A MODIFIED MEAN-VARIANCE-CONDITIONAL VALUE AT RISK MODEL
OF MULTI-OBJECTIVE PORTFOLIO OPTIMIZATION WITH AN
APPLICATION IN FINANCE

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To my beloved Parents, Family and my respected Supervisor

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ABSTRACT

This research focuses on the development of a portfolio optimization model based on the classic optimization method and a meta-heuristic algorithm. The main goal of a portfolio optimization model is to achieve maximum return with minimum investment risk by allocating capital based on a set of existing assets. Recently, mean-variance models have been improved to mean-variance-CVaR (MVC) model as a multi-objective portfolio optimization (MPO) problem which is difficult to be solved directly and optimally. In this work, a modified MVC model of portfolio optimization is constructed using the weighted sum method (WSM). In this method, each objective function of MVC model is given a weight. The optimization problem is then minimized as a weighted sum of the objective functions. The implementation of WSM enables the MVC model to be transformed from a multi-objective function to one with a single objective function. The modified MVC model is then solved using ant colony optimization (ACO) algorithm. This algorithm solves the MVC model by the number of ant colonies and the number of pheromone, a chemical creating trails for others to follow. The modified MVC model can be used in managing diverse investment portfolio, including stocks on the stock market and currency exchange. The applicability and effectiveness of the proposed method are demonstrated by solving a benchmark problem and a practical investment problem as examples. The data of practical examples are collected from the foreign currency exchange of Bank Negara Malaysia for the years 2012 and 2013. In conclusion, this thesis presented a hybrid optimization algorithm which utilizes a classical approach, WSM and a meta-heuristic approach, ACO to solve an MVC model of portfolio optimization.

ABSTRAK

Kajian ini memberi tumpuan kepada pembangunan model pengoptimuman portfolio berdasarkan kaedah pengoptimuman klasik dan algoritma meta-heuristik. Matlamat utama model pengoptimuman portfolio adalah untuk mencapai pulangan maksimum, dengan risiko pelaburan minimum dengan memperuntukkan modal berdasarkan satu set aset yang ada. Kebelakangan ini, model min-varians telah diperbaiki kepada model min-varians-CVaR (MVC) sebagai masalah pengoptimuman portfolio pelbagai-tujuan (MPO) yang sukar untuk diselesaikan secara langsung dan secara optimum. Dalam penyelidikan ini, satu model MVC terubahsuai untuk pengoptimuman portfolio dibina menggunakan kaedah hasiltambah wajaran (WSM). Dalam kaedah ini, setiap fungsi objektif pada model MVC diberikan satu pemberat. Masalah pengoptimuman tersebut kemudiannya diminimumkan sebagai hasiltambah wajaran fungsi objektif. Pelaksanaan WSM membolehkan model MVC ditukarkan dari sebuah fungsi pelbagai-tujuan kepada satu fungsi objektif tunggal. Model MVC terubahsuai kemudiannya diselesaikan dengan menggunakan algoritma pengoptimuman koloni semut (ACO). Algoritma ini menyelesaikan model MVC dengan jumlah koloni semut dan jumlah pheromone, bahan kimia yang menghasilkan jejak supaya dapat diikuti oleh yang lain. Model MVC terubahsuai boleh digunakan dalam menguruskan portfolio pelaburan pelbagai, termasuk saham di pasaran saham dan pertukaran mata wang. Kepenggunaan dan keberkesanan kaedah yang dicadangkan telah ditunjukkan dengan menyelesaikan satu masalah penanda aras dan masalah pelaburan praktikal sebagai contoh. Data dari contoh-contoh praktikal dikumpulkan dari pertukaran mata wang asing Bank Negara Malaysia bagi tahun 2012 dan 2013. Kesimpulannya, tesis ini membentangkan satu algoritma pengoptimuman hibrid yang menggunakan pendekatan klasik, WSM dan pendekatan meta-heuristik, ACO untuk menyelesaikan model MVC pengoptimuman portfolio.

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LIST OF ABBREVIATIONS

ACO	-	Ant Colony Optimization
ACOR	-	Ant Colony Optimization in continues space
MV	-	Mean Variance
WSM	-	Weighted Sum Method
MPO	-	Multi-Objective Portfolio Optimization
VaR	-	Value at Risk
CVaR	-	Conditional Value at Risk
DM	-	Decision Maker
PBO	-	Permutation Based Optimization
AWS	-	Adaptive Weighted Sum
BNM	-	Bank Negara Malaysia
RM	-	Ringgit Malaysia
MVSK	-	Mean-Variance-Skewness-Kurtosis
MVS	-	Mean-Variance-Skewness
MVC	-	Mean-Variance-CVaR
EDA	-	Estimation of Distribution Algorithm

LIST OF SYMBOLS

N -   The number of assets that are available

R_x -   Return depending on a decision vector x that belongs to a feasible set A

x_i -   Proportion of investment in i^{th} asset

Ω -   The feasible set of solutions

X -   A solution of problem

μ_i -   The expected mean of the ith asset

$\sigma^2(R_x)$ -   The variance belonging to R_x

K -   The number of assets to invest

δ_{ij} -   Covariance among the return of assets

δ -   Covariance matrix

α -   Confidence level

φ -   Overall maximum

x^* -   Best solution

$f(x^*)$ -   Pareto optimal

ω_i -   Weight of objective functions

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CHAPTER 1

INTRODUCTION

1.1   Overview

In finance, a portfolio refers to a collection of investments. Usually a person with a certain amount of fund wants to obtain higher income than interests paid by saving accounts or fixed deposits. The investor tries to choose various assets to purchase with the hope of getting a better return. These assets are commonly shares on the stocks markets. They may also be commodities (gold, iron, aluminum, cements, petroleum, coffee, palm oil, etc.), fixed properties (houses, shops, apartments, and condominiums), share holiday resorts and many others. In recent years, due to fast movement of money electronically and high rate of fluctuations among currencies, they have also become popular assets for investments.

An investor is usually interested in getting good return of his/her investment. He/she may be very ambitious, wanting top returns, and not concerned about possible losses. However, few investments can be assured of gains without possibilities of losses. Thus the investor tries to select assets which will probably yield the best returns with the minimum of losses. In general, assets with higher returns are usually speculative, and they tend to have higher risks of losses. On the other hand, assets which are considered safe in the sense that they are quite sure to gain values are usually less likely to gain much. Depending on the personality of the investors, some choose to go for high returns, even though the risk of losses is higher. However, many prefer to choose a safer route and are not willing to suffer losses.

The unwillingness to lose on investment is called risk aversion. The profile of the investor in terms of intended amount of gain and risk aversion determines the types of assets to invest. This profile sets the tone for selection of assets and apportioning the fund into each asset.

Many investors create their own portfolio based on their own judgments and inclinations. But serious investors with big funds usually seek the help of experts. The usual scenario is the investor (customer) engages a financial institution which plans out the portfolio. It is the institution's responsibility to form the portfolio with maximum return, taking into consideration the customer's profile of intended gain and risk aversion level. Traditionally, the portfolio was created rather arbitrarily, based on the previous knowledge and experience on the assets. Normally the institution trusted with investment will try to present the customer with different situations to select options related to their risk aversion (Mavrotas, 2009). Each version may have different forecasts for expected returns and risks.

It is necessary for a more formalized method to optimize the portfolio returns. Portfolio optimization was first introduced by Markowitz (1952) via a framework of return/ variance risk (Yu *et al*., 2011). The main problem is in finding a best solution to distribute a given fund on a set of existing assets. Maximization of return and minimizing risks are the two main aims. The user's risk aversion has a direct effect on the best solution. Two criteria are necessary for optimization of the portfolio: First is the set of solution to the portfolio optimization problem called "efficient frontier", or "pareto optimal". Second is the measure against the risk of the portfolio.

Markowitz's approach for portfolio optimization is based on the covariance measure, in which he proposed two criteria. The first is minimization of the risk, and the second is maximization of the expected return. These can be defined as follows:

$$\mu = E(R) = max \sum_{i=1}^{M} \mu_i x_i ,$$

$$\sigma^2 = \text{var(R)} = \min \sum_{i=1}^{M} \sum_{j=1}^{M} \delta_{ij} x_i x_j,$$

$$\text{s.t.} \quad \sum_{i=1}^{M} x_i = 1, x_i \geq 0, i = 1, \dots, M$$

where, M is the number of existing assets to invest, x_i is the proportion of budget invested in asset $i \in \{1, \dots, M\}$; $\boldsymbol{x} = [x_1, \dots, x_M]^T \in \mathbb{R}^M$ is the solution vector of M dimensions; μ_i denotes the expected return of asset $i \in \{1, \dots, M\}$; $\sum_{i=1}^{M} \sum_{j=1}^{M} \delta_{ij} x_i x_j$ is the variance among the returns of assets $i, j \in \{1, \dots, M\}$, and $\boldsymbol{\delta} = (\delta_{ij})_{i=1,\dots,M,j=1,\dots,M}$ shows the corresponding $M \times M$ covariance matrix (Ehrgott *et al.*, 2004).

1.2   Background of Problem

Numerous types of risk measures for portfolio optimization have been introduced since Markowitz's theory was proposed. The main aim of portfolio optimization model is maximum return on investment with lower risk. Two most popular types of risk measures for portfolio optimization are value at risk (VaR) and conditional value-at-risk (CVaR). VaR refers to the maximum of expected loss on an investment, related to specific time period and particular level of confidence. CVaR represents the expected loss conditional on exceeding a VaR threshold. These measures have been shown to be efficient for the models of optimization (Chen *et al.*, 2012).

The original method of modeling the risk for portfolio optimization is based on mean variance (MV). The expected value of returns and the value of a risk are measured as two statistical quantities that are computable via MV model. This is a practical model for decision making in finance (Aboulaich *et al.*, 2010).

Yu *et al*. (2011) used the weighted sum method (WSM) to model the mean-CVaR model of portfolio optimization. In practice, their model was considered as one of the multi-objective problem. The mean and CVaR are two objective functions of their model. They used multi-objective fuzzy programming (MFP) to solve their proposed model.

Actually using three objective functions in MVC to minimize the risk and to maximize the return of the portfolio selection increases the performance of their model. Also, the MVC model which includes three objective financial functions is useful for decision making in finance. The MV conditional value at risk (MVC) model added a parameter, the CVaR to the objective function. The MVC contains three parameters (Aboulaich *et al*., 2010):

1) the expected returns (E),
2) the variance (σ^2) and
3) the CVaR at a specified confidence level $\alpha \in (0,1)$

The MVC model based on Aboulaich *et al*. (2010) is as follows:

$$min\{CVaR, -E, var\},$$

$$s.t \quad x \in \left\{ (x_1, \dots, x_n) \left| \sum_{j=1}^{n} x_j = 1, x_j \geq 0, \forall j \in \{1, \dots, n\} \right. \right\}.$$

This model has one multi-objective system for portfolio optimization that consists of three objective functions. The first objective function is to minimize the CVaR of portfolio optimization. The second objective function is to maximize the expected value of return of the portfolio. The final objective function is to minimize the variance of returns. However, as a non-linear portfolio model, MVC is based on a rather complicated quadratic structure which is NP-hard problem. The MVC model was solved by using simulated annealing (SA).

1.3   Problem Statement

The main goal of this study is to modify the current model and simplify it by using WSM. The MVC model is defined in terms of mathematical structure which is used to explain the procedure via a recursive algorithm.

However, no attempts have been made to use meta-heuristic approaches to solve the MVC model of multi-objective portfolio optimization, as we intend to do here, particularly ACO. Hence, it gives us the motivation to develop a model of portfolio optimization based on the ACO method.

In this approach, each MVC model is first defined by a mathematical structure. Next, the simulated pseudo code of that model is presented based on ACO procedure, in which the pseudo code is transformed into ACO algorithm to solve the problem. The aim of implementing ACO algorithm consists of finding the pareto solutions based on the constraints of MVC model. This approach helps to increase the efficiency in finding the best solution.

1.4   Objectives of Study

The following are the objectives of this research:

1. To identify the variables of portfolio optimization model related to investment.
2. To develop a MVC model for multi-objective portfolio optimization via weighted sum method (WSM).
3. To solve the new model via ACO.
4. To apply the modified MVC model of portfolio optimization to investment.

1.5   Scope of the Study

This study presents the identification, analysis, and improvement of the MVC model. We first carry out a review on MVC model of portfolio optimization based on multi-objective approach.

Next, we employ WSM to MVC model of portfolio optimization in multi-objective approach to transform the model to one linear combination with specific coefficients. Then, the permutation based optimization (PBO) is used to solve the improved model.

In this study, the algorithm is run with MATLAB software version 2009. The test data is taken from two datasets. The first dataset is taken from Malaysian Exchange during 2012 and 2013. The second one is taken from China stoke market during 2010 and 2011.

Minimizing the CVaR, maximizing the return and minimizing the variance are done by choosing the best coefficients through PBO. ACO, as one of the meta-heuristic methods, is also utilized.

In addition, the thesis also includes

- Presentation of WSM as a classic optimization methods to develop the modified MVC model
- Application of ACO algorithm to solve the model and evaluating the proposed modified model
- Implementation of MATLAB simulation to evaluate the results of proposed model

1.6   Significance of Study

This study promotes the mathematical modeling and applications of MVC model of MPO. MVC model of portfolio optimization (which employs mean, variance and CVaR) is shown to give better results than current risk measure of portfolio (see Chapter 4 and 6). In addition, an important contribution of the research is the presentation of ACO as an alternative procedure in solving of the MPO (Chapter 5 and 6). The modified model is found to be suitable for currencies investment as shown in the examples presented later.

In general, there are two main purposes of this research, which are of theoretical and practical significance.

1.6.1   Contribution to Theory

- This study will add to the body of knowledge and the advancement of solution of MVC problem by using the WSM.

- This research develops a hybrid method of WSM and ACO to solve MVC in the multi-objective optimization approach.

1.6.2   Contribution to Practice

- This study modifies the mathematical model of MVC model for MPO.

- This research proposes a new solution method that is derived through the integration of WSM and ACO approach.

- The improved current model of portfolio optimization model can help extend financial activity, which helps to create the optimal set of assets (currencies) to investment.

1.7 Thesis Outline

This research is arranged into seven chapters. Chapter 2 discusses the current models and theoretical results, covered in the literature review of MPO in conventional finance. Chapter 3 presents the methodology of this research. This chapter also includes the overall research design and research framework of this research. Chapter 4 discusses the development of modified MVC model of portfolio through WSM. To achieve this aim, each objective function of MVC model is assigned a weighted coefficient. The optimization problem is then minimized as a weighted sum of the objectives. The application of WSM enables the MVC model to be transformed from a multi-objective function to a single objective function. Then the developed MVC model is solved via PBO. Also, Chapter 4 presents the comparison between the results of the MVC model based on PBO and LINGO software. Chapter 5 discusses the procedure of obtaining the solution for the new model via ACOR. This algorithm parameterizes the MVC model by the number of ant colonies and the number of pheromone trails. The solution is then compared with the result from LINGO. Chapter 6 presents the application, and comparison between the results of the MVC model based on PBO, LINGO and ACOR via illustrative examples. A practical example of investment in the stock market as a benchmark problem is also considered. The last chapter that is Chapter 7, presents the main contributions of the thesis, limitation of study and direction for future researches.

REFERENCES

Aboulaich, R., Ellaia, R. and El Moumen, S. (2010). The Mean-Variance-CVaR model for Portfolio Optimization Modeling using a Multi-Objective Approach Based on a Hybrid Method. *Mathematical Modelling of Natural Phenomena.* 5(07), 103-108.

Afshar, A., Sharifi, F. and Jalali, M. (2009). Non-dominated archiving multi-colony ant algorithm for multi-objective optimization: Application to multi-purpose reservoir operation. *Engineering Optimization.* 41(4), 313-325.

Aler, R., Handl, J. and Knowles, J. D. (2013). Comparing multi-objective and threshold-moving ROC curve generation for a prototype-based classifier. *Proceedings of the 15th annual conference on Genetic and evolutionary computation.* 06 − 10 July. Amsterdam, Netherland, 1029-1036.

Anagnostopoulos, K. and Mamanis, G. (2011). The mean–variance cardinality constrained portfolio optimization problem: An experimental evaluation of five multiobjective evolutionary algorithms. *Expert Systems with Applications.* 38(11), 14208-14217.

Armananzas, R. and Lozano, J. A. (2005). A multiobjective approach to the portfolio optimization problem. *The Proceedings of the 2005 IEEE Congress on Evolutionary Computation.* Lozano, Spain, 1388-1395.

Baixauli-Soler, J. S., Alfaro-Cid, E. and Fernández-Blanco, M. O. (2010). Several risk measures in portfolio selection: Is it worthwhile. *Revista Española de Financiación y Contabilidad.* 39(147).

Bawa, V. S., Brown, S. J. and Klein, R. W. (1979). *Estimation risk and optimal portfolio choice.* North-Holland Amsterdam.

Bertsekas, D. P. (2009). *Convex optimization theory.* Citeseer.

Blum, C. (2005). Ant colony optimization: Introduction and recent trends. *Physics of Life Reviews.* 2(4), 353-373.

Branke, J., Scheckenbach, B., Stein, M., Deb, K. and Schmeck, H. (2009). Portfolio optimization with an envelope-based multi-objective evolutionary algorithm. *European Journal of Operational Research.* 199(3), 684-693.

Briec, W., Kerstens, K. and Jokung, O. (2007). Mean-variance-skewness portfolio performance gauging: a general shortage function and dual approach. *Management Science.* 53(1), 135-149.

Brown, S. J. (1976). *Optimal portfolio choice under uncertainty: a Bayesian approach.* University of Chicago.

Cesarone, F., Scozzari, A. and Tardella, F. (2009). Efficient algorithms for mean-variance portfolio optimization with hard real-world constraints. *Giornale dell'Istituto Italiano degli Attuari.* 72, 37-56.

Chang, T.-H., Luo, Z.-Q., Deng, L. and Chi, C.-Y. (2008). A convex optimization method for joint mean and variance parameter estimation of large-margin CDHMM. *The Proceedings of the 2008 IEEE Conference on Acoustics, Speech and Signal Processing.* Las Vegas, USA, 4053-4056.

Chen, A. H., Fabozzi, F. J. and Huang, D. (2012). Portfolio revision under mean-variance and mean-CVaR with transaction costs. *Review of Quantitative Finance and Accounting.* 1-18.

Chen, L., Sun, H. and Wang, S. (2009). Solving continuous optimization using ant colony algorithm. *The Proceedings of the 2009 IEEE Second International Conference on Future Information Technology and Management Engineering.* Sanya, China, 92-95.

Chiam, S., Tan, K. and Al Mamum, A. (2008). Evolutionary multi-objective portfolio optimization in practical context. *International Journal of Automation and Computing.* 5(1), 67-80.

Chinchuluun, A. and Pardalos, P. M. (2007). A survey of recent developments in multiobjective optimization. *Annals of Operations Research.* 154(1), 29-50.

Chiranjeevi, C. and Sastry, V. (2007). Multi objective portfolio optimization models and its solution using genetic algorithms. *The Proceedings of the 2007 IEEE Conference on Computational Intelligence and Multimedia Applications.* Sivakasi, Tamil Nadu, 453-457.

Chopra, V. K. and Ziemba, W. T. (1993). The effect of errors in means, variances, and covariances on optimal portfolio choice. *The Journal of Portfolio Management.* 19(2), 6-11.

Chung, T., Zhang, S., Yu, C. and Wong, K. (2003). Electricity market risk management using forward contracts with bilateral options. *The Proceedings of the 2003 IEEE conference on Generation, Transmission and Distribution.* Kowloon, China, 588-594.

Cox, S. H., Lin, Y., Tian, R. and Zuluaga, L. F. (2012). Mortality portfolio risk management. *Journal of Risk and Insurance*. 1-38.

Crama, Y. and Schyns, M. (2003). Simulated annealing for complex portfolio selection problems. *European Journal of Operational Resea*rch. 150(3), 546-571.

Dahlgren, R., Liu, C.-C. and Lawarree, J. (2003). Risk assessment in energy trading. *IEEE Transactions on Power Systems*. 18(2), 503-511.

Deng, G. F. and Lin, W. T. (2010). Ant Colony Optimization for Markowitz Mean-Variance Portfolio Model. *Swarm, Evolutionary, and Memetic Computing*. 238-245.

Ding, Y., Gregov, S., Grodzevich, O., Halevy, I., Kavazovic, Z., Romanko, O., Seeman, T., Shioda, R. and Youbissi, F. (2006). *Discussions on normalization and other topics in multi-objective optimization*. Fields-MITACS, Fields Industrial Problem Solving Workshop.

Doerner, K., Gutjahr, W. J., Hartl, R. F., Strauss, C. and Stummer, C. (2004). Pareto ant colony optimization: A metaheuristic approach to multiobjective portfolio selection. *Annals of Operations Research*. 131(1), 79-99.

Dorigo, M. (2006). *Ant Colony Optimization and Swarm Intelligence: 5th International ANTS Workshop*, Brussels, Belgium, September 4-7, Springer-Verlag New York Incorporated.

Dorigo, M. and Birattari, M. (2010). *Ant colony optimization*, *Encyclopedia of Machine Learning*. Springer.

Dorigo, M. and Di Caro, G. (1999). Ant colony optimization: a new meta-heuristic. *The Proceedings of the 1999 IEEE conference on Evolutionary Computation.* Washington, USA, 1470-1477.

Ehrgott, M. (2009). Multiobjective (Combinatorial) Optimisation—Some Thoughts on Applications. *Multiobjective Programming and Goal Programming*. 267-282.

Ehrgott, M., Klamroth, K. and Schwehm, C. (2004). An MCDM approach to portfolio optimization. *European Journal of Operational Research.* 155(3), 752-770.

Eichner, T. (2010). Slutzky equations and substitution effects of risks in terms of mean-variance preferences. *Theory and Decision.* 69(1), 17-26.

Fletcher, R. (2013). *Practical methods of optimization.* John Wiley & Sons.

Gendreau, M. and Potvin, J.-Y. (2005). Metaheuristics in combinatorial optimization. *Annals of Operations Research.* 140(1), 189-213.

Ghoseiri, K. and Nadjari, B. (2010). An ant colony optimization algorithm for the bi-objective shortest path problem. *Applied Soft Computing.* 10(4), 1237-1246.

Grodzevich, O. and Romanko, O. (2006). Normalization and other topics in multi-objective optimization. *Proceedings of the Fields–MITACS Industrial Problems Workshop.* Toronto, Canada, 89-101.

Gupta, P., Mehlawat, M. K. and Saxena, A. (2008). Asset portfolio optimization using fuzzy mathematical programming. *Information Sciences.* 178(6), 1734-1755.

Haqiqi, K. F. and Kazemi, T. (2012). Ant Colony Optimization Approach to Portfolio Optimization. *International Conference on Economics, Business and Marketing Management.* Singapore, 292-296.

Jia, J., Fischer, G. W. and Dyer, J. S. (1998). Attribute weighting methods and decision quality in the presence of response error: a simulation study. Journal of Behavioral Decision Making. 11(2), 85-105.

Jubril, A. M. (2012). A nonlinear weights selection in weighted sum for convex multiobjective optimization. *Facta universitatis-series: Mathematics and Informatics.* 27(3), 357-372.

Khalidji, M., Zeiaee, M., Taei, A., Jahed-Motlagh, M. R. and Khaloozadeh, H. (2009). Dynamically Weighted Continuous Ant Colony Optimization for Bi-Objective Portfolio Selection Using Value-at-Risk. *The Proceedings of the Third Asia International Conference on Modelling & Simulation.* Bali, Malaysia, 230-235.

Kim, H. Y. and Viens, F. G. (2012). Portfolio optimization in discrete time with proportional transaction costs under stochastic volatility. *Annals of Finance.* 8(2), 405-425.

Kim, I. Y. and De Weck, O. (2006). Adaptive weighted sum method for multiobjective optimization: a new method for Pareto front generation. *Structural and Multidisciplinary Optimization*. 31(2), 105-116.

Kirytopoulos, K., Leopoulos, V., Mavrotas, G. and Voulgaridou, D. (2010). Multiple sourcing strategies and order allocation: an ANP-AUGMECON meta-model. *Supply Chain Management: An International Journal.* 15(4), 263-276.

Krokhmal, P., Palmquist, J. and Uryasev, S. (2002). Portfolio optimization with conditional value-at-risk objective and constraints. *Journal of Risk*. 4, 43-68.

Kumar, R., Mitra, G. and Roman, D. (2008). Long-Short Portfolio Optimisation in the Presence of Discrete Asset Choice Constraints and Two Risk Measures. *Available at SSRN 1099926*.

Lai, K. K., Yu, L. and Wang, S. (2006). Mean-variance-skewness-kurtosis-based portfolio optimization. *The Proceedings of First International Multi-Symposiums on Computer and Computational Sciences.* Hanzhou, China, 292-297.

Li, X., Qin, Z. and Kar, S. (2010). Mean-variance-skewness model for portfolio selection with fuzzy returns. *European Journal of Operational Research*. 202(1), 239-247.

Lin, D., Wang, S. and Yan, H. (2001). A multiobjective genetic algorithm for portfolio selection. *Proceedings of ICOTA*. 15-17.

Litterman, B. (2004). *Modern investment management: an equilibrium approach*. John Wiley & Sons.

Liu, F.-H. F. and Hai, H. L. (2005). The voting analytic hierarchy process method for selecting supplier. *International Journal of Production Economics*. 97(3), 308-317.

Liu, M. and Wu, F. F. (2006). Managing price risk in a multimarket environment. *IEEE Transactions on Power Systems*. 21(4), 1512-1519.

Maringer, D. and Kellerer, H. (2003). Optimization of cardinality constrained portfolios with a hybrid local search algorithm. *OR Spectrum*. 25(4), 481-495.

Markowitz, H. (1952). Selection Portfolio Diversification. *The Journal of Finance*. 7(1), 77-91.

Marler, R. T. and Arora, J. S. (2010). The weighted sum method for multi-objective optimization: new insights. *Structural and Multidisciplinary Optimization*. 41(6), 853-862.

Miettinen, K. (1999). *Nonlinear multiobjective optimization*. Springer.

Meucci, A. (2009). *Risk and asset allocation*. Springer.

Narzisi, G. and Mishra, B. (2011). Comparing de novo genome assembly: the long and short of it. *PloS one*. 6(4), 1-14.

Pindoriya, N., Singh, S. and Singh, S. (2010). Multi-objective mean–variance–skewness model for generation portfolio allocation in electricity markets. *Electric Power Systems Research*. 80(10), 1314-1321.

Radziukynienė, I. and Žilinskas, A. (2008). Evolutionary methods for multi-objective portfolio optimization. *Proceedings of the World Congress on Engineering*. London,UK, 1-5.

Rockafellar, R. T. and Uryasev, S. (2002). Conditional value-at-risk for general loss distributions. *Journal of Banking & Finance*. 26(7), 1443-1471.

Rockafellar, R. T. and Uryasev, S. (2000). Optimization of conditional value-at-risk. *Journal of Risk*. 2, 21-42.

Ruszczyński, A. P. (2006). *Nonlinear optimization*. Princeton University Press.

Roman, D., Darby-Dowman, K. and Mitra, G. (2006). Portfolio construction based on stochastic dominance and target return distributions. *Mathematical Programming*. 108(2), 541-569.

Roman, D., Darby-Dowman, K. and Mitra, G. (2007). Mean-risk models using two risk measures: a multi-objective approach. *Quantitative Finance*. 7(4), 443-458.

Sadjadi, S. J., Gharakhani, M. and Safari, E. (2012). Robust optimization framework for cardinality constrained portfolio problem. *Applied Soft Computing*. 12(1), 91-99.

Saeheaw, T., Charoenchai, N. and Chattinnawat, W. (2009). Application of Ant colony optimization for Multi-objective Production Problems. *World Academy of Science, Engineering and Technology*. 60, 654-659.

Sawik, B. (2009). A reference point approach to bi-objective dynamic portfolio optimization. *Decision Making in Manufacturing and Services*. 3(1-2), 73-85.

Stanimirovic, I. P., Zlatanovic, M. L. and Petkovic, M. D. (2011). On the linear weighted sum method for multi-objective optimization. *Facta Acta Universitatis*. 26, 49-63.

Steuer, R. E., Qi, Y. and Hirschberger, M. (2006). Portfolio optimization: new capabilities and future methods. *Journal of Buisiness Economics*. 76(2), 199-220.

Srinivas, N. and Deb, K. (1994). Muiltiobjective optimization using nondominated sorting in genetic algorithms. *Evolutionary Computation*. 2(3), 221-248.

Socha, K. and Dorigo, M. (2008). Ant colony optimization for continuous domains. *European Journal of Operational Research*. 185(3), 1155-1173.

Tanlapco, E., Lawarrée, J. and Liu, C.-C. (2002). Hedging with futures contracts in a deregulated electricity industry. *IEEE Transactions on Power Systems*. 17(3), 577-582.

Uryasev, S. (2000). Conditional value-at-risk: Optimization algorithms and applications. *The Proceedings of the 2000 IEEE conference on Computational Intelligence for Financial Engineering*. New York, USA, 49-57.

Wang, J. J., Jing, Y. Y., Zhang, C. F. and Zhao, J. H. (2009). Review on multi-criteria decision analysis aid in sustainable energy decision-making. *Renewable and Sustainable Energy Reviews*. 13(9), 2263-2278.

Xidonas, P., Mavrotas, G. and Psarras, J. (2010). Portfolio construction on the Athens Stock Exchange: A multiobjective optimization approach. *Optimization*. 59(8), 1211-1229.

Yu, X. (2012). The optimal portfolio model based on multivariate t distribution with linear weighted sum method. *E3 Journal of Business Management and Economics*. 3(1), 044-047.

Yu, X., Tan, Y., Liu, L. and Huang, W. (2012). The Optimal Portfolio Model Based on Mean-CvaR with Linear Weighted Sum Method. *The Proceedings of the Fifth IEEE International Joint Conference on the Computational Sciences and Optimization*. Harbin, China, 82-84.

Yu, X., Sun, H. and Chen, G. (2011). The Optimal Portfolio Model Based on Mean-CVaR. *Journal of Mathematical Finance*. 1(3), 132-134.

Zadeh, L. A. and Deoser, C. A. (1976). *Linear system theory*. Robert E. Krieger Publishing Company.

Zeiaee, M. and Jahed-Motlagh, M. R. (2009). A heuristic approach for value at risk based portfolio optimization. *The Proceedings of 14th International Computer Conference,* Tehran, Iran, 686-691.

Zhu, L., Coleman, T. F. and Li, Y. (2009). Min-max robust CVaR robust mean-variance portfolios. *Journal of Risk*. 11(3), 55.