SOLVING BURGER'S EQUANTION USING EXPLICIT FINITE DIFFERENCE METHOD AND METHOD OF LINE

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This dissertation is specially dedicated to:

My beloved family especially to my parents, lecturers
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ABSTRACT

Burgers' equation is a quasilinear differential equation can be solve either analytically or numerically. The analytical solutions use the Hopf-Cole transformation and reduced to diffusion equation. The focus of this research was to solve Burgers' equation numerically by using Finite Difference Method (FDM) and Method of Line (MOL) by using Fourth Order Runge-Kutta (RK4). The accuracy of MOL obtained solutions depends on the type of Ordinary Differential Equation (ODE) method used. The results obtained from both numerical method were compared between Hopf-Cole transformation analytical solutions. The simulations is coded by using MATLAB software. From the comparison, both methods shown to be good numerical approximation as the results obtained near to the exact solution. As the increase of spatial step size, the solutions obtained with be more accurate followed by individual methods' restrictions. Different time and viscosity coefficient also tested to observe the changes of Burgers' equation solutions.

ABSTRAK

Persamaan Burgers merupakan persamaan pembezaan quasilinear yang boleh diselesaikan secara analitikal atau berangka. Bagi mendapatkan penyelesaian analitikal, transformasi Hopf-Cole digunakan untuk menghasilkan persamaan resapan. Fokus kajian ini adalah untuk menyelesaikan persamaan Burgers secara kaedah berangka dengan menggunakan kaedah beza terhingga (FDM) dan kaedah garisan (MOL) yang menggunakan kaedah Runge-Kutta peringkat keempat (RK4). Ketepatan hasil pengiraan MOL bergantung kepada jenis kaedah yang digunakan untuk menyelesaikan masalah seperti persamaan pembezaan biasa (ODE). Seterusnya, perbandingan dibuat terhadap keputusan simulasi MATLAB untuk penyelesaian menggunakan kaedah berangka dan penyelesaian analitikal yang melalui transformasi Hopf-Cole. Berdasarkan perhatian didapati, penyelesaian berangka mampu memberi penyelesaian yang baik. Pengurangan panjang saiz langkah akan memberikan penyelesaian berangka yang lebih tepat. Masa dan pekali kelikatan yang berbeza juga diuji untuk melihat kesannya terhadap penyelesaian persamaan Burgers.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

FDM	-	Finite Difference Method
MOL	-	Method of Line
RK		Runge-Kutta
RK2	`	Second Order Runge-Kutta
RK3		Third Order Runge-Kutta
RK4		Fourth Order Runge-Kutta
PDE	-	Pertial Differential Eqution
ODE	-	Ordinary Differential Equation

LIST OF SYMBOLS

u_t	-	Unsteady term
uu_x	-	Convective term
vu_{xx}	-	Viscous term
Δx	-	Step size in term of x
Δy	-	Step size in term of y

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Background of the Study

Burgers' equation is a quasilinear differential equation shows the nonlinear convection and linear dissipation follows the evolution of time of the function $u(x,t)$ (Singh, 2016). The equation first discovered by Betemen (1915) used in the modelling of the motion of viscous fluid. In 1948, Burgers (1948) tried to formulate a simplest mathematical model that can related to turbulence. From that day onwards, the equation earned its name as Burgers' equation. In modern days, this equation are widely formulated in modelling, turbulence, gas fluid dynamics, traffic flows and so on. This equation also played as a model equation for the development in the computation in nonlinear equation.

Burgers' equation can be solved either analytically or numerically. The analytical or also known as exact solution often related to the Hopf-Cole transformation which is used to reduce the quasilinear equation into a diffusion equation. In recent years, beside Hopf-Cole transformation, various works have been produced on the findings of Burgers' equation analytical solution such as G'/G Expansion Method (Wang, Li and Zhang, 2008), Tanh Expansion Method (Malfliet and Hereman, 1996) and Method using Transformation from Sine-Gordon equation (Fu, Liu and Liu, 2002).

Simultaneously, the good numerical approximations on Burgers' equation also grew as time goes. Examples like Nyuyen and Reynen (1984) presented a space-time finite element approach to Burgers' equation, Kakuda and Tosaka (1990) used generalized boundary element method, Bar-Yoseph et al. (1995) used and discussed the space time spectral element method on Burgers's equation solution, Zhu et al. (2009) applied a cubic B-spline quasi interpolation to Burgers equation, Siraj et al.(2012) researched the numerical solution of Burgers' equations using meshless Method of Lines and many more.

In this research, the focus is to solve Burgers' equation numerically by using Finite Difference Method (FDM) and Method of Line (MOL) by using Fourth Order Runge-Kutta (RK4). The results obtained from both numerical method will be compared with Hopf-Cole transformation analytical solutions. The simulations is coded using MATLAB software.

1.2 Statements of the Problem

In reality, most of the physical problems existed in form of nonlinear partial differential equations. In this research, Burgers' equation is chosen due to the simplicity of one dimensional but contains the nonlinear properties.

Numerical method in other hand means the approximation of a solutions. Number of methods have been introduces in decades. Among the well-knowns, FDM and MOL are chosen to be used as the numerical approach to solved Burgers' equation. Although both come from different approach where FDM solved in partial differential equation (PDE) form meanwhile MOL transform a PDE into a system of Ordinary differential equation (ODE) and solved using various ODE solver methods. FDM is one of the classic method to solve PDE. However, methods of lines said to be more accurate and computational timewise compared to regular finite difference method (Sadika and Obiozor,2000). However, the accuracy of the solutions also depends on the methods used to solve the ODE after transformed by using MOL. Moreover, this method can achieved the numerical stability and convergence efficiently due to the separation of time and space discretization. There are several issues of concern discussed in this research.

1. How to numerically simulate Burgers' equation using MATLAB software?
2. How accurate are the numerical methods used to solve Burgers' equation?

1.3 Objectives of the Study

The following objectives is achieved from this research.

1. To review and understand the applications of numerical schemes on Burgers' equation.
2. To simulate numerical computational of Burgers' equation in MATLAB software.
3. To determine the accuracy of result obtained using numerical approach in Burgers' equation.
4. To observe different numerical approach in solving Burgers' equation.

1.4 Scope of the Study

This research focused on one dimensional nonlinear Burgers' equation that is linearized by the used of Hopf-Cole transformation and solved by using explicit FDM and MOL using RK4. The discretization will be carry out and code by using MATLAB software.

1.5 Significance of the Study

In recent years, the research of Burgers' equation had contributes various achievement especially in fluid dynamics field. Thus, the significant of this research are:

1. This research determined the accuracy of result obtained using numerical approach in Burgers' equation.
2. This research also provides extra information on the comparison between finite difference method and method of line in Burgers' equation.
3. This research provides other alternative to solve Burgers' equation by using two different approach of numerical methods, the pde solver and the ode solver.

1.6 Organization of the Research

This research is organised into five chapters. Chapter 1 presents the introduction, background of the study, statements of the problem, objectives, scope and the significance of study. The theoretical information and literature review related to the background of the study are discussed in Chapter 2. Literature review on the background of Burgers' equation and numerical methods that involved. On the other hand, Chapter 3 is about the mathematical formulation and algorithms. Based on the mathematical formulation, the equation is solved by using MATLAB software. Chapter 4 discussed about the results and discussion of this research. The last chapter of this research which is Chapter 5 concluded the whole thesis and some recommendations for future research.

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