shocks have a different impact on the conditional variance. Let S_{t-1}^{-} denote a dummy variable which takes the value of 1 when $\widehat{\varepsilon}_{t-1}$ is negative and 0 otherwise, where $\widehat{\varepsilon}$ are residuals from estimating a model for the conditional mean of the series under the assumption of conditional homoscedasticity. The tests examine whether the squared residuals can be predicted by $S_{t-1}^{-}, S_{t-1}^{-}\widehat{\varepsilon}_{t-1}$, and or $S_{t-1}^{+}\widehat{\varepsilon}_{t-1}$, where $S_{t-1}^{+} = 1 - S_{t-1}^{-}$.

The test statistics are computed as the t-ratio of the parameter γ_1 in the regression

$$\widehat{\varepsilon}_t^2 = \gamma_0 + \gamma_1 \widehat{w}_t + \xi_t$$

where $\widehat{w}_t$ is one of the three measures of asymmetry defined above and ξ_t the residual.

When $\widehat{w}_t = S_{t-1}^{-}$ in the regression the test is called Sign Bias (SB) as it tests whether the magnitude of the square of the current shock ε_t (and as the consequence the conditional variance h_t) depends on the sign of the lagged shock ε_{t-1}. In the case when $\widehat{w}_t = S_{t-1}^{-}\widehat{\varepsilon}_{t-1}$ or $\widehat{w}_t = S_{t-1}^{+}\widehat{\varepsilon}_{t-1}$ the tests are called Negative Size Bias(NSB) and Positive Size Bias(PSB), respectively, and these tests examine whether the effect of positive or negative shocks on the conditional variance also depends on their size.

2.7.3 Hansen (1996,2000) test for threshold effects

Hansen(2000) - proposed the method based on a threshold regression model where observations fall into classes or regimes that depend on the unknown value of the observed variable

$$y_{it} = \beta_0 + \beta_{a1} A_{it} I(A_{it} \leq \gamma) + \beta_{a2} A_{it} I(A_{it} > \gamma) + \beta_z z_{it} + u_{it}$$

where $I(\cdot)$ is the indicator function and z_{it} are other regressions.

Hansen(2000) recommends obtaining the least square estimate $\widehat{\gamma}$ as the value that minimizes the sum of squared errors $S_I(\gamma)$. The sum of the squared errors in turn depends on γ through the indicator function. Minimization problem here is the step procedure where each step occurs at the distinct values of the observed threshold value (A_{it}) For each of these values the threshold regression model is estimated and the sum of squared residuals obtained. The value $\widehat{\gamma}$ is the one that minimizes the function.

Hansen(2000) suggests bootstrapping to obtain the p-value of this test. First estimate the model under the null and alternative, this gives the actual values of the likelihood ratio test F_1

$$F_1 = \frac{S_0 - S_1(\widehat{\gamma})}{\widehat{\sigma}^2} \qquad \widehat{\sigma}^2 = \frac{1}{n(t-1)} S_1(\widehat{\gamma})$$

A bootstrap sample is created by drawing from the normal distribution of the residuals of the estimated threshold model. Regressors are hold fixed in the repeated bootstrap sample using the generated sample the model is estimated under the null (of no threshold) and alternative ($\widehat{\gamma}$) to obtain a new F_1. Repeat this procedure large number of times. The bootstrap estimate of the p-values for F_1 under the null is given by the percentage of draws for which the simulated statistic F_1 exceeds the actual one.

2.7.4 Test for dynamic correlation model - Engle and Sheppard (2001)

The null hypothesis is of the constant correlation against the alternative of dynamic conditional correlation

$$H_0 : R_t = \overline{R} \quad t \epsilon T$$

$$H_A : vech(R_t) = vech(\overline{R}) + \beta_1 vech(R_{t-1}) + ... + \beta_p vech(R_{t-p})$$

The testing procedure is as follows. Estimate the univariate GARCH processes and standardized the residuals for each series. Then estimate the correlation of the standardized residuals, and jointly standardized the vector of univariate standardized residuals by the symmetric square root decomposition of $\overline{R}$. Under the null of constant correlation, these residuals should be IID with the variance covariance matrix unit diagonal I_k (we consider k series). The artificial regression will be a regression of the outer products of the residuals on a constant and lagged outer products. Let

$$Y_t = vech^u[(\overline{R}^{-1/2} D_t^{-1} \varepsilon_t)(\overline{R}^{-1/2} D_t^{-1} \varepsilon_t)' - I]$$

where $(\overline{R}^{-1/2} D_t^{-1} \varepsilon_t)$ is a k by 1 vector of residuals jointly standardized under the null, and $vech^u$ is a modified $vech$ which only selects elements above the diagonal. The vector autoregression is

$$Y_t = \alpha + \beta_1 Y_{t-1} + ... + \beta_s Y_{t-s} + \eta_t$$

Under the null the constant and all the lagged parameters in the model should be zero. The test statistics is χ^2_{s+1} distributed.

2.8 Appendix 3: Results

2.8.1 The univariate GARCH models with the oil price returns as the explanatory variable - Specification 1

Table 13. Estimation results for each series of stock market returns for Specification 1. Specification 1 takes the oil price returns as the explanatory variable in the mean equation.

Specification 1					
	DJIA	*S&P*	*FTSE*	*DAX*	*NIKKEI*
c	0.0405* (3.11)	0.0323* (2.34)	0.0339* (2.71)	0.0432* (2.21)	0.0369* (2.03)
ϕ_1		0.0371* (2.22)	0.0230** (1.69)		
r_oil	−0.0131* (−3.12)	−0.0098* (−2.27)	0.0053 (0.90)	−0.0135** (−1.70)	−0.0132 (−1.55)
ω	0.0294* (6.28)	0.0235* (2.30)	0.0327** (1.86)	0.0630** (1.87)	0.0606* (3.56)
α	0.0177* (2.98)	0.0106** (1.88)	0.0564* (2.85)	0.0471* (3.24)	0.0448* (3.17)
γ	0.1143* (8.90)	0.1227* (4.20)	0.0809* (1.93)	0.0734* (2.34)	0.1047* (5.04)
β	0.9004* (173.33)	0.9072* (47.37)	0.8736* (19.47)	0.8884* (25.36)	0.8838* (46.86)
$Q(5)$	9.72 (0.08)	8.90 (0.11)	3.49 (0.62)	1.95 (0.85)	2.91 (0.71)
$Q(10)$	13.97 (0.17)	17.06 (0.07)	9.92 (0.44)	7.61 (0.66)	15.58 (0.11)
$Q(5)^2$	1.72 (0.88)	2.04 (0.84)	1.50 (0.91)	7.98 (0.15)	0.57 (0.98)
$Q(10)^2$	5.87 (0.82)	4.19 (0.93)	4.09 (0.94)	10.88 (0.36)	1.29 (0.99)

Table 13. Empirical estimation of the series of returns. We present the conditional mean equation and conditional volatility equation defined either as GJRGARCH model. The model is defined as $r_t = c + \phi_1 r_{t-1} + r\_oil * r\_oil_t + \varepsilon_t,\ \varepsilon_t = \sqrt{h_t} z_t,\ h_t = \omega + \alpha\varepsilon_{t-1}^2 + \gamma S_{t-1}^- \varepsilon_{t-1}^2 + \beta h_{t-1}$ (GJRGARCH). In parenthesis we report the t-statistics for the parameters and p-values for the Ljung-Box test statistics (Q(5) and Q(10) for standardized residuals and $Q(5)^2$ and $Q(10)^2$ for squared standardized residuals. * - statistically significant at 5% level, ** - at 10% level.

2.8.2 The univariate GARCH models with the lagged oil price returns as the explanatory variable - Specification 2

Table 14. Estimation results for each series of stock market returns for Specification 2. Specification. Specification 2 takes the lagged oil price returns as the explanatory variables.

Specification 2					
	$DJIA$	$S\&P$	$FTSE$	DAX	$NIKKEI$
c	0.0403* (3.07)	0.0320* (2.55)	0.0341* (2.72)	0.0427* (2.16)	0.0358* (1.97)
ϕ_1		0.0384* (2.32)	0.0229 (1.58)		
r_oil	0.0047 (0.90)	0.0046 (0.90)	0.0024 (0.43)	0.0052 (0.65)	−0.0039 (−0.40)
ω	0.0299* (4.51)	0.0237* (2.32)	0.0326** (1.85)	0.0615* (1.96)	0.0607* (3.56)
α	0.0176* (3.00)	0.0104** (1.86)	0.0568* (2.85)	0.0472* (3.13)	0.0442* (3.17)
γ	0.1154* (19.34)	0.1237* (4.32)	0.0802* (1.92)	0.0725* (2.28)	0.1051* (5.13)
β	0.8995* (185.0)	0.9069* (47.81)	0.8737* (19.42)	0.8895* (25.36)	0.8841* (47.46)
$Q(5)$	9.66 (0.08)	8.60 (0.12)	5.40 (0.36)	2.00 (0.84)	2.89 (0.71)
$Q(10)$	13.96 (0.17)	16.66 (0.08)	14.43 (0.15)	7.95 (0.63)	15.58 (0.11)
$Q(5)^2$	1.71 (0.88)	2.11 (0.83)	4.92 (0.42)	7.80 (0.16)	0.56 (0.98)
$Q(10)^2$	5.91 (0.82)	4.22 (0.93)	6.08 (0.80)	10.55 (0.39)	1.27 (0.99)

Table 14. Empirical estimation of the series of returns. We present the conditional mean equation and conditional volatility equation defined either as GJRGARCH model. The model is defined as $r_t = c + \phi_1 r_{t-1} + r\_oil * r\_oil_t + \varepsilon_t,\ \varepsilon_t = \sqrt{h_t} z_t$, $h_t = \omega + \alpha\varepsilon_{t-1}^2 + \gamma S_{t-1}^- \varepsilon_{t-1}^2 + \beta h_{t-1}$ (GJRGARCH). In parenthesis we report the t-statistics for the parameters and p-values for the Ljung-Box test statistics (Q(5) and Q(10) for standardized residuals and $Q(5)^2$ and $Q(10)^2$ for squared standardized residuals. * - statistically significant at 5% level, ** - at 10% level.

2.8.3 The univariate GARCH models with the dummy variable as the explanatory variable - Specification 3

Table 15. Estimation results for each series of stock market returns for Specification 3. Specification 3 takes dummy variable of the negative oil price returns (so decreases in oil prices) as the explanatory variable in the mean equation.

Specification 3					
	DJIA	*S&P*	*FTSE*	*DAX*	*NIKKEI*
c	0.0385* (2.25)	0.0353* (2.10)	0.0499* (2.82)	0.0365 (1.37)	0.0228 (0.84)
ϕ_1		0.0379* (2.26)	0.023** (1.58)		
r_oil	0.0037 (0.15)	-0.0069 (-0.29)	-0.0335 (-1.29)	0.0135 (0.34)	0.0295 (0.72)
ω	0.0298* (6.29)	0.0236* (2.33)	0.0328** (1.82)	0.0619* (2.07)	0.0610* (3.54)
α	0.0175* (2.92)	0.0104** (1.86)	0.0563* (2.86)	0.0472* (3.18)	0.0447* (3.15)
γ	0.1155* (9.37)	0.1227* (4.20)	0.0813** (1.71)	0.0729* (2.28)	0.1056* (5.07)
β	0.8996* (176.13)	0.9069* (48.13)	0.8732* (18.69)	0.8890* (26.55)	0.8833* (46.26)
$Q(5)$	9.74 (0.08)	8.54 (0.12)	5.27 (0.38)	2.02 (0.84)	2.87 (0.71)
$Q(10)$	13.99 (0.17)	16.59 (0.08)	14.68 (0.14)	7.86 (0.64)	15.51 (0.11)
$Q(5)^2$	1.73 (0.88)	2.11 (0.83)	4.90 (0.42)	7.82 (0.16)	0.57 (0.98)
$Q(10)^2$	5.89 (0.82)	4.21 (0.93)	6.11 (0.80)	10.65 (0.38)	1.29 (0.99)

Table 15. Empirical estimation of the series of returns. We present the conditional mean equation and conditional volatility equation defined either as GJRGARCH model. The model is defined as $r_t = c + \phi_1 r_{t-1} + r\_oil * r\_oil_t + \varepsilon_t,\ \varepsilon_t = \sqrt{h_t} z_t$, $h_t = \omega + \alpha\varepsilon_{t-1}^2 + \gamma S_{t-1}^{-}\varepsilon_{t-1}^2 + \beta h_{t-1}$ (GJRGARCH). In parenthesis we report the t-statistics for the parameters and p-values for the Ljung-Box test statistics (Q(5) and Q(10) for standardized residuals and $Q(5)^2$ and $Q(10)^2$ for squared standardized residuals. * - statistically significant at 5% level, ** - at 10% level.

2.8.4 The univariate GARCH models with the dummy variable as the explanatory variable - Specification 3

Table 16. Estimation results for each series of stock market returns for Specification 3. Specification 3 takes dummy variable of the positive oil price returns (so increases in oil prices) as the explanatory variable in the mean equation.

Specification 3					
	DJIA	*S&P*	*FTSE*	*DAX*	*NIKKEI*
c	0.0379* (2.03)	0.0225 (1.25)	0.0115 (0.50)	0.0611* (2.25)	0.0542* (2.06)
ϕ_1		0.0388* (2.73)	0.020 (1.28)		
r_oil	0.0038 (0.15)	0.0151 (0.61)	0.0441 (1.37)	−0.0360 (−0.92)	0.0346 (−0.86)
ω	0.0295* (3.91)	0.0235* (4.38)	0.0318** (1.86)	0.0622* (4.16)	0.0611* (3.53)
α	0.0171* (1.77)	0.0106* (1.92)	0.0462* (2.32)	0.0478* (4.20)	0.0448* (3.15)
γ	0.1154* (2.94)	0.1240* (3.64)	0.0728** (1.78)	0.0729* (2.80)	0.1057* (5.06)
β	0.9005* (42.12)	0.9076* (51.23)	0.8728* (19.11)	0.8885* (53.04)	0.8832* (46.02)
$Q(5)$	10.34 (0.11)	8.94 (0.14)	5.67 (0.39)	2.52 (0.81)	2.85 (0.72)
$Q(10)$	14.99 (0.18)	15.59 (0.09)	13.63 (0.17)	8.01 (0.61)	15.21 (0.13)
$Q(5)^2$	1.93 (0.87)	3.11 (0.75)	4.95 (0.41)	7.42 (0.19)	0.97 (0.96)
$Q(10)^2$	6.89 (0.75)	5.21 (0.91)	7.11 (0.70)	9.65 (0.42)	1.89 (0.99)

Table 16. Empirical estimation of the series of returns. We present the conditional mean equation and conditional volatility equation defined either as GJRGARCH model. The model is defined as $r_t = c + \phi_1 r_{t-1} + r\_oil * r\_oil_t + \varepsilon_t,\ \varepsilon_t = \sqrt{h_t} z_t$, $h_t = \omega + \alpha\varepsilon_{t-1}^2 + \gamma S_{t-1}^- \varepsilon_{t-1}^2 + \beta h_{t-1}$ (GJRGARCH). In parenthesis we report the t-statistics for the parameters and p-values for the Ljung-Box test statistics (Q(5) and Q(10) for standardized residuals and $Q(5)^2$ and $Q(10)^2$ for squared standardized residuals. * - statistically significant at 5% level, ** - at 10% level.

2.8.5 The univariate GARCH models with the dummy variable and the volatility of oil prices as the explanatory variable - Specification 4

Table 17. Estimation results for each series of stock market returns for Specification 4. Specification 4 takes the volatility of oil price returns as the explanatory variables.

Specification 4					
	DJIA	*S&P*	*FTSE*	*DAX*	*NIKKEI*
c	−0.0167 (−0.57)	−0.0237 (−0.88)	0.0206 (0.65)	−0.0069 (−0.17)	−0.0068 (−0.17)
ϕ_1		−0.0372* (−2.49)	0.0231 (1.42)		0.0221 (1.45)
r_oil	0.0262* (2.10)	0.0259* (2.28)	0.0059 (0.48)	0.0225 (1.22)	0.0176 (1.06)
ω	0.0310* (2.41)	0.0249* (2.17)	0.0328* (2.08)	0.0636* (2.02)	0.0663* (2.05)
α	0.0155* (3.00)	0.0079 (0.81)	0.0564* (3.13)	0.0472* (3.24)	0.0461* (2.75)
γ	0.0155 (1.58)	0.1275* (2.88)	0.0809* (2.11)	0.0738* (2.25)	0.1109* (3.14)
β	0.8984* (33.21)	0.9056* (31.42)	0.8734* (22.49)	0.8876* (25.93)	0.8775* (26.02)
$Q(5)$	9.76* (0.08)	9.12* (0.10)	5.31* (0.37)	2.07* (0.83)	2.68* (0.74)
$Q(10)$	14.08* (0.16)	17.39* (0.06)	14.23* (0.16)	7.98* (0.63)	15.11* (0.12)
$Q(5)^2$	1.76* (0.88)	2.15* (0.82)	4.89* (0.42)	7.69* (0.17)	0.58* (0.98)
$Q(10)^2$	6.18* (0.79)	4.49* (0.92)	6.06* (0.81)	10.47* (0.39)	1.29* (0.99)

Table 17. Empirical estimation of the series of returns. We present the conditional mean equation and conditional volatility equation defined either as GJRGARCH model. The model is defined as $r_t = c + \phi_1 r_{t-1} + r\_oil * r\_oil_t + \varepsilon_t,\ \varepsilon_t = \sqrt{h_t} z_t$, $h_t = \omega + \alpha\varepsilon_{t-1}^2 + \gamma S_{t-1}^{-}\varepsilon_{t-1}^2 + \beta h_{t-1}$ (GJRGARCH). In parenthesis we report the t-statistics for the parameters and p-values for the Ljung-Box test statistics (Q(5) and Q(10) for standardized residuals and $Q(5)^2$ and $Q(10)^2$ for squared standardized residuals. * - statistically significant at 5% level, ** - at 10% level.

2.8.6 Nonlinear models - SOPI as the explanatory variable

Table 18. Estimation results for each series of stock market returns with nonlinear transformation of oil prices as the explanatory variable - SOPI (scaled oil prices increases)

SOPI					
	DJIA	*S&P*	*FTSE*	*DAX*	*NIKKEI*
c	0.0421* (2.82)	0.0344* (2.39)	0.0269** (1.75)	0.0567* (2.49)	0.0232 (0.99)
ϕ_1		0.0378* (2.27)	0.0230 (1.40)		
r_oil	−0.0047 (−0.26)	−0.0063 (−0.35)	0.0190 (0.81)	−0.0367 (−0.97)	0.0369 (0.99)
ω	0.0298* (4.46)	0.0236* (4.02)	0.0328* (1.93)	0.0616* (1.90)	0.0599* (3.58)
α	0.0176* (3.00)	0.0104** (1.79)	0.0566* (3.08)	0.0468* (3.23)	0.0441* (3.15)
γ	0.1155* (19.61)	0.1235* (14.41)	0.0809* (1.93)	0.0726* (2.13)	0.1055* (5.16)
β	0.8997* (181.52)	0.9069* (140.98)	0.8732* (20.53)	0.8896* (25.07)	0.8843* (47.54)
$Q(5)$	9.74 (0.08)	8.65 (0.12)	5.32 (0.37)	2.03 (0.84)	2.76 (0.73)
$Q(10)$	13.98 (0.17)	16.70 (0.08)	14.45 (0.15)	7.84 (0.64)	15.30 (0.12)
$Q(5)^2$	1.72 (0.88)	2.10 (0.83)	4.75 (0.44)	7.83 (0.16)	0.62 (0.98)
$Q(10)^2$	5.88 (0.82)	4.20 (0.93)	5.91 (0.82)	10.68 (0.38)	1.39 (0.99)

Table 18. Empirical estimation of the series of returns. We present the conditional mean equation and conditional volatility equation defined either as GJRGARCH model. The model is defined as $r_t = c + \phi_1 r_{t-1} + r\_oil * SOPI_t + \varepsilon_t,\ \varepsilon_t = \sqrt{h_t} z_t,\ h_t = \omega + \alpha\varepsilon_{t-1}^2 + \gamma S_{t-1}^- \varepsilon_{t-1}^2 + \beta h_{t-1}$ (GJRGARCH). In parenthesis we report the t-statistics for the parameters and p-values for the Ljung-Box test statitics (Q(5) and Q(10) for standardized residuals and Q(5)^2 and Q(10)^2 for squared standardized residuals. * - statistically significant at 5% level, ** - at 10% level.

2.8.7 Nonlinear models - NOPI as the explanatory variable

Table 19. Estimated coefficients of $NOPI_t$ variable for each of the stock markets of interests (we present only r_oil from the model for each market).

NOPI						
	$NOPI_5$	$NOPI_6$	$NOPI_7$	$NOPI_8$	$NOPI_9$	$NOPI_{10}$
DJIA	−0.0045 (−0.24)	−0.0069 (−0.33)	−0.0131 (−0.57)	−0.0220 (−0.91)	−0.0180 (−0.71)	−0.0190 (−0.72)
S&P	−0.0057 (−0.32)	−0.0077 (−0.38)	−0.0131 (−0.60)	−0.0222 (−0.94)	−0.0199 (−0.80)	−0.0208 (−0.82)
FTSE	0.0105 (0.47)	0.0080 (0.32)	0.0085 (0.32)	0.0017 (0.06)	−0.0055 (−0.18)	−0.0016 (−0.05)
DAX	−0.0017 (−0.06)	−0.0028 (−0.08)	−0.0105 (−0.30)	−0.0096 (−0.26)	−0.0089 (−0.22)	−0.0058 (−0.13)
NIKKEI	−0.0005 (−0.02)	−0.0016 (−0.06)	0.0029 (0.10)	−0.0004 (−0.01)	−0.0058 (−0.17)	0.0044 (0.12)

Table 19. Empirical estimation of the series of returns. We present the conditional mean equation and conditional volatility equation defined either as GJRGARCH model. The model is defined as $r_t = c + \phi_1 r_{t-1} + r\_oil * NOPI_t + \varepsilon_t,\ \varepsilon_t = \sqrt{h_t} z_t$, $h_t = \omega + \alpha\varepsilon_{t-1}^2 + \gamma S_{t-1}^- \varepsilon_{t-1}^2 + \beta h_{t-1}$ (GJRGARCH). In parenthesis we report the t-statistics for the parameters and p-values for the Ljung-Box test statitics (Q(5) and Q(10) for standardized residuals and $Q(5)^2$ and $Q(10)^2$ for squared standardized residuals. * - statistically significant at 5% level, ** - at 10% level. For NOPI we present only the estimated parameters and the corresponding t-statistics. The rest of the result are available upon request.

Chapter 3

Changes in correlations between CEE stock markets and European stock markets

3.1 Introduction

The correlation of financial markets plays an important role in portfolio diversification. It is well documented that stock return correlations vary over time. De Santis and Gerard (1997) or Ang and Bekaert (1999) show that the correlations between financial markets change over time, declining in the bull markets and rising in the bear markets. This in an important implication for international investors since diversification sought by investing in international stock markets is likely to be lowest when it is the most desirable.

Significant long-run co-movements among different stock markets can be related to a range of reasons, including strong economic ties and policy coordination, advancement of international trading, financial innovations and technological progress, market deregulation and liberalization, multinational corporate activity, international capital flows, finally financial crises and contagion effects.

Long-run linkages among stock markets have important regional and global implications, as a domestic capital market cannot be adequately insulated from external shocks.

The cointegration analysis has emerges as a powerful technique for investigating interdependencies and common trends among international stock markets, providing a sound methodology for modelling long-run dynamics in a multivariate systems.

Correlation analysis of returns is crucial to the standard risk-return portfolio models. Correlation reflects co-movements in returns, which are liable to great instabilities over time. It is a short-run measure, and correlation based portfolio management strategies require often rebalancing. Cointegration on the other hand measures long-run co-movement, which may occur even during the periods with low correlation.

Hence portfolio management based on cointegrated financial assets may be more effective in the long-run, whereas strategies based only on volatility and correlation of returns cannot guarantee long-term performance.

Correlations tend to rise with the degree of international equity market integration. This pattern was observed among the European countries since the mid-1990s and was particularly strong after introducing the euro (see Cappiello *et al.* (2006), Hardouvelis *et al.* (2000), Kearney and Poti (2006)). The analysis of trends in correlations among European equity markets is of the special interest since the findings have relevance for the diversification effects of passive and active investment strategies.

Capiello *et al.* (2006) investigate the changes in correlations between European equity markets and show that the correlation between France, Germany, Italy and Great Britain has clearly increased since the introduction of Euro due to adoption of a common monetary policy and the consequent irrevocable fixing of exchange rates.

Kearney and Poti (2006) confirm a significant rise in the correlations among national stock market indexes that can be explained by a structural break shortly before the official adoption of Euro.

The figure below shows the estimated conditional correlation between the returns on the DAX index (Germany) and CAC index (France) obtained with the Dynamic Conditional Correlation (DCC)-GARCH Model of Engle (2002).

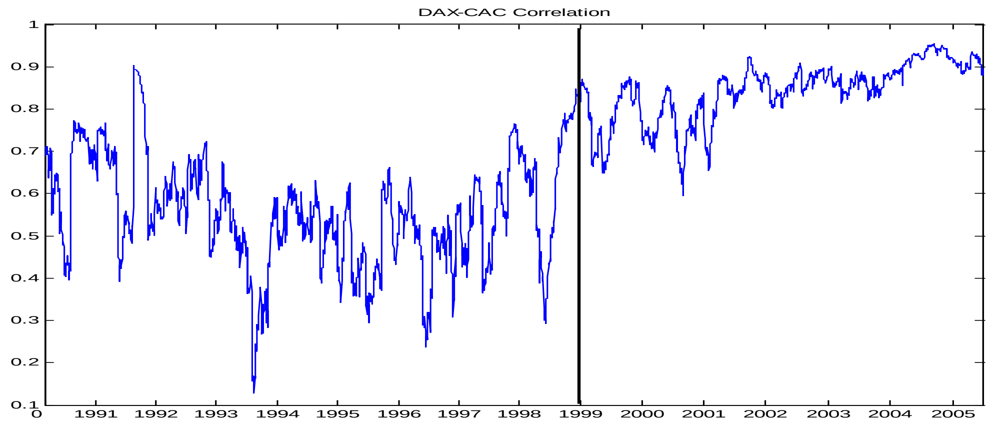


Figure 1. The dynamic correlation between DAX (Germany) and CAC (France).

We observe an increase in the correlation between the German and French stock market index from 1999 in line with the results of Capiello *et al.* (2006). They show that the average correlation between Germany and France has increased from 0.61 to 0.85 or between Germany and Italy from 0.44 to 0.81 when comparing pre- and post-Euro periods. We observe here a strong increase in correlation between both indexes especially after introducing Euro (the date indicated by the vertical bar) and also lower variation in the conditional volatility after 1999.

Central and Eastern European Countries (CEE) joined the European Union in May 2004. The EU enlargement is expected to have considerable implications for the economies of the new members and for the whole region as well. After the post-entry stabilization period those countries are expected to join the European Monetary Union and adopt the Euro as the sole currency.

Stock markets in Central and Eastern Europe countries, especially those in Warsaw, Prague and Budapest, underwent remarkable changes and development in terms of market capitalization, daily trade volumes and number of listed companies from the beginning of the economic transformation in the region. Strong international linkages of the economies and financial markets across Europe have positive effects for the CEE firms, improving their access to international financial markets.

From the international investors' perspective, it is important to assess links among CEE and European stock markets. The detection of any pattern in correlation between CEE and

European financial markets will give important information about the possible diversification strategies.

Although foreign investors were allowed to invest in these countries before entering the European Union (EU), some of them may have delayed the decision about entering into these markets due to perceived political, liquidity and economic risks. The entrance to the EU has alleviated these risks and increased the integration of CEE stock markets with the European stock exchanges. Such integration would lead to fall in the systematic risk and increase in stock prices.

Shortly after the entering the EU, in 2005 the Prague and Warsaw Stock Exchanges noticed a significant rise in the trading volume and the entrance of many foreign investors on these markets (see the Annual Report 2005 of Prague Stock Exchange or Annual Report 2005 of Warsaw Stock Exchange). In 2005 the Prague Stock Exchange quoted an increase in trading volume of 117.1% comparing to 2004. The Warsaw Stock Exchange reported a year of records: the highest turnover in the history (increase by 74%), the indexes setting continuously all-time highs, record market capitalization and the 41% share of foreign investors (higher by 8% than the institutional domestic investors) in the investors' structure.

The analysis of changes in correlations among European and CEE stock markets allows investigating the benefits from EU integration. The lower systematic risk should lower the cost of investment financing and contribute to the economic development. Correlation between stock markets defines the speed of transmission of shocks (contagion) between stock markets. We can assess as well possible market developments due to the future adopting of Euro as the common currency.

Although there are many studies about the integration of financial markets in the developed countries, there are relatively few studies about the CEE stock markets co-movements with mature international markets. The main problem for conducting such studies is mainly the lack of data and short time series.

The most of studies analyzing the interdependence between CEE stock markets and international stock markets concentrate on long-term dependence among them, using the cointegration as an econometric tool, not accessing the short-time implications.

MacDonald (2001) studies CEE stock market indexes as a group against three developed counties - USA, Germany and UK, and finds significant long-run co-movements.

Gilmore and McManus (2002) examine the short and long-term relationship between weekly US stock market returns and three Central European stock market returns (Poland, Hungary and Czech Republic) over the period 1995-2001. They show that these markets are not integrated, either individually or as a group with the US stock market. The results suggest that the relatively low correlations of these emerging markets with the US market are appropriate indicators of the benefits of international diversification for not only short-term but also long-term US investors.

Voronkowa (2004) investigates the cointegration among the CEE markets, British, German and French stock markets, and the USA, allowing for structural breaks in the model and confirms the presence of long-run linkages among the emerging stock markets and the mature markets.

Syriopoulous (2004) using error correction vector autoregression and daily data detects the presence of one cointegration vector, indicating a stationary long-run relationship between the US, the German and four Central European markets (Hungary, Poland, Czech Republic and Slovakia).

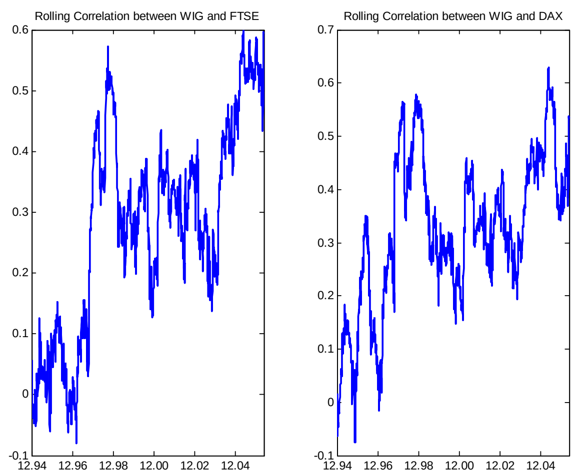
As we see the analysis based on the investigating the long-run relationship between CEE stock markets and international stock markets using the cointegration technique delivers different results.

In this paper we concentrate on short term co-movements and analyze changes in correlations among CEE stock markets and three European stock market indexes. One of the convenient ways of analyzing the changes in correlation is the Dynamic Conditional Correlation GARCH Model by Engle and Sheppard (2002) and the modification of this model to allow for asymmetric behaviour of the conditional correlations and also allowing for structural break in the conditional correlation.

The main questions we want to answer are if is there any changes in the correlation after entering the CEE countries into EU. We want to check if the correlations between stock markets have any structural breaks and detect the timing of such breaks. We also want to check the nature of both the conditional variance and conditional correlations allowing for the presence of asymmetry.

A simple time-varying estimator of the correlation between asset returns is the rolling correlation estimator. The correlation estimates are computed as unconditional sample estimates with a rolling window of a fixed size of N observations over the sample period T. The rolling estimator is attractive due to its simplicity. The main drawback of it is a strong dependence on the size of the window chosen. However, we can use the rolling correlation estimator to examine whether the correlation remain constant over time. If the correlations are constant the estimates over the rolling windows should remain approximately the same.

Figure 2 presents a rolling correlation estimator, constructed as mentioned above, between two major CEE stock market indexes - WIG from the Warsaw Stock Exchange and PX from the Prague Stock Exchange and the British FTSE and German DAX. These rolling correlations were computed over the period June 1994 to May 2006 using a window of six months.



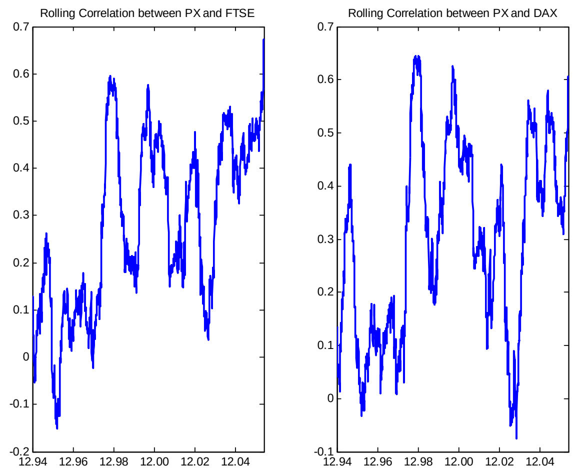


Figure 2. Changing correlation between WIG and PX and DAX and FTSE.

We observe that the correlations between CEE stock markets and European stock market indices are not constant over time and show a patter of dynamic changes. The correlation has significantly increased over the period of twelve years. The appropriate econometric model that captures all these dynamics will be of great benefit for portfolio managers that base their investment decisions on the level of correlation among different markets.

Using the Asymmetric Dynamic Correlation GARCH Model of Engle and Sheppard (2001) we estimate conditional correlation series between CEE stock markets and three important European stock indexes. We detect a structural break in the conditional correlation series some months before May 2004 - the date of the entrance of all the countries to the EU. The following increase in correlation of the Central and Eastern European financial markets and Western Europe is the sign of the higher integration of these markets with European stock markets. This is an evidence of the diminishing prospect of these stock markets as a diversification tool.

The paper is structured as follows - in the next section we discuss the models applied, following we discuss the data analyzed in this paper. Section 4 concentrates on the empirical results of the estimation and shows results of the structural break tests. Section 5 concludes and the appendix discusses the details of the tests applied and presents the news impact surfaces for the stock markets discussed.

3.2 Methodology

In this section we discuss the models applied in this paper. We specify the univariate conditional variance model for each series and propose model for conditional covariance matrix. We account especially for some of the stylized facts observed in the financial time series - volatility clustering and asymmetry.

3.2.1 Univariate models

Using ARMA filter we remove the deterministic part of the series obtaining the stochastic series with zero mean.

For the univariate models we use GARCH family models. These models capture in a convenient way the dynamics of volatility of financial time series. We account for the asymmetry observed in the series of stock markets returns (e.g. Capiello *et al.* 2006) that allows the positive and negative shocks to have different impact on volatility (the negative shocks - "bad news" increase stock market volatility stronger than the positive shocks).

Among different univariate GARCH models we use the Glosten, Jagannathan and Runkle (1993) model (thereafter GJR-GARCH model). The specification of the conditional variance is given as

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1}^2 d_{t-1} + \beta h_{t-1}$$

where the dummy variable $d_t = 1$ when $\varepsilon_t < 0$, and zero otherwise.

In this model good news ($\varepsilon_t > 0$) and bad news ($\varepsilon_t < 0$) have different effects on conditional variance - good news have an impact of α whereas the bad news have an impact of $\alpha + \gamma$.

The conditions for the non-negativeness of the conditional variance are $\omega > 0, (\alpha + \gamma)/2 \geq 0, \beta > 0$ and for stationarity $\alpha + \gamma/2 + \beta < 1$.

For the series that do not show the presence of asymmetry in the conditional variance we use the standard linear GARCH model of Bollerslev (1986) defined as

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}$$

where $\omega > 0, \alpha \geq 0, \beta \geq 0$ and $\alpha + \beta < 1$. These conditions guarantee the non-negativeness and the stationarity of the conditional volatility.

For the series of FTSE, DAX, WBI, WIG, PX and BUX we consider the asymmetric GJR-GARCH model and for two series of interest - SAX and SBI the linear (no asymmetry imposed) GARCH model since these series do not show any asymmetric effects in the conditional variance.

3.2.2 Multivariate model

The models of Engle (2002) and Tse and Tsui (2002) are the first to propose a generalization of the constant correlation models by making the variance-covariance matrix time dependent in order to account for another stylized fact observed in the financial time series namely the changing correlations between stock markets and the asymmetric response of conditional correlation to positive and negative shocks. A number of studies show that correlation between equity returns increases during bear markets and decreases when stock markets go up (see Ang and Bekaert (2001) or Longin and Solnik (2001)).

The Dynamic Conditional Correlation (DCC) GARCH model of Engle (2002) proves to be widely used by many researchers to investigate the multivariate systems due to the fact that the estimation can be divided into two steps and the conditions ensuring the positive definiteness of the conditional covariance are easy to implement. Capiello *et al.* (2006) propose the Asymmetric Dynamic Conditional Correlation (ADCC) GARCH model - the asymmetric extension of this model, presented below.

Engle and Sheppard (2001) propose the test for constant versus dynamic correlation framework that is applied in this paper and discussed in the appendix.

ADCC model assumes that the N-dimensional vector of series of interest (ε_t) is zero mean with the time varying variance covariance matrix given as

$$H_t = D_t R_t D_t$$

where D_t is the (N x N) diagonal matrix of the time-varying standard deviations from univariate GARCH model.

The specification of the univariate GARCH model is not limited to the standard GARCH(p, q), but can include any GARCH process with normally distributed errors that satisfies stationarity and non-negativity constraints (in our case as discussed earlier we propose GJR-GARCH and GARCH model for FTSE, DAX, WBI, WIG, PX , BUX and SAX, SBI respectively).

In this model R_t is defined as

$$R_t = diag(q_{11,t}^{-1/2}, ..., q_{NN,t}^{-1/2})Q_t diag(q_{11,t}^{-1/2}, ..., q_{NN,t}^{-1/2})$$

where the $(N\ x\ N)$ positive definite matrix $Q_t = (q_{ij,t})$ is given by

$$Q_t = (1 - \sum_{m=1}^{M} \delta_m - \sum_{n=1}^{N} \phi_n)\overline{Q} + \sum_{m=1}^{M} \varphi_m \overline{N} + \sum_{m=1}^{M} \delta_m z_{t-m} z_{t-m}^{'} + \sum_{m=1}^{M} \varphi_m n_{t-m} n_{t-m}^{'} + \sum_{n=1}^{N} \phi_n Q_{t-n}$$

We will consider the lowest possible order of the model - M and N equal to 1, so that dynamic correlation structure is given as

$$Q_t = (1 - \delta - \phi)\overline{Q} + \varphi\overline{N} + \delta(z_{t-1} z_{t-1}^{'}) + \varphi(n_{t-1} n_{t-1}^{'}) + \phi Q_{t-1}$$

where z_{t-1} are the univariate standardized residuals obtained in the first step of estimation defined as $z_t = \varepsilon_t/\sqrt{h_t}, n_{t-1} = z_{t-1} 1_{\{z_{t-1}<0\}}$ (indicator function that takes the value of one for negative returns), $\overline{N}$ is the unconditional covariance matrix of n_t, $\overline{Q}$ is the unconditional covariance matrix of the standardized residuals resulting from the first stage estimation, and δ and ϕ are nonnegative scalar parameters satisfying $\delta + \phi + \rho\varphi < 1$, where ρ is the maximum eigenvalue of $\overline{Q}^{-1/2}\overline{NQ}^{-1/2}$.

Notice that the ADCC model has a similar structure as the asymmetric conditional volatility GJR-GARCH model.

This model is a direct extension of the standard DCC-GARCH model proposed by Engle (2002) where the dynamic correlation structure is defined as

$$Q_t = (1 - \delta - \phi)\overline{Q} + \delta(z_{t-1} z_{t-1}^{'}) + \phi Q_{t-1}$$

3.2.3 News Impact Curves and Surfaces

News Impact Curves have been developed by Engle and Ng (1993) to represent the response of next period conditional volatility to a shock to the asset return. In this framework the shocks are defined as $\varepsilon = (\varepsilon_1, ..., \varepsilon_n)'$.

The News Impact Curves (NIC) for the univariate volatility model GJR-GARCH is given by

$$h_i(z) = \begin{cases} A_{h,i} + (\alpha_i + \gamma_i)\varepsilon_i^2 & \varepsilon_i < 0 \\ A_{h,i} + \alpha_i \varepsilon_i^2 & \varepsilon_i > 0 \end{cases}$$

where $\overline{h}_i$ is the unconditional variance of $\varepsilon_{i,t}$ and $A_{h,i} = \omega_i + \beta_i \overline{h}_i$.

For the linear GARCH model we have a symmetric NIC function

$$h_i(z) = \begin{cases} A_{h,i} + \alpha_i \varepsilon_i^2 & \varepsilon_i < 0 \\ A_{h,i} + \alpha_i \varepsilon_i^2 & \varepsilon_i > 0 \end{cases}$$

where $\overline{h}_i$ is the unconditional variance of $\varepsilon_{i,t}$ and $A_{h,i} = \omega_i + \beta_i \overline{h}_i$.

The News Impact Surfaces (NIS) present the reaction of conditional correlations to news in the multivariate setting. Jondaneu and Rockinger (2006) discuss the concept of NIC for higher moments. We consider shocks to both variables and evaluate their impact on the correlation between variables.

The news impact surface for correlation $\rho_{ij}(z)$ is given by

$$\frac{A_{\rho,ij} + (\delta + \varphi)\sqrt{\overline{h}_i}\sqrt{\overline{h}_j} z_i z_j}{\sqrt{(A_{\rho,ii} + (\delta + \varphi)\sqrt{\overline{h}_i} z_i)(A_{\rho,jj} + (\delta + \varphi)\sqrt{\overline{h}_j} z_j)}} \quad if \quad z_i, z_j \leq 0$$

$$\frac{A_{\rho,ij} + \delta\sqrt{\overline{h}_i}\sqrt{\overline{h}_j} z_i z_j}{\sqrt{(A_{\rho,ii} + \delta\sqrt{\overline{h}_i} z_i)(A_{\rho,jj} + \delta\sqrt{\overline{h}_j} z_j)}} \quad if \quad z_i, z_j > 0$$

$$\frac{A_{\rho,ij} + \delta\sqrt{\overline{h}_i}\sqrt{\overline{h}_j} z_i z_j}{\sqrt{(A_{\rho,ii} + (\delta + \varphi)\sqrt{\overline{h}_i} z_i)(A_{\rho,jj} + \delta\sqrt{\overline{h}_j} z_j)}} \quad if \quad z_i \leq 0, z_j > 0$$

$$\frac{A_{\rho,ij} + \delta\sqrt{\overline{h}_i}\sqrt{\overline{h}_j} z_i z_j}{\sqrt{(A_{\rho,ii} + \delta\sqrt{\overline{h}_i} z_i)(A_{\rho,jj} + (\delta+\varphi)\sqrt{\overline{h}_j} z_j)}} \quad if \quad z_i > 0, z_j \leq 0$$

where $A_{\rho,ij} = (1-\delta)cov_{ij} - \varphi cov\_n_{ij}$ and where cov_{ij} is the unconditional covariance between series i and j and cov_n_{ij} is the covariance between the standardized negative residuals.

3.2.4 Structural breaks

We can use the model proposed by Capiello *et al.* (2006) to impose the structural break in the conditional correlation. A straightforward extension of the model allowing for the structural break at unknown date τ is given as

$$Q_t = (1-\delta-\phi)\overline{Q} - d\widetilde{Q} + d\delta\widetilde{Q} + d\phi\widetilde{Q} + \delta(z_{t-1}z_{t-1}^{'}) + \phi Q_{t-1}$$

where we let d be 0 or 1, depending on whether $t > \tau < T$, $\overline{Q}$ is defined as $\overline{Q} = E\left[z_{t-1}z_{t-1}^{'}\right]$, $t < \tau$, and $\widetilde{Q} = \overline{Q} - E\left[z_{t-1}z_{t-1}^{'}\right]$, $t \geq \tau$.

We observe that the model with the structural break at the date τ nests the standard model presented in the section 2.2. We can therefore easily test for the break in the correlation using the likelihood ratio test statistic. Since the date of the break is unknown in this case the asymptotic distribution of the test statistic has a non-standard distribution.

Andrews (1993) and Andrews and Ploberger (1994) propose the tests for structural breaks for cases where the nuisance parameter is present under the alternative but not under the null hypothesis. Hansen (1997) discusses the numerical approximation to the asymptotic $p-$values for these test statistics.

They consider a function $F_n(\tau)$, where n is the number of breaks and $F(\tau)$ is the value of the likelihood ratio test statistics for the break at the date τ versus the model without the break. We assume that τ lies between two dates $T_1 = 0.05 * n$ and $T_2 = 0.95 * n$.

Andrews (1993) discusses the asymptotic properties of the test statistic

$$\sup_{T_1 \leq \tau \leq T_2} F_n = \sup F_n(\tau)$$

and reports asymptotic critical values. In this test, the date of the break τ that maximizes $F_n(\tau)$ will be the estimated date of the break.

Andrews and Ploberger (1994) propose two additional test statistics - $\exp F_n$ and $aveF_n$ that are calculated as

$$\exp F_n = \ln((1/n_\tau) * \sum_{\tau=T_1}^{T_2} \exp(0.5 * F_n(\tau)))$$

$$aveF_n = (1/(n_\tau)) \sum_{\tau=T_1}^{T_2} F_n(\tau)$$

where n_τ is the total number of breaks we consider.

The p-values associated with these statistics are calculated using the numerical approximation proposed by Hansen (1997).

3.3 Data

The financial time series we use in this study consist of continuously compounded returns of daily stock market closing prices (in dollars) for five major CEE stock markets, the German, British and Austrian stock markets.

In particular we include WIG20 from Poland, PX50 from Czech Republic, BUX from Hungary, SAX from Slovakia and SBI from Slovenia, DAX30 from Frankfurt, FTSE100 from London and WBI from Vienna. All the data are obtained from Datastream and cover the period from June 1994 to the end of June 2006 - the series start in June 1994 - the earliest possible time point we could find the data for all the markets.

The markets we consider are the main stock markets in the CEE region and represent well diversified stock market portfolios that adequately cover domestic market capitalization.

As the European stock markets we consider the two most important stock markets in the EU - the FTSE100 from London and DAX30 from Frankfurt. Additionally we consider the index of the Vienna Stock Exchange since the Austrian investors play an important role in the region.

Table 1 below shows the descriptive statistics of data. Notice that since we are interested in volatilities and correlations we consider zero mean series of stock market returns.

	St. Dev.	Skewness	Kurtosis	Aug Dickey Fuller test (p-value)
FTSE	1.0578	-0.1404	5.44	-35.66 (0.000)
DAX	1.4403	-0.1767	5.58	-56.74 (0.000)
WBI	0.9482	-0.2597	9.82	-58.92 (0.000)
WIG	2.0022	-0.1187	5.80	-56.23 (0.000)
PX	1.3143	-0.2354	5.09	-56.45 (0.000)
BUX	1.7859	-0.6401	14.82	-59.99 (0.000)
SAX	1.5260	-0.2524	8.60	-54.55 (0.000)
SBI	1.2876	-0.0166	15.53	-42.05 (0.000)

Table 1. Descriptive statistics of the data.

The CEE stock markets show higher values of standard deviation, which indicates higher fluctuations of the series.

The negative skewness apparent in all the markets implies that the distribution of the series has a fatter left tail, the kurtosis higher than 3 (which is the kurtosis of the normal distribution) indicates that we have higher mass of extreme returns than the one predicted by the normal distribution.

The Augmented Dickey Fuller test statistics for the series of returns show in all the cases stationary series (the critical value of the test at 5% level of significance is -2.86).

The following table shows the contemporaneous correlations for two cases - the first number is for the whole sample and the second one (in italics) the sample that spans from 01/05/2004 - the date of the entrance of the CEE countries into EU. All of the estimated correlations in both cases are statistically significant at 5% level of significance.

	$FTSE$	DAX	WBI	WIG	PX	BUX	SAX	SBI
$FTSE$	1.0000 (–)	–	–	–	–	–	–	–
DAX	0.6538 *0.8052*	1.0000 (–)	–	–	–	–	–	–
WBI	0.3358 *0.5857*	0.4097 *0.5899*	1.0000 (–)	–	–	–	–	–
WIG	0.2785 *0.5411*	0.3076 *0.4904*	0.2763 *0.5535*	1.0000 (–)	–	–	–	–
PX	0.3094 *0.5129*	0.3182 *0.4876*	0.3807 *0.6123*	0.3303 *0.6136*	1.0000 (–)	–	–	–
BUX	0.3276 *0.4476*	0.3524 *0.3997*	0.3752 *0.5107*	0.4310 *0.6951*	0.4078 *0.6197*	1.0000 (–)		–
SAX	0.0639 *0.2646*	0.0665 *0.2620*	0.1663 *0.2768*	0.0598 *0.2519*	0.1490 *0.3197*	0.0874 *0.2793*	1.0000 (–)	
SBI	0.0459 *0.2988*	0.0675 *0.2882*	0.2302 *0.3350*	0.0653 *0.2870*	0.1398 *0.3311*	0.1260 *0.1972*	0.1594 *0.3590*	1.0000 (–)

Table 2. Correlation between time series for the whole period of interests (the first number) and for the period from 01/05/2004, all the correlations statistically significant at 5% level.

The analysis of correlations shows a significant increase in the correlations between stock markets. The correlations are relatively high and positive. Some of the correlations have doubled (e.g. between WIG and FTSE a change from 0.2785 to 0.5411 or between PX and WBI from 0.3807 to 0.6123). Much stronger increase (almost 4 or 5 fold) we observe between relative smaller stock markets - SAX from Slovakia or SBI from Slovenia and all the European stock markets. The correlations among main CEE stock markets - WIG, PX and BUX have increased significantly as well.

3.4 Empirical Results

In this section we discuss the results of empirical estimation of the models. We present and evaluate the results for conditional variance and conditional correlation.

3.4.1 Specification of the model

In the first step we removed the mean and any deterministic features of the series applying an ARMA filter indicated by the lowest values of Schwarz Information Criterion. The lowest value of this criterion we obtain for FTSE - $ARMA(2,2)$, DAX - $ARMA(1,1)$, WBI - $ARMA(2,2)$, WIG - $ARMA(2,1)$, PX - $ARMA(2,1)$, BUX - $ARMA(2,2)$, $ARMA(2,2)$ and $ARMA(2,1)$

for SAX and SBI respectively. Applying these ARMA filters we obtain purely stochastic series with zero mean used to estimate conditional variance and covariance models.

Engle and Sheppard (2001) propose a test for constant versus dynamic correlation structure (presented in the appendix). The table below shows the results of this test for the lags $1-5$.

lag	1	2	3	4	5
$Engle\_Sheppard\_test$	64.3010 (0.0000)	70.0139 (0.0000)	83.2692 (0.0000)	85.2692 (0.0000)	95.0177 (0.0000)

Table 3. Engle & Sheppard test for constant correlation in the multivariate framework with corresponding p-values.

The results of the test show that at each lag we reject the hypothesis about the constant nature of the correlation matrix. All the $p-$values are zero. This supports the idea of using the Dynamic Conditional Correlation GARCH model and its asymmetric version.

3.4.2 Conditional variance

We estimate the models assuming the normal distribution of the errors. Although the errors can follow any other heavy-tailed distribution the Quasi Maximum Likelihood Theorem establishes that the QMLE estimates are consistent.

To recall we model the conditional variance either using GJR-GARCH model or the standard (no asymmetry) GARCH model. Only in the case of SAX and SBI we apply the linear GARCH model –the asymmetry in the conditional volatility specification was not statistically significant.

First we show the volatility part for each series of the stock market returns .

	$FTSE$	DAX	WBI	WIG	PX	BUX	SAX	SBI
ω	0.0178* (0.0053)	0.0211* (0.0066)	0.1227* (0.0416)	0.1165* (0.0536)	0.0630* (0.0216)	0.2817* (0.1088)	0.1364* (0.0680)	0.0396** (0.0210)
α	0.0141 (0.0100)	0.0382* (0.0144)	0.0661* (0.0254)	0.0635* (0.0176)	0.0826* (0.0169)	0.1276* (0.0411)	0.0720* (0.0233)	0.1227* (0.0296)
γ	0.0835* (0.0173)	0.0752* (0.0198)	0.1136* (0.0546)	0.0455* (0.0195)	0.0530* (0.0249)	0.1426** (0.0853)		
β	0.9256* (0.0143)	0.9131* (0.0125)	0.7354* (0.0711)	0.8821* (0.0345)	0.8568* (0.0270)	0.7127* (0.0816)	0.8713* (0.0463)	0.8619* (0.0360)
$Q(10)$	8.2825 (0.6013)	12.0677 (0.2806)	17.7463 (0.0594)	15.2017 (0.1249)	10.4359 (0.4031)	11.9244 (0.2902)	8.5058 (0.5796)	13.2831 (0.2083)
$Q(20)$	14.1270 (0.8240)	20.2664 (0.4414)	19.6539 (0.4798)	26.8546 (0.1394)	13.9799 (0.8315)	17.7665 (0.6028)	11.2538 (0.9394)	14.6605 (0.7955)

Table 4. The results of the estimation of the conditional variance equation, where the conditional volatility h_t is either GJR-GARCH(1,1) or GARCH(1,1), (in parenthesis the standard error, * significant at 5% level of significance,** significant

at 10%). $Q(10)$ and $Q(20)$ are the values of the Ljung-Box test statistic for the presence of correlation at lag 10 and 20 in the series of squared standardized residuals.

All the estimated parameters are statistically significant at 5% level of significance (except for γ for BUX, which is significant at 10% level of significance and α for FTSE not significant).

The parameters β show relatively high level of persistency (only in the case of WBI and BUX we observe lower value of this parameter, at the same time these markets show much higher impact of shocks on the conditional volatility). CEE stock markets show higher impact of shocks on the conditional volatility than the European stock markets.

The parameters γ capturing the imposed asymmetry are always statistically significant and have positive values indicating the presence of asymmetry in the conditional variance and illustrating the fact that negative shocks have a higher impact on conditional volatility than the positive ones. We observe that in case of CEE countries this asymmetric response to negative shocks is lower than in case of Western Europe (except for BUX where the parameter γ takes the highest value).

Specification testing

As presented in the Table 4 the values of the Ljung-Box statistic for squared standardized residuals show that there is no remaining autocorrelation in the series of squared standardized residuals which indicates that the models for univariate conditional volatility are correctly specified.

Wooldridge (1990, 1991) proposes the regression based diagnostics that can be applied to test for many possible misspecification. The details of the test for misspecification in the conditional volatility are presented in the appendix.

We check for misspecification in the conditional volatility equation. The results of the test for $Q = 4$ presents the Table 5 - the test statistics is χ^2 distributed with 4 degrees of freedom (critical value of 9.48)

i	$FTSE$	DAX	WBI	WIG	PX	BUX	SAX	SBI
$W_{ii}(Q)$	0.43 (0.9799)	2.18 (0.7026)	0.82 (0.9357)	0.51 (0.9725)	0.66 (0.9561)	0.25 (0.9928)	0.02 (0.9999)	0.03 (0.9999)

Table 5. Results of the test for no misspecification - no remaining correlation - in the conditional variance model.

The results of this test confirm that there is no remaining correlation in standardized residuals and the model for conditional volatility is correctly specified.

Using the framework proposed by Wooldridge we can also check if both asymmetric effects - the sign effect, which means that shocks of different sign have different impact on conditional volatility and size effect - not only the sign of shocks influences the conditional volatility but also the size of the shock has different impact, are correctly captured by the models we propose. We use the robust conditional moment tests of Wooldridge (1990, 1991) discussed in the appendix imposing following moment conditions

$$x_{1t-1} = I[z_{t-1} < 0] \qquad x_{2t-1} = I[z_{t-1} > 0]$$
$$x_{3t-1} = z_{t-1}^2 I[z_{t-1} < 0] \quad x_{4t-1} = z_{t-1}^2 I[z_{t-1} > 0]$$

In this setting the first two moments (x_{1t-1}, x_{2t-1}) account for the sign effect and the other two (x_{3t-1}, x_{4t-1}) for size effect. To be consistent with previous tests we take into account four lags and therefore the test statistic is χ^2 distributed with four degrees of freedom (critical value of 9.48 at 5% level of significance).

Table 6 shows the results of the misspecification test for each of the moments

i	$FTSE$	DAX	WBI	WIG	PX	BUX	SAX	SBI
x_{1t-1}	0.37 (0.5430)	0.10 (0.7518)	0.01 (0.9203)	0.04 (0.8414)	0.62 (0.4310)	0.01 (0.9203)	0.08 (0.7772)	0.11 (0.7401)
x_{2t-1}	0.05 (0.8230)	0.65 (0.4201)	0.04 (0.8414)	0.12 (0.7290)	0.09 (0.7641)	0.36 (0.5485)	0.07 (0.7914)	0.47 (0.4929)
x_{3t-1}	0.21 (0.6467)	0.01 (0.9203)	0.13 (0.7184)	0.14 (0.7082)	0.39 (0.5322)	0.54 (0.4624)	0.52 (0.4708)	0.62 (0.4310)
x_{4t-1}	0.90 (0.3427)	0.78 (0.3774)	0.34 (0.5598)	0.01 (0.9203)	0.02 (0.8875)	3.83 (0.0503)	0.12 (0.7290)	0.96 (0.3271)

Table 6. Results of the test for no misspecification with different moment conditions.

The results of the test prove that both the sign and size effects are accounted for by our models. All the estimated models pass the tests for misspecification at 5% level of significance.

Summarizing the models for conditional volatility correctly capture the volatility clustering and asymmetry found in the data.

News Impact Curves

Engle and Ng (1993) introduce the news impact curves in the univariate setting to show how the shock to volatility at time $t-1$ impacts the volatility at time t. For the standard GARCH model the NIC is symmetric whereas for asymmetric models (in out case GJR-GARCH) the NIC shows the volatility reacting stronger to negative shocks than to positive (the estimated parameter γ is positive and statistically significant).

Two plots below show the NIC for European and CEE stock markets. We observe much higher level of volatility in the CEE stock markets - already mentioned in the section 3 the CEE stock market series are characterized by much higher standard deviation. WBI and BUX are characterized by the highest level of asymmetry (γ parameters the highest). Among the CEE markets WIG, PX and BUX show higher level of volatility than SAX and SBI.

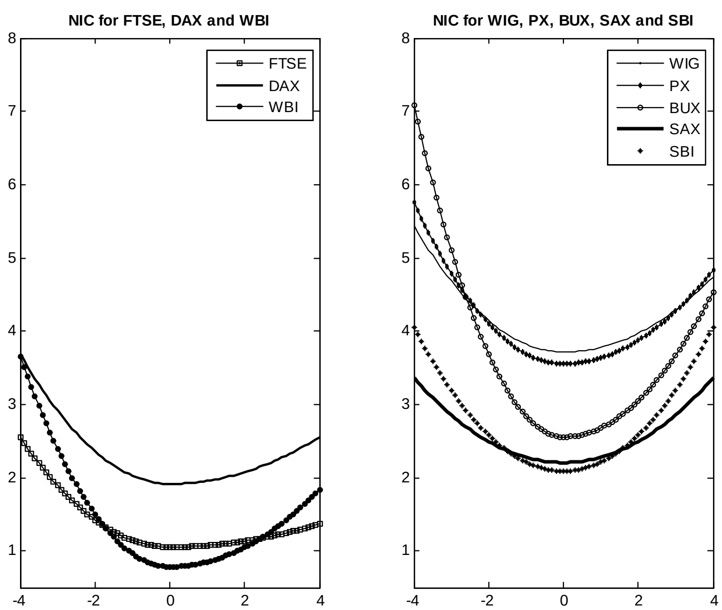


Figure 3. NIC for the estimated volatility models.

BUX has also the least persistent volatility (the lowest β) and at the same time the highest levels of parameters α and γ, which indicates the strongest reactions to shocks to the series.

3.4.3 Conditional Correlation

The conditional correlation equation is given as

$$Q_t = (1-\delta-\phi)\overline{Q} + \varphi\overline{N} + \delta(z_{t-1}z_{t-1}^{'}) + \varphi(n_{t-1}n_{t-1}^{'}) + \beta Q_{t-1}$$

and we obtain following parameters estimates

	Conditional_Correlation
δ	0.0063* (0.0011)
ϕ	0.9905* (0.0023)
φ	0.0008 (0.0009)

Table 7. Conditional correlation estimates.

The results show that the correlation dynamic is highly persistent (ϕ equal to 0.9905).

The parameter δ is statistically significant indicating that when market are simultaneously affected by shocks of the same sign, the next period correlation increases more than when the markets are affected by shocks of the opposite signs. This result is consistent with many other findings that the correlation increases after common (negative or positive) shocks (see Ang and Bekaert (2001)).

The parameter φ that accounts for asymmetry is not statistically significant. This result is contradictory to the results obtained by Cappiello *et al.* (2006) but in line with the results of Jondaneu and Rockinger (2006) who investigate the dynamics of correlation between S&P500, FTSE100, NIKKEI 225, DAX 30 and CAC 40 and also obtain the asymmetry parameter not statistically significant.

Specification testing

As in the case of conditional volatility we use the regression-based diagnostics suggested by Wooldridge (1990, 1991). We test if the conditional correlation is correctly specified and if there is no remaining dynamics that should be accounted for in the conditional correlation equation (please see details of the test in the appendix)

The result for the tests for misspecification in the conditional correlation shows the Table 8 (for $Q = 4$)

	$FTSE$	DAX	WBI	WIG	PX	BUX	SAX	SBI
$FTSE$	–	–	–	–	–	–	–	–
DAX	0.33 *0.9877*	–	–	–	–	–	–	–
WBI	0.56 *0.9684*	0.38 *0.9884*	–	–	–	–	–	–
WIG	1.42 *0.8407*	0.57 *0.7247*	0.74 *0.9437*	–	–	–	–	–
PX	1.19 *0.8797*	1.25 *0.8698*	1.09 *0.8958*	1.21 *0.8764*	–	–	–	–
BUX	1.60 *0.8087*	0.73 *0.9475*	1.95 *0.7449*	0.73 *0.9463*	3.85 *0.4266*	–	–	–
SAX	5.56 *0.2345*	5.27 *0.2606*	1.08 *0.8794*	1.81 *0.7706*	4.00 *0.4060*	6.60 *0.1585*	–	–
SBI	2.31 *0.6789*	3.57 *0.4676*	6.78 *0.1479*	4.32 *0.3644*	0.48 *0.9764*	3.61 *0.4613*	4.86 *0.3019*	–

Table 8. Results of the test for no misspecification in the conditional correlation model

For all estimated correlation series we accept at 5% level of significance the hypothesis about no misspecification in the conditional correlation series.

As in the case of conditional volatility in the case of conditional correlation we can check different moment conditions to investigate if there is any remaining asymmetry (both in the case of sign and size effect). We can define the following moments

$$x_{5t-1} = I[z_{i,t-1} < 0]I[z_{j,t-1} < 0]$$

$$x_{6t-1} = I[z_{i,t-1} > 0]I[z_{j,t-1} < 0]$$

$$x_{7t-1} = I[z_{i,t-1} < 0]I[z_{j,t-1} > 0]$$

$$x_{8t-1} = I[z_{i,t-1} > 0]I[z_{j,t-1} > 0]$$

$$x_{9t-1} = z_{i,t-1}z_{j,t-1}I[z_{i,t-1} < 0]I[z_{j,t-1} < 0]$$

$$x_{10t-1} = z_{i,t-1}z_{j,t-1}I[z_{i,t-1} > 0]I[z_{j,t-1} < 0]$$

$$x_{11t-1} = z_{i,t-1} z_{j,t-1} I[z_{i,t-1} < 0] I[z_{j,t-1} > 0]$$

$$x_{12t-1} = z_{i,t-1} z_{j,t-1} I[z_{i,t-1} > 0] I[z_{j,t-1} > 0]$$

In this setting the generalized residuals are defined as $u_{ij,t} = z_{i,t} z_{j,t} - \rho_{ij,t}$, where $\rho_{ij,t}$ is the estimated conditional correlation between series i and j. Here we estimate $8x28$ regression. Table 9 presents only the rejection rate for each of the before mentioned moments (that we expect to be below 10%)

	x_{5t-1}	x_{6t-1}	x_{7t-1}	x_{8t-1}	x_{9t-1}	x_{10t-1}	x_{11t-1}	x_{12t-1}
Rej_rate	0.035	0.071	0.071	0.00	0.035	0.214	0.178	0.035

Table 9. Results of the test for no misspecification with different moment conditions.

The overall rejection is 0.0802. This low rejection rate for different moment specifications shows that the conditional correlation is correctly specified.

News Impact Surfaces

The figures below present the News Impact Surfaces for WIG and PX. The NIS for BUX, SAX and SBI are presented in the appendix. The figures show the changes in correlations between stock markets due to the shocks to each of them. We impose both positive and negative shocks.

Empirical results show that the shocks of the same sign should increase the correlation between stock markets stronger than shocks of different sign.

The asymmetry, discussed in the financial literature and imposed in empirical model, should make the impact of negative shocks bigger than impact of positive shocks.

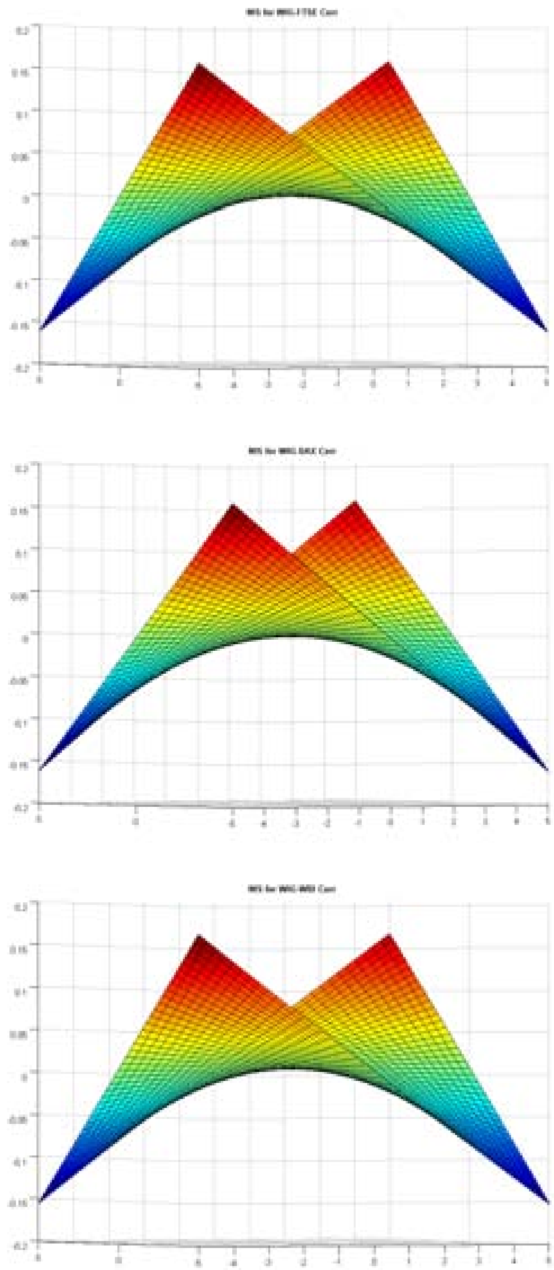


Figure 4. News Impact Surfaces for WIG with FTSE, DAX, WBI

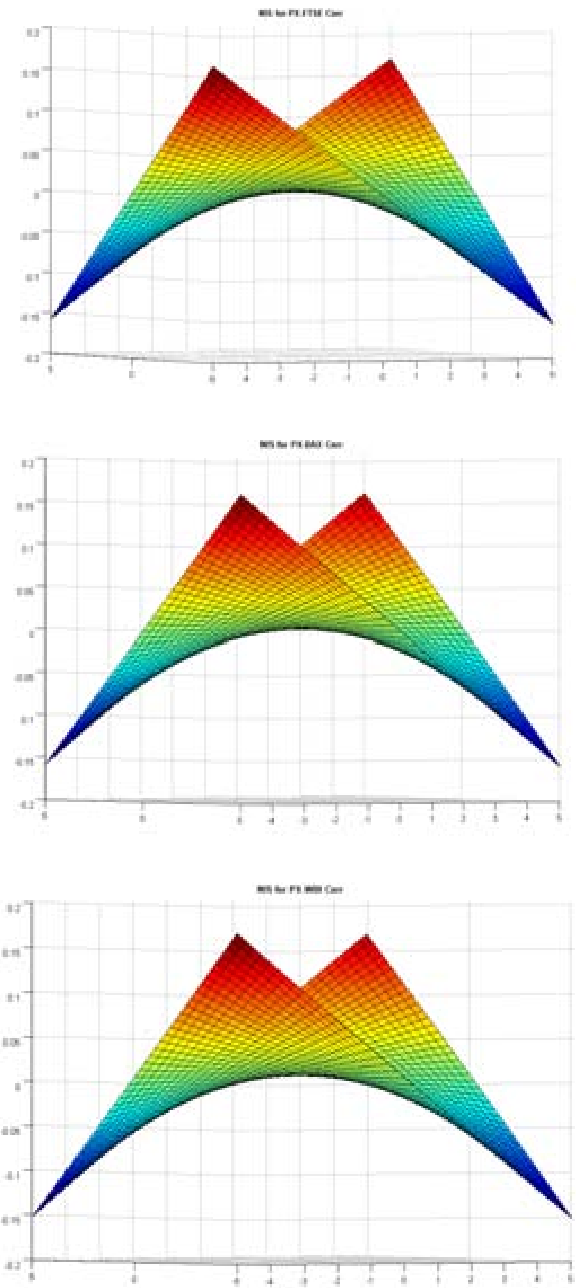


Figure 5. News Impact Surfaces for PX with FTSE, DAX, WBI

Shocks induced by combinations of same signs have greater impact on conditional correlations than other combinations. This pattern is characteristic and applies for the all the combinations of markets. The asymmetric effect in conditional correlations is very small, as the estimated parameter is small and statistically not significant.

This can be maybe due to the fact that the correlations are still rather small and the links between stock markets weaker as in the case of European or US stock markets (see Capiello *et al.* (2006)).

Structural breaks in the conditional correlation

To check the presence of structural breaks in conditional correlation we estimate the models that impose the structural break in conditional correlation as mention in the section 2.

We impose the timing of the break, τ, every month, starting from February 1995 and finishing in October 2005. We estimate 129 models with different timing of break.

The model with the imposed break in the conditional covariance matrix nests the standard model without the break so that for each potential point of break at time τ we define the test statistic as $F_n(\tau) = -2(l_r - l_u)$, where l_r and l_u are maximized valued of the loglikelihood function of the restricted and unrestricted model, respectively.

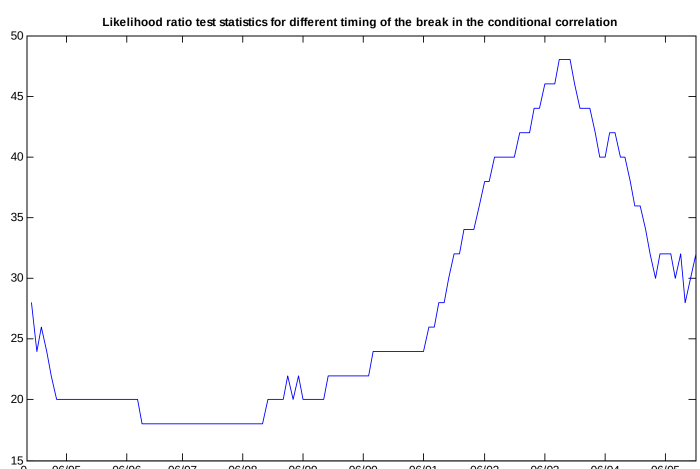


Figure 6. Likelihood ratio test statistics for the models with the break date at different points of time.

Figure 6 shows the highest levels of the test statistics for the break in the conditional covariance in the period between September 2003 and November 2003. Note that the CEE countries entered the European Union in May 2004.

Since the imposed break date is unknown, the likelihood ratio test statistics have non-standard distributions. We calculated the values of the test statistics proposed by Andrews (1993) and Andrews and Ploberger (1994). The asymptotic $p-values$ of the test numerically are approximated in the way proposed by Hansen (1997).

	$\sup Fn$	$ExpFn$	$AveFn$
$stat$	48.0	27.44	20.84
$p-value$	0.0000	0.0000	0.0000

Table 10. Structural break test.

The results of the likelihood ratio test with the test statistics calculated as proposed by Andrew (1993) and Andrew and Ploberger (1994) with the numeric approximations of asymptotic p-values as discussed by Hansen (1997) show that in the period September 2003 - November 2003 we have a break in the conditional correlation.

This can be explained by the direct impact of the entrance of CEE countries in the European Union and the inflow of the foreign investors to those countries which has an effect of increase in the correlations between financial markets in the CEE region and higher integration of these financial markets with the main stock markets in the Europe. The break was some months before the date of the May 2004 and it is evidence that the financial markets started to discount this fact some months in advance.

Below we present the series of the estimated conditional correlations. We present five panel for each of the market of CEE countries and their corresponding correlations with FTSE, DAX and WBI.

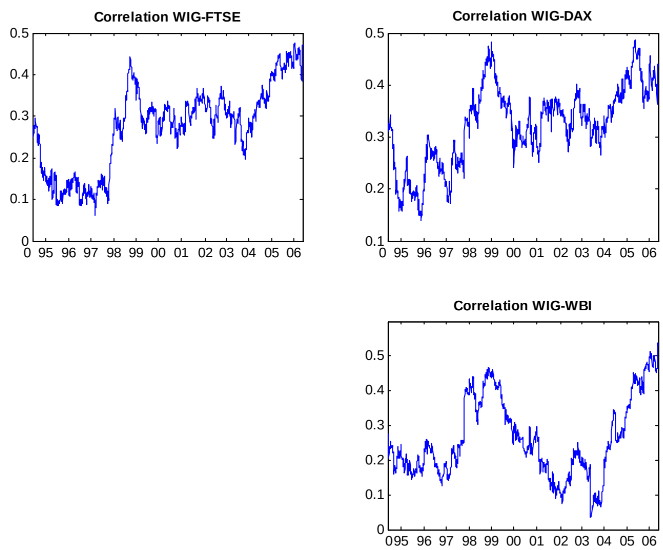


Figure 7. Estimated conditional correlation of WIG and FTSE, DAX and WBI

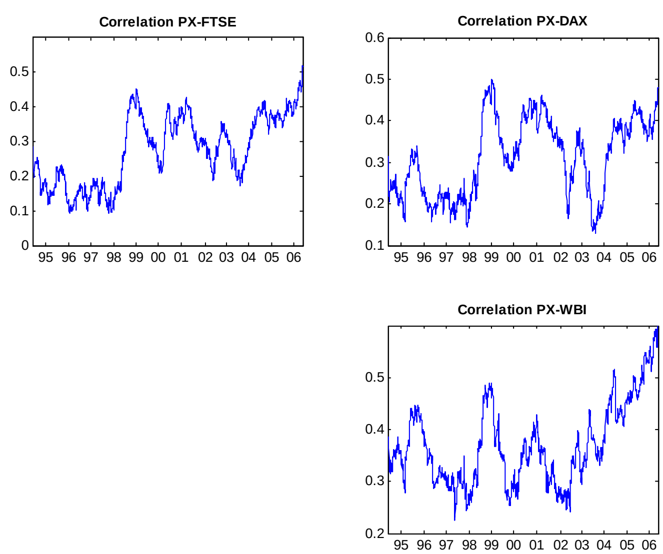


Figure 8. Estimated conditional correlations between PX and FTSE, DAX and WBI

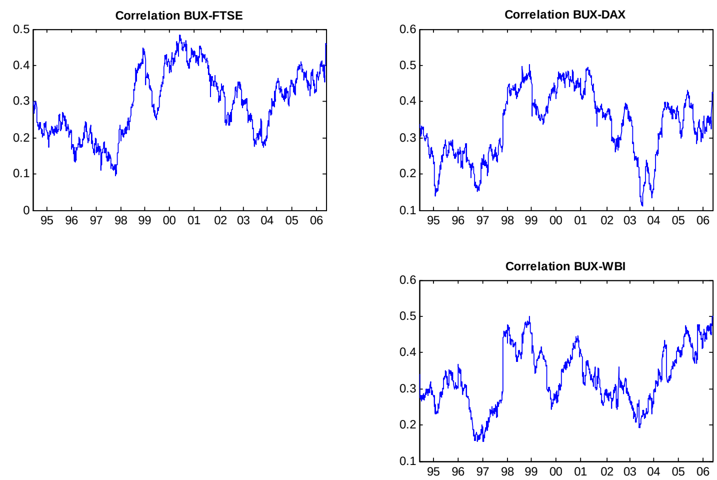


Figure 9. Estimated conditional correlations between BUX and FTSE, DAX and WBI

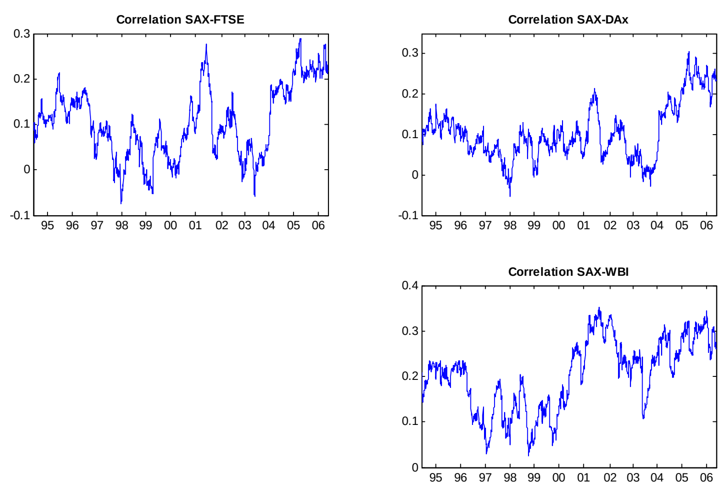


Figure 10. Estimated conditional correlations between SAX and FTSE, DAX and WBI

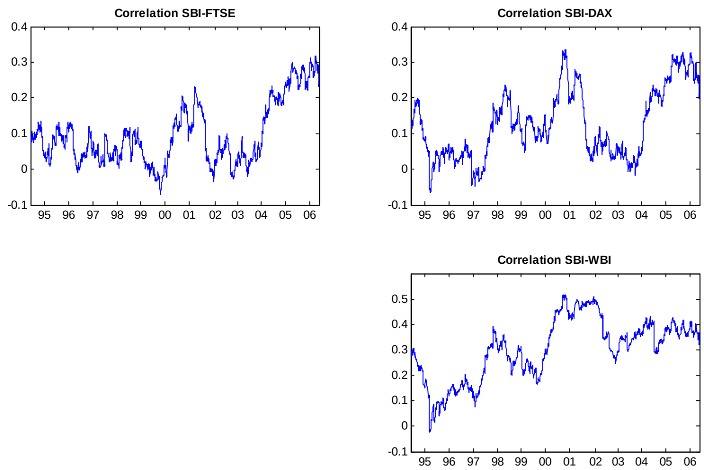


Figure 11. Estimated conditional correlations between SBI and FTSE, DAX and WBI

The series of estimated conditional correlations show the change in the behavior of the correlation between CEE and European markets after the detected break at the end of 2003. This is especially visible in the case of the bigger CEE markets, like Poland, Czech Republic and Hungary. SAX and SBI follow the same pattern. We observe an increasing patters in the correlation from the end of 2003 on.

The Russian crisis of 1998 is also reflected in the series of conditional correlations. Like in the case of other markets during the turmoil of 1998 the correlations increased significantly confirming the empirical fact that during the financial crisis the correlation between financial markets increases (see Campbell *et al.* (2002) or Solnik *et al.* (1996)) lowering the prospects of diversification in the moment when its benefits are the most needed. We find evidence of the increase in correlation between CEE countries and Western Europe in the time of Russian crisis. This result is in line with Yang *et al.* (2008).

The entrance of the Eastern European countries to European Union has positively contributed to the integration of the financial markets in the region and Western countries - the political, economical and juridical stability has encouraged new investors to enter these financial markets.

From one side the foreign investors brought liquidity and fresh capital, what we can see when analyzing the turnover of the stock markets in Warsaw in Prague in 2004 and 2005. But this higher integration of financial markets and increase in correlation among Eastern and Western stock markets has lowered the prospect of these financial markets as the good diversification tool for the portfolio managers.

We also have checked if the results change when we keep the stock market quotations in national currencies instead of changing them to dollar terms and have obtained the same results.

Finally we want to support our findings about the structural break in the conditional correlation due to the entrance of the CEE countries to the EU and the subsequent increasing tendency in the conditional correlations by investigating another very important market in the Central and Eastern Europe - Russia.

Russian stock market is the largest among the CEE stock markets in terms of market capitalization. RTS Stock Exchange was established in the middle of 1995 and is the biggest electronic trading floor in Russia. The official dollar denominated RTS index was first calculated in September 1995 and tracks 50 large-cap Russian stocks.

Using the same DCC model of Engle and Sheppard (2001) and time period 1995 -2006 we have obtained the following conditional correlations between RTS and FTSE and DAX.

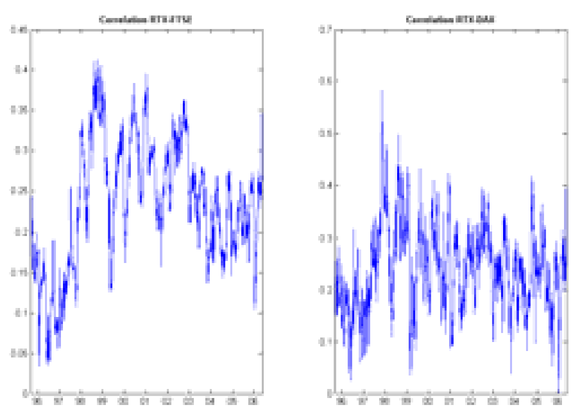


Figure 12. The conditional correlations between RTS and FTSE and DAX.

The estimated series of conditional correlation show different pattern of conditional correlations than the one observed in the CEE countries. Similarly to previous results we observe an increase in correlations around the Russian crisis of 1998. Following this period of turmoil we observe a declining tendency in the correlations between Russia and Western Europe. The series are also much more volatile - the parameter that governs the persistence in the conditional volatility is smaller than in the case of CEE countries (0.9804 for RTS-FTSE and 0.9353 for RTS-DAX) and the parameters that governs the impact of shocks on conditional correlations is higher for these correlations.

This supports our findings that the break in conditional correlations between CEE countries and European stock markets and increase after 2003 are more event specific and can be explained by the entrance of CEE countries to EU and not for the general tendency of increase in correlations among financial markets.

3.5 Conclusions

In this paper we investigate the changes in the correlations between Central Eastern European stock markets and main Western European stock markets (London, Frankfurt and Vienna).

We detect a structural break in the conditional correlation series around November 2003 - some months before May 2004, the date of the entrance of all the countries to the EU. The following increase in correlation of the Central and Eastern European financial markets and Western Europe is the sign of the higher integration of these markets with European stock markets. From the other side this is an evidence of the diminishing prospect of these stock markets as a diversification tool.

The next change in the level of conditional correlations between CEE countries and European stock markets we can expect at the time when these countries will adopt the Euro as the sole currency. Capiello *et al.* (2006) document the structural break in the correlations among bond and equity markets within Europe and the following increase in correlations among major markets in Europe due to the significant improvement in the economic conditions within the region (the lack of exchange rate risk). Savva *et al.* (2005) using Multivariate EGARCH model evidence the impact of Euro on the conditional correlation between French and German stock markets, not only by increasing the conditional correlation but also by making it remarkably constant. Therefore it would be interesting to analyze the changes in stock markets' correlation between the countries that had entered the European Union before the Euro adoption and than enjoyed the benefits of Euro introduction.

Another possible extension of our findings can be the similar analysis on the firm level. Kearney and Poti (2006) study the correlation between Euro area national stock market indexes and the correlation amongst a sample of blue chip European stocks. They confirm a significant rise in the correlations amongst national stock market indexes and the diminishing prospects of the Euro zone diversification, although diversification across individual stocks remains useful. In the same line we can analyze the CEE companies and their diversification prospects.

We may also consider the sector indices from the CEE countries or sector indices for the whole region and their correlation with other stock markets, which may lead us to detect other diversification opportunities.

Bibliography

[1] Ang, A. and G. Bekaert, "International Asset Allocation with Time-Varying Correlations", NBER Working Paper No. 7056, 1999.

[2] Andrews, D.W.K., "Tests for parameter instability and structural change with unknown change point", *Econometrica*, 61 (1993), 821-856.

[3] Andrews, D.W.K. and W. Ploberger, "Optimal Tests When a Nuisance Parameter is Present Only Under the Alternative", *Econometrica,* 62 (1994), 1383-1414.

[4] Balazs, E. and E. Kocenda, "Contagion Across and Integration of Central and Easter European Stock Markets: Evidence from Intraday Data", William Davidson Institute Working Paper No. 798, 2005.

[5] Bollerslev, T., "Generalized autoregressive heteroscedasticity", *Journal of Econometrics,* 31 (1986), 307-327.

[6] Cambell, R., Koedijk, K. and P. Kofman, "Increased correlation in Bear markets", *Financial Analyst Journal*, 58 (2002), 87-94.

[7] Capiello, L., Engle, R.F. and K. Sheppard, "Asymmetric Dynamics in the Correlations of Global Equity and Bond Returns", *Journal of Financial Econometrics*, 4 (2006), 537-572.

[8] De Santis, G. and B. Gerard, "International Asset Pricing and Portfolio Diversification with Time-Varying Risk", *Journal of Finance*, 52 (1997), 1881-1912.

[9] Engle, R.F., "Dynamic Conditional Correlation - a Simple Class of Multivariate GARCH models", *Journal of Business and Economic Statistics,* 20 (2002), 339-350.

[10] Engle, R.F. and V.K. Ng, "Measuring and Testing the Impact of News on Volatility", *Journal of Finance,* 48 (1993), 1749-1778.

[11] Engle, R.F. and K. Sheppard, "Theoretical and Empirical Properties of Dynamic Conditional Correlation Multivariate GARCH", NBER, Working Paper 8554, 2001.

[12] Gilmore, C.G. and McManus, G.M., "International Portfolio Diversification: US and Central European equity markets", *Emerging Markets Review*, 3 (2002), 69-83.

[13] Glosten, R.T., Jagannathan, R. and D. Runkle, "On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks", *Journal of Finance*, 48 (1993), 1779-1801.

[14] Hansen, B., "Approximate Asymptotic *P* Values for Structural - Change Tests", *Journal of Business and Economic Statistics,* 15 (1997), 60-66.

[15] Hardouvelis, G.A., Malliaropulos, D. and R. Priestley, "EMU and European Stock Market Integration", *Journal of Business,* 79 (2006), 365-392.

[16] Jondaneu, E. and M. Rockinger, "The Impact of News on Higher Moments", Working Paper No. 06-28, Swiss Finance Institute, November 2006.

[17] Kearney, C. and V. Poti, "Correlation Dynamics in European Equity Markets", *Research in International Business and Finance*, 20 (2006), 305-321.

[18] Longin, F. and B. Solnik, "Extreme Correlation of International Equity Markets", *Journal of Finance*, 56 (2001), 649-676.

[19] MacDonald, R., "Transformation of external shocks and capital market integration", in *The New Capital Markets in Central and Eastern Europe*, The Center for European Economic Research, Springer Verlag, 2001, 210-245.

[20] Savva, Ch.S., Osborn D.R. and L.Gill, "Spillovers and Correlations between US and major European Stock Markets: The Role of Euro", University of Manchester, Discussion Paper EDP-0515 (2005).

[21] Solnik, B., Boucrelle, C. and Y. Le Fur, "International market correlation and volatility", Financial Analyst Journal, 52 (1996), 17-34.

[22] Syriopoulos, T., "International portfolio diversification to Central European stock markets", *Applied Financial Economics,* 14 (2004), 1253-1268.

[23] Tse, Y and A. Tsui, "A Multivariate GARCH Model with Time-Varying Correlations," *Journal of Business and Economic Statistics*, 20 (2002), 351–362.

[24] Wooldridge, J.M., "A Unified Approach to Robust, Regression Based Specification Tests", *Econometric Theory,* 6 (1990), 17-43.

[25] Wooldridge, J.M., "On the Application of Robust, Regression Based Diagnostics to Models of Conditional Means and Conditional Variances", *Journal of Econometrics,* 47 (1991), 5-46.

[26] Voronkowa, S., "Equity market integration in Central European emerging markets: A cointegration analysis with shifting regimes", *International Review of Financial Analysis*, 13 (2004), 633-647.

[27] Yang, J., Hsiao, Ch., Li, Q. and Zijun Wang, "The emerging market crisis and stock market linkages: further evidence", *Journal of Applied Econometrics,* 21 (2006), 727-744.

3.6 Appendix 1: Tests

Test for dynamic correlation model - Engle and Sheppard (2001)

The null hypothesis is of the constant correlation against the alternative of dynamic conditional correlation

$$H_0 : R_t = \overline{R} \quad t \epsilon T$$

$$H_A : vech(R_t) = vech(\overline{R}) + \beta_1 vech(R_{t-1}) + ... + \beta_p vech(R_{t-p})$$

The testing procedure is as follows. Estimate the univariate GARCH processes and standardized the residuals for each series. Then estimate the correlation of the standardized residuals, and jointly standardized the vector of univariate standardized residuals by the symmetric square root decomposition of $\overline{R}$. Under the null of constant correlation, these residuals should be IID with the variance covariance matrix unit diagonal I_k (we consider k series). The artificial regression will be a regression of the outer products of the residuals on a constant and lagged outer products. Let

$$Y_t = vech^u[(\overline{R}^{-1/2} D_t^{-1} \varepsilon_t)(\overline{R}^{-1/2} D_t^{-1} \varepsilon_t)' - I]$$

where $(\overline{R}^{-1/2} D_t^{-1} \varepsilon_t)$ is a k by 1 vector of residuals jointly standardized under the null, and $vech^u$ is a modified $vech$ which only selects elements above the diagonal. The vector autoregression is

$$Y_t = \alpha + \beta_1 Y_{t-1} + ... + \beta_s Y_{t-s} + \eta_t$$

Under the null the constant and all the lagged parameters in the model should be zero. The test statistics is χ^2_{s+1} distributed.

Testing for misspecification in the conditional volatility - regression based test based on Wooldridge (1990, 1991).

Regression based test adapted to test the misspecification in the conditional volatility. We define $\widehat{\lambda}_{it} = (\widehat{z}^2_{i,t-1}, \widehat{z}^2_{i,t-2}, ..., \widehat{z}^2_{i,t-Q})'$ as the vector of indicator variables and $\nabla_\theta \widehat{h}_{it}$ as the scores of the estimated model. First we regress each element of $\widehat{\lambda}_{it}$ on the scores to obtain Q-element residuals $\widehat{r}_{it}$ and finally, we regress unity on the vector of Q regressors $\widehat{\phi}_{it}\widehat{r}_{it}$, where $\widehat{\phi}_{it} = \widehat{z}^2_{it} - 1$ (called generalized squared residuals). We calculate $W_{ii}(Q) = T - SSR$, where SSR is the sum of squares of the residuals of the last regression. If there is no model misspecification, $W_{ii}(Q)$ is asymptotically distributed as χ^2_Q.

Testing for misspecification in the conditional correlation - regression based test based on Wooldridge (1990, 1991).

We can use the regression-based diagnostics for the cross products of the standardized residuals from different equations. We define $\widehat{\lambda}_{ijt} = (\widehat{z}_{i,t-1}, \widehat{z}_{j,t-1}, \widehat{z}_{i,t-2}, \widehat{z}_{j,t-2}, ..., \widehat{z}_{i,t-Q}, \widehat{z}_{i,t-Q})'$ as the vector of indicator variables and $\nabla_\theta \widehat{h}_{it}$ as the scores of the estimated model. First we regress each element of $\widehat{\lambda}_{ijt}$ on scores to obtain Q-element residuals $\widehat{r}_{ijt}$ and finally, we regress unity on the vector of Q regressors $\widehat{\phi}_{ijt}\widehat{r}_{it}$, where $\widehat{\phi}_{ijt} = \widehat{z}_{it}\widehat{z}_{jt} - \widehat{\rho}_{ijt}$. We calculate $W_{ii}(Q) = T - SSR$, where SSR is the sum of squares of the residuals of the last regression. If there is no model misspecification, $W_{ij}(Q)$ is asymptotically distributed as χ^2_Q.

The misspecification tests - based on Wooldridge (1990,1991).

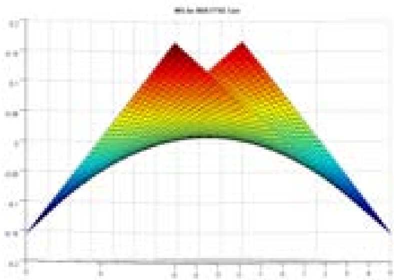
The robust conditional moment tests of Wooldridge (1990,1991) are a useful tool in detecting whether a variable is useful in predicting a *generalized* residuals (defined as $u_t = \varepsilon^2_{it} - h_{it}$). The resulting statistic tests if a set of moment conditions x_{gt-1} can predict the generalized residuals series. The test statistic is given by

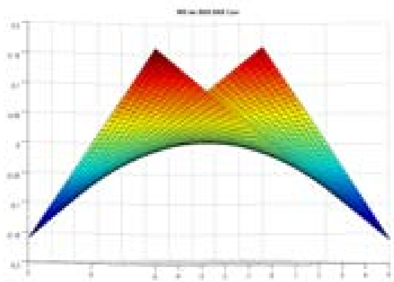
$$C = \left\{ \frac{1}{T} \sum_{t=1}^{T} u_{ij,t}\lambda_{g,t-1} \right\}^2 \left\{ \frac{1}{T} \sum_{t=1}^{T} u^2_{ij,t}\lambda^2_{g,t-1} \right\}^{-1}$$

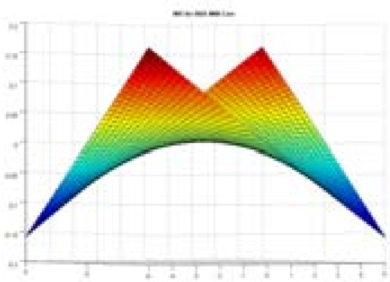
where $\lambda_{g,t-1}$ is the residual from a regression of the moment conditions on the scores of the likelihood. Under regularity conditions C is χ^2 distributed with one degree of freedom.

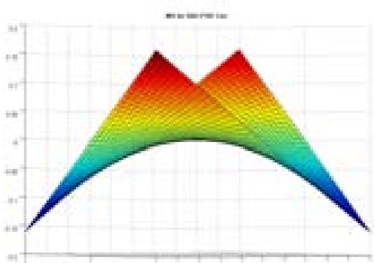
The test is simple to compute and consists of two regressions - the first one when the moments are regressed on the scores of estimated model, and the second where a vector of ones is regressed on the product of the generalized residuals and the residuals from the first regression. The moment conditions can be any function of any variable in the conditioning set.

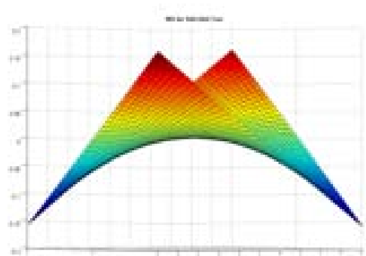
3.7 Appendix 2: News Impact Surfaces for BUX, SAX and SBI

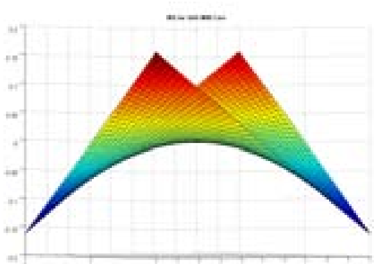


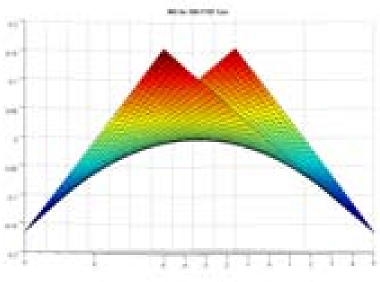


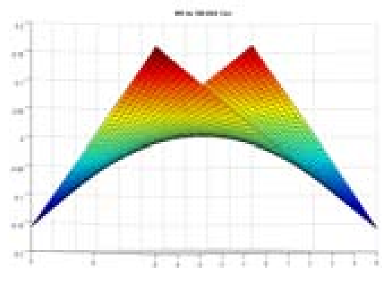


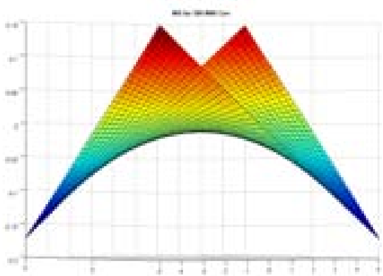












Chapter 4

Structural changes in the volatility of IBEX35

4.1 Introduction

Volatility of stock prices and its forecasting play an important role in many areas of business life.

Changes in volatility and especially an increase in the level of volatility of financial markets can impact the economic activity through many channels. Investors may link the higher volatility with higher risk and may alter or postpone their investments. Since shares are a part of household wealth, an increase in volatility may depress the consumer confidence and private consumption. The level of volatility in financial markets can also influence corporations' investment decisions and banks' willingness and ability to extend credit. Sharp changes in the level of financial market volatility can also be of concern to policy makers since it can threaten the viability of financial institutions and the smooth functioning of financial markets (see Becketti and Sellon (1989) for further discussion). Not to mention is the importance of volatility and volatility forecasting in asset pricing and risk management.

The volatility of financial variables is a dynamic process. The changes in volatility can be driven by the arrival of new information that alters the expected stock returns due to changes in local or global economic environment.

Another factor driving the volatility can be varying traded volume of financial instruments or sociological or psychological factors like panic or fears that drive the stock prices from their fundamental values.

Technological progress that allows the quicker and more precise carrying out the transactions on the stock markets is another reason that makes the volatility to increase.

Finally, the changes in volatility can be a consequence of stronger transmission of shocks due to increased interdependence and interconnectivity of stock markets coming from removing the barriers of trading in different markets (e.g. introduction of Euro significantly contributed to the increase in correlation among European stock markets (see Cappiello *et al.* (2006)).

Accurately modeling and forecasting time-varying volatility of financial time series is especially difficult when the underlying process experiences significant level changes which can lead to the structural breaks.

Well specified model requires that the parameters of the model are stable over time. The structural break test is a useful tool for investigating the stability of parameters of the model and detecting a possible turning point. Hillebrand (2005) shows that neglected structural breaks in the GARCH parameters induce upward biases in estimates of persistence of GARCH processes. Moreover, Stărică *et al.* (2005) show that long horizon forecasts of stock return volatility generated by GARCH (1,1) models assuming parameter stability are often inferior to forecasts that allow for frequent changes in unconditional variance of stock returns.

In this paper we concentrate on IBEX 35 ($IBEX$ thereafter), the index of the blue chips traded on the Spanish stock market. Initiated in 1992, the IBEX 35 is a market capitalization weighted index of the 35 most liquid Spanish stocks quoted on the Madrid Stock Exchange. We consider the daily quotations of the index over the period January 1992 - July 2011, which account for all the past quotations of the index and spans over periods of different behavior of the index.

We consider the IBEX 35 due to many reasons – first of all we are interested in the volatility of the Spanish stock market. Secondly, the Spanish index was characterized by the high volatility over the time of the recent financial meltdown that started in 2007. Finally, the integration of the Spanish stock market with the world economy and the exposure of Spanish companies to Latin America make the volatility of the index not only sensitive to the national economy but also induces a high exposure to the news coming from Eurozone, USA and Latin America.

To our knowledge this is the first paper that considers the whole history of the index with the daily data and also incorporates the period of the last economic and financial crisis.

The impact of financial and economic crisis on the volatility of stock markets has been documented in the academic literature.

Aggarwal *et al.* (1999) use the ICSS algorithm to identify the points of change in the variance of ten largest emerging market stock markets in Asia and Latin America, together with developed countries' stock markets, during 1985-1995. They find that the high volatility of emerging markets is characterized by frequent sudden changes in variance, majority of which are associated with important events in each country rather than the global event, with the October 1987 crash as the only global event that has significantly increased volatility of all the stock markets considered.

Kim *et al.* (2010) test for the stability of volatility processes in selected stock markets (Hong Kong, Japan, Korea, Singapore, Thailand, and the U.S.) for the period 1990-2005. Four stock markets (Hong Kong, Japan, Korea and Singapore) show one or more structural change in volatility. The structural shift is mainly the consequence of the Gulf war, Japan's economic recession, and corresponding policy changes. Three markets (Hong Kong, Korea, and Singapore) show additional structural change around the period of the Asian financial crisis in the mid-1990s. Interestingly, no structural break is detected in Thailand or the U.S, and the U.S. stock market is the most stable with the lowest volatility comparing to the other stock markets considered.

Cuñado *et al.* (2006) test whether the volatility of the six emerging market stock market indexes has changes over the period 1976-2004. They use the monthly data on stock returns for Argentina, Brazil, Chile, South Korea, Mexico and Thailand and detect the dates of structural breaks around the financial liberalization in these countries. The stabilization policies and opening of the financial markets in these countries resulted in lower volatility.

Cuñado *et al.* (2004) analyze the changes in volatility of the monthly returns of the Spanish stock market over 1941-2001. They detect the structural break around 1972, coinciding with the opening of the Spanish economy. They observe higher level of volatility and lower persistence from 1972 to 2001 mostly attributable to the increased growth of trading volume brought about by the economic development of the Spanish economy.

Gil-Alana *et al.* (2008) examine the stochastic volatility of the Spanish stock market over the period 2001-2006. They use the long memory model that takes into account the existence of the endogenous structural break. When a single break point is allowed they find a possible break around April 2003.

All these works show that indeed many stock markets underwent significant changes in the past and the structural breaks are empirical characteristics of financial time series.

In this paper we analyze the volatility of the index using Quadratic GARCH (QGARCH) model of Sentana (1995). This model not only allows for asymmetry in the conditional volatility but also makes this effect depending on the size of the shocks, which can be especially important during the period of financial crises. This model proves to be a useful tool in modelling conditional volatility. Franses and van Dijk (1996) employ random walk, GARCH, TARCH and QGARCH models to examine the volatility forecasting performance in five European stock markets. Using the weekly returns over the period 1986 to 1994 they find that QGARCH outperforms other forecasting models.

To investigate the possible structural break in the conditional volatility we use the ICSS algorithm of Inclan and Tiao (1994) modified by Sansó *et al.* (2004) to account for size distortion of the original ICSS algorithm when applied to the series that follow a dependent processes, like GARCH models for example. We will test for possible breaks in unconditional variance since a detected break implies a structural break in the GARCH process governing conditional volatility.

Rapach and Strauss (2008) apply modified ICSS test to daily US dollar exchange rate returns vis-à-vis the currencies of seven OECD countries and the daily returns for the trade-weighted US-dollar exchange rate and analyze in-sample and out-of-sample performance of the model with the detected breaks. They find significant evidence of structural break in the unconditional variance for seven of eight exchange rate return series and observe that the parameters of the estimated GARCH(1,1) models across the subsamples defined by the structural breaks detected by the modified ICSS algorithm, often differ notably. They conclude that taking structural breaks into account improves out-of-sample forecasting.

Rapach *et al.* (2008) test for structural break in the unconditional variance of daily returns for the S&P 500 index and ten sector indexes over 1989 – 2006 and find evidence of multiple variance breaks in almost all of the return series.

Using the before mentioned techniques we detect several structural breaks in the volatility of IBEX 35. The subsamples defined by the breaks differ in the persistence and the asymmetry of the impact of shocks on volatility. The last two months of observations are left for out-of-sample forecasting. We observe a better forecasting performance of the model with breaks than the benchmark, which is the QGARCH model estimated over the entire period of interest.

We also check if the observed structural break in the index volatility is rather a unique characteristic of the Spanish stock market or it is a feature experienced by other financial markets. We consider the Dow Jones Industrial Average index ($DJIA$ thereafter) and also detect several breaks detected by the modified ICSS algorithm. Some of the detected breaks coincide with the breaks in the volatility of IBEX 35. In both markets we observe similar increase in the volatility of the index from mid 2007 on.

The paper is organized as follows - in the next section we discuss the GARCH model used in this paper, structural break test applied to volatility of index returns and present the loss functions used to evaluate the forecasting performance. Section 3 discusses the evolution of stock index over the period of interests and presents the statistics of the data. Section 4 concentrates on the empirical results of the structural break tests and estimation of the models, discusses the forecasting performance of the models. In this section we also present the structural breaks detected for DJIA index. Section 5 concludes.

4.2 Methodology

In this section we discuss the volatility model applied in this paper, the news impact curve (NIC) used to show different impact of positive and negative shocks on the volatility, the structural break test used to detect the possible change in the volatility of the stock index and the loss functions used to evaluate the forecasting power of the volatility model.

4.2.1 GARCH model

Given the daily quotations of the index (P_t) we define continuously compounded returns as $r_t = 100 \ln(P_t/P_{t-1})$. We filter the series of returns using an appropriate ARMA filter eliminating the deterministic component of the series. The purely stochastic series of returns are defined as $r_t = \sigma_t \varepsilon_t$, where ε_t is an i.i.d. series with given distribution, mean of zero and unit variance.

The Generalized Autoregressive Conditional Heteroscedastic ($GARCH$) models, introduced by Engle (1982) and Bollerslev (1986), have been proposed to capture the empirical properties of financial time series like changing volatility and volatility clustering. The simplest $GARCH(1,1)$ model is defined as

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta_i\sigma_{t-1}^2$$

In the standard GARCH model the effect of the shock on volatility only depends on the size of the shocks - positive and negative shocks have the same impact on conditional volatility.

Black (1976) observes the tendency of stock market volatility to fall when there are "good news" and to rise when there are "bad news". Engle and Ng (1993) propose tests to examine this different impact of positive and negative returns on volatility (Sign Bias, Negative Size Bias and Positive Size Bias tests).

Most nonlinear GARCH models are motivated by the desire to capture the different effects of positive and negative shocks on conditional volatility or other types of asymmetry.

In this paper we use the $QGARCH$ model introduced by Sentana (1995), which not only allows for asymmetry in the conditional variance but also makes this effect depending on the size of the shock, which in time of high volatility should allow to capture the impact of several extreme market movements.

Sentana (1995) introduces the $GQGARCH$ (Generalized Quadratic GARCH) model defined as

$$\sigma_t^2 = \omega + \sum_{i=1}^{q}\gamma_i\varepsilon_{t-i} + \sum_{i=1}^{q}\alpha_{ii}\varepsilon_{t-i}^2 + 2\sum_{i=1}^{q}\sum_{j=1}^{q}\alpha_{ij}\varepsilon_{t-i}\varepsilon_{t-j} + \sum_{i=1}^{p}\beta_j\sigma_{t-j}^2$$

The GQGARCH model allows for asymmetry by introducing the lagged values of ε_t and the lagged values of the cross-product terms in the conditional variance specification.

In this study we focus on the simplest $QGARCH(1,1)$ specification given as

$$\sigma_t^2 = \omega + \gamma\varepsilon_{t-1} + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

which can be rewritten as

$$\sigma_t^2 = \omega + (\frac{\gamma}{\varepsilon_{t-1}} + \alpha)\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

Positivity of the variance is achieved if $\alpha, \beta \geq 0$ and $\gamma^2 < 4\alpha\omega$. The model is covariance stationary if $\alpha + \beta < 0$.

Asymmetry is introduced by parameter γ. If $\gamma < 0$ the effect of negative shocks on the conditional variance will be larger than the effect of positive shock of the same size. This effect in turns depends on the size of the shock.

The unconditional variance implied by QGARCH is $\sigma^2 = \omega/(1-(\alpha+\beta))$.

The news impact curves (NIC), introduced by Pagan and Schwert (1990) and discussed by Engle and Ng (1993), measure how new information is incorporated into volatility. The NIC for $QGARCH(1,1)$ discussed above is given as

$$NIC = \begin{cases} \omega + \beta\sigma^2 + \alpha\varepsilon_i^2 + \gamma\varepsilon_i & \varepsilon_i < 0 \\ \omega + \beta\sigma^2 + \alpha\varepsilon_i^2 + \gamma\varepsilon_i & \varepsilon_i > 0 \end{cases}$$

The nice feature of the GARCH models is that they can be easily estimated using the maximum likelihood technique. Using the normal distribution as the underlying distribution of the errors instead of the true distribution we obtain quasi maximum likelihood estimators of the parameters of the model, which as shown by Bollerslev and Wooldridge (1992), are consistent and asymptotically normal.

4.2.2 Structural break test

Structural break tests are important diagnostic tools in econometrics. In the similar way as the mean of economic variables, the heteroscedastic volatility can be affected by structural breaks in the underlying process. When modelling the time-varying volatility we require the parameters describing the data generating process to be stable over time. Otherwise the model can be miss-specified and the volatility forecasts can be affected.

Early work of Lamoureux and Lastreps (1990) questions the high level of persistence in the volatility estimated by GARCH models. They analyze 30 stock returns and demonstrate that the persistence was overstated because of the existence of deterministic structural shifts. Mikosch and Stărică (2004) and Hillebrand (2005) show that the neglected structural breaks in the GARCH parameters result in upward biases in estimates of the persistence of the GARCH process which can lead to poor estimates of the volatility.

Inclan and Tiao (1994) are the first to provide a method of detecting structural breaks in volatility. They propose the Iterative Cumulative Sums of Squares (ICSS) algorithm to detect multiple changes in variance. The ICSS algorithm uses cumulative sums of squares and searches for change points in unconditional volatility systematically at different moments of time.

The Inclan and Tiao (1994) test statistics is given by

$$IT = \sup_k \left|(T/2)^{0.5} D_k\right|$$

where $D_k = (C_k/C_T) - (k/T)$ and $C_k = \sum_{t=1}^{k} r_t^2$ for $k = 1, ..., T$ and where r_t is continuously compounded returns with conditional mean of zero. The value of k that maximizes the test statistic is the estimate of the break date. When r_t is distributed i.i.d. $N(0, \sigma_r^2)$, then the asymptotic distribution of the test statistic is given by $\sup_w |W^*(w)|$ where $W^*(w) = W(w) - wW(1)$ is a Brownian bridge and $W(w)$ is standard Brownian motion.

The most serious drawback of the IT test is that its asymptotic distribution is critically dependent on the assumption about the i.i.d. $N(0, \sigma_r^2)$ distribution of the returns. In fact, Andreou and Ghysels (2002), de Pooter and van Dijk (2004) and Sansó *et al.* (2004) show that IT statistic can be oversized for processes that follow different distribution, among them GARCH processes which depend on the past values.

To address this problem and allow the r_t to follow a variety of dependent processes, among them GARCH processes, a nonparametric adjustment based on the Bartlett Kernel is applied to the original IT statistic.

The AIT statistic (modified IT) is given by

$$AIT = \sup_k \left|T^{-0.5} G_k\right|$$

$$
\begin{aligned}
G_k &= \widehat{\lambda}^{-0.5}[C_k - (k/T)C_T] \\
\widehat{\lambda} &= \widehat{\gamma}_0 + 2\sum_{l=1}^{m}[1 - l(m+1)^{-1}]\widehat{\gamma}_l \\
\widehat{\gamma}_l &= T^{-1}\sum_{t=l+1}^{T}(r_t^2 - \widehat{\sigma}^2)(r_{t-l}^2 - \widehat{\sigma}^2) \\
\widehat{\sigma}^2 &= T^{-1}C_T
\end{aligned}
$$

and the lag truncation parameter m is selected using the procedure in Newey and West (1994).

Under general conditions, the asymptotic distribution of AIT is also given by $\sup_w |W^*(w)|$ and finite-sample critical values are generated via simulation.

To avoid the problem of size distortion for dependent processes we use the ICSS algorithm based on the AIT statistic and the 5% level of significance to test for multiple breaks in the unconditional variance of the stock index return series[1].

Once we detect at least one structural break, we estimate the $QGARCH$ models over the different regimes defined by the structural breaks and compare the out-of-sample performance of this model with the $QGARCH$ model estimated for the whole sample (no structural break).

4.2.3 Forecasting and forecast evaluation

As mentioned before forecasting future volatility based on the available information is an important and useful task in many areas of economic life. The expected future volatility of financial market returns is the main ingredient in assessing asset or portfolio risk and plays a key role in derivatives pricing models.

The family of GARCH model proves to be a useful tool for forecasting future volatility. As GARCH models specify the conditional variances as the explicit function of observed values, one-step ahead forecasts are easily obtained. More distant predictions are obtained by repeated substitution.

To select the best for model for conditional volatility based only on the in-sample estimation

[1] We implement the modified ICSS algorithm using the GAUSS procedures available from Andreu Sansó's page.

and model evaluation is only a part of the task. The out-of-sample forecasting ability of the GARCH models is an alternative approach to judge the adequacy of different volatility models.

Considering our $QGARCH(1,1)$ model

$$\sigma_t^2 = \omega + \gamma\varepsilon_{t-1} + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

We are interested in the forecast of σ_t^2 at future time s given all the available information at time t. We denote this forecast as $h_{t+s|t}$. We can evaluate the forecasts recursively as

$$h_{t+s|t} = \widehat{\omega} + \widehat{\alpha}\widehat{\varepsilon}_{t+s-1|t}^2 + \widehat{\beta}h_{t+s-1|t}$$

where $\widehat{\varepsilon}_{t+s|t}^2 = h_{t+s|t}$ for $s > 0$ by definition.

We can work out the formula for conditional forecast in the case of $QGARCH$ model and obtain

$$h_{t+s|t} = \widehat{\omega}\sum_{i=0}^{s-1}(\widehat{\alpha} + \widehat{\beta})^{s-1} + (\widehat{\alpha} + \widehat{\beta})^{s-1}h_{t+1|t}$$

Notice that this allows us to compute all the forecasts having estimated parameter models and $h_{t+1|t}$. $h_{t+1|t}$ is contained in the information set at time t and can be computed from all the available observations and using the estimated model for the conditional volatility.

The "true" and unobservable volatility is needed to evaluate the forecasting performances of the competing GARCH models. As the proxy for the volatility we use the realized volatility. Andersen and Bollerslev (1998) suggest that the high frequency data can be used to compute the unobserved volatility measure. Cumulative intraday squared returns provide a reduction in noise and a radical improvement in temporal stability relative to classical measure of volatility based on the squared daily returns.

Following the results from Andersen and Bollerslev (1998) we use a 1-hour squared returns of the index. The proxy for daily volatility $\widehat{\sigma}_t^2$ is defined as

$$\widehat{\sigma}_t^2 = \sum_{i=1}^{10} r_{i,i+1}^2$$

Comparing the forecasting performance of competing models is one of the most important aspects of forecasting process. In the economic literature different evaluation measures has been proposed. The statistical loss functions used in this study are

$$
\begin{aligned}
MSE &= \frac{1}{T}\sum_{s=1}^{T}(\widehat{\sigma}^2_{t+s} - h_{t+s|t})^2 \\
HMSE &= \frac{1}{T}\sum_{s=1}^{T}(\frac{\widehat{\sigma}^2_{t+s} - h_{t+s|t}}{h_{t+s|t}})^2 \\
MAE &= \frac{1}{T}\sum_{s=1}^{T}\left|\widehat{\sigma}^2_{t+s} - h_{t+s|t}\right| \\
QLIKE &= \frac{1}{T}\sum_{s=1}^{T}(\ln h_{t+s|t} + (\widehat{\sigma}^2_{t+s}h^{-1}_{t+s|t})) \\
LL_1 &= \frac{1}{T}\sum_{s=1}^{T}(\ln(\widehat{\sigma}^2_{t+s}) - \ln(h_{t+s|t}))^2 \\
LL_2 &= \frac{1}{T}\sum_{s=1}^{T}\left|\ln(\widehat{\sigma}^2_{t+s}) - \ln(h_{t+s|t})\right| \\
TIC &= \frac{\sqrt{\frac{1}{T}\sum_{s=1}^{T}(\widehat{\sigma}^2_{t+s} - h_{t+s|t})^2}}{\sqrt{\frac{1}{T}\sum_{s=1}^{T}(\widehat{\sigma}^2_{t+s})^2} + \sqrt{\frac{1}{T}\sum_{s=1}^{T}h^2_{t+s|t}}}
\end{aligned}
$$

MSE assumes that the forecasts face the quadratic loss and threats the prediction errors symmetrically in the same way as MAE does although the penalization in case of MSE is heavier. Bollerslev and Ghysels (1996) suggest that the accuracy should be evaluated using a heteroscedasticity adjusted MSE ($HMSE$). In this case, the forecast error is scaled by the actual volatility. The $QLIKE$ loss function, suggested by Bollerslev *et al.* (1994), corresponds to the loss function implied by the Gaussian likelihood.

The two logarithmic loss functions LL_1 and LL_2 proposed by Diebold and Lopez (1996) penalize the forecast errors asymmetrically. TIC (Theil Inequality Coefficient) is a scale invariant measure that always lies between zero and one, where zero indicates a perfect fit.

4.3 Data

We use daily quotation of the IBEX 35 over the period January 1992 - May 2011 to compute daily continuously compounded returns of the index, two months of observations (June and July 2011) are left to use for out-of-sample forecasting.

Figure 1 shows the evolution of the index whereas Figure 2 shows the evolution of returns of IBEX 35 over this period and the volatility computed as the rolling standard deviation of monthly intervals. We observe that the volatility was characterized by the changing behavior displaying periods of lower and higher volatility - the period of the last financial crisis that started in 2007 is marked by higher and more frequent picks in the daily returns, which are reflected in the increased volatility. Similar to the consequences of the dotcom bubble in 2001-2002 when the index lost almost 58% of its value. On the other hand we have period of relatively small volatility, for example 2004-2007, the period of stable economic growth of Spanish economy.

Figure 1. IBEX 35 daily quotations 1992 - 2011

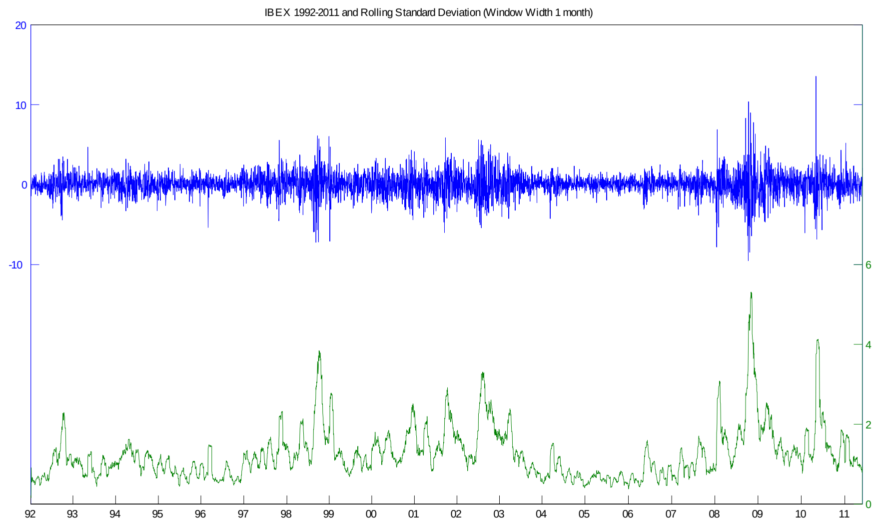


Figure 2. IBEX 35 returns and the rolling standard deviation over 1992-2011.

Table 1 reports summary statistics of the return series. Heteroscedastic and autocorrelation consistent standard errors for mean, standard deviation, skewness and excess kurtosis are computed as in West and Cho (1995) and are presented in parenthesis.

	$Mean$	$St.Dev.$	Max	Min	$Skew$	$ExKurt$	ADF_test
$IBEX$	0.015 (0.020)	1.416 (0.030)	13.59	-9.46	0.031 (0.221)	5.82 (1.544)	-70.24 (0.0000)

Table 1. The main statistics of the data.

The mean is very low and not significantly different from zero, although the series has the maximum returns of 13.59% (May 2010 as the European Union decided to create a bailout fund and help Greece) and minimum of -9.46% (October 2008, after the collapse of Lehman Brothers as the panic spread to others than financial sector).

The skewness is not statistically significant. Significant and high excess kurtosis indicates fat tails in the distribution.

The value of the Augmented Dickey Fuller Test (the critical value of -2.8619 at 5% level of significance) shows that the series of index returns is stationary.

We apply the ARMA filter to remove the deterministic part of the series of index returns. The lowest value of the Schwarz Information Criterion is obtained for $ARMA(1,1)$. Errors from the model will be used for estimation of volatility model and evaluation of the forecasts. The Modified Ljung-Box statistic at lag 20 of 21.6037 with the p-value=0.36239 gives no further evidence of autocorrelation in return series.

The ARCH-LM test of Engle (1982) applied to the series of residuals ε_t obtained from the mean equation is used to detect the heteroscedasticity in the volatility. For the lags 1 and 5 we obtain following values of ARCH-LM test statistic - $ARCH(1) = 195.95$ and $ARCH(5) = 143.40$ with the $p - value = 0.0000$ for both tests. Thus we have the presence of ARCH effects in conditional volatility.

We can further investigate the nature of the ARCH effects by the means of the Sign Bias and Negative Size Bias tests proposed by Engle and Ng (1993).

For the Sign Bias we obtain the value of the t-statistic for the parameter γ_1 of 3.8203, for the Negative Size Bias of -11.33. There is substantial evidence of asymmetric ARCH effects, coming especially from negative returns.

The discussed changes of the index over the history of the index motivate the investigation of the correct model for volatility of the index that will detect the points of possible break and will model the volatility in each subsample as a separate process.

4.4 Empirical Results

In this section we present and discuss the results of empirical estimation of the models. First, we discuss the results of the structural break test in volatility equation, estimation of the volatility models and present the result of volatility forecasting. We also discuss the behavior of DJIA index over the same period of time and discuss the results of applying the structural break test for this market.

4.4.1 Structural break test in volatility

We apply the modified ICSS algorithm to the series of continuously compounded returns of the IBEX 35 over the period January 1992 - May 2011.

Figure 3 shows the index return series and three-standard-deviation bands for each of the regimes defined by the structural break test.

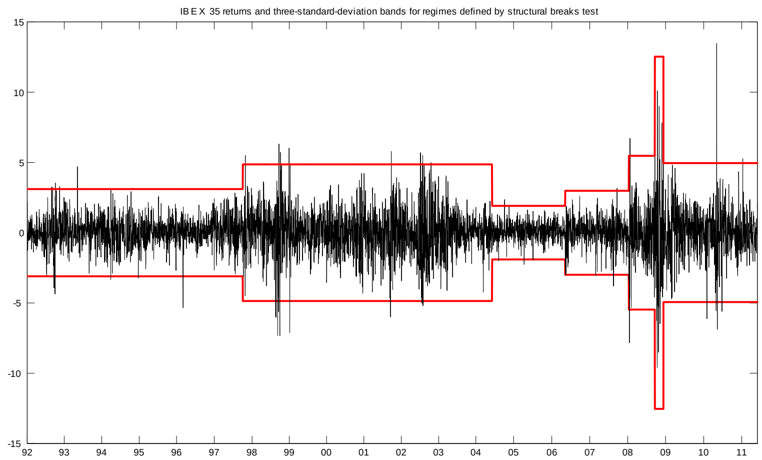


Figure 3. Index returns and regimes defined by modified ICSS algorithm.

The modified ICSS algorithm detects six breaks over the history of $IBEX35$.

Table 2 reports the estimates of the parameters of $QGARCH$ model for full sample of the index returns and each regime.

	ω	α	γ	β	$Q(5)^2$
$Full\_Sample$	0.0380* (0.0085)	0.0902* (0.0087)	−0.0929* (0.0139)	0.8897* (0.0117)	10.499 (0.062)
01.1992 − 09.1997	0.0906* (0.0467)	0.0882* (0.0227)	−0.0542** (0.0312)	0.8280* (0.0546)	3.031 (0.6952)
10.1997 − 05.2004	0.0574* (0.0143)	0.0798* (0.0120)	−0.1124* (0.0237)	0.8969* (0.0139)	10.289 (0.0674)
06.2004 − 05.2006	0.1005* (0.0442)	0.0344* (0.0144)	−0.1172* (0.0350)	0.7135* (0.1091)	2.603 (0.7608)
06.2006 − 12.2007	0.1993* (0.0557)	0.1057 (0.0472)	−0.2902* (0.0946)	0.7002* (0.0838)	2.697 (0.7465)
01.2008 − 08.2008	0.1710 (0.1155)	0.0535** (0.0323)	−0.1913* (0.0901)	0.8890* (0.0510)	3.651 (0.6007)
09.2008 − 11.2008	2.8926 (3.7692)	0.0447 (0.0305)	−0.7198 (0.5179)	0.7692* (0.2289)	4.801 (0.4406)
12.2008 − 05.2011	0.1714* (0.0617)	0.1081* (0.0376)	−0.2721* (0.0854)	0.8232* (0.0438)	10.361 (0.0656)

Table 2. The results of the estimation of the conditional variance equation, in parenthesis the standard error, * significant at 5% level of significance, ** significant at 10% level of significance, $Q(5)^2$ value of the Ljung-Box test statistic for the presence of correlation at lag 5 in the series of squared standardized residuals.

Table 3 reports the level of persistence, unconditional variance implied by the $QGARCH$ model and real sample variance and implied kurtosis for full sample of the index returns and each regime.

	$Persistence$	$Unconditional$ $Variance$	$\mathrm{Re}\,al$ $Variance$
$Full\_Sample$	0.9799	1.8974	2.0095
01.1992 – 09.1997	0.9165	1.0818	1.0692
10.1997 – 05.2004	0.9768	2.4773	2.6247
06.2004 – 05.2006	0.7480	0.3988	0.4010
06.2006 – 12.2007	0.8060	1.0274	0.9914
01.2008 – 08.2008	0.9426	2.9806	3.3043
09.2008 – 11.2008	0.0000	15.5543	17.370
12.2008 – 05.2011	0.9313	2.4965	2.7153

Table 3. Persistence, unconditional variance implied by the estimated model and real sample variance for each regime defined by the modified ICSS algorithm for the returns of the IBEX 35.

The first break in the volatility of $IBEX$ is located in September 1997. This subsample is characterized by the relatively low volatility and low persistence. The index was enjoying a period of stable growth, hitting for the first time the barrier of 5000 points and gaining in 1996 almost 42%, even after losing 28% of its value in 1994.

The second break is located in May 2004. This period of rather high volatility corresponds to several crises the index experienced over this period - in 1997 and 1998 the index was loosing the value as the consequence of the Asian crisis, 1999 - 2000 the index was enjoying the introduction of euro. As the consequence of dotcom bubble the index returned at the end of 2004 to the level of 1997.

The next subsample (June 2004 - May 2006) is a period of very low volatility and low persistence. The Spanish economy was enjoying a healthy growth over this period in the environment of low interest rates and cheap credit.

The fourth regime (June 2006 - December 2007) is a period of slightly higher but still relatively low volatility. In 2007 the subprime crisis already started in the USA and all these negative news contributed to increase in volatility which is than reflected in the next regime that spans from January to August 2008 and finished with the collapse of Lehman Brothers.

The next regime, very short from September to November 2008, is a period of high level of uncertainty due to the problems and then collapse of Lehman Brothers. Panic spread to the markets and the index in just five days lost 21% of its value. Over these just few days the index recorded many of its highest and lowest returns in the history of the index. After losing 9.46% in one day the next day the index gained more than 10% as the G-7 countries decided to help the banks with the liquidity issues. Looking at the estimated parameters we observe that only the β is significantly different from zero which comes from the fact that this subsample spans over only 3 months and due to few observations in this subsample we should consider this subsample as having a very high homoscedastic volatility.

Finally, right now we are in the period of high volatility but much lower as the volatility if the previous regime. In this regime IBEX 35 experienced the highest in the history one-day gain of almost 14% in May 2010 as the European Union decided to create a bailout fund to help the euro economies that are at the brink of default.

There are interesting contrasts to the full-sample parameter estimates when we estimate QGARCH(1,1) models for the sub-samples defined by the structural breaks in unconditional variance.

Persistence as measured by $\alpha + \beta$ falls for each sub-sample relatively to the level of persistence in the full sample model. The decreases in persistence are often significant. The persistence is for example only 0.748 for the period June 2004 - May 2006, which was characterized by the low volatility, comparing with the full sample model.

In the case of the period 09.2008 - 11.2008, where α and γ are not statistically significant we should consider this short-period regime as characterized by the conditional homoscedasticity (at a very high level).

These often sizable decreases in the persistence of the volatility processes relatively to the full-sample estimates are a likely manifestation of the upward biases in persistence that results from failing to account for structural breaks, which is in line with the results of Hillebrand (2005).

With respect to the leverage effect, the estimates for the sub-samples are often larger than those for the full sample, so that the leverage effect often becomes more pronounced for the sub-samples relative to the full sample.

The estimates of ω and the unconditional variance also vary considerably between sub-samples (see Table 3). The third regime (06.2004-05.2006) is characterized by the much lower unconditional variance of 0.3988 comparing to the unconditional variance estimated in the full sample model (1.8974), while the last two regimes, starting in September 2008, are characterized by the much higher unconditional volatility when comparing to the full sample model. In Table 3 we also present the sample variance of each regime and we observe that the unconditional variance implied by the estimated $QGARCH$ models does not differ much from the real sample variance, which in turn confirms that the estimated models correctly capture the dynamic of the underlying process.

Finally, the value of the Ljung-Box test statistic for the presence of correlation at lag 5 in the series of squared standardized residuals shows that there is no remaining autocorrelation in the series of squared standardized residuals and that we correctly model the behavior of conditional volatility.

Overall, the in-sample results show that failure to account for structural breaks in unconditional variance can mask important differences in $QGARCH$ parameters across various periods.

We also checked the presence of the structural break using other asymmetric GARCH model. We apply the model proposed by Glosten, Jagannathan and Runkle (1993) ($GJR-GARCH$), which allows for asymmetric effects of positive and negative shocks on volatility.

In this model the conditional volatility is defined as $\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \gamma I_{t-1}\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$, where I_{t-1} is a dummy variable that takes the value of one when $\varepsilon_{t-1} < 0$ and zero otherwise. The $GJR-GARCH$ specification allows for positive and negative shocks to have different effects on the conditional volatility. When $\gamma > 0$ the negative shocks at time t have the $\alpha + \gamma$ impact on conditional volatility at time $t+1$ while the positive shocks will have an impact of α.

We estimate the $GJR-GARCH$ model for each regime detected by the modified ICSS algorithm and observe similar results as in the case of $QGARCH$ model. The regimes of high and low volatility coincide with those detected by $QGARCH$ model. The persistence is lower for each sample comparing to the full sample model and asymmetric reaction to the negative shocks differ across regimes.

The news impact curves are a useful tool to show how new information is incorporated into volatility. We have computed the NIC for the model estimated for the full sample and for each regime detected by the modified ICSS test.

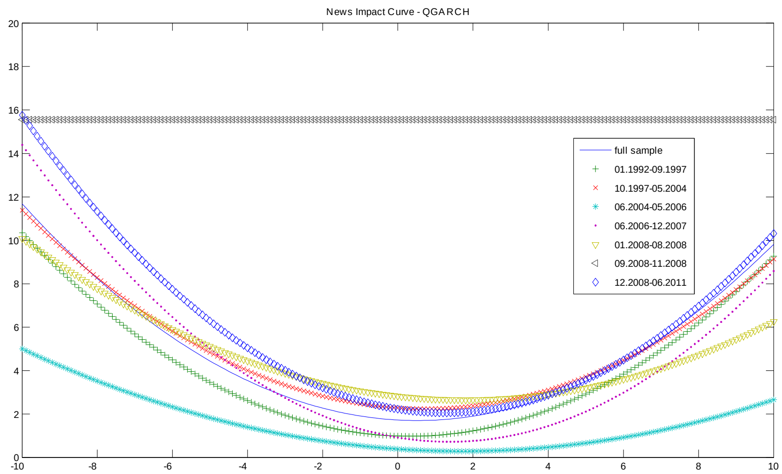


Figure 4. News Impact Curve for the full sample and for each regime detected by the modified ICSS algorithm.

The News Impact Curves show different reaction of the conditional volatility to positive and negative shocks for volatility regimes we have detected previously. Especially the last two regimes - the first 09.2008 - 11.2008 and the last one that starts in December 2008 show the highest reaction to negative shocks. These two regimes cover the period of the last financial crisis that severely impacted the Spanish stock market. On the other hand, the most tranquil regime (06.2004-05.2006) shows the smallest reaction to negative shocks.

4.4.2 Forecasting and forecast evaluation

Before computing the out-of-sample volatility forecasts we check first if there is a possible new break in the out-of-sample data. If there is no new break in the out-of-sample data we assure that the last regime applied to the out-o-sample data is the governing one.

We added the index observations for June and July 2011 to the in-sample data and run once again the modified ICSS algorithm. We obtain exactly the same points of breaks in the volatility and non-additional one. This allows us to apply the last detected regime to the out-of-sample data and evaluate the forecasting power of the model without the breaks and the model with the detected structural breaks in the volatility.

We have computed the out-of-sample volatility forecasts for one week, one month and two months. To evaluate the forecast we use the 1-hour ticks for the IBEX 35 for June - July 2011, which we have not used in estimation of the models. Figure 5 shows the evolution of IBEX 35 over these two months plotted using 1-hour observations.

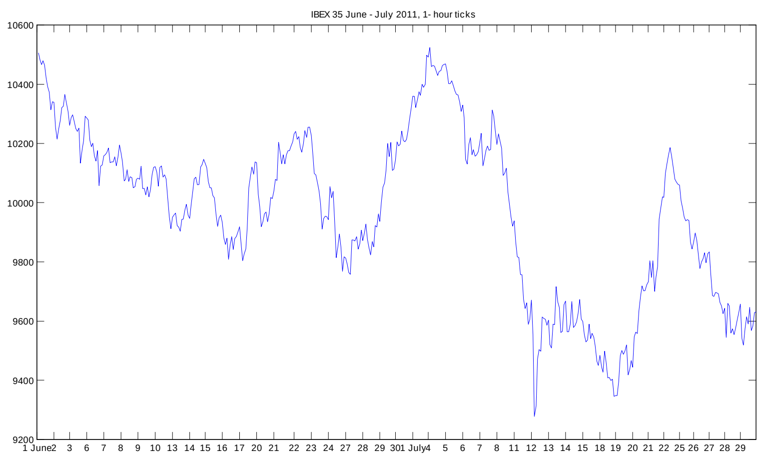


Figure 5. The evolution of IBEX 35, June - July 2011, 1-hour ticks.

To evaluate the out-of-sample forecasting performance of the models we use the loss functions discussed in section 2.3.

The tables below summarize the results of volatility forecasting. Table 4 presents the results for each loss function and Table 5 shows which model (full sample or model with detected breaks) has a lowest value of the loss function and thus has better forecasting performance.

Criterion	MSE	HMSE	MAE	QLIKE	LL1	LL2	TIC
1-week							
Full	0.8213	0.4658	0.8086	1.5705	0.3134	0.5318	0.2817
Break	0.6895	0.1924	0.7415	1.5373	0.3505	0.4862	0.2204
1-month							
Full	1.4697	0.7079	0.8564	1.6825	0.3190	0.4805	0.3350
Break	1.2343	0.2629	0.8798	1.6381	0.3623	0.4672	0.2562
2-months							
Full	6.0491	2.5132	1.3260	2.0159	0.5097	0.5748	0.5074
Break	5.0966	0.9048	1.2900	1.8779	0.4851	0.5537	0.4014

Table 4. Evaluation of the forecasting performance of the full sample and the model with the detected break.

Criterion	MSE	HMSE	MAE	QLIKE	LL1	LL2	TIC
1-week	Break	Break	Break	Break	Full	Break	Break
1-month	Break	Break	Break	Break	Full	Break	Break
2-months	Break	Break	Break	Break	Break	Break	Break

Table 5. Evaluation of the forecasting performance of the full sample and the model with the detected break.

The results of the out-of-sample forecasting show that the model with detected breaks produces better forecasts for each period than the full sample model. For 1-week and 1-month forecasting only one of the loss functions (LL1) prefers the full sample model. Yet, the majority of the loss functions for all the periods of forecasting select the model with the detected structural breaks as the preferred forecasting model.

Assuming that the economic stability will finally be restored in the markets, the volatility of the financial variables should return to the normal levels. It will be interesting to redo the exercise to detect another break after which the model inducing lower volatility should be used for forecasting purposes. Yet, we need to wait to have sufficient number of observations to be able to detect the next break in the process that governs the conditional volatility of the index.

4.4.3 Other financial markets

In the next step of our analysis we consider other financial markets to check if the detected breaks in the Spanish market are unique or if other financial markets also underwent the structural change.

In particular, we consider the most important world stock market index - Dow Jones Industrial Average - the leading index of the 30 American blue chip companies over the similar period January 1992 - May 2011. Computed first in 1896 this index is the oldest and most widely known index of the U.S. stock market.

Figure 6 shows the evolution of index over the period 1992 - 2011 and Figure 7 shows the continuously compounded returns of the index and rolling standard deviation with the window width of one month.

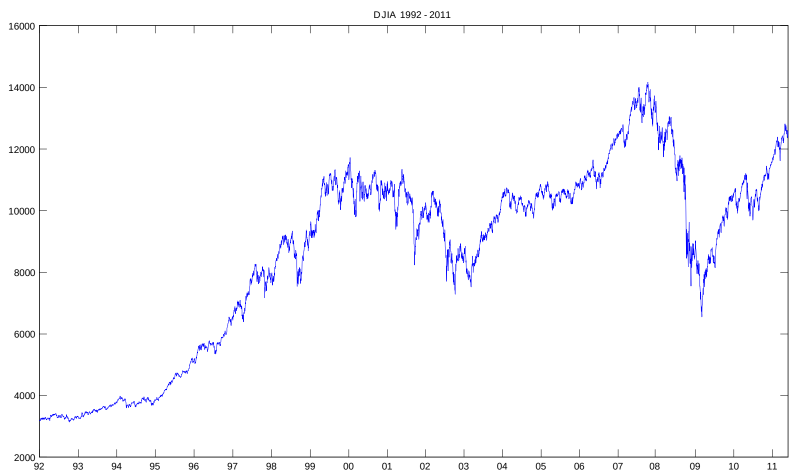


Figure 6. DJIA daily quotations 1992 - 2011.

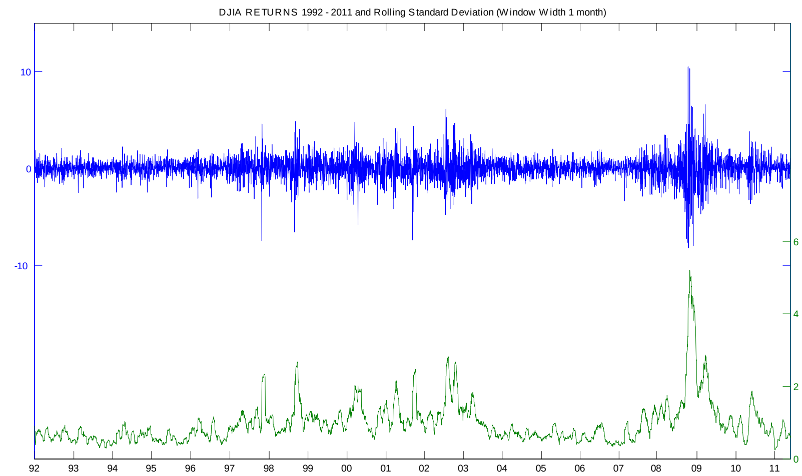


Figure 7. DJIA returns and the rolling standard deviation over 1992-2011.

Looking at these graphs we can see many similarities between IBEX 35 and DJIA - between 1992 - 2000 both indexes were enjoying growth of the index, between 2000 - 2003 both were decreasing although the changes in IBEX 35 were more significant, then both indexes returned to growth - IBEX 35 until 2008 and DJIA until mid 2007, after that both indexes severely suffered due to the last financial crisis, then DJIA started growing again whereas IBEX 35 initially enjoyed growth but then, from 2010 on, started to oscillate around 10,000 points.

Analyzing the rolling standard deviation we also detect many similarities - increased level of volatility around 1997 - 2003, then relatively tranquil period 2004 - 2007 and then return to high volatility period between 2007 - 2010, which is continuing in the case of IBEX 35 and then decreases in the case of DJIA.

Table 6 shows the summary statistics of the data

	$Mean$	$St.Dev.$	Max	Min	$Skew$	$ExKurt$	ADF_test
$DJIA$	0.0281 (0.0156)	1.1258 (0.028)	10.50	-8.20	-0.132 (0.221)	8.66 (1.671)	-22.14 (0.0000)

Table 6. The main statistics of the data.

The mean of the index is very low and almost zero although the series has the maximum returns around 10% and minimum around -8%. Over the period of the crisis stock markets experienced significant changes in the very short period of time.

Dow Jones experienced its highest and lowest peak in just two days. As CNN[2] reports, on Monday 13th of October 2008 the index saw its best one-day point gain ever and best one-day percentage gain since 1933. The advance was fueled by bets that the United States would follow Europe in pouring money directly into banks as part of the $750 billion bailout plan. Yet, as the House of Representatives initially rejected the government's $700 billion bank bailout plan the index lost 8% two days later.

We apply the modified ICSS algorithm to detect the points of the structural change in the volatility of DJIA and detect six points of the break that define seven regimes of the volatility. Figure 8 below shows the index return series and three-standard-deviation bands for each of the regimes defined by the structural break test.

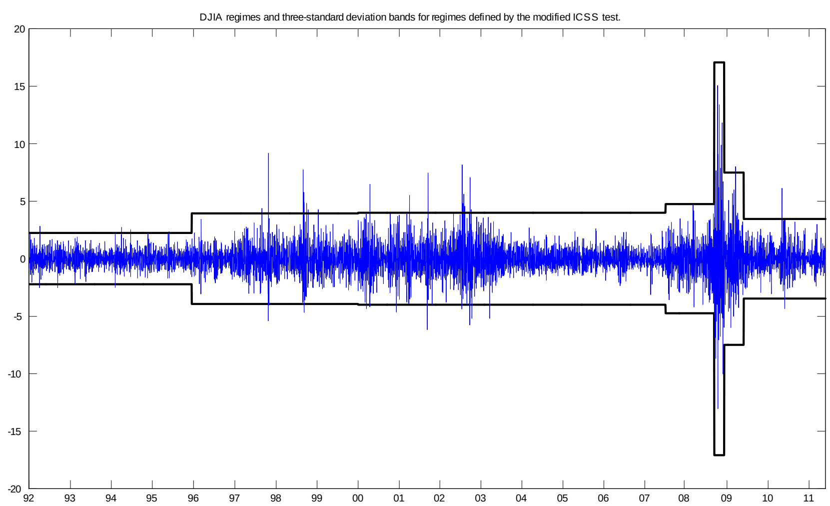


Figure 8. DJIA returns and regimes defined by modified ICSS test.

The modified ICSS algorithm detects seven regimes of different volatility.

The first regime - 01.1992 - 12.1995 is the period of low volatility and stable growth after the 1990 oil price shock. In November 1995 the index closed for the first time above 5,000 level.

The second regime 01.1996 - 12.1999 is characterized by higher level of volatility. Between 1996 - 1997 the index easily grew above 8,000 level in June 1997. However, in October the index

[2]CNNMoney.com, "Another huge Dow loss", October 15, 2008.

suffered losses due to the Asian and in 1998 Russian crisis. Then in May 1999, the index reached the 11,000 mark.

Next regime - 01.2000 - 06.2007 is relatively long regime of slightly higher volatility. Over this regime the index experienced first the dotcom bubble, then the September 11 attacks (the index lost 14.3% in one week but quickly regained the pre-attack level). Over 2002 - 2004 the index suffered and then was recovering from the larger bear market correction after a decade long bull - market. Low interest rates and cheap credit contributed to gains between 2006 - 2007.

The period 07.2007 - 08.2008 is the beginning of the current financial crisis and is marked by even higher volatility. This short regime can be seen as the final high of the cyclical bull, during which (in October 2007) the index closed at the all-time record level.

The following three months (09.2008 - 11.2008) are the months of the highest volatility, not seen before. The index suffered from the Lehman Brothers collapse in September 2008 and the record high oil prices. A series of bailout packages did not prevent further significant losses and contributed to extreme volatility in the market.

The next regime - 12.2008 - 05.2009 is a regime of still high volatility but much lower comparing to the level of volatility in the previous regime. In March 2009 the index lost almost 20% of its value in just two weeks but then started recovering mostly due to the optimism that the policies implemented by the government and FED will work out.

Finally, the last regime that started in June 2009 is already characterized by much lower level of volatility. The index recovered as the U.S. companies delivered healthy results in 2010 and the optimism spread to the market.

Table 7 reports the estimates of the parameters of $QGARCH$ model for full sample of the index returns and each regime.

	ω	α	γ	β	$Q(5)^2$
$Full\_Sample$	0.0227* (0.0051)	0.0804* (0.0085)	−0.0850* (0.0137)	0.8975* (0.0119)	7.667 (0.1755)
01.1992 − 12.1995	0.0763* (0.0313)	0.0526* (0.0250)	−0.0846* (0.0324)	0.7423* (0.0994)	2.641 (0.7551)
01.1996 − 12.1999	0.0800* (0.0277)	0.0761* (0.0237)	−0.1558* (0.0479)	0.8502* (0.0373)	4.606 (0.4657)
01.2000 − 06.2007	0.0218* (0.0033)	0.0789* (0.0145)	−0.0824* (0.0130)	0.8975* (0.0152)	4.633 (0.4623)
07.2007 − 08.2008	0.0759* (0.0367)	0.0289 (0.0229)	−0.0931* (0.0497)	0.9217* (0.0353)	6.951 (0.2242)
09.2008 − 11.2008	2.6380 (4.5055)	0.0351 (0.0709)	−0.6092 (0.4852)	0.8046* (0.2109)	9.544 (0.0892)
12.2008 − 05.2009	0.1355 (0.1784)	0.0221 (0.0286)	−0.1091* (0.0365)	0.9405* (0.0202)	6.798 (0.2360)
06.2009 − 05.2011	0.0698* (0.0208)	0.1000* (0.0232)	−0.1668* (0.0372)	0.8163* (0.0338)	8.733 (0.1201)

Table 7. The results of the estimation of the conditional variance equation, in parenthesis the standard error, * significant at 5% level of significance, ** significant at 10% level of significance, $Q(5)^2$ value of the Ljung-Box test statistic for the presence of correlation at lag 5 in the series of squared standardized residuals.

Table 8 reports the level of persistence, unconditional variance implied by the $QGARCH$ model and real sample variance for full sample of the index returns and each regime.

	$Persistence$	$Unconditional$ $Variance$	Real $Variance$
$Full\_Sample$	0.9780	1.0349	1.2640
01.1992 − 12.1995	0.7960	0.3722	0.3714
01.1996 − 12.1999	0.9263	1.0867	1.1357
01.2000 − 06.2007	0.9765	0.9308	1.1542
07.2007 − 08.2008	0.9506	1.5384	1.5502
09.2008 − 11.2008	0.0000	16.4717	17.6436
12.2008 − 05.2009	0.9627	3.6368	4.2302
06.2009 − 05.2011	0.9164	0.8353	0.8943

Table 8. Persistence, unconditional variance implied by the estimated model and real sample variance for each regime defined by the modified ICSS algorithm for the returns of the DJIA.

We can draw the similar conclusions as in the case of IBEX 35.

Persistence falls for each sub-sample relatively to the level of persistence in the full sample model. The decreases in persistence are often significant. The persistence is for example only 0.796 for the period June 2004 - May 2006, which was characterized by low volatility, comparing with the full sample model. This again is in line with results of Hillebrand (2005) among others.

In the case of the period 09.2008 - 11.2008, where α and γ are not statistically significant we should consider this short-period regime as characterized by the conditional homoscedasticity (at a very high level).

The estimates of ω and the unconditional variance also vary considerably between subsamples (see Table 8). The implied unconditional variance is similar to observed real sample variance which in turns confirms that we correctly model the underlying data generating process.

Interestingly, the timing of some of the regimes is similar between indexes, which would than suggest that both markets were influenced by the same events. The IBEX 35 regime 01.2008 - 08.2008 coincides with the DJIA regime 07.2007 - 08.2008, where in both cases we observe an increased level of volatility in the markets. This definitely was a consequence of the sub-prime crisis and the uncertainty spread to the markets. Then the next regime 09.2008-11.2008 occurs in both markets, as after the collapse of Lehman Brothers the panic spread to all the markets and the world stock indexes experienced the level of volatility not seen before. The next regime of still high but already much lower volatility still is present in the IBEX 35 as the Spanish economy was severely hit by the recession whereas DJIA entered in June 2009 in the period of much lower volatility and stable growth spurred by the optimism of investors.

4.5 Conclusions

This paper investigates the structural break in the volatility of IBEX 35. We investigate the volatility of the index over the period 1992 - 2011.

Applying the Quadratic GARCH model and the modified ICSS algorithm we detect several structural breaks in the volatility of the index. The detected regimes differ in the level of volatility, persistence of the shocks and the asymmetry in the reaction of the conditional volatility to negative news.

We observe three periods of low volatility - 01.1992 - 09.1997, 06.2004 - 05.2006, 06.2006 - 12.2007 and four regimes of high volatility - 10.1997 - 05.2004, 01.2008 - 08.2008, 09.2008 - 11.2008 and 12.2008 - 05.2011. The last three regimes cover the period of the last financial crisis, during which the Spanish economy and the Spanish stock market was severely depressed.

We also observe, as documented in academic literature, that subsamples detected by the structural break test are characterized by lower persistence and higher asymmetry then the full sample model that does not consider different regimes of volatility.

We check the forecasting power of the models for full sample and the model with the detected breaks. The break model shows a better forecasting performance than the full sample model.

We also consider the most important US stock index - Dow Jones Industrial Average and observe that this index also underwent several structural changes.

Interestingly some of the detected regimes coincide in timing in both markets, especially the last financial crisis is reflected in the volatility of both indexes. The very short period of three months after the collapse of Lehman Brothers in September 2008 is present in both markets. From June 2009 on the lower level of volatility returned to the DJIA index where the IBEX 35 is still governed by the rather high level of volatility.

It will be of great importance to redo this exercise once the stock markets return to the normal levels of volatility and we collect enough data to detect a new break point after which the volatility could be considered as staying on the normal long run levels. It can take time depending on how quickly the confidence in the markets will be restored and the economies return to the path of growth. Then, in the future will should be careful when analyzing the volatility and not to forget that during time of this financial crisis stock markets experienced a structural break in the volatility.

Bibliography

[1] Aggarwal, R., C. Inclan and R.Leal, “Volatility in Emerging Stock Markets”, *Journal of Financial and Quantitative Analysis*, 34 (1999), 33-55.

[2] Andrews, D.W.K., “Tests for Parameter Instability and Structural Change with Unknown Change Point”, *Econometrica*, 61 (1993), 821-856.

[3] Andrews, D.W.K. and W. Ploberger, “Optimal Tests when a Nuisance Parameter is Present Only Under the Alternative”, *Econometrica*, 62 (1994), 1383-1414.

[4] Andreo, E. and E. Ghysels, "Detecting multiple breaks in financial market volatility dynamics", *Journal of Applied Econometrics*, 17 (2002), 579-600.

[5] Becketti, S. and G.Sellon, "Has Financial market volatility increased?", Federal Reserve Bank of Kansas City, Economic Review 2 (1989), 17-30.

[6] Black, F., "Studies on stock price volatility changes", *Proceedings of the American Statistical Association, Business and Economic Statistics Section* (1976), 177-81.

[7] Bollerslev, T., "Generalized autoregressive heteroscedasticity", *Journal of Econometrics,* 31 (1986), 307-327.

[8] Bollerslev, T. and J. Wooldridge (1992), "Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time Varying Covariance", *Econometric Reviews*, 11, 143-172.

[9] Capiello, L., Engle, R.F. and K. Sheppard, "Asymmetric Dynamics in the Correlations of Global Equity and Bond Returns", *Journal of Financial Econometrics*, 4 (2006), 537-572.

[10] Cuñado, J.C., Gomez Biscarri, J. and F. Perez de Gracia Hidalgo, "Structural changes in volatility and stock market development: Evidence from Spain", *Journal of Banking & Finance*, 28 (2004), 1745-1773.

[11] Cuñado, J.C., Gomez Biscarri, J. and F. Perez de Gracia Hidalgo, "Changes in the Dynamic Behavior of Emerging Market Volatility: Revisiting the Effects of Financial Liberalization", *Emerging Markets Review*, 7 (2006), 261-278.

[12] de Pooter, M. and D. van Dijk, "Testing for changes in volatility in heteroscedastic series: a further examination", Econometric Institute Research Report 2004-38/A, Erasmus School of Economics, Rotterdam.

[13] Engle, R.F., "Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation", *Econometrica,* 20 (1982), 987-1007.

[14] Engle, R.F. and V.K. Ng, "Measuring and Testing the Impact of News on Volatility", *Journal of Finance,* 48 (1993), 1749-1778.

[15] Franses. P. H. and D. van Dijk, "Forecasting stock market volatility using (nonlinear) GARCH models", *Journal of Forecasting*, 15 (1996), 229-235.

[16] Gil-Alana, L., Cunado, J. and F. Perez de Gracia, "Stochastic volatility in the Spanish stock market: a long memory model with a structural break", *European Journal of Finance*, 14 (2008), 23-31.

[17] Glosten, R.T., Jagannathan, R. and D. Runkle, "On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks", *Journal of Finance,* 48 (1993), 1779-1801.

[18] Hansen, B.E., "Approximate Asymptotic P Values for Structural-Change Tests", *Journal of Business and Economic Statistics*, 15 (1997), 60-67.

[19] Hillebrand E., "Neglecting parameter changes in GARCH models", *Journal of Econometrics*, 129 (2005), 121-138.

[20] IMF, "World Economic Outlook Crisis and Recovery", *IMF*, April 2009.

[21] Inclan, C. and G.C. Tiao, "Use of Cumulative Sums of Squares for Retrospective Detection of Changes of Variance", *Journal of American Statistical Association*, 89 (1994), 913-923.

[22] Kim, J.W., Seo B. and D. Leatham, "Structural Change in Stock Price Volatility of Asian Financial Markets", *Journal of Economic Research,* 15 (2010), 1-27.

[23] Lamoureux, Ch.G. and W.D. Lastrapes, "Persistence in Variance, Structural Change and the GARCH model", *Journal of Business and Economic Statistics,* 8 (1990), 225-234.

[24] Mikosch, T. and C. Stărică, "Nonstationarities in Financial Time Series, the Long Range Dependence, and the IGARCh Effects", *Review of Economics and Statistics*, 86 (2004), 378-390.

[25] Newey, W.K., and W.D. West, "Automatic Lag Selection in Covariance Matrix Estimation", *Review of Economic Studies,* 61 (1994), 631-654.

[26] Pagan, A. and G.W. Schwert, "Alternative Models for Conditional Stock Volatility", *Journal of Econometrics,* 45 (1990), 267-290.

[27] Rapach, E. and J. Strauss,"Structural Breaks and GARCH Models of Exchange Rate Volatility", Journal of Applied Econometrics, 23 (2008), 65-90.

[28] Rapach, E., Strauss, J. and M. Wohar, "Forecasting Stock Return Volatility in the Presence of Structural Breaks", in Forecasting in the Presence of Structural Breaks and Model Uncertainty, Frontiers of Economics and Globalization, 3 (2008) (May 2008), 381-416.

[29] Sansó, A., Arragó, V. and J. Carrion-Silvestre, "Testing for change in the unconditional variance of financial time series", *Revista de Economía Financiera*, 4 (2004), 32-53.

[30] Sentana, E., "Quadratic ARCH Models", *Review of Economic Studies*, 62 (1995), 639-661.

[31] Smith, D.R., "Testing for Structural Breaks in GARCH Models", *Applied Financial Economics*, 18 (2008), 845-862.

[32] West, K.D. and D. Cho, "The predictive ability of several models of exchange rate volatility", *Journal of Econometrics*, 69 (1995), 367-391.