MATHEMATICAL MODELLING IN THE RIVER POLLUTION

AHMAD ZAKI BIN MOHAMAD AMIN

A dissertation submitted in partial fulfilment of
the requirements for the award of the degree of
Master of Science (Engineering Mathematics)

Faculty of Science
Universiti Teknologi Malaysia

JUN 2014

This is my present to my beloved parents Haji Mohamad Amin Bin Apah, Arwah Hajah Sulbah Binti Abdul Rahman, Ahmad Fauzi Mohamad Amin, Fadhil Mohamad Amin, Ahmad Fuad Mohamad Amin, Mastura Binti Mohamad Amin, and for those who are always believe and appreciate in the richness of learning.

ACKNOWLEDGEMENT

Alhamdulillah, all praise to Allah for giving me the strength health and patience throughout this whole process of completing my dissertation. This study is the result of the efforts of many individuals. Therefore, I would like to owe special thanks to a number of people who had helped through my study both direct and indirectly.

I would like to take this opportunity to express my greatest appreciation and honor gratitude to my supervisor Prof Madya. Dr. Shamsuddin bin Ahmad, co-supervisor, Dr. Yeak So Ho and all examiners for their continuous guidance, constructive comments, invaluable advices and their efforts and suggestions in helping me throughout these two semesters to complete my proposal together with my thesis. This study would have not been possible if not for everyone constant assistance and guidance.

The greatest appreciation goes to my family and friends for their love and care, encouragement and support to me while studying in Universiti Teknologi Malaysia (UTM) Johor Bahru. Thank you all for being part of my life.

ABSTRACT

Effective tools to simulate and predict pollutant transport in water environment especially river are water quality models, which can contribute to save the cost of labors and materials for a large number of chemical experiments to some degree. Due to special environmental pollution issues the water quality models become very important in some cases. Therefore, to identify water environment pollution, water quality model become an important tool to recognize the behaviours of pollutants in water environment. In this research, there are three main purposes to be resolved. The analytical solution of the model is found using Laplace transform method. For the graph of the solution, we interpret and discuss the concentration of the pollutant against time (t).

ABSTRAK

Alat yang berkesan untuk meramalkan aliran pencemaran dalam persekitaran air terutamanya sungai adalah model kualiti air, dimana ia boleh menyumbang kepada penjimatan kos tenaga kerja dan bahan-bahan untuk sebilangan besar experimen kimia. Oleh itu, untuk mengenalpasti pencemaran air dalam alam sekitar, model kualiti air menjadi alat yang penting untuk mengenal pasti tingkah laku bahan pencemaran dalam persekitaran air. Di dalam kajian ini, terdapat tiga tujuan utama untuk diselesaikan. Penyelesaian analisis model yang didapati adalah menggunakan kaedah Laplace. Dari graf penyelesian, kita akan mentafsirkan dan membincangkan kepekatan bahan pencemar terhadap masa (t).

TABLE OF CONTENTS

LIST OF TABLE

LIST OF FIGURES

LIST OF SYMBOLS

t	-	Time
x	-	Position
σ	-	Diffusion Coefficient
V_0	-	Velocity of the river
α	-	Substance discharge rate

CHAPTER 1

INTRODUCTION

1.0 Introduction

This chapter will briefly discuss on the dissertation flow from the study of background, problem statement and objectives of the study. Hence, it follows by the scope of study, significant of the study and study overview. Each subtopic is related with each other to make the readers easier to understand the dissertation.

1.1 Introduction of Water Quality Models.

Water quality models are effective tools to simulate and predict pollutants transport in water environment especially river. Using these models we can save the cost of labour and materials for a large number of experiments. Due to special environment pollution issues the water quality model become very important to recognize the behaviours of pollutants.

Water quality modelling involves the prediction of water pollution using mathematical simulation techniques. A typical water quality model consists of a collection of formulations representing the physical situation of pollutants in water. Laplace method is used to calculate the solution of the model and then interpreted and validated the solution accoding to the physical situation.

1.1.1 Pollutions

The existence of foreign substance in air and water affected the quality of air and water. Air pollution, water pollution and land pollution are other different kinds of pollution that were found in our environment. The reduction of the quality of air and water effects the living organisms in the medium by A. R. A. Baginda and Z. Zainudin (2009).

Water and air are discharged from industries contributes to water and air quality. The discharged material contains acids, alkalis, salts, poisons, oils and harmful bacteria. Other than industry, the mining and agricultural wastes also contribute to the pollution. Sewage disposal from domestic and farm often allowed to pollute the rivers. In the water pollution, there are point source and a nonpoint source pollutant. The pollutant solute transport can be modelled in term of partial differential equation known as advection-diffusion equation.

1.1.2 Water quality standards

The water quality standards especially rivers in Malaysia always concern by local authorities, government agencies as well as the public. Rivers in Malaysia generally considered to be polluted for example sungai klang in Selangor, sungai juru in Penang and sungai segget in Johor. River quality classification and monitoring is quite extensive. In fact, the water quality is at par, if not better, than many developed countries. At the moment, Malaysia has over 100 monitoring stations in 146 basins maintained by the Department of Environment (DOE). (Malaysia Environment Quality Report, 2006).

According to Malaysia Environment Quality Report (2006), in 1985 the government undertook a national study. The study was carried out in four phases with the intention of developing a national "benchmark" of water quality. Over 120 psycho-chemical and biological parameters were reviewed in the study. At the end, The Interim National Water Quality Standards (INWQS) defined six classes of polluted that is I, IIA, IIB, III, IV, V. (refer to table 1.1)

Table 1.1 Excerpts of INWQS by Malaysia Environment Quality Report (2006)

Parameters	Unit	Classes					
		I	IIA	IIB	III	IV	V
Ammoniacal Nitrogen	mg/i	0.1	0.3	0.3	0.9	2.7	>2.7
BaD.	mgi	1	3	3	6	12	>12
COD	mgI	10	25	25	50	100	> 100
DO	mg	7	5.7	5-7	3-5	<3	<1
pH		6.5-8.5	6,5.9.0	6.5-9.0	5-9	5-9	-
Color	TU C	15	150	150		-	
Elec. Conductivity	PS/cm	1000	1000			6000	-
Floatables		NV	NV	NV	-	-	-
Salinity		0.5	**1**	-	-	2	-
Taste		NOT	NOT	NOT	-	-	-
Total Suspended Solids	mg/I	25	50	50	150	300	300
Temperature	C	-	Normal + 2C	-	Normal + 2 C	-	-
Turbidity	NTU	5	50	50	-		
Fecal Coliform	counts/lOOml	10	100	400	5000 (20000)*	5000 (20000)*	
Total Coliform	counts/ 100 ml	100	5000	5000	50000	50000	>50000

Note :

NV=No Visible floatable materials/debris NOT=No objectionable taste

1.1.3 One Dimensional Advection Diffusion of River Pollution in Semi Infinite Media

Suppose we consider that a chemical substance is being poured at a constant rate into a straight, narrow and clean river that flows with a constant velocity. The concentration $u(x,t)$ of substance at a distance x downstream at time t is the solution of initial value problem (IVP).

$$\frac{\partial u}{\partial t} = \sigma \frac{\partial^2 u}{\partial x^2} - v_0 \frac{\partial u}{\partial x} - kc, \quad x > 0, \qquad t > 0$$

$$u(0,t) = \alpha = \text{Constant}, \; t > 0,$$

$$u(x,0) = 0, \;\; x > 0,$$

$$u(x,t) \to 0, \;\; x \to \infty$$

where σ is the diffusion coefficient, v_0= constant > 0 is the velocity of the river, α = constant > 0 is the substance discharge rate and the second term on the right-hand side in the PDE account for convection effect of the water flow on the substance.

Case 1: The river is slow, then the convection term is much smaller than the diffusion term. The PDE becomes

$$\frac{\partial u}{\partial t} = \sigma \frac{\partial^2 u}{\partial x^2}, \quad x > 0, \qquad t > 0$$

$$u(0,t) = \alpha = \text{Constant}, \, t > 0,$$

$$u(x,0) = 0, \;\; x > 0,$$

$$u(x,t) \to 0, \;\; x \to \infty$$

Case 2: The river is fast, then the PDE

$$\frac{\partial u}{\partial t} = -v_0 \frac{\partial u}{\partial x} - kc, \quad x > 0, \qquad t > 0$$

$$u(0,t) = \alpha = \text{Constant}, \, t > 0,$$

$$u(x,0) = 0, \;\; x > 0,$$

$$u(x,t) \to 0, \;\; x \to \infty$$

Case 3: We consider the initial value problem (IVP):

$$\frac{\partial u}{\partial t} = \sigma \frac{\partial^2 u}{\partial x^2} - v_0 \frac{\partial u}{\partial x} - kc, \quad x > 0, \qquad t > 0$$

$$u(0,t) = \alpha = \text{Constant}, \; t > 0,$$

$$u(x,0) = 0, \;\; x > 0,$$

$$u(x,t) \to 0, \;\; x \to \infty$$

1.2 Problem Statement

Environmental pollution issues related to water quality are very important. To study the water pollution we need to study the suitable mathematical model. For that purpose we consider the one dimensional advection diffusion equation of river flow.

Consider very long river with the substance already uniformly distributed in it from source of the river up to the observation point $x = 0$ assume that both diffusion and convection effects significantly. The concentration $u(x,t)$ of substance at a distance x downstream at time t is the solution of the initial value problem (IVBP):

$$\frac{\partial u}{\partial t} = \sigma \frac{\partial^2 u}{\partial x^2} - v_0 \frac{\partial u}{\partial x} - kc, \qquad x > 0, \qquad t > 0$$

$$u(0,t) = \alpha = constant, \qquad t > 0,$$

$$u(x,0) = 0, \qquad x > 0$$

$$u(x,t) \to 0, \qquad x \to \infty$$

1.3 Objectives

The objectives of this research are:

1. to derive the mathematical model of river pollution.
2. to solve the mathematical model of river pollution using Laplace method.
3. to analysis and interpret the solution of the mathematical model.

1.4 Scope

We only use mathematical modelling of river pollution in terms of Partial Differential Equation. The analytical solution of the model is calculated using Laplace transforms.

1.5 Significance of The Study

Using the mathematical model of river pollution we can calculated and predict the concentration of pollutant in the river using Laplace transforms and inverse Laplace transform.

1.6 Report Structure

The report of this dissertation is written as follow:

i. Chapter 2

Literature Review. All previous studies are assembled in this chapter, including the theories, models and method.

ii. Chapter 3

Mathematical Modelling. The derivation and solution of the model are explained clearly.

iii. Chapter 4

Analysis Techniques. The methods of Laplace transform method are used to get the solution of the models.

iv. Chapter 5

Results and Discussion. The solutions from previous chapter in terms of graphs are used to analyze the problems.

v. Chapter 6

Conclusion and Recommendations. This dissertation and recommendation are discussed.

REFERENCE

A. R. A. Baginda and Z. Zainudin (2009),"*Keynote Paper : Moving Towards Integrated River Basin Management (IRBM) in Malaysia*", The Institution of Engineers, Malaysia (IEM), Proceedings, 11th Annual IEM Water Resources Colloquium, ISBN 978-967- 5048-46-3.

Articles Department of Environment Malaysia (1985), "*Development of Water Quality Criteria and Standards for Malaysia*".

Articles Department of Environment Malaysia (2007), "*Malaysia Environmental Quality Report 2006*", In : Chapter 3 : River Water Quality, Sasyaz Holdings Sdn Bhd, pp. 24.

C. N. Sawyer, P. L. McCarty and G. F. Parkin, (2003) "*Chemistry for Environmental Engineering and Science : Fifth Edition*", In : Chapter 22 : Dissolved Oxygen, McGraw-Hill Professional., USA.

F. E. Cellier (1991), *Continuous System Modeling*, Springer - Verlag, New York.

Gerhand H. Jirka, (2004), *Advective Diffusion Equation*, Institute for Hydromechanics University of Karlsruhe

Guerrero J S P, Pimentel L C G, Skaggs T H and van Genuchten M Th 2009 *Analytical solution of the advection–diffusion transport equation using a change-of variable and integral transform technique*; Int. J. Heat Mass Transfer 52 3297–3304.

H. S Govinda Rao, (2009), *Advanced Engineering Mathematics*, Inderpuri, New Delhi – page 149

Jamaluddin. T, Zainal. A. A, (2012), *Mathematical Method, Lecture Note*, Universiti Teknologi Malaysia.

Dilip Kumar Jaiswal, Atul Kumar, Raja Ram Yadav (2011), *Analytical Solution to the One-Dimensional Advection-Diffusion Equation*, Journal of Water Resource and Protection, 76-84

Ogata and Banks 1961; Ogata 1964; Thormann and Mueller (1987), Solution of the Advection-Dispersion Equation: Continuous Load Of Finite Duration

Scott A.S, Gerhand H.J (2004), *Derivative of Advection Diffusion Equation*, Advection Diffusion Equation 29-31.