Universitat Politècnica de Catalunya

Càtedra UNESCO de Sostenibilitat

PhD Thesis

Topological Complexity of the Electricity Transmission Network. Implications in the Sustainability Paradigm

Martí Rosas i Casals

Directors
Dr. Ricard Solé
Dr. Sergi Valverde Castillo
(Complex Systems Laboratory, UPF)

Supervisor
Dr. Ricard Bosch i Tous
(Departament d'Enginyeria Elèctrica, UPC)

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Agraïments

És tardor de 1995. Un llibre platejat crida la meva atenció sobre d'un prestatge. Porta un títol suggerent: "*Complejidad. El caos como generador del orden*". L'autor, un tal Roger Lewin, és per mi un total desconegut. Diuen que un llibre, com un viatge, es comença amb inquietud i s'acaba amb melangia. Aquest el començo amb inquietud i l'acabo en menys de 24 hores. No hi ha lloc per la melangia. Sóc ple d'excitació i nerviosisme, plenament intoxicat per tot allò que s'hi tracta i del que en desconeixia l'existència: el límit del caos, explosions i extincions, biologia i ordinadors, el vel de la consciencia i la realitat del progrés. De les seves seductores pàgines en voleien els noms exòtics de llocs i personatges sorprenents: el Canyó del Chaco, Stuart Kauffman, Per Bak, James Lovelock, Chris Langton, Jack Cowan, Stephen Jay Gould, etc. I voleiant encara més amunt, per sobre de tots ells, un misteriós institut amb un nom força sacre: *Santa Fe Institute*.

És hivern de 2001. Curso els últims crèdits de doctorat d'un programa aliè al meu. L'assignatura porta l'atractiu nom de "Fenòmens crítics, caos i complexitat". Soc a l'aula el primer dia de curs. La porta s'obre. Entra, amb pas decidit i papers i llibres sota el braç, qui sembla que s'encarregarà de la matèria. Sense reserves haig de dir: qui és a punt de canviar la meva vida. Botes de pell, texans, camiseta negra i gorra de beisbol. Acaba el curs. Reconec clarament per primera vegada a què vull dedicar la meva vida a partir d'ara. Les primeres de moltes gràcies. Gràcies, Ricard S., per fer-ho possible.

La tornada a casa és llarga i tortuosa. Camino capcot i entro a l'estomac del metro de nou, pensant amb aquella part que no ha funcionat. Les estacions passen i es repeteixen. Totes elles. Una i altra vegada. Sense descans. Amago la cara entre les pàgines del llibre que es recolza sobre les meves mans mentre la ment delma per un mínim descans. Sóc, de nou, amb aquella part del meu món que em contempla amb comprensió i que ha ofert sempre la seva ajuda sense compromís. Gràcies, Ricard B., per creure en els nous camins, lluny de tot allò establert. Gràcies, Josep, pels teus cops de mà al llarg d'aquesta aventura. Gracies, Lluís, per fer-te càrrec i responsable d'allò on jo ja no arribava. I sobretot gràcies, Jaume, company, amic i confident de tantes qüestions vitals, per fer-me sempre costat. Espero que a partir d'ara pugui estar jo una mica més al vostre costat.

És tardor de 2009. Miro el sol reflectint-se sobre la mar Mediterrània tot pensant si els esdeveniments dels últims anys han estat realitat o tot just un somni. Moments. Espais. Finestres plenes de dibuixos, lletres i símbols. Dinars al voltant d'una taula baixa. Ordinadors i llistats de programació. Gràcies, Javier, per tots els teus consells i paraules. Cerveses i converses al voltant d'un cel molt obert. Gràcies Carlos, Josep i Bernat pels moments de gresca, reflexió i ciència sense cita prèvia. Hores i més hores entre llibres de segona mà i gratacels impossibles. Gats dormilegues carregats d'electricitat estàtica. Guerres d'estrelles de matinada, en una sala de reunions il·luminada per un estol d'espelmes. Escriptors solitaris passats a decoradors. Passadissos enmig del desert, plens de simfonies y contrapunts. Músics que són pintors. Capvespres màgics, pobles "bonitos" i roderes sobre la grava. Gràcies, Andreea, Sergi i Ricard S. de nou, per deixar-m'ho compartir amb vosaltres. Ha estat, i és, molt gran tot plegat.

El dia s'aixeca tímid i ple de boira. Els arbres regalimen rosada al ritme de l'estrèpit somort d'algú que talla fusta. El gall canta de bon matí. En Txomin s'arrauleix sota el cobert. Na Garbiñe estira les cames després de passejar amb en Platón. Na Natàlia seu a la cuina amb l'esmorzar davant, esperant un cafè que costa d'arribar. En Víctor llegeix el diari al menjador i pren notes en una agenda de tapes verdes. I na Helga neda entre coberts, estovalles i vapors, entre roba seca i roba mullada. De fet, ningú sap on és. Però ho reconec tot plegat com a llar. Moltes gràcies a tots per estar sempre al meu costat i per mantenir intacte aquest paradís particular, amagat entre cims boscosos i humits. Heu fet que la llunyania es transformés sempre en proximitat.

Els veig passejant pel parc. S'agafen de la mà i pugen per la costa. Es paren a parlar amb algú que els somriu i els saluda. D'un n'agafo la introspecció i la mecànica. De l'altra un cert estoïcisme i l'afició al setè art. D'ambdós la paciència, la capacitat de treball, la comprensió i l'amor incondicional pels que t'envolten. Gràcies pares, Paquita i Antonio, per deixar-me ser com sóc. Per respectar i recolzar totes les meves decisions. I per creure sempre amb mi, malgrat en certs moments posés impediments per tal que fos així.

Observo ara un sol roig eixint per l'horitzó d'una terra cremada. Penso en com de llarga serà avui la jornada per a ella. En tot allò que representa i significa a la meva vida. I que encara que potser a ella no li ho sembli, ho és tot. Ella és la petita ballarina a qui demano que no marxi lluny. A qui demano que m'ajudi a comptar els fanals de la carretera i que m'acotxi entre llençols de lli a la fi d'un dia esgotador. Gràcies, Selina, per fer-me conèixer els colors que hi ha entre el blanc i el negre. Gràcies, Selina, per trobar-me.

Totana, juny de 2009
Terrassa, setembre de 2009

The day when we shall know exactly what "electricity" is, will chronicle an event probably greater, more important than any other recorded in the history of the human race. The time will come when the comfort, the very existence, perhaps, of man will depend upon that wonderful agent.

Nikola Tesla[1] (1856 – 1943)

There is no doubt that the affluence [of wealth] recently acquired by the technological societies [...] has not brought about any comparable growth of human mental capacity to comprehend their over-all complexities.

Jagjit Singh[2] (1912 – 2002)

We live in these electric scabs.
These lesions once were lakes.
No one knows how to shoulder the blame.
Or learn from past mistakes.

Joni Mitchell (Bad Dreams)

[1] In Marc Seifer's *Wizard. The Life and Times of Nikola Tesla*, Citadel Press, New York, 1998.
[2] *Great Ideas in Information Theory, Language and Cybernetics*, Dover Publications Inc., New York, 1966.

Contents

Abstract

This PhD Thesis explores the structure, dynamics and evolution of the electricity transmission network from a complex systems perspective, its main objective being the definition of new criteria and tools to help designing a more efficient and sustainable transmission power grid. In doing so, two data sets have been explored and analyzed. On one hand, the Union for the Coordination of Transport of Electricity (UCTE) network, which associates most of the continental Europe national power grid operators in order to coordinate the production and demand of some annual 2 300 TWh of energy and 450 million customers from 24 countries. On the other hand, the *Gestionaire du Réseau du Transport d'Electricité* (RTE) transport network historical evolution, responsible for operating, maintaining and developing the biggest national network in Europe, the French electricity transmission network.

The results obtained so far show statistically significant dissimilarities in the structure of the power grids, clearly defining and enclosing particular dynamic behaviours that enable us to segregate European networks in two sets, namely fragile and robust. Fragile networks are characterized by meshed topologies and non random structures while robust ones share more randomly generated topologies. The consequences of these findings for the sustainability of infrastructure networks are significant in terms of cost and risk assessment. A topological model for the evolution of a power grid network is also presented. We suggest that global topological fragility increases when local connectivity schemes are adapted in order to increase local reliability. These outcomes appeal for new power grid design methods and tools capable to include these new topological aspects into efficiency and reliability assessments.

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Introduction. Power grids, complex systems and sustainability

We almost certainly agree that the harnessing of electricity can be considered the most important technological advance of the twentieth century. The electric power system has shaped the modern world from its very beginning. Electric generators and motors connected by an intricate network of power lines order the rhythm and pulse of the society we live in. Electricity allows innovation, to envision better futures and to enhance standards of living. In fact, following the opinion of Thomas P. Hughes[1], power systems can be considered cultural artifacts since all "Technological affairs contain a rich texture of technical matters, scientific laws, economic principles, political forces, and social concerns" (Hughes 1983). From this point of view, its presence is nowadays so intertwined with ours, and so much taken for granted, that we are only capable of sensing its absence.

The absence of electric supply is nowadays considered an extreme inconvenience. Chaos and despair are usually its inevitable consequences since in the almost endless list of power outages throughout history[2], there is not a single event that has not been characterized by some amount of looting, rioting and human and economic losses. It is therefore a priority for every developed nation to secure the electric chain, from production to consumption, in the most economic manner but, at the same time, with the most reliable technology available (Amin 2001). The task is one of enormous importance considering the expected expansion of highly liberalized markets, the many environmental constraints and the increasing social reluctance to new lines sitting and forecasting. Having this context in mind, some questions inevitably arise:

- Why do power networks still fail after years of continuing improvements? How is it possible that such a perfected engineering system collapses with not a warning sign?
- Which are, and had been during time, the influences between those different forces that shape the making of the grid?
- In a world of increasing population and energy consumption, can we define a pattern for the evolution of a future power grid without the actual environmental impact but with improved reliability indexes? Is it economically feasible or even possible to keep such a complex and vital system in a faultless state?

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[1] Thomas Parke Hughes (1929) is an American historian of technology, well known to introduce systems theory into the history of technology.

[2] http://en.wikipedia.org/wiki/List_of_power_outages. (Last visited, January 2009).

There is no single or easy answer to these questions. Power grids are now systems much more complex than those envisioned by pioneers such Thomas A. Edison and Nikola Tesla, or entrepreneurs such as Samuel Insull and George Westinghouse (Klein 2008). Motors, generators, substations and transformers are now connected by cable lines that span thousands of kilometers, crossing many countries, to serve energy and power in a globalized, networked world and economy. Extreme phenomena such as blackouts and cascading failures seem to appear more frequently in the last years and it seems the old reliability network criteria are not anymore an assurance for the safety of the system as a whole. New disturbances and energy injections due to renewable energies are transmitted through old frontiers and integrating this new energy system or the one that has to appear in the present one is not an easy task. Maybe it is time for new tools and conceptual frames to allow a different approach. The main objective in this PhD Thesis work is precisely to study the power grid from a different point of view, which is as a complex system, particularly as a complex network.

1.1 Power grid

According to the US National Academy of Engineers, the interconnected network for delivering electricity from suppliers to consumers that we call power grid can be considered the twentieth century's most beneficial innovation to our civilization (Constable and Somerville 2003). From a broader historical perspective, networks of energy, transportation and communication have constituted the very foundation of all kinds of societies. The study of these technological systems deserves attention in order to assure, essentially, structural integrity, efficiency and reliability of supply.

The term grid is used for an electricity network which may support all or some of three distinct operations: electricity generation, electric power transmission and electricity distribution[3]. It may be used to refer to an entire continent's electrical network, a regional transmission network or may be used to describe a local utility's transmission or distribution grid. Electricity might be provided by a simple distribution grid linking a central generator to homes, though the traditional paradigm for moving electricity around in developed countries is much more complex (Fig. 1.1.1). Generating plants are usually located near a source of power and away from heavily populated areas. Power generation can range from 200 MW in hydroelectric plants up until 1.500 MW for some nuclear power facilities. The generated electricity is stepped up to a higher voltage suited to connect the plant to the transmission network. The transmission network operates usually with voltages higher than 110 kV and with an upper limit that depends on national and continental constraints, but usually below 800 kV (Fig. 1.1.2, left). Most European grids operate at a maximum voltage of 400 kV. This bulk power transport network can cross national boundaries until it reaches its wholesale customer (usually the company that owns the local distribution network.) Upon arrival at the substation, the power will be stepped down from the transmission level voltage

[3] http://en.wikipedia.org/wiki/Grid_(electricity). (Last visited, January 2009).

to the distribution level voltage, which is lower that 110 kV. As it exits the substation it enters the distribution wiring. Finally, upon arrival at the service location, the power is stepped down again from the distribution voltage to the required service voltages.

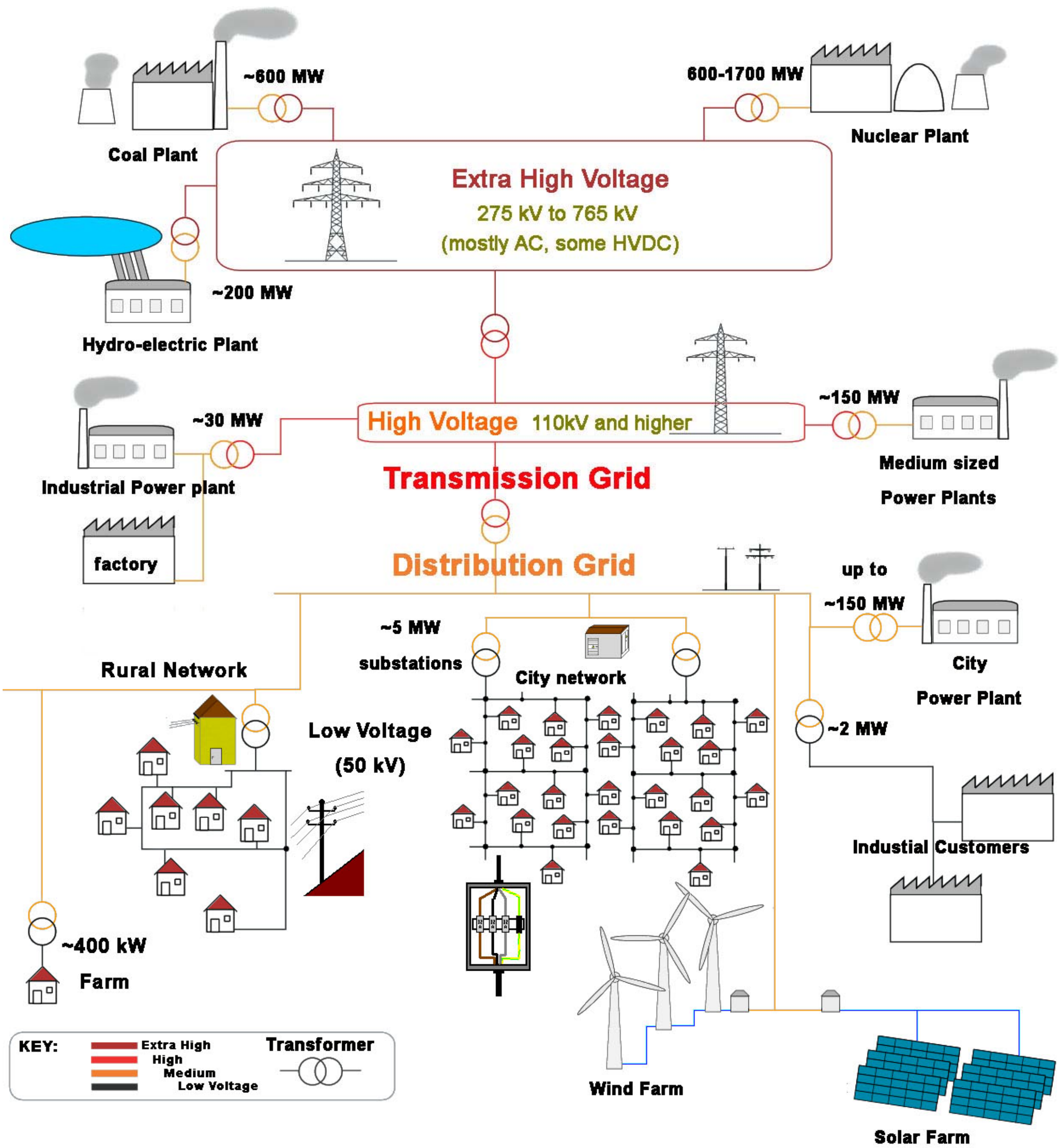

Fig. 1.1.1 Electricity network general layout. The transmission grid is usually defined for 110 kV and higher. Voltages and depictions of electrical lines are typical for European systems. (Image source: http://en.wikipedia.org/wiki/File:Electricity_grid_schema-_lang-en.jpg; last visited December 2008).

The topology of a distribution grid can vary widely, depending on the constraints of budget, requirements for system reliability, and the load and generation characteristics. Due to cost constraints, though, it usually uses a more radial and meshed structure (Fig. 1.1.2, right). This is usually a tree-shaped grid where power from a large supply substation radiates out into progressively lower voltage lines until the destination homes and businesses are reached. Though any power grid requires some level of redundancy in order to be reliable and secure, the expensive cost of more meshed topologies restricts their application to transmission grids. As a consequence, distribution grids tend to be meshed. This secures electric supply in case of a line failure: the power can be simply rerouted while the damaged line is repaired.

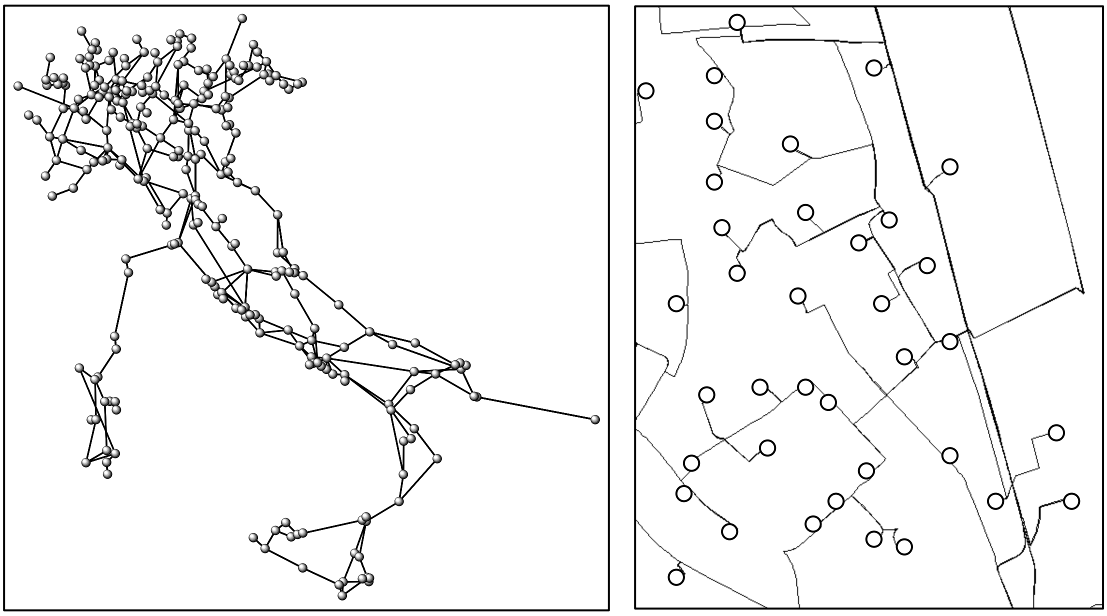

Fig. 1.1.2 The particular objectives followed in transmission and distribution grids force different structural patterns and layouts. Left: Italian (220 – 400 kV) transmission power grid, where each node is a substation or transformer. Right: Andorran local distribution grid topology (< 60 kV) where each node is a distribution feeder (also transformer) that sets the final voltage at domestic or industrial level.

The new trends in deregulation, markets, generation and demand equilibrium, and reliability pose the structure of the transportation grid at a stack. More power is needed and it has to be transported far away distances in lesser time. The traditional centralized model along with its distinctions is breaking down with the introduction of new technologies and renewable energy. In fact, the characteristics of power generation can in some new grids be entirely opposite of those named above. Generation can occur at low levels in dispersed locations, in highly populated areas, and not outside the distribution grids. Therefore, the structure of the actual grid seems to be not well suited to this new panorama. The electricity fluxes of the system are sensible to these processes and much more failures and instabilities are expected to appear. Although early work on distribution and communication networks already addressed some of these problems (Baran 1964), only recently an appropriate framework has been developed to face them from an alternative perspective.

1.1.1 Dynamics

In a stable functioning regime, power grid behaves dynamically as a great pool of sources and sinks where consumption continuously meet demand (Wood and Wollenberg 2005). The grid is usually forced to operate at a minimum cost. This means a delicate equilibrium between optimal power flow and problematic concerns such as economic dispatch, control of interconnected systems and phase synchronicity. Being an engineering formidable problem as it is, we must add, in recent years, several other problems. The grid is actually managed by different kinds of actors (often with different objectives) and increasingly adding huge quantities of heterogeneous components (spatially distributed and connected). It is, therefore, more difficult to ensure the security of the system as the interaction of its multiple components became more and more complex. Preventing its collapse due to unforeseen conditions is almost impossible.

The fact that the temporal behavior of the power grid is now much more intricate than it was fifty years ago can be easily perceived. Increasing frequency and size of malfunctions have raised general and media awareness about our real level of comprehension of the dynamics of these networks (Venkatasubramanian 2003; UCTE 2004; UCTE 2007). In recent years, both the North American and the (once almost faultless) European grids have experienced numerous examples of such malfunctions in the form of cascading failures and blackouts (Fig. 1.1.3). The explanations given by local, national and international electricity coordinating councils for most of these situations go from aspects related to low investment and maintenance, to those related to generation and demand inadequateness and, obviously, bad luck. But more than any, the most repeated explanation is that of a bad comprehension of the interdependencies present in the network. (Watts 2003; UCTE 2004)

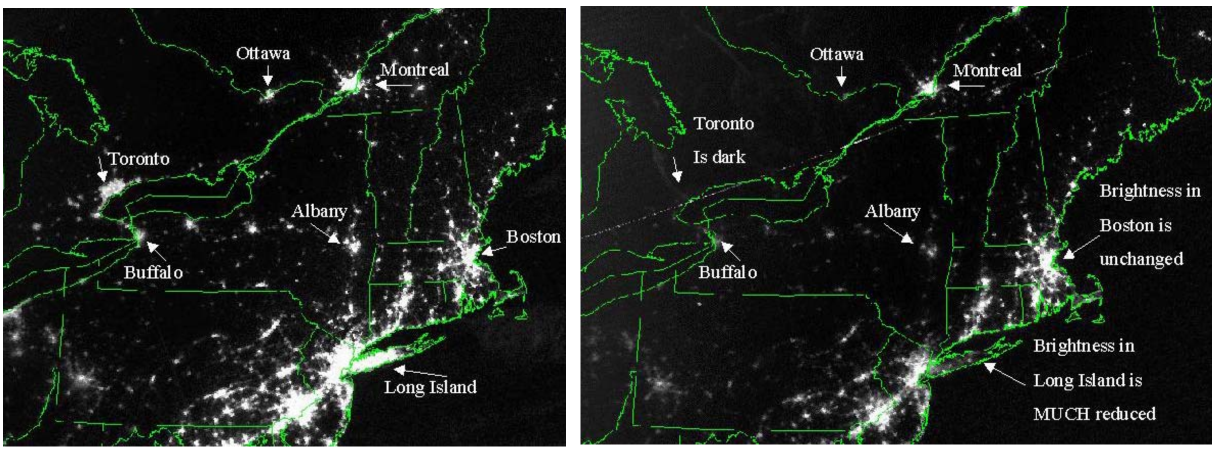

Fig. 1.1.3 Blackouts are result of unexpected power grid behaviours. Satellite images taken before (left) and after (right) of the 2003 historic blackout of the United States northeast, which left millions of people into darkness. (Image source: http://www.noaanews.noaa.gov/stories/s2015.htm ; last visited December 2008).

In our increasing electrified world transmission-operation entities required to provide grid access to many parties (both utility and non utility organizations) have a tremendous challenge: the task of developing operating schedules in some (as yet to be defined) "optimal" fashion within the physical constraints of the system, including those more often unexpected behaviors, while maintaining its reliability and security. But the global power grid is now a whole one, connected, complex and, in some sense, unknown.

The clear and perfectly engineered one-line diagrams presented in academic books and engineering faculties are now much more similar to a neural network, where relations between connections and functions, structure and dynamics, are less clearly defined. It is therefore necessary to adopt a different approach to study and deal with the intricacies of these systems. Advances in statistical physics, modeling and computational methods have stimulated the interest of the scientific community to study electric power grids as complex networks, one particularly successful way of studying complex systems. (Dorogovtsev and Mendes 2001)

1.2 Complex systems

There is no general accepted formal definition of *complex system* or even for system.[4] Very broadly, a *complex system*[5] can be defined as a large group of relatively simple components with no central control and where organization and *emergent* non trivial behaviour are exhibited (Kauffman 1995; Érdi 2008). With very few exceptions there is still a lack of effort for an epistemological reflection on its foundations, principles and limits. The epistemology of concepts such as "simple component" or "emergent non trivial behaviors" is at the core of the complexity's more formal debate and it is not the aim of this work to add more elements to its discussion. The former definition will essentially depend on the context (Morin 2006). For example, in brain and cognition sciences, the neuron can be seen as the simple component, while speech and thought are emergent behaviors (i.e., global characteristics not explained by the individual behavior of the components). On the other hand, social sciences consider the self as the simple component, while globally organized phenomena such as economy or wars can be seen as emergent behaviors. (Ball 2004; Buchanan 2007)

Over the last several years, and in spite of the usual controversy arising in the usage of new words and definitions, complex science, or *complexity* for short, has changed the way scientists approach all fields of life, from biology to medicine, from economics to engineering (Waldrop 1992; Lewin 1995; Solé and Manrubia 1996; Solé and Goodwin 2001; Érdi 2008). Words and concepts such as *self-organization*, *genetic algorithm*, *cellular automata*, *criticality*, *artificial life* or *chaos theory* are now widely accepted and used as new means of understanding the always changing reality. The history of these concepts in particular and *complex systems* research in general begins in the 1950's, with (a) the advent of von Bertalanffy's (and some others) *systems theory*, (b) the appearance of nonlinear phenomena in

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[4] Ludwig von Bertalanffy needed a whole book to define it: Bertalanffy, L.v. (1968), General Systems Theory, New York, Braziller.

[5] Words in italics are defined in the glossary (Appendix B).

scientific fields away from physics, like chemistry and biology, and (c) the study of *feedback* and derived concepts such as communication and control in living organisms, machines and organizations, also known as *cybernetics* (Fig. 1.2.1). From these early stages comes the idea of *threshold*, that is the fact that complication as well as organization below a certain minimum level is degenerative but beyond that level can become self-supporting and even increasing (Singh 1966; Kauffman 1995), a concept that turned up to be the cornerstone of much of the *complexity* science developments of the 1980's, especially in the *cellular automata* and *artificial life* fields, where complex behavior (associated to some specially complex patterns, neither random nor regular) seemed to appear suddenly, when a certain *threshold* in some control parameter was reached (Wolfram 2002). Though words like *self-organization*, *connectionism*, *adaptive system* or *complexity* itself, had already been used in the 1940s, it is not until the 1980's that the definitive impulse is achieved. From then on many books, journals, conferences, and even whole institutes devoted to the field have flourished everywhere, and even computer modeling of complex systems has become widely accepted as a valid scientific activity.

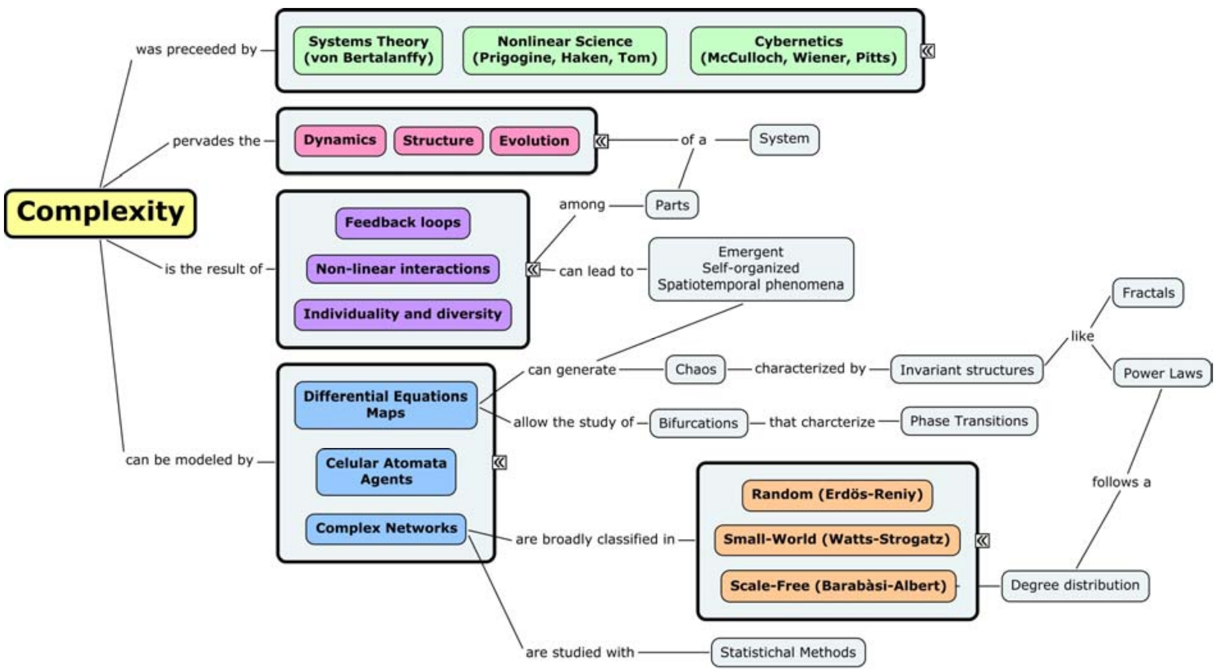

Fig. 1.2.1 Complexity is a difficult word to define. The conceptual map shown in this figure is a first attempt to highlight the various aspects involved in its characterization.

In this conceptual framework *complexity* pervades both the (a) structure (i.e., formal arrangement of the constituent parts), (b) dynamics (i.e., functional behavior) and (c) evolution (i.e., the way it has reached its actual formal and functional state) of any system. With the presence of *feedback* loops, nonlinear interactions and some level of heterogeneity in its composing elements, *complexity* is usually translated into some kind of *self-organization*, a concept emerged and used since the 1950's by mathematicians, physics, engineers, cyberneticians and neurologists, with a broad meaning of global organization, spatial and/or temporal, without any central control and related to concepts such as *criticality*, *phase transitions* and invariant *universalities*.

Though a consensus on the formal meaning of *complexity* has not been reached, there are, nonetheless, some widely accepted means, coming from well established scientific fields, to tackle some of the problems posed by complex systems, at least up to some level.[6] Historically, maybe the first one was the field of dynamics, particularly nonlinear dynamics. The flexibility offered by differential equations (or maps, in its discrete form) to explain the many behaviors that dynamic systems can exhibit, led eventually to the development of the theory of *deterministic chaos* (Strogatz 1994), a particularly well understood piece of the *complexity* puzzle. With the advent of computational capacity, simulations and models of real life phenomena arose. Particularly fruitful were *cellular automata*, which illustrate very well how complex behavior can suddenly arise once a particular threshold in the control parameters values is crossed. Their evolved cousins, *agent-based models*, have actually taken the lead (Miller and Page 2007). The rationale behind these two new both theoretical and computational tools is the coverage of a basic need a differential equation can not give, that is to account for the heterogeneity in the system's composing elements.

One way to analyze complex systems' behaviors is by determining the relations and dynamics between its inner components. Though differential equations, *cellular automata* and agents are particularly successful in addressing the physical properties of systems that are composed of many identical or non identical elements interacting through mainly local interactions, they face many difficulties when systems are composed of many non-identical elements that have diverse and multi-level interactions, local and non-local. Over the last ten years, and mainly due to advances in computational capacity and databases accessibility, modeling and computational methods have stimulated the interest of the scientific community to analyze *complex systems* as networks.

1.2.1 Complex networks

The mathematical study of networks arose from *graph theory*, which began as early as the eighteenth century with Euler's solution to the famous "Bridges of Königsberg" problem. Some influential work on the theory of random graphs was done in the 1950s but it is in the last ten years when the concept of complex network has been widely used and accepted. (Barabási 2002; Watts 2003; Solé 2009)

In its broadest sense, a network is a formal and functional representation of a *complex system*, where vertices are the elements of the system and edges represent the interactions between them. For example, living cells are supported by large molecular genetic networks, whose vertices are proteins, and edges represent the chemical interactions between them. Similarly, complex networks occur in social sciences, where vertices are individuals, organizations or countries and the edges characterize the social, economical or cultural

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[6] Up to the level that philosopher Edgar Morin has called *restricted complexity*. (Morin, op.cit.)

interactions between them (Wasserman and Faust 1994). Examples from nowadays informational society would be the internet, a network evolved from the 1970's whose vertices are computers and servers physically connected by cables (Fig. 1.2.2) or the world wide web, whose vertices are HTML documents connected by links pointing from one page to another. (Barabási *et al.* 2000; Pastor Satorras and Vespignani 2004)

Graph theory helps to unravel some of the networks' intricacies once the structure of a network is set and minimally understood. But networks are inherently difficult to understand mainly, and precisely, due to their structural complexity (Strogatz 2001). Connectivity (i.e. the elements' interaction map) can be sometimes very hard to find, due to both edge and node diversity. Besides, it usually varies in time: new nodes and links are born, old ones eventually die, and so do networks as a whole. These characteristics add dynamical complexity and favor the emergence of meta-complications where these various phenomena influence each other in unexpected ways. Due to these complications, the study of complex networks is still in its very beginnings and there are a lot of questions that network scientists are still trying to address, like for example:

- What structural and topological measures can be used to characterize the many properties of a network?
- What characteristics do different sets of real-world networks share, and why? How did these characteristics emerge? How have they evolved in time and why?
- How do these properties affect the dynamics of information (or disease, or other communications) spreading on such networks?
- Which level of resilience do networks have when some of their composing elements fail, be it by random choice or premeditated attack?

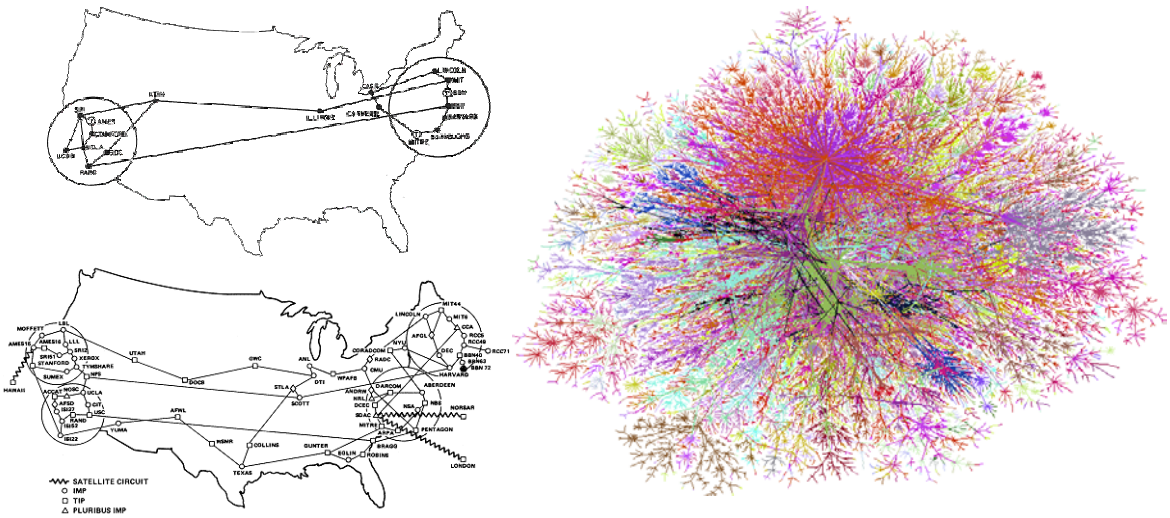

Fig. 1.2.2 A complex system viewed as a network. Left: ARPANET geographical maps in 1971 (above) and 1980 (below), the backbone of the actual internet. Right: part of the actual internet, retrieved from the Internet Mapping Project. (Source: http://www.visualcomplexity.com; last visited December 2008).

In answering these questions, scientists have developed models of dynamics, structure and evolution of real networks. Although these models can explain some of the most important phenomena observed in networks, there is still a long way to go in understanding the many networked systems around us. Three general models of networks have been intensely studied and fairly well developed so far: (a) *random*, (b) *small-world* and (c) *scale-free*, each characterized by the way in which networks are generated and by several statistical metrics.

(a) Random networks

Probably the oldest and most investigated model of a network is the binomial random graph or shortly *random graph* (Bollobás 2001). The model starts with a set of N nodes (or vertices) and no bonds. With connection probability p each pair of vertices is connected with a line (bond, edge or link), generating a random network (Fig. 1.2.3). The greatest discovery of this model was that many properties of these graphs like the appearance of trees[7], cycles[8] or a giant component[9], arise quite suddenly at a threshold value.

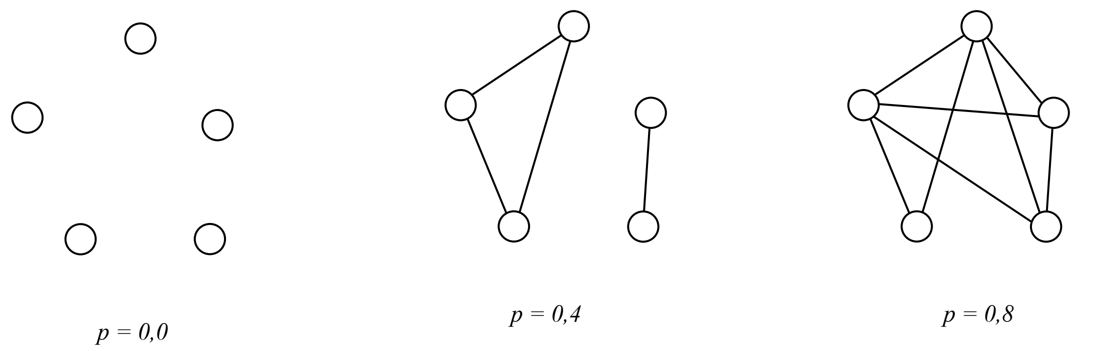

Fig. 1.2.3 Random graph model. Three different probabilities of connection p give rise to three different graphs.

Among the resulting statistics one of the most important is the *degree distribution*. In the study of graphs and networks, the *degree* k of a node in a network is the number of connections it has to other nodes. Therefore, the *degree distribution* $p(k)$ is the probability distribution of these degrees over the whole network. In a random graph with connection probability p the degree distribution can be approximated to a binomial degree distribution

[7] A connected graph without a cycle is a tree. A tree has the same number of links than nodes plus one. If a link is removed, the graph ceases to be connected. If a new link between two nodes is provided, a cycle is created.

[8] A chain where the initial and terminal node is the same and that does not use the same link more than once.

[9] A giant component is a connected subgraph that contains the majority of graph's nodes.

$$p(k) = \binom{N-1}{k} p^k (1-p)^{N-1-k} \tag{1.2.1}$$

which for large N can be replaced by a Poisson distribution like

$$p(k) \cong e^{(-pN)} \frac{(pN)^k}{k!} = e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!} \tag{1.2.2}$$

where $\langle k \rangle$ is the *mean degree* of the network. For an undirected graph with a number L of links, it can be written as

$$\langle k \rangle = \frac{2L}{N} \tag{1.2.3}$$

Thus, despite the fact that the position of the edges is random, a typical random graph is rather homogeneous, the majority of the nodes having the same number of edges $\langle k \rangle$.

(b) Small-world networks

One question arises immediately when the former model is used to explain real world networks and this is whether real world networks are really random or, on the contrary, they show not so random distributions and properties. In 1998, a one parameter model which interpolates between an ordered finite dimensional lattice and a random graph was introduced in order to explain the features of three real world networks taken from three different fields: biology (the neural network of the worm *C. elegans*,), technology (the power grid of the western United States) and society (the collaboration graph of film actors) (Watts and Strogatz 1998). These networks present peculiar features that make them stay away from the random model. These are related to two statistical properties known as the *average path length* and the *clustering coefficient*. The *average path length* ℓ is defined as the number of edges in the shortest path between two vertices, averaged over all pairs of vertices. The *clustering coefficient* C is defined as follows. Suppose that a vertex v has k_v neighbors; then at most $k_v(k_v-1)/2$ edges can exist between them (this occurs when every neighbor of v is connected to every other neighbor of v). Let C_v denote the fraction of these allowable edges that actually exist. C is then defined as the average of C_v over all v.[10] For the three real world networks under study, strong deviations from random ones were observed, especially as far as the clustering coefficient was concerned (Table 1.2.1): while ℓ is about the same order of magnitude for both networks (i.e., real and randomly generated one with the same N and $\langle k \rangle$), C differs in several orders of magnitude.

[10] For friendship networks, these statistics have intuitive meanings: ℓ is the average number of friendships in the shortest chain connecting two people; C_v reflects the extent to which friends of v are also friends of each other; and thus C measures the cliquishness of a typical friendship circle.

Network	ℓ_{actual}	ℓ_{random}	C_{actual}	C_{random}
Film actors	3,65	2,99	0,79	0,00027
Power grid	18,7	12,4	0,080	0,005
C. elegans	2,65	2,25	0,28	0,05

Table 1.2.1 Average path length ℓ and clustering coefficient C for three real networks, compared to random graphs with the same number of vertices N and mean degree $\langle k \rangle$. (Watts and Strogatz 1998)

Considering that a high clustering is much more characteristic of an ordered lattice rather than a random graph, the aim of the model was to find if there is a middle ground between these two extreme ordered and random states that could explain such behavior. The algorithm behind the model starts with a ring lattice with N nodes in which every node is connected to its first k neighbors, thus showing a large clustering and average path length (Fig. 1.2.4). It goes on with a randomly rewiring of each edge of the lattice with probability p such that self-connections and duplicate edges are excluded. For some values of p the rewiring process generates a number of shortcuts in the graph such that while ℓ has been strongly diminished, C has still values close to those of the lattice graph. As this model has its roots in social systems where most people are friends with their immediate neighbors and have one or two friends who are a long distance away, when this middle ground between order and randomness is reached, the graph is called a *small-world*. (Watts 1999)

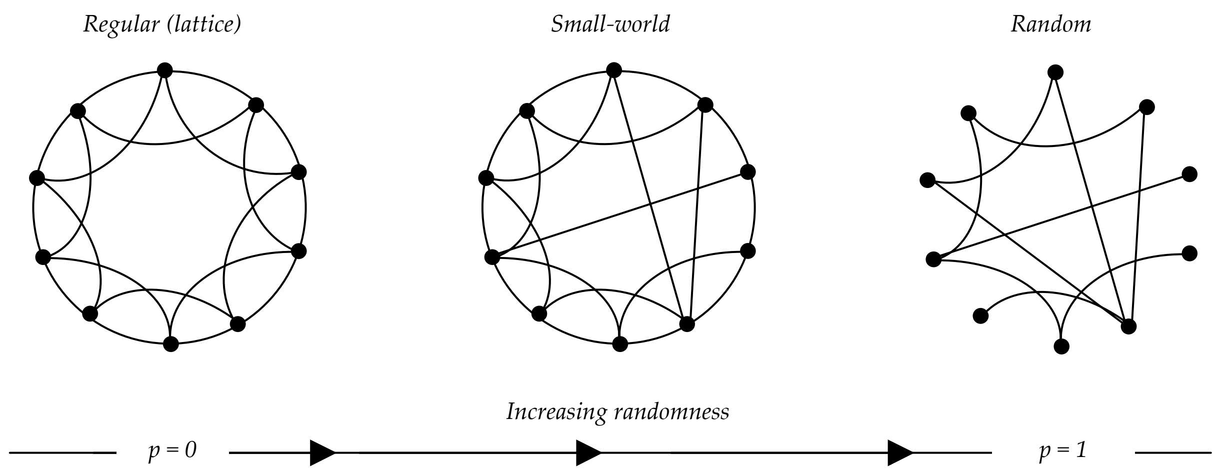

Fig. 1.2.4 Random rewiring procedure for interpolating between a regular ring lattice and a random network without altering the number of vertices or edges in the graph: with probability p an edge is reconnected to a vertex chosen uniformly at random over the entire ring. (Watts and Strogatz 1998)

Rather unexpectedly, at least from a first sight, though small-worlds stand between ordered lattices and random graphs, their degree distribution is mathematically explained by the same binomial distribution used for the random graph model and Eq. (1.2.2). The only difference lays on the variance: it does not exist for an ideal lattice while it has a typical value for a *random graph*. It happens that the authors of the classical small-world paper never thought to look for the *degree distribution* of the real networks in Table 1.2.1. If they had checked it, they would have noticed that their networks' distributions were far from being binomial, Poisson or even Gaussian.

(c) Scale-free networks

In fact, not just the former three networks but many networks in the real world have degree distributions that do not look anything like a Poisson distribution. Instead, they are very heterogeneous, in many cases following a *power law*, which was first discovered for the world wide network of web pages (Barabási and Albert 1999). A *power law* does not have a peak at its average value. Rather, it starts at its maximum value and then decreases relentlessly, with a characteristic exponent γ, all the way to infinity following[11]

$$p(k) \approx k^{-\gamma} \tag{1.2.4}$$

This is why they are also called *scale-free networks*, as there is no characteristic scale to define their degree distribution properly (i.e., although $\langle k \rangle$ exists, the variance is infinite). On the other hand the rate at which the *power law* decays is much lower than the decay rate for a normal distribution, implying a much greater likelihood of extreme and rare events characterized by nodes with extremely high k (Fig. 1.2.5). These nodes, called *hubs*, play the most important role in the network's connectivity. With their presence the network can usually maintain its functions, even though most of the not-so connected nodes fail. But on the contrary, when hubs disappear, most of the network's capacities in transmitting information and functions fail as well, and its structure can be easily broken down. (Albert *et al.* 2000)

The *scale-free model* tries to explain the origin of the *power law degree distribution* by means of two generic mechanisms common in many real networks: growth and *preferential attachment*. The model starts with a small number of nodes and at every time step a new node is added and connected with some probability. The probability that this new node is connected to an existing one depends on the degree of the latter: the higher the degree, the higher the probability of connection. This is also known as the *rich-get-richer mechanism* of network growth.

Scale-free networks seem to pervade the real world. Studies of real-world networks' degree distributions, including the World Wide Web, the Internet, infrastructural networks, networks of airline city connections, scientific collaboration networks, cellular networks and

[11] Strictly speaking, the most general form is given by $p(k) \approx k^{-\gamma} \exp(-k/k_c)$ where k_c is some characteristic cut-off.

several others, can be fit by equation (1.2.4), with γ values somewhere between 2 and 3, depending on the particular network. (Boccaletti *et al.* 2006)

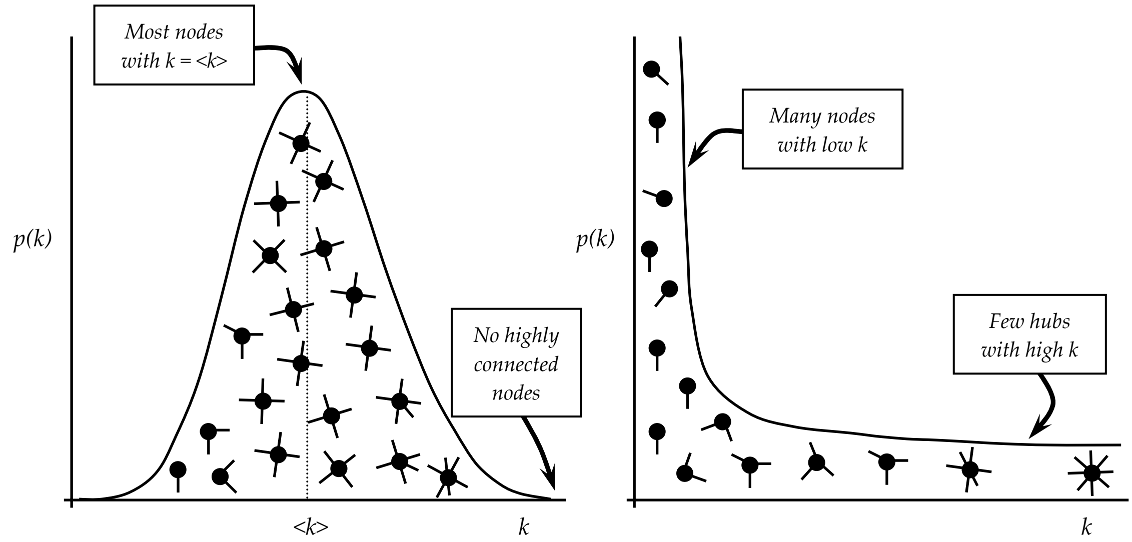

Fig. 1.2.5 Random and scale-free networks. The degree distribution of a random network (left) follows a bell curve: most nodes have the same number of links. In contrast, the power law degree distribution of a scale-free network (right) predicts that most nodes have only a few links, held together by a few highly connected hubs. Adapted from (Barabási 2002).

Power grids arise as natural objects of study under the conceptual frame of complex systems, particularly as complex networks. These are systems composed of multiple and diverse elements, such as transformers, generators, switching stations, etc., connected physically by electric cable lines. Each one of these composing elements is dynamically characterized by specific and well known physical laws that govern the grid's local and overall behavior. In spite of this, this behavior seems to be highly unpredictable sometimes, especially when unexpected emergent phenomena like blackouts and cascading failures arise. We believe complex networks conceptual frame is capable to find indeed a connection between dynamics and structure in power grids and it is our hope that this will help in understanding more thoroughly such unexpected phenomena.

1.3 Sustainability

Complex networks science appears as a new way to understand local and global phenomena for highly heterogeneous systems with non trivial connectivity distributions. Most of the technological networks around us can be considered complex and difficult to understand from a traditional point of view. However, engineering science tends to follow the same analytical path than that of the first pioneers with the telegraph and the idea of progress (Fig. 1.3.1): it considers and tends to analyze the growth and evolution of a networked system as something linear, a mere process of adding more and more elements, able to maintain their independent dynamics in spite of the context or substrate where they have landed, and ignoring some of the multiple *feedbacks* that take place in it. These *feedbacks* are not necessary and solely technical or economical: they are social and cultural. In fact, and due to social and cultural changes in the last twenty years, when somebody talks about infrastructures, talks about social and territorial conflicts as well. (Nel·lo 2003)

Fig. 1.3.1 "American progress" (John Gast, oil on canvas, 1872). In this allegorical painting "Divine Providence" watches over settlers on their journey west: she pushes bisons and Native Americans into darkness while pulling the telegraph cable, railroads and the light of the "western" society's new dawn. (Source: Museum of the American West, Autry National Centre, Los Angeles; http://www.autrynationalcenter.org/).

Several attempts offer new perspectives to point the power system to a new direction (although it is, as always, only a technological one). In Europe for example, the existing distribution systems, built about fifty years ago, are becoming obsolete and over the coming years will have to be progressively replaced. In response to climate change, recent years have witnessed a growth in renewable energies usage and with it the problem of incorporating them in the design of existing distribution networks. The EU-financed SmartGrids[12] and EU-DEEP[13] platforms aim to solve it by means of two ideas (Lethé 2008): (a) to favor a better interconnection of existing networks to create a vast European grid[14] and (b) to permit a two-way flow in which the consumer would, to a degree, be an active producer. Researchers

[12] http://www.smartgrids.eu/. (Last visited, January 2009).
[13] http://www.eudeep.com/. (Last visited, January 2009).
[14] "The bigger the network the greater the likelihood of being able to balance production and demand." (Hugues, op.cit.)

claim a number of advantages for their system: renewable energies could easily be included in the mini-networks as well as the main grid, without posing any problems in terms of low voltage or intermittence; this would result in decreased CO_2 emissions; and costs for the consumers would also fall as they would produce a part of their own electricity and could even sell any surplus. Similar projects with similar names can be encountered overseas.[15]

But electricity reaches consumers through local lines, local landscapes and different land mosaics. How can a global scale be accommodated to the local evaluation of the landscape fragmentation due to a line sitting? Which are, if any, the correct criteria to reflect the local particular impacts and effects of a global network layout? There exist automated methodologies of sitting new transmission lines that allow external groups to participate in the process and make decisions by utility professionals more transparent and credible (Houston and Johnson 2006). They use GIS software to map all geographic features in a study area, assign numerical suitability values to all features, assign engineering constraints, generate corridor alternatives using statistically sound algorithms, automatically generate alternative corridor reports and automatically create reports summarizing criteria used and values assigned.

All through these pages we have suggested the hypothesis of a bidirectional influence between structure and dynamics. If it exists, the aforementioned approaches will always be partial an inaccurate since they are unable to include global behaviors. Complex networks approach forces us to deal with a much broader parameter space. One that can consider ignored aspects of infrastructures networks such as local and global environmental impact, multiscale demand and generation management, planning criteria and even energy equity. In a nutshell: sustainability.

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2

Structure. Topology and fragility

The recent massive and comparative analysis of networks from different fields has produced, among many unexpected and surprising results, one singular thought: the need to move beyond reductionist approaches and try to understand the behavior of a system as a whole (Dorogovtsev and Mendes 2001; Albert and Barabási 2002; Boccaletti *et al.* 2006). Along this path, understanding the structure of the interactions between the components is the first unavoidable step. Much effort has been done in defining new metrics to characterize the topology of real networks. Some of them, such as the *degree*, *degree distribution*, *clustering coefficient* and *average path length* have been already presented. The main result has been the identification of a series of unifying principles and statistical properties common to most real networks that have allowed at the same time the development of models such as the *small-world* or the *scale-free network* one. Most real networks' structure, though, can not be explained solely with the aid of the former measures and models. They share characteristics of several models and need some other defining parameters.

A complete characterization of a network's topology is motivated by the expectancy that understanding and modeling the structure of a complex network would lead to a better knowledge of its evolutionary mechanisms and to a better comprehension of its dynamical and functional behavior (Solé *et al.* 2003). In fact a network is the result of the continuous evolution of the forces that form it and its topological growth is deeply rooted in their dynamics and the evolution of its dynamics over time. In other words: structure affects the function of a networked system and, conversely, its dynamics constrain the evolution of its topology. There exist indeed evidences pointing to the crucial role played by the network topology in determining the emergence of collective dynamical behavior such as synchronization (Strogatz 2000) or in governing the main features of relevant processes that take place in complex networks, such as the spreading of epidemics (Watts *et al.* 2005), information (Dodds *et al.* 2003) and rumors (Watts 2002). Moreover, a careful inspection of graph structure together with an appropriate model can help understanding how the network was originated. (Solé *et al.* 2003)

Some of the questions we are trying to address with the analysis of the structure of the topology of the power grid are the following:

- Which are the measures that best characterize the topology of a power grid? Are they useful in order to understand and classify these networked systems? Are they similar to those used in other investigations found in the literature?
- In a continental grid, formed by national grids constrained by differing historical, social, cultural and technological pathways, do different topologies exist? And if this is the case, are their dynamics affected by these differences?

- If topology and dynamics are deeply interwoven, can we find evidences of "weak" or conversely "robust" topologies? How different topologies stand the impact of different kinds of damages?

The answers to some of these questions are rather unexpected if not surprising. Although topologies are in general terms strikingly similar, the outcome of and relation with their dynamic processes are found to be completely different.

2.1 Topology of the UCTE power grid

Power grids have been in the line of fire of complex networks science from its very beginning. They have been, though, comparatively less investigated than cellular, ecological or social networks, perhaps due to the difficulties involved in finding suitable data or lack of very specific electric engineering knowledge. The first reference comes from Watts and Strogatz, who analyzed the Western States Power Graph (WSPG), the graph of the United States western power grid, by means of the following four strong simplifications (Watts and Strogatz 1998):

1. All transmission lines are assumed to be bidirectional; hence the resulting graph is undirected.
2. The nodes of the network (i.e., generators, transformers, substations, and so on) are treated as identical, featureless vertices.
3. All transmission lines are assumed to be identical (that is, unweighted, with no attributes associated to edges) ignoring the important fact that the voltages varies considerably, as we have presented in Chapter 1, and that different lines have significantly different carrying capacities, impedances and physical construction.
4. Only the transmission network is considered. This ignores an entire (and much larger) associated network, responsible for distributing power from the grid to individual house, offices, factories, etc.

Although the authors were clearly aware of the limits that these simplifications impose on the graph's utility for studying its dynamic properties or for any engineering purpose, they have been considered all the way until now in the literature. Limiting as it might sound, the search for the smallest set of assumptions in order to explain reality turns out to be what complex systems modeling is constantly looking for (Miller and Page 2007). Nonetheless, the aim of the research was not to characterize dynamics but to compare structures among different networks' topologies. The WSPG, formed by 4 941 nodes with mean degree $\langle k \rangle = 2{,}67$, appears to be a *small-world* network, with characteristic path length ℓ_{actual} similar to that of its random counterpart ℓ_{random}, but clustering coefficient C_{actual} one order of magnitude above its random counterpart C_{random} (Table 2.1.1).

	N	L	$\langle k \rangle$	ℓ_{actual}	ℓ_{random}	C_{actual}	C_{random}
WS Power Graph	4 941	6 597	2,67	18,7	12,4	0,080	0,005

Table 2.1.1 Statistical parameters of the WSPG compared to those of a random graph of the same size. The measures are: number of vertices *N*, number of links *L*, mean degree $\langle k \rangle$, characteristic path length ℓ and clustering coefficient *C*. The *random* subscript stands for the value obtained in randomly generated graph of the same size.

Watts and Strogatz groundbreaking work had to be complemented a year after by the analysis of the so called *degree distribution* measure, introduced in the literature by Barabási and Albert (Barabási and Albert 1999). Recall the *degree* of a vertex in a network is the number of edges incident on (i.e., connected to) that vertex. We defined *p(k)* to be the fraction of vertices in the network that have degree *k* (see Chapter 1). A plot of *p(k)* for any given network can be obtained by making a histogram of the degrees of vertices. This histogram is the degree distribution for the network which in a random graph is of the form shown in equation 1.2.1. An alternative, and usually more convenient, way of presenting degree data is to make a plot of the cumulative distribution function $P(k)$ defined as

$$P(k) = \sum_{k'=k}^{\infty} p(k') \tag{2.1.1}$$

which is the probability that the degree of a node is greater than or equal to *k*. Such a plot has the advantages that all the original data are represented and reduces the noise in the tail of the function.

Real-world networks are mostly found to be very unlike the random graph in their degree distributions. Far from having a Poisson distribution, the degrees of the vertices in most complex networks are highly skewed to the right meaning that their distribution has a long tail of values that are far above the mean. These functions, which can be *power law*, power law with cut-off, stretched exponential, log-normal, etc., have received the general *scale-free* nickname, a fact that bears important consequences as most of the network's dynamics strongly depends on this particular topological feature. (Amaral *et al.* 2000; Marquet *et al.* 2005; Clauset *et al.* 2009)

The first published degree distribution of a power grid was supposed to be *scale-free*, following a power law function of the form $P(k) \sim k^{-\gamma}$ with $\gamma = 4$ (Barabási and Albert 1999). But none of the subsequent later references would support this finding. The topological analysis of the Californian power grid by Amaral (Amaral *et al.* 2000) and the whole United States grid by Albert (Albert *et al.* 2004) detected both exponential cumulative degree functions of the form

$$P(k) \cong \exp(-\alpha k) \tag{2.1.2}$$

where α is the characteristic exponent of the function, that for the American power grid reaches $\alpha = 0{,}5$. This functional form classifies both power grids as single-scale networks. The cumulative degree distribution shows that the probability of having high-degree nodes is less than in a scale-free network.

Country	**UCTE countries short form**	N	L	$\langle k \rangle$	γ
Belgium	BE	53	58	2,18	1,005
Holland	NL	36	38	2,11	1,086
Germany	DE	445	560	2,51	1,237
Italy	IT	272	368	2,70	1,238
Austria	AT	70	77	2,20	1,409
Romania	RO	106	132	2,49	1,418
Greece	GR	27	33	2,44	1,457
Croatia	HR	34	38	2,23	1,594
Portugal	PT	56	72	2,57	1,606
UCTE		*2 783*	*3 762*	*2,70*	*1,630*
Poland	PL	163	212	2,60	1,641
Slovak Republic	SK	43	52	2,41	1,660
Bulgaria	BG	56	67	2,39	1,763
Switzerland	CH	147	186	2,53	1,850
Czech Republic	CZ	70	88	2,51	1,883
France	FR	667	899	2,69	1,895
Hungary	HU	40	47	2,35	1,946
Bosnia	BA	36	42	2,33	1,952
Spain	ES	474	669	2,82	2,008
Serbia	RS	65	81	2,49	2,199

Table 2.1.2 Summary of the basic features exhibited by some of the European power grids analyzed, ordered by increasing γ, the exponential degree distribution exponent (see text): number of nodes *N*, number of links *L* and mean degree $\langle k \rangle$.

The topological analysis of the European (or UCTE, Union for the Co-ordination of Transport of Electricity) power grid includes the *mean degree* and the cumulative *degree distribution* as main topological measures (Rosas-Casals *et al.* 2007). Table 2.1.2 shows a summary of the basic features of the UCTE power grid and different national grids. Though every single network contains hundreds of stations, substation, transformers and thousands of kilometers of energy transport lines, the results show a striking similarity in the mean degree values, very alike to those encountered in the aforementioned works of Watts and Albert. The relation between nodes and links is remarkably constant and goes around $\langle k \rangle \cong 2{,}7$. This would suggest that although every technical infrastructure has evolved and developed under different economical, political, historical and, luckily enough, environmental conditions and decisions, there should be some universal characteristics related to the spatial and technological constraints that rule the construction and evolution of such networks. This hypothesis is also supported by the observed cumulative distribution functions, which turn to be exponential of the form

$$P(k) = \beta \exp(-k/\gamma) \tag{2.1.3}$$

where β is a normalization constant and the value of the exponent γ goes from a minimum of $\gamma_{BE} = 1{,}005$ for Belgium to a maximum of $\gamma_{RS} = 2{,}199$ for Serbia. This variability in the value of γ will be shown, in the following sections, to be strongly tied to grid robustness. The graphical appearance of some of the cumulative *degree distribution* functions is shown in Fig. 2.1.1.

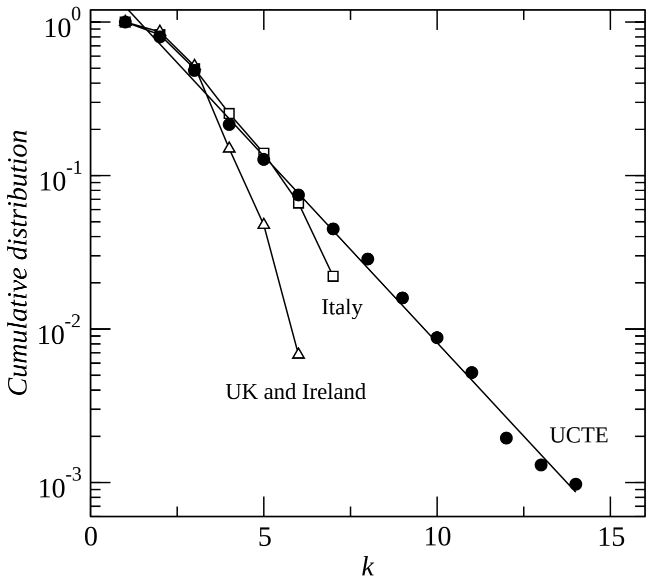

Fig. 2.1.1 Degree distribution for the UCTE power grid and two examples of national grids: UK and Ireland and Italy. These webs are homogeneous, having an exponential degree distribution that follows $P(k) = \beta \exp(-k/\gamma)$ though displaying different γ values.

Recall the small-world behaviour of a network can be characterized by the evaluation of two basic statistical properties: the *clustering coefficient* C, a measure of the average cliquishness of a node, and the *average path length* ℓ, a measure of the typical separation between two generic nodes in the network. In networked systems, short path lengths and high local clustering are signatures of the small-world phenomenon, indicating a neither completely regular nor random connectivity but small-world, which is highly clustered, like regular lattices, yet having small average path lengths, like random graphs. For the UCTE graph, the small-behaviour was found by means of deviation from two main predictions based on random graphs (Ferrer i Cancho *et al.* 2001):

1. The clustering coefficient over the average connectivity for a random graph follows an inverse scaling law with graph size:

$$C_{rand}/\langle k\rangle = 1/N \tag{2.1.4}$$

2. The average path length scales logarithmically as:

$$\ell_{rand}\log\langle k\rangle \approx \log(N) \tag{2.1.5}$$

The values of $C/\langle k\rangle$ and $\ell\log\langle k\rangle$ compared to those of $1/N$ and $\log(N)$ respectively are shown in Fig. 2.1.2. It can be seen that $C/C_{rand} > 1$ for most of the grids. Values of C/C_{rand} higher than one order of magnitude are achieved by the largest power grids while ℓ/ℓ_{rand} remains in the same order of magnitude for whatever size of the network. A similar pattern was observed in electronic circuits (Ferrer i Cancho *et al.* 2001) although the latter are more heterogeneous.

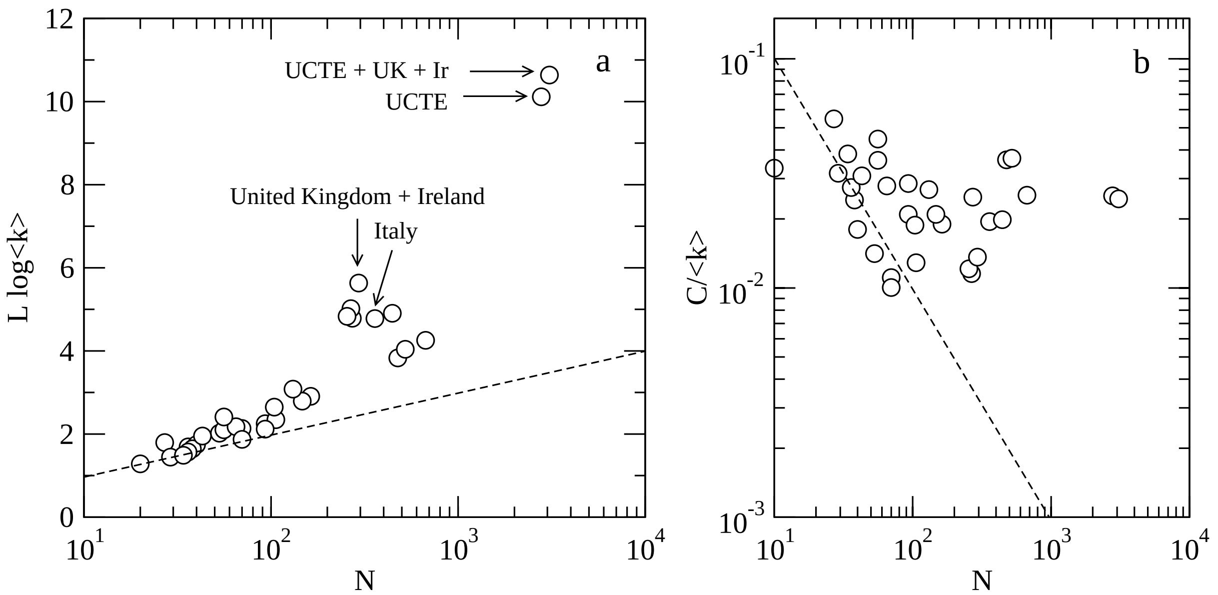

Fig. 2.1.2 Small world patterns as function of *N*, the network's size: (a) average shortest path and (b) clustering coefficient. Average path length is corrected by factor $\log\langle k\rangle$ and clustering by $\langle k\rangle$. Dashed lines signal the expected values for random graphs. It can be seen that larger networks involve larger deviations from the random cases.

From a structural point of view every country's grid has evolved in order to connect production sites with consumption sites within its own borders. In small countries, everything is at hand and long distance connections are not needed to expand the grid. On the contrary, in big countries (and consequently, with an increasing number of nodes) long distance and more meshed connections become more necessary: clustering and average path length increase but in dissimilar ways, being the former orders of magnitude higher than that of a randomly generated counterpart.

Topological complex networks characterization, such as small-world or scale-free, without a deeper structural analysis is nothing more than a label for network classification. This has been traditionally performed by means of static and dynamic robustness analysis.

2.2 Static robustness of the UCTE power grid

The ability of a network to avoid malfunctioning when a fraction of its constituents is damaged is usually known as *robustness*. It is a topic of obvious practical reasons for it affects directly the efficiency of any process running on top of the network. This was one of the first issues to be explored in the literature on complex networks (Albert *et al.* 2000; Cohen *et al.* 2000) and it can be encountered in two different variants:

1. *Static robustness*, meant as the act of deleting nodes without the need of redistributing any quantity that is being transported by the network.
2. *Dynamic robustness*, case in which the dynamics of redistribution of flows is taken into account.

At the same time, both variants can be implemented in different ways, depending on which property, or none at all, we choose in order to delete nodes or edges. Two main implementations have been considered so far:

1. *Tolerance to errors* (or random failures), understood as the ability of the system to maintain its connectivity properties after the random deletion of a fraction f of its nodes or edges.
2. *Tolerance to attacks* (or selective failures), understood as the ability of the system to maintain its connectivity properties when such a deletion process is targeted to a particular class of nodes like, for instance, the highly connected ones.

A network's capacity to maintain its connectivity will obviously depend on its original topology and the way we modify its structure by means of successive deleterious actions. For example, scale-free networks are extremely sensible to attacks (targeted deletion of nodes) but very resilient to error failures (random deletion) while random networks will react similarly, unaware of which kind of deletion are receiving. (Albert *et al.* 2000)

The first power grid whose robustness was sized was the North American one (Albert *et al.* 2004). The authors removed nodes randomly and in decreasing order of their degrees, for both generation nodes and transmission nodes, and monitored the connectivity loss, measured as the decrease of the ability of distribution substations to receive power from the generators. While the loss of generators increased the connectivity loss gradually, the loss of transmission substations increased the connectivity loss logarithmically, with a maximum connectivity loss of almost 60% with the loss of only 2% of the highly loads transmission substations. They concluded that the transmission highly connected *hubs*, while ensuring the connectivity of the power grid, are also its largest liability in case of power breakdowns. On the other hand, the first reference to European power grids was made by Crucitti *et al.* The authors studied and compared the topological properties of the Spanish, Italian and French power grids, finding those components that when removed affected the most the structure of the graph (Crucitti *et al.* 2005). It is, nonetheless, a poor analysis, since the 400 kV grid is the only considered and the proposed improvements are unrealistic nor physically feasible.

The numerical study of the robustness of the whole European power grids includes the static robustness under failures and attacks of every national power grid (Rosas-Casals *et al.* 2007). Two clearly differentiated responses are shown in Fig. 2.2.1, (a). Static tolerance to random (white circles) and selective (black circles) removal of a fraction f of nodes is measured by the relative size S_{inf} of the largest connected component (whiskers stand for the standard deviation of the realizations).

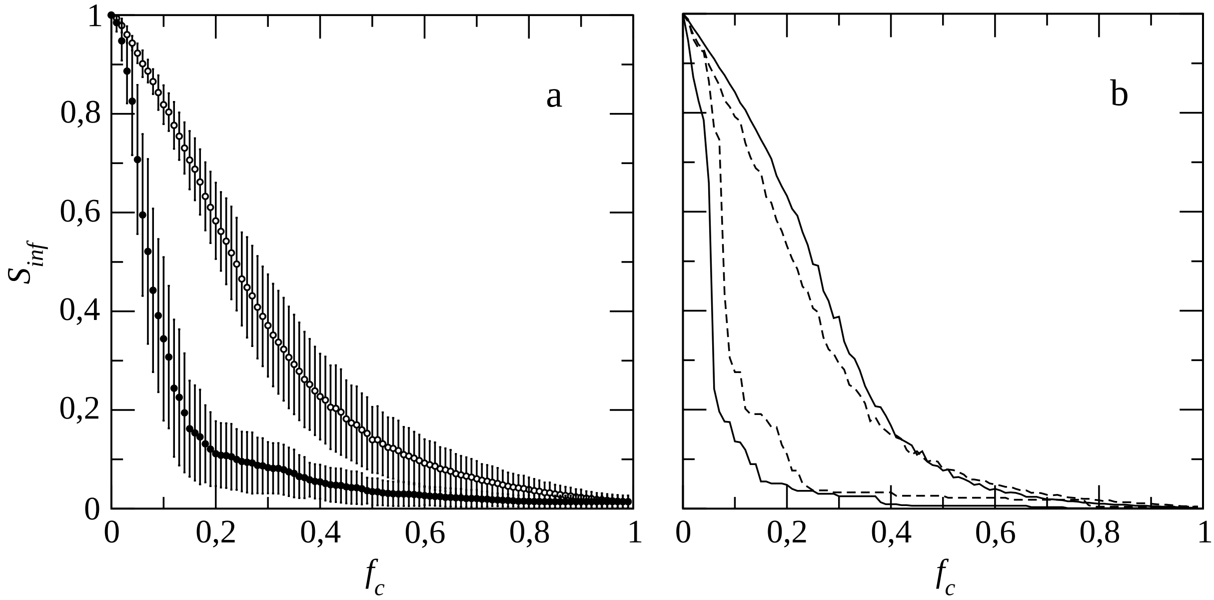

Fig. 2.2.1 Effects of attacks and failures on the topology of the EU power grids. (a) Static tolerance to random (white circles) and selective (black circles) removal of a fraction f of nodes. (b) Evolution of the static tolerance to random and selective node removal for Italy (dashed lines) and France (continuous lines).

As we can see, while random failures affect the structure of the grids in a rather decreasing but monotonous way, selective removal reduces drastically the size of the connected component. Although this fact agrees with results found in the literature, in Fig. 2.2.1, (b) a more surprising result is observed. It shows the evolution of the static tolerance to random and selective node removal for Italy (dashed lines) and France (continuous lines). Though in the case of random removal (failures) both networks exhibit a similar response, for the selective one (attacks) Italy is slightly more robust: for a fixed fraction of eliminated nodes, the relative size of the largest connected component in Italy always remains higher than that of France. This fact will be crucial in the following theoretical treatment of the deletion process.

2.2.1 Theoretical results

The theoretical treatment of the robustness of sparse[1] and uncorrelated[2] networks such as the European power grid can be approached using *percolation* and *mean field* theories (Solé and Montoya 2001; Solé *et al.* 2008). On one hand, the formulation of the critical fraction of nodes f_c to be eliminated in order to make a graph unable to percolate (that is to eliminate the possibility to go from one node to another following an edge) comes originally from graph theory (Molloy and Reed 1995) although its first practical application in complex networks was related to Internet (Cohen *et al.* 2000; Cohen *et al.* 2001). The critical fraction for random failures of nodes f_c^F is written as

$$f_c^F = 1 - \frac{1}{\frac{\langle k^2 \rangle}{\langle k \rangle} - 1} \tag{2.2.1}$$

where $\langle k \rangle$ is the mean degree and $\langle k^2 \rangle$ is the mean square degree, a value often found in graph theory analytical developments (Newman 2003) and simply defined as

$$\langle k^2 \rangle = \frac{\sum_{i=1}^{N} k_i^2}{N} \tag{2.2.2}$$

where N is the size of the network.

The study of random failures in complex networks can be exactly mapped into a standard percolation problem since equation (2.2.1) comes from the analytical treatment of a percolation model over a random graph. It fixes the fraction of nodes to be randomly erased in order to impede jumping from one node to another following a randomly token edge. Yet the same formalism can be extended with few modifications in order to find the critical fraction but for intentional attacks f_c^A defined implicitly as

$$1 + \left(\ln f_c^A - 1\right) f_c^A = \frac{1}{\frac{\langle k^2 \rangle}{\langle k \rangle} - 1} \tag{2.2.3}$$

---

[1] A dense graph is a graph in which the actual number of edges is close to the maximal number of edges. The opposite, a graph with only a few edges, is a sparse graph.

[2] A graph is uncorrelated when the degrees at the end points of any edge are completely independent.

On the other hand, recall UCTE power grids are characterized by a cumulative exponential degree distribution of the type

$$P(k) \approx \exp(-k/\gamma) \tag{2.2.4}$$

With equation (2.2.4) we assume a perfect cumulative exponential degree distribution from $k=1$ to infinity (i.e., the thermodynamic limit). The degree probability distribution (i.e., non-cumulative) can be written therefore as

$$p(k) \approx \left(-\frac{1}{\gamma}\right)\exp(-k/\gamma) \tag{2.2.5}$$

For an exponential function like (2.2.5) and generically written as $f(x)=\lambda\exp(-\lambda x)$, we have a mean that is $1/\lambda$. Since in our case $\lambda = 1/\gamma$ and our mean is the *mean degree* $\langle k\rangle$, we can thus write $\langle k\rangle = \gamma$. Similarly to the mean degree, that can be calculated (by definition) as

$$\langle k\rangle = \int_{k=1}^{\infty} k \cdot p(k)dk \approx \int_{k=1}^{\infty} k\left(-\frac{1}{\gamma}\right)\exp(-k/\gamma)dk = \gamma \tag{2.2.6}$$

we can obtain the mean field approximation for the mean square degree by doing

$$\langle k^2\rangle = \int_{k=1}^{\infty} k^2 \cdot p(k)dk \approx \int_{k=1}^{\infty} k^2\left(-\frac{1}{\gamma}\right)\exp(-k/\gamma)dk = 2\gamma^2 \tag{2.2.7}$$

and thus $\langle k^2\rangle = 2\gamma^2$. If we introduce these values in (2.2.1), we finally obtain

$$f_c = 1-\frac{1}{\frac{\langle k^2\rangle}{\langle k\rangle}-1} = 1-\frac{1}{\frac{2\gamma^2}{\gamma}-1} = 1-\frac{1}{2\gamma-1} \tag{2.2.8}$$

which is the mean field approximation to the theoretical critical fraction of nodes. Table 2.2.1 gives the summary of both treatments, considering the approximation made by $\langle k^2\rangle/\langle k\rangle = 2\gamma$ as a theoretical prediction and calculations exactly made with $\langle k\rangle$ and $\langle k^2\rangle$ as empirical results.

Fig. 2.2.2 (a) shows the evolution of the theoretical prediction. It increases smoothly as a function of γ from the limiting lower value $\gamma = 1$ for both random removal of nodes and intentional attacks. Here γ is the grid's characteristic exponent in (2.2.4) and Table 2.1.2, and f_c indicates the fraction of removed nodes required in order to break the network. The upper curve is the critical boundary for network percolation under random removal of nodes. Below it, a network experiencing such random failures would remain connected (i.e., with a giant component). The lower curve corresponds logically to the critical boundary for attacks, where a lower critical fraction is needed in order to break the network in many pieces.

	Theoretical prediction	Empirical estimation
Random failures	$f_c^F = 1 - \frac{1}{2\gamma - 1}$	$f_c^F = 1 - \frac{1}{\frac{\langle k^2 \rangle}{\langle k \rangle} - 1}$
Selective attacks	$1 + \left(\ln f_c^A - 1\right) f_c^A = \frac{1}{2\gamma - 1}$	$1 + \left(\ln f_c^A - 1\right) f_c^A = \frac{1}{\frac{\langle k^2 \rangle}{\langle k \rangle} - 1}$

Table 2.2.1 Theoretical predictions and empirical results for the static robustness analysis of the European power grid and for random failures and selective attacks.

In Fig. 2.2.2 (b) we display the empirical results of f_c for attacks from the thirty three EU power grids (circles) to be compared with the mean field prediction continuous line. The empirical results show a clear deviation for values of $\gamma \leq 1{,}5$. This finding has been a hallmark in our research, for it has permitted to broadly classify European power grids into two separate groups, namely: fragile and robust. The answer to the question of whether this deviation can be related to particular dynamics is left open until the following chapter.

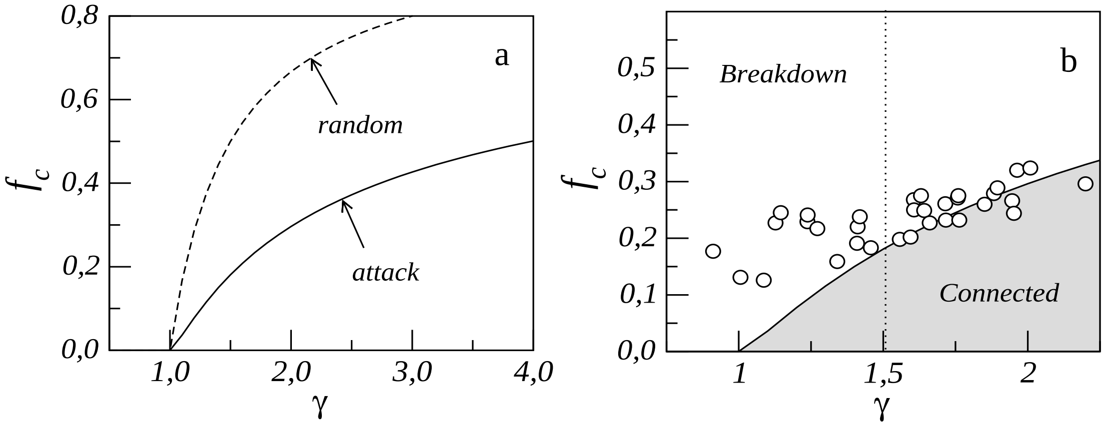

Fig. 2.2.2 Phase space for exponential uncorrelated networks under random removal of nodes and directed attack towards highly connected vertices. (a) Theoretical prediction. (b) Theoretical prediction vs. experimental results for the selective attacks scenario.

2.3 Dynamic robustness of the UCTE power grid

Recall dynamic robustness refers to the case in which the dynamics of redistribution of flows is taken into account. This is another important dimension to add to the problem, since it refers to modelling the dynamics of flows of the physical quantities of interest over the network. When it comes to modelling the dynamics, the situation is far more complicated since the components of a network may have different dynamical behaviours and flows are often a highly variable quantity, both in space and time. The usual shortcut to overcome these difficulties has been: (a) the assumption of a characteristic load of an element as a measure of its *robustness*; and (b) the association of this load to a topological measure defined on it. The most used topological measure has been the so called *betweenness* centrality of that element. Together with *degree* and *closeness* of a node (defined as the inverse of the average distance from all other nodes) *betweenness* is one of the standard measures of node centrality (Boccaletti *et al.* 2006). More precisely, the betweenness b_i of a node *i* is defined as

$$b_i = \sum_{j \neq k} \frac{n_{jk}(i)}{n_{jk}} \tag{2.3.1}$$

where n_{jk} is the number of shortest paths connecting nodes *j* and *k*, and $n_{jk}(i)$ is the number of shortest paths connecting nodes *j* and *k* and passing through node *i*. The dynamic robustness of the network is then evaluated in the following way: each element is characterized by a finite capacity (defined as the maximum load that the element can handle). Once a deletion has taken place, it changes the shortest paths between nodes and, consequently, the distribution of b_i, creating overloads on some other nodes. All the overloaded nodes are removed simultaneously from the network. This leads to a new redistribution of loads and subsequent overloads may occur. The new overloaded nodes are removed and the redistribution process continues until when, at a certain time, all the remaining nodes have a b_i value under or equal to its capacity.

Power grids seem to be an optimal candidate for this kind of analysis since cascading failures have been usually the prologue to huge blackouts (UCTE 2004). But at the same time, power system operation, flow and generation turns out to be one of the most mathematically complicated problems encountered in engineering nowadays. All the variables and processes involved in such calculations demonstrate that power grids do not concentrate flows depending on *betweenness* centrality nor the pool of sinks and sources are necessarily constrained by shortest paths (Wood and Wollenberg 2005). Therefore, most of the initial complex networks dynamic models encountered in the literature have only a qualitative role as explanatory theory. Although they provide some indications on the actions that can be performed in order to decrease undesired effects such as congestions, avalanches of node breakdown and cascading failures, these are not realistic neither sufficiently accurate to explain the power grid dynamics.

We acknowledge that a full characterization of a network cannot be fully accounted for without considering the interplay between structural and electrical aspects. Among the ongoing research paths opened by this PhD Thesis, other metrics are being considered and some more useful and different approaches are being used (see Chapter 5).

2.4 Main points in review

The main points in review in this chapter are the following:

- Transmission power grids' topologies are similar in terms of *mean degree* and *degree distribution*. This would suggest similar topological constraints, mostly associated with technological considerations and spatial limitations.
- Exponential cumulative *degree distributions* of the form $P(k) = \beta \exp(-k/\gamma)$ characterize European power grids, although with different characteristic exponent γ. This γ value characterizes as well their response to static robustness analysis: while for $\gamma > 1{,}5$ theoretical prediction and experimental results agree, experimental values are systematically higher than theoretical predictions for values of $\gamma \leq 1{,}5$. The implications of this finding will be outlined in the following chapter.
- Simple topological measures such as *betweenness* centrality can not be considered good candidates to unify topology and dynamics of power grids, since optimal power flow does not depend (at all) on shortest paths. Different approaches have to be considered.
- No topological metrics correlations have been found so far with other plausible welfare measures such as GDP, electric per capita consumption, population distribution, area, etc. At least at this transmission level, this fact would suggest that power grid's topology evolves in a rather autonomous way. This finding will be explored more thoroughly in Chapter 4.

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3

Dynamics. Blackouts, design and reliability

At 03.01 hours, on November 28th, 2003, a Swiss 380 kV electric transmission line tripped due to tree flashover. The many attempts to automatically re-close the so called "Lukmanier" line that connects Mettlen and Lavorgo substations were unsuccessful. At 03:25, the "San Bernardino" line, another Swiss 380 kV line connecting Sils and Soazza substations that took over the load of the tripped line, as is always the case in similar situations, tripped as well when the overheating of the conductors made it sag and touched a tree. By an almost simultaneous and automatic trip of the remaining interconnectors towards Italy and a series of cascading failures in generation units and relays, 60 million Italians were isolated from the European network in less than three minutes after the loss of the second line (UCTE 2004). Similarly, on the evening of November 4th, 2006, something went wrong again in the European power grid when a scheduled but erroneous switching off two power lines in Northern Germany left 15 million households in central Europe without electricity for an hour, half of them in France alone. (UCTE 2007)

These occurrences were traditionally considered to be a consequence of accidental faults and, accordingly, they were not common. But things have changed in recent years and power systems, as well as other critical infrastructures, seem to fail more frequently than desired. Besides random failures, the threat of malicious attacks has increased as well, transforming infrastructural vulnerability into a hot economical, social and political issue. (Perrow 2003)

As we have presented in the previous chapter, the applications of complex network concepts to power systems have been initially aimed at understanding its structure. But is there a way to relate the former results to the overall dynamical behavior of the power grid? The specific questions we are trying to answer are the following:

- We have seen that there exits a deviation between theoretical and empirical robustness behavior in UCTE power grids (Section 2.2), but do have major disturbances like blackouts any relation with the topological features that characterize their structure? Is there a way to characterize the former observed deviation?
- Is a pure connectivity approach like the one presented in the previous chapter suitable in order to grasp the many dynamical features of a power network? If not, does more suitable metrics exist in order to evaluate the performance of a power grid?

It is important here to stress the meaning of "dynamical behavior" (or simply dynamics) when dealing with complex networks, in contrast with its significance for the power generation, operation and control field. While complex networks dynamics is related to the flow of information, energy or matter through the networked system and the different temporal values that characterize the resulting feature vector, power systems dynamics is related to frequency, synchronization, swings and transient stability performance. In this chapter, and in all this work, whenever the word dynamics is used, it refers to the former complex networks acceptation.

3.1 Correlation between reliability and topology

In Section 2.2 a deviation from the theoretical predicted topological critical fraction for values of $\gamma \leq 1{,}5$ was presented. In this sense we would expect a correlation between the critical percolation fraction f_c, the exponent that characterizes the grids' cumulative degree distribution γ, and values that were directly related to the actual performance of the grid. Since daily dynamical flows of the European power grid are difficult, if not impossible, to obtain, we must rely on reliability parameters that indirectly relate dynamics and fragility. For the UCTE power grid, these parameters have been published in monthly statistical format since 2002.[1] These are the result of every major malfunction in the European power grid and, for each major event, they account for the following measures:

- *Energy Not Supplied (ENS)*. Measured in MWh, as loss of energy from the consumption side.
- *Total Loss of Power (TLP)*. Measured in MW, as loss of production from the generation side.
- *Restoration Time (RT)*. Measured in minutes. Note that since ENS and TLP are measured from different sides, RT can not be assumed as the ratio of ENS over TLP. It can be considered, therefore, an independent reliability measure.
- *Equivalent Time of Interruption (ETI)*. Defined as the duration of an interruption in minutes multiplied by the energy not supplied divided by the consumption for the last 12 months. Defined in this way, the ETI allows a direct comparison between Transport System Operators (TSOs) in terms of interruptions that occurred during a year.

The attempts to correlate these aforementioned network reliability measures and the structural topology for the European power grid were carried out in (Solé *et al.* 2008) and (Rosas-Casals and Corominas-Murtra 2009), and are shown in Fig. 3.1.1. Recall Table 2.1.2 offers a summary of the basic topological features exhibited by the European power grids and that we segregated them in two groups: robust ($\gamma < 1{,}5$) and fragile ($\gamma > 1{,}5$) power grids. Figure 3.1.1 (a) shows cumulated European power grid indexes for each group: percentage

[1] http://www.ucte.org/resources/publications/monthlystats/. (Last visited, January 2009).

size (i.e., number of nodes over the whole UCTE size, which is 2 783 nodes), energy share (i.e., cumulated electricity consumption over the UCTE energy consumption), and power share (i.e., national cumulated highest load over the UCTE power generation). The energy and power normalization has been done using national electricity consumption and highest load on the 3rd Wednesday of December respectively. For year 2008 (last year available), these cumulated values reached 2 392 TWh and 389 GW for the countries considered in Table 2.1.2, respectively. As we can see, grids in the fragile group, though represent two thirds of the UCTE size, share almost as much power and energy as grids in the robust group. Figure 3.1.1 (b) shows cumulated European power grid reliability indexes for each group: energy not supplied (ENS), total power loss (TPL), restoration time (RT) and equivalent interruption time (EIT). For each group, these values have been obtained as cumulated percentage of MWh (ENS), MW (TPL) and minutes (RT), over the whole UCTE cumulative value for the same time period. Equivalent time of interruption is normalized by definition.

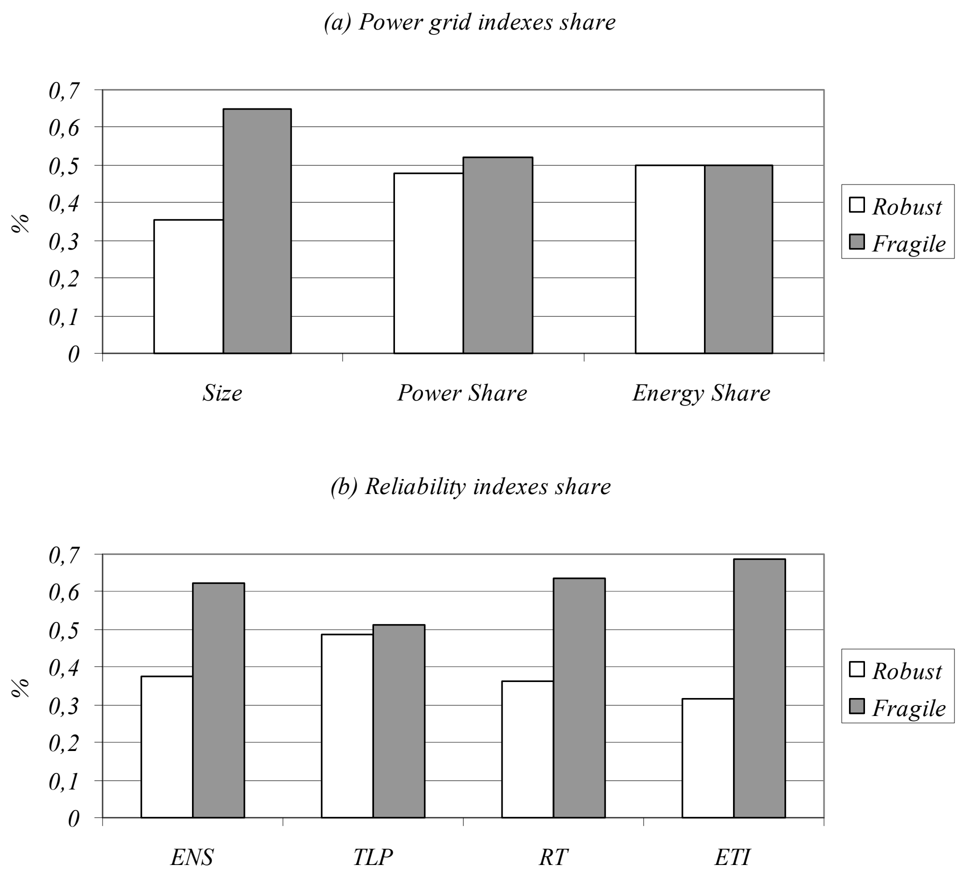

Fig. 3.1.1 Power grid indexes vs. reliability indexes (updated August 2008). Networks in the fragile group, though share almost as much power and energy as networks in the robust one, accumulate between 60% and 70% of the energy not supplied (ENS), restoration time (RT) and total equivalent interruption time (EIT). The total loss of power (TLP) is almost equivalent in both groups.

As we can see, except for the total power loss value, there is an obvious unbalanced situation, being the share of the grids in the fragile group much more significant than that of the robust one. Sadly, network reliability data have been published only from 2002 onwards.

Short time spans are sensible to extreme and rare events. In November 2006, 10 million people suffered the consequences of a major event triggered in the German power grid (16 724 MW loss). Without this single event, the share in total power loss (TPS) would be 60% for the fragile group and 40% for the robust one (where Germany is included).

Some other topological features have been found to increase as well with the fragility of the network (Rosas-Casals and Corominas-Murtra 2009):

(a) **Deviation from a random graph null model *degree distribution***. Calculation of the normalized deviation between the actual *mean degree* of every grid and that of the Poisson distribution that best fits the real *degree distribution* increases with increasing values of γ. Rather unexpectedly this fact would suggest a more fragile behavior as the network is less well fitted by the Poisson distribution that is as the network is less randomly designed.

(b) **Increased preponderance of star and triangle *motifs* in spite of linear ones**. Though global similarities may arise, networks might display very different local structure. This local structure can be characterized by patterns termed network *motifs* (or *subgraphs*) that appear at a much higher frequency than expected in randomized networks (Milo *et al.* 2002). A notable increase of interconnected local topologies in spite of linear ones is observed, as the fragility of the networks increases with γ. Fragility seems to increase as the elements of the grid become more interconnected and motifs such as stars and triangles began to appear.

(c) **Inhomogeneous patch size distribution**. Patch size distributions have been mainly used in landscape ecology as a fragmentation measure and a tool for environmental monitoring (Jaeger *et al.* 2007). Cable lines have been considered here as virtual spatial fragmentation limits and the distribution of the size of the resulting areas have been calculated considering political frontiers, seas and oceans as the very outmost limits of the patches for every country. Results for Spain (i.e., fragile set) and Germany (i.e., robust set), both with similar number of nodes, show absolute frequency distributions of patches notoriously different: while the German grid keeps this frequency almost constant for all orders of magnitude of patch size, the Spanish grid begins to strongly increase its frequency for patch size values lower than five hundred square kilometers. Although this is tentative measure and it has to be further explored, this fact would suggest a much messier and intricate grid for the Spanish case, heavily inhomogeneous at the spatial level and, consequently, much more difficult to control and prone to failures of different kind.

All these evidences show an increased fragility when the topology of the network deviates from a random one, maybe in search of a higher interconnectedness. Although aging infrastructures, excessive power delivered through increasing long distances and other possible causes may influence the increasing fragility of the power grids, it seems reasonable to think that maybe, on a topological basis, the application of the (N-X) contingency criteria, which favors connectivity and interconnectedness, though originally intended to avoid interruptions in power service, would difficult, at the same time, the islanding of disturbances.

The consequences of this finding are remarkable in planning long term overhead or underground transmission lines since it can not be therefore a question of identifying optimal route alternatives solely from local environmental, technical, etc., points of view but also considering global better topologies. A tendency to use less meshed network topologies without risking reliability should decrease overall transmission lines building and sitting costs on one hand, and operating costs on the other, for resistance losses would diminish as well.

3.2 Power grid major disturbances analysis

The research presented in the previous section has been focused in aggregated values: power grids have been segregated into fragile and robust sets and major events have been related accordingly. But major events are presented as temporal series of values whose frequency and appearance pattern can be also statistically analyzed. The statistical analysis of major disturbances in power networks from a complex systems perspective begins in the mid 1990's, when two distinct models emerged based on two general theories of systems failure (Fairley 2004). One, an optimization model, presumes that power engineers make conscious and rational choices to focus resources on preventing smaller and more common disturbances on the lines and large blackouts occur because the grid is not forcefully engineered to prevent them. The competing explanation views disturbances as a surprisingly constructive force in an unconscious feedback loop that operates over years or decades. The rationale of both theories lies deep in two more general visions of the world known as *highly optimized tolerance* (HOT) and *self-organized criticality* (SOC) respectively.

Keeping aside methodological considerations and deep, and even bitter, personal discussions between their defendants, it seems SOC (Jensen 1998; Bak 1999), rather than HOT (Carlson and Doyle 2002), theory has been much more widely accepted in the power systems community in order to try to unravel the laws governing complex systems like this. In physics SOC is a property of (classes of) dynamical systems which have a critical point as an attractor (Bak *et al.* 1987; Jensen 1998; Bak 1999; Solé and Goodwin 2001). Their macroscopic behavior thus displays the spatial and/or temporal scale-invariance characteristic of the critical point of a *phase transition*, but without the need to tune control parameters to precise values. This scale-invariance shows itself as a power law

$$f(x) = kx^{\alpha} \tag{3.2.1}$$

where k is a constant and α is known as the scaling parameter, the constant that characterizes the heaviness of the power law tail. As we can check, scaling by a constant c the independent value x in the former equation simply multiplies the original relation by the constant c^{α}. Thus, it follows that all power laws with a particular scaling exponent are equivalent up to constant factors, since each is simply a scaled version of the others. This behavior produces a straight-line on a log-log plot, a fact that is often called the necessary (but not sufficient) signature of a power law.

Since many natural and man made systems can be found to display power law behavior in one (or more) of their characteristic features (i.e., scale free networks and their power law degree distribution, fractal objects and their power law dimensional measurements, etc.) SOC concepts have enthusiastically, and sometimes inaccurately, been applied across fields as diverse as geophysics, physical cosmology, evolutionary biology and ecology, economics, quantum gravity, sociology, solar physics, plasma physics, neurobiology and, of course, power systems, where the chosen feature has been usually the frequency distribution of blackout sizes (Carreras *et al.* 2000). Time series of usual measures of blackout size like energy unserved, power loss or number of customers affected, have been shown to be algebraically (i.e., with power tail) distributed in North America (Chen *et al.* 2001), Sweden (Holmgren and Molin 2006), Norway (Bakke *et al.* 2006), New Zealand (Jordan *et al.* 2006) and China (Weng *et al.* 2006). Thus large blackouts are much more likely than expected purely by random eventuality and, when costs are considered, their risk is comparable to the cumulate risk of small blackouts. The idea that rules this methodology is the conjecture that (a) power systems tend to self-organize near a critical point and (b) that there may be some *universality* driving the inner depths of these systems.

This methodology avoids delving in the details of particular blackouts and, instead, it studies the statistics, dynamics and risk of series of blackouts by means of approximate global models. These models simulate an increase in power system load from zero (independent failures and negligible chance of large blackout) to emergency loading of all components (certain cascading failure). There is a critical loading (*phase transition*) in between these extremes at which there is a sharply increased chance of cascading failure and the appearance of power tails at this critical point. In order to *self-organize* the system to the critical point, two opposing forces have been suggested to act dynamically: (a) load growth, which reduces power system margins of operation, and (b) the engineering responses to blackouts, which tend to increase these margins. If these theory holds true, mitigation of blackout risk should take into account these counter-intuitive effects in complex self-organized critical systems, since, for example, suppressing small blackouts could lead the system to be operated closer to the edge and ultimately increase the risk of large blackouts.

3.2.1 Major events analysis for the UCTE power grid

The aforementioned methodology has been used on the UCTE power grid with two main objectives (Rosas-Casals and Solé 2009):

(a) To detect power law probability distributions in order to discuss if Europe's power grid has any of the SOC features previously defined, and

(b) To check probability distribution patterns for fragile and robust networks, that could have been hidden due to the coarse grained assumptions made in section 3.1.

In this case, three malfunction measures of the European power grid have been analyzed: energy not supplied (ENS), total loss of power (TLP) and restoration time (RT). The first goal here is to construct the cumulative distribution function of the data by a simple rank ordering of it, as it is shown in Fig. 3.2.1. The resulting function is then fitted to a power law, following well established statistical methods like those described in Ref. (Clauset et al. 2009).

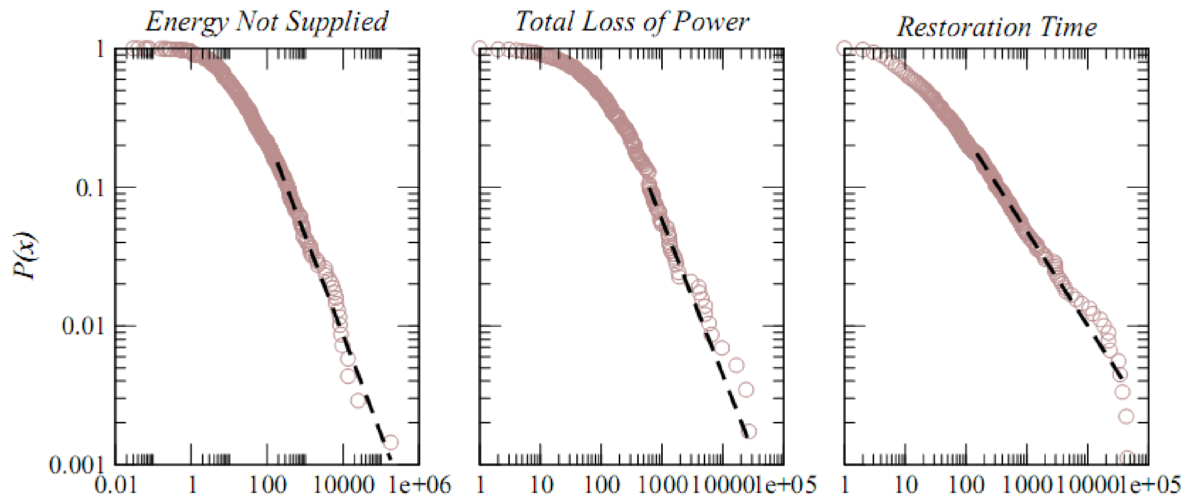

Fig. 3.2.1 Cumulative distribution functions $P(x)$ and their maximum likelihood power law fits for the UCTE reliability measures energy not supplied, total loss of power and restoration time.

We assume a quantity x follows a power law if it is drawn from a probability distribution $p(x) \propto x^{-\alpha}$, where α is the scaling parameter of the distribution. Since the probability density of a power law distribution diverges as $x \to 0$, there must exist a lower bound to the power law behavior (Newman 2005). We denote this lower bound as $x_{\min}$ and the number of events contained in the upper range as n_{tail}. Therefore, to fit a power law to any empirical data we must estimate these three parameters. And not only this: given an observed data set and a hypothesized power-law distribution from which the data are drawn, we would like to know whether our hypothesis is a plausible one, given the data. This can be finally done using a goodness-of-fit test, which generates a p-value that quantifies the plausibility of the hypothesis. In this case, if the resulting p-value is greater than 0,1 the power law is a plausible hypothesis for the data, otherwise it is rejected.

Table 3.2.1 summarizes these results. As we can see, the power law model is a plausible one for every data set considered (i.e., the p-value for the best fit is sufficiently large) and the scaling parameter values are similar to those encountered in the literature (Chen *et al.* 2001; Dobson *et al.* 2007). These are generic statistics on one side and results of the aforementioned statistical analysis on the other. Yet the power law model explains only a small amount of events: 15% for ENS ($n_{tail} = 104$), less than 10% for TLP ($n_{tail} = 57$) and 17% for RT ($n_{tail} = 157$).

					Maximum likelihood				
Data set	n	$\langle x \rangle$	σ	$x_{\max}$	$\hat{x}_{\min}$	$\hat{\alpha}$	n_{tail}	p	**Support for PL**
ENS	690	552	7004	180000	185 ± 72	1,7 ± 0,1	104 ± 120	0,24	Moderate
TLP	576	400	1790	26746	615 ± 244	2,1 ± 0,2	57 ± 96	0,36	Moderate
RT	897	510	3328	44640	150 ± 68	1,69 ± 0,07	157 ± 115	0,73	Ok

Table 3.2.1 UCTE major failures generic statistics and power law fits. For each measure we give the number of occurrences n, mean $\langle x \rangle$, standard deviation σ, maximum observed occurrence $x_{\max}$, lower bound to the power law behaviour $\hat{x}_{\min}$, scaling parameter value $\hat{\alpha}$, occurrences in the power law tail n_{tail} and p-value, p. The last column indicates the support for whether the observed data is well approximated by a power-law distribution. Estimated uncertainties for $\hat{x}_{\min}$, $\hat{\alpha}$ and n_{tail} are also given.

Although we believe that measures such as n_{tail} and $x_{\min}$ are fundamental to estimate the span of the power law behavior and to develop further quantitative models, these have not been considered in any of the aforementioned references. Only in the reanalysis of Ref. (Carreras *et al.* 2004) done in Ref. (Clauset *et al.* 2009) we have found an estimate for n_{tail} that gives an explanation for 28% of the events. It is likely that the limited span of available data in each set might have a sensible influence in the final power law fitting outcome. It is nonetheless evident from these results that:

(a) pure power law behavior can not be assumed for the whole data observed,
(b) there is no clear evidence for the existence of any critical point at this stage of the data span and
(c) there must be considerably more dynamics not explained by the power law model.

Our second objective was to analyze probability distribution patterns in order to differentiate fragile and robust networks. We have analyzed the probability distributions of the same malfunction measures of the European power grid (i.e., energy not supplied, total loss of power and restoration time) but segregated this time into the aforementioned two groups, fragile and robust. The results, obtained following again the methodology presented in Ref. (Clauset *et al.* 2009), are summarized in Table 3.2.2 and shown in Fig. 3.2.2.

One first remarkable result in Table 3.2.2 is the difference between mean values for each group. As we can see, although robust grids (subscript **r**) accumulate much less events than fragile ones (subscript **f**), $\langle x \rangle$ values for the robust group are significantly higher than those of the fragile. It seems malfunctions that strike the robust set imply higher risks and more important consequences than those that strike the fragile set, although events in the latter are more frequent. We have not found a plausible nor general agreed explanation to this phenomenon among consulted references and scholars' opinions.

As far as the segregated probability distributions of major events is concerned, the power law model turns now to be a plausible one only for some of the data sets: energy not supplied in the fragile group (ENS_f), total loss of power in the robust group (TLP_r) and restoration time for both groups. In the ENS_f case, it moderately (p = 0,30) explains, though, a 41% of the events, while in the TLP_r case it reaches the 23%. Fig. 3.2.2 shows a remarkable trait: cumulative distribution functions for the robust set (circles) present a higher probability of occurrence than that of the fragile set (stars) for the same measure (the only exception would be the upper tail of the TLP_r case).

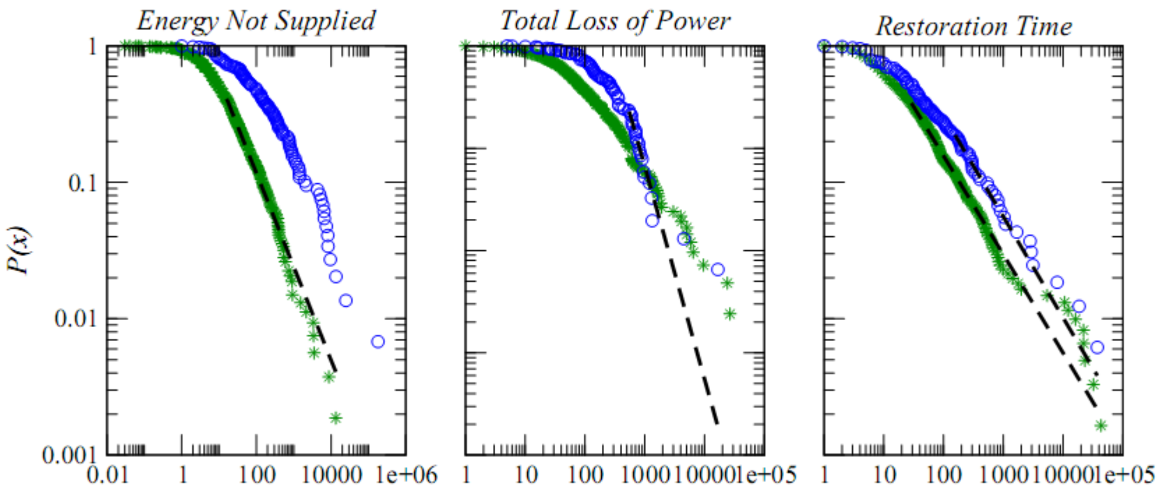

Fig. 3.2.2 Cumulative distribution functions $P(x)$ and their maximum likelihood power law fits for the same UCTE reliability measures as in Fig. 3.2.1 but segregated into fragile (circles) and robust (stars) sets.

Although not all distributions in Fig. 3.2.2 are power law, this qualitative behavior where robust set distribution stands with higher probability than fragile set has been observed in (Bakke *et al.* 2006) where a numerically and analytically validated model that simulates failures and avalanches in regular and random networks is presented. For a regular group of networks (square and triangular lattices) the scaling exponent of events distribution is found to be $\alpha = -2{,}0$ while for the randomly generated group of networks $\alpha = -1{,}5$. This finding agrees with the scaling parameters qualitatively observed in Fig. 3.2.2, where robust set distribution that topologically belongs to more randomly generated grids has a lower slope (i.e., scaling parameter α) than that of the fragile set, less randomly generated and with both more connected topologies and a higher slope.

Since no clear reasons are presented in (Bakke *et al.* 2006) to explain these different behaviors from a dynamical point of view, a more specific dynamical model able to connect topological and electrical facets is deeply needed in order to decipher these comportments. This is actually part of this PhD Thesis' ongoing research (see Chapter 5).

					Maximum likelihood				
Data set	n	$\langle x \rangle$	σ	$x_{\max}$	$\hat{x}_{\min}$	$\hat{\alpha}$	n_{tail}	p	**Support for PL**
ENS$_r$	147	944	15037	180000	-	-	-	0,03	None
ENS$_f$	534	117	751	13377	16 ± 23	1,68 ± 0,08	219 ± 56	0,30	Moderate
TLP$_r$	153	436	1401	16724	550 ± 227	3,0 ± 0,9	35 ± 36	0,60	Ok
TLP$_f$	416	390	1931	26746	-	-	-	0,00	None
RT$_r$	162	559	3344	37486	156 ± 62	1,7 ± 0,1	36 ± 27	0,83	Ok
RT$_f$	607	377	2842	43200	29 ± 22	1,72 ± 0,07	141 ± 58	0,62	Ok

Table 3.2.2 UCTE major events generic statistics and power law fits. For each measure we give the same values as in Table 3.2.1 segregated into fragile (subscript f) and robust (subscript r) sets. No values are given when the support for the power law model (p-value) is null or extremely low.

The case of the restoration time is remarkable since the power law model explains approximately 22% of events in both groups, with a high p-value and similar scaling parameters, alike to those observed for the whole UCTE power grid. Since restoration time values depend mostly on human factors, this behavior might be in accordance to some results found in the literature that relate human behavior with power law response time distributions. (Johansen 2004; Barabási 2005)

3.3 Main points in review

The main points in review in this chapter are the following:

- It seems there exists a positive correlation between topological features and dynamical major failures. Rather counterintuitively, evidences suggest an increase in fragility when the topology of the network deviates from a random one, maybe in search of a higher interconnectedness.
- With the actual data span UCTE grid can not be considered to be in a state of self-organized criticality. Power law fits are moderately supported and the number of events contained in the power law tail is very low. Strategies for optimal management and operation of these networks will have to consider the dynamical behavior not accounted for this particular heavy tailed function.
- Qualitative and quantitative differences exist in major failures distributions between robust and fragile sets. Particularly remarkable is the difference in mean values, where fragile networks mean stands always with lower figures than that of robust networks, although the former accumulate between 2,7 and 3,7 times much more events.
- Restoration time is the only measure that is particularly well supported by a power law model. This behavior is in accordance with findings where human temporal response distributions have been found to be fat tail distributed.

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4

Evolution. Time, space and constraints

In analyzing topology and dynamics of any networked system, we usually can only see the present outcome of a huge evolving process taking place in multiple spatial and temporal scales. Such process is driven by multiple and, usually, unknown forces. As these unknown forces shape and, at the same time, are being shaped by the evolving structure and dynamics of the network, its growth can be an intricate process. Here dynamics impels new topological forms that, at the same time, modify the flows of information between its constituents. Therefore, in order to fully comprehend a network we must deal with its evolution, as well as with its structure and topology. (Dorogovtsev and Mendes 2001)

Most network evolutionary models try to explain the observed structure by means of two processes: (a) an *ad hoc* growth process that introduces a new element at every time step; and (b) an attachment process that models the relations established among elements. One widely used and analyzed attachment process has been *preferential attachment*, already presented in Chapter 1, in which some quantity or wealth is distributed among a number of individuals or objects according to how much they already have, so that those who are already wealthy receive more than those who are not. Since under suitable circumstances this process ends up generating *power law* distributions, it was first suggested for the growth of the world wide web, a network particularly well known for having this kind of *degree distribution* (Barabási and Albert 1999). Following suit, similar growing models were developed, numerically and analytically, in order to explain the several power laws encountered in networks' degree distributions. (Boccaletti *et al.* 2006)

But many other networks in the real world do not present power law degree distribution neither they grow by means of any preferential attachment at all. Most spatial networks[1] for example, cannot go through a preferential attachment process since every new link implies the filling of a proportional real space in the node vicinity and such space is not unlimited. This fact thus makes the appearance of power law difficult.

On the other hand, what is really a drawback about network evolution is that no data of past evolution stages is usually available to corroborate the assumptions imposed by any model or to suggest the appearance of other intermediate evolutionary processes. The study of the evolution of networks is therefore an essential component of the complex networks research agenda, in order to shed light on some fundamental questions such as the following:

[1] Networks embedded in real Euclidean space (also known as geographical networks). See next section.

- As networked systems grow by adding elements and, at the same time, by coupling their dynamics to those already present, which is the level of interaction between elements and how this process modifies the internal information and energy flows?
- Would it be possible to find different growing processes at different growing stages? Would they be a function of the structure, substrate or dynamic processes already present in the system?
- Biologist have questioned the “continuous and inevitable” view of the evolution (Gould 1989). Even some structures found in biological networks are thought to have had no function at all in the beginning, but they were used and integrated as functional elements *a posteriori* (Solé and Valverde 2006). Where do, then, necessity, chance and contingency stand in these evolutionary processes? Which patterns are really necessary or even useful and at which evolutionary stage?

Some of these questions are more related to some types of networks than others. Biological networks are a particularly fruitful set, where the assumption of the former hypothesis has important consequences in evolutionary theory (Montoya *et al.* 2006; Solé and Valverde 2006). On the contrary, infrastructure networks in general (and power grids in particular) belong to the engineering field, where objectives and constraints are clear *a priori*. But is this last *a priori* assumption really true? Most technological networks evolve in time and space following well known driving forces such as economic and physical laws. Yet they have been continuously going through changes, spanning and crossing urban and natural systems from their early stages, adapting and being adapted by human societies, landscapes, territories and other constraints. Has this adaptive process the power to modify the initial objective functions? We begin in this chapter to delve into these questions introducing a particularly constrained subset of networks: the spatial network.

4.1 Spatial networks

The aforementioned *preferential attachment* model is only one of the several models evolved since the end of the 1990’s to try to explain the pervasiveness of the *power law* distribution observed in many natural and man-made systems.[2] But the truth is that many other networks are not scale-free but something completely different. A particular class of networks that do not use to hold the *power law* distinctive includes those embedded in the real space, that is networks whose nodes occupy a precise position in two or three-dimensional Euclidean space, and whose edges are real physical connections. The typical examples are neural networks (Sporns 2003), information and communication networks

[2] The first rigorous consideration of preferential attachment though seems to be that of Yule in 1925, who used it to explain the power-law distribution of the number of species per genus of flowering plants (Yule, G. U. (1925). "A Mathematical Theory of Evolution, based on the Conclusions of Dr. J. C. Willis, F.R.S." Philosophical Transactions of the Royal Society of London. Series B **213**: 21–87.) The process is sometimes called a "Yule process" in his honour.

(Pastor Satorras and Vespignani 2004), transportation systems ranging from rivers (Forman 1995), airports (Guimerà and Amaral 2004), streets (Porta *et al.* 2005), railway and subway (Latora and Marchiori 2001) networks, vascular plants (Noblin *et al.* 2007), ant networks of galleries (Buhl *et al.* 2004), electronic circuits (Ferrer i Cancho *et al.* 2001) and of course, power grids.

Most of the works in the literature have focused on the characterization of the topological properties of spatial networks while the spatial aspect has received less attention, if not neglected at all. Although there exist numerical and analytical two and three dimensional scale-free evolution network models, spatial power-law networks are extremely difficult to find in the real world since their topology is strongly constrained by their geographical embedding as we have said. The following is a list of the major characteristics and spatial constraints traditionally considered at work on such networks.

- *Fractal spatial distribution of nodes*. Networks with strong geographical constraints have been generally considered good example of networks with *fractal scaling*. But here scale plays an important role. The nodes of the Internet, for example, develop on a fractal support driven by the fractal nature of population patterns around the world (Yook *et al.* 2001). But the transport power grid, being equally geographically constrained, does not follow a fractal pattern until distribution and low voltage grids are considered. Hence, network distribution of nodes, be it fractal or non fractal, depends upon the scale at which they are delivering their final services and it can not be considered as a taxonomic characteristic.
- *Limited node degree*. As we have already mentioned, node degrees are constrained in spatial networks, since the number of edges that can be connected to a single node is limited by the physical space available to connect them. This is particularly evident in planar networks,[3] such as street patterns (Porta *et al.* 2005) or ant networks of galleries (Buhl *et al.* 2004).
- *Distance-dependence cost of edges*. In spatial networks distant nodes are less likely to be connected due to the distance-dependent cost of the edges. This fact has important consequences since long distance links are really difficult to appear and some behaviors, like the small-world one, can not be observed. Yet section 2.1 presented the appearance of the small-world behavior in the power grid as the grid size increases. This fact was mostly due to the increase in the clustering coefficient values compared to a random graph of the same size.
- *Trivial clustering-degree correlations*. It has been shown that some geographical networks (the Western United States power grid among them) have $C(k)$ independent of *k* (Ravasz and Barabási 2003). This result is different from what has been obtained for many other real networks which show a hierarchical behavior with $C(k)$ well approximated by $C(k) \approx k^{-1}$. In networks with strong geographical

[3] Planar graphs are those graphs forming vertices whenever two edges cross, whereas non-planar graphs can have edge cross and not form vertices.

constraints, hierarchy[4] is absent because of the limitations imposed by the link lengths on the topology. So it happens for the UCTE power grid. (Rosas-Casals *et al.* 2007)

These constraints imply severe difficulties to the modeling and study of the evolution of spatial networks. But since most of these networks are involved in the vital transport of goods, wealth and information, the effort made in developing useful spatial networks models has been remarkable.

4.1.1 Spatial networks models

The simplest way to generate geographical networks is based on two ideas: (a) to distribute N vertices at random in a two-dimensional space Ω and (b) link two any vertices i and j with a given probability which decays with their distance d_{ij} as

$$p(d_{ij}) = \beta \exp(-d_{ij} / \alpha D) \tag{4.1.1}$$

where D is the maximum distance between any two nodes, α is a parameter used to tune the ratio of short to long distance edges (i.e., fixes the length scale of the edges) and β controls the average degree of the network (Waxman 1988). Alternatively, the network development might start with a few nodes while new nodes and connections are added at each subsequent time step. This is known as spatial growth process and is able to generate a wide range of network topologies, including small-world and scale-free networks limited to a certain degree. (Kaiser and Hilgetag 2004)

Most spatial networks appear to show a preference for short edges over long ones, which is a natural effect of geography. However, highway networks, for example, have much shorter edges, lower degrees and larger diameter[5] than Internet or flights networks (Gastner and Newman 2006). These are all consequences of the planarity of the highway network. In fact, although each of these three networks is two-dimensional in a geographic sense (since it lives on the two-dimensional surface of the earth) it is possible to show that the road network is almost planar, while the other two networks are not. A simple one-parameter model explaining this feature can be constructed in terms of competing preferences for either short Euclidean (i.e., physical real) distances between nodes or short graph distances (i.e., hops between nodes) by assigning to each edge an effective length

$$D(i, j) = \lambda \sqrt{N} d_{ij} + (1 - \lambda) \tag{4.1.2}$$

---

[4] A network shows hierarchical topology when dependency of some kind between topological measures is observed.

[5] The diameter of a network is the longest shortest path existing between any two nodes.

where $0 \le \lambda \le 1$ determines the user's preference for measuring distance in terms of kilometers or hops: in a road network most travelers look for routes that are short in terms of kilometers while for airline travelers the number of changes among planes is often considered more important. Given the position of N nodes and the budget for building the network (consisting in the maximum total length of the wirings to be used) the model generates the network structure that connects all nodes and minimizes the mean distance between all node pairs by means of the distance defined by (4.1.2). The cases $\lambda = 0$ and $\lambda = 1$ produce networks strongly reminiscent of airlines and roads respectively. For intermediate values of λ the model finds a compromise between *hub* formation and local links.

As we will see in the next section, not only purely spatial constraints have to be considered in spatial network modeling. The aforementioned budget or the availability of resources in general turns out to be the most important issue when modeling the spatial growth of a network.

4.2 Spatial and temporal evolution of a power grid

In order to analyze the evolution of a technological network, it is necessary to find the suitable data set. For the power grid particularly, one such data set can be found for the French electricity transmission network and is partially shown in Fig. 4.2.1.[6]

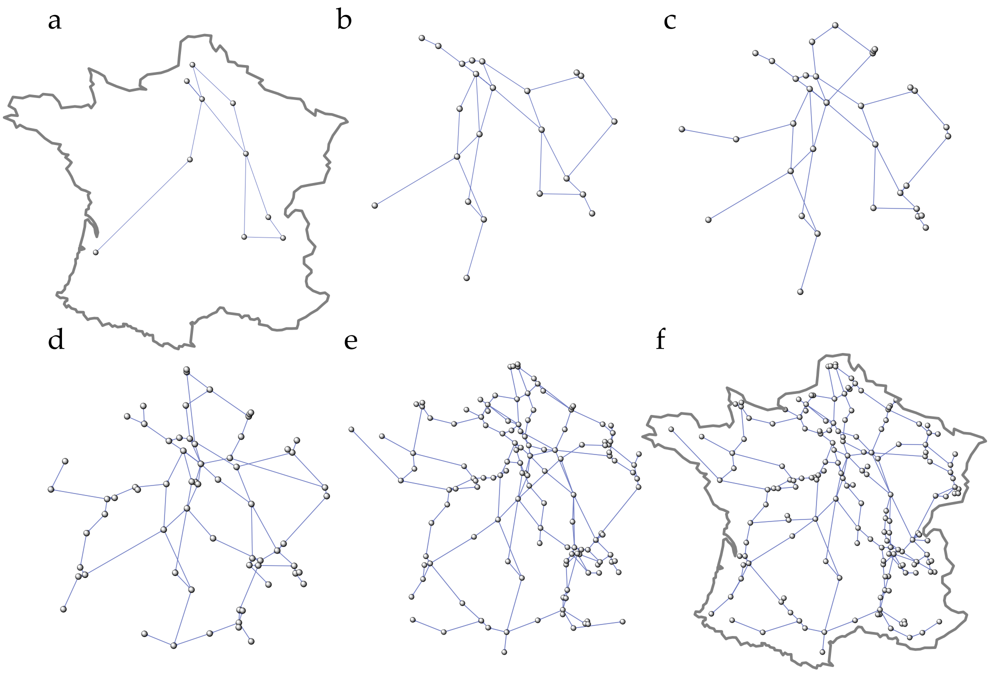

Fig. 4.2.1 Snapshots of the evolution of the French transmission power grid at several years. (a) 1962, (b) 1972, (c) 1976, (d) 1982, (e) 1992 and (f) 2005.

[6] http://www.rte-france.com/htm/fr/CEM_HTML/transport/historique-reseau-400kv.jsp. (Last visited, June 2009).

Evolved from 1962 until nowadays, this system might be though as the intricate result of many different societal, economical, political and lastly, environmental shaping processes. And yet it can be modeled as a planar graph whose spatial and temporal evolution follows a slightly modified version of equation (4.1.1).

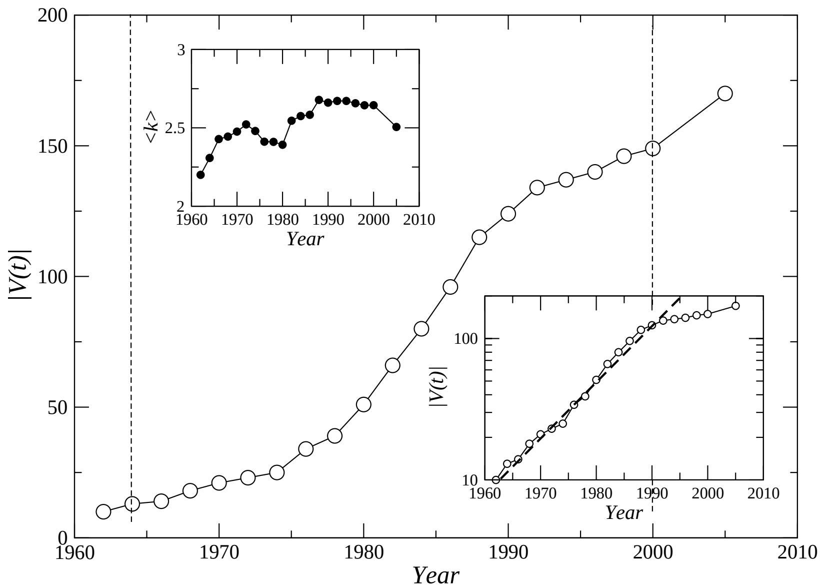

Fig. 4.2.2 Size (white circles) and mean degree (upper inset, black circles) evolution for the French transmission power grid. (Dashed vertical lines limit the studied time span, from 1964 to 2000).

Though data for this power grid can be obtained from year 1946 until year 2007, from 1946 to 1960 the French transport grid relayed on 220 kV technology and it was mainly formed by disconnected lines and substations. It is not until 1962 that a main 400 kV connected core of 10 substations and some 1.850 kilometers of electric lines is detected. From 1976 to 1980, the 400 kV power grid begins to noticeably and effectively grow, due to the increase in both, electricity consumption and nuclear power generation equipment.

			French Grid			Model		
Year	**Completion**	**Accum. Nodes**	$\langle k \rangle$	C	ℓ	$\langle k \rangle$	C	ℓ
1962	6%	10	2,20	0,00	2,38	2,2 ± 0,3	0,16 ± 0,08	2,2 ± 0,3
1972	13%	23	2,52	0,03	3,94	2,4 ± 0,2	0,09 ± 0,07	3,3 ± 0,2
1976	20%	34	2,41	0,02	4,67	2,3 ± 0,2	0,10 ± 0,06	4,1 ± 0,3
1982	40%	66	2,54	0,06	5,41	2,4 ± 0,1	0,11 ± 0,05	4,9 ± 0,4
1992	80%	134	2,67	0,10	7,01	2,3 ± 0,1	0,10 ± 0,03	6,2 ± 0,3
2000	100%	149	2,64	0,06	7,76	2,4 ± 0,1	0,09 ± 0,02	6,8 ± 0,5

Table 4.2.1 Network parameters evolution. For every year considered, the table shows the percentage of completion (number of nodes introduced with respect to final size), accumulated number of nodes for the RTE network, *mean degree* $\langle k \rangle$, *clustering coefficient* C and topological *average path length* ℓ. The results of the model have been averaged over 1000 realizations.

Figure 4.2.2 shows the size (cumulative number of nodes $|V(t)|$) and mean degree $\langle k \rangle$ of the very high voltage level French transport power grid through the years. The slightly S-shaped network size function follows the usual sigmoid growing process found in most technological networks. (Dupuy 1996)

Table 4.2.1 shows among other results and measures, the numerical evolution of $\langle k \rangle$. As we can see $\langle k \rangle$ is kept almost constant with a slight decrease at the point where the grid begins to increase (from 1975 to 1980) due to the fast addition of new lines from nodes with $k = 1$ with clearly one objective: to reach as much territory with as less time and cost as possible. From 1980 onwards $\langle k \rangle$ increases slightly due to the meshing process of the grid in order to attain a reliable (N −X) criteria (Willis 2004). Table 4.2.1 shows as well the evolution of two other characteristic topological measures already presented throughout these pages: the *clustering coefficient* C and the *average path length* ℓ.

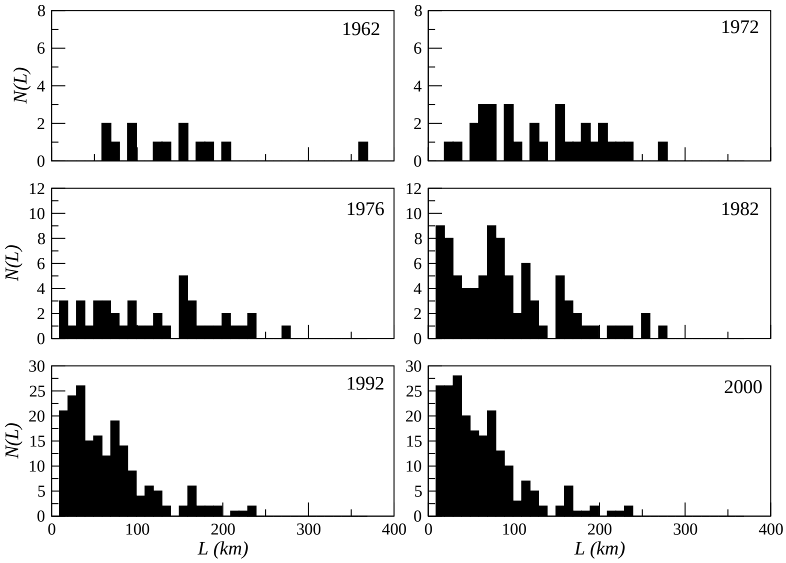

Fig. 4.2.3 Histograms for the lengths of edges L (in kilometers) of the evolution of the French transmission power grid at different years. The bias towards shorter edges appears once long transmission lines have been built, by mid 1970's, and the initially longest line (right extreme value in 1962) have been split into two in 1966.

Another simple way to describe the evolution of this network is the plot of the distribution of the lengths of the existing transmission lines at every time period considered. In Fig. 4.2.3 we show the evolution of the histogram of the lengths of edges of the French transport power grid at six characteristic years. From 1962 until 1976, the amount of edges increases notably but no characteristic mean is observed. In fact, almost every length is being used, even with the appearance of the longest line (354 km) before year 1972. This fact would suggest that nor economic neither technical factors would matter too much at the beginning of this growing process, other than a maximum spatial covering objective. From 1976 onwards, a clear tendency towards shorter lines appears, as it seems to happen in other types of spatial networks (Gastner and Newman 2004). After the covering process has been finished, there begins the meshing process in order to reassure the connectivity and (hopefully enough) the reliability of the grid.

The growing process of the model starts with a node randomly placed in a squared two dimensional space. At every time step t, a new node position is randomly chosen with coordinates in the interval [0, 1]. The subtle difference here with respect to (4.1.1) is that the probability of connecting a new node n_t with each existing node at a distance d at a time step t follows

$$p(d_{ij}, n_t) = \exp(-d_{ij} n_t) \tag{4.2.1}$$

The temporal increase in n_t allows the modeling of the evolution of the length of individual edges as shown in Fig. 4.2.4. The way equation (4.2.1) mimics the tendency observed in Fig. 4.2.3 is the following: at the beginning of the process, that is for lower values of n_t, the probability is almost unity for any distance d. As the network increases its number of nodes with time, that is higher values of n_t, the probability of establishing links between nodes at higher distances decreases exponentially. Table 4.2.1 shows the numerical evolution of $\langle k \rangle$, C and ℓ for the model compared with the French real measures. As we can see, the distribution of link lengths over time greatly resembles the real observed one, except in the last two final stages, where the exponential function used in the model arises more clearly. This slight deviation is due to a phenomenon that this model can not reflect and that is the split of long lines into many shorter ones. The capacity of this simple model to reflect the reality is nonetheless remarkable.

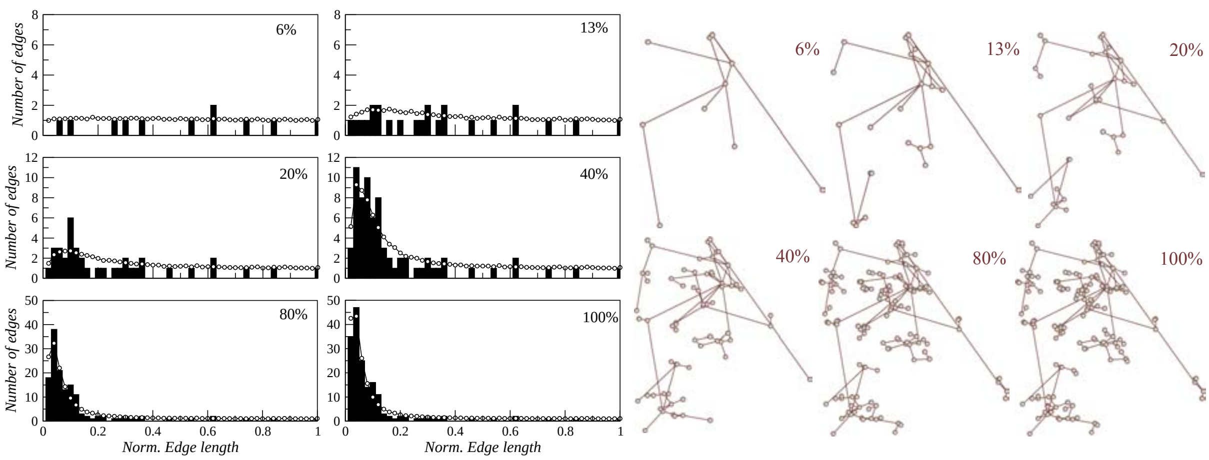

Fig. 4.2.4 Histograms of the lengths of edges (left) and snapshots (right) of the evolution of the spatial model at different stages of completion (shown as percentage of nodes introduced with respect to final size). White circles stand for mean values averaged over 1000 model realizations. Background histograms represent one sample.

The characterization of the topological change displayed by this network can not be completed without a tentative explanation of the characteristic *S*-shape observed in Fig. 4.2.2 for the network size evolution. In fact, many natural processes and complex system display a history-dependent progression from small beginnings that accelerates and approaches a climax over time. The simplest description of this process is the sigmoid curve which is produced by a mathematical function having an "S" shape. Often sigmoid function refers to the special case of the *logistic function* defined for each time step t like

$$f(t) = \frac{1}{1+\exp(-\varepsilon t)} \tag{4.2.2}$$

where ε is a characteristic parameter.

Long term engineering projects are usually characterized by uneven distribution of resources, with budgets normally distributed over time (Clark and Lorenzoni 1985; Humphreys and English 1993). For the French power grid Fig. 4.2.5 shows such distribution of costs when one considers a constant cost for nodes and a linear length dependent cost for links as it has been historically, and nowadays, the case for electric transmission networks construction (Landers *et al.* 1998; Willis 2004; Bosch 2008). Links (squares) and nodes (circles) share 80% and 20% of the total cost (stars) per two-year period respectively, regardless of any particular stage of the project completion. As the evolution of the expenditure over time is not constant, the accumulated cost follows a characteristic sigmoid function, similar (almost qualitatively exact) to that observed in Fig. 4.2.2 for the evolution of the size of the network.

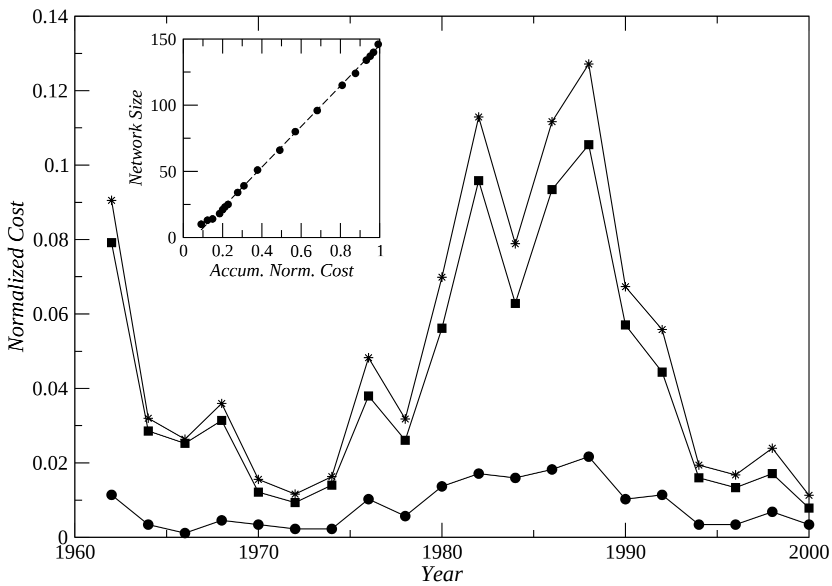

Fig. 4.2.5 Normalized costs distribution for the French power grid construction. Total normalized cost (stars) accounts for a constant cost for substations (circles) and linearly dependent one with distance for cables (squares). Inset: accumulated normalized cost and network size scale linearly over time. Normalization has been done over the total cost expenditure of the studied time span (i.e., 1964-2000).

This can also be observed in the inset where accumulated normalized costs scale linearly with network size. In order to consider this cost constraint, the spatial model has been coupled with a sigmoid like accumulated cost function that defines the maximum budget available for constructing new nodes and establishing new links during each two-year period. It takes the form

$$C_A(b) = \frac{1}{1+\exp(-(b-\mu)/\sigma)} \tag{4.2.3}$$

where b is the accumulated biennial period, and μ and σ are the function parameters adjusted in order to give the corrected time span. Equation (4.2.3) have been adjusted with $\mu = 10{,}57$ and $\sigma = 2{,}47$, for a time span of eighteen biennial periods from 1964 to 2000, in order to avoid the initial offset (ten nodes and some 1.850 km of lines, already in place) observed in year 1962. This offset comes from the last stages of the previous grid upgrade (that of 220 kV).

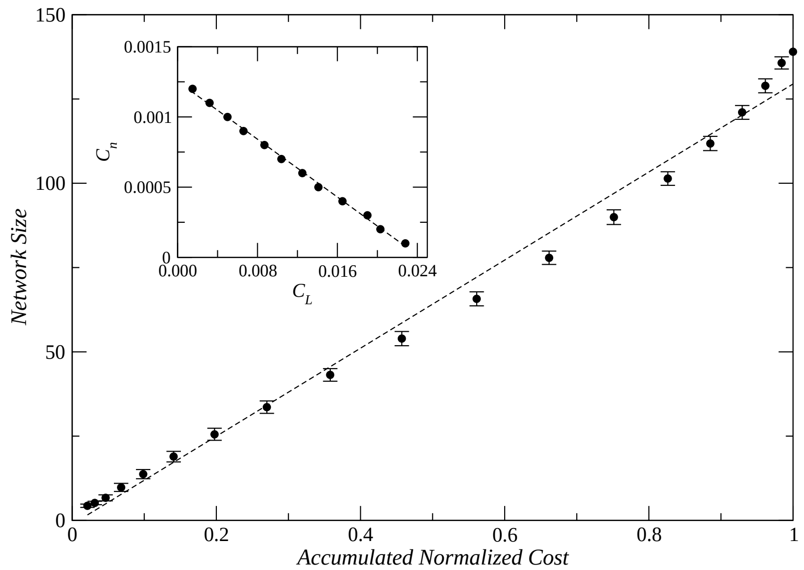

Fig. 4.2.6 Modelled cost and network size. Inset: linear relation between costs of nodes and links that give rise to optimal solutions for the model, which is eighteen biennial periods.

The cost function is coupled with the spatial model in the following way. At every time step t the model accumulates the total cost C_T of nodes and links introduced in the network and compares it with C_A. If $C_T \geq C_A$, we consider that two years have passed. Each biennial period includes nodes and links introduced in several time steps. If C_n is the cost of a node and C_L is the cost of a line per unit length, C_T can be written as

$$C_T = C_n \sum n_t + C_L \sum L_{n_t} \tag{4.2.4}$$

where L_{n_t} is the total length of electric cable (i.e., length of all links) associated with the introduction of the node n_t at time step t.

Equations (4.2.3) and (4.2.4) are coupled in time. No a priori analytic process exists to obtain for each $b = [0,1,...,18]$ (eighteen biennial periods, from 1964 to 2000) an optimal pair C_n and C_L other than to sweep the costs parameter space. Figure 4.2.6, inset, shows the relation between node and link costs that give rise to precisely eighteen biennial periods. This optimal relation follows a linear fitting ($R^2 = 0,9977$) of the form

$$C_n = 0,05C_L + 0,0013 \tag{4.2.5}$$

Fig. 4.2.6, principal, shows the mean and standard errors of 1000 realizations of one cost combination: $C_n = 0,001$ and $C_L = 0,005$. Similarly to Fig. 4.2.5, inset, both modeled accumulated normalized cost and network size scale almost linearly over time. Although a slight deviation can be observed, the approximation given by this simple model is conspicuous.

The historical development of the power grid as a network begins at the end of the nineteenth century in the United States of America, with the incandescent bulb lamp invention, the so called War of the Electric Currents between DC defenders and AC early adapters and finally, with the massive adoption and commercial use of electricity (Klein 2008). From the 1940's on, the power grid has been worldwide operated and controlled by participants of increasing different types and (usually) opposite interests like governments, utilities, end users, etc. And yet it is possible to devise relatively simple models to understand its evolution and growth. As far as we know, this is the first time where these aforementioned complexities have been relatively explained, in this particular case, in terms of tradeoffs between space filling and economic resources.

4.3 Main points in review

The main points in review in this chapter are the following:

- Power grids are examples of spatial networks, where nodes can be precisely located on Euclidean space and edges do have characteristic and measurable lengths. These facts highly coerce their topology and constrain their evolution.
- French power grid's topology particularly follows two clearly differentiated and consecutive evolving processes. A first global space filling process and a secondary local meshing process that increases connectivity at a local level. We assume that these processes are followed by power grids in general. Since meshing processes increase fragility, as we have stated all through these pages, it seems plausible to affirm that global fragility arises when local efficiency and reliability increases.

- Although historical and technological events of many kinds can participate in the molding and forming of a network such as the power grid, its growth can be modeled as a tradeoff between resources and a spatial filling process. This is an example of how complex outcomes are usually deeply rooted in simple assumptions.
- Complex networks can only be understood when man simultaneously consider the three dimensions that characterize its meaning. Namely: (1) structure, (2) dynamics and (3) evolution. It is our hope that this three-fold path will help us in finding new ways to tackle the optimal, organic (rather than hierarchical) design and modeling of this type of networks.

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5

Conclusions. Towards sustainability

At the beginning of this PhD Thesis, several questions were formulated. These were mainly concerned with failures, collapse and the definition of a reliable structure for a future power grid. Our goal during these pages has been to try to answer them by means of a new conceptual framework: complex networks theory. This framework has allowed us to detect evidences that relate topological structure with fragility and probability distributions of major events that suggest particular failure dynamics not observed until now in power grids. Although some answers have been partially answered, much more questions have appeared along the way. This is what usually happens when new concepts are applied to old objects.

Nikola Tesla's living goal was influenced by an evolutionary perspective and pragmatic considerations: he wanted to devise mechanical means to avoid needless tasks of physical labor so that humans could spend more time in creative endeavors. In this way, cultural evolution would proceed at ever faster rates (Seifer 1998). Technological evolution has proceeded indeed at an incredible rate since Tesla's times. If we consider technology as a form of and embodied in society's culture, then these were prophetical words. But the truth is that those mechanical means which Tesla spoke of have not evolved graciously in a neutral influence zone but rather the contrary. Although technology has given many advantages and well being to mankind, it has also predated and polluted the environment, and it has been developed mainly thanks to very inefficient processes, wasting resources all along its historical path.

Now that this work is finishing, we are still asking ourselves if a sustainable grid design process could ever be devised. As we have seen all this way through, topology, dynamics and long term design play a decisive role in the definition of an efficient, robust and sustainable power system. Defining the way these concepts have to mingle and interweave within the sustainable paradigm is the next and essential step in this research.

5.1 Power grids and the sustainability paradigm

The proper definition of a power grid from a sustainable point of view has to be built on economical, technological, environmental and, last but not least, social facets (WCED 1987; Cendra and Stahel 2006). Although much effort has been put on the optimization of the technological and economical sides of infrastructure projects (EU 2003; Butler 2007; EU 2008), environmental and especially social impacts require much more attention due to their potential capacity in destabilizing society's structure (Nel·lo 2003). As far as the power grid is concerned, the social opposition to sitting of new facilities[1] (even from renewable resources) or transmission lines[2] can be considered only the tip of the iceberg of a deeper and much more complex problem of energy equity and responsibility, where regions become attractive as consumers while others deserve no respect as merely producers of energy vectors and other commodities (Illich 1995; Augé 2001). From this point of view, sustainability assessments must deal with usually contradictory perspectives. On one side the economical and technological criteria that rule the effectiveness and reliability of infrastructures from a more local perspective; and on the other the social and environmental impacts which belong to a much more global and long term development panorama (Stahel *et al.* 2009). One way to tackle these contradictions has been paved by post-normal science in an attempt to characterize a methodology of inquiry that is appropriate for cases where "facts are uncertain, values in dispute, stakes high and decisions urgent" (Funtowicz and Ravetz 1993). The sustainability science rests in deciphering which priority do we give to which of these criteria to avoid long term inner contradictions among them. In this sense, the contribution of this PhD Thesis may offer new criteria to help elucidating uncertainties and favoring decisions in conflictive situations.

Although an infrastructure's topology (i.e., number of elements and their connectivity) depends highly on its substrate (i.e., the physical ground where it develops and grows), most technical considerations relay on prevailing economic and energy developing models. In this sense, the interaction between human and infrastructure systems exhibits complexities similar to those of coupled human and natural systems (Liu *et al.* 2007), such as nonlinear dynamics with thresholds (i.e., blackout or social response condition appearance) and heterogeneity (i.e., transport, distribution and low voltage network elements and lines), that make an adequate assessment of future *problematiques* [3] a rather difficult task. An "old" network like the power grid presents legacy effects as well, considered as those impacts of prior infrastructure-human coupling on later conditions, the most important among these being its own topological structure, based on economies of scale and centralized power production.

The simile, though, can not be wholly performed since coupled human and natural systems exhibit complexities actually completely absent in coupled infrastructure and human systems. One of these is resilience, defined originally as a system's capability to renew and sustain specified conditions or processes in spite of exogenous disturbances or changes in driving forces (Folke 2006). Resilience appears to be a perspective rather than a concept. It emerged historically from a stream of ecology that addressed ecosystems dynamics to become more recently a *motto* to explore social processes like knowledge-system integration and adaptive capacity and governance. It emphasizes the necessity to accept uncertainty and surprise as part of the game and learn to manage by change, rather than

[1] A Google search for Catalan words such as *plataforma*, *no*, *tèrmica*, *eòlica*, etc., is an easy way to realize the actual social opposition against energy infrastructure projects.

[2] http://www.nomat.org/. (Last visited, June 2009).

[3] The term *problematique* here is used in the sense proposed by the Club of Rome; that is problems of global and long term impact.

simply to react to it. Resilience can be quantified by means of studying temporal scales and their interrelations with spatial scales and spatial heterogeneity (i.e., slow and fast temporal variables). It stands in sharp contrast with engineering resilience, focused on maintaining efficiency of function, constancy of the system and a predictable world near a single steady state.[4] While the latter studies behavior near a stable equilibrium (i.e., a global steady state) the former deals with the boundary of a domain of attraction which is an unstable equilibrium, reflecting behavior of complex *adaptive systems* (Kauffman 1993; Holland 1995; Levin 1999). A complex *adaptive system* consists of heterogeneous collections of individual agents (i.e., functional groups) that interact locally, and evolve in their genetics, behaviors or spatial distributions based on the outcome of those interactions. The distribution of functional groups and their response diversity within and across scales enables regeneration and renewal following disturbance over a wide range of scales. The resilience of a complex adaptive system it is not therefore simply about resistance to change and conservation of existing structures but also about the opportunities that disturbances open in terms of recombination of evolved structures and process, renewal of the system and *emergence* of new trajectories.

In this sense resilience is an approach, a way of thinking that I personally consider as a valuable context for the integrative analysis of interacting social, natural and engineered systems like energy infrastructures. Many sustainable prospective scenarios developed during the last decade foresee highly heterogeneous and decentralized energy systems for the future to come, completely penetrated by renewable resources (Lovins 2002; Pacala and Socolow 2004; Lovins 2008). Due to the localized and unpredictable nature of renewable energy on one hand and energy markets liberalization processes undertaken by many countries on the other, power grids thus will have to manage increasing amounts of energy under severe economic, technical and environmental constraints, and considering different time scales. Interactions between these fast variables and slow response times, such as these involved in sitting new lines and facilities, can not be properly assessed without a comprehensive approach such as this. The resilience approach would have to pave the way for answering critical questions like, for example:

- How should distributed generation be localized and connected in order to properly inject the right amount of power at the right time and at the right voltage?
- Will it be possible to devise safe structural recombination processes, in terms of connections and disconnections, in order to isolate or reconnect parts of the grid when needed?
- How can we include risk assessment and evaluation when disturbance processes of any kind are taken place? And once risk is assessed, how can we effectively protect critical components and the global electric network from natural or malicious attacks?

[4] Resilience is the name given also to network fragility analysis of the type performed in Chapter 2.

Efforts have been made towards this direction, defining for example the necessary conditions for a grid to be self-healing (Amin 2001) or establishing the need for new communication means to speed up detection and technical response to local failures in order to avoid global disturbances.[5] These are indeed remarkable steps taken to surmount our increasing energetic system complexities. Yet these projects are still at fundamental development stages, being the aforementioned legacy effects derived from ancient economies of scale and centralized power production and distribution really hard barriers to overcome. Although these effects condition dialectics between innovative and traditional systems, heuristic models of nested adaptive renewal cycles put forth recently emphasize that disturbance is part of development, and that periods of gradual change and periods of rapid transition coexist and complement one another. (Gunderson and Holling 2001)

One citation at the beginning of this PhD Thesis states that " [...] the affluence [of wealth] recently acquired by the technological societies [...] has not brought about any comparable growth of human mental capacity to comprehend their over-all complexities" (Singh 1966). This was written by the Indian science popularizer Jagjit Singh (1912 – 2002) in 1966, a time when cybernetics, neural networks and pioneering concepts about cognition and knowledge emerged, posing the scientific world at the verge of a new and seemingly promising conceptual framework. Although this new framework has helped mankind to overcome problems of many kinds, particularly related to control and systems theory, it has not been the miraculous science that Singh, and many others, expected it to turn into. More than forty years since, we still face the same questions yet many more have appeared. And we keep on devising new frameworks to help us to understand the world we are living in.

The following paragraphs summarize implications for the sustainability paradigm that can be drawn from this PhD thesis's results. Although many complexities have still to be unveiled, it is my hope that these particular achievements can be useful to understand our current power grid and to devise means to define a more sustainable one in the near future.

- *Efficiency, cost and grid design*. All through these pages we have shown evidences that relate power grid's structural properties and its dynamic behavior. Networks with more meshed and connected topologies tend to accumulate more failures than that of their counterparts of the same size but with a more randomly generated connectivity. Planning overhead or underground transmission lines can not be therefore a question of identifying optimal route alternatives solely from a local environmental point of view but also considering global better topologies. The definition of a "better" or "optimal" topology is not an easy task. Although nowadays electric transmission line routing processes are based on several perspectives such as built environment (protecting people, places and cultural resources), engineering requirements (minimizing costs and schedule delays) and natural environment (protecting water resources, plants and animals), they tend nonetheless to identify the best route for an electric transmission line on a local

[5] http://www.smartgrids.eu/. (Last visited, January 2009).

level and to forget a more global topological perspective. Solely from the cost engineering point of view, a tendency to use less meshed network topologies should decrease overall transmission lines building and sitting costs on one hand, and operating costs on the other, for resistance losses would diminish as well.

Network	Average path length
Lattice	$\ell \approx N/2\langle k\rangle$
Random	$\ell \approx \log N/\log\langle k\rangle$
Scale Free	$\ell \approx \log N/\log\log N$

Table 5.1.1 Average path length ℓ evolution with size for three types of networks: lattice, random and scale free. N is the size of the network and $\langle k\rangle$ the mean degree. (Costa *et al.* 2007)

As Table 5.1.1 shows, *average path length* scales linearly with number of nodes in a lattice network while it scales only logarithmically in a random network. The scale free topology would be even better since it has a systematically shorter average path length than a random graph. But this is a difficult topology to attain in power grids due to the existing spatial limitation. Thus, and although it is a bit early to assure that this new definition of the power grid's topological facet would improve its overall efficiency, we believe the minimization of the average path length by means of more randomly designed topologies would add robustness to its structure and reduce the overall cost of the grid. As it has been stressed before, this topological optimization process should always be balanced with social and environmental long term impacts assessment, for "sustainable topologies" could stand in clear contradiction with, or even against, sustainable development. Recall that, in fact, no such thing as "sustainable" (or unsustainable) technology can be defined: sustainability is a property of the whole system, not solely of one of its parts. (Cendra *et al.* 2009)

- *Blackout risk assessment and distributed generation*. In Chapter 3 the analysis of the intrinsic dynamics of series of major failures for the UCTE has been presented. In contrast with some literature that considers power grids as self-organized systems our results make it difficult to accept the existence of such an equilibrium point near criticality for the European power grid. In fact we can clearly observe two regimes: a power law tail for $x \geq \hat{x}_{\min}$ and a logarithmic distribution of events for $x < \hat{x}_{\min}$. How this complex system dynamics impacts the assessment and mitigation of blackout risk is an extremely important although non trivial question (Dobson *et al.* 2002). Although extremely difficult to assess, risk can be simply defined, from an engineering point of view at least, as the product of the probability of an occurring event and its impact, losses or simply stated, cost (OECD 2003). In power grids, cost derived from major failures is the summation of direct (amount of power interrupted and duration) and indirect (social disorder, induced economic and environmental losses, etc.) costs. While there is no clear methodology to estimate the latter,[6] the former can be crudely expressed as a multiple of unserved energy. If we consider $p(E)$ as the probability of a major event with unserved energy E, and $c(E)$ as its associated cost, the risk R of a major failure can be written as

$$R \approx p(E)c(E) \tag{5.1.1}$$

For $x \geq \hat{x}_{\min}$ the UCTE data indicates a moderate power law scaling of major events frequency with unserved energy and total loss of power. This can be expressed as

$$p(E) \approx E^{-\alpha} \tag{5.1.2}$$

where α is the scaling exponent. Considering a value of $\alpha \cong 1{,}6$ for the UCTE and $c(E)$ roughly a linear function of E, (5.1.1) can be rewritten to express R as

$$R_{x \geq \hat{x}_{\min}} \approx E^{-0{,}6} \tag{5.1.3}$$

that indicates a moderate decrease in risk as blackout size increases. On the other hand for $x < \hat{x}_{\min}$ the UCTE data shows a logarithmic power law scaling of major events frequency with unserved energy and total loss of power. This can be expressed as

$$p(E) \approx a - \log E \tag{5.1.4}$$

where a is a higher limit for the maximum probability value. With the same considerations as before, R can be written as

$$R_{x < \hat{x}_{\min}} \approx Ea - E\log E \tag{5.1.5}$$

For values $a \leq 1$ and in the limit range $0 < x < \hat{x}_{\min}$, (5.1.5) indicates that risk peaks for blackouts of some intermediate size and decreases logarithmically for larger blackouts making the probability for blackouts with size higher than the risk peak vanishingly small. Thus $\hat{x}_{\min}$ separates two behaviors: one with blackout risk probability comparable to network size (i.e., power law regime) and another with blackout risk probability with a characteristic peak and logarithmic decay. Although risk assessment is difficult to assess, particularly for the electric power system (Gheorghe *et al.* 2006) and the assumption of a cost linearly dependent on E

[6] And they can be much higher than the direct costs.

is particularly bold if not inaccurate, our findings would suggest an appropriate scale for network size and distributed generation. This electrical paradigm has been highly discussed in the last years and no clear methodology exists to exactly define the limiting size or power generation of a distributed power grid (Willis and Scott 2000). Indentifying a boundary condition such as $\hat{x}_{\min}$ that separates different risk evaluation zones would help to define a network's size where blackouts do not have algebraic probability distributions. A word of much caution is in order here for two reasons. On one side we have seen that major events probability distributions in power grids strongly depend on the structure and the overall connectivity scheme of the composing elements. We can not therefore straightly conclude that a reduction in the number of buses[7] and lines in order to attain a size where major events' risk scale logarithmically will generate a similar failure probability distribution than the original one. The development of a good dynamic model to understand these deep correlations is capital (see *Extended topological analysis*, in section 5.2). On the other side, recall indirect costs can be much higher than direct ones. With a clear methodology to estimate them, the aforementioned results would surely vary.

- *Long term infrastructure design process*. The historical development of technological networks, from water pipes to Internet, through telegraph, telephone or electricity itself, follows always the same three-period trend (Dupuy 1996): (1) an initial period of slow motion and limited growing rate, when only a small amount of users are reached by or can afford the new development; (2) an intermediate period of exponential growth, when the innovation has been accepted and adapted by the general public; and (3) a last mature period when growing limits have been reached and no more development is needed, apart from that which improves the efficiency and secures structural integrity by means of connecting the network with other networks. This characteristic process is translated into a sigmoid-shaped curve that relates time and network's growth not only for an innovation implementation but also for technical enhancements, as it is the case shown in Chapter 4 for the French power grid voltage upgrade evolution. Sigmoid developing process characterizes the growth of many complex adaptive systems as well (Levin 1999). But while the latter base their evolution mainly on the outcomes of local interactions and aggregation processes, similar for example to those followed by cities (Batty and Longley 1994), the former usually relies on centrally managed decision schemes. These decision processes are guided by two consecutive objectives. The first is *global*: to reach as much territory with as less time and cost as possible by means of long transmission lines. The second is *local*: once this first objective has been achieved a meshing and connecting process begins in order to guarantee the grid's reliability by means of attaining the

[7] The electrical name given to a node in an electrical network.

minimum (N − 1) criteria (Willis 2004). We have stated that fragility seems to arise when meshing process are at play. And for the French grid this is basically what happens when the first global objective has been accomplished. From an evolutionary perspective thus global fragility would arise when local robustness is increased by means of more highly connected topologies. Recall the resilience of a complex adaptive system is not simply about conservation of existing structures but also about the opportunities that disturbances open in terms of renewal of the system and emergence of new trajectories. One such new trajectory would be the redefinition of the topology and size of a grid once a specified local connectivity level has been reached. If this embrittlement process could be devised sufficiently in advance, difficult decision processes such as islanding and distributed network design could be improved as well. This would have to influence the processes involved in the long term design of infrastructures in general, and electricity in particular.

5.2 Ongoing and future research

During the work on this PhD Thesis many collateral research paths have been devised. Some of them have been clearly discerned and I believe they will be fruitful in the near future. Some others, though promising, will require a considerable amount of effort for only clearing the weeds that stand at the beginning of our way. Yet all of them are too wide in scope to be properly treated within the space and time of doctorate studies.

The following items form the ongoing research paths that are actually completing the several results presented within these pages.

- *Reasons and impact of failures*. As we have stated in Chapter 3, every major failure in UCTE has a recorded cause. These can be broadly segregated into overloads, failures, external and other or unknown reasons. The percentage of major events due to the aforementioned reasons and for Europe as a whole, fragile and robust sets are shown in Fig. 5.2.1 (a). A first distinctive trait is that cumulated major events for the UCTE power grid motives other than overloads are notably more meaningful for the whole grid and for the groups considered. Failures, external impacts and even unknown reasons stand for more than 95% of the reasons triggering the European power grid main events, while overloads hold a mere 3,5%. This numbers would clearly support the view that considers major events in power grids the consequence of inadequate management, actions of inexperienced operators and outdated surveillance methods, rather than the lack of upgrading processes to meet an increasing demand. On the other hand, UCTE data segregates those parts of the system that have failed: production system or transmission system; and for this last, the particular element that has failed, being it a transformer, a substation or a line. Fig. 5.2.1 (b) shows data as a percentage of major events that have been caused by each part or element of the system. For the

UCTE as a whole, and in general terms, most of the major events are generated as malfunctions in the transmission network while the production system seems relatively secured. When we segregate the European grids into robust and fragile sets, remarkable dissimilarities arise. In fragile networks it is substations that suffer the most, while it is an extremely vulnerable production system that rules major failures in the robust group. Percentage of major events resulting from malfunctions in transformers and lines are similar in both groups.

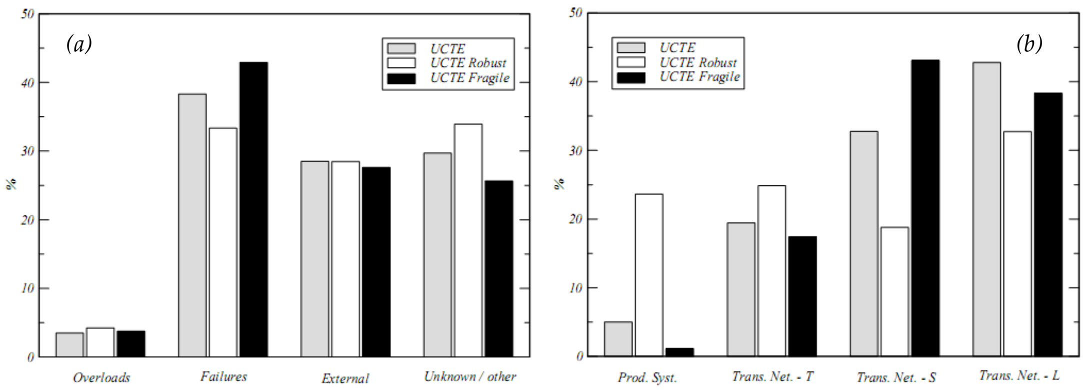

Fig. 5.2.1 Percentage of major events by (a) reason and (b) system. Reasons can be due to overloads, failures, external impacts and unknown reasons. Systems are two: production and transmission network. Within this last system, three elements are considered: transformers (T), substations (S) and lines (L). Percentages are given for robust and fragile sets and UCTE as a whole.

A possible explanation can be attributed to size (considered as the number of stations, substations and alike), since grids in the fragile group, sharing as much power and energy as grids in the robust group, need twice as much nodes. This fact would support the former hypothesis that relates robustness with random topologies, less meshed and less clustered and, therefore, with less substations and minor size. But this would be half of the story, since we can not give a plausible explanation for the production system vulnerability in the robust set. The explanation of these behaviors is now a main point in our research. It is also arresting the percentage of unknown reasons shown in Fig. 5.2.1 (a), which reaches a noticeable 30%. It is difficult to conceive an accurate failure assessment of a system when such a percentage of reasons are obscure.

- *Nested subgraphs and fragility*. Be it by the presence of *hubs*, some degree of assortativity[8] or another correlation among a network's observables, the internal organization of most complex systems displays some degree of hierarchical organization. This organization can be explored simplifying the graph topology by

[8] Assortativity refers to a preference for a network's nodes to attach to others that are similar or different in some way.

means of particularly destructive methods. One of these, known as *K-scaffold*, is the subgraph obtained by choosing all the nodes with degree $k \geq K$ and the nodes that, despite having degree $k < K$, are connected to a node e' whose $k' \geq K$ (Corominas-Murtra *et al.* 2007). In some cases, this methodology has defined a functionally meaningful subgraph, with evolutionarily and functionally related subsets of nodes (Rodriguez-Caso *et al.* 2005). Is the aim of this research to apply the *K-scaffold* methodology to the power grid with the objective of finding, if any, meaningful subsets of nodes characterized by significant empirical observables, like for example those nodes that have been stroke by a major failure more than two, three..., n times. The validation of this methodology in the case of an infrastructure network would signify a remarkable advance in detecting functionally related subsets of nodes or subgraphs.

- *Extended topological analysis*. As it has been stated in the previous chapters, the complex network approach, although most useful, can not wholly and properly capture the many physical properties and operational constraints of power systems. When a network bus or line disappears, topological fragility analysis does not take into account the instantaneous redirection of flows, power generation reallocation or variations in transmission losses. New metrics have been lately defined in order to subsume extended topological metrics that include power flow paths and operational limits dependencies. Among these, net-ability (Arianos *et al.* 2009; Bompard *et al.* 2009) and entropic degree (Bompard *et al.* 2009) seem most promising. The net-ability of a transmission grid is a measure of its ability to function properly under normal operating conditions. In calculating this ability, the maximum (real or apparent) power that can be allowed to flow over lines, the distribution of flow among them and their impedance are taken into account. By performing static fragility analysis similar to that presented in Chapter 2, critical transmission lines are labeled as those that cause a higher relative drop in net-ability. The entropic degree, on the other hand, expands the concept of degree presented in Chapter 2 to take into account (a) the strength of connection in terms of the weights (i.e., lengths) of the edges, (b) the number of edges connected with the vertex and (c) the distribution of weights among the edges. It can provide direct quantitative measurements of the importance of the buses. The aim of this research, done in collaboration with the electrical engineering faculty of the Politecnico di Torino in Italy, is to apply this new metrics to corroborate the fragility results obtained for the UCTE. In case of positive correlation, results would help us to validate the complex network topological approach here presented. On the contrary, it would spur the research on devising new efficient metrics to characterize power grid's robustness that include both topological and electrical concepts at the same time.

As we have said, some collateral research paths found on our main way seem promising though they will require some more effort to be properly characterized. The following entries define these long term research items that we hope will be useful and fruitful in the near future.

- *Spatial point patterns*. All data, and especially for geographical networks, have a more or less precise spatial and temporal label attached to them. Data that are close together in space (and time) are often more alike than those that are far apart. To analyze possible interdependencies there exist statistical tools that allow the identification of dynamical correlations between data. Although this research paradigm applies to some narrowly defined problems in the environmental sciences, it can also be applied to any process that typically exhibits strong spatial, temporal, and exogenous variability for which control may not be possible (Cressie 1993). This leads to a statistical methodology that is based fundamentally on hierarchical modeling: at each level of the hierarchy, simple conditional models are built (local modeling) and the result is a joint model that can be very complex but for which analysis is possible (global analysis). The development of a spatial statistical model that incorporates, for example, the population spatial variation into a stochastic generating mechanism would be an interesting tool for dynamic spatial load forecasting and long term power grid design. (Willis 2004)

- *Layered and multiscale networks*. Electricity transmission network is only a part of a major network that includes distribution and even a bigger one that includes low voltage and final consumption sites. The dynamical and topological relations established among these networks depend strongly on spatial and temporal scales that need to be properly defined. The main obstacle here is accessibility to reliable data since distribution and low voltage network's data are usually owned by local utilities and this is normally considered reserved and confidential. On the other hand, many complex systems may be described not by one, but by a number of complex networks mapped one on the other in a multilayer structure (Kurant and Thiran 2006). Metrics defined over graphs such as *degree* or *betweenness*, that characterizes a node by the number of shortest paths that pass through it, can be used together with flow patterns measures in order to define a layered complex system. The interactions and dependencies between these layers cause that what is true for a distinct single layer does not necessarily reflect well the state of the entire system. Recall that with this methodology the mapping between physical (i.e., real) and logical (i.e., flows) layers is still made by topological means. Thus, for electricity networks this would be an intermediate approach, more closely related to pure topological analysis than the extended topological analysis presented before.

- *Allometry*. Allometry is the study of the relationship between size and shape and has been particularly fruitful in differential growth rates statistical analysis of parts of a living organism's body (West *et al.* 1997). Allometric scaling relations[9] are characteristic of all living organisms. They can be derived from general models that describe how essential materials are transported through space-filling branching fractal networks. Although mainly observed in the structure of organisms evolved via natural selection, this optimal hierarchical branching networks have been found to characterize as well some engineered systems like microprocessors (Moses *et al.* 2008). On the other hand, it has been shown that many diverse properties of cities, from patent production and personal income to electrical cable length and consumption, are shown to be power law functions of population size with scaling exponents that fall into distinct universality classes (Bettencourt *et al.* 2007). As far as the electricity network is concerned, it is observed that (a) whereas household consumption scales approximately linearly with size and (b) economies of scale (i.e., sublinear scaling) are realized in electrical cable lengths, total electrical consumption scales superlinearly. This difference can only be reconciled if the distribution network is suboptimal (as it is observed with the superlinear scaling of resistive losses). The pace of social life in the cities does increase with population size. But different scaling relations arise depending on whether this growth is fueled by innovation and wealth creation (characteristic of social interactions) or material economies of scale (characteristic of infrastructure networks for example). This difference suggests that, as population grows, major innovation cycles must be generated at a continually accelerating rate to sustain growth and avoid collapse. We are asking whether there is a way to design power grids with optimal scaling relations and if biology can help us to establish the equilibrium between these two dynamics in engineered systems like transport, energy and information infrastructures. If it exists, the tradeoff between wealth creation and efficiency should be found in order to properly design sustainable networks since they are the primary determinant of urban growth.

5.3 One last thought

We are doomed to live with these, as Joni Mitchell poetically states, *electric scabs*. *These lesions* help mankind to evolve and survive. Our comfort and our very existence depend upon this *wonderful* agent electricity, as more than a century ago was foreseen by Tesla. At his very late years he stated:

> "*Day after day I asked myself what is electricity and found no answer. Eighty years have gone since and I still ask the same question, unable to answer it.*" (Seifer 1998)

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[9] Including the famous "three quarters" power law for metabolic rates.

It seems now quite a long ago when I decided to ask myself as well what electricity is. And as Tesla realized, I do not know anything about it either. The path that I have taken has not always been easy. And considering my last statement, one might think that it has been even fruitless. But in all these five years, happy and profitable days have exceedingly surmounted those less favored. I have been incredibly lucky to be in an also incredible pluridisciplinary research group, where ideas and opinions born from a particular discipline can move around from one area of knowledge to another, making the process an astounding productive one. This is quite uncommon and rare in a scientific world that clearly goes in the opposite direction, leading to speciation of disciplines and narrow-minded opinions. I have been also lucky to know, talk and live with scientist from other disciplines that clearly challenge the actual exacerbation of rational thought. And in doing so I have discovered that between black and white there lay many different shades of grey.

With this last thought I would like to stress a personal believe that has been taking form during these years. And this is the imperious need to work in transdiciplinary groups (Max-Neef 2005). As Max-Neef points out, none of the complex problems that surround us can be adequately tackled from the sphere of specific individual disciplines. And I think this PhD Thesis is quite a good example: new ideas coming from seemingly unrelated areas applied to old objects give rise to new and unexpected results. On the other hand, complex networks are still in its infancy. Many other measures can be (and will be) devised in order to characterize complex systems. For me this has been and I am sure will keep on being the most exciting journey. But it would not have been possible without being dragged into the particular research dynamics of the group were I have been.

Although transdisciplinarity implies an epistemological challenge rather difficult to delineate, for it is a more systemic and holistic manner of seeing the parts and the whole, its practical side is clearly defined. It seeks the coordination between hierarchical levels such as empirical, pragmatic or normative ones. It does not deny the necessity of disciplines but rather their contingency in order to appropriately tackle the problems of our post-normal era (Funtowicz and Ravetz 1993). *Contraria sunt complementa*[10] motto in Niels Bohr's (1885 – 1962) coat-of-arms can not have here a more precise meaning: although apparently in contradiction when faced alone, disciplinarity and transdisciplinarity complement each other as well. And both these approaches have to be considered, depending on the questions we plan to answer.

No doubt about it, this is another sustainable step to take if we want to conceive a better world and, quoting again Joni Mitchell's words, *learn from past mistakes*.

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[10] Opposites are complementary.

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Appendix A

Power grid data sets

The analysis of a network is mainly based in graph theory. In mathematics a *graph* is an abstract representation of a set of objects where some pairs of the objects are connected by links. The interconnected objects are represented by mathematical abstractions called *vertices*, and the links that connect some pairs of vertices are called *edges*. There exist many different types of graphs: directed (i.e., when edges point to vertices), undirected, weighted (i.e., when edges have *weights* in order to signal differences in fluxes or interaction intensities), etc. Besides, some of these graphs exist only in an abstract "network space", where the precise positions of the network nodes have no particular meaning. But many others, such as the Internet or the power grid, live in the real Euclidean space of everyday experience, with nodes and edges having well-defined positions and lengths. In this last sense, the definition of a power grid as a *geographical* (or *geometrical*) *graph* crosses different stages of completion. The three main steps, from raw to mathematically manageable data, include:

1. Obtaining physical and topological data of vertices and edges.
2. Introducing and referencing data in some Geographic Information System (GIS).
3. Programming and sequencing mathematical algorithms to analyze the graph.

Two real power grid datasets have been used so far: UCTE and RTE (see following sections). The process of converting them into graph entities has allowed to finally using them as the backbone of this PhD Thesis.

A.1 UCTE data set

The Union for the Co-ordination of Transmission of Electricity (UCTE) coordinates the operation and development of the electricity transmission grid from Portugal to Poland and from the Netherlands to Romania and Greece. Over more than fifty years, UCTE has been issuing all technical standards for a co-ordination of the international operation of high voltage grids, providing electricity supply for 430 million people in one of the biggest electrical synchronous interconnections worldwide. UCTE provides as well comprehensive statistics on electricity generation and transmission in the European mainland.

Although the UCTE Interconnected Network Map[1] shows plants, stations, existing high-voltage overhead lines and those under construction, for voltages of 110 kV to 400 kV and higher (if these lines cross national frontiers), it can not be acquired in digitalized form (at least, as far as we know). In order to be able to work with it, the whole map was digitalized

[1] http://www.ucte.org/resources/uctemap/. (Last visited, June 2009).

and introduced in geographical information software, where nodes and edges were referenced and correctly positioned in UTM coordinates.

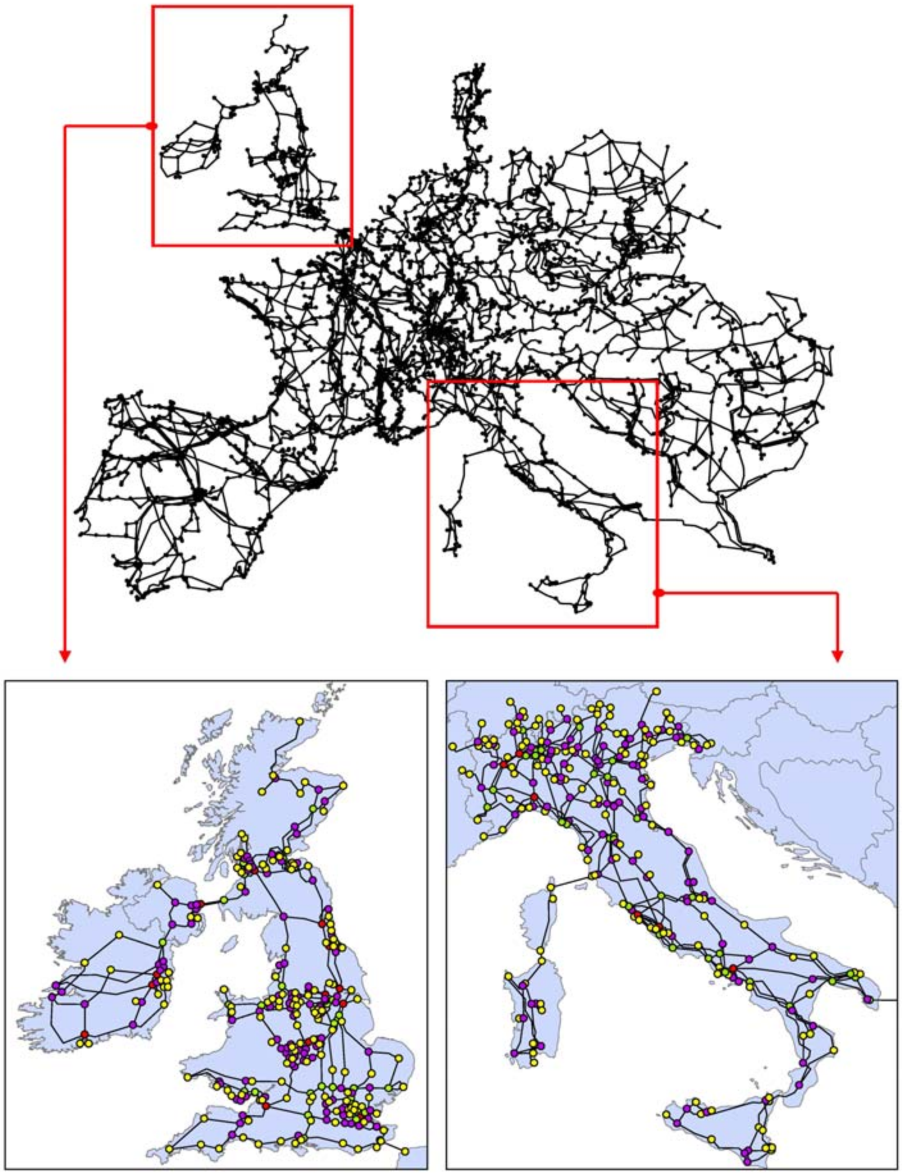

Fig. A.1 General GIS layout of the UCTE electricity network with snapshots for the United Kingdom and Ireland (bottom left) and Italy (bottom right). Colour indicates the degree of every node: 1 to 2 links, yellow; 3 to 4 links, purple; 5 to 6 links, green; and 7 to 8 links, red.

GIS packages allow the user to manipulate data in several ways. On one hand, plants, stations and transmission lines can be distinguished by means of their features, spatial relationships between features, and other thematic relationships (Fig. A.1). On the other hand, as GIS are built using formal objects and models that describe how they are located in space, a development platform can be used to create and manipulate all kind of data. This is a basic requirement in order to develop the algorithms needed to analyze the obtained graphs. The UCTE data contains more than 3 000 nodes act as stations, substations, transformers and generators, connected by some 200 000 km of high voltage lines (up to

4 300 edges approximately). Excel and Pajek[2] files containing geographical and topological data of the UCTE network can be found online at:

http://www.ct.upc.edu/termodinamica/UCTEdata

A.2 RTE data set[3]

Though included in the UCTE, the *Gestionnaire du Réseau de Transport d'Electricité*, the entity that manages the French power grid, offers the possibility to study its evolution in time, from year 1946 up to year 2006.[4] Similarly as with the UCTE graph, the different maps have been digitalized and introduced in the geographical information software, where nodes and edges were referenced and correctly positioned in UTM coordinates. Excel and Pajek files containing geographical and topological data of the RTE network can be found online at

http://www.ct.upc.edu/termodinamica/RTEdata

[2] http://vlado.fmf.uni-lj.si/pub/networks/pajek/

[3] http://www.rte-france.com/

[4] http://www.rte-france.com/htm/fr/CEM_HTML/transport/historique-reseau-400kv.jsp

Appendix B

Glossary[1]

Adaptive System. Set of interacting or interdependent entities, real or abstract, forming an integrated whole that together are able to respond to environmental changes or changes in the interacting parts.

Agent Based Model. Computational model for simulating the actions and interactions of autonomous individuals with a view to assessing their effects on the system as a whole.

Artificial Life. Field of study and an associated art form which examine systems related to life, its processes, and its evolution through simulations using computer models, robotics, and biochemistry.

Average Path Length. Network topology metric defined as the average number of steps along the shortest path for all possible pairs of network nodes.

Betweenness. Centrality measure of a vertex within a graph (also within an edge) that quantifies its appearance on shortest paths between other vertices.

Cellular Automaton. (Plural: cellular automata). Discrete model consisting of a regular grid of cells, each in one of a finite number of states, such as "On" and "Off".

Chaos Theory. Physics theory that deterministically describes the behavior of certain dynamical systems, that is, systems whose states evolve with time, that may exhibit dynamics that are highly sensitive to initial conditions (popularly referred to as the butterfly effect).

Closeness. Centrality measure of a vertex within a graph defined as the inverse of the average distance that separates it from all other nodes.

Clustering Coefficient. Network topology metric defined as the number of closed triplets (or 3 x triangles) over the total number of triplets (both open and closed) of a graph. It quantifies how close a node's neighbors are to being a clique (complete graph).

[1] Mainly retrieved and adapted from *Wikipedia, The Free Encyclopedia* (http://en.wikipedia.org/).

Complex System. Large group of relatively simple components, with no central control, that exhibit self-organization and emergent non trivial properties.

Complexity (***quality***). The quality of possessing *Emergent Properties*.

Complexity (***quantity***). The amount of information a particular system represents to the observer.

Connectionism. Set of approaches in the fields of artificial intelligence, cognitive psychology, cognitive science, neuroscience and philosophy of mind, that models mental or behavioral phenomena as the *Emergent* processes of interconnected networks of simple units. There are many forms of connectionism, but the most common forms use neural network models.

Criticality. Usually referred to *Self-organized Criticality*. A property of (classes of) dynamical systems which have a critical point as an attractor. Their macroscopic behavior thus displays the spatial and/or temporal scale-invariance characteristic of the critical point of a phase transition, but without the need to tune control parameters to precise values.

Cybernetics. Interdisciplinary study of the structure of regulatory systems, closely related to control theory and *Systems Theory*.

Degree. In graph theory, the number of edges incident to that vertex.

Degree Distribution. In graph theory, the probability distribution of degree over the whole network.

Deterministic Chaos. See *Chaos Theory*.

Emergence. In philosophy, systems theory and science, emergence is the way complex systems and patterns arise out of a multiplicity of relatively simple interactions. The phenomenon of *Emergent Properties*.

Emergent Properties. Properties of a system at the *Semantic Level* that are not *Entailed* at the *Syntactic Level*.

Entail. To logically imply something.

Feedback. The situation when output from (or information about the result of) an event or phenomenon in the past will influence the same event/phenomenon in the present or future.

Fractal scaling. Scale invariance characteristic of self-similar (i.e., fractal) objects.

Fragility. See *Robustness*.

Genetic Algorithm. A search technique used in computing to find exact or approximate solutions to optimization and search problems.

Graph. See *Graph Theory*.

Graph Theory. Mathematical study of graphs, that is mathematical structures consisting of atomic nodes linked by connections called edges used to model certain collections of objects.

Highly Optimized Tolerance. A method of generating power law behavior in systems by including a global optimization principle. It has been used to generate and describe internet-like graphs, forest fire models and may also apply to biological systems.

Hub. Highly connected node, usually in the context of *Scale-free Networks*.

Mean Degree. In graph theory, the average number of edges incident to vertexes.

Mean Field Theory. Physics theory based on replacing all interactions to any one body with an average or effective interaction. This reduces any multi-body problem into an effective one-body problem, allowing some insight into the behavior of the system at a relatively low cost.

Motif. Pattern (i.e., topological form) that recur within a network much more often than expected at random.

Percolation Theory. It describes the behavior of connected clusters when the probability of establishing a link is modified. A percolation threshold exists when we can find a path linking one node to any other.

Phase Transition. Process that transforms the properties of some medium by means of a controlling parameter. Phase transitions occur frequently and are found everywhere in the natural world.

Power Law. Polynomial relationship that exhibits the property of scale invariance.

Preferential Attachment. Any of a class of processes in which some quantity is distributed among a number of individuals or objects according to how much they already have.

Random Network (or Graph). A graph that is generated by some random process, usually obtained by starting with a set of n vertices and adding edges between them at random.

Rich-get-richer Mechanism. See *Preferential Attachment*.

Robustness. Quality of being able to withstand stresses, pressures, or changes in procedure, circumstance or, in graph theory, topology.

Scale-free Network (or Graph). A graph whose degree distribution follows a power law (at least asymptotically). A power law distribution is a common scale free distribution.

Self-organization. Process in which the internal organization of a system, normally an open system, increases in complexity without being guided or managed by an outside source.

Self-organized Criticality. See *Criticality*.

Semantic Level (or Space). Space of meanings for any description.

Small-world Network (or Graph). Type of mathematical graph in which most nodes are not neighbors of one another, but most nodes can be reached from every other by a small number of hops or steps.

Subgraph. See *Motif*.

Syntactic Level (or Space). Language in which a description is specified: letters of the alphabet, genetic code, laws of theoretical physics, as appropriate.

Systems Theory. Interdisciplinary field of science by which one can analyze and/or describe any group of objects that work in concert to produce some result.

Threshold. The quantitative point at which an action is triggered.

Universality. Property of *Phase Transitions* by which different systems often possess the same set of characteristic parameters, known as critical exponents.

Appendix C

Articles

C.1 *Topological vulnerability of the European power grid,* **International Journal of Bifurcation and Chaos (Volume 17, Issue 7, July 2007). Special Issue "Complex Networks' Structure and Dynamics".**

C.2 *Robustness of the European power grids under intentional attack*, **Physical Review E, Vol. 77, 026102 (2008).**

C.3 *Assessing power grid reliability by means of topological measures*, **Wessex Institute of Technology, Transactions on Ecology and the Environment, Vol. 121, p. 515-525 (2009).**

C.4 *Major events distribution in the European power grid*, **(2009). International Journal of Electrical Power and Energy Systems (submitted).**

International Journal of Bifurcation and Chaos, Vol. 17, No. 7 (2007) 2465–2475

TOPOLOGICAL VULNERABILITY OF THE EUROPEAN POWER GRID UNDER ERRORS AND ATTACKS

MARTÍ ROSAS-CASALS*, SERGI VALVERDE and RICARD V. SOLÉ†
ICREA-Complex Systems Lab.,
Universitat Pompeu Fabra,
Dr. Aiguader 80, 08003 Barcelona, Spain
**Càtedra UNESCO de Sostenibilitat,*
Universitat Politècnica de Catalunya,
EUETIT-Campus Terrassa, 08222 Barcelona, Spain
†Santa Fe Institute, 1399 Hyde Park Road,
NM 87501, USA

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We present an analysis of the topological structure and static tolerance to errors and attacks of the September 2003 actualization of the Union for the Coordination of Transport of Electricity (UCTE) power grid, involving thirty-three different networks. Though every power grid studied has exponential degree distribution and most of them lack typical small-world topology, they display patterns of reaction to node loss similar to those observed in scale-free networks. We have found that the node removal behavior can be logarithmically related to the power grid size. This logarithmic behavior would suggest that, though size favors fragility, growth can reduce it. We conclude that, with the ever-growing demand for power and reliability, actual planning strategies to increase transmission systems would have to take into account this relative increase in vulnerability with size, in order to facilitate and improve the power grid design and functioning.

Keywords: Complex networks; small world; power grid; fragility.

1. Introduction

Mostly evolved over the last hundred and fifty years, technical infrastructures, from telegraph [Standage, 1998] to Internet [Pastor Satorras & Vespignani, 2004], are the canvas where almost every aspect of our economy and society is portrayed. From a broader historical perspective, networks of energy, transportation and communication constitute the very foundation of all prospering societies, as the western culture actually knows them. Being usually managed by different kinds of actors (often with different objectives), formed by a huge quantity of heterogeneous components (spatially distributed and connected) characterized by complex interdependencies and relations, the study of these technological systems deserves attention in order to assure, essentially, structural integrity, efficiency and reliable supply.

In recent years, one particular kind of network has received much attention: the power grid. Hailed by the US National Academy of Engineers as the 20th century's engineering innovation most beneficial to our civilization, the role of the electric power has grown steadily in both scope and importance during this time and electricity is recognized as a key to societal progress throughout the world, driving economy prosperity, security and improving the quality of life [Willis, 2004]. With similar pace, though, increasing frequency and size of malfunctions have raised general awareness about our real level of comprehension of these networks. In recent years, both the North American and the (once almost faultless) European grid systems have experienced numerous examples of such malfunctions in the form of cascading failures and blackouts [Venkatasubramanian, 2003; UCTE, September, 2003]. The explanations given by local, national and international electricity coordinating councils for most of these situations go from aspects related to low investment and maintenance, to those related to generation and demand inadequateness and, obviously, bad luck. But more than any, the most repeated explanation is that of a bad comprehension of the interdependencies present in the network [Watts, 2003; UCTE, 2004].

In this sense, advances in statistical physics, modeling and computational methods have stimulated the interest of the scientific community to study electric power grids as complex networks. In complex network theory, one type of analysis of such interdependencies already mentioned is usually done under the *robustness* (or, in the contrary, *vulnerability*) epigraph [Boccaletti *et al.*, 2006]. It refers to the ability of a network to avoid malfunctioning when a fraction of its constitutive elements is damaged. In technical infrastructures, this turns to be a field of elementary practical reasons since it affects directly the efficiency of the processes taking place in the network and it can give hints about the resilience of the grid. The analysis of the robustness of a complex system has been done, traditionally, from two points of view: *static* and *dynamic*. In a static robustness analysis, nodes are deleted without the need of redistributing any quantity transported by the network [Albert *et al.*, 2000; Crucitti *et al.*, 2003]. In a dynamic robustness analysis, nodes are deleted and the flow or load carried by them must be distributed over the rest of the remaining network [Moreno *et al.*, 2002; Motter & Lai, 2002; Crucitti *et al.*, 2003; Kinney *et al.*, 2005]. At first glance, the theoretical approach to these two types of robustness seems quite similar, but while the static one can be analytically treated, the dynamic one must be, almost always, numerically solved.

In this letter, the static robustness of the European and most of the European countries and regions electricity transport power grids are investigated. Their tolerance to random loss (failures) and selective removal (attacks) of the most connected nodes is analyzed. In order to simplify its topological representation, a simple model of the power grid data is introduced. Final results and some features worth to notice are discussed in the last part of the letter.

2. European Power Grid Data

In this paper, the vulnerability of the September 2003 actualization of the Union for the Co-ordination of Transmission of Electricity (UCTE) map has been analyzed.[1] UCTE associates most of the continental Europe national power grid operators in order to coordinate the production and demand of some annual 2,300 TWh and 450 million customers from 24 countries. The map gives data from the transmission network (voltage levels from 110 kV to 400 kV) and ignores the much more extended distribution one. Nonetheless, it deals with more than 3,000 nodes (generators and substations) and some 200,000 km of transmission lines.

For more than fifty years UCTE has coordinated the international operation of high-voltage European countries' grids to ensure adequate balances between offer and demand through national frontiers. It operates one of the largest electric synchronous interconnections worldwide in order to optimize the use of installed capacities and reduce the economic cost of power outages. But more than this, the UCTE transmission network has been shaped by those national policies and decisions that, for the last one hundred years, have been seeking economic prosperity, security and quality of life of its inhabitants. From that point of view, and differently from previous examples considered in the literature [Watts, 1999; Albert *et al.*, 2004], those different power grids should be a good example of network evolution directed, at the same time, by technical, economical, political and, lastly, environmental decisions. Differentiated from country to country, we then would expect to find somehow different patterns and complex behavior for every country or territory considered.

[1] http://www.ucte.org

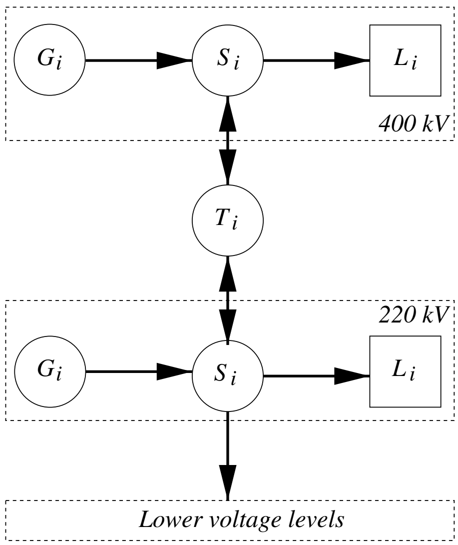


Fig. 1. An extremely simplified model for the transmission power grid: two voltage levels (400 kV and 220 kV for the European network) with generators G and loads L connected by switching stations S. Transformers T connect both tension levels in order to provide reliability, efficiency and control capacity.

In order to simplify the analysis of the structure of the European power grid, an idealized view has been adopted (Fig. 1). On one hand, transmission lines have been assumed bidirectional, as it should be in the electricity transport network, and identical, ignoring the voltage level variation between lines and other physical characteristics. Although we have different voltage levels, the transport network works as a whole, using transformers to increase or decrease voltage depending on time and space requirements, and it would not be suitable or realistic to split it into different voltage networks, as it has been done in some literature [Crucitti *et al.*, 2005]. On the other hand, although it is possible to distinguish four different kinds of elements, namely generators, transformers, switches (considered as stations or substations of any kind) and, finally, end line points, all these elements have been treated identically in order to avoid, at this initial point of the study, those difficulties involved in their differentiation and dynamical behavior characterization.

Bearing these assumptions in mind, five different data sets have been analyzed:

- UCTE as a whole.
- UCTE, United Kingdom and Ireland as a whole.
- UCTE, country by country, plus United Kingdom and Ireland.
- Geographically related regions (Iberian Peninsula, Ireland as an island and England as an island).
- Traditionally united or separated regions (formers Yugoslavia, Czechoslovakia and Federal and Democratic Republics of Germany).

Until this time, and as far as we know, no such analysis has been done for the European power grid and with such depth of detail. A thorough analysis of these data sets will surely give hints of historical and geographical constraints that might have shaped the structure of the power grid from country to country, and from time to time. For example, from a geographical point of view, although neither United Kingdom nor Ireland belong to the UCTE, their isolated geography might have strongly configured and constrained their national power grids. Similarly, although Germany is actually united, the former frontier between Federal and Democratic Republics is still "visible" in the form of a very few transmission lines connecting the east and the west of Germany.

3. Small-World Feature of the Power Grid

The different data sets have been obtained after introducing their topological values, i.e. geographical positions of stations, substations and longitudes of lines, in a geographical information system (GIS) (Fig. 2). The national power grid for every country has been obtained from a typical GIS query: the selection of the part of the UCTE's network constrained by every country's frontier. So far, data analyses of 33 different networks have been performed.

Using the formalism of graph theory, any of these networks can be described in terms of a graph Ω, defined as a pair, $\Omega = (W, E)$, where $W = \{w_i\}$, $(i = 1, \ldots, N)$ is the set of N nodes and $E = \{w_i, w_j\}$ is the set of edges or connections between nodes. Here, $\xi_{i,j} = \{w_i, w_j\}$ indicates that there is an edge (and thus a link) between nodes w_i and w_j. Two connected nodes are called adjacent, and the degree k of a given node is the number of edges connecting it with other nodes. In this case, the UCTE graph, Ω_{UCTE}, is defined as

$$\Omega_{\mathrm{UCTE}} = \bigcup_{i=1}^{n} \Omega_i \tag{1}$$

where Ω_i $(i = 1, \ldots, n)$ are the set of national power grids analyzed.

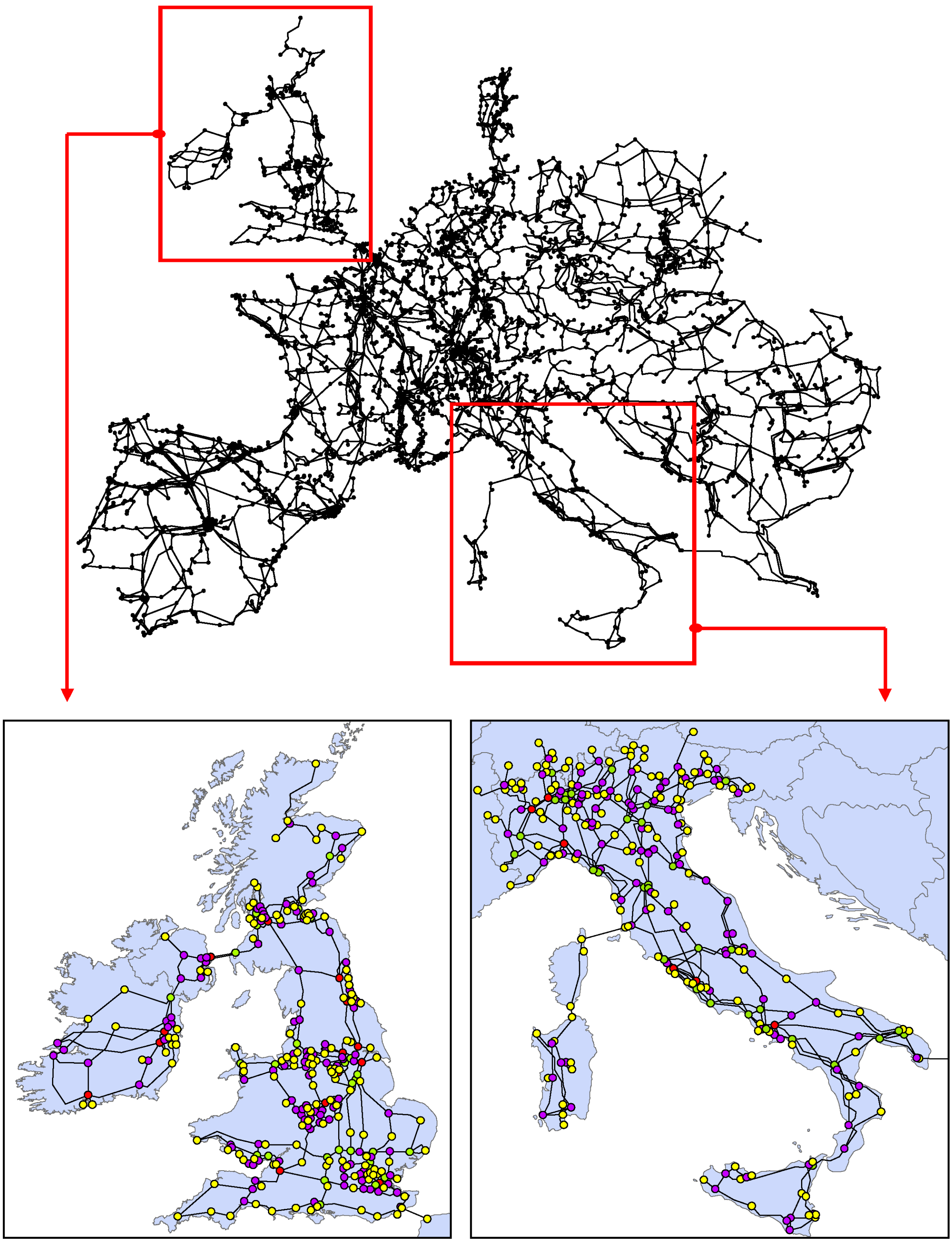


Fig. 2. The European electricity transport network (September 2003 actualization of the Union for the Co-ordination of Transmission of Electricity, UCTE, map) offers the upper topological image, where more than 3,000 nodes act as stations, substations, transformers and generators, connected by some 200,000 km of high voltage lines (up to 4,300 edges approximately). A closer look gives a more accurate perspective of some national power grids: United Kingdom and Ireland (bottom left), and Italy (bottom right). Color indicates the degree of every node: 1 to 2 links, yellow; 3 to 4 links, purple; 5 to 6 links, green; and 7 to 8 links, red.

As well as k, an additional property to be considered is the degree distribution $P(k)$. This is defined as the (normalized) probability that a node chosen uniformly at random has a degree k or, similarly, as the fraction of nodes in the graph having k edges. In this sense, it has been suggested that degree distributions can be classified in three types, namely exponential (gaussian or random), potential (scale-free) or some mixture of both, each one exhibiting different dynamic characteristics and adaptive behaviors [Amaral *et al.*, 2000]. Most of the real networks degree distributions follow a power law of the form $P(k) \approx k^{-\gamma}$ with the exponent γ being, mostly, between 2 and 3.

For the five different data sets presented in Sec. 2, the graph model used considers undirected and unweighted edges. Though every single network contains hundreds of stations, substations, transformers and thousands of km of energy transport lines, the results show a surprising unity in mean degree, very similar to those encountered in the literature [Watts, 1999; Albert *et al.*, 2004] for networks of the same size. The relation between nodes and links is constant and goes around $\langle k \rangle \cong 2,8$ for every network analyzed (Fig. 3). As it has been shown in the literature [Gastner & Newman, 2004], though a rigorous demonstration of planarity is still elusive, this result agrees with that of other so-called planar graphs like the US interstate highway network [Gastner & Newman, 2004], ant network of galleries [Buhl *et al.*, 2004] and urban networks [Buhl *et al.*, 2006]. This would suggest that, although every technical infrastructure has evolved and developed under different economical, political, historical and, luckily enough, environmental conditions and decisions, there should be some universal characteristics related, almost surely, to the spatial and technological constraints that rule the construction and evolution of such networks in order to give a common value for $\langle k \rangle$.

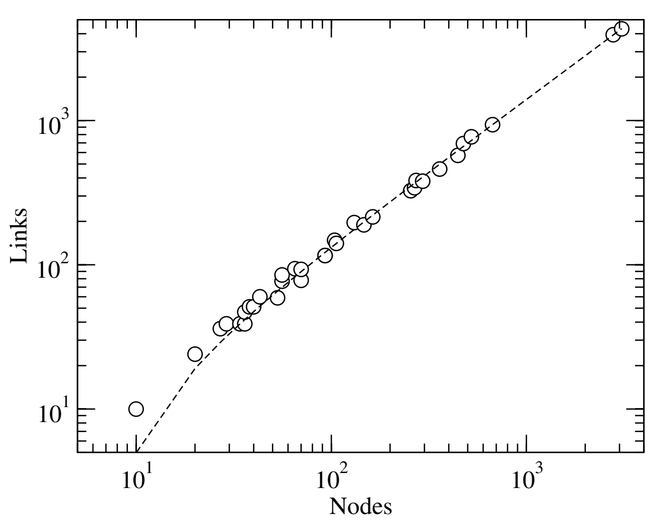


Fig. 3. Linear relation between nodes and links implies uniform mean degree value for every network analyzed, in spite of their different economical, political, historical and environmental evolution processes.

Every single network analyzed is uncorrelated (see below) and all of them follow an exponential cumulative probability degree distribution of the generic form

$$P_{k'>k}(k) = \int_{k'}^{\infty} P(k)dk \approx \exp\left(-\frac{k}{\gamma}\right) \tag{2}$$

The value adopted by the exponent γ of these single scaled distributions goes from a minimum of $\gamma_{\mathrm{UK}} = 0.91$ (United Kingdom, with $r^2 = 0.898$) to a maximum of $\gamma_{\mathrm{PT}} = 2.71$ (Portugal, with $r^2 = 0.989$). The γ exponent of the UCTE graph reaches a value of $\gamma_{\mathrm{UCTE}} = 1.78$, close to that of the North American power grid, as in [Albert *et al.*, 2004]. The mean value for the whole data sets analyzed is $\overline{\gamma} \cong 1.8$ [Fig. 4(a)].

The presence of degree correlations (namely, if connected nodes share common properties such as similar degrees) is conducted by measuring the average nearest neighbors connectivity of a node with degree k, i.e.

$$\langle k_{nn} \rangle = \sum_{k'} k' P_C(k'|k) \tag{3}$$

where $P_C(k'|k)$ is the conditional probability that a link belonging to a node with connectivity k points to a node with connectivity k' [Pastor Satorras *et al.*, 2001]. Since we have independence on k, i.e.

$$P_C(k'|k) = P_C(k') \approx k' P(k') \tag{4}$$

we thus have $\langle k_{nn} \rangle \approx$ constant [Fig. 4(b)]. Though no degree correlations has been found for the European power grid, for systems such as the Internet, such correlation exists and it is found that $\langle k_{nn} \rangle \approx k^{-\nu}$ with $\nu \cong 0,5$.

As well as the degree distribution, the small world (SW) feature has been used to characterize the topological structure of a network [Watts & Strogatz, 1998]. The mathematical characterization of the SW behavior is based on the evaluation of two basic statistical properties: the *clustering coefficient C*, a measure of the average cliquishness of

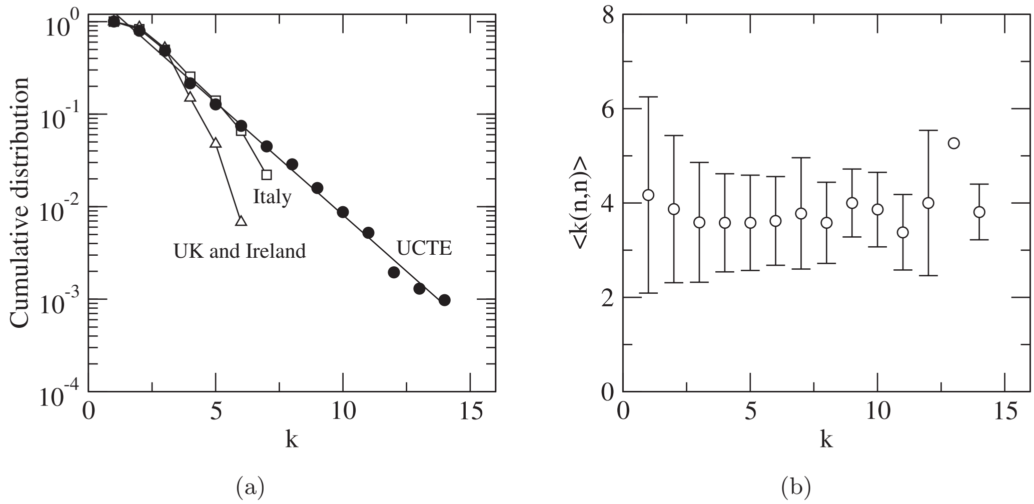


Fig. 4. Topological characteristics of the European power grid. (a) Cumulative degree distributions for the UCTE, (black dots, with exponential fitting), United Kingdom and Ireland (triangles) and Italy (squares) graphs. (b) Nearest neighbor degree correlation for the UCTE graph.

a node, and the *characteristic path length d*, a measure of the typical separation between two generic nodes in the network. On one hand, $\Gamma_i = \{j|\xi_{ij} = 1\}$ being the set of nearest neighbors of a node $w_i \in W$, the clustering coefficient for this node is defined as the number of connections between the components $w_j \in \Gamma_i$. By defining

$$\mathrm{Z}_i = \sum_{j=1}^{N} \xi_{ij} \left[\sum_{k\in\Gamma_i;j<k} \xi_{jk} \right], \tag{5}$$

we have

$$C_v(i) = \frac{Z_i}{\binom{|\Gamma_i|}{2}},$$

so that the clustering coefficient is the average over W,

$$C = \frac{1}{N}\sum_{i=1}^{N} C_v(i) \tag{6}$$

and measures the average fraction of pairs of neighbors of a node that are also neighbors of each other. On the other hand, $d_{\min}(i,j)$ being the minimum path length connecting two nodes $w_i, w_j \in W$ in Ω, we define the average path length of a given unit as

$$d_v(i) = \frac{1}{N}\sum_{j=1}^{N} d_{\min}(i,j) \tag{7}$$

and the path length for the graph as $d = \langle d_v(i)\rangle$. Characterized by small path lengths and high local clustering coefficient, the emergence of the SW phenomenon in some different real technological networks [Barabási & Albert, 1999; Watts, 1999; Ferrer i Cancho *et al.*, 2001; Albert *et al.*, 2004] indicates that their connection topology is neither completely regular nor completely random: small-worlds are indeed highly clustered, like regular lattices, yet having small characteristic path lengths, like random graphs.

Here, we use two predictions from random graph topologies in order to compare them against the observed topological patterns [Ferrer i Cancho *et al.*, 2001]: (1) the clustering coefficient over the average connectivity for a random graph follows an inverse scaling law with graph size:

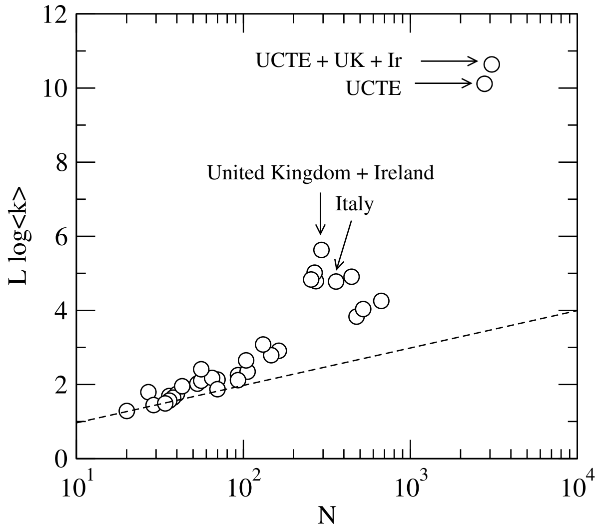
$$\frac{C^{\mathrm{rand}}}{\langle k\rangle} = \frac{1}{N} \tag{8}$$

and (2), the average path length scales logarithmically as

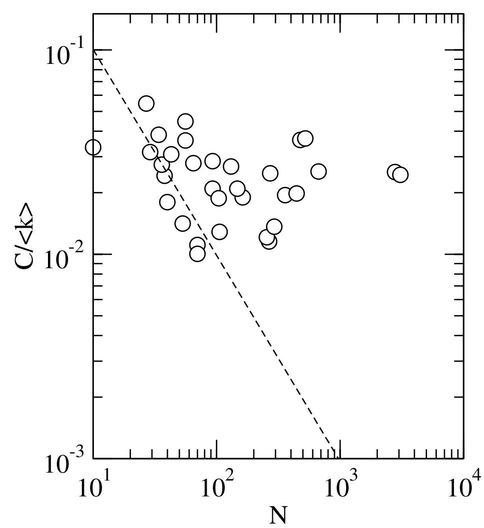
$$d^{\mathrm{rand}} \log\langle k\rangle \approx \log(N) \tag{9}$$

Figure 5 shows the values of $d\log\langle k\rangle$ and $C/\langle k\rangle$ compared to those of $1/N$ and $\log(N)$, respectively, for the 33 different power grids analyzed. It can be seen that $C/C_{\mathrm{rand}} > 1$ for most of the grids. Values of C/C_{rand} of more than one order of magnitude are achieved by the largest power grids while d/d_{rand} remains in the same order of magnitude for whatever size of the network. A similar pattern has been observed in electronic circuits [Ferrer i Cancho *et al.*, 2001].

From a structural point of view, every country's grid has evolved in order to connect production



(a)



(b)

Fig. 5. Small world patterns: (a) distance and (b) clustering for the power grids investigated. Real distance is corrected by a factor of $\log\langle k\rangle$ and clustering by $\langle k\rangle$. Dashed lines signal the expected values for random graphs. It can be seen that larger networks involve larger deviations from the random cases.

sites with consumption sites within its own borders. In small countries everything is at hand and long distance connections are not needed to expand the grid. On the contrary, in big countries (and consequently, with an increasing number of nodes) long distance connections become more and more necessary when connecting production and consumption. At the top UCTE level, the need to exchange energy between countries forces long distance connections to cross borders and to connect sites never connected before.

4. Static Tolerance to Errors and Attacks

The usual approach to the analysis of networks' static tolerance to errors and attacks seeks the relation between node deletion (without the need of redistributing any quantity transported by the network) and global connectivity (existence and relative size of the connected component, after such a deletion). An error simulation would be based on the random deletion of nodes while an attack simulation would be based on the deletion in decreasing order of the most connected (higher degree) ones. The experimental results for the 33 different power grid networks are shown in Fig. 6(b). Under random failure, simulations show a monotonical decrease of the relative network size of the connected component S with the increasing fraction f of nodes eliminated (orange circles). On the other hand, selective removal of the most connected nodes (blue dots), shows a much more dramatic size reduction of the connected component for the same fraction of nodes eliminated. This fact, in agreement with similar investigations done, for example, with ant galleries of networks [Buhl *et al.*, 2004] and street networks of urban settlements [Buhl *et al.*, 2006], clearly suggests different network behavior upon different forms of static deletion of nodes.

In addition to numerical results [Albert *et al.*, 2000; Motter & Lai, 2002; Crucitti *et al.*, 2003], the analytical approach to study tolerance to errors and attacks has been traditionally based in percolation theory. In this sense, the network percolates below a critical probability f_c related to the presence or absence of a specific number of edges. Its study can be then mapped into a standard percolation problem for errors and, with few modifications, for attacks as well [Boccaletti *et al.*, 2006]. Specifically for the static tolerance to errors, it has been shown [Molloy & Reed, 1998] that the condition for having a giant component S_∞ in a graph Ω is

$$\langle k^2\rangle - 2\langle k\rangle = \sum_k k(k-2)P(k) > 0 \qquad (10)$$

For randomly deleted nodes, it has been shown [Cohen *et al.*, 2000] that the critical fraction f_c is

$$f_c = 1 - \frac{1}{\left(\dfrac{\langle k^2\rangle}{\langle k\rangle} - 1\right)} \qquad (11)$$

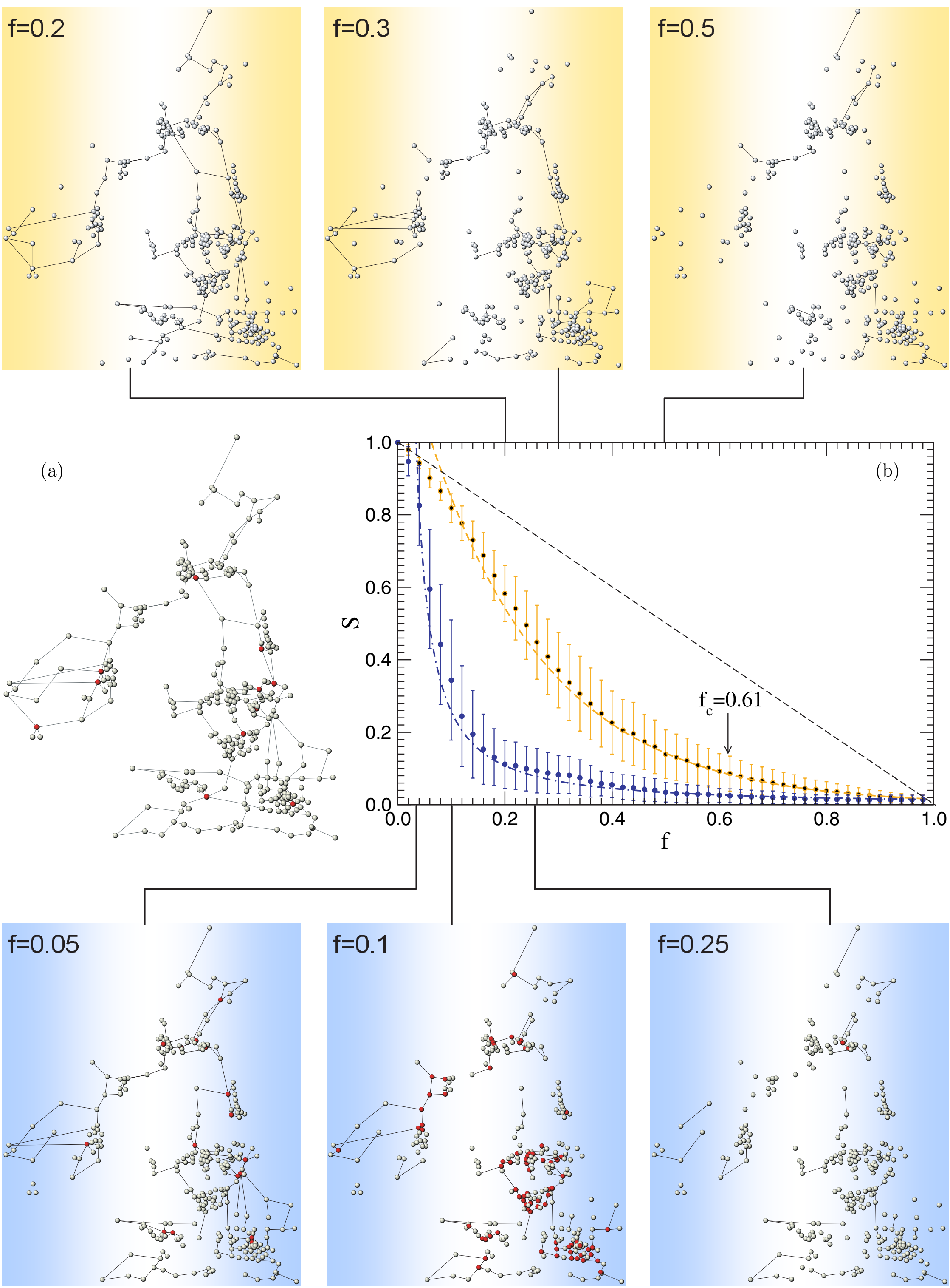


Fig. 6. Static tolerance to failures and attacks for the 33 networks analyzed (United Kingdom and Ireland power grid graph taken as an example). (a) United Kingdom and Ireland power grid original spatial graph, with the most connected nodes highlighted in red. (b) Static tolerance to random (orange) and selective (blue) removal of a fraction f of nodes, measured by the relative size S of the largest connected component for every network analyzed (with an analytically found critical fraction $f_c = 0.61$ for the random case). Whiskers stand for the standard deviation. For the sample power grid (United Kingdom and Ireland), snapshot figures illustrate three random (upper orange) and three selective (lower blue) experimental results. For the United Kingdom and Ireland power grid, a progressive random removal of nodes gives a completely disconnected graph when a fraction $f_c \cong 0.5$ is reached, while selective removal of the most connected ones causes the grid to reach this limit sooner, for $f_c \cong 0.25$ (in this last case, nodes highlighted in red note those prone to disappear at the next time step).

Considering the exponential degree distribution of the European power grid [Eq. (2)], we have $\langle k \rangle = \gamma$ and $\langle k^2 \rangle = 2\gamma^2$, and thus

$$f_c = 1 - \frac{1}{2\gamma - 1} \tag{12}$$

For $\overline{\gamma} \cong 1.8$, we have a predicted value $f_c \approx 0.61$. The experimental values of f_c for the random removal of nodes and for the different data sets analyzed are shown in Fig. 7.

As we can see, the value of the critical fraction remains quite invariable and independent of the network size, as it should be for exponential degree distribution networks, and in complete agreement with the predicted value of f_c. An equivalent study for the case of static tolerance to attacks will be presented elsewhere.

As it has been stated previously, the degree distributions of the different European power grids analyzed are exponential. That means that they are not like the highly skewed scale-free distributions typically found in other complex networks. In scale-free networks, the degree distribution follows a power law, where a very few nodes have many connections and most nodes have few connections. Instead, planar networks in general, and the European power grid in particular, display less skewed exponential or uniform degree distributions.

Networks with highly skewed link distribution characterized by power laws appear very sensitive to losing those highly connected nodes (or *hubs*), while being relatively robust to randomly losing the more highly abundant, less connected ones. In contrast, random networks with Poisson degree distribution, which are relatively unskewed, since nodes have similar number of connections, like power grids, should display similar responses to random and selective removal of nodes [Albert *et al.*, 2000]. The results presented insofar suggest, as it has been done for food webs [Solé & Montoya, 2001; Dunne *et al.*, 2002], that networks with exponential degree distributions would be, in fact, sensitive to different types of static node removals, more similar to scale-free networks than random or gaussian ones. In a nutshell: *exponential, but not that much.* In spite of this, these behaviors [Fig. 6(b)] seem to correlate well with an exponential function of the general form

$$S = \alpha \exp(-\beta f) \tag{13}$$

where S is the relative size of the connected component and f is the fraction of nodes removed.[2]

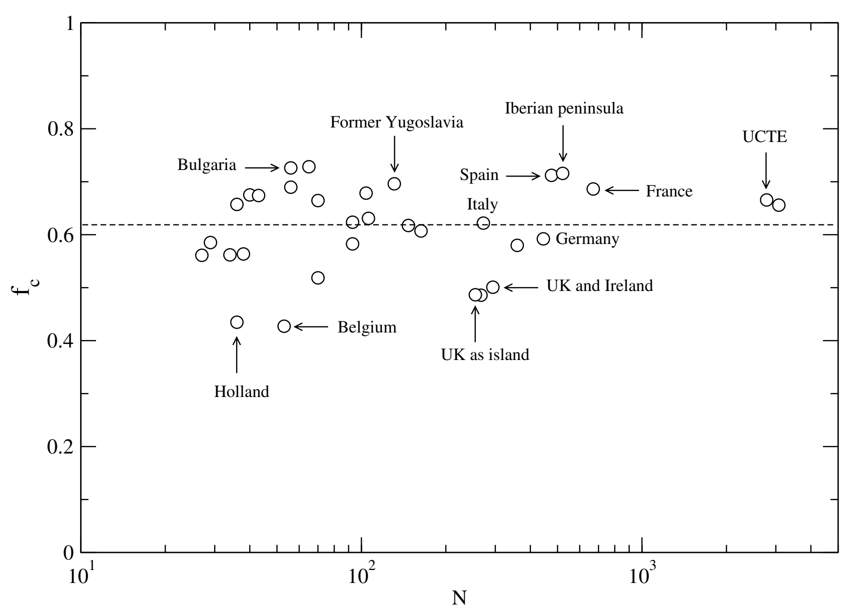


Fig. 7. Experimental values for the random removal critical fraction f_c for every data set analyzed, as a function of the network size N. All values move around the predicted critical fraction $f_c = 1 - [1/(2\gamma - 1)]$ (dashed line).

[2]The higher is the value of β, when the function is less skewed.

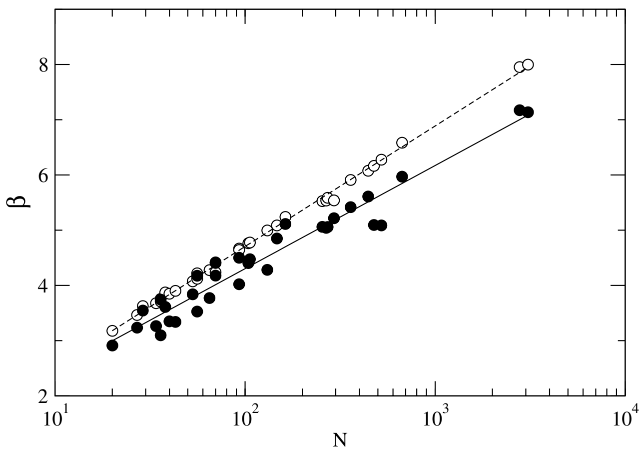


Fig. 8. Static tolerance to random (open circles) and selective (filled circles) removal of nodes, plotted as a function of the network size N and the β value of the exponential functions that correlate those behaviors from Fig. 4. For the selective case (filled circles), the subtle though obvious dispersion from the linear fitting could stand for its behavior different from that of a random removal, observed in Fig. 6(b).

The relation between β and the size of the networks has been plotted in log–linear axes, in Fig. 8. As can be seen, as the size of the network increases, the value of the β exponent that better fits Eq. (6), increases at the same time. Quite intuitively, as more and more elements are introduced in the network, more prone is the system to failures, whether they come from selective or random removal, and its fragility increases as well. The more counterintuitive result arises from the fact that the increase in the value of β is logarithmic with the size of the network. In the case of random failure, the results are very well correlated ($r^2 = 0.99$) by the logarithmic function $\beta_r = \alpha_r \ln(N) + \theta_r$, with $\alpha_r = 0.95$ and $\theta_r = 0.34$, where β is the exponent of the exponential function that fits the results of Fig. 6(b) and N is the size of the network. The results in the case of selective attack, though offer a different observed response to deletion of nodes than that of a random removal, are also very well correlated ($r^2 = 0.95$) by the function $\beta_s = \alpha_s \ln(N) + \theta_s$, with $\alpha_s = 0.81$ and $\theta_s = 0.56$.

5. Discussion

The robustness of real-world networks to the random loss of nodes ("errors") and its fragility to the selective loss of the most connected ones ("attacks") has been attributed to extremely skewed power-law distributions of links found in many small-world networks [Albert *et al.*, 2000]. Our study shows that these responses are not unique to small-world, scale-free networks. Every single power grid studied, which have less skewed exponential degree distributions and often lack typical small-world topology, display similar patterns of response to node loss. Moreover, the difference to network response to errors and attacks appears related only to network size and not to other topological measures of network complexity such as mean degree or betweenness centrality, for example (data not shown).

The evolution of both, the static tolerance to random and selective removal of nodes, plotted as a function of the network size and the exponent value of the exponential functions that correlate their node removal behavior (Fig. 4), shows two immediate facts worthy of notice: fragility increases with the size of the network and it is clearly logarithmic. We might think, rather intuitively, that when more elements are present in a system, the higher the probability that it fails. But as far as we observe the relation between the *relative* size S of the largest connected component and the *fraction* f of nodes deleted, the results of these static simulations should exhibit similar behaviors, quite independent of the size of the networks. On the other hand, if we consider that the increase in size of the networks is a sign of spatial or temporal evolutions, the logarithmic behavior of the fragility with size would suggest that, though size favors fragility, evolution can, relatively, reduce it.

Recent newsworthy wide-area electrical blackouts and failures have raised many questions about the specifics of such events and the vulnerability of interconnected power systems. With the ever-growing demand for power and reliability, actual planning strategies to increase transmission systems lack basic information about the grid's complexity. One possible way to prevent propagation of disturbances is to design the system to allow for intentional separation into stable islands or interrupt small amounts of load [Madani & Novosel, 2005]. If grid's resilience to attacks and failures is somehow related to its size and dimensions, an accurate power grid reliability analysis would have to take into account its relative increase in vulnerability in order to finally give a minimal definition of this *stable island*. From a spatial point of view, the definition of a *geographical stable island* would facilitate and improve the treatment of several different aspects related to power grid design and functioning, ranging from deregulation to spatial load forecasting and maintenance.

Acknowledgments

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Robustness of the European power grids under intentional attack

Ricard V. Solé,[1,2] Martí Rosas-Casals,[1,3] Bernat Corominas-Murtra,[1] and Sergi Valverde[1]
[1]*ICREA-Complex Systems Lab, Universitat Pompeu Fabra, Dr. Aiguader 80, 08003 Barcelona, Spain*
[2]*Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, New Mexico 87501, USA*
[3]*Catedra UNESCO de Sostenibilitat, Universitat Politecnica de Catalunya, EUETIT-Campus Terrassa, 08222 Barcelona, Spain*
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The power grid defines one of the most important technological networks of our times and sustains our complex society. It has evolved for more than a century into an extremely huge and seemingly robust and well understood system. But it becomes extremely fragile as well, when unexpected, usually minimal, failures turn into unknown dynamical behaviours leading, for example, to sudden and massive blackouts. Here we explore the fragility of the European power grid under the effect of selective node removal. A mean field analysis of fragility against attacks is presented together with the observed patterns. Deviations from the theoretical conditions for network percolation (and fragmentation) under attacks are analysed and correlated with non topological reliability measures.

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I. INTRODUCTION

The power grid defines, together with transportation networks and the Internet, the most important class of human-based web. It allows the success of advanced economies based on electrical power but it also illustrates the limitations imposed by environmental concerns, together with economic and demographic growth: The power grid reaches its limits with an ever growing demand [1]. A direct consequence of this situation is the fragility of this energy infrastructure, as manifested in terms of sudden blackouts and large scale cascading failures, mostly caused by localized, small scale failures, ocurring at an increasing frequency [2,3].

The fragility of the power grid is an example of a generalized feature of most complex networks, from the Internet to the genome [4–8]. Specifically, real networks are often characterized by a considerable resilience against random removal or failure of individual units but experience important shortcomings when the highly connected elements are the target of the removal. Such directed *attacks* have dramatic structural effects, typically leading to network fragmentation [9–12]. This behavior has been studied for skewed power-law distributions of links, which are found in many small-world networks [13,14]. But recent studies have shown that similar responses are not unique to small-world, scale-free networks: Power grids, having less skewed exponential degree distributions and often without small-world topology, display similar patterns of response to node loss [15].

An additional feature of the power grid is its spatial structure. The geographic character of this network implies that a number of constraints are expected to be at work. Other well known spatially extended nets include the Internet [16], street networks [17], railroad and subway networks [18], ant galleries [19], electric circuits [20], or cortical graphs [21].

One fundamental aspect concerning the analysis of complex networks is the increasing evidence of mutual influence between dynamical behavior and topological structure. The topology of human contact networks, for example, determines the emergence of epidemics [22]; similarly, the correct dynamics in cellular networks are rooted in the topology of the regulatory networks [23,24]. Here we present evidence of a plausible relation between topological and nontopological reliability measures for the power grid, suggesting that topology might be capturing the robustness (or fragility) of the real system, when dynamics are at work. This evidence has been obtained analyzing the resilience of 33 different power grids: (a) The 23 different EU countries, (b) four geographically related zones (Iberian Peninsula, Ireland as island, England as island, and United Kingdom and Ireland as a whole), (c) four traditionally united or separated regions (former Yugoslavia, Czechoslovaquia and Federal and Democratic Republics of Germany), (d) continental Europe, and (e) continental Europe plus United Kingdom and Ireland.

The paper is organized as follows. In Sec. II the data set on European power grids is presented and their basic topological features summarized. In Sec. III we present both analytical and numerical estimations of the boundaries for network collapse under attack, using a mean field theoretical approach. Two classes of networks are shown to be present. In Sec. IV, evidence for correlation between these two classes and nontopological reliability indexes is shown to exist. In Sec. V we summarize our findings and outline their implications.

II. POWER GRID DATA SETS

Europe's electricity transport network is nowadays the ensemble of more than twenty different national power grids coordinated, at its higher level, by the Union for the Coordination of Transmission Electricity, UCTE (http://www.ucte.org). The distribution and location of transmission lines, plants, stations, etc., can be found in the last version (July 2007) of the UCTE Map. The different data sets analyzed here have been obtained after introducing the topological values (i.e. geographical positions and longitudes) of more than 3000 generators and substations (nodes) and 200 000 km of transmission lines (edges) in a geographical information system (GIS). The national power grid for every country or region has been obtained from a typical GIS query: the selection of the part of the UCTE's network con-

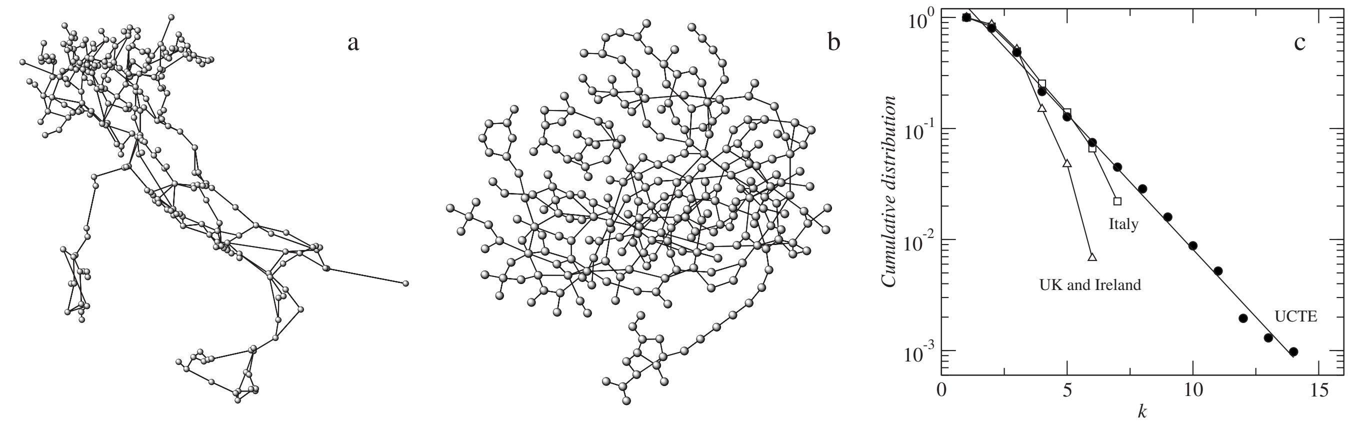

FIG. 1. Power grids define a spatial, typically planar graph with nodes including generators, transformers, and substations. Here we show (a) the geographical and (b) the topological organization of the Italian power grid. These webs are homogeneous, having an exponential degree distribution, $P(k)=\exp(-k/\gamma)/\gamma$, as shown in (c).

strained by every country's frontier. The power grid can then be formally described in terms of a graph $\Omega=(V,E)$. Here $V=\{v_i\}$ indicates the set of N nodes (transformers, substations or generators in our context). Figure 1 shows an example of such graphs with its geographical (a) and topological (b) structures, respectively. These nodes can be connected, and $E=\{e_{ij}\}$ indicates the set of actual links between pairs of nodes. Specifically, $e_{ij}=\{v_i,v_j\}$ indicates that energy is being transported between the nodes in the pair $\{v_i,v_j\}$. Our system can be analyzed at two main levels: The whole power grid Ω_{EU} including all countries within the EU and at the country level. If Ω_k indicates the kth power grid of one of the $n=33$ countries and regions involved, we have $\Omega_{\text{EU}}=\cup_{k=1}^{n}\Omega_k$.

The global organization of these webs has been previously analyzed [15], revealing a very interesting set of common regularities: (a) Most of them are small worlds (i.e., very short path lengths are typically present) and the larger webs display clustering coefficients much larger than expected from a random version of the network analysed; (b) they are very sparse, with an average of $\langle k\rangle=2.8$ over all the webs available (see Table I); (c) the link distribution is exponential: The probability of having a node linked to k other nodes is $P(k)=\exp(-k/\gamma)/\gamma$ [Fig. 1(c)]; and (d) these networks are weakly or not correlated. This exponential distribution is thus characterized by the constant γ which actually corresponds to the average degree (i.e., $\langle k\rangle=\gamma$).

Correlations were measured using the average nearest neighbor connectivity of a node with the degree k, i.e., the average $\langle k_{nn}\rangle=\Sigma_{k'}k'P(k'|k)$ where $P(k'|k)$ is the conditional probability that a link belonging to a node with connectivity k points to a node with connectivity k' [25]. For these webs, it was found that $\langle k_{nn}\rangle\approx$const, as expected if no correlations were present. This is a very useful property in our analysis, since makes mean field predictions valid in spite that we ignore the planar character of these networks, thus replacing the geographical pattern by a topological one. Nonetheless, as these webs are geographically embedded, some care needs to be taken (see [27] in connection with epidemic spreading).

III. ATTACKS IN EXPONENTIAL NETWORKS: MEAN FIELD THEORY

In our previous paper, we analyzed the effects of both random and selective removal of nodes on the EU grids [15]. Nonetheless, in that paper we were mostly interested in the average behavior of the networks analyzed (see Fig. 2). Here we want to extend these results to the analysis of the differences observed in EU power grids with the goal of interpreting the different patterns exhibited compared to the predictions from mean field theory on intentional attacks.

In order to compute the effect of random removal of nodes, we compute the percolation condition for the graph assuming it is sparse and uncorrelated. Let f be the fraction of removed nodes and $P(k)$ the link degree distribution of our graph. The damaged graph will be characterized by the following degree distribution $\mathbf{P}(k)$ [28]:

$$\mathbf{P}(k)=\sum_{i\geqslant k}^{\infty}\binom{i}{k}f^{i-k}(1-f)^kP(k). \tag{1}$$

Note that such an equation corresponds to the case when a fraction f of nodes are removed but it also holds when a fraction f of links are removed (or lead to unoccupied sites).

In order to study percolation properties, we use the standard generating function methodology. The two first generating functions of the damaged graph are

$$F_0(x)=\sum_{k}^{\infty}P(k)(1-f)x^k, \tag{2}$$

$$F_1(x)=\frac{1}{\langle k\rangle}\sum_{k}^{\infty}kP(k)(1-f)x^{k-1}. \tag{3}$$

The averages (i.e., the values at $x=1$) are $F_0(1)=F_1(1)=1-f$, respectively. Here $F_0(1)$ is the fraction of nodes from the original graph belonging to the damaged graph. Similarly, $F_1(1)$ is the relation among $\langle k\rangle$ and the average number of nodes from V that can be reached after deleting a fraction f of nodes. The generating function for the size of the compo-

TABLE I. A summary of the basic features exhibited by some of the European power grids analyzed, ordered by increasing γ, the exponential degree distribution exponent. The critical probability of node removal f_c is shown for both cases, theoretical and real, and random (errors) and selective (attacks) removal of nodes. The absolute difference $|\Delta f_c|$ between theoretical and observed critical probability diminishes as γ increases in general terms. Number of nodes N, number of links L, and mean degree $\langle k \rangle$ are also shown as reference. Countries in italics have been used to evaluate reliability indexes. EU results (i.e., results for the Ω_{EU} graph) are shown for comparative purposes.

		Errors			Attacks					
Country	γ	f_c^{theor}	f_c^{real}	$\|\Delta f_c\|$	f_c^{theor}	f_c^{real}	$\|\Delta f_c\|$	N	L	$\langle k \rangle$
Belgium	1,005	0,011	0,395	0,384	0,010	0,131	0,121	53	58	2,18
Holland	1,086	0,147	0,387	0,240	0,034	0,126	0,092	36	38	2,11
Germany	1,237	0,322	0,565	0,243	0,097	0,229	0,132	445	560	2,51
Italy	1,238	0,322	0,583	0,261	0,097	0,241	0,144	272	368	2,70
Austria	1,409	0,450	0,506	0,056	0,159	0,191	0,032	70	77	2,20
Rumania	1,418	0,455	0,579	0,124	0,162	0,238	0,076	106	132	2,49
Greece	1,457	0,477	0,492	0,015	0,174	0,183	0,009	27	33	2,44
Croatia	1,594	0,543	0,525	0,018	0,214	0,202	0,012	34	38	2,23
Portugal	1,606	0,548	0,595	0,047	0,217	0,250	0,033	56	72	2,57
EU	1,630	0,557	0,629	0,072	0,223	0,275	0,052	2783	3762	2,70
Poland	1,641	0,562	0,594	0,033	0,226	0,249	0,023	163	212	2,60
Slovakia	1,660	0,569	0,563	0,006	0,231	0,227	0,004	43	52	2,41
Bulgaria	1,763	0,604	0,570	0,034	0,256	0,232	0,024	56	67	2,39
Switzerland	1,850	0,629	0,610	0,020	0,275	0,260	0,015	147	186	2,53
Czech Republic	1,883	0,638	0,634	0,004	0,281	0,279	0,003	70	88	2,51
France	1,895	0,641	0,647	0,006	0,285	0,289	0,004	667	899	2,69
Hungary	1,946	0,654	0,617	0,036	0,295	0,266	0,029	40	47	2,35
Bosnia	1,952	0,655	0,588	0,067	0,295	0,244	0,052	36	42	2,33
Spain	2,008	0,668	0,689	0,020	0,307	0,324	0,017	474	669	2,82
Serbia	2,199	0,705	0,655	0,051	0,339	0,296	0,054	65	81	2,49

nents, other than the giant one, which can be reached from a randomly choosen node is

$$H_1(x) = f + xF_1[H_1(x)] \tag{4}$$

and the generating function for the size of the component to which a randomly choosen node belongs to is [26]

$$H_0(x) = f + xF_0[H_1(x)]. \tag{5}$$

Thus the average component size, other than the giant component, will be

$$\langle s \rangle = H_0'(1) = 1 - f + F_0'(1) \times H_1'(1). \tag{6}$$

After some algebra, we see that this leads to a singularity when $F_1'(1) = 1$. To ensure the percolation of the damaged graph, the following inequality has to hold:

$$\sum_k k(k-2)P(k) > \sum_k k(k-1)fP(k). \tag{7}$$

The above expression can be expressed as

$$\langle k^2 \rangle - 2\langle k \rangle > f(\langle k^2 \rangle - \langle k \rangle) \tag{8}$$

which leads to a critical probability of node removal f_c given by

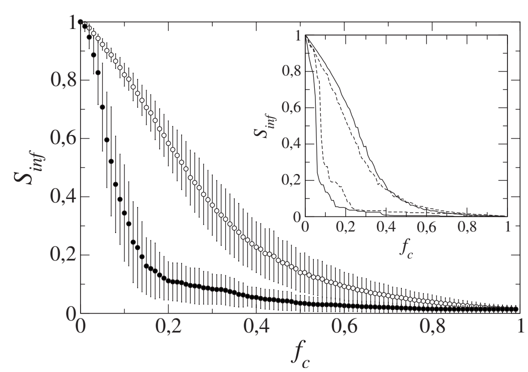

FIG. 2. Effects of attacks and failures on the topology of the EU power grids. Static tolerance to random (white circles) and selective (black circles) removal of a fraction f of nodes, measured by the relative size S_{inf} of the largest connected component. Whiskers stand for the standard deviation. Inset: Evolution of the static tolerance to random and selective node removal for Italy (dashed lines) and France (continous lines). Though in the case of random removal (failures) both networks exhibit a similar response, for the selective one (attacks), Italy behaves in a slightly stronger manner (i.e., for a fixed fraction of eliminated nodes, the relative size of the largest connected component in Italy always remains higher than that of France).

$$f_c = 1 - \frac{1}{\kappa_0 - 1}, \tag{9}$$

where $\kappa_0 = \langle k^2 \rangle / \langle k \rangle$, and in agreement with [28]. In our case, we have an analytic estimate $\kappa_0 = 2\gamma$. Using the average value $\langle \gamma \rangle = 1.9$, we obtain a predicted critical probability $f_c = 0.61$.

Although random removal is an interesting scenario, it considers chance events that are not correlated to network structure. Intentional attacks strongly deviate from random failures: Even a small fraction of removed nodes having large degrees has dramatic consequences. In order to predict the effects of such directed attacks on network structure, the critical probability associated to network breakdown can be computed. Here we follow the formalism developed by Cohen *et al.* [29]. Roughly speaking, this formalism enables us to *translate* an intentional attack into an equivalent random failure and study the problem in terms of standard percolation using Eq. (9). When the selective removal of the most connected nodes is considered, a fraction of order $\mathcal{O}(1/N)$ is removed by eliminating elements with a degree larger than a given $k=K$. This upper cutoff is then easily computed from the continuous approximation:

$$\sum_K^{\infty} P(k) \approx \int_K^{\infty} \frac{1}{\gamma} e^{-k/\gamma} dk = \frac{1}{N} \tag{10}$$

and the new cutoff $\tilde{K}$ can be obtained (again under a continuous approximation) from

$$\int_K^{\tilde{K}} \frac{1}{\gamma} e^{-k/\gamma} dk = \int_K^{\infty} \frac{1}{\gamma} e^{-k/\gamma} dk - \frac{1}{N} = p, \tag{11}$$

which gives (assuming K large enough) a new cutoff

$$\tilde{K} = -\gamma \ln p. \tag{12}$$

Following [29], we translate the problem of intentional attack to an equivalent random failure problem. The removal of a fraction f of nodes with the highest degree is then equivalent to the random removal of those links connecting the remaining nodes to those already removed. Thus, the probability that a specific link leads to a deleted node will be given by

$$\tilde{p} = \int_K^{\tilde{K}} \frac{kP(k)}{\langle k \rangle} dk, \tag{13}$$

$\langle k \rangle$ being the average degree of the undamaged graph. It is not difficult to show that this gives

$$\tilde{p} = \left(\frac{\tilde{K}}{\gamma} + 1 \right) e^{-\tilde{K}/\gamma}. \tag{14}$$

Using Eq. (12) it is straightforward to see that

$$\tilde{p} = (\ln p_c - 1) p_c, \tag{15}$$

where we assume that K is large enough to ignore the term $\exp(-K/\gamma)$. Thus an equivalent network with maximal degree $\tilde{K}$ has been built after a random removal of $\tilde{p}$ nodes due to the fact that the absence of correlations implies a random failure of links. In order to obtain the degree distribution of the damaged graph, such a failure can be introduced into Eq. (3). But this will be formally equivalent to the removal of $\tilde{p}$ nodes. Thus, to study stability properties, we only need the resulting probability $\tilde{p}$ to be introduced in the critical condition for percolation (9). Replacing $p_c = \tilde{p}$, we obtain

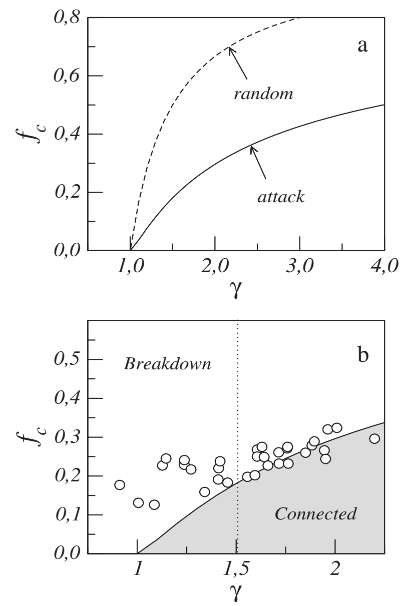

FIG. 3. (a) Phase space for exponential uncorrelated networks under random removal of nodes and directed attack towards highly connected vertices. Here γ is the average degree of the exponential network and f_c indicates the fraction of removed nodes required in order to break the network into many pieces. The upper curve is the critical boundary for network percolation under random removal of nodes. Below it, a network experiencing such random failures would remain connected (i.e., with a giant component). The lower curve corresponds to the critical boundary for attacks. In (b) we display the estimated values of $f_c(\gamma)$ for attacks from the thirty-three EU power grids (circles) to be compared with the mean field prediction (continuous line).

$$1 + (\ln p_c - 1) p_c = \frac{1}{2\gamma - 1}, \tag{16}$$

whose solutions (for each fixed γ) provide the conditions for network percolation under attacks. In Fig. 3 and Table I, we show the result of our calculations. As expected, a much lower value of f_c is required to break a power grid network through intentional attack.

Now we can compare this mean field prediction, evaluated as f_c^{theor}, with available data. Using the whole dataset of EU grids, we can estimate f_c^{real} for all EU countries. The result are shown, for both f_c's, in Fig. 3(b). As we can see, there is a very good agreement (given their small size) between observed (real) and predicted (theoretical) f_c values, but some nontrivial deviations are also obvious. We can see that aproximately for $\gamma > 1.5$ the expected f_c values are very similar to those predicted by theory. However, the power grids having lower exponents (when $\gamma < 1.5$) strongly deviate from the predicted values. These agreements and deviations are not due to some simple statistical trait, such as network size. As indicated in Table I, very large power grids are in both sides (i.e., the German and Italian grids are in the first group, whereas the Spanish and French ones belong to the second) and mixed with smaller ones. Although the effect of geography on the properties of some networks is important (see [27,30] for example), this last observation would suggest that the geographical embedding of these networks might have a small effect.

IV. CORRELATIONS WITH NONTOPOLOGICAL RELIABILITY MEASURES

The reliability of a power grid evaluates its ability to continuously meet demand under major events like overloads, general failures, external impacts and alike. At the engineering level, and due to the different dimensions of service quality involved in a power grid (i.e., consumers, companies, and regulators), reliability has been traditionally measured by different indexes as (a) the amount of energy not supplied, (b) the total loss of power, or (c) the equivalent time of interruption, which measures the number and duration of interruptions experienced by customers [31]. In this sense we would expect a correlation between the critical percolation fraction f_c, the exponent that characterizes the grids' cumulative degree distribution γ, and some of (if not all) these reliability indexes presented.

In order to explore the problem, three reliability indexes have been obtained from the UCTE monthly reliability measures [32]. They are related to four major events. Namely, overloads, general failures, external impacts and exceptional conditions, and finally other reasons (including unknown reasons). For every major event and transmission grid, the following indexes have been considered and normalized: (1) Energy not supplied, normalized by the gross UCTE electricity consumption; (2) total loss of power, normalized by the UCTE peak load on the third Wednesday of December; and (3) equivalent time of interruption (also known as average interruption time or AIT), which is the ratio between the total energy not supplied and the average power demand per year, measured in minutes per year (normalized by definition).

In order to avoid statistical deviations due to the limited historical data available (UCTE monthly statistics have been published only from January 2002 onwards), we have devided UCTE networks in two groups. Group 1 includes those countries whose critical breakdown probability f_c^{real} agrees with that predicted f_c^{theor} (i.e., countries with $\gamma > 1.5$). Group 2 includes those countries whose f_c^{real} deviates positively from f_c^{theor} (i.e., countries with $\gamma < 1.5$), with an expected more robust topology than that predicted.

Figure 4 gives the acummulated percentage values for the formerly presented reliability indexes and for each group of networks. As we can see, networks in group 1 (i.e., networks with $f_c^{\mathrm{real}} \cong f_c^{\mathrm{theor}}$) represent 63% of the whole UCTE nodes, they manage 48 and 51% of the UCTE energy and power, respectively, but acummulate 85, 68, and 79% of the UCTE average interruption time, power loss and energy not delivered, respectively. On the contrary, though networks in group 2 (i.e., networks with $f_c^{\mathrm{real}} > f_c^{\mathrm{theor}}$) represent a mere 33% of the whole UCTE nodes, they manage 46 and 44% of the UCTE energy and power respectively (similar to those of group 1) but, even so, they acummulate only 15, 32, and 21% of the UCTE average interruption time, power loss and energy not delivered, respectively. This fact would suggest a positive correlation between static topological robustness and nontopological reliability measures and, as a consequence, a clear diferentiation between two classes of networks in terms of their level of robustness.

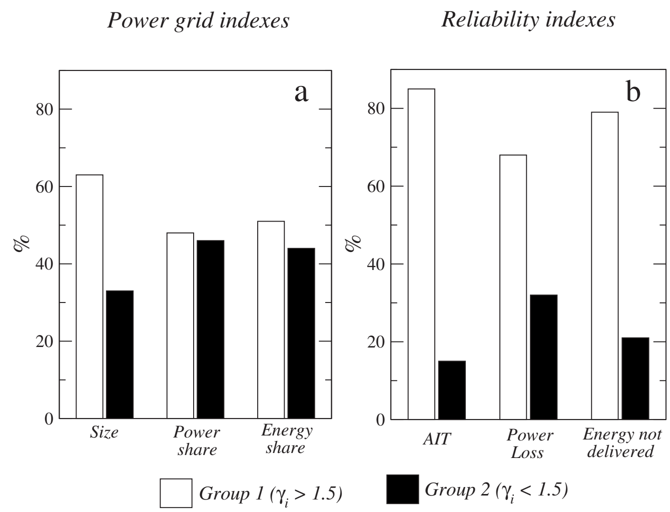

FIG. 4. Power grid indexes vs reliability indexes. (a) Networks in group 1 (i.e., $\gamma > 1.5$ and $f_c \cong f_{c,p}$), though representing two-thirds of the UCTE size, share almost as much power and energy as networks in group 2 (i.e., $\gamma < 1.5$ and $f_c > f_{c,p}$). (b) Nonetheless, these same networks of group 1 acummulate more than five times the average interruption time (AIT) of the latter, more than two times their power losses, and almost four times their undelivered energy.

V. DISCUSSION

In this paper, we have extended our previous work on the robustness of the European power grid under random failures with the intentional attacks scenario. A mean field theory approach has been used in order to analytically predict the fragility of the networks against selective removal of nodes and a significant deviation from predicted values has been found for power grids with an exponent $\gamma < 1.5$. For these networks, the real critical fraction f_c^{real} is higher than the theoretical one f_c^{theor} for the same γ. This suggests an increased robustness for these networks compared to those with $\gamma > 1.5$.

In order to evaluate the real existence of this two classes of networks, namely *robust* and *fragile*, real reliability measures from the Union for the Co-Ordination of Transport of Electricity (UCTE) have been used. It has been found that there seems to exist indeed a positive correlation between static topological robustness measures and real nontopological reliability measures. This correlation shows that networks in the *robust* class (i.e., networks with $f_c^{\mathrm{real}} > f_c^{\mathrm{theor}}$), though representing only 33% of the UCTE nodes under study and managing a similar amount of power and energy than that of the networks in the *fragile* class, acummulate much less percentage of the whole UCTE average interruption time, power loss, and energy not delivered. Due to the limited historical reliability data available, it is actually not possible to detect whether a network is more robust the higher f_c^{real} is, or simply due to the fact that $f_c^{\mathrm{real}} > f_c^{\mathrm{theor}}$. How this can be related with the internal topological structure of the networks and the subgraphs abundances is actually a main point under study and will be explored elsewhere.

This feature is of obvious importance. Up to this date and as far as we know, no such correlation between topological and dynamical features has been encountered in any study related to complex networks structure and dynamics. From the power industry point of view, constantly facing the challenge of meeting growing demands with security of supply at the lowest possible spenditure in infrastructures, the implications of this feature would permit new rather than traditional approaches to contingency-based planning criteria [33]. One of these traditional, and widely used, planning criteria is the so-called $(N-X)$ criterion. It assumes that no interruption of service can occur in a system with N units of equipment due to isolation of X outaged components. Without any topological feedback, the $(N-X)$ methodology (a) requires fast breaker operation to open any circuit pathway that has been faulted as well as to close the alternate path to service and (b) pushes the system to an increasing interconnection complexity as its utilization ratio (i.e., ratio between peak load and capacity of subtransmission lines and substation transformers) increases in time (aging infrastructures). Though aging infrastructures, excessive power delivered through increasing long distances and other possible causes may influence the increasing fragility of the power grids, it seems reasonable to think that, on a topological basis, the application of the $(N-X)$ contingency-based criteria, though originally intended to avoid interruptions in power service, would difficult, at the same time, the islanding of disturbances (i.e., the more connected an element is, the easier would be for a disturbance to reach). In other words, the same criteria that successfully has served to increase reliability in power systems through the late 20th century might now be responsible for the difficulties encountered in preventing perturbations, blackouts or isolating the different power grid elements.

Over the past years, and mainly due to economic imperatives, contingency-based planning criteria has been gradually pervaded by reliability-based planning criteria. In the latter, the prevention of likely contingencies of severe impact is considered much more effective than that of low probability and low impact. Nonetheless, this fact leaves the main conception of $(N-X)$ criteria still valid and at work in most of the ongoing grid's planning processes. Following the former discussion, we would suggest adding a third topology-based planning methodology, in order to take this fact into account.

ACKNOWLEDGMENTS

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Assessing European power grid reliability by means of topological measures

M. Rosas-Casals[1,2] & B. Corominas-Murtra[2]
[1]*Càtedra UNESCO de Sostenibilitat, Universitat Politècnica de Catalunya (UPC), EUETIT-Campus Terrassa, Edif. TR4, C. Colom, 1, 08222 Barcelona, Spain.*
[2]*ICREA-Complex Systems Lab., Universitat Pompeu Fabra – PRBB, C. Dr. Aiguader, 88, 08003 Barcelona, Spain.*

Abstract

Reliability assessment is crucial when dealing with complex systems, especially complex networks. Be they natural or man made, networks are able to sustain their functioning by means of a reliable set of components. The many functions a network can sustain are direct consequence of the topological structure that constraints and, at the same time, defines, the dynamical relation between its components. Therefore, some kind of relation between structure and dynamics should be expected to appear. In this paper, some of these relations that have been found for the European power grid are presented. Evidences for a critical relation between topology and dynamics are summarized, using some basic topological measures widely used in the developing complex networks paradigm. Finally, strategies for optimal management and operation of such networks are suggested.
Keywords: power grid, complex networks, reliability.

1 Introduction

The relation between structure and dynamics covers much of the literature devoted to complex networks science [1]. It is now obvious that the structure of a network affects and determines its collective dynamical behaviour and, at the same time, networks can modify their wirings in order to adapt certain dynamics to a required objective function [2]. When dynamic processes exceed the network's capability to handle them properly, there appear dramatic and usually unexpected effects such as congestions and jams [3] or cascading failures in infrastructure and organizational networks. [4, 5]

This last case is particularly relevant for power grids, where the most dramatic dynamical effects show themselves directly in form of blackouts and, indirectly, in form of huge economic and even human losses [6]. These major events, and the causes that generate them, are recorded and stored by public organizations. In Europe, this job is done by the Union for the Co-Ordination of Transport of Electricity (UCTE) and these events, given in total loss of power, energy not supplied, restoration time and equivalent time of interruption, are published monthly since 2002, and segregated by country and cause. [7]

One first attempt to correlate network reliability measures and structural topology for the European power grid can be found in [8]. Given real reliability measures from the UCTE, it is found that there seems to exist indeed a positive correlation between static topological robustness measures and real non-topological reliability measures such as energy not supplied, total loss of power and equivalent time of interruption. This fact leads the authors to classify the power grids of most of the European countries in two groups, namely *fragile* and *robust* grids, by means of a topological measure (see Section II below). They present both analytical and numerical estimations of the boundaries for network collapse under attack and failure, using a mean field theoretical approach.

The aim of the following sections is the exploration of some more different measures that relate this behaviour with the internal topological structure of the networks. The paper is organized as follows. In section II, some previous findings are summarized and updated in order to justify more broadly our subsequent work. In section III, the *mean degree* is proposed as a first evidence of relation between structure and dynamics. Section IV presents the motifs abundance as another segregation measure between *robust* and *fragile* networks. In Section V, we present the *patch size distribution* as a third and more tentative evidence for network robustness. Finally, Section VI summarizes our findings and outlines some proposed strategies for an improved grid design.

2 European power grid robustness update

The European power grid can be described in terms of a *graph* $\Omega = (V, E)$, where $V = \{v_i \ldots v_N\}$ indicates the set of N *nodes* (transformers, substations or generators in our context) connected by the set of actual *links* between pairs of nodes $E = \{e_{ij}\}$. Here, $e_{ij} = \{v_i, v_j\}$ indicates that there is an edge (and thus a link) between nodes v_i and v_j. Two connected nodes are called *adjacent*, and the *degree* k of a given node is the number of edges connecting it with other nodes. The mean of k over V is known as the *mean degree* $<k>$. Besides k

and $<k>$, an additional property widely used is the *cumulated degree distribution*. This is defined as the (normalized) probability that a node chosen uniformly at random has a degree k or higher (i.e., the fraction of nodes in the graph having k or more edges) [9]. All European countries' power grids have exponential cumulated degree distributions [10]. That is, the probability $P(k \geq K)$ of having a node linked to *k* or more other nodes follows

$$P(k \geq K) = C \exp(-k/\gamma) \tag{1}$$

where C is a normalization constant, k is the node degree and γ is a characteristic parameter. Table 1 offers a summary of the basic topological features exhibited by the European power grids segregated in two groups: *robust* ($\gamma < 1{,}5$) and *fragile* ($\gamma > 1{,}5$) power grids, as can be found in [8].

Group	Country	Short Form (from UCTE)	Exp. Deg. Dist. (γ)	Grid size (N)	Mean Degree $(<k>)$
Robust	Belgium	BE	1,005	53	2,18
	Holland	NL	1,086	36	2,11
	Germany	DE	1,237	445	2,51
	Italy	IT	1,238	272	2,70
	Romania	RO	1,418	106	2,49
	Greece	GR	1,457	27	2,44
Fragile	Portugal	PT	1,606	56	2,57
	Poland	PL	1,641	163	2,60
	Slovak Rep.	SK	1,660	43	2,41
	Switzerland	CH	1,850	147	2,53
	Czech Rep.	CZ	1,883	70	2,51
	France	FR	1,895	667	2,69
	Hungary	HU	1,946	40	2,35
	Spain	ES	2,008	474	2,82
	Serbia	RS	2,199	65	2,49

Table 1: Robust and fragile European grids, ordered by increasing γ, the exponential degree distribution characteristic parameter. Size (number of nodes N) and mean degree $<k>$ are also shown as reference. The analyzed cumulated grid size is 96% of the whole UCTE size.

For this paper and countries in Table 1, data from UCTE considered in [8] has been updated up until August, 2008. Figure 1(a) shows cumulated European power grid indexes for each group: percentage size (i.e., number of nodes over the whole UCTE size, which is 2 783 nodes), energy share (i.e., cumulated electricity consumption over the UCTE energy consumption), and power share (i.e., national cumulated highest load over the UCTE power generation). The energy and power normalization has been done using national electricity consumption and highest load on the 3rd Wednesday of December respectively. For year 2007 (last year available) these cumulated values reached 2 392 TWh and 389 GW for the countries considered in Table 1, respectively [11]. As we can see, grids in the *fragile* group (i.e., $\gamma > 1{,}5$), though represent two thirds of the UCTE size, share almost as much power and energy as grids in the *robust* group (i.e., $\gamma < 1{,}5$). Figure 1(b) shows cumulated European power grid reliability indexes for each group: energy not supplied (ENS), total power loss (TPL), restoration time (RT) and equivalent interruption time (EIT), which can be found in [7]. For each group, these values have been obtained as cumulated percentage of MWh (ENS), MW (TPL) and minutes (RT), over the whole UCTE cumulative value for the same time period. Equivalent time of interruption is normalized by definition.

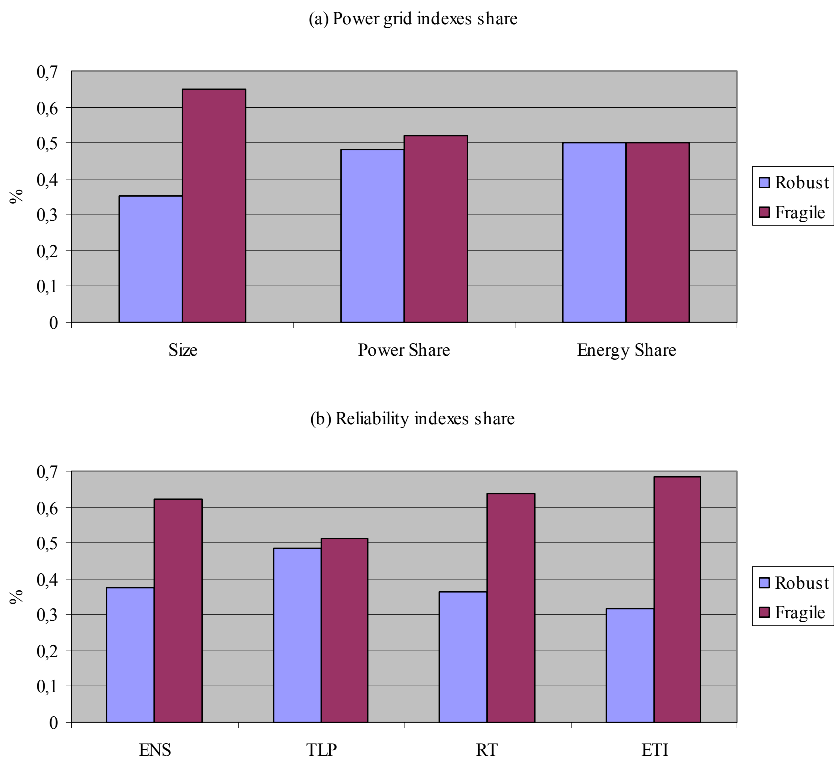

Figure 1: Power grid indexes vs. reliability indexes (updated August 2008). Networks in the *fragile* group, though share almost as much power and energy as networks in the *robust* one, accumulate between 60% and 70% of the energy not supplied (ENS), restoration time (RT) and total equivalent interruption time (EIT). The total loss of power (TLP) is almost equivalent in both groups.

As we can see, except for the total power loss value, there is an obvious unbalanced situation, being the share of the grids in the fragile group much more significant than that of the robust one. Sadly, network reliability data has been published only from 2002 onwards. This short statistical period is sensible to extreme and rare events. In November 2006, 10 million people suffered the consequences of a major event triggered in the German power grid (16 724 MW loss). Without this single event, the share in total power loss (TPS) would be 60% for the fragile group and 40% for the robust one (where Germany is included).

3 First evidence: mean degree deviation

Node degree has been widely used to evaluate structural properties and connectivity distribution of complex networks [12, 13]. The *degree distribution* (i.e., the fraction of nodes in the graph having precisely k edges), as opposed to the cumulated degree distribution formerly presented, is usually much more mathematically tractable. It has been stated that the cumulated degree distribution of the networks studied in this work follows an exponential function. By the very nature of the exponential function, it can be assumed that their degree distribution is also exponential.

Here, a first evidence of a correlated tendency between degree distribution and reliability indexes has been done comparing graphs in Table 1 with the simplest graph we can define, which is the Erdös – Rényi (ER) graph [14]. This graph is obtained as follows: given a set of N nodes, each pair of them is connected with constant probability $\langle k \rangle / N$, where $< k >$ is the mean degree. For large N, the probability that a vertex has k edges follows a Poisson distribution,

$$p(k) = \exp(-\langle k \rangle) \langle k \rangle^k / k! \tag{2}$$

The motivation to choose such a graph model is twofold: first, it is commonly accepted that the tail of a Poisson distribution decays qualitatively as an exponential function; second, the ER graph model stands for a generation algorithm with the smallest set of assumptions, thus being an interesting candidate for any null model. Therefore, equation (2) can be used in order to classify the robustness and fragility of the European power grids. To do so, we compare the actual mean degree $< k >$ of every grid with that of the Poisson distribution $< k' >$ that best fits the real degree distribution and calculate its normalized deviation as

$$\Delta \langle k \rangle = \frac{\langle k \rangle - \langle k' \rangle}{\langle k \rangle} \tag{3}$$

Deviations from the ER random graph behaviour are shown in Figure 2. For every country, ordered by its γ parameter, it presents a slightly exponential (broken line) increasing *mean degree normalized deviation* as γ increases. This fact would suggest a more fragile behaviour as the network is less well fitted by the Poisson distribution, i.e. rather unexpectedly, as the network is less randomly designed. Observed deviations might be explained for several reasons, mainly variations in the topology due to planarity and network motifs. Unlike random graphs, European power grids are almost *planar graphs* in the sense that they can be drawn in the plane in such a way that no two edges intersect [14]. This fact is still under investigation and results will be published elsewhere. The possible influence of network motifs is analyzed in the next section.

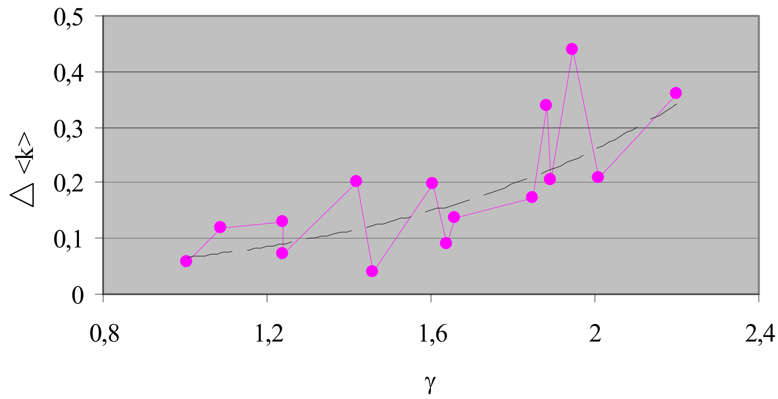

Figure 2: Mean degree normalized deviation (equation 3) as function of the γ parameter. Lines added for visual aid.

4 Second evidence: network motif analysis

Though global similarities may arise, networks might display very different local structure. This local structure can be characterized by patterns termed *network motifs*, or *subgraphs*, that appear at a much higher frequency than expected in randomized networks [15]. Functional or adaptive interpretations aside, network motifs can be used to characterize and compare the local structure of networks, even from different fields. [16]

Figure 3 shows the evolution of three particular four-node subgraph patterns: linear, star and star with triangle. We group the last two together as they represent high connectivity motifs. We observe a notable increase of interconnected local topologies in spite of linear ones, as the fragility of the networks increases with γ. Although in order to be synthetic only grids with more than one hundred nodes have been considered (i.e., Germany, Italy, Romania, Poland, Switzerland, France and Spain), this behaviour is followed by the rest of the grids as well: fragility seems to increase as the elements of the grid become more interconnected and motifs such as stars and triangles began to appear. Although aging infrastructures, excessive power delivered through increasing long distances and other possible causes may influence the increasing fragility of the power grids, it seems reasonable to think that maybe, on a topological basis, the application of the (N-X) contingency criteria, which favours connectivity and interconnectedness, though originally intended to avoid interruptions in power service, would difficult, at the same time, the islanding of disturbances. Nonetheless, a grid's dynamical model to certify this hypothesis is needed and already under development.

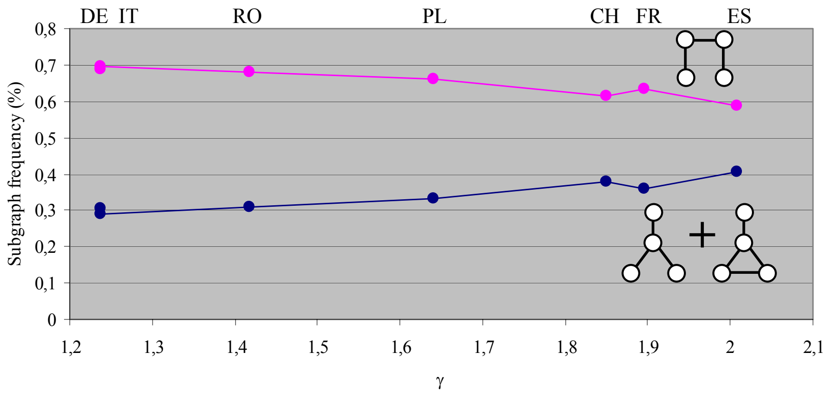

Figure 3: Subgraph abundances for power grids of size higher that 100 nodes as function of the exponent γ of the exponential degree distribution. These subgraphs display patterns of change with γ that are not independent of each other. (Upper part: UCTE's grids short form, from Table 1).

5 Third evidence: patch size distribution

For a last and more tentative, topological measure of the reliability of a power grid, we introduce here the *patch size distribution*. We compare the distribution of land patches enclosed by transportation cable lines for two different countries. The rationale behind this measure is basically inspired by concepts developed and used in (a) power distribution planning [17] and (b) landscape ecology. [18]

On one hand, the objective of transport and distribution planning is to provide an ordered and economical expansion of equipment and facilities to meet the utilities' future electrical demand, with an acceptable level of reliability. Considering the space as the substrate where the grid evolves and expands, we would expect a somehow regular distribution of substations and transformers, at least at a transport level, where the main objective is the distribution of bulk power in spite of population density or even geographical accidents. Nonetheless, power grids have evolved for a long time, usually without common long term planning criteria. It seems thus, that an optimal or even regular spatial distribution cannot be attained without redundancies and suboptimal designs.

On the other hand, and keeping forestry, agriculture and farming aside, the principal actors in the spatial processes that transform and change the land are technological infrastructures such as roads, railways and, in a lesser extent, energy transportation infrastructures such as the power grid. These processes of land fragmentation and transformation have important and sobering consequences in economics, biodiversity, conservation, global warming and society [19]. Quantification of fragmentation through spatial indexes is currently becoming a common practice in landscape ecology and related disciplines [20]. Recently, the *effective mesh size* has been proposed as a fragmentation measure and a tool for environmental monitoring. It has been used to evaluate the evolution of land fragmentation caused by transportation infrastructure and urban development. [21]

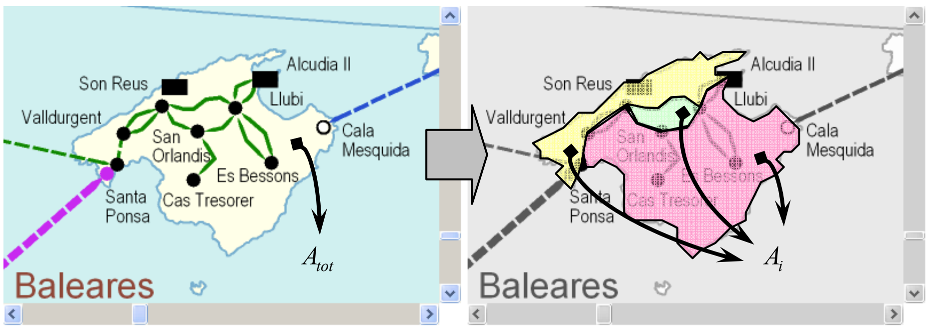

Figure 4: Patch size obtention example for the island of Mallorca. The whole area A_{tot} of the island can be fragmented into three smaller areas (or *patches*), with individual area A_i. In defining the patches, double lines are considered single lines and isolated nodes (i.e., Cas Tresorer and Es Bessons in the figure) can not be used, as they do not limit the area contained in the patch. (UCTE map snapshot from UCTE website, http://www.ucte.org/resources/uctemap/).

The effective mesh size expresses the probability that any two randomly chosen points in the region under observation may be connected (i.e., not separated by artificial barriers such as roads or urban areas). It is useful when the region under study is kept constant since it shows effectively the evolution of the land fragmentation over time. But it is of little use when the objective is to compare different regions at the same moment of time, since a common normalization factor can not be used.

The *patch size distribution* allows to overcome this last problem and to show the structure of the spatial distribution for different grids. We essentially consider cable lines as virtual spatial fragmentation limits and calculate the distribution of the size of the resulting areas (Figure 4). Political frontiers, seas and oceans would be the very outmost limits of the patches for every country.

Group	Country	Grid size (N)	Electricity consumption (TWh)	Served area (km^2)	Population (millions)
Robust	Germany	445	556	357 050	82
Fragile	Spain	474	261	493 519	42

Table 2: Comparative data for Germany and Spain (year 2007). For Spain, all data considered is peninsular. National electricity consumption for every year in the UCTE since 2002, can be found at [11]. Spanish electricity consumption and its segregation into peninsular and extra-peninsular data can be found at: http://www.ree.es/sistema_electrico/informeSEE.asp.

Here, two power grids of similar number of nodes but different robustness behaviour have been compared (Table 2): Germany, a *robust* grid with 445 nodes and Spain, a *fragile* one with 474 nodes. Though similar in size, Table 2 shows some striking differences in population (and therefore electricity consumption) and covered area: Germany's grid deals with more than two times electricity consumption than that of Spain, but in an area being 27% smaller.

Figure 5 shows the absolute frequency of patches as function of their area, in square kilometres. Both distributions span for over five orders of magnitude in A_i. But while the German grid keeps this frequency almost constant for all these orders of magnitude, the Spanish grid begins to strongly deviate for values of A_i lower than 500 km^2. Though the geography and area of Spain do obviously differ from that of Germany, a similar pattern but with different absolute frequency values would be expected. We insist this is a much more tentative measure and it has to be much further explored, but this fact would suggest a much messier and intricate Spanish grid, heavily inhomogeneous at the spatial level and, consequently, much more difficult to control and more prone to failures of different kind. We notice as well the inherent difficulties that arise in finding two grids with similar size, each one belonging to each group, i.e. fragile and robust.

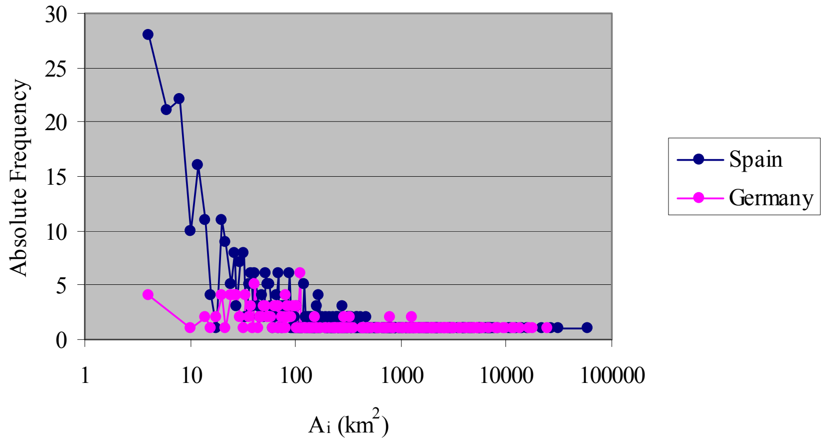

Figure 5: Absolute frequency of patches vs. patch size for Spain and Germany, in log-linear plot. The lower limit for A_i is 4 km^2.

6 Summary and discussion

The European electricity transmission system is a huge infrastructure formed by more than 200 000 km of transmission lines and almost 2 800 substations. As any other engineered system, it has been designed with a purpose: to deliver its 2 300 TWh of energy in an almost faultless way and satisfy demand with production instantly. It is, nonetheless and at the same time, a complex system where unexpected and seemingly lawless phenomena such as blackouts and cascading failures arise. The aim of this work has been the exploration of some evidences that relate the outcome of this unexpected behaviour (in form of reliability indexes) with the engineered part of the grid (i.e. its topological structure). Although reliability data has been recently published and it can be biased due to extreme and rare events, a notable correlation has been found between networks' cumulative degree distribution parameter γ and reliability indexes such as energy not delivered, total loss of power, restoration time and equivalent time of interruption. There are three main tendencies that tend to increase with the fragility of the networks: (a) a deviation from a random graph null model degree distribution, quantified by the *mean degree deviation* $\Delta\langle k\rangle$; (b) an increased preponderance of *star* and *triangle motifs* in spite of linear ones; and (c) an irregular *patch size distribution*. Evidences (a) and (b) would suggest an increased fragility when the topology of the network deviates from a random one, maybe in search of a higher interconnectedness. This would suggest that the same contingency criteria that favours connectivity, though originally intended to avoid interruptions in power service, would difficult, at the same time, the islanding of disturbances: i.e. the more connected an element is, the easier would be for a disturbance to reach. Evidence (c) has to be taken with caution, as more work is needed in order to fully understand how planar random graph topologies can generate such patch size distributions. It is obvious that strategies for optimal management and operation of these networks can not be separated from its dynamical behaviour. The relation between probability distributions of reliability indexes, reasons of main events (overloads, endogenous or exogenous failures, etc.) and network's topological fragility indexes, such as γ, are now questions under research. Engineers calculate, and calculation requires a theory or at least an organized framework [22]. It is our hope to define how these different factors constrain and are constrained by the real dynamics of the power grid in order to unravel the laws governing complex systems like this.

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Analysis of major failures in Europe's power grid

Martí Rosas-Casals*,a,b, Ricard Solé[b,c]

[a] Càtedra UNESCO de Sostenibilitat, Universitat Politècnica de Catalunya (UPC), EUETIT-Campus Terrassa, Edif. TR4, C. Colom, 1, 08222 Barcelona, Spain
[b] ICREA-Complex Systems Lab, Universitat Pompeu Fabra - PRBB, Dr. Aiguader 88, 08003 Barcelona, Spain
[c] Santa Fe Institute, 1399 Hyde Park Road, New Mexico 87501, USA

Abstract

Power grids are prone to failure. Time series of reliability measures such as total power loss or energy not supplied can give significant account of the underlying dynamical behavior of these systems, specially when the resulting probability distributions present remarkable features such as an algebraic tail, for example. In this paper, seven years (from 2002 to 2008) of Europe's transport of electricity network failure events have been analyzed and the best fit for this empirical data probability distribution is presented. With the actual span of available data and although there exists a moderate support for the power law model, the relatively small amount of events contained in the function's tail suggests that other causal factors might be significantly ruling the system's dynamics.

Key words: power grid, complex networks, time series
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1. Introduction

There has been in recent years an increasing awareness about infrastructure networks security and reliability [1, 2, 3]. Modern society's functional capacity relays on an optimal operation of infrastructure and information networks such as roads, railways, gas and oil pipes or Internet. Particularly

*Corresponding author
Email addresses: rosas@mmt.upc.edu (Martí Rosas-Casals), ricard.sole@upf.edu (Ricard Solé)

vital, and at the same time quite prone to failure, are electric power transmission networks. These are extremely complex engineered systems, composed of multiple and interconnected elements, whose reliability depends both on each component's behavior and, at the same time, on the many different dynamical interactions that span over and rule the overall connectivity of the system.

Although it is not always the case, a malfunction of a power transmission system shows usually itself as a blackout. This is a direct consequence of a cascading failure involving several of its composing and linking elements. This fact turns the study of the details of failures in power transmission networks from a traditional engineering point of view a hard task, if not an impossible one most of the times. In order to reduce the inherent complexity of this detailed approach, some new ways have been proposed in recent years. One of them is that of ignoring the details of particular failures and to focus on the study of global behaviors and dynamics of time series with approximate global models. Concepts such as criticality and self-organization have been applied to characterize blackout data, suggesting that the frequency of large blackouts is governed by non trivial distribution functions such as power laws and, consequently, that power systems are designed and operated near a critical point. (For a comprehensive review on this approach, see Ref.[4] and references therein).

This paper analyses for the first time, and as far as we know, the statistics of major electric transmission network events in the European power grid from this aforementioned complex systems approach. Following essentially the statistical analysis presented in Ref.[5], we estimate the basic parameters of the power-law model, then calculate the goodness-of-fit between the data and the power law and finally we compare the power law with alternative hypotheses via a likelihood ratio test. The paper is organized as follows. In section II European major events data is presented and explained. In section III blackout data is analyzed. Finally, section IV summarizes our main results.

2. UCTE major events data

European power network reliability data can be found in the Union for the Co-ordination of Transmission of Electricity (UCTE) web page, publicly available from 2002 onwards in monthly statistics format [6]. The UCTE is the association of Transmission System Operators (TSOs) in continental

Europe and manages data from 24 different European countries. Due to the complexity of events, sometimes involving more than one TSO, types of interruptions in the network and short time given to gather this information, UCTE major events data is somehow limited in its scope and does not provide a fully detailed account of some events. It is, nonetheless,the best documented source that has been found. For each major event, it summarizes the following information:

- **Country**.
- **Substations involved**.
- **Reason (R)**. Broadly classified into four groups: (1) overloads (also calculated brakes), (2) failures (false operation, failure in protection device or other element), (3) external (outside impacts and very exceptional weather and natural conditions) and (4) other or unknown reasons.
- **Energy Not Supplied (ENS)**. Measured in MWh, as loss of energy from the consumption side.
- **Total Loss of Power (TLP)**. Measured in MW, as loss of production from the generation side.
- **Restoration Time (RT)**. Measured in minutes. Note that since ENS and TLP are measured from different sides, RT can not be assumed as the ratio of ENS over TLP. It can be considered, therefore, an independent reliability measure.
- **Equivalent Time of Interruption (ETI)**. Defined as the duration of an interruption in minutes multiplied by the energy not supplied divided by the consumption for the last 12 months. Defined in this way, the ETI allows a direct comparison between TSOs in terms of interruptions that occurred during a year.

From 2002 to 2008, both years inclusive, 908 major events have been noticed. Due to the complexity of events some entries have zero value in one or more of their categories. While these zeroed values have not been considered, the rest of numerical values are effective measures of major events occurred in the UCTE power grid and, consequently, they have all been included in order to develop the analysis presented in this paper.

3. Probability distribution analysis

The study of the statistics and dynamics of series of events with approximate global models has been one of the most popular topics in the last twenty years, specially within the interdisciplinary study of complex systems. Probability distribution functions with a heavy tailed dependence in terms of event or object sizes seem to be ubiquitous statistical features of self-organized natural and social complex systems [7]. The appearance of algebraic distributions, specially power laws, is often thought to be the signature of hierarchy, robustness, criticality and universal subjacent mechanisms [8]. Electric power transmission networks have not escaped this captivation for power laws quest. Time series of usual measures of blackout size like energy unserved, power loss or number of customers affected, have been shown to be algebraically distributed in North America [9], Sweden [10], Norway [11], New Zealand [12] and China [13]. This apparent ubiquitous evidence have led to believe and try to demonstrate that power systems (a) tend to self-organize near a critical point and (b) that there may be some universality ruling the inner depths of these systems.

In spite of this *evidence*, most of the aforementioned literature relay on poorly performed statistical analysis and results can not be trusted. In some cases methodologies are not clearly explained (i.e., Ref.[11]) or simple visual inspection can clearly dismiss the analysis performed to rule in the power law hypothesis (i.e., Ref.[10] and Ref.[13]). In other cases, proper usage of statistical tools have given new results that limit the scope of the original analysis (i.e., Ref.[9] is dismissed as insufficiently substantiated in Ref.[16] and reanalyzed in Ref.[5], finding moderate support for the power law hypothesis and even some for an exponential distribution).

In this section we analyze the probability distributions of three malfunction measures of the European power grid: energy not supplied (ENS), total loss of power (TLP) and restoration time (RT). The results are summarized in Table 1 and shown in Figure 1. The methodology that has been used is that described in Ref.[5], where a maximum likelihood approach is proposed to estimate the heavy tailed function from the data and a significance test is constructed for testing the plausibility of the power law. Measures shown in Table 1 are generic statistics on one side and results of the aforementioned statistical analysis on the other. We assume a quantity x follows a power law if it is drawn from a probability distribution $p(x) \propto x^{-\alpha}$, where α is the scaling parameter of the distribution. Since the probability density of a

					Maximum likelihood				
Data set	n	$\langle x \rangle$	σ	x_{max}	$\hat{x}_{min}$	$\hat{\alpha}$	n_{tail}	p	**Support for PL**
ENS	690	552	7004	180000	185±72	1.7±0.1	104±120	0.24	Moderate
TLP	576	400	1790	26746	615±244	2.1±0.2	57±96	0.36	Moderate
RT	897	510	3328	44640	150±68	1.69±0.07	157±115	0.73	Ok

Table 1: UCTE major events generic statistics and power law fits. For each measure we give the number of occurrences n, mean $\langle x \rangle$, standard deviation σ, maximum observed occurrence x_{max}, lower bound to the power law behavior $\hat{x}_{min}$, scaling parameter value $\hat{\alpha}$, occurrences in the power law tail n_{tail} and p value p. The last column indicates the support for whether the observed data is well approximated by a power-law distribution. Estimated uncertainties for $\hat{x}_{min}$, $\hat{\alpha}$ and n_{tail} are also shown.

power law distribution diverges as $x \to 0$, there must exist a lower bound to the power law behavior [16]. We denote this lower bound as x_{min} and the number of events contained in the upper range as n_{tail}. Finally, the p-value denotes the significance test result: the power law is ruled out if $p \leq 0.1$. As we can see, the power law model is a plausible one for every data set considered (i.e., the p-value for the best fit is sufficiently large) and the scaling parameter values are similar to those encountered in the literature for the ENS and TLP distributions [4, 14]. Yet the power law model explains only a small amount of events: 15% for ENS ($n_{tail} = 104$), less than 10% for TLP ($n_{tail} = 57$) and 17% for RT ($n_{tail} = 157$) (even though it holds the better fit, with $p = 0.73$). We believe that measures such as n_{tail} and x_{min} are fundamental to estimate the span of the power law behavior and to develop further quantitative models, yet these values have not been considered in any of the aforementioned references. Only in the reanalysis of Ref.[9] done in Ref.[5] we have found an estimate for n_{tail} that gives an explanation for a 28% of the events.

We assume that the limited span of available data in each set might have a sensible influence in the final power law fitting outcome. It is nonetheless evident from these results that (a) power law behavior can not be assumed for the whole data observed, (b) we can not accept the existence of any critical point at this stage of the data span and (c) there must be right now considerably more dynamics not explained by the power law model.

In order to check if other distributions may be a better fit, we have performed log likelihood and p-value tests with respect to log-normal, exponen-

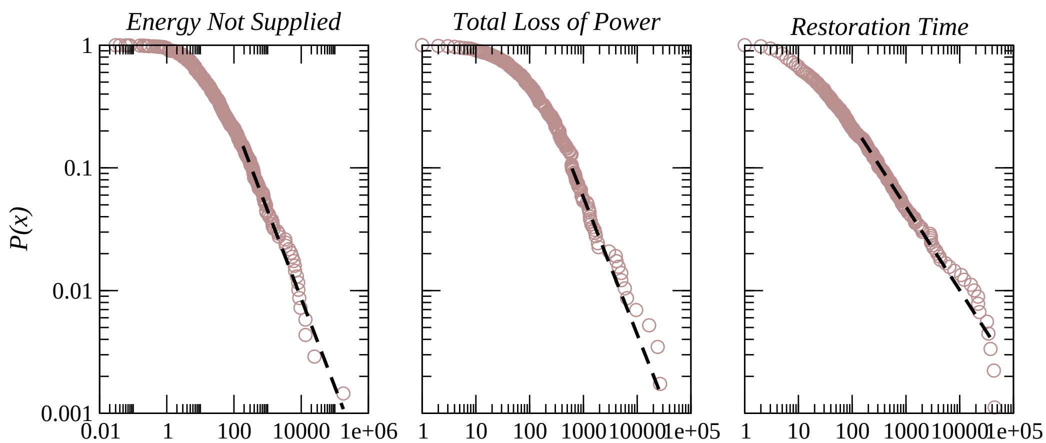

Figure 1: Cumulative distribution functions $P(x)$ and their maximum likelihood power law fits for the UCTE reliability measures energy not supplied, total loss of power and restoration time.

tial, stretched exponential (Weibull) and power law with cut off distributions. Results are shown in Table 2. Positive log likelihood values favor the power law hypothesis and p-values higher than 0.1 imply no significance on the results. Log-normal, Weibull and exponential distributions can be ruled out as $p \geq 0.1$ in the first two and $p = 0$ in the latter with positive LR values. Power law with cut off can be ruled out in the ENS and TLP data sets though it is a plausible option for the RT data set.

	log-normal		**exponential**		**Weibull**		**PL + cut off**	
Data set	***LR***	***p***	***LR***	***p***	***LR***	***p***	***LR***	***p***
ENS	-0.405	0.68	2.64	0.00	-0.393	0.69	-0.419	0.36
TLP	0.319	0.75	3.42	0.00	0.467	0.64	-0.08	0.68
RT	-0.382	0.70	7.57	0.00	-0.329	0.74	-1.7	0.06

Table 2: Test of power-law behavior. Positive values of the log likelihood ratios LR favors the power law model. Values of $p \geq 0.1$ imply though that result can not be trusted. The exponential distribution is definitely ruled out as possible model and only for the restoration time RT, the power law with cut off could be considered a valid model.

4. Summary and discussion

Power outages are considered unexpected phenomena in power grids. They appear without warning and, though widely investigated, there is not a common accepted theory that explains neither their pervasiveness nor their inner dynamics. The statistical overabundance of big blackouts has been explained using theories of systems failure able to reproduce their empirically found probability distribution. This distribution is considered a power law for most of the literature encountered, with the consequences that this algebraic tail involves (i.e., self-organization, criticality and universalities). In order to add one more reference to this field, in this paper we have analyzed seven years of disturbances data for the UCTE power grid and for three major event measures: energy not supplied, total loss of power and restoration time. Although evidences for self-organized criticality have been suggested for even five years of data [15], support for the power law hypothesis has been found moderate for two (ENS and TLP) of the three measures considered. Moreover, the amount of events explained by the power law hypothesis can be considered negligible. These facts make it difficult to accept the existence of an equilibrium point near criticality for the UCTE power grid, at least at this stage of data analysis, and it also suggests that most of the power grid dynamics should be explained by different models, other than power law.

There still exist many complexities not explained in this systems. Ongoing research is now focused in analyzing major events probability distributions in connection with (a) the reasons that trigger these major events and (b) the structure and topology of the power grids involved in these major malfunctions. Results will be published elsewhere.

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