KOSZUL CONNECTIONS OF FLAT EEG BUNDLES FOR DESCRIPTION OF BRAIN SIGNAL DYNAMICS

NIKI ANIS BIN AB KARIM

A thesis submitted in fulfilment of the
requirements for the award of the degree of
Doctor of Philosophy (Mathematics)

Faculty of Science
Universiti Teknologi Malaysia

MARCH 2016

THIS WORK IS DEDICATED TO

MY PARENTS

MY UNIVERSITY

MY COUNTRY

ACKNOWLEDGEMENT

I extend deepest thanks to my supervisor, Prof. Dr. Tahir bin Ahmad for showing great patience, wisdom, and openness during the course of this research. He has allowed me a lot of independence and personal space to explore ideas and solve problems, and his guidance has been essential through the years. He is as humble as he is inspirational, and there are so many reasons for him to be proud in leading the way for future educators and scientists like me.

I also offer gracious thanks to the thesis examiners, Prof. Dato' Dr. Abdul Razak bin Salleh and Assoc. Prof. Dr. Yusof bin Yaacob, and to the Chairman of the *viva voce* session, Prof. Dr. Zuhaimy bin Ismail, for their advice, wisdom and encouraging words.

Thanks also to the staff and educators at the Ibnu Sina Institute and the Mathematics Department of the Faculty of Science, Universiti Teknologi Malaysia for the life experiences and lessons, both in and out of the offices and lecture rooms, and for offering a conducive environment for my studies.

Immense gratitude goes to my parents, for nurturing a love for language and academia, for giving me a wicked sense of humor, and for being patient with my quirks and oddities. They are great sources of energy and inspiration.

Praises be to the Prophet Muhammad, for the gift of Revelation. Praises be to Allah the Wise and Merciful, for the gift of Creation. Verily He is the source of all things, and all return to Him.

ABSTRACT

The bio-electromagnetic inverse problem is the identification of electromagnetic sources based on signals recorded from Electroencephalography (EEG) or Magnetoencephalography (MEG) and physical equations with a minimum of priori information. Following initial applications in modelling epilepsy patients' electrical brain activity, extensive research has been done in processing massive amounts of signal data from patients while preserving as much information as possible. Improvements in analyzing EEG/MEG recordings have significant utility not just in epilepsy treatment and diagnosis, but neuro-cognitive research in general. The main objective of this research is to build structures holding visualized and flattened EEG data as differential geometric entities, and to extend these structures as a dynamical system dealing with the evolution of large amounts of point data about brain activity. One aspect of existing research involves EEG signal data recorded across a time interval and processed using fuzzy clustering techniques, resulting in point data sets representing areas of high electrical activity within the brain. Concepts in differential geometry are applied to these spaces as a dynamical and visualized approach to modelling the evolution of signal clusters in the brain over time. Initially, Flat EEG data sets are shown to be topological spaces, manifolds and vector spaces. Two vector bundle structures for the Flat EEG space are thus developed: one analogous to Minkowski space-time and the other based on the classical notion of spatial change over linear time. From there, Koszul connections were constructed for both vector bundles, and both are shown to have zero curvature. Having provided a continuous differential structure to Flat EEG data, the evolution of signal clusters as a discrete dynamical system is then interpolated into a continuous form, allowing an enhanced view of the brain's state changes over time.

ABSTRAK

Masalah songsangan bio-elektromagnet ialah mengenal pasti sumber elektromagnet berdasarkan isyarat rakaman Elektroensefalografi (EEG) atau Magnetoensefalografi (MEG) dan persamaan fizis bersama maklumat *priori* yang minimum. Berikutan daripada penggunaan awal dalam membina model aktiviti elektrik otak pesakit epilepsi, penyelidikan telah dilakukan demi memproses bilangan besar data isyarat pesakit sambil memelihara sebanyak mungkin maklumat asal. Penambahbaikan analisis rakaman EEG/MEG mempunyai kegunaan bukan sahaja dalam rawatan dan diagnosis epilepsi, malah juga penyelidikan neuro-kognitif secara am. Objektif utama penyelidikan ialah membina struktur menempatkan data EEG yang berbentuk visual dan rata sebagai objek geometri keterbezaan, dan melanjutkan struktur tersebut sebagai sistem dinamik berkenaan evolusi sejumlah besar data titik aktiviti otak. Satu aspek penyelidikan kini melibatkan data isyarat EEG yang dirakam pada suatu selang masa dan diproses menggunakan teknik kelompok kabur, menghasilkan set data titik mewakili kawasan aktiviti elektrik yang tinggi di dalam otak. Konsep geometri keterbezaan digunakan terhadap ruang set tersebut sebagai pendekatan dinamik dan visual bagi permodelan evolusi semasa kelompok isyarat dalam otak. Sebagai permulaan, data set EEG Rata dibuktikan sebagai ruang topologi, manifold dan ruang vektor. Dengan itu dua struktur berkas vektor untuk ruang EEG Rata dibina: satu setara dengan ruang-masa Minkowski dan satu lagi berdasarkan idea klasik perubahan ruang melawan masa linear. Dari situ, kaitan Koszul dibangunkan untuk kedua-dua berkas vektor, yang juga terbukti mempunyai kelengkungan sifar. Setelah menyediakan struktur keterbezaan lancar bagi data EEG Rata, evolusi kelompok isyarat tersebut dalam bentuk sistem dinamik diskret kemudian diberi interpolasi menjadi sistem dinamik berterusan, membolehkan pandangan lebih baik bagi perubahan keadaan otak terhadap masa.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF FIGURES

LIST OF SYMBOLS/NOTATIONS

M	–	Manifold
∇	–	Connection on a manifold
L_x	–	Lie differential operator
E	–	Vector bundle on a manifold
Tp(M)	–	Tangent space on a point p in manifold M
Hom(X,Y)	–	Set of homomorphisms from X to Y
L(X,Y)	–	Set of linear transforms from X to Y
End(X)	–	Set of endomorphisms from X to itself
S^2	–	Unit sphere manifold

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Introduction

The bio–electromagnetic inverse problem is the identification of electromagnetic sources based on signals recorded from Electroencephalography (EEG) or Magnetoencephalography (MEG) and physical equations with a minimum of priori information. Going from initial applications in modelling epilepsy patients' electrical brain signals, extensive research has been done in processing and transforming massive amounts of signal data from patients while preserving as much information as possible. Improvements in analysing EEG/MEG records have significant utility not just in epilepsy treatment and diagnosis, but neuro–cognitive research in general.

1.2 Background of Research

The field of signal processing presents the challenge of preserving data integrity as they are transformed into spaces with varying dimensions. One example of this in the field of medicine is the visualization of EEG signal data into two-dimensional arrays to aid diagnosis. Previous research were dependent on data recorded over instances within an interval. The data set is essentially discretised over time, even though EEG signal data is continuous. This is due to transformations required for the clustering process.

The question of the movement of clusters within space should involve differential geometrical structures. Vector spaces are simple, well-understood objects useful for modelling the spatial path of information over time. Neuroelectrical activity in the brain also exhibits the properties of a dynamical system.

1.3 Problem Statement

Analysis of Flat EEG signals has depended on discrete instances recorded over an interval. In order to aid the visualization and ease of interpretation of these massive sets of data by professionals, the dynamics of Flat EEG signals should be described as a set of vectors and thus rendered as an evolving system.

1.4 Research Objectives

The objectives of this research include

i. To construct vector bundles based on Flat EEG space.

ii. To construct connection(s) on the bundles.

iii. To express these geometric objects as dynamical systems.

iv. To approximate orbits of Flat EEG cluster data.

1.5 Research Scope

This research utilises mathematical theory based on differential geometry and dynamical systems on bioelectromagnetic field data of the brains of epilepsy patients during seizures, in order to be considered in conjunction with the Fuzzy Topographic Topological Mapping (FTTM) model.

1.6 Significance of Research

A differential structure for post-clustering Flat EEG is needed to stitch together disparate sets of instantaneous information into a continuous whole. This is helpful in order to aid the detection of epileptogenic foci by modelling the paths of the strong electrical signals in the brain.

1.7 Thesis Outline

Figure 1.1 shows a flowchart outlining all the chapters in this thesis and their main topics of interest.

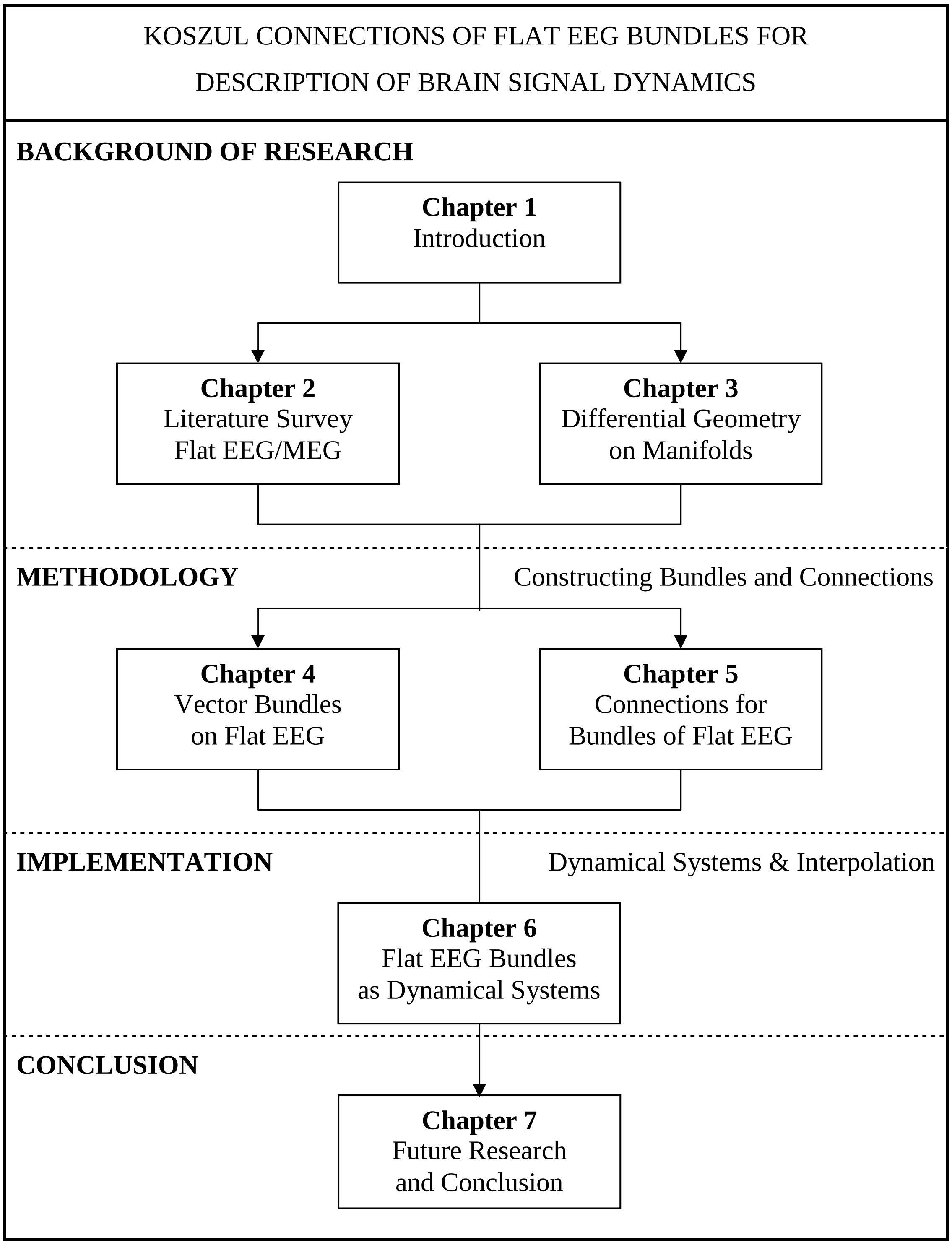

Figure 1.1 Thesis Outline

1.8 Context of Research

Figure 1.2 shows the relationship between the research in this thesis and other work previously done in similar fields. Further details and elaborations on this topic can be found in Chapter 2.

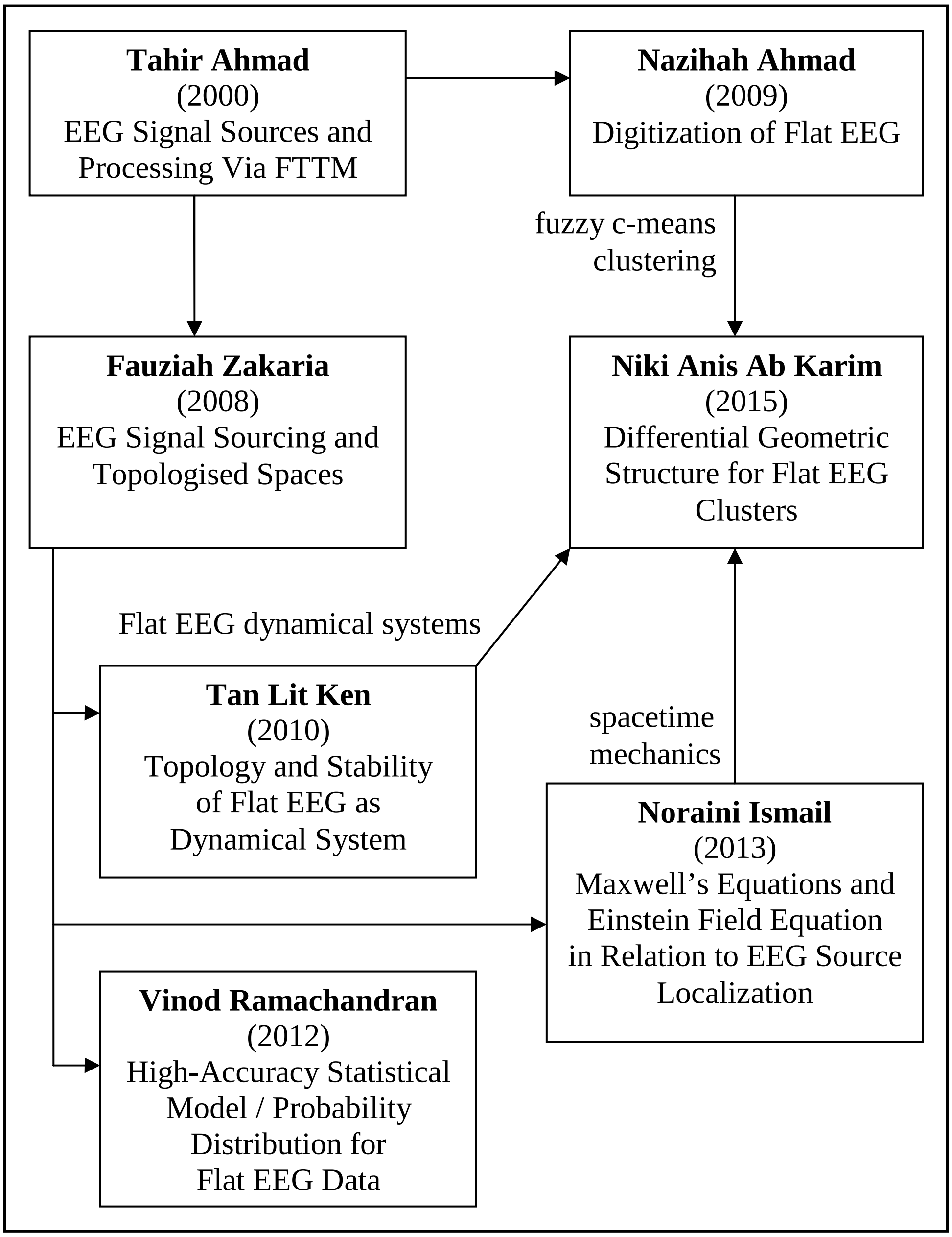

Figure 1.2 Context of Research

REFERENCES

1. Catherine Pouedras. This woman is undergoing an electroencephalogram (EEG) to diagnose Alzheimer's disease. On the computer screen at the right are the colored scans of the electrical activity in her brain. *Science Photo Library*. National Audubon Society Collection/Photo Researchers, Inc. Retrieved in 2013 from Encyclopedia of Mental Disorders: http://www.minddisorders.com/Del-Fi/Electroencephalography.html
2. Bullock T.H., Achimowicz J.Z., Duckrow R.B., Spencer S.S. and Iragui-Madoz V.J. Bicoherence of intracranial EEG in sleep, wakefulness and seizures. *Electroencephalography and Clinical Neurophysiology*. Department of Neurosciences, University of California, San Diego, La Jolla 92093-0201, USA. 1997. Dec;103(6):661-78.
3. Nazihah Ahmad. *Theoretical Foundation for Digital Space of Flat Electroencephalogram.* Doctorate thesis. Universiti Teknologi Malaysia. 2009.
4. Nicolson S.E., Mayberg H.S., Pennell P.B. and Nemeroff C.B. Persistent Auditory Hallucinations That Are Unresponsive to Antipsychotic Drugs. *The American Journal of Psychiatry.* American Psychiatric Association Publishing, Arlington, VA 22209, USA. 2006. 163(7), 1153-1159.
5. Wang Z.J., Pamela Wen-Hsin Lee and McKeown M.J. A Novel Segmentation, Mutual Information Network Framework for EEG Analysis of Motor Tasks. *BioMedical Engineering OnLine.* BioMed Central Ltd. 2009. 8(9) doi:10.1186/1475-925X-8-9.
6. Tan L.K. and Ahmad T. Flows of Continuous-Time Dynamical Systems with No Periodic Orbit as an Equivalence Class under Topological Conjugacy Relation. *Journal of Mathematics and Statistics.* Science Publications. 2011. 7(3), 207-215.

7. Ahmad T., Ahmad R.S., Li Yun Liau, Zakaria F., and Wan Abdul Rahman W.E.Z. Homeomorphisms of Fuzzy Topographic Topological Mapping (FTTM). *MATEMATIKA.* Department of Mathematical Sciences, Universiti Teknologi Malaysia. 2005. 21(1), 35–42.

8. Ahmad T. and Tan L. K. (2010) Topological Conjugacy Between Seizure and Flat Electroencephalography. *American Journal of Applied Sciences.* Science Publications. 2010. 7(11), 1470-1476.

9. Ahmad T., Zakaria, F., Abdullah J.M. and Mustapha F. (2005) Dynamical System of an Epileptic Seizure. *Asian Conference on Sensors and the International Conference on New Techniques in Pharmaceutical and Biomedical Research – Proceedings.* IEEE Explore, Kuala Lumpur. 2005. DOI: 10.1109/ASENSE.2005.1564510 78-80.

10. Fauziah Zakaria. *Dynamic Profiling of EEG Data During Seizure Using Fuzzy Information Space.* Doctorate thesis. Universiti Teknologi Malaysia. 2008.

11. Tan L.K. and Ahmad T. Structural Stability of Flat Electroencephalography. *Life Science Journal.* Marsland Press / Zhengzhou University. 2014. 11(8), 165-170.

12. Ahmad T. and Ramachandran V. Hyperspherical Manifold for EEG Signals of Epileptic Seizures. *Journal of Applied Mathematics.* Hindawi Publishing Corporation. 2012. Article ID 926358. http://dx.doi.org/10.1155/2012/926358

13. Ahmad T., Saeidiasl L., Alias N., Ismail N. Relationship between Maxwell’s Equations and Einstein Field Equation Base on EEG Source Localization in the Brain. *Research Journal of Applied Sciences, Engineering and Technology.* Maxwell Science Publication. 2013. 6(9):1582-1587.

14. Abraham, R. and Marsden, J.E. *Foundations Of Mechanics (2nd Edition).* Benjamin/Cummings Publishing, Massachusetts, USA. 1978.

15. Spivak, M. *Calculus On Manifolds.* W.A. Benjamin, Inc., Massachusetts, USA. 1965.

16. Kobayashi, S., and Nomizu, K. *Foundations Of Differential Geometry, Volume I.* Interscience, New York, USA. 1963.

17. Tahir Ahmad. Satu Kaitan Koszul Mendatar Untuk Berkas Tangen Unit Sfera (TS^2, π, S_2). *MATEMATIKA.* Department of Mathematical Sciences, Universiti Teknologi Malaysia. 2000. 16(1), 41-46.

18. Varadarajan V.S. *Vector Bundles and Connections in Physics and Mathematics: Some Historical Remarks. A Tribute to C.S. Seshadri: A Collection of Articles on Geometry and Representation Theory.* Birkhäuser Basel, Switzerland. 2003. 502-536.

19. Bozhidar Z.I. Links between Connections, Parallel Transports and Transports Along Paths in Differentiable Fiber Bundles. *International Journal of Geometric Methods In Modern Physics*. (0219-8878). 2005. 2(5), 823-838.

20. Jost, J. *Riemannian Geometry and Geometric Analysis.* Universitext. 2008. 113-177. DOI: 10.1007/978-3-540-77341-2_3

21. Voronov, T. *Lecture Notes in Differential Geometry.* University of Manchester, UK. 2009. Retrieved in 2012 from: http://www.maths.manchester.ac.uk/~tv/Teaching/Differential%20Geometry/2008-2009/#lectures

22. Ilinski, K. Physics of Finance. *Proceeding of Budapest's Conference on Econophysics.* 1997. arXiv:hep-th/9710148

23. Amari S. Differential-Geometrical Methods in Statistics. *Lecture Notes in Statistics Volume 28.* Springer-Verlag, New York, USA. 1985.

24. Amari S., Nagaoka H., and Harada D. *Methods of Information Geometry: Translations of Mathematical Monographs.* American Mathematical Society, Providence, RI, USA. 2007.