BAYESIAN AND FUZZY KRIGING INTERPOLATION TECHNIQUES FOR SPATIAL ESTIMATION IN MINING FIELD

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A thesis submitted in fulfilment of the
requirements for the award of the degree of
Doctor of Philosophy (Mathematics)

Faculty of Science
Universiti Teknologi Malaysia

MARCH 2014

This dissertation is dedicated to my family for their endless support and encouragement.

ACKNOWLEDGEMENT

"In the Name of Allah, Most Gracious, Most Merciful"

First and foremost, *Alhamdulillah*, it is with the assistance and grace of *Allah Almighty* that I was able to finish this dissertation.

I would like to express my sincere appreciation to my supervisor Prof. Dr. Muhammad Hisyam Lee for his great advice and generous help during the period of my study and also, who had the patience and wisdom to guide me in order to overcome all the academic obstacles that I faced during my study. My overwhelming gratitude to my evaluators, I am also grateful for their helpful suggestions.

A special thanks to my parents and my siblings for their unlimited moral support, to everyone in my extended family, and for their lessons on how to be patient and strong. I thank them very much for always being there for me and I ask Allah the almighty to grant them Paradise.

Last but not least, I would like sincerely to thank all the lecturers, staff, friends and my fellow postgraduate friends for their emotional support and cognitive, thanks for all the care and concern. I wish you more and brighter success in this world and the Hereafter.

ABSTRACT

The focus of this research is in the area of spatial estimation. Such a study is very important in order to improve the spatial prediction performance. Many techniques of prediction that are based on the regionalized variables, and the surface trend change from linear to quadratic or cubic that produces inaccurate results in the prediction process. In this thesis, Bayesian and fuzzy kriging methods are suggested to solve the problem of uncertainty, which requires obtaining a minimum error in the prediction process. This study aims to improve the mixed approaches among methods of spatial prediction that are used for evaluation of prediction. The study also finds the performance of variation interpolation methods of minerals needed to develop the relationship between Bayesian techniques and fuzzy kriging and apply the results for further modeling a spatial relationship. This spatial prediction assumes stationary property. The findings of this study are mathematical models of covariance functions. The variogram and cross variogram functions are computed for all compass directions for the phenomena under the study and its parameters are estimated. Another aspect is to obtain Bayesian predictor, kriging predictor, and Bayesian kriging variance which represent the minimum variance of prediction. In addition, the constraints weights of linear prediction were computed. The practical side of this study includes the applications of the Bayesian and fuzzy kriging techniques on real spatial data with their locations in the mining fields of Australia, Canada, and Colombia. All the computations were carried out by using Matlab software. In conclusion, this study uses two different methods (Bayesian and fuzzy kriging techniques) for incorporating the spatial autocorrelation in order to improve the accuracy of uncertainty and estimation with minimum error. The approach combines more than one prediction methods to determine a model which is based on a cross validation that satisfies the best optimal prediction.

ABSTRAK

Tumpuan kajian ini adalah dalam bidang penganggaran ruang. Kajian ini sangat penting untuk mempertingkatkan prestasi ramalan ruang. Pelbagai teknik ramalan berdasarkan pembolehubah regionalisasi dan perubahan jalan permukaan daripada linear kepada kuadratik atau kubik menghasilkan keputusan yang tidak tepat dalam proses ramalan. Kaedah Bayesian dan kriging kabur dicadangkan untuk menyelesaikan masalah ketidaktentuan yang memerlukan supaya ralat minimum dalam proses ramalan dapat diperoleh matlamat kajian ini adalah untuk meningkatkan pendekatan gabungan antara kaedah bagi ramalan ruang yang digunakan dalam penilaian ramalan. Prestasi bagi kaedah interpolasi variasi juga ditemukan bagi mineral yang diperlukan untuk membangunkan hubungan antara teknik Bayesian dan kriging kabur dan melanjutkan dapatan bagi pemodelan atau hubungan ruang. Ramalan ruang ini mengandaikan ciri pegun. Dapatan kajian ini adalah model bermatematik bagi fungsi kovarians. Fungsi variogram dan variogram bersilang dikira untuk semua arah kompas bagi fenomena dalam kajian dan parameternya dianggarkan. Aspek lain adalah untuk mendapatkan peramal Bayesian, peramal kriging dan varian kriging Bayesian yang mewakili varians minimum bagi ramalan. Tambahan lagi, kekangan pemberat bagi ramalan linear dikira. Bahagian praktikal kajian ini termasuk penggunaan teknik Bayesian dan kriging kabur bagi data ruang sebenar dengan lokasi di kawasan perlombongan di Australia, Canada dan Colombia. Semua pengiraan dilakukan menggunakan perisian Matlab. Kesimpulannya, kajian ini menggunakan dua kaedah yang berbeza (Teknik Bayesian dan kriging kabur) untuk menggabungkan auto korelasi ruang bagi meningkatkan ketepatan bagi ketidaktentuan dan mendapatkan penganggaran dengan ralat yang minimum. Pendekatan yang digunakan menggabungkan lebih daripada satu kaedah ramalan untuk menentukan suatu model berasaskan pengesahan bersilang bertujuan memenuhi ramalan optima terbaik.

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LIST OF SYMBOLS

(x)	-	Locations x of Spatial Variable
(x') or $(x+h)$	-	Locations $(x+h)$ of Spatial Variable
$Z(x)$	-	Regionalized Variable of location (x)
$Z(x_o)$	-	Regionalized Variable of location (x_o)
$Z(x+h)$	-	Regionalized Variable of location $(x+h)$
Z , or Z_β	-	Vector of Real Data
$\bar{Z}$, $\bar{d}$	-	Sample Mean
h	-	Euclidian Distance. or Lag
(x, y)	-	Points of location
$N(\Delta x, \Delta y)$, or $N(h)$	-	Number of pairs of observations
λ_i	-	The Weights in Estimation of Ordinary Kriging
λ'	-	The Weights Transpose
λ_β	-	The Weights Vector
ω_i	-	The Weight in Estimation of Universal Kriging
$\gamma(\mathrm{x}, h)$	-	Semivariogram Function
$2\gamma(\mathrm{x}, h)$	-	Variogram Function
$\hat{\gamma}(h)$	-	Estimator of Semivariogram Function
γ^*_{ij}	-	Cross Semivariogram Function
$C(h)$, ϕ	-	Covariance Function
$C_{12}(h)$	-	Cross Covariance Function
C^{-1}, K^{-1}	-	Inverse Matrix
ϕ^{-1}	-	Covariance Inverse

C_o	-	Nugget Effect
$C+C_o$	-	Sill
a	-	Range
$\hat{z}(x)$	-	Predictor Values of Variable
z^b	-	Variable of Bayes
μ	-	Mean of Stationary Spatial Process
$\mu(x)$, $\mu_m(x)$	-	Mean of Non-Stationary Spatial Process
σ^2 , σ_E^2	-	Variance Parameter
$\sigma^2{}_{OK}$	-	Ordinary Kriging Variance
$\sigma^2{}_{uK}$	-	Universal Kriging Variance
$\sigma^2{}_{COK}$	-	COKriging Variance
LP	-	Lagrange Multiple
P	-	Dimensions
P(A)	-	Probability of Event A
P(x)	-	Probability of Event x
P(m)	-	Probability of Event m
P(D)	-	Probability of Event D
P($x\|A$)	-	Probability x Condition A
P($A\|x$)	-	Probability A Condition x
P(x, A)	-	Joint Probability of x and A
P($m\|D$)	-	Probability m Condition D
P($D\|m$)	-	Marginal Likelihood
P(A $\cup$ B)	-	Probability of Event Union
P(A $\cap$ B)	-	Probability of Event Intersection
$\rho(\psi, d)$	-	The Matérn Covariance
ρ	-	Correlation Coefficient
$\rho_{12}(h)$	-	Cross Correlation Coefficient

$\varepsilon(\mathrm{x})$	-	Random Error Vector
β	-	Vector Unknown Parameter
$\widehat{\beta}_{BK}$	-	Posterior of Bayesian Kriging
$\forall$	-	For all variables
D	-	Domain (or Region)
D_2	-	Special Distance
R	-	Real Numbers
R^p	-	Real Numbers R in P Dimension Space
∞	-	Infinity
$\sum$	-	Summation
$\sum\sum$	-	Double Summation
$\int$	-	Integral
F	-	Information Function
F^T	-	Matrix Transpose
θ	-	Angle (or Trend)
$K_u(\bullet)$	-	Bessel Function
$\tilde{A}$	-	Fuzzy Set
$\mu_A(x)$	-	Membership Function
$\mu_{\tilde{A}}(x)$	-	Membership Function of Fuzzy Set $\tilde{A}$
M^-	-	Left Spreads
M^m	-	Model Value
M^+	-	Right Spreads
$\mu_{\tilde{A}}(x)$	-	Membership Function of Fuzzy Set $\tilde{A}$
$\tilde{O}^{\beta}$	-	Trend Parameter
$\tilde{C}^{\beta}$	-	Covariance Matrices
Δ	-	Change in or Difference

$\mid \; \mid$	-	Absolute Value
$\lVert \; \rVert$	-	Euclidian Norm on R^p
$\sim$	-	Approximate
$\approx$	-	Approximate to Equal to
$\triangleq$	-	Equal by Definition
$\rightarrow$	-	Go to or Lead
$\begin{pmatrix} & \\ & \end{pmatrix}$	-	Matrix
$\{\ ,\ \}$	-	The Set of
$(\ ), [\ ]$	-	Brackets
(a,b)	-	Open Interval
$[a,b]$	-	Closed Interval
$\oplus$	-	Direct Sum
$\mathrm{E}(Z \mid \bullet)$	-	Conditional Expectation of Z
$\mathrm{E}(\bullet)$	-	Expectation Value
$\mathrm{Var}(\bullet)$	-	Variance
I	-	Identity Matrix
$f(\bullet)$	-	Non Linear Function

LIST OF ABBREVIATIONS

SK	-	Simple Kriging
OK	-	Ordinary Kriging
UK	-	Universal Kriging
COK	-	Cokriging
COV	-	Covariance
KED	-	Kriging with the External Drift
RK	-	Regression Kriging
RF	-	Random Function
BLUE	-	Best Linear Unbiased Estimator
ME	-	Mean Error
MAE	-	Average Absolute Error
MSE	-	Mean Square Error
KRMSE	-	Kriged Random Mean Square Error
CORMSE	-	Cokriged Mean Square Error

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CHAPTER 1

INTRODUCTION

1.1 Overview

Geostatistics is centrally focused areas of applied statistics. This branch of statistics is concerned with the spatial or temporal outlook of the data studied and the corresponding contour distribution of that data. Using geology, which is at the root of geostatistics, was pioneered by two researchers Krige (1951) and Matheron (1963). It was originally meant to project changes in the quality of ore from particular mines.

In the mid-nineties, Krige, a seasoned engineer working in South African mines, developed a model for interpolating true random spatial variables of a sample space. Krige's spatial statistic model was later modified by Matheron, a French mathematician create the best kriging. The trend of kriging was evident in the research conducted by Journel and Huijbregts (1978), Ripley (1981), Lam (1983), Davis (1986), and Cressie (1990) in their geostatistical investigation of various data sets.

1.2 Research Background

The spatial statistics theory has its roots from the theory of regionalized random spatial variables. This spatial statistics theory is concerned with spatial data such as deposits of mineral ore, oil reserves and data on rainfall distribution, epidemics, or data on various types of pollution. These spatial variables may be generated from geographic locations on the earth's surface, underground or from the atmosphere.

Spatial interpolation has close connections to geology and it entails a series of mathematical computational methods. It is also; concerned with the spatial phenomena. It uses methods that emphasis regionalized random variables so as to generate spatial distribution. Studies of spatial statistics based on regionalized variables theory have successfully formulated mathematical models for determining the nature of the spatial distribution through use of functions such as a variogram. A variogram is one of the most commonly employed functions used for modeling in spatial statistics.

A variogram functions is employed to determine existing variations in observed phenomenon. The function was employed in studies by Journel (1992), Cressie (1993), and Chiles and Defflar (1999). The variogram function uses spatial prediction. Spatial prediction uses the kriging method to estimate the Best Linear Unbiased Estimator (BLUE) for a set of spatial real data.

There are various types of kriging including simple kriging, ordinary kriging (which is the most commonly used), anisotropic kriging (uses accounting geometric variance), universal kriging (uses local accounting trends), cokriging (where there is more than one variables). Each of these kriging techniques types employs either a semivariogram or a variogram function.

The kriging school of thought uses geostastistical interpolation to estimate the value of unknown parameters using available location data (Cressie, 1985; Burrough, 1986). Some of the key studies that employed spatial interpolation models are Burrough and Mcdonnel (2000), and Cressie (2003). Other works that employed universal kriging, include Martinez and Zinck (2004) and Gooavaerts (1997a) who estimated parameters using approaches such as maximum likelihood estimation, least squares quadratic method, and Bayesian. Bayesian kriging is; where model parameters are generated with the aid of regionalized random variables.

1.3 Significance of the study

The significance of the current study lies in the fact that it will further enhance understanding of contour maps using regionalized random variables; and outline situations where the kriging technique for estimating the parameters of experimental functions and knowledge properties can be used.

1.4 Research Questions

The spatial estimation of interpolation methods was based on the following research questions:

i. Does mineral ore data satisfy the stationary assumptions?
ii. Are isotropic variations accounted for?
iii. Is the robust estimator trend required?
iv. In which method do different variogram estimation techniques, contribute to variations in the estimated covariance parameters?
v. How can a reasonable estimate of the nugget variance be achieved?
vi. How can we obtain a mathematical model that fits with covariance functions?
vii. Which model provides the best spatial estimation performance?

1.5 Problem Statement

Spatial interpolation techniques are an essential input for the development of spatial prediction models that use a stochastic process. In practice, location estimation using geostatistics or spatial statistics is associated with some level of precision error and uncertainty. This can be attributed to the fact that the surface trend changes from linear to quadratic or cubic thereby resulting in the inaccuracy. As such, the current research seeks to solve this problem by developing prediction models for enhancing the spatial prediction performance with minimal prediction error. Therefore, Bayesian kriging with fuzzy is proposed as a mixed approach to solve this prediction problem. The main motivation is to jointly handle different types of uncertain information such as uncertainty in the variogram parameters and uncertainty property in the variogram model. Kriging possesses advantages over the interpolation method as it has the ability to determine an uncertainty estimate for the value of the regionalized variable.

1.6 Objectives of the research

The objectives of the current research are as follows:

i. To analyze the spatial data using regionalized variables in the mining industry.
ii. To make a comparison between various types of kriging models (ordinary, universal, universal cokriging, Bayesian) to know the best performance.
iii. To establish the ability and accuracy of fuzzy method for enhancing the spatial prediction.
iv. To combine kriging techniques with the Bayesian fuzzy kriging so as to observe their output.
v. To identify the uncertainty features present in the mining industry and to analyze the effect of these features on the prediction process.
vi. To establish a mathematical model and to compare it with the covariance functions; and study the performance of the variation interpolation model.
vii. To enhance the performance of the spatial prediction models and to ascertain the environment effect of mineral ores in the study area.

1.7 Scope of the study

The current study was concerned with the analysis of data generated from metal ore mines and how the contour maps of the data are distributed in the study area. The idea was to allow for predictions that use spatial data and the theory of random spatial process to explore for minerals such as gold, silver, nickel, lead, zinc and copper.

1.8 Thesis Organization

The thesis is organized into five chapters. Chapter 1 includes an introduction of spatial statistics and gives an idea of research background, significance of the study problem statement, objectives of the research and the scope of the study. Chapter 2 provides a review of the literature on kriging which includes the origins of kriging, applications of the kriging, universal kriging, and cokriging techniques, applications of the Bayesian and fuzzy approach, and other applications of interpolation methods. Chapter 3 contains the research methodology that starts with introduction of geostatistics and goes on defining regionalized variables, the experimental variogram function, and spatial predictions. Chapter 4 illustrates the data analysis and the results based on the real spatial data in different areas of Colombia, Australia, and Britain, by using interpolation methods for predictions. Finally, Chapter 5 is reserved for conclusions and recommendations for future work.

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