MAGNETOHYDRODYNAMICS NATURAL CONVECTION BOUNDARY LAYER FLOW OF DUSTY FLUID PAST A VERTICAL STRETCHING SHEET

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To my beloved family members,

mama and papa

Kila, Boboy, Bob, anje, ayang, cinta

Aziela, mira, Rab

andam

Supervisors,

Dr Anati Ali and PM Dr Sharidan Shafie

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Nor, Lim, Erry, Asma, Cheman, Kak wawi, Tira

Friends,

Thank you for supporting me until the end...

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ABSTRACT

Research on magnetohydrodynamics in natural convection boundary layer flow of dusty fluid has become a great interest in the fluid mechanic dynamics due to its importance in many engineering applications. However, the present of solid particles in the form of dust, ash or soot in dusty fluid either naturally or deliberately, suspended in the electrically conducting fluid may influence the fluid flow characteristics. Therefore, in this study the effect of suspended particles on magnetohydrodynamics flow in a viscous fluid past a vertical stretching sheet is investigated. Specific cases with different effects are considered such as slip effect, thermal radiation, convective boundary condition and Hall effect. The governing non-linear partial differential equations of the problems are transformed into a set of non-linear ordinary differential equations by using a suitable similarity transformation. The obtained equations are solved numerically by Keller-box method. The numerical results of velocity profile, temperature profile, skin friction and Nusselt number affected by fluid-particle interaction, magnetic, slip velocity, slip thermal, radiation and Hall parameters as well as Biot number, Grashof number and Prandtl number for particle and fluid phases are presented graphically and analyzed in detail. This study shows that, the presence of suspended particles in a fluid caused the momentum and thermal boundary layer thickness to become thinner. The magnetic parameter plays the role in decreasing the velocity profile and momentum boundary layer thickness. Also, magnetic parameter affects the increment of the temperature profile and thermal boundary layer thickness. In addition, increasing slip velocity parameter reduces the velocity profile and skin friction. However, increasing slip thermal parameter decreases temperature profile and Nusselt number. Furthermore, radiation parameter is observed to increase the velocity profile, temperature profile and thermal boundary layer thickness. Lastly, the Hall parameter increases the velocity profile but decreases the temperature profile. In all cases studied, the velocity and temperature profiles for fluid phase are always higher than the dust phase.

ABSTRAK

Penyelidikan terhadap hidrodinamik magnet di dalam aliran olakan bebas bagi bendalir berdebu telah menjadi kajian yang amat menarik dalam bidang dinamik bendalir kerana kepentingannya dalam banyak aplikasi kejuruteraan. Namun, dengan kehadiran zarah pepejal berbentuk habuk, abu atau jelaga di dalam bendalir berdebu sama ada secara semulajadi atau sengaja, yang terampai di dalam aliran cecair berelektrik boleh mempengaruhi ciri-ciri aliran bendalir. Oleh itu, dalam kajian ini kesan zarah terampai ke atas aliran hidrodinamik magnet di dalam bendalir likat yang melintasi lembaran regangan menegak disiasat. Beberapa kes tertentu dengan kesan yang berbeza dipertimbangkan seperti kesan gelincir, sinaran terma, keadaan sempadan berolakan dan kesan Hall. Persamaan menakluk pembezaan separa tak linear bagi semua masalah diubah menjadi satu set persamaan pembezaan biasa tak linear dengan menggunakan penjelmaan serupa yang sesuai. Persamaan yang diperoleh diselesaikan secara berangka dengan menggunakan kaedah kotak-Keller. Keputusan berangka bagi profil halaju, profil suhu, geseran kulit dan nombor Nusselt yang terjejas akibat oleh parameter-parameter interaksi cecair-zarah, magnetik, gelincir halaju, gelincir haba, sinaran dan Hall serta nombor Biot, nombor Grashof dan nombor Prandtl untuk fasa bendalir dan fasa zarah dipersembahkan secara graf dan dikaji secara terperinci. Kajian ini menunjukkan bahawa, kehadiran zarah terampai di dalam bendalir menyebabkan ketebalan lapisan sempadan momentum dan haba menjadi semakin nipis. Parameter magnetik memainkan peranan bagi mengurangkan profil halaju dan ketebalan lapisan sempadan momentum. Parameter magnetik juga meningkatkan profil suhu dan ketebalan aliran sempadan haba. Tambahan lagi, parameter gelinciran halaju mengurangkan profil halaju dan geseran kulit. Manakala, pertambahan nilai parameter gelinciran haba, menurunkan profil suhu dan nombor Nusselt. Tambahan pula, parameter sinaran diperhatikan dapat meningkatkan profil halaju, profil suhu serta ketebalan lapisan sempadan haba. Akhir sekali, parameter Hall meningkatkan profil halaju tetapi mengurangkan profil suhu. Secara keseluruhannya, profil halaju dan profil suhu untuk fasa bendalir adalah sentiasa lebih tinggi daripada fasa zarah.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

B	-	magnetic force
B_0	-	uniform magnetic field
b	-	induced magnetic field
Bi	-	Biot number
c	-	stretching rate
c_p	-	specific heat of fluid
c_m	-	specific heat of solid particles
C_{fx}	-	skin friction in x-direction
C_{fz}	-	skin friction in z-direction
e	-	electron charge
E	-	electric field
F, f	-	dimensionless stream function
$\mathbf{F}_b$	-	body force
$\mathbf{F}_p$	-	fluid-particle interaction force
g	-	gravitational force
Gr	-	Grashof number
h_f	-	heat transfer coefficient
J	-	current density
k	-	thermal conductivity
M	-	Magnetic parameter

m_p	-	mass of each particle
m_e	-	mass of electron
m	-	hall parameter
n	-	number density of particle
N	-	mass concentration of particle phase
n_e	-	number density of electron
Nr	-	radiation parameter
Nux	-	local Nusselt number
p	-	pressure
p_h	-	hydrostatic pressure
p_d	-	dynamic pressure
p_e	-	pressure of electronic
Pr	-	Prandtl number
q_w	-	convection heat transfer coefficient
$\mathbf{q}_r$	-	radiative heat flux
Q_p	-	heat transfer from a particle to fluid
R	-	heat additive
r_p	-	radius of the particle
Re	-	Reynold number
S	-	slip velocity parameter
St	-	slip temperature parameter
t_e	-	collision time of electron
T	-	temperature of the fluid
T_p	-	temperature of solid particles
T_f	-	temperature of the hot fluid

T_w	-	temperature at the surface
T_∞	-	temperature of the fluid in the free stream
U_w	-	mean velocity of stretching sheet
u	-	velocity component of fluid phase in x-direction
u_p	-	velocity component of dust phase in x-direction
V	-	volume
$\mathbf{V}$	-	velocity of the flow of fluid phase
$\mathbf{V}_p$	-	velocity of the flow of dust phase
v	-	velocity component of fluid phase in y-direction
v_p	-	velocity component of dust phase in y-direction
w	-	velocity component of fluid phase in z-direction
w_p	-	velocity component of dust phase in z-direction
μ	-	dynamic viscosity coefficient
υ	-	kinematic viscosity coefficient
τ_v	-	relaxation time of velocity
τ_T	-	relaxation time of temperature
τ_w	-	wall shear stress
θ	-	dimensionless of temperature for fluid phase
θ_p	-	dimensionless of temperature for dust phase
σ	-	electric conductivity
$\sigma*$	-	Stefan-Boltzmann constant
β	-	fluid-particle interaction parameter
β^*	-	thermal expansion
$k*$	-	Rosseland mean absorption coefficient
ω_e	-	electron frequency

ρ	-	density of the fluid
ρ_p	-	density of the dust particles
ρ_∞	-	density of the ambient fluid
ψ	-	stream function
γ	-	specific heat ratio of the mixture
λ	-	convective parameter

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Introduction

In this chapter the strength of this thesis are explained concisely. The background of the research is presented in Section 1.2. The problem statements as well as objective and scope of the research are given in Section 1.3, 1.4 and 1.5 respectively. Consequently, the significance of the research is presented in Section 1.6. Lastly, Section 1.6 gives the outline of the whole thesis.

1.2 Research Background

Most living things in this world made of fluid. In average sixty five percent of our body is fluid. In fact, the air we breathe every day is also considered as a fluid. In 1687, Newton (Anderson and Wendt, 1995) stated that

"A fluid is anybody whose parts yield to any force impressed on it, and by yielding, is easily moved among themselves"

Theoretically, fluids consist of two different types of matter. It could be a liquid or a gas. In fluid mechanics, fluids can be categorized as viscous or inviscid flows. Viscous flow is the fluid flow in which its frictional effect is important. Conversely, inviscid flow is the situation where the effect of friction on the fluid flow is unimportant. These two types of fluids are restricted only for fluids in pure form. However, it is difficult to find fluids in pure form; without any impurities such as foreign bodies and dust particles. The fluid flow in simultaneous different phases (ie. solid, liquid or gas) is known as multiphase flows. The simplest model of multiphase flow is two phase flow which only involves two types of phases such as solid-liquid flow, solid-gas flow, liquid-gas flow and liquid-liquid flow.

Noting that, this research only focuses on two phase flow in which the fluid contains the distribution of solid spherical particle. These flows not only have different velocities but also different temperature between the fluid and the particle. Normally, most of the particles in such flow are never in uniform size. Since the particle velocity is influenced by the particle size, hence the particle phase would have many velocities. By reducing the complexity, the average size of particles is measured and therefore only one velocity for particles phase is considered in most case studies.

The importance of dusty fluid flow in several imperative processes in engineering and industrial applications has attracted the interest of many engineers as well as scientists to investigate the phenomenon. Their applications is obviously can be found in sedimentation, pipe flow, fluidized bed, gas purification and transport processes (Singleton, 1965). The experimental study published by Sproull (1961) on the motion of the air flow carrying small dust particles. Many examinations have been done either experimentally or theoretically regarding the problem of dusty fluid flow among researchers in order to clarify the phenomenon between fluid and dust particles. In general, this solid liquid two phase flow system can be described by several theories which are macroscopic or microscopic method, continuum theory and kinetic theory. In a study by Marble (1963), indicated that in the fluid-particle flow system, the mutual interaction of fluid with the particle cloud are governed by four similarity parameters; i) the velocity equilibration parameter, ii) the thermal equilibration parameter, iii) the momentum interaction parameter and iv) the thermal interaction parameter. It is noted that each of parameters has its own physical significance.

In dusty fluid flow, dust particles have independent motion from the fluid flow and affect the flow behavior. Hence, both fluid and particle phases are separately governed by continuum theory via conservation law. The interaction between dust particles and fluid appears as fluid-particle motion. The motion of fluid phase is governed by Navier-Stokes equation and motion of particle phase described by a separate set of Euler's equation. The equation for each phase contains a term describing the interaction between the fluid and the particle namely, Stokes drag. Noting that, the term Stokes drag described the condition of laminar flow with the low Reynold number involved with small particle size with low velocity (Collinson and Roper, 1995). The mathematical analysis of this two phase flows is more challenging than the fluids flow in pure form. In the most cases, it is found that one of half the fluid particle mixture is particles and the density of the particle is larger by a thousand times than the density of fluid which implies the volume fraction of the particle is neglected.

In natural phenomenon, dusty fluid can be observed in atmospheric flow during haze, rain erosion, smoke, flow of mud in river and air pollution. In fact, the importance of the transport of solid particles by fluids is extensively wide in many industries for example petroleum, cement processes, steel manufacture and metallurgical. Furthermore, in engineering and technical fields, dusty fluid flows have been applied in solid rocket exhaust nozzles, fluidized beds, environmental pollutions, purification of crude oil, nuclear reactor cooling, sedimentation, paint spraying and many others applications (Rudinger, 2012). There are many processes in industry as well as at home that involve two phase solid-liquid flow phenomena. For instance, preparing coffee in a coffeemaker where hot water mixture passes through a bed of coffee beans particles. In almost a similar process, but in a larger scale, can be found in the chemical industry such as when the fluid flow through a bed of solid particles for filtration purpose or chemical interaction (see Ni and Beckermann (1991), Johansen (2002), Micale *et al*. (2004) and Khopkar *et al*. (2006)).

Nowadays, theoretical study of dusty fluid boundary layer on stretching sheet is quite prominent among researchers. This is due to its essential contributions especially in engineering processes as well as in industries; mainly in manufacturing development. The production of a quality product is highly emphasized in the demand of industries. In producing a high quality of the resulting sheeting material (product), the knowledge of the flow properties of the ambient fluid, speed of the collection and the rate of heat transfer at the stretching surface become important. The study of boundary layer flow and the rate of heat transfer past a stretching sheet are initiated by Crane (1970). Thereafter, a huge amount of investigations which were extended based on Crane (1970) with various physical problems has been reported especially related to stretching sheet in dusty fluid. In pursuing the expansion of the dusty fluid theory, many researchers have done exploration in various methods with different physical situations. The geometry of the stretching sheet can be horizontal, vertical or inclined. This research focused only on vertical stretching sheet. If there is a difference between the temperatures of stretching sheet with the ambient temperature in a vertical surface, hence the fluid flow will experience a natural convection flow. The hot fluids will have a lower density than the cooler fluids, as a result the hotter fluids will rise.

Furthermore, there is condition where the others effects such as Magnetohydrodynamics (MHD), Hall effect, radiation, convective boundary condition and slip effect are significant to observe along with the phenomenon of natural convection of dusty fluid over a vertical stretching sheet.

1.2.1 Magnetohydrodynamics

The advantages of the effect of Magnetohydrodynamics (MHD) on natural convection flow past a stretching sheet is expressive in development of the technology in industry and engineering applications. MHD can be defined as the study of dynamics of matter moving in an electromagnetic field, especially where currents established in the matter by induction modify the field, so that the field and dynamics equations are coupled. For example, plasmas, liquid metals, and salt water or electrolytes. Fundamentally, the magnetic fields induces currents in a moving conductive fluid, which then tends to form a drag forces on the fluid flow called Lorentz force. Lorentz force will consequently affect the fluid flow. Investigation on the effects of this buoyancy force together with MHD is totally essential in various fields which include construction of cooling systems with liquid metals, petroleum industry, refinement of crude oil, polymer technology, centrifugal separation of matter from fluid, the cooling reactors, and solidification processing for metallic alloys and semiconductors. The study of dusty fluid is relevant with these devices or processes as the resulting effects from corrosion and wear activities which produce solid particles in the form of ash or soot suspended in a conducting fluid.

1.2.2 Hall Effect

The above studies investigate the situation where the electrical conductivity of the fluid is assumed to be uniform and experience a low strength magnetic field. Nonetheless, the situation where the magnetic field is substantially large in ionized fluid with low density leads to the occurrence of phenomenon of Hall Effect. This study is meaningful as it produces substantial changes to the fluid flow properties. In 1879 Edwin Herbert Hall discovered the existence of Hall Effect (Hannaway,1980). He found that a potential, proportional to the current and to the magnetic field is developed across the material in a direction perpendicular to both the current and the magnetic field when an electric current is applied through a medium in the presence of magnetic field. In other words, Hall effect is an advancement of the Lorentz force when it react on the electrons moving through a conductor, then an electrical potential difference is developed between the two sides of the conductor.

In the view of the impact of Hall Effect on dusty fluid problem, Gireesha *et al.* (2016) inspect the Hall effect on dusty fluid past a permeable stretching sheet with the presence of suction or injection. The Hall effect current is seen to increase the boundary layer thickness of transverse and axial velocity of the fluid and dust phases.

1.2.3 Thermal Radiation

In industry, many technological processes take places at high temperature. In this situation the effect of thermal radiation become critical and its effects cannot be ignored. Thermal radiation is the third mode of heat transfer. It can be defined as energy emitted by matter that is at a finite temperature in the form of electromagnetic waves (Bergmen and Incropera, 2011). Radiation not only occurs from a solid surface but also occur from liquids and gasses. It attributes to a variation in the electron configurations of the constituent atom or molecules. The energy of the radiation field is transported by electromagnetic waves or photon. In addition, the energy transfer by radiation does not require material medium unlike others modes such conduction and convection. Due to its importance, many researchers investigate the impact of thermal radiation on boundary layer of dusty fluid. The knowledge of this study is very helpful in development of engineering application as well as two phase theory.

1.2.4 Convective Boundary Condition

The boundary layer and heat transfer problem is normally considered the boundary condition with constant surface temperature or a constant surface heat flux. However, there might be a situation where a constant surface temperature or a constant heat flux may not be relevant in which the surface heat transfer depends on the surface temperature. For example engineering device such as in heat exchanger where the conduction in a solid tube wall is highly influenced by the convection in the fluid flowing over it. Hence, Aziz (2009) initiated the study of laminar thermal boundary layer over a flat plat with convective surface boundary condition. Apart from this work, there is extensive amount of researches which have studied the convective boundary condition on various fluid flow problems with different orientation, condition and type of fluids. In concerning the dusty fluid problem, Ramesh *et al.* (2015) explored the effect of convective boundary condition of boundary layer flow of fluid containing dust particles past a stretching sheet.

1.2.5 Slip Effect

Most of fluid flow problem especially in Navier-Stokes theory is often considered condition of the no slip boundary condition. No slip condition states that at a solid boundary, the velocity of fluid equal to the velocity of solid boundary. Nevertheless, there is phenomenon where the no slip boundary condition is not valid that is when the fluid is particulate such as emulsions, suspensions, foams and polymer solutions. In such phenomenon, partial slip velocity may occur on the stretching surface where the velocity of fluid at the surface interface may be in conflict with the velocity of the surface.

In Prandtl boundary layer theory, based on Navier-Stokes allows boundary layer structure, in which, this region of the flow is independent of the Mach number as well as Knudsen number. Noting that, Knudsen number represents the ratio of the molecular mean free path length to a representative physical length scale. It is used to specify the formulation of fluid dynamic to model certain situations, either statistical mechanics or continuum mechanics should be chosen. If the Knudsen number is near and less than one, the continuum assumption of fluid mechanics is a good approximation and known as hydrodynamic regime. On the other hand, if the Knudsen number is near and greater than one, the statistical method should be used and known as Knudsen regime. Increasing in Knudsen number may decrease the boundary layer thickness and in time a slip-like phenomenon occurs, and slip boundary condition are used to account the phenomenon (Naoufel *et al.*, 2004).

1.3 Problem Statement

Research on MHD natural convection flow and heat transfer due to stretching sheet has risen considerably due to the occurrence of magnetic effect in various industrial applications such as extrusion of plastic and rubber sheets, polymer processing and metallurgy (Tamizharasi and Kumaran, 2011). For a long term, the impact from corrosion and wear activities may produce solid particles in the form of ash or soot suspended in a conducting fluid. Moreover, these resulting solid particles may influence the behavior of flow as well as the temperature of the fluid. Therefore, the study of two phase flow or dusty fluid in which solid particles distributed in a conducting fluid are very imperative and motivating to investigated. Furthermore, the phenomenon of natural convection flow of dusty fluid affected by several important effects such as MHD, Hall effect, radiation, convective surface and slip interesting to be investigated. Therefore, this study of dusty fluid with various effects leads us to the following research questions:

1. How is the mathematical model of the dusty fluid problem formulated?
2. How do dusty fluid models compared with existing Navier-Stokes models describe the nature of boundary layer flow of dusty fluid past a vertical stretching surface?
3. How do the effects of fluid particle suspension, MHD, Hall effect, slip, radiation and convective surface influence the boundary layer flow and heat transfer characteristics?

1.4 Objective of the Research

This research aims to investigate theoretically the effect of the presence of dust particles distributed in a fluid on natural convection flow past a vertical stretching sheet. This involves the mathematical formulation which included the derivation of the continuity, momentum and energy equations for fluid and dust phases and the transformation of the resulting governing equations into non-dimensional ordinary differential equations by using similarity transformation. The solution of the non-dimensional ordinary differential equation is obtained numerically by using implicit finite difference scheme, known as Keller-box method. The main objectives of this study are:

1. To investigate the effect of MHD on natural convection flow of dusty fluid.
2. To investigate the slip effect on MHD natural convection flow of dusty fluid.
3. To investigate the effect of thermal radiation on MHD natural convection flow of dusty fluid with convective boundary condition.

4. To investigate the effect of Hall current on MHD natural convection flow of dusty fluid.
5. To investigate the slip effect and Hall current on MHD natural convection flow of dusty fluid with thermal radiation and convective boundary condition.

1.5 Scope of the Research

The two dimensional incompressible fluid models is taken into consideration in this study. The energy dissipation is neglected due to its small magnitude (Rivin, 1999) and only laminar boundary layer MHD dusty fluid flow is considered. This study focused on MHD boundary layer flow of dusty fluid past a vertical stretching sheet. The usual boundary conditions in which no slip condition and constant surface temperature is considered in this study. In addition, the occurrence where the partial slip velocity and convective surface boundary condition is also considered. Furthermore, the MHD natural convection past a stretching sheet is applied in dusty fluid. The Keller-box method is used in order to solve the coupled ordinary differential equation with the help of FOTRAN software. The research framework of this study is shown in Figure 1.2.

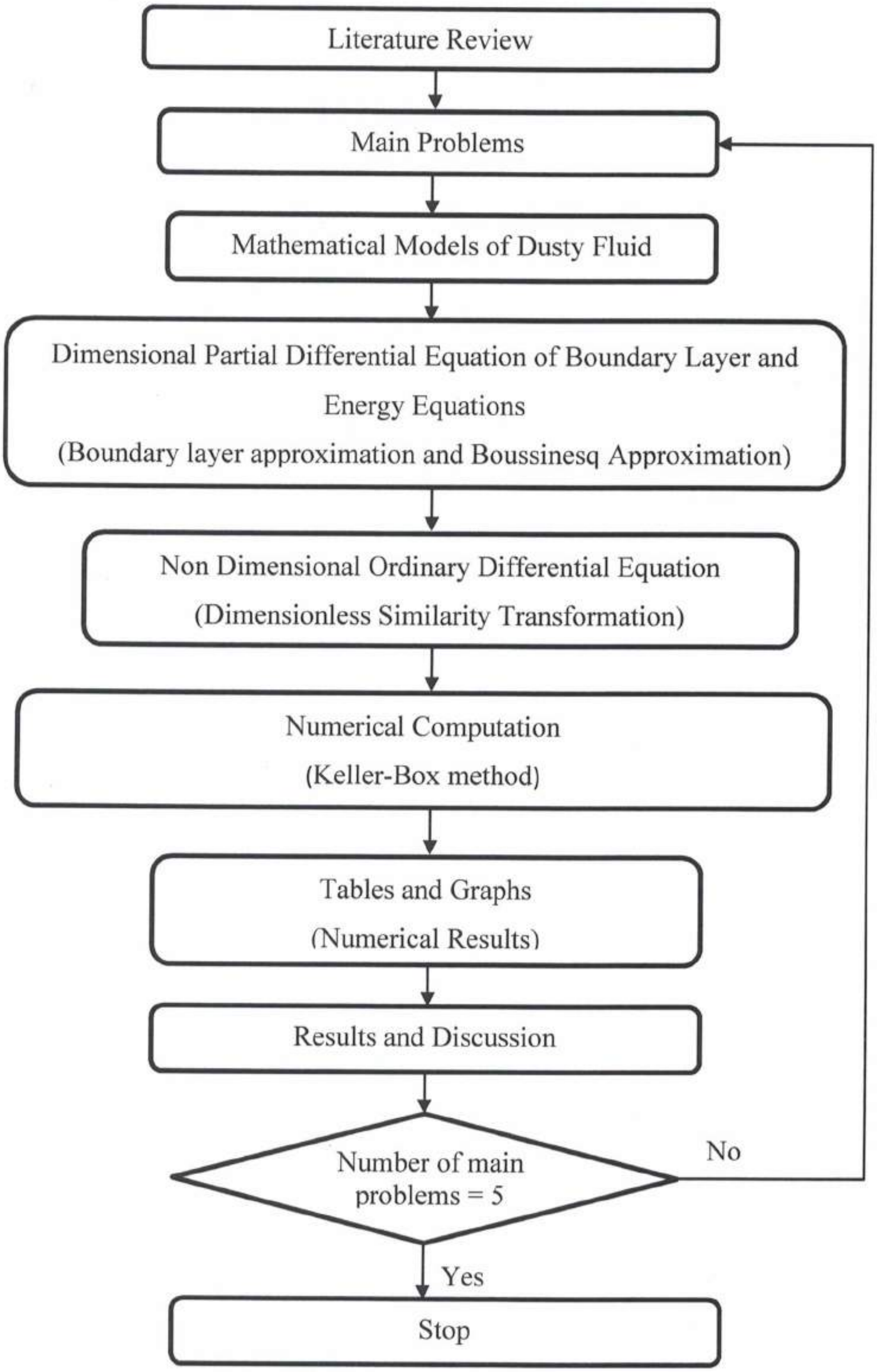

Figure 1. 1 Research Framework.