EXACT SOLUTIONS OF UNSTEADY FREE CONVECTION FLOW OF CASSON, NANO, AND MICROPOLAR FLUIDS OVER AN OSCILLATING VERTICAL PLATE

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A thesis submitted in fulfilment of the
requirements for the award of the degree of
Doctor of Philosophy (Mathematics)

Faculty of Science
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JANUARY 2017

To My Beloved Mother & Father

ACKNOWLEDGEMENT

All praise and glory to Almighty Allah (Subhanahu Wa Taalaa) the most merciful, the most beneficent, Who gave me courage and patience to carry out this work. Affectionate love to Prophet Muhammad (Peace Be Upon Him) for being constant source of guidance.

Years ago my mother used to say, “I wish my daughter (Asma) to be a doctor”. Although she meant some other doctor, but I am sure she would be the most happiest and proud person for my title of PhD. My great and lovely father, I really don’t want to through a piece of word ‘thanks’ for you because it never expresses my feeling for you, I just wish you to be with me and rest of the family healthier and happier ever, Ameen.

I express my sincere appreciation and gratitude to my supervisor Assoc. Prof. Dr. Sharidan Shafie, for his constant help, guidance and valuable suggestions. He has been helping me out and supported me throughout this research and on several other occasions. Without his efforts it is not possible for me to complete this difficult task. I am also indebted to my co-supervisor Asst. Prof. Dr. Ilyas Khan for his assistance, passionate encouragement and constructive critiques of this research work.

I would like to acknowledge with great gratitude, Sardar Bahadur Khan Women’s University Quetta, Pakistan and Higher Education Commission Pakistan for granting me ample opportunity to conduct this study and financially support my stay abroad.

There are many people deserving appreciation and acknowledgements who have contributed in one way or the other to make this thesis possible. My greatest gratitude goes to my grandfather for his love, care and precious prayers.

I would like to express my gratitude to my dearest friends Nabila, Hanifa and Usman for their support, devotion and encouragement through this entire process. Special thanks goes to my fellows Abid, Lim, Sharena, Athirah, Qushairi, Noraihan and Arshad for their help, guidance and company. Thank you all for the priceless memories.

I express the most wholehearted gratitude to my sisters Sabika, Madiha, Monazza, Amara and Fareeha for their everlasting love, prayers and moral support at every step of this journey. I also thank for heart-warming kindness from my brothers in-law Sami and Umar. I cannot pen down this acknowledgement note without according my deep love and affection to my lovely nieces Warisha, Daneen, Raheema and my nephews Abdul Rehman, Amaanullah, Abdul Basit, Safiullah and Abdul Wadood.

May the Almighty Allah richly bless all of you… Ameen!

Asma Khalid

ABSTRACT

Fluid-mechanics is an ancient science that is incredibly alive today. Therefore, the modern technologies require a deeper understanding of the behaviour of real fluids. Based on the relationship between shear stress and the rate of strain, fluids can be categorized as Newtonian fluids and non-Newtonian fluids. Various non-Newtonian fluid models have been used to investigate the behaviour of fluid motion, because of their universal nature. Solution corresponding to Newtonian and non-Newtonian fluids problem have received considerable attention due to their numerous applications in industries. This thesis is devoted to study the unsteady free convection flow of Newtonian fluid (nanofluids) and non-Newtonian fluids (Casson and micropolar fluids) over an oscillating vertical plate. Specifically, free convection flows of Casson fluids and micropolar fluids were studied with and without magnetohydrodynamic and porosity effects. Whereas studied in nanofluids also considered ramped wall temperature. Laplace transform was used to solve the partial differential equations governing the motion. The expressions of the obtained solutions for velocity, temperature and concentration were presented in simple forms. Skin friction, Nusselt number and Sherwood number were also calculated. The analytical results were plotted and discussed for magnetic, porosity, radiation, nanoparticle volume friction, Casson and microrotation parameters as well as Prandtl, Grashof and modified Grashof numbers. For Casson fluid, it was observed that velocity decreases with increasing values of Casson parameter as Casson fluid exhibits yield stress. In case of nanofluids, it was found that fluid velocity was greater for isothermal temperature as compared to ramped wall temperature of the plate. However, for micropolar fluid, microrotations increases near the plate and decreases far away from the plate due to an increase in viscosity parameter. The results showed that for long time interval, the oscillations have similar amplitudes and phase shift that persists for all times. For verification, the obtained solutions were recovered as special cases. The existing solutions in the literature were also reduced to their limiting cases of the present results. The exact solutions obtained in this thesis serve as a benchmark to verify approximate methods, whether asymptotic, experimental or numerical.

ABSTRAK

Mekanik bendalir merupakan sains purba yang masih berkembang sehingga ke hari ini. Oleh itu, teknologi moden memerlukan pemahaman yang lebih mendalam berkenaan kelakuan bendalir sebenar. Berdasarkan hubungan antara tegasan ricih dan kadar terikan, bendalir boleh dikategorikan sebagai bendalir Newtonan dan bendalir bukan Newtonan. Pelbagai model bendalir bukan Newtonan telah digunakan untuk mengkaji tingkah laku gerakan bendalir, disebabkan oleh sifat serba boleh mereka. Penyelesaian yang berkaitan dengan masalah bendalir Newtonan dan bendalir bukan Newtonan telah mendapat banyak perhatian kerana pelbagai kegunaannya dalam industri. Tesis ini adalah dikhaskan untuk mengkaji aliran tak mantap olakan bebas bendalir Newtonan (bendalir nano) dan bendalir bukan Newtonian (Casson dan mikrokutub) melintasi plat menegak berayun. Secara khususnya, aliran olakan bebas bagi bendalir Casson dan bendalir mikrokutub telah dikaji dengan dan tanpa kesan hidrodinamik magnet dan keliangan. Manakala, kajian terhadap bendalir nano juga mempertimbangkan suhu tanjakan dinding. Penjelmaan Laplace telah diguna bagi menyelesaikan persamaan pembezaan separa yang menakluk gerakan. Ungkapan bagi penyelesaian halaju, suhu dan kepekatan yang diperoleh telah dibentangkan dalam bentuk yang mudah. Geseran kulit, nombor Nusselt dan nombor Sherwood juga telah dikira. Keputusan secara analitik ini, diplot dan dibincangkan untuk parameter-parameter magnet, keliangan, radiasi, isipadu pecahan partikel nano, Casson dan mikroputaran berserta juga nombor-nombor Prandtl, Grashof dan Grashof terubah suai. Untuk bendalir Casson, diperhatikan bahawa halaju berkurangan dengan peningkatan nilai-nilai parameter Casson dengan keadaan bendalir Casson mempamerkan tekanan alah. Dalam kes bendalir nano, didapati bahawa halaju bendalir adalah lebih besar untuk suhu isoterma berbanding dengan suhu tanjakan. Walau bagaimanapun, untuk bendalir mikrokutub, mikroputaran meningkat berhampiran plat dan berkurangan berada jauh dari plat disebabkan oleh peningkatan dalam parameter kelikatan. Keputusan yang diperoleh menunjukkan bahawa untuk tempoh masa yang lama, ayunan mempunyai amplitud yang sama dan anjakan fasa yang berterusan untuk setiap masa. Untuk penentusahan, penyelesaian yang diperoleh diturunkan sebagai kes-kes khas. Penyelesaian sedia ada di dalam kajian terdahulu juga diturunkan kepada menghadkan kes bagi penyelesaian yang didapati sekarang. Penyelesaian tepat yang diperoleh dalam tesis ini menyediakan suatu penanda aras untuk mengesahkan kaedah anggaran, sama ada secara asimptot, eksperimen atau berangka.

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Al_2O_3 - Aluminium oxide

Ag - Silver

Fe_3O_4 - Iron oxide

TiO_2 - Titanium dioxide

Cu - Copper

H_2O - Water

LIST OF SYMBOLS

Roman Letters

B_0 - Magnitude of applied magnetic field

B - Total magnetic field

$(C_p)_s$ - Heat capacity of solid nanoparticles

$(C_p)_f$ - Heat capacity of base fluids

$(C_p)_{nf}$ - Heat capacity of nanofluids

C_m - Wall couple stress

D - Diffusion flux vector

D - Mass diffusivity

D_1 - Thermal diffusivity

d / dt - Material time derivative

E - Total electric field

e - Internal energy per unit volume

$\dot{e}_{ij}$ - $\left(i, j\right)^{th}$ component of the deformation rate

erf - Error function

erfc - Complementary error function

exp - Exponential function

F - Force

f - Constant shear stress

Gr - Thermal Grashof number

Gm - Modified Grashof number

g - Gravitational acceleration

$H(t)$ - Heaviside function

$\mathbf{I}$ - Identity tensor
$\mathbf{i}$ - Cartesian unit vector in the $x-$direction
$\mathbf{j}$ - Cartesian unit vector in the $y-$direction
j - Microinertia per unit mass
$\mathbf{k}$ - Cartesian unit vector in the $z-$direction
K - permeability parameter
k_1 - Dimensionless permeability parameter
k - Thermal conductivity
k_s - Thermal conductivity of solid nanoparticles
k_f - Thermal conductivity of base fluids
k_{nf} - Thermal conductivity of nanofluids
$\mathscr{L}$ - Laplace transform
$\mathscr{L}^{-1}$ - Inverse Laplace transform
M - Magnetic parameter
N - Microrotations
N_r - Radiation parameter
Nu - Nusselt number
n - Microelement
$\Pr$ - Prandtl number
p - Pressure
p_h - Hydrostatic pressure
p_d - Dynamic pressure
p_y - Yield stress
$\mathbf{q}_r$ - Radiant flux vector
q_r - Magnitude of radiant heat flux
$\mathbf{q}''$ - Heat conduction per unit area
q'' - Magnitude of heat conduction per unit area
q - Laplace transform parameter
Re - Reynold’s number
Sh - Sherwood number

Sc	-	Schmidt number
$\mathbf{T}$	-	Cauchy stress tensor for Newtonian fluids
$\mathbf{T}_{ij}$	-	Cauchy stress tensor for Casson fluids
T	-	Temperature
t	-	Time
t_0	-	Characteristic time
u	-	Velocity in $x-$direction
$u_{\cos}$	-	Velocity for cosine oscillations
$u_{\sin}$	-	Velocity for sine oscillations
u_c	-	Convective part of velocity
u_m	-	Mechnical part of velocity
U_0	-	Reference velocity
$\mathbf{V}$	-	Velocity vector field
V	-	Magnitude of velocity
x	-	Dimensionless coordinate axis along the plate
y	-	Dimensionless coordinate axis normal to the plate

Greek Letters

α	-	Vortex viscosity
β	-	Microrotaion parameter
β_T	-	Volumetric coefficient of thermal expansion
β_C	-	Volumetric coefficient of expansion for concentration
β_s	-	Volumetric coefficient of thermal expansion of solid nanoparticles
β_f	-	Volumetric coefficient of thermal expansion of base fluids
β_{nf}	-	Volumetric coefficient of thermal expansion of

nanofluids

∇ - Vector operater Del

η - Spin gradient parameter

ϕ - Volume fraction of solid nanoparticles

γ - Casson parameter

$\hbar_{i=1,2,3}$ - Spin gradient viscosity

μ - Dynamic viscosity

μ_B - Plastic dynamic viscosity

μ_s - Dynamic viscosity of solid nanoparticles

μ_f - Dynamic viscosity of base fluids

μ_{nf} - Dynamic viscosity of solid nanofluids

ν - Kinematic viscosity

ω - Oscillating parameter

ωt - Phase angle

π_1 - Product of deformation rate with itself

π_c - Critical value of the product

ρ - Density

ρ_s - Density of solid nanoparticles

ρ_f - Density of base fluids

ρ_{nf} - Density of nanofluids

σ - Electrical conductivity

σ_1 - Stefan-Boltzmann constant

σ_{nf} - Electrical conductivity of nanofluids

τ - Skin friction

$\underline{\tau}$ - Shear stress

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CHAPTER 1

INTRODUCTION

1.1 Introduction

This chapter discusses the main area of this research which emphasise on Newtonian fluids as well as non-Newtonian fluids, along with some basic terminologies of fluid mechanics. It consists of a brief introduction of the research background, problem statement, research objectives, scope of the study and the significance of the present research.

1.2 Research Background

In the eighteenth and early nineteenth centuries, scientists imagined that all bodies contained an invisible fluid which they called caloric (Lienhard, 2008). Caloric was assigned a variety of properties, some of which proved to be contradictory with nature, like it had weight and it could not be created nor destroyed. But its most important characteristic was that it flowed from hot bodies into cold ones. It was a very useful way to think about heat transfer.

In thermodynamics, heat transfer is the energy interaction in a medium or between media due to temperature difference. Heat is not a storable quantity and is defined as energy in transit due to a temperature difference (Cengel, 2004). The science of heat transfer is used to understand the mechanism of heat transfer process and to predict that, at which rate heat transfer has taken place. It may also be used to predict the amount of energy required to change a system from one equilibrium state to another. In the study of heat transfer, one of the significant variable is temperature, and it is necessary to express the net buoyancy force in terms of a temperature difference, that represents the variation of the density of a fluid with temperature at constant pressure. Heat transfer has broad applications in nature and in industry, particularly heating and cooling of earth's surface, formation of rain and snow, climatic changes are some of the natural facts wherein heat transfer plays a vital role and the survival of living beings is feasible due to the utmost heat source, the sun. (Ghoshdastidar, 2004). Generally, there are three basic modes of heat transfer namely conduction, convection and radiation.

1.2.1 Conduction

Conduction is heat transfer by means of molecular agitation within a material without any motion of the material as a whole (Lienhard, 2008). When one part of body is at higher temperature then the other, heat transfer take place from higher temperature body to the lower temperature body. In this case, the energy is said to be transferred by conduction. Higher temperatures are associated with higher molecular energies and a transfer of energy from the more energetic to the less energetic molecules must occur when neighbouring molecules have a collision. In the presence of temperature gradient, energy transferred by conduction must occur in the direction of decreasing temperature.

1.2.2 Convection

Convection is the transfer of thermal energy from one place to another by the movement of fluids or gases. The convection mode of heat transfer is divided into three types which are known as free, mixed and forced convections (Ghoshdastidar, 2004). If the fluid motion is induced by some external resources such as fluid machinery pump, blower and vehicle motion, the convection is called as forced and the process is generally known as forced convection flow. While, if the motion in the fluid is induced by body forces such as gravitational or centrifugal forces, this kind of flow is said to be free or natural convection. On the other hand, mixed convection flow occurs when free and forced convection mechanisms simultaneously and significantly contribute to the heat transfer (Cengel, 2004).

Free convection has attracted a great deal of attention from researchers because of its presence both in nature and engineering applications. In nature, convection cells formed from air raising above sunlight-warmed land or water are major feature of all weather systems. Convection is also seen in the sea-wind formation, oceanic currents, and in rising plume of hot air from fire. In engineering applications, convection is commonly visualized in the configuration of microstructures during the cooling of molten metals and fluid flows around covered heat-dissipation fins, and solar ponds.

1.2.3 Radiation

Radiation is a form of electromagnetic energy transmission and is independent of any medium between the emitter and receiver of such energy (Ghoshdastidar, 2004). However, radiative heat transfer depends on a temperature difference for the transfer of energy to take place. Radiative heat and mass transfer have many applications in manufacturing industries for the combustion and furnace design, gas turbines and different driving devices for air craft, nuclear power plant, food processing as well as for several heath applications. Therefore, the study of radiative heat and mass transfer by free convection in a magnetohydrodynamics (MHD) fluid through a porous medium is currently undergoing a period of great magnification and demarcation of the subject matter and has attracted the interest of researchers (Anuradha and Priyadharshini, 2014).

1.2.4 Mass Transfer

Free convection flows occur not only due to temperature difference, but also due to concentration difference or the combination of these two. If a multi-component system with a concentration gradient, one constituent of the mixture gets transported from the region of higher concentration to the region of lower concentration till the concentration gradient reduces to zero. This phenomenon of the transport of mass as a result of concentration gradient is called mass transfer (Cengel, 2004; Bergman *et al.*, 2011). Mass transfer is also used in different scientific disciplines for different purposes. For example, in engineering it is used for physical process that involves diffusive and convective transport of chemical species within physical system. Heat and mass transfer phenomena is essential part of science and technology. In practical situations, such as condensation, evaporation and chemical reactions, where the heat transfers phenomena is always accomplished by the mass transfer phenomena.

1.2.5 Boundary Layer Theory

In 1904 at the International Mathematical Congress in Heidelberg, when Prandtl give a lecture entitled “On fluid flow with very little friction”. He proposed that, the viscosity of a fluid plays a vital role in a thin layer adjacent to the surface, which he called the boundary layer (Herbert, 2004). In other words, in a simple flow situation the effect of viscosity and the wall is limited to a thin layer adjacent to the wall and that frictional effects experienced only in a boundary layer, a thin region near the surface. Outside the boundary layer flow, the flow is inviscid that studied for the previous two centuries. With this idea, the understanding of fluid flow was extensively increased and with an order of magnitude analysis, this assumption can simplify the Navier Stokes’ equation significantly.

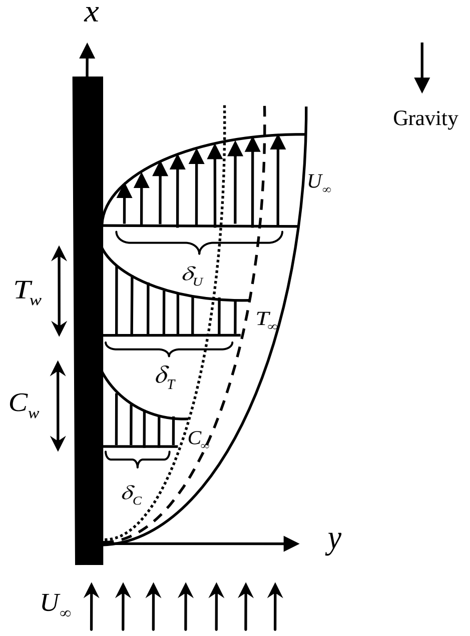

Figure 1.1 Boundary layer over a flat plate.

The physical configuration of the flow is shown in Figure 1.1. The thermal, concentration and velocity boundary layers are shown by δ_T, δ_C and δ_U respectively. The flow associated to the flat plate, the boundary layer is very thin compared to the size of the plate. The velocity changes extremely over very short distance normal to the surface of a body absorbed in a flow (Anderson, 2005).

1.2.6 Magnetohydrodynamics Heat and Mass Transfer Flow

The influence of magnetic field is observed in several natural and human-made flows. Magnetic fields are commonly applied in industry to pump, heat, levitate and stir liquid metals. There is the terrestrial magnetic field which is maintained by fluid flow in the earth's core, the solar magnetic field which originates sunspots and solar flares, and the galactic magnetic field which is thought to control the configuration of stars from interstellar clouds (Shercliff, 1965). So MHD is the study of the contact between magnetic fields and moving conducting fluids.

The laws of magnetism and fluid flow are the innovations of twentieth-century. Hannes Alfvén (1908-1995) was the first to present the term magnetohydrodynamics and won the Nobel prize for his work on magnetohydrodynamics (Goossens, 2012). Some early pioneering work has been done by J. Hartmann, through inventing the electromagnetic pump in 1918 (Molokov *et al.*, 2007). He also considered a systematic theoretical as well as experimental investigation of the flow of mercury in a homogeneous magnetic field. This is the reason that the term ‘Hartmann flow’ is now used to represent duct flows in the presence of a magnetic field.

The study of the interplay of electromagnetic fields and electrically conducting fluids caught the attention of researchers. As a result many standard problems of fluid mechanics were reexamined under the influence of magnetic field. The study of channel flow heat transfer has applications in the fields of power generation and propulsion in devices as a MHD power generator and pump. Despite the fact that the consideration of MHD makes the problem complicated, yet the present study incorporates the topic for its relevance in the entire research work.

1.2.7 Heat and Mass Transfer Flow in a Porous Medium

Porous medium is a material consisting of a solid matrix with an interconnected empty space. The porosity of a porous medium is characterized as the portion of the total volume of the medium that is occupied by empty space (Nield and Bejan, 2006). The flows though porous media occur in many industrial and natural situations, like membrane separation process, forced flow oil from sand stone reservoirs, seepage of rain waste through permeable ground into aquifer, wetting and drying process and powder technology. From the last few decades, researchers are keen interested in thermal convection problems in porous medium, this is because of their numerous applications in manufacture and process industries. The detailed discussion on the convection flow through porous medium is given in the books as Pop and Ingham (2001) and Ingham and Pop (2005). Keeping in mind the above facts, present study also investigates the free convection flows of Newtonian and non-Newtonian fluids over an infinite vertical plate embedded in a porous medium.

1.2.8 Newtonian Fluids

Fluids that obey the Newton's law of viscosity are known as Newtonian Fluids. In Newtonian fluid, viscosity is entirely dependent upon the temperature and pressure of the fluid and the relation between the shear stress and the shear rate is linear, passing through the origin, the constant of proportionality being the coefficient of viscosity, mathematically

$$\underline{\tau} = \mu \frac{du}{dy}, \tag{1.1}$$

where $\underline{\tau}$ is the shear stress exerted by the fluid, μ is the dynamic viscosity of the fluid and du/dy is the shear strain or deformation rate perpendicular to the direction of shear. Equation (1.1) is known as Newton's law of viscosity and for which μ has a constant value are known as Newtonian fluids (White, 2006). Simply, this means that the fluid continues to flow regardless of the forces acting on it. For example, water is Newtonian, because it continues to exemplify fluid properties no matter how fast it is stirred or mixed.

Newtonian fluids describe by Navier Stokes equations are extensively studied in the literature for the past few decades. Largely, this is due to the fact that they are relatively simple and their solutions are convenient (Soundalgekar, 1977; Das *et al.*, 1994; Chaudhary and Jain, 2006; Fetecau *et al.*, 2008; Rubbab *et al.*, 2013). However, Newtonian fluids which have a linear relationship between the stress and the rate of strain are limited in view of their applications. They do not explain several phenomena observed for the fluids in industry and other technological applications. For example, many complex fluids such as blood, soap, clay coating, certain oils and greases, elastomers, suspensions and many emulsions are noteworthy due to their various applications in industry. Unfortunately, Navier Stokes equations are no more convincing to describe such fluids. In literature, they are known as non-Newtonian fluids. These fluids are described by a non-linear relationship between the stress and the rate of strain. Present study contained the heat transfer flow of nanofluid with ramped wall temperature over an oscillating vertical plate.

1.2.9 Non-Newtonian Fluids

In recent years, non-Newtonian fluids have received great importance due to their numerous applications. The non-Newtonian behavior of a fluid is described by the power law model as given by

$$\underline{\tau} = k_0 \left(\frac{du}{dy} \right)^{\kappa}, \kappa \neq 1, \tag{1.2}$$

where k_0 is the flow consistency index and κ is called flow behaviour index. More, exactly, a non-Newtonian fluid is a fluid whose flow properties differ in any way from those of Newtonian fluids. Most commonly the viscosity of a non-Newtonian fluid is not independent of shear rate or shear rate history. Many polymer solutions and molten polymers are non-Newtonian fluids. Examples of non-Newtonian fluids includes substances such as ketchup, custard, toothpaste, starch suspensions, paint, blood and shampoo. In non-Newtonian fluids, the relation between the shear stress and the shear rate is different, and can even be time-dependent. Therefore a constant coefficient of viscosity cannot be defined.

Due to great diversity in the physical structure of non-Newtonian fluids, many models have been proposed to describe their rheological behaviour. Amongst them the second grade fluid, third grade fluid, fourth grade fluid, Maxwell fluid, Oldroyd fluid, Burgers fluid, generalized Burgers fluid, Walters'-B liquid and Power law fluid are very famous. However, recently some other non-Newtonian fluids have become very popular in the literature such as Casson fluids and micropolar fluids, due to their distinct characteristics.

1.2.10 Laplace Transform Technique

The distinct nature of fluid dynamics problems, especially the problems related with non-Newtonian fluid dynamics makes it complex to find exact solutions. In this situation some of problems can be dealt for analytical solutions. This is the cause that all the times researchers are impressed by finding exact solutions to more complex problems. Therefore, exact solutions are important not only because they provide the solutions for fundamental flows but also they serve as accuracy standard for approximate methods, whether numerical or experimental.

Various analytical techniques are available exact solutions. Amongst them, the Laplace transform technique is beneficial particularly for initial value problems for finding exact solutions of Newtonian and non-Newtonian fluids. This transform was first introduced by Laplace, a French mathematician, in the year (1790) in his work on probability theory. A detailed discussion on Laplace transform technique and on its necessary and suffcient conditions are presented in the book of Rao (1995). There are large number of applications of Laplace transforms in the field of science and technology, such as signal analysis or central energy. In present work, the Laplace transform technique has been used for finding the exact solutions of the problems. Indeed, the Laplace transform technique converts linear differential equations into algebraic equations while using given boundary conditions. It transforms the functions of time $f\left(t\right)$ to the functions of complex angular frequency. Mathematically,

$$\begin{aligned}\mathscr{L}\left\{f(t)\right\} &= \int_0^\infty e^{-qt} f\left(t\right) dt,\\ &= \overline{F}\left(q\right),\end{aligned} \tag{1.3}$$

where q is Laplace transform parameter. The inverse Laplace transform is represented by $f\left(t\right) = \mathscr{L}^{-1}\left\{\overline{F}\left(q\right)\right\}$. Moreover, for initial value problems, optimum results can be obtained by using Laplace transform technique (Dyke, 1999). In some problems, it is difficult to find the inverse Laplace transform of a function, which is product of two transformed functions. In such situation, Convolution theorem gives the inverse Laplace transform of that function.

Convolution theorem is defined as follow.

If $\overline{F}(q)$ be a composition of two Laplace transformed functions $\overline{G}(q)$ and $\overline{\aleph}(q)$ (Anumaka, 2012) given by

$$\overline{G}(q) = \mathscr{L}\{g(t)\}, \quad \overline{\aleph}(q) = \mathscr{L}\{h(t)\}, \tag{1.4}$$

therefore,

$$\mathscr{L}^{-1}\{\overline{F}(q)\} = f(t) = \int_0^t g(s)h(t-s)ds. \tag{1.5}$$

In this thesis, Laplace transforms technique is used to determine the exact solutions of the problems given in Section 1.4.

1.3 Problem Statement

Many researchers are engaged in analyzing heat and mass transfer due to free convection. Most of them are interested in finding numerical solutions. It is due to the fact that exact solutions most of the times are not possible to obtain. Therefore, exact solutions to such problems are very rare in the literature but of great interest for the researchers. It is because exact solutions can be used as a check of correctness for the solutions that are obtained numerically or experimentally. This is the main reason that researchers are motivated recently to find exact solutions for unsteady free convection problems of Newtonian and non-Newtonian fluids. Towards obtaining the exact solutions of Newtonian fluid (nanofluids) and non-Newtonian fluids (Casson and micropolar fluids) this study will explore the following questions.

1. How do the mathematical models for nanofluids, Casson fluids and micropolar fluids can be developed?

2. How do these fluids behave in the problem of unsteady free convection flow over an oscillating vertical plate with constant wall temperature?
3. How does the mathematical model behave in the problem involving heat and mass transfer?
4. How does the presence of non-Newtonian fluid parameters together with MHD, porosity and other parameters affect the fluid motion and heat transfer?
5. How does the micropolar material parameter influence the wall shear stress as well as fluid velocity and microrotation profiles?
6. How do the exact solutions for complicated free convection flow for the proposed fluid models can be obtained?

1.4 Research Objectives

The objective of this study is to investigate theoretically the unsteady free convection flow for three different types of fluids, which are Casson, nano and micropolar fluids. This investigation includes the formulation of the appropriate governing equations with some suitable initial and boundary conditions based on the constituted suitable physical models.

Specifically, the objective of this study is to find the exact solutions by using the Laplace transform technique for the following problems.

1. Unsteady free convection flow of Casson fluid over an oscillating vertical plate with constant wall temperature.
2. Unsteady MHD free convection flow of Casson fluid over an oscillating vertical plate with constant wall temperature embedded in a porous medium.
3. Unsteady free convection flow of nanofluids over an oscillating vertical plate with ramped wall temperature.
4. Unsteady MHD free convection flow of ferrofluids over an oscillating vertical plate with ramped wall temperature embedded in porous medium.
5. Unsteady free convection flow of micropolar fluid with heat and mass transfer over an oscillating vertical plate with constant wall temperature.

6. Unsteady MHD free convection flow of micropolar fluid with heat and mass transfer over an oscillating vertical plate with constant wall temperature embedded in a porous medium.

1.5 Scope of the Study

This study will focus on the unsteady MHD flow of Newtonian and non-Newtonian fluids with either heat or heat and mass transfer together. Two different driving forces will be considered, which are responsible for inducing the motion into the fluid. These are buoyancy force and oscillating boundary condition.

The first two problems emphasise on free convection flow of Casson fluids when the plate obeys the oscillating wall condition. The third and fourth problems focus on the free convection flow of nanofluids together with oscillating boundary condition which also allow the plate to induce ramped wall temperature. This ramped behavior of temperature at the wall will be responsible for the comparative study of ramped and isothermal motion and heat transfer. The fifth problem highlights the combined effects of heat and mass transfer on the micropolar fluids placed over a vertical plate oscillating in its own plane. Sixth problem extends the idea of Problem 5, when micropolar fluid is electrically conducting and passing through a porous medium.

All fluids are studied in the absence and the presence of MHD and porosity effects. In all these problems, the governing linear partial differential equations are solved for exact solutions by using the Laplace transform technique. Expressions for skin friction, rate of heat transfer and rate of mass transfer are evaluated and also computed in tabular forms. For the validation purpose, the obtained solutions are reduced to some published results in the literature. There is no verification of the solution compared to the experimental results. Graphical results are provided for various embedded parameters and discussed. Two computational software MATHEMATICA and MATHCAD are used for this purpose. More exactly, the

MATHEMATICA software is used for the computation of tabular results whereas the MATHCAD software is used for plotting.

1.6 Significance of Study

The significance of the study are as follows

1. To build a better understanding of the MHD and heat transfer characteristics past an oscillating vertical plate with constant wall temperature and through a porous medium.
2. Accurate exact solutions for mathematical models involving isothermal and ramped wall temperatures.
3. Enhance understanding of the flow of the non-Newtonian fluid induced by an oscillating vertical plate embedded in a porous medium.
4. These results can be used as the basis for fluid flow problems frequently occurring in engineering and applied sciences.
5. The obtained results will assist scientists and engineers. These exact solutions can be used as a check of correctness for the solutions of more complex mathematical models obtained through numerical schemes.

1.7 Research Methodology

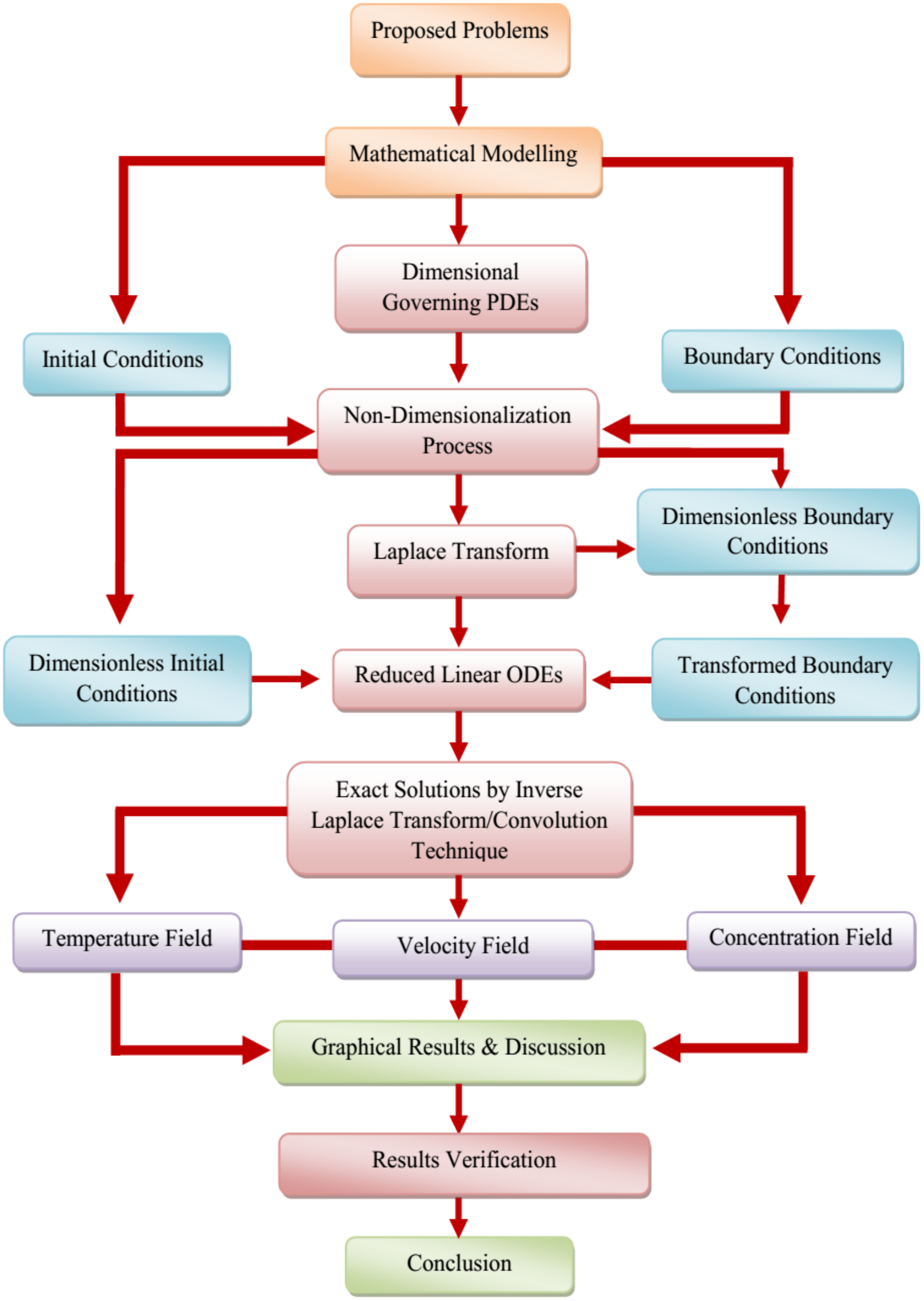

Figure 1.2 Operational framework.

1.8 Thesis Outlines

This thesis consists of 9 chapters. Chapter 1 starts with the research background which describes the general introduction succeeded by problem statements, objectives of research, scope of study, research methodology and significance of the present research. Chapter 2 covers a detailed literature review concerning the problems identified in the objectives of research. Chapter 3 begins with the problem regarding the unsteady free convection flow of Casson fluid with constant wall temperature. The flow in the fluid is induced by an oscillating infinite vertical plate. Both cosine and sine oscillations of the plate are considered. Using constitutive relations, the governing equations of the problem are formulated. Dimensionless variables are used to simplify the dimensional governing equations as well as appropriate initial and boundary conditions. Exact solutions of the dimensionless governing equations are obtained via of Laplace transform method. Some special cases are discussed. It is found that the general solutions obtained in this chapter reduce to some well known solutions in the literature, as limiting cases. Finally, the influence of important flow parameters on velocity and temperature are shown by graphs. Skin friction and Nusselt number are computed and shown in tables.

Chapter 4 includes the unsteady MHD free convection flow of Casson fluid over an oscillating vertical plate embedded in a porous medium with constant temperature. The fluid is electrically conducted under the influence of a transverse uniform magnetic field. Expressions for velocity and temperature are obtained. Similar to Chapter 3, both cosine and sine oscillations of the plate are considered. The graphical results for various embedded flow parameters are analyzed through graphs. The obtained solutions are reduced to the existing solutions in the literature. Moreover, the expressions for skin friction and Nusselt number are determined.

The focus in Chapter 5 is on the unsteady free convection flow of nanofluids over an oscillating vertical plate with ramped wall temperature. By taking into account, the physical properties of nanofluids, the problem is modeled in terms of linear partial differential equations. Initial and boundary conditions for velocity are same as in previous chapters. However, in case of temperature, both cases of ramped and isothermal wall are considered. Exact solutions for velocity and temperature obtained via Laplace transform method. Both cases of ramped and isothermal temperature are discussed. The obtained solutions are reduced to the existing solutions in the literature. Furthermore, results of skin friction and Nusselt number are also evaluated. Graphs are sketched and the effects of pertinent flow parameters are discussed.

Chapter 6 extends the idea of Chapter 5, by taking into account the effects of MHD and porosity. More exactly, the nanofluid is taken electically conducting in the presence of a uniform magnetic field and passing through a porous medium. The influence of thermal radiation in heat equation is also considered. The governing equations along with appropriate initial and boundary conditions are made dimensionless and then solved by Laplace transform. As special cases, the obtained solutions are reduced to known solutions from the literature. Results for velocity and temperature are plotted graphically and discussed. Skin friction and Nusselt number are computed in tables.

Chapter 7 investigates the unsteady free convection flow of micropolar fluid over an infinite oscillating vertical flat plate with wall couple stress. This chapter begins with the mathematical formulation of the problem to model the governing equation for micropolar fluid. The governing equations along with appropriate initial and boundary conditions are made dimensionless and then solved by the Laplace transform technique. The obtained solutions are reduced to the existing solutions in the literature. The expressions for velocity, micrororations, temperature and concentration are sketched and discussed in detail. Furthermore, skin friction, wall couple stress, Nusselt number and Sherwood number are also determined.

Chapter 8 is a continuation of previous chapter, which includes the unsteady MHD free convection flow of micropolar fluid over an oscillating vertical plate in a porous medium with wall couple stress. More precisely, the micropolar fluid is taken electically conducting in the presence of a uniform magnetic field and passing through a porous medium. The governing equations along with appropriate initial and boundary conditions are made dimensionless and the then solved by the Laplace transform technique. Specifically, in this chapter is to find the inverse Laplace transform and convolution technique is used. Expressions for velocity, micorotations, temperature and concentrations are obtained. The graphical results for various embedded flow parameters are analyzed through graphs. The obtained solutions are reduced to the existing solutions in the literature. Moreover, the expressions for skin friction and wall couple stress are computed.

In Chapter 9 the summary of this research and suggestions for future research are presented. References are listed at the end.

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