

# The Measurement of Environmental Economic Inefficiency with Pollution-generating Technologies

Juan Aparicio<sup>a</sup>, Magdalena Kapelko<sup>b</sup>, José L. Zofío<sup>c,\*</sup>

<sup>a</sup> Center of Operations Research (CIO). Universidad Miguel Hernández, Elche, Spain.

<sup>b</sup> Department of Logistics, Institute of Applied Mathematics, Wrocław University of Economics, Wrocław, Poland.

<sup>c</sup> Department of Economics. Universidad Autónoma de Madrid, Madrid, Spain.  
Erasmus Research Institute of Management, Erasmus University, Rotterdam, The Netherlands.

## Abstract

This study introduces the measurement of environmental inefficiency from an economic perspective that integrates, in addition to marketed good outputs, the negative environmental externalities associated with bad outputs. We develop our proposal using the latest by-production models that consider two separate and parallel technologies: a standard technology generating good outputs, and a polluting technology for the by-production of bad outputs (Murty et al., 2012). While research into environmental inefficiency incorporating undesirable or bad outputs from a technological perspective is well established, no attempts have been made to extend it to the economic sphere. Our model defines an economic inefficiency measure that accounts for suboptimal behavior in the form of foregone private revenue and social cost excess (environmental damage). We show that economic inefficiency can be consistently decomposed according to technical and allocative criteria, considering the two separate technologies and market prices, respectively. We illustrate the empirical implementation of our approach on a set of established and complementary models using a dataset on agriculture at the level of US states.

**Keywords:** Environmental economic inefficiency, Pollution-generating technologies, Technical and allocative efficiency measurement, Data envelopment analysis, US agriculture.

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\* Corresponding author: J.L. Zofío. Voice: +34 914972406; E-mail: [jose.zofio@uam.es](mailto:jose.zofio@uam.es), [jzofio@rsm.nl](mailto:jzofio@rsm.nl).

## 1. Introduction

Measuring the environmental inefficiency of production units is an increasingly important topic of recent economic research. Environmental inefficiency assessment integrates marketed (desirable, intended, or good) outputs with negative environmental externalities into inefficiency modeling (the production of so-called undesirable, unintended, detrimental, or bad outputs). Such analysis is important from the perspective of sustainable production because it provides valuable insights for firms on how to adopt environmentally friendly strategies, and for policy makers to improve the design of pollutant-abatement instruments, accounting for environmental challenges.

Since the seminal work of Pittman (1983), the literature on modeling production technologies that account for bad outputs has developed into two main frameworks: one involving parametric methods (such as stochastic frontier analysis, SFA; Aigner et al., 1977), and one based on nonparametric methods (such as data envelopment analysis, DEA; Charnes et al., 1978; Banker et al., 1984). The present study relies on data envelopment techniques because they are flexible and do not impose restrictive assumptions on the parametric specification of the technology, nor on the distribution of environmental inefficiency.<sup>1</sup> Using these alternative frameworks, many different approaches have been proposed to assess environmental efficiency of production units. Lauwers (2009) classified these approaches into three groups. The first group concerns environmentally adjusted production efficiency models, in which undesirable outputs are incorporated into the production technology. In general, two main branches of studies within this group can be distinguished: (i) treating bad outputs as strong (free) disposable inputs (Haynes et al., 1993; Hailu and Veeman, 2001)<sup>2</sup> or (ii) treating bad outputs as weakly disposable outputs and assuming the null-jointness of both bad and good outputs (Färe et al., 1986; Färe et al., 1989).<sup>3</sup> The second group of studies consists of frontier eco-efficiency models (Korhonen and Luptacik, 2004; Kuosmanen and Kortelainen, 2005), which do not follow axiomatic production efficiency frameworks, but relate aggregate ecological outcomes with economic outcomes only. In other words, eco-efficiency is measured either through minimization of environmental outcomes given economic outcomes (for

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<sup>1</sup> See Tyteca (1996) for an exposition of early models within the non-parametric approach based on the output, input, and hyperbolic distance functions, which were subsequently implemented in a parametric framework by Cuesta, Lovell, and Zofio (2009).

<sup>2</sup> Free disposability of inputs implies that a reduction (increase) in inputs cannot increase (decrease) the output.

<sup>3</sup> Weak disposability of bad outputs implies that their production can only be reduced at the expense of reducing other (good) outputs. Null-jointness implies that if zero bad outputs are produced, then zero good outputs are produced as well; that is, there is no “free-lunch” in desirable outputs.

example, value added) or the alternative maximization of economic outcomes given the environmental outcomes. The third group of studies is based on the introduction of the materials balance principle into production models (Lauwers and Van Huylenbroeck, 2003; Coelli et al., 2007). The materials balance principle states that flows into and out of the environment are equal, linking the raw materials used in the production system to outputs, both intended and residual ones.

While these three groups of approaches are currently in use, their principles have been heavily debated. The branches of studies assuming bad outputs as free disposable inputs or weakly disposable outputs have confronted each other (see, for example, the discussion between Hailu and Veeman (2001), Färe and Grosskopf (2003) and Hailu (2003)). Further, the main criticisms of these studies are inconsistency with physical laws or violating the materials balance principle (Coelli et al., 2007; Murty et al., 2012). Eco-efficiency models have been criticized mainly for their incomplete characterization of the production process (Dakpo et al., 2016). Finally, critics of the materials balance approach have noted that it does not specify how bad outputs are generated, focuses mainly on material inputs, and requires all variables to be measured in the same measurement unit (Førsund, 2009; Hoang and Rao, 2010; Murty et al., 2012). As a result, many subsequent extensions, as well as empirical applications, have followed one of these three diverging approaches (see, for example, Reinhard et al., 2000; Mahlberg and Sahoo, 2001 for the first approach; Pérez Urdiales et al., 2016; Picazo-Tadeo et al., 2011 for the second approach; and Welch and Barnum, 2009; and Hampf and Rødseth, 2015 for the third approach).

Dakpo et al.'s (2016) recent survey of environmental efficiency studies extended the Lauwers (2009) classification into the fourth, most recent, category of by-production models, which are based on the idea of defining two subtechnologies in parallel: one that generates good outputs and a second that generates bad outputs. This approach was introduced by Murty et al. (2012) and, as a consistent and relatively new approach, its empirical applications are flourishing (e.g., Dakpo et al., 2017; Arjomandi et al., 2018; Ray et al., 2018) as are its extensions (e.g., Serra et al., 2014; Lozano, 2015; Dakpo, 2016; Førsund, 2018).

Regardless the modeling approach under the four listed categories, a common feature of all previous studies is that they are only capable of measuring technical efficiency by focusing on the technological side of the production process, while neglecting the measurement of environmental efficiency from an economic perspective. The determination of economic efficiency is important from a managerial standpoint focused on market-oriented performance. Managers are interested in increasing performance not only in physical terms by taking

advantage of the best technology available, but also by realizing the economic gains associated with allocative efficiency improvements; that is, the choice of optimal output and input mixes, leading to either maximum profit, revenue, or minimum cost. In the current framework including undesirable outputs, economic efficiency not only relates to the private objectives listed above, but must be extended to the social cost associated to the by-production of undesirable outputs. Indeed, the economic damage associated with their production, represented by a social cost function, shows how their production is detrimental to the economy. Yet, the existing models fail to take this step forward and internalize the negative economic effects associated to their by-production. In other words, they only consider the technological side, while it still remains an externality from an economic perspective.

This study enhances these models by introducing a measure of environmental economic inefficiency that includes undesirable outputs and implements them from theoretical and empirical perspectives. To fill in the gap in the literature we postulate a comprehensive framework that is consistent with the economic behavior of organizations in their attempt to maximize revenue, but also accounts for the environmental inefficiency that results from the failure to minimize the economic cost associated to environmental damage. This results in the definition of an “environmental profit function” that maximizes the difference between private (market) revenue less social (environmental) cost, using the prices of good and bad outputs.<sup>4</sup> Hence, we develop a framework that is capable of balancing private gains (revenue) and social losses (cost) into a measure of economic inefficiency that can be decomposed according to technical and allocative criteria. Furthermore, within our framework we show how to decompose overall profit inefficiency into desirable (marketed output) inefficiency, and eco-damage inefficiency.

In this regard, we define the DEA programs that allow the empirical implementation of our novel approach.<sup>5,6</sup> Our point of departure is the by-production model introduced by Murty

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<sup>4</sup> The model can be easily enhanced to include the minimization of inputs cost, but instead we keep the definition of “environmental profit inefficiency” as a trade-off between private revenue and social cost.

<sup>5</sup> Brännlund et al. (1995) measured profit inefficiency under a quota system and the production of undesirable outputs by DEA models. However, they did not use prices for weighting the negative externalities and do not decompose profit inefficiency into its drivers, something that we will do in this paper. Additionally, we note that Pham and Zelenyuk (2018) defined revenue inefficiency in the banking industry accounting for nonperforming loans (NPLs), which are modeled as undesirable outputs under the approach of weak disposability. However, the model is internal to the firm (that is, private revenue), as it does not include environmental indicators, while they do not implement it empirically.

<sup>6</sup> Also, Coelli et al. (2007) used the materials balance approach to estimate both environmental efficiency and cost efficiency separately, but they did not relate them to estimate an overall measure of cost efficiency incorporating environmental factors. The studies of Welch and Barnum (2009), Nguyen et al. (2012) and Hoang and Alauddin (2012) are similar to that of Coelli et al. (2007). Although other studies invoke the concept of revenue inefficiency in the context of production with undesirable outputs, they do so for the purpose of estimating shadow prices

et al. (2012), as it represents the most recent extension of previous approaches and can arguably be seen as a generalization that, by considering two independent technologies for desirable and undesirable outputs, avoids some of their inconsistencies (namely, the multiplicity of optimal combinations of desirable and undesirable outputs for a given level of inputs, and erroneously signed marginal rates of transformation – shadow prices – between outputs and inputs). Nevertheless, our model could be easily particularized for previous approaches.<sup>7</sup> We also consider recent qualifications of the original by-production model by Dakpo (2016) and Førsund (2018).<sup>8</sup>

We demonstrate the practical usefulness of our newly developed methodology through an application to state-level data of the United States agricultural sector. Agriculture involves the production of not only good outputs such as primary food commodities, but also of bad outputs related with, for example, the need for fuel, the usage of pesticides, fertilizers and other agriculture chemicals, or the management of manure (Skinner et al., 1997; Reinhard et al., 1999, 2000). Examples of bad outputs associated to these polluting inputs in agriculture are greenhouse gas emissions, pesticide and nitrogen leaching and runoff, risk to human health and fish from exposure to pesticides and fertilizers, etc. (see Ball et al., 2001; Kellog et al., 2002; Dakpo et al., 2017). In the empirical application we are capable of considering two of these bad outputs: CO<sub>2</sub> emissions and pesticide exposures.

The remainder of this paper is structured as follows. The next section reviews the by-production models of technical inefficiency and introduces their mathematical underpinnings. The subsequent section develops our extension allowing the measurement of economic (profit) inefficiency. We then discuss our empirical application, briefly commenting the dataset and presenting the results. Conclusions are drawn in the final section.

## 2. The by-production models

Pitmann (1983) and Färe et al. (1986) initiated the asymmetric modeling of outputs when measuring efficiency depending on their nature, increasing those that are market-oriented

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between good and bad outputs. Examples of such studies are Färe et al. (2005, 2006). In addition, productivity change analyses accounting for bad outputs have been also undertaken, but without considering the economic side (Ball et al., 2005).

<sup>7</sup> Details on the characteristics of the by-production approach are presented in the next section.

<sup>8</sup> Although we are aware of other methodological developments that rely on the by-production model, such as Serra et al. (2014) or Lozano (2015), we have not considered them since their general idea is to mix the by-production approach with other efficiency frameworks, and not the modification of the model *per se*. Hence, if applied, their results would not be comparable to those of the original by-production model.

while reducing those that are detrimental to the environment. A key question is how to axiomatically model the production technology when calculating technical efficiency through distance functions. Most particularly, as commented in the introduction to this paper, should the axioms underlying the production technology reflect their strong or weak disposability, and eventually, be modeled as outputs or as if they were inputs? Among the existing approaches for dealing with undesirable outputs and efficiency, the by-production model introduced by Murty and Russell (2002) and Murty et al. (2012) is currently considered a preferred option (for applications in agriculture see, for example, Serra et al., 2014, and Dakpo et al., 2017).

The by-production approach posits that complex production systems are made up of several independent processes (Frisch, 1965). In this model, the technology can be separated into sets of sub-technologies; one for the production of good outputs and one for the generation of bad outputs. The “global” technology implies interactions between several separate sub-technologies. Førsund (2018) and Murty and Russell (2018) recently classified the by-production approach among the multi-equation modeling approaches and argued that an important advantage of this approach is that it represents pollution-generating technologies by accounting for the Material Balance Principle, thereby satisfying the laws of thermodynamics. Additionally, as Murty et al. (2012) remarked, the by-production model avoids two inconsistencies of previous approaches. In particular, several technical efficiency combinations of good and bad outputs, with varying levels of bad output, could be possible when holding (polluting and non-polluting) input quantities fixed. However, in the absence of abatement activities implemented by the firm, this type of combination is contrary to the phenomenon of by-production, since by-production implies that, at fixed levels of inputs, there is only one level of pollution at the frontier of the production possibility set. Moreover, it is possible to observe a negative trade-off between the inputs associated with pollution, like fuel, and their associated bad output, such as CO<sub>2</sub>, which represents a clear inconsistency (more fuel but less CO<sub>2</sub>). These are the reasons why the by-production approach is utilized in the current study to introduce the concept of environmental economic inefficiency taking market prices into account.

In order to briefly review the standard by-production approach, let us formally define  $x \in R_+^n$  as a vector of inputs,  $y \in R_+^m$  as a vector of good outputs,  $z \in R_+^{m'}$  as a vector of pollutants, and let us assume that  $p$  DMUs have been observed. Murty et al. (2012) presented their model by splitting the input vector into two groups: non-polluting inputs,  $x_1 \in R_+^{n_1}$  and

pollution-generating inputs,  $x_2 \in R_+^{n_2}$ , with  $n_1 + n_2 = n$ .<sup>9</sup> The first set could comprise land, labor, and so on, while the second set, in the context of our empirical application on agriculture, consists of inputs like fuel, fertilizers, and pesticides, which produce certain pollutants as by-products, such as CO<sub>2</sub> emissions and pesticide exposures. In this way, the ‘global’ technology, denoted by  $T$ , is the intersection of two sub-technologies,  $T_1$  and  $T_2$ . Whereas  $T_1$  is the standard production technology with only good outputs,  $T_2$  represents the production of bad outputs. In the model by Murty et al. (2012), both technologies are linked through the level of the polluting inputs.

In the non-parametric framework of DEA, the two sub-technologies may be expressed mathematically under variable returns to scale (VRS) as:

$$T_1 = \left\{ (x_1, x_2, y, z) \geq 0 : \sum_{d=1}^p \lambda_d x_{1d} \leq x_1, \sum_{d=1}^p \lambda_d x_{2d} \leq x_2, \sum_{d=1}^p \lambda_d y_d \geq y, \sum_{d=1}^p \lambda_d = 1, \lambda_d \geq 0 \right\}, \quad (1)$$

$$T_2 = \left\{ (x_1, x_2, y, z) \geq 0 : \sum_{d=1}^p \mu_d x_{2d} \geq x_2, \sum_{d=1}^p \mu_d z_d \leq z, \sum_{d=1}^p \mu_d = 1, \mu_d \geq 0 \right\}. \quad (2)$$

With  $T = T_1 \cap T_2$ .

Note that the sub-technologies are defined with two different intensity variables:  $\lambda$  and  $\mu$ . Additionally, as Murty et al. (2012) highlighted,  $T_1$  satisfies the standard free-disposability property of inputs (pollutant and non-pollutant) and the good output. On the pollution side, the bad outputs satisfy the assumption of costly disposability, which implies the possibility of observing inefficiency in the generation of pollution.

Regarding the measurement of technical efficiency, Murty et al. (2012) showed that some conventional approaches, like the hyperbolic and directional distance function defined on  $T = T_1 \cap T_2$ , are inadequate in the context of by-production. We use the term “output-oriented” in this context because these distance functions measure efficiency with respect to both good and bad outputs simultaneously. In this way, the weakness is due to the fact that the two aforementioned measures use the same coefficient (decision variable) for determining efficiency both in  $T_1$  for the good outputs and  $T_2$  for the bad outputs. This implies that it is possible to reach the efficiency frontier for some of the sub-technologies, but the observation can fall short of achieving the frontier of the other one. For consistency, efficiency in the by-

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<sup>9</sup> Ayres and Kneese (1969) proposed these two same groups when introducing the materials balance principle to economists.

production approach requires models that project the assessed observations onto *both* the efficient frontier of  $T_1$  and the efficient frontier of  $T_2$ .

The abovementioned drawbacks of standard approaches motivated Murty et al. (2012) to propose a different measure for dealing with good and bad outputs under by-production. For DMU<sub>0</sub>, this measure is good-output-specific and bad-output-specific, and is based on the index previously defined by Färe et al. (1985):

$$\begin{aligned}
\min \quad & \frac{1}{2} \left[ \underbrace{\frac{1}{m} \sum_{j=1}^m \theta_j}_{\text{standard efficiency}} + \underbrace{\frac{1}{m'} \sum_{k=1}^{m'} \gamma_k}_{\text{environmental efficiency}} \right] \\
s.t. \quad & \sum_{d=1}^p \lambda_d x_{id} \leq x_{i0}, \quad i = 1, \dots, n \\
& \sum_{d=1}^p \lambda_d y_{jd} \geq y_{j0} / \theta_j, \quad j = 1, \dots, m \\
& \sum_{d=1}^p \lambda_d = 1, \\
& \sum_{d=1}^p \mu_d x_{id} \geq x_{i0}, \quad i = n_1 + 1, \dots, n \\
& \sum_{d=1}^p \mu_d z_{kd} \leq \gamma_k z_{k0}, \quad k = 1, \dots, m' \\
& \sum_{d=1}^p \mu_d = 1, \\
& \theta_j \leq 1, \quad j = 1, \dots, m \\
& \gamma_k \leq 1, \quad k = 1, \dots, m' \\
& \lambda_d \geq 0, \mu_d \geq 0, \quad d = 1, \dots, p
\end{aligned} \tag{3}$$

The optimal value of (3) coincides with the mean of the standard good-output-oriented efficiency and the environmental bad-output-oriented efficiency. Note also that the above model is separable. In this case, this means that the optimal value can be determined as the

mean of a model that minimizes  $\frac{1}{m} \sum_{j=1}^m \theta_j$  on  $T_1$  and a model that minimizes  $\frac{1}{m'} \sum_{k=1}^{m'} \gamma_k$  on  $T_2$ :



$$\begin{array}{ll}
\min & \frac{1}{m} \sum_{j=1}^m \theta_j \\
s.t. & \sum_{d=1}^p \lambda_d x_{id} \leq x_{i0}, \quad i = 1, \dots, n \\
& \sum_{d=1}^p \lambda_d y_{jd} \geq y_{j0} / \theta_j, \quad j = 1, \dots, m \\
& \sum_{d=1}^p \lambda_d = 1, \\
& \theta_j \leq 1, \quad j = 1, \dots, m \\
& \lambda_d \geq 0, \quad d = 1, \dots, p \\
\min & \frac{1}{m'} \sum_{k=1}^{m'} \gamma_k \\
s.t. & \sum_{d=1}^p \mu_d x_{id} \geq x_{i0}, \quad i = n_1 + 1, \dots, n_2 \\
& \sum_{d=1}^p \mu_d z_{kd} \leq \gamma_k z_{k0}, \quad k = 1, \dots, m' \\
& \sum_{d=1}^p \mu_d = 1, \\
& \gamma_k \leq 1, \quad k = 1, \dots, m' \\
& \mu_d \geq 0, \quad d = 1, \dots, p
\end{array} \tag{4}$$

It is worth mentioning that the recent paper by Førsund (2018) argued that non-pollution causing inputs should also be included in technology  $T_2$  given that substitution between the two groups of causing inputs can help mitigate the pollution. Dakpo et al. (2017) indicated that some additional constraints must be added to the by-production approach of Murty et al. (2012) in order to guarantee that the projection points for input dimensions are the same in  $T_1$  and  $T_2$ . In particular, the condition that should be incorporated to model (3) would be:

$\sum_{d=1}^p \lambda_d x_{id} = \sum_{d=1}^p \mu_d x_{id}, \forall i$ . Hereafter, we use  $T^M$  to denote the production possibility set defined as the intersection of  $T_1$  and  $T_2$  in (1) and (2), respectively, as a way of highlighting that the definition of this technology corresponds to the original proposal of Murty et al. (2012). In the same way, we use  $T^D$  to denote the production possibility set defined from the original by-production approach but incorporating the constraints  $\sum_{d=1}^p \lambda_d x_{id} = \sum_{d=1}^p \mu_d x_{id}, \forall i$ , as pointed out by Dakpo et al. (2017). Finally, we will utilize  $T^{MF}$  to denote the production possibility set defined by Murty et al. (2012) but incorporating non-polluting inputs in technology  $T_2$ . Likewise,  $T^{DF}$  denotes the production possibility set à la Dakpo et al. (2017) but again considering non-polluting inputs in the definition of technology  $T_2$ .

To introduce our economic inefficiency model we extend the state-of-the-art of by-production approach (Murty et al. 2012, Dakpo et al. 2017 and Førsund, 2018) by incorporating information on market prices. To do that, we resort to duality theory following Chambers et al. (1998), and, more recently, Aparicio et al. (2015), Aparicio et al. (2016a), and Aparicio et al. (2016b). In particular, we recall relevant duality results concerning the directional distance function. Consequently, we start out by defining this type of measure from an output-oriented

perspective in the context of by-production. Under the viewpoint introduced by Murty et al. (2012), we need a measure that allows us to project the assessed observations onto the efficient frontiers of  $T_1$  and  $T_2$  simultaneously. In this way, the “by-production” directional output-oriented distance function for the Murty et al. (2012) approach with directional vector  $g = (0, y_0, z_0)$  is defined as follows:

$$\begin{aligned} \bar{B}(x_0, y_0, z_0; T^M) = & \max \quad \delta^{T_1} \beta^{T_1} + \delta^{T_2} \beta^{T_2} \\ \text{s.t.} \quad & \sum_{j=1}^p \lambda_{j0} x_{ij} \leq x_{i0}, \quad i = 1, \dots, n_1 \quad (5.1) \\ & \sum_{j=1}^p \lambda_{j0} x_{ij} \leq x_{i0}, \quad i = n_1 + 1, \dots, n_2 \quad (5.2) \\ & -\sum_{j=1}^p \lambda_{j0} y_{rj} + \beta^{T_1} y_{r0} \leq -y_{r0}, \quad r = 1, \dots, m \quad (5.3) \\ & \sum_{j=1}^p \lambda_{j0} = 1, \quad (5.4) \quad (5) \\ & -\sum_{j=1}^p \mu_{j0} x_{ij} \leq -x_{i0}, \quad i = n_1 + 1, \dots, n_2 \quad (5.5) \\ & \sum_{j=1}^p \mu_{j0} z_{kj} + \beta^{T_2} z_{k0} \leq z_{k0}, \quad k = 1, \dots, m' \quad (5.6) \\ & \sum_{j=1}^p \mu_{j0} = 1, \quad (5.7) \\ & \beta^{T_1}, \beta^{T_2}, \lambda_{j0}, \mu_{j0} \geq 0 \quad (5.8) \end{aligned}$$

The exogenous coefficients  $\delta_1 \geq 0$  and  $\delta_2 \geq 0$ ,  $\delta_1 + \delta_2 = 1$ , are weights that are pre-fixed by the corresponding decision maker (manager, politician, regulator, etc.) to reflect the relative importance of the standard (traditional) way of producing versus the new and clean paradigm for generating goods and services. Additionally, its linear dual is:

$$\begin{aligned}
\bar{B}(x_0, y_0, z_0; T^M) = \min & \sum_{i=1}^{n_1} v_{i0}^1 x_{i0} + \sum_{i=n_1+1}^{n_2} v_{i0}^1 x_{i0} - \sum_{r=1}^m u_{r0}^1 y_{r0} + \alpha_0^1 + \\
& - \sum_{i=n_1+1}^{n_2} v_{i0}^2 x_{i0} + \sum_{k=1}^{m'} u_{k0}^2 z_{k0} + \alpha_0^2 \\
s.t. & \sum_{i=1}^{n_1} v_{i0}^1 x_{ij} + \sum_{i=n_1+1}^{n_2} v_{i0}^1 x_{ij} - \sum_{r=1}^m u_{r0}^1 y_{rj} + \alpha_0^1 \geq 0, \quad j=1, \dots, p \quad (6.1) \\
& \sum_{r=1}^m u_{r0}^1 y_{r0} \geq \delta^{T_1}, \quad (6.2) \quad (6) \\
& - \sum_{i=n_1+1}^{n_2} v_{i0}^2 x_{ij} + \sum_{k=1}^{m'} u_{k0}^2 z_{kj} + \alpha_0^2 \geq 0, \quad j=1, \dots, p \quad (6.3) \\
& \sum_{k=1}^{m'} u_{k0}^2 z_{k0} \geq \delta^{T_2}, \quad (6.4) \\
& v_{i0}^1, v_{i0}^2, u_{r0}^1, u_{k0}^2 \geq 0, \quad (6.5) \\
& \alpha_0^1, \alpha_0^2 \text{ free} \quad (6.6)
\end{aligned}$$

Finally, to complete this opening section, we recall the first additive measure and decomposition of economic inefficiency proposed in the literature. We refer to the Nerlovian profit inefficiency measure, which can be decomposed into technical inefficiency (the directional distance function) and a residual term interpreted as allocative inefficiency (Chambers et al., 1998).

In the standard production context, considering private revenue and cost only, and given a vector of input and output prices  $(w, p) \in R_+^{m+s}$  and technology  $T$ , the profit function  $\Pi$  is

defined as  $\Pi_T(w, p) = \max_{x, y} \left\{ \sum_{r=1}^s p_r y_r - \sum_{i=1}^m w_i x_i : (x, y) \in T \right\}$ . Profit inefficiency *à la* Nerlove for

DMU<sub>0</sub> is defined as optimal profit (that is, the value of the profit function at market prices)

minus observed profit, both normalized by the value of a reference vector  $g = (g^x, g^y) \in R_+^{m+s}$

$$: \frac{\Pi_T(W, P) - \left( \sum_{r=1}^s p_r y_{r0} - \sum_{i=1}^m w_i x_{i0} \right)}{\sum_{r=1}^s p_r g_r^y + \sum_{i=1}^m w_i g_i^x}. \text{ Additionally, Chambers et al. (1998) showed that profit}$$

inefficiency may be decomposed into technical inefficiency and allocative inefficiency, where

technical inefficiency corresponds to the directional distance function

$$\bar{D}_T(x_0, y_0; g^x, g^y) = \max \{ \beta : (x_0 - \beta g^x, y_0 + \beta g^y) \in T \} :$$

$$\frac{\Pi_T(w, p) - \left( \sum_{r=1}^s p_r y_{r0} - \sum_{i=1}^m w_i x_{i0} \right)}{\sum_{r=1}^s p_r g_r^y + \sum_{i=1}^m w_i g_i^x} = \bar{D}_T(x_0, y_0; g^x, g^y) + AI_T^N(x_0, y_0; w, p; g^x, g^y) \quad (7)$$

### 3. Measuring economic inefficiency with by-production models in DEA

#### 3.1. Economic inefficiency model considering Murty et al.'s (2012) technology

We will first introduce some notation and definitions. Given a fixed level of input  $x_0 = (x_{10}, \dots, x_{n0}) \in R_+^n$  and a fixed level of bad output  $z_0 = (z_{10}, \dots, z_{m'0}) \in R_+^{m'}$ , let us also define as  $r(x_0, z_0, q, T)$  the maximum feasible revenue given the output price vector  $q = (q_1, \dots, q_m) \in R_{++}^m$  :

$$r(x_0, z_0, q, T) = \sup_y \left\{ \sum_{r=1}^m q_r y_r : (x_0, y, z_0) \in T = T_1 \cap T_2 \right\} = \sup_y \left\{ \sum_{r=1}^m q_r y_r : (x_0, y, z_0) \in T_1 \cap T_2 \right\}. \quad (8)$$

Under Murty et al.'s (2012) approach, this optimization problem can be always solved independently on  $T_1$  and  $T_2$ . Therefore, as for  $T_1$ , maximum feasible revenue given the output price vector  $q = (q_1, \dots, q_m) \in R_{++}^m$  may be determined by:

$$r(x_0, z_0, q, T^M) = \sup_y \left\{ \sum_{r=1}^m q_r y_r : (x_0, y, z_0) \in T^M \right\} = \sup_y \left\{ \sum_{r=1}^m q_r y_r : (x_0, y, z_0) \in T_1 \right\}. \quad (9)$$

Next, we explicitly show how the value of  $r(x_0, z_0, q, T^M)$  can be calculated in DEA under the by-production framework (see Ray, 2004):

$$\begin{aligned}
r(x_0, z_0, q, T^M) &= \max_{\lambda, y} \sum_{r=1}^s q_r y_r \\
\text{s.t.} \quad & \sum_{j=1}^p \lambda_j x_{ij} \leq x_{i0}, \quad i = 1, \dots, n_1 \quad (10.1) \\
& \sum_{j=1}^p \lambda_j x_{ij} \leq x_{i0}, \quad i = n_1 + 1, \dots, n_2 \quad (10.2) \\
& -\sum_{j=1}^p \lambda_j y_{rj} + y_r \leq 0, \quad r = 1, \dots, m \quad (10.3) \\
& \sum_{j=1}^p \lambda_j = 1, \quad (10.4) \\
& \lambda_j \geq 0, \quad j = 1, \dots, p \quad (10.5) \\
& y_r \geq 0, \quad r = 1, \dots, m \quad (10.6)
\end{aligned} \tag{10}$$

The dual program of (10) is (11):<sup>10</sup>

$$\begin{aligned}
\min_{c, d, \psi} &= \sum_{i=1}^{n_1} c_i x_{i0} + \sum_{i=n_1+1}^{n_2} c_i x_{i0} + \psi \\
\text{s.t.} \quad & \sum_{i=1}^{n_1} c_i x_{ij} + \sum_{i=n_1+1}^{n_2} c_i x_{ij} - \sum_{r=1}^m d_r y_{jr} + \psi \geq 0, \quad j = 1, \dots, p \quad (11.1) \\
& d_r \geq q_r, \quad r = 1, \dots, m \quad (11.2) \\
& c_i \geq 0 \quad (11.3)
\end{aligned} \tag{11}$$

To evaluate economic loss due to revenue inefficiency, in the context of the directional output distance functions, Färe and Primont (2006) proved that a normalized measure of

revenue inefficiency, in particular the ratio  $\frac{r(x_0, q, T) - \sum_{r=1}^m q_r y_{r0}}{\sum_{r=1}^m q_r g_r}$ , may be decomposed into

technical inefficiency,  $\bar{D}_o(x^0, y^0; g)$ , plus a residual term interpreted as allocative inefficiency in the Farrell tradition, where  $r(x_0, q, T)$  and  $\bar{D}_o(x^0, y^0; g)$  denote the ‘standard’ revenue function and directional output distance function, respectively, and  $g$  is the corresponding reference directional vector.

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<sup>10</sup> Actually, the dual program of model (10) has an additional set of non-negativity constraints for the decision variables  $d_r$ ,  $r = 1, \dots, m$ . However, this set of constraints is redundant if we consider (11.2) and  $q_r > 0$ ,  $r = 1, \dots, m$ .

Likewise, we can introduce cost efficiency following the same rationale, and based on the cost function. However, in our context we are interested in “social/environmental” cost functions rather than private costs, representing a measure of the (monetary) minimal damage caused by the production of undesirable outputs. The cost function represents a “monetized metric” of the ecological footprint such as the social cost of carbon (SCC); for example, the damage per ton of CO<sub>2</sub> (see Pearce et al., 1996). Correspondingly, an observation is economically inefficient in environmental terms if, given the amount of undesirable outputs produced, it causes larger damage than that represented by the minimum “social/environmental” cost function (either as a result of technical or allocative inefficiencies). Let us assume that it is possible to observe or estimate prices for the undesirable outputs:  $w = (w_1, \dots, w_m) \in R_{++}^m$ . Under Murty et al.’s (2012) approach, the eco-damage function will be non-parametrically determined directly from  $T_2$  as follows.

$$D(x_0, y_0, w, T^M) = \min_{\mu, z} \sum_{k=1}^{m'} w_k z_k$$

*s.t.*

$$\sum_{j=1}^p \mu_j x_{ij} \geq x_{i0}, \quad i = n_1 + 1, \dots, n_2 \quad (12.1)$$

$$-\sum_{j=1}^p \mu_j z_{kj} + z_k \geq 0, \quad k = 1, \dots, m' \quad (12.2)$$

$$\sum_{j=1}^p \mu_j = 1, \quad (12.3)$$

$$\mu_j \geq 0, \quad j = 1, \dots, p \quad (12.4)$$

$$z_k \geq 0, \quad k = 1, \dots, m' \quad (12.5)$$

(12)

The dual program of (12) is (13):

$$\max_{e, f, \chi} \sum_{i=n_1+1}^{n_2} e_{i0} x_{i0} - \chi_0$$

*s.t.*

$$\sum_{i=n_1+1}^{n_2} e_{i0} x_{ij} - \sum_{k=1}^{m'} f_{k0} z_{kj} - \chi_0 \leq 0, \quad j = 1, \dots, p \quad (13.1)$$

$$f_{k0} \leq w_k, \quad r = 1, \dots, m \quad (13.2)$$

$$e_{i0}, f_{k0} \geq 0 \quad (13.3)$$

(13)

We now derive, by duality, a normalized measure of economic inefficiency and show how it can be decomposed into (desirable) revenue inefficiency and eco-damage inefficiency. In order to do that, we first prove the following technical proposition.

**Proposition 1.** Let  $\delta^{T_1}, \delta^{T_2} > 0$ . Then,

$$\inf_{t,h} \left\{ r(x_0, z_0, t, T^M) - \sum_{r=1}^m t_r y_{r0} + \sum_{k=1}^{m'} h_k z_{k0} - D^{T_2}(x_0, y_0, h, T^M) : \min \left\{ \frac{\sum_{r=1}^m t_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} h_k z_{k0}}{\delta^{T_2}} \right\} \geq 1 \right\} \\ \geq \bar{B}(x_0, y_0, z_0; T^M).$$

*Proof.* Let  $x_0 \in R_+^n$ ,  $y_0 \in R_+^m$ ,  $z_0 \in R_+^{m'}$  and let  $t \in R_+^m$ ,  $h \in R_+^{m'}$  such that

$$\min \left\{ \frac{\sum_{r=1}^m t_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} h_k z_{k0}}{\delta^{T_2}} \right\} \geq 1. \text{ Let } (c_0^*, d_0^*, \psi_0^*) \text{ be an optimal solution of (11) and let } (e_0^*, f_0^*, \chi_0^*)$$

be an optimal solution of (13) when  $x_0 \in R_+^n$ ,  $y_0 \in R_+^m$ ,  $z_0 \in R_+^{m'}$  and  $t \in R_+^m$  (acting as  $q$ ),  $h \in R_+^{m'}$  (acting as  $w$ ) are taken as arguments. We will prove that  $(v_0^1, u_0^1, \alpha_0^1, v_0^2, u_0^2, \alpha_0^2) = (c_0^*, t, \psi_0^*, e_0^*, h, \chi_0^*)$  is a feasible solution of (6). Constraints (6.5) and

(6.6) are trivially satisfied. Regarding (6.1),  $\sum_{i=1}^{n_1} c_{i0}^* x_{ij} + \sum_{i=n_1+1}^{n_2} c_{i0}^* x_{ij} - \sum_{r=1}^m t_r y_{rj} + \psi_0^* \underset{\text{by (11.2)}}{\geq}$

$\sum_{i=1}^{n_1} c_{i0}^* x_{ij} + \sum_{i=n_1+1}^{n_2} c_{i0}^* x_{ij} - \sum_{r=1}^m d_{r0}^* y_{rj} + \psi_0^* \underset{\text{by (11.1)}}{\geq} 0$ . As for (6.2),  $\frac{\sum_{r=1}^m t_r y_{r0}}{\delta^{T_1}} \geq 1$  since

$\frac{\sum_{r=1}^m t_r y_{r0}}{\delta^{T_1}} \geq \min \left\{ \frac{\sum_{r=1}^m t_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} h_k z_{k0}}{\delta^{T_2}} \right\} \geq 1$ . Therefore,  $\sum_{r=1}^m t_r y_{r0} \geq \delta^{T_1}$ . In the same way, it is

possible to prove that (6.3) and (6.4) are also satisfied. In particular, constraint (6.3) holds by (13.1) and (13.2). Consequently,  $(c_0^*, t, \psi_0^*, e_0^*, h, \chi_0^*)$  is a feasible solution of (6). Regarding the objective function of (6) evaluated at this point,  $\bar{B}(x_0, y_0, z_0; T^M) \leq$

$$\sum_{i=1}^{n_1} c_{i0}^* x_{i0} + \sum_{i=n_1+1}^{n_2} c_{i0}^* x_{i0} - \sum_{r=1}^m t_r y_{r0} + \psi_0^* - \sum_{i=n_1+1}^{n_2} e_{i0}^* x_{i0} + \sum_{k=1}^{m'} h_k z_{k0} + \chi_0^* =$$

$r(x_0, z_0, t, T^M) - \sum_{r=1}^m t_r y_{r0} + \sum_{k=1}^{m'} h_k z_{k0} - D(x_0, y_0, h, T^M)$ , since models (10) and (11) have the

same optimal value and models (12) and (13) also have the same optimal value. In this way,

$\bar{B}(x_0, y_0, z_0; T^M)$  is a lower bound of the set

$$\left\{ \sum_{i=1}^{n_1} c_{i0}^*(t) x_{i0} + \sum_{i=n_1+1}^{n_2} c_{i0}^*(t) x_{i0} - \sum_{r=1}^m t_r y_{r0} + \psi_0^*(t) - \sum_{i=n_1+1}^{n_2} e_{i0}^*(h) x_{i0} + \sum_{k=1}^{m'} h_k z_{k0} + \chi_0^*(h) : \forall (t, h) \in S_0 \right\},$$

where  $(c_0^*(t), d_0^*(t), \psi_0^*(t))$  is any optimal solution of (11) when  $q = t$ ,

$(e_0^*(h), f_0^*(h), \chi_0^*(h))$  is any optimal solution of (13) when  $w = h$ , and

$$\left\{ \sum_{i=1}^{n_1} c_{i0}^*(t) x_{i0} + \sum_{i=n_1+1}^{n_2} c_{i0}^*(t) x_{i0} - \sum_{r=1}^m t_r y_{r0} + \psi_0^*(t) - \sum_{i=n_1+1}^{n_2} e_{i0}^*(h) x_{i0} + \sum_{k=1}^{m'} h_k z_{k0} + \chi_0^*(h) : \forall (t, h) \in S_0 \right\}$$

$$= \left\{ r(x_0, z_0, t, T^M) - \sum_{r=1}^m t_r y_{r0} + \sum_{k=1}^{m'} h_k z_{r0} - D(x_0, y_0, h, T^M) : \forall (t, h) \in S_0 \right\}, \quad \text{with}$$

$$S_0 = \left\{ (q, w) \in R_+^{m+m'} : \min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\} \geq 1 \right\}.$$

the greatest lower bound of that set, we see that

$$\inf_{t, h} \left\{ r(x_0, z_0, t, T^M) - \sum_{r=1}^m t_r y_{r0} + \sum_{k=1}^{m'} h_k z_{r0} - D(x_0, y_0, h, T^M) : (t, h) \in S_0 \right\} \geq \bar{B}(x_0, y_0, z_0; T^M),$$

which is the inequality that we were seeking. ■

Let  $(q, w) \in R_{++}^{m+m'}$  be market prices for good and bad outputs, respectively. Then,

$$(\tilde{q}, \tilde{w}) = \frac{(q, w)}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}} \in S_0 = \left\{ (q, w) \in R_+^{m+m'} : \min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\} \geq 1 \right\}.$$

Consequently, applying Proposition 1, we get

$$r(x_0, z_0, \tilde{q}, T^M) - \sum_{r=1}^m \tilde{q}_r y_{r0} + \sum_{k=1}^{m'} \tilde{w}_k z_{r0} - D(x_0, y_0, \tilde{w}, T^M) \geq$$

$$\inf_{t, h} \left\{ r(x_0, z_0, t, T^M) - \sum_{r=1}^m t_r y_{r0} + \sum_{k=1}^{m'} h_k z_{r0} - D(x_0, y_0, h, T^M) : (q, h) \in S_0 \right\} \quad (14)$$

$$\geq \bar{B}(x_0, y_0, z_0; T^M).$$

Finally, given that  $r(x_0, z_0, t, T^M)$  is a function homogeneous of degree +1 in  $t$  and

$D(x_0, y_0, h, T^M)$  is a function homogeneous of degree +1 in  $h$ , then



$$\frac{\left[ r(x_0, z_0, q, T^M) - \sum_{r=1}^m q_r y_{r0} \right] + \left[ \sum_{k=1}^{m'} w_k z_{k0} - D(x_0, y_0, w, T^M) \right]}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}} \geq \bar{B}(x_0, y_0, z_0; T^M). \quad (15)$$

Note that the left-hand side of (15) may be interpreted as a (normalized) measure of economic environmental inefficiency. Additionally, following Farrell's tradition, the right-hand side can be interpreted as (environmental) technical inefficiency and the residual term associated with closing the inequality could be interpreted as allocative inefficiency. Moreover, it is possible to decompose the left-hand side of (15) into

$$\begin{aligned} & \frac{r(x_0, z_0, q, T^M) - \sum_{r=1}^m q_r y_{r0} + \sum_{k=1}^{m'} w_k z_{k0} - D(x_0, y_0, w, T^M)}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}} = \\ & \underbrace{\hspace{10em}}_{\text{Overall Inefficiency}} \\ & = \underbrace{\frac{r(x_0, z_0, q, T^M) - \sum_{r=1}^m q_r y_{r0}}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}}}_{\text{(Good) Revenue Inefficiency}} + \underbrace{\frac{\sum_{k=1}^{m'} w_k z_{k0} - D(x_0, y_0, w, T^M)}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}}}_{\text{Eco-Damage Inefficiency}}. \end{aligned} \quad (16)$$

However, note that the normalization term used in (15) and (16) – that is,

$$\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\} - \text{depends on two different terms, in contrast to what happens with}$$

respect to the Nerlovian profit inefficiency measure in (7). By analogy with the standard approach based on the directional distance function, we suggest resorting to an endogenous

value for  $\delta^{T_1}$  and, therefore, also for  $\delta^{T_2} = 1 - \delta^{T_1}$ , such that  $\frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}} = \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}}$ . It is easy to

check that this value is  $\delta^{T_1^*} = \frac{\sum_{r=1}^m q_r y_{r0}}{\left( \sum_{r=1}^m q_r y_{r0} + \sum_{k=1}^{m'} w_k z_{k0} \right)}$ .

### 3.2. Economic inefficiency model considering Dakpo et al.'s (2012) approach

We now turn to Dakpo et al.'s (2017) approach. In this case, the projection points in the two subtechnologies for the input dimensions must coincide. The “by-production” directional output distance function under the Dakpo et al. approach is as follows:<sup>11</sup>

$$\begin{aligned} \bar{B}(x_0, y_0, z_0; T^D) = \max \quad & \delta^{T_1} \beta^{T_1} + \delta^{T_2} \beta^{T_2} \\ \text{s.t.} \quad & \sum_{j=1}^p \lambda_{j_0} x_{ij} \leq x_{i_0}, \quad i = 1, \dots, n_1 \quad (17.1) \\ & \sum_{j=1}^p \lambda_{j_0} x_{ij} \leq x_{i_0}, \quad i = n_1 + 1, \dots, n_2 \quad (17.2) \\ & -\sum_{j=1}^p \lambda_{j_0} y_{rj} + \beta^{T_1} y_{r_0} \leq -y_{r_0}, \quad r = 1, \dots, m \quad (17.3) \\ & \sum_{j=1}^p \lambda_{j_0} = 1, \quad (17.4) \\ & -\sum_{j=1}^p \mu_{j_0} x_{ij} \leq -x_{i_0}, \quad i = n_1 + 1, \dots, n_2 \quad (17.5) \\ & \sum_{j=1}^p \mu_{j_0} z_{kj} + \beta^{T_2} z_{k_0} \leq z_{k_0}, \quad k = 1, \dots, m' \quad (17.6) \\ & \sum_{j=1}^p \mu_{j_0} = 1, \quad (17.7) \\ & -\sum_{j=1}^p \lambda_{j_0} x_{ij} + \sum_{j=1}^p \mu_{j_0} x_{ij} \leq 0, \quad i = n_1 + 1, \dots, n_2 \quad (17.8) \\ & \beta^{T_1}, \beta^{T_2}, \lambda_{j_0}, \mu_{j_0} \geq 0. \quad (17.9) \end{aligned} \tag{17}$$

Its linear dual is:

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<sup>11</sup> Constraints (17.2) and (17.5) imply that  $-\sum_{j=1}^p \lambda_{j_0} x_{ij} + \sum_{j=1}^p \mu_{j_0} x_{ij} \geq 0$ , for all  $i = n_1 + 1, \dots, n_2$ . This inequality, together with (17.8), implies  $\sum_{j=1}^p \lambda_{j_0} x_{ij} = \sum_{j=1}^p \mu_{j_0} x_{ij}$  for all  $i = n_1 + 1, \dots, n_2$ , which coincides with the constraint related to Dakpo et al.'s (2017) approach. We prefer to include (17.8) instead of  $\sum_{j=1}^p \lambda_{j_0} x_{ij} = \sum_{j=1}^p \mu_{j_0} x_{ij}$ , for all  $i = n_1 + 1, \dots, n_2$ , because, in this way, the corresponding dual decision variables in model (16) are directly non-negative.

$$\begin{aligned}
\bar{B}(x_0, y_0, z_0; T^D) &= \min && \sum_{i=1}^{n_1} v_{i0}^1 x_{i0} + \sum_{i=n_1+1}^{n_2} v_{i0}^1 x_{i0} - \sum_{r=1}^m u_{r0}^1 y_{r0} + \alpha_0^1 + \\
&&& - \sum_{i=n_1+1}^{n_2} v_{i0}^2 x_{i0} + \sum_{k=1}^{m'} u_{k0}^2 z_{k0} + \alpha_0^2 \\
s.t. &&& \sum_{i=1}^{n_1} v_{i0}^1 x_{ij} + \sum_{i=n_1+1}^{n_2} v_{i0}^1 x_{ij} - \sum_{r=1}^m u_{r0}^1 y_{rj} + \alpha_0^1 + \\
&&& - \sum_{i=n_1+1}^{n_2} \gamma_{i0} x_{ij} \geq 0, \quad j = 1, \dots, p \\
&&& \sum_{r=1}^m u_{r0}^1 y_{r0} \geq \delta^{T_1}, \\
&&& - \sum_{i=n_1+1}^{n_2} v_{i0}^2 x_{ij} + \sum_{k=1}^{m'} u_{k0}^2 z_{kj} + \alpha_0^2 + \sum_{i=n_1+1}^{n_2} \gamma_{i0} x_{ij} \geq 0, \quad j = 1, \dots, p \\
&&& \sum_{k=1}^{m'} u_{k0}^2 z_{k0} \geq \delta^{T_2}, \\
&&& v_{i0}^1, v_{i0}^2, u_{r0}^1, u_{k0}^2, \gamma_{i0} \geq 0, \\
&&& \alpha_0^1, \alpha_0^2 \text{ free.}
\end{aligned} \tag{18}$$

In this context we now define a new support function, representing profit in Dakpo et al.'s model, as  $\Gamma(x_0, q, w, T^D)$ :

$$\begin{aligned}
\Gamma(x_0, q, w, T^D) &= \max && \sum_{r=1}^m q_r y_r - \sum_{k=1}^{m'} w_k z_k \\
s.t. &&& \sum_{j=1}^p \lambda_{j0} x_{ij} \leq x_{i0}, \quad i = 1, \dots, n_1 \\
&&& \sum_{j=1}^p \lambda_{j0} x_{ij} \leq x_{i0}, \quad i = n_1 + 1, \dots, n_2 \\
&&& - \sum_{j=1}^p \lambda_{j0} y_{rj} + y_r \leq 0, \quad r = 1, \dots, m \\
&&& \sum_{j=1}^p \lambda_{j0} = 1, \\
&&& - \sum_{j=1}^p \mu_{j0} x_{ij} \leq -x_{i0}, \quad i = n_1 + 1, \dots, n_2 \\
&&& \sum_{j=1}^p \mu_{j0} z_{kj} - z_k \leq 0, \quad k = 1, \dots, m' \\
&&& \sum_{j=1}^p \mu_{j0} = 1,
\end{aligned} \tag{19}$$

$$-\sum_{j=1}^p \lambda_{j0} x_{ij} + \sum_{j=1}^p \mu_{j0} x_{ij} \leq 0, \quad i = n_1 + 1, \dots, n_2 \quad (19.8)$$

$$y_r, z_k, \lambda_{j0}, \mu_{j0} \geq 0, \quad (19.9)$$

which maximizes the difference between private revenue and eco-damage costs in our by-production context.

The linear dual of (19) is:

$$\begin{aligned} \Gamma(x_0, q, w, T^D) = \min & \sum_{i=1}^{n_1} c_{i0} x_{i0} + \sum_{i=n_1+1}^{n_2} c_{i0} x_{i0} + \psi_0 + \\ & - \sum_{i=n_1+1}^{n_2} e_{i0} x_{i0} + \chi_0 \\ \text{s.t.} & \sum_{i=1}^{n_1} c_{i0} x_{ij} + \sum_{i=n_1+1}^{n_2} c_{i0} x_{ij} - \sum_{r=1}^m d_{r0} y_{rj} + \psi_0 - \sum_{i=n_1+1}^{n_2} a_{i0} x_{ij} \geq 0, \quad (20.1) \\ & j = 1, \dots, p, \quad d_{r0} \geq q_r, \quad (20.2) \\ & - \sum_{i=n_1+1}^{n_2} e_{i0} x_{ij} + \sum_{k=1}^{m'} f_{k0} z_{kj} + \chi_0 + \sum_{i=n_1+1}^{n_2} a_{i0} x_{ij} \geq 0, \quad (20.3) \\ & j = 1, \dots, p, \quad f_{k0} \leq w_k, \quad (20.4) \\ & c_{i0}, d_{r0}, e_{i0}, f_{k0}, a_{i0} \geq 0 \quad (20.5) \\ & \psi_0, \chi_0 \text{ free.} \quad (20.6) \end{aligned} \quad (20)$$

**Proposition 2.** Let  $\delta^{T_1}, \delta^{T_2} > 0$ . Then,

$$\inf_{t, h} \left\{ \Gamma(x_0, t, h, T^D) - \sum_{r=1}^m t_r y_{r0} + \sum_{k=1}^{m'} h_k z_{k0} : \min \left\{ \frac{\sum_{r=1}^m t_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} h_k z_{k0}}{\delta^{T_2}} \right\} \geq 1 \right\} \geq \bar{B}(x_0, y_0, z_0; T^D).$$

*Proof.* Following the same steps than in Proposition 1, we get the desired result. ■

Applying Proposition 2, with market prices  $(q, w)$ , we get the following inequality.

$$\frac{\Gamma(x_0, q, w, T^D) - \left( \sum_{r=1}^m q_r y_{r0} - \sum_{k=1}^{m'} w_k z_{k0} \right)}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}} \geq \bar{B}(x_0, y_0, z_0; T^D). \quad (21)$$

The left-hand side in (21) may be interpreted as a measure of economic environmental inefficiency, which could be decomposed into technical inefficiency (the right-hand side in (21)) and a residual term, interpreted as allocative inefficiency.

### 3.3. Economic inefficiency model considering Førsund's (2018) proposal

Finally, it is possible to incorporate Førsund's (2018) proposal, adapting Murty et al. (2012) and Dakpo et al. (2017). To do this, it is sufficient to include the non-polluting inputs in the subtechnology  $T_2$ . The results of Proposition 1 and 2 are valid for  $\bar{B}(x_0, y_0, z_0; T^{MF})$  and  $\bar{B}(x_0, y_0, z_0; T^{DF})$ . Hence

$$\begin{aligned} \bar{B}(x_0, y_0, z_0; T^{MF}) = \max \quad & \delta^{T_1} \beta^{T_1} + \delta^{T_2} \beta^{T_2} \\ \text{s.t.} \quad & \sum_{j=1}^p \lambda_{j0} x_{ij} \leq x_{i0}, \quad i = 1, \dots, n_1 \quad (22.1) \\ & \sum_{j=1}^p \lambda_{j0} x_{ij} \leq x_{i0}, \quad i = n_1 + 1, \dots, n_2 \quad (22.2) \\ & -\sum_{j=1}^p \lambda_{j0} y_{rj} + \beta^{T_1} y_{r0} \leq -y_{r0}, \quad r = 1, \dots, m \quad (22.3) \\ & \sum_{j=1}^p \lambda_{j0} = 1, \quad (22.4) \quad (22) \\ & -\sum_{j=1}^p \mu_{j0} x_{ij} \leq -x_{i0}, \quad i = 1, \dots, n \quad (22.5) \\ & \sum_{j=1}^p \mu_{j0} z_{kj} + \beta^{T_2} z_{k0} \leq z_{k0}, \quad k = 1, \dots, m' \quad (22.6) \\ & \sum_{j=1}^p \mu_{j0} = 1, \quad (22.7) \\ & \beta^{T_1}, \beta^{T_2}, \lambda_{j0}, \mu_{j0} \geq 0, \quad (22.8) \end{aligned}$$

and

$$\begin{aligned}
D(x_0, y_0, w, T^{MF}) &= \min_{\mu, z} \sum_{k=1}^{m'} w_k z_r \\
&\text{s.t.} \\
&\sum_{j=1}^p \mu_j x_{ij} \geq x_{i0}, \quad i = 1, \dots, n \quad (23.1) \\
&-\sum_{j=1}^p \mu_j z_{kj} + z_k \geq 0, \quad k = 1, \dots, m' \quad (23.2) \\
&\sum_{j=1}^p \mu_j = 1, \quad (23.3) \\
&\mu_j \geq 0, \quad j = 1, \dots, p \quad (23.4) \\
&z_k \geq 0, \quad k = 1, \dots, m' \quad (23.5)
\end{aligned} \tag{23}$$

with

$$\frac{r(x_0, z_0, q, T^M) - \sum_{r=1}^m q_r y_{r0} + \sum_{k=1}^{m'} w_k z_{r0} - D(x_0, y_0, w, T^{MF})}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}} \geq \bar{B}(x_0, y_0, z_0; T^{MF}). \tag{24}$$

The left-hand side may be interpreted as a measure of economic environmental inefficiency. In particular, it is possible to decompose it into

$$\begin{aligned}
&\frac{r(x_0, z_0, q, T^M) - \sum_{r=1}^m q_r y_{r0} + \sum_{k=1}^{m'} w_k z_{r0} - D(x_0, y_0, w, T^{MF})}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}} = \\
&\underbrace{\frac{r(x_0, z_0, q, T^M) - \sum_{r=1}^m q_r y_{r0} + \sum_{k=1}^{m'} w_k z_{r0} - D(x_0, y_0, w, T^{MF})}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}}}_{\text{Overall Inefficiency}} \\
&= \underbrace{\frac{r(x_0, z_0, q, T^M) - \sum_{r=1}^m q_r y_{r0}}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}}}_{\text{(Good) Revenue Inefficiency}} + \underbrace{\frac{\sum_{k=1}^{m'} w_k z_{r0} - D(x_0, y_0, w, T^{MF})}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}}}_{\text{Eco-Damage Inefficiency}}.
\end{aligned} \tag{25}$$

Regarding Dakpo et al.'s (2017) model, including Førsund's (2018) extension, we have:

$$\bar{B}(x_0, y_0, z_0; T^{DF}) = \max \quad \delta^{T_1} \beta^{T_1} + \delta^{T_2} \beta^{T_2}$$

$$s.t. \quad \sum_{j=1}^p \lambda_{j0} x_{ij} \leq x_{i0}, \quad i = 1, \dots, n_1 \quad (26.1)$$

$$\sum_{j=1}^p \lambda_{j0} x_{ij} \leq x_{i0}, \quad i = n_1 + 1, \dots, n_2 \quad (26.2)$$

$$-\sum_{j=1}^p \lambda_{j0} y_{rj} + \beta^{T_1} y_{r0} \leq -y_{r0}, \quad r = 1, \dots, m \quad (26.3)$$

$$\sum_{j=1}^p \lambda_{j0} = 1, \quad (26.4)$$

$$-\sum_{j=1}^p \mu_{j0} x_{ij} \leq -x_{i0}, \quad i = 1, \dots, n \quad (26.5)$$

$$\sum_{j=1}^p \mu_{j0} z_{kj} + \beta^{T_2} z_{k0} \leq z_{k0}, \quad k = 1, \dots, m' \quad (26.6)$$

$$\sum_{j=1}^p \mu_{j0} = 1, \quad (26.7)$$

$$-\sum_{j=1}^p \lambda_{j0} x_{ij} + \sum_{j=1}^p \mu_{j0} x_{ij} \leq 0, \quad i = n_1 + 1, \dots, n_2 \quad (26.8)$$

$$\beta^{T_1}, \beta^{T_2}, \lambda_{j0}, \mu_{j0} \geq 0. \quad (26.9)$$

(26)

And

$$\Gamma(x_0, q, w, T^{DF}) = \max \quad \sum_{r=1}^m q_r y_r - \sum_{k=1}^{m'} w_k z_k$$

$$s.t. \quad \sum_{j=1}^p \lambda_{j0} x_{ij} \leq x_{i0}, \quad i = 1, \dots, n_1 \quad (27.1)$$

$$\sum_{j=1}^p \lambda_{j0} x_{ij} \leq x_{i0}, \quad i = n_1 + 1, \dots, n_2 \quad (27.2)$$

$$-\sum_{j=1}^p \lambda_{j0} y_{rj} + y_r \leq 0, \quad r = 1, \dots, m \quad (27.3)$$

$$\sum_{j=1}^p \lambda_{j0} = 1, \quad (27.4)$$

(27)

$$-\sum_{j=1}^p \mu_{j0} x_{ij} \leq -x_{i0}, \quad i = 1, \dots, n \quad (27.5)$$

$$\sum_{j=1}^p \mu_{j0} z_{kj} - z_k \leq 0, \quad k = 1, \dots, m' \quad (27.6)$$

$$\sum_{j=1}^p \mu_{j0} = 1, \quad (27.7)$$

$$-\sum_{j=1}^p \lambda_{j0} x_{ij} + \sum_{j=1}^p \mu_{j0} x_{ij} \leq 0, \quad i = n_1 + 1, \dots, n_2 \quad (27.8)$$

$$y_r, z_k, \lambda_{j0}, \mu_{j0} \geq 0, \quad (27.9)$$

which results in the following inequality:

$$\frac{\Gamma(x_0, q, w, T^{DF}) - \left( \sum_{r=1}^m q_r y_{r0} - \sum_{k=1}^{m'} w_k z_{k0} \right)}{\min \left\{ \frac{\sum_{r=1}^m q_r y_{r0}}{\delta^{T_1}}, \frac{\sum_{k=1}^{m'} w_k z_{k0}}{\delta^{T_2}} \right\}} \geq \bar{B}(x_0, y_0, z_0; T^{DF}). \quad (28)$$

Inequalities (24) and (28) make it possible to define technical and allocative terms as drivers of the corresponding measure of economic environmental inefficiency. In the empirical application we solve the models corresponding to Murty et al. (2012) and Dakpo et al. (2017), enhanced with Førsund's (2018) proposal. This represents a total of four models.

## 4. Empirical application

### 4.1. Dataset and variables

The empirical illustration relies on state-level data in the United States that comes from multiple agencies. The main source of data is the US Department of Agriculture's (USDA) Economic Research Service (ERS), which compiled the data necessary to calculate agricultural productivity in the US, and, in particular, the price indices and implicit quantities of farm outputs and inputs for each of the 48 continental states for 1960–2004. The dataset has been validated and used extensively in previous research (for example, in Ball et al., 1999; Zofio and Lovell, 2001; Huffman and Evenson, 2006; Sabasi and Shumway, 2018). A critical review of the data in light of recent developments can be found in Shumway et al. (2015; 2016). To illustrate our models, we consider the most recent year available in the dataset (2004) and assume that the production process is characterized by the following three non-polluting inputs



(capital services excluding land, land service flows, and labor services), two polluting inputs (energy and pesticides), and two good outputs (livestock and crops).<sup>12</sup> All these variables are calculated as implicit (real) quantity indices, expressed in thousands of dollars, at constant prices of 1996, using the first state (Alabama) as reference benchmark. An index of relative real output (alternatively, real input) is obtained by dividing the nominal output (input) value ratio for the two states by the corresponding output (input) price index. The details on the method of construction of all variables are contained in the following webpage of USDA-ERS: <https://www.ers.usda.gov/data-products/agricultural-productivity-in-the-us/methods/>.

As for the undesirable output production generated by energy consumption, we consider carbon dioxide (CO<sub>2</sub>) emissions from the agricultural sector associated with fuel combustion, also for 2004 (expressed in tons of CO<sub>2</sub> equivalents), obtained from the US Environmental Protection Agency (EPA). Since these data are given in overall terms for the whole country, we further disaggregate it by state, using for that purpose the share that each state has in farm production expenses for gasoline, fuels, and oils, as reported by the US Department of Agriculture.<sup>13</sup> The price of CO<sub>2</sub> emissions is proxied by the market clearing price set in the state of California (price of carbon emissions expressed in thousands of dollars per ton of CO<sub>2</sub> equivalents), since a general market for CO<sub>2</sub> for the whole US does not exist.<sup>14</sup> In particular, this is the price of carbon for tradable allowances with a futures contract that originates from the Californian greenhouse gases trading market under the Cap and Trade Program. We consider the average 2012 price and deflate it to 2004 using the consumer price index in absence of a suitable deflator (US Bureau of Labor Statistics).<sup>15</sup>

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<sup>12</sup> The ERS dataset also contains data on one more output: other farm-related output. However, we do not take it into account since it is a residual to capture additional farm income and therefore usually consists of a very marginal fraction of the total farm output (rarely above 5 percent of the total farm output) and is therefore negligible. In addition, the ERS dataset provides the information on inputs of agricultural chemicals, fertilizers, and other intermediate inputs. Since we cannot relate these inputs with their associated bad outputs for which data are unavailable, we drop these inputs in our analysis.

<sup>13</sup> In particular, we use the data on expenses in gasoline, fuels and oils from the Census of Agriculture as reported by the US Department of Agriculture, expressed in thousands of dollars. Since the census was not conducted in 2004, we take the average of the values reported for 2002 and 2007 censuses, which consists of the closest approximation of the 2004 data.

<sup>14</sup> On the contrary, a general US market for sulfur dioxide (SO<sub>2</sub>) has existed for many years (see <https://www.epa.gov/airmarkets/so2-allowance-auctions>).

<sup>15</sup> California's GHG emissions program is the second largest in the world after the European Union's Emissions Trading System. From its beginning in 2012 it covered the power and industrial facilities, and it expanded to natural gas and transportation fuels in 2015, allowing it to cover approximately 85 percent of California's GHG emissions. Another program exists in the US, called the Regional Greenhouse Gas Initiative (RGGI), in which nine states participate. However, it is much narrower than the Californian program since it applies only to some power plants and only to CO<sub>2</sub> emissions. Because of this, we decided to exclusively use the data from Californian program in our research. Also, mixing data on prices from both Californian and RGGI initiatives would not be appropriate since the two sets of values are incompatible.

The measure of bad output related to pesticides is the number of pesticide exposures per state for 2004 obtained from the Centers for Disease Control and Prevention at the US Department of Health & Human Services. As the approximation of the price of this bad output we use the cost of hospitalized treatment of pesticide-related poisonings (in thousands of dollars) as estimated in Pimentel (2005). Because this cost is provided for 1995, we further inflate it to 2004 prices using the price index for medical services as obtained from the US Bureau of Labor Statistics.

Tables 1 and 2 summarize the descriptive statistics of input-output quantities and their corresponding prices, respectively, for the US states in 2004. Data in Table 1 indicates relatively large variations for input-output variables among states, as evidenced by large standard deviations with respect to their means, and resulting in relatively large coefficients of variation. Regarding the prices summarized in Table 2, smaller variation in the sample is generally observed. For both bad outputs (CO<sub>2</sub> emissions and pesticide exposures), standard deviation is equal to 0 as we use a single price for all states. Obviously, there is some variability of these prices per state, but we are unable to capture that with the available data.

Table 1. Descriptive statistics of input-output data (implicit real quantities), 2004.

Variable	Mean	SD	Coefficient of variation
Non-pollution-generating inputs			
Capital services (thousand \$)	541,067.83	450,804.23	0.83
Land service flows (thousand \$)	650,678.34	738,001.84	1.13
Labor services (thousand \$)	1,292,827.06	1,186,855.19	0.92
Pollution-generating inputs			
Energy (thousand \$)	166,646.86	144,483.99	0.87
Pesticides (thousand \$)	164,921.16	163,857.64	0.99
Good outputs			
Livestock and products (thousand \$)	2,103,075.52	1,997,443.05	0.95
Crops (thousand \$)	2,819,480.05	3,419,130.27	1.21
Bad outputs			
CO <sub>2</sub> emissions (tons of CO <sub>2</sub> equivalents)	996,394.52	914,820.59	0.92
Pesticide exposures (number)	2,458.71	2,256.45	0.92

Notes: SD=Standard deviation.

Table 2. Descriptive statistics of prices for input-output data, 2004.

Variable	Mean	SD	Coefficient of variation
Non-pollution-generating inputs			
Capital services (indices relative to Alabama)	1.10	0.03	0.03
Land service flows (indices relative to Alabama)	1.13	0.64	0.56
Labor services (indices relative to Alabama)	1.16	0.32	0.28
Pollution-generating inputs			
Energy (indices relative to Alabama)	1.44	0.17	0.12
Pesticides (indices relative to Alabama)	1.14	0.25	0.22
Good outputs			
Livestock and products	1.24	0.20	0.16
Crops (indices relative to Alabama)	1.07	0.16	0.15
Bad outputs			
CO <sub>2</sub> emissions (thousand \$ per ton of CO <sub>2</sub> equivalents)	0.01	0	0
Pesticide exposures (thousand \$)	6.22	0	0

Notes: SD=Standard deviation. Prices of inputs and good outputs vary across states and are expressed in relative terms with respect to the first state, Alabama, considering 1996 as the base year. Prices of bad outputs are unique for all states and are deflated to the 2004 reference year.

## 4.2. Results

### 4.2.1. Technical, allocative and profit frontiers.

When solving our four reference economic models – that is, Murty et al. (2012) and Dakpo et al. (2017), each complemented with Førsund’s (2018) proposal – it is relevant to determine, from a technological perspective, the number of observations that are efficient, thereby defining the frontier of the global *by-production* technology  $T$ , consisting of both the *intended production*  $T_1$  technology, (1) (hereafter, conventional or standard technology) and the *pollution-generating* technology  $T_2$ , (2) (hereafter, polluting technology). Table 3 shows that the number of observations defining the production frontier is greater in the conventional technology  $T_1$  than in the polluting technology  $T_2$ , except in the case of the Murty et al. (2012) model incorporating Førsund’s proposal. Nevertheless, the number of observation jointly defining the by-production technology by being efficient in both  $T_1$  and  $T_2$  is greatly reduced. Interestingly, all states that are efficient according to Murty et al.’s model are also efficient in the other three models. These are California, Delaware, Iowa, Illinois, Rhode Island, and Vermont, whose different production scales indicate that they represent alternative most productive scale sizes, serving as benchmark for the remaining states.

Table 3. Number of efficient US states.

Model	Technical			Allocative	Profit
	$T$	$T_1$	$T_2$		
Murty et al.	6	19	11	4	4
Dakpo et al.	13	30	16	6	6
Murty & Førstund	10	19	24	4	4
Dakpo & Førstund	19	30	28	15	15

Clearly, not all technically efficient states are allocative-efficient and thereby achieve profit efficiency. Again considering Murty et al.'s results, Delaware and Illinois fail to maximize profit at the existing input and output prices, which means they are allocative-inefficient. Approximately 10 percent of US states (four or six out of 48) are fully efficient, except in Dakpo et al.'s approach, where the inclusion of the restriction ensuring that the optimal polluting inputs are quantitatively the same in  $T_1$  and  $T_2$ , enhanced with the inclusion of non-polluting inputs in  $T_2$  following Førstund, increases the number of profit-efficient states to 31.25 percent.

#### 4.2.2. Technical inefficiency: results within and between models

Departing from this general portrait of inefficiency frequencies at the technical, allocative, and overall profit inefficiency levels, we now focus on the technological side, with Figure 1 portraying the average absolute technical efficiency values in  $T_1$ ,  $T_2$  and their global by-production aggregate  $T$ , across the four models (left panel).

Several features are worth highlighting:

i) Technical inefficiency in the conventional technology  $T_1$  differs substantially on average between Murty et al.'s models (5) and (22), and Dakpo et al.'s models (17) and (26), but it is equal *within* each type of model. That is, as Førstund's assumption includes non-polluting inputs in  $T_2$ , the characterization of  $T_1$  is the same and therefore remains unaffected by this assumption.

ii) Average  $T_1$  inefficiency in Murty et al.'s models is about 50 percent greater than Dakpo et al.'s models: 0.1814 vs. 0.1190. This result is also expected since the introduction of the additional constraint in Dakpo et al.'s models, ensuring that optimal polluting input quantities are the same in  $T_1$  and  $T_2$ , results in a tighter envelopment of the observed data, and hence in

lower inefficiency values. Nevertheless, a 50 percent difference is quite remarkable from an empirical perspective.

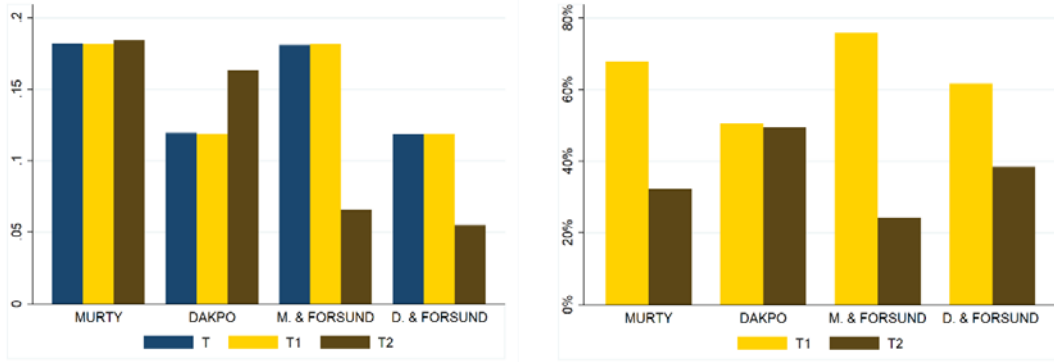
iii) Technical inefficiency in the residual (polluting) technology  $T_2$  differs across the four models. However, while there are not significant differences between Murty et al.'s (5) and Dakpo (17) et al.'s models (0.1844 vs. 0.1632), what makes a difference is the introduction of Førsund's proposal including non-polluting inputs in  $T_2$ . Indeed, the average technical inefficiency in Murty et al.'s model (5) is three times greater than that for the same model enhanced with Førsund's assumption (22): 0.1844 vs. 0.0659. The difference between Dakpo et al.'s model (17) and that enhanced with Førsund's proposal (26) is similar to the difference above: 0.1632 vs. 0.0551. Thus, the inclusion of non-polluting outputs in  $T_2$  – which partly results from the inclusion of the additional set of constraints – also has remarkable effects on reducing technical inefficiency.

iv) Regarding the global by-production technical inefficiency  $T$ , its values closely follow those of the conventional technology  $T_1$  as a result of the weighting scheme that, as long as the profit inefficiency normalizing constraint is the same for all observations, tends to favor a higher weight of  $\delta^{T_1}$ . In the particular case of the current empirical application, based on US agriculture data, the  $\delta^{T_1}$  – as long as the condition  $\sum_{r=1}^m q_r y_{r0} / \delta^{T_1} = \sum_{k=1}^{m'} w_k z_{k0} / \delta^{T_2}$ ,  $\delta^{T_1} + \delta^{T_2} = 1$ , is satisfied, so the normalizing value  $\min \left\{ \sum_{r=1}^m q_r y_{r0} / \delta^{T_1}, \sum_{k=1}^{m'} w_k z_{k0} / \delta^{T_2} \right\}$  is common to all states and models – corresponds to  $\delta^{T^*} \simeq 0.99$ ; see the denominator in expressions (15), (21), (24) and (28).<sup>16</sup>

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<sup>16</sup> We acknowledge that this practically implies a “business as usual” evaluation of technical inefficiency in the by-production model. However, our proposed model is general enough to accommodate other weights, as would be the case if other (subjective) weights were chosen, or in other applications where the difference in economic value between private revenue and social cost would not be that large (as could be seen if average private revenue and social cost were calculated using the mean quantity and price values presented in Table 1). One should keep in mind that our proposal to choose delta simply reflects the empirical balance between the former economic values (that is, private benefit and social cost), since a weight equal to 0.5 would simply imply that both monetary valuations are equal.

Figure 1. Average technical inefficiencies by models (absolute and percentage values).



The correlations between the technical efficiency scores using Spearman’s definition show that the conventional and environmental efficiency performance are weakly or even negatively correlated in most cases (see Table 4). This should come as no surprise given that observations do have market incentives to perform better in the conventional side of the production process  $T_1$  (that is, to maximize output revenue), but these incentives are weak or absent in the case of environmental cost minimization. Since the production of undesirable outputs (CO<sub>2</sub> emissions and pesticide exposures) is not normally internalized by the economic system, productive efficiency in  $T_2$  is not tightly pursued, which means that a negative correlation between both rankings is a likely outcome. This can be seen clearly in the unmodified Murty et al. and Dakpo et al. models, where  $T_2$  inefficiencies are greater on average. Nevertheless, we note that the variability in the rankings is so high that none of these correlations are significant at the standard confidence levels. It is also worth remarking that the correlations between models’ rankings for the polluting technology  $T_2$  are generally smaller than for the standard technology  $T_1$  (except for the Murty et al. and Dakpo et al. models enhanced with Førsund’s proposal, whose correlation is  $\rho(M\&F^{T_2}, D\&F^{T_2}) = 0.871$ ).

Table 4. Spearman's correlation matrix between technical inefficiencies.

		Conventional technology ( $T_1$ )				Polluting Technology ( $T_2$ )			
		Murty T1	Dakpo T1	M. & F. T1	D. & F. T1	Murty T2	Dakpo T2	M. & F. T2	D. & F. T2
Conventional technology ( $T_1$ )	Murty T1	1.000*							
	Dakpo T1	0.764*	1.000*						
	M. & F. T1	1.000*	0.764*	1.000*					
	D. & F. T1	0.764*	1.000*	0.764*	1.0000*				
Polluting technology ( $T_2$ )	Murty T2	-0.142 <sup>ns</sup>	0.090 <sup>ns</sup>	-0.142 <sup>ns</sup>	0.090 <sup>ns</sup>	1.000*			
	Dakpo T2	-0.079 <sup>ns</sup>	0.108 <sup>ns</sup>	-0.080 <sup>ns</sup>	0.108 <sup>ns</sup>	0.901*	1.000*		
	M. & F. T2	-0.132 <sup>ns</sup>	-0.133 <sup>ns</sup>	-0.132 <sup>ns</sup>	-0.133 <sup>ns</sup>	0.557*	0.549*	1.000*	
	D. & F. T2	0.012 <sup>ns</sup>	-0.006 <sup>ns</sup>	0.012 <sup>ns</sup>	-0.006 <sup>ns</sup>	0.430*	0.542*	0.871*	1.000*

Notes: Murty et al.: (5), Dakpo et al.: (17), Murty et al. & Førsund: (22), Dakpo et al. & Førsund: (26).

\*  $p < 0.01$ ; <sup>ns</sup> Non-significant at 10% level.

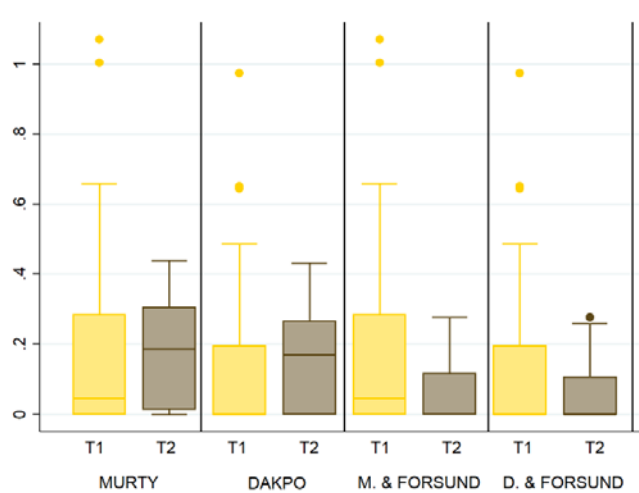
Besides focusing on absolute values, we can gain information on the weight that each technology ( $T_1$  and  $T_2$ ) has on the by-production technology  $T$ . The right-hand panel of Figure 1 shows the average percentage weight that each one of them has on the aggregate result. On average, for US agriculture, both  $T_1$  and  $T_2$  inefficiencies account for a significant share of aggregate by-production inefficiency, their values being driven by the large number of efficient observations in either technology, resulting in a null contribution to aggregate inefficiency, and explaining the relative balanced average percentage values for the four models, regardless of  $\delta^{T_1}$ . This is particularly the case for the Dakpo et al. approach, where  $T_2$  technical inefficiency is rather large in absolute values. Nevertheless, since mean values provide only a rough first approximation to inefficiency results, we now study their different distributions.

A visual comparison of the values of the technical inefficiency scores for  $T_1$  and  $T_2$  is presented in Figure 2, where box-plots of the different distributions make it possible to identify extreme values. The different boxes (grouped in pairs by models) represent the intervals between the first and third quartiles of the ranking distribution (that is, the interquartile range (IQR) between Q1 and Q3), with its median represented by the horizontal line within it (the median can then be compared to the mean values presented in Figure 1). The dispersion in the rankings within this interquartile range is relatively low, particularly for the polluting technologies  $T_2$ , incorporating Førsund's assumption. It is also reassuring that only a few outliers were identified. In particular, for the Murty et al. model the states of Louisiana (0.994) and Montana (1.066) are the most inefficient, lying outside the area below the whisker equal

to one a half times the IQR. In Dakpo et al.’s model, Montana (0.970) is the worst-performing state. As for these extreme states, the ranking and values do not change when Førsund’s assumption is considered, suggesting robustness in the results.

Still focusing on the box-plots, it is relevant to test whether the distributions of the conventional and polluting technologies,  $T_1$  and  $T_2$ , are equal *within* each one of the four models. For this purpose, we have performed the test proposed by Simar and Zelenyuk (2006). Their method adapts the nonparametric test for the equality of two densities developed by Li (1996). For this test we use algorithm II with 1000 replications, which computes the Li statistic on the bootstrapped estimates of the DEA scores, and where the null values of the efficient firms (resulting in the truncation of the efficiency scores) are smoothed by adding a small noise.<sup>17</sup> The obtained results reject the null hypothesis of equality of densities for all models at the 5 percent level of significance, which means that  $T_1$  and  $T_2$  inefficiencies are statistically different in each of the four models.<sup>18</sup>

Figure 2. Box plots of the technical inefficiency distributions by models.



Alternatively, it is also interesting to test if the  $T_1$  and  $T_2$  inefficiency distributions are different *between* models. Figure 3 depicts their kernel distributions in each one of the four models (left and right panels for  $T_1$  and  $T_2$ , respectively). When plotting these distributions we follow the procedure proposed by Simar and Zelenyuk (2006), which in short: (i) uses Gaussian

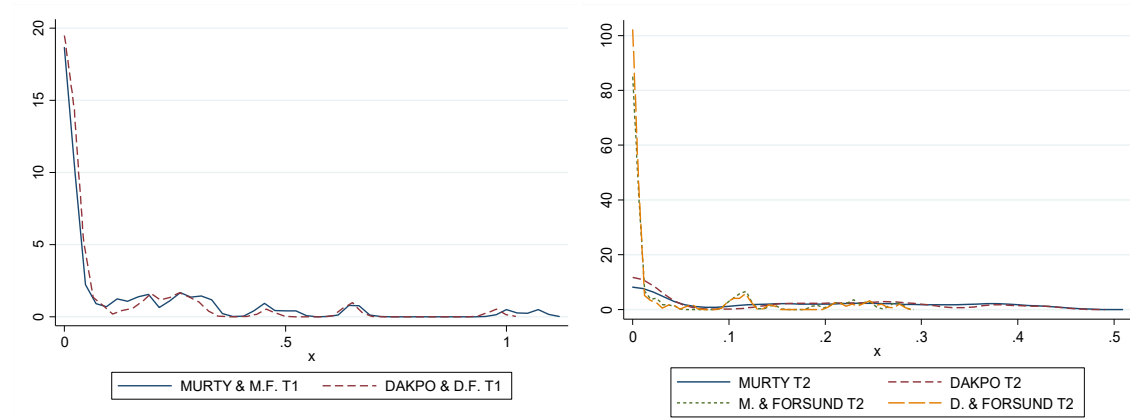
<sup>17</sup> Simar and Zelenyuk (2006) developed their algorithm for radial distance functions, in which the efficiency values equal to one are smoothed. We adapted their algorithm to our additive context by smoothing the inefficiency scores equal to zero.

<sup>18</sup> The level of significance changes to 1 percent when the hypothesis is tested for Dakpo et al.’s model enhanced with Førsund’s assumption.



kernels, (ii) employs the reflection method to overcome the issue of a zero-bounded support of the inefficiency scores (Silverman, 1986), and (iii) determines the bandwidths using Sheather and Jones's (1991) method. As commented,  $T_1$  distributions are the same across pairwise models; that is, Murty et al.'s model (5), and Dakpo et al.'s model (17), are equal to their respective Førsund's extensions, (22) and (26) (that is,  $M^{T_1} = M\&F^{T_1}$ ,  $D^{T_1} = D\&F^{T_1}$ ). When comparing Murty et al.'s and Dakpo et al.'s models ( $M^{T_1}$  vs.  $D^{T_1}$ ) and Murty et al.'s enhanced with Førsund's assumption model and Dakpo et al.'s model ( $M\&F^{T_1}$  vs.  $D^{T_1}$ ), the null hypotheses of the equality of densities cannot be rejected at the 10 percent level, so these models are not statistically different. However, the comparison between Murty et al.'s and Dakpo et al.'s models, both enhanced with Førsund's assumption ( $M\&F^{T_1}$  vs.  $D\&F^{T_1}$ ), and Murty et al.'s model and Dakpo et al.'s model enhanced with Førsund's assumption ( $M^{T_1}$  vs.  $D\&F^{T_1}$ ), shows that the null hypotheses of equality of densities is rejected at the 5 percent and 10 percent levels, respectively.

Figure 3. Technical inefficiencies kernel distributions ( $T_1$  and  $T_2$ ) by models



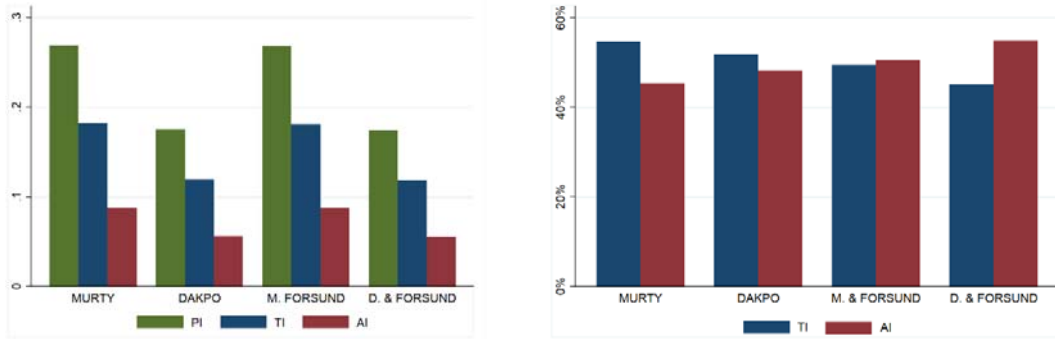
As for the differences in the polluting technology  $T_2$  (right panel), the adapted Li test returns that the Murty et al.'s (5) and Dakpo et al.'s models (17) are not statistically different among themselves ( $M^{T_2}$  vs.  $D^{T_2}$ ); this result extends to their Førsund's versions: (22) and (26) ( $M\&F^{T_2}$  vs.  $D\&F^{T_2}$ ). However, when comparing Murty et al.'s model to its Førsund's extension ( $M^{T_2}$  vs.  $M\&F^{T_2}$ ), Dakpo et al.'s model to its Førsund's extension ( $D^{T_2}$  vs.  $D\&F^{T_2}$ ), Murty et al.'s model to Dakpo et al.'s in its Førsund's version ( $M^{T_2}$  vs.  $D\&F^{T_2}$ ), as well as Murty et al.'s in its Førsund's version model to Dakpo et al.'s model ( $M\&F^{T_2}$  vs.  $D^{T_2}$ ), the results of the adapted Li test show that that null hypotheses of the equality of densities are

rejected at the 1 percent level. Detailed results on these bilateral tests for both  $T_1$  and  $T_2$  are presented in Appendix A.1. Therefore, as for technical inefficiencies, we can conclude that  $T_1$  and  $T_2$  results are statistically different within models, but in many cases not between models.

#### 4.2.3. Profit inefficiency: technical and allocative inefficiencies between models

We now discuss the economic efficiency dimension of the agricultural sector at the state level. The left-hand panel of Figure 4 portrays absolute average values of the profit, technical, and allocative inefficiencies of the four technological models. Average profit inefficiency is about 50 percent greater in Murty et al.’s model (15) than in Dakpo et al.’s model (21): 0.269 vs. 0.176, respectively. Despite the different technological characterization of the polluting technology when incorporating Førsund’s extensions to the profit inefficiency definition, (25) and (28), the difference in results between the former and the latter models is marginal. Consequently, while the difference *within* each type of model is minimal (that is,  $M^{PI}$  vs.  $M\&F^{PI}$ , and  $D^{PI}$  vs.  $D\&F^{PI}$ ), the differences *between* models remain the same at the 50 percent level. Regarding the difference between technical and allocative inefficiencies in absolute terms, the former doubles the latter in absolute terms on average.

Figure 4. Average values of technical and allocative inefficiencies (AI and TI) by models.



We have also calculated Spearman’s correlations between the different pairs of models. Table 5 reports the coefficients for the profit inefficiencies (PIs), allocative inefficiencies (AIs), and the aggregate by-production technical inefficiencies (TIs), thus complementing Table 4’s presentation of the correlations for the standard and polluting technologies,  $T_1$  and  $T_2$ . The ranking correlation *between* Murty et al. and Dakpo et al.’s profit inefficiencies is rather high at  $\rho(M^{PI}, D^{PI}) = 0.705$ , similar to that between their Førsund extensions:  $\rho(M\&F^{PI}, D\&F^{PI}) = 0.749$ . Nevertheless, the ranking compatibility is much greater *within* models. Indeed, the correlation between profit inefficiency defined under each type of technological model (either

*à la* Murty et al. or Dakpo et al.) and their corresponding Førsund variations is almost perfect at  $\rho (M^{PI}, M\&F^{PI}) = 0.999$  and  $\rho (D^{PI}, D\&F^{PI}) = 0.969$ . Also, reading vertically the profit inefficiency columns (PIs) in Table 5, we learn that they correlate more with their technical inefficiencies components (TIs) than with their allocative inefficiencies (AIs), particularly for the Murty et al. models.

The magnitudes of these correlation coefficients extend to the aggregate by-production technical inefficiencies (TIs), as the correlation *between* the Murty et al. and Dakpo et al. models is  $\rho (M^{TI}, D^{TI}) = 0.681$ , increasing to  $\rho (M\&F^{TI}, D\&F^{TI}) = 0.757$  if their Førsund specifications are considered. Correlation increases again *within* models – that is, the coefficient for the Murty et al. model and its Førsund extension is  $\rho (M^{TI}, M\&F^{TI}) = 0.983$  – while that for the Dakpo et al. approach and Førsund extension is  $\rho (D^{TI}, D\&F^{TI}) = 0.932$ . Similarly, the alternative allocative inefficiency rankings (AIs) present rather comparable results, both *between* and *within* models.

Finally, as presented in the lower central panel of Table 5, technical and allocative inefficiencies correlate mildly, albeit positively. Particularly for the Dakpo et al. models:  $\rho (D^{TI}, D^{AI}) = 0.777$  and  $\rho (D\&F^{TI}, D\&F^{AI}) = 0.697$ . This suggests that the economic performance of US states from both technological and allocative perspectives go hand by hand. Indeed, as none of the coefficients are negative, it is possible to dismiss the idea that realized technological and allocative behavior follow opposite ways. This result is expected because profit inefficiency includes marketed outputs, and farms have an incentive to perform well technologically by attaining the maximum feasible quantities of desirable outputs given the technology (that is, livestock and crops), and also to choose their optimal relative quantities (output mix) in order to maximize revenue at market prices. On the contrary, although the social side of the profit inefficiency definition, represented by the environmental (monetary) cost damage, is less binding (to the extent that farmers do not explicitly aim to minimize CO<sub>2</sub> emissions and pesticide exposures, and therefore their social cost), its economic values are notably smaller than their marketed private revenue counterparts. Hence the technical and allocative efficiency levels correlate positively, as they are both dominated by the private side of economic performance, actively pursued by the economic agents. We also stress that, in this case, most of the correlations are statistically significant at the 1 percent level, particularly within models.

Table 5. Spearman's correlation matrix between profit inefficiencies (PI, TI and AI)

		Profit inefficiency (PI)				Technical inefficiency (TI)				Allocative inefficiency (AIs)			
		Murty PI	Dakpo PI	M. & F. PI	D. & F. PI	Murty TI	Dakpo TI	M. & F. TI	D. & F. TI	Murty AI	Dakpo AI	M. & F. AI	D. & F. AI
Profit Inefficiency (PI)	Murty PI	1.000*											
	Dakpo PI	0.705*	1.000*										
	M. & F. PI	0.999*	0.700*	1.000*									
	D. & F. PI	0.749*	0.969*	0.749*	1.000*								
Technical Inefficiency (TI)	Murty TI	0.900*	0.677*	0.900*	0.702*	1.000*							
	Dakpo TI	0.625*	0.945*	0.620*	0.893*	0.681*	1.000*						
	M. & F. TI	0.896*	0.665*	0.899*	0.722*	0.983*	0.657*	1.000*					
	D. & F. TI	0.684*	0.881*	0.683*	0.907*	0.740*	0.932*	0.757*	1.000*				
Allocative Inefficiency (AI)	Murty AI	0.648*	0.416*	0.648*	0.452*	0.397*	0.265**	0.392*	0.280**	1.000*			
	Dakpo AI	0.585*	0.898*	0.580*	0.869*	0.500*	0.777*	0.491*	0.696*	0.579*	1.000		
	M. & F. AI	0.639*	0.409*	0.639*	0.449*	0.390*	0.258**	0.386	0.278**	0.998*	0.579*	1.000*	
	D. & F. AI	0.592*	0.842*	0.593*	0.875*	0.489*	0.702*	0.512	0.697	0.624*	0.963*	0.630*	1.000*

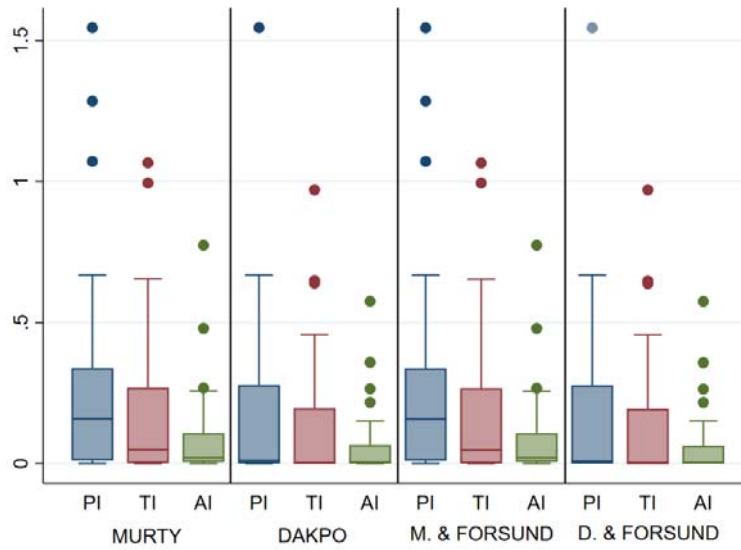
Notes: Murty et al.: (15), Dakpo et al.: (21), Murty et al. & Førsund: (25), Dakpo et al & Førsund: (28).

\*  $p < 0.01$ ; \*\*  $p < 0.1$

Also, since profit efficiency is the aggregate resulting from adding technical and allocative inefficiencies ( $PI = TI + AI$ ), it is relevant to highlight its sources in percentage terms. The left-hand panel in Figure 4 shows that, on average, TI doubles AI in value. Specifically, in the model characterizing the technology following Murty et al., average TI amounts to 0.182, while AI amounts to 0.087. This 50 percent difference is also observed for the economic model based on Dakpo et al.'s technological characterization: 0.119 vs. 0.056. The same holds for the difference between both types of models assuming the Førsund extension in the polluting technology. However, once the percentage shares of the technical and allocative inefficiencies have been calculated at the individual state level, and the results averaged, the right-hand panel of Figure 4 shows a balanced picture, with both sources of inefficiency weighting about 50 percent each. This suggests that the individual distributions of technical and allocative inefficiencies exhibit relatively large dispersions.

For this reason, we study the characteristics of these distributions by resorting to their box-plot representations. One can corroborate the differences *between* Murty et al.'s and Dakpo et al.'s models, and the similarities *within* the same type of model when comparing the former to their Førsund extensions. Focusing initially on the three US states with the worst economic performance, lying above one and half times the interquartile range (IQR) of Murty et al.'s model, Montana presents a profit inefficiency value of 1.545 (five times greater than the mean at 0.260), resulting from the addition of technical inefficiency, 1.066 (whose mean value is 0.182), and allocative inefficiency, 0.479 (0.087). The second and third worst-performing states are North Dakota ( $1.285 = 0.511 + 0.774$ ) and Louisiana ( $1.071 = 0.994 + 0.077$ ), respectively. These results illustrate the high variability in the relative values of the technical and allocative components of overall profit inefficiency across the sample. Nevertheless, it is observed that the IQR for allocative inefficiency,  $Q_3^{AI} - Q_3^{AI} = 0.104$ , is about half of that observed for technical inefficiency,  $Q_3^{TI} - Q_3^{TI} = 0.284$ . As for the worst-performing states outside one and a half times the IQR in the Dakpo et al. model, only one (Montana again) incurs the highest profit inefficiency ( $1.545 = 0.970 + 0.575$ ); in this case, about nine times greater than the profit inefficiency mean at 0.167. A similar gap between the technical and allocative IQRs can also be observed in this model. We do not comment further on the results of each type of model enhanced with Førsund's proposal since they closely follow those already presented.

Figure 5. Box plots of the profit inefficiency distributions by models.



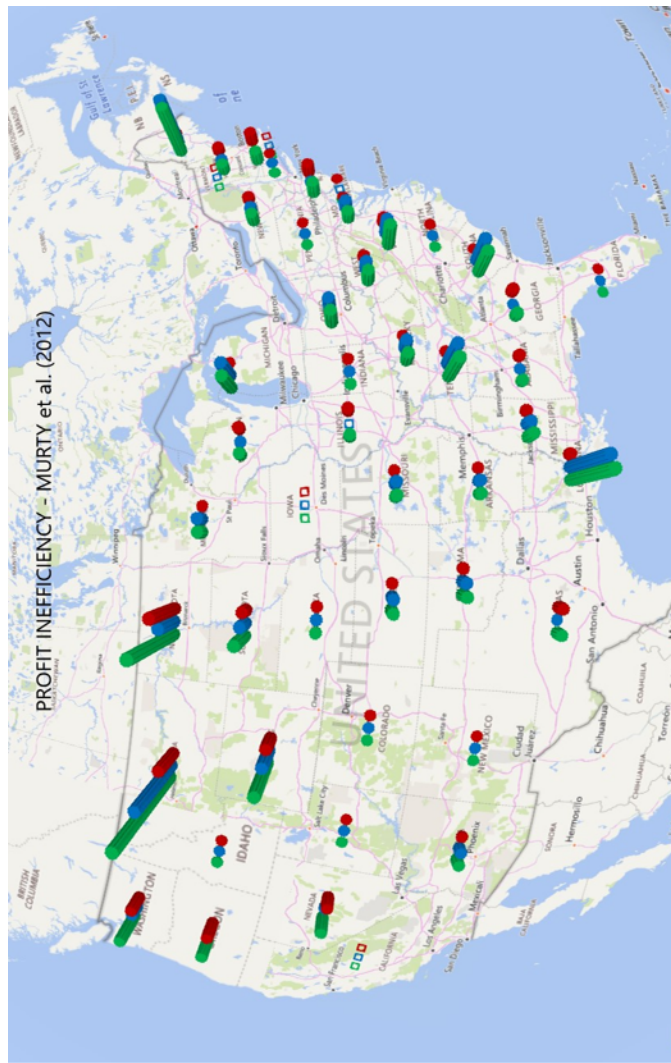
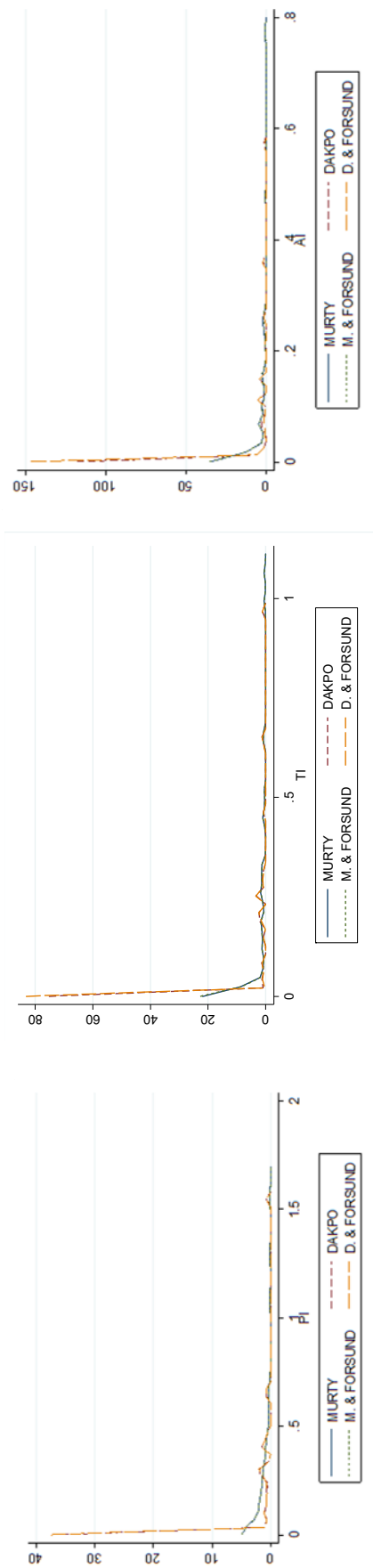
We conclude the empirical section by checking whether these distributions are statistically different. As before, we first test whether the technical and allocative efficiencies components of profit inefficiency are different from each other *within* the same model ( $M^{TI}$  vs.  $M^{AI}$ ,  $D^{TI}$  vs.  $D^{AI}$ , etc.) following the method proposed by Simar and Zelenyuk (2006). In all models except Dakpo et al.'s, the null hypothesis testing the equality of the densities cannot be rejected, even at the 10 percent level of significance. Hence, technical and allocative inefficiencies are not statistically different in the three models. This can be visually corroborated by comparing the TI and AI distributions in the box-plots corresponding to each model in Figure 5, or comparing the technical and allocative distributions presented in the left and central panels of Figure 6.

As for the differences in profit, technical and allocative inefficiencies *between* models (M vs. D, M vs. M&F, etc.), the test of Simar and Zelenyuk (2006) returns varied results on the (in)existence of statistical differences between these distributions, which can be visually anticipated in Figure 6. The comparison of profit, technical, and allocative inefficiencies for Murty et al.'s model with Dakpo et al.'s model reveals significant differences between them (depending on the models being compared at the 5 percent or 10 percent level of significance) ( $M^{PI}$  vs.  $D^{PI}$ ,  $M^{TI}$  vs.  $D^{TI}$ , and  $M^{AI}$  vs.  $D^{AI}$ ); this result extends to their Førsund's versions ( $M\&F^{PI}$  vs.  $D\&F^{PI}$ ,  $M\&F^{TI}$  vs.  $D\&F^{TI}$ , and  $M\&F^{AI}$  vs.  $D\&F^{AI}$ ). Hence we conclude that the choice of model is not neutral when comparing economic performance across states. On the contrary, it turns out that these distributions are the same for Murty et al.'s model and its counterpart extended with Førsund

assumption ( $M^{PI}$  vs.  $M\&F^{PI}$ ), as well as for Dakpo et al.'s model and its Førsund extension ( $D^{PI}$  vs.  $D\&F^{PI}$ ). This is also a remarkable result, implying that, for the whole US agricultural sector, profit inefficiency, including its technical and allocative terms, can be equally measured *irrespective* of whether the global underlying production technology  $T$  incorporates Førsund's proposal or not. The detailed results on the test for profit, technical, and allocative inefficiencies are presented in Appendix A.2.

Finally, Map 1 illustrates the profit, technical, and allocative inefficiency results for all 48 states in the sample. As all four models are equally representative, we map the results of the economic model characterizing the global technology following Murty et al. (2012). We visually confirm the existence of several geographical clusters, particularly of large profit inefficiencies in the Pacific Northwest states of Washington, Oregon, Montana, and the Dakotas. This suggests that the agricultural characteristics of their production processes, mainly focused in livestock production, along with market prices, are hampering their economic performance. On the contrary, we could not observe a significant clustering of the economically efficient states: California, Delaware, Iowa and Vermont. While there seems to be visual evidence calling for the application of spatial regression analyses on the inefficiency results, as well as their explanation in terms of the technological specialization and market orientation of the different states, these extensions fall beyond the scope of the current application, which is intended to illustrate the new economic models.

Figure 6. Profit inefficiency kernel distributions (PI, TI and AI). Map 1. Profit Inefficiency: Murty et al.'s (2012) model.





#### 4. Conclusions

This paper introduces the theory and practice of environmental economic inefficiency measurement. Environmental economic inefficiency represents the ability of firms to maximize the difference between private revenue and “social/environmental” cost given the production technology and market prices. This objective can be likened to a “profit” function that economically weighs private gains and social losses by internalizing the damage associated with the production of undesirable outputs. Resorting to duality theory enabled us to demonstrate how this (supporting) economic function relates to a technical counterpart represented by the directional distance function, effectively extending the analytical framework of Chamber et al. (1998) to the field of environmental economics. Since the directional distance function can be regarded a measure of technical efficiency, the gap between technical and optimal economic performance can be attributed to allocative inefficiencies. Hence, profit inefficiency can be consistently decomposed into its technical and allocative sources.

The new model departs from one of the most recent proposals characterizing the production technology in the presence of undesirable outputs; the so-called by-production model put forward by Murty et al. (2012). This analytical framework differentiates between two separate sub-technologies, one corresponding to the conventional (privately oriented) approach and one characterizing the production of pollutants only. Our model makes it possible to assign different weights to each technology in order to account for the modeler or stakeholder preferences (managerial, political, legal/regulatory, etc.). Although the new economic model could be developed adopting other technological characterizations, the by-production approach overcomes prior limitations and is becoming increasingly popular among practitioners. Moreover, it is subject to continuous qualifications such as those recently introduced by Dakpo et al. (2017) and Førsund (2018).

We develop our new model within the data envelopment analysis framework, which allows us to illustrate its empirical viability using a real-life data set on US agriculture for 2004. The production technology is characterized by five inputs – capital, land, labor, energy, and pesticide (the latter two of which are polluting inputs) – and two outputs: livestock and crops. We implement four models corresponding to the original proposal by Murty et al. (2012), a modified version corresponding to Dakpo et al.’s (2017)

qualification that ensures that the projections points for input dimensions are the same in the conventional and polluting technologies, and their corresponding modifications that incorporate Førsund's (2018) proposal of bringing non-contaminating inputs into the polluting technology, so as to allow for substitution effects. The main empirical findings are the following:

- Technical inefficiencies between the Murty et al. and Dakpo et al. models do not generally differ in the case of the conventional technology, either looking at Spearman's correlations or Li tests. On the contrary, although they also yield similar results regarding the polluting technology, they differ statistically from their Førsund extensions. We confirm that technical inefficiencies in the conventional and environmental technologies are unrelated, with the latter being larger than the former. This simply reflects the fact that farmers do not have market incentives to perform better in the environmental side of the production process (that is, reducing social costs), as opposed to the conventional side, where falling short from the production frontier results in lower (private) revenue. In passing, we note that these empirical results comparing technical efficiencies for alternative models are novel, since they had not been confronted until now.
- As for the new economic inefficiency framework, statistical differences can be found across the alternative models. Profit inefficiency is generally larger in Murty et al.'s model than in Dakpo et al.'s. This result extends to their technical and allocative components. Our results show that technical inefficiency is generally larger than allocative inefficiency, suggesting that there is more room for economic improvements by taking advantage of the existing technology than by reallocating the relative demand for inputs and outputs given their market prices (that is, the relative specialization in input usage and output production). As for the extension of these two models with Førsund's proposal, no statistically significant differences emerge.

We conclude from these results that, as expected, the analytical approach chosen to evaluate environmental economic efficiency is highly dependent on the technological model upon which it is based. Choosing alternative models leads to significant differences in the magnitude of technical and allocative inefficiency, which may question the credibility of results given their lack of robustness, and lead to contradictions and faulty managerial and policy decision making. Therefore, caution should be exerted when implementing the new analytical framework, which nevertheless opens the door to a

whole new range of models capable of internalizing the social cost of environmental damage when assessing economic performance. This is a key extension in the measurement of environmental efficiency that was not available until now.

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Appendix A.1. Results of Simar and Zelenyuk (2006) adapted Li test (test statistic and significance level) for T1 and T2

Conventional technology (T1)				
	Murty	Dakpo	M. & F.	D. & F.
Murty	-	4.91	-1.05	7.07*
Dakpo		-	5.14	-1.11
M. & F.			-	7.40**
D. & F.				-

Polluting Technology (T2)				
	Murty	Dakpo	M. & F.	D. & F.
Murty	-	-0.58	17.01***	18.53***
Dakpo		-	10.81***	11.82***
M. & F.			-	-1.04
D. & F.				-

Notes: Murty et al.: (5), Dakpo et al.: (17), Murty et al. & Førsund: (22), Dakpo et al. & Førsund: (26).

\*\*\* Denotes statistically significant differences between models at the critical 1 percent level.

\*\* Denotes statistically significant differences between models at the critical 5 percent level.

\* Denotes statistically significant differences between models at the critical 10 percent level.

Appendix A.2. Results of Simar and Zelenyuk (2006) adapted Li test (test statistic and significance level) for profit, technical and allocative inefficiencies

Profit inefficiency (PI)				
	Murty	Dakpo	M. & F.	D. & F.
Murty	-	15.02**	-0.87	15.34**
Dakpo		-	15.02**	1.00
M. & F.			-	15.34**
D. & F.				-

Technical inefficiency (TI)				
	Murty	Dakpo	M. & F.	D. & F.
Murty	-	17.03*	-0.85	16.69**
Dakpo		-	17.00*	6.22
M. & F.			-	16.69**
D. & F.				-

Allocative inefficiency (AI)				
	Murty	Dakpo	M. & F.	D. & F.
Murty	-	12.14**	-1.00	13.02**
Dakpo		-	12.08**	1.97
M. & F.			-	13.02**
D. & F.				-

Notes: Murty et al.: (5), Dakpo et al.: (17), Murty et al. & Førsund: (22), Dakpo et al. & Førsund: (26).

\*\* Denotes statistically significant differences between models at the critical 5 percent level.

\* Denotes statistically significant differences between models at the critical 10 percent level.

## References

- Aigner, D., Lovell, C.A.K. and Schmidt, P. (1977). Formulation and estimation of stochastic frontier production function models. *Journal of Econometrics*, 6(1): 21–37.
- Aparicio, J., Borrás, F., Pastor, J. T. and Vidal, F. (2015). Measuring and decomposing firm' s revenue and cost efficiency: The Russell measures revisited. *International Journal of Production Economics*, 165, 19–28.
- Aparicio, J., Borrás, F., Pastor J.T and Zofío, J.L. (2016a). Loss Distance Functions and Profit Function: General Duality Results. In Juan Aparicio, C. A. Knox Lovell and Jesus T. Pastor (eds.) *Advances in Efficiency and Productivity*, Springer: NY, 76–91.
- Aparicio, J., Pastor, J. T. and Vidal, F. (2016b). The weighted additive distance function. *European Journal of Operational Research*, 254(1), 338–346.
- Arjomandi, A., Dakpo, K.H. and Seufert, J.H. (2018). Have Asian airlines caught up with European airlines? A by-production efficiency analysis. *Transportation Research Part A: Policy and Practice*, 116: 389–403.
- Ayres, R.U. and Kneese, A.V. (1969). Production, consumption, and externalities. *The American Economic Review*, 59(3): 282–297.
- Ball, V.E., Gollop, F.M., Kelly-Hawke, A. and Swinand, G.P. (1999). Patterns of state productivity growth in the U.S. farm sector: Linking state and aggregate models. *American Journal of Agricultural Economics*, 81(1): 164–179.
- Ball, V.E., Färe, R., Grosskopf, S. and Nehring, R. (2001). Productivity of the U.S. agricultural sector: The case of undesirable outputs. In: *New Developments in Productivity Analysis* (edited by Hulten, C.R., Dean, E.R. and Harper, M.J.), pages 541–586. Chicago: University of Chicago Press.
- Ball, E., Färe R., Grosskopf, S. and Zaim, O. (2005). Accounting for externalities in the measurement of productivity growth: the Malmquist cost productivity measure. *Structural Change and Economic Dynamics*, 16: 374–394.
- Banker, R.D., Charnes, A. and Cooper, W.W. (1984). Some models for estimating technical and scale inefficiencies in Data Envelopment Analysis. *Management Science*, 30(9): 1078–1092.
- Brännlund, R., Färe, R. and Grosskopf, S. (1995). Environmental regulation and profitability: an application to Swedish pulp and paper mills. *Environmental and resource Economics*, 6(1): 23–36.
- California Cap and Trade Program. Prices of CO<sub>2</sub>. <http://calcarbondash.org/>.
- Centers for Disease Control and Prevention. U.S. Department of Health & Human Services. Environmental Public Health Tracking Network. Pesticide Exposures. Accessed from Environmental Public Health Tracking Network: [www.cdc.gov/ephttracking](http://www.cdc.gov/ephttracking)
- Charnes, A., Cooper, W. W. and Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research*, 2(6): 429–444.
- Chambers, R. G., Chung, Y. and Färe, R. (1998). Profit, Directional Distance Functions, and Nerlovian Efficiency. *Journal of Optimization Theory and Applications* 98:2, 351–64.
- Coelli, T.J., Lauwers, L. and Van Huylenbroeck, G., 2007. Environmental efficiency measurement and the materials balance condition. *Journal of Productivity Analysis*, 28: 3–12.
- Cuesta, Rafael, C.A. Knox Lovell and José L. Zofío (2009). Environmental Efficiency Measurement with Translog Distance Functions. *Ecological Economics*, 68: 2232–2242.
- Dakpo, K.H. (2016). On modeling pollution-generating technologies: a new formulation of the by-production approach. Working Papers 245191, Institut National de la

- recherche Agronomique (INRA), Departement Sciences Sociales, Agriculture et Alimentation, Espace et Environnement (SAE2).
- Dakpo, K.H., Jeanneaux, P. and Latruffe, L. (2016). Modeling pollution-generating technologies in performance benchmarking: Recent developments, limits and future prospects in the nonparametric framework. *European Journal of Operational Research*, 250: 347–359.
- Dakpo, K.H., Jeanneaux, P. and Latruffe, L. (2017). Greenhouse gas emissions and efficiency in French sheep meat farming: A nonparametric framework of pollution adjusted technologies. *European Review of Agricultural Economics*, 44: 33–65.
- Färe, R., Grosskopf, S. and Lovell, C.A.K. (1985) The Measurement of Efficiency of Production. Kluwer Nijhof Publishing.
- Färe R., Grosskopf S. and Pasurka C. (1986). Effects on relative efficiency in electric power generation due to environmental controls. *Resources and Energy*, 8: 167–184.
- Färe R., Grosskopf S., Lovell, C.A.K. and Pasurka, C. (1989). Multilateral productivity comparisons when some outputs are undesirable: A nonparametric approach. *The Review of Economics and Statistics*, 71: 90–98.
- Färe R., Grosskopf, S. and Weber, W.L. (2006). Shadow prices and pollution costs in U.S. agriculture. *Ecological Economics*, 56: 89–103.
- Färe R., Grosskopf, S., Noh, D.-W. and Weber, W. (2005). Characteristics of a polluting technology: theory and practice. *Journal of Econometrics*, 126: 469–492.
- Färe, R. and Grosskopf, S. (2003). Nonparametric productivity analysis with undesirable outputs: Comment. *American Journal of Agricultural Economics*, 85(4): 1070–74.
- Frisch, R. (1965). Theory of production. D. Reidel Publishing Company, Dordrecht.
- Førsund, F.R. (2009). Good modeling of bad outputs: Pollution and multiple-output production. *International Review of Environmental and Resource Economics*, 3: 1–38.
- Førsund, F.R. (2018). Multi-equation modelling of desirable and undesirable outputs satisfying the materials balance. *Empirical Economics*, 54(1): 67–99.
- Hailu, A. (2003). Nonparametric productivity analysis with undesirable outputs: Reply. *American Journal of Agricultural Economics*, 85(4): 1075–77.
- Hailu, A. and Veeman, T.S. (2001). Non-parametric productivity analysis with undesirable outputs: An application to the Canadian pulp and paper industry. *American Journal of Agricultural Economics*, 83: 605–616.
- Hampf, B. and Rødseth, K.L. (2015). Carbon dioxide emission standards for U.S. power plants: An efficiency analysis perspective. *Energy Economics*, 50: 140–153.
- Haynes, K.E., Ratick, S., Bowen, W.M. and Cummings-Saxton, J. (1993). Environmental decision models: U.S. experience and new approaches to pollution management. *Environment International*, 19: 261–275.
- Hoang, V.-N. and Alauddin, M. (2012). Input-orientated Data Envelopment Analysis framework for measuring and decomposing economic, environmental and ecological efficiency: An Application to OECD agriculture. *Environmental and Resource Economics*, 51: 431–452.
- Hoang, V.-N. and Rao, D.S.P. (2010). Measuring and decomposing sustainable efficiency in agricultural production: A cumulative exergy balance approach. *Ecological Economics*, 69:9, 1765–1776.
- Huffman, W.E. and Evenson, R.E. (2006). Do formula or competitive grant funds have greater impacts on state agricultural productivity. *American Journal of Agricultural Economics*, 88(4): 783–798.
- Kellog, R.L., Nehring, R.F., Grube, A., Goss, D.W. and Plotkin, S. (2002). Environmental indicators of pesticide leaching and runoff from farm fields. In *Agricultural*

- Productivity: Measurement and Sources of Growth, edited by Ball, V.E. and Norton, G.W., pages 213-256. Kluwer Academic Publishers: Boston/Dordrecht/London.
- Korhonen P.J. and Luptacik M. (2004). Eco-efficiency analysis of power plants: An extension of data envelopment analysis. *European Journal of Operational Research*, 154: 437–446.
- Kuosmanen, T. and Kortelainen, M. (2005). Measuring eco-efficiency of production with data envelopment analysis. *Journal of Industrial Ecology*, 9(4): 59–72.
- Lauwers L. (2009). Justifying the incorporation of the materials balance principle into frontier-based eco-efficiency models. *Ecological Economics*, 68: 1605–1614.
- Lauwers, L.H. and Van Huylenbroeck, G. (2003). Materials balance based modelling of environmental efficiency. Contributed paper selected for presentation at the 25th International Conference of Agricultural Economists, August 16-22, 2003, Durban, South Africa.
- Li, Q. (1996). Nonparametric testing of closeness between two unknown distribution functions. *Econometric Reviews*, 15: 261–274.
- Lozano, S. (2015). A joint-inputs network DEA approach to production and pollution-generating technologies. *Expert Systems with Applications*, 42: 7960–7968.
- Mahlberg, B. and Sahoo, B.K. (2011). Radial and non-radial decompositions of Luenberger productivity indicator with an illustrative application. *International Journal of Production Economics*, 131: 721–726.
- Murty, S. and Russell, R.R. (2002). On modeling pollution-generating technologies, Department of Economics, University of California, Riverside, Discussion Papers Series, No. 02-14, 2002.
- Murty, S., Russell, R.R. and Levkoff, S.B. (2012). On modeling pollution-generating technologies. *Journal of Environmental Economics and Management*, 64: 117–135.
- Murty, S. and Russell, R.R. (2018). Modeling emission-generating technologies: reconciliation of axiomatic and by-production approaches. *Empirical Economics*, 54(1): 7–30.
- Nguyen, T.T., Hoang, V.-N. and Seo, B. (2012). Cost and environmental efficiency of rice farms in South Korea. *Agricultural Economics*, 43: 369–378.
- Pearce, D., Cline, W., Achanta, A., Fankhauser, S., Pachauri, R., Tol, R., and Vellinga, P. (1996). The social cost of climate change: greenhouse damage and the benefits of control, in Bruce, J., Lee, H., and Haites, E. (eds.), *Climate Change 1995: Economic and Social Dimensions of Climate Change*, Cambridge: Cambridge University Press, 179–224.
- Pérez Urdiales, M., Oude Lansink, A. and Wall, A. (2016). Eco-efficiency among dairy farmers: The importance of socio-economic characteristics and farmer attitudes. *Environmental and Resource Economics*, 64(4): 559–574.
- Pham, M.D. and Zelenyuk, V. (2018). Slack-based directional distance function in the presence of bad outputs: Theory and application to Vietnamese banking. *Empirical Economics*, 54: 153–187.
- Picazo-Tadeo, A. J., Gómez-Limón, J. A. and Reig-Martínez, E. (2011). Assessing farming eco-efficiency: a data envelopment analysis approach. *Journal of Environmental Management*, 92: 1154–1164.
- Pimentel (2005). Environmental and economic costs of the application of pesticides primarily in the United States. *Environment, Development and Sustainability*, 7: 229–252.
- Pittman, R.W. (1983). Multilateral productivity comparisons with undesirable outputs. *The Economic Journal*, 93: 391–883.

- Ray, S.C. (2004). Data envelopment analysis: theory and techniques for economics and operations research. Cambridge University Press.
- Ray, S.C., Mukherjee, K. and Venkatesh, A. (2018). Nonparametric measures of efficiency in the presence of undesirable outputs: a by-production approach. *Empirical Economics*, 54(1): 31–65.
- Reinhard, S., Lovell, C.A.K. and Thijssen, G.J. (1999). Econometric estimation of technical and environmental efficiency: An application to Dutch dairy farms. *American Journal of Agricultural Economics*, 81: 44–60.
- Reinhard, S., Lovell, C.A.K. and Thijssen, G.J. (2000). Environmental efficiency with multiple environmentally detrimental variables; estimated with SFA and DEA. *European Journal of Operational Research*, 121: 287–303.
- Sabasi, D. and Shumway, C.R. (2018). Climate change, health care access and regional influence on components of U.S. agricultural productivity. *Applied Economics*, 50: 6149–6164.
- Serra, T., Chambers, R.G. and Oude Lansink, A. (2014). Measuring technical and environmental efficiency in a state-contingent technology. *European Journal of Operational Research*, 236: 706–717.
- Sheather, S. and Jones, M. (1991). A reliable data-based bandwidth selection method for kernel density estimation. *Journal of the Royal Statistical Society. Series B*, 53: 683–690.
- Shumway, C.R., Fraumeni, B.M., Fulginiti, L.E., Samuels, J.D. and Stefanou, S.E. (2015). Measurement of U.S. agricultural productivity: A 2014 review of current statistics and proposals for change. Working paper series WP 2015-12. School of Economic Sciences. Washington State University.
- Shumway, C.R., Fraumeni, B.M., Fulginiti, L.E., Samuels, J.D. and Stefanou, S.E. (2016). U.S. agricultural productivity: A review of USDA Economic Research Service methods. *Applied Economic Perspectives and Policy*, 38(1): 1–29.
- Silverman, B. W. (1986). *Density Estimation for Statistics and Data Analysis*. London: Chapman and Hall.
- Simar, L. and Zelenyuk, V. (2006). On testing equality of distributions of technical efficiency scores. *Econometric Reviews*, 25(4): 497–522.
- Skinner, J.A., Lewis, K.A., Bardon, K.S., Tucker, P., Catt, J.A. and Chambers, B.J. (1997). An overview of the environmental impact of agriculture in the U.K. *Journal of Environmental Management*, 50(2): 111–128.
- Tyteca, D. (1996) On the measurement of the environmental performance of firms—A literature review and a productive efficiency perspective. *Journal of Environmental Management*, 46, 281–308.
- U.S. Bureau of Labor Statistics. Consumer price index and price index for medical services. <https://www.bls.gov/>
- U.S. Department of Agriculture (USDA). Census of Agriculture. <https://www.nass.usda.gov/AgCensus/>
- U.S. Department of Agriculture. Economic Research Service (ERS). Data on Agricultural Productivity in the U.S. <https://www.ers.usda.gov/data-products/agricultural-productivity-in-the-us/>
- U.S. Environmental Protection Agency (EPA). Greenhouse gas inventory. <https://www3.epa.gov/climatechange/ghgemissions/inventoryexplorer/>
- Welch, E. and Barnum, D. (2009). Joint environmental and cost efficiency analysis of electricity generation. *Ecological Economics*, 68: 2336–2343.
- Zofio, J.L., Knox Lovell, C.A. 2001. Graph efficiency and productivity measures: an application to US agriculture. *Applied Economics*, 33(11): 1433–1442.