Investigation and modelling of direct toppling using a three-dimensional Distinct Element approach with incorporation of point cloud geometry

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Abstract

Block toppling instability can be a common problem in natural rock masses, especially in mining environments where excavation activity may trigger discontinuity-controlled instability by modifying the natural slope geometry. Traditional investigations of block toppling failure consider classic kinematic analyses and simplified two-dimensional limit equilibrium methods. This approach is still the most commonly adopted, but the simple two-dimensional conceptual model may often oversimplify the instability mechanisms, ignoring potential critical factors specifically related to the three-dimensional geometry. This paper uses a three-dimensional distinct element method approach applied to an example case study, identifying the critical parameters that influence direct toppling instability in an open pit environment. Terrestrial laser scanning was used to obtain detailed three-dimensional geometrical information of the slope face geometry for subsequent stability analyses. A series of sensitivity analyses on critical parameters such as friction angle, discontinuity shear and normal stiffness, discontinuity spacing and orientation was performed, using simple conceptual three-dimensional numerical modelling. Results of the analyses revealed the importance of undertaking three-dimensional analyses for direct toppling investigations that allow identification of critical parameters. A three-dimensional distinct element analysis was then performed using a more realistic complex volumetric mesh model of the case study slope which confirmed the previous modelling results but also identified unstable blocks in high slope angle areas, providing useful information for life of mine design. The paper highlights the importance of slope geometry and fracture network orientation on potential slope instability mechanisms.

Keywords Slope Stability, Mining, Toppling, 3DEC, Three-dimensional distinct element modelling

1 Introduction

Discontinuity-controlled instability mechanisms are unfortunately a common phenomenon in nature, especially when human activity causes alteration or disturbance of the natural system. This is the case in mining, where perturbation resulting from excavation activity may cause instability of anthropogenic or natural rock slopes causing potential safety problems. This can be not only due to induced changes in stresses, but also due to the generation of a new or modified slope geometry that may create the conditions necessary for a discontinuity-controlled failure to develop. In this regard, if the discontinuity setting or rock fracture network of an area is known, simple kinematic analyses can provide useful indications of the likelihood of potential discontinuity-controlled instability to develop for particular slope orientations.

Indeed, kinematic analyses based, for example, on the Markland test (Markland 1972) for planar sliding, wedge sliding, direct and flexural toppling are still commonly used, and can be considered as an essential "starting point" for a stability analysis. Nevertheless, one of the main limitations of such an approach based on stereographic analysis is the inability to locate a potentially unstable block at a specific location or area of the slope face. Francioni et al. 2015 proposed a GIS (Geographic Information System) based approach to overcome problems related to complex slope geometry and location of possible unstable areas on a rock slope.

However, given the intrinsic simplification of kinematic techniques, more complex stability analysis approaches are often necessary for obtaining reliable results to be used, for example, in Life of Mine (LoM) design planning or calculations of Factor of Safety (FoS) and potential risk of instability. In this regard, as described by Stead et al. (2006), classical Limit Equilibrium Methods (LEMs) can be used for two- and three-dimensional stability analyses by using a simplified approach that includes a limited number of variables (representative geometry, material/joint shear strength, material unit weights, groundwater and external loading/support condition). This approach allows for rapid FoS calculations but requires an assumption of the instability mechanisms a priori, and does not consider the effects of such factors as in-situ stresses, strains and failure of intact material (Stead et al. 2006).

Nevertheless, two- and three-dimensional LEM techniques are still commonly adopted for rock mass stability analyses (Bretas et al. 2012; Zhou and Cheng 2015; Dong-ping et al. 2016; Jiang et al. 2017; Salvini et al. 2018; Vanneschi et al. 2018). Their limitations can be overcome by using more complex numerical modelling techniques. These include, for example, Finite Element Methods (FEMs) for continuum modelling and the use of Distinct Element Methods (DEMs) for discontinuum modelling. The key advantages of these methods are the ability to consider intact rock deformation and movements, complex hydro-mechanical and dynamic analyses, and additional insight into identification of potential instability mechanisms. However, as the number of variables increase, model uncertainty can also increase, which may influence the reliability of the results obtained.

Despite these limitations, the potential benefits of using numerical modelling has resulted in their widespread adoption for engineering-geological investigations with both two-dimensional (Shimizu et al. 2010; Bahrani et al. 2014; Francioni et al. 2015; Salvini et al. 2015a; Khanal et al. 2017) or three-dimensional (Francioni et al. 2014; Karampinos et al. 2015; Shreedharan and Kulatilake 2016; Spreafico et al. 2016) approaches. As a consequence, as more complex problems are analyzed, the reliance on the quality and quantity of input data becomes ever more crucial. In this regard, recent developments in remote sensing techniques and associated data acquisition have provided a framework for improved use of remotely captured data in slope stability investigations. For example, Francioni et al. (2018) recently highlighted the advantages and limitations of different geomatic techniques for engineering-geological investigation, and the optimized use of the relative high resolution geometrical data to reduce model uncertainty in slope stability analyses.

The importance of geomatics, from photogrammetry to laser scanning techniques, is also highlighted by recent literature where remote sensing approaches are used for geotechnical investigations, from geometrical reconstruction (Vanneschi et al. 2014; Geyer et al. 2015; Svennevig et al. 2015) to discontinuity mapping and rock mass characterization (Khoshelham et al. 2011; Riquelme et al. 2015; Salvini et al. 2017), monitoring systems (Salvini et al. 2015b; Bovenga et al. 2017; Huang et al. 2017), rockfall runout analyses (Pedrazzini et al. 2012; Salvini et al. 2013; Hsu et al. 2016) and numerical modelling analyses (Francioni et al. 2015; Havaej et al. 2016; Spreafico et al. 2016).

The present study investigates discontinuity-controlled instability in the form of a direct toppling mechanism affecting a portion of an open pit. Toppling failure has been described by Hoek and Bray (1981) as a 'rotation of columns or blocks of rocks about some fixed base', while Goodman and Bray (1976) recognized different toppling mechanisms, which are: flexural toppling (where closely spaced and pervasive discontinuities affect a rock mass in absence of a basal plane, therefore toppling failure occurs when the intact rock strength is exceeded), direct toppling (where closely spaced and pervasive discontinuities affect a rock mass in presence of a basal plane, therefore toppling blocks are free to generate a rotating moment without necessity of developing new fractures) and direct-flexural toppling (which is a mix of the two mechanisms). Figure 1 shows a simplified example of a slope affected by block instability, where both direct toppling and sliding mechanisms are present.

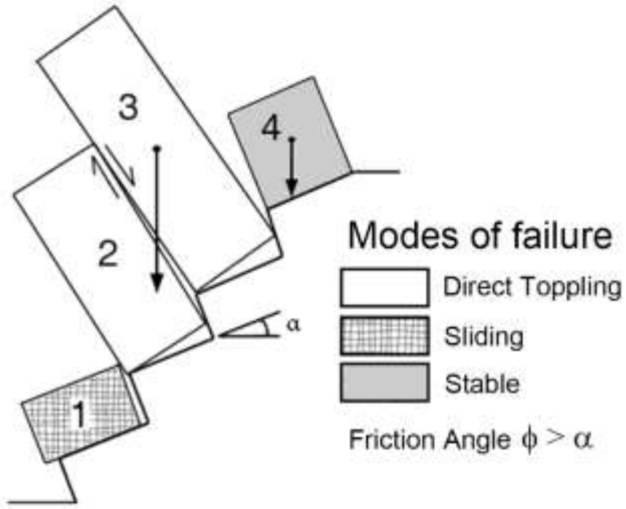

Fig. 1 Simplified model of direct toppling and sliding instability (modified from Wyllie 1980)

Figure 1 shows that, in the absence of other external forces, direct toppling develops if the centre of gravity of a block acts outside its base, as in the case of blocks 2 and 3 (shear developed along discontinuity is a consequence of this). The centre of gravity of block 4 acts within its' base and therefore the block does not have the potential for generating a rotating moment. Similarly, block 1 does not have the potential to topple, but sliding instability may occur because of the additional shear force transmitted by blocks 2 and 3. Several previous research projects have been undertaken to analyse toppling failure mechanisms through numerical modelling methods, especially using two-dimensional codes (Pritchard and Savigny 1990; Coggan and Pine 1996; Deangeli and Ferrero 2000; Merrien-Soukatchoff et al. 2001; Nichol et al. 2002; Tosney et al. 2004). More recently, complex large-scale toppling instability was also investigated by Gu and Huang (2016), adopting a two-dimensional DEM approach, and by Spreafico et al. (2017), adopting a two-dimensional FEM-DFN-Voronoi approach. A three-dimensional DEM analysis of discontinuity-controlled mechanisms at a bench scale was undertaken by Havaej et al. (2016) in reference to basal wedge sliding instability. Brideau and Stead (2010, 2012) analysed direct toppling mechanisms through simplified conceptual models, demonstrating the importance of performing three-dimensional analyses that, according to their results, lead to significantly different results from two-dimensional toppling investigations. Nevertheless, review of previous literature suggests the use of a three-dimensional DEM approach for direct toppling investigations is still infrequent, and the majority of stability analyses are performed using traditional two-dimensional approaches. This paper demonstrates the benefits and application of a three-dimensional approach for improved understanding of direct toppling failure.

In order to provide an improved understanding of the geometrical and geotechnical conditions behind the development of direct toppling instability in an open pit environment, and for providing guidelines for mine design purposes, a remote sensing survey was undertaken in a section of an open pit (Test bench area) previously identified to be kinetically unstable through engineering-geological investigations. The stability investigation presented uses the remotely captured three-dimensional data as input for detailed three-dimensional DEM analyses in order to identify the critical parameters influencing slope stability, and to analyse possible solutions for reducing the risk level hypothesized.

2 Engineering-geological setting of the case study

Melbur Pit is located to the far west of the St. Austell granite (Cornwall, UK, Fig. 2b) within the 285-280 Ma biotite granite domain. Four distinct categories of alteration of kaolinised granite were determined by Hencher et al. (1990): Grade II (Slightly altered granite), Grade III (Medium altered granite), Grade IV (Highly altered granite) and Grade V (Completely altered granite). A geological map of Melbur Pit (Carter 2015), shown in Fig. 2a, provides an indication of the spatial distribution of the different alteration grades.

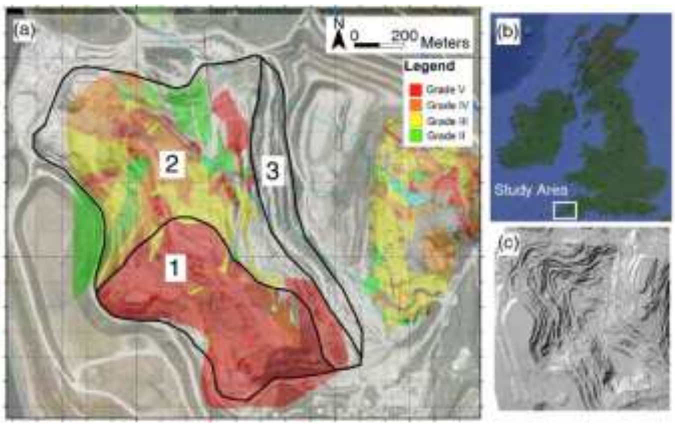

Fig. 2 (a) Geological map of Melbur Pit (modified from Carter 2015), (b) study area location, (c) hillshade map of Melbur Pit

In Fig. 2a the area is also sub-divided into three geotechnical domains: Zones 1, 2 and 3. Zone 1 is predominantly characterized by Grade V material, which may be susceptible to rotational failure through the completely altered material. Zone 2 is characterized by mixed alteration grade material and may be at risk from both structurally controlled failure (planar, wedge, and toppling) and failure through the rock mass. Zone 3 is characterized by low alteration grade material, generally dry, where the dominant failure mode is discontinuity or structurally controlled.

Previous studies (Carter 2015; Keverne et al. 2015) indicate that three principal discontinuity set orientations are present within Melbur Pit: J1 (sub-vertical, East dipping), J2 (sub-vertical, North-West dipping), and J3 (low angle, West dipping). The studies also indicated that on a bench scale it was observed that discontinuity-controlled instability mechanisms may be possible and influenced by the granite alteration grade within Melbur Pit (Carter 2015). In view of this, a specific GIS-based kinematic analysis was carried out to identify the spatial distribution of potential discontinuity-controlled failure mechanisms within the less altered regions of the Melbur Pit. The study combined geological investigation of the area (excluding areas with alteration IV and V materials, since they are more susceptible to rock mass-controlled or rotational failures), discontinuity sets orientation (dip and dip directions), slope bench face angles and directions (calculated using LiDAR data), and safety limit angles for every possible slope direction (calculated through traditional kinematic analyses using Markland test) in ArcGIS. This provided a basis for identification of zones where discontinuity-controlled instability may develop.

3 Methods of analysis

3.1 Geometrical characterization of the slope

Detailed geometrical data of the slope bench was acquired through a Terrestrial Laser Scanning (TLS) survey, using a Leica HDS6000. Multiple laser scan positions were undertaken to cover the majority of the test area, minimizing blinding and occlusion. Local control between the independent laser scan positions was established using Leica 'tilt and turn' targets which remained static between at least two adjoining setups and were used to constrain the laser scans. The target locations provide common points in the subsequent registration process and an initial estimate for the ICP (Iterative Closest Point) alignment, resulting in a coherent point cloud referenced to a local common coordinate system. In addition, during the onsite survey, various static targets were positioned in the scene and co-ordinated onto OSGB36 using conventional survey methods. The geo-referenced targets were then identified in the laser scans and used to apply a spatial transformation to locate the point cloud in a real world coordinate system.

Once registered and geo-referenced the point cloud was exported so that discontinuity mapping could be undertaken. The orientation of 776 joints was calculated by creating 'patches' that best fit the identified discontinuity planes on the point cloud. In addition, measurements were taken between the discontinuity sets to establish spacing. Following this, the data was then exported into Dips 7.0 (RocScience Inc. 2017) for kinematic analysis and discontinuity set characterization for this section of the open pit.

3.2 Stability analysis

Given the three-dimensional nature of the potential direct toppling instability for the present case study, it was decided to overcome limitations of the two- three-dimensional LEM approach by using the three-dimensional DEM code 3DEC 5.2 (Itasca Consulting Group Inc. 2017a) for the investigation. The code uses an explicit time-stepping system to solve equations of motion (details in Cundall 1988; Itasca Consulting Group Inc. 2017b), simulating the response of a discontinuum model to static or dynamic loading. The model in 3DEC is composed of a number of blocks separated by discontinuities, and is particularly suitable for modelling large displacements and block rotations, allowing consideration of the effects of discontinuity Shear (Ks) and Normal (Kn) stiffness. In addition, blocks can be considered to be either rigid or deformable. In this case, because of the discontinuity-controlled nature of the problem, together with the relatively shallow depth of the test bench under study, rigid blocks with no imposed stresses were assumed for the analysis.

The model loading strategy adopted for the analysis was to solve the model by initially applying high geotechnical properties to the joints (i.e. 89° friction angle), in order to reach an initial equilibrium state, which allows calculation of the in-situ stress state. After the model was brought to equilibrium, the behaviour of the slope face was studied by reducing the geotechnical properties of the discontinuities to their real values. In this context, a static or quasi-static solution is reached in 3DEC when the rate of change of kinetic energy in a model approaches a negligible value, which can be accomplished by damping the equations of motion. The default mode (global damping) is useful for obtaining initial conditions, because it attempts to quickly bring the model to equilibrium. However, this solution may not be accurate if movement in the model are expected, as in the case of joint or discontinuity sliding/toppling (Itasca Consulting Group Inc. 2017b). Therefore, the damping strategy for this work was to reach the initial equilibrium state of the model by using the global damping scheme, and then to apply the local damping mode (which may be preferred for analyses where a different amount of damping is required in different regions of the model) for the direct toppling analysis.

The strength properties assumed for DEM analyses are summarized in Table 1 (the Mohr-Coulomb strength criteria was adopted for this study; Coulomb, 1776; Mohr, 1900).

Table 1 Material and discontinuity properties used in the stability analyses

Material property	
Density (kg/m3)	2700
Young's modulus (GPa)	40
Poisson's ratio	0.2
Joint properties	
Shear Stiffness (Pa/m)	5e7
Normal Stiffness (Pa/m)	5e8
Friction angle (°)	35
Cohesion (MPa)	0
Tensile Strength (MPa)	0

The initial DEM simulations were based on simplified slope geometries determined from TLS point cloud measurements. This initial simplified approach was chosen in order to improve the understanding of the influence of different parameters on the proposed failure mechanism. This can be achieved by using simple models that allow a high level of control on the model behaviour and make interpretation of the results easier, in addition to shorter run-times. Once the main failure mechanism(s), and the controlling influences are understood, complexity can then be added to the model; with the consequent exponential increase in simulation time. Using this philosophy, a more representative slope geometry was then incorporated within 3DEC by importing a complex meshed model created from the TLS point cloud. The meshed model was developed in Rhinocheros 5.0 (McNeel and Associates 2017) by using the plug-in Griddle (Itasca Consulting Group Inc. 2017c), specifically designed for creating volumetric three-dimensional mesh to be used in 3DEC. This final meshed model may therefore be considered more representative of the geometrical setting or three-dimensional slope profile of the test bench slope face.

4 Results

4.1 Geometrical characterization

The final TLS point cloud of the test bench is shown in Fig. 3. The registration process indicated that the final accuracy achieved for the plano-altimetric alignment was sub-centimetre.

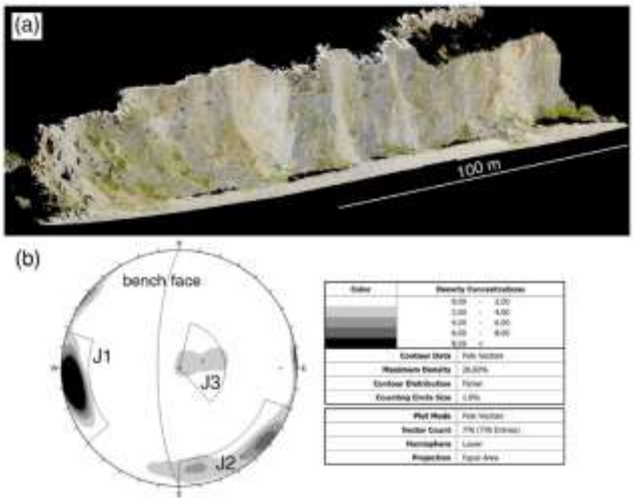

Fig. 3 TLS point cloud of the Test bench (**a**) and stereonet showing discontinuity sets identified and bench orientation (**b**)

Figure 3 also shows a stereonet of identified discontinuities using Dips 7.0, indicating the discontinuity sets (whose properties are illustrated in Table 2; orientation and approximated spacing) identified on the point cloud, and average bench or slope face orientation (Dip/Dip Direction 75°/270°).

Table 2 Orientation and Spacing of major discontinuity sets identified within test bench in Melbur Pit

System	Dip (°)	Dip Direction (°)	Spacing (m)
J1	81	082	1
J2	83	321	2
J3	15	261	2

Results of the kinematic analysis showed that planar and wedge sliding are unlikely to occur in Melbur Pit, whereas some regions of the pit are influenced by the potential for direct toppling failure. Figure 4 shows example results of the direct toppling-GIS analysis, where highlighted pixels represent areas that have the necessary combined attributes for instability.

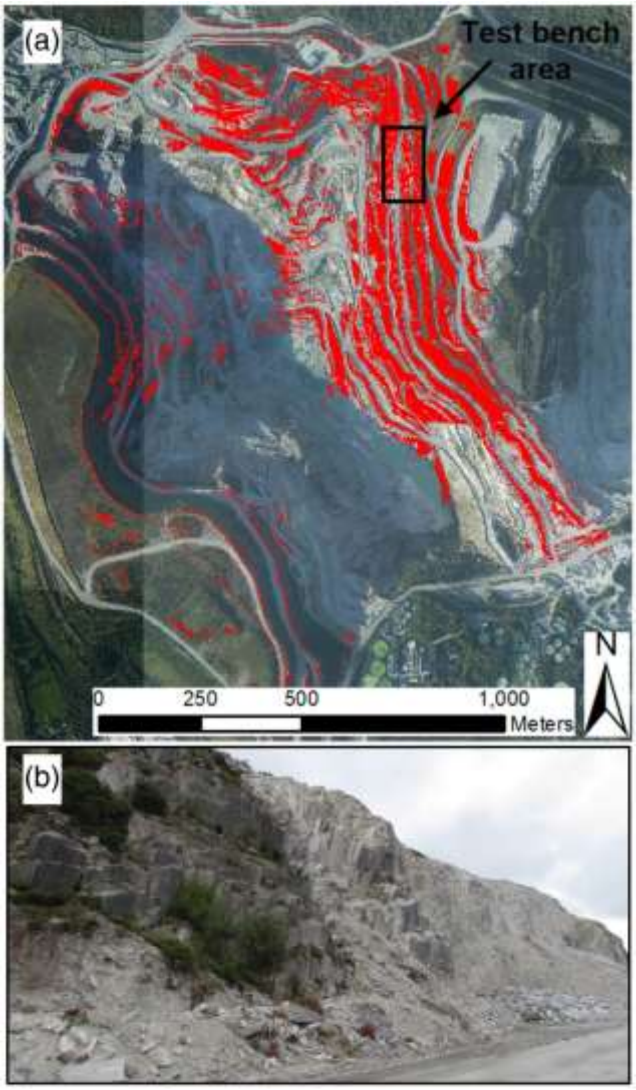

Fig. 4 Melbur Pit with indication (highlighted pixels) of possible direct toppling source areas calculated with a GIS-based approach (a). Test bench area highlighted with a black rectangle (b)

The results highlight that, according to traditional kinematic analysis, the East slope of Melbur Pit has the necessary geometrical conditions for direct toppling instability to occur. Therefore, further analyses described in this paper have been undertaken for the 'Test Area' shown in Fig. 4, located within the identified critical sectors that are susceptible to direct toppling instability.

According to kinematic analyses, and supported by direct field observations, it can be confirmed that J1, J2, and J3 provide the potential for direct toppling instability, with the centre of gravity of the formed blocks lying outside the outline of the base of the block, resulting in the critical development of an overturning moment. Field observations suggest that flexural toppling, even if kinematically and geometrically possible, is unlikely to occur in this kind of competent rock. This is because for flexural toppling to occur the intact rock should be subjected to failure induced by inter-layer slip, but in this case, the low angle system J3 has a primary control on the instability mechanism and potential release of a failed block. For this reason, the following stability analyses are focused on analysis of direct toppling.

To summarize, the model created for the subsequent analyses is based on the following characteristics determined from the TLS point cloud: 3 discontinuity sets (J1, 1 m spacing, that represent the rear release surface; J2, 2 m spacing, that represents the lateral release surface; J3, 2 m spacing, that represents the basal release surface), average slope angle of 75°, 20 m bench height, 15 m bench width, vertical and fixed lateral boundaries (this assumption is adopted for representing the geometry of an open-pit bench, where no free lateral boundaries are present).

4.2 Distinct Element numerical modelling

The initial 3DEC analyses were performed using simplified geometry (Fig. 5) based on the TLS data highlighted in section 4.1. Several History points were placed within the model to monitor horizontal displacement and horizontal velocity. For clarity, only the results for History 1 are provided in this paper. History 1 was chosen as this point is close to the surface of the model and provides good representation of the model behaviour. The location of monitoring points further into the pit wall had reduced modelled displacement until the Step-Path surface (shown Fig. 6) is reached. The modelled slope is stable beyond the step-path surface. Figure 5 also shows the orientation of the plane used for the cross section (provided in Fig. 6). In this case the vertical plane has been oriented using a dip direction of 321°, in order to be parallel to the lateral release surface (J2), which in this case corresponds to the direction of toppling.

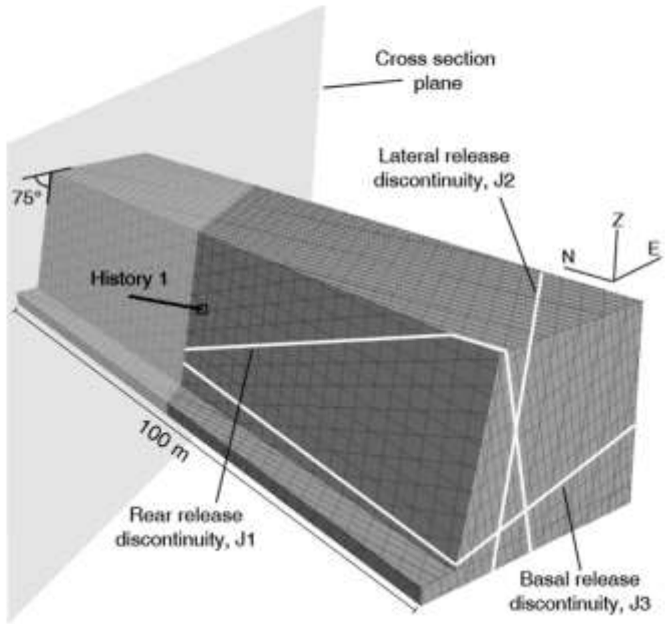

Fig. 5 Simplified three-dimensional model of the Test bench used in 3DEC

Fig. 6 shows the result of the first deterministic 3DEC analysis, using the parameters provided in Table 1.

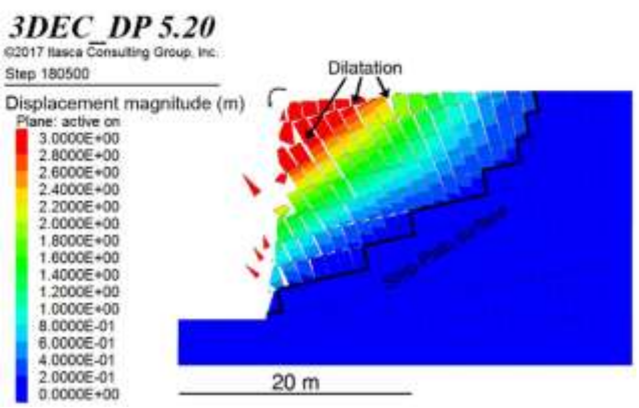

Fig. 6 Cross section of 3DEC deterministic analysis with calculated absolute displacement contours

Examination of displacement contours along the cross section shown in Fig. 6 shows that the geometric configuration and modelled discontinuities lead to direct toppling failure. Figure 7 shows the recorded horizontal displacements and velocity plot for History 1. The figure allows identification of the point at which the block starts to rotate and fall. This corresponds to an accelerating trend in the velocity with direct consequences on observed displacements. It must be specified that the time indicated in the figures refers to the software simulation time (i.e. number of steps), and does not refer to real time.

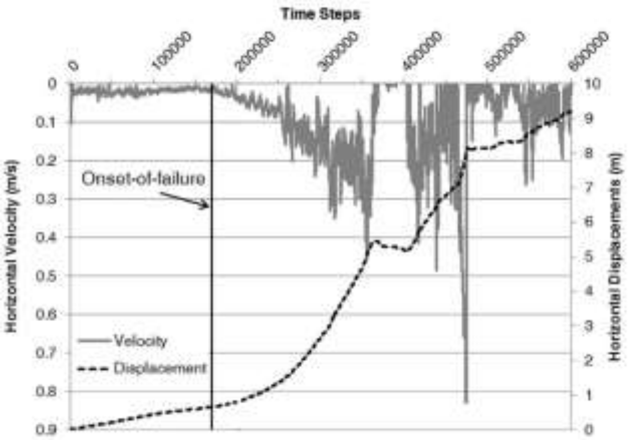

Fig. 7 Result of 3DEC deterministic analysis with indication of recorded horizontal velocity and displacement for History 1

4.2.1 Influence of discontinuity properties on model behaviour

A series of 3DEC simulations were carried out to investigate the influence of discontinuity parameters (only the most representative ones have been reported) such as friction (phi), Kn, and Ks on model behaviour. During this initial investigation all discontinuities were assigned the same properties. Figure 8 shows results of sensitivity analysis undertaken on the discontinuity friction angle, where friction angle was varied from 25° to 40°.

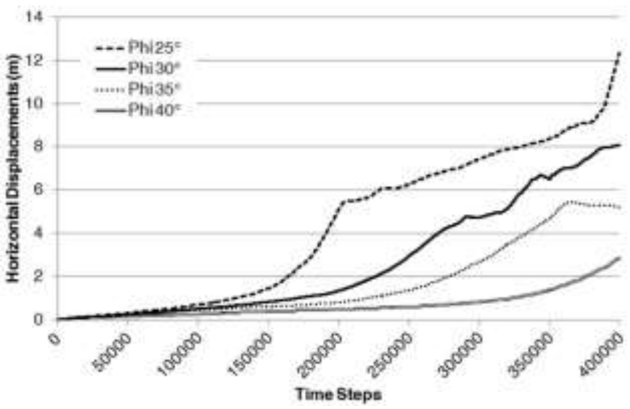

Fig. 8 Recorded horizontal displacements of 3DEC sensitivity analysis where discontinuity friction angle is varied from 25° to 40°

From Fig. 8 it is evident that discontinuity friction angle has an influence on the modelled final horizontal displacements magnitude, but the underlying toppling failure always occurs for the range of friction angles simulated, even if with increased delay for higher friction angles.

Other important factors investigated were the influence of Kn and Ks along discontinuities. This sensitivity analysis is particularly important since Kn and Ks parameters are not considered in traditional LEM analysis. The difficulty is that there is a high degree of uncertainty in identifying the correct Kn and Ks values to be used in numerical modelling (Brideau and Stead 2012). This is especially true in this case study where the granites may be affected by a variable degree of alteration. The results of the sensitivity analysis suggest that the major influence on the modelled results is related to the magnitude of Ks values, as relatively small changes can significantly affect the results. Variation of Kn values do not have a significant influence on the modelled results. Figure 9 shows that an increase from 5e7 Pa to 1e8 Pa for Ks is sufficient to prevent direct toppling failure.

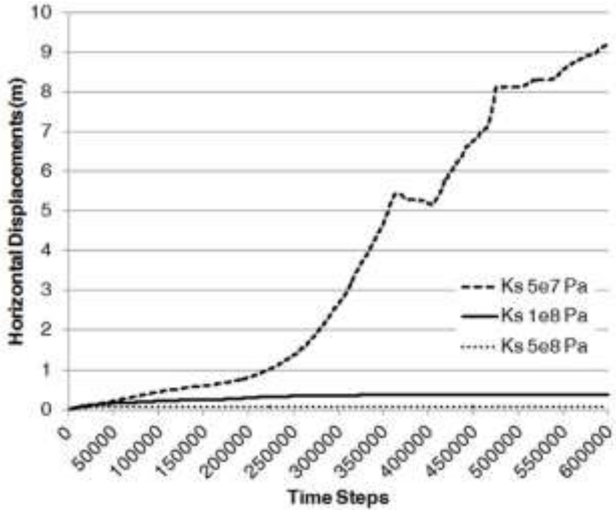

Fig. 9 Recorded horizontal displacements of 3DEC sensitivity analysis where discontinuity shear stiffness is varied from 5e7 Pa to 5e8 Pa

4.2.2 Influence of geometrical variability on model behaviour

Following the evaluation of discontinuity properties, the next series of 3DEC simulations was performed using different geometrical settings. Conservative (or relatively low) discontinuity geotechnical properties were adopted to assess the impact of model geometry on the potential for direct toppling to occur. The discontinuity properties used for all the sets were a friction angle of 25°, Kn of 5e9 Pa and Ks of 5e7 Pa.

The first series of analyses investigated the influence of discontinuity spacing on the simulation (Fig. 10). The aspect ratio of the original spacing was kept constant, but with multiplication factors of 1.5 and 2 used in the analyses. Three different spacing configurations, shown in Table 3, were analysed.

Table 3 Different spacing configurations analyzed

Configuration	Spacing J1 (m)	Spacing J2 (m)	Spacing J3 (m)
1	1	2	2
2	1.5	3	3
3	2	4	4

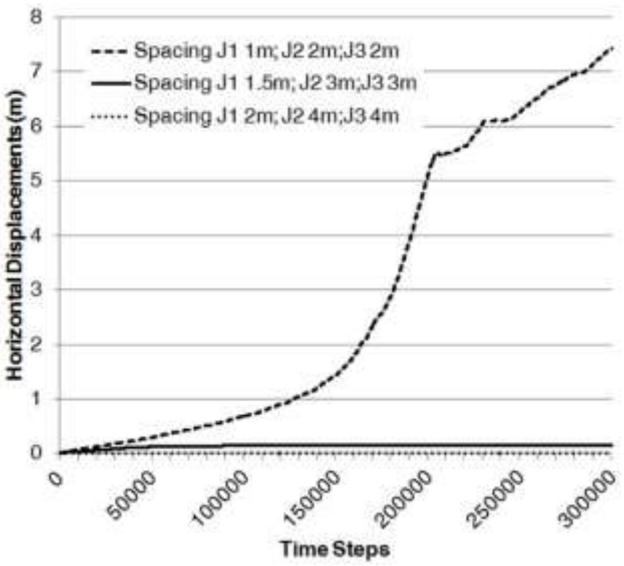

Fig. 10 Recorded horizontal displacements of 3DEC sensitivity analysis with variable discontinuity spacing configurations (1, 2 and 3)

Results show that an increase of the spacing by a factor of 1.5 is sufficient to prevent significant displacements within the model, even when low discontinuity properties are used.

Given the dynamic variability of natural systems, it was considered important to verify the influence of changing more than one geometrical attribute at a time. Therefore, the dip angle of the rear release surface (J1) and the bench slope angle were varied for the different spacing configurations. It was found that a change of the rear release surface (J1 from 83° to 70°) did not have an influence on the modelled results (not shown), since no significant displacements were detected for spacing configurations 2 and 3. However, an increase in the bench slope angle was found to affect the simulation results. From Fig. 11, when spacing configuration 2 is used, direct toppling can occur if the bench slope angle is increased to 85°.

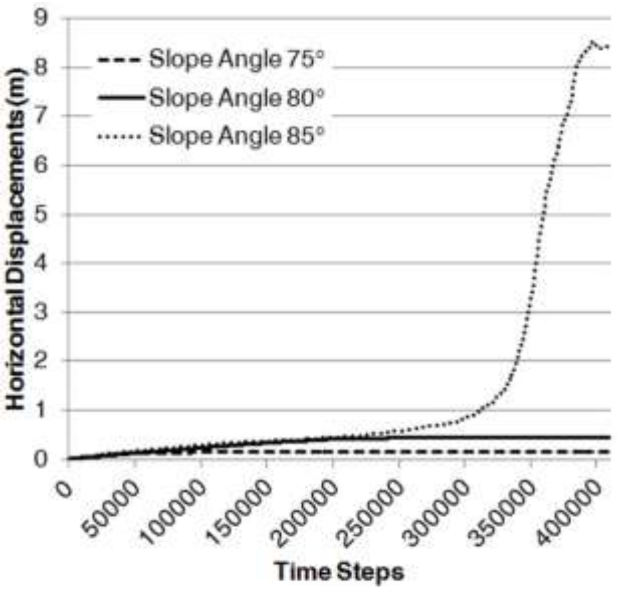

Fig. 11 Recorded horizontal displacements of 3DEC sensitivity analysis where slope angle is varied from 75° to 85° (using spacing configuration 2)

When spacing configuration 3 is used, no significant displacements were detected, even if a combination of rear release surface angle of 70° and a slope angle of 85° is used (results not shown). The modelled results suggest that bench slope angle may have a significant influence on direct toppling instability. This is an important aspect, since it may have important implications for LoM excavation design. Therefore, a new 3DEC simulation using the original spacing configuration (configuration 1- J1/1m, J2/2m, J3/2m) was performed, but the bench slope face angle was decreased from 75° to 70° (Fig. 12).

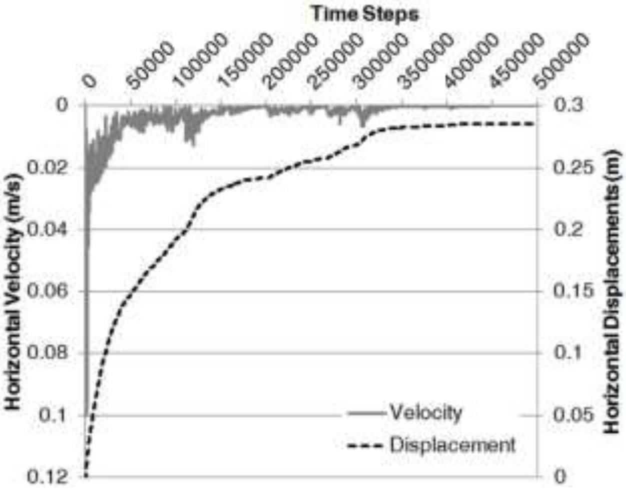

Fig. 12 Recorded horizontal displacements of 3DEC deterministic analysis with indication of horizontal velocity and displacement using spacing configuration 1 (J1/1m, J2/2m, J3/2m) and slope angle of 70°

Figure 12 clearly shows that a decrease of the bench slope face angle from 75° to 70° results in no significant modelled displacements, which corresponds to a fully stable model. It should be also emphasized that in this case low geotechnical properties were used (friction angle of 25°, Kn of 5e9 Pa and Ks of 5e7 Pa).

To conclude the sensitivity analysis, the effect of discontinuity persistence was also evaluated. 3DEC simulates the presence of rock bridges as the probability that a given block lying in the path of a joint is split (i.e., if p=0.5, then 50 % of the blocks will be split, on average; Itasca Consulting Group Inc. 2017b). Results (Fig. 13) indicate that the model is very sensitive to this parameter, and the use of a p value of 0.9 is sufficient to prevent instability within the model.

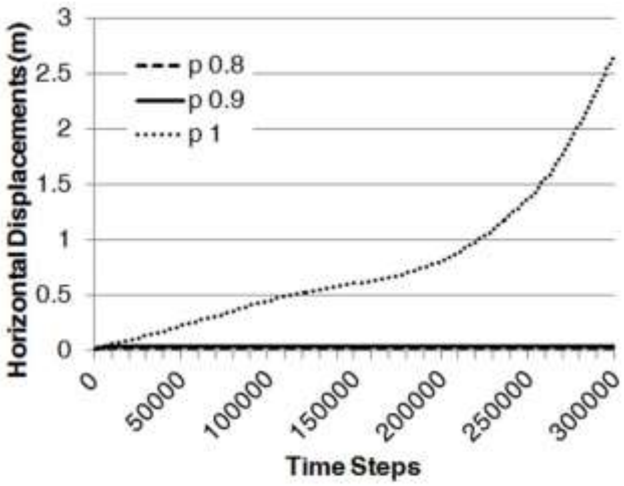

Fig. 13 Recorded horizontal displacements of 3DEC sensitivity analysis where persistence p value is varied from 0.8 to 1 (using spacing configuration 1)

4.2.3 Incorporation of representative slope face geometry from remote sensing

Previous results for the idealised slope geometry highlighted that bench slope face angle may have a critical influence on the stability of the bench under study. Therefore, it was decided to perform a 3DEC analysis incorporating a more realistic representation of the slope face geometry obtained from the TLS and point cloud illustrated in section 4.1 (Fig. 3). In order to do this, the plug-in Griddle for Rhinoceros 5.0 was used to create a volumetric mesh that was imported into 3DEC. In respect of geotechnical properties, the same assumptions from section 4.2.2 were adopted. The results of the 3DEC simulation using the more realistic representation of the slope face geometry is shown in Fig. 14.

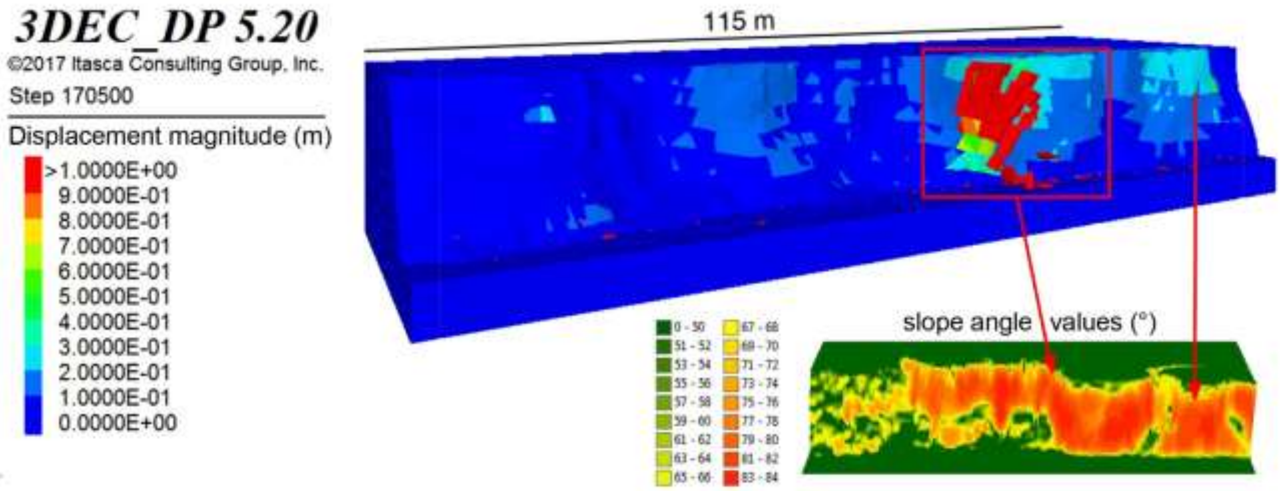

Fig. 14 3DEC simulation showing displacement magnitudes using bench face geometry from terrestrial laser scanning point cloud data. Lower right inset map indicates bench face slope angle values (°) calculated in ArcGis

The results indicate that direct toppling does not occur evenly throughout the model, but it is concentrated in regions that have a higher slope angle and instability occurs where rock blocks are confined by only one lateral discontinuity. Results of horizontal displacements with reference to the area highlighted by a rectangle in Fig. 14 are shown in Fig. 15.

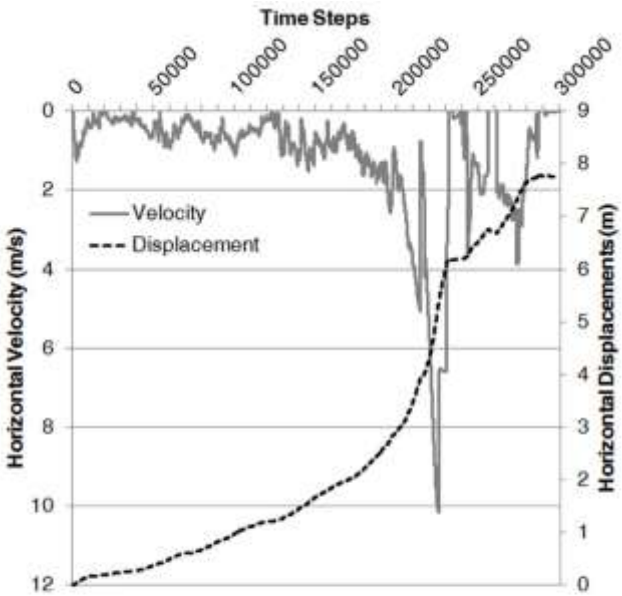

Fig. 15 Result of 3DEC deterministic analysis using more realistic face geometry with indication of recorded horizontal velocity and displacement

5 Discussion

The stability analyses undertaken in this study provide improved understanding of discontinuity-controlled instability mechanisms affecting the Test bench area in Melbur Pit. From previous engineering-geological investigation of the area (Carter 2015; Keverne et al. 2015; Vanneschi et al. 2017) it was found that direct toppling is the likely dominant failure mechanism, therefore further investigation was carried out combining different methods of analysis. A detailed TLS survey of a selected location was performed in order to obtain accurate geometrical data for characterization of both the bench slope face profile and discontinuity orientations for use in subsequent analyses. As direct toppling is a three-dimensional problem, controlled by 3 discontinuities sets (one basal plane and 2 sets forming block edges) a three-dimensional DEM analysis was undertaken. Initial deterministic 3DEC analyses performed on an idealised slope face geometry (represented in Figs. 5 and 7) indicate the possibility for direct toppling to develop within the model. This is evident from Fig. 7, where horizontal displacement history results are shown. It is interesting to note that after onset of failure, velocity and displacements do not show a smooth increase, but two interruptions occurred around time steps 360,000 and 470,000. This is likely due to interactions with adjacent blocks that caused a certain degree of confinement and interlocking. In addition, Fig. 6 shows rock mass dilation due to the rotation of blocks developed after onset of failure. From the same image it is also possible to ascertain that the observed mechanism is pure toppling, with no evidence of sliding blocks. It is also interesting to note the development of a step-path basal failure surface. This agrees with modelled results of previous 3DEC direct toppling/sliding investigation by Brideau and Stead (2012) that demonstrated the possibility of stepped-failure surfaces to develop in models with fully persistent discontinuity sets. Step-path surfaces are also common in slopes where rock bridges limit the persistence of the joints (Jennings 1970; Einstein et al. 1983; Goodman 2003).

A series of sensitivity analyses were then performed on discontinuity properties, the first of which was focused on evaluation of the influence of discontinuity friction angle. The results show that discontinuity friction angle has an influence on the modelled final horizontal displacements magnitude, but the underlying toppling failure always occurs for the range of friction angles simulated, even if with increased delay for higher friction angles. The second sensitivity analysis indicated that the model is very sensitive to Ks values. Such behaviour was also noted by Brideau and Stead (2012) and confirmed here, where a small increase of Ks leads to a stable condition within the model. Such factors should be carefully considered in development of numerical models as Ks, together with Kn, is one of the most uncertain parameters to determine in geotechnical investigations, and this may have implications for interpretation of results from numerical modelling investigations.

Given the uncertainty related to geometrical setting in rock slopes, further sensitivity analyses were performed to investigate the influence of discontinuity spacing and dip angle of the toppling joint (J1), that according to LEM results would have the most influence on the model. Results shown in Fig. 10, indicate that an increase in the spacing aspect ratio by a factor of 1.5 is sufficient to prevent direct toppling to develop within the model. This is due to the change of block size, with the centre of gravity acting within, or close to, the base of the new blocks. In addition, the inclination of the base surface (J3 set) is too low and the ratio of the shear to normal forces acting on the base does not exceed its friction angle, preventing sliding failure to develop. Therefore, neither direct toppling nor sliding failure can cause instability within the model. This is especially true for spacing configuration 3 (J1/2m, J2/2m, J3/4m), where no instability is reached, even when changing both the toppling joint dip and slope angle, and using low Ks, Kn, and friction (phi) values. The modelling suggests that spacing configuration 2 (J1/1.5m, J2/3m, J3/3m) appears to be close to instability, as the sensitivity analysis depicted in Fig. 13 shows that an increase of the slope angle from 75° up to 85°

can trigger direct toppling failure. This is an interesting result, as it indicates that slope face angle may be a critical influential parameter, with possible important consequences on LoM decisions. In order to investigate the influence of this parameter, a 3DEC deterministic analysis (Figs. 6 and 7) was subsequently performed on configuration 1, but decreasing the slope angle value from 75° down to 70°. Results (shown in Fig. 12) clearly indicate that if a lower bench face angle is considered, a fully stable model is obtained.

The importance of bench slope face angle was also confirmed by the 3DEC simulation performed using a more realistic or representative three-dimensional slope face geometry (shown in Figs. 14 and 15) obtained from TLS point cloud data. The analysis indicated that zones of the slope characterized by high bench face angles are more susceptible to failure by direct toppling. In addition, instability first develops where blocks are only confined by one lateral discontinuity. The 3DEC analysis provided good representation of the observed instability mechanisms occurring within the Test bench face area in Melbur Pit East face, as shown in Fig. 16.

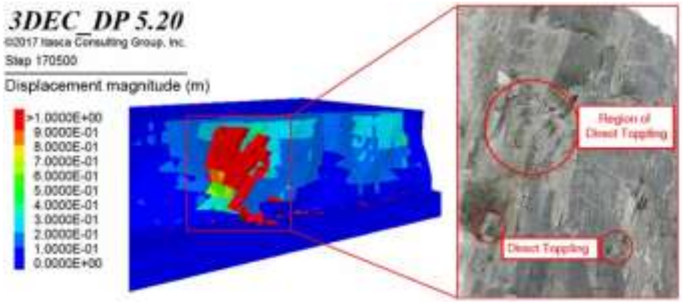

Fig. 16 Comparison between results of 3DEC analysis and direct field observation

It is well known that persistence is one of the most influential parameters in geotechnics, and one of the most difficult to determine/obtain. Several Authors have investigated the effect of rock bridges in numerical modelling (Nichol et al. 2002; Elmo et al. 2011; Brideau and Stead 2012; Sturzenegger and Stead 2012; Salvini et al. 2017) but, as recently reported by Tuckey and Stead (2016), an accepted standard for incorporation of rock bridges into slope stability analysis does not yet exist. In this case study, results of the persistence sensitivity analysis showed that a decrease of the joint persistence by 10% leads to a fully stable model. Therefore, the model appears to be very sensitive to this important parameter. The reason for this behaviour is explained in Fig. 17.

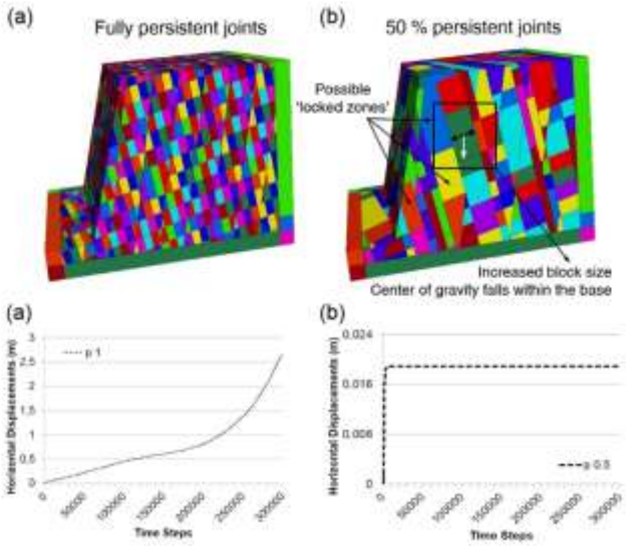

Fig. 17 Increase in block size due to different persistence configurations in 3DEC and consequences on numerical modeling results; comparison between a fully discontinuity persistent model (a) and a 50% persistent discontinuity model (b)

Figure 17 clearly shows the difference in block size when using the p option in 3DEC and the consequences on the analysis results. As an example, a fully discontinuity persistent model (a) is compared to a model with 50% persistent discontinuities (b). This means that in the second model only half of the possible blocks are actually formed. The consequence is similar to an increase of discontinuity spacing/block size. This means that, as previously described, there are a number of blocks in the bench that do not have the potential for toppling, given that the centre of gravity is more likely to act within their base. In addition, persistence in this case is independent of "depth", and there is the same probability to find a non-persistent joint at surface or at depth. These conditions may create 'locked zones' within the slope, preventing block failures. This is not always correct in open pit environments, since excavation activity (i.e., blasting) can cause damage to the rock mass and influence the distribution of persistence values within the bench rock mass.

To summarize, it was found that modelled bench slope face angle in 3DEC has a major influence on the stability of the three-dimensional model, on a par with the effects of discontinuity Ks and spacing. While the latter cannot be changed, being intrinsic properties of the rock mass, the bench slope angle can be modified by the design engineer for safety purposes. Given the potentially unstable nature of the slope face and potential hazard identified from direct toppling a critical section of the East slope at Melbur Pit adjacent to a haul road underwent remediation through controlled blasting to reduce the overall bench slope angle, described in Keverne et al. (2015).

The general failure mechanism for the slope orientation has been identified and useful information has been obtained for optimization of LoM excavation bench design. However, further work should be undertaken to investigate the effects of persistence on model behaviour in relation to the discrete fracture network. The slope modelled in this case study was essentially dry (to simulate site conditions), but the detrimental effects of water pressure should also be considered on the underlying failure mechanism in future modelling. The approach adopted during this investigation could be extended to other discontinuity-controlled instability mechanisms and could be undertaken at different scales (to identify bench, inter-ramp and overall slope instability mechanisms).

To conclude, several previous research projects have been undertaken to analyse toppling failure mechanisms, but the majority are still carried out using traditional LEM method or two-dimensional numerical analyses. This work, following the recent research by Brideau and Stead (2010, 2012), highlights the importance of analysis of direct toppling mechanisms through use of three-dimensional numerical modelling, by incorporation of three-dimensional point cloud data to provide realistic or more representative consideration of slope face geometry.

6 Conclusion

The case study presented in this paper provided an investigation into discontinuity-controlled instability of a bench section of an active open pit. The study used geometrical information from remote mapping (TLS) to characterise the three-dimensional geometry of a section of an open pit that was susceptible to direct toppling. The three-dimensional point cloud data was used to determine discontinuity orientation and spacing within the rock mass and provide geometrical input data for incorporation into three-dimensional DEM analyses (using 3DEC).

The results of the modelling indicate bench slope face angle is one of the critical parameters influencing the stability of the bench when considering the potential for direct toppling. The modelling undertaken was able to correctly identify the observed direct toppling failure mechanism for the slope orientation considered, and provide useful information for optimization of LoM excavation bench design.

In summary, the following observations can be made:

- 3DEC analyses confirm the high sensitivity of the model to changes in discontinuity Ks values previously discussed by Brideau and Stead (2012).
- 3DEC sensitivity analyses confirm that discontinuity spacing and bench slope face angle are the geometrical parameters that have the greatest influence on the stability analysis results. For the case study modelled reducing the bench slope face angle to 70^{o} from 75^{o} was sufficient to prevent direct toppling on the slope.
- 3DEC investigations performed using more realistic representation of the bench slope face geometry, through use of point cloud data from TLS and subsequent incorporation of a mesh generated within Griddle, was able to reproduce the direct toppling instability observed in the field, providing confidence in and validation of the model used.
- Further work is required to fully explore the effects of rock bridges (by using the ‘p’ option within 3DEC). Initial results suggest that a fully stable model is obtained with only 10% rock bridges within the model. Given the importance of such a parameter further research is still required to evaluate the effect of variable discontinuity persistence on model behaviour.

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