OPTIMAL DESIGNS OF THE DOUBLE SAMPLING $\bar{X}$ CHART BASED ON PARAMETER ESTIMATION

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The notations and abbreviations used in this thesis are listed as follows:

SQC	Statistical quality control
SPC	Statistical process control
SAS	Statistical Analysis Software
DS	Double sampling
TS	Triple sampling
VP	Variable parameters
VSI	Variable sampling interval
VSS	Variable sample size
DSVSI	Combined DS and VSI
EWMA	Exponentially weighted moving average
CSEWMA	Combined Shewhart-EWMA
CUSUM	Cumulative sum
ARMA (1, 1)	First-order autoregressive moving average
RL	Run length
$Var(\text{RL})$	Variance of the run length
$E(\text{RL})$	Expected value of the run length
ARL	Average run length
ARL_0	In-control ARL
ARL_1	Out-of-control ARL
SDRL	Standard deviation of the run length
SDRL_0	In-control SDRL

SDRL_1	Out-of-control SDRL
MRL	Median run length
MRL_0	In-control MRL
MRL_1	Out-of-control MRL
ASS	Average sample size
ASS_0	In-control ASS
ASS_1	Out-of-control ASS
AEQL	Average extra quadratic loss
EARL	Expected value of the ARL
ℓ_{ζ}	$(100\zeta)^{\text{th}}$ percentage point of the run length distribution, where $0<\zeta<1$
IQR	Interquartile range
FSR	False signal rate
FAP	False alarm probability
MGF	Moment generating function
μ	Process mean
σ	Process standard deviation
μ_0	In-control mean
μ_1	Out-of-control mean
σ_0^2	In-control variance
σ_0	In-control standard deviation
$\hat{\mu}_0$	Estimator of μ_0
$\hat{\sigma}_0$	Estimator of σ_0

S_p	Pooled sample standard deviation
S	Sample standard deviation
S^2	Sample variance
$\bar{S}$	Average sample standard deviation
R	Sample range
$\bar{R}$	Average sample range
$\overline{MR}$	Average moving range
$N(\mu,\ \sigma^2)$	Normal distribution having mean μ and variance σ^2
δ	Magnitude of a standardized mean shift
δ_{opt}	Desired standardized mean shift, for which a quick detection is required
δ^*	Desired standardized mean shift used in minimizing the ASS
m	Number of Phase-I samples
n	Phase-I sample size
X	Quality characteristic of a Phase-I process
$X_{i,j}$	The j^{th} observation in the i^{th} sample of a Phase-I process, where $i = 1$, 2, …, m and $j = 1$, 2, …, n
$\bar{X}_i$	The i^{th} sample mean of quality characteristic *X*, in the Phase-I process
$\bar{\bar{X}}$	Sample grand average of quality characteristic *X*, in the Phase-I process
Y	Quality characteristic of a Phase-II process

$Y_{i,j}$	The j^{th} observation at the i^{th} sampling time of a Phase-II process, where $i=1,\ 2,\ \ldots$ and $j=1,\ 2,\ \ldots,\ n$, for the Shewhart $\bar{X}$ and EWMA $\bar{X}$ charts
$\bar{Y}_i$	Sample mean of the Shewhart $\bar{X}$ and EWMA $\bar{X}$ charts or combined-sample mean of the DS $\bar{X}$ chart at the i^{th} sampling time, of a Phase-II process
Z_{ς}	$(1-\varsigma)100^{\text{th}}$ percentage point of the standard normal distribution
c_4	S chart's constant
p	Fraction nonconforming
np	Number of nonconforming units
c	Count of non conformities
u	Count of non conformities per unit of inspection
k	Reference value for the CUSUM statistics
CL	Center line
UCL	Upper control limit
LCL	Lower control limit
$UCL_{\text{Bon}\_\bar{X}}$	Upper control limit of the Bonferroni-adjusted $\bar{X}$ chart
$LCL_{\text{Bon}\_\bar{X}}$	Lower control limit of the Bonferroni-adjusted $\bar{X}$ chart
UCL_{Bon_S}	Upper control limit of the Bonferroni-adjusted S chart
LCL_{Bon_S}	Lower control limit of the Bonferroni-adjusted S chart
iid	Independently and identically distributed
cdf	Cumulative distribution function
pmf	Probability mass function

pdf	Probability density function
$\Phi(\cdot)$	cdf of the standard normal distribution
$\Phi^{-1}(\cdot)$	Inverse cdf of the standard normal distribution
$\phi(\cdot)$	pdf of the standard normal distribution
$f_N(\cdot \mid a,b)$	pdf of the normal distribution with mean a and variance b
$f_\gamma(\cdot \mid c,d)$	pdf of the gamma distribution with parameters c and d
$\chi^2(df)$	Chi-square distribution with df degrees of freedom
$f_{\mathrm{RL}}(\ell)$	pmf of the run length
$F_{\mathrm{RL}}(\ell)$	cdf of the run length
W	Random variable defined as $(\hat{\mu}_0 - \mu_0)\frac{\sqrt{n}}{\sigma_0}$
U	Random variable defined as $(\hat{\mu}_0 - \mu_0)\frac{\sqrt{mn}}{\sigma_0}$
V	Random variable defined as $\frac{\hat{\sigma}_0}{\sigma_0}$
$f_W(w \mid m)$	pdf of the random variable W
$f_U(u)$	pdf of the random variable U
$f_V(v \mid m,n)$	pdf of the random variable V
τ	Desired in-control ARL
τ'	Desired in-control MRL
ε	Desired out-of-control ARL corresponding to a shift δ^*
ε'	Desired out-of-control MRL corresponding to a shift δ^*

The notations and abbreviations used for the Shewhart $\bar{X}$ chart in this thesis are as follows:

$UCL_{\bar{X}}$	Upper control limit of the Shewhart $\bar{X}$ chart with known parameters
$LCL_{\bar{X}}$	Lower control limit of the Shewhart $\bar{X}$ chart with known parameters
$\widehat{UCL}_{\bar{X}}$	Upper control limit of the Shewhart $\bar{X}$ chart with estimated parameters
$\widehat{LCL}_{\bar{X}}$	Lower control limit of the Shewhart $\bar{X}$ chart with estimated parameters
K	A multiplier controlling the width of the Shewhart $\bar{X}$ chart's control limits
α	Type-I error probability
β	Type-II error probability
$\hat{\beta}$	Conditional Type-II error, given fixed values of $\hat{\mu}_0$ and $\hat{\sigma}_0$
$n_{\bar{X}}$	Sample size of the Shewhart $\bar{X}$ chart with known parameters
n_{est}	Sample size of the Shewhart $\bar{X}$ chart with estimated parameters

The notations and abbreviations used for the DS $\bar{X}$ chart in this thesis are presented as follows:

n_1	First sample size
n_2	Second sample size
L	Control limit of the DS $\bar{X}$ chart based on the first sample
L_1	Warning limit of the DS $\bar{X}$ chart based on the first sample
L_2	Control limit of the DS $\bar{X}$ chart based on the combined samples

I_1	Interval $\left[-L_1,\ L_1\right]$ of the DS $\bar{X}$ chart
I_2	Intervals $\left[-L,\ -L_1\right)\cup\left(L_1,\ L\right]$ of the DS $\bar{X}$ chart
I_2^*	Intervals $\left[-L+\delta\sqrt{n_1},\ -L_1+\delta\sqrt{n_1}\right)\cup\left(L_1+\delta\sqrt{n_1},\ L+\delta\sqrt{n_1}\right]$ of the DS $\bar{X}$ chart
I_3	Intervals $\left(-\infty,\ -L\right)\cup\left(L,\ +\infty\right)$ of the DS $\bar{X}$ chart
I_4	Interval $\left[-L_2,\ L_2\right]$ of the DS $\bar{X}$ chart
$Y_{1i,j}$	The j^{th} observation in the first sample at the i^{th} sampling time, for $j =$ 1, 2, …, n_1 and $i = 1, 2, \ldots$
$\bar{Y}_{1i}$	Sample mean of the first sample at the i^{th} sampling time, for $i = 1$, 2, …
$Y_{2i,j}$	The j^{th} observation in the second sample at the i^{th} sampling time, for $j = 1, 2, \ldots, n_2$ and $i = 1, 2, \ldots$
$\bar{Y}_{2i}$	Sample mean of the second sample at the i^{th} sampling time, for $i = 1$, 2, …
Z_{1i}	The standardized random variable of the first sample for the case of known parameters, at the i^{th} sampling time, for $i = 1, 2, \ldots$
Z_{2i}	The standardized random variable of the second sample for the case of known parameters, at the i^{th} sampling time, for $i = 1, 2, \ldots$
Z_i	The standardized random variable of the combined samples for the case of known parameters, at the i^{th} sampling time, for $i = 1, 2, \ldots$
$\hat{Z}_{1i}$	The standardized random variable of the first sample for the case of estimated parameters, at the i^{th} sampling time, for $i = 1, 2, \ldots$

$\hat{Z}_{2i}$ The standardized random variable of the second sample for the case of estimated parameters, at the i^{th} sampling time, for $i = 1, 2, \ldots$

$\hat{Z}_{i}$ The standardized random variable of the combined samples for the case of estimated parameters, at the i^{th} sampling time, for $i = 1, 2, \ldots$

$F_{\hat{Z}_{1i}}\left(z \middle| \hat{\mu}_0, \hat{\sigma}_0\right)$ The conditional cdf of $\hat{Z}_{1i}$, given the fixed values of $\hat{\mu}_0$ and $\hat{\sigma}_0$

$f_{\hat{Z}_{1i}}\left(z \middle| \hat{\mu}_0, \hat{\sigma}_0\right)$ The conditional pdf of $\hat{Z}_{1i}$, given the fixed values of $\hat{\mu}_0$ and $\hat{\sigma}_0$

P_{a1} Probability that the process is declared as in-control "by the first sample" for the DS $\bar{X}$ chart with known parameters

P_{a2} Probability that the process is declared as in-control "after taking the second sample" for the DS $\bar{X}$ chart with known parameters

P_{a} Probability that the process is considered as in-control for the DS $\bar{X}$ chart with known parameters

P_{2} Probability of requiring the second sample for the known-parameter case

P_{4} Conditional probability of $Z_i \in I_4$ given that $Z_{1i} = z$, where $z \in I_2^*$

$\hat{P}_{a1}$ Conditional probability that the process is declared as in-control "by the first sample" for fixed values of $\hat{\mu}_0$ and $\hat{\sigma}_0$

$\hat{P}_{a2}$ Conditional probability that the process is declared as in-control "after taking the second sample" for fixed values of $\hat{\mu}_0$ and $\hat{\sigma}_0$

$\hat{P}_{a}$ Conditional probability that the process is considered as in-control for fixed values of $\hat{\mu}_0$ and $\hat{\sigma}_0$

$\hat{P}_2$ Conditional probability of requiring the second sample for fixed values of $\hat{\mu}_0$ and $\hat{\sigma}_0$

$\hat{P}_4$ Conditional probability of $\hat{Z}_i \in I_4$ given $\hat{Z}_{1i} = z$, $\hat{\mu}_0$ and $\hat{\sigma}_0$, where $z \in I_2$

The notations and abbreviations used for the EWMA chart are as follows:

λ Smoothing constant of the EWMA chart

$Z_{i\ (\mathrm{EWMA})}$ Plotting statistic of the EWMA $\bar{X}$ chart

K_{EWMA} A multiplier controlling the width of the EWMA $\bar{X}$ chart's control limits

P Transition probability matrix

p Number of transient states in **P**

Q Transition probability matrix for the transient states

$Q_{i,j}$ Transition probability for entry (i, j) in matrix **Q**

I Identity matrix

1 A vector with each of its elements equal to unity

q Initial probability vector

v_i i^{th} factorial moment of the RL

n_{EWMA} Sample size of the EWMA $\bar{X}$ chart

LIST OF PUBLICATIONS

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1. Khoo, M.B.C., **Teoh, W.L.**, Castagliola, P. and Lee, M.H. (2012). Optimal designs of the double sampling $\bar{X}$ chart with estimated parameters. *International Journal of Production Economics*. Under revision. [ISSN: 0925-5273][2011 impact factor: 1.760; 2011 5-year impact factor: 2.384]

2. **Teoh, W.L.**, Khoo, M.B.C., Castagliola, P. and Chakraborti, S. (2012). Optimal design of the double sampling $\bar{X}$ chart with estimated parameters based on median run length. *Computers and Industrial Engineering*. Under review. [ISSN: 0360-8352] [2011 impact factor: 1.589; 2011 5-year impact factor: 1.872]

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4. **Teoh, W.L.**, Khoo, M.B.C., Castagliola, P. and Lee, M.H. (2012). The exact run length distribution and design of the Shewhart $\bar{X}$ chart with estimated parameters based on median run length. *Journal of Applied Statistics*. Under revision. [Print ISSN: 0266-4763; Online ISSN: 1360-0532][2011 impact factor: 0.405; 2011 5-year impact factor: 0.479]

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1. **Teoh, W.L.** and Khoo, M.B.C. (2012). *A preliminary study on the double sampling $\bar{X}$ chart with unknown parameters*. Proceedings of the 2012 IEEE and the 2012 International Conference on Innovation, Management and Technology Research, ICIMTR 2012, Hotel Equatorial, Malacca, Malaysia, pp 80-84. Published. [ISBN:978-1-4673-0654-6]

2. Khoo, M.B.C., **Teoh, W.L.**, Liew, J.Y. and Teh, S.Y. (2013). *A Study on the double sampling $\bar{X}$ chart with estimated parameters for minimizing the in-control average sample size*. Proceedings of the International Conference on Computer Science and Computational Mathematics, ICCSCM 2013, Citrus Hotel Kuala Lumpur, Kuala Lumpur, Malaysia. Submitted.

REKA BENTUK OPTIMUM CARTA $\bar{X}$ PENSAMPELAN GANDA DUA BERDASARKAN PENGANGGARAN PARAMETER

ABSTRAK

Carta kawalan yang dilihat sebagai alat yang paling berkuasa dan paling mudah dalam Kawalan Proses Berstatistik (SPC) digunakan secara meluas dalam industri pembuatan dan perkhidmatan. Carta $\bar{X}$ pensampelan ganda dua (DS) mengesan anjakan min proses yang kecil hingga sederhana dengan berkesan, di samping mengurangkan saiz sampel. Aplikasi lazim carta $\bar{X}$ DS biasanya disiasat dengan anggapan bahawa parameter-parameter proses adalah diketahui. Walau bagaimanapun, parameter-parameter proses biasanya tidak diketahui dalam aplikasi praktikal; justeru, parameter-parameter ini dianggarkan daripada data Fasa-I yang terkawal. Dalam tesis ini, kesan penganggaran parameter terhadap prestasi carta $\bar{X}$ DS diperiksa. Dengan mempertimbangkan penganggaran parameter, sifat-sifat panjang larian carta $\bar{X}$ DS diperoleh. Oleh sebab bentuk dan kepencongan taburan panjang larian berubah dengan magnitud anjakan min proses, bilangan sampel Fasa-I dan saiz sampel, ukuran prestasi yang digunakan secara meluas, iaitu purata panjang larian (ARL), tidak harus digunakan sebagai ukuran tunggal prestasi carta. Oleh hal yang demikian, ARL, sisihan piawai panjang larian (SDRL), median panjang larian (MRL), persentil taburan panjang larian dan purata saiz sampel (ASS) disyorkan untuk menilai carta $\bar{X}$ DS berdasarkan panganggaran parameter yang dicadangkan ini dengan berkesan. Idea utama tesis ini terdiri daripada cadangan empat reka bentuk optimum yang baru untuk carta $\bar{X}$ DS berasaskan ARL dan MRL dengan parameter-parameter yang dianggarkan. Secara khususnya, reka bentuk optimum baru yang dicadangkan ini ialah carta $\bar{X}$ DS berasaskan ARL

dengan parameter-parameter yang dianggarkan untuk meminimumkan (i) ARL terluar kawal (ARL_1) dan (ii) ASS terkawal (ASS_0), serta carta $\bar{X}$ DS berasaskan MRL dengan parameter-parameter yang dianggarkan untuk meminimumkan (iii) MRL terluar kawal (MRL_1) dan (iv) ASS_0 . Tambahan pula, bagi memudahkan pelaksanaan, tesis ini membekalkan parameter-parameter carta optimum yang direka khas untuk carta $\bar{X}$ DS berdasarkan penganggaran parameter. Parameter-parameter carta optimum diperoleh berdasarkan bilangan sampel Fasa-I yang biasanya digunakan dalam amalan. Program-program pengoptimuman untuk reka bentuk optimum carta $\bar{X}$ DS berdasarkan penganggaran parameter juga dibekalkan dalam tesis ini. Program-program pengoptimuman ini memudahkan pengamal dalam menentukan parameter-parameter carta optimum untuk situasi yang dikehendaki oleh mereka, diikuti dengan penggunaan carta optimum yang dicadangkan dengan serta-merta untuk data mereka sendiri. Selain itu, garis panduan empirikal tentang pembinaan carta optimum $\bar{X}$ DS berdasarkan penganggaran parameter diberikan dalam tesis ini.

OPTIMAL DESIGNS OF THE DOUBLE SAMPLING $\bar{X}$ CHART BASED ON PARAMETER ESTIMATION

ABSTRACT

Control charts, viewed as the most powerful and simplest tool in Statistical Process Control (SPC), are widely used in manufacturing and service industries. The double sampling (DS) $\bar{X}$ chart detects small to moderate process mean shifts effectively, while reduces the sample size. The conventional application of the DS $\bar{X}$ chart is usually investigated assuming that the process parameters are known. Nevertheless, the process parameters are usually unknown in practical applications; thus, they are estimated from an in-control Phase-I dataset. In this thesis, the effects of parameter estimation on the DS $\bar{X}$ chart's performance are examined. By taking into consideration of the parameter estimation, the run length properties of the DS $\bar{X}$ chart are derived. Since the shape and the skewness of the run length distribution change with the magnitude of the process mean shift, the number of Phase-I samples and sample size, the widely applicable performance measure, i.e. the average run length (ARL) should not be used as a sole measure of a chart's performance. For this reason, the ARL, the standard deviation of the run length (SDRL), the median run length (MRL), the percentiles of the run length distributions and the average sample size (ASS) are recommended to effectively evaluate the proposed DS $\bar{X}$ chart with estimated parameters. The key idea of this thesis consists of proposing four new optimal designs for the ARL-based and MRL-based DS $\bar{X}$ chart with estimated parameters. In particular, these newly developed optimal designs are the ARL-based DS $\bar{X}$ chart with estimated parameters obtained by minimizing (i) the out-of-control ARL (ARL_1) and (ii) the in-control ASS (ASS_0), as well as the MRL-based DS $\bar{X}$

chart with estimated parameters obtained by minimizing (iii) the out-of-control MRL (MRL_1) and (iv) ASS_0. Furthermore, for the ease of implementation, this thesis provides specific optimal chart parameters specially designed for the DS $\bar{X}$ chart with estimated parameters, based on the number of Phase-I samples commonly used in practice. Crucially, optimization programs for optimally designing the DS $\bar{X}$ chart with estimated parameters are available in this thesis. These optimization programs facilitate the practitioners in determining the optimal chart parameters for their desired situations, followed by applying the proposed optimal chart to their own data instantaneously. Also, empirical guidelines on the construction of the optimal DS $\bar{X}$ chart with estimated parameters are given in this thesis.

CHAPTER 1
INTRODUCTION

1.1 Statistical Process Control (SPC)

Customers' satisfaction is very important in the world today. Improving quality and productivity of a production process are the key factors leading to a successful and competitive business. Statistical Process Control (SPC) is a collection of powerful statistical techniques that is used to reduce variability in the key parameters, to ensure improvement in the process performance and to maintain a higher quality control in the production process (Smith, 1998). Garrity (1993) claimed that SPC is not only the better way, but also the only way of running a thriving business. The attractiveness of SPC is rooted in its valuable tools that lead to many process improvements and thus, allowing the manufacturing of higher quality and uniformity outputs with fewer defects to rework and less scrap. SPC also enables a significant reduction in machine downtime, an increase in profit, a lower average production cost, as well as an improved competitive position (Smith, 1998). In view of these appealing properties, SPC is adopted to solve problems in production, inspection, engineering, service, management and accounting.

The existence of variations in any manufacturing processes is inevitable. The process variations can be classified into two categories, i.e. common causes of variation and assignable causes of variation. Gitlow *et al*. (1995) stated that the common causes of variation are inherent in a process, whereas the assignable causes of variation lie outside the system and thus, it is not part of the chance causes. In addition, Shewhart (1931) recognized that the common causes of variation are uncontrollable and are due to unidentifiable sources; hence, such causes cannot be rectified from the process without very expensive measures. Contrarily, the assignable causes of variation arise from identifiable sources, which can be systematically detected and eliminated from the process. There are three sources contributing to this variation, which comprise defective raw materials, operator errors, as well as improper adjustments of machines (Montgomery, 2009). A process is in a state of statistical control if only common causes of variation are present in the process. If the process is operating under both the common and assignable causes, it is considered unstable and out of statistical control (Gupta & Walker, 2007).

SPC consists of seven important statistical tools which are used to achieve process stability and improve process capability by reducing process variations. These tools are known as the "Magnificent Seven", which include the Pareto chart, check sheet, cause-and-effect diagram, defect concentration diagram, histogram, control chart and scatter diagram (Montgomery, 2009). Among these tools, the control chart is an excellent and irreplaceable process monitoring technique adopted in manufacturing and service processes, for keeping a process predictable (see Thompson & Koronacki, 2002; Gupta & Walker, 2007; Montgomery, 2009).

A control chart is a graphical tool for controlling, analyzing and understanding a process; thus, it assures the production of conforming products by that particular process (Ledolter & Burrill, 1999). It is a time-sequence plot of crucial product characteristics with "decision lines" added. Moreover, Ryan (2000) stated that statistical principles are employed in the construction of a control chart. Specifically, it is based on some statistical distributions. Control charts are classified into two main types, i.e. variables control charts and attributes control charts. A variables control chart is used to monitor characteristics that can be expressed in terms of continuous values and numerical measurements. This type of control chart allows for a continuous reduction in process variations and a never-ending process improvement (Gitlow *et al*., 1995). An attribute control chart, on the other hand, is used to monitor characteristics that are in the form of discrete counts. Therefore, the inspected items are categorized as either conforming or nonconforming units. This type of control chart is generally used for defects prevention so that a zero-defects process will be achieved (Gitlow *et al*., 1995).

Knowledge about process variations is the foundation of a control chart's analysis. To reduce variation in a process and to attain a stable process, the common steps in constructing a control chart in practice can be illustrated as follows (Xie *et al*., 2002):

Step 1. Collect a sequence of measurements representing a quality characteristic from a process over time.

Step 2. Estimate the process mean μ and set it as the center line *CL* of the chart.

Step 3. Estimate the process standard deviation σ.

Step 4. Establish the upper control limit *UCL* and the lower control limit *LCL*, based on the "± 3" standard deviation width from the *CL*.

Step 5. Plot the sequence of measurements on the chart and then connect the consecutive points with straight-line segments.

Step 6. If any sample point falls outside the control limits, the process is classified as out-of-control. Then find and eliminate the assignable cause(s) corresponding to this behaviour.

Step 7. Revise and modify the *CL*, *UCL* and *LCL*, if necessary. Then reconstruct the revised chart.

Step 8. Continue plotting whenever a new measurement is acquired.

In practice, control charts are generally divided into Phase-I and Phase-II applications. In Phase-I analysis, control charts are employed retrospectively to analyze a set of collected process data, define the in-control state of the process and assess process stability. Once an in-control reference Phase-I dataset is ensured, the process parameters are estimated from this dataset. In Phase-II analysis, control charts are adopted prospectively to detect any changes in the process being monitored. It must be emphasized that process monitoring is the main goal of Phase-II analysis so that an out-of-control process can be brought into statistical control (Jensen *et al*., 2006; Montgomery, 2009). In the literature, most of the SPC control charts are used for Phase-II process monitoring, which is also the focus of this thesis.

1.2 Problem Statement

Traditionally, the evaluation and development of Phase-II control charts are based on the indispensable assumption of known parameters. In practical applications, it is rarely the case that the process parameters are known to the practitioners and hence, they are usually estimated from an in-control reference Phase-I dataset. When parameters are estimated based on a small number of Phase-I samples, the variability of the estimators gives rise to some undesirable and unexpected control chart's performance. Quesenberry (1993) pointed out that a chart with parameters estimated from only a few number of Phase-I samples will produce a greater number of false alarms and thus, ultimately resulting in a higher production cost. Therefore, there is a need for more research in the area of control charts with estimated parameters as identified by Woodall and Montgomery (1999) and Jensen *et al*. (2006).

Hawkins *et al*. (2003) hypothesized that the charts with a desirable property of being sensitive to smaller shifts are more severely affected by parameter estimation. In view of this fact, Jensen *et al*. (2006) accentuated that future research is essential to be conducted on the development of new charting procedures for these types of control charts with estimated parameters. The DS $\bar{X}$ chart is eminent for its remarkable improvement in statistical efficiency, in terms of sensitizing the detection of small and moderate mean shifts while reducing the sample size, thus leading to the reduction of sampling and inspection costs. To the best of the author's knowledge, all the existing literature on the DS type charts depend on the fundamental assumption of known process parameters. To solve the above problems, it is vital to develop new theoretical and optimization methods for the DS $\bar{X}$ chart, correctly accounting for parameter estimation.

It is known that when parameters are estimated, the run length no longer follows a geometric distribution, but its distribution is highly right-skewed. In this regard, the dependence on the average run length (ARL), as a sole measure of performance of a control chart with estimated parameters, has been subjected to criticisms in recent years (see Quesenberry, 1993; Jones *et al*., 2004, Jensen *et al*., 2006; Bischak & Trietsch, 2007). Yet, to date, there has been little literature focusing on other performance measures in evaluating a control chart with estimated parameters. Thus, for a thorough study on the effects of parameter estimations on a control chart's performance, it is crucial that various performance measures, including the ARL, standard deviation of the run length (SDRL), median run length (MRL), percentiles of the run length distribution and average sample size (ASS), are used to effectively evaluate the control charts with estimated parameters.

In real-world applications, quality practitioners are interested to use only a small number of Phase-I samples to estimate the process parameters and proceed to the Phase-II process monitoring at an earlier stage. For optimum and effective implementation, practitioners also desire to find out the control chart's optimal parameters. In order to establish a more economically feasible process monitoring, it is important to develop optimization algorithms for a control chart with estimated parameters, especially devoted to the practical number of Phase-I samples.

1.3 Objectives of the Thesis

The primary objectives of this thesis are as follows:

(i) To derive the run length properties of the DS $\bar{X}$ chart with estimated parameters.

(ii) To examine the impact of parameter estimation on the performances of the ARL-based and MRL-based DS $\bar{X}$ chart.

(iii) To develop two new optimization algorithms by minimizing (a) the out-of-control average run length (ARL_1) and (b) the in-control average sample size (ASS_0), for the ARL-based DS $\bar{X}$ chart with estimated parameters.

(iv) To develop two new optimization algorithms by minimizing (a) the out-of-control median run length (MRL_1) and (b) the ASS_0, for the MRL-based DS $\bar{X}$ chart with estimated parameters.

(v) To provide new optimal-parameter combinations, for both the ARL-based and MRL-based DS $\bar{X}$ chart with estimated parameters, particularly dedicated to the number of Phase-I samples used in industries.

(vi) To provide some empirical guidelines for quality practitioners to construct the optimal ARL-based and MRL-based DS $\bar{X}$ chart with estimated parameters.

Note that the optimal designs of the DS $\bar{X}$ chart with estimated parameters for minimizing ASS_0 require the application of some properties of the Shewhart $\bar{X}$ chart with estimated parameters. Because of this reason, the secondary objectives of this thesis are stated as follows:

(i) To derive the run length properties of the Shewhart $\bar{X}$ chart with estimated parameters.

(ii) To provide new charting constants for the ARL-based and MRL-based Shewhart $\bar{X}$ chart with estimated parameters.

(iii) To develop new design models for both the ARL-based and MRL-based Shewhart $\bar{X}$ chart with known and estimated parameters. These design models are part of the optimal designs of the ARL-based and MRL-based DS $\bar{X}$ charts with estimated parameters for minimizing ASS_0.

1.4 Organization of the Thesis

Chapter 1 presents an overview of SPC, as well as highlights the problem statement and objectives of this research. In Chapter 2, we turn to the literature review on the development of the DS type charts and control charts with estimated parameters. The operation of the DS $\bar{X}$ chart is briefly outlined in this chapter as well. Short explanations on the intuitive appealing measures of a control chart's performance, are also provided. Additionally, Chapter 2 demonstrates the run length properties of the univariate control charts with known parameters. It introduces three main control charts, including the Shewhart $\bar{X}$ chart, the DS $\bar{X}$ chart and the EWMA $\bar{X}$ chart.

Chapter 3 details the derivations of the run length properties of the Shewhart $\bar{X}$ and DS $\bar{X}$ charts with estimated parameters. In Chapter 4, the designs of the Shewhart $\bar{X}$ chart with known and estimated parameters are suggested. Note that this chapter is mainly aimed at facilitating the optimal designs of the DS $\bar{X}$ chart with estimated parameters by minimizing ASS_0, which will be detailed in Chapters 5 and 6.

The ARL-based and MRL-based DS $\bar{X}$ charts with estimated parameters are presented in Chapters 5 and 6, respectively. The comparative studies of these charts for both the cases with known and estimated parameters are discussed. When estimates are used in place of known parameters, the problems for the bewildering measurement of ARL as the sole criterion of the DS $\bar{X}$ chart, is shown numerically and graphically in Chapter 6. Chapter 5 focuses on the two new optimal designs of the ARL-based DS $\bar{X}$ chart with estimated parameters, for minimizing (a) ARL_1 and (b) ASS_0; whereas Chapter 6 focuses on another two new optimal designs of the MRL-based DS $\bar{X}$ chart with estimated parameters, for minimizing (a) MRL_1 and (b) ASS_0. By considering the number of Phase-I samples used in practice, specific optimal-parameter combinations are tabulated in both Chapters 5 and 6. To illustrate a successful application of the DS $\bar{X}$ chart with estimated parameters, both chapters conclude with an example. Finally, the foremost contributions of this research and some recommendations for future research are summarized in Chapter 7.

Numerous programs and optimization programs written in the ScicosLab software, Statistical Analysis Software (SAS) and MATLAB software are provided in Appendices A to G. These computer programs are used to compute the run length properties (ARLs, SDRLs, MRLs, percentiles of the run length distribution and ASSs) and the optimal parameters of the charts under study, i.e. the DS $\bar{X}$ chart, the Shewhart $\bar{X}$ chart and the EWMA $\bar{X}$ chart. Also, additional results for the DS $\bar{X}$ chart with estimated parameters are tabulated in Appendices D to G.

CHAPTER 2
LITERATURE REVIEW

2.1 Introduction

In this chapter, we review the relevant literature on this study. Despite the availability of advanced computer systems, the Shewhart $\bar{X}$ chart is still extensively used in most of today's manufacturing industries (Yang *et al*., 2012). Nevertheless, it is relatively insensitive towards small and moderate process mean shifts. The DS type charts are widely investigated in the literature as alternative methods to improve the Shewhart type charts' performances. The development of the DS type charts is discussed in Section 2.2. In addition, the operation of the DS $\bar{X}$ chart is demonstrated in Section 2.3.

As aforementioned in the problem-statement section of Chapter 1, the statistical properties of the control charts with estimated parameters are receiving growing attention among researchers. The related studies on control charts with estimated parameters are discussed in Section 2.4.

Section 2.5 defines and provides some explanations on the ARL, SDRL, percentiles of the run length distribution, MRL and ASS, which are used as the performance or design criteria of a control chart. In Section 2.6, the run length properties of three univariate control charts with known parameters are provided. These charts include the Shewhart $\bar{X}$ chart, the DS $\bar{X}$ chart and the EWMA $\bar{X}$ chart.

2.2 Development of the Double Sampling (DS) Type Control Charts

Croasdale (1974) first introduced the DS $\bar{X}$ chart, where rejection is prohibited at the first sample. The first sample provides information as to whether the second sample needs to be taken, but the process is then sentenced based on only the information provided by the second sample. By applying the concept of double sampling plans, Daudin (1992) suggested a modified DS $\bar{X}$ chart, where the information from both samples is used to determine the out-of-control decision at the combined-sample stage. This DS procedure incorporates both the ideas of the variable sampling interval (VSI) and variable sample size (VSS). Unlike the VSI procedure, two successive samples are taken in the DS procedure without any intervening time and thus, both the first and second samples are taken from the same population. Both Croasdale (1974) and Daudin (1992) suggested the minimization of the ASS_0. Irianto and Shinozaki (1998) showed that Daudin's procedure is better than that of Croasdale's.

The advantage of the DS $\bar{X}$ chart is that it maintains the simplicity in computing the Shewhart $\bar{X}$ chart's statistics, while improving the effectiveness in process monitoring without increasing the sample size (Torng *et al*., 2009a). Daudin (1992) and Costa (1994) showed that some of the properties of the DS chart are superior to those of the Shewhart, EWMA, CUSUM, VSI and VSS charts. Compared with the Shewhart $\bar{X}$ chart, Daudin (1992) concluded that the DS $\bar{X}$ chart has a notable gain in statistical efficiency, for detecting small and moderate mean shifts. Furthermore, the sample size dramatically decreases to nearly 50% when the process is in-control, via the use of the DS $\bar{X}$ chart. As stated by Daudin (1992), the DS $\bar{X}$ chart outperforms the VSI $\bar{X}$ chart when the time needed to collect and measure the samples can be neglected. Moreover, the DS $\bar{X}$ chart surpasses both the EWMA and CUSUM charts in detecting moderate and large process mean shifts (Daudin, 1992). Although both the EWMA and CUSUM charts are more sensitive to detect small process mean shifts, the control procedures of these two charts are not as simple as that of the DS $\bar{X}$ chart (Torng *et al*., 2010). When the detection of small and moderate mean shifts are the goal of a process monitoring, Costa (1994) pointed out that the sample size of the DS $\bar{X}$ chart is more economical than that of the VSS $\bar{X}$ chart. In addition, when the incoming quality is either excellent or poor, the DS plans have an attractiveness of a lower total sample size and thus, save costs. This is because the lot is either accepted or rejected on the first sample (Ledolter & Burrill, 1999; Gupta & Walker, 2007). Therefore, Torng *et al*. (2009b) stated that the DS scheme is an appropriate choice for process monitoring with destructive testing or higher inspection costs.

Furthermore, the DS chart is a good option when greater efficiency is required for small shifts and protection against large shifts is also vital (He & Grigoryan, 2002). By virtue of the merits and motivation in using the DS scheme, there is a rich literature evolving around the DS type charts. Research works on the DS scheme can be categorized into the DS $\bar{X}$ type, DS S type and other DS type control charts.

2.2.1 DS $\bar{X}$ Type Control Charts

Concerning the DS $\bar{X}$ type control charts, Irianto and Shinozaki (1998) developed a statistical design model to minimize the ARL_1. By employing the optimization model proposed by Daudin (1992), He *et al*. (2002) designed the DS and triple sampling (TS) $\bar{X}$ charts with genetic algorithm. Hsu (2004) commented on the work done by He *et al*. (2002) as the latter only considered the ASS_0. When comparing various charts' performances, Hsu (2004) stated that the ASS for both the in-control and out-of-control situations should be taken into consideration.

Carot *et al*. (2002) presented a combined DS and VSI $\bar{X}$ chart, called the DSVSI $\bar{X}$ chart. Due to the effectiveness of this DSVSI $\bar{X}$ chart, for both small and moderate mean shifts, Lee *et al*. (2012b) studied the economic design of this chart. Costa and Claro (2008) applied the DS $\bar{X}$ control chart to monitor a first-order autoregressive moving average (ARMA (1, 1)) process model. They found that the DS $\bar{X}$ chart is quicker in detecting process mean shifts when the correlation levels within subgroup are small to moderate.

The DS $\bar{X}$ chart and the DSVSI $\bar{X}$ chart under non-normality were studied by Torng and Lee (2009) and Torng *et al*. (2010), respectively. The performances of these two charts were compared with that of the Shewhart $\bar{X}$ chart and the variable parameters (VP) $\bar{X}$ chart. The comparison results revealed that the DS $\bar{X}$ chart's performance is as good as the VP $\bar{X}$ chart and it is more sensitive toward small mean shifts than the Shewhart $\bar{X}$ chart (Torng & Lee, 2009); whereas, the DSVSI $\bar{X}$ chart has the best overall performance for monitoring small mean shifts (Torng *et al*., 2010). By means of genetic algorithm, Torng *et al*. (2009a) and Torng *et al*. (2009b) developed economic design models of the DS $\bar{X}$ chart, for independent and correlated data, respectively.

Irianto and Juliani (2010) proposed a method to estimate the DS $\bar{X}$ chart's limits by optimizing the risks of the producer and customer. With the implementation of this optimization procedure, they claimed that there is a higher capability in producing an out-of-control signal. Using the Markov chain approach, Costa and Machado (2011) compared the performances of the VP $\bar{X}$ and DS $\bar{X}$ charts, in the presence of correlation. The synthetic DS $\bar{X}$ chart suggested by Khoo *et al*. (2011) provides a significant improvement in the detection speed, compared with the synthetic $\bar{X}$, DS $\bar{X}$ and EWMA $\bar{X}$ charts, though the detection of small shifts is better accomplished with the EWMA $\bar{X}$ chart.

2.2.2 DS *S* Type Control Charts

The relevant literature on the DS *S* control charts was studied by He and Grigoryan (2002) for agile manufacturing. Agile manufacturing refers to an operational strategy that enables a quick respond to customer's needs and market changes, while still controlling the quality and costs of the production process. He and Grigoryan (2003) enhanced their earlier work by developing an improved DS *S* chart that does not require the normality assumption of the sample standard deviations. Optimization models to minimize the ASS_0 using genetic algorithm were constructed in both papers. When the aim is to detect small standard deviation shifts, they exhibited that the DS *S* chart and the improved DS *S* chart are more economically preferable to the traditional *S* chart. Hsu (2007) claimed that the conclusion made by He and Grigoryan (2002) is questionable since the out-of-control average sample size $\left(\text{ASS}_1\right)$ is not taken into consideration when comparing the performances of the DS *S* chart with the Shewhart *S* chart. To circumvent this problem, Lee *et al*. (2010) modified the design model of He and Grigoryan (2003) and applied this new suggested model on the destructive testing process. They found that the DS *S* chart has extraordinary performance for detecting the shifts in the process standard deviation and reducing the sample size. To improve the efficiency in the detection of small standard deviation shifts, Lee *et al*. (2012a) extended the idea of Carot *et al*. (2002) to propose the DSVSI *S* chart.

2.2.3 Other DS Type Control Charts

Focusing on other DS type control charts, He and Grigoryan (2006) constructed a joint statistical design of the DS $\bar{X}$ and S charts for simultaneously monitoring the mean and variability. The proposed joint DS chart has a better ARL performance for all ranges of shifts, compared to the joint standard, two-stage sampling and VSS $\bar{X}$ and R charts. In comparison with the combined EWMA and CUSUM schemes, as well as the omnibus EWMA scheme, the proposed scheme outperform these schemes over certain shift ranges. Furthermore, the DS *np* chart for attributes suggested by Rodrigues *et al*. (2011), offers a faster detection of increases in the process fraction nonconforming.

2.3 The Operation of the Double Sampling $\bar{X}$ Chart

Without loss of generality, let us assume that the measurements of a quality characteristic Y taken from a Phase-II process, are independent and follow an identical normal $N\left(\mu_0,\ \sigma_0^2\right)$ distribution, where μ_0 and σ_0^2 are the in-control mean and variance, respectively. The operation of the Daudin's (1992) DS $\bar{X}$ chart can be viewed as a two-stage Shewhart $\bar{X}$ chart, where the second sample will only be observed if the first sample falls within the warning regions of the chart's first-sample stage. Let $L_1 > 0$ and $L \geq L_1$ be the warning and control limits, based on the first sample, respectively; whereas $L_2 > 0$ is the control limit, based on the combined samples. Then the intervals in Figure 2.1 are defined as $I_1 = \left[-L_1,\ L_1\right]$, $I_2 = \left[-L,\ -L_1\right) \cup \left(L_1,\ L\right]$, $I_3 = \left(-\infty,\ -L\right) \cup \left(L,\ +\infty\right)$ and $I_4 = \left[-L_2,\ L_2\right]$. With the aid of the DS $\bar{X}$ chart's graphical view in Figure 2.1, the operation of the chart is explicitly illustrated as follows:

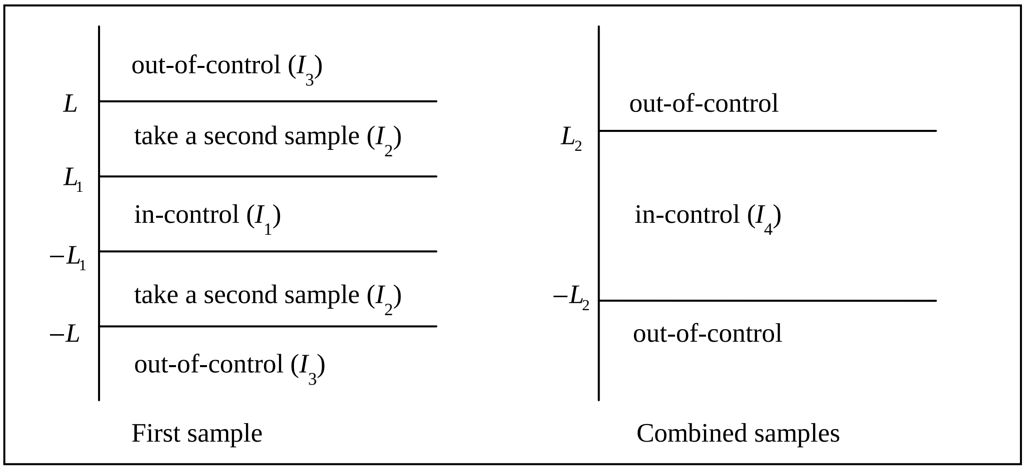

Figure 2.1. Graphical view of the DS $\bar{X}$ chart's operation

Step 1. Determine the limits L, L_1 and L_2.

Step 2. Take a first sample of size n_1 and compute the sample mean $\bar{Y}_{1i} = \frac{1}{n_1}\sum_{j=1}^{n_1} Y_{1i,j}$, where $Y_{1i,j}$, with $j = 1, 2, \ldots, n_1$, are the Phase-II observations at the i^{th} sampling time of the first sample.

Step 3. If $Z_{1i} = \frac{\left(\bar{Y}_{1i} - \mu_0\right)\sqrt{n_1}}{\sigma_0} \in I_1$, the process is regarded as in-control. Then the control flow goes back to Step 2.

Step 4. If $Z_{1i} \in I_3$, the process is deemed as out-of-control. Then the control flow advances to Step 8.

Step 5. If $Z_{1i} \in I_2$, take a second sample of size n_2 from the same population as the first sample. Then calculate the second sample mean $\bar{Y}_{2i} = \frac{1}{n_2}\sum_{j=1}^{n_2} Y_{2i,j}$, where $Y_{2i,j}$, with $j = 1, 2, \ldots, n_2$, are the Phase-II observations at the i^{th} sampling time of the second sample.

Step 6. Compute the combined-sample mean $\bar{Y}_i = \dfrac{n_1\bar{Y}_{1i} + n_2\bar{Y}_{2i}}{n_1 + n_2}$ at the i^{th} sampling time.

Step 7. If $Z_i = \dfrac{\left(\bar{Y}_i - \mu_0\right)\sqrt{n_1 + n_2}}{\sigma_0} \in I_4$, the process is proclaimed as in-control; otherwise, the process is declared as out-of-control and the control flow proceeds to Step 8.

Step 8. Issues an out-of-control signal at the i^{th} sampling time, where corrective actions are taken to investigate and eliminate the assignable cause(s). Then return to Step 2.

2.4 Control Charts with Estimated Parameters

In recent years, a great deal of research interest has arisen in the area of control charts with estimated parameters. A thorough literature review on the impact of parameter estimation for different types of control charts' properties can be found in Jensen *et al*. (2006). Research works on the control charts with estimated parameters can be grouped into the $\bar{X}$ type, individuals X type, dispersion type, EWMA and CUSUM type, as well as attribute type control charts.

2.4.1 $\bar{X}$ Type Control Charts

Many published SPC literature focuses on the $\bar{X}$ type control charts. Among these research works, an influential paper was presented by Quesenberry (1993). From his comprehensive simulation study, he recommended the use of $m \geq \dfrac{400}{n-1}$ Phase-I samples, for the $\bar{X}$ chart with a sample size n, so that the chart performs similarly like the corresponding $\bar{X}$ chart with known parameters. Compared with the known-parameter case, he also summarized that the values of the in-control and out-of-control ARLs and SDRLs are larger for the case of estimated parameters. Furthermore, Quesenberry (1993) observed that a higher in-control ARL $\left(\mathrm{ARL}_0\right)$ for the case of estimated parameters is not an indication of a better performance, because there is an increased number of short run lengths and extremely long run lengths in such a case. Del Castillo (1996a) showed that improved run length performance of the $\bar{X}$ chart can be obtained by using the pooled standard deviation $\left(S_p\right)$ to estimate the control limits, rather than the average sample range $\left(\bar{R}\right)$ and the average standard deviation $\left(\bar{S}\right)$. Del Castillo (1996b) provided a C program to compute the run length distribution and ARL of the $\bar{X}$ chart with estimated process variance.

By investigating the $\bar{X}$ chart based on the estimators $\bar{R}$, $\bar{S}$ and the adjusted version of S_p, Chen (1997) concluded that for small process shifts, the out-of-control performance of the $\bar{X}$ chart is significantly affected by parameter estimation compared to that of large shifts. Also, the effects of parameter estimation are greater on the SDRL than the ARL; hence, increasing the sample size *n* has a favorable effect of getting the SDRL to be closer to the corresponding SDRL of the known-parameter case (Chen, 1997). Chakraborti (2000) evaluated the expressions for the exact run length distribution and the ARL of the Shewhart $\bar{X}$ chart using simulation. He agreed with the conclusions presented by Chen (1997), but he recommended a much larger number of Phase-I samples *m*, i.e. around 500 to 1000, for the Shewhart $\bar{X}$ chart with a sample size $n = 5$.

Wu *et al*. (2002) studied seven robust estimators of σ_0 via simulation, to obtain the control limits for the Shewhart $\bar{X}$ chart. These estimators include the S_p, $\bar{S}$, $\bar{R}$, an estimator based on the absolute deviations from the mean and three estimators based on deviations from the median. For normally distributed data, they found that the ARL performance for all the seven estimators is comparable to one another. By means of the conditional probability method, Yang *et al*. (2006) derived the analytical formulae for calculating the false signal rate (FSR) and then studied the effects of parameter estimation on the FSR for the $\bar{X}$ chart with supplementary rules.

The performance of the $\bar{X}$ chart with estimated parameters is commonly studied from the perspective of the ARL. Since the run length distribution is generally highly skewed, Chakraborti (2007), and Bischak and Trietsch (2007) argued that the ARL is an ambiguous representation of the run length. Chakraborti (2007) suggested the use of the percentiles of the run length distribution, including the median (MRL) and the interquartile range (IQR) to examine the entire run length distribution of the Shewhart $\bar{X}$ chart with estimated parameters. Meanwhile, Bischak and Trietsch (2007) suggested working with the rate of false signals, which focuses on the behavior of the $\bar{X}$ chart with estimated limits during extended use.

In view of the additional variability of the estimates on the process, it is now well accepted that accurate parameter estimation is vital to a Phase-II control chart's performance. Therefore, by taking the number of Phase-I samples *m* and sample size *n* into account, some authors have proposed new or optimal charting parameters for designing the $\bar{X}$ type control charts with estimated parameters. Their works include the Shewhart $\bar{X}$ chart (Nedumaran & Pignatiello, 2001; Chakraborti, 2006), runs rules $\bar{X}$ chart (Zhang & Castagliola, 2010), synthetic $\bar{X}$ chart (Zhang *et al*., 2011), VSI $\bar{X}$ chart (Zhang *et al*., 2012) and VSS $\bar{X}$ chart (Castagliola *et al*., 2012).

2.4.2 Individuals X Type Control Charts

Concerning individuals X control chart, Quesenberry (1993) recommended using about 300 observations to estimate control limits, which will behave like the known limits. When parameters are estimated, the recommended sample size for the individuals X chart is much higher than the number of samples required for the $\bar{X}$ chart. Rigdon *et al*. (1994) suggested using the average moving range $\left(\overline{MR}\right)$, which is a short-term estimate of the process variability, to estimate the standard deviation of a process, as opposed to the long-term estimate of variability, such as the sample standard deviation (S). Similar conclusions to Quesenberry (1993) were obtained by Rigdon *et al*. (1994), but they recommended a smaller sample size, i.e. at least 100 observations are required in a Phase-I process.

Maravelakis *et al*. (2002) examined the individuals charts with estimated parameters for process dispersion. When parameters are estimated, the simulated results reveal that the marginal in-control and out-of-control ARL and SDRL values are higher than that of the known-parameter case. When detecting an increase in variation, they suggested to collect at least 300 Phase-I observations, that is consistent with the sample size recommended by Quesenberry (1993).

Albers and Kallenberg (2004a) considered using exceedance probabilities and the ARL as the performance criteria to evaluate the individuals chart. They suggested applying the corrected control limits to reduce the requirement on the number of Phase-I samples and to obtain a sufficiently small exceedance probability.

Their works were extended by Albers and Kallenberg (2004b), who studied extensively the out-of-control performance of the individuals X chart. When implementing the corrected control chart, Albers and Kallenberg (2004b) claimed that the recommended sample size of at least 300 observations (Quesenberry, 1993; Maravelakis *et al*., 2002) can be reduced to 40 observations. Braun and Park (2008) examined ten σ estimators for the individuals charts, in the presence of non-normal and out-of-control conditions. Via simulation, they showed that after screening, the Boyles' dynamic linear model estimator, which appears to be considerably robust, is often the best way to estimate σ.

2.4.3 Control Charts for Dispersion

Chen (1998) discussed the run length distribution of the *R*, *S* and S^2 charts with estimated σ. In his study, the *R*, *S* and S^2 charts were based on the estimators $\bar{R}$, $\bar{S}$ and S_p, respectively. For all the three charts, he concluded that parameter estimations decrease the in-control ARL (ARL_0) and increase the out-of-control ARL (ARL_1). For all the three charts with the Phase-I sample sizes $4 \le n \le 10$, he recommended using at least 75 Phase-I samples, so that a better performance for detecting changes in the standard deviation is obtained. Similar study and conclusions on the *S* chart were presented by Maravelakis *et al*. (2002). Owing to the fact that they employed the estimator $\bar{S}$ rather than S_p, they suggested a larger number of Phase-I samples, i.e. $m \ge 100$, each having sample size $n \ge 20$ to estimate the process parameters.

Zhang *et al*. (2005) considered an ARL-unbiased S^2 chart and two types of ARL-biased S^2 charts. They ascertained that the sample size required to achieve an adequate performance, is slightly smaller for the ARL-biased charts than that of the ARL-unbiased chart. Castagliola *et al*. (2009) extended the works by Chen (1998) to derive the exact run length distributions of the R, S and S^2 charts with estimated parameters. Crucially, they provided a new design and charting parameters for the S^2 chart with estimated parameters, which allow estimation from a small practical number of Phase-I samples.

Schoonhoven *et al*. (2011) analyzed and designed the standard deviation control chart with estimated parameters when the Phase-I data are contaminated or uncontaminated. Also, they considered 12 different estimators to estimate the in-control Phase-I σ and then derived the Phase-II control limits. By incorporating a simple screening method into an estimation approach, they suggested a robust estimation procedure based on the mean absolute deviation from the median, which has a better performance than the traditional estimators.

2.4.4 EWMA and CUSUM Type Control Charts

There are some recent researches concerning the EWMA type control charts with estimated parameters. For instance, Jones *et al*. (2001) investigated the marginal and conditional run length distributions of the EWMA $\bar{X}$ chart with estimated parameters. Similarly, like the performances of other types of control charts with estimated parameters, they showed that parameter estimation results in substantially more frequent false alarms and a reduction in the sensitivity of the EWMA $\bar{X}$ chart, for detecting process mean shifts. For the EWMA $\bar{X}$ chart with a smoothing constant $\lambda = 0.5$, Jones *et al*. (2001) proposed using 200 Phase-I samples, each having five observations; while for $\lambda = 0.1$ and $\lambda = 0.2$, 400 and 300 Phase-I samples of five observations each, respectively, are required. It is obvious that the

Phase-I sample-size recommendations strongly depend on the value of λ used. The recommended large number of Phase-I sample size, m is impractical in many industrial situations. In this situation, by using smaller m values, Jones (2002) suggested a new design and charting parameters for the EWMA $\bar{X}$ chart with estimated parameters so that the chart has a specific ARL_0 value as that of the known-parameter case.

Zhang and Chen (2002) proved that the standard EWMA $\bar{X}$ chart with known parameters is ARL-unbiased. Then using the results derived from the case of known parameters, they discussed the impact of the estimated process mean on the EWMA chart's ARL performance. They demonstrated that the EWMA $\bar{X}$ chart with estimated variance is ARL-unbiased; whereas, it is ARL-biased when the process mean is estimated. A modified EWMA chart with estimated parameters for monitoring the process standard deviation was proposed by Maravelakis and Castagliola (2009). By using the Markov chain and integral equation approaches, they derived the exact run length distribution of the proposed chart. The main contributions of this paper deal with the optimal design and charting parameters of the EWMA S^2 chart with estimated parameters, specially accounted for the Phase-I m and n.

Capizzi and Masarotto (2010) investigated the effects of parameter estimation on the performance of the combined Shewhart-EWMA (CSEWMA) $\bar{X}$ chart. When parameters are estimated, the comparative studies showed that the performance of the CSEWMA chart is similar to that of the EWMA chart and better than the Shewhart chart. They also gave sample-size recommendations required to achieve the desired level of in-control performance. The properties of the exponential EWMA chart with estimated parameters were investigated by Ozsan *et* *al*. (2010). For large values of λ, as well as small values of *n* and mean shifts δ, they showed that the performance of the chart is significantly worse than that of the known-parameter case. When small δ and $\lambda \leq 0.20$ are considered, $n \geq 200$ observations are needed to improve the marginal out-of-control performance of the chart.

Concerning the CUSUM type control charts with estimated parameters, Bagshaw and Johnson (1975) studied the CUSUM chart's ARL performance when the σ is estimated from $n = 10$ observations. Hawkins and Olwell (1998) quantified the impact of parameter estimation in the case of the individuals CUSUM chart. They noted that a one-sided CUSUM chart is dramatically affected by parameter estimation than a two-sided CUSUM chart. Moreover, a CUSUM chart with a small reference value *k*, which is specially designed for detecting small shifts, is more sensitive to random errors due to parameter estimation than the CUSUM chart with a large *k*. They claimed that 100 Phase-I observations are insufficient to stabilize the ARL performance of the CUSUM chart.

Using a similar approach as shown by Jones *et al*. (2001) for the EWMA chart, Jones *et al*. (2004) studied the run length distribution of the CUSUM $\bar{X}$ chart with estimated parameters. Generally, when parameters are estimated, similar conclusions as shown by the EWMA chart (Jones *et al*., 2001) are obtained for the CUSUM chart (Jones *et al*., 2004). In addition, the run length distribution of the one-sided CUSUM chart with estimated parameters is highly skewed compared with that of the two-sided chart. On the contrary, Castagliola and Maravelakis (2011) derived and discussed the run length properties of a CUSUM S^2 chart with estimated process variance using the Markov chain and integral equation approaches. The CUSUM chart for monitoring the process dispersion is also severely impacted