Violations of Betweenness and Choice Shifts in Groups

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Abstract: In decision theory, the betweenness axiom postulates that a decision maker who chooses an alternative *A* over another alternative *B* must also choose any probability mixture of *A* and *B* over *B* itself and can never choose a probability mixture of *A* and *B* over *A* itself. The betweenness axiom is a weaker version of the independence axiom of expected utility theory. Numerous empirical studies documented systematic violations of the betweenness axiom in revealed individual choice under uncertainty. This paper shows that these systematic violations can be linked to another behavioral regularity—choice shifts in a group decision making. Choice shifts are observed if an individual faces the same decision problem but makes a different choice when deciding alone and in a group.

JEL Classification Codes: D81, D72

Keywords: Betweenness; Choice Shift; Quasi-Concave Preference; Quasi-Convex Preference; Risky Shift; Cautious Shift; Expected Utility Theory

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1. Introduction

Systematic violations of expected utility theory, such as the Allais (1953) paradox and the common ratio effect (Kahneman and Tversky, 1979), motivated the development of generalized non-expected utility models (*e.g.*, Starmer, 2000). A typical approach was to weaken the axioms of expected utility theory, most notably the independence axiom. One of the weaker versions of the independence axiom is the betweenness axiom (*e.g.*, Dekel, 1986), which can be summarized as follows. A decision maker who chooses an alternative *A* over another alternative *B* must also choose any probability mixture of *A* and *B* over *B* itself and can never choose a probability mixture of *A* and *B* over *A* itself. In other words, if *A* is revealed preferred to *B*, then *A* must be also revealed preferred to any probability mixture of *A* and *B* and the mixture must be revealed preferred to *B*. Thus, in terms of revealed preferences, a probability mixture of two choice alternatives is in between the alternatives themselves, which explains the name of the axiom.

The betweenness axiom can be violated in two ways. A decision maker, who chooses a probability mixture of *A* and *B* over alternative *A* as well as over alternative *B*, reveals a quasi-concave preference or a preference for randomization. A decision maker, who chooses alternative *A* as well as alternative *B* over a probability mixture of *A* and *B*, reveals a quasi-convex preference or an aversion to randomization. Early experimental tests of the betweenness axiom found both types of violation. Systematic quasi-concave preferences were documented in Becker, DeGroot and Marschak (1963), Chew and

Waller (1986), Prelec (1990) and Blavatskyy (2013a, p.63). Systematic quasi-convex preferences were documented in Conlisk (1987) and Gigliotti and Sopher (1993).

Camerer (1989) found that subjects tend to reveal quasi-concave preferences in the proximity of the horizontal edge of the Marschak-Machina probability triangle (Machina, 1982) and quasi-convex preferences in the proximity of the hypotenuse of the probability triangle. Note that lotteries located near the hypotenuse of the probability triangle are relatively risky compared to lotteries located on the same indifference curve but near the horizontal edge of the probability triangle. Camerer and Ho (1994) and Bernasconi (1994) also provide strong experimental evidence of a “squiggle” pattern of betweenness violations (a quasi-concave preference for a relatively safe probability mixture and a quasi-convex preference for, or rather aversion to, a relatively risky probability mixture).

In this paper we link systematic violations of the betweenness axiom in revealed individual choice under uncertainty to another behavioral regularity—choice shifts in a group decision making[3] Choice shifts are observed if an individual faces the same decision problem but makes a different choice when deciding alone and in a group (*e.g.*, Davis et al. 1992). There are two types of choice shifts. An individual exhibits a risky shift when she chooses a safer alternative when deciding alone but votes for a riskier alternative in a group.[4] An individual exhibits a cautious shift when she chooses a riskier alternative when deciding alone but votes for a safer alternative in a group.

[3] Despite the fact that group decision-making is ubiquitous in social and economic life, economists have been a long time silent on this subject. It can be related to the literature on household behavior that attempts to model households as collective decision units (*cf*. Bourguignon and Chiappori, 1992).

[4] A traditional explanation for a risky shift is the "diffusion of responsibility" when decisions are taken within a group (*e.g*., Wallach *et al* (1962, 1964)). Specifically, an individual choosing a relatively risky option may experience an *ex post* regret/guilt and this feeling is diluted within group-decision making.

Baker et al. (2008) and Shupp and Williams (2008) found evidence of risky shifts for relatively safe lotteries and cautious shifts for relatively risky lotteries. Masclet et al. (2009) also documented cautious shifts for relatively risky lotteries. We shall demonstrate that such empirical evidence is consistent with empirical evidence on violations of the betweenness axiom and both behavioral regularities might be the two sides of the same coin.

Eliaz et al. (2006) already linked choice shifts in groups to another well-known behavioral regularity in individual decision making—the Allais paradox (Allais, 1953). Eliaz et al. (2006, p.1322, footnote 4) note that preferences that are consistent with choice shifts must violate the betweenness axiom. Eliaz et al. (2006) consider one prominent family of such preferences—those represented by rank-dependent utility with a concave probability weighting function (e.g., Abdellaoui, 2002). In this framework, an individual exhibits the Allais paradox if and only if she reveals a specific pattern of choice shifts in groups (Theorem 1 in Eliaz et al., 2006).

In this paper we show that an individual exhibits choice shifts in group decision making if and only if she violates the betweenness axiom, that is, as already mentioned above, a weaker version of the independence axiom. Our result generalizes the result of Eliaz et al. (2006) since we do not restrict our analysis to a specific class of preferences. If preferences are not represented by rank-dependent utility, Theorem 1 in Eliaz et al. (2006) does not apply, *i.e*. an individual, who exhibits the Allais paradox, may not necessarily violate the betweenness axiom (*e.g*., Chew, 1983) and therefore she may exhibit no choice shift. For example, if individual preferences are captured by a disappointment aversion theory, a decision maker may exhibit the Allais paradox without violating the betweenness axiom. Thus, such an individual exhibits no choice shifts.

Unlike Eliaz et al. (2006), we do not assume rank-dependent utility representation of preference in this paper. Yet, our results are consistent with a rank-dependent utility representation with an inverse S-shaped probability weighting function. It is worthwhile to note that an inverse S-shaped probability weighting function is strongly supported by empirical evidence, *e.g.*, Wu and Gonzalez (1996), Wakker (2010). In contrast, Eliaz et al. (2006) considered only rank-dependent utility with a concave probability weighting function. Such an assumption about the probability weighting function is arguably less descriptively accurate (see, however, Blavatskyy (2013)).

The remainder of the paper is organized as follows. Section 2 defines the first behavioral regularity—violations of the betweenness axiom in individual choice under uncertainty. Section 3 defines the second behavioral regularity—choice shifts in a group decision making. Section 4 presents our main result linking the two regularities. Section 5 concludes with a general discussion.

2. Violations of the Betweenness Axiom

Let $p[\boldsymbol{s}]+(1-p)[\boldsymbol{r}]$ denote a probability mixture of a relatively safer lottery $\boldsymbol{s}$ and a relatively riskier lottery $\boldsymbol{r}$ for some mixing probability $p \in (0,1)$. The literature documenting violations of the betweenness axiom can then be summarized as follows. People tend to choose a mixture $p[\boldsymbol{s}]+(1-p)[\boldsymbol{r}]$ over lottery $\boldsymbol{s}$ as well as lottery $\boldsymbol{r}$ when probability p is close to one. People tend to choose lottery $\boldsymbol{s}$ as well as lottery $\boldsymbol{r}$ over a mixture $p[\boldsymbol{s}]+(1-p)[\boldsymbol{r}]$ when probability p is close to zero. This pattern can be generated by several prominent non-expected utility theories such as, for example, rank-dependent utility (Quiggin, 1981) and cumulative prospect theory (Tversky and Kahneman, 1992) with inverse S-shaped probability weighting function (*cf*. Figure 1). Blavatskyy (2006) shows that betweenness violations can be an artifact of random errors (even when an individual maximizes expected utility).

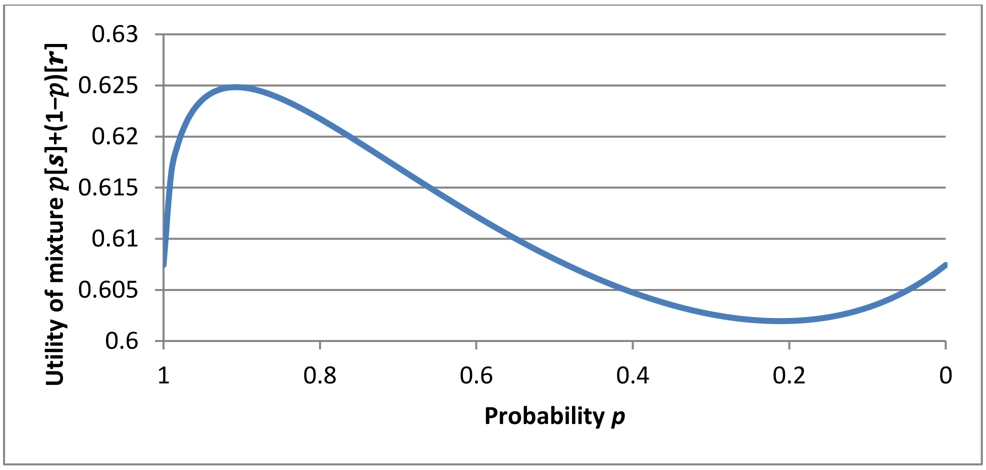

Figure 1: Utility of mixture $p[\boldsymbol{s}]+(1-p)[\boldsymbol{r}]$ as a function of probability p according to rank-dependent utility theory with probability weighting function $w(p)=p^{\gamma}/(p^{\gamma}+(1-p)^{\gamma})^{1/\gamma}$, $\gamma=0.61$ (a median estimate in Tversky and Kahneman, 1992). Lottery $\boldsymbol{r}$ yields outcome x, $u(x)=1$, with probability 0.8 and zero otherwise ($u(0)=0$). Lottery $\boldsymbol{s}$ yields outcome y, u(y)=w(0.8)/w(0.95)=0.766, with probability 0.95 and zero otherwise.

We consider a "squiggle" pattern of betweenness violations: there exist a probability $\bar{p} \in [0, 1]$ such that an individual has strictly quasi-concave preferences over probability mixtures $p[\boldsymbol{s}] + (1-p)[\boldsymbol{r}]$ when $p > \bar{p}$ and strictly quasi-convex preferences—when $p < \bar{p}$. This is formalized in the following Assumption 1 (as usual, preference relation $\boldsymbol{s} \succcurlyeq \boldsymbol{r}$ denotes that lottery $\boldsymbol{s}$ is chosen over lottery $\boldsymbol{r}$, the symmetric part of $\succcurlyeq$ is denoted by $\sim$ and the asymmetric part of $\succcurlyeq$ is denoted by $\succ$). In the following and through the paper we assume rational and continuous preferences.

Assumption 1 *An individual violates betweenness if there are two lotteries* **s** *and* **r** *as well as a probability* $\bar{p} \in [0, 1]$ *such that* $\boldsymbol{s} \sim \boldsymbol{r} \sim \bar{p}[\boldsymbol{s}] + (1-\bar{p})[\boldsymbol{r}]$ *and*

a) the individual has strictly quasi-concave preferences over mixtures $p[\boldsymbol{s}] + (1-p)[\boldsymbol{r}]$ *when* $p \geq \bar{p}$*, i.e., for all* $\bar{p} \leq \alpha < \beta < \gamma \leq 1$*, mixture* $\beta[\boldsymbol{s}] + (1-\beta)[\boldsymbol{r}]$ *is strictly preferred over the worst of* $\alpha[\boldsymbol{s}] + (1-\alpha)[\boldsymbol{r}]$ *and* $\gamma[\boldsymbol{s}] + (1-\gamma)[\boldsymbol{r}]$;

b) the individual has strictly quasi-convex preferences over mixtures $p[\boldsymbol{s}] + (1-p)[\boldsymbol{r}]$ *when* $p \leq \bar{p}$*, i.e., for all for* $0 \leq \alpha < \beta < \gamma \leq \bar{p}$*, the best of* $\alpha[\boldsymbol{s}] + (1-\alpha)[\boldsymbol{r}]$ *and* $\gamma[\boldsymbol{s}] + (1-\gamma)[\boldsymbol{r}]$ *is strictly preferred over mixture* $\beta[\boldsymbol{s}] + (1-\beta)[\boldsymbol{r}]$

Probability $\bar{p}$ denotes the *crossing point*: an individual chooses lottery $\boldsymbol{r}$ as well as lottery $\boldsymbol{s}$ over a mixture $p[\boldsymbol{s}] + (1-p)[\boldsymbol{r}]$ when p is below the threshold $\bar{p}$ and she chooses the mixture over lottery $\boldsymbol{r}$ as well as lottery $\boldsymbol{s}$ when p is above the threshold $\bar{p}$. Eliaz et al. (2006) considered only quasi-convex preferences, *i.e*. they allowed for betweenness violations when $\bar{p} = 1$. We generalize the analysis of Eliaz et al. (2006) and allow for the violations of the betweenness axiom when $\bar{p} \leq 1$.

3. Choice Shifts

Stoner (1961) was the first study to document risky shifts—a group tends to choose a riskier option compared to the individual choices of the group's members in isolation. Subsequently, several studies also documented cautious shifts—a group tends to choose a safer option compared to the individual choices of the group's members in isolation (*e.g.*, Nordhoy (1962), Stoner (1968)). Bateman and Munro (2005) found that groups make consistently more risk averse choices than individual members of the group and mentioned anecdotal evidence from their subjects that fear of "recrimination" made the subjects reluctant to choose risky options within a group. Rockenbach *et al*. (2007) found that groups were significantly less risk taking than independent individuals (*i.e*. not members of the group). Shupp and Williams (2008) found that groups were significantly more risk averse than individual members of the group as well as independent individuals in the higher-risk lotteries. Shupp and Williams (2008) found that groups were less risk averse than independent individuals (but not individual members of the group) in the lower-risk lotteries.

On the other hand, Bone et al. (1999) found that groups reveal the common ratio effect to a similar extent as individuals. Harrison *et al*. (2012) found no evidence that subjects systematically reveal different risk attitudes in a group (with no prior knowledge about the risk preferences of others) compared to individual decisions. Kerr *et al*. (1996) provide an extensive review of the literature in social psychology on differences in groups and individual decision making. To sum up, no clear consensus has been reached in the literature: although group shifts appear to be a consistent and robust phenomenon, both risky and cautious shifts have been identified in the literature (*cf*. Davis *et al*. (1992) for a discussion).

When voting between a safe lottery $\boldsymbol{s}$ and a risky lottery $\boldsymbol{r}$ in a group an individual *de facto* chooses between two compound lotteries $\boldsymbol{r}^*$and $\boldsymbol{s}^*$:

$$\boldsymbol{r}^* = a\,[\boldsymbol{r}] + (1-a)\big[b[\boldsymbol{s}] + (1-b)[\boldsymbol{r}]\big]$$

$$\boldsymbol{s}^* = a\,[\boldsymbol{s}] + (1-a)\big[b[\boldsymbol{s}] + (1-b)[\boldsymbol{r}]\big],$$

where $a \in (0,1)$ is the probability that an individual is pivotal (an outcome depends on her decision) and $b \in [0,1]$ is the probability that the group choses $\boldsymbol{s}$ when an individual is not pivotal. This framework allows us to capture various possible voting rules. For instance, when a unanimous vote is required to replace ***s*** with ***r*** then *b*=1; when a unanimous vote is required to replace ***r*** with ***s*** then *b*=0.

Definition 1 *A decision maker reveals a risky shift when* $\boldsymbol{s} \sim \boldsymbol{r}$ *and* $\boldsymbol{r}^* \succ \boldsymbol{s}^*$*; a decision maker reveals a cautious shift when* $\boldsymbol{s} \sim \boldsymbol{r}$ *and* $\boldsymbol{s}^* \succ \boldsymbol{r}^*$***.***

Lotteries $\mathbf{r}^*$ and $\mathbf{s}^*$ can be simplifies as follows:

$$\boldsymbol{r}^* = p_r[\boldsymbol{s}] + (1-p_r)[\boldsymbol{r}]$$

$$\boldsymbol{s}^* = p_s[\boldsymbol{s}] + (1-p_s)[\boldsymbol{r}]$$

where $p_r = (1-a)b$ and $p_s = (1-a)b + a$. Thus, both lotteries $\mathbf{r}^*$ and $\mathbf{s}^*$ are probability mixtures of lotteries $\boldsymbol{r}$ and $\boldsymbol{s}$. An individual, who respects the betweenness axiom, chooses between probability mixtures $\mathbf{r}^*$ and $\mathbf{s}^*$ in the same manner as she chooses between lotteries $\boldsymbol{r}$ and $\boldsymbol{s}$. Yet, a revealed preference between probability mixtures $\mathbf{r}^*$ and $\mathbf{s}^*$ may differ from a revealed preference between lotteries $\boldsymbol{r}$ and $\boldsymbol{s}$ when an individual violated the betweenness axiom. Thus, choice shifts are linked to the violations of betweenness and the next theorem formally characterizes this relationship.

4. Main Result

Theorem 1 *The following two statements are equivalent:*

1) *A decision maker violates the betweenness axiom;*
2) *There exist* $b_1(a), b_2(a) \in [0,1]$, $b_2(a) > b_1(a)$ *such that for all* $a \in (0,1)$
   a) *an individual exhibits a risky shift when either*
      i. $a \leq \bar{p}$ and $b < b_1(a)$ *or*
      ii. $a \leq 1 - \bar{p}$ and $b > b_2(a)$;
   b) *an individual exhibits a cautious shift when either*
      i. $\bar{p} \geq a > 1 - \bar{p}$ and $b > b_1(a)$, *or*
      ii. $\bar{p} < a \leq 1 - \bar{p}$ and $b < b_2(a)$;
      iii. $a \leq \min\{\bar{p}, \; 1 - \bar{p}\}$ *and* $b_1(a) < b < b_2(a)$
      iv. $a > \max\{\bar{p}, \; 1 - \bar{p}\}$

Proof is presented in the Appendix.

Whether an individual exhibits a risky or a cautious shift depends on her expectation to be pivotal (probability *a*) and her expectation how the group chooses when she is not pivotal (probability *b*). For example, when an individual has high expectations to be pivotal, *i.e*. when $a > max\{\bar{p}, 1 - \bar{p}\}$, then she always reveals a cautious shift. Also, when an individual is not quite sure how the group will chose when she is not pivotal, *i.e*. when $b_1 < b < b_2$, then she always reveals a cautious shift. On the other hand, when an individual has low expectations to be pivotal, she may reveal a risky shift or a cautious shift (depending on her expectations how the group chooses when she is not pivotal).

Eliaz et al. (2006) considered only quasi-convex preferences, *i.e*., when the crossing probability $\bar{p} = 1$. In this case, probability *a* always satisfies condition $a < \bar{p} = 1$ and it can never satisfy condition $a < 1 - \bar{p} = 0$. Theorem 1 then implies that an individual exhibits a risky shift when $b < b_1$ and a cautious shift when $b > b_1$, for some threshold $b_1 \in [0,1]$. This corresponds to the main result of Eliaz et al. (2006, p. 1324, Theorem 1).

For an individual with quasi-concave preferences the crossing probability is $\bar{p} = 0$. In this case, probability a always satisfies condition $a < 1 - \bar{p} = 1$ and it can never satisfy condition $a < \bar{p} = 0$. Theorem 1 then implies that an individual exhibits a risky shift when $b > b_2$ and a cautious shift when $b < b_2$, for some threshold $b_2 \in [0,1]$.

Consider an example when a unanimity vote is required to change the status quo. In this case probability a is simply the probability that all other group members chose the same lottery (otherwise there is already a disagreement among other group members and a unanimity vote cannot be achieved no matter what our individual decides). It is easy to see that $b = 1$ when the status quo is a safer lottery $\boldsymbol{s}$ and $b = 0$ when the status quo is a riskier lottery $\boldsymbol{r}$. Eliaz et al. (2006, p. 1326) show that a choice shift to the status quo occurs for all values of probability a (under the assumption of quasi-convex preferences). In our setup, the decision to revert to the status quo depends on the value of probability a.

Consider the case when the status quo is lottery $\boldsymbol{s}$ ($b = 1$)**.** According to Theorem 1, an individual reveals a risky shift (away from the status quo) if probability a is low enough, *i.e.* $a < 1 - \bar{p}$. An individual, who is indifferent between lotteries $\boldsymbol{r}$ and $\boldsymbol{s}$ when choosing alone, may prefer to vote for a risky lottery $\boldsymbol{r}$ in a group if she believes that the chance of her being pivotal is rather low. The intuition is similar to the pseudo-endowment effect (Prelec, 1990).

Now consider the case when the status quo is lottery $\boldsymbol{r}$ ($b = 0$). According to Theorem 1, an individual reveals a cautious shift (away from the status quo) if probability a is high enough, *i.e.* $a > \bar{p}$. Intuitively, when $b = 0$ and an individual decides to vote for a safer lottery in a group, she effectively faces a compound lottery $\boldsymbol{s}^* = a\,[\boldsymbol{s}] + (1 - a)[\boldsymbol{r}]$. For high values of probability a this compound lottery lies in the neighborhood of $\boldsymbol{s}$. Since an individual has quasi-concave preferences in the neighborhood of $\boldsymbol{s}$, she prefers compound lottery $\boldsymbol{s}^* = a\,[\boldsymbol{s}] + (1 - a)[\boldsymbol{r}]$ over $\boldsymbol{r}^* = \boldsymbol{r}$. In other words, a cautious shift (away from the status quo) is observed.

5. Conclusion

This paper offers a new decision-theoretical explanation why decisions made in a group may differ from decisions taken alone even when the decision problem is the same. When deciding in a group, a decision maker faces compound lotteries that are probability mixtures of simple lotteries that she faces in individual choice. Under standard microeconomic assumption (the betweenness axiom) preferences over compound lotteries should be consistent with those over simple lotteries. Yet, empirical evidence strongly suggests that the betweenness axiom is often violated. In such a case, the preferences revealed over compound lotteries may not correspond to the preferences revealed over simple lotteries, *i.e*. an individual may exhibit a choice shift. This paper shows that a typical, so-called "squiggle", pattern of betweenness violations implies a specific pattern of choice shifts: an individual exhibits a risky shift when her expectation to be pivotal is sufficiently low and her expectation how the group will decide when she is not pivotal is sufficiently close to zero or one. On the other hand, an individual exhibits a cautious shift when she has a high expectation to be pivotal.

Theoretical results presented in this paper have several testable implications. For instance, if a household is viewed as a small group where each member of the household has a relatively high chance to be pivotal, then individual members of the household exhibit a cautious shift, which can be reflected in a relatively low investment on the stock market. Thus, our decision-theoretical explanation of choice shifts predicts a lower investment on the stock market of households composed of several individuals compared to households composed of only one individual. In other words, an equity premium puzzle on the household level can be a consequence of the fact that individual members of the household violate expected utility theory (and, in particular, the betweenness axiom).

Our main result (theorem 1) can be also applied to decision making within a large committee/jury where each member has a relatively low chance to be pivotal. In this case, individual members exhibit a risky shift when the positions of other members are already known and a cautious shift—when the positions of other members are not really known.

A natural extension of our work is to consider an equilibrium model of choice shifts. Given a specific voting rule and the distribution of risk preferences, we can construct Bayesian Nash equilibrium. However, the existence of Nash equilibrium crucially depends on the assumption of quasi-concavity of preferences (*e.g.*, Crawford, 1990) and it may not be satisfied when decision makers exhibit "squiggle" pattern of betweenness violations. In such a case the notion of *equilibrium in beliefs* as proposed by Crawford (1990) could be applied.

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Appendix

Proof of Theorem 1: We prove first that part 1 of the theorem implies part 2.

Notice that $p_s = (1-a)b + a > (1-a)b = p_r$. Since $\bar{p}[\boldsymbol{s}] + (1-\bar{p})[\boldsymbol{r}] \sim \boldsymbol{s}$ then by setting $\alpha = \bar{p}$, $\beta = p$ and $\gamma = 1$ in part a) of Assumption 1 we immediately obtain that mixture $p[\boldsymbol{s}] + (1-p)[\boldsymbol{r}]$ is strictly preferred over $\boldsymbol{s}$ $for\ all\, p \in (\bar{p}, 1)$. Since $\bar{p}[\boldsymbol{s}] + (1-\bar{p})[\boldsymbol{r}] \sim \boldsymbol{r}$ then part b) of Assumption 1 implies that $p[\boldsymbol{s}] + (1-p)[\boldsymbol{r}] \prec \boldsymbol{r}$ $for\ all\, p \in (0, \bar{p})$. Thus, if $p_s > \bar{p} > p_r$ then we must have $\boldsymbol{s}^* \succ \boldsymbol{s} \sim \boldsymbol{r} \succ \boldsymbol{r}^*$ and an individual exhibits a cautious shift. This evidences proves statement *b) iv*. Therefore, an individual can exhibit a risky shift only in two cases: either when $\bar{p} \geq p_s > p_r$ or when $p_s > p_r \geq \bar{p}$.

Consider first the case when $\bar{p} \geq p_s > p_r$. Since $p_s = p_r + a$ then this case is only possible when $a \leq \bar{p}$. According to part b) of Assumption 1, an individual has strictly quasi-convex preferences on the interval $[0, \bar{p}]$ *i.e.,* for all for $0 \leq \alpha < \beta < \gamma \leq \bar{p}$, the best of $\alpha[\boldsymbol{s}] + (1-\alpha)[\boldsymbol{r}]$ and $\gamma[\boldsymbol{s}] + (1-\gamma)[\boldsymbol{r}]$ is strictly preferred over mixture $\beta[\boldsymbol{s}] + (1-\beta)[\boldsymbol{r}]$. Quasi-convexity implies that the set $L = \{p \in [0, \bar{p}]: s^* \succcurlyeq p[s] + (1-p)[r]\}$ is convex, *i.e*. an interval. Moreover, since $\boldsymbol{s}^* \succcurlyeq \boldsymbol{s}^*$ then this interval *L* must contain probability p_s. More specifically, interval *L* must be of the form either $[c, p_s]$ or $[p_s, c]$ for some threshold $c \in [0, \bar{p})$. An individual then exhibits a risky shift $\boldsymbol{r}^* \succ \boldsymbol{s}^*$ if and only if $p_r \in [0, max\{c, p_s\}]\backslash L$, which is an interval of the form $[0, min\{c, p_s\})$, for some threshold $c \in [0, \bar{p})$. Thus, an individual exhibits a risky shift when $a \leq \bar{p}$ and $p_r < min\{c, p_s\}$. The latter inequality can be rewritten as $p_r = (1-a)b < min\{c, p_s\}$ or $b < min\{c, p_s\}/(1-a) \equiv b_1(a)$. This evidence proves part a) i. and b) i. of the theorem.

Now consider the second case when $p_s > p_r \geq \bar{p}$. Since $p_s = p_r + a$ then this case is only possible when $a \leq 1 - \bar{p}$. According to part a) of Assumption 1, an individual has strictly quasi-concave preferences on the interval $[\bar{p}, 1]$, *i.e.,* for all $\bar{p} \leq \alpha < \beta < \gamma \leq 1$, mixture $\beta[\boldsymbol{s}] + (1-\beta)[\boldsymbol{r}]$ is strictly preferred over the worst of $\alpha[\boldsymbol{s}] + (1-\alpha)[\boldsymbol{r}]$ and $\gamma[\boldsymbol{s}] + (1-\gamma)[\boldsymbol{r}]$. Quasi-concavity implies that the set $U = \{p \in [\bar{p}, 1]: p[s] + (1-p)[r] \succcurlyeq r^*\}$ is convex, *i.e.* an interval. Since $\boldsymbol{r}^* \succcurlyeq \boldsymbol{r}^*$ then this interval *U* must contain probability p_r. More specifically, interval *U* must be of the form either $[\, p_r, d]$ or $[d, p_r]$ for some threshold $d \in [\bar{p}, 1]$. An individual then exhibits a risky shift $\boldsymbol{r}^* \succ \boldsymbol{s}^*$ if and only if $p_s \in [\, min\{d, p_r\},1]\backslash U$, which is an interval of the form $(max\{d, p_r\},1]$, for some threshold $d \in [\bar{p}, 1]$. Thus, an individual exhibits a risky shift if $a \leq 1-\bar{p}$ and $p_s > max\{d, p_r\}$. The latter inequality can be rewritten as $p_s = (1-a)b + a > max\{d, p_r\}$ or $b > (max\{d, p_r\} - a)/(1-a) \equiv b_2(a)$. This evidence proves part a) ii. and b) ii. of the theorem.

To finish the proof that part 1 implies part 2 we need to show that $b_2(a) > b_1(a)$. Fix a value of a s.t. $a < \bar{p}$ and $a < 1-\bar{p}$. First note that $r^* \sim s^*$ can happen only in two cases: $\bar{p} \geq p_s > p_r$ and $p_s > p_r \geq \bar{p}$. When $\bar{p} \geq p_s > p_r$ we observe the risk shift only for all $b < b_1(a)$ and for $b = b_1(a)$ we have that $r^* \sim s^*$. Note that in this case we have $p_r = (1-a)b_1(a)$ so that inequality $\bar{p} > p_r$ can be rewritten as

(1) $$\bar{p} > (1-a)b_1(a)$$

When $p_s > p_r > \bar{p}$ we observe the risk shift only for all $b > b_2(a)$ and for $b = b_2(a)$ we have that $r^* \sim s^*$. Note that in this case we have $p_r = (1-a)b_2(a)$ so that inequality $p_r > \bar{p}$ can be rewritten as

(2) $$(1-a)b_2(a) > \bar{p}$$

Inequalities (1) and (2) can hold simultaneously only if $b_2(a) > b_1(a)$. Finally we have to consider the cases in which either $a \leq \bar{p}$ or $a \leq 1-\bar{p}$ or both are not satisfied. Note that $a \leq \bar{p}$ implies $0 < b_1(a) < 1$ and $a \leq 1-\bar{p}$ implies $0 < b_2(a) < 1$. Suppose that $a \geq \bar{p}$ in this case $b_1(a) = 0$. Suppose that $a \geq 1-\bar{p}$ in this case $b_2(a) = 1$. Then

$b_2(a) > b_1(a)$ holds for all possible values of a. This evidence together with the previous ones proves b) iii. of the theorem.

Now we prove that part 2 of the theorem implies part 1.

We claim that the part 2 of the theorem implies $p[\boldsymbol{s}] + (1-p)[\boldsymbol{r}] \succ \boldsymbol{r}\ \forall p \in (\bar{p}, 1)$ and $p[\boldsymbol{s}] + (1-p)[\boldsymbol{r}] \prec \boldsymbol{r}\ \forall p \in (0, \bar{p})$. Consider an individual who is indifferent between $\boldsymbol{s}$ and $\boldsymbol{r}$. Assume $\bar{p} < a < 1$. In this case, when $b = 0$, part 2 of the theorem implies a cautious shift, i.e. $\boldsymbol{s}^* \succ \boldsymbol{r}^*$, $\forall a \in (\bar{p}, 1)$. Note that for $b = 0$, $\boldsymbol{r}^* = \boldsymbol{r}$. By contradiction, suppose that there exists a value $p^* \in (\bar{p}, 1)$ such that $p^*[\boldsymbol{s}] + (1-p^*)[\boldsymbol{r}] \preccurlyeq \boldsymbol{r}$ and $p^*[\boldsymbol{s}] + (1-p^*)[\boldsymbol{r}] \preccurlyeq \boldsymbol{s}$. Suppose $b = 0$ and $a = p^* > \bar{p}$. In such a case $\boldsymbol{r}^* = [\boldsymbol{r}]$ and $\boldsymbol{s}^* = p^*[\boldsymbol{s}] + (1-p^*)[\boldsymbol{r}]$. By initial assumption we have then $\boldsymbol{r}^* \succcurlyeq \boldsymbol{s}^*$, which contradicts part 2 of the theorem, that in such case states $\boldsymbol{s}^* \succ \boldsymbol{r}^*$.

Assume $1 - \bar{p} < a < 1$. In this case, when $b = 1$, part 2 of the theorem implies a cautious shift, i.e. $\boldsymbol{s}^* \succ \boldsymbol{r}^*$, $\forall a \in (1-\bar{p}, 1)$. Note that for $b = 1$, $\boldsymbol{s}^* = \boldsymbol{s}$. By contradiction, suppose that there exists a value $p^* \in (0, \bar{p})$ such that $p^*[\boldsymbol{s}] + (1-p^*)[\boldsymbol{r}] \succcurlyeq \boldsymbol{r}$ and $p^*[\boldsymbol{s}] + (1-p^*)[\boldsymbol{r}] \succcurlyeq \boldsymbol{s}$. Suppose $b = 1$ and $a = 1 - p^* > 1 - \bar{p}$. In such a case $\boldsymbol{s}^* = [\boldsymbol{s}]$ and $\boldsymbol{r}^* = p^*[\boldsymbol{s}] + (1-p^*)[\boldsymbol{r}]$. By initial assumption we have then $\boldsymbol{r}^* \succcurlyeq \boldsymbol{s}^*$, which contradicts part 2 of the theorem, that in such case states $\boldsymbol{s}^* \succ \boldsymbol{r}^*$.

The evidences above prove our claim.

Now we claim that the theorem implies the existence of $\bar{p} \in [0, 1]$ *such that* $\boldsymbol{s} \sim \boldsymbol{r} \sim \bar{p}[\boldsymbol{s}] + (1-\bar{p})[\boldsymbol{r}]$. Assume an individual who is indifferent between $\boldsymbol{s}$ and $\boldsymbol{r}$. Consider the set $U = \{p[s] + (1-p)[r]: p[s] + (1-p)[r] \succcurlyeq \boldsymbol{r}\}$. Using the above considerations for $\bar{p} < a < 1$ we can state that this set is closed only if lottery $\bar{p}[s] + (1-\bar{p})[r] \succcurlyeq \boldsymbol{r}$. Consider the set of $L = \{p[s] + (1-p)[r]: \boldsymbol{s} \succcurlyeq p[s] + (1-p)[r]\}$.

Using the above considerations for $1 - \bar{p} < a < 1$ we can state that this set is closed only if lottery $\boldsymbol{s} \succcurlyeq \bar{p}[s] + (1 - \bar{p})[r]$**.** Continuity of preferences implies that sets L and U are closed that happen only if $s \sim r \sim \bar{p}[s] + (1 - \bar{p})[r]$.

Now we show that statement part 2 of the theorem implies that preferences are quasi-concave for $p > \bar{p}$ and quasi-convex for $p < \bar{p}$. By definition of quasi-concavity preferences are strictly quasi-concave for $p \in [\bar{p}, 1]$ if and only if are single-peaked on this interval. Suppose preferences are not quasi-concave for $p > \bar{p}$ i.e. there exist probabilities $p, q \in [\bar{p}, 1]$, $p \neq q$, such that $p[\boldsymbol{s}] + (1 - p)[\boldsymbol{r}] \sim q[\boldsymbol{s}] + (1 - q)[\boldsymbol{r}]$ and for some $\alpha \in (0, 1)$ $(\alpha \cdot p + (1 - \alpha) \cdot q)[\boldsymbol{s}] + (1 - \alpha \cdot p - (1 - \alpha) \cdot q)[\boldsymbol{r}] \preccurlyeq p[\boldsymbol{s}] + (1 - p)[\boldsymbol{r}]$. It follows that preferences are not single peaked in $p \in [\bar{p}, 1]$ and therefore there are either 1) at least two separated intervals on p where is possible to find a value of $a < 1 - \bar{p}$ such that $s^* \succ r^*$ or 2) an interval on p where is possible to find a value of $a < 1 - \bar{p}$ such that $s^* \sim r^*$. In both case a violation of the part 2 of the theorem.

Using the similar arguments we can prove that part 2 of the theorem implies strictly quasi-convex preferences for $p < \bar{p}$.

QED.