

DYNAMIC FRACTURE PROCESS OF SOLDER/INTERMETALLIC
INTERFACE IN LEAD-FREE SOLDER INTERCONNECTS USING
COHESIVE ZONE MODEL

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To my beloved father and mother

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ABSTRACT

Solder joint reliability (SJR) is an important requirement in electronics packaging. Most of the failures in a package are found in solder joints and interconnections. Brittle solder/intermetallic (IMC) interface fracture is the dominant failure mode in cases of impact loading and fast mechanical fatigue loading. In this study, the response of a single solder specimen subjected to cyclic shear deformation and a typical ball grid array (BGA) package undergoing board-level drop test is investigated. The finite element (FE) analysis of the single reflowed solder specimen and the BGA package is employed to understand the mechanics of the solder joints and the brittle solder/IMC fracture process. Inelastic behavior of the solder joints is described using unified inelastic strain model (Anand model) with optimized model parameters. The brittle solder/IMC interface fracture is demonstrated using cohesive zone model (CZM). The accuracy of interface fracture description depends on the CZM model prescribed in the analysis. The CZM model is modified further to ensure better predictive capability especially in cyclic loading. FE results for single solder specimen under shear fatigue test simulation shows that the CZM parameters degraded as the number of cycles is increased. Rapid damage progression occurs at the beginning of cycle and propagated slowly for subsequent cycles. For a board-level drop test simulation, the critical solder joint is located the farthest away from the center of the board. The highest stress and inelastic strain are confined to a small edge region at solder/IMC interfaces. Damage initiated from the outer peripheral solder and propagated into the inner peripheral solder joint.

ABSTRAK

Kebolehpercayaan sambungan pateri (SJR) adalah salah satu faktor penting dalam pembungkusan elektronik. Kebanyakan kegagalan pakej terletak di sambungan pateri dan penyambungannya. Kegagalan rapuh di permukaan pateri/lapisan sebatian antara logam (IMC) lebih dominan bagi kes hentaman beban dan kes mekanikal lesu. Dalam kajian ini, tindak balas spesimen pateri tunggal yang dikenakan dengan beban kitar ricih dan tipikal pakej *ball grid array* (BGA) semasa hentaman *board* dikaji. Analisis unsur terhingga (FE) untuk pateri tunggal dan pakej BGA digunakan untuk memahami peri laku mekanik di sambungan pateri dan proses patah di permukaan pateri/IMC. Terikan tak anjal pada sambungan pateri ditunjukkan dengan menggunakan model penyatuan terikan tidak anjal (model Anand) dengan parameter-parameter model yang dioptimumkan. Patah rapuh di permukaan pateri/IMC ditunjukkan dengan menggunakan *cohesive zone model* (CZM). Ketepatan patah di permukaan bergantung kepada peri laku model CZM yang digunakan dalam analisis. Model CZM diperbaiki lagi bagi memperolehi keupayaan ramalan yang lebih baik terutama sekali di dalam beban kitaran. Keputusan FE untuk simulasi spesimen pateri tunggal yang dikenakan beban kitar ricih menunjukkan parameter-parameter CZM berkurangan apabila kitaran meningkat. Kerosakan bercambah dengan cepat pada permulaan kitaran dan secara perlahan-lahan untuk kitaran seterusnya. Untuk simulasi hentaman *board*, keputusan menunjukkan bahawa sambungan pateri kritikal terletak di kedudukan paling jauh dari pusat *board*. Magnitud tertinggi tegasan dan terikan tak anjal terletak pada tepi sudut kecil antara permukaan pateri/IMC. Kerosakan bermula dari sempadan luaran pateri dan bercambah masuk kebahagian dalam sambungan pateri.

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LIST OF ABBREVIATIONS

ABQ	-	Abaqus
BGA	-	Ball Grid Array
BK	-	Benzeggagh-Kenane
CTE	-	Coefficient thermal expansion
CZM	-	Cohesive zone model
DBSTR	-	Ductile to brittle transition strain rates
ENIG	-	Electroless Nickel Immersion Gold
FCBGA	-	Flip Chip Ball Grid Array
FE	-	Finite element
IC	-	Integrated circuit
IMC	-	Intermetallics compound
OSP	-	Organic Solderability Perservative
PBGA	-	Plastic Ball Grid Array
PCB	-	Printed circuit board
SAC	-	Sn-Ag-Cu
SAC405	-	Sn4.0Ag0.5Cu
SJR	-	Solder joint reliability
SMT	-	Surface Mount Technology
UMAT	-	User material subroutine

LIST OF SYMBOLS

E	-	Young's modulus
σ_y	-	Yield stress
ν	-	Poisson's ratio
T_L	-	Liquidus melting temperature
T_S	-	Solidus melting temperature
ε_T	-	Total strain
ε_e	-	Elastic strain
ε_{in}	-	Total inelastic strain
ε_p	-	Time-independent plastic strain
ε_{Cr}	-	Time-dependent creep strain
ε_{vp}	-	Time-dependent visco-plastic strain
$\dot{\varepsilon}_p$	-	Inelastic strain rate
σ	-	Stress
T	-	Temperature
s	-	Anand model- internal variable
s_0	-	Anand model- initial value of internal variable
Q/R	-	Anand model- activation energy
A	-	Anand model- pre-exponential factor
ξ	-	Anand model- stress multiplier
m	-	Anand model- strain rate sensitivity of stress
h_0	-	Anand model- hardening coefficient
\hat{s}	-	Anand model- coefficient for deformation resistance saturation value

n	-	Anand model- strain rate sensitivity of saturation value
a	-	Anand model- strain rate sensitivity of hardening coefficient
s^*	-	Anand model- saturation value of internal variable
δ	-	Separation
δ^f	-	Separation at failure
δ_{pp}	-	Separation at failure for perfectly plastic criterion
δ_{pro}	-	Separation at failure for progressive softening criterion
δ_{lin}	-	Separation at failure for linear softening criterion
δ_{Ne}	-	Separation at failure for Needleman criterion
δ_{reg}	-	Separation at failure for regressive softening criterion
G_C	-	Critical fracture energy
P	-	Force
t	-	Thickness
A	-	Area
K_p	-	Penalty Stiffness
δ_0	-	Separation at damage initiation
D	-	Damage
N	-	Cohesive element maximum normal stress direction
S	-	Cohesive element maximum shear stress direction
G_I	-	Mode I strain energy release rate
G_{II}	-	Mode II strain energy release rate
G_{IC}	-	Mode I critical strain energy release rate
G_{IIC}	-	Mode II critical strain energy release rate
η	-	BK mixed-mode parameter
K_n	-	Cohesive element penalty stiffness at normal direction
K_s	-	Cohesive element penalty stiffness at shear direction
U_x	-	Displacement in X-axis
U_y	-	Displacement in Y-axis
UR_x	-	Rotation about X-axis
UR_z	-	Rotation about Z-axis
UR_y	-	Rotation about Y-axis
UR_z	-	Rotation about axis-Z

σ_{vm}	-	von Mises stress
τ_{13}	-	Shear stress at direction-13
τ_{23}	-	Shear stress at direction-23
σ_{33}	-	Normal stress at direction-33
φ	-	Damage

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CHAPTER 1

INTRODUCTION

1.1 Background of Study

Microelectronic industries concern the reliability of microelectronic products in term of product specification, design and etc. One of the major aspects is solder joint reliability (SJR). Solder joint is not only for electrical connection but it also provides mechanical strength between electronic components and board. The emerging of new technologies provides more challenges to solder joint connections. A standard reliability tests is needed to make sure that the connections of the solder joints met certain criteria. There are several reliability tests available to evaluate the performance of solder joints. The typical reliability and life-cycle tests carried out for electronic packages include mechanical tests like drop tests, bending tests and flexural testing. The external load during handling, transporting and shipping contributed flexural of the board. This causes mechanical fatigue of the solder joints in package assemblies. There are two types of failures often seen in the solder joints namely; the ductile solder failure and the brittle solder/intermetallic (IMC) interface fracture. For high strain rate conditions such as impact and mechanical fatigue, the brittle failure is dominated.

The aim of the research is to examine the brittle solder/IMC interface fracture during mechanical fatigue and impact loading condition. The mechanic behavior of solder/IMC interface is demonstrated using finite element method. The analyses capitalize on previous work with determination of solder joint material parameters especially on solder joint inelastic behavior. After that, it continued by determination of solder/IMC interface methodology and parameters extraction from experimental tests. The current study is a continuation of previous projects with conjunction of refinement Anand model and modifying cohesive zone model (CZM) for cyclic loading. All of these features are performed using SAC405 solder material.

The work is on-going research in SJR sponsored by Intel Technology, Penang, Malaysia. These includes the establishment of new CZM for improving interface damage prediction during cyclic loading and development of solder joint life prediction models utilizing the new CZM model in the current study to interpret solder/IMC interface fracture.

1.2 Problem Definition

Fatigue, or failure resulting from the application of cyclical stresses, is the third category of solder joint failures. It is often considered to be the largest and most critical failure category, since it is encountered in many different situations that are difficult to control. Therefore, understanding the fatigue behavior and its deformation mechanisms at the solder joints are important. The issue is largely raised by mechanical stresses in the solder connections associated with mechanical fatigue loading during handling, transporting and shipping. The stresses occur at the solder joint is due to relative motion between board and the package during fatigue loading. The inelastic strain from localized stresses contributed interface cracking at solder/IMC interface.

Progression of interface cracking is depends on local strain rate experienced at the solder joint. Higher localized straining rates increasing tendency of interface fracture. CZM concept can be used to model fracture process at solder/IMC interfaces. But, the current CZM is limited to demonstrate interface failure under cyclic loading. New CZM formulation is needed to get better representation of interface crack under such loading. This study will be focused on cohesive zone interface between SAC405 solder and Cu_6Sn_5 intermetallic compound under impact and mechanical fatigue loading.

1.3 Objectives

The objectives of the project are:

1. To develop cohesive zone model (CZM) for solder/IMC interface fracture for cyclic loading.
2. To describe the mechanics of solder/IMC interface fracture under cyclic shear loading.
3. To quantify solder/IMC interface fracture process in BGA package with Pb-free solders under impact loading conditions.

1.4 Scope of Study

The scope of this study covers the followings;

1. Sn-4Ag-0.5Cu (SAC405) solder joint with Cu_6Sn_5 IMC layer as a demonstrator solder material.
2. Formulation of CZM for cyclic loading to described solder/IMC interface fracture under cyclic loading.
3. FE simulations of solder joint reliability using commercial Abaqus version 6.9EF software:
 - Case A – Single solder shear fatigue test simulation
 - Case B – Board-level drop test simulation with a BGA package

1.5 Significance of Study

This study addressing brittle type failure process at solder/IMC interfaces under impact and mechanical cyclic condition. The extended cohesive zone model (CZM) for cyclic loading acknowledge the degradation of CZM parameters and damage accumulation until separation occur between solder/IMC interfaces. This model is expected to give better prediction to represent solder/IMC interface fracture process.

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