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# **Mathematical modeling and multi-objective optimization of technological parameters in Hard Turning operation using RSM and Genetic Algorithmic Approach**

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# **Résumé:**

*Dans le présent article de recherche, les effets des paramètres d'usinage tels que le rayon de coupe, la vitesse de coupe, la vitesse d'avance et la profondeur de passe sur la rugosité de surface, la force de coupe et le taux d'enlèvement de matière en tournage dur de finition de l'acier trempé au chaud X38CrMoV5-1 [AISI H11] traité à 50 HRC à l'aide d'outils céramiques mélangés revêtus CC6050 ont été expérimentalement étudiés. La conception de l'expérience factorielle était basée sur le réseau orthogonal L36 de Taguchi. La méthodologie de la surface de réponse (RSM) et l'analyse de la variance (ANOVA) ont été utilisées pour vérifier la validité de plusieurs modèles de régression linéaire et pour déterminer le paramètre significatif affectant les facteurs de réponse. Une technique d'optimisation multi-objectifs basée sur la méthode RSM et l'approche de l'algorithme génétique (GA) a ensuite été appliquée pour trouver des combinaisons optimales des conditions de coupe afin de minimiser la qualité de la surface, la force de coupe et le maximum de productivité. Les résultats expérimentaux révèlent que le paramètre d'usinage le plus important pour la rugosité de surface est l'avance suivi par le rayon de bec de l'outil. Cependant, la vitesse de coupe affecte considérablement et le taux d'enlèvement de matière. Le (GA) a permis l'optimisation des conditions de coupe pour une rugosité de surface et une force de coupe minimale et un taux d'enlèvement de matière maximal.*

# **Abstract :**

*In the present research work, the effects of machining parameters such as cutting radius, cutting speed, feed rate and depth of cut on the surface roughness, cutting force, and material removal rate in finish hard turning of hardened hot work steel X38CrMoV5-1 [AISI H11] treated at 50 HRC using coated mixed ceramic tools CC6050 were experimentally investigated. The factorial experiment design was based on Taguchi's L36* 

*orthogonal array. The response surface methodology (RSM) and analysis of variance (ANOVA) were used to check the validity of multiple linear regression models and to determine the significant parameter affecting the responses factors. A multi-objective optimization technique based on the RSM method and the Genetic algorithm approach (GA) was then applied to find optimal combinations of the cutting conditions to minimize the surface quality, cutting force and maximum the productivity. The experimental results reveal that the most significant machining parameter for surface roughness is the feed followed by cutting radius. However, the cutting speed affects considerably the metal removal rate. The (GA) allowed the optimization of the cutting conditions for minimal surface roughness, cutting force and maximal material removal rate.* 

**Keywords:** Hard turning, AISI H11 steel, Ceramic, ANOVA, RSM, Genetic Algorithmic, Multi-objective optimization.

# **1 Introduction**

 The current study investigates the influence of cutting parameters (cutting radius (r, mm), cutting speed (Vc, m/min), feed rate (f, mm/rev) and depth of cut (ap, mm)) in relation with on the surface roughness (Ra,  $\mu$ m), cutting force (Fc, N) and material removal rate (MRR, cm3/min) on machinability. The processing conditions are turning of hardened hot work steel (AISI H11) with CC6050, ceramic tool coated with TiN using response surface methodology (RSM) and ANOVA. A multi-objective optimization technique based on the RSM method and the Genetic Algorithmic approach (GA) was then applied to find optimal combinations of the cutting conditions.

# **2 Experimental conditions and procedures**

## **2.1 Material, work piece and tool**

In the present investigation, the workpiece material used in this study is grade AISI H11 steel, hot work steel bars with dimensions of 400 mm length and 75 mm in diameter. The reference chemical composition (in wt. %) is given in Table 2. It is hardened to 50 HRC. The cutting insert used namely, TiN coated mixed ceramic CC6050.

The machine used in the current work is the lathe 'TOS TRENCIN; model SN40C'. The lathe is equipped with 6.6 kW spindle power and a maximum spindle speed of 2000 rpm. The cutting conditions for finish hard turning under higher parametric condition are shown in Table 1.

A Surf test 201 Mitutoyo roughness meter was selected to measure different criteria of surface roughness (Ra) for each cutting condition. In addition, four measurements were made using a 3D surface topography with optical platform of metrology modular Altisurf 500. The cutting forces were measured in real time with a Kistler three component dynamometer model 9257 B linked via a multichannel charge amplifier (type 5011 B) to high impedance cable. The metal removal rate (MRR) is calculated using equation (1).

$$
MRR = Vc.f.ap
$$

#### (1)

# **2.2 Design of experiment**

The response surface methodology (RSM) is the procedure to determine the relationship between the independent process parameters with the desired response and to explore the effect of these parameters on responses, including six steps [8]. These are, in the order, (1) define the independent input variables and the desired responses with the design constants, (2) adopt an experimental design plan, (3) perform regression analysis with the multiple linear model of RSM, (4) calculate the statistical analysis of variance (ANOVA) for the independent input variables in order to find which parameter significantly affects the desired response, then, (5) determine the situation of the multiple linear model of RSM and decide whether the model of RSM needs screening variables or not and finally, (6) optimize and conduct confirmation experiment and verify the predicted performance characteristics. [1-8]

<b>Symbol</b>	Control	Unit	<b>Symbol of</b>	Levels			
	factor		factors	<b>Level 1</b>	<b>Level 2</b>	Level 3	
ap	Depth of cut	mm			0.3	0.5	
	Feed rate	mm/rev		0.08	0.14	0.20	
Vc	Cutting speed	m/min		100	150	200	
	Cutting radius	Mm		0.8	1.2		

**Table 1.** Cutting parameters and their levels for turning.

**Table 2**. Chemical composition of AISI H11 steel.

Composition		ີ	Mo _____		$\sim$ n,	Mn	∼		$\gamma$ ther components	H
(Wł $\frac{0}{0}$	$\sim$ ∪.∪ أ	⊃.∠0	ı .	0.50	1.01	$\sim$ $\mathsf{U} \cdot \mathsf{U} \rightarrow \mathsf{U}$	0.002	$\Omega$ $\Omega$ 1. ΠQ v.v	942 $\cdot$ v	90.31

**Table 3** L36 (21×33) orthogonal array, experimental results for the surface roughness, cutting force and metal removal rate



# **3 RESULTS AND DISCUSSION**

## **3.1 Analysis of variance ANOVA**

Table 4 shows the results of ANOVA. This analysis was out for a 5% significance level, i.e., for a 95% confidence level. In addition to degree of freedom, mean of squares (MS), sum of squares (SS), F-value and probability (Prob.) associated with each factor level were presented. The last but one column of tables shows the factor contribution (percentage; Cont. %) on the total variation, indicating the degree of influence on the result. [1-8]

From the analysis of Table 4, it can be apparent seen that the model is significant and the feed rate (f, mm/rev) is the most important factor affecting Ra. Its contribution is 89.953 %. This is because its increase generates helicoid furrows, the result of tool shape and helicoid movement tool-workpiece. These furrows become deeper and broader when the feed rate increases. The next largest factor influencing Ra is cutting radius (r, mm) with 4.898 % contribution.

Concerning now tangential force, as shown in Table 5, the percentage contributions of factors A, B, C and D on the Fc are (0.23, 0.38,15.62 and 81.56) % respectively. In this case, the most effective parameter for the tangential force is factor D; namely, the depth of cut, because increasing depth of cut increases the chip volume removed. The next largest factor influencing Fc is feed rate (C) with 15.65 %, respectively. The cutting speed and tool nose radius do not present any statistical significance on the tangential force.

Table 6 shows the results of ANOVA model for material remove rate (MRR). It can be apparent seen that the model, cutting speed (B), feed rate (C), depth of cut (D), AD, BC, BD, and CD has a P-value less than 0.05, it means that only this factors (terms) has a significant effect on the response (material remove rate (MRR). All other terms are insignificant. As shown in Table 6, the percentage contributions of factors B, C and D on the MRR are (9.29), (15.23) and (51.17) % respectively. In this case, the most effective parameter for the material remove rate is the depth of cut. The next largest factor influencing MRR is feed rate (C). The other factors do not present any statistical significance on the material remove rate.

<b>Rapic + Allarysis of variance for Ra</b>								
<b>Source</b>	<b>SS</b>	DF	MS	<b>F-value</b>	Prob.	Cont. $%$	<b>Remarks</b>	
<b>Model</b>	6.038464	10	0.60384643	76.23911	< 0.0001		Significant	
$A-r$ , mm	0.179108	1	0.17910833	22.61346	< 0.0001	4.898	Significant	
$B-Vc$ , m/min	0.002216		0.00221681	0.279885	0.6014	0.061	Insignificant	
$C-f$ , mm/rev	3.289100		3.28910046	415.2680	< 0.0001	89.953	Significant	
$D$ - <i>ap</i> , mm	0.003504	1	0.00350417	0.442421	0.5120	0.096	Insignificant	
AВ	0.083535	Ι.	0.08353521	10.54680	0.0033	2.285	Significant	
AC	0.051658	1	0.05165824	6.522152	0.0171	1.413	Significant	
AD	0.003139		0.00313962	0.396395	0.5347	0.086	Insignificant	
BC	0.030636	Ι.	0.03063683	3.868077	0.0604	0.838	Insignificant	
<b>BD</b>	0.005318	$\mathbf{I}$	0.0053184	0.671478	0.4203	0.145	Insignificant	
CD	0.008257		0.00825726	1.042527	0.3170	0.226	Insignificant	
<b>Residual</b>	0.198010	25	0.00792043					
<b>Lack of Fit</b>	0.188110	16	0.01175692	10.68810	0.0005		Significant	
<b>Pure Error</b>	0.009900	9	0.0011					
<b>Cor Total</b>	6.236475	35				100		

**Table 4** Analysis of variance for Ra

**Table 5** ANOVA result for tangential force (Fc).

<b>Source</b>	SS	DF	<b>MS</b>	<b>F-value</b>	Prob.	Cont. $%$	<b>Remarks</b>		
(a) CC6050									
Model	83350.822	10	8335.0822	51.6559099	< 0.0001		Significant		
$A-r$ , mm	180.15543		180.15543	1.116496795	0.3008	0.23	Insignificant		
$B-Vc$ , m/min	306.50024		306.50024	1.899507211	0.1803	0.38	Insignificant		
$C-f$ , mm/rev	12496.522		12496.522	77.44605472	< 0.0001	15.62	Significant		
$D$ -ap, mm	65252.167		65252.167	404.394337	< 0.0001	81.56	Significant		
AВ	134.81677		134.81677	0.835514632	0.3694	0.17	Insignificant		
AC	76.881684		76.881684	0.476467203	0.4964	0.10	Insignificant		
AD	0.0021559		0.0021559	1.33612E-05	0.9971	0.01	Insignificant		
BС	317.61405		317.61403	1.968383923	0.1729	0.40	Insignificant		
<b>BD</b>	1.4316313		1.4316313	0.008872405	0.9257	0.01	Insignificant		
CD	1243.6552		1243.6552	7.707439863	0.0103	1.55	Significant		
<b>Residual</b>	4033.9441	25	161.35776						





#### **Table 6** ANOVA table for MRR

### **3.2 Mathematical modeling**

The functional relationship between the dependent variables (Ra, Fc and MRR) and the investigated independent variables (cutting speed, feed rate, depth of cut and cutting radius) were obtained by multiple linear regressi This correlation can be represented by the following equations:

variables (cutting speed, feed rate, depth of cut and cutting radius) were obtained by multiple linear regressions.  
This correlation can be represented by the following equations:  

$$
Ra = -0.996 + 1.287r + 3.935 \times 10^{-3}Vc + 8.677f - 0.830ap - 7.447 \times 10^{-3}r \times Vc - 4.880r \times f + 0.301r \times ap + 0.019Vc \times f + 1.994 \times 10^{-3}Vc \times ap + 2.070f \times ap
$$

 $(R^2 = 96.82\%)$ 

$$
Fc = -34.415 - 5.055r - 0.120Vc + 689.979f + 143.563ap + 0.299r \times Vc - 188.271r \times f - 0.250r \times ap - 1.953Vc \times f + 0.032Vc \times ap + 803.512f \times ap
$$
\n(3)

 $(R^2 = 95.62\%)$ 

$$
MRR = +3.565 + 2.512r - 0.042Vc - 41.053f - 14.132ap - 0.0016r \times Vc - 1.415r \times f
$$
  
-6.863r \times ap + 0.3Vc \times f + 0.147Vc \times ap + 143.867f \times ap

 $(R^2 = 99.97\%)$ 

#### **3.3 Effect of machining parameters on response surface**

In order to investigate the influences of machining parameters on the surface roughness criteria, tangential force and material remove rate, 3D surface and contour graphs were plotted based on the model equations (Eqs. (2) to (4)) in Figs. (1a to 1f) and Figs. (2a to 2f), respectively. The 3D response surface plots were generated considering two machining parameters at a time, while the other parameters were kept at the middle levels.

From interaction plot Fig. 1a it can be observed that, at constant cutting radius, the surface roughness sharply increases with increase in feed rate, this is because its increase generates helicoid furrows, the result of tool shape and helicoid movement tool-workpiece. These furrows are deeper and broader as the feed rate increases. On the other hand, surface roughness has a tendency to decrease with an increase in cutting radius at constant feed rate. Fig. 1b shows the relations of depth of cut (ap)–cutting speed (Vc). The figure indicate that for a given depth of cut, the surface roughness decrease with increase in cutting speed. On the other hand, depth of cut has less effect on surface roughness.

Fig. 1c shows the interaction effects of depth of feed rate  $(f)$  – cutting radius  $(r)$  on tangential force. The surface plot illustrates that when feed rate increase at constant cutting and depth of cut, the tangential force increases. As it can be deduced from this figure, the cutting radius have not statistically significant. On the other hand, the relationship between the tangential force and both depth of cut and feed rate is plotted in Fig. 1d. As it was expected, the tangential force increases with the increase of depth of cut and feed rate due to the enlargement of cutting action area. Additionally, it reaches its maximum value at high levels of depth of cut and feed rate.

Figures (1e and 1f) shows the interaction effects of cutting speed (Vc), feed rate (f) and depth of cut (ap) on material remove rate (MRR). Form the analyses of this figure, it can be clearly seen that the material remove rate increases with increase in feed rate, cutting speed and depth of cut. The three variable and there interaction has a significant effect on MRR.



**Fig. 1**. 3D surface plots for interaction effects of machining parameters on *Ra, Fc and MRR*.





**Fig. 2**. Contour Graphs plots for interaction effects of machining parameters on Ra, Fc and MRR.

# **3.4 3D surface topography**

The representative examples of 3D images of hard turned surfaces are visualized by means of two isometric views and contour maps at (a)  $r = 0.8$  mm and (b)  $r = 1.2$  mm. It must be noted that the both 3D profiles have represented pure roughness values, i.e. the turned surface topography in Figs 3a and 3b shows well-defined peaks and valleys, this is mainly because when the turning operation process uses a single cutting edge generates helicoids furrows the result of tool shape and helicoids movement tool–workpiece. The 2D surface profiles of the hard-turned surfaces along the feed direction are shown in Figs. 3a and 3b. It must be noted that all the 2D profiles have represented pure roughness values, i.e. the waviness components have been filtered out.



**Fig. 3**. 3D topography for turning with; CC6050 at (a)  $r = 0.8$  mm and (b)  $r = 1.2$  mm.

# **4 Optimization of Responses using Genetic Algorithm Approach**

The object of multi-response optimization is to determine the conditions on the independent variables that lead to optimal or nearly optimal values of the response variables. [9, 10]

The genetic algorithm is a method for solving both constrained and unconstrained optimization problems that is based on natural selection, the process that drives biological evolution. The genetic algorithm repeatedly modifies a population of individual solutions. At each step, the genetic algorithm selects individuals at random from the current population to be parents and uses them to produce the children for the next generation. Over successive generations, the population "evolves" toward an optimal solution. You can apply the genetic algorithm to solve a variety of optimization problems that are not well suited for standard optimization algorithms, including problems in which the objective function is discontinuous, nondifferentiable, stochastic, or highly nonlinear. The genetic algorithm can address problems of mixed integer programming, where some components are restricted to be integer-valued.

The genetic algorithm uses three main types of rules at each step to create the next generation from the current population:

- Selection rules select the individuals, called parents, that contribute to the population at the next generation.
- Crossover rules combine two parents to form children for the next generation.
- **Mutation rules apply random changes to individual parents to form children.**

An integration of GA and RSM methods is applied to find the optimal process parameters in hard turning process. The RSM is used to establish the nonlinear relationships between the hard-turning process parameters and the performance characteristics. Finally, the GA approach is applied to find the optimal process parameters using the overall utility function as the fitness function. The detailed procedures of the integrated GA and RSM methods are stated as follows:

- **Step 1.** Chromosome representation and generating the initial populations: The process parameters are used to create the solution space for the GA approach.
- **Step 2.** Calculate the fitness value.
- Step 3. Constraints: Due to the limitations on the machine and cutting tool and due to the safety of machining the cutting parameters are limited with the bottom and top permissible limit. There are several factors limiting the process parameters. Those factors originate usually from technical specifications and organizational considerations. The following limitations are taken into account as given by Eqs.  $(5)-(8)$ .



- Step 4. Selection The elitist (elitist count = 2 individuals) and roulette wheel selection operators are used in this research for selecting a parent individual for the next generation when copying the best chromosome to the new population.
- Step 5. Crossover: The crossover operator is used to create a pair of offspring chromosomes. For each selected pair, a Heuristic crossover operation with ration of 1.2 is applied for exchanging the parent string segments and recombining them to produce two resulting offspring individuals.
- **Step 6.** Mutation In this study, the adaptive feasible mutation operation for playing a role of random local search which searches regardless of the direction of learning to obtain the better solution. By using the crossover and mutation operations, new Npop (offspring) populations are created.
- **Step 7.** Hybrid function: Hybrid function enables to specify another minimization function that runs after the genetic algorithm terminates. In this study, the pattern search as hybrid function is used to improve the value of the fitness function.

The results of the multi-response optimization Genetic Algorithm approach appear in the following table 6 containing both objective function values (for simultaneously minimal surface roughness and maximal material removal rate) and the value of the variables. Figure 3 represent the average spread and the projection of the pareto front (2D) generated by the proposed optimization technique which represent the space of solutions.

<b>Index</b>	Ra	<b>MRR</b>	Vc	f	ap	r
1	0.1119	1.5579	197.4417	0.0814	0.1231	1.1987
$\boldsymbol{2}$	1.4005	19.2475	195.0345	0.2000	0.4983	0.8489
3	0.4993	12.1658	196.6832	0.1237	0.4960	1.1551
4	0.1478	5.7411	196.9811	0.0830	0.3448	1.1948
5	0.9668	17.0946	196.5153	0.1795	0.4961	1.1217
6	0.2086	8.6782	196.8715	0.0894	0.4739	1.1950
7	0.5898	13.0186	195.6204	0.1360	0.4908	1.1635
8	0.2739	9.6031	196.6059	0.0958	0.4915	1.1670
9	0.8452	16.0340	196.5962	0.1679	0.4960	1.1558
10	0.1774	7.7693	195.9795	0.0848	0.4480	1.1859
1	0.1119	1.5579	197.4417	0.0814	0.1231	1.1987
12	0.3647	10.0096	195.9536	0.1072	0.4664	1.1593
13	0.9806	17.6762	195.3803	0.1873	0.4983	1.1744
14	0.7575	14.6918	195.5389	0.1540	0.4934	1.1277
15	0.9136	16.1819	195.7927	0.1702	0.4938	1.0959
16	1.0432	18.4596	195.5395	0.1966	0.4974	1.1834
17	1.2526	18.6620	195.1788	0.1952	0.4978	0.9562
18	0.1335	4.1207	197.2680	0.0814	0.2623	1.1834

**Table 6** Pareto front points generated by the proposed optimization technique



**Fig. 3**. Pareto front points and average spread

# **5 Conclusion**

Based on the above results for the hard turning of AISI H11 steel with 50 HRC using coated ceramic tool, the following conclusions are made:

- This study shows that the surface roughness Ra is strongly influenced by the feed rate. Its contribution is 89.953 %. Additionally.
- Cutting force components varied almost linearly, with the feed and depth of cut but showed different behavior with cutting speed. Initially, the cutting forces decreased with the increase in cutting speed but almost unaltered in higher cutting speed range.
- The linear mathematical models developed for surface roughness using regression analysis technique are very useful for predicting new experiments. Close correlation between predicted and measured values was established.
- 3D visualization confirmed some characteristic features of surfaces produced with both inserts tested, i.e. peaks and valleys.
- The multi-objective optimization technique based on the RSM method and the Genetic algorithm approach (GA) allowed the optimization of the cutting conditions at the three cases of optimizations for minimal surface roughness and maximal material removal rate.

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