A coupled viscoelastic-viscoplastic-damage model for short fibre reinforced composites

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Abstract:

A constitutive model of viscous behaviour of short-fibre reinforced composites (SFRC) where complex distributions of fibre orientations are taken into account is proposed in this work. The approach considered for the computation of composite macroscopic behaviour is based on an additive decomposition of the state potential. The SFRC is assimilated to an assembly of several fibre media embedded in a polymeric matrix medium. One of the main assets of this approach is the possibility to model reinforcement with complex distributions of fibre orientations. Moreover, this decomposition allows the implementation of complex behaviour laws coupled with damage models. The polymeric matrix behaviour is typically strain-rate sensitive, i.e. viscoelastic-viscoplastic. This property has to be taken into account when the modelling of the composite behaviour over a large range of strain rate is intended. Therefore, a viscoelastic constitutive model, based on a generalised Maxwell model, and a viscoplastic correction scheme, based on an overstress approach, are implemented for matrix material. The developed constitutive model is then coupled to two damage laws. The first one is introduced in the framework of Continuum Damage Mechanics in order to model the ductile damage behaviour of the matrix material. The second one deals with fibre/matrix interfacial degradation through an interfacial debonding law. In order to identify the parameters involved in the present model, experimental tests are performed (case of polypropylene reinforced with short glass fibres). Micro-computed tomography is used for the characterisation of the fibres distribution of orientation. The efficiency of the proposed model is demonstrated by comparisons between numerical and experimental responses in different loading conditions, including dynamic loadings.

Key words: Short-fibre reinforced composites, viscoelasticity-viscoplasticity, anisotropic damage, fibre/matrix interfacial debonding

1 Introduction

Thermoplastics reinforced with short fibres are more and more appealing for a wide range of industrial applications, in particular in automotive industry for parts possibly subjected to severe loading

conditions and high loading rate (e.g. crash…). The complex reinforcement characteristics in terms of orientation, mechanical and geometrical properties and the complex behaviour of polymeric matrix are the main features of short-fibre reinforced composites (SFRC) behaviour. Indeed, the composite response up to failure is governed by numerous interdependent phenomena, such as strain-rate dependency, complex anisotropy, ductile damage of the matrix, fibre debonding. Extended homogenization methods have been developed in order to model non-linear behaviour of SFRC [1-4].
However, these methods become difficult to implement if aiming taking all these interdependent features into account. This paper presents an alternative approach for the modelling of viscous behaviour of SFRC with complex distributions of fibre orientation and for a wide range of strain rate. According to the approach proposed by Notta-Cuvier et al. [5], complex fibre orientations can be easily dealt with thanks to the consideration of the SFRC as an assembly of the matrix medium and several fibre media, each of them being characterized by a different unit orientation vector and corresponding volume fraction. Strain-rate sensitivity of the polymeric matrix is introduced through the implementation of a viscoelastic-viscoplastic model of behaviour [6]. Viscoelastic constitutive laws are based on generalised linear Maxwell model with parameters experimentally identified by Dynamic Mechanical Analyses (DMA). The modelling of the viscoplasticity is based on an overstress approach. Identification of viscoplastic parameters is performed based on quasi-static and dynamic tests over a large range of strain rate and using the $\mathrm{SE\dot{E}}$ method, developed by Lauro et al. [7]. An anisotropic ductile damage model is coupled to the matrix constitutive model and fibre-matrix interfacial debonding is also introduced [8-9].
Real distributions of fibre orientation are characterised based on micro-computed tomography scans, leading to an accurate representation of the actual reinforcement orientations. Thus, the coupled influence of strain rate and anisotropy of SFRC behaviour can be modelled.
Finally, the anisotropy of the SFRC considered in this work is experimentally investigated for different loading directions with respect to the injection flow direction. Tests at different loading rates have been considered for the validation of the developed model.

2 Constitutive model

The composite material considered here is made of short fibres assumed to be uniformly dispersed in the matrix medium. Fibres which have the same orientation, geometrical characteristics and material behaviour are grouped into the same family. Each fibre family is therefore characterised by its own orientation vector, volume fraction and geometry. Behaviour of fibre families and matrix material are numerically determined before the 3D stress state of the composite material is computed via an additive decomposition of the specific free energy potential.

2.1 Matrix behaviour model

In order to predict the rate-dependent behaviour of thermoplastic matrix, a coupled viscoelastic (VE)-viscoplastic (VP) scheme is considered here. A first assumption is that the matrix total strain, ε, is subdivided into VE and VP parts, i.e. $\varepsilon = \varepsilon^{ve} + \varepsilon^{vp}$. It is worth noting that this decomposition is valid in the framework of small deformation, which is consistent with composite behaviour. Indeed, although unreinforced polymeric matrix can exhibit high level of deformation, this assumption remains valid when dealing with reinforced matrix, which shows consequently decreased strain at break.

The VE part of the response is modelled using the linear Wiechert model (i.e. Generalized Maxwell model) and matrix Cauchy stress tensor, σ_M, at an instant t, is linearly related to the history of VE strain, ε^{ve}, via Boltzmann's hereditary integral:

$$\sigma_M(t) = \int_{-\infty}^{t} R^{ve}(t-\zeta) : \frac{\partial \varepsilon^{ve}(\zeta)}{\partial \zeta} d\zeta \tag{1}$$

where R^{ve} is the fourth order relaxation tensor expressed by:

$$R^{ve}(t) = R_\infty + \sum_{i=1}^{N} R_i(t) \tag{2}$$

N is the number of Maxwell elements. R_∞ and R_i are the fourth order long term elasticity tensor and viscoelasticity tensor of the i^{th} Maxwell element, $i \in \{1, \dots, N\}$, respectively:

$$\begin{gathered} G_\infty = 2G_\infty I_{dev} + K_\infty 1 \otimes 1 \\ R_i(t) = 2G_i \exp\left(-\frac{t}{\tau_i^{dev}}\right) I_{dev} + K_i \exp\left(-\frac{t}{\tau_i^{vol}}\right) 1 \otimes 1 \end{gathered} \tag{3}$$

1 and I_{dev} are respectively the second order identity and the deviatoric projection identity tensors.
This response is updated, when necessary, using a pressure sensitive viscoplastic scheme in the framework of non-associated viscoplasticity. Viscoplastic flow occurs as soon as the first invariant of the matrix Cauchy stress tensor, I_1, and the second invariant of the stress tensor, I_2, reach a critical combination given by Raghava's yield surface expression: I_1^2

$$f(I_1, I_2, R) = \frac{(\eta - 1)I_1 + \sqrt{(\eta - 1)^2 {I_1}^2 + 12\eta I_2}}{2\eta} - \sigma_t - R(\kappa) \geq 0 \tag{4}$$

where $I_1 = \mathrm{tr}\big(\sigma_M(t)\big)$ and $I_2 = \frac{1}{2} S_M(t) : S_M(t)$, with S_M the matrix deviatoric stress tensor. η is the pressure dependency parameter defined as the ratio between the initial yield stresses in compression and tension. $R(\kappa)$ is an isotropic hardening function, for instance defined by:

$$R(\kappa) = h_1 \exp(h_2 \kappa^2)\big(1 - \exp(-h_3 \kappa)\big) \tag{5}$$

where h_1, h_2 and h_3 are material parameters and κ is the equivalent viscoplastic strain.
The viscoplastic strain rate tensor is derived from a viscoplastic potential of dissipation, ψ_M^{vp}, Eq. (6), and expressed using the normality rule, Eq. (7).

$$\psi_M^{vp}(I_1, I_2) = \sqrt{3I_2 + \frac{1}{3} a^+ \langle I_1 \rangle^2 + a^- \langle -I_1 \rangle^2} \tag{6}$$

$$\dot{\varepsilon}^{vp} = \dot{\lambda} \frac{\partial \psi_M^{vp}}{\partial \sigma_M} = \dot{\lambda} n \tag{7}$$

where n is the viscoplastic flow direction tensor. $\dot{\lambda}$ is the viscoplastic multiplier rate calculated using the theory of overstress-based viscoplasticity, according to which the static yield surface, f, is extended to a dynamic yield surface, F^{vp}, defined as follows:

$$F^{vp} = f - \sigma^{vp} = f - \left(\sigma_t + R(\kappa)\right)\left(\frac{\dot{\kappa}}{\dot{\kappa}_0}\right)^m \tag{8}$$

where σ^{vp} is the viscous overstress as postulated in Perzyna's model [10]. m and $\dot{\kappa}_0$ are material parameters that express the strain rate sensitivity, respectively. Finally, the rate form of the viscoplastic multiplier is obtained as follows:

$$\dot{\lambda} = \frac{\dot{\kappa}_0}{\sqrt{\frac{2}{3} n : n \left(\frac{\sigma^{vp}}{2\eta\left(\sigma_t + R(\kappa)\right)^{1/m}}\right)}} \tag{9}$$

2.2 Stress state of the composite material

2.2.1 Fibre stress state

In SFRC, the load applied to the polymeric matrix is transferred to embedded fibres through the interface. As already stated, the presence of fibres with variable characteristics in the composite material is modelled by the coexistence of N_{fam} families. Each family i ($i \in \{1, \ldots, N_{fam}\}$) is characterised by its elastic properties, its vector of orientation in global coordinates system, $\vec{a}^i$, and therefore matrix of orientation, A^i, defined by $A^i = \vec{a}^i \otimes \vec{a}^i$, i.e. $A^i_{kl} = a^i_k \otimes a^i_l$, $\forall$ k, l, its geometric properties and its volume fraction, v^i_F, so that $\sum_{i=1}^{N_{fam}} v^i_F = v_F = 1 - v_M$, where v_F and v_M are respectively the total volume fraction of fibres and matrix in the composite material. A 1D linear elastic stress state is attributed to the fibres with a local iso-strain state between the fibres and the matrix, in the direction of fibre axis. The fibre tensor of deformation gradient, F^i_F, is defined as the projection of the total deformation gradient tensor, F, applied to the composite material, in the direction of fibre orientation (Eq. (10)).

$$F^i_F = FA^i \qquad \forall\, i \in \{1, \ldots, N_{fam}\} \tag{10}$$

By construction, the right Cauchy-Green tensor C^i_F (defined as $C^i_F = {F^i_F}^T F^i_F$, $\forall$ i) of each fibre family i has a unique eigenvalue different from zero, called λ^i_F, with associated eigenvector $\vec{a}^i$. λ^i_F stands for the square of the ratio of the fibres current length by initial length. As a consequence, the 1D fibres strain, $\varepsilon^{0,i}_F$, is simply expressed from λ^i_F as follows:

$$\varepsilon^{0,i}_F = \frac{1}{2}\ln\left(\lambda^i_F\right) \qquad \forall\, i \in \{1, \ldots, N_{fam}\} \tag{11}$$

The average stress in a fibre is computed based on a modified shear lag model based on original work of Bowyer and Bader [11]. According to this approach, the average 1D-stress state of each fibre family, $\sigma^{0,i}_F$, $\forall$ i, can be computed using Eq. (12).

$$\sigma_F^{0,i} = E_F^i \varepsilon_F^{0,i}\left(1 - \frac{E_F^i r^i}{2L^i\tau^i}\left|\varepsilon_F^{0,i}\right|\right) \qquad \text{if } \left|\varepsilon_F^{0,i}\right| \leq \frac{L^i\tau^i}{2r^i}$$
$$\sigma_F^{0,i} = \mathrm{sign}\left(\varepsilon_F^{0,i}\right)\ \frac{L^i\tau^i}{2r^i} \qquad \text{otherwise} \tag{12}$$

Where L^i and r^i are fibre length and radius, respectively, E_F^i is the elastic modulus and τ^i is the interfacial shear strength for fibre family i, $\forall$ i $\in \{1, \ldots, N_{fam}\}$. The other components of fibres 3D stress tensor are computed based on modified iso-stress conditions in transverse directions to fibre axis.

2.2.1 Composite stress state

The stress tensor applied to the composite material can be determined as a combination of the contribution of all fibre and matrix media. In practice, the state potential of the composite material, here the Helmholtz free energy, is assumed to be additively split into a part specific to the matrix medium and other parts specific to each fibre family (Eq. (13)).

$$\rho\phi_c = v_M\rho_M\phi_M + \sum_{i=1}^{N_{fam}} v_F^i\rho_F^i\,\phi_F^i \tag{13}$$

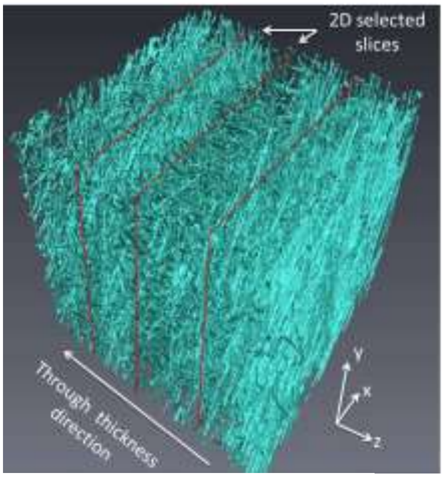
where ρ, ρ_M and ρ_F^i are the density of the composite material, the matrix material and the fibre family i, respectively. ϕ_M and ϕ_F^i are the state potentials of the matrix and the fibre family i, respectively. A particular solution of the Clausius-Duhem inequality can be expressed by Eq. (14), thus giving the expression of composite macroscopic stress tensor.

$$\sigma_c = v_M\sigma_M + \sum_{i=1}^{N_{fam}} v_F^i A^i \sigma_F^i A^i \tag{14}$$

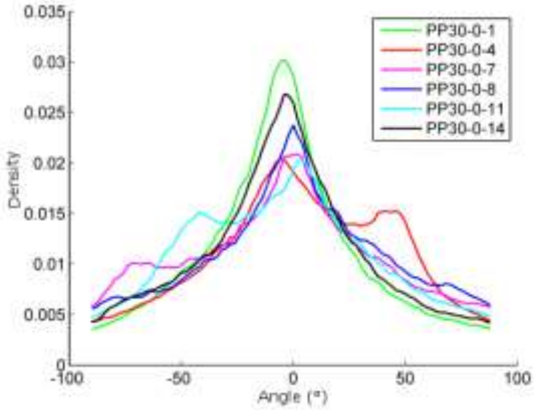
where σ_c and σ_F^i are, respectively, the Cauchy stress tensors of the composite and of the fibre medium i.

3 Experimental characterisation and identification of the viscoelastic-viscoplastic model parameters

The material under investigation is an injection-moulded polypropylene reinforced with 30 wt. % short glass fibres. First, the matrix material (polypropylene, PP) is subjected to various tests (Dynamic Mechanical Analysis (DMA), tension and compression at different loading speeds) so that viscoelastic and viscoplastic parameters of the matrix constitutive model can be identified. Note that viscoplastic parameters are determined following the SE$\dot{\text{E}}$ method [7]. Then, tensile tests were performed for the composite material at different strain rate and different loading angles, θ, with respect to the injection flow direction (IFD) [11]. In parallel, the microstructure of the SFRC is investigated using micro-computed tomography, so that actual distribution of fibre orientation is characterised (Fig. 1) and used as input for the implemented model.



(a) A 3D representation



(b) Directionality histograms for different scanned specimens cut in injection flow direction

Fig. 1. Reconstructed microstructure of PP-30GF by micro-computed tomography

Numerical simulations of the tensile tests of the composite material are carried out, using experimentally identified parameters. Simulations of quasi-static and dynamic numerical tests (at strain rates equal to 5.55 10^{-4} s^{-1}, 5.55 10^{-3} s^{-1}, 3.33 10^{-2} s^{-1}, 0.5 s^{-1}, 5 s^{-1} and 50 s^{-1}) for different loading angles, θ, with respect to the injection flow direction show that the model is able to reproduce the coupled effect of strain rate dependency and anisotropy induced by complex fibre orientations (Fig. 2, examples of $\theta = 0°$ and $\theta = 90°$).

4 Damage modelling

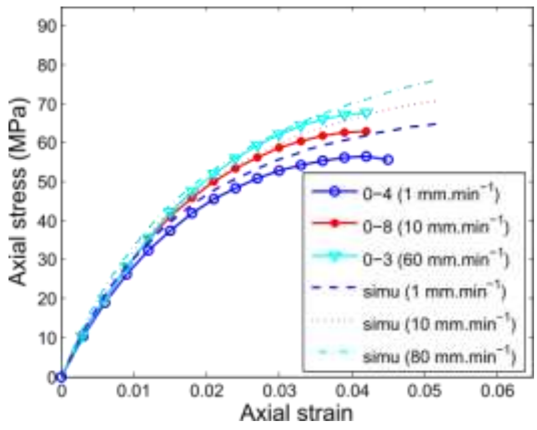
The development of damage phenomena is addressed in this work with an anisotropic ductile damage model for the matrix material and an interfacial debonding model for the fibre-matrix interface.

4.1 Matrix ductile damage

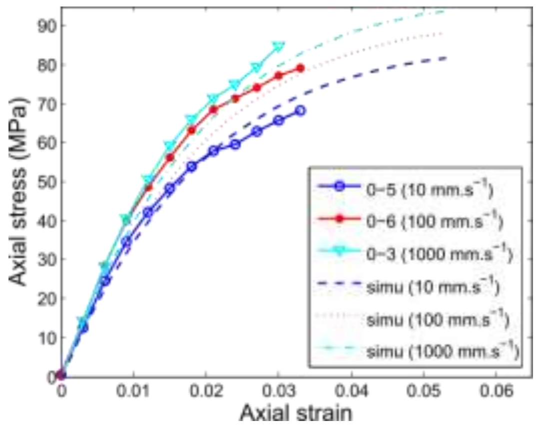
Ductile damage of the matrix material is introduced in the framework of continuum damage mechanics, following the concept of real and effective stress [12].
To deal with anisotropic damage, caused by complex distributions of orientation of fibres embeddeb in the matrix [8], a 4th-order damage tensor, D, is introduced in order to link the actual and effective Cauchy stress tensors, Eq. (15).

$$\sigma_M = D\,\tilde{\sigma}_M \qquad (15)$$

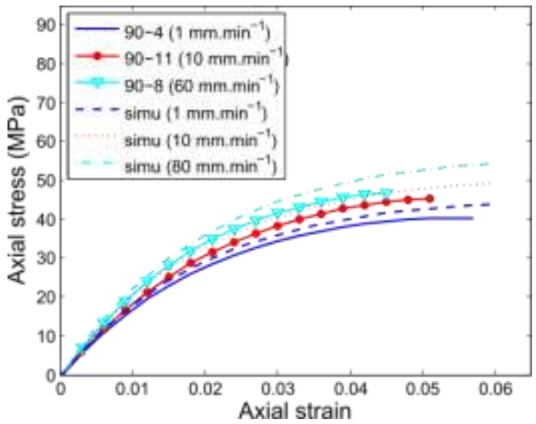
where σ_M and $\tilde{\sigma}_M$ are respectively the actual and effective matrix Cauchy stress tensors.



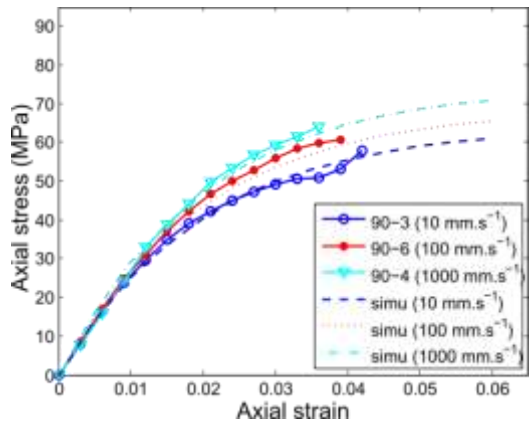
(a) Quasi-static tests at 0°



(b) Dynamic tests at 0°



(c) Quasi-static tests at 90°



(d) Dynamic tests at 90°

Fig. 2. Comparison of experimental and numerical data for tests of composite specimens cut at 0° and 90° with respect to injection flow direction

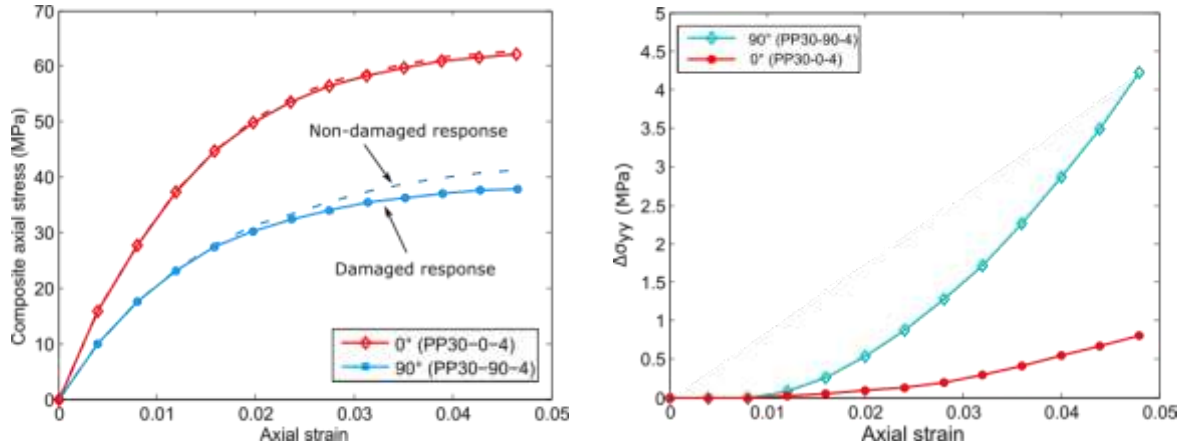
In the formulation of the tensor D, the main assumption is that short fibres prevent the damage in their direction of orientation [8]. This is illustrated in Fig. 3, where it can be seen that damage effect is more pronounced in the case of transversally oriented fibres (90°) compared to the case of fibres with low angle of orientation (0°) with respect to the loading direction.

4.2 Fibre/matrix interfacial debonding

For the interfacial damage modelling, microcracks are assumed to develop and propagate from each fibre tip as soon as fibre strain reaches a threshold value, ε_{th} [9], based on direct SEM observations by Sato et al. [13]. For a given fibre medium i, the area of microcracks extends from fibre tips along fibre side and load transmission is prevented over a fibre length equal to δ^i, defined in Eq. (16). The 1D stress state is then actualised for a fibre length $L^i - 2\delta^i$ (see Fig. 4).

$$\begin{aligned} \delta^i &= a\left(\frac{\varepsilon_F^{0,i} - \varepsilon_{th}}{\varepsilon_{th}}\right)^b \frac{L^i}{2} && \text{if } \varepsilon_F^{0,i} \leq \varepsilon_{th} \\ \delta^i &= 0 && \text{otherwise} \end{aligned} \qquad (16)$$

where a and b are constant parameters.

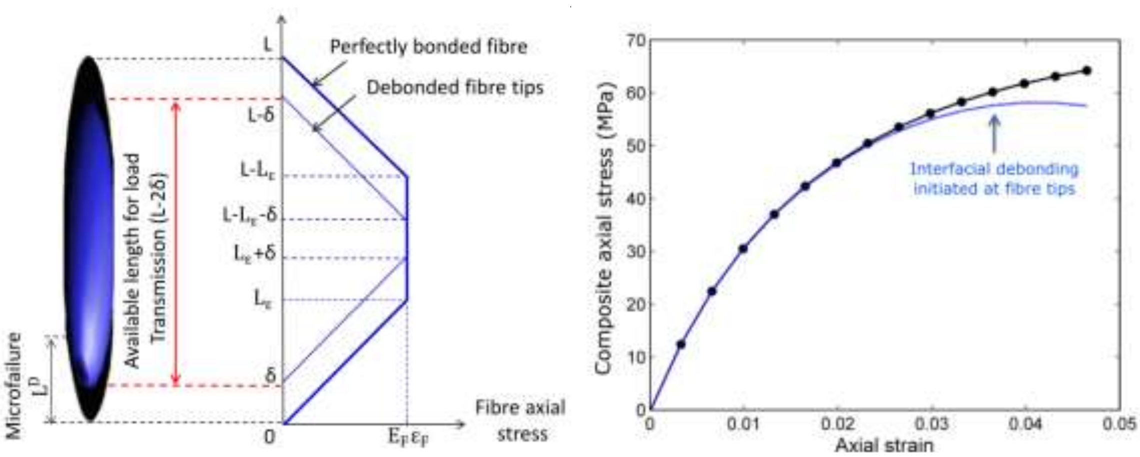


(b) Composite stress-strain response of "non-damaged" and "damaged" matrix for specimens cut at 0° and 90° with respect to injection flow direction

(a) Difference between the effective and real composite axial stress ($\Delta\sigma_{yy}$) for specimens cut at 0° and 90° with respect to injection flow direction

Fig. 3. Anisotropy of the matrix damage for the PP-30GF composite with different distributions of fibre orientation

Tensile tests are simulated with the developed constitutive model by introducing the fibre-matrix interfacial damage mechanism. A schematic representation of interfacial debonding effect on the load transmission is given in Fig. 4a. As can be seen in Fig. 4b (example of a specimen cut in injection flow direction), softening in the stress-strain curves caused by fibre-matrix interfacial damage is reproduced by the current model.



(a) Effect of fibre/matrix debonding on load transmission

(b) Composite stress-strain response with debonded and non-debonded fibres

Fig. 4. Modelling of fibre/matrix interfacial debonding and impact on composite axial behaviour

5 Conclusion

This work presents a modelling of SFRC behaviour based on an original approach that allows to deal with material complexity in an efficient way. In particular, the strain rate dependency of the composite behaviour (i.e. viscoelasticity and/or viscoplasticity) and the complex fibre orientation are addressed. Model accuracy is verified in terms of sensitivity to the strain rate conjugated with the anisotropy induced by the fibres distribution of orientation. The model ability to reproduce the experimentally observed behaviour of a 30 wt. % short glass fibre reinforced polypropylene is evaluated through an implementation in explicite FE code. All the parameters of the matrix constitutive model are experimentally identified from various tests (DMA, tension and compression at different loading speeds). Numerical responses arising from simulations carried out with the constitutive model are in agreement with the experiments, thus validating the model and its implementation. Actual material behaviour is strongly dependent on the reinforcement properties and the damage mechanisms that may develop in it, namely anisotropic damage of reinforced matrix and fibre/matrix debonding. An extension of the implemented constitutive model to damaged behaviour is performed with the introduction of an anisotropic ductile damage model for the matrix material and debonding model for the fibre-matrix interface. Their identification and evaluation are the object of an ongoing work.

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Références

[1] M. Berveiller, A. Zaoui, An extension of the self-consistent scheme to plastically-flowing polycrystals, J Mech Phys Solids (1979) 325-344

[2] A. Molinari, Averaging models for heterogeneous viscoplastic and elasto-viscoplastic materials. J Eng Mater Technol (2002) 62-70

[3] R. Masson, A. Zaoui, Seld-consistent estimates for the rate-dependent elastoplastic behavior of polycrystalline materials. J Mech Phys Solids (1999) 1543-1568

[4] O. Pierard, I. Doghri, An enhanced affine formulation and the corresponding numerical algorithms for the mean-field homogenization of elasto-viscoplastic composites. Int J Plast (2006) 131-157

[5] D. Notta-Cuvier, F. Lauro, B. Bennani, R. Balieu, An efficient modelling of inelastic composites with misaligned short fibres. International Journal of Solids and Structures (2013) 2857-2871

[6] M. Nciri, D. Notta-Cuvier, F. Lauro, F. Chaari, Y. Maalej, B. Zouari, Modelling and characterization of dynamic behaviour of short-fibre-reinforced composites. Composite Structures (2016) 857-871

[7] F. Lauro, B. Bennani, D. Morin, AF. Epee, The SEĖ method for determination of behaviour laws of strain rate dependent material: Application to polymer material. International Journal of Impact Engineering (2010) 715-722

[8] D. Notta-Cuvier, F. Lauro, B. Bennani, R. Balieu, Epee, Damage of short-fibre reinforced materials with anisotropy induced by complex fibres orientations. Mechanics of Materials. (2014) 193-206

[9] D. Notta-Cuvier, F. Lauro, B. Bennani, Modelling of progressive fibre/matrix debonding in short-fibre reinforced composites up to failure. International Journal of Solids and Structures. (2015) 140-150

[10] P. Perzyna, Fundamental problems in viscoplasticity. Advances in Applied Mechanics. (1966) 243-377

[11] D. Notta-Cuvier, M. Nciri, F. Lauro, R. Delille, F. Chaari, F. Robache, G. Haugou, Y. Maalej, Coupled influence of strain rate and heterogeneous fibre orientation on the mechanical efficiency of polypropylene reinforcement by short glass fibres. Mechanics of Materials. (2016) 186-197

[12] J. Lemaitre, JL. Chaboche, Mechanics of Solid Materials. Cambridge University Press: England (1996)

[13] N. Sato, T. Kurauchi, S. Sato, O. Kamigaito, Microfailure behaviour of randomly dispersed short fibre reinforced thermoplastic composites obtained by direct SEM observation. J. Mater. (1991) 3891-3898