Efficient Transmission in Multi-user Relay Networks with Node Clustering and Network Coding

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Abstract

This paper investigates the communication problem in a class of multi-user dual-hop networks in which multiple source terminals desire to distribute their independent messages to multiple destinations through the assistance of multiple relay terminals. We consider an efficient transmission strategy that combines network coding and non-orthogonal transmission techniques to balance the achievability of spatial diversity and channel utilization. Specifically, in addition to applying a class of finite-field network codes in the relays, we divide the sources and the relays into clusters such that terminals within each cluster can access the same channel resource. The achievable error performance under Nakagami-m fading environment is derived and is shown to significantly outperform the conventional transmission methods.

I. Introduction

An important application scenario for future wireless communication systems is the content distribution service where a number of geographically closed content generators intend to broadcast their messages to ambient mobile devices. Potential examples include sensor networks under the framework of Internet of things (IoT), vehicular networks in intelligent transportation systems (ITS) [1], [2], and advertisement broadcasting or file sharing in 5G device-to-device (D2D) networks [3], to name a few. In many practical situations, these applications operate in harsh signal propagation environments, e.g., when all terminals are located indoor or moving with high-speed. It is hence hard to guarantee a satisfactory performance, especially when the source transmission powers are constrained but the number of desirable destinations is large. Deploying relaying and network coding techniques are widely considered as an effective solution to address the problem [4]–[7].

To avoid multi-user interference, most existing network-coding based transmission schemes in multi-user relay networks demand all transmitting terminals to share the available channel resources orthogonally. High reliability of message delivery is achieved with the cost of relatively low efficiency of channel utilization. From an information-theoretic point-of-view, it is well known that in multiple-access (MAC) channels non-orthogonal transmission outperforms orthogonal transmission in terms of achievable rate region [8]. Recently, non-orthogonal multiple access (NOMA) has been deemed as a key technology in future 5G systems to attain high spectral efficiency and massive connectivity [9]. Potential applications have been widely discussed in cellular downlink MIMO, D2D, vehicular, and IoT systems [10]–[12].

The feasibility of taking the advantages of non-orthogonal transmission in network-coded relaying schemes has been proposed and initially studied in [13]. A cellular uplink transmission setup is considered such that multiple relays are deployed to assist in the message transmission from a number of sources to a single destination. In addition to applying a class of maximum-distance-separable finite-field network codes (MDS-FFNCs) [4] in the relays to achieve full diversity, multiple terminals (either sources or relays) are allowed to share the same (time) channel resource. In other words, terminals are clustered and those within the same cluster are non-orthogonally activated. Under Rayleigh fading, it is shown that the proposed method achieves better diversity-multiplexing tradeoff (DMT) [14] than the conventional approach that orthogonalizes all transmitting terminals.

However, the information-theoretic DMT concept only implies error performance when the operating signal-to-noise ratio (SNR) approaches infinity. It does not reflect whether the performance advantage still remains in the practical finite-SNR regime, especially when network topology are more complex and the channel characteristics are more general. This paper aims to address this question. In particular, we consider message distribution within a class of multi-user relay networks with multiple sources, multiple relays, and multiple destinations, in a Nakagami-m fading environment. The aforementioned scheme is adopted such that network coding and non-orthogonal transmission techniques are combined to facilitate the sources to broadcast their messages to the destinations. Deriving the explicit expression of the system error probability demands the probability that a particular number of messages can be correctly decoded in a MAC channel, which can be involved in general [15]. To this end, we provide the result for two-user and three-user MAC channels. Using a number of example networks, we show that properly clustering the network nodes can provide significant performance gains over the conventional method that orthogonalizes all sources and relays, not only in the high-SNR regime but also in the finite-SNR regime. The system error probability expression derived in this paper also allows conducting further system design (e.g., power/rate control and clustering/relay selection) to optimize performance such as spectral or energy efficiency.

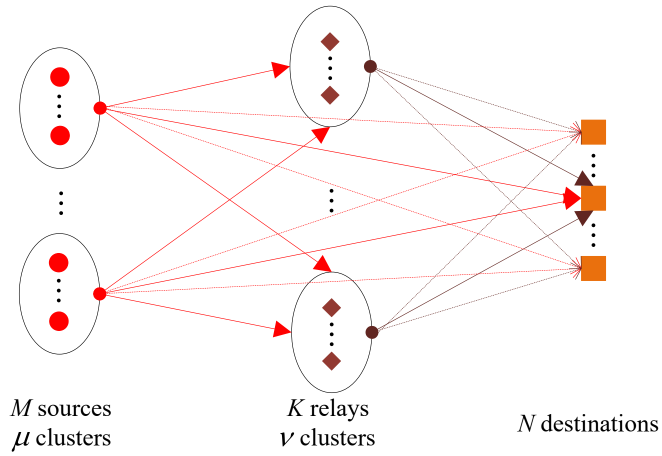


Fig. 1. System model.

II. System model and transmission scheme

We consider a class of wireless single-antenna multi-user relay networks, in which $M \geq 2$ independent information sources intend to broadcast their messages to $N \geq 1$ independent destinations, with the help of $K \geq 1$ half-duplex decode-and-forward (DF) relays, as shown in Fig. 1. The network can represent message distribution in a wide range of practical application scenarios in D2D, V2V, and IoT systems.

We denote the set of source messages by $\mathcal{I} = \{I_1, I_2, \cdots, I_M\}$, in which I_i is the message generated by the ith source S_i. The complete set $\mathcal{I}$ is expected to be attained by all the destinations $\mathrm{D}_1, \cdots, \mathrm{D}_N$. Therefore, an error event is define as the situation that at least one destination cannot fully recover $\mathcal{I}$. The probability of such an event is termed *system error probability* P_{err}.

Each source message is encoded using a sufficiently strong channel code with rate R bits per message such that decoding error occurs only if the channel is in outage, i.e., the mutual information between the transmitter-receiver pair is smaller than R. Wireless signal propagation within the network is assumed to experience narrow-band Nakagami-m slow-fading. We denote the channel fading coefficient from transmitter a to receiver b by $h_{b,a}$ and the channel power gain $|h_{b,a}|^2$ is modeled as a Gamma-distributed random variable with integer shape parameter $m_{b,a}$ and rate parameter $\frac{m_{b,a}}{\Omega_{b,a}}$, i.e., $|h_{b,a}|^2 \sim \text{Gamma}\left(m_{b,a}, \frac{m_{b,a}}{\Omega_{b,a}}\right)$ [16]. To facilitate presentation, we use $f(x;\alpha,\beta) = \frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{(\alpha-1)!}$ and $F(x;\alpha,\beta) = 1 - \sum_{i=0}^{\alpha-1} \frac{(\beta x)^i e^{-\beta x}}{i!}$ to respectively denote the probability density function (PDF) and cumulative density function (CDF) of a Gamma random variable with integer shape parameter α and rate parameter β. By this means, the PDF and CDF of $|h_{b,a}|^2$ can be represented by $f(x; m_{b,a}, \frac{m_{b,a}}{\Omega_{b,a}})$ and $F(x; m_{b,a}, \frac{m_{b,a}}{\Omega_{b,a}})$. Throughout the paper, we assume that $h_{b,a}$ remains fixed for the whole transmission period and is known at only the receiver b. Hence transmitter-side power control according to channel state information is not possible. Each transmitter a transmits with a fixed power level ρ_a.

The message transmission in the considered network is carried out in a (time) slotted fashion. A number of transmission schemes can be applied. The simplest approach adopts the repetition coding concept [8], and demands each relay to repeat each source message by forwarding the same codeword. It allows each destination to apply maximum ratio combing (MRC) to its received $K+1$ messages to achieve spatial diversity gain. However, in the considered M-source network, a total of $M(K+1)$ time slots have to be consumed, which is unnecessarily large and leads to low spectral efficiency.

The network coding technique operated in the high-order finite field is known to be an effective solution to this problem. Instead of individually forwarding multiple source messages, which consumes multiple time slots, each relay can combine all source messages to form a new message so that a single time slot suffices to forward it. Let $\hat{I}_j$ denote the new message generated by the jth relay. Then the relationship between $\hat{I}_j$ and the source messages can be expressed as $\hat{I}_j = \sum_{i=1}^{M} \gamma_{j,i} I_i$, where the summation is conducted in a proper finite field, and $\gamma_{j,i}$ denotes coding coefficients. The complete encoding process can be written as a matrix form:

$$\begin{bmatrix} I_1 \\ \vdots \\ I_M \\ \hat{I}_1 \\ \vdots \\ \hat{I}_K \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ \gamma_{1,1} & \gamma_{1,2} & \cdots & \gamma_{1,M} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{K,1} & \gamma_{K,2} & \cdots & \gamma_{K,M} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_M \end{bmatrix}. \tag{1}$$

In order to guarantee full diversity gain, the MDS-FFNC strategy [4] demands the coding coefficients to be designed such that the encoding matrix in (1) has full rank. (Such network codes always exist if the coding field size is sufficiently large.)

By this means, any M out of the $M+K$ messages in the left hand side of (1) suffice to recover the complete set $\mathcal{I}$. For networks with $N=1$ and Rayleigh fading, full diversity gain is proven to be achievable, with much higher spectral efficiency than the repetition-coded solution [4], [13].

If we allow the relays in the considered network to apply the MDS-FFNC and orthogonalize all transmitting terminals, a total of $M+K$ time slots are needed to complete the transmission of $\mathcal{I}$ from the sources to the destinations, through the relays. In other words, the first M slots are assigned to the M sources, and the remaining K slots are reserved for the K relays. This method has low decoding complexity. But one may argue that its spectral efficiency is not sufficiently high since all terminals use orthogonal channels to transmit information. As an evidence, for networks with $N=1$ and Rayleigh fading, when the SNR approaches infinity, [13] shows that properly[1] allowing some terminals to transmit messages non-orthogonally can achieve better performance in terms of DMT.

In this paper, we adopt the similar concept in the considered network. We aim to show that the advantage of combing network coding and non-orthogonal transmission techniques remains in a more general network setup with multiple destinations and Nakagami-m fading. More importantly, the performance gain does not require a sufficiently high SNR. However, to reveal it, one needs to properly consider all system parameters and carefully design the clustering strategy.

Specifically, we divide the M sources into μ $(1 \leq \mu \leq M)$ *clusters* $\mathcal{S}_1, \cdots, \mathcal{S}_\mu$, and divide the K relays into ν $(1 \leq \nu \leq K)$ clusters $\mathcal{R}_1, \cdots, \mathcal{R}_\nu$, as shown in Fig. 1. We permit all terminals within the same cluster to share the same time slot non-orthogonally to reduce channel usage, but demand different clusters to orthogonally use different time slots to avoid unnecessarily large inter-user interference. Now, to complete the desired transmission requires a total of only $\mu+\nu$ time slots, each of which is assigned to a single cluster. This means, at time slot 1, all sources in $\mathcal{S}_1$ broadcast their messages together. Each relay and destination applies successive interference cancellation (SIC) to its received signals to recover the source messages. At time slot 2, all sources in $\mathcal{S}_2$ broadcast their messages together. The process continues until all sources complete transmission using μ time slots.

The remaining ν time slots are reserved to the ν relay clusters respectively. Albeit the fact that each relay is capable of encoding and forwarding only a subset of $\mathcal{I}$, to minimize system complexity we consider the worst-case scenario in which a relay is activated only if it recovers all the M source messages from its received signals in the first time slots. Otherwise, it remains silent without participating in the message transmission process. Use $\hat{\mathcal{R}}_i$ to denote the set of activated relays in the relay cluster $\mathcal{R}_i$. Now at time slot $\mu+1$, each relay in $\hat{\mathcal{R}}_1$ combines all the source messages in $\mathcal{I}$ using its MDS-FFNC coefficients and forwards its network-coded message to the destinations, each of which applies SIC to its received signal. At time slot $\mu+2$, all relays in $\hat{\mathcal{R}}_2$ are activated to forward their network-coded messages. The process continues similarly until the $(\mu+\nu)$th time slot. If the total number of messages that one destination can recover is at least M, the destination can fully obtain the source message set $\mathcal{I}$. If all destinations recover $\mathcal{I}$ without error, then the transmission is deemed to be successful.

We term this scheme *cluster FFNC relaying*. Clearly, the aforementioned approach that orthogonalizes all sources and relays, which we term *orthogonal FFNC relaying*, serves as a special case with $\mu=M$ and $\nu=K$. We aim to derive the system error probability P_{err} of the cluster FFNC relaying and show that the scheme can provide better performance than the orthogonal FFNC relaying. To this end, we need to characterize the decoding behaviors of every relay and destination in each time slot. Obviously, in each slot, the activated node cluster and every receiver form a MAC channel. In the next section, we will temporarily divert our presentation from the considered multi-user relay network to a single-hop MAC channel and elaborate the method to calculate the probability that the receiver correctly decodes a particular number of transmitter messages. The result will be used in Section IV to derive P_{err}.

III. Error Performance of MAC Channels

In this section, we focus on a single-hop MAC channel [8] formed by a set of L simultaneously activated transmitters $\mathcal{L}=\{a_1,\cdots,a_L\}$ and a common receiver b. Assume that a_l $(1 \leq l \leq L)$ encodes and transmits its message using unit-power capacity-achieving Gaussian random codeword x_{a_l} with rate R bits per message and power ρ_{a_l}. Let h_{b,a_l} denote channel coefficient between a_l and b. The received signal at b is

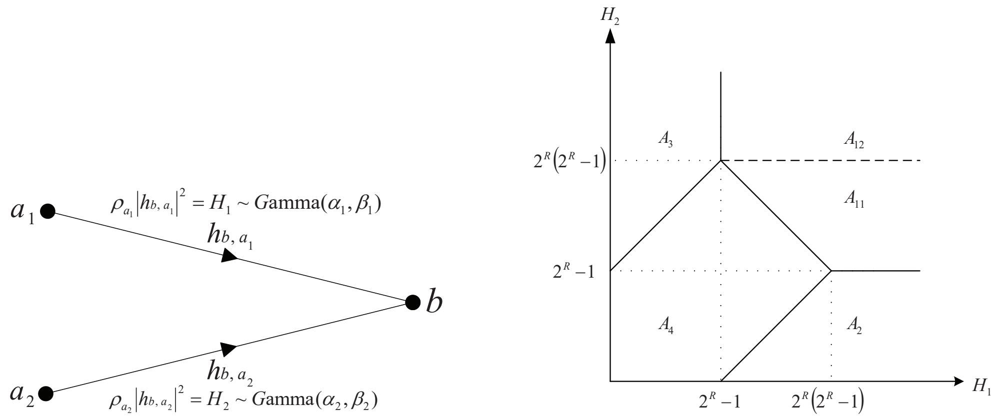
$$y_b = \sum_{a_l \in \mathcal{L}} \sqrt{\rho_{a_l}} h_{b,a_l} x_{a_l} + \epsilon_b,$$

where ϵ_b denotes unit-power additive white Gaussian noise.

In order to recover the transmitted messages, b applies SIC to y_b. Let $|\mathcal{A}|$ and $\bar{\mathcal{A}}$ denote the cardinality and complementary set of set $\mathcal{A}$, respectively. If for a given subset $\mathcal{A} \subseteq \mathcal{L}$, the following two events $E_{1,\mathcal{A}}$ and $E_{2,\bar{\mathcal{A}}}$ occur

$$E_{1,\mathcal{A}} : \log_2 \left(1 + \frac{\sum_{a_j \in \mathcal{J}} \rho_{a_j} |h_{b,a_j}|^2}{1 + \sum_{a' \in \bar{\mathcal{A}}} \rho_{a'} |h_{b,a'}|^2} \right) > |\mathcal{J}|R, \quad \forall \mathcal{J} \subseteq \mathcal{A},$$

[1]Note that activating all sources (relays) simultaneously may not be a good solution. This is different from the case in a single-hop MAC channel, where allowing all transmitters to use the same channel always attains better DMT than activating subset by subset.



(a) MAC channel (b) Error regions

Fig. 2. 2-user MAC channel

$$E_{2,\bar{\mathcal{A}}} : \log_2\left(1 + \frac{\sum_{a_q \in \mathcal{Q}} \rho_{a_q} |h_{b,a_q}|^2}{1 + \sum_{a' \in \bar{\mathcal{Q}} \cap \bar{\mathcal{A}}} \rho_{a'} |h_{b,a'}|^2}\right) < |\mathcal{Q}|R, \quad \forall \mathcal{Q} \subseteq \bar{\mathcal{A}},$$

then b can successfully decode all nodes in $\mathcal{A}$, but cannot decode the remaining nodes (i.e., those in $\bar{\mathcal{A}}$) [15].

For example, consider a 2-user MAC channel with $\mathcal{L} = \{a_1, a_2\}$. The event that b is able to correctly recover only the message from a_1 occurs when $\mathcal{A} = \{a_1\}$, $\bar{\mathcal{A}} = \{a_2\}$, the event $E_{1,\{a_1\}}$ is $\log_2\left(1 + \frac{\rho_{a_1}|h_{b,a_1}|^2}{1+\rho_{a_2}|h_{b,a_2}|^2}\right) > R$, and the event $E_{2,\{a_2\}}$ is $\log_2\left(1 + \rho_{a_2}|h_{b,a_2}|^2\right) < R$.

The probability of occurring the events $E_{1,\mathcal{A}}$ and $E_{2,\bar{\mathcal{A}}}$ is termed *individual error probability* and is denoted by $Q^{\mathcal{A}}_{\mathcal{L},b}$. Consequently, the probability that b recovers *exactly* s $(0 \le s \le L)$ transmitting messages from its received signal, denoted as $Q^{[s]}_{\mathcal{L},b}$, can be calculated by the sum of $Q^{\mathcal{A}}_{\mathcal{L},b}$ over all realizations of $\mathcal{A}$ given $|\mathcal{A}| = s$. For instance, in a two-user MAC channel, the probability that b decodes exactly one transmitter message can be found by $Q^{[1]}_{\{a_1,a_2\},b} = Q^{\{a_1\}}_{\{a_1,a_2\},b} + Q^{\{a_2\}}_{\{a_1,a_2\},b}$.

Although the above paragraphs provide the method to calculate the individual error probability in a MAC channel, in practice finding the closed-form expressions of $Q^{\mathcal{A}}_{\mathcal{L},b}$ and $Q^{[s]}_{\mathcal{L},b}$ can be involved in large networks, even for Rayleigh fading [15]. In what follows, we provide the results for the two-user case, when the channels are Nakagami-m faded and the channel fading power gains are modeled as $|h_{b,a_l}|^2 \sim \text{Gamma}\left(m_{b,a_l}, \frac{m_{b,a_l}}{\Omega_{b,a_l}}\right)$ with integer shape parameters m_{b,a_l} and rate parameters $\frac{m_{b,a_l}}{\Omega_{b,a_l}}$ $(l \in \{1,2\})$. In the appendix, we provide the results in a three-user network for the special case $m_{b,a_l} = m$ and $\Omega_{b,a_l} = \Omega$ for all $l \in \{1,2,3\}$. These results will be used in Section IV in a few simple example networks to demonstrate the key objective of the paper: combing network coding and non-orthogonal transmission techniques can lead to better performance than the orthogonal FFNC relaying.

Now, consider a two-user MAC channel, as shown in Fig. 2(a). To simplify the presentation, let $H_i = \rho_{a_i}|h_{b,a_i}|^2 \sim \text{Gamma}(\alpha_i, \beta_i)$, where the parameters $\alpha_i = m_{b,a_i}$ and $\beta_i = \frac{\alpha_i}{\Omega_{b,a_i}\rho_{a_i}}$. Define $\eta_1 = 2^R - 1$ and $\eta_2 = 2^{2R} - 1$. Then following the above analysis, the probability that b can correctly decode the messages of both a_1 and a_2 is $Q^{\{a_1,a_2\}}_{\{a_1,a_2\},b} = \Pr\{H_1 > \eta_1, H_2 > \eta_1, H_1 + H_2 > \eta_2\}$. From Fig. 2(b), we can see that the conditions $H_1 > \eta_1, H_2 > \eta_1, H_1 + H_2 > \eta_2$ are represented by the union of two sub-regions A_{11} and A_{12}. We denote the probabilities corresponding to these sub-regions as $\mathcal{P}\{A_{11}\}$ and $\mathcal{P}\{A_{12}\}$, respectively. Using the PDF $f(x;\alpha,\beta)$ and CDF $F(x;\alpha,\beta)$ of Gamma random variables, we have

$$\begin{aligned}
\mathcal{P}\{A_{11}\} &= \int_{\eta_1}^{\eta_2-\eta_1} \int_{\eta_2-x_2}^{\infty} f(x_1;\alpha_1,\beta_1|x_2)\mathrm{d}x_1 f(x_2;\alpha_2,\beta_2)\mathrm{d}x_2 \\
&= \sum_{i=0}^{\alpha_1-1} \sum_{j=0}^{i} \binom{i}{j} \frac{\beta_2^{\alpha_2}\beta_1^i e^{-\beta_1\eta_2}(-1)^j \eta_2^{i-j}}{i!(\alpha_2-1)!} \cdot \int_{\eta_1}^{\eta_2-\eta_1} x_2^{\alpha_2-1+j} e^{-(\beta_2-\beta_1)x_2}\mathrm{d}x_2
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}\{A_{12}\} &= \int_{\eta_2-\eta_1}^{\infty} \int_{\eta_1}^{\infty} f(x_1;\alpha_1,\beta_1|x_2)\mathrm{d}x_1 f(x_2;\alpha_2,\beta_2)\mathrm{d}x_2 \\
&= \left(1 - F(\eta_2-\eta_1;\alpha_2,\beta_2)\right)\left(1 - F(\eta_1;\alpha_1,\beta_1)\right).
\end{aligned}$$

The value $\mathcal{P}\{A_{11}\}$ depends on the relationship between β_1 and β_2. For the case $\beta_1 = \beta_2$, we have

$$\begin{aligned}
Q^{[2]}_{\{a_1,a_2\},b} &= Q^{\{a_1,a_2\}}_{\{a_1,a_2\},b} = \mathcal{P}\{A_{11}\} + \mathcal{P}\{A_{12}\} \\
&= \sum_{i=0}^{\alpha_1-1} \sum_{j=0}^{i} \binom{i}{j} \frac{\beta_1^i \beta_2^{\alpha_2} e^{-\beta_1\eta_2}\eta_2^{i-j}\left((\eta_2-\eta_1)^{\alpha_2+j} - \eta_1^{\alpha_2+j}\right)}{(-1)^j i!(\alpha_2-1)!(\alpha_2+j)} + \left(1 - F(\eta_2-\eta_1;\alpha_2,\beta_2)\right)\left(1 - F(\eta_1;\alpha_1,\beta_1)\right). \quad (2)
\end{aligned}$$

Otherwise, if $\beta_1 \neq \beta_2$ we can use equation $F(x;\alpha,\beta)=\int_0^x \frac{\beta^\alpha x^{\alpha-1}e^{-\beta x}}{(\alpha-1)!}\mathrm{d}x$ to express $\mathcal{P}\{A_{11}\}$ and have

$$\begin{aligned}Q^{[2]}_{\{a_1,a_2\},b} &= Q^{\{a_1,a_2\}}_{\{a_1,a_2\},b}=\mathcal{P}\{A_{11}\}+\mathcal{P}\{A_{12}\}\\ &=\sum_{i=0}^{\alpha_1-1}\sum_{j=0}^{i}\binom{i}{j}\frac{(-1)^j\beta_1^i\beta_2^{\alpha_2}e^{-\beta_1\eta_2}\eta_2^{i-j}(\alpha_2+j-1)!}{i!(\alpha_2-1)!\,(\beta_2-\beta_1)^{\alpha_2+j}}\cdot\big(F(\eta_2-\eta_1;\alpha_2+j,\beta_2-\beta_1)-F(\eta_1;\alpha_2+j,\beta_2-\beta_1)\big)\\ &\quad+\big(1-F(\eta_2-\eta_1;\alpha_2,\beta_2)\big)\big(1-F(\eta_1;\alpha_1,\beta_1)\big). \end{aligned}\tag{3}$$

As mentioned earlier, the probability that b recovers exactly one message can be calculated by

$$Q^{[1]}_{\{a_1,a_2\},b}=Q^{\{a_1\}}_{\{a_1,a_2\},b}+Q^{\{a_2\}}_{\{a_1,a_2\},b}.\tag{4}$$

Clearly, the probability that b can correctly decode only a_1 is $Q^{\{a_1\}}_{\{a_1,a_2\},b}=\Pr\{H_2<\eta_1,H_1>\eta_1(H_2+1)\}$. From Fig. 2(b) we can see that the event corresponds to the region A_2. Therefore, we have

$$\begin{aligned}Q^{\{a_1\}}_{\{a_1,a_2\},b} &= \int_0^{\eta_1}\int_{\eta_1(x_2+1)}^{\infty} f(x_1;\alpha_1,\beta_1|x_2)\mathrm{d}x_1 f(x_2;\alpha_2,\beta_2)\mathrm{d}x_2\\ &= \sum_{i=0}^{\alpha_1-1}\sum_{j=0}^{i}\binom{i}{j}\frac{\beta_1^i\beta_2^{\alpha_2}e^{-\beta_1\eta_1}\eta_1^i}{i!(\alpha_2-1)!}\cdot\int_0^{\eta_1}x_2^{\alpha_2-1+j}e^{-(\beta_1\eta_1+\beta_2)x_2}\mathrm{d}x_2\\ &= \sum_{i=0}^{\alpha_1-1}\sum_{j=0}^{i}\binom{i}{j}\frac{\beta_2^{\alpha_2}\beta_1^i\eta_1^i e^{-\beta_1\eta_1}(\alpha_2+j-1)!}{i!(\alpha_2-1)!\,(\beta_1\eta_1+\beta_2)^{\alpha_2+j}}\cdot F(\eta_1;\alpha_2+j,\beta_1\eta_1+\beta_2). \end{aligned}\tag{5}$$

By symmetry, the probability that b correctly decodes only a_2 but not a_1, which corresponds to the region A_3, is

$$Q^{\{a_2\}}_{\{a_1,a_2\},b}=\sum_{i=0}^{\alpha_2-1}\sum_{j=0}^{i}\binom{i}{j}\frac{\beta_1^{\alpha_1}\beta_2^i\eta_1^i e^{-\beta_2\eta_1}(\alpha_1+j-1)!}{i!(\alpha_1-1)!\,(\beta_2\eta_1+\beta_1)^{\alpha_1+j}}\cdot F(\eta_1;\alpha_1+j,\beta_2\eta_1+\beta_1).\tag{6}$$

Finally, the event that b is able to decode neither a_1 nor a_2 corresponds to the region A_4 in Fig. 2(b). Its occurring probability can be obtained simply by

$$Q^{[0]}_{\{a_1,a_2\},b}=1-Q^{\{a_1\}}_{\{a_1,a_2\},b}-Q^{\{a_2\}}_{\{a_1,a_2\},b}-Q^{\{a_1,a_2\}}_{\{a_1,a_2\},b}.\tag{7}$$

In summary, equations (2), (3), and (5)-(7) characterize the individual error probabilities in the two-user network. Equations (2)-(4) and (7) provide the expressions of the probabilities that b decodes a certain number of messages.

If we consider a special case where both transmitter messages are sent with the same power level and experience similar fading environments, i.e., $m_{b,a_1}=m_{b,a_2}=m$ and $\frac{m_{b,a_1}}{\Omega_{b,a_1}\rho_{a_1}}=\frac{m_{b,a_2}}{\Omega_{b,a_2}\rho_{a_2}}=\beta$, we can attain

$$Q^{[2]}_{\{a_1,a_2\},b}=\sum_{i=0}^{m-1}\sum_{j=0}^{i}\frac{\beta^{m+i}\eta_2^j e^{-\beta\eta_2}\left((\eta_2-\eta_1)^{m+i-j}-\eta_1^{m+i-j}\right)}{(-1)^{i-j}j!(i-j)!(m+i-j)(m-1)!}+\sum_{i=0}^{m-1}\sum_{j=0}^{m-1}\frac{\beta^{i+j}(\eta_2-\eta_1)^i\eta_1^j e^{-\beta\eta_2}}{i!j!},\tag{8}$$

$$Q^{[1]}_{\{a_1,a_2\},b}=\sum_{i=0}^{m-1}\sum_{j=0}^{i}\binom{i}{j}\frac{2\beta^{m+i}\eta_1^i e^{-\beta\eta_1}(m+i-j-1)!}{i!(m-1)!\,(\beta\eta_1+\beta)^{m+i-j}}\cdot\left(1-\sum_{l=0}^{m+i-j-1}\frac{((\beta\eta_1+\beta)\eta_1)^l}{l!}e^{-(\beta\eta_1+\beta)\eta_1}\right),\tag{9}$$

$$Q^{[0]}_{\{a_1,a_2\},b}=1-Q^{[1]}_{\{a_1,a_2\},b}-Q^{[2]}_{\{a_1,a_2\},b}.\tag{10}$$

If we set $m=1$, i.e., channels are Rayleigh faded, it is easy to show the above results become the same as those in [15].

In the next section, we will use the individual error probabilities for the two-user and three-user (provided in the appendix) MAC channels to calculate the achievable system error probability of our cluster FFNC relaying scheme.

IV. System performance and numerical results

Now we focus back to the considered multi-user relay network and start presenting the performance analysis of our cluster FFNC relaying scheme. We will first elaborate the approach to derive the system error probability. Afterwards, the individual error probability for MAC channels presented in the above section will be used in a few simple example networks to numerically show the performance advantages over orthogonal FFNC relaying.

As defined in Section II, message distribution is deemed to be successful if all the N destinations are able to correctly recover all the M source messages. It is not difficult to see that the decoding behaviors of different destinations are actually dependent. In order to calculate the system error probability P_{err}, i.e., the probability that at least one destination fails to recover the complete source message set $\mathcal{I}$, we define two binary indicator vectors to represent the status of the relay and destination terminals respectively. The first is a K-dimensional vector $\boldsymbol{\pi} = [\pi_1^{[1]}, \cdots, \pi_1^{[|\mathcal{R}_1|]}, \cdots, \pi_\nu^{[1]}, \cdots, \pi_\nu^{[|\mathcal{R}_\nu|]}]$ indicating the decoding status of the K relays. If the ith relay in $\mathcal{R}_v$ ($v \in \{1, \cdots, \nu\}$), represented by $\mathrm{R}_v^{[i]}$, can fully recover $\mathcal{I}$ and thus is activated to assist in the source message delivery process, the element $\pi_v^{[i]} = 1$. Otherwise, $\pi_v^{[i]} = 0$. The set of all possible realizations of $\boldsymbol{\pi}$ is denoted by set $\mathcal{W}_\pi$, i.e.,

$$\mathcal{W}_\pi = \left\{\boldsymbol{\pi} : \pi_v^{[i]} \in \{0, 1\}, 1 \le v \le \nu, 1 \le i \in |\mathcal{R}_v|\right\}. \tag{11}$$

The second indicator vector is defined as an N-dimensional vector $\boldsymbol{\tau} = [\tau_1, \cdots, \tau_N]$ indicating the decoding status of the N destinations: $\tau_n = 1$ if D_n ($n \in \{1, \cdots, N\}$) recovers $\mathcal{I}$ and $\tau_n = 0$ otherwise.

Use $\Pr\{\boldsymbol{\pi}\}$ to denote the probability that a particular realization of $\boldsymbol{\pi} \in \mathcal{W}_\pi$ occurs. The conditional probability that at least one element in $\boldsymbol{\tau}$ is zero, i.e., the desired message transmission in the considered network fails, given $\boldsymbol{\pi}$ is denoted by $P_{\mathrm{err}|\boldsymbol{\pi}}$. Then the overall system error probability of our cluster FFNC relaying scheme can be derived using the following expression:

$$P_{\mathrm{err}} = \sum_{\boldsymbol{\pi} \in \mathcal{W}_\pi} \Pr\{\boldsymbol{\pi}\} P_{\mathrm{err}|\boldsymbol{\pi}}. \tag{12}$$

Let us start from $\Pr\{\boldsymbol{\pi}\}$. The event $\pi_v^{[i]} = 1$ occurs only if $\mathrm{R}_v^{[i]}$ correctly decodes all source signals during the first μ time slots, Clearly, at each time slot t ($t \in \{1, 2, \cdots, \mu\}$), the sources in $\mathcal{S}_t$ and $\mathrm{R}_v^{[i]}$ represent a $|\mathcal{S}_t|$-user MAC channel. The probability that $\mathrm{R}_v^{[i]}$ can correctly recover all these $|\mathcal{S}_t|$ source messages can be calculated using the MAC channel individual error probabilities as $Q_{\mathcal{S}_t, \mathrm{R}_v^{[i]}}^{[|\mathcal{S}_t|]}$. This leads to

$$\Pr\left\{\pi_v^{[i]} = 1\right\} = \prod_{t=1}^{\mu} Q_{\mathcal{S}_t, \mathrm{R}_v^{[i]}}^{[|\mathcal{S}_t|]}, \tag{13}$$

for all $v \in \{1, \cdots, \nu\}$ and $i \in \{1, \cdots, |\mathcal{R}_v|\}$. Using $\Pr\left\{\pi_v^{[i]} = 0\right\} = 1 - \Pr\left\{\pi_v^{[i]} = 1\right\}$, we attain

$$\Pr\{\boldsymbol{\pi}\} = \prod_{v=1}^{\nu} \prod_{i=1}^{|\mathcal{R}_v|} \left(\prod_{t=1}^{\mu} Q_{\mathcal{S}_t, \mathrm{R}_v^{[i]}}^{[|\mathcal{S}_t|]}\right)^{\pi_v^{[i]}} \left(1 - \prod_{t=1}^{\mu} Q_{\mathcal{S}_t, \mathrm{R}_v^{[i]}}^{[|\mathcal{S}_t|]}\right)^{1-\pi_v^{[i]}}. \tag{14}$$

When the relay activation status $\boldsymbol{\pi}$ is decided, whether the destinations are able to attain $\mathcal{I}$ becomes independent. Given $\boldsymbol{\pi}$, we denote the cluster of activated relay nodes (i.e., those with $\pi_v^{[i]} = 1$) that transmit signals at time slot $\mu + v$ ($v \in \{1, 2, \cdots, \nu\}$) by $\hat{\mathcal{R}}_{v|\boldsymbol{\pi}}$. The probability that the nth destination D_n can successfully decode the messages of s_t ($0 \le s_t \le |\mathcal{S}_t|$) nodes in $\mathcal{S}_t$, and those of $s_{\mu+v}$ ($0 \le s_{\mu+v} \le |\hat{\mathcal{R}}_{v|\boldsymbol{\pi}}|$) nodes in $\hat{\mathcal{R}}_{v|\boldsymbol{\pi}}$ can be calculated using the MAC channel individual error probability as $Q_{\mathcal{S}_t, \mathrm{D}_n}^{[s_t]}$ and $Q_{\hat{\mathcal{R}}_{v|\boldsymbol{\pi}}, \mathrm{D}_n}^{[s_{\mu+v}]}$, respectively. After all source clusters and relay clusters completing their transmissions, the probability that D_n can fully recover $\mathcal{I}$ (conditioned on $\boldsymbol{\pi}$) is the probability that it correctly decodes at least M signals from all the $\sum_{t=1}^{\mu} |\mathcal{S}_t| + \sum_{v=1}^{\nu} |\hat{\mathcal{R}}_{v|\boldsymbol{\pi}}|$ transmitted signals it received. Define vector $\mathbf{s} = [s_1, \cdots, s_{\mu+\nu}]$. Then this probability is

$$\Pr\{\tau_n = 1\} = \sum_{\mathbf{s} \in \mathcal{W}_{s|\boldsymbol{\pi}}} \prod_{t=1}^{\mu} Q_{\mathcal{S}_t, \mathrm{D}_n}^{[s_t]} \prod_{v=1}^{\nu} Q_{\hat{\mathcal{R}}_{v|\boldsymbol{\pi}}, \mathrm{D}_n}^{[s_{\mu+v}]}, \tag{15}$$

where $\mathcal{W}_{s|\boldsymbol{\pi}}$ is the set of all potential realizations of decoding status that allow a destination to recover $\mathcal{I}$:

$$\mathcal{W}_{s|\boldsymbol{\pi}} = \left\{\mathbf{s} : \sum_{t=1}^{\mu+\nu} s_t \ge M, 0 \le s_t \le |\mathcal{S}_t|, 0 \le s_{\mu+v} \le |\hat{\mathcal{R}}_{v|\boldsymbol{\pi}}|\right\}.$$

Now, $P_{\mathrm{err}|\boldsymbol{\pi}}$, the conditional probability of communication failure in the considered system can be found as

$$P_{\mathrm{err}|\boldsymbol{\pi}} = 1 - \prod_{n=1}^{N} \left(\sum_{\mathbf{s} \in \mathcal{W}_{s|\boldsymbol{\pi}}} \prod_{t=1}^{\mu} Q_{\mathcal{S}_t, \mathrm{D}_n}^{[s_t]} \prod_{v=1}^{\nu} Q_{\hat{\mathcal{R}}_{v|\boldsymbol{\pi}}, \mathrm{D}_n}^{[s_{\mu+v}]}\right)^{\tau_n}. \tag{16}$$

Armed with such knowledge, we can substitute (14) and (16) into (12) to attain the system error probability P_{err}.

The above analysis is suitable for applying our cluster FFNC relaying scheme in any arbitrary network topology and clustering strategy, with any channel fading characteristics and system parameters ρ_a and R. As long as the expressions of $Q_{\mathcal{S}_t, \mathrm{R}_v^{[i]}}^{[|\mathcal{S}_t|]}$ in (13), and $Q_{\mathcal{S}_t, \mathrm{D}_n}^{[s_t]}$ and $Q_{\hat{\mathcal{R}}_{v|\boldsymbol{\pi}}, \mathrm{D}_n}^{[s_{\mu+v}]}$ in (15) are obtained, then calculating P_{err} can be conducted, through enumerations over the sets $\mathcal{W}_\pi$ and $\mathcal{W}_{s|\boldsymbol{\pi}}$.

In what follows, we use some simple examples to numerically show the performance of our cluster FFNC relaying scheme. It should be noted that when the number of concurrently transmitting nodes in a MAC channel is large, the exact expressions of individual error probabilities can be hard to attain. However, using the results presented in Section III (i.e., MAC channels

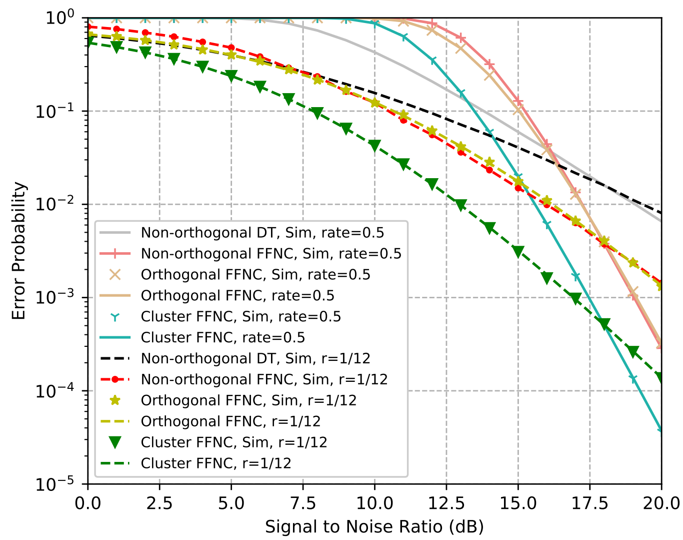


Fig. 3. Performance comparison in a network with $M = 4$, $K = 2$, $N = 3$.

with two or three users) suffices to demonstrate the advantages of combining network coding and non-orthogonal transmission techniques. When clustering strategies with large clusters are taken into consideration, one can use asymptotic performance (such as DMT[2]) to show the performance gains and identify the optimal clustering solution.

The first example network has $M = 4$ sources, $K = 2$ relays, and $N = 3$ destinations. To simplify presentation, we consider the case such that all channels experience similar fading characteristics such that the whole network has the same Nakagami-m fading parameters $m = 2$ and $\Omega = 1$. If relays are not used for helping the sources, one can non-orthogonally activate all the four sources to broadcast their messages to the destinations. This method, termed *non-orthogonal direct transmission (DT)*, uses a single time slot to complete transmission and is known to outperform the approach that orthogonalizes the sources. This reflects advantage of non-orthogonal transmission. But the achievable diversity gain is relatively low, which leads to a relatively slowly decreasing error probability when the SNR increases.

Applying the orthogonal FFNC relaying (whose system error probability can also be derived following the above method with $Q^{[1]}_{\{a_1\},b} = 1 - F(\eta_1; m, \frac{m}{\rho_{a_1}\Omega})$) in this example network requires 6 time slots to deliver messages to the destinations (if repetition-coding is applied in the relays, then 12 time slots would be needed). For fair comparison, we fix the *average transmission rate* of different transmission schemes to be the same as $\bar{R}$ bits per channel use (i.e., per time slot). Then if a scheme demands T time slots to complete transmission, then its message rate R should be set to $R = T\bar{R}$ bits per message to maintain $\bar{R}$. Fig. 3 illustrates the error probability comparison of the orthogonal FFNC and non-orthogonal DT (through simulation, since the closed-form error probability expression is unknown) schemes. Again, for fair comparison, we fix the average received SNR to be ρ. At each time slot, if A terminals are activated simultaneously, then each node transmits with power $\frac{\rho}{A}$. In Fig. 3, we set $\bar{R} = \frac{1}{2}$. This means that for the non-orthogonal DT scheme each source transmits with rate $R = \frac{1}{2}$ and power $\frac{\rho}{4}$, while for the orthogonal FFNC relaying scheme each source and relay terminal transmits with rate $R = 3$ and power ρ. From the figure, the diversity gain of deploying relays can be clearly seen. However, for relatively low SNR, the non-orthogonal DT scheme performs better, because the channel usage of the orthogonal FFNC relaying is large which demands a large R to maintain the average rate.

Now, we apply our cluster FFNC relaying scheme by dividing the sources into two 2-source clusters and treating the relays as a single cluster, i.e., $\mu = 2$, $\nu = 1$, and $|\mathcal{S}_1| = |\mathcal{S}_2| = |\mathcal{R}_1| = 2$. Since all channels have the same fading statistics, the individual error probabilities for source-relay, source-destination, and relay-destination links are the same. Hence we simplify the MAC channel error probability expression $Q^{[s]}_{\mathcal{L},b}$ with Nakagami-m fading parameters m and Ω, and node transmit power $\rho_a = \frac{\rho}{|\mathcal{L}|}$, by $Q^{[s]}_{|\mathcal{L}|}$. For instance, $Q^{[2]}_2$ means the probability that the receiver in a two-user MAC channel can decode both source messages. And $Q^{[1]}_1 = 1 - F(2^R - 1; m, \frac{m}{\rho\Omega})$ denotes the correct decoding probability for a point-to-point link.

Following the analysis shown earlier in this section, it is easy to see that $\Pr\{\boldsymbol{\pi} = [0,0]\} = (1 - (Q^{[2]}_2)^2)^2$, $\Pr\{\boldsymbol{\pi} = [1,0]\} = \Pr\{\boldsymbol{\pi} = [0,1]\} = (Q^{[2]}_2)^2(1 - (Q^{[2]}_2)^2)$, and $\Pr\{\boldsymbol{\pi} = [1,1]\} = (Q^{[2]}_2)^4$ are the probabilities of all potential relay decoding status. The associated conditional error probabilities at each destination are $\Pr\{\tau_n = 1|\boldsymbol{\pi} = [0,0]\} = (Q^{[2]}_2)^2$,

[2]The achievable infinite-SNR DMT in a general multi-source multi-relay multi-destination network with arbitrary clustering strategy and Nakagami-m fading, as well as the finite-SNR DMT for certain special cases, will be presented in our full journal-version paper.

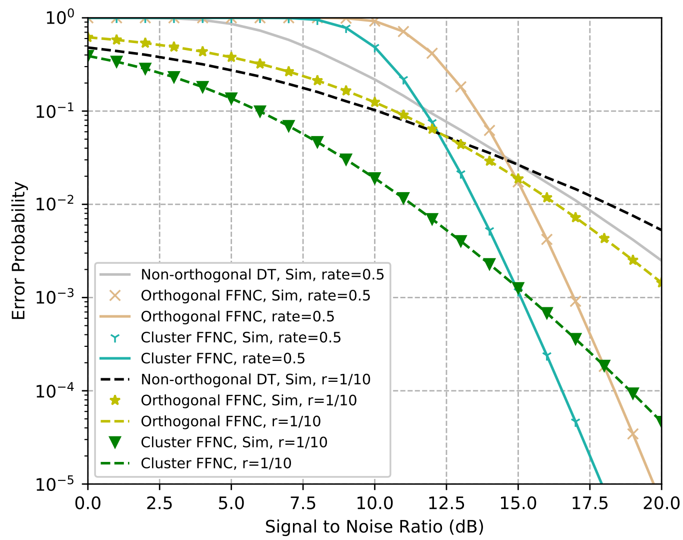


Fig. 4. Performance comparison in a network with $M = 3$, $K = 3$, $N = 3$.

$\Pr\{\tau_n = 1|\boldsymbol{\pi} = [0,1]\} = \Pr\{\tau_n = 1|\boldsymbol{\pi} = [1,0]\} = (Q_2^{[2]})^2 + 2Q_2^{[2]}Q_2^{[1]}Q_1^{[1]}$, and $\Pr\{\tau_n = 1|\boldsymbol{\pi} = [1,1]\} = (Q_2^{[2]})^2 + 2Q_2^{[2]}Q_2^{[1]}(Q_2^{[2]} + Q_2^{[1]}) + (2Q_2^{[2]}Q_2^{[0]} + (Q_2^{[1]})^2)Q_2^{[2]}$. In Fig. 3 we show P_{err} by both analytical and simulation results, with $R = 3\bar{R} = \frac{3}{2}$. It can be seen that they match nicely, which proves the accuracy of the results presented in both the previous section and this section. The performance advantage over the orthogonal FFNC relaying can be clearly observed. The error probability achieved by setting all four sources as a single cluster (i.e., $\mu = 1$ and $\nu = 1$) is also displayed (through simulation). This approach is termed non-orthogonal FFNC relaying, since it requires all sources (and relays) to non-orthogonally use the channel. Different from single-hop MAC channels, simply demanding all terminals to access the same channel may not necessarily be the best choice.

To more clearly see the advantage of our cluster FFNC relaying scheme, we follow the concept of finite-SNR DMT [17] and allow the average transmission data rate to scale with SNR as $\bar{R} = r\log(1+\rho)$. Setting $r = \frac{1}{12}$, we can see that the slope of the error probability of the cluster FFNC relaying scheme becomes larger than the orthogonal FFNC relaying scheme. This implies a better tradeoff between diversity and multiplexing gains in the finite-SNR regime. We conjecture that performance advantages of combining network coding and non-orthogonal transmission can be found using other metrics such as the tradeoff between spectral and energy efficiencies.

In Fig. 4, we consider a different network structure with $M = 3$ sources, $K = 3$ relays, and $N = 3$ destinations. We treat the sources as a single cluster and relays as a single cluster and apply our cluster FFCN relaying scheme, i.e., $\mu = 1$, $\nu = 1$, and $|\mathcal{S}_1| = |\mathcal{R}_1| = 3$. Two time slots are consumed to complete the transmissions, while the orthogonal FFNC relaying needs six time slots. Following the analysis provided earlier, we can attain the probabilities of different relay decoding status $\Pr\{\boldsymbol{\pi} = [0,0,0]\} = (1 - Q_3^{[3]})^3$, $\Pr\{\boldsymbol{\pi} = [0,0,1]\} = Q_3^{[3]}(1 - Q_3^{[3]})^2$, $\Pr\{\boldsymbol{\pi} = [0,1,1]\} = (Q_3^{[3]})^2(1 - Q_3^{[3]})$, $\Pr\{\boldsymbol{\pi} = [1,1,1]\} = (Q_3^{[3]})^3$, and the conditional error probabilities $\Pr\{\tau_n = 1|\boldsymbol{\pi} = [0,0,0]\} = Q_3^{[3]}$, $\Pr\{\tau_n = 1|\boldsymbol{\pi} = [0,0,1]\} = Q_3^{[3]} + Q_3^{[2]}Q_1^{[1]}$, $\Pr\{\tau_n = 1|\boldsymbol{\pi} = [0,1,1]\} = Q_3^{[3]} + Q_3^{[2]}(Q_2^{[1]} + Q_2^{[2]}) + Q_3^{[1]}Q_2^{[2]}$, and $\Pr\{\tau_n = 1|\boldsymbol{\pi} = [1,1,1]\} = Q_3^{[3]} + Q_3^{[2]}(Q_3^{[1]} + Q_3^{[2]} + Q_3^{[3]}) + Q_3^{[1]}(Q_3^{[2]} + Q_3^{[3]}) + Q_3^{[0]}Q_3^{[3]}$. The system error probability can be derived, and is verified through simulations. From the figure, we can attain similar observations as in Fig. 3. The advantages of our cluster FFNC relaying scheme are clear.

V. Conclusion

We have investigated an efficient transmission scheme for multi-user relay networks where multiple sources intend to distribute their messages to multiple ambient destinations. Multiple relays applying a class of finite-field network coding are deployed to assist in the transmission process. By dividing the sources and relays into proper clusters, non-orthogonal transmission can be combined with the relaying and network coding techniques to further enhance system performance. We have provided the method to derive the system error performance, and used example networks to show that it can notably outperform the conventional approach that orthogonalizes all terminals. We conjecture that the performance gain of combining network coding and non-orthogonal transmission can be further enhanced through additional system designs such as power control, rate adaptation, and clustering selections. These will be treated as meaningful future works.

APPENDIX

A. Individual error probability in 3-user MAC channel

In this section, we present the individual error probabilities in the three-user MAC channel with transmitter set $\mathcal{L} = \{a_1, a_2, a_3\}$ and a common receiver b. Due to page length limit, we provide only a sketch of the derivation approach of the individual error probabilities. Afterwards, the results for a special case, where all three transmitters send their messages with the same power level in a similar fading environment such that $m_{b,a_i} = m$ and $\frac{m_{b,a_i}}{\Omega_{b,a_i}\rho_{a_i}} = \beta$, $\forall i \in \{1,2,3\}$, are shown.

The analysis follows the similar approach in the two-user case. We can also use an event figure similar to Fig. 2(b) (but with three dimensions) to identify the integration region for the probability of each error event. More specifically, for $i \in \{1,2,3\}$, define $\eta_i = 2^{iR} - 1$, and letting $H_i = \rho_{a_i}|h_{b,a_i}|^2$ leads to $H_i \sim \text{Gamma}(\alpha_i, \beta_i)$, where $\beta_i = \frac{\alpha_i}{\Omega_{b,a_i}\rho_{a_i}}$. The conditions that lead to the event that b can correctly decode all three transmitters can be expressed as $E_{1,\{a_1,a_2,a_3\}}: \log_2(1+H_i) > R, \log_2(1+H_i+H_j) > 2R, \log_2(1+H_1+H_2+H_3) > 3R$ for all $i,j \in \{1,2,3\}$ and $i \neq j$. This means, we have $Q_{\mathcal{L},b}^{\{a_1,a_2,a_3\}}$ (or $Q_{\mathcal{L},b}^{[3]}$) as

$$Q_{\mathcal{L},b}^{\{a_1,a_2,a_3\}} = \Pr\Big\{H_1 > \eta_1, H_2 > \eta_2, H_3 > \eta_3, H_1+H_2 > \eta_2, H_1+H_3 > \eta_2, H_2+H_3 > \eta_2, H_1+H_2+H_3 > \eta_3\Big\}.$$

It can be shown that the error event corresponds to six non-overlapping sub-regions in the error event figure. Let $f_{\mathbf{H}}(\mathbf{x}) = f(x_1;\alpha_1,\beta_1|x_2,x_3)f(x_2;\alpha_2,\beta_2|x_3)f(x_3;\alpha_3,\beta_3)$. We can calculate $Q_{\{a_1,a_2,a_3\},b}^{\{a_1,a_2,a_3\}}$ via the following integrations:

$$\begin{aligned}
Q_{\mathcal{L},b}^{\{a_1,a_2,a_3\}} \;=\; & \int_{\eta_1}^{\eta_2-\eta_1}\int_{\eta_2-x_3}^{\eta_3-\eta_2}\int_{\eta_3-x_2-x_3}^{\infty} f_{\mathbf{H}}(\mathbf{x})\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3 + \int_{\eta_1}^{\eta_2-\eta_1}\int_{\eta_3-\eta_2}^{\infty}\int_{\eta_2-x_3}^{\infty} f_{\mathbf{H}}(\mathbf{x})\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3 \\
& + \int_{\eta_2-\eta_1}^{\eta_3-\eta_2}\int_{\eta_1}^{\eta_3-\eta_1-x_3}\int_{\eta_3-x_2-x_3}^{\infty} f_{\mathbf{H}}(\mathbf{x})\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3 + \int_{\eta_2-\eta_1}^{\eta_3-\eta_2}\int_{\eta_3-\eta_1-x_3}^{\infty}\int_{\eta_1}^{\infty} f_{\mathbf{H}}(\mathbf{x})\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3 \\
& + \int_{\eta_3-\eta_2}^{\infty}\int_{\eta_1}^{\eta_2-\eta_1}\int_{\eta_2-x_3}^{\infty} f_{\mathbf{H}}(\mathbf{x})\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3 + \int_{\eta_3-\eta_2}^{\infty}\int_{\eta_2-\eta_1}^{\infty}\int_{\eta_1}^{\infty} f_{\mathbf{H}}(\mathbf{x})\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3.
\end{aligned} \tag{17}$$

Similarly, the probability that b can decode a_1 and a_2 but not a_3, i.e., $Q_{\{a_1,a_2,a_3\},b}^{\{a_1,a_2\}}$, can be derived as

$$\begin{aligned}
Q_{\mathcal{L},b}^{\{a_1,a_2\}} &= \Pr\left\{\frac{H_1}{1+H_3} > \eta_1, \frac{H_2}{1+H_3} > \eta_1, \frac{H_1+H_2}{1+H_3} > \eta_2, H_3 < \eta_1\right\} \\
&= \int_0^{\eta_1}\int_{\eta_1(1+x_3)}^{(\eta_2-\eta_1)(1+x_3)}\int_{\eta_2(1+x_3)-x_2}^{\infty} f_{\mathbf{H}}(\mathbf{x})\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3 + \int_0^{\eta_1}\int_{(\eta_2-\eta_1)(1+x_3)}^{\infty}\int_{\eta_1(1+x_3)}^{\infty} f_{\mathbf{H}}(\mathbf{x})\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3.
\end{aligned} \tag{18}$$

By symmetric, the probabilities $Q_{\mathcal{L},b}^{\{a_1,a_3\}}$ and $Q_{\mathcal{L},b}^{\{a_2,a_3\}}$ can be attained using the similar method. These lead to $Q_{\mathcal{L},b}^{[2]} = Q_{\mathcal{L},b}^{\{a_1,a_2\}} + Q_{\mathcal{L},b}^{\{a_1,a_3\}} + Q_{\mathcal{L},b}^{\{a_2,a_3\}}$.

Furthermore, the probability that b can decode only a_1 but neither a_2 nor a_3, i.e., $Q_{\mathcal{L},b}^{\{a_1\}}$, can be derived as $Q_{\mathcal{L},b}^{\{a_1\}} = \Pr\Big\{\frac{H_1}{1+H_2+H_3} > \eta_1, \frac{H_2}{1+H_3} < \eta_1, \frac{H_3}{1+H_2} < \eta_1, H_2+H_3 < \eta_2\Big\}$, i.e.,

$$Q_{\mathcal{L},b}^{\{a_1\}} = \int_0^{\eta_1}\int_0^{\eta_1(1+x_3)}\int_{\eta_1(1+x_2+x_3)}^{\infty} f_{\mathbf{H}}(\mathbf{x})\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3 + \int_{\eta_1}^{\eta_2-\eta_1}\int_{\frac{x_3}{\eta_1}-1}^{\eta_2-x_3}\int_{\eta_1(1+x_2+x_3)}^{\infty} f_{\mathbf{H}}(\mathbf{x})\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3. \tag{19}$$

Similar analysis can be applied also to $Q_{\mathcal{L},b}^{\{a_2\}}$ and $Q_{\mathcal{L},b}^{\{a_3\}}$. As a result, $Q_{\mathcal{L},b}^{[1]} = Q_{\mathcal{L},b}^{\{a_1\}} + Q_{\mathcal{L},b}^{\{a_2\}} + Q_{\mathcal{L},b}^{\{a_3\}}$.

For instance, consider the case $m_{b,a_1} = m_{b,a_2} = m_{b,a_3} = m$ and $\frac{m_{b,a_1}}{\Omega_{b,a_1}\rho_{a_1}} = \frac{m_{b,a_2}}{\Omega_{b,a_2}\rho_{a_2}} = \frac{m_{b,a_3}}{\Omega_{b,a_3}\rho_{a_3}} = \beta$. We can have

$$\begin{aligned}
Q_{\mathcal{L},b}^{[3]} \;=\; & \sum_{i=0}^{m-1}\sum_{j=0}^{i}\sum_{l=0}^{j} \frac{\beta^{2m+i}(-1)^{i-l}e^{-\beta\eta_3}\eta_3^l}{((m-1)!)^2 l!(i-j)!(j-l)!\sigma_1}\Bigg(\frac{\eta_1^{\sigma_1}}{\sigma_2}\Big((\eta_2+1)^{\sigma_1}\big((\eta_2-\eta_1)^{\sigma_2}-\eta_1^{\sigma_2}\big)-\big((\eta_3-\eta_2)^{\sigma_2}-(\eta_2-\eta_1)^{\sigma_2}\big)\Big) \\
& \qquad + \sum_{q=0}^{\sigma_1}\frac{\sigma_1!(-1)^{\sigma_1-q}\eta_2^q}{q!(\sigma_1-q)!\sigma_3}\Big(-\eta_2^q\big((\eta_2-\eta_1)^{\sigma_3}-(\eta_1)^{\sigma_3}\big)+(\eta_1+1)^q\big((\eta_3-\eta_2)^{\sigma_3}-(\eta_2-\eta_1)^{\sigma_3}\big)\Big)\Bigg) \\
& + \sum_{i=0}^{m-1}\sum_{j=0}^{i}\binom{i}{j}\frac{\beta^{m+i}(-1)^{i-j}}{i!(m-1)!\sigma_1}\Bigg(2e^{-\beta\eta_2}\eta_2^j\Big((\eta_2-\eta_1)^{\sigma_1}-\eta_1^{\sigma_1}\Big)\Big(1-F(\eta_3-\eta_2;m,\beta)\Big) \\
& \qquad + \Big(1-F(\eta_1;m,\beta)\Big)e^{\beta(\eta_3-\eta_1)}(\eta_3-\eta_1)^j\Big((\eta_3-\eta_2)^{\sigma_1}-(\eta_2-\eta_1)^{\sigma_1}\Big)\Bigg) \\
& + \Big(1-F(\eta_1;m,\beta)\Big)\Big(1-F(\eta_2-\eta_1;m,\beta)\Big)\Big(1-F(\eta_3-\eta_2;m,\beta)\Big),
\end{aligned} \tag{20}$$

$$
\begin{aligned}
Q^{[2]}_{\mathcal{L},b} \;=\;& \sum_{i=0}^{m-1}\sum_{j=0}^{i}\sum_{l=0}^{m+i}\binom{i}{j}\binom{m+i}{l}\frac{3\beta^l(-1)^{i-j}\eta_2^j e^{-\beta\eta_2}}{i!((m-1)!)^2\sigma_1(\eta_2+1)^{\sigma_4}}(\sigma_4-1)!\Big((\eta_2-\eta_1)^{\sigma_1}-\eta_1^{\sigma_1}\Big)F(\eta_1;\sigma_4,\beta(\eta_2+1))\\
&+\sum_{i=0}^{m-1}\sum_{j=0}^{m-1}\sum_{l=0}^{i+j}\binom{i+j}{l}\frac{3\beta^l e^{-\beta\eta_2}(\eta_2-\eta_1)^i\eta_1^j}{i!j!(m-1)!(\eta_2+1)^{\sigma_2+i}}(\sigma_2+i-1)!F(\eta_1;\sigma_2+i,\beta(\eta_2+1)), \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
Q^{[1]}_{\mathcal{L},b} \;=\;& \sum_{i=0}^{m-1}\sum_{j=0}^{i}\sum_{l=0}^{j}\binom{i}{j}\binom{j}{l}\frac{3\beta^l\eta_1^i e^{-\beta\eta_1}(\sigma_1-1)!(\sigma_2-1)!}{i!((m-1)!)^2(\eta_1+1)^{\sigma_4}}F(\eta_1;\sigma_2,\beta(\eta_1+1))\\
&-\sum_{i=0}^{m-1}\sum_{j=0}^{i}\sum_{q=0}^{\sigma_1-1}\sum_{l=0}^{q+j}\binom{i}{j}\binom{q+j}{l}\frac{\beta^l\eta_1^i}{i!q!}\cdot\frac{3e^{-\beta\eta_2}(\eta_2-\eta_1)^q(\sigma_1-1)!(\sigma_2+q-1)!}{((m-1)!)^2(\eta_1+1)^{\sigma_1}(\eta_2+1)^{\sigma_2+q}}F(\eta_1;\sigma_2+q,\beta(\eta_2+1))\\
&+\sum_{i=0}^{m-1}\sum_{j=0}^{i}\sum_{l=0}^{\sigma_1-1}\sum_{q=0}^{l}\sum_{s=0}^{j}\binom{i}{j}\binom{l}{q}\binom{j}{s}\frac{3\beta^{m+j+l}(\sigma_1-1)!\eta_1^i}{i!l!((m-1)!)^2(\eta_1+1)^{\sigma_1-l}}\\
&\cdot\left(\frac{\eta_1^{-q}e^{\beta}(\sigma_5-1)!}{(-1)^{l-q}(\beta\sigma_6)^{\sigma_5}}\Big(F(\eta_2-\eta_1;\sigma_5,\beta\sigma_6)-F(\eta_1;\sigma_5,\beta\sigma_6)\Big)-\frac{e^{-\beta\eta_3}(-1)^q\eta_2^{l-q}}{\sigma_5}\Big((\eta_2-\eta_1)^{\sigma_5}-\eta_1^{\sigma_5}\Big)\right), \qquad (22)
\end{aligned}
$$

where $\sigma_1=m+i-j$, $\sigma_2=m+j-l$, $\sigma_3=2m+i-q-l$, $\sigma_4=2m+i-l$, $\sigma_5=m+q+s$, and $\sigma_6=2+\eta_1+\frac{1}{\eta_1}$.

Finally, the probability that b is unable to decode any message can be simply found by $Q^{[0]}_{\mathcal{L},b}=1-Q^{[1]}_{\mathcal{L},b}-Q^{[2]}_{\mathcal{L},b}-Q^{[3]}_{\mathcal{L},b}$.

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