PARALLELIZATION OF MULTIDIMENSIONAL HYPERBOLIC PARTIAL DIFFERENTIAL EQUATION ON DÉTENTE INSTANTANÉE CONTRÔLÉE DEHYDRATION PROCESS

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To my dear husband, abah, mak, and family.

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ABSTRACT

The purpose of this research is to propose some new modified mathematical models to enhance the previous model in simulating, visualizing and predicting the heat and mass transfer in dehydration process using instant controlled pressure drop (DIC) technique. The main contribution of this research is the mathematical models which are formulated from the regression model (Haddad *et al.*, 2007) to multidimensional hyperbolic partial differential equation (HPDE) involving dependent parameters; moisture content, temperature, and pressure, and independent parameters; time and dimension of region. The HPDE model is performed in multidimensional; one, two and three dimensions using finite difference method with central difference formula is used to discretize the mathematical models. The implementation of numerical methods such as Alternating Group Explicit with Brian (AGEB) and Douglas-Rachford (AGED) variances, Red Black Gauss Seidel (RBGS) and Jacobi (JB) method to solve the system of linear equation is another contribution of this research. The sequential algorithm is developed by using Matlab R2011a software. The numerical results are analyzed based on execution time, number of iterations, maximum error, root mean square error, and computational complexity. The grid generation process involved a fine grained large sparse data by minimizing the size of interval, increasing the dimension of the model and level of time steps. Another contribution is the implementation of the parallel algorithm to increase the speedup of computation and to reduce computational complexity problem. The parallelization of the mathematical model is run on Matlab Distributed Computing Server with Linux operating system. The parallel performance evaluation of multidimensional simulation in terms of execution time, speedup, efficiency, effectiveness, temporal performance, granularity, computational complexity and communication cost are analyzed for the performance of parallel algorithm. As a conclusion, the thesis proved that the multidimensional HPDE is able to be parallelized and PAGEB method is the alternative solution for the large sparse simulation. Based on the numerical results and parallel performance evaluations, the parallel algorithm is able to reduce the execution time and computational complexity compared to the sequential algorithm.

ABSTRAK

Kajian ini dilakukan bertujuan untuk mengemukakan model baru yang diubah untuk menambah baik model sebelum ini dalam menjalankan simulasi, menggambarkan dan meramal pemindahan haba dan jisim dalam proses pengeringan menggunakan teknik kawalan segera kejatuhan tekanan (DIC). Sumbangan utama kajian ini ialah model matematik diformulasikan daripada model regresi (Haddad *et al*., 2007) kepada persamaan pembezaan separa hiperbolik (HPDE) pelbagai dimensi yang melibatkan parameter bersandar; kandungan air, suhu, dan tekanan, dan parameter tidak bersandar; masa dan rantau dimensi. Model HPDE dilaksanakan dalam pelbagai dimensi; satu, dua dan tiga dimensi menggunakan kaedah beza terhingga dengan rumus beza tengah telah digunakan untuk mendiskretasikan model matematik tersebut. Pelaksanaan kaedah berangka seperti Kaedah Tak Tersirat Kumpulan Berselang-seli dengan variasi Brian (AGEB), dan variasi Douglas-Rachford (AGED), Gauss Seidel Merah Hitam (RBGS), dan Jacobi (JB), dalam menyelesaikan sistem persamaan linear juga merupakan satu lagi sumbangan dalam kajian ini. Algoritma berjujukan dibangunkan menggunakan perisian Matlab R2011a. Keputusan berangka dianalisis berdasarkan masa pelaksanaan, bilangan lelaran, ralat maksima, ralat punca min kuasa dua, dan pengiraan kompleksiti. Proses penjanaan grid melibatkan butiran halus data yang besar dan jarang iaitu dengan meminimumkan saiz selang ruang, meningkatkan dimensi model dan peringkat paras masa. Antara sumbangan lain ialah algoritma selari dicadangkan untuk meningkatkan kecepatan pengiraan dan mengurangkan masalah pengiraan kompleksiti. Model matematik yang diselarikan dilaksanakan menggunakan Pengkomputeran Teragih Matlab dengan sistem operasi Linux. Penilaian prestasi selari bagi model pelbagai dimensi berdasarkan masa pelaksanaan, kecepatan, kecekapan, keberkesanan, prestasi sementara, granulariti, pengiraan kompleksiti dan kos komunikasi dianalisis untuk prestasi algoritma selari. Sebagai kesimpulan, kajian ini membuktikan HPDE pelbagai dimensi dapat diselarikan dan kaedah PAGEB merupakan penyelesaian alternatif bagi simulasi yang besar dan jarang. Berdasarkan keputusan berangka dan penilaian prestasi selari, algoritma selari dapat mengurangkan masa pelaksanaan dan kompleksiti pengiraan berbanding dengan algoritma berjujukan.

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LIST OF ABBREVIATIONS

1D_PAGEB	-	1D Parallel Alternating Group Explicit with Brian variant
1D_PAGED	-	1D Parallel Alternating Group Explicit with Douglas-Rachford variant
1D_PJB	-	1D Parallel Jacobi
1D_PRBGS	-	1D Parallel Red Black Gauss Seidel
1D_SAGEB	-	1D Sequential Alternating Group Explicit with Brian variant
1D_SAGED	-	1D Sequential Alternating Group Explicit with Douglas-Rachford variant
1D_SJB	-	1D Sequential Jacobi
1D_SRBGS	-	1D Sequential Red Black Gauss Seidel
2D_PAGEB	-	2D Parallel Alternating Group Explicit with Brian variant
2D_PAGED	-	2D Parallel Alternating Group Explicit with Douglas-Rachford variant
2D_PJB	-	2D Parallel Jacobi
2D_PRBGS	-	2D Parallel Red Black Gauss Seidel
2D_SAGEB	-	2D Sequential Alternating Group Explicit with Brian variant
2D_SAGED	-	2D Sequential Alternating Group Explicit with Douglas-Rachford variant
2D_SJB	-	2D Sequential Jacobi
2D_SRBGS	-	2D Sequential Red Black Gauss Seidel
3D_PAGEB	-	3D Parallel Alternating Group Explicit with Brian variant
3D_PAGED	-	3D Parallel Alternating Group Explicit with Douglas-Rachford variant
3D_PJB	-	3D Parallel Jacobi
3D_PRBGS	-	3D Parallel Red Black Gauss Seidel

3D_SAGEB	-	3D Sequential Alternating Group Explicit with Brian variant
3D_SAGED	-	3D Sequential Alternating Group Explicit with Douglas-Rachford variant
3D_SJB	-	3D Sequential Jacobi
3D_SRBGS	-	3D Sequential Red Black Gauss Seidel
AGE	-	Alternating Group Explicit
AGEB	-	Alternating Group Explicit with Briant variant
AGED	-	Alternating Group Explicit with Douglas-Rachford variant
API	-	Application Programming Interface
CPU	-	Central Processing Unit
DIC	-	Détente Instantanée Contrôlée
DPCS	-	Distributed Parallel Computing System
FDM	-	Finite Difference Method
FEM	-	Finite Element Method
FVM	-	Finite Volume Method
HPDE	-	Hyperbolic Partial Differential Equation
JB	-	Jacobi
LAN	-	Local Area Network
MDCS	-	Matlab Distributed Computing Server
MIMD	-	Multiple Instruction Multiple Data
MISD	-	Multiple Instruction Single Data
MPI	-	Message Passing Interface
PCT	-	Parallel Computing Toolbox
PCW	-	Parallel Command Window
PPE	-	Parallel performance evaluations
PVM	-	Parallel Virtual Machine
RBGS	-	Red Black Gauss Seidel
RMSE	-	Root Mean Square Error
SIMD	-	Single Instruction Multiple Data
SISD	-	Single Instruction Single Data
SLE	-	System of Linear Equations

LIST OF SYMBOLS

C_p	-	Specific heat capacity
D_o	-	Diffusivity
h_m	-	Mass transfer coefficient
M	-	Moisture content
M_o	-	Initial moisture content
Ni	-	Total grid on the x-axis
Nj	-	Total grid on the y-axis
Nk	-	Total grid on the z-axis
P	-	Pressure
P_o	-	Initial pressure
p	-	Number of workers
r	-	Acceleration parameter
t	-	Time
T	-	Temperature
T_o	-	Initial temperature
V	-	Velocity
Δx	-	Step size at x-axis
Δy	-	Step size at y-axis
Δz	-	Step size at z-axis
Δt	-	Time step size
ε	-	Tolerance
ρ	-	Density

LIST OF APPENDICES

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CHAPTER 1

INTRODUCTION

1.1 Background of Research

Food dehydration is one of the most ancient and efficient preservation methods. Numerous food products are routinely preserved using dehydration techniques, which include grains, marine products, meat products, as well as fruits and vegetables. There are several other food preservation techniques such as storing, freezing, pickling, and canning. Some of the storage techniques require low temperatures and are difficult to maintain throughout the distribution chain (Sagar and Suresh Kumar, 2010). Meanwhile, for pickling and canning, chemical preservative is added to extend the shelf life (Silva and Lidon, 2016). On the contrary, the dehydration involves heat, mass transfer phenomena and frequently used in most food processing industries (Cohen and Yang, 1995; Kristiawan *et al.*, 2011). It is a suitable alternative for post-harvest tasks.

Dehydration is a process of removing the water vapor from food into the surrounding area under controlled conditions that cause minimum changes in the food properties (Chen and Mujumdar, 2008; Potter and Hotchkiss, 1998). The purposes of dehydration are to extend the life of the food product, decrease weight for transportation, enhance storage stability and minimize the packaging requirements. Besides, it is necessary to remove the moisture content to a certain level in order to prevent the growth of bacteria, yeast, and molds thus slowing down or stopping food spoilage (Mujumdar and Law, 2010). The conventional dehydration techniques found in the food processing industry are freeze, hot air, osmotic, solar, and vacuum (George *et al.* 2004). Unfortunately, these conventional dryers have several limitations such as high operating cost, low quality and slow process. Table 1.1 shows the advantages and disadvantages of these conventional dehydration techniques.

Table 1.1 : Summary of the conventional dehydration techniques

Drying techniques	**Characteristic**	**Advantages**	**Disadvantages**	**References**
Freeze	The frozen water is removed from food without going through liquid phase.	Highest quality product, minimal reduction in shape, color and structure.	High operating costs.	Marques *et al.* (2006), Ratti (2001), Shishehgarha *et al.* (2007)
Hot air	The food is in contact with hot air.	Product extends the life of a year.	Low quality compared to the original food.	Ratii (2001)
Osmotic	The food is soaked in hypertonic solution.	High quality, little energy, reduces process temperature, short drying time.	A slow process because depends on the cell membrane permeability and architecture.	Ahmed *et al.* (2016), Amami *et al.* (2007)
Solar	The food is dried using solar light.	Simple, low cost.	Large space, labor-intensive, difficult to control, slow process, bacterial contamination.	Sagar and Suresh Kumar (2010)
Vacuum	The food is operated under low pressure and temperature.	High quality product, low energy consumption	A slow process.	Saberian *et al.* (2014), Thorat *et al.* (2012)

Based on the limitations from Table 1.1, the conventional dehydration techniques have been improved to enhance the quality of end drying products in terms of color, flavor, nutritional value and texture (Alves-Filho, 2007; Chen and Mujumdar, 2008; Fernandes *et al.* 2011; Mujumdar, 2006). Some of the novel dehydration techniques are microwave, fluidized-bed, ultrasonic and microwave-augmented freeze (Cohen and Yang, 1995; Falade and Omojola, 2010; Fernandes *et al.*, 2011; Jangam, 2011; Mujumdar and Law, 2010; Sagar and Suresh Kumar, 2010).

1.2 DIC Technique

Another alternative of dehydration is Détente Instantanée Contrôlée (DIC) technique. DIC is known as instant control pressure drop technique. This technique has the potential to be the most commonly used dehydration methods for high value products. DIC is developed by the Laboratory for Mastering Agro-Industrial Technologies (LMTAI) research team (Allaf *et al.*) since 1988 (Allaf *et al.*, 1999; Setyopratomo *et al.*, 2009) from the University of La Rochelle, France. It is based on the high temperature short time heating (HTST) and followed by an instant pressure drop. DIC consists of three main parts which are processing chamber, vacuum reservoir and valve. The products are treated in the processing chamber at high temperature (up to 170°C) and at high pressure (up to 8×10^5 Pa) with steam. The volume of vacuum tank is at least 50 times greater than the processing chamber. The DIC layout diagram is shown in Figure 1.1 (Haddad and Allaf, 2007). Figure 1.1 shows the vacuum pump, vacuum tank with cooling liquid jacket; instant pressure-drop valve, DIC reactor with heating jacket; and steam boiler. Table 1.2 shows the value of parameters used in DIC technique such as pressure, water content and processing time.

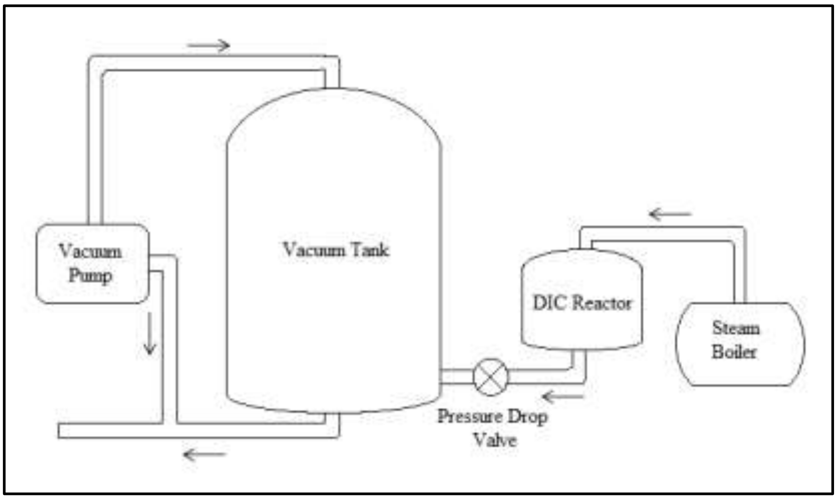


Figure 1.1 Schematic diagram of the DIC reactor

Table 1.2 : Value of parameters for pressure, water content and processing time

No	Parameter	Symbol	Value	References
1	Pressure	Pa	4-7x10^5	Haddad and Allaf (2007), Haddad *et al.* (2007), Setyopratomo *et al.* (2012)
			4-5x10^5	Haddad *et al.* (2008)
2	Initial water content	g water/100g dry matter	30-50	Haddad and Allaf (2007), Haddad *et al.* (2007)
3	Time	s	40-60	Haddad and Allaf (2007), Haddad *et al.* (2007)
			10-20	Haddad *et al.* (2008)
			30-45	Setyopratomo *et al.* (2012)

The temperature and pressure changes are presented in Figure 1.2 where stage (a) is at atmospheric pressure. Then, a vacuum condition is created within the reactor to get the greatest contact between steam and materials surface by opening the discharge valve (Figure 1.2(b)). Steam is injected to the materials to create a pressurized atmosphere (Figure 1.2(c)). The materials are left in contact with high pressure for a few seconds (Figure 1.2(d)). Then, a sudden pressure drop in the reactor is created by opening the discharge valve in less than a second (Figure 1.2(e)) which is called as instantaneous pressure drop since the value of $\frac{\Delta P}{\Delta t}$ is higher than 5×10^5 Pa s^{-1}. This instantaneous pressure drop induces rapid auto-vaporization of the moisture from the material and lead to texture change which results in higher porosity. Besides, it also intensifies functional behavior of drying product (Setyopratomo *et al.*, 2009). The material is maintained in vacuum condition (Figure 1.2(f)). The final step is returning the reactor to atmospheric pressure (Figure 1.2(g)). DIC increases the material porosity and surface area and reduces the diffusion resistance of moisture during the final dehydration step.

This technique has been successfully used for various products including: fruit swell drying and vegetables drying (Djilali *et al.*, 2016; Haddad *et al.*, 2008; Louka and Allaf, 2002; Tellez-Perez *et al.*, 2015), texturing and drying various biological products by instant auto vaporization (Haddad and Allaf, 2007; Kristiawan *et al.*, 2011; Louka and Allaf, 2004; Louka *et al.*, 2004; Nouviaire *et al.*, 2008; Setyopratomo *et al.*, 2012), and microbiological decontamination (Setyopratomo *et al.*, 2009), post harvesting or steaming paddy rice (Pilatowski *et al.*, 2010) and essential oil extraction (Amor *et al.*, 2008; Besombes *et al.*, 2010). Besides, some experiments have been done to investigate the effect of the DIC technique on Lupin (Haddad *et al.*, 2007); soybean (Haddad and Allaf, 2007); glucose polymer (Rezzoug *et al.*, 2000); and milk (Mounir *et al.*, 2010).

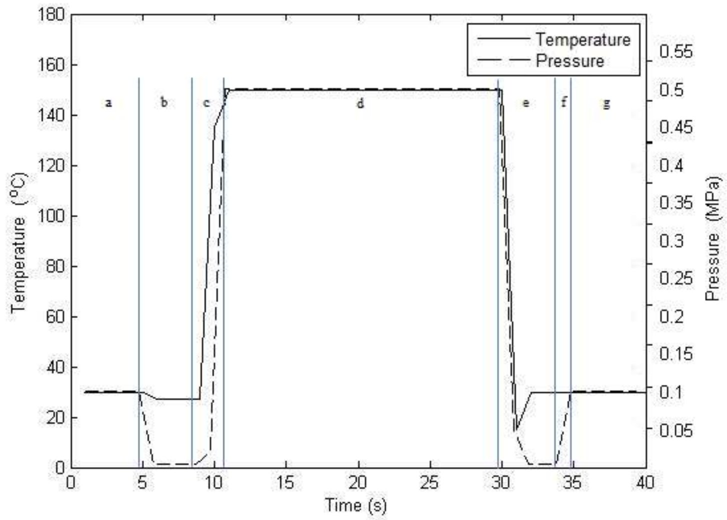


Figure 1.2 Temperature and pressure changes during DIC treatment

1.2.1 Mathematical Model in DIC Technique

Researches on the experiment and mathematical model have been done in order to understand the dehydration mechanism since it is a very complex process. The development of mathematical models is to predict, design and control water losses, weight reduction, dehydration rates and temperature behavior. It is also able to perform an optimal strategy for dryer process control. Parameters during dehydration can range from a very simple to complicated parameter in order to upgrade the quality of dehydration technology.

The mathematical model in drying method can be classified as empirical, semi-empirical and theoretical models depending on the different applications (Vijayaraj *et al.*, 2006). The empirical and semi-empirical model take into account the external resistance to moisture transfer meanwhile the theoretical model considers the internal resistance to moisture transfer between the food product and air (Midilli *et al.*, 2002; Panchariya *et al.*, 2002). Theoretical models require assumptions of geometry of food, its mass diffusivity and conductivity (Demirtas *et al.*, 1998; Wang *et al.*, 2007). The fundamental of drying process is not taken into consideration for empirical model and this model presents a direct relationship between average time and drying time using regression analysis (Ozdemir and Devres, 1999).

In DIC literature, most researches focused on statistical method of regression model (Haddad and Allaf, 2007; Haddad *et al.*, 2007; Mounir *et al.*, 2010; Setyopratomo *et al.*, 2012). The regression model estimated the relationships among a dependent variable and one or more independent variables. Haddad and Allaf (2007) and Haddad *et al.* (2007) demonstrated the efficiency of DIC in drying the soybean trypsin inhibitors and phytate content, respectively. The steam pressure, treatment time, and initial water content were the DIC operating parameters that were taken into consideration. The results obtained show the reduction of trypsin inhibitors and phytate content were affected due to these operating parameters which was in a quadratic form. Besides, it is found that pressure and treatment time gave high impact to the reduction of the trypsin inhibitors. The regression model presented a good fit to the observed data but it is limited to a certain experiment (Kaushal and Sharma, 2014). When the experiment is implemented under different conditions, the model did not provide good simulation of dehydration process. Besides, the regression model neglects the fundamental of dehydration process where the parameters involved have no physical meaning (Simal *et al.*, 2005).

Based on the limitations from the regression model, parabolic PDE is shown to be fit with the regression model. The parabolic PDE or Fick's law of diffusion equation is proposed to analyze the effect of DIC technique on the drying kinetics of drying materials but neglected the effects of possible shrinkage (Abdulla *et al.*, 2010; Kamal *et al.*, 2012; Mounir *et al.*, 2011; Mounir *et al.*, 2012; Pilatowski *et al.*, 2010; Setyopratomo *et al.*, 2009; Setyopratomo *et al.*, 2012). However, most of the researchers only discussed the fundamental of the dehydration model in DIC technique without solving the equation (Haddad *et al.*, 2008; Mounir *et al.*, 2012). Some of the authors solved the model using Crank (1975) solution according to the geometry of the solid matrix to solve the diffusion equation for mass transport of water within the drying material (Abdulla *et al.*, 2010; Mounir *et al.*, 2011; Mounir *et al.*, 2014; Setyopratomo *et al.*, 2009; Setyopratomo *et al.*, 2012; Tellez-Perez *et al.*, 2012). Meanwhile, other authors (Albitar *et al.*, 2011; Kamal *et al.*, 2012) solved the PDE model with the logarithmic transformation. Zarguili *et al.* (2009) solved the first order partial differential equation (PDE) of mass transfer equation by using integration method. Only a few researchers in DIC technique solved the model using numerical methods.

The existing parabolic model does not involve the main parameter in DIC technique which is pressure. Besides, based on the simulation results obtained in Chapter 3, the diffusion is found to be a very slow process which contradicts to the DIC technique where it involves high temperature high pressure process. Therefore, a new modified mathematical model based on the hyperbolic PDE (HPDE) is proposed. This model is relevant based on Meszaros *et al.* (2004) and Reverbi *et al.*

(2008) where they stated that hyperbolic heat and mass transfer is an alternative model because the classical parabolic equation is impossible to solve the extreme condition such as high temperature. The HPDE model is able to integrate between the dependent parameters; moisture content, temperature, and pressure, and independent parameters; time and dimension of region in order to simulate, visualize, and predict the heat and mass transfer during the dehydration process using DIC technique. Further details on the formulation of the HPDE model will be discussed in Chapter 3.

Numerical methods are able to solve a complex system of PDE which is almost impossible to be solved analytically. The Finite Element (FEM), Finite Volume (FVM) and Finite Difference methods (FDM) are some alternative numerical methods to solve the PDE (Peiro and Sherwin, 2005). For the other applications of drying, the FDM has been widely used to solve the heat and mass transfer models (Braud *et al.*, 2001; Karim and Hawlader, 2005; Liu *et al.*, 2014; Naghavi *et* al, 2010; Rovedo *et al.*, 1995; Simal *et al.*, 2000). The FDM scheme is chosen because this method is simple to formulate a set of discretized equations from the transport differential equations in a differential manner (Botte *et al.*, 2000; Chandra and Singh, 1994). Besides, this method is straightforward in determining the unknown values (Incopera and DeWitt, 1996). Thus, due to this reason, the mathematical model in this research will be solved using FDM scheme. Further details of FDM will be discussed in Chapter 2.

A large sparse data of system of linear equations (SLE) is governed by the FDM to present the actual region of the dehydration proses for numerical simulation. The grid generation process involved a fine grained of the large sparse data by minimizing the size of interval, increasing the dimension of the model and level of time steps. However, using only one CPU will take too high execution time to compute for the solution. Therefore, parallelization in solving a large sparse data is a great important process. The objective is to speed up the computation and increase the efficiency by using massively parallel computers. Thus, it is important to design the parallel algorithm before implementing on the DPCS. The strategy to design the parallel algorithm is illustrated in Figure 1.3.

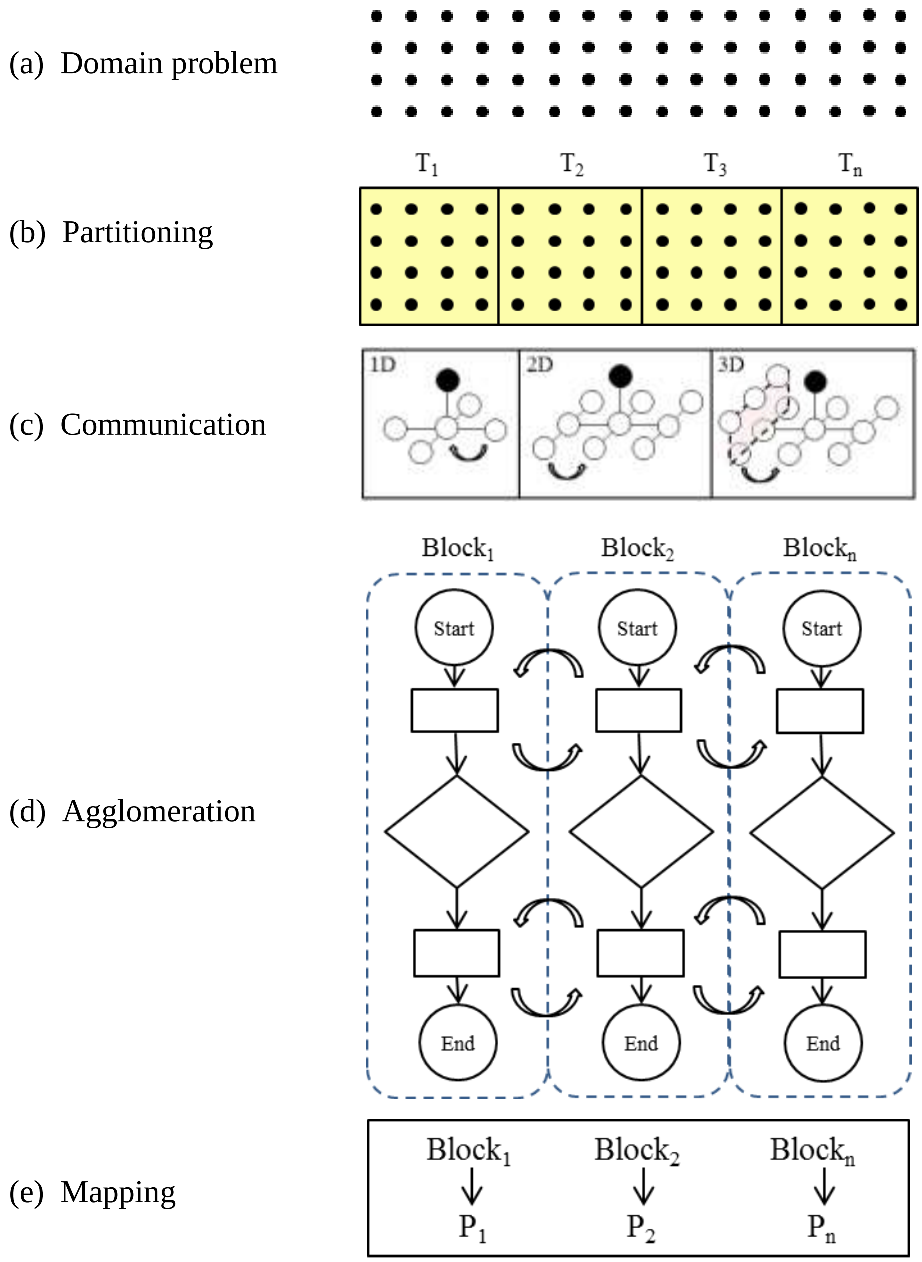


Figure 1.3 Parallel algorithm design

The domain depends on the problem where it can be in 1D, 2D or 3D domain (Figure 1.3(a)). The domain problem is partitioned column-wise distribution into equal sized tasks, $T_1, T_2, ..., T_n$ where n is number of processors involved in the parallel algorithm (Figure 1.3(b)). Then, the tasks are connected to each other through local and global communication (Figure 1.3(c)). The local communication involves communication by sending and receiving data between the neighboring points where the data is sent by point for 1D, by line for 2D and by surface for 3D. Meanwhile global communication requires communication with other tasks. The number of tasks is combined into a set of tasks; $Block_1$, $Block_2$, …, $Block_n$ to improve the performance of parallelization. This strategy is called as agglomeration (Figure 1.3(d)). Lastly, each block is assigned to a processor (Figure 1.3(e)). Static mapping is implemented because it is easier to design and implemented on the distributed parallel computing architecture compared to dynamic mapping which is more complicated in message passing program.

The hardware computational tool to support the parallel algorithm is based on distributed parallel computing system (DPCS). The software tool to support DPCS is based on Matlab Distributed Computing Server (MDCS) version 7.12 (R2011a). The MDCS consists of a heterogeneous computing system contains 8 computers with Intel Core Duo CPUs under Fedora 8 featuring a 2.6.23 based Linux kernel operating system, connected with internal network 10/100/1000 NIC. The DPCS and MDCS are discussed further in Chapter 2.

1.3 Statement of Problem

The existing mathematical model in dehydration process using DIC technique is focused on the statistical method of regression model. However, this model limits to certain experiment (Kaushal and Sharma, 2014). Besides, this model neglects the fundamental of dehydration process where the parameters involved have no physical meaning. Thus, the dehydration process cannot be predicted using the regression model. The second problem is some of the researchers in DIC technique only discussed the fundamental of the dehydration model in DIC technique without produced any solution to the mathematical model. The third problem is some of them solved the PDE analytically which involves too many parameters. Therefore, it is almost impossible to be solved and it is time consuming.

Based on these limitations, the main aim of this research is to formulate a new modified mathematical model based on HPDE from the regression model obtained from Haddad *et al.* (2007). The HPDE is able to integrate between the dependent parameters; moisture content, temperature, and pressure, and independent parameters; time and dimension of region in order to simulate, visualize, and predict the heat and mass transfer during the dehydration process using DIC technique. The mathematical model performs in multidimensional problem and FDM is used to discretize the mathematical model. Numerical methods such as Jacobi (JB), Red Black Gauss Seidel (RBGS), Alternating Group Explicit with Douglas-Rachford (AGED), and Brian (AGEB) variances are used to solve the SLE. A large sparse matrix from the SLE is obtained from the discretization, thus, it performs high execution time using a single CPU. Therefore, a DPCS is implemented on MDCS to reduce the computational time and increase the speedup performance.

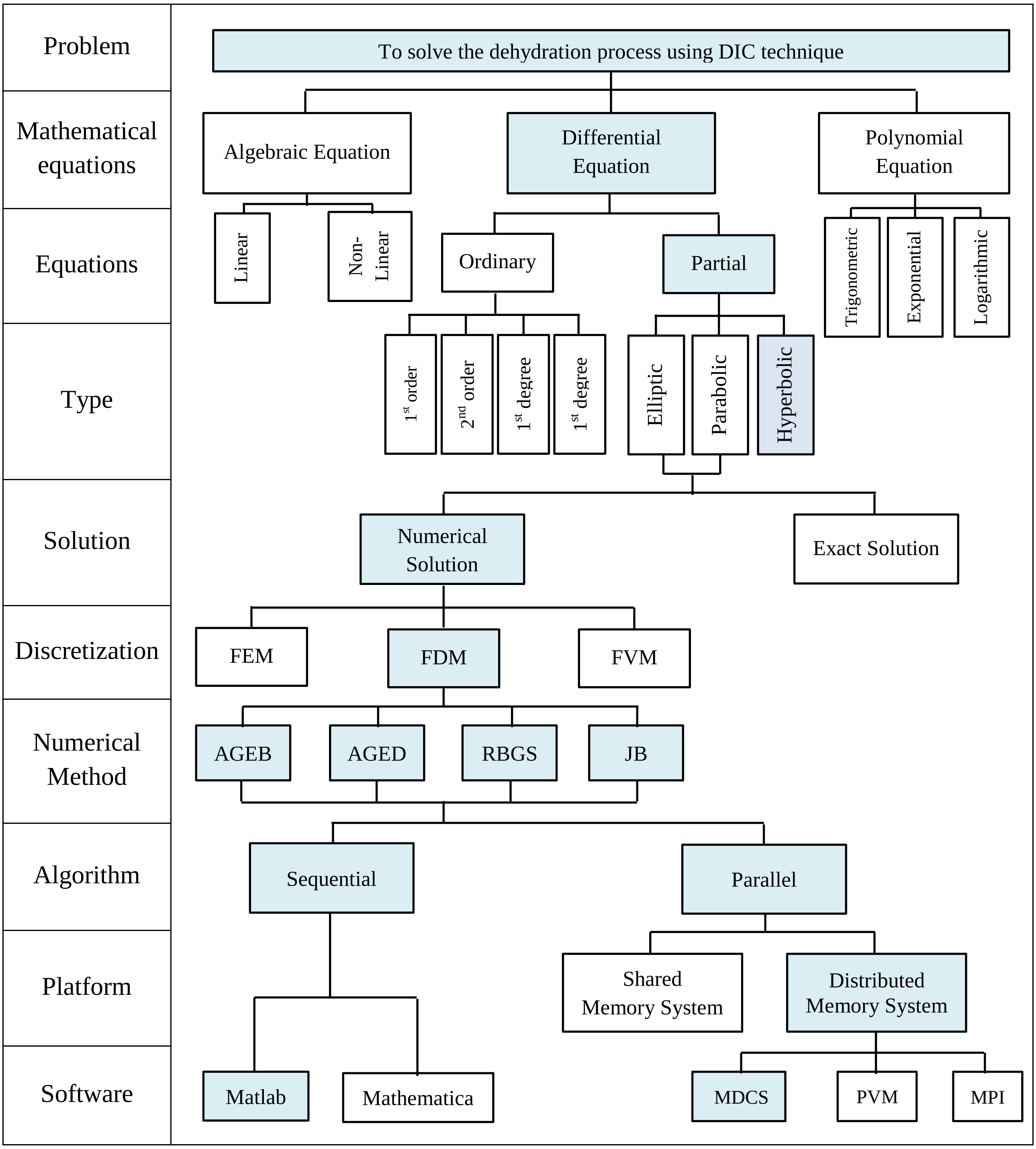
1.4 Objectives of Research

This section explains the objectives of this research which are:

a) To formulate the regression model from Haddad *et al.* (2007) to a new modified mathematical model of heat and mass transfer in DIC technique and discretized using FDM to approximate the solution of the mathematical model.
b) To solve the SLE in (a) using some numerical methods such as AGEB, AGED, RBGS, JB methods.
c) To develop sequential and parallel algorithms from (b) using MDCS.
d) To analyze the results in (c) based on the numerical results for sequential algorithm and PPE for parallel algorithm.

1.5 Scope of Research

The main research problem of this thesis is to solve the dehydration process involved using DIC technique. Based on the limitations of the existing mathematical model in DIC technique, a new modified mathematical model based on the HPDE is formulated from the regression model. HPDE model is chosen because this model is able to integrate between the time, dimension of region, moisture content, temperature and pressure. The HPDE is discretized using FDM based on the central difference formula. Then, the SLE obtained from the discretization is solved using some numerical methods such as AGEB, AGED, RBGS and JB methods where JB is the benchmark for the other numerical methods. The numerical methods are solved using the sequential algorithm on the Matlab software. Since it involves a large sparse matrix which results in high execution time and high computational complexity, thus, the parallel algorithm is implemented on the MDCS. The scope of this research is illustrated in the table below:


Problem
Mathematical equations
Equations
Type
Solution
Discretization
Numerical Method
Algorithm
Platform
Software
To solve the dehydration process using DIC technique
Algebraic Equation
Differential Equation
Polynomial Equation
Linear
Non-Linear
Ordinary
Partial
Trigonometric
Exponential
Logarithmic
1st order
2nd order
1st degree
1st degree
Elliptic
Parabolic
Hyperbolic
Numerical Solution
Exact Solution
FEM
FDM
FVM
AGEB
AGED
RBGS
JB
Sequential
Parallel
Shared Memory System
Distributed Memory System
Matlab
Mathematica
MDCS
PVM
MPI

1.6 Significance of Research

The first significance of this research is the HPDE is the alternative model to simulate, visualize, predict and control the independent and dependent parameters of dehydration process. The second significance is the implementation of the numerical methods such as AGEB, AGED, RBGS and JB methods are suitable to solve the multidimensional HPDE. The third significance is the parallel implementation to solve the large sparse data for the multidimensional HPDE on DPCS successfully reduces the computational time and increases the performance of speedup. The numerical results and PPE are the indicators to measure the performance of multidimensional HPDE and the large sparse simulation. From the numerical results and PPE, AGEB is the best method to solve the HPDE model followed by AGED, RBGS and JB methods. It is also found that the parallel algorithm is performed better than the sequential algorithm.

1.7 Thesis Organization

In this thesis, there are seven chapters including the introduction and conclusion parts. Chapter 1 comprises a description of the research problem statement on DIC technique. The dehydration process is described based on the previous literature review on mathematical model developed in DIC technique. This chapter also discusses the research objectives, scope and the significance of the research.

Chapter 2 discusses the literature review on the FDM and the basic scheme for PDE. The numerical methods such as JB, RBGS, AGED, and AGEB, and its algorithm procedure are presented in this chapter. It follows by explaining the numerical analysis based on the consistency, convergence, stability, numerical errors and computational complexity. Finally, this chapter will discuss the platform of DPCS to support the MDCS and PPE based on speedup, efficiency, effectiveness, temporal performance, granularity and communication cost to measure the parallel algorithm.

The new modified mathematical model development in DIC technique is covered in Chapter 3. In this chapter, the formulation of the hyperbolic partial differential equation (HPDE) from the regression model from Haddad *et al.* (2007) is presented. The simulations of the mathematical models are analyzed and shown through graphical representation using Matlab 7.12 (R2011a) software. The HPDE is visualized in multidimensional model which are in 1D, 2D and 3D model.

The contribution of Chapter 4 is the numerical results and parallel performance evaluations of sequential and parallel algorithms for 1D HPDE model. The SLE for 1D model is obtained from FDM and it is solved using some numerical methods such as 1D_SJB, 1D_SRBGS, 1D_SAGED, and 1D_SAGEB. These numerical methods are compared according to execution time, number of iteration, maximum error and root mean square error. Then, these numerical methods are parallelized to improve the performance of the sequential algorithm. The parallel performances for these methods: 1D_PJB, 1D_PRBGS, 1D_PAGED and 1D_PAGEB are measured based on speedup, efficiency, effectiveness, temporal performance and granularity.

The 1D model is then upgraded into 2D because it reflects the real physical phenomena. The numerical results and parallel performance evaluations for 2D HPDE model are the main contribution for Chapter 5. The 2D HPDE model is discretized using FDM with central difference formula and numerical methods such as 2D_SJB, 2D_SRBGS, 2D_SAGED, and 2D_SAGEB are used to solve the SLE. The numerical results are compared based on execution time, number of iteration, maximum error and RMSE. Meanwhile, the parallelization of these numerical methods such as 2D_PJB, 2D_PRBGS, 2D_PAGED, and 2D_PAGEB are analyzed based on speedup, efficiency, effectiveness, temporal performance and granularity.

Furthermore, the contribution of Chapter 6 focuses on the numerical results and parallel performance of the sequential and parallel algorithms for 3D HPDE model. The discretization of the model is based on the FDM. It is then solved by some numerical methods which are 3D_ SJB, 3D_SRBGS, 3D_SAGED, and 3D_SAGEB. The numerical methods are parallelized into 3D_PJB, 3D_PRBGS, 3D_PAGED, and 3D_PAGEB. The PPE of these methods are measured using speedup, efficiency, effectiveness, temporal performance and granularity.

Lastly, Chapter 7 concludes the research findings based on every chapter in the thesis. Some general remarks on the recommendation for future research are discussed.

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