Crack repulsion and attraction in Linear Elastic Fracture Mechanics

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Résumé :

Nous proposons une nouvelle méthode de détermination du chemin pris par une fissure se propageant, basée sur un calcul par élements finis des paramètres mécaniques pour ensuite appliquer le principe de symétrie locale. A l'aide de celle ci, nous examinons la possibilité pour une paire de fissures parallèles mais non alignées (EP-cracks) de propagager l'une vers l'autre en s'attirant ou en se repoussant. La forte variation de l'angle initial de déviation avec les distances séparant les pointes internes des fissures montre une très forte influence des paramètres géométriques définissant la configuration initale. Cette sensibilité est confirmée lors de l'étude de chemins de propagation complets : des variations apparemment minimes de la configuration initiale peuvent provoquer des changements spectaculaires dans la forme finale de la fissure. Ces résultats, en accord avec de précdentes observations expérimentales, mettent en avant les limites de la mécanique élastique linéaire de la rupture pour l'étude de fissures en interaction.

Abstract :

By means of a new crack paths computation scheme, based on finite elements analysis and the use of the principle of local symmetry, we examine the possibility of a pair of parallel and offset cracks in a plate (known as EP-cracks) to repulse or attract one another as they propagate inward. The variations of the initial kink angle with the tip to tip separation distances get more abrupt as the inner tips get closer, demonstrating the acute influence of geometric parameters in the case of EP-cracks. This sensitivity is confirmed by the shape of whole crack paths : a seemingly small change of initial geometry can lead to a completely different final path. These results, when compared to experimental data, emphasize the limit of linear elastic fracture mechanics (LEFM) to compute the paths of interacting cracks.

Mots clefs : LEFM, Finite Element Method, EP-cracks, Crack Interaction, PLS

1   Introduction

The simplest case of crack interaction is sometimes referred in the literature as *en-passant* cracks pairs (EP-cracks) : two initially straight, parallel and offset cracks which are submitted to far-field opening stress. Even in this relatively elementary problem, the cracks' behavior is ambiguous. A purely attractive behavior, according which the cracks propagate straight ahead until they attract each other after overlapping, has been observed many times, for a wide variety of settings and material properties, to the point that it has been deemed as universal [1]. However, observations in geological settings [2, 3] and during recent laboratory controlled experiments [4] showed that cracks can significantly repulse one another before overlapping. In some cases, this discrepancy has been imputed to the limit of LEFM [4] or simply ignored [1]. There is a need to carefully examine the case of EP-cracks in order to determine whether the LEFM framework is sufficient to predict crack repulsion.

To determine the cracks' bifurcation directions, we choose to complete this framework by the principle of local symmetry (PLS). Indeed, out of all the classic fracture criteria, the PLS is the only one that is physically acceptable for isotropic, homogeneous materials [5]. Hence, we make the assumption that any crack propagates in order to situate itself in pure opening mode.

In this communication, we briefly present the computation scheme we developed to determine crack paths under the assumptions of both the LEFM and the PLS. We then present our results concerning the initial kink angle of EP-crack pairs as well as their whole propagation paths. It appears that, while the theoretical framework LEFM+PLS is sufficient to predict some crack repulsion, the high sensitivity to initial and boundary conditions of this problem makes it unrealistic to neglect plastic damage if one wishes to make quantitative bifurcation prediction.

2   Computation scheme

2.1   Kink angle determination

In order to apply the PLS bifurcation criterion for a given mixed mode fracture problem, we need to determine the kink angle θ formed between the tip of the original crack and its extension, that will result in a pure opening mode at the newly extended crack tip. In other words, we need to establish K_{II}^*, the sliding stress intensity factor (SIF) at the tip of the crack extension, as a function of θ and find its zeros.

For an infinitesimally small extension, the SIF at the end of the extension, K_I^* and K_{II}^*, are dependent only on the SIF *before extension*, K_I and K_{II}, and the kink angle θ [5] :

$$\begin{bmatrix} K_I^*(\theta) \\ K_{II}^*(\theta) \end{bmatrix} = \begin{bmatrix} M_{11}(\theta) & M_{12}(\theta) \\ M_{21}(\theta) & M_{22}(\theta) \end{bmatrix} \cdot \begin{bmatrix} K_I \\ K_{II} \end{bmatrix} \tag{1}$$

The coefficient of the matrix M in equation 1 are universal (they do not depend on geometry or loading) and are known as polynomial expressions of θ. The SIF in the initial configuration K_I and K_{II} can be easily retrieved from a FEM computation. Therefore, determining the initial kink angle satisfying the PLS simply amounts to solving :

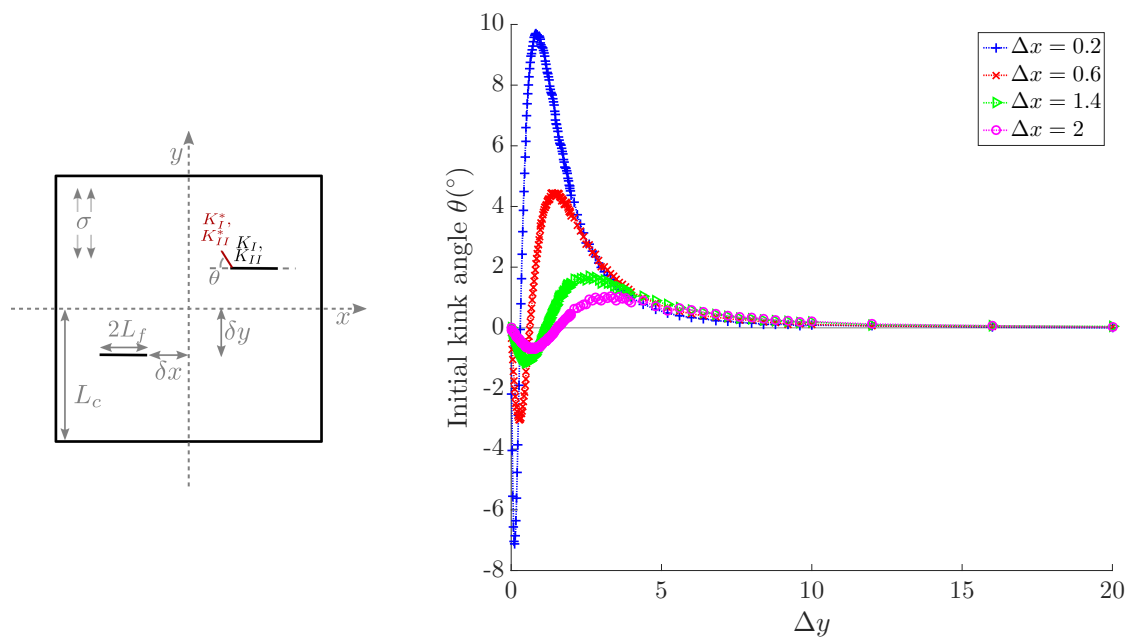$$M_{21}(\theta)K_I + M_{22}(\theta)K_{II} = 0 \tag{2}$$

FIGURE 1 – a- Initial geometry considered to study the initial kink angle of EP-crack pairs. b- Evolution of the inner kink angle with tip to tip spacing

This first step was successfully tested against the experimental data presented in [6], in which a Plexiglas plate notched with an inclined crack is loaded in pure traction.

2.2 Full crack paths determination

Starting with a FEM computation we saw in section 2.1 that we can easily retrieve the kink angle formed between a crack tip and its extension. From there, we add a small segment of arbitrary length at the tip of the original crack, and determine the new SIF with another FEM computation. Repeating this procedure as necessary, we can approximate full crack paths as a succession of straight segments.

This method, while crude, successfully predicted the experimental crack paths in [7], in which a plate, notched with three holes and an emergent crack, is loaded in bending. In this test case, as well as in the application case presented in section 3.2, the influence of increment's length on the final computed path was negligible.

3 Results : application to the case of EP-crack pairs

3.1 Evolution of the initial kink angle

We applied the computation scheme presented in section 2 to investigate the case of EP-cracks. To study how crack interaction affects the initial kink angle while minimizing the impact of boundary conditions on the cracks' propagation paths, we examine the case of a square plate notched with cracks significantly smaller than the length of the plate's side (ratio of 100), as depicted schematically in Fig. 1 a. Rigid body movements are restrained by clamping the mid point on the plate's left side, and only allowing displacement along x for the opposing point. The plate's horizontal sides are pulled apart by imposing a stress in the σ_{yy} direction. We only consider an ideally elastic, isotropic and homogeneous material for all scales considered.
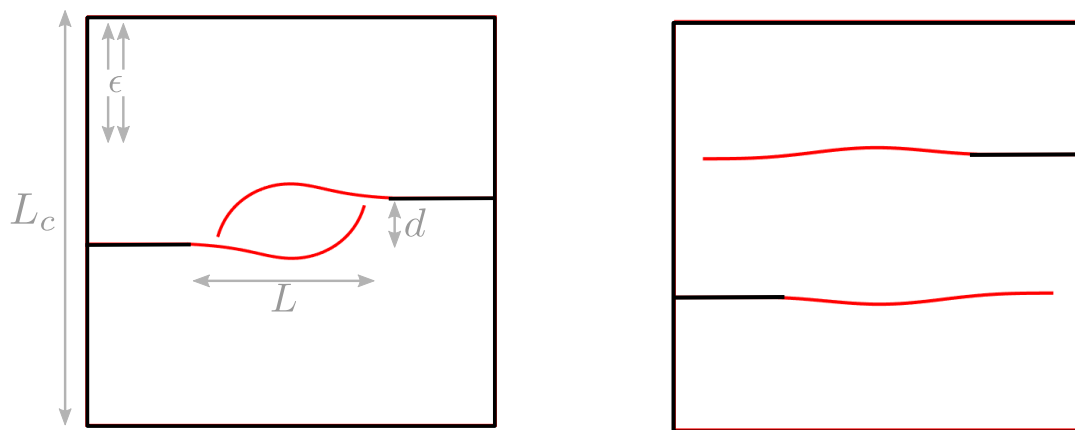
FIGURE 2 – Propagation paths (red) computed for two sets of EP-crack pairs (black) :

While three parameters are necessary to define our geometry (the cracks' half length L_f, the vertical and horizontal spacing between the inner crack tips δy and δx), θ is determined by only two : the scaled inner tips separations : $\Delta x = \delta x / L_f$ and $\Delta y = \delta y / L_f$.

We present in Fig. 1 b the evolution of the initial kink angle θ as a function of Δy, for a given strictly positive Δx :

For cracks standing far away from each other, interaction is limited and θ is close to zero. Then, as Δy diminishes, the cracks' behavior grows more and more attractive : θ increases until it reaches a maximum value. With Δy still shrinking, the cracks' behavior changes from attractive to repulsive : θ decreases until it reaches a negative minimum value. It is interesting to note that θ always goes back to zero for perfectly aligned cracks. This qualitative behavior is independent of Δx. However, the amplitude of θ's variation is strongly influenced by the horizontal tip to tip separation : the closer the cracks are, the greater the extreme values of θ are.

3.2 Crack paths shape

We applied our method to numerically reproduce the experiments presented in [4] : a square plate of side length L_C is notched with two collinear cracks separated horizontally by a distance L and vertically by a distance d. In the experiment, the sides of the plate are slowly pulled apart so that the cracks propagate quasi-statically.

Fixing the value of L and varying the value of d we were able to identify two types of crack paths (See Fig. 2) :
  — Type A : for small values of d, the cracks deviate away from each other at first before turning as the inner tips overlap : the behavior is initially repulsive before becoming attractive.Propagation stops when the cracks intersect.
  — Type B : for larger values of d, the interaction between the two cracks is weak and mostly attractive, even though the cracks do not intersect. Propagation stops when the cracks reach the plate's sides.

In the second case, crack-crack interaction seems overshadowed by the interaction between each crack and the sides of the plate. This assumption was confirmed by computation in rectangular plates : lengthening plates in the horizontal direction increases the threshold value of d above which type B is exhibited.

For type A paths, we find good agreement with observation in geological settings [2, 3] : the turning point of the cracks corresponds to the inner tips overlapping, the cracks intersect at an angle close to 90°, and the aspect ratio of the released central part is distributed around 2.

4   Conclusion

We presented a fast and accurate computation scheme to determine crack paths in the context of linear elastic fracture mechanics, under the assumption of the principle of local symmetry.

Applying it to study the case of EP-cracks we showed that, contrary to what was previously thought, it is possible to predict repulsive crack paths in the LEFM+PLS framework. However, the complex dependency of θ with Δx and Δy entails significant repulsion only for a very narrow $(\Delta x, \Delta y)$ domain. This, combined with the sensitivity to boundary conditions observed in the determination of complete crack paths, indicates that even small variations in the stress field around the cracks can result in strong fluctuations of the final crack shape. It is then unreasonable to neglect the changes in the stress field induced by plastic deformation around the crack tips, at least when it comes to making precise prediction or when using materials with a significant plastic process zone. These results corroborate the conclusions of [4], whose experiments displayed EP-cracks pairs propagating into two types of materials and exhibiting different amount of deviation, even though both materials had the same macroscopic properties (Young's modulus, Poisson ratio) and only one disparity : the shape of the process zone at cracks' tips.

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