Author:
Chen, Xu

Title:
Coopetition models and applications

Coopetition models and applications

Xu Chen

School of Economics, Finance and Management

UNIVERSITY OF BRISTOL

A dissertation submitted to the University of Bristol in accordance with the requirements for awards of the degree of DOCTOR OF PHILOSOPHY in the Faculty of Social Science and Law

February 2019

Word Count: 51,019

Abstract

Focusing on coopetition, a concept defined as simultaneous pursuit of cooperation and competition (Brandenburger and Nalebuff 1996), this doctoral research explores the strategic choices between competition and coopetition by the two rival firms through three different contextual settings namely: production coopetition, green R&D coopetition, and service coopetition. Use the game theoretical approach, the focus of this doctoral research is on firms' optimal coopetition strategy in different business settings and management applications considering key issues including the internal operational factors, external market and policy environment, and inter-firm relationships. This doctoral research contributes to the coopetition literature by presenting models and applications of production coopetition, low carbon technology licensing coopetition, and service coopetition between rival firms and filling an important gap in the literature. Through modeling the firms' decision behaviors and consequential performances in three different coopetition applications, the research helps to understand the economic principle underlining firms' strategic decision on coopetition. It is the trade-off between the benefits gained from cooperation and financial loss incurred when facing a strengthened competitor that determines firms' strategic decision on coopetition. The examination of three coopetition applications generates a wide range of outcomes that are not captured from traditional models and provides valuable insights of firms' coopetition behavior. These research insights provide strategic guidance for businesses in different market environments to pursue coopetition. The knowledge of the underlying economic principle that governs coopetition decisions will be helpful for managers make the right strategic and operational decisions to enhance their competitive advantages.

Acknowledgements

I have been extremely fortunate in having Professor Xiaojun Wang as my supervisor. In the past four years, I was warmly cared, helped and instructed by him. Without his help, I would not successfully come to the conclusion of my Ph.D study. He set me an example by his industriousness and conscientiousness as an outstanding scholar. I hereby extend my heartfelt appreciation to Prof. Wang.

Besides, I would like to express my gratitude to Department of Management, School of Economics, Finance and Management, University of Bristol for giving me this valuable opportunity to further my study.

My thanks also go to my wife and daughter for their understanding and support during my Ph.D journey!

Author’s declaration

I declare that the work in this dissertation was carried out in accordance with the requirements of the University's *Regulations and Code of Practice for Research Degree Programmes* and that it has not been submitted for any other academic award. Except where indicated by specific reference in the text, the work is the candidate's own work. Work done in collaboration with, or with the assistance of, others, is indicated as such. Any views expressed in the dissertation are those of the author.

SIGNED: DATE:

Table of Contents

List of Tables

Table number **Description** **Page number**

List of Figures

Statement of Contributions

A research paper developed from the first study presented in Chapter 3 “Production coopetition models and applications” has been accepted for publication by *Production and Operations Management* with joint authorship of my Ph.D supervisor: Professor Xiaojun Wang and an external PhD advisor Professor Yusen Xia, as:

Chen, X., Wang, X. and Xia, Y. 2018. Production coopetition strategies for competing manufacturers that produce partially substitutable products. *Production and Operations Management*, Doi: 10.1111/POMS.12998, forthcoming

The author of this dissertation acted as sole researcher in terms of modelling and analysis and the corresponding author of the paper. The statement of contributions of the authors is as follows and this declaration is jointly authorized by the signature of the parties below:

Name of the author	**Contribution**	**Signature**
Xu Chen	Conceptualisation, literature review, analytical modelling, numerical analysis, inference, and writing.	[redacted]
Xiaojun Wang	Supervision and commentary.	[redacted]
Yusen Xia	Advice and commentary.	[redacted]

Statement of Contributions

Part of literature review presented in Chapter 2 has been developed into a research paper and accepted for publication by *Omega: an International Journal of Management Science* with joint authorship of my Ph.D supervisor: Professor Xiaojun Wang and an external collaborator Luo Zheng as:

Chen, X., Luo, Z. and Wang, X. 2018. *Compete or cooperate: Intensity, dynamics, and optimal strategies*, *Omega: an International Journal of Management Science,* Doi.org/10.1016/j.omega.2018.07.002.

The author of this dissertation acted as the first and corresponding author of the paper. The statement of contributions of the authors is as follows and this declaration is jointly authorized by the signature of the parties below:

Name of the author	**Contribution**	**Signature**
Xu Chen	Conceptualisation, literature review, analytical modelling, inference, and writing	[redacted]
Zheng Luo	Numerical experiment	[redacted]
Xiaojun Wang	Supervision and commentary	[redacted]

Chapter 1 Introduction

1.1 Background

With the rapid technological advancement and the development of global economy, more and more firms have recognized the importance of cooperating with the rival firms to gain competitive advantages. For example, in the aviation industry, airlines form an alliance with rival airlines to share resources and gain efficiency benefits (Oum et al. 2004). Furthermore, despite patent battles and lawsuits between the two market leaders in the smartphone industry, Apple and Samsung recently announced collaboration on future projects; Samsung will be the main supplier of chips and displays for the latest Apple products (Kang 2016). In the automotive industry, Ford offers technology licensing of its electrified vehicle technology to other automakers despite being arch rival in the hybrid and electric vehicle (HEV) market (Atiyeh 2015). In the online retailing sector, marketplace firms such as Amazon and JD.com invest heavily in their distribution and logistics infrastructure and provide delivery service to merchants selling at online marketplaces (Lopez 2017). While there is a cooperative relationship in delivery service provisions, these merchants and marketplace firms are also competing for consumers' demands at the same time. It is clear from the examples of different industrial sectors that rival firms cooperate in various aspects such as service, production and technology investment to gain competitive advantages.

This phenomenon is known as coopetition, a concept defined as simultaneous pursuit of cooperation and competition by firms (Brandenburger and Nalebuff 1996; Dowling et al. 1996; Bengtsson and Kock 2000; Gnyawali et al. 2006; Chen 2008). Since the seminal work by Brandenburger and Nalebuff (1996), coopetition has attracted growing interests from among academics. So far, the research on coopetition has been carried out in different theoretical fields including innovations (e.g. Quintana-García and Benavides-Velasco 2004; Cassiman et al. 2009; Gnyawali and Park 2009; 2011; Ritala and Hurmelinna-Laukkanen 2009; 2013; Mention 2011), strategic alliance (e.g. Khanna et al. 1998; Das and Teng 2000; Dussauge et al. 2000; Garrette et al. 2009; Oxley et al. 2009; Rai 2016), new product development (Fernandez et al. 2014; Yami and Nemeh 2014; Bouncken et al. 2018); international business (e.g. Luo 2004; 2005; 2007; Kim and Parkhe 2009), marketing (e.g. Luo et al. 2006; Bello et al. 2010) and supply chain management (e.g. Bakshi and Kleindorfer 2009; Li et al. 2011; Wilhelm 2011), and on different levels of unit analysis ranging from cross-functional units at the intra-organization level (e.g. Tsai 2002; Luo et al. 2006; Chiambaretto et al. 2018) to value chain horizontal rival firms (e.g. Luo et al. 2007; Garrette et al. 2009; Kumar 2010; Luo et al. 2016) or partners within a supply chain (e.g. Bakshi and Kleindorfer 2009; Wilhelm 2011; Lacoste 2012) at the inter-firm level, and intra-network (e.g. Gnyawali et al. 2006; Schiavone and Simoni 2011) or inter-network(e.g. Peng and Bourne 2009; Schiavone and Simoni 2011) at the network level.

The existing coopetition literature regards it as the most advantageous relationship between competitors (Bengtsson and Kock 2000) and argues that firms can achieve greater performance and gain financial benefits through obtaining valuable resources from the coopetitive relationships and strengthen their own competitive capabilities (Lado et al. 1997; Gnyawali and Madhavan 2001; Gnyawali et al. 2006; Gnyawali and Park 2009). Chen and Miller (2012) states that the benefits associated to the pursuit of a coopetition strategy are high, especially when companies seek to explore new markets or develop technological capabilities. Coopetition is also regarded as a risky relationship that is detrimental to alliance performance and results in failures (Park and Russo 1996; Kim and Parkhe 2009; Ritala 2012). The reasons behind these diverse arguments are not fully understood, and highlight a clear gap in the literature.

Ritala (2012) points out that the relationship between the coopetition parties and firm-specific factors as well as the embedded market and economic context all have significant impacts on the success of a coopetition strategy. This argument, to some extent, provides explanation to why coopetition strategies are often adopted in highly competitive and dynamic market environments. For example, in the aviation industry, in which there is an intense market competition, rival airlines often form an alliance to improve resource efficiency and increase their competitiveness in relation to other airlines or alliances. Moreover, the constant pressures of rapid technological development, short product life cycles, high R&D expenditure, and fierce competition drive many firms in the high-tech industry (e.g., Apple and Samsung, Microsoft and Google) to collaborate with their fiercest competitors. One natural question arising from these different business settings is whether the cooperation among competing firms is desirable from the perspectives of all participating firms, consumers and other stakeholders. Furthermore, the nature of competition (or cooperation) and the dynamics of coopetition among the participating firms by also are changed by pursuing competition and cooperation simultaneously. The benefits from cooperation may diminish over time when market categories mature, which gives no economic incentives for rival firms continue cooperating (Gnyawali and Park 2009; Mathias et al. 2018). The constantly changing business environment and firms' enhanced operational and technical capability and resulted competitiveness through coopetition may also require them to re-evaluate their coopetition strategies.

1.2 Research questions

Although coopetition has become a heated topic both in practice and in academia, it is clear from the above discussion that there are some critical research questions demanding clear answers. The observations from real-world business examples and the relevant academic literature motivated this doctorial research to explore these important issues regarding coopetition in various business settings. The focus of this doctoral research is therefore on firms' optimal coopetition strategy in different business settings and management applications considering key issues including the internal operational factors, external market and policy environment, and inter-firm relationships. In particular, this doctoral research investigates the following central questions:

Q1: What is the underlining economic principle that governs firms' strategic decision on coopetition?

This central research question leads to further detailed questions. For instance, how do competing firms choose to compete or cooperate with their rivals under different coopetition applications? What is the nature of coopetition dynamics? How does the external market competition affect firms' strategic decision on coopetition, and reciprocally, what impact does coopetition have on the nature of the market competition?

Q2: How do the internal, external and inter-firm specific factors affect the firms' coopetition decisions?

This central research questions leads to further detailed questions. For instance, what the specific factors that determine firms' decision on coopetition in various coopetition applications? What is the most influential one among these internal, external and inter-firm specific factors that determines firms' decision on coopetition?

Q3: What impact does coopetition have on the competing firms and other stakeholders such as consumers and environment?

This central research questions leads to further detailed questions. For instance, what impact does coopetition have on firms' financial performance? How does coopetition affect consumers' welfare? How does the low carbon technology licensing coopetition affect the environmental performance?

1.3 Research method

1.3.1 Research framework

This dissertation systematically explores the strategic choices of two competing firms regarding competition and coopetition in three different scenarios: production coopetition, green technology coopetition and distribution service coopetition. The research framework is developed according to three different coopetition applications, relevant models within different applications, and the research questions addressed in each study in the context of these coopetition applications as illustrated in Figure 1.1.

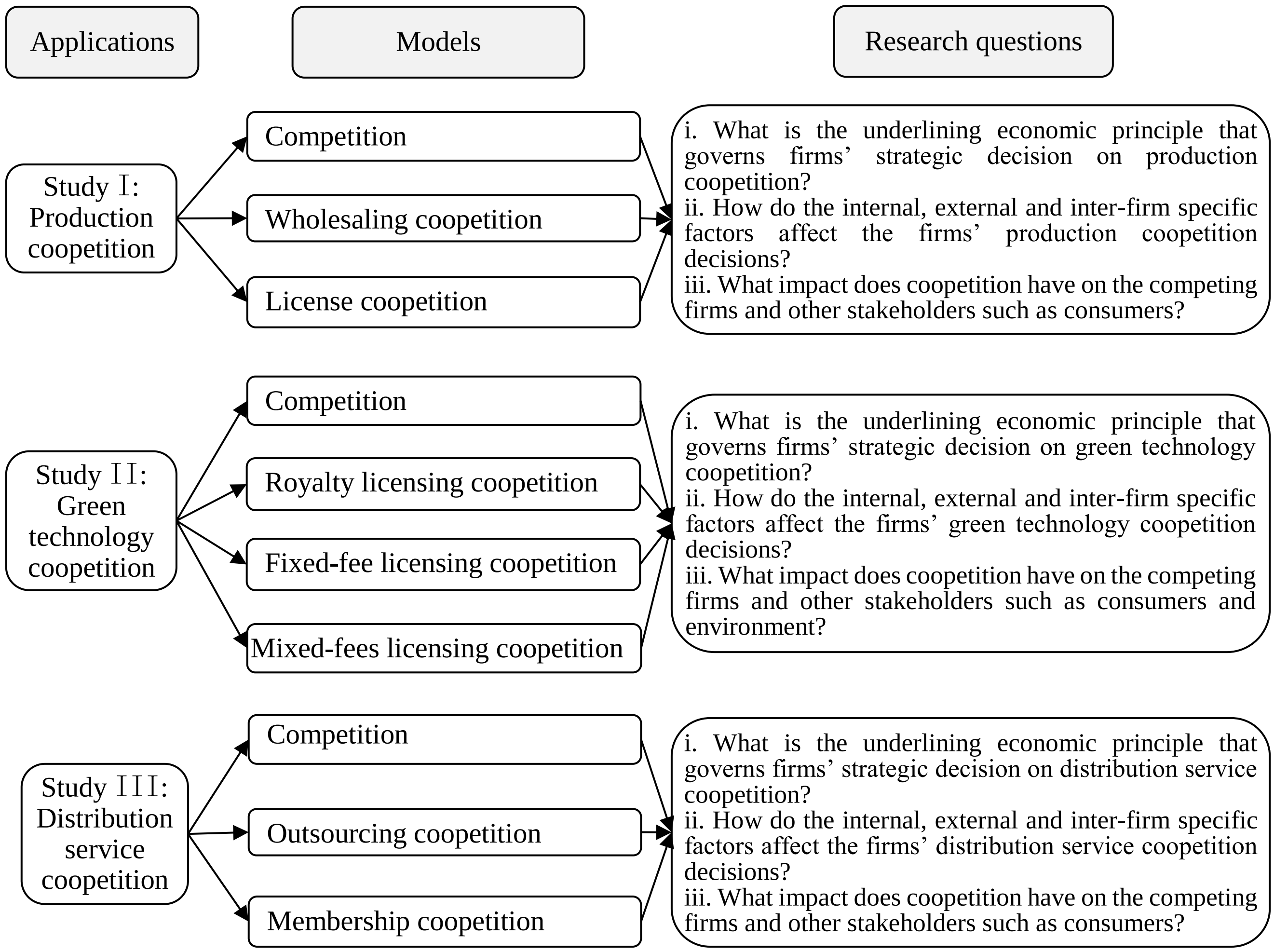


Figure 1.1 Research framework

In Study I, the production coopetition scenario is considered, where the two manufacturers collaborate on component production through either wholesaling or licensing while simultaneously competing for end-customer demand. The study investigates the strategic choice between purely competition and production coopetition (wholesaling coopetition or license coopetition) and explore how various internal and external factors influence firms' strategic decision on coopetition and the consequential effect on the manufacturers' economic performance individually and collectively. This study addresses the research questions on (i) what is the underlining economic principle that governs firms' strategic decision on production coopetition? (ii) how do the internal, external and inter-firm specific factors affect the firms' production coopetition decisions? and (iii) what impact does coopetition have on the competing firms and other stakeholders such as consumers?

In Study II, the green technology coopetition scenario is considered, where the two manufacturers collaborate on green technology investment through royalty licensing (royalty licensing coopetition), or fixed-fee licensing (fixed-fee licensing coopetition), or mixed-fees licensing (mixed-fees licensing coopetition) while still simultaneously engaging market completion for end-customer demand. The study investigates the strategic choice between competition and green technology coopetition (royalty licensing coopetition, or fixed-fee licensing coopetition, or mixed-fees licensing coopetition), and explore how various internal and external factors affect firms' strategic decision on coopetition and the economic and environmental performances. This study addresses the research questions on (i) what is the underlining economic principle that governs firms' strategic decision on green technology coopetition? (ii) how do the internal, external and inter-firm specific factors affect the firms' green technology coopetition decisions? and (iii) what impact does coopetition have on the competing firms and other stakeholders such as consumers and environment?

In Study III, the distribution service coopetition scenario is considered, in which the online retailer and e-marketplace firm collaborating on distribution service through outsourcing (outsourcing coopetition) or membership (membership coopetition) despite being market rival competing for end-customer demand. The study investigates the strategic choice between competition and distribution service coopetition (outsourcing coopetition, or membership coopetition) and explore how various internal and external factors influence firms' strategic decision on coopetition and the consequential effects on the firms' economic performance and consumer welfare. This study addresses the research questions on (i) what is the underlining economic principle that governs firms' strategic decision on distribution service coopetition? (ii) how do the internal, external and inter-firm specific factors affect the firms' distribution service coopetition decisions? and (iii) what impact does coopetition have on the competing firms and other stakeholders such as consumers?

1.3.2 Research Strategy

Analytical modelling is adopted as the main research strategy for the development of coopetition models. Analytical models are mathematical models that can be solved using classical techniques ranging from algebraic manipulation to calculus methods (Oakshott 1997). In developing analytical models, mathematical concepts such as functions, matrices, and equations are used to describe the most important characteristics of the entity that is being modelled (Carter and Prince 2000). When models are created, a real-world problem is transformed from its initial context into a mathematical concept. The mathematical problem is then solved using mathematical or statistical techniques such as calculus and numerical solution techniques. Analytical models serve as a powerful tool for the study of interrelationships among the important variables by setting aside unimportant variables. In doing so, some assumptions have to be made about the real-world system. Although analytical models can be improved by making fewer assumptions, it increases to the complexity of the model and the difficulty of solving the model (Oakshott 1997).

In this dissertation, each study is carried out following four stages, those are, characterizing, modelling, solving and application. In the characterizing stage, the research settings are specified that reflect the key characteristics of the external and internal environment. For instance, the demand functions are adopted that characterize the nature of competition (e.g. single element Cournot competition or dual elements price and service competition) and use the level of product substitution to measure the intensity of competition between the competing firms. In the modelling stage, non-cooperative game theory is adopted to model the competition scenarios and the cooperative game theory is applied to model the coopetition scenarios. Cooperative games are games with competition between firms due to external enforcement of cooperative behavior through contractual arrangements (e.g. supplier-buyer agreement, technology licensing agreement, or delivery service agreement). According to Brandenburger and Stuart (2007), cooperative game theory offers a broader prediction about possible outcomes on the basis of some fundamental characteristics of the game, in contrast, non-cooperative game theory typically presents an exact outcome depending on the game specified rules. Bargaining games are also employed to characterize the inter-firm power relationship between the rival firms, which is a new tendency adopted to examine the effect of different power relationship on firms' strategic and operational decisions and the consequential performances (Nagarajan and Sošic 2009; Feng and Lu 2013; Shi et al. 2013). In the solving stage, optimisation theory is utilized to derive the firms' optimal solutions and the resulted optimal financial performances (e.g. profit) and/or environmental performance (e.g. carbon emissions). At last, in the application stage, numerical analysis and industry examples are used to analyse the effects of various internal, inter-firm, and external factors on the success of coopetition strategies and discuss the managerial insights of the analysis results. More detail about the research method will be further discussed in the chapters corresponding to each of three studies.

1.4 Dissertation organization

The rest of this doctoral dissertation is organized as follows. Chapter 2 reviews the relevant literature on coopetition. It starts with an induction of the concept of coopetition and followed with a discussion of rationales for rival firms to engage in cooperation. After a brief review of coopetition in industrial organization literature, the chapter also discusses some important issues of coopetition including the dynamics of coopetition and the intensity of competition and cooperation. Overall, the literature review presented in the chapter provides a theoretical background of coopetition and more detailed literature that relevant to individual coopetition applications will be presented in the later chapters.

Chapter 3 presents the first study that focuses on production coopetition between rival manufacturers that produce substitutable products. In this study, there is a complex relationship between these two rival firms. One of the studied firm has an option to purchase a key component or technology licensing of manufacturing this key component from the other firm. At the same time, the other firm have an option of selling this key component or licensing the technology to its market rival. Both firms also have option of not pursuing the collaboration opportunities. Based on this complex relationship, two coopetition models are developed and then compared to a benchmark competition model. Through a comparison of the equilibria of these competition and coopetition models, the research findings indicate that the underlying economic principle that determines firms' optimal choice on the coopetition.

Chapter 4 presents the second study that focuses on low carbon technology licensing coopetition between rival firms under cap-and-trade policies. We investigate the effects of low carbon technology licensing on the economic and environmental performance of two rival manufacturers under a cap-and-trade policy. We model alternative contractual arrangement of technology licensing through either royalty, fixed fee or mixed fee and evaluate the performances of four model settings (i.e., pure competition, royalty licensing, fixed-fee licensing and mixed licensing) from the perspectives of different stakeholders including manufacturers, customers and policy makers. The research findings show that the contractual choice on low carbon technology licensing is determined by the trade-off between the benefits gained from technology licensing and the consequential losses incurred from competition with a strengthened competitor, which is influenced by a combination of factors including internal operational and technological capability, interfirm power relationship, external market characteristics and the carbon emission control policy. Among them, the interfirm power relationship is more influential in determining the optimal contractual decision. Finally, the analysis show that it is critical for governments to develop appropriate carbon emissions control policies to promote the agenda of a sustainable, low-carbon economy.

Chapter 5 presents the third study that focuses on delivery service coopetition of an e-tailer and a marketplace firm. As more retailers are selling online, e-tailers face a dilemma between investing in their own distribution/logistics operations or using the logistics service provided by marketplace firms, e.g., Amazon or JD.com. Inspired by this problem, we consider a competitive setting in which an e-tailer and a marketplace firm (e.g., Amazon or JD.com) sell partially substitutable products. The e-tailer may choose to contract with the marketplace firm to use its delivery service. For the e-tailer, service cooperation improves the service level, which results in increased customer demand but comes at some expense, such as a unit delivery rate when outsourcing its delivery service or a membership fee with a lower unit delivery rate when obtaining a membership. For the marketplace firm, providing delivery services will generate additional revenue income. However, for both firms, the delivery service cooperation will have a negative impact on their profitability when they face a strengthened competitor in the competition for customer demand. The optimal decisions for both the e-tailer and marketplace firm are analyzed, and the system equilibria is characterized. The research finding indicates that a firm’s decision regarding coopetition strategies is mainly determined by the inter-firm power relationship in the cooperation contract negotiation and the degree of product substitution. At the same time, other factors, such as customers’ willingness to pay for a delivery service and the difference in the delivery service level between the two firms, also affect the magnitude of benefit and loss from service coopetition, which has an impact on whether coopetition results in a win-win outcome for the two firms.

Chapter 6 concludes this doctoral research by summarizing the main research findings. The contribution to the knowledge of field and the contribution to the managerial practices are discussed.

Finally, the limitations of this doctoral research are critically discussed, and directions of future research are suggested by outlining how these limitations can be remedied via future research.

Chapter 2 Literature review

This chapter presents literature review on coopetition. Since more specific literature review related to individual studies will be present in the corresponding chapters, the review presented in this chapter mainly concentrates on more general issues of coopetition including the concept of coopetition, the rationale of coopetition, the coopetition in industrial organization literature, the coopetition dynamics, the coopetition intensity, and the coopetition at different level. The research gaps filled in this research are outlined at the end.[1]

2.1 The concept of coopetition

Coopetition has become an important topic in the management and economic literature in the last two decades. Different definitions have been given to coopetition by academics. Among them, Brandenburger and Nalebuff (1996) give a broad definition that regards coopetition as a value net consisting of a firm's stakeholders including suppliers, customers, competitors, and complementors. Their interdependence involves both competing and collaborating elements, with rivalry as well as collaborative mechanisms, in the course of maximizing profit for individual firms (Brandenburger and Nalebuff 1996). In contrast, a narrow definition is provided by Bengtsson and Kock (2000) that considers coopetition as a dyadic relationship concerning firms' simultaneous engagement in competition and cooperation. Over the last two decades, academics have come up with different definitions and conceptualizations of coopetition with their respective levels. They are closely related to the Actor and the Activity Schools of Thought that Bengtsson and Raza-Ullah (2016) use to brand the broad or narrow definitions respectively. The simultaneous competitive and cooperative relationships are the focus of the Activity School of Thought, whereas the underlying principle of the Actor School of Thought is "value-net", through which, actors cooperate to make a bigger pie and then compete to divide it up (Bengtsson and Raza-Ullah 2016).

Coopetition has attracted rising interests from practitioners and academics. The research on coopetition has been carried out in different management fields including innovations, strategic

[1] Part of literature review presented in the chapter has been developed into a research paper and accepted for publication as: Chen, X., Luo, Z. and Wang, X. 2018. Compete or cooperate: Intensity, dynamics, and optimal strategies. *Omega: an International Journal of Management Science*, https://doi.org/10.1016/j.omega.2018.07.002.

alliance, new product development, international business, marketing, and supply chain management as discussed in the previous chapter. With the background of the growing interest in coopetition, several pieces of comprehensive systematic reviews (Stein 2010; Bouncken et al. 2015; Bengtsson and Raza-Ullah 2016; Dorn et al. 2016) have been recently conducted in attempts for a better understanding of the coopetition phenomenon and recommendations for strengthening this research inquiry in near future. For example, Bengtsson and Raza-Ullah (2016) integrate key critical themes into a framework consisting of Driver, Process, and Outcomes with an aim of providing a richer and more comprehensive perspective of the coopetition phenomenon. In another systematic review of coopetition studies, Dorn et al. (2016) analyze and synthesize coopetition research and highlight five multilevel research areas: (1) nature of the relationship, (2) governance and management, (3) output of the relationship, (4) actor characteristics, and (5) environmental characteristics, for future research avenues. More detail about the concept of coopetition can be found in these comprehensive literature reviews (Stein 2010; Bouncken et al. 2015; Bengtsson and Raza-Ullah 2016; Dorn et al. 2016). Readers can refer to these literature reviews for further information about the concept of coopetition.

2.2 The rationale of coopetition

Why do firms want to cooperate with their market rivals? There are various reasons for firms to adopt a coopetition strategy. The most common reason is to obtain the financial benefits through increasing the total value between the alliance partners by collaborating with each other. According to Brandenbruger and Nalebuff (1996), coopetition embraces the logic that firms cooperate in order to increase the size of the business pie, and then compete with each other in dividing it up. From resource dependence theory and the resource-based view, firms may wish to improve the efficiency of the existing resource utilization in serving their current market or capturing a greater share (Ritala 2012; Dorn et al. 2016). A typical example of this motivation is the airlines: alliances are often developed between the rival airlines to share each other’s resources in order to gain efficient benefits and gain competitive advantages over airlines outside the alliance (Oum et al. 2004; Garrette et al. 2009).

The main drivers of coopetition are classified by Bengtsson and Raza-Ullah (2016) in the systematic review work into three categories: external, relation-specific, and internal drivers. The external drivers are often environmental conditions and industrial specific characteristics that incentivize firms engaging coopetition (Sahaym et al. 2007; Ritala 2012; Bengtsson and Johansson

2014). The relation-specific drivers are characteristics related to partner and relationship that facilitate coopetitive formation (Khanna et al. 1998; Luo et al. 2008; Peng and Bourne 2009; Gnyawali and Park 2011). And finally, the internal drivers are specific motives, resources and capabilities that encourage firms to be reactive or proactive at pursuing co-opetitive strategies (Luo 2007; Gnyawali and Park 2009; Ritala et al. 2014).

In the attempt to specify the conditions, under which, coopetition is likely to emerge, Dorn et al. (2016) categorize inter-form coopetition antecedents into three different aspects: (i) market conditions e.g. environmental aspects, regulatory bodies, and laws; (ii) dyadic factors e.g. power relationship between the competing entities, (iii) individual factors e.g. willingness and capabilities that are specific to the involved entities. Although a firm's internal circumstances, particularly its past participation and prior experience of coopetition is one of the most crucial factors for endorsing and forming coopetitive relationships (Gnyawali and Park 2011; Schiavone and Simoni 2011), market conditions in particular the nature of competition and the dyadic relationship between the coopetitive parties are more typical motivations for coopetition. This partially explains why coopetition is more common in sectors such as the airline industry, the high-tech industry, and the financial industry than other sectors.

2.3 Coopetition in industrial organization literature

The classical economics approaches consider competition as the driving force for commercial activity, which drives down prices for consumers and raise the level of innovations (Walley 2007). In microeconomics, industrial organization models are developed focusing on industrial structure and performance and the analysis results shows that a larger number of firms in an industry leads to a higher level of competition (Barney 1986). The industrial organization models are the dominant political ideology of the 1990s in Western Europe (Palmer 2000), which have also influenced the legislative framework that tends to favour a competitive market environment and encourage competitive activities by limiting monopolistic power (Walley 2007). Only in the mid-1990s, managers, academics, and policy makers started feeling that there is a need for a new conceptualization to overwhelm the crystallized vision that privileges competition as the overly dominating paradigm (Dagnino and Padula 2009). In addition, with the rapid development of emerging economies, firms begin to realize the potential benefit and strategic importance of cooperation due to an increasing pressure for an integration of the global value chain stemming from a necessity for improved efficiency and productivity. Alternatively, governmental authorities, in some cases, have "forced" market rivals to work together to achieve resource efficiency when it points toward an improvement of economic welfare (Mariani 2007).

Coopetition also shares some common features of the collusion, an act of working together to make decisions about price and quantity, in economics as both concepts take place within an industrial setting when rival companies cooperate for their mutual benefit. Rival firms cooperate with each other in collusion, and, for that reason, collusion satisfies a narrow definition of coopetition (Rusko 2011). Some academics even regard coopetition as "just another form of collusion" (Walley 2007, p. 15). Others disagree by highlighting that not every instance of cooperation constitutes anticompetitive collusion (Hunt 1997). Nevertheless, the two concepts can be distinguished. For instance, coopetition is a business strategy that has to take place under the legal framework for involved firms to gain competitive advantages. For example, in the air transportation industry, rival airlines often form an alliance to improve resource efficiency and increase their competitiveness in relation to other airlines or alliances (Oum et al. 2004; Garrette et al. 2009). Furthermore, in the high-tech industry, the pressures of rapid technological advancement, fierce market competition, short product lifecycle, and high R&D expenditure drive many technology companies to cooperate with their fiercest competitors on upstream value chain activities including R&D and resources sharing on production (Cassiman et al. 2009; Gnyawali and Park 2011). In contrast, collusion often exists within the market structure of oligopoly, in which the decision to collude by a few firms can make a significant impact on the market as a whole (Eckbo 1983, Green and Porter 1984; Bresnahan 1987). Moreover, collusion sometimes violates the legal framework that governs competition because they are situated in downstream value chain activities like pricing. The benefit of collusion goes to firms by a way of increasing firms' surplus through rises in price and power of monopoly, and consumers are often penalized by reducing consumer surplus, which leads to a decrease in total surplus or social welfare (Rusko 2011). The coopetitive relationship has the potential for collusion, but whether the actual collusion exists must be determined by referencing to its impact on consumers' welfare (Walley 2007).

2.4 Coopetition dynamics

One unique feature of coopetition is the relationship that contains of both competition and cooperation elements simultaneously (Brandenburger and Nalebuff 1996; Bengtsson and Kock 2000). This simultaneous pursuit of competition and cooperation can lead to conflicts between counterparts and activities due to the rising internal disagreement (Bengtsson and Kock 2000). The cooperation encourages collective interests, common benefits, and goodwill behavior, whereas the competition highlights zero-sum game, individual benefits, and opportunistic behaviour (Khanna et al. 1998; Das and Teng 2000). As Raza-Ullah et al. (2014) suggested, competition and cooperation are paradoxical forces resulting in ambivalent emotions within organizations. The conflicting logics of competition and cooperation bring tensions (Das and Teng 2000; Bello et al. 2010; Dorn et al. 2016). Consequently, the involving actors may experience the tensions stemming from coopetition and the associated ambivalent emotions, and eventually put this coopetitive relationship in jeopardize (Gynawali and Park 2011).

Many studies argue that an optimal combination of competitive and cooperative forces requires a balanced relationship (Bengtsson and Kock 2000; Das and Teng 2000; Quintana-García and Benavides-Velasco 2004; Chen 2008; Cassiman et al. 2009; Peng and Bourne 2009; Dorn et al. 2016). For instance, Das and Teng (2000) suggest that the balance between competition and coopetition is instrumental to the stability of a strategic alliance. Luo (2004) points out that coopetition and the paradox-solving Yin-Yang philosophy are closely related and the author also claims that the Yin-Yang philosophy naturally fosters coopetition. Similarly, Chen (2008) re-conceptualizes the coopetitve relationship through the Chinese “middle way” philosophy and an integration of the paradox perspective. The author argues that the competing and cooperating opposite forces may be interdependent in nature and the combination of the two forces forms a totality. Peng and Bourne (2009) claim that the complimentary sets of resources are more likely to balance competition and cooperation than the distinctly different sets of sources between the two firms, and at the network level, it is easier to achieve such a balance if there are compatible but different network structures. Park et al. (2014) come up with the concept of “balance” in coopetition. In the context of business innovation in the semiconductor industry, they investigate the impact of competition and cooperation balance on firms’ performance and find that an optimal coopetition balance has a positive effect on innovation performance. Although the existing coopetition literature encourages research exploring a balance of competitive and cooperative forces, the main challenge is to find out what the optimal balance is and how such a balance can be achieved (Dorn et al. 2016).

Moreover, the coopetitive relationship between firms is dynamic and the balance of competition and cooperation may change over time, which add complexity to this already challenging problem (Peng et al. 2012; Dahl 2014; Park et al. 2014; Dorn et al. 2016). Dahl (2014) illustrates that the interplay of competitive and cooperative elements of this coopetitive relationship is the root cause of coopetition dynamics. Under this context, it is not surprising that coopetition is regarded by many scholars to have the potential impacting on an industry's competitive dynamics (Gnyawali and Madhavan 2001; Bengtsson et al. 2010; Ritala 2012). For instance, one firm's market power might be strengthened relatively through cooperation, and as a consequence, it increases the intensity of market competition (Peng et al. 2012). Furthermore, firms' behavior could change from cooperative to somewhat competitive in a multilateral alliance while other parties reduce their input resources towards the relationship (Ritala and Tidström 2014). It would be noticeably more challenging to sustain the dynamic balance if it also requires external factors and motives to establish such a balance.

2.5 Coopetition intensity

As stated in earlier discussion, the market competition is often one of the main drivers for firms' strategic decision of engaging coopetition. In fact, the intensity of market competition within the sector also has significant influence on the benefits that firms can gain from a coopetition strategy. Ritala (2012) finds from an empirical study of coopetition strategy and its impact on firms' performance in Finland that market uncertainty, network externalities and competitive intensity all have an impact on the success of coopetition strategy to a certain extent. Interesting, coopetition can be an effective strategy in either a highly competitive market environment that involves numerous rival firms offering substitutive products (Dussauge et al. 2000), or in a less competitive environment that only involves a limited number of competitors offering similar products (Peng and Bourne 2009). Oxley et al. (2009) argue that an alliance with competitors help the involved businesses become more profitable by softening the competition intensity of the market, and at the same time, such an alliance contributes to business performance improvement due to the enhanced competitiveness among the partnering firms in the competition with other firms. The arguments of Oxley et al. (2009) partially explain why firms can benefit from coopetition no matter a high or low competition intensity.

Despite the importance of coopetition intensity to the firms' strategic decision on coopetition and the success of the coopetition strategy, there is also a concern of measuring competition and cooperation intensity from the methodological perspective. For example, Luo et al. (2016) acknowledge that without incorporating the intensity of coopetition is one of the research limitations in their investigation of the role of coopetition in achieving low carbon manufacturing. They call for an incorporation of the intensity of coopetition in the modelling in the examination of the impacts of coopetition strategy on firms' decisions and performances. Bengtsson and Raza-Ullah (2016) also call for the development of new measurement scales for coopetition that measure the intensities of competition and cooperation and the similarity levels of both when highlighting the directions of future research on coopetition. Our research is going to fulfil this research gab by systematically looking at how competition intensity affects firms' strategic decision on coopetition, and reciprocally, how coopetition has an impact on the nature of the market competition through different coopetition models and applications.

2.6 Coopetition at different levels

Coopetition has been studied at different levels of unit analysis including cross-functional units at the intra-firm level (e.g. Tsai 2002; Luo et al. 2006), value chain horizontal rival firms (e.g. Luo et al. 2007; Garrette et al. 2009; Kumar 2010; Luo et al. 2016) or partners within a supply chain (e.g. Bakshi and Kleindorfer 2009; Wilhelm 2011; Lacoste 2012) at the inter-firm level, and intra-network (e.g. Gnyawali et al. 2006; Schiavone and Simoni 2011) or inter-network(e.g. Peng and Bourne 2009; Schiavone and Simoni 2011) at the network level. For instance, Tsai (2002) draws on a social network perspective and studies the effectiveness of coordination mechanisms on knowledge sharing in the context of intra-organisational units embedded with competing and collaborating ties among these units. Also, at the intra-organisational level, Luo et al. (2006) investigate how cross-functional coopetition affects performance outcomes through enhanced market learning.

Nevertheless, most studies tend to consider the interaction between competition and cooperation in inter-firm networks as built around dyads (Chen and Miller 2015). There is also an emphasis on individual actions and agency that has been a defining feature of scholarship in this area and has provided the core basis for inter-organizational investigation in the field (Das and Teng 1998). Among these studies, the dyadic comparison of firms' positions and resources has been central to analysis, which in turn has become the mainstay of exploring competition and cooperation dynamics (Chen 2008). In contrast, other studies have shown that the relationship between competition and cooperation is more significant when the level of analysis is expanded beyond the dyad to the network (Madhavan et al. 2004). Here, a different understanding needs to be considered. Firms can be interconnected with other firms through a wide array of social and economic relationships including cooperative and competitive ones, each of which can constitute a network, for example producer–supplier relationships (Podolny and Page 1998; Gulati and Gargiulo 1999; Tsai 2000). However, the network perspective emphasizes how structure determines economic and strategic action (Uzzi 1996; Granovetter 2005; Gulati 1995). Thus, it is claimed that any dyad is embedded in many possible extra-dyadic relations, and the structure of these relations influences the dyadic relationships (Krackhardt and Kilduff 2002).

This focus on the structural nature of network relations to explore competition and cooperation in inter-firm networks is not new, and a number of interesting studies have recently surfaced in the literature (Tsai 2002; Madhavan et al. 2004). The perspectives they offer are complex, and many of the studies persistently conclude that the dynamics of the interactions between competitive and cooperative networks is not well understood (Gimeno 2004). Thus, despite extensive research into these processes so crucial to inter-firm competitive and cooperative relations, key questions remain about the dynamics of multifaceted inter-firm relationships (Shipilov and Li 2008).

2.7 Summary

Despite the increasing importance of coopetition for today's interfirm dynamics, many scholars argue that coopetition is an important theme that is under researched and demands escalating attention (Brandenburger and Nalebuff 1996; Dagnino 2009). Furthermore, although the notion of coopetition as an important topic has gained an increasing interest in the management literature, the majority of coopetition studies applies conceptual or qualitative approaches demonstrating coopetition research still in its infancy (Bouncken et al. 2015; Bengtsson and Raza-Ullah 2016; Dorn et al. 2016). There is great potential for theory building on competition and cooperation through research focusing on game theory with its focus on extra-dyadic relations, little research has examined how they jointly influence and constrain organizational and strategic actions (Uzzi 1996), giving rise to a gap in the literature. Besides predominately being qualitative from the methodological perspective, the existing coopetition studies have also been limited in terms of research contexts, which raises question mark from a validity and generalizability point of view (Bouncken et al. 2015; Bengtsson and Raza-Ullah 2016; Dorn et al. 2016). Addressing these research gaps is important because the application of game theory to examining the interaction between competition and cooperation in inter-firm networks through applications in the context of various industry sectors may provide novel predictions that have not been observed from existing theoretical and methodological perspectives.

Chapter 3 Production coopetition models and applications

An earlier version of this chapter has been accepted for publication as below. The authors' contribution statement has been provided at the start of this dissertation.

"Chen, X., Wang, X. and Xia, Y. 2018. Production coopetition strategies for competing manufacturers that produce partially substitutable products. Productions & Operations Management, forthcoming, DOI: 10.1111/poms.12998".

3.1 Introduction

In numerous industries, firms purchase components or raw materials from upstream suppliers while competing with these same suppliers in the downstream market. For example, in the smartphone market, Google supplies the Android system to other smartphone vendors such as Samsung and Huawei. In addition, Google launched Pixel to compete in the smartphone market (Gibbs 2016). Furthermore, despite being sworn rivals in the hybrid and electric vehicle (HEV) market, Ford has offered to license its electric vehicle technology to other automakers (Atiyeh 2015). In the pharmaceutical sector, Dr. Reddy's Laboratories, an Indian multinational pharmaceutical company, licensed and supplied its products to GlaxoSmithKline in various emerging markets to expand their market (Pitelis et al. 2015). This shift in the competitive paradigm has not exclusively occurred in the smartphone, automobile, and pharmaceutical industries. These types of relationships have become common in high-tech industries such as PC, TV, and medical devices, which are characterized by short product life cycles, rapid technical advancement, high research and development (R&D) expenses, and fierce competition. These pressures often drive numerous firms to collaborate with their fiercest competitors on upstream activities such as R&D and production resources (Cassiman et al. 2009; Gnyawali and Park 2011; Mantovani and Ruiz-Aliseda 2016).

With the rapid technological advancement and the development of emerging economies, firms have realized the importance of cooperation because of increasing pressure to integrate the global value chain that stems from a need for improved efficiency and productivity. In certain cases, legislative bodies have "forced" competitors to collaborate to achieve an efficient use of resources when doing so leads to improved economic welfare (Mariani 2007). Thus, the notion of competition has evolved to coopetition, which is a concept that refers to interdependence in which competition and cooperation simultaneously occur between two or more firms; however, each firm focuses on increasing the size of the total pie for division (Brandenburger and Nalebuff 1996; Mantovani and Ruiz-Aliseda 2016).

A natural question that arises in these settings is whether the cooperation between rival firms is desirable from the perspectives of firms and consumers. Intuitively, the supplier benefits from a new revenue stream and the buyer will take advantage of reduced component costs and concentrate on its core operations. However, decisions concerning such a strategic engagement are more involved when it is embedded within a competitive relationship between market rivals. Supply chain cooperation enhances each firm’s competitiveness through increased efficiency or an additional revenue stream; however, this strategy could have a negative implication when each firm competes for customer demand. In this case, cooperation might have adverse effects on the firms. To help firms make the optimal strategic decision, it is essential to have a comprehensive understanding of the fundamental economics that govern coopetitive relationships between competing firms. Should firms purchase from or supply key components to their primary market rivals? What is the optimal unit component price when supplying to or purchasing from a firm’s rival? Should firms license their key technologies to fierce market rivals? How do these coopetitive relationships affect the firms and consumers?

To investigate these issues, we consider a complex relationship between two manufacturing firms who produce partially substitutable products and compete for end-customer demand. The substitutable products (e.g., smartphones or tablet computers) require a key component (e.g., chips or displays) that can be manufactured by either of the two firms with different manufacturing costs. The manufacturers can produce the component in house, or they can purchase it from a market rival at a lower cost. Alternatively, a manufacturer can pay a fixed license fee plus a royalty based on a rate to its market rival to adopt the rival’s technology for manufacturing the component at a lower cost. The scenario in which both manufacturers make the component in-house is referred to as the competition model, and the cases in which one manufacturer opts to procure the component from or pay licensing fees to the rival manufacturer are referred to as the coopetition models. We seek to understand the dynamic relationship between the embedded competition and cooperation elements and how the strategic movement of coopetition affects individual firms’ operational decision and financial performance by analyzing the equilibriums of the competition and coopetition models and examining manufacturers’ pricing strategies and consequences of total sales and profitability.

Through a comparison of the equilibria of two coopetition models and the benchmark competition model, the research finds that the optimal coopetition strategy is determined by a combination of internal, inter-firm, and external factors including the degree of product substitution, the inter-firm power relationship in the negotiation of a cooperation contract (i.e., wholesale price and license fees) and the difference in production efficiency between the two manufacturers. Fundamentally, it is the trade-off between the benefit (gain from the production cooperation) and the losses (incur from market competition with a cooperation strengthened competitor) that determines firms' strategic decision on coopetition (e.g., competition vs. coopetition or wholesaling vs. licensing). The extent of benefit and loss depends on a combination of important external and internal factors including the degree to which their products are substitutable, power relationship in the contract negotiation, maximum retail prices and cost difference in component production.

This study makes several contributions. First, our research contributes to the coopetition literature by investigating production coopetition between two rival firms and filling a significant gap in the literature. This problem differs from conventional supply chain cooperation and/or outsourcing problems in which the cooperation and competition elements primarily concentrate on a vertical supplier-buyer relationship in the supply chain. In contrast, our study explores how interaction of horizontal market competition and vertical supply chain cooperation affects firms' performance individually and collectively. This exploration enables us to derive the structured optimal solutions for the firms and enhances our understanding of the nature of coopetitive behavior by analyzing the dynamic relationship between the competing and cooperating forces. This study contributes to the continuing debates concerning the efficacy of coopetition (Brandenburger and Nalebuff 1996; Gnyawali and Madhavan 2001) and the role of an agentic or structural perspective in understanding the dynamics of simultaneous competition and cooperation for an inter-firm relationship (Das and Teng 2000; Peng and Bourne 2009; Dorn et al. 2016). Second, our analysis provides notable results that are new. For example, the optimal strategy for coopetition is determined by not only the intensity of market competition (Tsay and Agrawal 2000; Peng and Bourne 2009; Ritala 2012) but also the joint effect of external market characteristics, the power relationship between manufacturers in the negotiation of the cooperation contract (i.e., wholesale price and license fees) and the difference in production efficiency between them. By examining the coopetition effect on firms' retail prices and individual and collective profits, we identify the decision region for stable and unstable coopetition.

The remainder of this chapter is organized as follows: after reviewing relevant studies in Section 3.2, the Cournot competition, wholesaling coopetition (WC), and license coopetition (LC) models are presented in Section 3.3. Section 3.4 examines the effect of coopetition on the retail prices and maximum profits of two manufacturing firms by comparing the equilibrium results of the three different models. Section 3.5 discusses the selection of a coopetition strategy. Section 3.6 extends the analysis to the asymmetric-manufacturer case and the case of both partial and perfect substitutes, and examines their effect on the selection decision. Section 3.7 discusses the managerial relevance and insights of our research findings. Finally, we draw conclusions and provide suggestions for future studies in Section 3.8.

3.2 Literature review

This study is related to several streams of research: competition, cooperation and coopetition. Numerous prior studies have been conducted concerning competition, cooperation and coopetition, which is defined as a dyadic relationship involving firms' simultaneous engagement in competition and cooperation (Brandenburger and Nalebuff 1996; Bengtsson and Kock 2000). Studies on coopetition in the existing literature have been applied in various management applications as shown in the previous chapter, few studies have been conducted regarding production coopetition.

Why do firms cooperate with their rivals? Using game theory, Brandenbruger and Nalebuff (1996) explain that coopetition embraces the logic that firms cooperate to increase their size of the business pie and then compete with each other to divide it. From resource dependence theory and the resource-based view, firms can seek to improve the efficiency of their use of existing resources when serving their current market share or capturing a larger share (Ritala 2012; Dorn et al. 2016). Typical examples of this motive include the airline industry, in which alliances are often developed between rival airlines to gain efficiency benefits by sharing resources (Oum et al. 2004; Garrette et al. 2009). Bengtsson and Raza-Ullah (2016) classify the drivers of coopetition into three categories: external, relationship-specific, and internal. External drivers include environmental conditions and industrial characteristics that force firms to engage in coopetition (Ritala 2012; Bengtsson and Johansson 2014). Relationship-specific drivers include partner and relationship characteristics that facilitate coopetition (Peng and Bourne 2009; Gnyawali and Park 2011). Internal drivers include specific motives, resources and capabilities that motivate firms to be proactive or reactive in pursuing coopetitive strategies (Gnyawali and Park 2009; Ritala et al. 2014).

Since the seminal study conducted by D'Jacquemin and Aspremont (1988) concerning cooperative and non-cooperative R&D, studies concerning coopetition have explored various management fields including innovation (Gnyawali and Park 2011; Ritala and Hurmelinna-Laukkanen 2013), strategic alliances (Dussauge et al. 2000; Rai 2016), international business (Kim and Parkhe 2009), marketing (Luo et al. 2006; Bello et al. 2010) and supply chain management (Bakshi and Kleindorfer 2009; Wilhelm 2011). In addition, prior studies incorporated various levels of analysis including cross-functional units at the intra-firm level (Tsai 2002; Luo et al. 2006) or rival firms at the inter-firm level (Garrette et al. 2009; Luo et al. 2016). This concept has been extended to networks at the intra-network (Gnyawali et al. 2006; Schiavone and Simoni 2011) and inter-network levels (Peng and Bourne 2009; Schiavone and Simoni 2011). Interestingly, few studies have analyzed coopetition at the production stage of the value chain.

Most of these studies adopt conceptual or empirical approaches such as case studies or surveys. Despite the call for game theory approaches by Brandenburger and Nalebuff (1996) in their study concerning coopetition, very few studies (Bakshi and Kleindorfer 2009; Carfì and Schiliro 2012; Luo et al. 2016) have applied game theory to coopetitive decision problems. Bakshi and Kleindorfer (2009) analyze the choice of risk mitigation strategies by supply chain participants using the Harsanyi-Selten-Nash bargaining framework and determine that coopetition is superior to competition in the context of managing supply chain security. At the macroeconomic level, Carfì and Schiliro (2012) apply the complex construct of coopetition to address climate change challenges and demonstrate that a coopetitive strategy can deliver win-win solutions for participating countries that seek to implement green economies. At the microeconomic level, Luo et al. (2016) employ a game theory model to examine the role of coopetition in low-carbon manufacturing and determine that coopetition is a viable strategy that can increase profits and reduce the firms' total carbon emissions. Mantovani and Ruiz-Aliseda (2016) develop a game theory model in which firms cooperate to enhance the quality of innovation ecosystems. They examine the advantages and disadvantages of coopetition strategies for participating firms and society. In contrast to these studies, we examine coopetition for production, which is an upstream supply chain activity, in the context of two manufacturers who produce substitutive products and can simultaneously engage in supplier-buyer cooperation and a licensing arrangement for one key component of their finished products.

More relevant to the setting of this work, Venkatesh et al. (2006) examine the optimal choice among three distribution strategies: sole entrant, co-optor, or component supplier for proprietary component manufacturers (PCMs). The authors show that although each strategy has its unique domain of optimality, the co-optor strategy, in which a PCM opts to sell to customers directly and to sell supplies to its competitor, is the most widely optimal for PCMs. Xu et al. (2010) extend the work of Venkatesh et al. (2006) by examining the effect of horizontal differentiation and capability advantage on the optimal choice of distribution strategy. The above two studies only adopt a PCM's perspective on whether to supply a proprietary component to be assembled in the competitor's end product. From the perspective of original equipment manufacturers (OEMs), Pun (2015) examines outsourcing decisions of two competing OEMs in which firms can outsource either to each other or to third-party suppliers and finds that more cooperation between competitors can be harmful. Using a similar setting, Pun and Ghamat (2016) examine how competition affects component commonality and R&D joint-venture decisions when outsourcing to competitors. Different from the above research, we use the concept of coopetition to examine how cooperation decisions between competitors affect firms and consumers. In addition, in contrast to the works of Venkatesh et al. (2006) and Xu et al. (2010), who model competition between PCM and OEM based on location, and the works of Pun (2015) and Pun and Ghamat (2016), who model competition based on price, we model the end market competition as quantity based.

The studies closest to me are those of Wang et al. (2013) and Yang et al. (2017). Wang et al. (2013) adopt the Cournot competition model and use a similar setting. Different from their focus on a production outsourcing relationship between an OEM and a contract manufacturer, our research concentrates on the evaluation of the buyer-supplier coopetition strategy along with purely competition and licensing agreement strategies. Yang et al. (2017) also employ the concept of coopetition and the Cournot competition model to analyze the optimal distribution strategies for a supplier with limited supply capacity when selling to a competing buyer. Different from Yang et al. (2017) that consider an established supplier-buyer relationship and examine how the competition brought by supplier's direct-selling channel affects their relationship and performances, we consider the case of an established market rivalry between two manufacturers and examine how cooperation in the form of wholesaling or licensing agreement affects market competition and consequential firm decisions and performance. In addition, different from both Wang et al. (2013) and Yang et al. (2017), who assume end-market demand to be symmetric, we consider both symmetric and asymmetric cases in our analysis.

3.3 The models and equilibrium analysis

3.3.1 The model

We consider two competitive manufacturers who produce partially substitutable products and compete in the market. When making the products, the manufacturers incur two types of costs: a component cost and a product production cost. Prior to presenting the models, we introduce the notations in Table 3.1 as follows.

Table 3.1 Notations

c_1, c_2	Unit component cost for manufacturers 1 and 2
Δc	Difference in the unit component cost between manufacturers; $\Delta c = c_2 - c_1$, where $c_2 > c_1$
m	Manufacturer's unit production cost
q_1, q_2	Demand for manufacturers 1 and 2
r	Manufacturer 1's royalty fee for the component, where $0 < r < \Delta c$
M	Manufacturer 1's fixed license fee for the component, $M > 0$
p_1, p_2	Unit retail price for manufacturers 1 and 2
w	Manufacturer 1's unit component wholesale price, where $c_1 < w < c_2$
$\delta_1,\ \delta_2$	Maximum unit profit for manufacturers 1 and 2; $\delta_1 = \alpha - m - c_1 > 0$, $\delta_2 = \alpha - m - c_2 > 0$
$\pi_1^n(q_1)$, $\pi_2^n(q_2)$	Profit for manufacturers 1 and 2 using the competition model
$\pi_1^c(q_1, w)$, $\pi_2^c(q_2)$	Profit for manufacturers 1 and 2 using the wholesaling coopetition model
$\pi_1^l(q_1),\ \pi_2^l(q_2)$	Profit for manufacturers 1 and 2 using the license coopetition model
π^n	Manufacturers' total profit using the competition model; $\pi^n = \pi_1^n(q_1) + \pi_2^n(q_2)$
π^c	Manufacturers' total profit using the wholesaling coopetition model; $\pi^c = \pi_1^c(q_1) + \pi_2^c(q_2)$
π^l	Manufacturers' total profit using the license coopetition model; $\pi^l = \pi_1^l(q_1) + \pi_2^l(q_2)$
θ	Manufacturer 1's negotiation/bargaining power, $0 \le \theta \le 1$

In alignment with prior studies (e.g., Wang et al. 2013; Shang et al. 2016; Yang et al. 2017), we use the following demand function:

$$p_i = \alpha - q_i - \beta(q_i + q_j),\ \ i, j = 1,2 \ \text{ and } \ i \neq j.$$

This type of linear inverse demand function is commonly used in the economics, marketing, and operations fields to investigate product competition (Farahat and Perakis 2011; Wang et al. 2013; Yang et al. 2017). Each manufacturer's retail price decreases its production quantity and the competitor's production quantity. For this study, α represents the manufacturer's maximum retail price. β ($\beta \geq 0$) is a parameter that is interpreted as the degree of product substitution of manufacturer j's product over that of manufacturer i. It measures the cross-effect of the change in manufacturer i's product demand caused by a change in that of manufacturer j. A low value of β indicates a low degree of product substitution. If $\beta = 0$, it corresponds to the case of independent products and products are not substitutable. In contrast, a high value of β corresponds to the case of high degree of substitution. A high degree of product substitution often leads to more intense market competition (Wang et al. 2013; Qing et al. 2017).

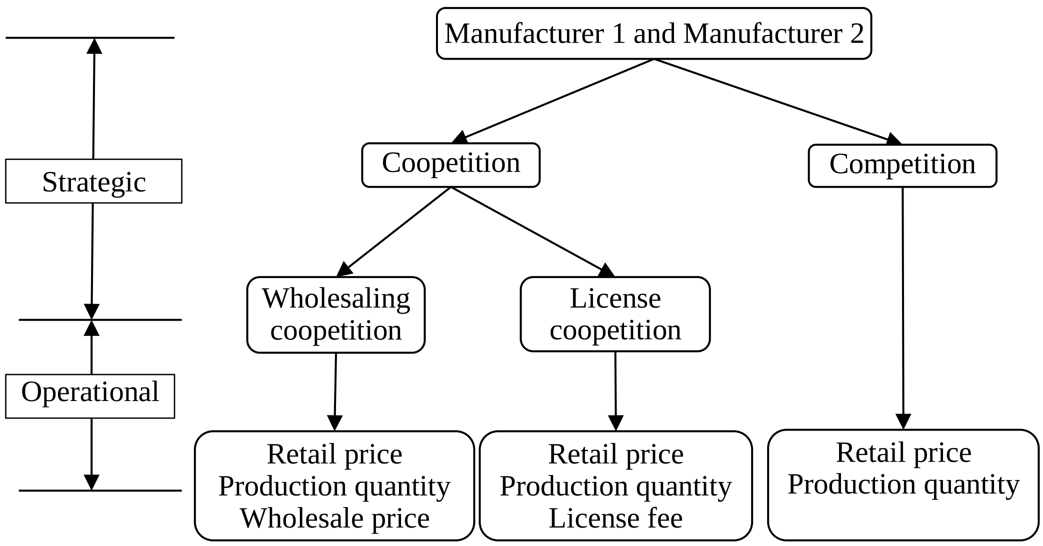


Figure 3.1 The framework

We consider three models for the relationship between the two manufacturers (as illustrated in Figure 3.1): Cournot competition, wholesaling coopetition (WC), and license coopetition (LC). For the Cournot competition model, manufacturers produce their own component, and the two firms have a competitive relationship in which they compete in quantities by simultaneously choosing production quantities. Both firms are economically rational and act strategically to maximize profits based on their competitors' decisions. For the WC model, manufacturer 2 purchases components from manufacturer 1; the two manufacturers compete in the downstream retail market but have a supplier-buyer cooperative relationship in the upstream component production. For the LC model, manufacturer 2 obtains a license from manufacturer 1 by paying a fixed licensing fee and royalty rate; the two manufacturers compete in the downstream retail market and have a cooperative relationship in the form of a license agreement for producing upstream components.

3.3.2 Competition model

First, we explore the Cournot competition model as a benchmark so that we can compare the equilibria of the WC and LC models with the equilibria of the benchmark model to examine the effect of coopetition on manufacturers' performance. In the competition model, the two manufacturers independently and simultaneously determine their production quantities to maximize their profits, and manufacturer 1's profit $\pi_1^n(q_1)$ is calculated as follows:

$$\pi_1^n(q_1) = [\alpha - q_1 - \beta(q_1 + q_2) - m - c_1]q_1. \tag{3-1}$$

The first part of this formula represents manufacturer 1's marginal unit profit, and the second part represents manufacturer 1's market demand.

Similarly, for the competition model, manufacturer 2's profit $\pi_2^n(q_2)$ is calculated as follows:

$$\pi_2^n(q_2) = [\alpha - q_2 - \beta(q_1 + q_2) - m - c_2]q_2. \tag{3-2}$$

Table 3.2 lists the optimal production quantities (q_1^n, q_2^n) for the two manufacturers based on equations (1) and (2). The derivation of these optimal solutions is provided in the Appendix.

By examining Table 3.2, we can derive the effect of the market competition on the manufacturers' optimal retail prices and maximum profits. Here, we mainly focus on the effect of the degree of product substitution, β, a parameter that is associated to market competition (Wang et al. 2013; Qing et al. 2017).

Lemma 3.1: *(1)* p_2^n, $\pi_1^n(q_1^n)$ *and* $\pi_2^n(q_2^n)$ *decrease in* β*; (2) if* $0 < \Delta c \le \frac{\delta_1}{5}$ *or* $\Delta c > \frac{\delta_1}{5}$ *and* $0 < \beta < \beta^N$*, then* p_1^n *decreases in* β*; if* $\Delta c > \frac{\delta_1}{5}$ *and* $\beta > \beta^N$*, then* p_1^n *increases in* β*.*[2]

[2] The form of β^N is listed in the proof of lemma 3.1 in the Appendix A. Its value depends upon the maximum unit profit for manufacturer 1 and 2 (δ_1, δ_2) and the difference between the unit component cost for the two manufacturers (Δc).

Table 3.2 Optimal solutions for the three models

Models	Competition model ($i=n$)	WC model ($i=c$)	LC model ($i=l$)
q_1^i	$\frac{(2+\beta)\delta_1+\beta\Delta c}{(2+\beta)(2+3\beta)}$	$\frac{\delta_1(8+3\beta^2+\beta(14+\theta)-\beta T_a)}{2(1+\beta)(8+16\beta+3\beta^2)}$	$\frac{(4+6\beta+\beta^2)\delta_1}{2(1+\beta)(4+8\beta+\beta^2)}$
p_1^i	$m+c_1+(1+\beta)q_1^n$	$m+c_1+(1+\beta)q_1^c$	$m+c_1+(1+\beta)q_1^l$
q_2^i	$\frac{(2+\beta)\delta_2-\beta\Delta c}{(2+\beta)(2+3\beta)}$	$\frac{\delta_1(2-\theta+T_a)}{8+16\beta+3\beta^2}$	$\frac{2\delta_1}{4+8\beta+\beta^2}$
p_2^i	$m+c_2+(1+\beta)q_2^n$	$m+w^c+\frac{(2+4\beta+\beta^2)\delta_1(2-\theta+T_a)}{2(1+\beta)(8+16\beta+3\beta^2)}$	$m+c_1+\frac{\delta_1(4+12\beta+8\beta^2+\beta^3)}{2(1+\beta)(4+8\beta+\beta^2)}$
w^i	/	$c_1+\frac{\delta_1(3\beta^3+4(2+\theta)+8\beta(3+\theta)+2\beta^2(9+\theta)-2(2+4\beta+\beta^2)T_a)}{2(1+\beta)(8+16\beta+3\beta^2)}$	/
r^i	/	/	$\frac{\beta(2+\beta)^2\delta_1}{2(1+\beta)(4+8\beta+\beta^2)}$
M^i	/	/	$\frac{\delta_1^2(16(1+\beta)^2-(32+96\beta+76\beta^2+16\beta^3+\beta^4)(1-\theta))}{4(1+\beta)(4+8\beta+\beta^2)^2}$

Where $T_a=\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2}$.

Lemma 3.1 indicates that for the competition model, higher degree of production substitution (β) negatively affect manufacturers' profitability, which is consistent with the classic economic theory that intense matket competition harms firms' financial performance because such competition can lead to a price war between rival competitors. Surprisingly, whereas high degree of product substitution certainly drives down the retail price of manufacturer 2, the effect of β on manufacturer 1's optimal retail price is more complicated. This effect depends upon the relationship of β with a critical threshold β^N, and the difference in the unit component cost between the two manufacturers (Δc) and its relationship with manufacturer 1's maximum unit profit (δ_1), as shown in Lemma 3.1. This dependency exists because manufacturer 1 has the advantage of a lower unit-component cost. A larger cost advantage can offset the manufacturer's pressure to engage in a price war with its rival competitor despite high degree of product substitution. For instance, Huawei, one of the leading smartphone manufacturers in the Chinese Smartphone market, has the advantage of production cost over their rivals for some key components because of their R&D and production capability. Interestingly, they adopt a more conventional pricing policy when engaging in low-end product competition. In contrast, they often do not engage in a price war with rivals for the high-end product range, which is often viewed as more-intense market competition.

3.3.3 Wholesaling coopetition model

For the WC model, a supplier-buyer cooperative relationship exists between the two rival manufacturers. Manufacturer 2 purchases components from manufacturer 1 while competing for the same market. It is common to have this type of relationship in the PC and electronics industries, in which manufacturers engage supplier-buyer cooperation and market competition simultaneously (Wang et al. 2013; Yang et al. 2017). Therefore, the two manufacturers' decision sequence is described as follows. First, manufacturers negotiate the wholesale price (w) for the component. Second, manufacturer 2 decides its order quantity (q_2) for the component from manufacturer 1. Third, manufacturer 1 decides the production quantity $q_1 + q_2$, where q_1 represents manufacturer 1's demand. Finally, when the end-consumers' demand is realized, the two manufacturers obtain their revenue/profits accordingly.

For the WC model, manufacturer 1's profit $\pi_1^c(q_1)$ is calculated as follows:

$$\pi_1^c(q_1) = [\alpha - q_1 - \beta(q_1 + q_2) - m - c_1]q_1 + (w - c_1)q_{2.} \tag{3-3}$$

The first part of the formula represents the profit from product sales, and the second part represents the profit from wholesaling the component to manufacturer 2.

Similarly, for the WC model, manufacturer 2's profit $\pi_2^c(q_2)$ is calculated as follows:

$$\pi_2^c(q_2) = [\alpha - q_2 - \beta(q_1 + q_2) - m - w]q_2 \tag{3-4}$$

Following the literature (e.g., Nagarajan and Bassok 2008; Chen et al. 2016), we introduce parameter θ to measure the negotiation power of manufacturer 1. Correspondingly, the negotiation power of manufacturer 2 will be $1 - \theta$. With extreme negotiation powers, the bargaining over the wholesaling model is equivalent to the standard Stackelberg or Vertical Nash games. The wholesale price negotiation process for the WC model is as follows:

$$\max_w \pi^{cw}(w) = \max_w [\pi_1^c(q_1(w))]^\theta [\pi_2^c(q_2(w))]^{1-\theta} \tag{3-5}$$

Manufacturer 1's optimal production quantity (q_1^c), optimal retail price (p_1^c) and optimal component wholesale price (w^c) and manufacturer 2's optimal order quantity (q_2^c) and optimal retail price (p_2^c) in the WC model are provided in Table 3.2. With respect to the effect of β on manufacturers' optimal retail prices and maximum profits, we present the following lemma.

Lemma 3.2: *For the WC model, (1) p_2^c, $\pi_1^c(q_1^c)$ and $\pi_2^c(q_2^c)$ decrease in β; (2) if $\beta > \beta^A$ and $\theta^y < \theta < 1$, then p_1^c increases in β; if $0 < \beta < \beta^A$, or $\beta > \beta^A$ and $\theta^c < \theta < \theta^y$, then p_1^c decreases in β; (3) if $\theta^t < \theta < min\{\theta^s, 1\}$, then w^c decreases in β; if $\theta^c < \theta < \theta^t$, or $\beta > \beta^C$ and $\theta^s < \theta < 1$, then w^c increases in β.*[3]

This lemma indicates that the two manufacturers' profits and manufacturer 2's retail price are decreasing functions of the degree of product substitution (β) for the WC model. This finding is similar to the competition model, which means that the buyer-supplier cooperation between the two competing manufacturers does not affect how the market competition factor impacts their financial performance. Different from the classic economic theory, the effect of the degree of product substitution on manufacturer 1's optimal retail price and component wholesale price is more complex for the WC model. Depending upon the relationship between β and β^A and the relationships between θ and the corresponding critical thresholds (θ^c and θ^y), manufacturer 1's optimal retail price can be an increasing or decreasing function of β. Similarly, depending upon the relationship between β and β^C and the relationships between θ and the corresponding critical thresholds (θ^c, θ^s, and θ^t), manufacturer 1's optimal wholesale price can be a decreasing or increasing function of

[3] The forms of θ^y, θ^s, θ^t and θ^c are listed in the proof of lemma 3.2 in the Appendix A. Their values depend upon the degree of product substitution (β).

β because high degree of product substitution will drive both manufacturers' retail prices down. At the same time, manufacturer 1 is able to set a higher wholesale price due to its possessing a greater negotiation power than that of manufacturer 2, and the revenue generated from component sales must be incorporated by manufacturer 1 in setting its optimal retail price. It implies that manufacturers must consider the inter-firm power relationship and market competition factor when deciding the wholesale prices for key components and setting retail prices for their products when engaging wholesale coopetition.

3.3.4 License coopetition model

For the LC model, a cooperative relationship exists in the form of a licensing arrangement between the two rival manufacturers. This type of relationship is common in the pharmaceutical and technological industries, in which a firm licenses its innovation to a potential competitor (Simonet 2002; Ziedonis 2007). In the context of this study, manufacturer 1 licenses manufacturer 2 to use its technology to produce the component while competing for the same market. Therefore, the two manufacturers' decision sequence is described as follows. First, the manufacturers negotiate the fixed license fee (M) and the royalty rate (r) for the component. Second, the two manufacturers independently and simultaneously determine their production quantities to maximize their profits. Finally, when the end-consumers' demand is realized, the two manufacturers obtain their revenues and profits accordingly.

For the LC model, manufacturer 1's profit $\pi_1^l(q_1)$ is calculated as follows:

$$\pi_1^l(q_1) = [\alpha - q_1 - \beta(q_1 + q_2) - m - c_1]q_1 + rq_2 + M \tag{3-6}$$

The first part of the formula represents the profit from product sales, and the second and the third parts represent the royalty fee and fixed license fee that are received from manufacturer 2.

For the LC model, manufacturer 2's profit $\pi_2^l(q_2)$ is calculated as follows:

$$\pi_2^l(q_2) = [\alpha - q_2 - \beta(q_1 + q_2) - m - c_1]q_2 - rq_2 - M \tag{3-7}$$

The first part of the formula represents the unit marginal profit of manufacturer 2, and the second and the third parts represent the royalty fee and fixed license fee paid to manufacturer 1.

Assuming that manufacturer 1's negotiation power is θ, we can model the negotiation process of the licensing fees for the LC model as follows:

$$\max_{r,M} \pi^{cl}(r, M) = \max_{r,M} [\pi_1^l(q_1(r, M))]^\theta [\pi_2^l(q_2(r, M))]^{1-\theta} \tag{3-8}$$

The optimal production quantities (q_1^l , q_2^l) and optimal retail prices (p_1^l, p_2^l) for both manufacturers in the LC model are provided in Table 3.2.

Similar to Lemma 3.2, we can derive Lemma 3.3 concerning the effect of β on the manufacturers' optimal retail prices and maximum profits.

Lemma 3.3: *For the LC mode, (1)* $\pi_1^l(q_1^l)$, $\pi_2^l(q_2^l)$, *and* M^l *decrease in* β; r^l *increases in* β. *(2) If* $0 < \beta < \beta^X = 2$, *then* p_1^l *and* p_2^l *decrease in* β ; *if* $\beta^X < \beta < \beta^Y \approx 3.7587$, *then* p_1^l *increases in* β *and* p_2^l *decreases in* β; *if* $\beta > \beta^Y$, *then* p_1^l *and* p_2^l *increase in* β.

Similar to the WL model, the licensing cooperation does not change the effect of the degree of product substitution on the two manufacturers' financial performance. Interestingly, Lemma 3 indicates that in the LC model, the optimal fixed license fee is a decreasing function of β, whereas the optimal royalty rate is an increasing function of β. A low fixed-license fee helps break the ice of intense competition and engages the rival firms in the license cooperation. The royalty rate often constitutes the main part of the licensing agreement cost, and firms tend to charge a higher royalty rate when agreeing on licensing with their fiercest market rivals. Revenue (or cost) from the licensing agreement has a knock-on effect on the optimal retail price of manufacturer 1 (or manufacturer 2). For manufacturer 1, the revenue from a licensing agreement mitigates the pressure of a decreasing retail price from the intense market competition. For manufacturer 2, the cost of the licensing agreement must be a factor of setting the optimal retail price. For example, the incremental licensing revenue has helped technology giant Nokia, which has licensing agreements with all major smartphone manufacturers to compensate for the declines from tough competition in the telecom market (Rogers 2018). The tradeoff between the cooperation and competition forces will determine how the two manufacturers' optimal retail prices are influenced by the market competition factor. More specifically, the competition force overtakes the cooperation force in influencing two manufacturers' pricing decisions when β is less than the threshold β^X. In contrast, the cooperation force overtakes the competition force in influencing the pricing decisions when β is greater than the threshold β^Y . When β is between the two thresholds ($\beta^X < \beta < \beta^Y$), it affects the two manufacturers' optimal retail prices differently, as illustrated in Lemma 3.3.

3.4 Effects of coopetition

3.4.1 Effects of wholesaling coopetition

In this section, we examine the effect of the WC strategy on optimal retail prices and maximum profits for both manufacturers by comparing the derived equilibrium solutions for the Cournot competition model and the WC model.

3.4.1.1 Effect of wholesaling coopetition on optimal retail prices

First, we present the effect of WC on manufacturers' optimal retail prices.

Lemma 3.4: *If* $0 < \Delta c < \Delta c^B$ *and* $\theta^e < \theta < 1$, *then* $p_1^c > p_1^n$ *and* $p_2^c > p_2^n$; *if* $\Delta c^B < \Delta c < \Delta c^H$, *or* $0 < \Delta c < \Delta c^B$ *and* $\theta^c < \theta < \theta^e$, *then* $p_1^c < p_1^n$ *and* $p_2^c < p_2^n$.[4]

Lemma 3.4 implies that the wholesaling coopetition can drive up or down the prices of both manufacturers depending upon the differences in the unit component cost between manufacturers (Δc), manufacturer 1's negotiation power (θ) and their corresponding critical thresholds. More specifically, with a large value of Δc, wholesaling coopetition leads to a decrease of the optimal retail prices and therefore benefits the customers. With a small value of Δc, the effect of wholesaling coopetition on optimal retail prices is determined by other external market characteristics (i.e., β) and internal operational capability (i.e., δ_1).

3.4.1.2 Effect of wholesaling coopetition on maximum profits

Next, we explore the effect of WC on the manufacturers' maximum profits.

Proposition 3.1: (1) *If* $0 < \Delta c < \Delta c^H$ *and* $max\{\theta^p, \theta^c\} < \theta < min\{\theta^q, 1\}$, *then WC is the better strategy; otherwise, competition is the better strategy.*

(2) When WC is a better strategy than competition is, if $\Delta c^A < \Delta c < \Delta c^H$ *and* $\theta^f < \theta < min\{\theta^g, 1\}$, *then WC delivers Pareto improvement; otherwise, if* $0 < \Delta c < \Delta c^K$ *and* $max\{\theta^g, \theta^f\} < \theta < 1$, *then* $\pi_1^c(q_1^c) > \pi_1^n(q_1^n)$ *and* $\pi_2^c(q_2^c) < \pi_2^n(q_2^n)$; *if* $0 < \Delta c < \Delta c^H$ *and* $\theta^c < \theta < min\{\theta^g, \theta^f\}$, *then* $\pi_1^c(q_1^c) < \pi_1^n(q_1^n)$ *and* $\pi_2^c(q_2^c) > \pi_2^n(q_2^n)$.

(3) For the Pareto improvement WC strategy, $p_1^c < p_1^n$ *and* $p_2^c < p_2^n$.[5]

This proposition implies that whether the wholesaling coopetition increases or decreases the manufacturers' maximum profits compared with the competition model is decided by the degree of product substitution (β), manufacturers' negotiation power relationship (θ), and internal operational

[4] The forms of θ^e, θ^c, Δc^H and Δc^B are listed in the proof of Lemma 3.4 in the Appendix A. The value of θ^e depends upon the maximum unit profit for manufacturer 1 (δ_1), the difference in the unit component cost between manufacturers (Δc), and the degree of product substitution (β). The value of θ^c depends upon β. The values of both Δc^H and Δc^B depend upon δ_1 and β.

[5] The values of Δc^H, Δc^A and Δc^K depend upon the maximum unit profit for manufacturer 1 (δ_1) and the degree of product substitution (β). The value of θ^c depends upon β. The values of θ^p, θ^q and θ^g depend upon δ_1, β and the difference between the unit component costs of the two manufacturers (Δc).

capabilities (Δc and δ_1). This relationship is illustrated in Figure 3.2, which is divided into three decision regions. The characteristics of each region are discussed next.

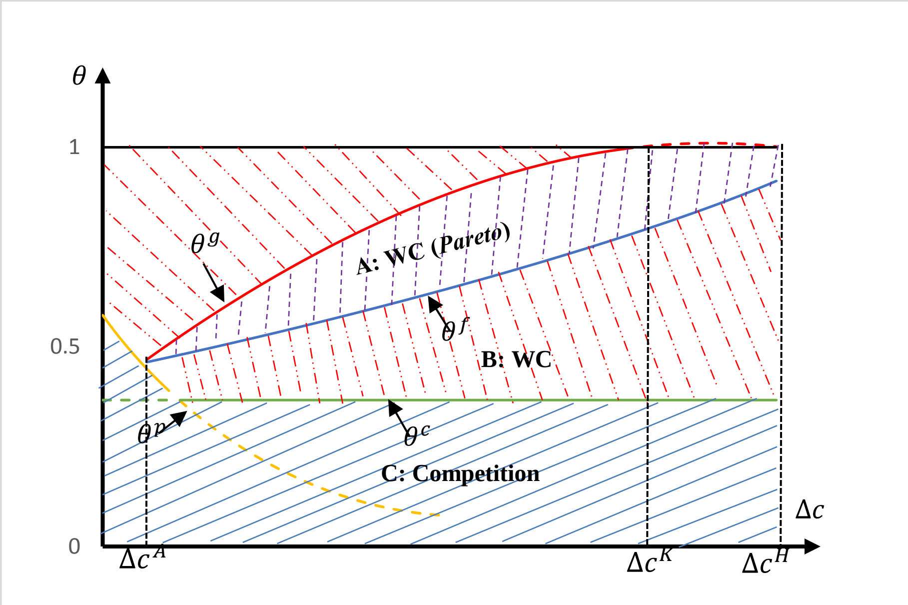


Figure 3.2 Effect of the WC strategy on manufacturers' profits

Region A outlines the decision region in which both manufacturers' maximum profits for the WC model are greater than are those under the competition model, which implies that the wholesaling coopetition can lead to *Pareto improvement*. Consequently, such a coopetitive relationship will be embraced by both parties. From part (3) of Proposition 3.1, we know that in this *Pareto improvement* region, both manufacturers' retail prices are lower than those under the competition model, which is beneficial for consumers. Therefore, we can conclude that in this situation, wholesaling coopetition positively affects individual firms and consumers.

Region B specifies the conditions under which one of the two manufacturers will earn less profit in the WC model than in the competition model despite the total profit between the two being greater in the WC model than in the competition model ($\pi^c > \pi^n$). In this situation, the manufacturer, who suffers profit loss through wholesaling coopetition, has no incentive to continue the buyer-supplier cooperative relationship. In this case, the wholesaling coopetition can only continue the cooperation if the better-off manufacturer is willing to redistribute the profit gain between the two parties. A *Pareto* improvement can only be realized through a further cooperation mechanism such as a profit sharing contract.

Region C specifies the conditions under which competition is the optimal strategy. More specifically, if manufacturer 1's negotiation power is less than θ^c, then $w^r < c_1$. It is not realistic for a firm to sell components to rival firms at a wholesale price that is lower than its production cost;

therefore, the wholesaling coopetition is infeasible. In addition, if manufacturer 1's negotiation power meets the condition of $\theta^c < \theta < \theta^p$, then the total profit of the two firms will be less in the WC model than in the competition model ($\pi^c < \pi^n$). In this situation, competition is also the optimal strategy.

3.4.2 Effects of license coopetition

In this section, we examine the effect of the LC strategy on the optimal retail prices and maximum profits for both manufacturers by comparing the derived equilibrium solutions for the Cournot competition model and the LC model.

3.4.2.1 Effect of license coopetition on optimal retail prices

In the following, Lemma 3.5 presents the effect of LC on manufacturers' optimal retail prices.

Lemma 3.5: *If* $0 < \Delta c < \Delta c^Y$, *then* $p_1^l > p_1^n$ *and* $p_2^l > p_2^n$; *if* $\Delta c^Y < \Delta c < \Delta c^H$, *then* $p_1^l < p_1^n$ *and* $p_2^l < p_2^n$.[6]

Lemma 3.5 implies that, similar to wholesale coopetition, license coopetition can drive up or down the optimal retail prices of both manufacturers compared with the competition model. Again, this finding shows the difference between coopetition and collusion from consumers' point of view. Different from wholesale coopetition, the effect of license coopetition on the manufacturers' optimal retail prices is predominantly determined by Δc, δ_1 and β; manufacturer 1's negotiation/bargaining power (θ) has no effect.

3.4.2.2 Effect of license coopetition on maximum profits

To determine the effect of license coopetition on manufacturers' maximum profits, we derive the total profit of both manufacturers in the LC model and compare it with that in the competition model. Therefore, we propose the following:

Proposition 3.2: *(1) If* $0 < \Delta c < \Delta c^H$ *and* $\theta^j < \theta < 1$, *then LC is the better strategy; otherwise, competition is the better strategy.*

(2) When LC is the better strategy, if $\Delta c^P < \Delta c < \Delta c^H$ *and* $max\{\theta^j, \theta^k\} < \theta < \theta^l$, *then LC delivers Pareto improvement; otherwise, if* $0 < \Delta c < \Delta c^H$ *and* $max\{\theta^j, \theta^l\} < \theta < 1$, *then*

[6] The mathematical forms of Δc^H and Δc^Y are listed in the proof of Lemma 3.5 in the Appendix A. Their values depend upon δ_1 and β.

$\pi_1^l(q_1^l) > \pi_1^n(q_1^n)$ *and* $\pi_2^l(q_2^l) < \pi_2^n(q_2^n)$; *if* $\Delta c^J < \Delta c < \Delta c^H$ *and* $\theta^j < \theta < \theta^k$, *then* $\pi_1^l(q_1^l) < \pi_1^n(q_1^n)$ *and* $\pi_2^l(q_2^l) > \pi_2^n(q_2^n)$.

(3) For the Pareto improvement LC strategy, $p_1^l < p_1^n$ *and* $p_2^l < p_2^n$.[7]

This proposition implies that whether the license coopetition is beneficial to the manufacturers is determined by the degree of product substitution (β), manufacturers' negotiation power relationship (θ), and internal operational capabilities (Δc and δ_1). This relationship is further illustrated in Figure 3.3, which is divided into three decision regions. Similar to the WC model, each decision region is discussed individually.

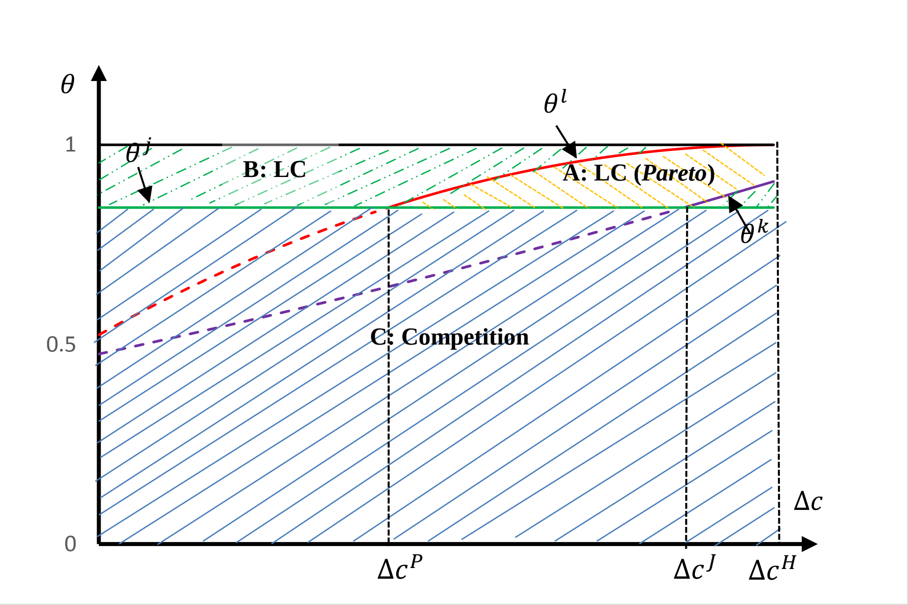


Figure 3.3 Effect of the LC strategy on manufacturers' profits

Region A highlights the decision region in which both manufacturers' maximum profits for the LC model are greater than those under the competition model, which implies that license coopetition can achieve *Pareto improvement*. Consequently, such a coopetitive relationship will be embraced by both parties. From Proposition 3.2 (3), we also know that the license coopetition leads to lower retail prices compared with competition. Therefore, we can conclude that in this situation, license coopetition positively affects individual firms and consumers.

Region B specifies the conditions under which one of the two manufacturers will incur profit loss in the LC model compared with the competition model, despite an increase in the total profit. In this case, the manufacturer incurring a profit loss has no incentive to engage in licensing cooperation.

[7] The mathematical forms of Δc^H, Δc^P, Δc^J, θ^j, θ^k and θ^l are listed in the proof of Proposition 3.2 in the Appendix A. The values of Δc^H, Δc^P and Δc^J depend upon the maximum unit profit for manufacturer 1 (δ_1) and the degree of product substitution (β). The value of θ^j depends upon β. The values of θ^k and θ^l depend upon δ_1, β and Δc.

Nevertheless, because the total profit of two manufacturers in the LC model is greater than that in the competition model ($\pi^l > \pi^n$), the better-off manufacturer has the capacity to persuade its counterpart to continue cooperating if it is willing to redistribute the profit gained from coopetition. *Pareto* improvement can be realized through further cooperation.

Region C describes the decision region in which competition is the optimal strategy. In this region, license coopetition will generate less profit than competition. Furthermore, if manufacturer 1's negotiation power is less than θ^j, then $M^l < 0$. In other words, manufacturer 1 will receive a negative fixed-licensing fee, which is not realistic. These results explain to some extent why firms in the automotive, smartphone and PC industries have license agreement with other vendors in the industry but not with their fiercest rivals (BBC 2014; Nokia 2016).

3.5 Selection of a coopetition strategy

In this section, we explore the optimal coopetition strategy considering different internal operational factors and external market circumstances. Proposition 3.3 summarizes the optimal strategy among competition, wholesaling coopetition and license coopetition.

Proposition 3.3: *(1) If* $0 < \Delta c < \Delta c^H$ *and* $\theta^j < \theta < 1$, *then LC is the optimal strategy.*

(2) If $0 < \Delta c < \Delta c^H$ *and* $max\{\theta^p, \theta^c\} < \theta < \theta^j$, *then WC is the optimal strategy.*

(3) Otherwise, competition is the optimal strategy.[8]

This proposition indicates that the optimal strategic decision on coopetition depends upon manufacturer 1's negotiation power (θ), the difference in the two manufacturers' unit component cost (Δc), and their relationships with the corresponding thresholds (θ^j, θ^p, θ^c and Δc^H). Note that these thresholds are determined by the maximum unit profit for manufacturer 1 (δ_1) and the degree of product substitution (β). This finding supports the view of Bengtsson and Raza-Ullah (2016) that external, relationship-specific, and internal drivers motivate firms to engage in coopetition. In the context of this research, the combination of the external market characteristic (β), inter-firm power relationship (θ), and internal operational resources and capabilities (Δc and δ_1) governs firms' strategic decisions on coopetition. The relationship between these external, relationship-specific, and internal factors and the manufacturers' optimal strategy is further illustrated in Figure 3.4, which

[8] The forms of Δc^H, θ^j, θ^p and θ^c are listed in the proof of Proposition 3.3 in the Appendix A. Δc^H depends upon the maximum unit profit for manufacturer 1 (δ_1) and the degree of product substitution (β). θ^j and θ^c depend upon β. θ^p depends upon the difference between two manufacturers' unit component costs (Δc).

highlights three decision regions. Each region is discussed as follows:

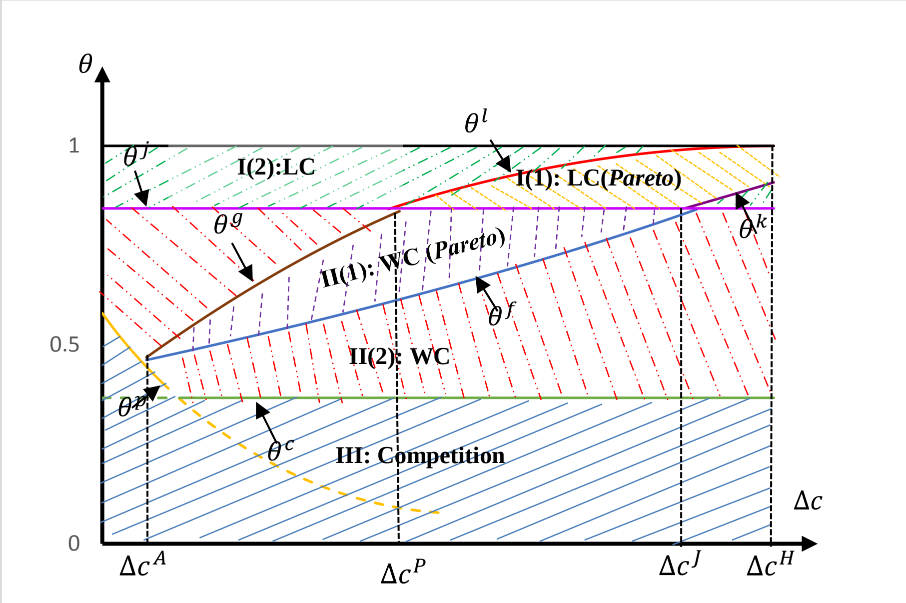


Figure 3.4 Selection of coopetition strategies ($\boldsymbol{\theta}, \Delta \boldsymbol{c}$)

In **Region I,** license coopetition is the optimal strategy for both manufacturers; the license agreement results in a larger profit than does competition or wholesaling coopetition ($\pi^l > \{\pi^c, \pi^n\}$). In other words, firms are more likely to benefit from license coopetition when they license technology to rival firms with less negotiation power. In **Region II,** wholesaling coopetition is the optimal strategy for both manufacturers because it leads to a greater profit than does competition or license coopetition ($\pi^c > \{\pi^l, \pi^n\}$). However, in both **Regions I** & **II**, situations exist such that further cooperation such as a profit-sharing contract would be required to ensure that both firms benefit from the coopetitive relationship, as discussed in Propositions 3.1 & 3.2. In **Region III,** competition is the optimal strategy for both manufacturers because the financial gains in the upstream key component production through either license coopetition or wholesaling coopetition cannot compensate for the losses that are incurred in the downstream market competition when facing a competitor strengthened due to coopetition. For numerous firms across various sectors, competition remains the most commonly adopted strategy when engaging with market rivals.

3.5.1 Effect of product substitution on strategy selection

The results in Figure 3.4 show that although Δc has less influence on the selection of a coopetition strategy, Δc significantly affects whether the strategy can achieve *Pareto improvement* without further cooperation in both licensing and wholesaling coopetition. It is more likely to achieve a win-win outcome from coopetition if there is a large difference between the two manufacturers' unit component costs. More importantly, the optimal strategic choice is primarily determined by the negotiation power of manufacturer 1 (θ). Furthermore, the degree of product substitution (β) affects those critical thresholds θ^j and θ^c upon which the decision regions of optimal coopetition strategy depend. Therefore, further analysis is performed to analyze how the external market competition (β) and relationship-specific negotiation power (θ) affect the selection of the optimal coopetition strategy. Here, β depends upon the nature of the product/service and the characteristics of the industry, and θ is subject to the technical difficulty of component production and the availability of an alternative component supply in the market. We fix the value of Δc (i.e., Δc = 0.5) and plot the optimal strategic choice corresponding to different values of β and θ. We start the analysis with comparable values of β and θ $\{\beta, \theta \in (0, 1)\}$, and the result is illustrated in Figure 3.5(a). If there is a low level of market competition, firms will benefit more by engaging in license coopetition when manufacturer 1 has more negotiation power than does manufacturer 2; conversely, competition is the optimal strategy when manufacturer 1 has less negotiation power. If the market competition intensifies further, wholesale competition will be more beneficial when manufacturer 1 has more or similar power compared with manufacturer 2; otherwise, competition is the optimal strategy when manufacturer 1 has less power. From Figure 3.5(a), it is also clear that only the two critical thresholds θ^j and θ^c, whose values depend upon β and θ, have influenced the decision on strategy selection. To further scrutinize the effect of the key parameters on coopetition strategy selection, we extend the value range of the degree of product substitution to $\beta \in (0, 10)$; the analysis result is illustrated in Figure 3.5(b).

Figure 3.5(b) shows clearly that the external market attributes (i.e., β and θ) profoundly influence the strategic choice of coopetition. Although the result in Figure 3.5(b) mirrors that in Figure 3.5(a) when the degree of product substitution is low, it also shows that when the degree of product substitution increases further to higher levels, it is more beneficial for manufacturers to choose competition only unless manufacturer 1 has negotiation power superior to that of manufacturer 2. The licensing or wholesale coopetition strategy has often been adopted in the smartphone and electronic vehicle, in which there is often high degree of product substitution among rival firms. Our analysis result also shows that coopetition particularly licensing agreement can be beneficial to firms when the degree of product substitution is low. This finding partially explains that there are more licensing agreements between firms with low degree of product substitution and more wholesale cooperation between firms with high degree of product substitution (BBC 2014; Kang 2016; Nokia 2016). This result supports the views in the existing literature that, in highly competitive market environments where there are numerous rival firms offering substitutive products (Dussauge et al. 2000), or in a less competitive environment where there are only a limited number of competitors offering similar products (Peng and Bourne 2009), coopetition can be an effective strategy. However, the selection of optimal coopetition strategy (e.g. wholesale or license) is not only determined by the degree of product substitution and inter-firm power relationship but also influenced by the production capability difference, which will be further discussed in the following section.

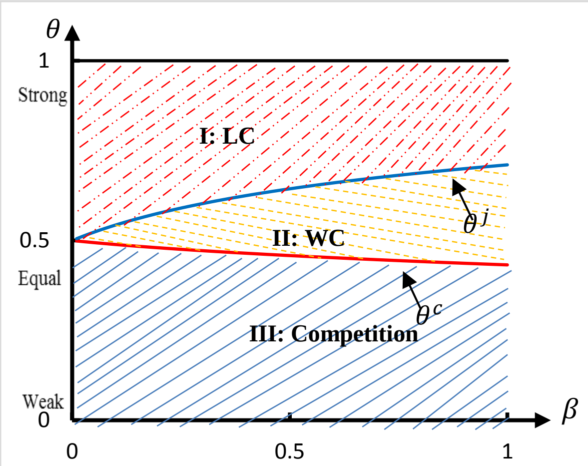


Figure 3.5(a) $\boldsymbol{\beta}$**: 0 → 1**

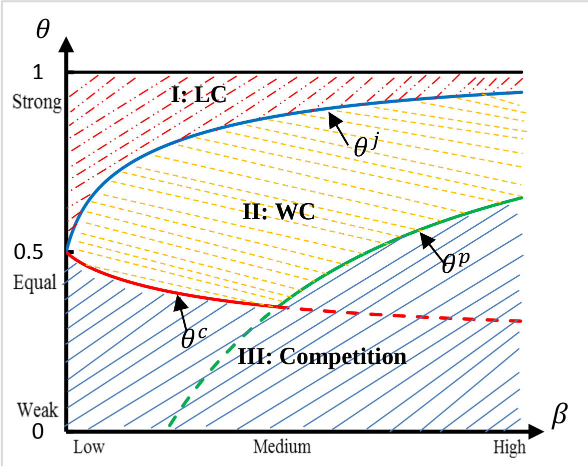


Figure 3.5(b) $\boldsymbol{\beta}$**: 0 → 10**

Figure 3.5 Effect of $\boldsymbol{\beta}$ **on selection of coopetition strategies** $(\Delta \boldsymbol{c} = \mathbf{0.5})$

3.5.2 Effect of component cost difference on strategy selection

The above results are based on the assumption of fixing the value of Δc (i.e., Δc = 0.5). However, as discussed previously, internal operational capability is one of the main drivers for firms to pursue a coopetition strategy. To examine the robustness of our results, further analysis is performed with a range of different values for Δc (i.e., $\Delta c = 0.1$, $\Delta c = 0.25$, $\Delta c = 0.4$, and $\Delta c = 0.5$[9]); the results are displayed in Figure 3.6.

[9] The critical threshold Δc^H, which defines the feasible region of maximum Δc value, is determined by δ_1 and β. Because $\beta \in (0, 10)$ is specified in the analysis of Section 5.1, we derive the maximum feasible value of Δc as 0.54 through inputting $\beta = 10$ in the mathematical expression of Δc^H. Therefore, the values of Δc considered in the analysis cover a reasonable range.

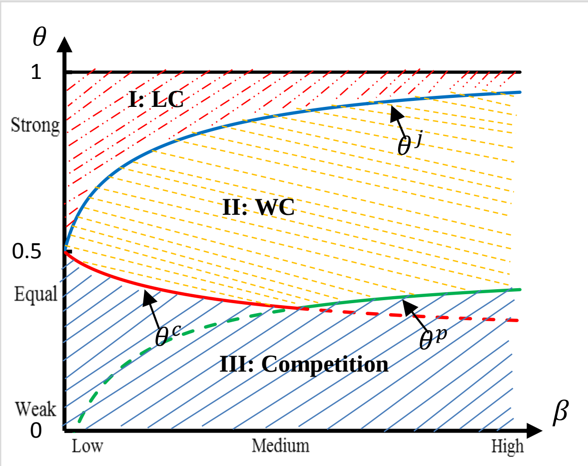


Figure 3.6(a) $\Delta c = 0.1$

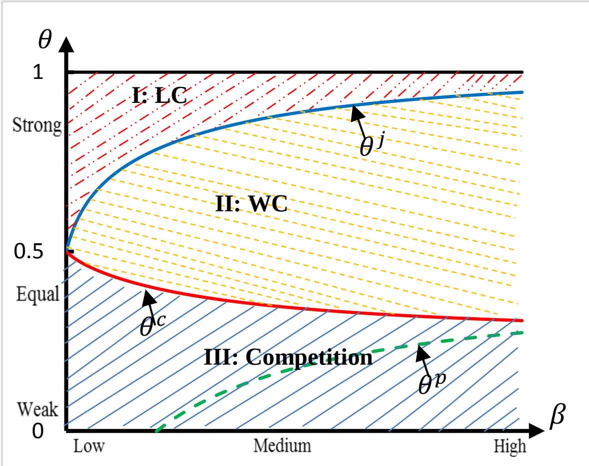


Figure 3.6(b) $\Delta c = 0.25$

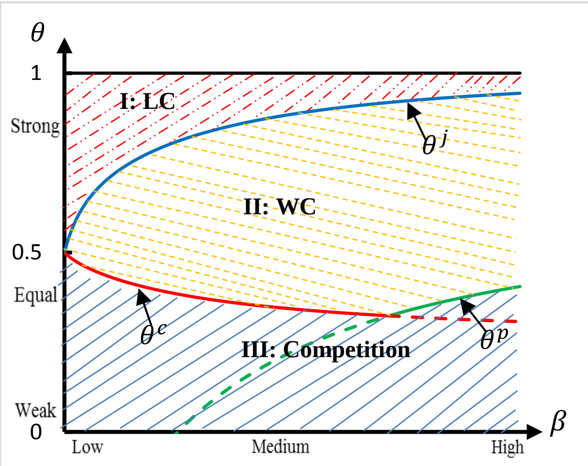


Figure 3.6(c) $\Delta c = 0.4$

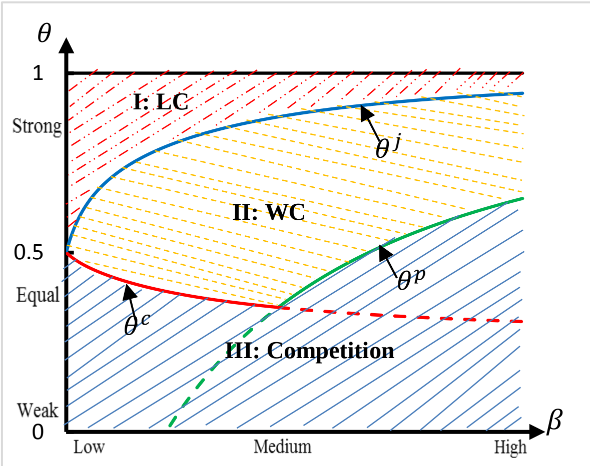


Figure 3.6(d) $\Delta c = 0.5$

Figure 3.6 Effect of Δc on selection of coopetition strategies (β: $0 \rightarrow 10$)

It is clear that Δc only affects the strategic choice between competition and wholesaling coopetition; it does not affect the decision on license coopetition. Whether to pursue license coopetition is decided by the relationship between θ and the threshold θ^j, which is dependent upon β. For instance, automakers, PC manufacturers, and pharmaceutical firms license technologies and patents to many other firms, but often not to their fiercest market rival, because of the market competition factor and their inter-firm relationship as discussed previously. The effect of Δc on strategy selection is primarily expressed through its influence on another critical threshold, θ^p. Interestingly, we find that when the value of Δc is small (i.e., $\Delta c = 0.1$), a further increase of Δc will move the intersection between the thresholds θ^c and θ^p rightwards, which means that wholesaling coopetition is more likely to be the preferred optimal strategy over competition. When the value of Δc increases to a certain extent (i.e., $\Delta c = 0.25$), $\theta^c > \theta^p$; therefore, there is no intersection between the two thresholds. When the value of Δc increases further (i.e., Δc = 0.4 and Δc = 0.5), the intersection between the two thresholds reappears and moves leftwards, which means competition is more likely to be the preferred optimal strategy over wholesale coopetition. These results reinforce the findings of Proposition 3.3 that the relationship between θ and θ^p determines the strategic choice between wholesaling coopetition and competition when the value of Δc is either small or large. In contrast, when the value of Δc is in the middle, the same strategic choice is determined by the relationship between θ and θ^c. Therefore, we can conclude that firms must incorporate the external market competition, inter-firm relationship characteristics and internal operational resources and capabilities to make an optimal strategic decision on coopetition.

3.6 The extended models

3.6.1 The asymmetric case

In the previous sections, we assume a symmetric case in which $\alpha_1 = \alpha_2 = \alpha$. Here, α_1 and α_2 represent the maximum retail prices of manufacturers 1 and 2, respectively. In this section, we consider the scenario in which $\alpha_1 \neq \alpha_2$. Then, the demand function $p_i = \alpha_i - q_i - \beta(q_i + q_j)$, $i, j = 1,2$ and $i \neq j$. Based on this demand function, the optimal solutions for the competition, WC and LC models are provided in Table 3.3. The derivation of these optimal solutions is provided in the Appendix.

Comparing the optimal solutions in Table 3.3 to those in Table 3.2, it is clear that α_1, α_2 significantly affect manufacturers' optimal operational decisions. Consequently, they will affect manufacturers' profits in the competition, WC and LC models and the values of important critical thresholds that determine manufacturers' optimal decision regions on coopetition strategy selection. Therefore, to verify whether the structural results presented in the symmetric case still hold in the asymmetric-manufacturer case, a numerical example is provided here to demonstrate the effect of the asymmetric-manufacturer case (i.e., $\alpha_1 \neq \alpha_2$) on the selection of coopetition strategies. We assume that $\delta_1 = 1$ and $\beta = 4$. In Figure 3.7, we specify that $\alpha_2 - \alpha_1 = 0.1$, which means that $\alpha_2 > \alpha_1$. In Figure 3.8, we specify that $\alpha_2 - \alpha_1 = -0.1$, which means that $\alpha_2 < \alpha_1$.

Table 3.3 Optimal solutions for the three models ($\alpha_1 \neq \alpha_2$)

Models	Competition model ($i = n$)	WC model ($i = c$)	LC model ($i = l$)
q_1^i	$\frac{(2+\beta)\delta_1 + \beta(\Delta c - \Delta\alpha)}{(2+\beta)(2+3\beta)}$	$\frac{(2+6\beta+5\beta^2+\beta^3)(\beta(1+\beta)\Delta\alpha(-2+\theta)+\delta_1(8+3\beta^2+\beta(14+\theta))) - \beta T_l}{2(1+\beta)^2(2+4\beta+\beta^2)(8+16\beta+3\beta^2)}$	$\frac{(4+6\beta+\beta^2)\delta_1 - 2\beta(1+\beta)\Delta\alpha}{2(1+\beta)(4+8\beta+\beta^2)}$
p_1^i	$m + c_1 + (1+\beta)q_1^n$	$m + c_1 + (1+\beta)q_1^c$	$m + c_1 + (1+\beta)q_1^l$
q_2^i	$\frac{(2+\beta)\delta_2 - \beta(\Delta c - \Delta\alpha)}{(2+\beta)(2+3\beta)}$	$\frac{(2+6\beta+5\beta^2+\beta^3)(\delta_1+\Delta\alpha+\beta\Delta\alpha)(2-\theta)+T_l}{(1+\beta)(2+4\beta+\beta^2)(8+16\beta+3\beta^2)}$	$\frac{2(\delta_1+\Delta\alpha+\beta\Delta\alpha)}{4+8\beta+\beta^2}$
p_2^i	$m + c_2 + (1+\beta)q_2^n$	$m + c_1 + \frac{1}{2(1+\beta)^2(8+16\beta+3\beta^2)}((1+\beta)(\Delta\alpha(12+36\beta+28\beta^2+4\beta^3+2\theta+6\beta\theta+5\beta^2\theta+\beta^3\theta)$ $+ \delta_1(3\beta^3+2(6+\theta)+4\beta(8+\theta)+\beta^2(20+\theta))) - T_l)$	$m + c_1 + \frac{\delta_1(4+12\beta+8\beta^2+\beta^3)+4(1+3\beta+2\beta^2)\Delta\alpha}{2(1+\beta)(4+8\beta+\beta^2)}$
w^i	/	$c_1 + \frac{1}{2(1+\beta)^2(8+16\beta+3\beta^2)}((1+\beta)\big(3\beta^3+4(2+\theta)+8\beta(3+\theta)+2\beta^2(9+\theta)\big)\delta_1$ $+ 2((1+\beta)^2\big(\beta^2(1+\theta)+2(2+\theta)+4\beta(2+\theta)\big)\Delta\alpha - T_l)$	/
r^i			$\frac{\beta((2+\beta)^2\delta_1 - 4\beta(1+\beta)\Delta\alpha)}{2(1+\beta)(4+8\beta+\beta^2)}$
M^i			$\frac{8\delta_1\Delta\alpha\big(4\theta+8\beta\theta+\beta^2(3+\theta)\big)+4(1+\beta)\Delta\alpha^2(4\theta+8\beta\theta+\beta^2}{4(4+8\beta+\beta^2)^2}$ $+ \frac{\delta_1^2(16(1+\beta)^2-(32+96\beta+76\beta^2+16\beta^3+\beta^4)(1-\theta))}{4(1+\beta)(4+8\beta+\beta^2)^2}$

Where $T_l = ((2+6\beta+5\beta^2+\beta^3)^2(2(1+\beta)\delta_1\Delta\alpha(-2+\theta)^2+(1+\beta)^2\Delta\alpha^2(-2+\theta)^2+\delta_1^2\big((12+16\beta+3\beta^2)(1-\theta)+\theta^2\big)))^{\frac{1}{2}}$

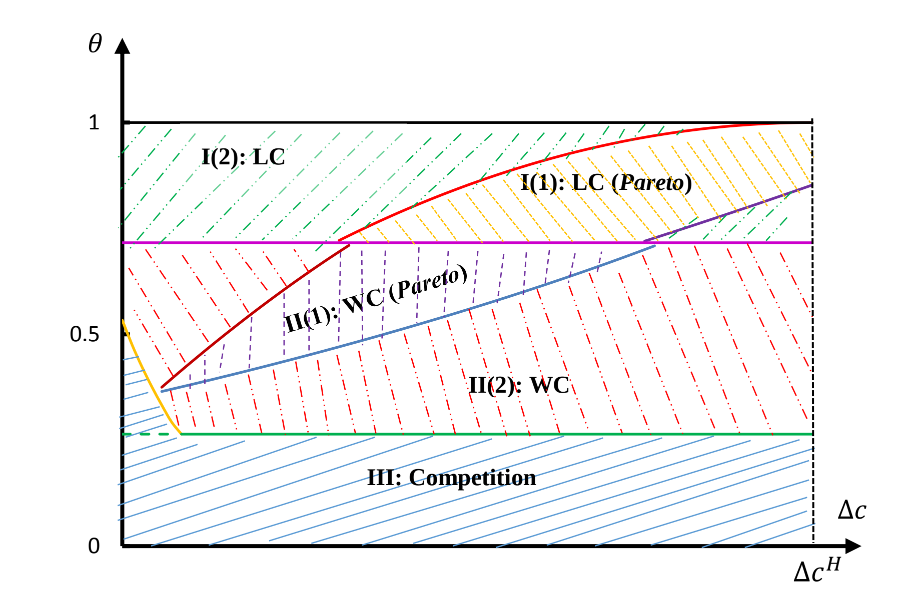


Figure 3.7 Selection of coopetition strategies ($\alpha_2 > \alpha_1$)

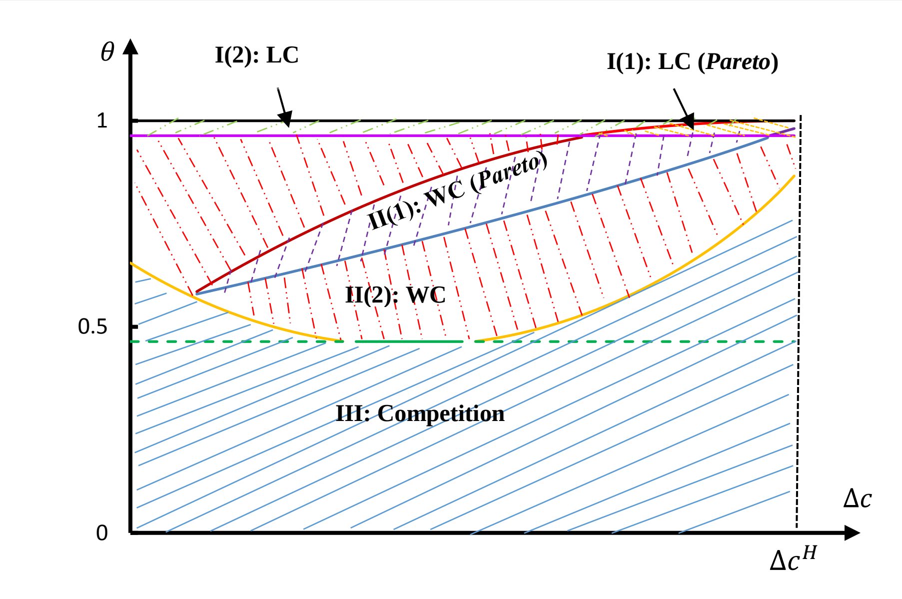


Figure 3.8 Selection of coopetition strategies ($\alpha_2 < \alpha_1$)

From Figures 3.7 and 3.8, we obtain that the structure of an optimal strategic decision on coopetition is similar to the scenario with the same maximum retail price for the two manufacturers. At the same time, the critical points are affected by the difference between the maximum retail prices of manufacturers ($\Delta\alpha$). That is, a positive $\Delta\alpha$ results in a larger decision region for LC strategy and a smaller region for competition strategy; conversely, a negative $\Delta\alpha$ leads to a smaller decision region for LC strategy and a larger region for competition strategy. In other words, if manufacturer 1 has a higher maximum retailer price than manufacturer 2 does, it is less likely that license coopetition is the optimal strategic decision. Clearly, α_1, α_2 affect manufacturers' optimal operational decisions (e.g., retail prices, wholesale price, fixed license fee, and royalty rate) and the values of important critical thresholds that influence manufacturers' optimal decisions on coopetition strategy. Nevertheless, the structural results presented in the previous sections still hold when two manufacturers are asymmetric.

3.6.2 The case of both partial and perfect substitutes

In this section, we extend the analysis of the partially substitutable products case to the case that includes the scenarios of partial and perfect substitutes. We adopt the demand function, $p_i = \alpha - q_i - \beta q_j$, $i, j = 1,2$ and $i \neq j$, $0 < \beta \leq 1$, that is used in Wang et al. (2013). Here, $0 < \beta < 1$ corresponds to the scenario of partial substitutes, and the limiting value, $\beta = 1$, corresponds to the case of perfect substitutes. Based on the new demand function, the optimal solutions for the competition, WC and LC models are presented in Table 3.4.

Comparing the optimal solutions in Table 3.4 to those in Table 3.2, it is clear that the optimal solutions are presented in different mathematical formations due to a different expression of β in the new demand function. We then repeat the same analysis of Section 3.3, 3.4 and 3.5 to examine how different internal operational factors and external market circumstances affect the selectin of coopetition strategies with the new demand function. The results are illustrated in Figure 3.9 and Figure 3.10, which correspond to the scenarios of partial and perfect substitutes respectively.

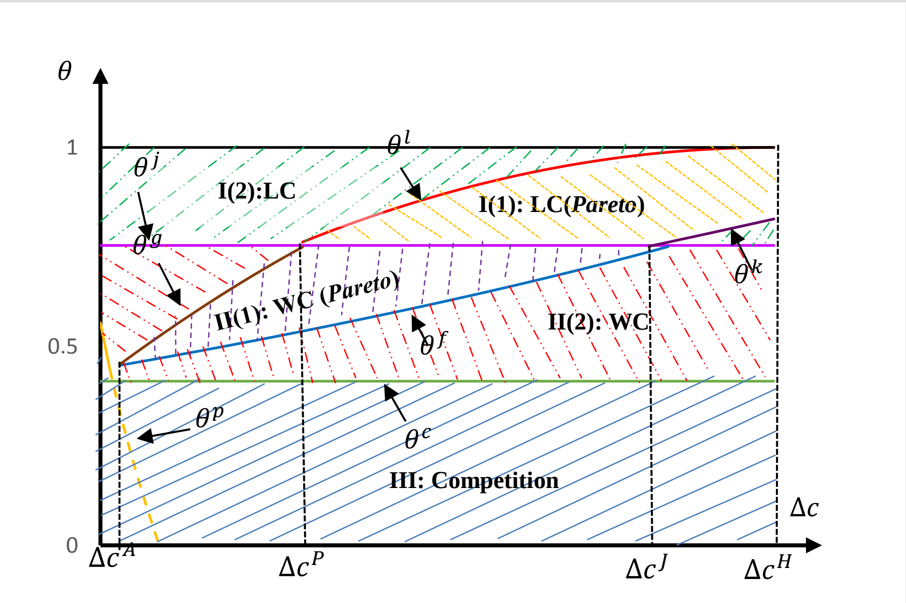


Figure 3.9 Selection of coopetition strategies ($\boldsymbol{\theta}, \Delta c, \boldsymbol{\beta} = \mathbf{0.5}$)

Table 3.4 Optimal solutions for the three models for the general substitutable product case

Models	Competition model ($i=n$)	WC model ($i=c$)	LC model ($i=l$)
q_1^i	$\frac{\beta\Delta c+(2-\beta)\delta_1}{4-\beta^2}$	$\frac{\delta_1(8-2\beta-3\beta^2+(\beta-\beta^2)\theta-\beta T_a)}{2(8-5\beta^2)}$	$\frac{(4-2\beta-\beta^2)\delta_1}{2(4-3\beta^2)}$
p_1^i	$m+c_1+q_1^n$	$m+c_1+q_1^c$	$m+c_1+q_1^l$
q_2^i	$\frac{-2\Delta c+(2-\beta)\delta_1}{4-\beta^2}$	$\frac{\delta_1(2-\theta+T_a))}{(8-5\beta^2)}$	$\frac{2(1-\beta)\delta_1}{4-3\beta^2}$
p_2^i	$m+c_2+q_2^n$	$m+w^c+\frac{(2-2\beta-\beta^2+\beta^3)(2-\theta)\delta_1+\beta T_a}{2(8-5\beta^2)}$	$m+c_1+\frac{(4-4\beta^2+\beta^3)\delta_1}{2(4-3\beta^2)}$
w^i	/	$c_1+\frac{\delta_1(8-6\beta^2+\beta^3+(4-4\beta-2\beta^2+2\beta^3)\theta-2(2-\beta^2)T_a)}{2(8-5\beta^2)}$	/
r^i	/	/	$\frac{(2-\beta)^2\beta\delta_1}{2(4-3\beta^2)^2}$
M^i	/	/	$\frac{(-16+36\beta^2-24\beta^3+3\beta^4+(32-32\beta-20\beta^2+24\beta^3-3\beta^4)\theta)\delta_1^2}{4(4-3\beta^2)^2}$

Where $T_a=\sqrt{((12-8\beta-\beta^2)(1-\theta)+(1-2\beta+\beta^2)\theta^2)}$.

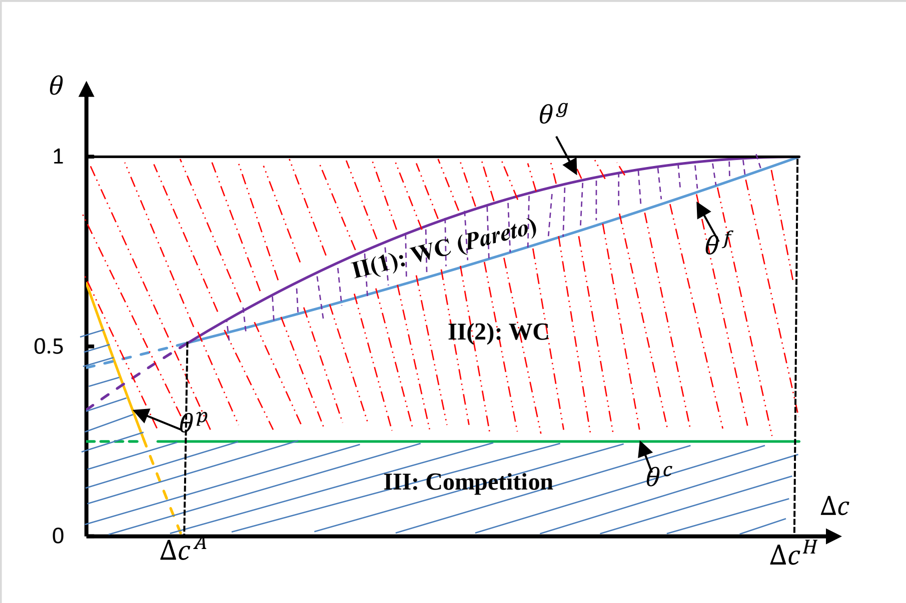


Figure 3.10 Selection of coopetition strategies ($\boldsymbol{\theta}, \Delta \boldsymbol{c}, \boldsymbol{\beta} = \mathbf{1}$)

From Figure 3.9, it is clear that the selection of coopetition strategies maintains the same structural result for the scenario of partial substitutes ($0 < \beta < 1$) regardless of the new demand function. From Figure 3.10, interestingly, although the selection decision between competition and WC is similar to the scenario of partial substitutes, license coopetition is no longer an option for optimal selection of coopetition strategies for the scenario of perfect substitutes ($\beta = 1$). It means that firms should not consider license coopetition if their products are perfectly substitutable. This is due to that θ^j, whose relationship with θ determines the optimal choice between LC and WC, depends upon β. The value of θ^j equals 1 when $\beta = 1$. Perfect substitutes often indicate an intense market competition. This finding is also consistent to the industrial practice that firms do not license key technology to rival firms when there is an intense market competition.

3.7 Managerial relevance and insights

Our research findings are beneficial to firms in industries such as high tech (e.g., smartphone, automobile, PC, and medical devices) that are characterized by rapid technological development and short product life cycles, particularly for those firms currently engaging in some form of cooperation (i.e., buyer-supplier relationships and license agreements) with their competitors or have an intention to do so. In this dynamic and competitive market environment, firms must compete with more-sophisticated strategies rather than simply focusing on product or price. Coopetition has become a viable strategic option as shown in the smartphone, automobile, and pharmaceutical industries. However, firms face a dilemma when cooperating with their competitors. As illustrated in this chapter, production coopetition either through wholesaling or license agreements, on the one hand, provides an extra revenue stream or reduces production cost for the two manufacturers; on the other hand, it incurs a loss in the competition with an enhanced rival for customer demand. Whether firms should opt for coopetition depends upon the tradeoff between the conflicting cooperating and competing forces, which is determined by a combination of external, relationship-specific, and internal factors. Our research comprehensively examines how these factors affect firms' optimal strategy selection decisions and suggests a broad set of decision outcomes that have not been captured in previous studies (Luo et al. 2016; Yang et al. 2017). Based on the findings, we propose a decision framework as illustrated in Table 3.5 to provide some strategic guidance for firms' optimal decisions concerning coopetition strategies.

The decision framework systematically outlines how the external, relationship-specific and internal factors (i.e., $\beta, \theta, and\ \Delta c$**)** affect the strategy selection, which will be useful for firms in a similar business environment to make important strategic decisions. Here, β measures the cross-effect of the change in one manufacturer's product demand caused by a change in that of the other manufacturer. A high degree of product substitution tends to intensify the market competition between two manufacturers. θ characterizes the inter-firm power relationship between manufacturers in the negotiation of the wholesaling or licensing agreement. For instance, firms with superior component production capability should not supply key components or license relevant technology to rival firms when they hold less negotiation/bargaining power, despite the conditions of degree of product substitution and difference in their operational capabilities. When there is more-balanced negotiation power between the rival firms, they should consider wholesaling coopetition if there is a medium level of product substitution between rival firms and opt for competition only if the product substitution level is low or high. Note that this situation is the only one in which the operational capability difference (Δc) will also play a role in influencing the optimal strategic choice between wholesaling coopetition and competition, as discussed in Section 5.2. When they have a more dominant negotiation power, license coopetition should be selected if the degree to which their products are substitutable is low and, conversely, wholesaling coopetition should be chosen.

Table 3.5 Strategic guidance on coopetition

Parameters		Optimal strategic decision				
Product substitution rate ($\boldsymbol{\beta}$)		Low		Medium		High
Manufacturer 1's negotiation power ($\boldsymbol{\theta}$)	Strong	LC		LC/WC		WC
	Similar	C	C/WC	WC	WC/C	C
	Low	C	…	C	…	C
Operational capability difference ($\Delta \boldsymbol{c}$)		Small		Medium		High

Note: C, LC, and WC refer to Competition, License Coopetition and Wholesaling Coopetition, respectively.

Considering the dynamic nature of competition and cooperation dualism (Dorn et al. 2016), coopetition itself will affect the nature of market competition and interfirm relationships. With changing market dynamics, power relationships, and internal operational capacities, firms should regularly examine their optimal coopetition strategy because any change in these factors could alter the outcome of their original strategy selection. With a better understanding of the underlying economic principle that governs the coopetition decision, our research findings could support firms in making correct strategic and operational decisions and improve their business competitiveness.

3.8 Conclusions

This study systematically examines the effect of two coopetition strategies on the performance of two rival manufacturers. By comparing the two manufacturers' prices and profits for competition, wholesaling coopetition, and license coopetition models, we derive notable results that provide a richer representation of firms' strategic behavior concerning coopetition. Our study provides a broader set of decision outcomes that have not been reported by other studies concerning coopetition. Coopetition in the context of wholesaling or license cooperation and pure competition does not necessarily increase profits. Whether the economic effect from the coopetition strategy is positive or negative is determined by the external market characteristics, inter-firm power relationship, and the difference between the rival firms' capabilities and efficiencies of their internal operations. Specifically, we demonstrate the following:

- The optimal decision for the coopetition strategies (e.g., competition vs. coopetition or wholesaling vs. licensing) is determined by the tradeoff between the benefit that is gained from the production cooperation and the losses that are caused by market competition when faced with a strengthened competitor. The benefits of cooperation and the losses incurred from competition are determined by a combination of important external and internal factors including the degree to which their products are substitutable (β), manufacturers' negotiation power (θ), maximum retail prices (α_1, α_2) and cost difference in component production (Δc). These factors depend upon the internal operational and technological capabilities of the involved firms, relationship-specific characteristics, and the external market environment. Essentially, the optimal choice of the coopetition strategy is governed by the dynamic relationship between the cooperating and competing forces, which is also subject to changes in internal operational capabilities and/or the external market environment over time.

- An enduring coopetitive relationship requires that the firms achieve a win-win outcome. When either wholesaling coopetition or license coopetition is the optimal strategy, situations exist in which one of the manufacturers is worse off despite an increase in the total profit between the two manufacturers. In those situations, a further operational mechanism (i.e., profit-sharing contracts) could be designed to achieve a win-win outcome. Furthermore, the difference between the two manufacturers' unit component costs (Δc) profoundly affects whether wholesaling and license coopetition deliver a *Pareto* improvement. A *Pareto improvement* will more likely be achieved if the two competing firms cooperate on an operation function in which there is a substantial difference in efficiency/capability between the two firms.
- We identify that *Pareto improvement* in both wholesaling and license coopetition leads to increased profits for both manufacturers and decreased retail prices as shown in Propositions 3.1 and 3.2. Therefore, coopetition can positively affect individual firms and consumers. This situation is different from collusion, in which firms increase producers' surplus by raising prices and consumers are penalized by the decreasing consumer surplus, which leads to a decrease in social welfare (Rusko 2011). In this case, coopetition is an economically sustainable strategy that benefits both firms and consumers.

Chapter 4 Green technology coopetition models and applications

4.1 Introduction

Climate change is still the most critical global challenge as highlighted by the recent special report on global warming by the Intergovernmental Panel on Climate Change (IPCC). The authors of this landmark report call for urgent and unprecedented changes to reach the target of keeping temperature increase below 1.5°C above pre-industrial levels in order to reduce the risks to human well-being, ecosystems and sustainable development (IPCC 2018). To achieve national carbon emissions reduction targets, many governments have implemented various emissions control policies such as mandatory carbon emission capacity, carbon emission tax, cap-and-trade. Among these policies, cap and trade is one of the most influential emissions trade schemes and has been widely adopted by many places worldwide, including the European Union, New Zealand, and California, as well as pilot programs in China and Kazakhstan (İşlegen and Reichelstein 2011; Grubb 2012; Newell et al. 2014). For example, as a key part of meeting the European Union’s (EU) emissions reduction target, the EU Emission Trading System (ETS) was implemented in 2005, and it is the largest multi-country, multi-sector greenhouse gas emissions trading system worldwide (Grubb 2012). Despite only being in the development stage, China’s ETS pilots have covered 743 MT of CO2 emitted by more than two thousand firms, second only to the EUETS (Zhang et al. 2014).

Meanwhile, consumers have become more aware of environmental issues, and purchasing low-carbon products is an overwhelming trend among the public (Olsen et al. 2014; Wang et al. 2017). The scrutiny from the media and NGOs has also made firms more mindful in managing their reputational risks (Castka and Corbett 2016a; Castka and Corbett 2016b). Increased pressures from different stakeholders have led firms to incorporate a range of sustainability practices into their products, processes and supply chains (Klassen and Vachon 2003; Caro et al. 2013; Drake et al. 2016). An increasing number of companies have been investing in low-carbon technologies and innovations to make their products and processes more carbon efficient to gain competitive advantages. Another remarkable shift toward low-carbon technologies is that many rival firms form strategic alliance in the global fight against climate change. In 2016, a $1 billion fund was created in technologies investment by the Oil and Gas Climate Initiative, a group that comprises ten of the world's largest oil companies that would reduce carbon emissions from oil and natural gas (Pandey 2016). Furthermore, major U.S. companies, including Facebook and Microsoft, have formed an alliance, the Renewable Energy Buyers Alliance, to promote the development of 60 gigawatts of renewable energy by 2025 (Shallenberger 2016). A Greenpeace report published in 2017 praised technology giants Apple, Facebook, and Google for using an increasing amount of renewable energy to power their data centers (Greenpeace 2017).

Wide access to low carbon technologies is crucial to achieve carbon emissions reduction targets. One significant strategic response from the industry sector is low carbon technology licensing among the industrial competitors. For instance, in the automotive industry, Ford offers to license its electrified vehicle technology to other automakers despite being sworn rivals in the hybrid and electric vehicle (HEV) market (Atiyeh 2015). In February 2017, Lenovo, the world's leading PC manufacturer, announced the breakthrough of an innovative low-temperature solder manufacturing process that will reduce carbon emissions by 35%, compared with traditional manufacturing processes (Lenovo Newsroom 2017). The CEO of Lenovo also expressed in the news report that Lenovo would license this technology to other manufacturers. Technology licensing, defined as technology owner (licensor) selling the rights of using its technology for a fixed fee and/or royalty to a sourcing firm (licensee), has become a popular form of interfirm technology transfer and commercialization (Khoury et al. 2018). However, there is a dilemma embedded in technology licensing especially the two trading parties are market rivals (Fosfuri 2006; Wu 2018). From the licensor's perspective, it is the trade-off between the revenue increase from the licensing payments and the reduced profit margin and/or reduced market share implied by increased competition from the licensee. From the licensee's perspective, it is the trade-off between the cost of license payment and the increase profit margin and/or market share implied by licensed technology enhanced market competitiveness.

There is an emerging stream of literature that discusses the importance of technology licensing contractual choices between the licensor and licensee. Previous research addressing this general question has focused on competitively sensitive issues such as the coordination between contractual partners (Gulati et al 2005), experience and signaling value (Kotha et al. 2018) and the contracts governing these agreements (Ariño et al. 2014). There are also mixed views regarding those commonly used technology licensing contractual arrangements: royalty, upfront fixed-fee or a mixture of royalty and upfront fixed-fee (Bagchi and Mukherjee 2014; Hong et al. 2017). Furthermore, few studies have investigated the licensing contractual issues in the context of low carbon technologies, which adds the environmental dimension to this already complex problem. Furthermore, despite a growing number of studies that have acknowledged the benefits of environmental collaboration between competitors (Klassen and Vachon 2003; Caro et al. 2013; Luo et al. 2016), very little attention has been paid to explore the effectiveness of the licensing contractual design and inter-firm relationship (e.g. bargain power and differentiation) in accelerating green technology adoption for low-carbon economy. Our research aims to fill this gap by addressing the following key questions:

- Should firms license low carbon technologies to their fierce market rivals? If so, which is the best licensing contract arrangement among royalty, fixed fee and a mixture of royalty and fixed fee?
- How does the alternative contractual designs of low carbon technology licensing affect economic, environmental and social performance?
- How to design government policies to promotion technology diffusion for a low carbon economy.

To answer these questions, we focus on two rival manufacturers that produce substitutable products with different carbon emissions efficiencies of their production processes. In addition to a purely competitive relationship, one manufacturer can adopt its rival firm's (green innovator) low carbon technology to reduce its unit carbon emissions through different forms of licensing contractual agreement including royalty payment (Sen 2005; San Martín and Saracho 2010), a fixed license fee (Sen and Tauman 2007; Sen and Stamatopoulos 2016) and a mixture of royalty and fixed fee (Kim and Lee 2014; Khoury et al. 2018). Through analysis of the equilibriums for four different game theoretical models, we provide some key insights. First, the contractual choice on low carbon technology licensing is determined by the trade-off between the benefits gained from technology licensing and the consequential losses incurred from competition with a strengthened competitor. This decision is influenced by a combination of factors including internal operational and technological capability, interfirm power relationship, external market characteristics and the carbon emission control policy. Among them, the interfirm power relationship is more influential in determining the optimal decision on the low carbon technology licensing. Second, although firms' decision on whether and how to license their low carbon technologies is mainly determined by their economic benefit, these decisions also have profound impact on the environment and consumers. In general, licensing through mixed fees or royalty fees improve environmental performance when they increase economic benefits collectively, consumers may have to pay extra prices for mixed-fees licensing but not necessarily for royalty-fee licensing. Furthermore, firms' optimal decision may change overtime according to the alteration of the internal operational and technological capability, external market and policy environment, or interfirm power relationship.

Several contributions are made in the second study of this doctorial research. First, our research contributes to the green technology licensing literature (Kim and Lee 2014, 2016; Hu et al. 2017) by providing a better understanding of how various contractual arrangement of low carbon technology licensing can contribute to low-carbon manufacturing. Our systematic examination, in a structured manner, provides manufacturing firms with strategic guidance on whether/how to engage low carbon technology licensing with rival firms considering their unique internal operational and technological capabilities, interfirm relationship, market competition, and policy circumstances. Second, our research complements the coopetition literature by extending its applications to low-carbon manufacturing in the context of low carbon technology licensing (Luo et al. 2016; Hafezalkotob 2017). We argue that to sustainable coopetitive relationship requires an improvement in both economic and environmental performance as well as a win-win outcome for individual firms and consumers. Finally, our research also makes important practical and policy contributions. For manufacturing firms, our findings could support them in making optimal strategic and operational decisions regarding low carbon technology licensing and improving their competitiveness. For policy makers, our findings could help them to develop appropriate carbon emissions control policies that support a sustainable, low-carbon economy.

The remainder of this chapter is structured as follows. Section 4.2 provides a review of relevant research streams. Subsequently, the competition and coopetition models and equilibrium analysis are presented in Section 4.3. We examine the impacts of Royalty Licensing (RL) coopetition, Fixed-fee Licensing (FL) coopetition and Mixed Licensing (ML) coopetition on the manufacturers, environment and consumers in Sections 4.4, 4.5 and 4.6, respectively. In Section 4.7, we analyze the optimal selection of coopetition strategies from the manufacturers' perspective. Section 4.8 extends the analysis to the case of asymmetric-manufacturer and the case of both partial and perfect substitutions, and examines their effect on the coopetition decision respectively. Finally, we discuss the key findings in Section 4.9.

4.2 Literature review

Our study is related to three streams of research: (1) technology licensing; (2) technology licensing in green cooperation; and (3) coopetition in the low-carbon economy.

There is an emerging stream of literature that discusses the importance of technology licensing contractual choices between the licensor and licensee. Technology licensing is often arranged by means of a royalty, a fixed fee, or even combination of the two, and there is ongoing debate in the literature about which is superior (Wang 1998; 2002; Sen 2005). There are different views in the existing technology licensing literature regarding contractual features such as royalty versus fixed-fee license (Bagchi and Mukherjee 2014; Hong et al. 2017) and exclusive versus nonexclusive license (Aulakh et al. 2010; Khoury et al. 2017). For instance, Bagchi and Mukherjee (2014) examined the two popular licensing schemes (royalty and fixed-fee) used by a technology innovator and multiple licensees and found that the innovator and consumers can benefit more from royalty-based licensing than that under fixed-fee licensing. Wu (2018) followed the finding of Bagchi and Mukherjee (2014) and consider technology licensing with a pure royalty policy in the investigation of the effects of price competition and licensing on product innovation decisions. In contrast, Hong et al. (2017) found in their examination of technology licensing in the context of a closed-loop supply chain that fixed-fee licensing is superior for the licensor than royalty-based licensing. These differences can be explained by the licensing dilemma heighted in Fosfuri's (2006) empirical investigation of the determinants of the rate of technology licensing, in which, the author argued that technology license holder must balance the trade-off between the revenue from licensing payments and the lower price-cost margin and/or reduced market share triggered by increased competition from the licenses. This is in line with the view of those technology licensing studies (Aulakh et al. 2010, 2013; Khoury et al. 2017) on the contractual choice between exclusivity and nonexclusively that the trade-off between expected revenues and the associated costs determines the licensor's strategic choice of contracting with one versus multiple licensees. Different to the above studies, we explore the licensing contractual issues between two rival firms in the context of low carbon technologies, which incorporating the environmental performance into this already complex problem.

Regarding licensing of low-carbon technology, evidence from the wind power industry has shown that licensing is the most direct channel of technology diffusion (Dechezleprêtre and Glachant 2014). In the context of technology licensing under carbon emissions control policies, Harrori (2017) explored the optimal solutions of carbon emissions taxation policies under a royalty contract of low-carbon technology licensing. The paper demonstrated that the optimal social welfare could be achieved by the combination of emissions taxes and R&D subsidies. More relevant to this research, Kim and Lee (2014, 2016) studied the different patent licensing agreements of eco-technology between an innovator and oligopolistic polluting firms on the welfare performances under the carbon taxation. Their analysis of different arrangements including royalties, fixed fees, and auction licensing contracts, showed that a non-exclusive contract would increase welfare, depending on the level of emission taxation and the gap in production cost. However, the inverse demand function adopted in their study is over simplified and does not consider important factors (e.g., price elasticity of demand, product substitution level, and power relationship in the contractual negotiation) that could make a substantial impact on firms' decisions and performances. Different to their studies, we incorporate these factors and investigate the technology licensing agreement between the two rival manufacturers under the cap-and-trade policy. The complexity of this kind of bilateral coopetitive relationship lies in the fact that on the hand, both parties benefit from the license cooperation as the licensee gets extra revenue income and the license receiver reduces its carbon emission, which can transform into an improvement of economic performance; and on the other hand, the green cooperation may also result in financial loss in the market competition with a strengthened competitor. It is the simultaneous competing and cooperating forces, the trade-off between the resulted financial benefits and losses, and the impact of the strategic (competition vs coopetition) and operational (e.g. alternative forms of licensing) choice on the environment and consumers that make this study different and worthy of investigation.

The simultaneous competing and cooperating forces involved in technology licensing between rival firms is also closely associated with the notion of coopetition, which is described as simultaneously pursuing competition and cooperation between two or more firms (Brandenburger and Nalebuff 1996; Mantovani and Ruiz-Aliseda 2016). Several studies explored the impact of vertical cooperation between a manufacturer and a downstream retailer or upstream suppliers in the context of low-carbon economy (Park et al. 2015; Hafezalkotob 2017; Ji et al. 2017). For instance, Park et al. (2015) considered three different market competition settings and showed that competitive settings affect the effectiveness of different emissions regulations, e.g., carbon taxes or cap and trade, in improving social welfare. Hafezalkotob (2017) applied the coopetition concept to the supply chain level and examine the best response strategies (i.e., competition, coopetition and cooperation) of chains under different government policies. Ji et al. (2017) investigated cooperation between a manufacturer and a retailer with online and offline shops under a cap-and-trade policy and examined initial carbon allowance allocation rules by modeling supply chain firms' emissions reduction behaviors and profits as well as social welfare. The above studies mainly focused on vertical cooperation between supply chain parties. Note that in contrast to the above study, we explore a horizontal cooperation in green technology between two rival manufacturers.

Among the most relevant studies, Carfì and Schiliro (2012) applied the complex construct of coopetition at the macroeconomic level to address the challenges of climate change. Their study proved that coopetition is able to deliver win-win outcomes for participating countries in seeking the implementation of low-carbon economies. At the microeconomic level, Luo et al. (2016) investigated the role of coopetition in delivering low-carbon manufacturing under a cap-and-trade policy. Their analysis showed that the coopetitive strategy is a viable strategy for increasing profits and reducing total carbon emissions by participating firms. However, in their study, the cooperative relationship was articulated as a joint decision on green investment and pricing between two rival manufacturers. It is a special form of coopetition requiring a high degree of trust between the engaging firms. A joint pricing decision between two rival manufacturers can be regarded as a collusive behavior to gain an unfair market advantage. Furthermore, in contrast with the work of Luo et al. (2016), rival firms make their pricing decisions independently in this paper, and coopetition is explicitly expressed and modeled in our study, wherein a manufacturer (green innovator) licenses green technology to a rival manufacturer in the form of a fixed-fee, royalty or a combination of two. These forms of technology licensing have been widely adopted in many industries such as automotive and steel production. Another closely related research is Hu et al. (2017), who extended the investigation of the effects of technology sharing strategies to the upstream supplier and found that open technologies intensify future competition between the rival manufacturers but can induce supplier investments. Their study focuses on technology in the context of electronic vehicle technology but does not take into account the carbon emission control policies such as cap-and-trade policy considered in this study. Furthermore, open technology policy can be considered as one specific case of technology licensing, in which licensing fee is assumed to be zero. In practice, many companies still charge licensing fees through royalty or fixed fee when they open technologies to rival firms or supply chain partners.

4.3 The models and equilibrium analysis

4.3.1 The models

Two rival manufacturers are considered in this study that produce substitutable products and compete in the same market. The manufacturers operate in a market regulated by cap and trade. It is common in Europe and some parts of China and the U.S. (e.g., California) that major carbon emitters, such as power plants and steel makers, are regulated by cap-and-trade policies (Grubb 2012; Barrieu and Fehr 2014). Under the policy, on the one hand, manufacturers can buy shortage quotas from the outside market if they exceed the initial carbon emissions allowance cap imposed by the government. On the other hand, if manufacturers emit less carbon than the cap, they can sell surplus quotas to the outside market. We assume that the two manufacturers have different unit carbon emissions from their production processes. Without loss of generality, we assume that manufacturer 1 is a green technology innovator and generates fewer unit product carbon emissions from production processes, and manufacturer 2 emits more unit product carbon emissions from production. Before outlining the models, the notations are presented in Table 4.1 as follows.

Table 4.1 Notations

Notation	Description
q_1, q_2	Demand for manufacturers 1 and 2
c	Unit production cost for manufacturer 1 and 2
p_1, p_2	Unit product price for manufacturers 1 and 2
e_1, e_2	Unit carbon emissions from production processes for manufacturers 1 and 2, $e_1 < e_2$
Δe	Difference in unit carbon emissions from production between manufacturers: $\Delta e = e_2 - e_1 > 0$
T_1, T_2	Total carbon emissions for manufacturers 1 and 2
T	Total carbon emissions for both manufacturers, that is, $T = T_1 + T_2$

K	Carbon emissions cap, $K > 0$
λ_0	Unit carbon emission trade price, $\lambda_0 > 0$
E_1, E_2	Carbon emissions trading quantities with the outside market for manufacturers 1 and 2. $E_i > 0$ indicates that manufacturers buy their shortage quotas from the outside market, $E_i < 0$ indicates that manufacturers sell their remaining quotas to the outside market, $i = 1, 2$
δ_1	Maximum marginal profit for manufacturers 1, that is, $\delta_1 = \alpha - c - \lambda_0 e_1 > 0$
λ	Royalty rate
M	Fixed license fee
π^n	Manufacturers' total profits in the competition model, that is, $\pi^n = \pi_1^n(q_1) + \pi_2^n(q_2)$
π^r	Manufacturers' total profits in the royalty licensing coopetition model, that is, $\pi^r = \pi_1^r(q_1) + \pi_2^r(q_2)$
π^f	Manufacturers' total profits in the fixed-fee licensing coopetition model, that is, $\pi^f = \pi_1^f(q_1) + \pi_2^f(q_2)$
θ	Manufacturer 1's market power, $0 \le \theta \le 1$

We use the following demand function, which is widely adopted in the marketing and operations management literature (e.g., Padmanabhan and Png 1997; Cai 2010; Shang et al. 2016).

$$p_i = \alpha - \beta(q_i + q_j), \quad i, j = 1,2 \text{ and } i \neq j.$$

Here, α represents manufacturers' maximum product price and β measures the price elasticity of demand.

The research framework is illustrated in Figure 4.1, in which four models are considered representing four different relationships between two manufacturers: competition, royalty licensing (RL) coopetition, fixed-fee licensing (FL) coopetition and mixed licensing (ML) coopetition. These forms of the licensing arrangement are common in practice (Sen 2005; Sen and Stamatopoulos 2016). We assume two economically rational firms who act strategically to maximize their own profits. For the benchmark competition model, manufacturers produce their products with their own technologies, and there is only a competitive relationship that is production quantity competition by simultaneously choosing production quantities. For the RL coopetition model, FL coopetition model and ML coopetition model, manufacturer 2 obtains from manufacturer 1 for a license to use its green technology in the production process by only paying a royalty rate, only a fixed fee and both a royalty rate and a fixed fee, respectively. The two manufacturers compete with each other for market demand, but they have a cooperative relationship in an agreement to use green technology in their production.

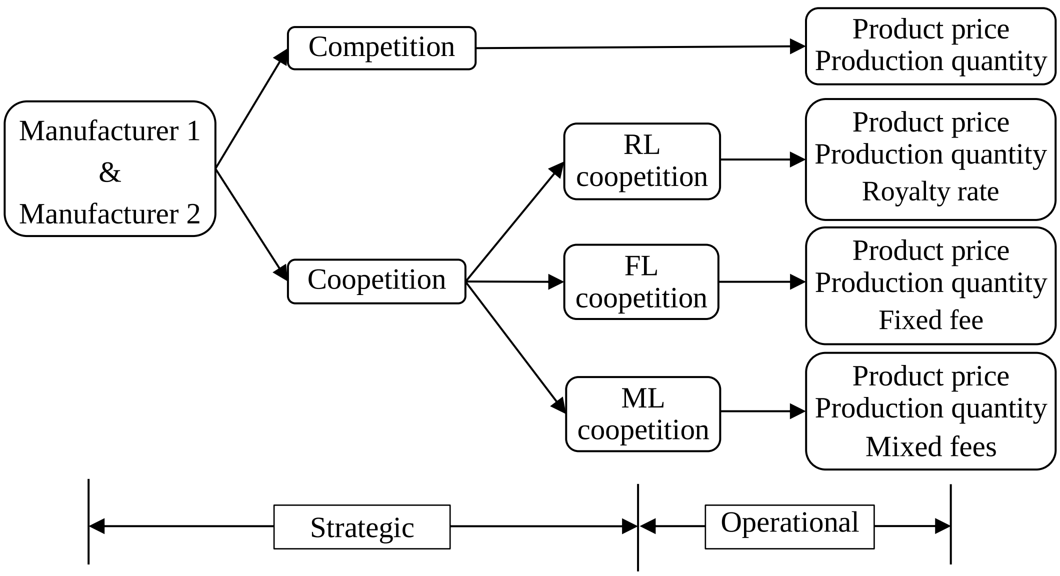


Figure 4.1 The framework

4.3.2 Competition model

First, the competition model is presented as a benchmark. In the competition model, two independent manufacturers simultaneously decide on their production quantities to maximize their own profits. For the competition model, manufacturer 1's profit $\pi_1^n(q_1)$ is

$$\pi_1^n(q_1) = [\alpha - \beta(q_1 + q_2) - c]q_1 - \lambda_0 E_1 \tag{4-1}$$

The first part of Equation (4-1) is manufacturer 1's profit from product sales, and the second part represents manufacturer 1's cost/revenue of buying/selling carbon emissions quotas from/to the outside market.

Similarly, manufacturer 2's profit $\pi_2^n(q_2)$ for the competition model is

$$\pi_2^n(q_2) = [\alpha - \beta(q_1 + q_2) - c]q_2 - \lambda_0 E_2 \tag{4-2}$$

The decision problem faced by manufacturer 1 is

$$\max_{q_1} \pi_1^n(q_1)$$

$$s.t \quad q_1 e_1 - E_1 = K$$

Similarly, the decision problem faced by manufacturer 2 is

$$\max_{q_2} \pi_2^n(q_2)$$

$$s.t \quad q_2 e_2 - E_2 = K$$

For the competition model, the decision problems faced by manufacturers 1 and 2 are

$$\left.\begin{array}{c} \max\limits_{q_1} \ \pi_1^n(q_1) \\ s.t \ \ q_1 e_1 - E_1 = K \\ \max\limits_{q_2} \pi_2^n(q_2) \\ s.t \ \ q_2 e_2 - E_2 = K \end{array}\right\}$$

Table 4.2 lists the optimal production quantities (q_1^n, q_2^n) for the two manufacturers in the competition model. The derivation of the corresponding optimal solutions is provided in the Appendix.

4.3.3 Royalty licensing coopetition model

For the RL coopetition model, there is a cooperative relationship between the two rival manufacturers. More specifically, manufacturer 1 licenses its green technology to manufacturer 2 while competing for the same market. As a result, manufacturer 2 reduces its unit carbon emissions from its production process to the same level as manufacturer 1 and pays manufacturer 1 a royalty fee at the rate of λ per unit. For example, Ford offers to license its electrified vehicle technology to rival automakers today, and only a decade earlier, Ford had to pay royalties to license hybrid technology from Toyota (Atiyeh 2015). The decision sequence of the two manufacturers is described as follows: they negotiate the royalty rate ($\lambda > 0$) for licensing green technology to manufacturer 2, and once they agree, the production quantities $q_i \geq 0$ are determined independently and simultaneously by manufacturer 1 and 2 to maximize their own profits. Then, the two manufacturers receive their revenues and profits accordingly when demand from end consumers is realized.

For the RL coopetition model, manufacturer 1's profit $\pi_1^r(q_1)$ is

$$\pi_1^r(q_1) = [\alpha - \beta(q_1 + q_2) - c]q_1 + \lambda q_2 - \lambda_0 E_1 \tag{4-3}$$

The first part of Equation (4-3) is the profit from product sales, the second part is the royalty rate paid by manufacturer 2 and the last part is the cost/revenue of trading carbon emissions with the outside market.

Similarly, for the RL coopetition model, manufacturer 2's profit $\pi_2^r(q_2)$ is

$$\pi_2^r(q_2) = [\alpha - \beta(q_1 + q_2) - c]q_2 - \lambda q_2 - \lambda_0 E_2 \tag{4-4}$$

The royalty rate negotiation process for the RL coopetition model as follows

$$\max_{\lambda} \pi^r(\lambda) = \max_{\lambda}[\pi_1^r(q_1(\lambda))]^{\theta}[\pi_2^r(q_2(\lambda))]^{1-\theta} \tag{4-5}$$

The decision problems faced by manufacturers 1 and 2 are

$$\left.\begin{array}{c}\max\limits_{\lambda}\pi^r(\lambda)\\ s.t\ \ q_1e_1-E_1=K\\ q_2e_1-E_2=K\end{array}\right\}\rightarrow\left.\begin{array}{c}\max\limits_{q_1}\pi_1^r(q_1)\\ s.t\ \ q_1e_1-E_1=K\\ \max\limits_{q_2}\pi_2^r(q_2)\\ s.t\ \ q_2e_1-E_2=K\end{array}\right\}$$

Manufacturer 1's optimal production quantity (q_1^r) and optimal royalty rate (λ^r) and manufacturer 2's optimal order quantity (q_2^r) for the RL coopetition model are shown in Table 4.2.

4.3.4 Fixed-fee licensing coopetition model

For the FL coopetition model, there is also a cooperative relationship in the form of a green technology licensing arrangement. Compared to RL coopetition model, the difference is that manufacturer 1 and manufacturer 2 negotiate the fixed fee ($M > 0$) for licensing the green technology while still competing in the same market. For example, the Haier Group, one of world's leading manufacturers of consumer electronics and home appliances, increases company revenues through fees to license its low-carbon technologies to rival firms (SIPO 2016). In this FL coopetition model, manufacturers' decision sequences are similar to that of the RL coopetition model, except that manufacturer 1 announces a fixed fee (M) for licensing the green technology in the first stage.

For the FL coopetition model, manufacturer 1's profit $\pi_1^f(q_1)$ is

$$\pi_1^f(q_1)=[\alpha-\beta(q_1+q_2)-c]q_1+M-\lambda_0E_1 \tag{4-6}$$

The first part of Equation (4-6) is the profit from product sales, the second part is the fixed licensing fee received from manufacturer 2 and the third part is the cost/revenue of trading carbon emissions with the outside market.

Similarly, for the FL coopetition model, manufacturer 2's profit $\pi_2^f(q_2)$ is

$$\pi_2^f(q_2)=[\alpha-\beta(q_1+q_2)-c]q_2-M-\lambda_0E_2 \tag{4-7}$$

The fixed fee negotiation process for the FL coopetition model as follows

$$\max_M\pi^f(M)=\max_M\left[\pi_1^f(q_1)\right]^{\theta}\left[\pi_2^f(q_2)\right]^{1-\theta} \tag{4-8}$$

The decision problems faced by manufacturers 1 and 2 are

$$\left.\begin{array}{c}\max\limits_{M}\pi^f(M)\\ s.t\ \ q_1e_1-E_1=K\\ q_2e_1-E_2=K\end{array}\right\}\rightarrow\left.\begin{array}{c}\max\limits_{q_1}\ \pi_1^f(q_1)\\ s.t\ \ q_1e_1-E_1=K\\ \max\limits_{q_2}\pi_2^f(q_2)\\ s.t\ \ q_2e_1-E_2=K\end{array}\right\}$$

The optimal production quantities (q_1^f, q_2^f) and optimal fixed fee (M^f) for the FL coopetition model can be found in Table 4.2.

4.3.5 Mixed licensing coopetition model

For the ML coopetition model, we consider that manufacturer 1 licenses the green technology to manufacturer 2 by two-part tariff strategy so that we can compare the profit of the RL, FL coopetition models with that of ML coopetition model. Similarly, compared to RL, FL coopetition model, there still exists a cooperative relationship in a licensing arrangement and competitive relationship in downstream market. At this time, the operational decisions made by two manufacturers are considered as follows: in the first stage, they negotiate the royalty rate and fixed fee of licensing technology. Then the next stage is the same sequence of events discussed for the RL and FL coopetition models.

For the ML coopetition model, manufacturer 1's profit $\pi_1^l(q_1)$ is

$$\pi_1^l(q_1) = [\alpha - \beta(q_1 + q_2) - c]q_1 + \lambda q_2 + M - \lambda_0 E_1 \tag{4-9}$$

The first part of Equation (4-9) is the profit from product sales, the second and the third part represent the profit from the royalty rate and fixed fee respectively paid by manufacturer 2. The last part is the cost/revenue of trading carbon emissions with the outside market.

Similarly, for the ML coopetition model, manufacturer 2's profit $\pi_2^l(q_2)$ is

$$\pi_2^l(q_2) = [\alpha - \beta(q_1 + q_2) - c]q_2 - \lambda q_2 - M - \lambda_0 E_2 \tag{4-10}$$

The two-part tariff negotiation process for the ML coopetition model as follows

$$\max_{\lambda,M} \pi^l(\lambda, M) = \max_{\lambda,M} [\pi_1^l(\lambda, M)]^\theta [\pi_2^l(\lambda, M)]^{1-\theta} \tag{4-11}$$

The decision problems faced by manufacturers 1 and 2 are

$$\left.\begin{array}{c} \max\limits_{\lambda,M} \pi^l(\lambda, M) \\ s.t \quad q_1 e_1 - E_1 = K \\ q_2 e_1 - E_2 = K \end{array}\right\} \rightarrow \left.\begin{array}{c} \max\limits_{q_1} \ \pi_1^l(q_1) \\ s.t \ \ q_1 e_1 - E_1 = K \\ \max\limits_{q_2} \pi_2^l(q_2) \\ s.t \ \ q_2 e_1 - E_2 = K \end{array}\right\}$$

The optimal production quantities (q_1^l, q_2^l) and optimal license fee (λ^l, M^l) for the ML coopetition model are provided in Table 4.2.

Table 4.2 Optimal decisions of the four models

Models	Competition model ($i=n$)	RL coopetition model ($i=r$)		FL coopetition model ($i=f$)	ML coopetition model ($i=l$)
	$0<\Delta\mathrm{e}<\Delta\mathrm{e}^n$	$\frac{4}{9}<\theta<\theta_0$	$\theta_0 \le \theta \le 1$	$\frac{1}{2}<\theta\le 1$	$\theta_1<\theta\le 1$
q_1^i	$\frac{\delta_1+\Delta\mathrm{e}\lambda_0}{3\beta}$	$\frac{5\delta_1-\sqrt{5}\sqrt{(1-\theta)\delta_1^2+(4-9\theta)K\beta\lambda_0}}{10\beta}$	$\frac{\delta_1}{2\beta}$	$\frac{\delta_1}{3\beta}$	$\frac{\delta_1}{2\beta}$
q_2^i	$\frac{\delta_1-2\Delta\mathrm{e}\lambda_0}{3\beta}$	$\frac{\sqrt{(1-\theta)\delta_1^2+(4-9\theta)K\beta\lambda_0}}{\sqrt{5}\beta}$	0	$\frac{\delta_1}{3\beta}$	0
λ^i	/	$\frac{5\delta_1-3\sqrt{5}\sqrt{(1-\theta)\delta_1^2+(4-9\theta)K\beta\lambda_0}}{10}$	$\frac{1}{2}\delta_1$	/	$\frac{1}{2}\delta_1$
M^i	/	/		$\frac{(\delta_1^2+9K\beta\lambda_0)(2\theta-1)}{9\beta}$	$\frac{(8K\beta\lambda_0+\delta_1^2)\theta-\delta_1^2-4K\beta\lambda_0}{4\beta}$

Where $\Delta e^n=\frac{\delta_1}{2\lambda_0}$, $\theta_0=\frac{\delta_1^2+4K\beta\lambda_0}{\delta_1^2+9K\beta\lambda_0}$ and $\theta_1=\frac{\delta_1^2+4K\beta\lambda_0}{\delta_1^2+8K\beta\lambda_0}$.

4.4 Competition vs. Royalty licensing coopetition

In this section, the effects of the RL coopetition strategy on optimal maximum profits, retail prices, and total carbon emissions for both manufacturers are examined by a comparison of the derived equilibrium solutions for the Cournot competition model and the RL coopetition model.

4.4.1 Effect of RL coopetition on maximum profits

First, we explore the effect of RL coopetition on the manufacturers' maximum profits and present the following proposition.

Proposition 4.1: ***(1) If*** $\boldsymbol{0 < \Delta e < \Delta e^m}$ ***and*** $\boldsymbol{\frac{4}{9} < \theta \leq 1}$***; or*** $\boldsymbol{\Delta e^m < \Delta e < \Delta e^n}$ ***and*** $\boldsymbol{\theta_3 < \theta \leq 1}$***, then RL coopetition is the preferred strategy; otherwise, competition is the preferred strategy.***

(2) When RL coopetition is the preferred strategy, if $\boldsymbol{0 < \Delta e < \Delta e^n}$ ***and*** $\boldsymbol{\theta_4 < \theta < \theta_2}$***, then RL coopetition strategy realizes a Pareto improvement.***

(3) For the Pareto improvement RL coopetition strategy, $\boldsymbol{p^r < p^n}$***.*** [10]

This proposition indicates that the relationship between the two manufacturers' maximum profits in the RL coopetition model and in the competition model is determined by the differences in the unit carbon emissions from production between manufacturers (Δe) and manufacturer 1's bargaining power factor (θ), as illustrated in Figure 4.2.

Figure 4.2 specifies three decision regions. In **Region I(1),** both manufacturers' maximum profits for the RL coopetition model are greater than are those for the competition model, which implies that the royalty licensing coopetition can lead to *Pareto improvement*. As a result, both firms will embrace such a cooperative relationship. From part (3) of Proposition 4.1, it is clear that in the *Pareto improvement* region, consumers can also benefit from the RL coopetition as retail prices of both retailers are lower than those in the competition model. Therefore, royalty licensing coopetition has a positive impact on the engaging firms and consumers.

In **Region I(2),** the conditions are specified for the case that one of the two manufacturers will be worse off in the RL coopetition model despite an increase in the total profit ($\pi^r > \pi^n$) as compared

---

[10] Where $\Delta e^m = \frac{2\delta_1}{5\lambda_0}$, $\Delta e^n = \frac{\delta_1}{2\lambda_0}$, $\theta_2 = \frac{4(\delta_1^2+9K\beta\lambda_0+5\Delta e\delta_1\lambda_0-5\Delta e^2\lambda_0^2)}{9(\delta_1^2+9K\beta\lambda_0)}$, $\theta_3 = \frac{4(\delta_1^2+9K\beta\lambda_0-10\Delta e\delta_1\lambda_0+25\Delta e^2\lambda_0^2)}{9(\delta_1^2+9K\beta\lambda_0)}$, $\theta_4 = \frac{4(\delta_1^2+9K\beta\lambda_0+2\Delta e\delta_1\lambda_0+\Delta e^2\lambda_0^2)}{9(\delta_1^2+9K\beta\lambda_0)}$ and $\theta_5 = -\frac{4}{9e_1^2(\delta_1^2+9K\beta\lambda_0)}[20\Delta e^4\lambda_0^2 + 20\Delta e^3\lambda_0(e_1\lambda_0 - \delta_1) + 5\Delta e^2(\delta_1^2 - 4e_1\delta_1\lambda_0 + e_1^2\lambda_0^2) + 5(\delta_1 - e_1\lambda_0)e_1\delta_1\Delta e - e_1^2(\delta_1^2 + 9K\beta\lambda_0)]$..

to the competition model. Under this circumstance, the manufacturer experienced decrease in profit through royalty licensing coopetition, has no intention to hold on the cooperative relationship. In this case, the cooperation through the royalty licensing can only continue if the increase of the total profit can be more balanced distributed between the two parties. A *Pareto* improvement can only be realized if a further cooperation mechanism such as a profit sharing contract is considered.

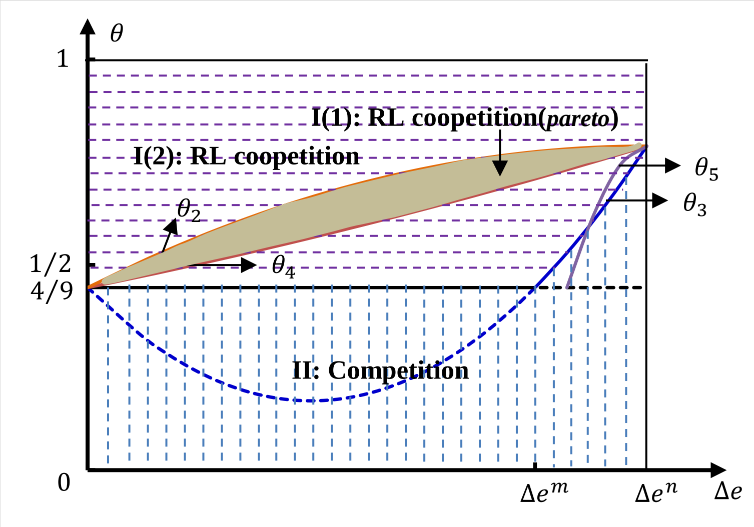


Figure 4.2 Effect of the RL coopetition strategy

In **Region II,** it outlines the conditions under which competition is the preferred strategy. For instance, if manufacturer 1's negotiation power is less than $\frac{4}{9}$, then $\lambda^r < 0$. It is not realistic for manufacturer 1 to license technology to its rival through a negative royalty rate, therefore, royalty licensing coopetition is not feasible. Moreover, if manufacturer 1's negotiation power is in the range of $\frac{4}{9} < \theta < \theta_3$, then the total profit of the two manufacturers will be less in the RL coopetition model than in the competition model ($\pi^r < \pi^n$). In such a case, competition is again the preferred strategy.

It is clear from the analysis that interfirm power relationship (e.g., θ) is a more dominant factor in determining whether firms should license their low carbon technology to their market rivals through royalty. It is more likely for rival firms to benefit from the licensing coopetition when the license holder has more power over the licensee in the licensing contractual negotiation. It is better for the smaller firms to hold on their technological advantage as the revenue generated from technology licensing may not weigh off the loss incurred in the market competition with the licensee. This finding is supported by industrial practices that it is often the industrial leaders such as Ford, Toyota, and Lenovo license their low carbon technologies to market rivals (Atiyeh 2015; Lenovo Newsroom 2017). In contrast, less powerful firms are more likely to hold on their key technologies to enhance market competitiveness. Interestingly, when the technological gap (Δe) between the two firms exceeds a critical value ($\Delta e > \Delta e^m$), further increase of this technological gap will also increase the threshold of licensor's negotiation power (θ) that determines royalty licensing coopetition as an optimal strategy.

4.4.2 Effect of RL coopetition on optimal retail prices

Next, the effect of RL coopetition on manufacturers' optimal retail prices is presented as the following lemma.

Lemma 4.1: ***If*** $\boldsymbol{0 < \Delta e < \Delta e^n}$ ***and*** $\boldsymbol{\frac{4}{9} < \theta < \theta_2}$***, then*** $\boldsymbol{p^r < p^n}$***; if*** $\boldsymbol{0 < \Delta e < \Delta e^n}$ ***and*** $\boldsymbol{\theta_2 < \theta \leq 1}$***, then*** $\boldsymbol{p^r > p^n}$***.***[11]

Lemma 4.1 indicates that the RL coopetition strategy can push up or down both manufacturers' retail prices depending on the differences in the unit carbon emissions of production between manufacturers (Δe) and manufacturer 1's negotiation power (θ). More specifically, if Δe is smaller than this critical threshold (Δe^n), then with a small value of θ ($\frac{4}{9} < \theta < \theta_2$), RL coopetition results in a decrease of the optimal retail prices and therefore it is beneficial to the customers. With a large value of θ, then the retail prices in RL coopetition model are higher than that in competition model, which is harmful to consumers.

4.4.3 Effect of RL coopetition on the total carbon emissions for both manufacturers

Finally, we obtain the effect of RL coopetition on the manufacturers' total carbon emissions.

Corollary 4.1: ***If*** $\boldsymbol{0 < \Delta e < \Delta e^n}$ ***and*** $\boldsymbol{max\{\frac{4}{9}, \theta_5\} < \theta \leq 1}$***, then*** $\boldsymbol{T^r < T^n}$***; otherwise*** $\boldsymbol{T^r > T^n}$***.***[12]

Within the feasible region ($0 < \Delta e < \Delta e^n$), whether the RL cooperation model or the competition model makes less total carbon emissions from two manufacturers is primarily determined by manufacturer 1's market power θ and its relationship with critical threshold, $max\{\frac{4}{9}, \theta_5\}$. The value of threshold, θ_5, is influenced by a combination of operational, market and policy related

[11] Where $\Delta e^n = \frac{\delta_1}{2\lambda_0}$ and $\theta_2 = \frac{4(\delta_1^2 + 9K\beta\lambda_0 + 5\Delta e\delta_1\lambda_0 - 5\Delta e^2\lambda_0^2)}{9(\delta_1^2 + 9K\beta\lambda_0)}$.

[12] Where $\Delta e^n = \frac{\delta_1}{2\lambda_0}$ and $\theta_5 = -\frac{4}{9e_1^2(\delta_1^2 + 9K\beta\lambda_0)}[20\Delta e^4\lambda_0^2 + 20\Delta e^3\lambda_0(e_1\lambda_0 - \delta_1) + 5\Delta e^2(\delta_1^2 - 4e_1\delta_1\lambda_0 + e_1^2\lambda_0^2) + 5(\delta_1 - e_1\lambda_0)e_1\delta_1\Delta e - e_1^2(\delta_1^2 + 9K\beta\lambda_0)]$.

factors including maximum marginal profit of licensor (δ_1), unit carbon emissions of manufacturers 1 and 2 (e_1, e_2), the price elasticity of demand (β), carbon emissions cap (K), and unit carbon emission trade price (λ_0). If the bargaining power of manufacturer 1 exceeds the critical threshold ($max\{\frac{4}{9}, \theta_5\} < \theta \leq 1$) in the licensing contractual negotiation, then RL coopetition leads to less total carbon emissions compared to the competition model, which is beneficial for the environment. Conversely, manufacturers will emit more carbon in the RL cooperation model than in the competition model, which has a negative impact on the environment.

4.5 Competition vs. Fixed fee licensing coopetition

In this section, the effects of the FL coopetition strategy on maximum profits, retail prices, and total carbon emissions for both manufacturers are examined by a comparison of the derived equilibrium solutions for the Cournot competition model and the FL coopetition model.

4.5.1 Effect of FL coopetition on maximum profits

Next, the effect of FL coopetition on the manufacturers' maximum profits is explored through the following proposition.

Proposition 4.2: ***(1) If*** $\mathbf{0 < \Delta e < \Delta e^m}$ ***and*** $\mathbf{\frac{1}{2} < \theta \leq 1}$***, then FL coopetition increases the total profit as compared to competition; otherwise, competition delivers better economic performance.***

(2) When FL coopetition generate more total profits for the two manufacturers, if $\mathbf{0 < \Delta e < \Delta e^m}$ ***and*** $\mathbf{\theta_6 < \theta < \theta_7}$***, then FL coopetition strategy achieves a Pareto improvement.***

(3) For the Pareto improvement FL coopetition strategy, $\mathbf{p^f < p^n}$***.***[13]

The above proposition indicates that whether the FL coopetition contributes to the improvement in the manufacturers' economic performance is determined by the difference of manufacturers' technology (Δe) and manufacturers' negotiation power (θ). This relationship is further illustrated in Figure 4.3, which also includes three decision regions. Similar to the RL model, each decision region is discussed individually.

[13] Where $\Delta e^m = \frac{2\delta_1}{5\lambda_0}$, $\theta_6 = \frac{\delta_1^2+9K\beta\lambda_0+2\Delta e\delta_1\lambda_0+\Delta e^2\lambda_0^2}{2(\delta_1^2+9K\beta\lambda_0)}$ and $\theta_7 = \frac{\delta_1^2+9K\beta\lambda_0+4\Delta e\delta_1\lambda_0-4\Delta e^2\lambda_0^2}{2(\delta_1^2+9K\beta\lambda_0)}$.

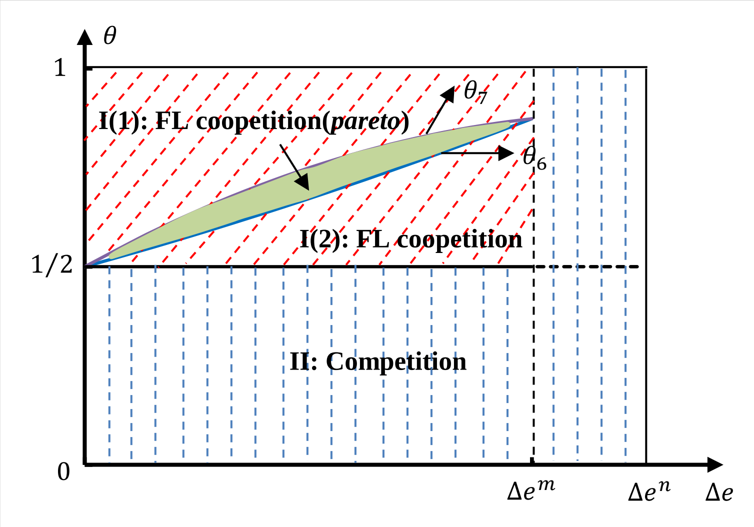


Figure 4.3 Effect of the FL coopetition strategy

Region I(1) specifies an *Pareto improvement* decision region in which both manufacturers will experience a profit increase under the FL model as compared to the competition model. From Proposition 4.2 (3), it is also clear that FL coopetition results in lower retail prices compared to competition if $\frac{1}{2} < \theta \leq 1$. Therefore, intuitively, fixed-fee licensing coopetition has a positive impact on individual manufactures and consumers. In **Region I(2),** although there is an increase of the total profit, one of the two manufacturers will experience profit loss in the FL coopetition model as compared to the competition model. In this circumstance, the worse-off manufacturer has no intention to continue engaging-in fixed-fee licensing cooperation unless the total profit increase gained from coopetition can be more fairly shared between the two manufacturers. *Pareto* improvement can be realized if the better-off manufacturer is willing to do so through further cooperation such as profit-sharing contract.

Region II describes the scenario where competition is the optimal strategy for the two manufacturers. In this case, there is a decrease of total profit in the FL coopetition as compared to competition. If manufacturer 1's negotiation power is less than $\frac{1}{2}$, then $M^f < 0$. In other words, a negative fixed-licensing fee occurs, which is not realistic for manufacturer 1 to do so. It means the manufacturer should only consider licensing its low carbon technology to the rival firm through fixed-fee if they hold more power in the licensing contractual negotiation. Furthermore, if the technological gap (Δe) between the two firms exceeds a critical value ($\Delta e > \Delta e^m$). Financially, it is better for the two firms to only compete no matter the power relationship between them.

4.5.2 Effect of FL coopetition on optimal retail prices

Next, the effect of FL coopetition on manufacturers' optimal retail prices is presented through the following lemma.

Lemma 4.2: ***If*** $\boldsymbol{0 < \Delta e < \Delta e^n}$ ***and*** $\boldsymbol{\frac{1}{2} < \theta \leq 1}$***, then*** $\boldsymbol{p^f < p^n}$***.***

Lemma 4.2 implies that when FL coopetition is the optimal strategy, the optimal retail prices of both manufacturers also decrease as compared to the competition model, which is beneficial to consumers.

4.5.3 Effect of FL coopetition on the total carbon emissions for both manufacturers

Finally, we obtain the effect of FL coopetition on the manufacturers' total carbon emissions.

Corollary 4.2: ***If*** $\boldsymbol{max\{0, \Delta e^a\} < \Delta e < \Delta e^n}$ ***and*** $\boldsymbol{\frac{1}{2} < \theta \leq 1}$***, then*** $\boldsymbol{T^f > T^n}$***; otherwise*** $\boldsymbol{T^f < T^n}$***.***[14]

This corollary shows that the impact of FL coopetition on environmental performance is more complicated. Within the feasible region of FL coopetition ($\frac{1}{2} < \theta \leq 1$), if the unit product carbon emissions of manufacturer 1 is higher than the threshold $\frac{\delta_1}{\lambda_0}$ (i.e., $e_1 > \frac{\delta_1}{\lambda_0}$), then the total carbon emissions in FL coopetition is always more than that in competition. Initiatively, the FL coopetition can only improve the environmental performance if the licensor has enough technological advantage in low carbon manufacturing. Even if the unit product carbon emissions of manufacturer 1 is lower than the threshold $\frac{\delta_1}{\lambda_0}$ (i.e., $e_1 < \frac{\delta_1}{\lambda_0}$), FL coopetition can only deliver an improved environmental performance ($T^f < T^n$) if the difference in unit carbon emissions between the two manufacturers is small ($0 < \Delta e < \Delta e^a$) but not high ($\Delta e^a < \Delta e < \Delta e^n$), which is surprising.

4.6 Competition vs. Mixed licensing coopetition

In this section, the effects of the ML coopetition strategy on maximum profits, retail prices, and the total carbon emissions for both manufacturers are examined by a comparative analysis of the derived equilibrium solutions for the Cournot competition model and the ML coopetition model.

---

[14] Where $\Delta e^a = \frac{\delta_1 - e_1\lambda_0}{2\lambda_0}$ and $\Delta e^n = \frac{\delta_1}{2\lambda_0}$.

4.6.1 Effect of ML coopetition on maximum profits

First, the total profit of both manufacturers in the ML coopetition model is derived and compared with that in the competition model. We obtain the following proposition.

Proposition 4.3: ***If*** $\boldsymbol{0 < \Delta e < \Delta e^n}$ ***and*** $\boldsymbol{\theta_1 < \theta \leq 1}$***, then ML coopetition generates more profit; otherwise, competition is the better strategy economically.***[15]

This proposition indicates that the relationship between the two manufacturers' maximum profits in the ML coopetition model and in the competition model is primarily determined by the manufacturer 1's bargaining power (θ), as illustrated in Figure 4.4.

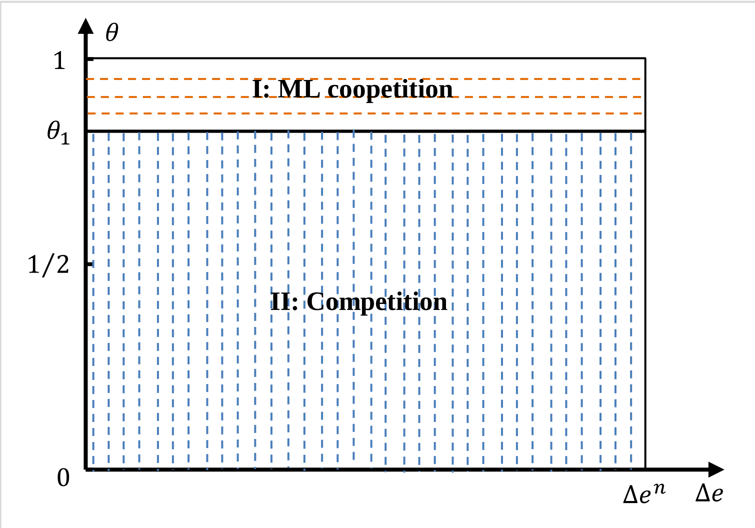


Figure 4.4 Effect of the ML coopetition strategy

Region (I) shows if the negotiation power of manufacturer 1 exceeds a threshold ($\theta_1 < \theta \leq 1$), then the total profit between the two manufacturers is greater in the ML coopetition model than in the competition model ($\pi^l > \pi^n$). The value of critical threshold, θ_1, is influenced by a combination of operational, market and policy related factors including maximum marginal profit of licensor *(*δ_1), the price elasticity of demand *(*β*)*, carbon emissions cap (*K*), and unit carbon emission trade price *(*λ_0*)*. However, differing to the RL and FL coopetition models, ML coopetition cannot achieve a Pareto improvement, which means one of the two manufacturers will loss out despite an increase of the total profit between the two. **Region (II)** indicates competition is the optimal strategy if $0 < \theta < \theta_1$, which incurs a negative fixed-licensing fee. In this case, manufacturer 1 is not willing to license the technology to the rival through ML coopetition strategy.

---

15 Where $\Delta e^n = \frac{\delta_1}{2\lambda_0}$ and $\theta_1 = \frac{\delta_1^2+4K\beta\lambda_0}{\delta_1^2+8K\beta\lambda_0}$.

4.6.2 Effect of ML coopetition on optimal retail prices

The effect of ML coopetition on manufacturers' optimal retail prices is presented through the following lemma.

Lemma 4.3: ***If*** $\boldsymbol{0 < \Delta e < \Delta e^n}$ ***and*** $\boldsymbol{\theta_1 < \theta \leq 1}$***, then*** $\boldsymbol{p^l > p^n}$**.**

Lemma 4.3 implies that when ML coopetition is the optimal strategy, it also drives up the optimal retail prices of both manufacturers compared with the competition model, which is harmful to consumers.

4.6.3 Effect of ML coopetition on the total carbon emissions for both manufacturers

Similarly, in this part, I obtain the effect of ML coopetition on the manufacturers' total carbon emissions.

Corollary 4.3: ***If*** $\boldsymbol{0 < \Delta e < \Delta e^n}$ ***and*** $\boldsymbol{\theta_1 < \theta \leq 1}$***, then*** $\boldsymbol{T^l < T^n}$**.**

From this corollary, it is clear while ML coopetition generate more total profits for the two manufacturing ($\theta_1 < \theta \leq 1$), it also leads to better environmental performance. Interestingly, the unit carbon emissions of manufacturer 2 (e_2) and the emissions gap between them (Δe) have no impact on this finding. Furthermore, Δe^n is a decreasing function of λ_0, and θ_1 is a decreasing function of K. Therefore, from policy makers' point of view, it is better to set a lower unit carbon emission trade price (λ_0) or/and higher carbon emissions cap (K). Such a cap-and-trade policy will increase the possibility of adopting ML coopetition by the manufacturers. While the manufacturers enjoy profit increase, it also leads to reduced total carbon emissions. However, consumers have to pay extra prices for improved environmental performance.

4.7 Selection of optimal strategies

In this section, we explore the optimal coopetition strategy considering all the licensing contractual options discussed in previous sections including competition, RL coopetition, FL coopetition and ML coopetition. Since firms' strategic decision is often driven by the economic benefit, here we mainly focus on the economic performance of alternative coopetition models and derive the following proposition.

Proposition 4.4: ***(1) If*** $\boldsymbol{0 < \Delta e < \Delta e^m}$ ***and*** $\boldsymbol{\frac{4}{9} < \theta \leq \theta_1}$***; or*** $\boldsymbol{\Delta e^m < \Delta e < \Delta e^n}$ ***and*** $\boldsymbol{\theta_3 <}$

$\theta \le \theta_1$, then RL coopetition is the preferable strategy;

(2) If $0 < \Delta e < \Delta e^n$ and $\theta_1 < \theta \le 1$, then ML coopetition strategy is the preferable strategy.

(3) Otherwise, competition is the preferable strategy.[16]

The above proposition shows that the optimal strategic decision on coopetition is determined by manufacturer 1's negotiation power (θ) and the difference in unit carbon emissions from production between manufacturers (Δe) and their relationship with relevant critical thresholds (θ_1, θ_2, θ_3, θ_4 and Δe^m). Note that these thresholds are dependent on the maximum unit profit for manufacturer 1 (δ_1), carbon emissions cap (K) and unit carbon emission trade price (λ_0). The relationship is further illustrated in Figure 4.5, which outlines three decision regions.

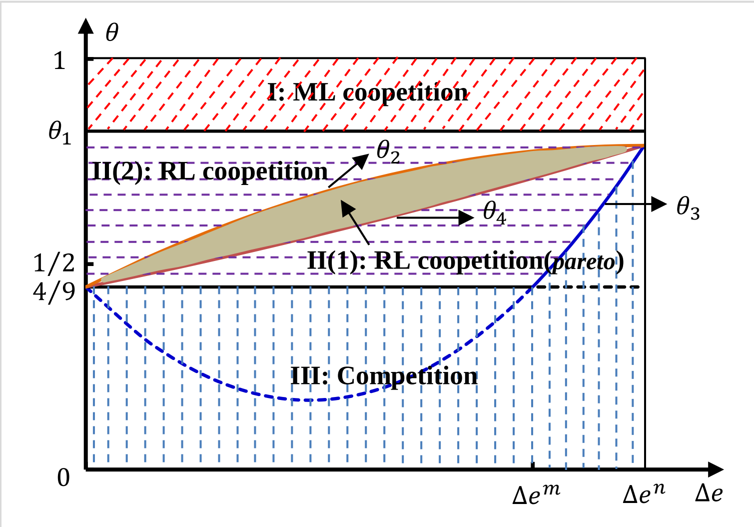


Figure 4.5 Selection of coopetition strategies

In **Region I**, ML coopetition is the preferable strategy when the licensor has dominant power in the licensing contractual negotiation ($\theta > \theta_1$). In this case, ML coopetition also guarantees a reduction of total carbon emissions. Although ML coopetition is the optimal strategy from the view of the total profit, it cannot guarantee a win-win outcome for the two manufacturers. It can only achieve a Pareto improvement if the better off firm is willing to re-distribute the profit gain to compensate the other. In **Region II,** when the negotiation power of manufacturer 1 reduces to a certain range ($max\left\{\frac{4}{9}, \theta_3\right\} < \theta \le \theta_1$), RL coopetition is the optimal strategy as it results in a greater profit than does competition or another coopetition strategies ($\pi^r > \{\pi^f, \pi^l, \pi^n\}$). However, in this region, there is the situation ($\theta_4 < \theta < \theta_2$) that Pareto improvement can be achieved without further

16 Where $\Delta e^m = \frac{2\delta_1}{5\lambda_0}$, $\Delta e^n = \frac{\delta_1}{2\lambda_0}$, $\theta_1 = \frac{\delta_1^2+4K\beta\lambda_0}{\delta_1^2+8K\beta\lambda_0}$ and $\theta_3 = \frac{4(\delta_1^2+9K\beta\lambda_0-10\Delta e\delta_1\lambda_0+25\Delta e^2\lambda_0^2)}{9(\delta_1^2+9K\beta\lambda_0)}$.

cooperation mechanism (e.g. profit sharing). In **Region III,** when the negotiation power of manufacturer 1 decreases further, competition is the optimal strategy because the financial gains in the technology licensing through coopetition strategies cannot compensate for the losses that are incurred in the market competition with strengthened competitor. For many firms especially smaller firms, it is better not to license their low carbon technologies when they have less power in the licensing contractual negotiation.

4.8 The extended models

4.8.1 The asymmetric case

In the previous analysis, a symmetric case is considered, in which $\alpha_1 = \alpha_2 = \alpha$. Here, α_1 and α_2 represent the maximum retail prices of manufacturers 1 and 2, respectively. In the following analysis, a more general scenario is considered, in which $\alpha_1 \neq \alpha_2$. Then, the demand function $p_i = \alpha_i - \beta(q_i + q_j)$, $i, j = 1,2$ and $i \neq j$. Based on this demand function, the optimal solutions for the competition, RL, FL and ML models can be derived as described in Table 4.3. Readers can refer to the appendix for the derivation procedures of these optimal solutions.

It is clear from the comparison of the optimal solutions presented in Table 4.3 and Table 4.2 that α_1, α_2 have a notable impact on the optimal operational decisions of both manufacturers. As a consequence, manufacturers' optimal profits in the competition, RL coopetition, FL coopetition and ML coopetition models are affected by α_1, α_2 and the same applies to the values of important critical thresholds that determine the optimal decision regions on low carbon technology licensing contractual choice. Therefore, to verify the research findings, numerical analysis is presented here to demonstrate the effect of the asymmetric manufacturer case (i.e., $\alpha_1 \neq \alpha_2$) on the selection of green technology coopetition strategies. For simplicity, let $\Delta\alpha = \alpha_2 - \alpha_1$ and $\delta_1 = \alpha_1 - c - \lambda_0 e_1$. We assume that $K = 15$, $\delta_1 = \frac{21}{2}$, $\lambda_0 = \frac{1}{2}$ and $\beta = 1$. In Figure 4.6, we specify that $\alpha_2 - \alpha_1 = 0.1$, which means $\alpha_2 > \alpha_1$. In Figure 4.7, we specify that $\alpha_2 - \alpha_1 = -0.1$ which means $\alpha_2 < \alpha_1$.

Table 4.3 Optimal solutions for the four models ($\alpha_1 \neq \alpha_2$)

Models	Competition ($i=n$)	RL coopetition ($i=r$)	FL coopetition ($i=f$)	ML coopetition ($i=l$)
	$max\{0, \Delta e^d\} < \Delta e < \Delta e^n$	$\theta_{10} < \theta \le 1$	$\theta_{11} < \theta \le 1$	$0 < \Delta\alpha < \frac{1}{4}\delta_1$ and $\theta_1 < \theta \le 1$
q_1^i	$\frac{\delta_1 + \Delta e\lambda_0 - \Delta\alpha}{3\beta}$	$\frac{\delta_1 + \lambda^r - \Delta\alpha}{3\beta}$	$\frac{\delta_1 - \Delta\alpha}{3\beta}$	$\frac{\delta_1 - 2\Delta\alpha}{2\beta}$
q_2^i	$\frac{\delta_1 - 2\Delta e\lambda_0 + 2\Delta\alpha}{3\beta}$	$\frac{\delta_1 - 2\lambda^r + 2\Delta\alpha}{3\beta}$	$\frac{\delta_1 + 2\Delta\alpha}{3\beta}$	$\frac{2\Delta\alpha}{\beta}$
λ^i	/	$\Phi(\lambda) = 0$	/	$\frac{1}{2}\delta_1 - 2\Delta\alpha$
M^i	/	/	$\frac{(5\Delta\alpha^2 + 2\Delta\alpha\delta_1 + 2\delta_1^2 + 18K\beta\lambda_0)\theta - \Delta\alpha^2 + 2\Delta\alpha\delta_1 - \delta_1^2 - 9K\beta\lambda_0}{9\beta}$	$\frac{(\delta_1^2 + 8K\beta\lambda_0 + 4\Delta\alpha^2)\theta + 12\Delta\alpha^2 - \delta_1^2 - 4K\beta\lambda_0}{4\beta}$

Where $\Delta e^d = \frac{\Delta\alpha - \delta_1}{\lambda_0}$, $\Delta e^n = \frac{\delta_1 + 2\Delta\alpha}{2\lambda_0}$, $\Phi(\lambda) = -40\lambda^3 + \lambda^2(72\Delta\alpha + 24\Delta\alpha\theta + 60\delta_1) + \lambda[(-48\Delta\alpha^2 - 24\Delta\alpha\delta_1 - 18\delta_1^2 - 162K\beta\lambda_0)\theta - 24\Delta\alpha^2 - 72\Delta\alpha\delta_1 - 12\delta_1^2 + 72K\beta\lambda_0] + (24\Delta\alpha^3 + 24\Delta\alpha^2\delta_1 + 24\Delta\alpha\delta_1^2 + 9\delta_1^3 + 108K\beta\Delta\alpha\lambda_0 + 81K\beta\delta_1\lambda_0)\theta - 8\Delta\alpha^3 + 12\Delta\alpha^2\delta_1 - 4\delta_1^3 - 72K\beta\Delta\alpha\lambda_0 - 36K\beta\delta_1\lambda_0$, $\lambda^r(\theta_{10}) = 0$, $\theta_{11} = \frac{(\Delta\alpha - \delta_1)^2 + 9K\beta\lambda_0}{5\Delta\alpha^2 + 2\delta_1(\Delta\alpha + \delta_1) + 18K\beta\lambda_0}$ and $\theta_1 = \frac{\delta_1^2 + 4K\beta\lambda_0 - 12\Delta\alpha^2}{\delta_1^2 + 8K\beta\lambda_0 + 4\Delta\alpha^2}$.

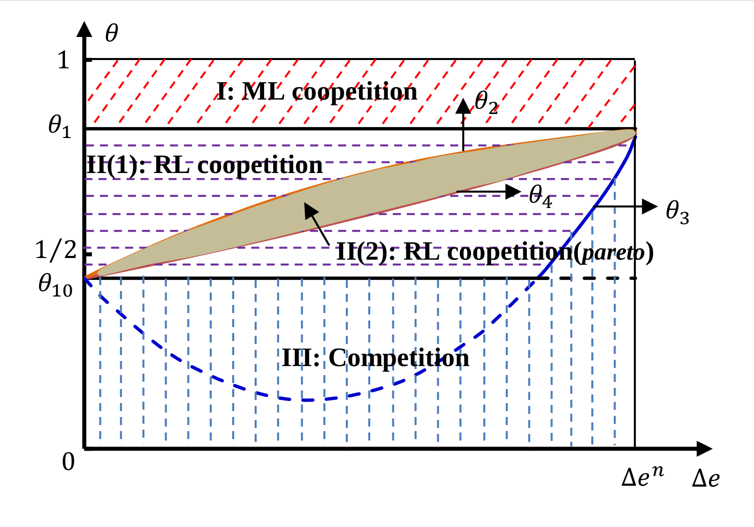


Figure 4.6 Selection of coopetition strategies ($\alpha_2 > \alpha_1$)

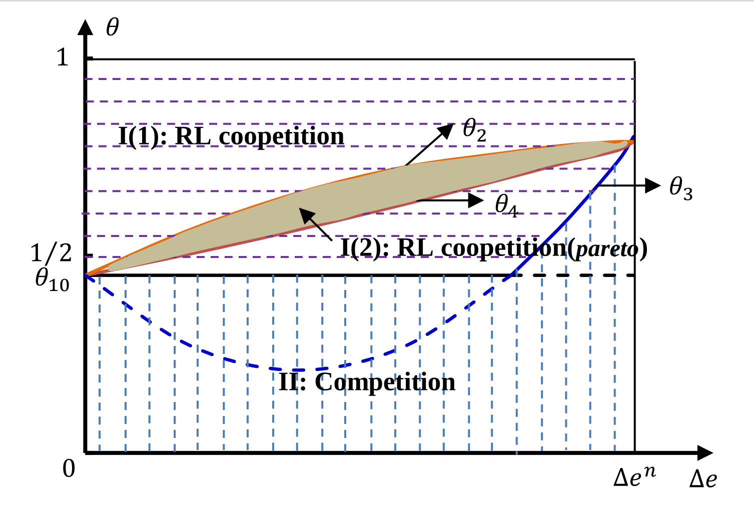


Figure 4.7 Selection of coopetition strategies ($\alpha_2 < \alpha_1$)

From Figure 4.6 and Figure 4.7, we obtain that the structural results in the asymmetric case is similar to the symmetric case when the maximum retail price of manufacturer 2 is higher than that of manufacture 1 (a positive $\Delta\alpha$). Meanwhile, $\Delta\alpha$ has an influence on the critical points. That is, a positive $\Delta\alpha$ leads to a larger decision region for ML coopetition strategy and a smaller region for competition strategy[17]. Interestingly, ML coopetition strategy is not an option for optimal selection of coopetition strategies any more when the maximum retail price of manufacturer 2 is lower than that of manufacture 1 (a negative $\Delta\alpha$)[18].

[17] If $\alpha_2 > \alpha_1$, then $\theta_{10} = 0.438496 < \frac{4}{9}$. If $\alpha_2 < \alpha_1$, then $\theta_{10} = 0.450353 > \frac{4}{9}$.

[18] If $\Delta\alpha < 0$, then $q_2^l = \frac{2\Delta\alpha}{\beta} < 0$, which is not feasible.

4.8.2 The case of both partial and perfect substitutes

In this sub-section, the analysis is extended from the case of perfect substitutable products to the case that includes the scenarios of partial and perfect substitutions. A new demand function is adopted as, $p_i = \alpha - \beta q_i - \gamma q_j$, $i, j = 1{,}2$ and $i \neq j$. γ is a parameter that measures the cross-effect of the change in manufacturer i's customer demand caused by a change in that of manufacturer j. Here, $0 < \gamma < \beta$ describes the scenario of partial substitutes, and the limiting value, $\gamma = \beta$, refers to the case of perfect substitutes. On the basis of the new demand function, the optimal solutions for the competition, RL coopetition, FL coopetition and ML coopetition models are derived and presented in Table 4.4.

Through the comparison of the optimal solutions presented in Table 4.4 and Table 4.2, it is not surprise to find that the optimal solutions are expressed in different mathematical formations due to a parameter γ in the new demand function. We then repeat the same analysis of Section 4.4, 4.5, 4.6 and 4.7 to examine how the level of product substitution affects the selectin of coopetition strategies. The results are illustrated in Figure 4.8 and Figure 4.9.

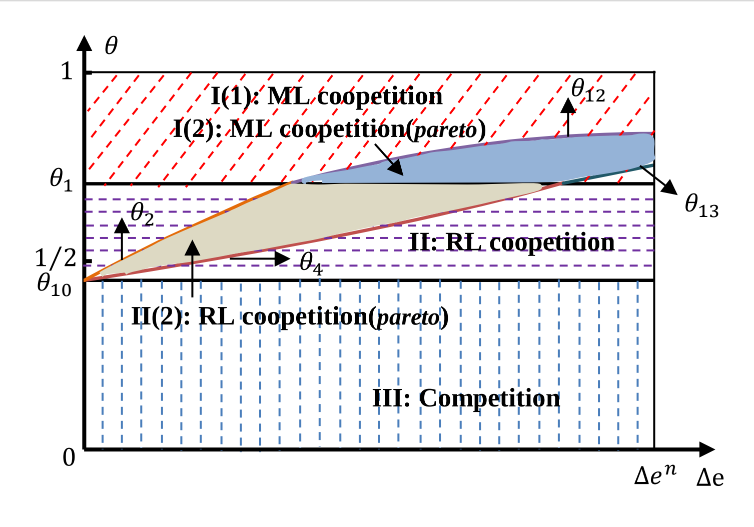


Figure 4.8 Selection of coopetition strategies ($\gamma = 0.7\beta$)

Table 4.4 Optimal solutions for the four models for the general substitutable product case

Models	Competition $(i=n)$	RL coopetition $(i=r)$	FL coopetition $(i=f)$	ML coopetition $(i=l)$
	$0<\Delta\mathrm{e}<\Delta\mathrm{e}^n$	$\theta_{10}<\theta\le 1$	$\frac{1}{2}<\theta\le 1$	$\theta_1<\theta\le 1$
q_1^i	$\frac{2\beta\delta_1-\gamma(\delta_1-\Delta\mathrm{e}\lambda_0)}{4\beta^2-\gamma^2}$	$\frac{(2\beta-\gamma)\delta_1+\gamma\lambda^r}{4\beta^2-\gamma^2}$	$\frac{\delta_1}{2\beta+\gamma}$	$\frac{(4\beta^2-2\beta\gamma-\gamma^2)\delta_1}{2\beta(4\beta^2-3\gamma^2)}$
q_2^i	$\frac{2\beta(\delta_1-\Delta\mathrm{e}\lambda_0)-\gamma\delta_1}{4\beta^2-\gamma^2}$	$\frac{(2\beta-\gamma)\delta_1-2\beta\lambda^r}{4\beta^2-\gamma^2}$	$\frac{\delta_1}{2\beta+\gamma}$	$\frac{2(\beta-\gamma)\delta_1}{4\beta^2-3\gamma^2}$
λ^i	/	$\Phi(\lambda)=0$	/	$\frac{(2\beta-\gamma)^2\gamma\delta_1}{2\beta(4\beta^2-3\gamma^2)}$
M^i	/	/	$\frac{(2\theta-1)[\beta\delta_1^2+(4K\beta^2+4K\beta\gamma+K\gamma^2)\lambda_0]}{(2\beta+\gamma)^2}$	$K\lambda_0(2\theta-1)+\frac{f(\theta)}{4\beta(4\beta^2-3\gamma^2)^2}$

Where $\Delta\mathrm{e}^n=\frac{(2\beta-\gamma)\delta_1}{2\beta\lambda_0}$, $\lambda^r(\theta_{10})=0$, $\theta_1=\frac{(16\beta^4-36\beta^2\gamma^2+24\beta\gamma^3-3\gamma^4)\delta_1^2+4K\beta(4\beta^2-3\gamma^2)^2\lambda_0}{(4\beta^2-3\gamma^2)((8\beta^2-8\beta\gamma+\gamma^2)\delta_1^2+8K\beta(4\beta^2-3\gamma^2)\lambda_0)}$, $f(\theta)=\delta_1^2\{4\beta^2\gamma^2(9-5\theta)+3\gamma^3(\gamma-8\beta)(1-\theta)-16\beta^3[2\gamma\theta-\beta(2\theta-1)]\}$ and $\Phi(\lambda)=8\beta^4(8\beta^2-3\gamma^2)\lambda^3+4\delta_1\lambda^2\beta^3(2\beta-\gamma)[2\theta(\gamma^2+\beta\gamma-2\beta^2)+5\gamma^2-16\beta^2-4\beta\gamma]+2\beta\lambda(\gamma-2\beta)^2[3\theta(4\beta^2-\gamma^2)(\beta\delta_1^2+4K\beta^2\lambda_0+4K\beta\gamma\lambda_0+K\gamma^2\lambda_0)+2\beta(2\beta^2\delta_1^2+2\beta\gamma\delta_1^2-\gamma^2\delta_1^2-2K\lambda_0\beta(2\beta+\gamma)^2)]+\delta_1(\gamma-2\beta)^3[(8\beta^2+2\beta\gamma-\gamma^2)\theta-4\beta^2][K\lambda_0(\gamma+2\beta)^2+\beta\delta_1^2]$.

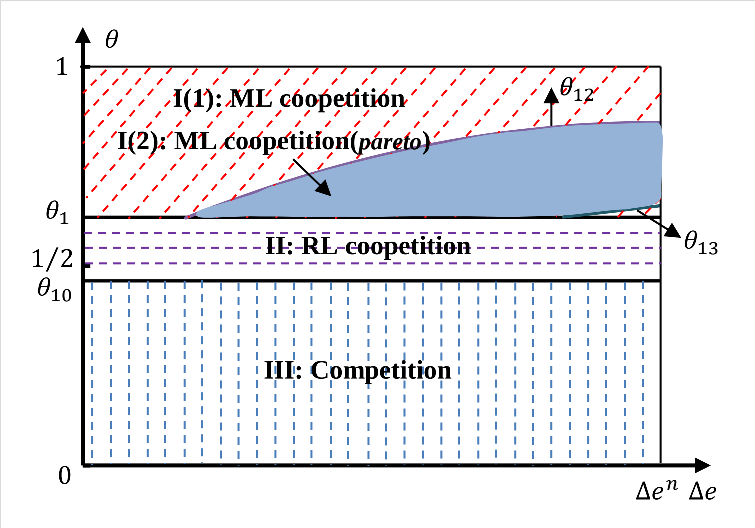


Figure 4.9 Selection of coopetition strategies ($\gamma = 0.4\beta$)

It is clear from Figure 4.8 and Figure 4.9 that the size of these decision regions depends on the critical thresholds, like θ_1 and θ_{10}. For instance, when substitution level γ decreases, which means lower intensity of market competition, then θ_1 decreases and θ_{10} increases. As a result, it extends the two decision regions where ML coopetition and competition are the optimal strategies, respectively. In other words, a lower product substitution level increases the possibility of competition as the optimal strategy when the technology license holder has less negotiation power than the licensee; and increases the possibility of ML coopetition as the optimal strategy when the holder has more negotiation power than the licensee. The decrease of product substitution rate also leads to a smaller region for RL coopetition as the optimal strategy when the two manufacturers have a more balanced power relationship in the licensing contract negotiation.

Furthermore, the product substitution level also affects the decision regions that coopetition strategies result in Pareto improvement. For instance, from Figure 4.5, we can see that the RL coopetition strategy can realize a Pareto improvement but for ML coopetition strategy when $\gamma = \beta$. From Figure 4.8, when $\gamma = 0.7\beta$, then both ML coopetition and RL coopetition strategies can deliver a Pareto improvement. From Figure 4.9, when the substitution level decrease (e.g. $\gamma = 0.4\beta$), then only ML coopetition strategy can achieve a Pareto improvement. From the analysis results, clearly, the structural results presented in the perfect substitutes case still hold in the scenario of partial substitutes. However, the product substitution level, an important indicator of market competition, affects the values of critical thresholds that determine manufacturers' optimal decision regions on green technology coopetition strategy as well as the decision regions for Pareto improvement.

4.9 Conclusions

This research evaluates the effects of the contractual choice regarding low carbon technology licensing on the economic and environmental performance of two rival manufacturers (e.g., Ford vs. Toyota in the automotive industry or Lenovo vs. Dell in the PC industry) under a cap-and-trade policy. The licensing payment can be arranged through either a fixed fee, royalties or mixed fees. Through a comparison of the equilibriums of the competition, RL coopetition, FL coopetition and ML coopetition models, we emphasize the economic principles that govern firms' behaviors toward technology licensing contractual choice. The study further examines the impact of coopetition/competition decisions on the environment and consumers, and it analyzes how a cap-and-trade policy can be designed to promote a sustainable low-carbon economy. Our analysis provides several important insights.

The contractual choice on low carbon technology licensing is governed by the relationship between the benefits gained from licensing cooperation on low carbon technologies and the losses incurred from competition with a strengthened market rival because of the cooperation. This decision is determined by a combination of factors including internal operational and low carbon capability (e.g., maximum marginal profit of licensor δ_1, manufacturers' unit carbon emissions e_1, e_2), interfirm power relationship (θ), external market characteristics (e.g., the price elasticity of demand β, product substitution level, $\frac{\gamma}{\beta}$, manufacturers' maximum retail prices, α_1, α_2), and the carbon emission control policy (e.g., carbon emissions cap, K, and unit carbon emission trade price λ_0). Interestingly, among these factors, the interfirm power relationship plays a more prominent role in determining the optimal contractual decision on the low carbon technology licensing. More specifically, mixed fee licensing is preferable choice if the licensor has a dominant power in the contractual negotiation, and in contrast, no licensing agreement is preferable choice if the licensor has less negotiation power as compared to the licensee. Technology licensing through royalty fee should be considered if licensor's negotiation power is the between. Interestingly, fixed-fee is not a viable option as compared to others.

While firms' decision on whether and how to license their low carbon technologies to rival firms is mainly determined by their economic benefit, the licensing decisions have profound impact on individual firms, environment and consumers. For example, depending on the interfirm power relationship (θ) manufacturers' maximum retail prices (α_1, α_2), there are decision regions that Pareto improvement can be achieved when mixed-fees or royal fee licensing is the optimal strategy. When the mixed-fees licensing produces better economic performance as compared to competition and other licensing options, it also guarantees an improvement of environmental performance. However, when royalty fee licensing is optimal choice, an improvement in environmental requires additional condition that the bargaining power of manufacturer 1 must exceed a critical threshold ($max\{\frac{4}{9}, \theta_5\} < \theta \leq 1$). In addition, while the mixed-fees licensing will certainly push up the retail prices, whether royalty licensing has a negative or positive impact on the retail prices is determined by manufacturer 1's negotiation power.

However, the optimal decision on low carbon technology licensing is dynamic that is influenced by the internal operational and technological capability, external market and policy environment, and interfirm power relationship. The changes in the internal capabilities, the external environment and/or interfirm power balance over time will affect the firms' strategic decisions about low carbon technology licensing. For example, internal and external technology development over time has enabled Ford to transition from the stage of licensing patents from Toyota, when it developed the first Escape hybrid a decade ago, to the current stage of offering electrified vehicle technology licenses to rival automakers. Furthermore, our research findings also have important policy implications. In general, technology licensing between rival firms will lead to reduced total carbon emissions. Therefore, it is critical for governments to create a supportive policy environment that incentivizes technology licensing. For instance, a cap-and-trade policy with a high carbon emissions cap and/or lower unit carbon emission trade price is more likely to encourage firms to adopt mixed licensing technology. In addition, it is important to protect the green innovator's bargaining power in the technology licensing negotiation. It is more likely to promote the green technology licensing among the rival firms in such a policy environment.

Chapter 5 Delivery service coopetition models and applications

5.1 Introduction

Online retailing has grown substantially in the past decade, and this growth is expected to continue in the foreseeable future. According to the United States (U.S.) Census Bureau (2018), consumers spent $453.46 billion online for retail purchases in 2017, a 16.0% increase compared to $390.99 billion in 2016. Much of the gains were from the internet giant Amazon, which was responsible for approximately 44% of all U.S. e-commerce sales in 2017 (Zaroban 2018). While online retailing has enjoyed rapid growth with many successful business cases around the world, such as Amazon, eBay, and JD.com, the online retail competition is also becoming as fierce as ever. When online marketplaces such as Amazon, Alibaba and JD.com continue increasing their market shares, many conventional brick-and-mortar retailers, e.g., Walmart and Tesco, have also expanded their businesses online to delve into this ever-increasing market. In addition, while marketplace firms such as Amazon and Alibaba provide online retailing platforms to merchants (i.e., sellers or retailers) for selling products, Amazon and JD.com are also directly competing with these merchants in selling substitutable products themselves.

Online marketplaces provide consumers choice and convenience. When attempting to find bargains and shopping convenience for their desired products, the quality of delivery service, e.g., timeliness, flexibility and reliability of delivery, is one of the major factors that influence many consumers' purchasing decisions. As Collier and Bienstock (2006) stated, the delivery of their purchased goods is the most important aspect of the quality of the customers' online retail experience. In response, marketplace firms have invested heavily to improve their distribution and logistics capabilities to develop a logistics infrastructure capable of delivering goods to consumers where and how they want it. Among the marketplace firms, Amazon and JD.com are among the industry leaders in providing distribution and logistics service in their associated e-commerce markets and invest massively in the area to further strengthen their market position. For instance, according to Amazon's CFO Brian Olsavsky, much of the 51% year-over-year growth in capital expenditures came from the investment in fulfillment centers, with 23 new warehouses being added in the second half of 2016

(Lopez 2017). Meanwhile, JD.com, Alibaba's biggest competitor in China's online shopping market, has invested heavily in its own distribution and logistics capacity, such as warehouses and delivery trucks, to ensure good service (Bloomberg News 2016). For merchants selling at online marketplaces, they have the option of providing their own delivery services or opting for a 3rd party logistics service including delivery services provided by marketplaces such as Amazon and JD.com. Outsourcing distribution and logistics operation enables these merchants to improve the quality of the delivery service and concentrate on their core business at the expense of service charges paid to marketplace firms. For marketplace firms, such services to these merchants selling on their online platforms have become an important revenue stream. Using Amazon as an example, 3rd party logistics has become one of the fastest growth areas for the company because it provides fulfilment services for an increasing array of merchants selling goods via Amazon (Hook 2017). Revenue from 3rd party logistics services rose 38% to $7 billion in the 2nd quarter of 2017, representing more than one-sixth of Amazon's sales (Hook 2017). Whereas there are some obvious benefits to cooperating in delivery service provisions, it is not clear how the nature of competition is affected by the delivery service cooperation because e-tailers and marketplace firms are also competing for consumers' demands at the same time. This type of market setting is referred to coopetition. As discussed in the earlier chapters, the concept of coopetition describes the interdependence where competition and cooperation simultaneously take place between two or more firms and where each firm concentrateing on increasing the size of the total pie for division (Brandenburger and Nalebuff 1996; Mantovani and Ruiz-Aliseda 2016). A natural question that arises in this particular setting is whether cooperation between the e-tailer and marketplace firm is desirable from the perspective of the firms and consumers. These observations motivated us to systematically analyze the impact that delivery service coopetition has on the e-tailer, the marketplace firm and consumers. In particular, we are interested in investigating the following questions:

- Should the e-tailer and marketplace firm cooperate in delivery service? If yes, which is the better option for delivery service cooperation: outsourcing or membership?
- How does the coopetitive relationship affect firms' profitability and consumer welfare?
- How does the external market environment, inter-firm relationship, and internal operational capability affect the optimal decision on delivery service coopetition?

To answer these research questions, we consider a setting in which a marketplace firm such as

Amazon or JD.com operates an online platform through which e-tailers can sell their products to end consumers directly. Meanwhile, the marketplace firm also sells partially substitutable products in competing for end-customer demand. However, the marketplace firm (e.g., Amazon) has superior distribution and logistics capability than the e-tailer. Therefore, assuming the unit delivery cost is the same, this marketplace firm is able to provide higher level of delivery service than the e-tailers selling through the marketplace firm's platform. For the e-tailer (e.g., merchants on Amazon), it can use its own delivery service or outsources its delivery service to the marketplace firm with a unit outsourcing price. Alternatively, the e-tailer can pay a fixed membership fee to the marketplace firm (e.g., Amazon) with a lower rate of unit delivery service charge. Service delivery cooperation, through either outsourcing or membership format, will help the e-tailer improve its delivery service to the same level as the marketplace firm (e.g., Amazon). We refer to the case where both firms provide their own delivery services as the competition model and the cases where the e-tailer opts to pay a unit outsourcing fee or a fixed membership fee with a lower unit rate to the marketplace firm as the outsourcing coopetition model and the membership coopetition model, respectively. We seek to understand the underlying principles that govern firms' cooperation behavior and how the coopetition decision affects individual firms' profitability and consumers' welfare by comparing the equilibria of the competition and coopetition models.

This dissertation makes several contributions. First, despite rapid growth in online retailing and strategic significance of delivery service for the sector, few studies have so far employed the notion of coopetition to examine how delivery service cooperation impacts the online retailing eco-system. By modeling the firms' decision behaviors and consequential performances, our analysis helps to show that firms' strategic decision on coopetition is determined by the trade-off between the benefits gained from cooperation and financial loss incurred when facing a strengthened competitor that determines firms' strategic decision on coopetition. In this context, for the marketplace firm, delivery service cooperation generates additional revenue streams; for the e-tailer, cooperation helps increase demand, which is stimulated by the improved delivery service. At the same time, both retailers could incur losses when facing a strengthened competitor as a consequence of the service cooperation. While the cooperation benefits are influenced by the degree of product substitution, the customers' willingness to pay for a delivery service, and the difference between the two firms' delivery service capabilities, the losses incurred in the demand competition are dependent on both the price and service competition factors, including the degree of product substitution and customers' willingness to pay for delivery service. Our study reveals the interactive dynamic relationship between competition and coopetition that has not been captured by other coopetition studies that only consider a single competition factor (Gnyawali and Park 2011; Ryan et al. 2012). Building on our research findings, a decision framework is developed to help marketplace firms and e-tailers make important strategic decisions concerning coopetition.

The rest of this chapter is organized as follows. After reviewing relevant research streams in Section 5.2, we present the competition, outsourcing coopetition (OC), and membership coopetition (LC) models in Section 5.3. After that, we examine the impact of outsourcing coopetition and membership coopetition on the profits and consumer surplus of the e-tailer and marketplace firm through a comparison of the equilibrium results of the competition and coopetition models in Sections 5.4 and 5.5, respectively. Section 5.6 analyses the optimal selection of coopetition strategies and discuss the managerial implications. Then, we extend our model to an asymmetric case, in which the marketplace firm and e-tailer have different maximum retail prices in Section 5.7. Finally, we draw conclusions by highlighting the main insights in Section 5.8.

5.2 Literature review

Given the background of rapid growth in online retailing, there is an increasing number of studies on the various aspects of managing online retailing operations in the marketing and operations management literature. These studies have concentrated on various issues of online retailing, including coordination (Tsay and Agrawal 2004; Cao and Li 2015), pricing (Gümüş et al. 2013; Fisher et al. 2018), information sharing (Gallino and Moreno 2014), product returns (Ofek et al. 2011; Griffis et al. 2012), and channel structure (Bernstein et al. 2008; Yoo and Lee 2011). More details about this area of research can be found in the literature review work of Grieger (2003) and Wang et al. (2008). To highlight our contributions, the review here mainly concentrates on three lines of inquiry: price and service competition, service cooperation, and coopetition.

There is often a fierce price competition in online retailing because of the increased price transparency. It only takes a few clicks for consumers to find out price information. As a result, many e-tailers employ a competition-based pricing strategy and constantly monitor their competitors' prices and this information to decide their own prices (Fisher et al. 2018). Among the studies on price competition in the online retailing setting, Ba et al. (2008) developed an oligopoly model with a general cost structure to adverse price effect in the online market, where e-tailers sell identical products with different service offerings. Ellison and Ellison (2009) examined the price competition between a group of e-tailers with a price search engine, and their analysis indicated that the convenience of price search makes demand tremendously price-sensitive for some products. They also argued that retailers deliberately create more confusing websites to prevent consumers from figuring out the total price. Gümüş et al. (2013) analyzed two price partitioning strategies: the PS strategy, where the product price includes an item price and a separate shipping & handling surcharge, and the ZS strategy, where the price already includes the shipping cost in online retailing. Their empirical analyses show that PS retailers charge lower product prices but higher total prices than ZS retailers. More recently, motivated by e-tailers' pricing practices, Moon et al. (2018) investigated the value of intertemporal pricing and introduced a randomized price markdown policy, which benefits e-tailers by combining price commitment with exploiting heterogeneity in consumers' monitoring costs. In their field experiments of competition-based dynamic pricing in online retailing, Fisher et al. (2018) found that consumers' engagement in price comparison is the most critical factor for e-tailers' response to competitor price changes, and such responses should be differentiated according to the competitor's market significance.

While price is an important factor for consumers buying online, other factors associated with the online buying experience such as convenience, customer service, delivery and product return are also critical in gaining customer orders (Ahuja et al. 2003; Forman et al. 2009). Pan et al. (2002) pointed out in their empirical study on online markets that while e-tailers' service quality partially influenced pricing, market characteristics, such as the number of competitors, are stronger drivers for their pricing decisions. In the investigation of online store choice decisions of multi-channel grocery shoppers, Melis et al. (2015) found from their empirical study that consumers tend to choose an online store that belongs to the same chain but may switch to other online stores based on the online buying experience. Chen et al. (2008) investigated the manufacturer's problem of managing direct online retailing channel and conventional retail channel considering service competition. In their study, delivery lead time and product availability are used to measure the service of the online and offline retailing channels, respectively. Despite the growing number of studies (Tsay and Agrawal 2000; Bernstein and Federgruen 2004; Pekgün et al. 2017) that consider both price and service competition in modeling market dynamics and firms' behavior, few have explored the similar problem in the context of online retailing. Among them, considering the demand that is sensitive to price and service time in an online duopoly market, Ding et al. (2018) examined the impact of service time on online retailing competition and illustrated the dynamic relationship between the two competing elements. Different from the work of Ding et al. (2018), we not only consider both the price and service competition but also explore how cooperation in the service affects the nature of competition and the performance of rival firms.

More relevant to this study, Ryan et al. (2012) studied a channel conflict between a marketplace firm such as Amazon, which operates the marketplace system and sells products, and an e-tailer, which can sell similar products to consumers through its own website and/or the marketplace system. They investigated the problem of whether the marketplace firm and e-tailer should contract with each other to cooperate by analyzing the optimal decisions from both firms' perspective of view and characterizing the system equilibrium. In this dissertation, we also consider a similar setting of Ryan et al. (2012) to consider whether a marketplace firm such as Amazon or JD.com operates an online marketplace through which merchants can sell their products directly to consumers. However, different to the work of Ryan et al. (2012), we consider the conditions under which an e-tailer should choose to contract with the marketplace firm to use its delivery service because marketplace firms (e.g., Amazon) often have superior distribution and logistics capabilities because of the economic scale and significant investment in the area (Lopez 2017). Under which conditions should the e-tailer choose the contractual agreement: outsourcing or membership? We also consider the problem from the marketplace firm's perspective in examining whether to offer a delivery service contract to the e-tailer and how the firm would set the delivery service contract parameters, e.g., unit delivery service charges and membership fee.

5.3 The models and equilibrium analysis

5.3.1 The model

Following the research setting specified in the last section, the marketplace firm (e.g., Amazon or JD.com) and e-tailer sell partially substitutable products and provide the e-tailing service to consumers at the expense of two types of costs: purchasing and delivery costs. As this study focuses on the delivery service coopetition behavior, we assume that the two firms have the same unit wholesale price of partially substitutable products when purchasing from suppliers but they have different efficiencies in distribution and logistics; that is, with the same distribution and logistics cost, they provide different levels of delivery service. Because marketplace firms (e.g., Amazon and JD.com) often have superior distribution and logistics capacities and capabilities, we assume that the delivery service efficiency of the marketplace firm is higher than that of the e-tailer. Prior to presenting the models, we introduce the notations in Table 5.1 as follows.

Table 5.1 Notations

w	Unit wholesale cost for the marketplace firm and e-tailer.
p_1, p_2	Unit retail price for the marketplace firm and e-tailer.
c	Unit delivery cost for the marketplace firm and e-tailer.
s_1, s_2	Delivery service levels of the marketplace firm and e-tailer, $s_1 > s_2$.
Δs	Difference in the delivery service levels between the marketplace firm and e-tailer, $\Delta s = s_1 - s_2$.
m	The marketplace firm's unit price for outsourcing the delivery service, $m > c$.
T	The marketplace firm's fixed membership fee for the delivery service, $T > 0$.
u	The marketplace firm's unit price for membership delivery service, $u > 0$
q_1, q_2	Demand for the marketplace firm and e-tailer.
$\pi_1^n(p_1), \pi_2^n(p_2)$	Profit for the marketplace firm and e-tailer in the competition model.
$\pi_1^o(p_1), \pi_2^o(p_2)$	Profit for the marketplace firm and e-tailer in the outsourcing coopetition model.
$\pi_1^m(p_1), \pi_2^m(p_2$	Profit for the marketplace firm and e-tailer in the membership coopetition model.
π^n	The total profit in the competition model; $\pi^n = \pi_1^n(p_1) + \pi_2^n(p_2)$.
π^o	The total profit in the outsourcing coopetition model; $\pi^o = \pi_1^o(p_1, m) + \pi_2^o(p_2)$.
π^m	The total profit in the member coopetition model; $\pi^m = \pi_1^m(p_1) + \pi_2^m(p_2)$.
θ	Marketplace firm's negotiation/bargaining power; $0 \le \theta \le 1$.

In alignment with prior studies (e.g., Choi 1996; Tsay and Agrawal 2000; Liu et al. 2012), we use the following demand function $q_i = \alpha - p_i + \beta(p_j - p_i) + \tau[s_i - \beta(s_j - s_i)]$, $i, j = 1,2$ and $i \neq j$. Here, α represents the firms' maximum retail price. β ($\beta \ge 0$) is a parameter that is interpreted as the degree of product substitution of firm j's product over that of firm i, which is a measure of the intensity of the market competition. τ ($\tau \ge 0$) is a measure of the consumers' willingness to pay for a delivery service. The marginal profit per unit for firm i is $p_i - w - c > 0$,

$i = 1,2$, so $p_i > w + c$. When there is no competition, that is, $\beta = 0$, then $q_1 = \alpha - p_1 + \tau s_1 > 0$. Because $p_1 > w + c$, let $\delta = \alpha - c - w + \tau s_1$, then $\delta > 0$.

The sequence of events and decisions in our research is illustrated in Figure 5.1. It leads to three models representing three different relationships between two firms: competition, outsourcing coopetition (OC), and membership coopetition (MC). We assume that both firms are economically rational and act strategically to maximize their own profits. For the competition model, the two firms purchase, sell and deliver products independently. There is only a competitive relationship between the two firms. They compete with each other in retail price and service level for customer demand. For the OC model, the e-tailer outsources its delivery service to the marketplace firm by paying a unit outsourcing delivery price, which is higher than the unit delivery cost of the marketplace firm. Two firms compete with each other for market demand but have a cooperative relationship in the form of a delivery service outsourcing contract. For the MC model, the e-tailer obtains a membership from the marketplace firm to use its delivery service by paying a fixed membership fee and a lower unit delivery service rate to the marketplace firm. Similar to OC, while the two firms compete in the retail market, there is a cooperative relationship between them in the form of a delivery service membership contract. The two types of delivery service contracts are commonly provided by marketplace firms such as Amazon and JD.com. In the coopetition situation, the marketplace firm obtains an additional revenue source, while the e-tailer improves its delivery service level but has to pay an extra cost.

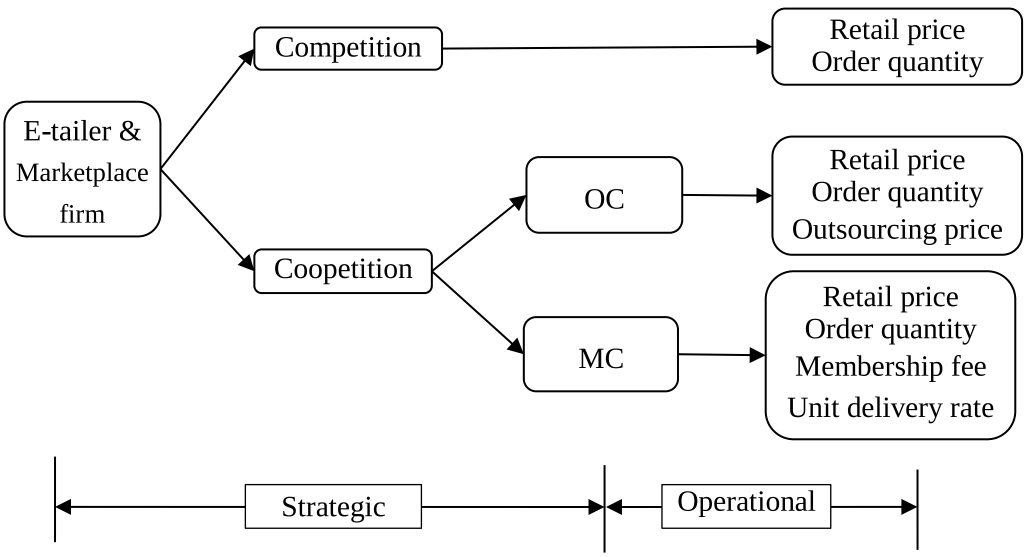


Figure 5.1 The framework

5.3.2 Competition model

First, the competition model is explored as a benchmark, in which two firms independently and simultaneously determine their unit retail price to maximize their own profits. For the competition model, the marketplace firm's profit $\pi_1^n(p_1)$ is

$$\pi_1^n(p_1) = (p_1 - w - c)\{\alpha - p_1 + \beta(p_2 - p_1) + \tau[s_1 - \beta(s_2 - s_1)]\} \tag{5-1}$$

The first part of Equation (5-1) represents the marketplace firm's marginal unit profit, and the second part represents its market demand. Similarly, the e-tailer's profit $\pi_2^n(p_2)$ for the competition model is

$$\pi_2^n(p_2) = (p_2 - w - c)\{\alpha - p_2 + \beta(p_1 - p_2) + \tau[s_2 - \beta(s_1 - s_2)]\} \tag{5-2}$$

Table 5.2 lists the marketplace firm's optimal retail price (p_1^n) and the e-tailer's optimal retail price (p_2^n) in the competition model.

5.3.3 Outsourcing coopetition model

For the OC model, there is a cooperative relationship between the competing firms. More specifically, the marketplace firm provides the distribution and logistics service for the e-tailer while competing for the same market. As a result, the e-tailer improves its delivery service to the same level as the marketplace firm, that is, $s_2 = s_1$, and pays the marketplace firm for the outsourced delivery service at m per unit. The two firms' decision sequence is described as follows. First, the marketplace firm and the e-tailer negotiate the unit outsourcing price (m) for the delivery service. Second, two firms independently and simultaneously determine their unit retail price. Finally, the two firms meet the consumers' demand and obtain their revenues accordingly.

For the OC model, the marketplace firm's profit $\pi_1^o(p_1)$ is

$$\pi_1^o(p_1) = (p_1 - w - c)[\alpha - p_1 + \beta(p_2 - p_1) + \tau s_1] + (m - c)[\alpha - p_2 + \beta(p_1 - p_2) + \tau s_1] \tag{5-3}$$

The first part of Equation (5-3) represents the profit from product sales, and the second part represents the profit from outsourcing the delivery service to the e-tailer. Similarly, for the OC model, the e-tailer's profit $\pi_2^o(p_2)$ is

$$\pi_2^o(p_2) = (p_2 - w - m)[\alpha - p_2 + \beta(p_1 - p_2) + \tau s_1] \tag{5-4}$$

The outsourcing price negotiation process for the OC model is as follows

$$\max_m \pi^{om}(m) = \max_m [\pi_1^o(p_1)]^\theta [\pi_2^o(p_2)]^{1-\theta} \tag{5-5}$$

The marketplace firm's optimal retail price (p_1^o), optimal outsourcing price (m^o) and the e-tailer's optimal retail price (p_2^o) in the OC model are provided in Table 5.2.

5.3.4 Membership coopetition model

For the MC model, there is also a cooperative relationship in the form of a delivery service membership contract. Compared to the OC model, the difference is that the e-tailer pays the marketplace firm a fixed membership fee plus a unit rate as a membership delivery service charge to use its delivery service while still competing in the same market. In this MC model, e-tailer's decision sequences are similar to that of the OC model, except that the marketplace firm and the e-tailer negotiate the unit delivery service fee (u) and the fixed fee (T) for membership in the first stage.

For the MC model, the marketplace firm's profit $\pi_1^m(p_1)$ is

$$\pi_1^m(p_1) = (p_1 - w - c)[\alpha - p_1 + \beta(p_2 - p_1) + \tau s_1] + uq_2 + T \tag{5-6}$$

The first part of Equation (5-6) is the profit from product sales, and the second and third parts represent the profit from the unit delivery service rate and fixed membership fee, respectively, paid by the e-tailer.

Similarly, for the MC model, the e-tailer's profit $\pi_2^m(p_2)$ is

$$\pi_2^m(p_2) = (p_2 - w - c)[\alpha - p_2 + \beta(p_1 - p_2) + \tau s_1] - uq_2 - T \tag{5-7}$$

The membership price negotiation process for the MC model is as follows

$$\max_{u,T} \pi^{mT}(u,T) = \max_{u,T}[\pi_1^m(p_1)]^{\theta}[\pi_2^m(p_2)]^{1-\theta} \tag{5-8}$$

The marketplace firm's optimal retail price (p_1^m), unit delivery rate (u^m), fixed membership fee (T^m), and the e-tailer's optimal retail price (p_2^m) in the MC model are provided in Table 5.2.

Table 5.2 Optimal solutions of the three models

Models	Competition model ($i=n$)	OC model ($i=o$)	MC model ($i=m$)
p_1^i	$c+w+\frac{\tau\Delta s\beta(1+\beta)+\delta(2+3\beta)}{(2+\beta)(2+3\beta)}$	$c+w+\frac{\delta}{4(8+32\beta+41\beta^2+18\beta^3)}\{16+9\beta^3(4+\theta)+6\beta^2(15+2\theta)+\beta(68+6\theta)+3\beta A\}$	$c+w+\frac{\delta(4+6\beta+9\beta^2)}{2(4+8\beta+9\beta^2)}$
p_2^i	$c+w+\frac{\delta(2+3\beta)-\tau\Delta s(2+4\beta+\beta^2)}{(2+\beta)(2+3\beta)}$	$c+w+\frac{\delta}{4(1+2\beta)(8+24\beta+25\beta^2+9\beta^3)}\{9\beta^4(4+\theta)+4(6+\theta)+16\beta(7+\theta)+6\beta^3(23+4\theta)+4\beta^2(47+7\theta)-(2+4\beta+3\beta^2)A\}$	$c+w+\frac{\delta(4+12\beta+18\beta^2+9\beta^3)}{2(4+12\beta+17\beta^2+9\beta^3)}$
m^i	/	$c+\frac{1}{4(8+40\beta+73\beta^2+59\beta^3+18\beta^4)}\{16\delta+80\delta\beta+144\delta\beta^2+114\delta\beta^3+36\delta\beta^4+8\delta\theta+32\delta\beta\theta+50\delta\beta^2\theta+36\delta\beta^3\theta+9\delta\beta^4\theta-\delta(4+8\beta+3\beta^2)A\}$	/
u^i	/	/	$\frac{\delta\beta(2+3\beta)^2}{8+24\beta+34\beta^2+18\beta^3}$
T^i	/	/	$\frac{\delta^2}{4(1+\beta)(4+8\beta+9\beta^2)^2}[-16-96\beta-284\beta^2-456\beta^3-405\beta^4-162\beta^5+(32+160\beta+396\beta^2+552\beta^3+441\beta^4+162\beta^5)\theta]$

Where $A=[(48+256\beta+500\beta^2+432\beta^3+144\beta^4)(1-\theta)+(4+16\beta+28\beta^2+24\beta^3+9\beta^4)\theta^2]^{\frac{1}{2}}$

5.4 Effects of outsourcing coopetition

In this section, the effects of the OC strategy on maximum profits and consumer surplus for both firms are examined by comparing the derived equilibriums for the competition model and the OC model.

5.4.1 Effect of outsourcing coopetition on maximum profits

First, we explore the effect of OC on both firms' maximum profits and present the following proposition.

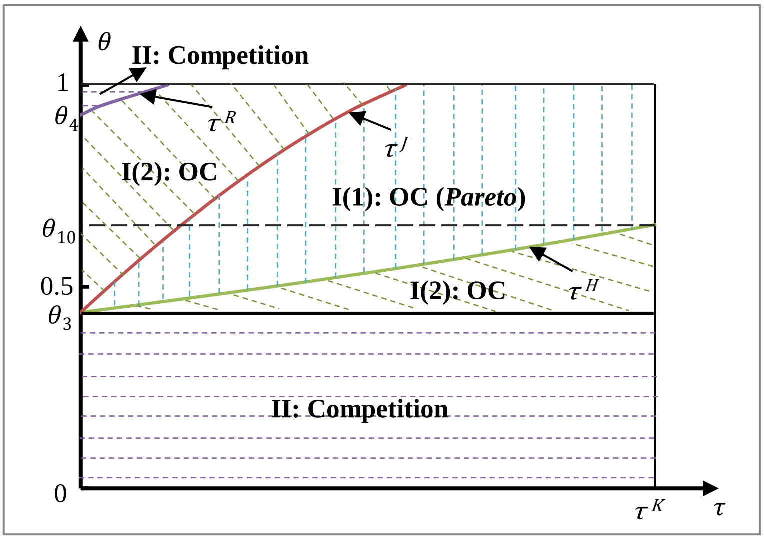
Proposition 5.1:

(1) The difference between the two firms' maximum total profits in the OC and competition models is decided by the degree of product substitution (β), the marketplace firm's bargaining power factor (θ) and the consumers' willingness to pay for service products (τ), and their relationships with the corresponding thresholds: β_1, β_2, θ_3, θ_4, τ^K, τ^R, and τ^T. [19]

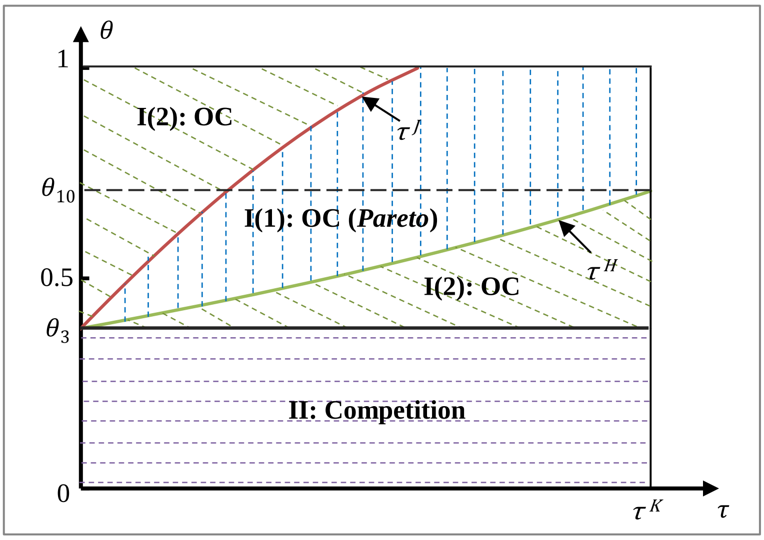
(2) When OC is a better strategy than competition, and if $\tau^J < \tau < \tau^H$, then OC delivers Pareto improvement.

More specifically, when the degree of product substitution is low ($0 < \beta < \beta_1$), if $\theta_3 < \theta < \theta_4$ and $0 < \tau < \tau^K$, or $\theta_4 < \theta < 1$ and $\tau^R < \tau < \tau^K$; or when the degree of product substitution is medium ($\beta_1 < \beta < \beta_2$), if $\theta_3 < \theta < 1$ and $0 < \tau < \tau^K$; or when the degree of product substitution is high ($\beta > \beta_2$), if $\theta_3 < \theta < \theta_5$ and $0 < \tau < \tau^T$, or $\theta_5 < \theta < 1$ and $0 < \tau < \tau^K$, then OC is the better strategy; otherwise, competition is the better strategy. These conditions are further illustrated in Figure 5.2, which is divided into several decision regions. Each region is discussed as follows.

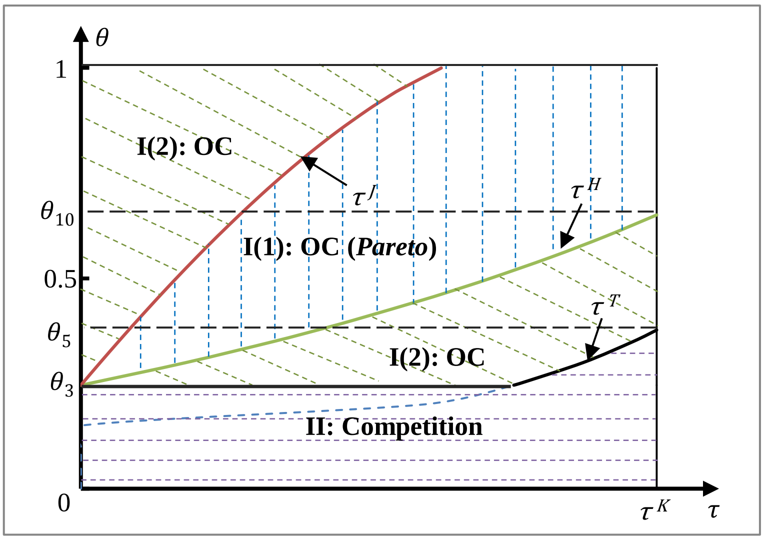
[19] Where $\beta_1 \approx 0.6996$, $\beta_2 \approx 1.8754$, $\theta_3 = \frac{4(1+\beta)}{8+10\beta+3\beta^2}$, $\tau^K = \frac{(2+3\beta)(\alpha-c-w)}{(2+4\beta+\beta^2)\Delta s-(2+3\beta)s_1}$, $\theta_4 = \frac{4(16+128\beta+472\beta^2+936\beta^3+1049\beta^4+630\beta^5+153\beta^6)}{128+736\beta+2064\beta^2+3424\beta^3+3536\beta^4+2190\beta^5+693\beta^6+81\beta^7}$, The forms of θ_5, τ^R, τ^T, τ^H and τ^J are listed in the proof of Proposition 5.1 in the Appendix A, where the value of θ_5 depends on the degree of product substitution (β), and the values of τ^R, τ^T, τ^H and τ^J depend on β, the marketplace firm's negotiation power over the e-tailer (θ) and the difference in the delivery service levels between the two firms (Δs).



(a) $\boldsymbol{0 < \beta < \beta_1 \approx 0.6996}$ $\boldsymbol{(\beta = 0.5)}$



(b) $\boldsymbol{\beta_1 \approx 0.6996 < \beta < \beta_2 \approx 1.8754}$ $\boldsymbol{(\beta = 1)}$



(c) $\boldsymbol{\beta > \beta_2 \approx 1.8754}$ $\boldsymbol{(\beta = 3)}$

Figure 5.2 Effect of OC strategy on firms' profits

Outsourcing coopetition is the optimal strategy (e.g., Region I) and generates more total profit when the marketplace firm's negotiation power is greater than the associated critical threshold ($\theta > \theta_3$). In contrast, competition is the more favorable strategy (e.g., Region II) when the marketplace firm's negotiation power is less than the critical threshold ($0 < \theta < \theta_3$). These results partially explain that smaller e-tailors often outsource delivery services to marketplace firms such as Amazon or JD.com, and in contrast, larger and more powerful e-tailers provide their own delivery service or outsource their delivery services to 3rd party logistics providers instead of marketplace firms. Interestingly, it is more likely that OC is the optimal strategy (e.g., a larger decision region as an optimal strategy) if there is a high degree of product substitution (β). As a high level of product substitution often brings more intense market competition (Liu et al. 2012; Qing et al. 2017), this result supports the views of the current literature that coopetition can be an effective strategy in the competitive market environment where there is a high degree of product substitution (Dussauge et al. 2000), or in a less competitive environment, where there is a low degree of product substitution (Peng and Bourne 2009).

Furthermore, a consumer's willingness to pay for delivery services (τ) has less influence on the selection of optimal coopetition strategy except in two cases, compared to the degree of product substitution (β) and interfirm power relationship in a delivery service contract negotiation (θ). In the first case where there is a low degree of product substitution ($0 < \beta < \beta_1$), competition is the optimal strategy if consumers' willingness to pay high prices for delivery services is small ($0 < \tau < \tau^R$) and the marketplace firm has overwhelming negotiation power ($\theta_4 < \theta < 1$). Intuitively, e-tailers have less incentive to outsource their delivery service if they have less bargaining power in negotiating delivery service fees with the marketplace firm and if customers are not willing to pay a higher price for delivery service in addition to a low degree of product substitution. In the second case, where there is a high degree of product substitution ($\beta > \beta_2$), competition is a more favorable choice for the two firms if consumers' willingness to pay high prices for delivery services increases ($\tau^T < \tau < \tau^K$) and the marketplace firm's negotiation power is higher than the critical threshold, θ_3.

When OC is the optimal strategy, there is also a decision region (Region I(1)) under which both the marketplace firm and e-tailer obtain greater profit than those under the competition model. It implies that outsourcing coopetition can lead to *Pareto improvement*, and such a coopetitive relationship should be embraced by both firms. There is also a decision region (Region I(2)) under which one of the two firms will earn less profit in the OC model than in the competition model despite an increase in the total profit. In this situation, the firm, which incurs profit loss through outsourcing coopetition, is not willing to continue this cooperative relationship unless the better-placed firm wishes to redistribute the profit gain between the two firms through further cooperation mechanisms, such as a profit sharing contract. The decision regions of *Pareto improvement* are determined by the critical thresholds of τ^R, τ^T, τ^H and τ^J, and their values are dependent on the degree of product substitution (β), the interfirm power relationship (θ) and the difference in the delivery service levels between the two firms (Δs).

5.4.2 Effect of outsourcing coopetition on consumer surplus

Next, we present the effect of OC on both firms' consumer surplus.

Lemma 5.1:

(1) If $\theta_3 < \theta < \theta_6$ and $0 < \tau < \tau^C$, or $\theta_6 < \theta < 1$ and $0 < \tau < \tau^K$, then $CS_1^o > CS_1^n$; otherwise, $CS_1^o < CS_1^n$[20].

(2) The e-tailer's consumer surplus in the OC model is always higher than that in the competition model.

This lemma implies that the difference between the consumer surplus in the OC model and the competition model is also decided by the marketplace firm's bargaining power factor (θ) and the consumers' willingness to pay for delivery services (τ). This relationship is illustrated in Figure 5.3.

For the marketplace firm's customers, the consumer surplus may increase or decrease depending on the relationship between the key parameters (θ and τ) and their corresponding critical thresholds (θ_6 and τ^C), as illustrated in Figure 5.3. This can be explained by the fact that the delivery service provided by the marketplace firm remains at the same level, and at the same time, the optimal product price charged to its customers may change because of the outsourcing coopetition. On the one hand, the e-tailer may increase its product price due to an improved delivery service. As a result, the marketplace firm can increase the retail price to enhance its profit margin and, consequently, its consumer surplus decreases. On the other hand, an improved delivery service for e-tailers can also

[20] Where $\theta_3 = \frac{4(1+\beta)}{8+10\beta+3\beta^2}$, $\theta_6 = \frac{52+220\beta+288\beta^2+108\beta^3}{60+264\beta+372\beta^2+180\beta^3+27\beta^4}$ and $\tau^K = \frac{(2+3\beta)(\alpha-c-w)}{(2+4\beta+\beta^2)\Delta s-(2+3\beta)s_1}$. The forms of τ^C are listed in the proof of Lemma 5.1 in the Appendix A, where the value of τ^C depends on the degree of product substitution (β), the bargaining power factor (θ) and the difference in the delivery service levels between the two firms (Δs).

intensify market competition, and both firms drive down their retail prices for differentiation. Therefore, it will increase the consumer surplus for both firms.

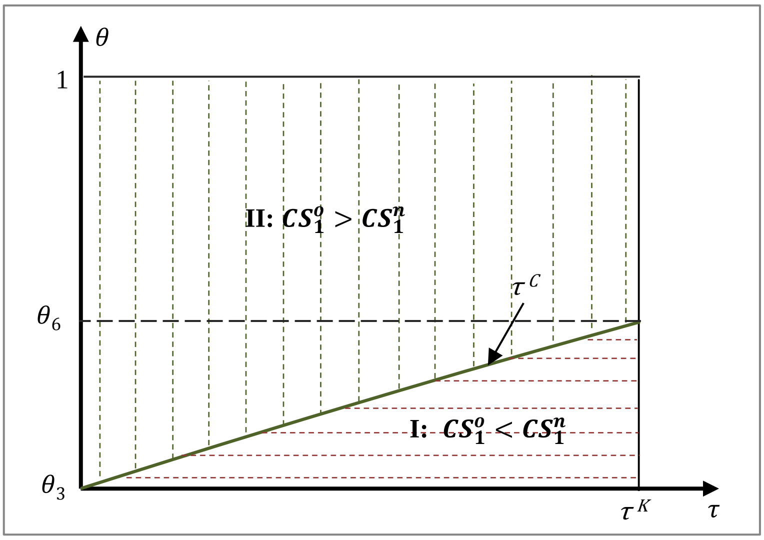


Figure 5.3 Effect of OC strategy on the marketplace firm's consumer surplus

For the e-tailer's consumers, the consumer surplus in the OC model is always higher than that in the competition model in the feasible region ($\theta_3 < \theta < 1$ and $0 < \tau < \tau^K$). This is because outsourcing coopetition leads to an improvement in delivery service for the e-tailer. However, its optimal product price will not increase to the same extent due to the market competition. So, OC always benefits the e-tailer's consumers.

5.5 Effects of membership coopetition

In this section, the effects of the MC strategy on maximum profits and consumer surplus for both firms are examined through a comparison of the derived equilibrium solutions in the competition model and the MC model.

5.5.1 Effect of membership coopetition on maximum profits

To determine the effect of membership coopetition on firms' maximum profits, we derive both firms' profits collectively and individually in the MC model and compare them with those in the competition model. The comparison results enable to derive the following proposition:

Proposition 5.2:

(1) If $\theta_0 < \theta < 1$ and $0 < \tau < \tau^K$, then MC is the preferred strategy; otherwise, competition is the preferred strategy.

(2) When MC is the preferred strategy, if $\tau^M < \tau < \tau^K$, then MC delivers Pareto

improvement.[21]

This proposition implies that the relationship between the two firms' maximum profits in the MC model and in the competition model is determined by the marketplace firm's bargaining power factor (θ) and the consumers' willingness to pay for delivery services (τ). This relationship is illustrated in Figure 5.4.

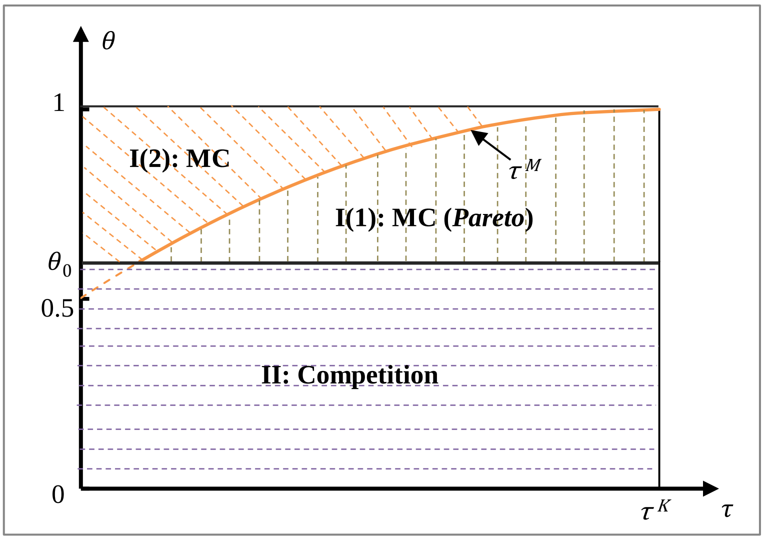


Figure 5.4 Effect of MC strategy on firms' profits

Interestingly, whether to engage in membership coopetition or competition is primarily determined by the relationship between the membership contract negotiation power factor (θ) and its corresponding critical threshold (θ_0), whose value is determined by the degree of product substitution (β). Within the feasible region ($0 < \tau < \tau^K$), coopetition is the optimal strategy with a large value of θ ($\theta_0 < \theta < 1$), and competition is the better strategy inversely, as displayed in Figure 5.4. This coincides with the industry practices where it is more likely for small e-tailers to gain membership and use marketplace firms' delivery service than large and more powerful e-tailers. Marketplace firms such as Amazon and JD.com often have greater bargaining power than e-tailers in the membership contract negotiation. In addition, there is also a decision region for *Pareto Improvement* Region I(1) when both firms' maximum profits in the MC model are larger than those in the competition model, which is mainly determined by the relationship between consumers' willingness to pay for delivery services (τ) and its corresponding threshold (τ^M). The value of τ^M is subject to a combination of key parameters, including α, β, θ, Δs, c, and w. The intuition is that it is more likely to result in

[21] Where $\theta_0 = \frac{16+96\beta+284\beta^2+456\beta^3+405\beta^4+162\beta^5}{32+160\beta+396\beta^2+552\beta^3+441\beta^4+162\beta^5}$, $\tau^M = \frac{F(\alpha-c-w)}{2(4+8\beta+9\beta^2)(2+6\beta+5\beta^2+\beta^3)\Delta s-Fs_1}$, $F = 8 + 24\beta + 34\beta^2 + 18\beta^3 - (2+\beta)\sqrt{(32+160\beta+396\beta^2+552\beta^3+441\beta^4+162\beta^5)(1-\theta)}$ and $\tau^K = \frac{(2+3\beta)(\alpha-c-w)}{(2+4\beta+\beta^2)\Delta s-(2+3\beta)s_1}$.

Pareto Improvement for MC with stronger consumer willingness to pay for a delivery service ($\tau > \tau^M$).

5.5.2 Effect of membership coopetition on consumer surplus

In the following, Lemma 5.2 presents the effect of MC on both firms' consumer surplus.

Lemma 5.2:

(1) If $\boldsymbol{0 < \beta < \beta_3,\ \theta_0 < \theta < 1}$ ***and*** $\boldsymbol{0 < \tau < \tau^D}$***, or*** $\boldsymbol{\beta > \beta_3,\ \theta_0 < \theta < 1}$ ***and*** $\boldsymbol{0 < \tau < \tau^K}$***, then*** $\boldsymbol{CS_1^m > CS_1^n}$***; otherwise,*** $\boldsymbol{CS_1^m < CS_1^n}$***.***[22]

(2) The e-tailer's consumer surplus in the MC model is always higher than that in the competition model.

This lemma implies that within the feasible region ($\theta_0 < \theta < 1$ and $0 < \tau < \tau^K$), MC always has a positive effect on the consumer surplus for the e-tailer. The explanation is similar to Lemma 5.1 in which membership coopetition improves the e-tailer's delivery service, but its product price does not increase to the same extent due to the competition. However, the effect of MC on the consumer surplus for the marketplace firm is more complicated and is determined by the relationship of the degree of product substitution (β), the consumers' willingness to pay for the delivery service (τ) and their relationship with the corresponding critical thresholds.

More specifically, for the marketplace firm, MC always has a positive effect on consumer surplus when there is a relatively high degree of product substitution ($\beta > \beta_3$) because a high level of product substitution leads to intense market competition (Liu et al. 2012; Qing et al. 2017), and an identical delivery service level will further intensify the price competition between the two firms and therefore drive down the retail prices. Consequently, its customer will benefit from an increased consumer surplus from MC. When there is a relatively low degree of product substitution ($0 < \beta < \beta_3$), its effect on consumer surplus is further dependent on the consumers' willingness to pay for delivery service (τ) and the associated threshold (τ^D) as illustrated in Figure 5.5. The value of τ^D is subject to a combination of key parameters, including $\alpha,\ \beta,\ \theta,\ s_1\ \Delta s,\ c,$ and w. This is the critical point in whether the marketplace firm will increase or decrease its product retail price as a consequence of membership coopetition. It is more likely to increase the price when the consumers' willingness to

---

[22] Where $\beta_3 \approx 0.7413$, $\theta_0 = \frac{16+96\beta+284\beta^2+456\beta^3+405\beta^4+162\beta^5}{32+160\beta+396\beta^2+552\beta^3+441\beta^4+162\beta^5}$, $\tau^D = \frac{3\beta(\alpha-c-w)(2+3\beta)^2}{(8+24\beta+34\beta^2+18\beta^3)\Delta s-3\beta(2+3\beta)^2 s_1}$, $\tau^K = \frac{(2+3\beta)(\alpha-c-w)}{(2+4\beta+\beta^2)\Delta s-(2+3\beta)s_1}$.

pay for delivery services is high ($\tau > \tau^D$).

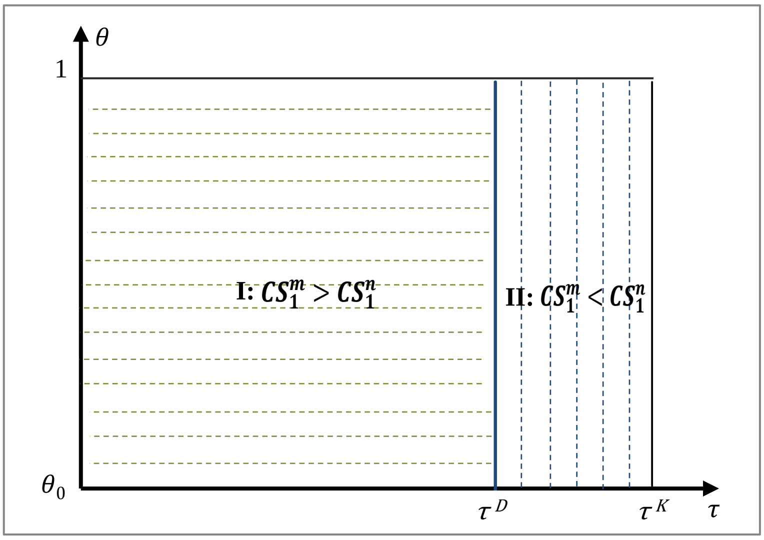


Figure 5.5 Effect of MC strategy on the marketplace firm's consumer surplus ($0 < \beta < \beta_3,\ \theta_0 < \theta < 1$)

5.6 Selection of a coopetition strategy and managerial insights

5.6.1 Selection of a coopetition strategy

In this section, we attempt to explore firms' optimal strategy regarding competition and coopetition. Despite the importance of consumer surplus, the main reason for firms to compete or cooperate with their rivals is to maximize their profits. Therefore, we evaluate the optimal selection of competition and coopetition strategies by analyzing the firms' total profits in the three different models and derive the following proposition.

Proposition 5.3:

(1) When $0 < \beta < \beta_2$, if $\theta_3 < \theta < \theta_0$ and $0 < \tau < \tau^K$, or when $\beta > \beta_2$, if $\theta_3 < \theta < \theta_5$ and $0 < \tau < \tau^T$, or $\theta_5 < \theta < \theta_0$ and $0 < \tau < \tau^K$, then OC is the optimal strategy.[23]

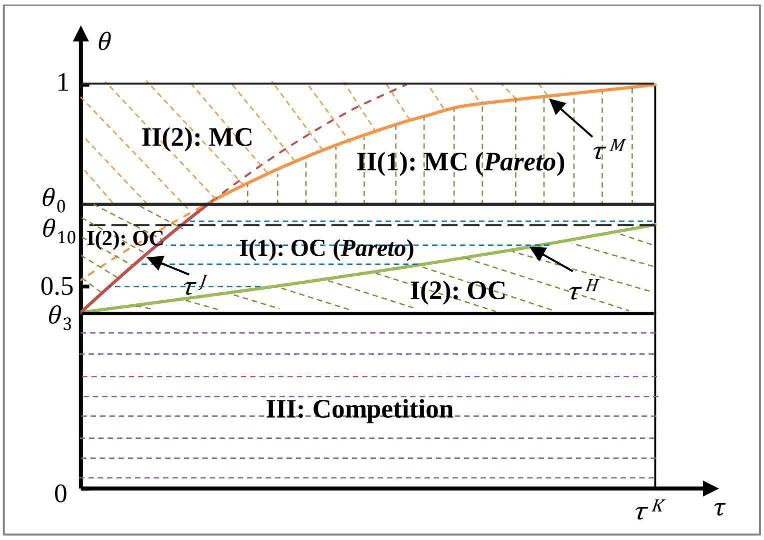
(2) When $\theta_0 < \theta < 1$ and $0 < \tau < \tau^K$, MC is the optimal strategy.

(3) Otherwise, competition is the optimal strategy.

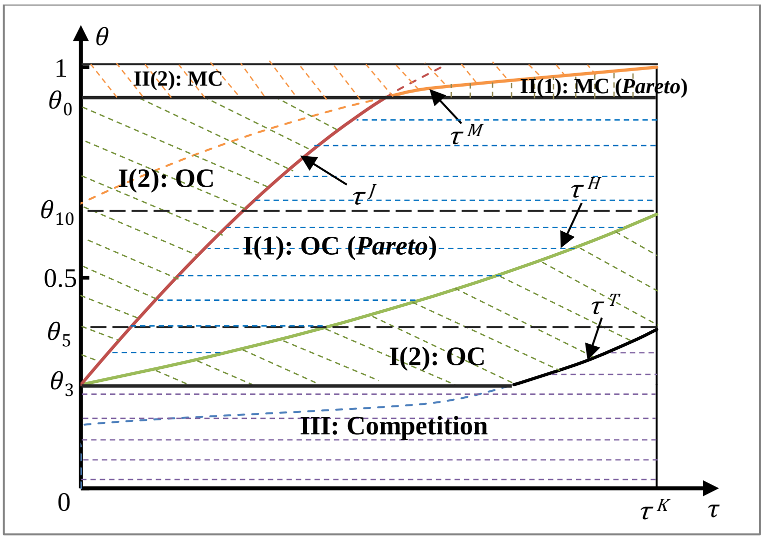
From Proposition 5.3, we can find that the optimal strategic choice among the competition, OC and MC models is determined by product substitution level (β), the marketplace firm's bargaining power factor (θ) and the consumers' willingness to pay for delivery services (τ) and their relationships

[23] Where $\beta_2 \approx 1.8754$, $\theta_3 = \frac{4(1+\beta)}{8+10\beta+3\beta^2}$, $\theta_0 = \frac{16+96\beta+284\beta^2+456\beta^3+405\beta^4+162\beta^5}{32+160\beta+396\beta^2+552\beta^3+441\beta^4+162\beta^5}$, $\tau^K = \frac{(2+3\beta)(\alpha-c-w)}{(2+4\beta+\beta^2)\Delta s-(2+3\beta)s_1}$. The forms of θ_5 and τ^T are listed in the proof of Proposition 5.3, where the value of θ_5 depends on the degree of product substitution (β), and the value of τ^T depends on β, the bargaining power factor (θ) and the difference in the delivery service levels between the two firms (Δs).

with the corresponding thresholds, as illustrated in Figure 5.6.



(a) $0 < \beta < \beta_2 \approx 1.8754$ ($\beta = 0.5$)



(b) $\beta > \beta_2 \approx 1.8754$ ($\beta = 3$)

Figure 5.6 Selecting the coopetition strategies

From Proposition 5.3, it is clear that among all the key parameters, the marketplace firm's negotiation power factor (θ) plays the most significant role in the decision on optimal coopetition strategy, although the degree of product substitution (β) also has some influence. More specifically, the optimal strategic selection is predominately decided by the relationship between θ and its corresponding thresholds (θ_0, θ_3, and θ_5). The degree of product substitution (β) affects the value of these critical thresholds and therefore impacts the selection of coopetition strategy. For instance, a large value of β increases the value of θ_0 but decreases the value of θ_3. As a consequence, the decision region of OC as the optimal strategy expands, and the decision regions of competition and MC as the optimal strategy shrink. Moreover, in a situation with a high degree of product substitution and willingness to pay for delivery services (e.g., $\beta > \beta_2$ and $\tau^T < \tau$), competition is the optimal strategy if $\theta_3 < \theta < \theta_5$, which is different to the simulation when there is low degree of product substitution ($0 < \beta < \beta_2$). Here, apart from this case, the consumers' willingness to pay for delivery services (τ) has no impact on the optimal coopetition strategy. Nevertheless, the relationships between τ and the corresponding critical thresholds determine whether the selected strategy delivers *Pareto* improvement. Further cooperation mechanisms (e.g., profit sharing or contract rebate) would be required to sustain the coopetitive relationship and achieve a win-win outcome.

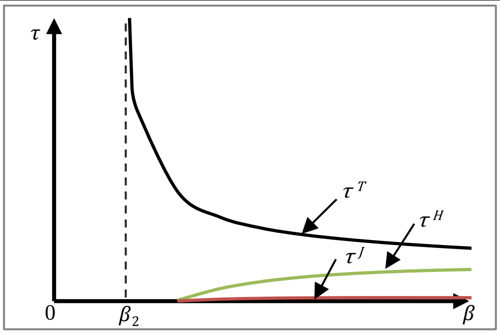
5.6.2 Sensitivity analysis

Sensitivity analysis was performed and focused on how the market characteristics (e.g., the degree of product substitution, β, and customers' willingness to pay for delivery service, τ) and the operational capability (e.g., the difference in the delivery service levels between the marketplace firm and e-tailer, Δs) influence the optimal decision on coopetition strategy selection. As to the effect of the degree of product substitution on the two critical decision thresholds, θ_3 and θ_0, the following lemma is obtained.

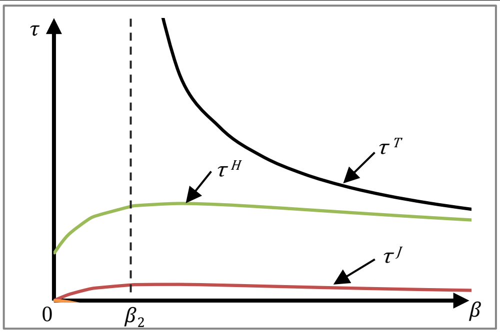
Lemma 5.3: ***θ_3 decreases in β, and θ_0 increases in β.***

This lemma means that when the degree of product substitution is high, then the value of θ_3 is small. It indicates that it is more likely for firms to choose coopetition strategy when there is a high level of market competition intensity. On the other hand, when the degree of product substitution is high, then the value of θ_0 is large. It indicates that it is more likely for the firms to select outsourcing competition between the two coopetition strategies, which further supports the finding of the optimal coopetition strategy selection illustrated in Figure 5.6.

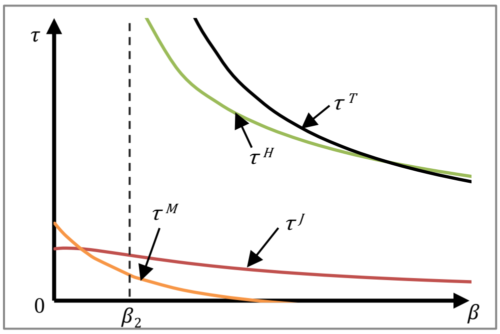
As shown in an earlier analysis, the degree of product substitution also has an impact on the values of other critical thresholds (e.g., τ^R, τ^T, τ^H and τ^J) that determine influence on the optimal strategy selection decisions or *Pareto* improvement zone. However, the values of these thresholds are also influenced by other parameters (e.g., θ and Δs). A numerical analysis was conducted to see how β affects these critical thresholds and the results are displayed in Figure 5.7.



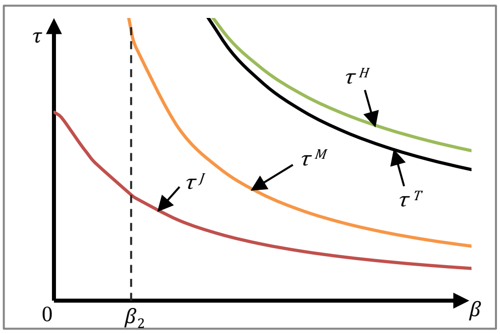
(a) $\boldsymbol{\theta = 0.25}$



(b) $\boldsymbol{\theta = 0.5}$



(c) $\boldsymbol{\theta = 0.75}$



(d) $\boldsymbol{\theta = 1}$

Figure 5.7 Effect of the degree of product substitution on $\boldsymbol{\tau^i}$ $\boldsymbol{(i = J, H, T, M)}$

From Figure 5.7, we observe that, first, an increase of the degree of product substitution will decrease the value of τ^T, but it only applies to the situation when $\beta > \beta_2$, which means that it is more likely that firms will benefit from coopetition when there is a high degree of product substitution. Second, β has an impact on τ^M only if the marketplace firm's negotiation power (θ) is high (e.g., Figure 5.7(c) and (d)) and τ^M is a decreasing function of β. This can be explained by the fact that τ^M only affects the *Pareto* improvement region when the marketplace firm has dominant negotiation power and MC is the optimal strategy. A high degree of production substitution will increase the probability of *Pareto* improvement from MC. In addition, the effects of β on the other two thresholds, τ^J and τ^H, that affect the *Pareto* improvement region of OC are more complicated depending on the marketplace firm's negotiation power (θ) and the critical threshold β_2.

As to the difference in the delivery service level between the two firms on strategy selection mainly affects the thresholds τ^T, τ^H, τ^J *and* τ^M but not θ_3 and θ_0. Therefore, the following lemma is obtained.

Lemma 5.4: ***The thresholds of the Pareto improvement regions ($\boldsymbol{\tau^H, \tau^J}$, and $\boldsymbol{\tau^M}$) and the threshold that determines the optimal strategy between OC and competition ($\boldsymbol{\tau^T}$) all decrease in $\boldsymbol{\Delta s}$.***

This lemma shows that when the difference in the delivery service level between the firms is large, then τ^T is small, which means that the decision region for competition as the optimal strategy expands, and therefore, it is more likely that firms prefer competition over coopetition if other parameters remain the same. On the other hand, when the delivery service level difference between firms is large, then τ^M is small, which means that the decision region for membership coopetition as the optimal strategy expands; therefore, firms are more likely to achieve a win-win situation when MC is the optimal strategy. The effect of the delivery service level difference on the OC strategy is more complex. When the value of Δs increases, then both τ^H and τ^J decrease. However, the *Pareto* improvement area when OC is the optimal strategy may expand or shrink depending on the extent to which τ^H and τ^J decrease, respectively.

5.6.3 Managerial implications

The above findings are beneficial for marketplace firms and e-tailers to address the dilemma of whether to compete or cooperate on delivery services with their competitors. As highlighted in our analysis, delivery service coopetition (either through membership or outsourcing) provides additional revenue stream for the marketplace firm and improves the delivery service level for the e-tailer. At the same time, both firms incur a loss when competing for customer demand with cooperation-enhanced rivals. Whether firms should engage coopetition is dependent on the trade-off between financial gain and loss, which is influenced by a combination of external, internal and relationship-specific factors. A decision framework is proposed (as illustrated in Table 5.3) to offer some strategic guidance for marketplace firms and e-tailers to make important strategic decisions.

The framework thoroughly outlines how important factors, including market characteristics related to the degree of product substitution (β), consumer characteristic-related consumers' willingness to pay for delivery services (τ) and inter-firm relationship-related marketplace firm's negotiation power (θ), affect the strategic decision on coopetition. As shown in Table 5.3, the decision on coopetition strategies is mainly determined by a marketplace firm's negotiation power and the degree of product substitution. More specifically, MC should be selected if the marketplace firm has dominant power in delivery service contract negotiation (e.g., a high value of θ); competition is the optimal strategy for the two firms if the e-tailer has dominant power (e.g., a low value of θ), and OC should be chosen if the two firms have similar power in the delivery service contract negotiation.

Here, β, which measures the cross-effect of the change in one firm's demand caused by a change in that of the other, mainly affects the two critical thresholds, θ_0 and θ_3, which specify the optimal decision region regarding coopetition. For instance, by comparing Tables 5.3(a) and 5.3(b), an increase of β (e.g., $\beta > \beta_2$) will increase the value of θ_0 and decrease the value of θ_3. As a result, it extends the design region where OC is the optimal strategy.

Table 5.3 Strategic guidance on coopetition

5.3(a) Strategic guidance on coopetition ($0 < \beta < \beta_2$)

Parameters		**Optimal strategic decision**				
Marketplace firm's negotiation power (θ)	Strong	MC	→	MC	→	MC
		OC	→	OC	→	OC
	Similar	OC	→	OC	→	OC
		OC/C	→	OC/C	→	OC/C
	Low	C	…	C	…	C
Consumers' willingness to pay for delivery services (τ)		Small		Medium		High

5.3(b) Strategic guidance on coopetition ($\beta > \beta_2$)

Parameters		**Optimal strategic decision**				
Marketplace firm's negotiation power (θ)	Strong	MC	→	MC	→	MC
		OC/MC	→	OC/ MC	→	OC/MC
	Similar	OC	→	OC	→	C/OC
		C/OC	→	C/OC	→	C/OC
	Low	C	…	C	…	C
Consumers' willingness to pay for delivery services (τ)		Small		Medium		High

Note: C, MC, MC(P), OC, and OC(P) refer to competition, membership coopetition, membership coopetition with Pareto improvement, outsourcing coopetition, and outsourcing coopetition with Pareto improvement, respectively.

In addition, although consumers' willingness to pay for delivery services (τ) and the difference in the delivery service level between the two firms (Δs) have a limited impact on coopetition strategy selection, both factors have a significant impact on whether MC or OC strategies deliver a win-win outcome for both firms. Further cooperation will be required to sustain the coopetitive relationship if one party is worse off despite an increase in overall profit.

5.7 An extended model: the asymmetric case

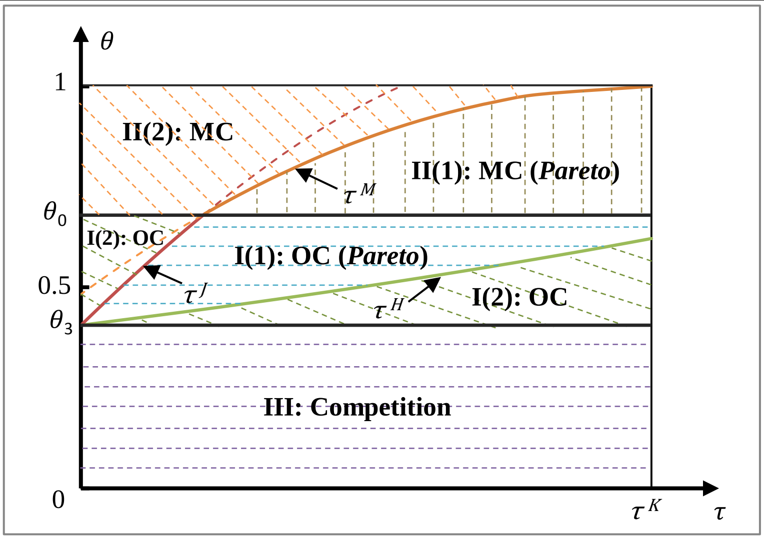
The analysis in the previous sections assumes a symmetric case in which $\alpha_1 = \alpha_2 = \alpha$. Here, α_1 and α_2 represent the maximum retail prices of marketplace firm and e-tailer, respectively. In this section, we consider a scenario in which $\alpha_1 \neq \alpha_2$. Then, the demand function $p_i = \alpha_i - q_i - \beta(q_i + q_j) + \tau[s_i - \beta(s_j - s_i)]$, $i, j = 1,2$ and $i \neq j$. Similar to 5.3.1, let $\delta_1 = \alpha_1 - c - w + \tau s_1$ and $\delta_2 = \alpha_2 - c - w + \tau s_1$; then $\delta_1 > 0$. Using this demand function, the optimal solutions for the competition, OC and MC models are obtained and presented in Table 5.4. Readers can refer to the appendix for the derivation of these optimal solutions.

Through the comparison of the optimal solutions in Table 5.4 and Table 5.2, we can conclude that α_1, α_2 significantly affect the firms' optimal operational decisions. As a consequence, firms' profits in the competition, OC and MC models will be affected by α_1, α_2 so as the values of important critical thresholds that determine firms' optimal decision regions on coopetition strategy selection. Therefore, numerical analysis is presented here to verify whether the structural results presented in the symmetric case still hold in the asymmetric case (i.e., $\alpha_1 \neq \alpha_2$). It is assumed that $\delta_1 = 1$ and $\beta = 4$. In Figure 5.8, we specify that $\alpha_2 - \alpha_1 = 0.1$, which means that $\alpha_2 > \alpha_1$. In Figure 5.9, we specify that $\alpha_2 - \alpha_1 = -0.1$, which means that $\alpha_2 < \alpha_1$.

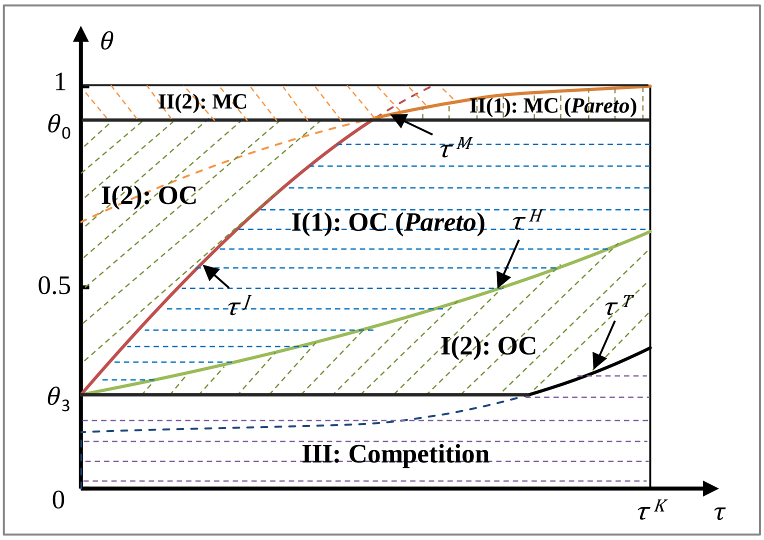
Table 5.4 Optimal solutions of the three models ($\alpha_1 \neq \alpha_2$)

Models	Competition model ($i=n$)	OC model ($i=o$)	MC model ($i=m$)
p_1^i	$c+w+\frac{\tau\Delta s\beta(1+\beta)+\delta_1(2+3\beta)-\beta\Delta\alpha}{(2+\beta)(2+3\beta)}$	$c+w+\frac{\delta(4+8\beta+3\beta^2)}{4(1+2\beta)(8+16\beta+9\beta^2)}\{\delta_1(16+9\beta^3(4+\theta)+6\beta^2(15+2\theta)+\beta(68+6\theta))-\beta[3A_1+\Delta\alpha(20+6\theta+9\beta^2(2+\theta)+4\beta(10+3\theta))]\}$	$c+w+\frac{-\Delta\alpha\beta(2+4\beta+9\beta^2)+\delta_1(4+14\beta+21\beta^2+18\beta^3)}{8+32\beta+50\beta^2+36\beta^3}$
p_2^i	$c+w+\frac{\delta_1(2+3\beta)-\tau\Delta s(2+4\beta+\beta^2)}{(2+\beta)(2+3\beta)}-\frac{2(1+\beta)\Delta\alpha}{(2+\beta)(2+3\beta)}$	$c+w-\frac{1}{4(1+2\beta)(8+24\beta+25\beta^2+9\beta^3)}\{A_1(2+4\beta+3\beta^2)+\Delta\alpha(9\beta^4(2+\theta)+28\beta^2(5+\theta)+4(6+\theta)+16\beta(6+\theta)+8\beta^3(11+3\theta))-\delta_1(9\beta^4(4+\theta)+4(6+\theta)+16\beta(7+\theta)+6\beta^3(23+4\theta)+4\beta^2(47+7\theta))\}$	$c+w+\frac{-\Delta\alpha(4+16\beta+30\beta^2+28\beta^3+9\beta^4)}{8+40\beta+82\beta^2+86\beta^3+36\beta^4}+\frac{\delta_1(4+20\beta+42\beta^2+45\beta^3+18\beta^4)}{8+40\beta+82\beta^2+86\beta^3+36\beta^4}$
m^i	/	$c+\frac{1}{4(1+2\beta)^2(8+24\beta+25\beta^2+9\beta^3)}\{\Delta\alpha(-16-96\beta-224\beta^2-256\beta^3-146\beta^4-36\beta^5-8\theta-48\beta\theta-114\beta^2\theta-136\beta^3\theta-81\beta^4\theta-18\beta^5\theta)+\delta_1(16+112\beta+304\beta^2+402\beta^3+264\beta^4+72\beta^5+8\theta+48\beta\theta+114\beta^2\theta+136\beta^3\theta+81\beta^4\theta+18\beta^5\theta)-(4+16\beta+19\beta^2+6\beta^3)A_1\}$	/
u^i	/	/	$\frac{\beta[\delta_1(1+2\beta)(2+3\beta)^2-\Delta\alpha\beta(8+16\beta+9\beta^2)]}{8+40\beta+82\beta^2+86\beta^3+36\beta^4}$
T^i	/	/	$\frac{1}{4(1+3\beta+2\beta^2)(4+8\beta+9\beta^2)^2}P(\Delta\alpha)$

Where $A_1=\left[\Delta\alpha^2(2+4\beta+3\beta^2)^2(-2+\theta)^2+{\delta_1}^2\left((48+256\beta+500\beta^2+432\beta^3+144\beta^4)(1-\theta)+(4+16\beta+28\beta^2+24\beta^3+9\beta^4)\theta^2\right)+2\delta_1\Delta\alpha\left((16+96\beta+208\beta^2+196\beta^3+72\beta^4)(1-\theta)+(4+16\beta+28\beta^2+24\beta^3+9\beta^4)\theta^2\right)\right]^{\frac{1}{2}}$ and $P(\Delta\alpha)=\Delta\alpha^2(2+4\beta+3\beta^2)^2(9\beta^2(-1+\theta)+4\theta+8\beta\theta)-2\delta_1\Delta\alpha(1+2\beta)(81\beta^5(-1+\theta)+16\theta+16\beta(-1+5\theta)+36\beta^4(-5+6\theta)+28\beta^2(-3+7\theta)+16\beta^3(-11+17\theta))+{\delta_1}^2(1+2\beta)[-16+162\beta^5(-1+\theta)+32\theta+32\beta(-3+5\theta)+24\beta^3(-19+23\theta)+9\beta^4(-45+49\theta)+4\beta^2(-71+99\theta)]$.

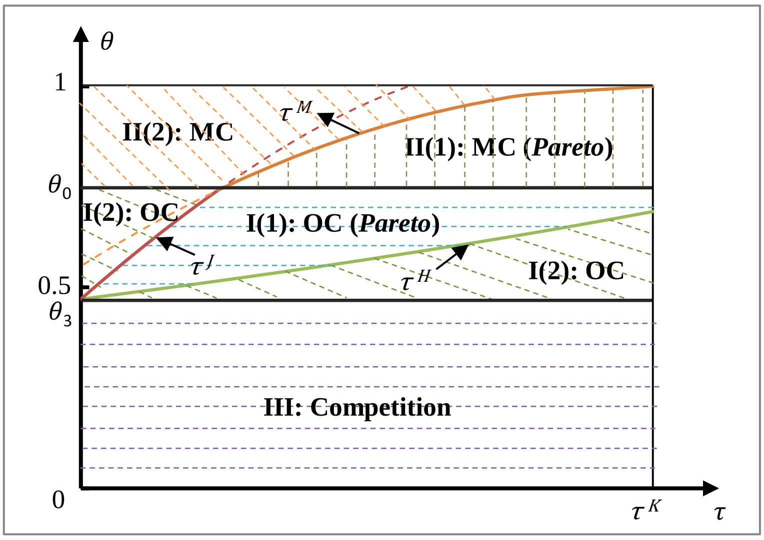


(a) $0 < \beta < \beta_2 \approx 1.8754$ $(\beta = 0.5)$

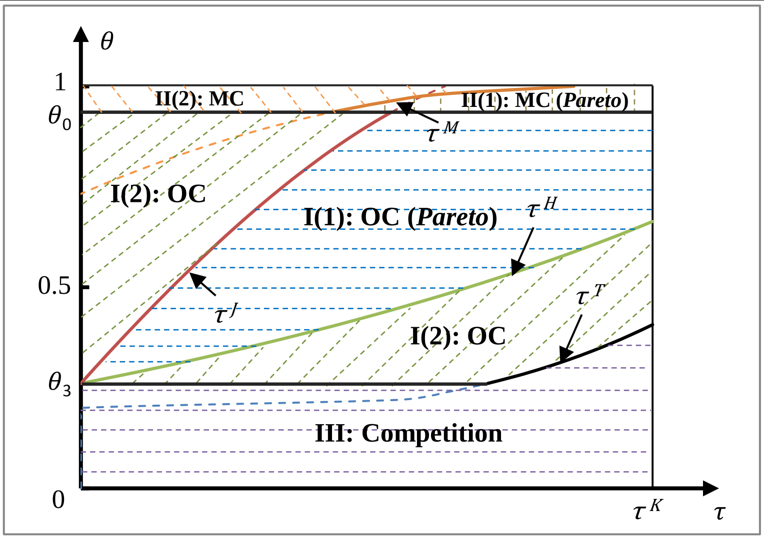


(b) $\beta > \beta_2 \approx 1.8754$ $(\beta = 3)$

Figure 5.8 Selection of coopetition strategies ($\alpha_2 > \alpha_1$)



(a) $0 < \beta < \beta_2 \approx 1.8754$ $(\beta = 0.5)$



(b) $\beta > \beta_2 \approx 1.8754$ ($\beta = 3$)

Figure 5.9 Selection of coopetition strategies ($\alpha_2 < \alpha_1$)

It is clear from Figures 5.8 and 5.9 that the structural results of optimal strategic decisions on delivery service coopetition is similar to the symmetric case where the marketplace firm and e-tailer have the identical maximum retail price. Nevertheless, the critical thresholds that specify decision regions of the associated optimal coopetition strategy are influenced by the difference between the maximum retail prices of the two firms ($\Delta\alpha$). For instance, when the marketplace firm has a higher maximum retail price than the e-tailer ($\alpha_1 > \alpha_2$), it increases the value of the critical threshold, θ_3. As a result, it extends the decision region where competition is the optimal strategy. At the same time, it also increases the value of the critical threshold, θ_0, and consequently, it reduces the decision region where MC is the optimal strategy. In other words, membership coopetition is less likely to be the optimal strategic decision. Noticeably, α_1, α_2 affect firms' optimal operational decisions (e.g., retail prices, delivery service level, unit price for outsourcing delivery service, and membership fee and unit price for membership delivery service) and the critical thresholds that determine firms' optimal decisions on coopetition strategy. Nevertheless, the structural results of the selection of the optimal delivery service coopetition strategy presented in the previous sections still hold in the asymmetric case.

5.8 Conclusions

This research is inspired by the problem faced by many online retailers of whether to invest in one's own distribution and logistics operation or use the delivery service provided by marketplace firms like Amazon or JD.com. From marketplace firms' perspectives, whether they should offer the delivery service to all merchants selling on their platform, even to their fieriest market rivals, is the question. If yes, how do they determine the key parameters, e.g., unit delivery rate or membership fee, in setting up the service delivery cooperation contract? As a fast-growing industry sector, these are common business problems faced by many e-tailers and marketplace firms. Following this enquiry, we investigated the problem of whether the marketplace firm and e-tailer should contract with each other for delivery service cooperation and how to set up the cooperation contract by analyzing the system equilibria from the perspectives of both firms individually and collectively. The analysis results lead to the following managerial insights.

- The underlying principle that governs the firms' delivery service coopetition behavior is the trade-off between the benefits gained from cooperation and the financial loss incurred when facing a strengthened competitor. For the e-tailer, the benefit of delivery service cooperation comes from demand increase induced from the improved delivery service level. For the marketplace firm, the benefit of cooperation comes from extra revenue income as a delivery service provider. The benefits gained from cooperation will consequently strengthen both firms' positions when competing with each other for customer demand, which has a negative impact on each firm's profits. The trade-off between the benefit and loss from cooperation determines the competing firms' decision on the delivery service cooperation. The interplay of competitive and cooperative elements of the relationship is the cause of coopetition dynamics, which further influences the competitive dynamics within the online retailing industry (Ritala 2012; Dahl 2014).
- Not only do our results demonstrate that the strategic decision on coopetition is driven by external, relation-specific, and internal factors (Bengtsson and Raza-Ullah 2016); they also illustrate how the external market-related product substitution rate and consumer's willingness to pay delivery service, inter-firm power relationship in the cooperation contract negotiation, and internal distribution/logistics capability affect firms' service coopetition behavior. While the marketplace firm's power in the cooperation contract negotiation and its relationship with critical thresholds (e.g., θ_0 and θ_3) have the most significant impact on the coopetition strategy selection decision, the degree of product substitution, which is an important indicator for market competition, affects the values of these critical thresholds. Moreover, although the coopetition strategy selection decision is not dependent on consumers' willingness to pay for delivery services (τ) and the

difference in the delivery service level between the two firms, the two factors play a critical role in determining whether MC or OC strategies deliver win-win outcomes for both firms. These internal and external factors may also change over time and, as a result, further impact firms' optimal strategic choice between competition and coopetition.

- Moreover, we reveal how service coopetition impacts the consumer surplus of the online retailing eco-system. For instance, coopetition, either membership or outsourcing coopetition, will improve the consumer surplus of the e-tailer's customers. Nevertheless, the impact of coopetition on the marketplace firm's customers' consumer surplus is a more complex subject in the relationship between the degree of product substitution (β) and the consumers' willingness to pay for delivery services (τ) and their corresponding critical thresholds. This is also one of the key differences between collusion and coopetition, where consumers are penalized by the decreasing consumer surplus in collusion and coopetition can lead to a 'win-win-win' outcome for participating firms and consumers (Rusko 2011).

Chapter 6 Conclusions and future research

6.1 Introduction

The final chapter reflects on issues of coopetition models and applications examined in this doctoral research. It begins with a summary of the main research findings in each study. It is then followed by a discussion on the theoretical contributions to knowledge highlighting how this doctoral research collectively contributes to the knowledge of the field. Managerial implications of this doctoral research are discussed highlight how firms can learn from the insights derived in this research. Finally, the dissertation is concluded by critically discussing the research limitations and suggesting the directions of future research avenues. Each of these elements will be further elaborated in the following sections.

6.2 Research Findings

This dissertation explores the strategic choices between competition and coopetition by the two rival firms through the three different contextual settings: production coopetition, green R&D coopetition, and service coopetition. The main research findings of each study are summarized as below.

In the first study (**Chapter 3**), the research examines production coopetition strategies for competing manufacturers (e.g., Apple or Samsung) that produce substitutable products. There is a complex relationship between these two rival firms. More specifically, despite being the market rival, each of these firms has an option to purchase (or sell) a key component from (or to) the other. Two coopetition models are developed in this study. In the wholesaling coopetition model, the manufacturers compete for end-customer demand but collaborate on component production through buyer-supplier cooperation. In the license coopetition model, the manufactures collaborate on component production through licensing agreement while competing with each for end-customer demand. Through a comparison of the equilibria of two coopetition models and the benchmark competition model, the research findings highlights that the optimal coopetition strategy is determined by a combination of internal, inter-firm, and external factors including the degree of product substitution, the inter-firm power relationship in the negotiation of a cooperation contract (i.e., wholesale price and license fee) and the difference in production efficiency between the two manufacturers, which is in line with the argument of Bengtsson and Raza-Ullah (2016) that the strategic decision on coopetition is driven by external, relation-specific, and internal factors. Fundamentally, it is the trade-off between the benefit (gain from the production cooperation) and the loss (incur from market competition with a cooperation strengthened competitor) that determines firms' strategic decision on coopetition (e.g., competition vs. coopetition or wholesaling vs. licensing). The extent of benefit and loss depends on a combination of important external and internal factors including the degree to which their products are substitutable, power relationship in the contract negotiation, maximum retail prices and cost difference in component production. Essentially, the dynamic relationship between the cooperation and competition forces governs firms' choice of coopetition strategies, subjecting to the changes in internal operational capabilities and/or the external market environment over time (Brandenburger and Nalebuff 1996; Dahl 2014). An enduring coopetitive relationship requires a win-win outcome for all parties engaged in coopetition. When coopetition delivers superior total profit (either wholesaling coopetition or license coopetition), there exists *Pareto improvement* that results in improved profits for both manufacturers and decreased retail prices. Under the same condition, there are also situations that one of the manufacturers is worse off and a further operational mechanism (i.e., profit-sharing contracts) is required to sustain the coopetitive relationship.

In the second study (**Chapter 4**), the research examines low carbon technology licensing coopetition strategies between rival firms under cap-and-trade policies. Wide access to low carbon technologies is crucial to achieve carbon emissions reduction targets in the battle of Climate Change Challenge, and technology licensing has become central form of interfirm technology transfer and commercialization (Khoury et al. 2018). However, there is a dilemma embedded in technology licensing especially when the two trading parties are market rivals (Fosfuri, 2006; Wu 2018). From the licensor's perspective, it is the trade-off between the revenue increase from the licensing payments and the reduced profit margin and/or reduced market share implied by increased competition from the licensee. From the licensee's perspective, it is the trade-off between the cost of license payment and the increased profit margin and/or market share implied by licensed technology enhanced market competitiveness. We investigate the effects of low carbon technology licensing on the economic and environmental performance of two rival manufacturers under a cap-and-trade policy, one of the most influential emissions trade schemes that have been widely adopted by many places worldwide. We model alternative contractual arrangement of technology licensing through either royalty, fixed fee or mixed fee and evaluate the performances of four model settings (i.e., pure competition, royalty licensing coopetition, fixed-fee licensing coopetition and mixed licensing coopetition) from the perspectives of different stakeholders including manufacturers, customers and policy makers. Similar to the previous two studies, the research findings show that the contractual choice on low carbon technology licensing is determined by the trade-off between the benefits gained from technology licensing and the consequential losses incurred from competition with a strengthened competitor, which is influenced by a combination of factors including internal operational and low carbon capability (e.g., maximum marginal profit of licensor and manufacturers' unit carbon), interfirm power relationship, external market characteristics (e.g., the price elasticity of demand, product substitution level, and manufacturers' maximum retail prices), and the carbon emission control policy (e.g., carbon emissions cap and unit carbon emission trade price). Again, this supports the argument of Bengtsson and Raza-Ullah (2016) that the strategic decision on coopetition is driven by external, relation-specific, and internal factors. Interestingly, among these factors, the interfirm power relationship plays a more prominent role in determining the optimal contractual decision on the low carbon technology licensing. More specifically, mixed fee licensing is preferable choice if the licensor has a dominant power in the contractual negotiation, and in contrast, no licensing agreement is preferable choice if the licensor has less negotiation power as compared to the licensee. Technology licensing through royalty fee should be considered if licensor's negotiation power is the between. Fixed-fee is not a viable option as compared to others. While firms' decision on whether and how to license their low carbon technologies to rival firms is mainly determined by their economic benefit, the licensing decisions have profound impact on individual firms, environment and consumers. Furthermore, the optimal decision on low carbon technology licensing is dynamic that is influenced by the internal operational and technological capability, external market and policy environment, and interfirm power relationship. The changes in the internal capabilities, the external environment and/or interfirm power balance over time will affect the firms' strategic decisions about low carbon technology licensing coopetition. Finally, our analysis shows that it is critical for governments to develop appropriate carbon emissions control policies to promote the agenda of a sustainable, low-carbon economy.

In the third study (**Chapter 5**), the research examines the delivery service coopetition strategies of an e-tailer and a marketplace firm for substitutable products. As more retailers are selling online, e-tailers face a dilemma between investing in their own distribution/logistics operations or using the logistics service provided by marketplace firms (e.g., Amazon or JD.com). Service cooperation improves the service level of e-tailers, which results in increased customer demand but comes at some expense (e.g. service charge or membership fee). Providing delivery services will generate additional revenue income for marketplace firm. However, the delivery service cooperation will have a negative impact on both firms' profitability when they face strengthened competitors. Inspired by this problem, we consider a competitive setting in which an e-tailer and a marketplace firm (e.g., Amazon or JD.com) sell partially substitutable products. The e-tailer may choose to contract with the marketplace firm to use its delivery service. We analyze the optimal decisions for both the e-tailer and marketplace firm and characterize the system equilibria. We find that a firm's decision regarding coopetition strategies is mainly determined by the inter-firm power relationship in the cooperation contract negotiation and the degree of product substitution. Similar to the previous study, the underlying principle that governs the firms' delivery service coopetition behavior is the trade-off between the benefits gained from cooperation and the financial loss incurred when facing a strengthened competitor. For the e-tailer, the benefit of delivery service cooperation comes from demand increase induced from the improved delivery service level. For the marketplace firm, the benefit of cooperation comes from extra revenue income as a delivery service provider. The benefits gained from cooperation will consequently strengthen both firms' positions when competing with each other for customer demand, which has a negative impact on each firm's profits. The trade-off between the benefit and loss from cooperation determines the competing firms' decision on the delivery service cooperation. Not only do our results demonstrate that the strategic decision on coopetition is driven by external, relation-specific, and internal factors (Bengtsson and Raza-Ullah 2016); they also illustrate how the external market-related product substitution rate and consumer's willingness to pay delivery service, inter-firm power relationship in the cooperation contract negotiation, and internal distribution/logistics capability affect firms' service coopetition behavior. Factors including customers' willingness to pay for a delivery service and the difference in the delivery service level between the two firms affect the magnitude of benefit and loss from service coopetition, which has an impact on whether coopetition results in a win-win outcome for the two firms. These internal and external factors may also change over time and, as a result, further impact firms' optimal strategic choice between competition and coopetition. The interplay of competitive and cooperative elements of the relationship is the cause of coopetition dynamics, which further influences the competitive dynamics within the online retailing industry (Ritala 2012; Dahl 2014). Moreover, we reveal how service coopetition impacts the consumer surplus of the online retailing eco-system. For instance, coopetition, either membership or outsourcing coopetition, will improve the consumer surplus of the e-tailer's customers, and the impact of coopetition on the marketplace firm's customers' consumer surplus is a more complex. This finding highlights one of the key differences between collusion and coopetition that consumers are often penalized by the decreasing consumer surplus in collusion and coopetition can lead to a 'win–win–win' outcome for participating firms and consumers (Rusko 2011).

6.3 Contribution

This study makes several theoretical contributions. First, our research contributes to the coopetition literature by presenting models and applications of production coopetition, low carbon technology licensing coopetition, and service coopetition between rival firms and filling an important gap in the literature. More specifically, the research problem studied in Chapter 3 differs from traditional supply chain cooperation that mainly focuses on a vertical supplier-buyer supply chain relationship (Yang et al. 2017). This study, in contrast, explores how firms' individual and collective performance are affected by the interaction of horizontal market competition and vertical supply chain cooperation. The low carbon technology licensing coopetition studied in Chapter 4 extended the coopetition application to low carbon manufacturing under the cap-and trade policy (Luo et al. 2016; Hafezalkotob 2017). Such an application does not only consider the external market environment, relation-specific power relationship, and internal operational factors that argued by Bengtsson and Raza-Ullah (2016) as the main drivers of the strategic decision on coopetition, but also incorporate the policy environment (e.g., cap-and-trade) in the analysis. The exploration of these coopetition models and applications enables to produce the structured optimal solutions that improve the understanding of coopetitive behavior of firms in various business setting. For the service coopetition studied in Chapter 5, despite rapid growth in online retailing and strategic significance of delivery service for the sector, few studies have so far employed the notion of coopetition to examine how delivery service cooperation impacts the online retailing eco-system (Pekgün et al. 2017; Ding et al. 2018). In the context of online retailing, for the marketplace firm, delivery service cooperation generates additional revenue streams; for the e-tailer, cooperation helps increase demand, which is stimulated by the improved delivery service. At the same time, both retailers could incur losses when facing a strengthened competitor as a consequence of the service cooperation.

Second, through modeling the firms' decision behaviors and consequential performances in three different coopetition applications, our analysis helps to understand the economic principle underlining firms' strategic decision on coopetition. It is the trade-off between the benefits gained from cooperation and financial loss incurred when facing a strengthened competitor that determines firms' strategic decision on coopetition. More specifically, for the first study, the optimal strategy for production coopetition is determined by the intensity of market competition (Tsay and Agrawal 2000; Peng and Bourne 2009; Ritala 2012) as well as the joint effect of external market characteristics, the power relationship between manufacturers in the cooperation contract negotiation and the difference in production efficiency between engaging firms. The second study also supports the same economic principle that governs firms' strategic decision on coopetition. In addition, while the cooperation benefits are influenced by the degree of product substitution, the customers' willingness to pay for a delivery service, and the difference between the two firms' delivery service capabilities, the losses incurred in the demand competition are dependent on both the price and service competition factors. This trade-off between the benefits gained from cooperation and financial loss incurred when facing a strengthened competitor is determined by many external, relation-specific, and internal factors that Bengtsson and Raza-Ullah (2016) argued as the main drivers for firms' coopetition decision. While the third study also supports the same economic principle that governs firms' strategic decision on coopetition, it highlights that the interfirm power relationship plays a more prominent role than other internal and external drivers in determining the optimal contractual decision on the low carbon technology licensing coopetition. The study also argues that to achieve sustainable coopetitive relationship requires an improvement in both economic and environmental performance as well as a win-win outcome for individual firms and consumers.

Third, our analysis provides valuable insights of firms' coopetition behavior. The examination of three coopetition applications generates a broader set of decision outcomes that have not been captured from traditional models. Considering the dynamic nature of the competition and cooperation dualism (Luo 2007; Dahl 2014; Dorn et al. 2016), our exploration into the dynamics of coopetition helps researchers and managers understand how the decrease or increase of competition intensity level will have an impact on the benefits of coopetition strategies, and how the nature of competitions is influenced by changing organizational or environmental conditions caused by their coopetition decisions. Moreover, the market competition intensity, inter-firm power relationships, and dynamics of coopetition captured and explored in our analytical modelling are an important supplement to the existing studies (Luo 2007; Ritala 2012; Dorn et al. 2016) that suggest these areas as key issues to advance coopetition research. Coopetition decision frameworks are developed in these studies in an attempt of providing strategic guidance on firms' decisions on coopetition strategies. The decision frameworks proposed in the doctoral dissertation give a richer representation of firms' strategic behavior towards coopetition and contribute to the continuing debates concerning the efficacy of coopetition (Brandenburger and Nalebuff 1996; Gnyawali and Madhavan 2001). This research builds on a body of work that have recognized that any approach to understand inter-firm behavior must include agency and well as structural action (Granovetter 2005).

6.4 Managerial implications

The findings of this doctoral research provide important managerial implications that can be utilized as strategic guidance for firms to pursue coopetition in different business settings. The systematic examination of various coopetition models and applications enables to derive the structured optimal solutions for the involved business organizations and provides a better understanding of effects of coopetition in the different business environments. The coopetition does not necessarily lead to profit increase and the strategic decision on coopetition is affected by many factors (e.g. the external market and policy environment, the internal operational capabilities, and the inter-firm power relationship). With the rapid economic development, it is critical for firms to choose the appropriate strategies to cope with fierce market competition. These new strategic choices also bring new challenges for firms' operational decisions such as pricing policies and service levels. Therefore, our research makes some practical managerial contributions to businesses that are currently operating or planing to operate in similar market environments. Our analysis results can be used a strategic guidance for firms to decide how to choose the coopetition or competition strategy according to their operational capabilities, the market and policy environment. In addition, with a better understanding of the underlying economic principle that governs firm's coopetition decisions, this research will be helpful for managers to make the right strategic and operational decisions to enhance their competitive advantages. More specifically:

The research findings in the first study are particularly beneficial to firms in the smartphone, automobile, PC, and medical devices industries that are currently engaging in some kind of cooperation with their competitors such as wholesaling of licensing arrangements or have an intention to pursuing such opportunities. In this competitive and ever-changing market environment, it is critical for firms to adopt more-sophisticated strategies in the market competition rather than simply focusing on product or price. As seen in the smartphone, automobile, and pharmaceutical industries, coopetition has emerged as a viable strategic option. However, whether firms should pursuing coopetition strategy will depend upon their internal, external and relationship specific factors that determine the tradeoff between the benefit and loss from cooperating and competing forces. The decision framework is proposed, which systematically outlines how the external, relationship-specific and internal factors affect the strategy selection. It can be used as strategic guidance by firms in a similar business environment to make important strategic and operational decisions.

The research findings in the second study also make important practical and policy contributions. For manufacturing firms, the findings could support them in making optimal strategic and operational decisions regarding low carbon technology licensing and improving their competitiveness. For policy makers, the findings could help them to develop appropriate carbon emissions control policies that support a sustainable, low-carbon economy. In general, green technology licensing between rival firms will lead to reduced total carbon emissions. Therefore, it is critical for governments to create a supportive policy environment that incentivizes technology licensing. For instance, a cap-and-trade policy with a high carbon emissions cap and/or lower unit carbon emission trade price is more likely to encourage firms to adopt mixed licensing technology. In addition, it is important to protect the green innovator’s bargaining power in the technology licensing negotiation. It is more likely to promote the green technology licensing among the rival firms in such a policy environment.

The research findings in the third study are particularly beneficial for marketplace firms and e-tailers operating in the online retailing environment to address the dilemma of whether to compete or cooperate on delivery services with their competitors. Delivery service coopetition provides additional revenue stream for the marketplace firm and improves the delivery service level for the e-tailer. At the same time, both firms incur a loss when competing for customer demand with cooperation-enhanced rivals. Whether firms should engage coopetition is dependent on the trade-off between financial gain and loss, which is influenced by a combination of external, internal and relationship-specific factors. A decision framework thoroughly outlining how important factors including market characteristics related to the degree of product substitution, consumer characteristic-related consumers' willingness to pay for delivery services and inter-firm relationship-related marketplace firm's negotiation power, affect the strategic decision on coopetition is proposed to offer some strategic guidance for marketplace firms and e-tailers to make important strategic decisions on coopetition.

6.5 Limitations and future research

Similar to many other studies using modeling approaches, several assumptions are made in this doctoral research. Several useful directions of future research can emerge by relaxing these assumptions. For instance, deterministic demand functions are adopted in all the three coopetition applications. Although these forms of demand functions have been widely adopted in similar studies (Wang et al. 2013; Qing et al. 2017; Yang et al. 2017), demand uncertainty is often one of the important factors for firms to engage in coopetition as pointed out by Ritala (2012). One future research extension is to apply stochastic demand function to explore how the results captured in the three studies might be affected by stochastic demand. In the first study, manufacturer 2 is assumed to have the ability to produce the common component at the same quality level as manufacturer 1 with a higher production cost. Nevertheless, the quality of manufacturer 2's product might be compromised with the key component and consequently have a negative impact on customer demand. It will be beneficial to incorporate the quality aspect into the coopetition models.

Furthermore, we identify the non-Pareto improvement decision regions in the production coopetition, service coopetition and technology licensing coopetition models. In these non-Pareto improvement regions, the production, delivery service, or technology licensing coopetition increases the overall profit of the two firms but damages the profit of one firm. One future research extension is to explore further cooperation mechanisms, e.g., profit sharing contract (Abhishek et al. 2013) or revenue sharing contract (Raza 2018), to ensure a win-win outcome for both firms.

Finally, all three studies only consider two firms that engage in a dyadic coopetitive relationship. In practice, there are often more than two competitors operating in the same marketplace. It would be interesting to see how additional dimensions of competition brought by multiple players would affect firms' behaviors towards coopetition and associated economic, environmental and social performance. Furthermore, firms' strategic decisions on production coopetition, R&D coopetition or service coopetition could also be influenced by their supply chain positions and their relationships with upstream suppliers and downstream customers (Wilhelm 2011; Hu et al. 2017). Therefore, another important future research avenue is to incorporate upstream and/or downstream supply chain parties in the analysis. Incorporating more companies in a complex network setting would clearly lead to changes of market competition and interfirm relationships and therefore influence the decision outcome of firms' optimal coopetition strategies.

References

Abhishek, V., Hajek, B. and Williams, S. 2013. Auctions with a profit sharing contract. *Games and Economic Behavior*. 77(1): 247-270.

Ahuja, M., Gupta, B. and Raman, P. 2003. An empirical investigation of online consumer purchasing behavior. *Communications of the ACM*, 46(12): 145-151.

Ariño, A., Reuer, J. J., Mayer, K. J. and Jané, J. 2014. Contracts, negotiation, and learning: An examination of termination provisions. *Journal of Management Studies*, 51(3): 379-405.

Atiyeh, C. 2015. Ford offers to license its hybrid and electric vehicle patents to other automakers, MSN, [ONLINE] Available at: http://www.msn.com/en-us/autos/news/ford-offers-to-license-its-hybrid-and-electric-vehicle-patents-to-other-automakers/ar-BBxRs7Y, accessed on 20/06/2017.

Aulakh, P. S., Jiang, M. S. and Pan, Y. 2010. International technology licensing: Monopoly rents, transaction costs and exclusive rights. *Journal of International Business Studies*, 41(4): 587-605.

Aulakh, P. S., Jiang, M. S. and Li, S. 2013. Licensee technological potential and exclusive rights in international licensing: A multilevel model. *Journal of International Business Studies*, 44(7): 699-718.

Ba, S., Stallaert, J. and Zhang, Z. 2008. Oligopolistic price competition and adverse price effect in online retailing markets. *Decision Support Systems*, 45(4): 858-869.

Bakshi, N., and Kleindorfer, P. 2009. Coopetition and investment for supply-chain resilience. *Production and Operations Management*, 18(6): 583-603.

Bagchi, A. and Mukherjee, A. 2014. Technology licensing in a differentiated oligopoly. *International Review of Economics & Finance*, 29: 455-465.

Barney, J. B. 1986. Types of Competition and the Theory of Strategy: Toward an Integrative Framework. *Academy of Management Review*, 11 (4): 791-800.

Barrieu, P. and Fehr, M. 2014. Market-consistent modeling for cap-and-trade schemes and application to option pricing. *Operations Research*, 62(2): 234-249.

BBC. 2014. Google and Samsung sign global patent deal, BBC news. [ONLINE] Available at: http://www.bbc.co.uk/ news/business-25908364, accessed on 20/06/2017.

Bello, D. C., Katsikeas, C. S., and Robson, M. J. 2010. Does accommodating a self-serving partner in an international marketing alliance pay off? *Journal of Marketing*, 74(6): 77-93.

Bengtsson, M. and Kock, S. 2000. “Coopetition” in business networks - to cooperate and compete simultaneously. *Industrial Marketing Management*, 29(5): 411-426.

Bengtsson, M., Eriksson, J., and Wincent, J. 2010. Co-opetition dynamics – an outline for further inquiry, *Competitiveness Review: An International Business Journal*, 20: 194-214.

Bengtsson, M. and Johansson, M. 2014. Managing coopetition to create opportunities for small firms. *International Small Business Journal*, 32(4): 401-427.

Bengtsson, M. and Raza-Ullah, T. 2016. A systematic review of research on coopetition: toward a multilevel understanding. *Industrial Marketing Management*, 57: 23-39.

Bernstein, F. and Federgruen, A. 2004. A general equilibrium model for industries with price and service competition. *Operations Research*, 52(6): 868-886.

Bernstein, F., Song, J. S. and Zheng, X. 2008. “Bricks-and-mortar” vs. “clicks-and-mortar”: An equilibrium analysis. *European Journal of Operational Research*, 187(3): 671-690.

Bloomberg News. 2016. JD.com signals stepped-up spending on areas outside e-commerce, https://www.bloomberg.com/news/articles/2016-10-20/jd-com-signals-stepped-up-spending-on-areas-outside-e-commerce, accessed 27/09/2017

Brandenburger, A. M. and Nalebuff, B. J. 1996. Co-opetition. Doubleday, New York.

Brandenburger, A. and Stuart, H. 2007. Biform games. *Management Science*, 53(4): 537-549.

Bresnahan, T. F. 1987. Competition and collusion in the American automobile industry: The 1955 price war. *The Journal of Industrial Economics*, 35(4): 457-482.

Bouncken, R. B., Gast, J., Kraus, S. and Bogers, M. 2015. Coopetition: a systematic review, synthesis, and future research directions. *Review of Managerial Science*, 9(3): 577-601.

Bouncken, R. B., Fredrich, V., Ritala, P. and Kraus, S. 2018. Coopetition in new product development alliances: advantages and tensions for incremental and radical innovation. *British Journal of Management*, 29(3), 391-410.

Cai, G. G. 2010. Channel selection and coordination in dual-channel supply chains. *Journal of Retailing*, 86(1): 22-36.

Cao, L. and Li, L. 2015. The impact of cross-channel integration on retailers’ sales growth. *Journal of Retailing*, 91(2): 198-216.

Carfì, D. and Schiliro, D. 2012. A coopetitive model for the green economy. *Economic Modelling*, 29(4): 1215-1219.

Caro, F., Corbett, C. J., Tan, T. and Zuidwijk, R. 2013. Double counting in supply chain carbon footprinting. *Manufacturing & Service Operations Management*, 15(4): 545-558.

Carter, M.W. and Prince, C.C. 2000. Operations research: a practical introduction, CRC Press, Florida

Cassiman, B., Di Guardo, M.C. and Valentini, G. 2009. Organising R&D projects to profit from innovation: Insights from co-opetition. *Long Range Planning*, 42(2): 216-233.

Castka, P. and Corbett, C. J. 2016a. Adoption and diffusion of environmental and social standards: The effect of stringency, governance, and media coverage. *International Journal of Operations & Production Management,* 36(11): 1504-1529.

Castka, P. and Corbett, C. J. 2016b. Governance of eco-labels: Expert opinion and media coverage. *Journal of Business Ethics*, 135(2): 309-326.

Chen, M. J. 2008. Reconceptualizing the competition–cooperation relationship: A transparadox perspective. *Journal of Management Inquiry*, 17(4): 288-305.

Chen, M. J., and Miller, D. 2012. Competitive dynamics: Themes, trends, and a prospective research platform. *The Academy of Management Annals*, 6(1): 135-210.

Chen, K.Y., Kaya, M., and Özer, Ö. 2008. Dual sales channel management with service competition, *Manufacturing & Service Operations Management*, 10(4): 654-675.

Chen, L., Gilbert, S. M. and Xia, Y. 2016. Product line extensions and technology licensing with a strategic supplier. *Production and Operations Management*, 25(6): 1121-1146.

Chen, M. J. and Miller, D. 2015. Reconceptualizing competitive dynamics: A multidimensional framework. *Strategic Management Journal*, 36(5): 758-775.

Chen, X., Wang, X. and Jiang, X. 2016. The impact of power structure on retail service supply chain with an O2O mixed channel. *Journal of the Operational Research Society*, 67(2): 294-301.

Chen, X., Wang, X. and Xia, Y. 2019. Production coopetition strategies for competing manufacturers that produce partially substitutable products. *Production and Operations Management*, Doi: 10.1111/ POMS.12998, forthcoming.

Chiambaretto, P., Massé, D. and Mirc, N. 2018. "All for One and One for All?"-Knowledge broker roles in managing tensions of internal coopetition: The Ubisoft case. *Research Policy*. https://doi.org/10.1016/ j.respol.2018.10.009

Choi, S. C. 1996. Price competition in a duopoly common retailer channel. *Journal of Retailing*, 72(2): 117-134.

Collier, J. E. and Bienstock, C. C. 2006. How do customers judge quality in an e-tailer? *MIT Sloan Management*

Review, 48(1), 35-40.

Dagnino, G. B. and Padula, G. 2009. Coopetition strategy: Theory, experiments and cases, Routledge, London

Dagnino, G.B. 2009. Coopetition strategy: a new kind of interfirm dynamics for value creation. (pp. 45-63). In Coopetition Strategy Theory, experiments and cases, edited by Dagnino, G.B. and Rocco, E., Routledge, London.

Dahl, J. 2014. Conceptualizing coopetition as a process: an outline of change in cooperative and competitive interactions. *Industrial Marketing Management*, 43(2): 272-279.

Das, T. K. and Teng, B. S. 1998. Between trust and control: Developing confidence in partner cooperation in alliances. *Academy of Management Review*, 23(3): 491-512.

Das, T. K. and Teng, B. S. 2000. Instabilities of strategic alliances: an internal tensions perspective. *Organization Science*, 11(1), 77-101.

D'Aspremont, C. and Jacquemin, A. 1988. Cooperative and noncooperative R&D in duopoly with spillovers. *The American Economic Review*, 78(5): 1133-1137.

Dechezleprêtre, A. and Glachant, M. 2014. Does foreign environmental policy influence domestic innovation? Evidence from the wind industry. *Environmental and Resource Economics*, 58(3): 391-413.

Ding, Y., Gao, X., Huang, C., Shu, J. and Yang, D. 2018. Service competition in an online duopoly market. *Omega*, 77: 58-72.

Dorn, S., Schweiger, B. and Albers, S. 2016. Levels, phases and themes of coopetition: a systematic literature review and research agenda. *European Management Journal*, 34(5): 484-500.

Dowling, M. J., Roering, W. D., Carlin, B. A. and Wisnieski, J. 1996. Multifaceted relationships under coopetition description and theory. *Journal of Management Inquiry*, 5(2): 155-167.

Drake, D. F., Kleindorfer, P. R. and Van Wassenhove, L. N. 2016. Technology choice and capacity portfolios under emissions regulation. *Production and Operations Management*, 25(6): 1006-1025.

Dussauge, P., Garrette, B. and Mitchell, W. 2000. Learning from competing partners: outcomes and durations of scale and link alliances in Europe, North America and Asia, *Strategic Management Journal*, 21: 99-126.

Eckbo, B. E. 1983. Horizontal mergers, collusion, and stockholder wealth. Journal of financial Economics, 11(1): 241-273.

Ellison, G. and Ellison, S. F. 2009. Search, obfuscation, and price elasticities on the internet. *Econometrica*, 77(2): 427-452.

Farahat, A. and Perakis, G. 2011. A comparison of Bertrand and Cournot profits in oligopolies with differentiated products. *Operations Research*, 59(2): 507-513.

Fernandez, A. S., Le Roy F. and Gnyawali, D. R. 2014. Sources and management of tension in coopetition case evidence from telecommunications satellites manufacturing in Europe. *Industrial Marketing Management*, 43: 222-235.

Feng, Q. and Lu, L. X. 2013. Supply chain contracting under competition: bilateral bargaining vs. Stackelberg. *Production and Operations Management*, 22(3): 661-675.

Fisher, M., Gallino, S. and Li, J. 2018. Competition-based dynamic pricing in online retailing: A methodology validated with field experiments. *Management Science*, 64(6): 2496-2514.

Forman, C., Ghose, A. and Goldfarb, A. 2009. Competition between local and electronic markets: How the benefit of buying online depends on where you live. *Management Science*, 55(1): 47-57.

Fosfuri, A. 2006. The licensing dilemma: understanding the determinants of the rate of technology licensing. *Strategic Management Journal*, 27(12): 1141-1158.

Gallino, S. and Moreno, A. 2014. Integration of online and offline channels in retail: The impact of sharing reliable inventory availability information. *Management Science*, 60(6): 1434-1451.

Garrette, B., Castañer, X., and Dussauge, P. 2009. Horizontal alliances as an alternative to autonomous production: product expansion mode choice in the worldwide aircraft industry 1945-2000. *Strategic Management Journal*, 30: 885-894.

Gibbs, S. 2016. Google launches Pixel phone in direct bid to take on Apple's iPhone, Guardian, [ONLINE] Available at: https://www.theguardian.com/technology/2016/oct/04/google-launch-pixel-xl-iphone, accessed on 20/06/ 2017.

Gimeno, J. 2004. Competition within and between networks: The contingent effect of competitive embeddedness on alliance formation. *Academy of Management Journal*, 47(6): 820-842.

Gnyawali, D. R. and Madhavan, R. 2001. Cooperative networks and competitive dynamics: a structural embeddedness perspective. *Academy of Management Review*, 26: 431-445.

Gnyawali, D. R., He, J. and Madhavan, R. 2006. Impact of co-opetition on firm competitive behavior: an empirical examination. *Journal of Management*, 32(4): 507-530.

Gnyawali, D. R. and Park, B.J. R. 2009. Co-opetition and technological innovation in small and medium-sized enterprises: A multilevel conceptual model. *Journal of small business management*, 47(3): 308-330.

Gnyawali, D. R. and Park, B. J. R. 2011. Co-opetition between giants: Collaboration with competitors for technological innovation. *Research Policy*, 40(5): 650-663.

Granovetter, M. 2005. The impact of social structure on economic outcomes. *Journal of Economic Perspectives*, 19(1): 33-50.

Green, E. J. and Porter, R. H. 1984. Noncooperative collusion under imperfect price information. *Econometrica: Journal of the Econometric Society*, 52(1): 87-100.

Greenpeace. 2017. Clicking clean: who is winning the race to build a green Internet, the 2017 Report, [http://www.greenpeace.org/usa/global-warming/click-clean/] Accessed 11/08/2017

Grieger, M. 2003. Electronic marketplaces: A literature review and a call for supply chain management research. *European journal of operational research*, 144(2): 280-294.

Griffis, S. E., Rao, S., Goldsby, T. J. and Niranjan, T. T. 2012. The customer consequences of returns in online retailing: An empirical analysis. *Journal of Operations Management*, 30(4): 282-294

Grubb, M. 2012. Emissions trading: Cap and trade finds new energy. *Nature*, 491(7426): 666-667.

Gulati, R. 1995. Social structure and alliance formation patterns: A longitudinal analysis. *Administrative Science Quarterly*, 40(4): 619-652.

Gulati, R., Lawrence, P. R. and Puranam, P. 2005. Adaptation in vertical relationships: Beyond incentive conflict. *Strategic Management Journal*, 26(5): 415-440.

Gümüş, M., Li, S., Oh, W. and Ray, S. 2013. Shipping fees or shipping free? A tale of two price partitioning strategies in online retailing. *Production and Operations Management*, 22(4): 758-776.

Hafezalkotob, A. 2017. Competition, cooperation, and coopetition of green supply chains under regulations on energy saving levels. *Transportation Research Part E: Logistics and Transportation Review*, 97: 228-250

Hattori, K. 2017. Optimal combination of innovation and environmental policies under technology licensing. *Economic Modelling*, 64: 601-609.

Hong, X., Govindan, K., Xu, L. and Du, P., 2017. Quantity and collection decisions in a closed-loop supply chain with technology licensing. *European Journal of Operational Research*, 256(3): 820-829.

Hook, L. 2017. Amazon sticks with investment plans despite blow to profits, Financial Times https://www.ft.com/content/7ee5e70c-730a-11e7-aca6-c6bd07df1a3c, accessed on 06/09/2017

Hu, B., Hu, M. and Yang, Y. 2017. Open or Closed? Technology Sharing, Supplier Investment, and Competition. *Manufacturing and Service Operations Management*. 19(1): 132-149.

Hunt, S. D. 1997. Competing Through Relationships: Grounding Relationship Marketing in Resource-Advantage Theory. *Journal of Marketing Management*, 13 (5): 431-445.

IPCC. 2018. Intergovernmental Panel on Climate Change Special Report on Global Warming of 1.5 °C (SR15), http://www.ipcc.ch/, accessed on 23/11/2018

İşlegen, Ö. and Reichelstein, S. 2011. Carbon capture by fossil fuel power plants: An economic analysis. *Management Science*, 57(1): 21-39.

Ji, J., Zhang, Z. and Yang, L. 2017. Comparisons of initial carbon allowance allocation rules in an O2O retail supply chain with the cap-and-trade regulation. *International Journal of Production Economics*, 187: 68-84.

Kang, J. 2016. Samsung will be Apple's top supplier for iPhones again in 2017, Forbes, [ONLINE] Available at: https://www.forbes.com/sites/johnkang/2016/12/16/samsung-will-be-apples-top-supplier-for-iphones-again-in-2017/#431ebb1e1fb0, accessed on 20/06/2017.

Khanna, T., Gulati, R. and Nohria, N. 1998. The dynamics of learning alliances: Competition, cooperation, and relative scope. *Strategic Management Journal*, 19(3): 193-210.

Kim, J. and Parkhe, A. 2009. Competing and cooperating similarity in global strategic alliances: an exploratory examination. *British Journal of Management*, 20: 363-376.

Kim, S. L. and Lee, S. H. 2014. Eco-technology licensing under emission tax: royalty vs. fixed-fee. *Korean Economic Review*, 30: 273-300.

Kim, S. L. and Lee, S. H. 2016. The licensing of eco-technology under emission tax: fixed-fee vs. auction. *International Review of Economics and Finance*, 45: 343-357.

Khoury, T. A., Pleggenkuhle-Miles, E. G. and Walter, J. 2018. Experiential Learning, Bargaining Power, and Exclusivity in Technology Licensing. *Journal of Management*, forthcoming.

Klassen, R.D. and Vachon, S. 2003. Collaboration and evaluation in the supply chain: The impact on plant-level environmental investment. *Production and Operations Management*, 12(3): 336-352.

Kotha, R., Crama, P. and Kim, P. H. 2018. Experience and signaling value in technology licensing contract payment structures. *Academy of Management Journal*, 61(4): 1307-1342.

Krackhardt, D., and Kilduff, M. 2002. Structure, culture and Simmelian ties in entrepreneurial firms. *Social Networks*, 24(3): 279-290.

Kumar, M. V. S. 2010. Are joint ventures positive sum games? The relative effects of cooperative and noncooperative behaviour. *Strategic Management Journal*, 32: 32-54.

Lacoste, S. 2012. "Vertical coopetition": the key account perspective. *Industrial Marketing Management*, 41(4): 649-658.

Lado, A. A., Boyd, N. G., and Hanlon, S. C. 1997. Competition, cooperation, and the search for economic rents: A syncretic model. *Academy of Management Review*, 22: 110-141.

Lenovo Newsroom. 2017. Lenovo™ announces breakthrough, innovative PC manufacturing process, [http://news.lenovo.com/news-releases/lenovo-announces-breakthrough-innovative-pc-manufacturing-process.htm] Accessed 11/08/2017

Li, Y., Liu, Y. and Liu, H. 2011. Co-opetition, distributor's entrepreneurial orientation and manufacturer's knowledge acquisition: Evidence from China. *Journal of Operations Management*, 29(1): 128-142.

Liu, Z. Anderson, T. and Cruz, J. 2012. Consumer environmental awareness and competition in two-stage supply chains. *European Journal of Operational Research*, 218: 602-613.

Lopez, E. 2017. UPS, Amazon invest heavily in logistics networks to keep up with e-commerce, Supply Chain Dive, http://www.supplychaindive.com/news/UPS-Amazon-logistics-networks-results-warehouse-automation-air/441526/, accessed 07/09/2017

Luo, X., Slotegraaf, R. J. and Pan, X. 2006. Cross-functional "coopetition": The simultaneous role of cooperation and competition within firms. *Journal of Marketing*, 70(2): 67-80.

Luo, X., Rindfleisch, A., and Tse, D. K. 2007. Working with rivals: the impact of competitor alliances on financial performance. *Journal of Marketing Research*, 44(1): 73-83.

Luo, Y. 2004. Coopetition in international business. Copenhagen: Copenhagen Business School Press.

Luo, Y. 2005. Toward coopetition within a multinational enterprise: a perspective from foreign subsidiaries. *Journal of World Business*, 40(1): 71-90.

Luo, Y. 2007. From foreign investors to strategic insiders: Shifting parameters, prescriptions and paradigms for MNCs in China. *Journal of world Business*, 42(1): 14-34.

Luo, Y., Shenkar, O. and Gurnani, H. 2008. Control–cooperation interfaces in global strategic alliances: a situational typology and strategic responses. *Journal of International Business Studies*, 39(3): 428-453.

Luo, Z., Chen, X. and Wang, X. 2016. The role of co-opetition in low carbon manufacturing. *European Journal of Operational Research*, 253(2): 392-403.

Mantovani, A. and Ruiz-Aliseda, F. 2016. Equilibrium innovation ecosystems: the dark side of collaborating with complementors. *Management Science*, 62(2): 534-549.

Madhavan, R., Gnyawali, D. R. and He, J. 2004. Two's company, three's a crowd? Triads in cooperative-competitive networks. *Academy of Management Journal*, 47(6): 918-927.

Mariani, M. M. 2007. Coopetition as an emergent strategy: empirical evidence from an Italian consortium of opera houses. *International Studies of Management & Organization*, 37(2): 97-126.

Mathias, B. D., Huyghe, A., Frid, C. J. and Galloway, T. L. 2018. An identity perspective on coopetition in the craft beer industry. *Strategic Management Journal*, 39(12): 3086-3115.

Melis, K., Campo, K., Breugelmans, E. and Lamey, L. 2015. The impact of the multi-channel retail mix on online store choice: Does online experience matter? *Journal of Retailing*, 91(2): 272-288.

Mention, A. L. 2011. Co-operation and co-opetition as open innovation practices in the service sector: Which influence on innovation novelty? *Technovation*, 31(1): 44-53.

Moon, K., Bimpikis, K. and Mendelson, H. 2018. Randomized markdowns and online monitoring. *Management Science*, 64(3): 1271-1290.

Nagarajan, M. and Sošic, G. 2009. Coalition stability in assembly models. *Operations Research*, 57(1): 131-145.

Nagarajan, M. and Bassok, Y. 2008. A bargaining framework in supply chains: The assembly problem. *Management Science*, 54(8): 1482-1496.

Nokia Corporation. 2016. Nokia and Samsung expand their intellectual property cross license. Stock Exchange Release, [ONLINE] Available at: http://www.nokia.com/en_int/news/releases/2016/07/13/nokia-and-samsung-expand-their-intellectual-property-cross-license, accessed on 20/06/2017.

Newell, R.G., Pizer, W.A. and Raimi, D. 2014. Carbon market lessons and global policy outlook. *Science*, 343(6177): 1316-1317.

Oakshott, L. 1997. Business modelling and simulation Prentice Hall, Essex.

Ofek, E., Katona, Z. and Sarvary, M. 2011. “Bricks and clicks”: The impact of product returns on the strategies of multichannel retailers. *Marketing Science*, 30(1): 42-60.

Olsen, M.C., Slotegraaf, R.J. and Chandukala, S.R. 2014. Green claims and message frames: how green new products change brand attitude. *Journal of Marketing*, 78(5): 119-137.

Oum, T. H., Park, J. H., Kim, K. and Yu, C. 2004. The effect of horizontal alliances on firm productivity and profitability: evidence from the global airline industry. *Journal of Business Research*, 57(8): 844-853.

Oxley, J. E., Sampson, R. C., and Silverman, B. S. 2009. Arms race or détente? How interfirm alliance announcements change the stock market valuation of rivals. *Management Science*, 55: 1321-1337.

Padmanabhan, V. and Png, I. P. 1997. Manufacturer's return policies and retail competition. *Marketing Science*, 16(1): 81-94.

Palmer, A. 2000. Co-operation and competition: a darwinian synthesis of relationship marketing. *European Journal of Marketing*, 34 (5): 687-704.

Pan, X., Ratchford, B. T. and Shankar, V. 2002. Can price dispersion in online markets be explained by differences in e-tailer service quality? *Journal of the Academy of Marketing Science*, 30(4): 433-445.

Pandey, A. 2016. Climate change: World's biggest oil firms to invest $1 billion in low-carbon technology. International Business Times, [http://www.ibtimes.com/climate-change-worlds-biggest-oil-firms-invest-1-illion-low-carbon-technology-2442344] Accessed 13/08/2017

Park, B. J., Srivastava, M. K., and Gnyawali, D. R. 2014. Walking the tight rope of coopetition: impact of competition and cooperation intensities and balance on firm innovation performance. *Industrial Marketing Management*, 43(2): 210-221.

Park, S. H. and Russo, M. V. 1996. When competition eclipses cooperation: an event history analysis of joint venture failure. *Management Science*, 42: 875-890.

Park, S. J., Cachon, G.P., Lai, G. and Seshadri, S. 2015. Supply chain design and carbon penalty: monopoly vs. monopolistic competition. *Production and Operations Management*, 24(9): 1494-1508.

Pekgün, P., Griffin, P.M. and Keskinocak, P. 2017. Centralized versus decentralized competition for price and lead-time sensitive demand. *Decision Sciences*, 48(6): 1198-1227.

Peng, T. J. A. and Bourne, M. 2009. The coexistence of competition and cooperation between networks: implications from two Taiwanese healthcare networks. *British Journal of Management*, 20(3): 377-400.

Peng, T. J. A., Pike, S., Yang, J. C. H., and Roos, G. 2012. Is cooperation with competitors a good idea? An example in practice. *British Journal of Management*, 23(4): 532-560.

Pitelis, C.N., Panagopoulos, A. and Desyllas, P. 2015. Fostering coopetition through market extension and co-creation in the global pharmaceutical sector. *Journal of Organic Chemistry*, 80(8): 4017-4029.

Podolny, J. M., and Page, K. L. 1998. Network forms of organization. *American Journal of Sociology*, 24: 57-76.

Pun, H. 2015. The more the better - optimal degree of supply-chain cooperation between competitors. *Journal of Operational Research Society*, 66(12): 2092-2101.

Pun, H. and Ghamat, S. 2016. The value of partnership under competition: when competitors may be R&D joint-venture and supply-chain partners for a critical component. *International Journal of Production Economics*, 177: 1-11.

Qing, Q., Deng, T. and Wang, H., 2017. Capacity allocation under downstream competition and bargaining. *European Journal of Operational Research*, 261(1): 97-107.

Quintana-García, C. and Benavides-Velasco, C. A. 2004. Cooperation, competition, and innovative capability: a panel data of European dedicated biotechnology firms. *Technovation*, 24(12): 927-938.

Rogers, A. 2018. Could Nokia Beat the Estimates in 4Q17? Pre-release Earning Reports, Market Realist, [ONLINE] Available at: https://marketrealist.com/2018/01/behind-nokias-and-huaweis-patent-licensing-agreement, accessed on 19/04-2018

Rai, R. K. 2016. A co-opetition-based approach to value creation in interfirm alliances construction of a measure and examination of its psychometric properties. *Journal of Management*, 42: 1663-1699.

Raza, S. 2018. Supply chain coordination under a revenue-sharing contract with corporate social responsibility and partial demand information. International Journal of Production Economics, 205: 1-14.

Raza-Ullah, T., Bengtsson, M., and Kock, S. 2014. The coopetition paradox and tension in coopetition at multiple levels. *Industrial Marketing Management*, 43(2): 189-198.

Ritala, P. 2012. Coopetition strategy–when is it successful? Empirical evidence on innovation and market performance. *British Journal of Management*, 23(3): 307-324.

Ritala, P. and Hurmelinna-Laukkanen, P. 2009. What's in it for me? Creating and appropriating value in innovation-related coopetition. *Technovation*, 29(12): 819-828.

Ritala, P. and Hurmelinna-Laukkanen, P. 2013. Incremental and radical innovation in coopetition-the role of absorptive capacity and appropriability. *Journal of Product Innovation Management*, 30(1): 154-169.

Ritala, P., Golnam, A. and Wegmann, A. 2014. Coopetition-based business models: The case of Amazon.com. *Industrial Marketing Management*, 43(2): 236-249.

Ritala, P. and Tidström, A. 2014. Untangling the value-creation and value appropriation elements of coopetition strategy: a longitudinal analysis on the firm and relational levels. *Scandinavian Journal of Management*, 30: 498-515.

Rusko, R. 2011. Exploring the concept of coopetition: A typology for the strategic moves of the Finnish forest industry. *Industrial Marketing Management*, 40(2): 311-320.

Ryan, J. K., Sun, D. and Zhao, X. 2012. Competition and coordination in online marketplaces. *Production and Operations Management*, 21(6): 997-1014.

Sahaym, A., Steensma, H. K. and Schilling, M. A. 2007. The influence of information technology on the use of loosely coupled organizational forms: An industry-level analysis. *Organization Science*, 18(5): 865-880.

Schiavone, F. and Simoni, M. 2011. An experience-based view of co-opetition in R&D networks. *European Journal of Innovation Management*, 14(2): 136-154.

Sen, D. 2005. Fee versus royalty reconsidered. *Games and Economic Behavior*, 53(1): 141-147.

Sen, D. and Tauman, Y. 2007. General licensing schemes for a cost-reducing innovation. *Games and Economic Behavior*, 59(1): 163-186.

Sen, D. and Stamatopoulos, G. 2016. Licensing under general demand and cost functions. *European Journal of Operational Research,* 253(3): 673-680.

Shallenberger, K. 2016. Major US companies launch renewable energy buyers alliance, online source, http://www.utilitydive.com/news/major-us-companies-launch-renewable-energy-buyers-alliance/ 419184/ [Accessed 09/08/2017]

Shang, W., Ha, A.Y. and Tong, S. 2016. Information sharing in a supply chain with a common retailer. *Management Science*, 62(1): 245-263

Shi, R., Zhang, J. and Ru, J. 2013. Impacts of power structure on supply chains with uncertain demand. *Production and Operations Management*, 22(5): 1232-1249.

Shipilov, A. and Li, S. 2008. Can you have your cake and eat it too? Structural holes' influence on status accumulation and market performance in collaborative networks. *Administrative Science Quarterly*, 53: 73-108.

Simonet, D. 2002. Licensing agreements in the pharmaceutical industry. *Journal of Medical Marketing.* 2(4): 329-341

SIPO. 2016. Haier Group: the use of patented technology to achieve industrial value, State Intellectual Property office, Patent news, [http://www.sipo.gov.cn/ztzl/ndcs/zgzlznew/qqzl/201611/ t20161123_1302952.html] Accessed 11/08/2017

Stein, H. D. 2010. Literature overview on the field of co-opetition. *Business: Theory and Practice*, 11(3): 256-265.

The U.S. Census Bureau. 2018. Advance Monthly Retail Trade Survey, https://census.gov/retail/index.html, Accessed on 09/07/2018

Tsai, W. 2000. The formation of intraorganizational linkages. *Strategic Management Journal*, 21(9): 925-939.

Tsai, W. 2002. Social structure of "coopetition" within a multiunit organization: Coordination, competition, and intraorganizational knowledge sharing. *Organization Science*, 13(2): 179-190.

Tsay, A. A., and Agrawal, N. 2000. Channel dynamics under price and service competition. *Manufacturing & Service Operations Management,* 2(4): 372-391.

Tsay, A. A. and Agrawal, N. 2004. Channel conflict and coordination in the e‐commerce age. *Production and Operations Management*, 13(1): 93-110.

Uzzi, B. 1996. The sources and consequences of embeddedness for the economic performance of organizations: The network effect. *American Sociological Review*, 61: 674-698.

Venkatesh, R., Chintagunta, P. and Mahajan, V. 2006. Research note - sole entrant, co-optor, or component supplier: optimal end-product strategies for manufactures of proprietary component brands. *Management Science*, 52(4): 613-622.

Walley, K. 2007. Coopetition. An introduction to the subject and an agenda for research. *International Studies and Management & Organization,* 37(2): 11-31.

Wang, S., Zheng, S., Xu, L., Li, D. and Meng, H. 2008. A literature review of electronic marketplace research: Themes, theories and an integrative framework. *Information Systems Frontiers*, 10(5): 555-571.

Wang, X. H. 1998. Fee versus royalty licensing in a Cournot duopoly model. *Economics letters*, 60(1): 55-62.

Wang, X. H. 2002. Fee versus royalty licensing in a differentiated Cournot duopoly. *Journal of Economics and Business*, 54(2): 253-266.

Wang, Y., Niu, B. and Guo, P. 2013. On the advantage of quantity leadership when outsourcing production to a competitive contract manufacturer. *Production and Operations Management*, 22(1): 104-119.

Wilhelm, M. M. 2011. Managing coopetition through horizontal supply chain relations: Linking dyadic and network levels of analysis. *Journal of Operations Management*, 29(7): 663-676.

Wu, C. H., 2018. Price competition and technology licensing in a dynamic duopoly. *European Journal of Operational Research*, 267(2): 570-584.

Xu, Y., Gurnani, H. and Desiraju, R. 2010. Strategic supply chain structure design for a proprietary component manufacturer. *Production and Operations Management*, 19(4): 371-389.

Yami, S. and A. Nemeh. 2014. Organizing coopetition for innovation: the case of wireless telecommunication sector in Europe. *Industrial Marketing Management*, 43: 250-260.

Yang, Z., Hu, X., Gurnani, H. and Guan, H. 2017. Multichannel distribution strategy: selling to a competing buyer with limited supplier capacity. *Management Science*. In press. https://doi.org/10.1287/ mnsc.2016.2702

Yoo, W. S. and Lee, E. 2011. Internet channel entry: A strategic analysis of mixed channel structures. *Marketing Science*, 30(1): 29-41.

Zaroban, S. 2018. U. S. e-commerce sales grow 16.0% in 2017, News, https://www.digitalcommerce360.com/ article/us-ecommerce-sales/, Accessed on 09/07/2018

Zhang, D., Karplus, V. J., Cassisa, C. and Zhang, X. 2014. Emissions trading in China: Progress and prospects. *Energy policy*, 75: 9-16.

Ziedonis, A. A. 2007. Real options in technology licensing. *Management Science,* 53(10): 1618-1633.

Appendix A: Proofs

Chapter 3

Derivation of Table 3.2:

(1) Cournot competition model: From (3-1), we obtain $\frac{d^2\pi_1^n(q_1)}{dq_1^2} = -2(1+\beta) < 0$ and $\pi_1^n(q_1)$ is a concave function of q_1. Similarly, from (3-2), we obtain $\frac{d^2\pi_2^n(q_2)}{dq_2^2} = -2(1+\beta) < 0$ and $\pi_2^n(q_2)$ is a concave function of q_2. $\frac{d\pi_1^n(q_1)}{dq_1} = \frac{d\pi_2^n(q_2)}{dq_2} = 0$ shows that $q_1^n = \frac{(2+\beta)\delta_1+\beta\Delta c}{(2+\beta)(2+3\beta)}$ and $q_2^n = \frac{(2+\beta)\delta_2-\beta\Delta c}{(2+\beta)(2+3\beta)}$. Recall that $p_i = \alpha - q_i - \beta(q_i + q_j)$, we obtain $p_1^n = m + c_1 + (1+\beta)q_1^n$ and $p_2^n = m + c_2 + (1+\beta)q_2^n$.

(2) WC model: For a given w, from (3-3), we obtain $\frac{d^2\pi_1^c(q_1)}{dq_1^2} = -2(1+\beta) < 0$ and $\pi_1^c(q_1)$ is a concave function of q_1. $\frac{d\pi_1^c(q_1)}{dq_1} = 0$ shows that $q_1 = \frac{-m+\alpha-c_1-\beta q_2}{2(1+\beta)}$. Replace q_1 in (3-4) and we obtain $\frac{d^2\pi_2^c(q_2)}{dq_2^2} = \frac{-2-4\beta-\beta^2}{1+\beta} < 0$, then $\pi_2^n(q_2)$ is a concave function of q_2. $\frac{d\pi_2^c(q_2)}{dq_2} = 0$ shows that $q_2(\mathrm{w}) = \frac{-2m-2w+2\alpha-m\beta-2w\beta+\alpha\beta+\beta c_1}{2(2+4\beta+\beta^2)}$ and $q_1(\mathrm{w}) = \frac{-m+\alpha-c_1-\beta q_2(\mathrm{w})}{2(1+\beta)}$.

Replace $q_1(w)$ and $q_2(w)$ in (3-5), we obtain $ln\pi^{cw}(w) = \theta\, ln\pi_1^c\big(q_1(w)\big) + (1-\theta)ln\pi_2^c\big(q_2(w)\big)$ and $\frac{1}{\pi^{cw}(w)}\frac{d\pi^{cw}(w)}{dw} = \theta\frac{1}{\pi_1^c(q_1(w))}\frac{d\pi_1^c(q_1(w))}{dw} + (1-\theta)\frac{1}{\pi_2^c(q_2(w))}\frac{d\pi_2^c(q_2(w))}{dw}$, then $\frac{d\pi^{cw}(w)}{dw} = \pi^{cw}(w)[\theta\frac{1}{\pi_1^c(q_1(w))}\frac{d\pi_1^c(q_1(w))}{dw} + (1-\theta)\frac{1}{\pi_2^c(q_2(w))}\frac{d\pi_2^c(q_2(w))}{dw}]$. $\frac{d\pi^{cw}(w)}{dw} = 0$ shows that there are three real roots: $w_1 = c_1 + \frac{\delta_1(3\beta^3+4(2+\theta)+8\beta(3+\theta)+2\beta^2(9+\theta)-2(2+4\beta+\beta^2)\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2})}{2(1+\beta)(8+16\beta+3\beta^2)}$, $w_2 = c_1 + \frac{(2+\beta)\delta_1}{2(1+\beta)}$, $w_3 = c_1 + \frac{\delta_1(3\beta^3+4(2+\theta)+8\beta(3+\theta)+2\beta^2(9+\theta)+2(2+4\beta+\beta^2)\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2})}{2(1+\beta)(8+16\beta+3\beta^2)}$. Recall $q_2^n = \frac{(2+\beta)\delta_2-\beta\Delta c}{(2+\beta)(2+3\beta)} > 0$, then $0 < \Delta c < \Delta c^H = \frac{(2+\beta)\delta_1}{2(1+\beta)}$ and $w_2 = c_1 + \Delta c^H > c_2$, so $w_3 - w_2 > 0$ and $w_3 > w_2$. Similarly, $w_2 > w_1$. So $w_3 > w_2 > w_1$.

$(3\beta^3 + 4(2+\theta) + 8\beta(3+\theta) + 2\beta^2(9+\theta))^2 - \left(2(2+4\beta+\beta^2)\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2}\right)^2 = (8+16\beta+3\beta^2)F_1(\theta)$, where $F_1(\theta) = -16 - 48\beta - 44\beta^2 - 12\beta^3 - \beta^4 + 4(8+26\beta+26\beta^2+9\beta^3+\beta^4)\theta$. Bacause $4(8+26\beta+26\beta^2+9\beta^3+\beta^4) > 0$, then $F_1(\theta)$ increases in θ. There is one root for $F_1(\theta)$, $\theta^c = \frac{(4+6\beta+\beta^2)^2}{4(1+\beta)(4+\beta)(2+4\beta+\beta^2)} > 0$ and $1-\theta^c > 0$, so $0 < \theta^c < 1$. If $\theta^c < \theta < 1$, then $w_1 > c_1$; if $0 < \theta < \theta^c$, then $w_1 < c_1$. Recall $c_1 < w < c_2$, so if $\theta^c < \theta < 1$, then $w^c = w_1 = c_1 + \frac{\delta_1(3\beta^3+4(2+\theta)+8\beta(3+\theta)+2\beta^2(9+\theta)-2(2+4\beta+\beta^2)\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2})}{2(1+\beta)(8+16\beta+3\beta^2)}$. Replace w^c in $q_1(w)$ and $q_2(w)$ and we obtain $q_1^c = \frac{\delta_1(8+3\beta^2+\beta(14+\theta)-\beta\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2})}{2(1+\beta)(8+16\beta+3\beta^2)}$ and $q_2^c = \frac{\delta_1(2-\theta+\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2}))}{8+16\beta+3\beta^2}$. Recall that $p_i = \alpha - q_i - \beta(q_i + q_j)$ and we obtain $p_1^c = m + c_1 + (1+\beta)q_1^c$ and $p_2^c = m + w^c + \frac{(2+4\beta+\beta^2)\delta_1(2-\theta+T_a)}{2(1+\beta)(8+16\beta+3\beta^2)}$.

(3) LC model: From (3-6), (3-7), we obtain $\frac{d^2\pi_1^l(q_1)}{dq_1^2} = \frac{d^2\pi_2^l(q_2)}{dq_2^2} = -2(1+\beta) < 0$, then $\pi_1^l(q_1)$ is a concave function of q_1 and $\pi_2^l(q_2)$ is a concave function of q_2. $\frac{d\pi_1^l(q_1)}{dq_1} = \frac{d\pi_2^l(q_2)}{dq_2} = 0$ shows that $q_1(r) = \frac{2\alpha+(r+\alpha)\beta-c_1(2+\beta)-m(2+\beta)}{(2+\beta)(2+3\beta)}$ and $q_2(r) = -\frac{2(c_1+m+r-\alpha)+(c_1+m+2r-\alpha)\beta}{(2+\beta)(2+3\beta)}$.

Replace $q_1(r)$ and $q_2(r)$ in (3-8), we obtain $ln\pi^{cl}(r,M) = \theta\, ln\pi_1^l(q_1(r,M)) + (1-\theta)ln\pi_2^l(q_2(r,M))$. Then $\frac{\partial\pi^{cl}(r,M)}{\partial r} = \pi^{cl}(r,M)[\theta\frac{1}{\pi_1^l(q_1(r,M))}\frac{\partial\pi_1^l(q_1(r,M))}{\partial r} + (1-\theta)\frac{1}{\pi_2^l(q_2(r,M))}\frac{\partial\pi_2^l(q_2(r,M))}{\partial r}]$ and $\frac{\partial\pi^{cl}(r,M)}{\partial M} = \pi^{cl}(r,M)[\theta\frac{1}{\pi_1^l(q_1(r,M))}\frac{\partial\pi_1^l(q_1(r,M))}{\partial M} + (1-\theta)\frac{1}{\pi_2^l(q_2(r,M))}\frac{\partial\pi_2^l(q_2(r,M))}{\partial M}]$. $\frac{\partial\pi^{cl}(r,M)}{\partial r} = \frac{\partial\pi^{cl}(r,M)}{\partial M} = 0$ shows that there are three real roots: $r_1 = \frac{(2+\beta)(2\beta+\beta^2-2\sqrt{8+8\beta+\beta^2}-3\beta\sqrt{8+8\beta+\beta^2})\delta_1}{2(1+\beta)(4+8\beta+\beta^2)} < 0$, $M_1 = \frac{(1+\beta)(12+8\beta+\beta^2+4\sqrt{8+8\beta+\beta^2})\delta_1^2}{(4+8\beta+\beta^2)^2}$; $r_2 = \frac{\beta(2+\beta)^2\delta_1}{2(1+\beta)(4+8\beta+\beta^2)}$, $M_2 = \frac{\delta_1^2(-16-64\beta-60\beta^2-16\beta^3-\beta^4+(32+96\beta+76\beta^2+16\beta^3+\beta^4)\theta)}{4(1+\beta)(4+8\beta+\beta^2)^2}$; $r_3 = \frac{(2+\beta)(2\beta+\beta^2+2\sqrt{8+8\beta+\beta^2}+3\beta\sqrt{8+8\beta+\beta^2})\delta_1}{2(1+\beta)(4+8\beta+\beta^2)} > \Delta c^H$, $M_3 = -\frac{(1+\beta)(-12-8\beta-\beta^2+4\sqrt{8+8\beta+\beta^2})\delta_1^2}{(4+8\beta+\beta^2)^2}$. Recall $q_2^n = \frac{(2+\beta)\delta_2-\beta\Delta c}{(2+\beta)(2+3\beta)} > 0$, so $0 < \Delta c < \Delta c^H = \frac{(2+\beta)\delta_1}{2(1+\beta)}$ and $0 < r < \Delta c^H$. Then $r^l = r_2 = \frac{\beta(2+\beta)^2\delta_1}{2(1+\beta)(4+8\beta+\beta^2)}$, $M^l = M_2 = \frac{\delta_1^2(16(1+\beta)^2-(32+96\beta+76\beta^2+16\beta^3+\beta^4)(1-\theta))}{4(1+\beta)(4+8\beta+\beta^2)^2}$. Replace r^l in $q_1(r)$ and $q_2(r)$, we obtain $q_1^l = \frac{(4+6\beta+\beta^2)\delta_1}{2(1+\beta)(4+8\beta+\beta^2)}$ and $q_2^l = \frac{2\delta_1}{4+8\beta+\beta^2}$. Recall that $p_i = \alpha - q_i - \beta(q_i+q_j)$ and we obtain $p_1^l = m + c_1 + (1+\beta)q_1^l$ and $p_2^l = m + c_1 + \frac{\delta_1(4+12\beta+8\beta^2+\beta^3)}{2(1+\beta)(4+8\beta+\beta^2)}$.

Proof of Lemma 3.1: $q_2^n > 0$ implies that $\alpha - m > \frac{-\beta c_1+2(1+\beta)c_2}{2+\beta}$.

(1) From Table 3.2, we obtain $\frac{dp_2^n}{d\beta} = \frac{-1}{(2+3\beta)^2}(\alpha-m) + \frac{(4+8\beta+5\beta^2)c_1-4\beta(1+\beta)c_2}{(2+\beta)^2(2+3\beta)^2} < \frac{2(1+\beta)(c_1-c_2)}{(2+\beta)^2(2+3\beta)} < 0$ and p_2^n decreases in β. Similarly, from Table 3.2 and (3-1), we obtain $\frac{d\sqrt{\pi_1^n(q_1^n)}}{d\beta} < \frac{(4+6\beta+3\beta^2)(c_1-c_2)}{2\sqrt{1+\beta}(2+\beta)^2(2+3\beta)} < 0$ and $\pi_1^n(q_1^n)$ decreases in β. Similarly, from Table 3.2 and (3-2), we obtain $\frac{d\sqrt{\pi_2^n(q_2^n)}}{d\beta} < \frac{2\sqrt{1+\beta}(c_1-c_2)}{(2+\beta)^2(2+3\beta)} < 0$ and $\pi_2^n(q_2^n)$ decreases in β.

(2) From Table 3.2, we obtain $\frac{dp_1^n}{d\beta} = \frac{F(\beta)}{(2+\beta)^2(2+3\beta)^2}$, where $F(\beta) = [5(c_2-c_1)-(\alpha-m-c_1)]\beta^2 + (-4\alpha+4m-4c_1+8c_2)\beta + 4(\alpha-m-c_2)$.

1) If $5(c_2-c_1)-(\alpha-m-c_1) = 0$, then $\Delta c = \frac{\delta_1}{5}$, so $F(\beta) = -12(c_2-c_1)\beta + 4(\alpha-m-c_2)$ and

$F(\beta)$ decreases in β. Let $F(\beta)=0$ and we obtain $\beta^0=-\frac{\alpha-m-c_2}{3(c_2-c_1)}<0$. Since $\beta>0>\beta^0$, then $F(\beta)<0$, $\frac{dp_1^n}{d\beta}<0$ and p_1^n decreases in β.

2) If $5(c_2-c_1)-(\alpha-m-c_1)\neq 0$, then $\Delta=16(c_2-c_1)(2\alpha-2m-c_2-c_1)>0$. Let $F(\beta)=0$ and we obtain two real roots : $\beta^N=2\frac{-[2(c_2-c_1)-(\alpha-m-c_1)]+\sqrt{(c_2-c_1)(2\alpha-2m-c_2-c_1)}}{5(c_2-c_1)-(\alpha-m-c_1)}$ and $\beta^*=2\frac{-[2(c_2-c_1)-(\alpha-m-c_1)]-\sqrt{(c_2-c_1)(2\alpha-2m-c_2-c_1)}}{5(c_2-c_1)-(\alpha-m-c_1)}$.

If $5(c_2-c_1)-(\alpha-m-c_1)<0$, then $\Delta c<\frac{\delta_1}{5}$, so $2(c_2-c_1)-(\alpha-m-c_1)<0$ and $\beta^N<0$. Since $(c_2-c_1)(2\alpha-2m-c_2-c_1)-[2(c_2-c_1)-(\alpha-m-c_1)]^2=[5(c_2-c_1)-(\alpha-m-c_1)](\alpha-m-c_2)<0$, then $-[2(c_2-c_1)-(\alpha-m-c_1)]-\sqrt{(c_2-c_1)(2\alpha-2m-c_2-c_1)}>0$ and $\beta^*<0$. Recall that $\beta>0$, then $F(\beta)<0$, $\frac{dp_1^n}{d\beta}<0$ and p_1^n decreases in β.

If $5(c_2-c_1)-(\alpha-m-c_1)>0$, then $\Delta c>\frac{\delta_1}{5}$, so $(c_2-c_1)(2\alpha-2m-c_2-c_1)-[2(c_2-c_1)-(\alpha-m-c_1)]^2=[5(c_2-c_1)-(\alpha-m-c_1)](\alpha-m-c_2)>0$, $\sqrt{(c_2-c_1)(2\alpha-2m-c_2-c_1)}>|2(c_2-c_1)-(\alpha-m-c_1)|$ and $\beta^N>0>\beta^*$. Recall that $\beta>0$ and we obtain that if $0<\beta<\beta^N$, then $\frac{dp_1^n}{d\beta}<0$ and p_1^n decreases in β; if $\beta>\beta^N$, then $\frac{dp_1^n}{d\beta}>0$ and p_1^n increases in β.

Therefore, if $0<\Delta c\leq\frac{\delta_1}{5}$ or $\Delta c>\frac{\delta_1}{5}$ and $0<\beta<\beta^N$, then p_1^n decreases in β; however, if $\Delta c>\frac{\delta_1}{5}$ and $\beta>\beta^N$, then p_1^n increases in β, where $\beta^N=\frac{2[2\Delta c-\delta_1-\sqrt{\Delta c(\delta_1+\delta_2)}]}{\delta_1-5\Delta c}$.

Proof of Lemma 3.2: Recall $q_2^n=\frac{(2+\beta)\delta_2-\beta\Delta c}{(2+\beta)(2+3\beta)}>0$, then $0<\Delta c<\Delta c^H=\frac{(2+\beta)\delta_1}{2(1+\beta)}$. Recall $c_1<w<c_2$, then $\theta^c=\frac{(4+6\beta+\beta^2)^2}{4(1+\beta)(4+\beta)(2+4\beta+\beta^2)}<\theta<1$.

(1) From Table 3.2, we obtain $\frac{dp_2^c}{d\beta}=\frac{\delta_1(F_1(\theta)-F_2(\theta))}{2(1+\beta)^2(8+16\beta+3\beta^2)^2\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2}}$, where $F_1(\theta)=64+288\beta+472\beta^2+314\beta^3+64\beta^4+3\beta^5-64\theta-288\beta\theta-472\beta^2\theta-314\beta^3\theta-64\beta^4\theta-3\beta^5\theta+16\theta^2+60\beta\theta^2+70\beta^2\theta^2+24\beta^3\theta^2+3\beta^4\theta^2$ and $F_2(\theta)=(32+136\beta+164\beta^2+48\beta^3+3\beta^4+16\theta+60\beta\theta+70\beta^2\theta+24\beta^3\theta+3\beta^4\theta)\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2}$. Recall $\theta^c<\theta<1$, then $F_1(\theta)>0$ and $F_2(\theta)>0$. $(F_1(\theta))^2-\left(F_2(\theta)\right)^2=(8+16\beta+3\beta^2)^2F_3(\theta)$, where $F_3(\theta)=-128-800\beta-1612\beta^2-1280\beta^3-376\beta^4-48\beta^5-2\beta^6-128\theta-544\beta\theta-1016\beta^2\theta-1048\beta^3\theta-544\beta^4\theta-96\beta^5\theta-5\beta^6\theta+224\theta^2+1176\beta\theta^2+2328\beta^2\theta^2+2080\beta^3\theta^2+799\beta^4\theta^2+114\beta^5\theta^2+4\beta^6\theta^2+32\beta\theta^3+136\beta^2\theta^3+200\beta^3\theta^3+$

$118\beta^4\theta^3+30\beta^5\theta^3+3\beta^6\theta^3$. Since $\frac{d^2F_3(\theta)}{d\theta^2}=448+2352\beta+4656\beta^2+4160\beta^3+1598\beta^4+228\beta^5+8\beta^6+192\beta\theta+816\beta^2\theta+1200\beta^3\theta+708\beta^4\theta+180\beta^5\theta+18\beta^6\theta>0$, then $F_3(\theta)$ is a convex function. $F_3(\theta=\theta^c)<0$ and $F_3(\theta=1)<0$, so if $\theta^c<\theta<1$, then $\frac{dp_2^c}{d\beta}<0$ and p_2^c decreases in β.

From Table 3.2 and (3-3), we obtain $\frac{d\pi_1^c(q_1^c)}{d\beta}=\frac{-\delta_1^2\theta F_4(\theta)}{4(1+\beta)^2(8+16\beta+3\beta^2)^2\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2}}$, where $F_4(\theta)=448+1248\beta+1128\beta^2+354\beta^3+36\beta^4-(448+1248\beta+1128\beta^2+354\beta^3+36\beta^4)\theta+(48+76\beta+18\beta^2)\theta^2$. Since $48+76\beta+18\beta^2>0$, then $F_4(\theta)$ is a convex function. $\Delta>0$ means that there are two real roots for $F_4(\theta)$: $\theta_3=\frac{224+624\beta+564\beta^2+177\beta^3+18\beta^4-\sqrt{28672+185600\beta+484992\beta^2+657984\beta^3+498120\beta^4+213012\beta^5+50985\beta^6+6372\beta^7+324\beta^8}}{2(24+38\beta+9\beta^2)}$ and $\theta_4=\frac{224+624\beta+564\beta^2+177\beta^3+18\beta^4+\sqrt{28672+185600\beta+484992\beta^2+657984\beta^3+498120\beta^4+213012\beta^5+50985\beta^6+6372\beta^7+324\beta^8}}{2(24+38\beta+9\beta^2)}>\theta_3$. Recall $\theta^c<\theta<1$, then $\theta_3-1>0$ and $F_4(\theta)>0$. So $\frac{d\pi_1^c(q_1^c)}{d\beta}<0$ and $\pi_1^c(q_1^c)$ decreases in β.

Similarly, from Table 3.2 and (3-4), we obtain $\frac{d\pi_2^c(q_2^c)}{d\beta}=\frac{-\delta_1^2(2-\theta+\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2})F_5(\theta)}{2(1+\beta)^2(8+16\beta+3\beta^2)^3\sqrt{12+16\beta+3\beta^2-12\theta-16\beta\theta-3\beta^2\theta+\theta^2}}$, where $F_5(\theta)=320+1408\beta+2312\beta^2+1772\beta^3+654\beta^4+120\beta^5+9\beta^6+(-320-1408\beta-2312\beta^2-1772\beta^3-654\beta^4-120\beta^5-9\beta^6)\theta+(48+168\beta+186\beta^2+70\beta^3+9\beta^4)\theta^2$. Since $48+168\beta+186\beta^2+70\beta^3+9\beta^4>0$, then $F_5(\theta)$ is a convex function. $\Delta>0$ means that there are two real roots for $F_5(\theta)$: $\theta_5=\frac{1}{2(48+168\beta+186\beta^2+70\beta^3+9\beta^4)}(320+1408\beta+2312\beta^2+1772\beta^3+654\beta^4+120\beta^5+9\beta^6-(40960+415744\beta+1833984\beta^2+4613632\beta^3+7311616\beta^4+7633248\beta^5+5359424\beta^6+2555760\beta^7+830772\beta^8+182016\beta^9+25848\beta^{10}+2160\beta^{11}+81\beta^{12})^{\frac{1}{2}})$ and $\theta_6=\frac{1}{2(48+168\beta+186\beta^2+70\beta^3+9\beta^4)}(320+1408\beta+2312\beta^2+1772\beta^3+654\beta^4+120\beta^5+9\beta^6+(40960+415744\beta+1833984\beta^2+4613632\beta^3+7311616\beta^4+7633248\beta^5+5359424\beta^6+2555760\beta^7+830772\beta^8+182016\beta^9+25848\beta^{10}+2160\beta^{11}+81\beta^{12})^{\frac{1}{2}})>\theta_5$. Recall $\theta^c<\theta<1$, then $\theta_5-1>0$ and $F_5(\theta)>0$. So $\frac{d\pi_2^c(q_2^c)}{d\beta}<0$ and $\pi_2^c(q_2^c)$ decreases in β.

(2) From Table 3.2, we obtain $\frac{dp_1^c}{d\beta}=\frac{\delta_1(F_6(\theta)+(-8+3\beta^2)(2-\theta)\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2})}{2(8+16\beta+3\beta^2)^2\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2}}$, where $F_6(\theta)=-96-192\beta-140\beta^2-24\beta^3+96\theta+192\beta\theta+140\beta^2\theta+24\beta^3\theta-8\theta^2+3\beta^2\theta^2$. If $-8+3\beta^2<0$, then $0<\beta<2\sqrt{\frac{2}{3}}$, so $F_6(\theta)$ is a concave function; if $-8+3\beta^2>0$, then $\beta>2\sqrt{\frac{2}{3}}$, so $F_6(\theta)$ is a convex function. $\Delta>0$ means that there are two real roots for $F_6(\theta)$: $\theta_7=\frac{2\left(-(24+48\beta+35\beta^2+6\beta^3)+\sqrt{2(192+960\beta+1888\beta^2+1872\beta^3+953\beta^4+219\beta^5+18\beta^6)}\right)}{-8+3\beta^2}$ and $\theta_8=$

$-\frac{2\left(24+48\beta+35\beta^2+6\beta^3+\sqrt{2(192+960\beta+1888\beta^2+1872\beta^3+953\beta^4+219\beta^5+18\beta^6)}\right)}{-8+3\beta^2}$.

If $0<\beta<2\sqrt{\frac{2}{3}}$, $\theta_7-1>0$, then $\theta_7>1$ and $\theta_8>\theta_7>1$. So $F_6(\theta)<0$. $(-8+3\beta^2)(2-\theta)\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2})<0$. So if $0<\beta<2\sqrt{\frac{2}{3}}$, then $\frac{dp_1^c}{d\beta}<0$.

If $\beta>2\sqrt{\frac{2}{3}}$, then $\theta_7<1$. Similarly, $\theta_7>\theta^c$ and $\theta_8<\theta^c<\theta_7<1$. If $\beta>2\sqrt{\frac{2}{3}}$ and $\theta_7<\theta<1$, then $F_6(\theta)>0$ and $(-8+3\beta^2)(2-\theta)\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2}>0$. So if $\beta>2\sqrt{\frac{2}{3}}$ and $\theta_7<\theta<1$, then $\frac{dp_1^c}{d\beta}>0$; if $\beta>2\sqrt{\frac{2}{3}}$ and $\theta^c<\theta<\theta_7$, then $F_6(\theta)<0$ and $(-8+3\beta^2)(2-\theta)\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2}>0$. $((-8+3\beta^2)(2-\theta)\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2})^2-\left(F_6(\theta)\right)^2=(8+16\beta+3\beta^2)^2(1-\theta)F_7(\theta)$, where $F_7(\theta)=-96-128\beta-52\beta^2+96\theta+128\beta\theta+52\beta^2\theta-8\theta^2+3\beta^2\theta^2$, $-8+3\beta^2>0$, $F_7(\theta)$ is a convex function. $\Delta>0$ means that there are two real roots for $F_7(\theta)$: $\theta_9=-\frac{2\left(24+32\beta+13\beta^2+4\sqrt{(1+\beta)^2(24+32\beta+13\beta^2)}\right)}{-8+3\beta^2}<0$ and $\theta^y=\frac{2(-24-32\beta-13\beta^2+4\sqrt{(1+\beta)^2(24+32\beta+13\beta^2)})}{-8+3\beta^2}$. $\theta_7-\theta^y>0$, $\theta^y-\theta^c>0$. So if $\beta>2\sqrt{\frac{2}{3}}$ and $\theta^c<\theta<\theta^y$, then $\frac{dp_1^c}{d\beta}<0$; if $\beta>2\sqrt{\frac{2}{3}}$ and $\theta^y<\theta<1$, then $\frac{dp_1^c}{d\beta}>0$.

Therefore, if $\beta>\beta^A=2\sqrt{\frac{2}{3}}$ and $\theta^y<\theta<1$, then $\frac{dp_1^c}{d\beta}>0$, p_1^c increases in β; if $0<\beta<\beta^A$, or $\beta>\beta^A$ and $\theta^c<\theta<\theta^y$, then $\frac{dp_1^c}{d\beta}<0$, p_1^c decreases in β.

(3) From Table 3.2, we obtain $\frac{dw^c}{d\beta}=\frac{\delta_1(F_8(\theta)-F_9(\theta))}{2(1+\beta)^2(8+16\beta+3\beta^2)^2\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2}}$, where $F_8(\theta)=128+576\beta+944\beta^2+628\beta^3+128\beta^4+6\beta^5-128\theta-576\beta\theta-944\beta^2\theta-628\beta^3\theta-128\beta^4\theta-6\beta^5\theta+32\theta^2+120\beta\theta^2+140\beta^2\theta^2+48\beta^3\theta^2+6\beta^4\theta^2$ and $F_9(\theta)=(16\beta+24\beta^2-3\beta^4+32\theta+120\beta\theta+140\beta^2\theta+48\beta^3\theta+6\beta^4\theta)\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2}$. Recall $\theta^c<\theta<1$, then $F_8(\theta)>0$ and $F_9(\theta)>0$. $(F_8(\theta))^2-(F_9(\theta))^2=(8+16\beta+3\beta^2)^2F_{10}(\theta)$, where $F_{10}(\theta)=256+1280\beta+2576\beta^2+2528\beta^3+1124\beta^4+144\beta^5+\beta^6-512\theta-2752\beta\theta-5696\beta^2\theta-5488\beta^3\theta-2284\beta^4\theta-240\beta^5\theta+7\beta^6\theta+192\theta^2+1088\beta\theta^2+2272\beta^2\theta^2+2064\beta^3\theta^2+679\beta^4\theta^2-24\beta^5\theta^2-20\beta^6\theta^2+64\theta^3+368\beta\theta^3+824\beta^2\theta^3+896\beta^3\theta^3+484\beta^4\theta^3+120\beta^5\theta^3+12\beta^6\theta^3$, $\frac{d^2F_{10}(\theta)}{d\theta^2}=384+2176\beta+4544\beta^2+4128\beta^3+1358\beta^4-48\beta^5-40\beta^6+(384+2208\beta+4944\beta^2+5376\beta^3+2904\beta^4+720\beta^5+72\beta^6)\theta$, $384+2208\beta+4944\beta^2+5376\beta^3+2904\beta^4+720\beta^5+72\beta^6>0$, we obtain $\theta_{11}=\frac{-192-1088\beta-2272\beta^2-2064\beta^3-679\beta^4+24\beta^5+20\beta^6}{12(1+\beta)^2(16+60\beta+70\beta^2+24\beta^3+3\beta^4)}$. Recall $\theta^c<\theta<1$, then $\theta^c-\theta_{11}>0$. So if $\theta^c<\theta<1$, $\frac{d^2F_{10}}{d\theta^2}>$

0, $F_{10}(\theta)$ is a convex function.

According to the Cardano formula, three real roots exist for $F_{10}(\theta)$: $\theta_{12} = \frac{-1}{12(1+\beta)^2(16+60\beta+70\beta^2+24\beta^3+3\beta^4)}(192+1088\beta+2272\beta^2+2064\beta^3+679\beta^4-24\beta^5-20\beta^6+2T_b)$, $\theta^s = \frac{1}{12(1+\beta)^2(16+60\beta+70\beta^2+24\beta^3+3\beta^4)}(-192-1088\beta-2272\beta^2-2064\beta^3-679\beta^4+24\beta^5+20\beta^6+T_b+\sqrt{3}T_c)$, $\theta^t = \frac{1}{12(1+\beta)^2(16+60\beta+70\beta^2+24\beta^3+3\beta^4)}(-192-1088\beta-2272\beta^2-2064\beta^3-679\beta^4+24\beta^5+20\beta^6+T_b-\sqrt{3}T_c)$, where $T_b = (135168+1511424\beta+7453696\beta^2+21257728\beta^3+38632576\beta^4+46472128\beta^5+37226720\beta^6+19493488\beta^7+6396009\beta^8+1234320\beta^9+131876\beta^{10}+7080\beta^{11}+148\beta^{12})^{\frac{1}{2}}\mathrm{Cos}[\frac{1}{3}\mathrm{ArcCos}[(49545216+829292544\beta+6396346368\beta^2+30107009024\beta^3+96534497280\beta^4+222884100096\beta^5+381932101120\beta^6+493395573504\beta^7+483235517760\beta^8+358003910880\beta^9+198755612304\beta^{10}+81369151896\beta^{11}+24020765539\beta^{12}+4975533960\beta^{13}+701347650\beta^{14}+64731924\beta^{15}+3684450\beta^{16}+115488\beta^{17}+1504\beta^{18})(135168+1511424\beta+7453696\beta^2+21257728\beta^3+38632576\beta^4+46472128\beta^5+37226720\beta^6+19493488\beta^7+6396009\beta^8+1234320\beta^9+131876\beta^{10}+7080\beta^{11}+148\beta^{12})^{-\frac{3}{2}}]]$, $T_c = (135168+1511424\beta+7453696\beta^2+21257728\beta^3+38632576\beta^4+46472128\beta^5+37226720\beta^6+19493488\beta^7+6396009\beta^8+1234320\beta^9+131876\beta^{10}+7080\beta^{11}+148\beta^{12})^{\frac{1}{2}}\mathrm{Sin}[\frac{1}{3}\mathrm{ArcCos}(49545216+829292544\beta+6396346368\beta^2+30107009024\beta^3+96534497280\beta^4+222884100096\beta^5+381932101120\beta^6+493395573504\beta^7+483235517760\beta^8+358003910880\beta^9+198755612304\beta^{10}+81369151896\beta^{11}+24020765539\beta^{12}+4975533960\beta^{13}+701347650\beta^{14}+64731924\beta^{15}+3684450\beta^{16}+115488\beta^{17}+1504\beta^{18})(135168+1511424\beta+7453696\beta^2+21257728\beta^3+38632576\beta^4+46472128\beta^5+37226720\beta^6+19493488\beta^7+6396009\beta^8+1234320\beta^9+131876\beta^{10}+7080\beta^{11}+148\beta^{12})^{-\frac{3}{2}}]]$.

If $0<\beta<\beta^C\approx 3.1163$, then $\theta^s>1$; if $\beta>\beta^C$, then $\theta^s<1$. $F_{10}(\theta=\theta^c)>0$. Therefore, if $\theta^t<\theta<min\{\theta^s,1\}$, then w^c decreases in β; if $\theta^c<\theta<\theta^t$, or $\beta>\beta^C$ and $\theta^s<\theta<1$, then w^c increases in β.

Proof of Lemma 3.3: Recall $q_2^n=\frac{(2+\beta)\delta_2-\beta\Delta c}{(2+\beta)(2+3\beta)}>0$, then $0<\Delta c<\Delta c^H=\frac{(2+\beta)\delta_1}{2(1+\beta)}$. Recall $M^l=\frac{\delta_1^2(16(1+\beta)^2-(32+96\beta+76\beta^2+16\beta^3+\beta^4)(1-\theta))}{4(1+\beta)(4+8\beta+\beta^2)^2}>0$, then $\theta^j=\frac{16+64\beta+60\beta^2+16\beta^3+\beta^4}{32+96\beta+76\beta^2+16\beta^3+\beta^4}<\theta<1$.

(1) From Table 3.2, we obtain $\frac{dM^l}{d\beta}=\frac{\delta_1^2F_1(\theta)}{4(1+\beta)^2(4+8\beta+\beta^2)^3}$, where $F_1(\theta)=64+480\beta+864\beta^2+584\beta^3+$

$168\beta^4+24\beta^5+\beta^6-(256+1056\beta+1488\beta^2+872\beta^3+216\beta^4+24\beta^5+\beta^6)\theta$, $-(256+1056\beta+1488\beta^2+872\beta^3+216\beta^4+24\beta^5+\beta^6)<0$ means that $F_1(\theta)$ decreases in β. We obtain $\theta_1=\frac{64+480\beta+864\beta^2+584\beta^3+168\beta^4+24\beta^5+\beta^6}{256+1056\beta+1488\beta^2+872\beta^3+216\beta^4+24\beta^5+\beta^6}$. Recall $\theta^j<\theta<1$, $\theta_1-\theta^j<0$, $\theta_1<\theta^j$. Then $\frac{dM^l}{d\beta}<0$, M^l decreases in β.

From Table 3.2 and (3-6), we obtain $\frac{d\pi_1^l(q_1)}{d\beta}=-\frac{(64+136\beta+84\beta^2+16\beta^3+\beta^4)\delta_1^2\theta}{4(1+\beta)^2(4+8\beta+\beta^2)^2}<0$ and $\pi_1^l(q_1)$ decreases in β. Similarly, from Table 3.2 and (3-7), we obtain $\frac{d\pi_2^l(q_2^l)}{d\beta}=\frac{(64+136\beta+84\beta^2+16\beta^3+\beta^4)\delta_1^2(-1+\theta)}{4(1+\beta)^2(4+8\beta+\beta^2)^2}<0$ and $\pi_2^l(q_2^l)$ decreases in β. From Table 3.2, we obtain $\frac{dr^l}{d\beta}=\frac{(2+\beta)(8+12\beta+6\beta^2+5\beta^3)\delta_1}{2(1+\beta)^2(4+8\beta+\beta^2)^2}>0$ and r^l increases in β.

(2) From Table 3.2, we obtain $\frac{dp_1^l}{d\beta}=\frac{(-2+\beta)(2+\beta)\delta_1}{(4+8\beta+\beta^2)^2}$. Therefore, if $0<\beta<\beta^X=2$, then $\frac{dp_1^l}{d\beta}<0$ and p_1^l decreases in β; if $\beta>\beta^X$, then $\frac{dp_1^l}{d\beta}>0$ and p_1^l increases in β. From Table 3.2, we obtain $\frac{dp_2^l}{d\beta}=\frac{\beta(-8-12\beta+\beta^3)\delta_1}{2(1+\beta)^2(4+8\beta+\beta^2)^2}$. If $0<\beta<\beta^Y\approx 3.7587$, then $\frac{dp_2^l}{d\beta}<0$ and p_2^l decreases in β; if $\beta>\beta^Y$, then $\frac{dp_2^l}{d\beta}>0$ and p_2^l increases in β.

Therefore, if $0<\beta<\beta^X$, then p_1^l and p_2^l decrease in β; if $\beta^X<\beta<\beta^Y$, then p_1^l increases in β and p_2^l decreases in β; if $\beta>\beta^Y$, then p_1^l and p_2^l increase in β.

Proof of Lemma 3.4: Recall $q_2^n=\frac{(2+\beta)\delta_2-\beta\Delta c}{(2+\beta)(2+3\beta)}>0$, then $0<\Delta c<\Delta c^H=\frac{(2+\beta)\delta_1}{2(1+\beta)}$. Recall $c_1<w<c_2$, then $\theta^c=\frac{(4+6\beta+\beta^2)^2}{4(1+\beta)(4+\beta)(2+4\beta+\beta^2)}<\theta<1$.

From Table 3.2, we obtain $p_1^c-p_1^n=\frac{\beta F_1(\theta)}{2(2+\beta)(2+3\beta)(8+16\beta+3\beta^2)}$, where $F_1(\theta)=-2(8+24\beta+19\beta^2+3\beta^3)\Delta c+(2+\beta)\delta_1(4+10\beta+3\beta^2+2\theta+3\beta\theta)-(4+8\beta+3\beta^2)\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2)}$, $\frac{dF_1(\theta)}{d\theta}=\frac{\beta\delta_1(12+16\beta+3\beta^2-2\theta+2\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2)}}{4(8+16\beta+3\beta^2)\sqrt{12+16\beta+3\beta^2-12\theta-16\beta\theta-3\beta^2\theta+\theta^2}}>0$, $F_1(\theta)$ increases in θ. There is one root for $F_1(\theta)$: $\theta^e=\frac{2((2+\beta)^2(2+4\beta+3\beta^2)\delta_1^2+2(8+32\beta+40\beta^2+19\beta^3+3\beta^4)\delta_1\Delta c-2(1+\beta)^2(8+16\beta+3\beta^2)\Delta c^2)}{(2+\beta)(2+3\beta)\delta_1(8\delta_1+10\beta\delta_1+3\beta^2\delta_1-4\Delta c-4\beta\Delta c)}$. $1-\theta^e=\frac{((2+\beta)\delta_1-2(1+\beta)\Delta c)((8+24\beta+16\beta^2+3\beta^3)\delta_1-2(8+24\beta+19\beta^2+3\beta^3)\Delta c)}{(2+\beta)(2+3\beta)\delta_1((8+10\beta+3\beta^2)\delta_1-4(1+\beta)\Delta c)}$, we obtain $\Delta c^B=\frac{(8+24\beta+16\beta^2+3\beta^3)\delta_1}{2(8+24\beta+19\beta^2+3\beta^3)}<\Delta c^H$, if $0<\Delta c<\Delta c^B$, then $1>\theta^e$; if $\Delta c^B<\Delta c<\Delta c^H$, then $1<\theta^e$. $\theta^e-\theta^c=\frac{((160+640\beta+896\beta^2+552\beta^3+152\beta^4+15\beta^5)\delta_1-4(1+\beta)^2(32+72\beta+28\beta^2+3\beta^3)\Delta c)(8\Delta c+24\beta\Delta c+\beta^3(\delta_1+4\Delta c)+2\beta^2(\delta_1+10\Delta c))}{4(1+\beta)(2+\beta)(4+\beta)(2+3\beta)(2+4\beta+\beta^2)\delta_1((8+10\beta+3\beta^2)\delta_1-4(1+\beta)\Delta c)}$. Recall $0<\Delta c<\Delta c^H$, then $\theta^e>\theta^c$.

Therefore, if $0<\Delta c<\Delta c^B$ and $\theta^e<\theta<1$, then $p_1^c>p_1^n$; if $\Delta c^B<\Delta c<\Delta c^H$, or $0<\Delta c<\Delta c^B$ and $\theta^c<\theta<\theta^e$, then $p_1^c<p_1^n$.

$p_2^c - p_2^n = \frac{(2+4\beta+\beta^2)F_1(\theta)}{2(1+\beta)(2+\beta)(2+3\beta)(8+16\beta+3\beta^2))}$. If $0<\Delta c<\Delta c^B$ and $\theta^e<\theta<1$, then $p_2^c>p_2^n$; if $\Delta c^B<\Delta c<\Delta c^H$, or $0<\Delta c<\Delta c^B$ and $\theta^c<\theta<\theta^e$, then $p_2^c<p_2^n$.

Therefore, if $0<\Delta c<\Delta c^B$ and $\theta^e<\theta<1$, then $p_1^c>p_1^n$ and $p_2^c>p_2^n$; if $\Delta c^B<\Delta c<\Delta c^H$, or $0<\Delta c<\Delta c^B$ and $\theta^c<\theta<\theta^e$, then $p_1^c<p_1^n$ and $p_2^c<p_2^n$.

Proof of Proposition 3.1: Recall $q_2^n = \frac{(2+\beta)\delta_2-\beta\Delta c}{(2+\beta)(2+3\beta)}>0$, then $0<\Delta c<\Delta c^H=\frac{(2+\beta)\delta_1}{2(1+\beta)}$. Recall $c_1<w<c_2$, then $\theta^c=\frac{(4+6\beta+\beta^2)^2}{4(1+\beta)(4+\beta)(2+4\beta+\beta^2)}<\theta<1$.

(1) From Table 3.2, (3-1), (3-2), (3-3) and (3-4), we obtain $\pi^c-\pi^n = \frac{F(\theta)+2(2+\beta)^2(2+3\beta)^2\delta_1^2(8+16\beta+4\beta^2+4\theta+8\beta\theta+\beta^2\theta)\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2}}{4(1+\beta)(2+\beta)^2(2+3\beta)^2\left(8+\beta(16+3\beta)\right)^2}$, where $F(\theta)=-2((2+\beta)^2(128+768\beta+1736\beta^2+1832\beta^3+898\beta^4+168\beta^5+9\beta^6)\delta_1^2-4(16+56\beta+62\beta^2+25\beta^3+3\beta^4)^2\delta_1\Delta c+2(4+8\beta+5\beta^2)(8+24\beta+19\beta^2+3\beta^3)^2\Delta c^2)+((4+8\beta+3\beta^2)^2(32+128\beta+144\beta^2+40\beta^3+3\beta^4)\delta_1^2)\theta+(-2(2+\beta)^2(2+3\beta)^2(4+\beta(8+\beta))\delta_1^2)\theta^2$. $-2(2+\beta)^2(2+3\beta)^2\left(4+\beta(8+\beta)\right)\delta_1^2<0$ means that $F(\theta)$ is a concave function. $\Delta=(2+\beta)^2(2+3\beta)^2(8+16\beta+3\beta^2)\delta_1^2F_1(\Delta c)$, where $F_1(\Delta c)=(2+\beta)^2(-512-1536\beta+3072\beta^2+17792\beta^3+26816\beta^4+17568\beta^5+4980\beta^6+612\beta^7+27\beta^8)\delta_1^2+64(2+3\beta+\beta^2)^2(32+128\beta+148\beta^2+40\beta^3+3\beta^4)\delta_1\Delta c-32(1+\beta)^2(128+768\beta+1776\beta^2+1984\beta^3+1072\beta^4+224\beta^5+15\beta^6)\Delta c^2$, $-32(1+\beta)^2(128+768\beta+1776\beta^2+1984\beta^3+1072\beta^4+224\beta^5+15\beta^6)\Delta c^2<0$, $F_1(\Delta c)$ is a concave function. $\Delta>0$ means that there are two real roots for $F_1(\Delta c)$: $\Delta c_1 = \frac{1}{8(1+\beta)^2(4+8\beta+\beta^2)(8+16\beta+3\beta^2)(4+8\beta+5\beta^2)}((1024+7168\beta+20352\beta^2+30336\beta^3+25728\beta^4+12576\beta^5+3416\beta^6+464\beta^7+24\beta^8)\delta_1-T_d)$ and $\Delta c_2=\frac{1}{8(1+\beta)^2(4+8\beta+\beta^2)(8+16\beta+3\beta^2)(4+8\beta+5\beta^2)}((1024+7168\beta+20352\beta^2+30336\beta^3+25728\beta^4+12576\beta^5+3416\beta^6+464\beta^7+24\beta^8)\delta_1+T_d)>\Delta c^H$, where $T_d=(2(4+12\beta+11\beta^2+3\beta^3)^2(16384+163840\beta+768000\beta^2+2228224\beta^3+4334592\beta^4+5697536\beta^5+4954752\beta^6+2760960\beta^7+952656\beta^8+199616\beta^9+24584\beta^{10}+1632\beta^{11}+45\beta^{12})\delta_1^2))^{\frac{1}{2}}$. $\left((1024+7168\beta+20352\beta^2+30336\beta^3+25728\beta^4+12576\beta^5+3416\beta^6+464\beta^7+24\beta^8)\delta_1\right)^2-(T_d)^2=-2(1+\beta)^2(2+\beta)^2(4+8\beta+\beta^2)(8+16\beta+3\beta^2)(4+8\beta+5\beta^2)(-512-1536\beta+3072\beta^2+17792\beta^3+26816\beta^4+17568\beta^5+4980\beta^6+612\beta^7+27\beta^8)\delta_1^2$. If $0<\beta<0.2944$, then $\Delta c_1>0$; if $\beta>0.2944$, then $\Delta c_1<0$. If $\beta>0.2944$, or $0<\beta<0.2944$ and $\Delta c_1<\Delta c<\Delta c^H$, then $\Delta>0$; if $0<\beta<0.2944$ and $0<\Delta c<\Delta c_1$, then $\Delta<0$, $F(\theta)<0$.

If $\Delta > 0$, there are two real roots for $F(\theta)$: $\theta_4 = \frac{(128+768\beta+1696\beta^2+1696\beta^3+764\beta^4+144\beta^5+9\beta^6)\delta_1 - T_e}{4(2+\beta)(2+3\beta)(4+8\beta+\beta^2)\delta 1}$ and $\theta_5 = \frac{(128+768\beta+1696\beta^2+1696\beta^3+764\beta^4+144\beta^5+9\beta^6)\delta_1 + T_e}{4(2+\beta)(2+3\beta)(4+8\beta+\beta^2)\delta_1} > 1$, where $T_e = ((8+16\beta+3\beta^2)((2+\beta)^2(-512-1536\beta+3072\beta^2+17792\beta^3+26816\beta^4+17568\beta^5+4980\beta^6+612\beta^7+27\beta^8)\delta_1^2+64(2+3\beta+\beta^2)^2(32+128\beta+148\beta^2+40\beta^3+3\beta^4)\delta_1\Delta\mathrm{c}-32(1+\beta)^2(128+768\beta+1776\beta^2+1984\beta^3+1072\beta^4+224\beta^5+15\beta^6)\Delta\mathrm{c}^2))^{\frac{1}{2}}$. $1-\theta_4 = \frac{T_e-(64+512\beta+1376\beta^2+1568\beta^3+752\beta^4+144\beta^5+9\beta^6)\delta_1}{4(2+\beta)(2+3\beta)(4+8\beta+\beta^2)\delta_1}$, $(T_e)^2 - \left((64+512\beta+1376\beta^2+1568\beta^3+752\beta^4+144\beta^5+9\beta^6)\delta_1\right)^2 = 8(4+8\beta+\beta^2)F(\Delta\mathrm{c})$, where $F(\Delta\mathrm{c}) = (2+\beta)^2(-160-800\beta-1344\beta^2-792\beta^3-26\beta^4+60\beta^5+9\beta^6)\delta_1^2+8(16+56\beta+62\beta^2+25\beta^3+3\beta^4)^2\delta_1\Delta\mathrm{c}-4(4+8\beta+5\beta^2)(8+24\beta+19\beta^2+3\beta^3)^2\Delta\mathrm{c}^2$, $-4(4+8\beta+5\beta^2)(8+24\beta+19\beta^2+3\beta^3)^2 < 0$, $F(\Delta\mathrm{c})$ is a concave function. There are two roots for $F(\Delta\mathrm{c})$: $\Delta c_3 = \frac{(512+3584\beta+10240\beta^2+15488\beta^3+13480\beta^4+6872\beta^5+1994\beta^6+300\beta^7+18\beta^8)\delta_1 - T_f}{2(1+\beta)^2(8+16\beta+3\beta^2)^2(4+8\beta+5\beta^2)}$ and $\Delta c_4 = \frac{(512+3584\beta+10240\beta^2+15488\beta^3+13480\beta^4+6872\beta^5+1994\beta^6+300\beta^7+18\beta^8)\delta_1 + T_f}{2(1+\beta)^2(8+16\beta+3\beta^2)^2(4+8\beta+5\beta^2)} > \Delta c^H$, where $T_f = ((32+160\beta+292\beta^2+236\beta^3+81\beta^4+9\beta^5)^2(96+384\beta+608\beta^2+576\beta^3+354\beta^4+96\beta^5+9\beta^6)\delta_1^2))^{\frac{1}{2}}$. $((512+3584\beta+10240\beta^2+15488\beta^3+13480\beta^4+6872\beta^5+1994\beta^6+300\beta^7+18\beta^8)\delta_1)^2 - \left(T_f\right)^2 = -(1+\beta)^2(2+\beta)^2(8+16\beta+3\beta^2)^2(4+8\beta+5\beta^2)(-160-800\beta-1344\beta^2-792\beta^3-26\beta^4+60\beta^5+9\beta^6)\delta_1^2$. If $0 < \beta < 3.7386$, then $\Delta\mathrm{c}_3 > 0$; if $\beta > 3.7386$, then $\Delta\mathrm{c}_3 < 0$. Therefore, if $\beta > 3.7386$, or $0 < \beta < 3.7386$ and $\Delta\mathrm{c}_3 < \Delta\mathrm{c} < \Delta c^H$, then $\theta_4 < 1$; if $0 < \beta < 3.7386$ and $0 < \Delta\mathrm{c} < \Delta\mathrm{c}_3$, then $\theta_4 > 1$.

If $\beta > 3.7386$ and $\theta_4 < \theta < 1$, or $0 < \beta < 3.7386$ and $\Delta\mathrm{c}_3 < \Delta\mathrm{c} < \Delta c^H$ and $\theta_4 < \theta < 1$, then $F(\theta) > 0$, $\pi^c > \pi^n$; if $\beta > 3.7386$ and $\theta^c < \theta < \theta_4$, or $0 < \beta < 3.7386$ and $\Delta\mathrm{c}_3 < \Delta\mathrm{c} < \Delta c^H$ and $\theta^c < \theta < \theta_4$, or $0 < \beta < 3.7386$ and $0 < \Delta\mathrm{c} < \Delta\mathrm{c}_3$, then $F(\theta) < 0$.

If $F(\theta) < 0$, then $(2(2+\beta)^2(2+3\beta)^2\delta_1^2(8+16\beta+4\beta^2+4\theta+8\beta\theta+\beta^2\theta)\sqrt{12+16\beta+3\beta^2-12\theta-16\beta\theta-3\beta^2\theta+\theta^2})^2 - F(\theta)^2 = H(\theta)$. $\Delta > 0$ means that there are two roots for $H(\theta)$: $\theta^p = \frac{2(T_y-2T_g)}{(2+\beta)^2(2+3\beta)^2(4+8\beta+\beta^2)\delta_1^2\left((2+\beta)^2(64+192\beta+204\beta^2+84\beta^3+9\beta^4)\delta_1^2-32(2+3\beta+\beta^2)^2\delta_1\Delta\mathrm{c}+16(1+\beta)^2(4+8\beta+5\beta^2)\Delta\mathrm{c}^2\right)}$ and $\theta^q = \frac{2(T_y+2T_g)}{(2+\beta)^2(2+3\beta)^2(4+8\beta+\beta^2)\delta_1^2\left((2+\beta)^2(64+192\beta+204\beta^2+84\beta^3+9\beta^4)\delta_1^2-32(2+3\beta+\beta^2)^2\delta_1\Delta\mathrm{c}+16(1+\beta)^2(4+8\beta+5\beta^2)\Delta\mathrm{c}^2\right)}$, where $T_y = (4+8\beta+3\beta^2)^2\delta_1^2((2+\beta)^2(64+384\beta+840\beta^2+840\beta^3+382\beta^4+64\beta^5+3\beta^6)\delta_1^2-4(2+3\beta+\beta^2)^2(32+128\beta+144\beta^2+40\beta^3+3\beta^4)\delta_1\Delta\mathrm{c}+2(1+\beta)^2(128+768\beta+1760\beta^2+1952\beta^3+1052\beta^4+224\beta^5+15\beta^6)\Delta\mathrm{c}^2)$ and $T_g = ((4+8\beta+3\beta^2)^2\delta_1^2((2+\beta)^2(40+168\beta+242\beta^2+136\beta^3+21\beta^4)\delta_1^2 -$

$4(2+3\beta+\beta^2)^2(8+16\beta+3\beta^2)\delta_1\Delta c+2(1+\beta)^2(32+128\beta+180\beta^2+104\beta^3+15\beta^4)\Delta c^2)^2(16\beta^2\delta_1^2+32\beta^3\delta_1^2+24\beta^4\delta_1^2+8\beta^5\delta_1^2+\beta^6\delta_1^2+8(2+3\beta+\beta^2)^2(4+8\beta+\beta^2)\delta_1\Delta c-4(1+\beta)^2(16+64\beta+88\beta^2+48\beta^3+5\beta^4)\Delta c^2)^{\frac{1}{2}}$.

$1-\theta^q=$, where $T_z=(4+8\beta+3\beta^2)^2\delta_1^2((2+\beta)^2(128+512\beta+736\beta^2+480\beta^3+148\beta^4+28\beta^5+3\beta^6)\delta_1^2+8(2+3\beta+\beta^2)^2(16+96\beta+140\beta^2+40\beta^3+3\beta^4)\delta_1\Delta c-4(1+\beta)^2(64+512\beta+1408\beta^2+1760\beta^3+1032\beta^4+224\beta^5+15\beta^6)\Delta c^2)$. $(T_z)^2-(4T_g)^2=(2+\beta)^2(2+3\beta)^2(4+8\beta+\beta^2)\delta_1^2((2+\beta)\delta_1-2(1+\beta)\Delta c)((-8-12\beta-2\beta^2+\beta^3)\delta_1+2(4+12\beta+13\beta^2+5\beta^3)\Delta c)H_1(\Delta c)H_2(\Delta c)$., where $H_1(\Delta c)=(2+\beta)^2(64+192\beta+204\beta^2+84\beta^3+9\beta^4)\delta_1^2-32(2+3\beta+\beta^2)^2\delta_1\Delta c+16(1+\beta)^2(4+8\beta+5\beta^2)\Delta c^2$ and $H_2(\Delta c)=(2+\beta)^2(-64-320\beta-512\beta^2-240\beta^3+64\beta^4+60\beta^5+9\beta^6)\delta_1^2+8(16+56\beta+62\beta^2+25\beta^3+3\beta^4)^2\delta_1\Delta c-4(4+8\beta+5\beta^2)(8+24\beta+19\beta^2+3\beta^3)^2\Delta c^2$. Recall $0<\Delta c<\Delta c^H$, then $H_1(\Delta c)>0$. $-4(4+8\beta+5\beta^2)(8+24\beta+19\beta^2+3\beta^3)^2<0$ means that $H_2(\Delta c)$ is a concave function. $\Delta=16(1+\beta)^2(2+\beta)^2(2+3\beta)^2(8+16\beta+3\beta^2)^2(192+768\beta+1152\beta^2+896\beta^3+404\beta^4+96\beta^5+9\beta^6)\delta_1^2>0$, implies that two real roots exist for $H_2(\Delta c)=0$: $\Delta c^M=\frac{(2+\beta)(32+112\beta+124\beta^2+50\beta^3+6\beta^4+(2+3\beta)\sqrt{192+768\beta+1152\beta^2+896\beta^3+404\beta^4+96\beta^5+9\beta^6})}{2(1+\beta)(8+16\beta+3\beta^2)(4+8\beta+5\beta^2)}\delta_1$ and $\Delta c^N=\frac{(2+\beta)(32+112\beta+124\beta^2+50\beta^3+6\beta^4-(2+3\beta)\sqrt{192+768\beta+1152\beta^2+896\beta^3+404\beta^4+96\beta^5+9\beta^6})}{2(1+\beta)(8+16\beta+3\beta^2)(4+8\beta+5\beta^2)}\delta_1$. $\Delta c^M-\Delta c^H>0$ and $\Delta c^N-\Delta c^H<0$ implies that $\Delta c^N<\Delta c^H<\Delta c^M$. $(32+112\beta+124\beta^2+50\beta^3+6\beta^4)^2-((2+3\beta)\sqrt{192+768\beta+1152\beta^2+896\beta^3+404\beta^4+96\beta^5+9\beta^6})^2=-(4+10\beta+3\beta^2)(4+8\beta+5\beta^2)(-16-40\beta-16\beta^2+10\beta^3+3\beta^4)$. If $0<\beta<\beta^K\approx 2.2281$, then $0<\Delta c^N<\Delta c^H<\Delta c^M$. If $\Delta c^N<\Delta c<\Delta c^H$ and $0<\beta<\beta^K$, then $H_2(\Delta c)>0$; if $0<\Delta c<\Delta c^N$ and $0<\beta<\beta^K$, then $H_2(\Delta c)<0$.

$(-8-12\beta-2\beta^2+\beta^3)\delta_1+2(4+12\beta+13\beta^2+5\beta^3)\Delta c=0$ shows that $\Delta c^G=-\frac{(-8-12\beta-2\beta^2+\beta^3)\delta_1}{2(4+12\beta+13\beta^2+5\beta^3)}$. If $0<\beta<\beta^H\approx 4.8284$, then $\Delta c^G>0$. $\Delta c^G-\Delta c^N>0$, if $0<\beta<\beta^K$ and $\Delta c^N<\Delta c<\Delta c^G$ or $\beta^K<\beta<\beta^H$ and $0<\Delta c<\Delta c^G$, then $\theta^q>1$; if $0<\beta<\beta^K$ and $0<\Delta c<\Delta c^N$ or $\Delta c^G<\Delta c<\Delta c^H$, or $\beta^K<\beta<\beta^H$ and $\Delta c^G<\Delta c<\Delta c^H$, or $\beta>\beta^H$, then $\theta^q<1$.

$\theta^p-\theta^c=$

$$\frac{(4+8\beta+3\beta^2)^2\delta_1^2(H_3(\Delta c)-16(1+\beta)(4+\beta)(2+4\beta+\beta^2)T_g)}{4(1+\beta)(2+\beta)^2(4+\beta)(2+3\beta)^2(2+4\beta+\beta^2)(4+8\beta+\beta^2)\delta_1^2((2+\beta)^2(4+3\beta)(16+3\beta(2+\beta)(6+\beta))\delta_1^2-32(1+\beta)^2(2+\beta)^2\delta_1\Delta c+16(1+\beta)^2(4+\beta(8+5\beta))\Delta c^2}$$

, where $H_3(\Delta c)=(20480\beta+162816\beta^2+559616\beta^3+1095168\beta^4+1351424\beta^5+1099072\beta^6+597536\beta^7+215536\beta^8+50048\beta^9+7024\beta^{10}+524\beta^{11}+15\beta^{12})\delta_1^2-32(2+3\beta+\beta^2)^2(192+1536\beta+4736\beta^2+7232\beta^3+5848\beta^4+2522\beta^5+581\beta^6+67\beta^7+3\beta^8)\delta_1\Delta c+16(1+\beta)^2(768+7680\beta+32192\beta^2+74496\beta^3+104928\beta^4+93032\beta^5+51740\beta^6+17526\beta^7+3453\beta^8+359\beta^9+15\beta^{10})\Delta c^2)$,

$16(1+\beta)^2(768+7680\beta+32192\beta^2+74496\beta^3+104928\beta^4+93032\beta^5+51740\beta^6+17526\beta^7+3453\beta^8+359\beta^9+15\beta^{10})>0$ means that $H_3(\Delta c)$ is a convex function. $\Delta=-64(1+\beta)^2(2+\beta)^2(2+3\beta)^2(192+1536\beta+4736\beta^2+7232\beta^3+5848\beta^4+2522\beta^5+581\beta^6+67\beta^7+3\beta^8)(-3072-19456\beta-48384\beta^2-57984\beta^3-29376\beta^4+4416\beta^5+12752\beta^6+6112\beta^7+1284\beta^8+116\beta^9+3\beta^{10})\delta_1^2$. If $0<\beta<1.6872$, there are two real roots for $H_3(\Delta c)$: $\Delta c_5=\frac{(3072+33792\beta+159488\beta^2+427520\beta^3+724608\beta^4+816928\beta^5+626960\beta^6+329384\beta^7+117396\beta^8+27660\beta^9+4088\beta^{10}+340\beta^{11}+12\beta^{12})\delta_1-T_h}{4(1+\beta)^2(4+8\beta+5\beta^2)(192+1536\beta+4736\beta^2+7232\beta^3+5848\beta^4+2522\beta^5+581\beta^6+67\beta^7+3\beta^8)}$

and $\Delta c_6=\frac{(3072+33792\beta+159488\beta^2+427520\beta^3+724608\beta^4+816928\beta^5+626960\beta^6+329384\beta^7+117396\beta^8+27660\beta^9+4088\beta^{10}+340\beta^{11}+12\beta^{12})\delta_1+T_h}{4(1+\beta)^2(4+8\beta+5\beta^2)(192+1536\beta+4736\beta^2+7232\beta^3+5848\beta^4+2522\beta^5+581\beta^6+67\beta^7+3\beta^8)}$

, where $T_h=(-(4+12\beta+11\beta^2+3\beta^3)^2(-589824-8454144\beta-53723136\beta^2-199811072\beta^3-482521088\beta^4-790325248\beta^5-883036160\beta^6-643397120\beta^7-245485824\beta^8+37913088\beta^9+115075072\beta^{10}+78166944\beta^{11}+31393072\beta^{12}+8357016\beta^{13}+1503860\beta^{14}+179326\beta^{15}+13367\beta^{16}+549\beta^{17}+9\beta^{18})\delta_1^2)^{\frac{1}{2}})$. Recall $0<\Delta c<\Delta c^H$, we obtain $\Delta c_5>0$; if $0<\beta<1.1161$ and $\Delta c_6>\Delta c^H$; if $\beta>1.1161$, $\Delta c_6<\Delta c^H$. So if $0<\Delta c<\Delta c_5$ or $1.1161<\beta<1.6872$ and $\Delta c_6<\Delta c<\Delta c^H$, or $\beta>1.6872$, then $H_3(\Delta c)>0$. If $0<\beta<1.1161$ and $\Delta c_5<\Delta c<\Delta c^H$, or $1.1161<\beta<1.6872$ and $\Delta c_5<\Delta c<\Delta c_6$, then $\theta^p<\theta^c$.

$(H_3(\Delta c))^3-\left(16(1+\beta)(4+\beta)(2+4\beta+\beta^2)T_g\right)^2=(2+\beta)^2(2+3\beta)^2(4+8\beta+\beta^2)\delta_1^2((2+\beta)^2(64+192\beta+204\beta^2+84\beta^3+9\beta^4)\delta_1^2-32(2+3\beta+\beta^2)^2\delta_1\Delta c+16(1+\beta)^2(4+8\beta+5\beta^2)\Delta c^2)((2+\beta)^2(25600+209920\beta+727040\beta^2+1389312\beta^3+1606784\beta^4+1162112\beta^5+525760\beta^6+144944\beta^7+22832\beta^8+1776\beta^9+45\beta^{10})\delta_1^2-32(1+\beta)^4(64+176\beta+128\beta^2+34\beta^3+3\beta^4)^2\delta_1\Delta c+16(1+\beta)^4(4+8\beta+5\beta^2)(32+72\beta+28\beta^2+3\beta^3)^2\Delta c^2)H_4(\Delta c)$, where $H_4(\Delta c)=(64\beta^3+240\beta^4+320\beta^5+192\beta^6+52\beta^7+5\beta^8)\delta_1^2-32(4+14\beta+16\beta^2+7\beta^3+\beta^4)^2\delta_1\Delta c+16(4+8\beta+5\beta^2)(2+6\beta+5\beta^2+\beta^3)^2\Delta c^2$. Recall $0<\Delta c<\Delta c^H$, then $(2+\beta)^2(25600+209920\beta+727040\beta^2+1389312\beta^3+1606784\beta^4+1162112\beta^5+525760\beta^6+144944\beta^7+22832\beta^8+1776\beta^9+45\beta^{10})\delta_1^2-32(1+\beta)^4(64+176\beta+128\beta^2+34\beta^3+3\beta^4)^2\delta_1\Delta c+16(1+\beta)^4(4+8\beta+5\beta^2)(32+72\beta+28\beta^2+3\beta^3)^2\Delta c^2>0$ and $(2+\beta)^2(64+192\beta+204\beta^2+84\beta^3+9\beta^4)\delta_1^2-32(2+3\beta+\beta^2)^2\delta_1\Delta c+16(1+\beta)^2(4+8\beta+5\beta^2)\Delta c^2>0$. $16(4+8\beta+5\beta^2)(2+6\beta+5\beta^2+\beta^3)^2>0$ means that $H_4(\Delta c)$ is a convex function. $\Delta=-64(1+\beta)^2(2+\beta)^2(2+3\beta)^2(2+4\beta+\beta^2)^2(-64-256\beta-384\beta^2-272\beta^3-84\beta^4-4\beta^5+\beta^6)\delta_1^2$, if $0<\beta<\beta^J\approx 12.5904$, then $\Delta>0$; if $\beta>\beta^J$, then $\Delta<0$, $H_4(\Delta c)>0$.

$\Delta c^U=\frac{1}{4(1+\beta)^2(2+4\beta+\beta^2)^2(4+8\beta+5\beta^2)}((64+448\beta+1296\beta^2+2016\beta^3+1840\beta^4+1008\beta^5+324\beta^6+$

$56\beta^7 + 4\beta^8)\delta_1 - T_i)$, $\Delta c^V = \frac{1}{4(1+\beta)^2(2+4\beta+\beta^2)^2(4+8\beta+5\beta^2)}((64 + 448\beta + 1296\beta^2 + 2016\beta^3 + 1840\beta^4 + 1008\beta^5 + 324\beta^6 + 56\beta^7 + 4\beta^8)\delta_1 + T_i)$, where $T_i = (-(8 + 40\beta + 74\beta^2 + 62\beta^3 + 23\beta^4 + 3\beta^5)^2(-64 - 256\beta - 384\beta^2 - 272\beta^3 - 84\beta^4 - 4\beta^5 + \beta^6)\delta_1^2)^{\frac{1}{2}}$. Recall $0 < \Delta c < \Delta c^H$, then $\Delta c^U > 0$. $\Delta c^V - \Delta c^H = \frac{1}{4(1+\beta)^2(2+4\beta+\beta^2)^2(4+8\beta+5\beta^2)}(-(32\beta + 224\beta^2 + 632\beta^3 + 920\beta^4 + 736\beta^5 + 320\beta^6 + 70\beta^7 + 6\beta^8)\delta_1 + T_i)$, $(T_i)^2 - \left((32\beta + 224\beta^2 + 632\beta^3 + 920\beta^4 + 736\beta^5 + 320\beta^6 + 70\beta^7 + 6\beta^8)\delta_1\right)^2 = -(1+\beta)^2(2+\beta)^3(2 + 3\beta)^2(2 + 4\beta + \beta^2)^2(4 + 8\beta + 5\beta^2)(-8 - 12\beta + 2\beta^2 + \beta^3)\delta_1^2$. If $0 < \beta < \beta^M \approx 2.9623$, then $\Delta c^V > \Delta c^H$; if $\beta^M < \beta < \beta^J$, then $\Delta c^V < \Delta c^H$. if $0 < \beta < \beta^J$ and $0 < \Delta c < \Delta c^U$, or $\beta^M < \beta < \beta^J$ and $\Delta c^V < \Delta c < \Delta c^H$, then $H_4(\Delta c) > 0$; if $0 < \beta < \beta^M$ and $\Delta c^U < \Delta c < \Delta c^H$, or $\beta^M < \beta < \beta^J$ and $\Delta c^U < \Delta c < \Delta c^V$, then $H_4(\Delta c) < 0$.

$\Delta c_5 - \Delta c^U > 0$ and $\Delta c_6 - \Delta c^V < 0$, so if $0 < \beta < \beta^J$ and $0 < \Delta c < \Delta c^U$,or $\beta^M < \beta < \beta^J$ and $\Delta c^V < \Delta c < \Delta c^H$, or $\beta > \beta^J$, then $\theta^p > \theta^c$; if $0 < \beta < \beta^M$ and $\Delta c^U < \Delta c < \Delta c^H$, or $\beta^M < \beta < \beta^J$ and $\Delta c^U < \Delta c < \Delta c^V$, then $\theta^p < \theta^c$.

Therefore, if $0 < \beta < \beta^K$ and $\Delta c^N < \Delta c < \Delta c^G$ or $\beta^K < \beta < \beta^H$ and $0 < \Delta c < \Delta c^G$, then $\theta^q > 1$; if $0 < \beta < \beta^K$ and $0 < \Delta c < \Delta c^N$ or $\Delta c^G < \Delta c < \Delta c^H$, or $\beta^K < \beta < \beta^H$ and $\Delta c^G < \Delta c < \Delta c^H$, or $\beta > \beta^H$, then $\theta^q < 1$; if $0 < \beta < \beta^J$ and $0 < \Delta c < \Delta c^U$, or $\beta^M < \beta < \beta^J$ and $\Delta c^V < \Delta c < \Delta c^H$, or $\beta > \beta^J$, then $\theta^p > \theta^c$; if $0 < \beta < \beta^M$ and $\Delta c^U < \Delta c < \Delta c^H$, or $\beta^M < \beta < \beta^J$ and $\Delta c^U < \Delta c < \Delta c^V$, then $\theta^p < \theta^c$.

If $\beta > 3.7386$, or $0 < \beta < 3.7386$ and $\Delta c_3 < \Delta c < \Delta c^H$, then $\theta^q - \theta_4 > 0$, $\theta^q > \theta_4$. $\Delta c_3 - \Delta c^N > 0$, $\Delta c_3 > \Delta c^N$. Similarly, $\Delta c_3 - \Delta c^G < 0$, $\Delta c_3 < \Delta c^G$. Therefore, $\Delta c^N < \Delta c_3 < \Delta c^G$. Recall if $\beta > 3.7386$ and $\theta_4 < \theta < 1$, or $0 < \beta < 3.7386$ and $\Delta c_3 < \Delta c < \Delta c^H$ and $\theta_4 < \theta < 1$, then $F(\theta) > 0$, $\pi^c > \pi^n$. Therefore, if $0 < \beta < \beta^K$ and $0 < \Delta c < \Delta c^N$, then $\theta^q < min\{1, \theta_4\}$.

If $0 < \beta < \beta^K$ and $0 < \Delta c < \Delta c^N$ and $max\{\theta^p, \theta^c\} < \theta < \theta^q$, or $\beta > \beta^K$ and $max\{\theta^p, \theta^c\} < \theta < 1$, or $0 < \beta < \beta^K$ and $\Delta c^N < \Delta c < \Delta c^H$ and $max\{\theta^p, \theta^c\} < \theta < 1$, then WC is the better strategy; otherwise, competition is the better strategy.

Therefore, if $0 < \Delta c < \Delta c^H$ and $max\{\theta^p, \theta^c\} < \theta < min\{\theta^q, 1\}$, then WC is the better strategy; otherwise, competition is the better strategy.

(2) From Table 3.2, (3-1) and (3-3), we obtain $\pi_1^c(q_1^c) - \pi_1^n(q_1^n) = \frac{F_1(\theta)}{4(1+\beta)(2+\beta)^2(2+3\beta)^2(8+16\beta+3\beta^2)}$, where $F_1(\theta) = (-8\beta(1+\beta)^2(16 + 40\beta + 22\beta^2 + 3\beta^3)\delta_1\Delta c - 4\beta^2(1+\beta)^2(8 + 16\beta + 3\beta^2)\Delta c^2 + (2 + \beta)^2\delta_1^2\big(-32 + 48\theta + 3\beta^4(-4 + 9\theta) + 16\beta(-8 + 13\theta) + 4\beta^3(-22 + 45\theta) + 4\beta^2(-43 + 78\theta)\big) + (-32 -$

$128\beta-176\beta^2-96\beta^3-18\beta^4)\delta_1^2\theta^2+2(2+\beta)^2(2+3\beta)^2\delta_1^2\theta\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2})$. $\frac{dF_1(\theta)}{d\theta}>0$ and $F_1(\theta)$ increases in θ. There is one real root for $F_1(\theta)$: $\theta^f=\frac{4((12+16\beta+3\beta^2)(4+12\beta+11\beta^2+3\beta^3)^2\delta_1^2\left((2+\beta)\delta_1+\beta\Delta c\right)^2-2T_j}{(2+\beta)^2(2+3\beta)^2\delta_1^2\left((2+\beta)^2(64+240\beta+328\beta^2+180\beta^3+27\beta^4)\delta_1^2+32\beta(1+\beta)^2(2+\beta)\delta_1\Delta c+16\beta^2(1+\beta)^2\Delta c^2\right)}$, where $T_j=((1+\beta)^4(4+8\beta+3\beta^2)^2\delta_1^2\left((2+\beta)\delta_1+\beta\Delta c\right)^4((2+\beta)^2(16+80\beta+140\beta^2+92\beta^3+15\beta^4)\delta_1^2-8\beta(1+\beta)^2(16+40\beta+22\beta^2+3\beta^3)\delta_1\Delta c-4\beta^2(1+\beta)^2(8+16\beta+3\beta^2)\Delta c^2))^{\frac{1}{2}}$. $\theta^f-\theta^c>0$ and $\theta^f>\theta^c$. Similarly, $1-\theta^f=\frac{(2+\beta)^4(2+3\beta)^2(16+80\beta+140\beta^2+92\beta^3+15\beta^4)\delta_1^4-8\beta(2+\beta)^3(2+5\beta+3\beta^2)^2(8+16\beta+3\beta^2)\delta_1^3\Delta c-4\beta^2(8+16\beta+3\beta^2)(4+12\beta+11\beta^2+3\beta^3)^2\delta_1^2\Delta c^2+8T_j}{4(1+\beta)(2+\beta)^2(4+\beta)(2+3\beta)^2(2+4\beta+\beta^2)\delta_1^2\left((2+\beta)^2(64+240\beta+328\beta^2+180\beta^3+27\beta^4)\delta_1^2+32\beta(1+\beta)^2(2+\beta)\delta_1\Delta c+16\beta^2(1+\beta)^2\Delta c^2\right)}$. Recall $0<\Delta c<\Delta c^H$, $(2+\beta)^4(2+3\beta)^2(16+80\beta+140\beta^2+92\beta^3+15\beta^4)\delta_1^4-8\beta(2+\beta)^3(2+5\beta+3\beta^2)^2(8+16\beta+3\beta^2)\delta_1^3\Delta c-4\beta^2(8+16\beta+3\beta^2)(4+12\beta+11\beta^2+3\beta^3)^2\delta_1^2\Delta c^2>0$, then $\theta^f<1$, $\theta^c<\theta^f<1$.

Therefore, if $\theta^c<\theta<\theta^f$, then $\pi_1^c(q_1^c)<\pi_1^n(q_1^n)$; if $\theta^f<\theta<1$, then $\pi_1^c(q_1^c)>\pi_1^n(q_1^n)$.

From Table 3.2, (3-2) and (3-4), we obtain $\pi_2^c(q_2^c)-\pi_2^n(q_2^n)=\frac{F_3(\theta)}{2(1+\beta)(2+\beta)^2(2+3\beta)^2(8+16\beta+3\beta^2)^2}$, where $F_3(\theta)=8(1+\beta)^3(2+\beta)(8+16\beta+3\beta^2)^2\delta_1\Delta c-512\Delta c^2-4096\beta\Delta c^2-13696\beta^2\Delta c^2-24832\beta^3\Delta c^2-26440\beta^4\Delta c^2-16672\beta^5\Delta c^2-5936\beta^6\Delta c^2-1056\beta^7\Delta c^2-72\beta^8\Delta c^2-(2+\beta)^2\delta_1^2\big(128\theta+768\beta\theta+9\beta^6(-1+3\theta)+12\beta^5(-5+24\theta)+8\beta^2(-3+223\theta)+8\beta^3(-11+251\theta)+2\beta^4(-56+561\theta)\big)+(2(2+\beta)^2(2+3\beta)^2(2+4\beta+\beta^2)\delta_1^2)\theta^2+2(2+\beta)^2(2+3\beta)^2(2+4\beta+\beta^2)\delta_1^2(2-\theta)\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2})$. Then $\frac{dF_3(\theta)}{d\theta}<0$ and $F_3(\theta)$ decreases in θ. There is one real root for $F_3(\theta)$: $\theta^g=\frac{(2+4\beta+\beta^2)(4+8\beta+3\beta^2)^2\delta_1^2\left((2+\beta)^2(-24-56\beta-32\beta^2+4\beta^3+3\beta^4)\delta_1^2+8(1+\beta)^3(32+48\beta+22\beta^2+3\beta^3)\delta_1\Delta c-8(1+\beta)^4(16+16\beta+3\beta^2)\Delta c^2\right)+2T_k}{(2+\beta)^2(2+3\beta)^2\left(2+\beta(4+\beta)\right)\delta_1^2\left(\beta(2+\beta)^2\left(24+\beta(62+48\beta+9\beta^2)\right)\delta_1^2+32(1+\beta)^3(2+\beta)\delta_1\Delta c-32(1+\beta)^4\Delta c^2\right)}$, where $T_k=(2(2+4\beta+\beta^2)(4+12\beta+11\beta^2+3\beta^3)^2\delta_1^2((2+\beta)^3(24+104\beta+156\beta^2+92\beta^3+15\beta^4)\delta_1^3-2(2+\beta)^2(56+288\beta+560\beta^2+508\beta^3+207\beta^4+27\beta^5)\delta_1^2\Delta c+24(1+\beta)^4(16+40\beta+22\beta^2+3\beta^3)\delta_1\Delta c^2-16(1+\beta)^5(8+16\beta+3\beta^2)\Delta c^3)^2)^{\frac{1}{2}}$.

Similarly, $1-\theta^g=\frac{2\left((4+12\beta+11\beta^2+3\beta^3)^2(24+80\beta+82\beta^2+28\beta^3+3\beta^4)\delta_1^2\left((2+\beta)\delta_1-2(1+\beta)\Delta c\right)^2-T_k\right)}{(2+\beta)^2(2+3\beta)^2\left(2+\beta(4+\beta)\right)\delta_1^2\left(\beta(2+\beta)^2\left(24+\beta(62+48\beta+9\beta^2)\right)\delta_1^2+32(1+\beta)^3(2+\beta)\delta_1\Delta c-32(1+\beta)^4\Delta c^2\right)}$. $((4+12\beta+11\beta^2+3\beta^3)^2(24+80\beta+82\beta^2+28\beta^3+3\beta^4)\delta_1^2\left((2+\beta)\delta_1-2(1+\beta)\Delta c\right)^2)^2-(T_k)^2=(1+\beta)^2(2+\beta)^2(2+3\beta)^2(2+4\beta+\beta^2)\delta_1^2((2+\beta)\delta_1-2(1+\beta)\Delta c)^2((2+\beta)^2(96\beta\delta_1^2+344\beta^2\delta_1^2+464\beta^3\delta_1^2+290\beta^4\delta_1^2+84\beta^5\delta_1^2+9\beta^6\delta_1^2+32(1+\beta)^3(2+\beta)\delta_1\Delta c-32(1+\beta)^4\Delta c^2)F_6(\Delta c)$, where $F_6(\Delta c)=48+304\beta+740\beta^2+864\beta^3+487\beta^4+114\beta^5+9\beta^6)\delta_1^2-4(1+\beta)^3(2+\beta)(8+16\beta+$

$3\beta^2)^2\delta_1\Delta c+4(1+\beta)^4(8+16\beta+3\beta^2)^2\Delta c^2$. Recall $0<\Delta c<\Delta c^H$, then $96\beta\delta_1^2+344\beta^2\delta_1^2+464\beta^3\delta_1^2+290\beta^4\delta_1^2+84\beta^5\delta_1^2+9\beta^6\delta_1^2+32(1+\beta)^3(2+\beta)\delta_1\Delta c-32(1+\beta)^4\Delta c^2>0$. $4(1+\beta)^4(8+16\beta+3\beta^2)^2>0$ means that $F_6(\Delta c)$ is a convex function, we obtain $\Delta=32(1+\beta)^4(2+\beta)^2(2+3\beta)^2(2+\beta(4+\beta))(8+\beta(16+3\beta))^2\delta_1^2>0$, $\Delta c^K=\frac{(2+\beta)[(8+24\beta+19\beta^2+3\beta^3)-(3\beta+2)\sqrt{4+8\beta+2\beta^2}]\delta_1}{2(1+\beta)^2(8+16\beta+3\beta^2)}$ and $\Delta c^J=\frac{(2+\beta)[(8+24\beta+19\beta^2+3\beta^3)+(3\beta+2)\sqrt{4+8\beta+2\beta^2}]\delta_1}{2(1+\beta)^2(8+16\beta+3\beta^2)}$. $(8+24\beta+19\beta^2+3\beta^3)^2-\left((3\beta+2)\sqrt{4+8\beta+2\beta^2}\right)^2=48+304\beta+740\beta^2+864\beta^3+487\beta^4+114\beta^5+9\beta^6>0$ implies that $\Delta c^J>\Delta c^K>0$. $\Delta c^J-\Delta c^H=\frac{(2+\beta)(2+3\beta)\sqrt{2+4\beta+\beta^2}\delta_1}{\sqrt{2}(1+\beta)^2(8+16\beta+3\beta^2)}>0$ and $\Delta c^K-\Delta c^H=-\frac{(2+\beta)(2+3\beta)\sqrt{2+4\beta+\beta^2}\delta_1}{\sqrt{2}(1+\beta)^2(8+16\beta+3\beta^2)}<0$ implies that $0<\Delta c^K<\Delta c^H<\Delta c^J$. Therefore, if $\Delta c^K<\Delta c<\Delta c^H$, then $\theta^g>1$ and if $0<\Delta c<\Delta c^K$, then $\theta^g<1$. Similarly, we obtain $\theta^g>\theta^c$. If $\Delta c^K<\Delta c<\Delta c^H$, then $\theta^g>1$ and if $0<\Delta c<\Delta c^K$, then $\theta^c<\theta^g<1$.

Therefore, if $\Delta c^K<\Delta c<\Delta c^H$ and $\theta^c<\theta<1$, or $0<\Delta c<\Delta c^K$, then $\theta^c<\theta<\theta^g$, then $\pi_2^c(q_2^c)>\pi_2^n(q_2^n)$; if $0<\Delta c<\Delta c^K$ and $\theta^g<\theta<1$, then $\pi_2^c(q_2^c)<\pi_2^n(q_2^n)$.

$\theta^g-\theta^f=\Big((2+\beta)^2(2+3\beta)^2\delta_1^2((2+\beta)^2(64+240\beta+328\beta^2+180\beta^3+27\beta^4)\delta_1^2+32\beta(1+\beta)^2(2+\beta)\delta_1\Delta c+16\beta^2(1+\beta)^2\Delta c^2)\left(\beta(2+\beta)^2\left(24+\beta(62+48\beta+9\beta^2)\right)\delta_1^2+32(1+\beta)^3(2+\beta)\delta_1\Delta c-32(1+\beta)^4\Delta c^2\right)\Big)^{-1}F_1(\Delta c)$, where $F_1(\Delta c)=F_2(\Delta c)+8(2+4\beta+\beta^2)(\beta(2+\beta)^2(24+62\beta+48\beta^2+9\beta^3)\delta_1^2+32(1+\beta)^3(2+\beta)\delta_1\Delta c-32\Delta c^2-128\beta\Delta c^2-192\beta^2\Delta c^2-128\beta^3\Delta c^2-32\beta^4\Delta c^2)T_j+2((2+\beta)^2(64+240\beta+328\beta^2+180\beta^3+27\beta^4)\delta_1^2+32\beta(1+\beta)^2(2+\beta)\delta_1\Delta c+16\beta^2(1+\beta)^2\Delta c^2)T_k$, where $F_2(\Delta c)=-(2+4\beta+\beta^2)(4+8\beta+3\beta^2)^2\delta_1^2((2+\beta)^4(1536+10496\beta+30176\beta^2+46848\beta^3+41952\beta^4+21448\beta^5+5820\beta^6+720\beta^7+27\beta^8)\delta_1^4-8(1+\beta)^2(2+\beta)^3(832+4960\beta+12240\beta^2+15768\beta^3+11120\beta^4+4170\beta^5+765\beta^6+54\beta^7)\delta_1^3\Delta c+4(2+3\beta+\beta^2)^2(1664+11520\beta+34944\beta^2+59296\beta^3+60536\beta^4+37552\beta^5+13596\beta^6+2556\beta^7+189\beta^8)\delta_1^2\Delta c^2+512\beta(1+\beta)^5(2+\beta)^2\delta_1\Delta c^3+512\beta^2(1+\beta)^6\Delta c^4)$. Replace $\Delta c=0$ in $F_1(\Delta c)$, we obtain $F_1(\Delta c=0)<0$. Similarly, replace $\Delta c=\Delta c^K$ in $F_1(\Delta c)$, we obtain $F_1(\Delta c=\Delta c^K)>0$.

If $0<\Delta c<\Delta c^K$, $8(2+4\beta+\beta^2)(\beta(2+\beta)^2(24+62\beta+48\beta^2+9\beta^3)\delta_1^2+32(1+\beta)^3(2+\beta)\delta_1\Delta c-32\Delta c^2-128\beta\Delta c^2-192\beta^2\Delta c^2-128\beta^3\Delta c^2-32\beta^4\Delta c^2)T_j+2((2+\beta)^2(64+240\beta+328\beta^2+180\beta^3+27\beta^4)\delta_1^2+32\beta(1+\beta)^2(2+\beta)\delta_1\Delta c+16\beta^2(1+\beta)^2\Delta c^2)T_k>0$. $\frac{d^2F_2(\Delta c)}{d\Delta c^2}=-8(1+\beta)^2(2+\beta)^2(2+3\beta)^2(2+4\beta+\beta^2)\delta_1^2((2+\beta)^2(1664+11520\beta+34944\beta^2+59296\beta^3+60536\beta^4+37552\beta^5+13596\beta^6+2556\beta^7+189\beta^8)\delta_1^2+384\beta(1+\beta)^3(2+\beta)^2\delta_1\Delta c+768\beta^2(1+\beta)^4\Delta c^2)<0$ and $F_2(\Delta c)$ is a concave function.

From $F_1(\Delta c = 0) < 0$, $F_1(\Delta c = \Delta c^K) > 0$ and $F_2(\Delta c)$ is a concave function. Obviously, we obtain there must exist Δc^A. if $0 < \Delta c < \Delta c^A$, then $\theta^g < \theta^f$; if $\Delta c^A < \Delta c < \Delta c^K$, then $\theta^g > \theta^f$.

Therefore, in WC strategy zone, if $\Delta c^A < \Delta c < \Delta c^H$ and $\theta^f < \theta < min\{\theta^g, 1\}$, then WC is WC strategy in region A, Figure 3.2; otherwise, WC is WC strategy in region B, Figure 3.2.

(3) Therefore, if $0 < \Delta c < \Delta c^K$ and $max\{\theta^g, \theta^f\} < \theta < 1$, then $\pi_1^c(q_1^c) > \pi_1^n(q_1^n)$ and $\pi_2^c(q_2^c) < \pi_2^n(q_2^n)$; if $0 < \Delta c < \Delta c^H$ and $\theta^c < \theta < min\{\theta^g, \theta^f\}$, then $\pi_1^c(q_1^c) < \pi_1^n(q_1^n)$ and $\pi_2^c(q_2^c) > \pi_2^n(q_2^n)$.

(4) From Lemma 3.4, we obtain $\Delta c^B = \frac{(8+24\beta+16\beta^2+3\beta^3)\delta_1}{2(8+24\beta+19\beta^2+3\beta^3)}$ and $\Delta c^K - \Delta c^B > 0$. $\theta^e - \theta^g = \frac{F_6(\Delta c) - 2((8+10\beta+3\beta^2)\delta_1 - 4(1+\beta)\Delta c)T_k}{(2+\beta)^2(2+3\beta)^2(2+4\beta+\beta^2)\delta_1^2\left((8+10\beta+3\beta^2)\delta_1-(4+4\beta)\Delta c\right)\left(\beta(2+\beta)^2\left(24+\beta(62+48\beta+9\beta^2)\right)\delta_1^2+32(1+\beta)^3(2+\beta)\delta_1\Delta c-32(1+\beta)^4\Delta c^2\right)}$,

where $F_6(\Delta c) = ((8 + 32\beta + 42\beta^2 + 20\beta^3 + 3\beta^4)\delta_1((2 + \beta)^4(192 + 976\beta + 1920\beta^2 + 1880\beta^3 + 984\beta^4 + 270\beta^5 + 27\beta^6)\delta_1^4 - 4(2 + \beta)^3(272 + 1576\beta + 3672\beta^2 + 4464\beta^3 + 3086\beta^4 + 1251\beta^5 + 288\beta^6 + 27\beta^7)\delta_1^3 + 4(2 + 3\beta + \beta^2)^2(608 + 2752\beta + 4576\beta^2 + 3532\beta^3 + 1356\beta^4 + 288\beta^5 + 27\beta^6)\delta_1^2\Delta c^2 - 32(1 + \beta)^5(160 + 448\beta + 340\beta^2 + 96\beta^3 + 9\beta^4)\delta_1\Delta c^3 + 128(1 + \beta)^6(8 + 16\beta + 3\beta^2)\Delta c^4)$. $\frac{d^2F_6(\Delta c)}{d\Delta c^2} = -4(1 + \beta)(2 + \beta)(2 + 3\beta)(2 + 4\beta + \beta^2)\delta_1(-2(2 + \beta)^2(608 + 3360\beta + 7328\beta^2 + 8108\beta^3 + 4888\beta^4 + 1644\beta^5 + 315\beta^6 + 27\beta^7)\delta_1^2 + 48(1 + \beta)^4(160 + 448\beta + 340\beta^2 + 96\beta^3 + 9\beta^4)\delta_1\Delta c - 384(1 + \beta)^5(8 + 16\beta + 3\beta^2)\Delta c^2)$. Recall $0 < \Delta c < \Delta c^B$, then $\frac{d^2F_6(\Delta c)}{d\Delta c^2} > 0$ and $F_6(\Delta c)$ is a convex function. If $\Delta c = 0$, then $\frac{dF_6(\Delta c)}{d\Delta c} = -4(1 + \beta)(2 + \beta)^4(2 + 3\beta)(2 + 4\beta + \beta^2)(272 + 1304\beta + 2368\beta^2 + 2096\beta^3 + 990\beta^4 + 261\beta^5 + 27\beta^6)\delta_1^4 < 0$; if $\Delta c = \Delta c^B$, then $\frac{dF_6(\Delta c)}{d\Delta c} = -\frac{1}{(8+16\beta+3\beta^2)^2}4(1 + \beta)(2 + \beta)^4(2 + 3\beta)^4(2 + 4\beta + \beta^2)(576 + 2304\beta + 3672\beta^2 + 3056\beta^3 + 1434\beta^4 + 324\beta^5 + 27\beta^6)\delta_1^4 < 0$. Therefore, $F_6(\Delta c)$ decreases in Δc. $F_6(\Delta c = \Delta c^B) = \frac{8(1+\beta)^2(2+\beta)^5(2+3\beta)^5(2+4\beta+\beta^2)(12+16\beta+3\beta^2)^2\delta_1^5}{(8+16\beta+3\beta^2)^3} > 0$, then $F_6(\Delta c) > 0$. Recall $0 < \Delta c < \Delta c^H$, then $2((8 + 10\beta + 3\beta^2)\delta_1 - 4(1 + \beta)\Delta c)T_k > 0$. $F_6^2(\Delta c) - \left(2\left((8 + 10\beta + 3\beta^2)\delta_1 - 4(1 + \beta)\Delta c\right)T_k\right)^2 = \beta^2(2 + \beta)^2(2 + 3\beta)^2(2 + 4\beta + \beta^2)\delta_1^2((2 + \beta)\delta_1 - 2(1 + \beta)\Delta c)^2(\beta(2 + \beta)^2(24 + 62\beta + 48\beta^2 + 9\beta^3)\delta_1^2 + 32(1 + \beta)^3(2 + \beta)\delta_1\Delta c - 32(1 + \beta)^4\Delta c^2)F_7(\Delta c)$, where $F_7(\Delta c) = (2 + \beta)^4(1920 + 14080\beta + 42432\beta^2 + 68064\beta^3 + 62944\beta^4 + 33936\beta^5 + 10224\beta^6 + 1512\beta^7 + 81\beta^8)\delta_1^4 - 4(2 + \beta)^3(2816 + 22656\beta + 76704\beta^2 + 142576\beta^3 + 159280\beta^4 + 110096\beta^5 + 46752\beta^6 + 11682\beta^7 + 1539\beta^8 + 81\beta^9)\delta_1^3\Delta c + 4(16 + 56\beta + 62\beta^2 + 25\beta^3 + 3\beta^4)^2(96 + 264\beta + 244\beta^2 + 84\beta^3 + 9\beta^4)\delta_1^2\Delta c^2 - 192(1 + \beta)^5(16 + 40\beta + 22\beta^2 + 3\beta^3)^2\delta_1\Delta c^3 + 128(1 + \beta)^6(8 + 16\beta + 3\beta^2)^2\Delta c^4$. Recall $0 < \Delta c < \Delta c^B$, then $\beta(2 + \beta)^2(24 + 62\beta + 48\beta^2 + 9\beta^3)\delta_1^2 + 32(1 + \beta)^3(2 + \beta)\delta_1\Delta c - 32(1 + \beta)^4\Delta c^2 > 0$. Similarly, recall $0 < \Delta c < \Delta c^B$, then $\frac{dF_7(\Delta c)}{d\Delta c} =$

$-4(2+\beta)^3(2816+22656\beta+76704\beta^2+142576\beta^3+159280\beta^4+110096\beta^5+46752\beta^6+11682\beta^7+1539\beta^8+81\beta^9)\delta_1^3+8(16+56\beta+62\beta^2+25\beta^3+3\beta^4)^2(96+264\beta+244\beta^2+84\beta^3+9\beta^4)\delta_1^2\Delta c-576(1+\beta)^5(16+40\beta+22\beta^2+3\beta^3)^2\delta_1\Delta c^2+512(1+\beta)^6(8+16\beta+3\beta^2)^2\Delta c^3<0$, $\frac{d^2F_7(\Delta c)}{d\Delta c^2}=8((16+56\beta+62\beta^2+25\beta^3+3\beta^4)^2(96+264\beta+244\beta^2+84\beta^3+9\beta^4)\delta_1^2-144(1+\beta)^5(16+40\beta+22\beta^2+3\beta^3)^2\delta_1\Delta c+192(1+\beta)^6(8+16\beta+3\beta^2)^2\Delta c^2)>0$. $F_7(\Delta c)$ decrease in Δc. $F_7(\Delta c=\Delta c^B)>0$, then $F_7(\Delta c)>0$. Therefore, if $0<\Delta c<\Delta c^B$, then $\theta^e>\theta^g$.

Recall if $\Delta c^B<\Delta c<\Delta c^H$, or $0<\Delta c<\Delta c^B$ and $\theta^c<\theta<\theta^e$, then $p_1^c<p_1^n$ and $p_2^c<p_2^n$. Therefore, in WC strategy zone in region A, Figure 3.2, $p_1^c<p_1^n$ and $p_2^c<p_2^n$.

Proof of Lemma 3.5: Recall $q_2^n=\frac{(2+\beta)\delta_2-\beta\Delta c}{(2+\beta)(2+3\beta)}>0$, so $0<\Delta c<\Delta c^H=\frac{(2+\beta)\delta_1}{2(1+\beta)}$.

From Table 3.2, we obtain $p_1^l-p_1^n=\frac{\beta F_1(\Delta c)}{2(2+\beta)(2+3\beta)(4+8\beta+\beta^2)}$, where $F_1(\Delta c)=4\beta\delta_1+4\beta^2\delta_1+\beta^3\delta_1-(8+24\beta+18\beta^2+2\beta^3)\Delta c$. $-8-24\beta-18\beta^2-2\beta^3<0$, then $F_1(\Delta c)$ decreases in Δc and there is one real root for $F_1(\Delta c)=0$: $\Delta c^Y=\frac{\beta(2+\beta)^2\delta_1}{2(1+\beta)(4+8\beta+\beta^2)}$. $\Delta c^H-\Delta c^Y=\frac{(2+\beta)(2+3\beta)\delta_1}{(1+\beta)(4+8\beta+\beta^2)}>0$, then $0<\Delta c^Y<\Delta c^H$. If $0<\Delta c<\Delta c^Y$, then $p_1^l>p_1^n$; if $\Delta c^Y<\Delta c<\Delta c^H$, then $p_1^l<p_1^n$. From Table 3.2, we obtain $p_2^l-p_2^n=\frac{(2+4\beta+\beta^2)F_1(\Delta c)}{2(1+\beta)(2+\beta)(2+3\beta)(4+8\beta+\beta^2)}$. Similarly, if $0<\Delta c<\Delta c^Y$, then $p_2^l>p_2^n$; if $\Delta c^Y<\Delta c<\Delta c^H$, then $p_2^l<p_2^n$.

So, if $0<\Delta c<\Delta c^Y$, then $p_1^l>p_1^n$ and $p_2^l>p_2^n$; if $\Delta c^Y<\Delta c<\Delta c^H$, then $p_1^l<p_1^n$ and $p_2^l<p_2^n$.

Proof of Proposition 3.2: Recall $q_2^n=\frac{(2+\beta)\delta_2-\beta\Delta c}{(2+\beta)(2+3\beta)}>0$, then $0<\Delta c<\Delta c^H=\frac{(2+\beta)\delta_1}{2(1+\beta)}$. Recall $c_1<w<c_2$, then $\theta^c=\frac{(4+6\beta+\beta^2)^2}{4(1+\beta)(4+\beta)(2+4\beta+\beta^2)}<\theta<1$. Recall $M^l=\frac{\delta_1^2(-16-64\beta-60\beta^2-16\beta^3-\beta^4+(32+96\beta+76\beta^2+16\beta^3+\beta^4)\theta)}{4(1+\beta)(4+8\beta+\beta^2)^2}>0$, then $\theta^j=\frac{16+64\beta+60\beta^2+16\beta^3+\beta^4}{32+96\beta+76\beta^2+16\beta^3+\beta^4}<\theta<1$.

(1) From Table 3.2, (3-1), (3-2), (3-6) and (3-7), we obtain $\pi^l-\pi^n=\frac{G_1(\Delta c)}{4(1+\beta)(2+\beta)^2(2+3\beta)^2(4+8\beta+\beta^2)}$, where $G_1(\Delta c)=(16\beta^2+32\beta^3+24\beta^4+8\beta^5+\beta^6)\delta_1^2+8(2+3\beta+\beta^2)^2(4+8\beta+\beta^2)\delta_1\Delta c-4(1+\beta)^2(16+64\beta+88\beta^2+48\beta^3+5\beta^4)\Delta c^2$.$-4(1+\beta)^2(16+64\beta+88\beta^2+48\beta^3+5\beta^4)<0$, then $G_1(\Delta c)$ is a concave function. Δ_1= $16(1+\beta)^2(2+\beta)^2(2+3\beta)^2(4+8\beta+\beta^2)((2+\beta)^4\delta_1^2>0$ means that there are two real roots exist for $G_1(\Delta c)=0$: $\Delta c^T=\frac{\delta_1(2(1+\beta)(2+\beta)^2(4+8\beta+\beta^2)-\sqrt{(2+\beta)^6(2+3\beta)^2(4+8\beta+\beta^2)})}{2(1+\beta)(4+8\beta+\beta^2)(4+8\beta+5\beta^2)}$ and $\Delta c^S=\frac{\delta_1(2(1+\beta)(2+\beta)^2(4+8\beta+\beta^2)+\sqrt{(2+\beta)^6(2+3\beta)^2(4+8\beta+\beta^2)})}{2(1+\beta)(4+8\beta+\beta^2)(4+8\beta+5\beta^2)}$.

$(2(1+\beta)^2(2+\beta)^2(4+8\beta+\beta^2)\delta_1)^2-(1+\beta)^2(2+\beta)^6(2+3\beta)^2\big(4+\beta(8+\beta)\big)\delta_1^2)=-\beta^2(1+$

$\beta)^2(2+\beta)^4(4+8\beta+\beta^2)(4+8\beta+5\beta^2)\delta_1^2<0$, then $\Delta c^T<0$; $\Delta c^S-\Delta c^H=\frac{-\beta(1+\beta)(2+\beta)(2+3\beta)(4+\beta(8+\beta))\delta_1+\sqrt{(1+\beta)^2(2+\beta)^6(2+3\beta)^2(4+\beta(8+\beta))\delta_1^2}}{2(1+\beta)^2(4+8\beta+\beta^2)(4+8\beta+5\beta^2)}>0$, then $\Delta c^S>\Delta c^H$. So $\Delta c^T<0<\Delta c^H<\Delta c^S$. Therefore, $\pi^l>\pi^n$.

Therefore, if $0<\Delta c<\Delta c^H$ and $\theta^j<\theta<1$, then LC is the better strategy; otherwise, competition is the better strategy.

(2) From Table 3.2, (3-1) and (3-6), $\pi_1^l(q_1^l)-\pi_1^n(q_1^n)=\frac{F_1(\theta)}{4(1+\beta)(2+\beta)^2(2+3\beta)^2(4+8\beta+\beta^2)}$, where $F_1(\theta)=-4(1+\beta)^2(4+8\beta+\beta^2)((2+\beta)\delta_1+\beta\Delta c)^2+\left((8+8\beta+\beta^2)(4+8\beta+3\beta^2)^2\delta_1^2\right)\theta$. $(8+8\beta+\beta^2)(4+8\beta+3\beta^2)^2\delta_1^2>0$, then $F_1(\theta)$ increases in θ. We obtain $\theta^k=\frac{4(1+\beta)^2(4+8\beta+\beta^2)(2\delta_1+\beta\delta_1+\beta\Delta c)^2}{(2+\beta)^2(2+3\beta)^2(8+8\beta+\beta^2)\delta_1^2}$, $1-\theta^k=\frac{F_2(\Delta c)}{(2+\beta)^2(2+3\beta)^2(8+8\beta+\beta^2)\delta_1^2}$, where $F_2(\Delta c)=(2+\beta)^2(16+64\beta+88\beta^2+44\beta^3+5\beta^4)\delta_1^2-8\beta(1+\beta)^2(8+20\beta+10\beta^2+\beta^3)\delta_1\Delta c-4\beta^2(1+\beta)^2(4+8\beta+\beta^2)\Delta c^2$. $-4\beta^2(1+\beta)^2(4+8\beta+\beta^2)<0$, then $F_2(\Delta c)$ is a concave function. $\Delta=16\beta^2(1+\beta)^2(2+\beta)^2(2+3\beta)^2(4+8\beta+\beta^2)(8+8\beta+\beta^2)\delta_1^2>0$ means that there are two real roots for $F_2(\Delta c)$: $\Delta c_1=-\frac{16\beta\delta_1+72\beta^2\delta_1+116\beta^3\delta_1+82\beta^4\delta_1+24\beta^5\delta_1+2\beta^6\delta_1}{2\beta^2(1+\beta)^2(4+8\beta+\beta^2)}-\frac{\sqrt{(32+96\beta+76\beta^2+16\beta^3+\beta^4)(4\beta+12\beta^2+11\beta^3+3\beta^4)^2\delta_1^2}}{2\beta^2(1+\beta)^2(4+8\beta+\beta^2)}<0$ and $\Delta c_2=-\frac{16\beta\delta_1+72\beta^2\delta_1+116\beta^3\delta_1+82\beta^4\delta_1+24\beta^5\delta_1+2\beta^6\delta_1}{2\beta^2(1+\beta)^2(4+8\beta+\beta^2)}+\frac{\sqrt{(32+96\beta+76\beta^2+16\beta^3+\beta^4)(4\beta+12\beta^2+11\beta^3+3\beta^4)^2\delta_1^2}}{2\beta^2(1+\beta)^2(4+8\beta+\beta^2)}>\Delta c^H$. Recall $0<\Delta c<\Delta c^H$, then $\theta^k<1$. $\theta^k-\theta^j=\frac{\beta F_3(\Delta c)}{(2+\beta)^2(2+3\beta)^2(4+8\beta+\beta^2)(8+8\beta+\beta^2)\delta_1^2}$, where $F_3(\Delta c)=-(2+\beta)^3(32+128\beta+168\beta^2+74\beta^3+5\beta^4)\delta_1^2+8(2+\beta)(4+12\beta+9\beta^2+\beta^3)^2\delta_1\Delta c+4\beta(4+12\beta+9\beta^2+\beta^3)^2\Delta c^2$. $4\beta(4+12\beta+9\beta^2+\beta^3)^2>0$, then $F_3(\Delta c)$ is a convex function. $\Delta=16(1+\beta)^2(2+\beta)^2(2+3\beta)^2(4+8\beta+\beta^2)^2(16+64\beta+60\beta^2+16\beta^3+\beta^4)\delta_1^2>0$ means that there are two real roots for $F_3(\Delta c)$: $\Delta c_3=-\frac{(64+416\beta+1056\beta^2+1328\beta^3+868\beta^4+282\beta^5+40\beta^6+2\beta^7)\delta_1}{2\beta(1+\beta)^2(4+8\beta+\beta^2)^2}-\frac{\sqrt{(16+64\beta+60\beta^2+16\beta^3+\beta^4)(16+80\beta+144\beta^2+112\beta^3+35\beta^4+3\beta^5)^2\delta_1^2}}{2\beta(1+\beta)^2(4+8\beta+\beta^2)^2}<0$ and $\Delta c^J=-\frac{(64+416\beta+1056\beta^2+1328\beta^3+868\beta^4+282\beta^5+40\beta^6+2\beta^7)\delta_1}{2\beta(1+\beta)^2(4+8\beta+\beta^2)^2}+\frac{\sqrt{(16+64\beta+60\beta^2+16\beta^3+\beta^4)(16+80\beta+144\beta^2+112\beta^3+35\beta^4+3\beta^5)^2\delta_1^2}}{2\beta(1+\beta)^2(4+8\beta+\beta^2)^2}$. Recall $0<\Delta c<\Delta c^H$, then $\Delta c^J-\Delta c^H<0$ and $\Delta c^J<\Delta c^H$. So, if $0<\Delta c<\Delta c^J$, then $\theta^k<\theta^j<1$; if $\Delta c^J<\Delta c<\Delta c^H$, then $\theta^j<\theta^k<1$.

Therefore, if $0<\Delta c<\Delta c^J$ and $\theta^j<\theta<1$, or $\Delta c^J<\Delta c<\Delta c^H$ and $\theta^k<\theta<1$, then $\pi_1^l(q_1^l)>\pi_1^n(q_1^n)$; if $\Delta c^J<\Delta c<\Delta c^H$ and $\theta^j<\theta<\theta^k$, then $\pi_1^l(q_1^l)<\pi_1^n(q_1^n)$.

From Table 3.2, (3-2) and (3-7), $\pi_2^l(q_2^l)-\pi_2^n(q_2^n)=\frac{F_4(\theta)}{4(1+\beta)(2+\beta)^2(2+3\beta)^2(4+8\beta+\beta^2)}$, where $F_4(\theta)=(2+$

$\beta)^2(16+64\beta+88\beta^2+44\beta^3+5\beta^4)\delta_1^2+16(1+\beta)^3(8+20\beta+10\beta^2+\beta^3)\delta_1\Delta c-16(1+\beta)^4(4+8\beta+\beta^2)\Delta c^2-((8+8\beta+\beta^2)(4+8\beta+3\beta^2)^2\delta_1^2)\theta$. $-(8+8\beta+\beta^2)(4+8\beta+3\beta^2)^2\delta_1^2<0$, then $F_4(\theta)$ decreases in θ . We obtain $\theta^l=\frac{(2+\beta)^2(16+64\beta+88\beta^2+44\beta^3+5\beta^4)\delta_1^2+16(1+\beta)^3(8+20\beta+10\beta^2+\beta^3)\delta_1\Delta c-16(1+\beta)^4(4+8\beta+\beta^2)\Delta c^2}{(2+\beta)^2(2+3\beta)^2(8+8\beta+\beta^2)\delta_1^2}$, $1-\theta^l>0$. $\theta^l-\theta^j=\frac{F_5(\Delta c)}{(2+\beta)^2(2+3\beta)^2(4+8\beta+\beta^2)(8+8\beta+\beta^2)\delta_1^2}$, where $F_5(\Delta c)=4(1+\beta)^2(2+\beta)\delta_1(-\beta(2+\beta)^2(8+\beta(14+\beta))\delta_1+4(1+\beta)(4+\beta(8+\beta))^2\Delta c)-16(1+\beta)^4(4+\beta(8+\beta))^2\Delta c^2$. $-16(1+\beta)^4\left(4+\beta(8+\beta)\right)^2<0$, then $F_5(\Delta c)$ is a concave function. $\Delta=512(1+\beta)^6(2+\beta)^2(2+4\beta+\beta^2)(4+8\beta+\beta^2)^2(4+12\beta+\beta^2)\delta_1^2>0$ means that there are two real roots for $F_5(\Delta c)$: $\Delta c^P=\frac{\beta(2+\beta)^2\delta_1}{2(1+\beta)(4+8\beta+\beta^2)}$ and $\Delta c_6=\frac{(2+\beta)(8+14\beta+\beta^2)\delta_1}{2(1+\beta)(4+8\beta+\beta^2)}$. Recall $0<\Delta c<\Delta c^H$, then $\Delta c_6-\Delta c^H=\frac{(2+\beta)(2+3\beta)\delta_1}{(1+\beta)(4+8\beta+\beta^2)}>0$ and $\Delta c^H-\Delta c^P=\frac{(2+\beta)(2+3\beta)\delta_1}{(1+\beta)(4+8\beta+\beta^2)}>0$, so $0<\Delta c^P<\Delta c^H<\Delta c_6$. If $0<\Delta c<\Delta c^P$, then $\theta^l<\theta^j<1$; if $\Delta c^P<\Delta c<\Delta c^H$, then $\theta^j<\theta^l<1$.

Therefore, if $\Delta c^P<\Delta c<\Delta c^H$ and $\theta^j<\theta<\theta^l$, then $\pi_2^l(q_2^l)>\pi_2^n(q_2^n)$; if $0<\Delta c<\Delta c^P$ and $\theta^j<\theta<1$, or $\Delta c^P<\Delta c<\Delta c^H$ and $\theta^l<\theta<1$, then $\pi_2^l(q_2^l)<\pi_2^n(q_2^n)$.

$\Delta c^J-\Delta c^P=-\frac{(64+416\beta+1072\beta^2+1392\beta^3+956\beta^4+334\beta^5+53\beta^6+3\beta^7)\delta_1}{2\beta(1+\beta)^2(4+8\beta+\beta^2)^2}+\frac{\sqrt{(16+64\beta+60\beta^2+16\beta^3+\beta^4)(16+80\beta+144\beta^2+112\beta^3+35\beta^4+3\beta^5)^2\delta_1^2}}{2\beta(1+\beta)^2(4+8\beta+\beta^2)^2}$. $(16+64\beta+60\beta^2+16\beta^3+\beta^4)(16+80\beta+144\beta^2+112\beta^3+35\beta^4+3\beta^5)^2\delta_1^2-\left((64+416\beta+1072\beta^2+1392\beta^3+956\beta^4+334\beta^5+53\beta^6+3\beta^7)\delta_1\right)^2=4\beta(1+\beta)^2(2+\beta)^4(2+3\beta)^2(4+8\beta+\beta^2)^2\delta_1^2>0$, then $\Delta c^J>\Delta c^P$. Recall $0<\Delta c<\Delta c^H$, then $\theta^l-\theta^k>0$ and $\theta^l>\theta^k$.

Therefore, in LC strategy zone, if $\Delta c^P<\Delta c<\Delta c^H$ and $max\{\theta^j,\theta^k\}<\theta<\theta^l$, then LC is LC strategy in region A, Figure 3.3; otherwise , LC is LC strategy in region B, Figure 3.3.

(3) Therefore, if $0<\Delta c<\Delta c^H$ and $max\{\theta^j,\theta^l\}<\theta<1$, then $\pi_1^l(q_1^l)>\pi_1^n(q_1^n)$ and $\pi_2^l(q_2^l)<\pi_2^n(q_2^n)$; if $\Delta c^J<\Delta c<\Delta c^H$ and $\theta^j<\theta<\theta^k$, then $\pi_1^l(q_1^l)<\pi_1^n(q_1^n)$ and $\pi_2^l(q_2^l)>\pi_2^n(q_2^n)$.

(4) From Lemma 3.5, we obtain $\Delta c^Y=\frac{\beta(2+\beta)^2\delta_1}{2(1+\beta)(4+8\beta+\beta^2)}$ and $\Delta c^P-\Delta c^Y=0$. Therefore, in LC strategy zone in region A, Figure 3.3, $p_1^l<p_1^n$ and $p_2^l<p_2^n$.

Proof of Proposition 3.3: Recall $q_2^n=\frac{(2+\beta)\delta_2-\beta\Delta c}{(2+\beta)(2+3\beta)}>0$, then $0<\Delta c<\Delta c^H=\frac{(2+\beta)\delta_1}{2(1+\beta)}$. Recall $c_1<w<c_2$, then $\theta^c=\frac{(4+6\beta+\beta^2)^2}{4(1+\beta)(4+\beta)(2+4\beta+\beta^2)}<\theta<1$. Recall $M^l=\frac{\delta_1^2(-16-64\beta-60\beta^2-16\beta^3-\beta^4+(32+96\beta+76\beta^2+16\beta^3+\beta^4)\theta)}{4(1+\beta)(4+8\beta+\beta^2)^2}>0$, then $\theta^j = \frac{16+64\beta+60\beta^2+16\beta^3+\beta^4}{32+96\beta+76\beta^2+16\beta^3+\beta^4} < \theta < 1$. $\theta^j - \theta^c > 0$ and $\theta^j > \theta^c$.

(1) $\pi^l - \pi^c = \frac{-\delta_1^2}{4(1+\beta)(4+8\beta+\beta^2)(8+16\beta+3\beta^2)^2}(64\beta^5(-1+\theta) + 3\beta^6(-1+\theta) - 2\beta^4(250-238\theta+\theta^2) - 32\beta^3(52-45\theta+\theta^2) - 128\beta(10-6\theta+\theta^2) - 32(8-4\theta+\theta^2) - 48\beta^2(47-34\theta+3\theta^2) + 2(4+8\beta+\beta^2)(8+16\beta+4\beta^2+4\theta+8\beta\theta+\beta^2\theta)\sqrt{(12+16\beta+3\beta^2)(1-\theta)+\theta^2})$. Similarly, $\pi^l \geq \pi^c$. If $\theta^m = \frac{16+64\beta+68\beta^2+16\beta^3+\beta^4}{32+96\beta+76\beta^2+16\beta^3+\beta^4}$, then $\pi^l = \pi^c$; $\theta^m - \theta^j = \frac{8\beta^2}{(4+8\beta+\beta^2)(8+8\beta+\beta^2)}$. So if $\theta^j < \theta < 1$, then $\pi^l \geq \{\pi^c, \pi^n\}$.

(2) From Proposition 3.1 and 3.2, we obtain $\theta^j - \theta^p = ((2+\beta)^2(2+3\beta)^2(4+8\beta+\beta^2)\delta_1^2(8+8\beta+\beta^2)((2+\beta)^2(64+192\beta+204\beta^2+84\beta^3+9\beta^4)\delta_1^2 - 32(2+3\beta+\beta^2)^2\delta_1\Delta c + 16(1+\beta)^2(4+8\beta+5\beta^2)\Delta c^2))^{-1}(\delta_1^2(\beta^2(2+\beta)^4(2+3\beta)^2(-320-704\beta-336\beta^2+256\beta^3+252\beta^4+52\beta^5+3\beta^6)\delta_1^2 + (8(2+\beta)^4(2+5\beta+3\beta^2)^2(192+1024\beta+1968\beta^2+1536\beta^3+484\beta^4+64\beta^5+3\beta^6)\delta_1)\Delta c - 4(4+12\beta+11\beta^2+3\beta^3)^2(768+5632\beta+17024\beta^2+27008\beta^3+24064\beta^4+11808\beta^5+2944\beta^6+344\beta^7+15\beta^8)\Delta c^2)) + 4(8+8\beta+\beta^2)T_g)$. Similarly, $\theta^j > \theta^p$.

Therefore, (1) if $0 < \Delta c < \Delta c^H$ and $\theta^j < \theta < 1$, then LC is the optimal strategy. (2) If $0 < \Delta c < \Delta c^H$ and $max\{\theta^p, \theta^c\} < \theta < \theta^j$, then WC is the optimal strategy. (3) If $0 < \Delta c < \Delta c^H$ and $0 < \theta < max\{\theta^p, \theta^c\}$, competition is the optimal strategy.

Derivation of Table 3.3:

(1) Cournot competition model: From (3-1), we obtain $\frac{d^2\pi_1^n(q_1)}{d{q_1}^2} = -2(1+\beta) < 0$ and $\pi_1^n(q_1)$ is a concave function of q_1. Similarly, from (3-2) we obtain $\frac{d^2\pi_2^n(q_2)}{d{q_2}^2} = -2(1+\beta) < 0$ and $\pi_2^n(q_2)$ is a concave function of q_2. Set $\Delta\alpha = \alpha_2 - \alpha_1$. $\frac{d\pi_1^n(q_1)}{dq_1} = \frac{d\pi_2^n(q_2)}{dq_2} = 0$ shows that $q_1^n = \frac{(2+\beta)\delta_1+\beta(\Delta c-\Delta\alpha)}{(2+\beta)(2+3\beta)}$ and $q_2^n = \frac{(2+\beta)\delta_2-\beta(\Delta c-\Delta\alpha)}{(2+\beta)(2+3\beta)}$. Recall that $p_i = \alpha_i - q_i - \beta(q_i + q_j)$, we obtain $p_1^n = m + c_1 + (1+\beta)q_1^n$ and $p_2^n = m + c_2 + (1+\beta)q_2^n$.

(2) Recall $q_1^n = \frac{(2+\beta)\delta_1+\beta(\Delta c-\Delta\alpha)}{(2+\beta)(2+3\beta)} > 0$ and $q_2^n = \frac{(2+\beta)\delta_2-\beta(\Delta c-\Delta\alpha)}{(2+\beta)(2+3\beta)} > 0$, we obtain if $-\frac{(2+\beta)\delta_1}{2(1+\beta)} < \Delta\alpha < \frac{(2+\beta)\delta_1}{\beta}$, then $0 < \Delta c < \frac{(2+\beta)\delta_1}{2(1+\beta)} + \Delta\alpha$; if $\Delta\alpha > \frac{(2+\beta)\delta_1}{\beta}$, then $-\frac{(2+\beta)\delta_1}{\beta} + \Delta\alpha < \Delta c < \frac{(2+\beta)\delta_1}{2(1+\beta)} + \Delta\alpha$. Set $\Delta\alpha^s = -\frac{(2+\beta)\delta_1}{2(1+\beta)}$, $\Delta\alpha^q = \frac{(2+\beta)\delta_1}{\beta}$, $\Delta c^H = \frac{(2+\beta)\delta_1}{2(1+\beta)} + \Delta\alpha$ and $\Delta c^V = -\frac{(2+\beta)\delta_1}{\beta} + \Delta\alpha$. Therefore, if $\Delta\alpha^s < \Delta\alpha < \Delta\alpha^q$, then $0 < \Delta c < \Delta c^H$; if $\Delta\alpha > \Delta\alpha^q$, then $\Delta c^V < \Delta c < \Delta c^H$.

WC model: From (3-3), we obtain $\frac{d^2\pi_1^c(q_1)}{dq_1^2} = -2(1+\beta) < 0$ and $\pi_1^c(q_1)$ is a concave function of q_1. $\frac{d\pi_1^c(q_1)}{dq_1} = 0$ shows that $q_1 = \frac{-m+\alpha_1-c_1-\beta q_2}{2(1+\beta)}$. Replace q_1 in (3-4) and we obtain $\frac{d^2\pi_2^c(q_2)}{dq_2^2} = \frac{-2-4\beta-\beta^2}{1+\beta} < 0$ and

$\pi_2^c(q_2)$ is a concave function of q_2. $\frac{d\pi_2^c(q_2)}{dq_2}=0$ shows that $q_2(w)=\frac{2\alpha_2+(c_1-\alpha_1+2\alpha_2)\beta-2w(1+\beta)-m(2+\beta)}{2(2+4\beta+\beta^2)}$. So $q_1(w)=\frac{-m+\alpha_1-c_1-\beta q_2(w)}{2(1+\beta)}$.

Replace $q_1(w)$ and $q_2(w)$ in (3-5), we obtain $ln\pi^{cw}(w)=\theta\, ln\pi_1^c\big(q_1(w)\big)+(1-\theta)ln\pi_2^c\big(q_2(w)\big)$ and $\frac{d\pi^{cw}(w)}{dw}=\pi^{cw}(w)[\theta\frac{1}{\pi_1^c(q_1(w))}\frac{d\pi_1^c(q_1(w))}{dw}+(1-\theta)\frac{1}{\pi_2^c(q_2(w))}\frac{d\pi_2^c(q_2(w))}{dw}]$.

$\theta\frac{1}{\pi_1^c(q_1(w))}\frac{d\pi_1^c(q_1(w))}{dw}+(1-\theta)\frac{1}{\pi_2^c(q_2(w))}\frac{d\pi_2^c(q_2(w))}{dw}=0$ shows that there are three real roots: $w_1=c_1+\frac{1}{2(1+\beta)^2(8+16\beta+3\beta^2)}((1+\beta)\big(3\beta^3+4(2+\theta)+8\beta(3+\theta)+2\beta^2(9+\theta)\big)\delta_1+2((1+\beta)^2\big(\beta^2(1+\theta)+2(2+\theta)+4\beta(2+\theta)\big)\Delta\alpha-((2+6\beta+5\beta^2+\beta^3)^2(2(1+\beta)\delta_1\Delta\alpha(-2+\theta)^2+(1+\beta)^2\Delta\alpha^2(-2+\theta)^2+\delta_1^2(12-16\beta(-1+\theta)-3\beta^2(-1+\theta)-12\theta+\theta^2)))^{\frac{1}{2}})$, $w_2=c_1+\frac{(2+\beta)\delta_1}{2(1+\beta)}+\Delta\alpha=c_1+\Delta c^H>c_2$, $w_3=c_1+\frac{1}{2(1+\beta)^2(8+16\beta+3\beta^2)}((1+\beta)\big(3\beta^3+4(2+\theta)+8\beta(3+\theta)+2\beta^2(9+\theta)\big)\delta_1+2(1+\beta)^2(\beta^2(1+\theta)+2(2+\theta)+4\beta(2+\theta))\Delta\alpha+((2+6\beta+5\beta^2+\beta^3)^2(2(1+\beta)\delta_1\Delta\alpha(-2+\theta)^2+(1+\beta)^2\Delta\alpha^2(-2+\theta)^2+\delta_1^2(12-16\beta(-1+\theta)-3\beta^2(-1+\theta)-12\theta+\theta^2)))^{\frac{1}{2}})$. $w_3-w_2>0$, $w_3>w_2$. Similarly, $w_2>w_1$. So $w_3>w_2>w_1$.

Set $F_1(\theta)=(1+\beta)\big(3\beta^3+4(2+\theta)+8\beta(3+\theta)+2\beta^2(9+\theta)\big)\delta_1+2((1+\beta)^2\big(\beta^2(1+\theta)+2(2+\theta)+4\beta(2+\theta)\big)\Delta\alpha=8(1+\beta)\delta_1+6\beta(1+\beta)(4+\beta)\delta_1+3\beta^2(1+\beta)(4+\beta)\delta_1+8(1+\beta)^2\Delta\alpha+16\beta(1+\beta)^2\Delta\alpha+2\beta^2(1+\beta)^2\Delta\alpha+(2(1+\beta)(2+\beta(4+\beta))(\delta_1+\Delta\alpha+\beta\Delta\alpha))\theta$. If $\Delta\alpha>\Delta\alpha^t=-\frac{\delta_1}{1+\beta}$, then $2(1+\beta)\big(2+\beta(4+\beta)\big)(\delta_1+\Delta\alpha+\beta\Delta\alpha)>0$ and $F_1(\theta)$ increases in θ; if $\Delta\alpha^s<\Delta\alpha<\Delta\alpha^t$, then $2(1+\beta)\big(2+\beta(4+\beta)\big)(\delta_1+\Delta\alpha+\beta\Delta\alpha)<0$ and $F_1(\theta)$ decreases in θ. There is one root for $F_1(\theta)$: $\theta^u=-\frac{(8+3\beta(2+\beta)(4+\beta))\delta_1+2(1+\beta)(4+\beta(8+\beta))\Delta\alpha}{2(2+\beta(4+\beta))(\delta_1+\Delta\alpha+\beta\Delta\alpha)}$. If $\Delta\alpha>\Delta\alpha^t$, then $\theta^u<0$, recall $0<\theta<1$, so $F_1(\theta)>0$; if $\Delta\alpha^s<\Delta\alpha<\Delta\alpha^t$, then $\theta^u>1$, recall $0<\theta<1$, so $F_1(\theta)>0$. $((1+\beta)\big(3\beta^3+4(2+\theta)+8\beta(3+\theta)+2\beta^2(9+\theta)\big)\delta_1+2\big((1+\beta)^2\big(\beta^2(1+\theta)+2(2+\theta)+4\beta(2+\theta)\big)\Delta\alpha\big)^2-(2+6\beta+5\beta^2+\beta^3)^2\big(2(1+\beta)\delta_1\Delta\alpha(-2+\theta)^2+(1+\beta)^2\Delta\alpha^2(-2+\theta)^2+\delta_1^2(12-16\beta(-1+\theta)-3\beta^2(-1+\theta)-12\theta+\theta^2)\big)=(1+\beta)^2(8+16\beta+3\beta^2)(-((4+6\beta+\beta^2)\delta_1-2\beta(1+\beta)\Delta\alpha)^2+(4(2+6\beta+5\beta^2+\beta^3)((4+\beta)\delta_1^2+(4+\beta)\delta_1\Delta\alpha+2(1+\beta)\Delta\alpha^2))\theta$. Set $F_2(\theta)=(1+\beta)^2(8+16\beta+3\beta^2)(-((4+6\beta+\beta^2)\delta_1-2\beta(1+\beta)\Delta\alpha)^2+(4(2+6\beta+5\beta^2+\beta^3)((4+\beta)\delta_1^2+(4+\beta)\delta_1\Delta\alpha+2(1+\beta)\Delta\alpha^2))\theta$. $4(2+6\beta+5\beta^2+\beta^3)\big((4+\beta)\delta_1^2+(4+\beta)\delta_1\Delta\alpha+2(1+\beta)\Delta\alpha^2\big)>0$ means that $F_2(\theta)$ increases in θ. There is one root for $F_2(\theta)$: $\theta^v=\frac{(4\delta_1+6\beta\delta_1+\beta^2\delta_1-2\beta\Delta\alpha-2\beta^2\Delta\alpha)^2}{4(1+\beta)(2+4\beta+\beta^2)(4\delta_1^2+\beta\delta_1^2+4\delta_1\Delta\alpha+\beta\delta_1\Delta\alpha+2\Delta\alpha^2+2\beta\Delta\alpha^2)}>0$. $1-\theta^v=$

$\frac{((2+\beta)\delta_1+2(1+\beta)\Delta\alpha)((8+24\beta+18\beta^2+3\beta^3)\delta_1+2(4+12\beta+9\beta^2+\beta^3)\Delta\alpha)}{4(1+\beta)(2+4\beta+\beta^2)((4+\beta)\delta_1^2+(4+\beta)\delta_1\Delta\alpha+2(1+\beta)\Delta\alpha^2)}$. Recall $\Delta\alpha > \Delta\alpha^s$, then $\theta^v < 1$. So $0 < \theta^v < 1$. If $\theta^v < \theta < 1$, then $w_1 > c_1$; if $0 < \theta < \theta^v$, $w_1 < c_1$. Recall $c_1 < w < c_2$, so if $\theta^v < \theta < 1$, then $w^c = w_1 = c_1 + \frac{1}{2(1+\beta)^2(8+16\beta+3\beta^2)}((1+\beta)\big(3\beta^3 + 4(2+\theta) + 8\beta(3+\theta) + 2\beta^2(9+\theta)\big)\delta_1 + 2((1+\beta)^2\big(\beta^2(1+\theta) + 2(2+\theta) + 4\beta(2+\theta)\big)\Delta\alpha - (((2+6\beta+5\beta^2+\beta^3)^2(2(1+\beta)\delta_1\Delta\alpha(-2+\theta)^2 + (1+\beta)^2\Delta\alpha^2(-2+\theta)^2 + \delta_1^2(12-16\beta(-1+\theta) - 3\beta^2(-1+\theta) - 12\theta + \theta^2)))^{\frac{1}{2}})$.

Replace w^c in q_1 and q_2, we obtain $q_1^c = \frac{1}{2(1+\beta)^2(2+4\beta+\beta^2)(8+16\beta+3\beta^2)}(((2+6\beta+5\beta^2+\beta^3)(\beta(1+\beta)\Delta\alpha(-2+\theta) + \delta_1(8+3\beta^2+\beta(14+\theta))) - \beta((2+6\beta+5\beta^2+\beta^3)^2(2(1+\beta)\delta_1\Delta\alpha(-2+\theta)^2 + (1+\beta)^2\Delta\alpha^2(-2+\theta)^2 + \delta_1^2(12-16\beta(-1+\theta) - 3\beta^2(-1+\theta) - 12\theta + \theta^2)))^{\frac{1}{2}})$ and $q_2^c = \frac{1}{(1+\beta)(2+4\beta+\beta^2)(8+16\beta+3\beta^2)}((2+6\beta+5\beta^2+\beta^3)(\delta_1 + \Delta\alpha + \beta\Delta\alpha)(2-\theta) + ((2+6\beta+5\beta^2+\beta^3)^2(2(1+\beta)\delta_1\Delta\alpha(-2+\theta)^2 + (1+\beta)^2\Delta\alpha^2(-2+\theta)^2 + \delta_1^2(12-16\beta(-1+\theta) - 3\beta^2(-1+\theta) - 12\theta + \theta^2)))^{\frac{1}{2}})$.

Recall that $p_i = \alpha_i - q_i - \beta\big(q_i + q_j\big)$ and we obtain $p_1^c = m + c_1 + (1+\beta)q_1^c$ and $p_2^c = m + c_1 + (\frac{1}{2(1+\beta)^2(8+16\beta+3\beta^2)}((1+\beta)(\Delta\alpha(12+36\beta+28\beta^2+4\beta^3+2\theta+6\beta\theta+5\beta^2\theta+\beta^3\theta) + \delta_1(3\beta^3+2(6+\theta)+4\beta(8+\theta)+\beta^2(20+\theta))) - ((2+6\beta+5\beta^2+\beta^3)^2(2(1+\beta)\delta_1\Delta\alpha(-2+\theta)^2 + (1+\beta)^2\Delta\alpha^2(-2+\theta)^2 + \delta_1^2(12-16\beta(-1+\theta) - 3\beta^2(-1+\theta) - 12\theta + \theta^2)))^{\frac{1}{2}})$.

(3) LC model: From (3-6), (3-7) we obtain $\frac{d^2\pi_1^l(q_1)}{dq_1^2} = \frac{d^2\pi_2^l(q_2)}{dq_2^2} = -2(1+\beta) < 0$, then $\pi_1^l(q_1)$ is a concave function of q_1 and $\pi_2^l(q_2)$ is a concave function of q_2. $\frac{d\pi_1^l(q_1)}{dq_1} = \frac{d\pi_2^l(q_2)}{dq_2} = 0$ shows that a $q_1(r) = \frac{2\delta_1+\beta(r+\delta_1-\Delta\alpha)}{(2+\beta)(2+3\beta)}$ and $q_2(r) = \frac{-2r(1+\beta)+(2+\beta)\delta_1+2(1+\beta)\Delta\alpha}{4+8\beta+3\beta^2}$.

Replace $q_1(r)$ and $q_2(r)$ in (3-8), we obtain $ln\pi^{cl}(r,M) = \theta\, ln\pi_1^l\big(q_1(r, M)\big) + (1-\theta)ln\pi_2^l(q_2(r,M))$, then $\frac{\partial\pi^{cl}(r,M)}{\partial r} = \pi^{cl}(r,M)[\theta\frac{1}{\pi_1^l(q_1(r, M))}\frac{\partial\pi_1^l(q_1(r, M))}{\partial r} + (1-\theta)\frac{1}{\pi_2^l(q_2(r,M))}\frac{\partial\pi_2^l(q_2(r,M))}{\partial r}]$ and $\frac{\partial\pi^{cl}(r,M)}{\partial M} = \pi^{cl}(r,M)[\theta\frac{1}{\pi_1^l(q_1(r, M))}\frac{\partial\pi_1^l(q_1(r, M))}{\partial M} + (1-\theta)\frac{1}{\pi_2^l(q_2(r,M))}\frac{\partial\pi_2^l(q_2(r,M))}{\partial M}]$. $\frac{\partial\pi^{cl}(r,M)}{\partial r} = \frac{\partial\pi^{cl}(r,M)}{\partial M} = 0$ shows that there are three real roots: $r_1 = \frac{\beta((2+\beta)^2\delta_1-4\beta(1+\beta)\Delta\alpha)-(2+\beta)(2+3\beta)\sqrt{(8+8\beta+\beta^2)\delta_1^2+8(1+\beta)\delta_1\Delta\alpha+4(1+\beta)^2\Delta\alpha^2}}{2(1+\beta)(4+8\beta+\beta^2)} < 0$, $M_1 = \frac{(1+\beta)((12+8\beta+\beta^2)\delta_1^2+16(1+\beta)\delta_1\Delta\alpha+8(1+\beta)^2\Delta\alpha^2+4(\delta_1+\Delta\alpha+\beta\Delta\alpha)\sqrt{(8+8\beta+\beta^2)\delta_1^2+8(1+\beta)\delta_1\Delta\alpha+4(1+\beta)^2\Delta\alpha^2})}{(4+8\beta+\beta^2)^2}$; $r_2 = \frac{\beta((2+\beta)^2\delta_1-4\beta(1+\beta)\Delta\alpha)}{2(1+\beta)(4+8\beta+\beta^2)}$, $M_2 = \frac{8\delta_1\Delta\alpha(4\theta+8\beta\theta+\beta^2(3+\theta))}{4(4+8\beta+\beta^2)^2} + \frac{4(1+\beta)\Delta\alpha^2(4\theta+8\beta\theta+\beta^2(3+\theta))}{4(4+8\beta+\beta^2)^2} +$

$\frac{\delta_1^2(16(1+\beta)^2-(32+96\beta+76\beta^2+16\beta^3+\beta^4)(1-\theta))}{4(1+\beta)(4+8\beta+\beta^2)^2}$; $r_3 = \frac{\beta((2+\beta)^2\delta_1-4\beta(1+\beta)\Delta\alpha)+(2+\beta)(2+3\beta)\sqrt{(8+8\beta+\beta^2)\delta_1^2+8(1+\beta)\delta_1\Delta\alpha+4(1+\beta)^2\Delta\alpha^2}}{2(1+\beta)(4+8\beta+\beta^2)} > \Delta c^H$, $M_3 = \frac{(1+\beta)((12+8\beta+\beta^2)\delta_1^2+16(1+\beta)\delta_1\Delta\alpha+8(1+\beta)^2\Delta\alpha^2-4(\delta_1+\Delta\alpha+\beta\Delta\alpha)\sqrt{(8+8\beta+\beta^2)\delta_1^2+8(1+\beta)\delta_1\Delta\alpha+4(1+\beta)^2\Delta\alpha^2})}{(4+8\beta+\beta^2)^2}$. Recall if $\Delta\alpha^s < \Delta\alpha < \Delta\alpha^q$, then $0 < \Delta c < \Delta c^H$; if $\Delta\alpha > \Delta\alpha^q$, then $\Delta c^V < \Delta c < \Delta c^H$, and $0 < r < \Delta c^H$, then $r^l = r_2 = \frac{\beta((2+\beta)^2\delta_1-4\beta(1+\beta)\Delta\alpha)}{2(1+\beta)(4+8\beta+\beta^2)}$, $M^l = M_2 = \frac{8\delta_1\Delta\alpha(4\theta+8\beta\theta+\beta^2(3+\theta))}{4(4+8\beta+\beta^2)^2} + \frac{4(1+\beta)\Delta\alpha^2(4\theta+8\beta\theta+\beta^2(3+\theta))}{4(4+8\beta+\beta^2)^2} + \frac{\delta_1^2(16(1+\beta)^2-(32+96\beta+76\beta^2+16\beta^3+\beta^4)(1-\theta))}{4(1+\beta)(4+8\beta+\beta^2)^2}$.

Replace r^l in $q_1(r)$ and $q_2(r)$, we obtain $q_1^l = \frac{(4+6\beta+\beta^2)\delta_1-2\beta(1+\beta)\Delta\alpha}{2(1+\beta)(4+8\beta+\beta^2)}$ and $q_2^l = \frac{2(\delta_1+\Delta\alpha+\beta\Delta\alpha)}{4+8\beta+\beta^2}$. Recall that $p_i = \alpha_i - q_i - \beta(q_i + q_j)$ and we obtain $p_1^l = m + c_1 + (1+\beta)q_1^l$ and $p_2^l = m + c_1 + \frac{\delta_1(4+12\beta+8\beta^2+\beta^3)+4(1+3\beta+2\beta^2)\Delta\alpha}{2(1+\beta)(4+8\beta+\beta^2)}$.

Chapter 4

Derivation of Table 4.2

(1) Competition model

Replacing $q_1e_1 - E_1 = K$ in $\pi_1^n(q_1)$, we obtain $\pi_1^n(q_1) = [\alpha - \beta(q_1+q_2) - c]q_1 - \lambda_0(q_1e_1 - K)$, $\frac{d\pi_1^n(q_1)}{dq_1} = -c + \alpha - 2\beta q_1 - \beta q_2 - \lambda_0 e_1$ and $\frac{d^2\pi_1^n(q_1)}{dq_1^2} = -2\beta < 0$, so $\pi_1^n(q_1)$ is a concave function of q_1. Similarly, replacing $q_2e_2 - E_2 = K$ in $\pi_2^n(q_2)$, we obtain $\pi_2^n(q_2) = [\alpha - \beta(q_1+q_2) - c]q_2 - \lambda_0(q_2e_2 - K)$, $\frac{d\pi_2^n(q_2)}{dq_2} = -c + \alpha - \beta q_1 - 2\beta q_2 - \lambda_0 e_2$, and $\frac{d^2\pi_2^n(q_2)}{dq_2^2} = -2\beta < 0$, so $\pi_2^n(q_2)$ is a concave function of q_2. Let $\frac{d\pi_1^n(q_1)}{dq_1} = \frac{d\pi_2^n(q_2)}{dq_2} = 0$ and we obtain $q_1^n = \frac{\delta_1+\Delta e\lambda_0}{3\beta}$ and $q_2^n = \frac{\delta_1-2\Delta e\lambda_0}{3\beta}$. Recall that $p_i = \alpha - \beta(q_1+q_2)$, we obtain $p_1^n = p_2^n = c + \lambda_0 e_1 + \beta q_1^n$.

(2) RL coopetition model

Replacing $q_1e_1 - E_1 = K$ in $\pi_1^r(q_1)$, we obtain $\pi_1^r(q_1) = [\alpha - \beta(q_1+q_2) - c]q_1 + \lambda q_2 - \lambda_0(q_1e_1 - K)$, $\frac{d\pi_1^r(q_1)}{dq_1} = -c + \alpha - 2\beta q_1 - \beta q_2 - \lambda_0 e_1$ and $\frac{d^2\pi_1^r(q_1)}{dq_1^2} = -2\beta < 0$, so $\pi_1^r(q_1)$ is a concave function of q_1. Replacing $q_2e_1 - E_2 = K$ in $\pi_2^r(q_2)$, we obtain $\pi_2^r(q_2) = [\alpha - \beta(q_1+q_2) - c]q_2 - \lambda q_2 - \lambda_0(q_2e_1 - K)$, $\frac{d\pi_2^r(q_2)}{dq_2} = -c + \alpha - \lambda - \beta q_1 - 2\beta q_2 - \lambda_0 e_1$ and $\frac{d^2\pi_2^r(q_2)}{dq_2^2} = -2\beta < 0$, so $\pi_2^r(q_2)$ is a concave function of q_2. Let $\frac{d\pi_1^r(q_1)}{dq_1} = \frac{d\pi_2^r(q_2)}{dq_2} = 0$, then we obtain $q_1(\lambda) = \frac{\delta_1+\lambda}{3\beta}$ and $q_2(\lambda) = \frac{\delta_1-2\lambda}{3\beta}$.

Replacing $q_1(\lambda)$ and $q_2(\lambda)$ in (4-3) and (4-4), we obtain $\pi_1^r(q_1(\lambda))=\frac{-5\lambda^2+5\lambda\delta_1+\delta_1^2+9K\beta\lambda_0}{9\beta}$ and $\pi_2^r(q_2(\lambda))=\frac{4\lambda^2-4\lambda\delta_1+\delta_1^2+9K\beta\lambda_0}{9\beta}$. From (4-5), we get $ln\pi^r(\lambda)=\theta\, ln\pi_1^r(q_1(\lambda))+(1-\theta)ln\pi_2^r(q_2(\lambda))$ and $\frac{1}{\pi^r(\lambda)}\frac{d\pi^r(\lambda)}{d\lambda}=\theta\frac{1}{\pi_1^r(q_1(\lambda))}\frac{d\pi_1^r(q_1(\lambda))}{d\lambda}+(1-\theta)\frac{1}{\pi_2^r(q_2(\lambda))}\frac{d\pi_2^r(q_2(\lambda))}{d\lambda}$, then $\frac{d\pi^r(\lambda)}{d\lambda}=\pi^r(\lambda)[\theta\frac{1}{\pi_1^r(q_1(\lambda))}\frac{d\pi_1^r(q_1(\lambda))}{d\lambda}+(1-\theta)\frac{1}{\pi_2^r(q_2(\lambda))}\frac{d\pi_2^r(q_2(\lambda))}{d\lambda}]$. From $\frac{d\pi^r(\lambda)}{d\lambda}=0$ and $\pi^r(\lambda)>0$, we get $\theta\frac{1}{\pi_1^r(q_1(\lambda))}\frac{d\pi_1^r(q_1(\lambda))}{d\lambda}+(1-\theta)\frac{1}{\pi_2^r(q_2(\lambda))}\frac{d\pi_2^r(q_2(\lambda))}{d\lambda}=0$. It is equivalent to solve $\Phi(\lambda)=0$, where $\Phi(\lambda)=\theta\pi_2^r(q_2(\lambda))\frac{d\pi_1^r(q_1(\lambda))}{d\lambda}+(1-\theta)\pi_1^r(q_1(\lambda))\frac{d\pi_2^r(q_2(\lambda))}{d\lambda}$. After replacing $\pi_1^r(q_1(\lambda))$ and $\pi_2^r(q_2(\lambda))$ into $\Phi(\lambda)$, we get $\Phi(\lambda)=-\frac{1}{81\beta^2}(2\lambda-\delta_1)(20\lambda^2-20\lambda\delta_1-4\delta_1^2+9\theta\delta_1^2-36K\beta\lambda_0+81K\beta\theta\lambda_0)$. Let $\phi(\lambda)=20\lambda^2-20\lambda\delta_1-4\delta_1^2+9\theta\delta_1^2-36K\beta\lambda_0+81K\beta\theta\lambda_0$ and the discriminant of $\phi(\lambda)$ is $\Delta=720[(1-\theta)\delta_1^2+(4-9\theta)K\beta\lambda_0]$. Thus, if $0\le\theta<\theta_0$, then $\Delta>0$; if $\theta_0\le\theta\le 1$, then $\Delta\le 0$, where $\theta_0=\frac{\delta_1^2+4K\beta\lambda_0}{\delta_1^2+9K\beta\lambda_0}$. Next, we will discuss the optimal value in two cases.

Case 1: If $0\le\theta<\theta_0$, then there are three real roots for $\Phi(\lambda)=0$: $\lambda_1=\lambda_2-\frac{3\sqrt{5}\sqrt{(1-\theta)\delta_1^2+(4-9\theta)K\beta\lambda_0}}{10}$, $\lambda_2=\frac{1}{2}\delta_1$ and $\lambda_3=\lambda_2+\frac{3\sqrt{5}\sqrt{(1-\theta)\delta_1^2+(4-9\theta)K\beta\lambda_0}}{10}$. Obviously, at this time, we can get $\lambda_1<\lambda_2<\lambda_3$. Recall $q_2(\lambda)=\frac{\delta_1-2\lambda}{3\beta}\ge 0$ and we get $0<\lambda\le\frac{1}{2}\delta_1$.

According to the second derivative of $\pi^r(\lambda)$, that is $\frac{d^2(\pi^r(\lambda))}{\mathrm{d}\lambda^2}|_{\lambda=\lambda_1}=-\frac{80[(1-\theta)\delta_1^2+(4-9\theta)K\beta\lambda_0]}{9\theta(1-\theta)(\delta_1^2+9K\beta\lambda_0)^2}\pi^r(\lambda_1)<0$ and $\frac{d^2(\pi^r(\lambda))}{\mathrm{d}\lambda^2}|_{\lambda=\lambda_2}=\frac{80[(1-\theta)\delta_1^2+(4-9\theta)K\beta\lambda_0]}{9K\beta\lambda_0(\delta_1^2+4K\beta\lambda_0)}\pi^r(\lambda_2)>0$, so if $\lambda_1>0$, then $\lambda^r=\lambda_1$. Now, we aim to compare λ_1 and 0. Since $(5\delta_1)^2-\left[3\sqrt{5}\sqrt{(1-\theta)\delta_1^2+(4-9\theta)K\beta\lambda_0}\right]^2=5(9\theta-4)(\delta_1^2+9K\beta\lambda_0)$ and $\theta_0-\frac{4}{9}=\frac{5\delta_1^2}{9(\delta_1^2+9K\beta\lambda_0)}>0$, we get if $0\le\theta\le\frac{4}{9}$, then $\lambda_1\le 0$, so we consider there is no optimal value for RL coopetition model under this condition; if $\frac{4}{9}<\theta<\theta_0$, then $\lambda_1>0$, so $\lambda^r=\lambda_1$. Replacing $\lambda^r=\lambda_1$ in $q_1(\lambda)$ and $q_2(\lambda)$, we get $q_1^r=\frac{5\delta_1-\sqrt{5}\sqrt{(1-\theta)\delta_1^2+(4-9\theta)K\beta\lambda_0}}{10\beta}=\frac{\delta_1+\lambda^r}{3\beta}$ and $q_2^r=\frac{\sqrt{(1-\theta)\delta_1^2+(4-9\theta)K\beta\lambda_0}}{\sqrt{5}\beta}=\frac{\delta_1-2\lambda^r}{3\beta}$. Recall $p_i=\alpha-\beta(q_1+q_2)$, we obtain $p_1^r=p_2^r=c+\lambda_0e_1+\beta q_1^r$.

Case 2: If $\theta_0\le\theta\le 1$, then there is one real root $\lambda_2=\frac{1}{2}\delta_1$ for $\Phi(\lambda)=0$. At this time, $\frac{d^2(\pi^r(\lambda))}{\mathrm{d}\lambda^2}|_{\lambda=\lambda_2}=\frac{80[(1-\theta)\delta_1^2+(4-9\theta)K\beta\lambda_0]}{9K\beta\lambda_0(\delta_1^2+4K\beta\lambda_0)}\pi^r(\lambda_2)\le 0$. Thus, the optimal royalty rate for RL coopetition model is $\lambda^r=\lambda_2$. Replacing $\lambda^r=\lambda_2$ in $q_1(\lambda)$ and $q_2(\lambda)$, we get $q_1^r=\frac{\delta_1}{2\beta}$ and $q_2^r=0$. Recall $p_i=\alpha-\beta(q_1+q_2)$, we obtain $p_1^r=p_2^r=c+\lambda_0e_1+\beta q_1^r$.

In summary, we get $\lambda^r = \begin{cases} \lambda_1^r, & \frac{4}{9} < \theta < \theta_0 \\ \lambda_2^r, & \theta_0 \le \theta \le 1 \end{cases}$, $(q_1^r, q_2^r) = \begin{cases} \left(\frac{\delta_1+\lambda^r}{3\beta}, \frac{\delta_1-2\lambda^r}{3\beta}\right), & \frac{4}{9} < \theta < \theta_0 \\ \left(\frac{\delta_1}{2\beta}, 0\right), & \theta_0 \le \theta \le 1 \end{cases}$ and $p_1^r = p_2^r = c + \lambda_0 e_1 + \beta q_1^r$ for all $\frac{4}{9} < \theta \le 1$.

(3) FL coopetition model

Replacing $q_1 e_1 - E_1 = K$ in $\pi_1^f(q_1)$, we obtain $\pi_1^f(q_1) = [\alpha - \beta(q_1+q_2) - c]q_1 + M - \lambda_0(q_1 e_1 - K)$, $\frac{d\pi_1^f(q_1)}{dq_1} = -c + \alpha - 2\beta q_1 - \beta q_2 - \lambda_0 e_1$ and $\frac{d^2\pi_1^f(q_1)}{dq_1^2} = -2\beta < 0$, so $\pi_1^f(q_1)$ is a concave function of q_1. Replacing $q_2 e_1 - E_2 = K$ in $\pi_2^f(q_2)$, we obtain $\pi_2^f(q_2) = [\alpha - \beta(q_1+q_2) - c]q_2 - M - \lambda_0(q_2 e_1 - K)$, $\frac{d\pi_2^f(q_2)}{dq_2} = -c + \alpha - \beta q_1 - 2\beta q_2 - \lambda_0 e_1$ and $\frac{d^2\pi_2^f(q_2)}{dq_2^2} = -2\beta < 0$, so $\pi_2^f(q_2)$ is a concave function of q_2. Let $\frac{d\pi_1^f(q_1)}{dq_1} = \frac{d\pi_2^f(q_2)}{dq_2} = 0$, then we obtain $q_1^f = q_2^f = \frac{\delta_1}{3\beta}$, so we get $p^f = c + \lambda_0 e_1 + \beta q_1^f$.

Replacing q_1^f and q_2^f in (4-6) and (4-7), we obtain $\pi_1^f(M) = \frac{9M\beta + \delta_1^2 + 9K\beta\lambda_0}{9\beta}$ and $\pi_2^f(M) = \frac{\delta_1^2 + 9K\beta\lambda_0 - 9M\beta}{9\beta}$. From (4-8), we get $ln\pi^f(M) = \theta\, ln\pi_1^f(M) + (1-\theta) ln\pi_2^f(M)$ and $\frac{1}{\pi^f(M)}\frac{d\pi^f(M)}{dM} = \theta\frac{1}{\pi_1^f(M)}\frac{d\pi_1^f(M)}{dM} + (1-\theta)\frac{1}{\pi_2^f(M)}\frac{d\pi_2^f(M)}{dM}$, then $\frac{d\pi^f(M)}{dM} = \pi^f(M)[\theta\frac{1}{\pi_1^f(M)}\frac{d\pi_1^f(M)}{dM} + (1-\theta)\frac{1}{\pi_2^f(M)}\frac{d\pi_2^f(M)}{dM}]$. From $\frac{d\pi^f(M)}{dM} = 0$ and $\pi^f(M) > 0$, we get $\theta\frac{1}{\pi_1^f(M)}\frac{d\pi_1^f(M)}{dM} + (1-\theta)\frac{1}{\pi_2^f(M)}\frac{d\pi_2^f(M)}{dM} = 0$ and derive a root $M^f = \frac{(\delta_1^2 + 9K\beta\lambda_0)(2\theta-1)}{9\beta}$. Since $\frac{d^2(\pi^f(M))}{\mathrm{d}M^2}|_{M=M^f} = \frac{81\beta^2}{4(-1+\theta)\theta(\delta_1^2+9K\beta\lambda_0)^2}\pi^f(M^f) < 0$, so M^f is the optimal fixed fee for FL coopetition model.

(4) ML coopetition model

Replacing $q_1 e_1 - E_1 = K$ in $\pi_1^l(q_1)$, we obtain $\pi_1^l(q_1) = [\alpha - \beta(q_1+q_2) - c]q_1 + \lambda q_2 + M - \lambda_0(q_1 e_1 - K)$, $\frac{d\pi_1^l(q_1)}{dq_1} = -c + \alpha - 2\beta q_1 - \beta q_2 - \lambda_0 e_1$ and $\frac{d^2\pi_1^l(q_1)}{dq_1^2} = -2\beta < 0$, so $\pi_1^l(q_1)$ is a concave function of q_1. Similarly, replacing $q_2 e_1 - E_2 = K$ in $\pi_2^l(q_2)$, we obtain $\pi_2^l(q_2) = [\alpha - \beta(q_1+q_2) - c]q_2 - \lambda q_2 - M - \lambda_0(q_2 e_1 - K)$, $\frac{d\pi_2^l(q_2)}{dq_2} = -c + \alpha - \lambda - \beta q_1 - 2\beta q_2 - \lambda_0 e_1$, and $\frac{d^2\pi_2^l(q_2)}{dq_2^2} = -2\beta < 0$, so $\pi_2^l(q_2)$ is a concave function of q_2. Let $\frac{d\pi_1^l(q_1)}{dq_1} = \frac{d\pi_2^l(q_2)}{dq_2} = 0$ and we obtain $q_1(\lambda) = \frac{\delta_1+\lambda}{3\beta}$ and $q_2(\lambda) = \frac{\delta_1 - 2\lambda}{3\beta}$.

Replacing $q_1(\lambda)$ and $q_2(\lambda)$ in (4-9) and (4-10), we obtain $\pi_1^l(\lambda, M) = \frac{-5\lambda^2 + 5\lambda\delta_1 + \delta_1^2 + 9K\beta\lambda_0 + 9M\beta}{9\beta}$ and $\pi_2^l(\lambda, M) = \frac{4\lambda^2 - 4\lambda\delta_1 + \delta_1^2 + 9K\beta\lambda_0 - 9M\beta}{9\beta}$. From (4-11), we get $ln\pi^l(\lambda, M) = \theta\, ln\pi_1^l(\lambda, M) + (1-\theta) ln\pi_2^l(\lambda, M)$. According to the first partial derivative of $\pi^l(\lambda, M)$, that is $\frac{\partial\pi^l(\lambda,M)}{\partial\lambda} = \pi^l(\lambda, M)[\theta\frac{1}{\pi_1^l(\lambda,M)}\frac{\partial\pi_1^l(\lambda,M)}{\partial\lambda} + (1-\theta)\frac{1}{\pi_2^l(\lambda,M)}\frac{\partial\pi_2^l(\lambda,M)}{\partial\lambda}]$ and $\frac{\partial\pi^l(\lambda,M)}{\partial M} = \pi^l(\lambda, M)[\theta\frac{1}{\pi_1^l(\lambda,M)}\frac{\partial\pi_1^l(\lambda,M)}{\partial M} + (1-\theta)\frac{1}{\pi_2^l(\lambda,M)}\frac{\partial\pi_2^l(\lambda,M)}{\partial M}]$, we get three roots by $\frac{\partial\pi^l(\lambda,M)}{\partial\lambda} = \frac{\partial\pi^l(\lambda,M)}{\partial M} = 0$. $\lambda_1 = \frac{1}{2}(\delta_1 - 3\sqrt{\delta_1^2 + 8K\beta\lambda_0})$ and $M_1 = \frac{\delta_1^2 + 9K\beta\lambda_0}{\beta}$; $\lambda_2 = \frac{\delta_1}{2}$ and $M_2 =$

$\frac{-\delta_1^2-4K\beta\lambda_0+\theta(8K\beta\lambda_0+\delta_1^2)}{4\beta}$; $\lambda_3=\frac{1}{2}(\delta_1+3\sqrt{\delta_1^2+8K\beta\lambda_0})$ and $M_3=\frac{\delta_1^2+9K\beta\lambda_0}{\beta}$. Recall $q_2(\lambda)=\frac{\delta_1-2\lambda}{3\beta}\geq 0$, we get $0<\lambda\leq\frac{1}{2}\delta_1$. Since $\lambda_1<0$ and $\lambda_3>\frac{1}{2}\delta_1$, then we omit (λ_1,M_1) and (λ_3,M_3). For another, $M_2=\frac{-\delta_1^2-4K\beta\lambda_0+\theta(8K\beta\lambda_0+\delta_1^2)}{4\beta}>0$ indicates $\theta_1<\theta\leq 1$, where $\theta_1=\frac{\delta_1^2+4K\beta\lambda_0}{\delta_1^2+8K\beta\lambda_0}$.

$\frac{\partial^2(\pi^l(\lambda,M))}{\partial\lambda^2}|_{\lambda=\lambda_2,M=M_2}=\frac{-8}{9(\delta_1^2+8K\beta\lambda_0)}\pi^l(\lambda,M)<0$, $\frac{\partial^2(\pi^l(\lambda,M))}{\partial M^2}|_{\lambda=\lambda_2,M=M_2}=\frac{-16\beta^2}{(1-\theta)\theta(\delta_1^2+8K\beta\lambda_0)^2}\pi^l(\lambda,M)<0$ and $\frac{\partial^2(\pi^l(\lambda,M))}{\partial\lambda\partial M}|_{\lambda=\lambda_2,M=M_2}=0$. Thus, if $\theta_1<\theta\leq 1$, then (λ_2,M_2) is the optimal value for ML coopetition model. Replacing $\lambda^l=\lambda_2$ and $M^l=M_2$ in $q_1(\lambda)$ and $q_2(\lambda)$, we get $q_1^l=\frac{\delta_1}{2\beta}$ and $q_2^l=0$. Recall $p_i=\alpha-\beta(q_1+q_2)$, we obtain $p_1^l=p_2^l=c+\lambda_0e_1+\beta q_1^l$.

Proof of Proposition 4.1

(1) From Table 4.2, for competition model, we have to satisfy $p^n>c+\lambda_0e_1$, $p^n-(c+\lambda_0e_2)=\beta q_1^n-\Delta\mathrm{e}\lambda_0=\frac{1}{3}(\delta_1-2\Delta\mathrm{e}\lambda_0)>0$ and $q_2^n=\frac{\delta_1-2\Delta\mathrm{e}\lambda_0}{3\beta}>0$, which means $0<\Delta\mathrm{e}<\Delta\mathrm{e}^n$, where $\Delta\mathrm{e}^n=\frac{\delta_1}{2\lambda_0}$. Under above conditions, we can easily get $\pi_1^n(q_1^n)>0$ and $\pi_2^n(q_2^n)>0$. For RL coopetition model, we discuss the conditions we should satisfy under $\frac{4}{9}<\theta<\theta_0$ and $\theta_0\leq\theta\leq 1$ respectively, where $\theta_0=\frac{\delta_1^2+4K\beta\lambda_0}{\delta_1^2+9K\beta\lambda_0}$. Firstly, if $\frac{4}{9}<\theta<\theta_0$, then we can easily get $p^r-(c+\lambda_0e_1+\lambda_1^r)=\frac{\sqrt{(1-\theta)\delta_1^2+(4-9\theta)K\beta\lambda_0}}{\sqrt{5}}>0$, $\lambda_1^r=\frac{5\delta_1-3\sqrt{5}\sqrt{(1-\theta)\delta_1^2+(4-9\theta)K\beta\lambda_0}}{10}>0$ and $q_2^r=\frac{\delta_1-2\lambda_1^r}{3\beta}=\frac{\sqrt{(1-\theta)\delta_1^2+(4-9\theta)K\beta\lambda_0}}{\sqrt{5}\beta}>0$. At this time, we can easily get $\pi_1^r(q_1^r)>0$ and $\pi_2^r(q_2^r)>0$. Secondly, if $\theta_0\leq\theta\leq 1$, then $\lambda_2^r=\frac{\delta_1}{2}>0$.

Now, we should choose the strategy that is more profitable. Next, we will compare the profit of these strategies.

Case 1: If $0<\Delta\mathrm{e}<\Delta\mathrm{e}^n$ and $\frac{4}{9}<\theta<\theta_0$, then we can get $\pi^r(\lambda_1^r)-\pi^n=\frac{1}{180\beta}[-4\delta_1^2-36K\beta\lambda_0+40\Delta\mathrm{e}\delta_1\lambda_0-100\Delta\mathrm{e}^2\lambda_0^2+\theta(9\delta_1^2+81K\beta\lambda_0)]$. Let $F_1(\theta)=-4\delta_1^2-36K\beta\lambda_0+40\Delta\mathrm{e}\delta_1\lambda_0-100\Delta\mathrm{e}^2\lambda_0^2+\theta(9\delta_1^2+81K\beta\lambda_0)$, we obtain a positive root $\theta_3=\frac{4(\delta_1^2+9K\beta\lambda_0-10\Delta\mathrm{e}\delta_1\lambda_0+25\Delta\mathrm{e}^2\lambda_0^2)}{9(\delta_1^2+9K\beta\lambda_0)}$. Firstly, $\theta_0-\theta_3=\frac{5(\delta_1-2\Delta\mathrm{e}\lambda_0)(\delta_1+10\Delta\mathrm{e}\lambda_0)}{9(\delta_1^2+9K\beta\lambda_0)}>0$. For another, $\theta_3-\frac{4}{9}=\frac{20\Delta\mathrm{e}\lambda_0(5\Delta\mathrm{e}\lambda_0-2\delta_1)}{9(\delta_1^2+9K\beta\lambda_0)}$. Let $\Delta\mathrm{e}^m=\frac{2\delta_1}{5\lambda_0}$ and we get if $0<\Delta\mathrm{e}<\Delta\mathrm{e}^m$, then $\theta_3<\frac{4}{9}$; if $\Delta\mathrm{e}^m<\Delta\mathrm{e}<\Delta\mathrm{e}^n$, then $\theta_3>\frac{4}{9}$. Therefore, if $0<\Delta\mathrm{e}<\Delta\mathrm{e}^m$ and $\frac{4}{9}<\theta<\theta_0$; or $\Delta\mathrm{e}^m<\Delta\mathrm{e}<\Delta\mathrm{e}^n$ and $\theta_3<\theta<\theta_0$ then $\pi^r>\pi^n$; if $\Delta\mathrm{e}^m<\Delta\mathrm{e}<\Delta\mathrm{e}^n$ and $\frac{4}{9}<\theta<\theta_3$, then $\pi^r<\pi^n$.

Case 2: If $0<\Delta\mathrm{e}<\Delta\mathrm{e}^n$ and $\theta_0\leq\theta\leq 1$, then we can easily get $\pi^r(\lambda_2^r)-\pi^n=\frac{(\delta_1-2\Delta\mathrm{e}\lambda_0)(\delta_1+10\Delta\mathrm{e}\lambda_0)}{36\beta}>0$.

In summary, if $0 < \Delta e < \Delta e^m$ and $\frac{4}{9} < \theta \leq 1$; or $\Delta e^m < \Delta e < \Delta e^n$ and $\theta_3 < \theta \leq 1$, then RL coopetition is the better strategy. If $0 < \Delta e < \Delta e^n$ and $0 \leq \theta \leq max\{\frac{4}{9}, \theta_3\}$, then competition is the better strategy.

(2) Firstly, considering $0 < \Delta e < \Delta e^n$ and $\frac{4}{9} < \theta < \theta_0$, we get $\pi_1^r(\lambda_1^r) - \pi_1^n = \frac{1}{36\beta}[-4\delta_1^2 - 36K\beta\lambda_0 - 8\Delta e\delta_1\lambda_0 - 4\Delta e^2\lambda_0^2 + \theta(9\delta_1^2 + 81K\beta\lambda_0)]$. Define $F_2(\theta) = -4\delta_1^2 - 36K\beta\lambda_0 - 8\Delta e\delta_1\lambda_0 - 4\Delta e^2\lambda_0^2 + \theta(9\delta_1^2 + 81K\beta\lambda_0)$, we get a positive root $\theta_4 = \frac{4(\delta_1^2+9K\beta\lambda_0+2\Delta e\delta_1\lambda_0+\Delta e^2\lambda_0^2)}{9(\delta_1^2+9K\beta\lambda_0)}$. According to $\theta_0 - \theta_4 = \frac{(\delta_1-2\Delta e\lambda_0)(5\delta_1+2\Delta e\lambda_0)}{9(\delta_1^2+9K\beta\lambda_0)} > 0$ and $\theta_4 - \frac{4}{9} = \frac{4\Delta e\lambda_0(2\delta_1+\Delta e\lambda_0)}{9(\delta_1^2+9K\beta\lambda_0)} > 0$, we get if $\frac{4}{9} < \theta < \theta_4$, then $\pi_1^r(\lambda_1^r) < \pi_1^n$; if $\theta_4 < \theta < \theta_0$, then $\pi_1^r(\lambda_1^r) > \pi_1^n$.

$\pi_2^r(\lambda_1^r) - \pi_2^n = -\frac{-4\delta_1^2-36K\beta\lambda_0-20\Delta e\delta_1\lambda_0+20\Delta e^2\lambda_0^2+\theta(9\delta_1^2+81K\beta\lambda_0)}{45\beta}$. Let $F_3(\theta) = -4\delta_1^2 - 36K\beta\lambda_0 - 20\Delta e\delta_1\lambda_0 + 20\Delta e^2\lambda_0^2 + \theta(9\delta_1^2 + 81K\beta\lambda_0)$, we get a root $\theta_2 = \frac{4(\delta_1^2+9K\beta\lambda_0+5\Delta e\delta_1\lambda_0-5\Delta e^2\lambda_0^2)}{9(\delta_1^2+9K\beta\lambda_0)}$. According to $\theta_0 - \theta_2 = \frac{5(\delta_1-2\Delta e\lambda_0)^2}{9(\delta_1^2+9K\beta\lambda_0)} > 0$ and $\theta_2 - \frac{4}{9} = \frac{20\Delta e\lambda_0(\delta_1-\Delta e\lambda_0)}{9(\delta_1^2+9K\beta\lambda_0)} > 0$, we have if $\frac{4}{9} < \theta < \theta_2$, then $\pi_2^r(\lambda_1^r) > \pi_2^n$; if $\theta_2 < \theta < \theta_0$, then $\pi_2^r(\lambda_1^r) < \pi_2^n$.

Since $\theta_4 > \frac{4}{9}$, $\theta_2 - \theta_4 = \frac{4\Delta e\lambda_0(\delta_1-2\Delta e\lambda_0)}{3(\delta_1^2+9K\beta\lambda_0)}$ and $\theta_4 - \theta_1 = \frac{16\Delta e\lambda_0(\delta_1-2\Delta e\lambda_0)}{3(\delta_1^2+9K\beta\lambda_0)}$, we obtain if $0 < \Delta e < \Delta e^n$, then $\theta_2 > \theta_4 > \frac{4}{9}$ and $\theta_2 > \theta_4 > \theta_1$. Therefore, in RL strategy zone, if $0 < \Delta e < \Delta e^n$ and $\theta_4 < \theta < \theta_2$, then $\pi_1^r(\lambda_1^r) > \pi_1^n$ and $\pi_2^r(\lambda_1^r) > \pi_2^n$.

Secondly, considering $0 < \Delta e < \Delta e^n$ and $\theta_0 \leq \theta \leq 1$, we obtain $\pi_1^r(\lambda_2^r) - \pi_1^n = \frac{(\delta_1-2\Delta e\lambda_0)(5\delta_1+2\Delta e\lambda_0)}{36\beta} > 0$ and $\pi_2^r(\lambda_2^r) - \pi_2^n = -\frac{(2\Delta e\lambda_0-\delta_1)^2}{9\beta} < 0$.

In summary, if $0 < \Delta e < \Delta e^n$ and $\theta_4 < \theta < \theta_2$, then RL coopetition strategy achieves a Pareto improvement.

(3) In stable RL coopetition strategy, from Lemma 1, we can easily get $p^r < p^n$.

Proof of Lemma 4.1

(1) From Table 4.2, we should satisfy $0 < \Delta e < \Delta e^n$ for competition model and $\frac{4}{9} < \theta \leq 1$ for RL coopetition model. Recall $p^n = c + \lambda_0 e_1 + \beta q_1^n$ and $p^r = c + \lambda_0 e_1 + \beta q_1^r$, we get $p^r - p^n = \beta(q_1^r - q_1^n)$. Now we discuss this from the following two cases.

Case 1: $0 < \Delta e < \Delta e^n$ and $\frac{4}{9} < \theta < \theta_0$. $q_1^r - q_1^n = \frac{1}{30\beta}[5\delta_1 - 10\Delta e\lambda_0 - 3\sqrt{5}\sqrt{(1-\theta)\delta_1^2 + (4-9\theta)K\beta\lambda_0}]$. Since $(5\delta_1 - 10\Delta e\lambda_0)^2 - [3\sqrt{5}\sqrt{(1-\theta)\delta_1^2 + K\beta(4-9\theta)\lambda_0}]^2 =$

$5[(9\delta_1^2+81K\beta\lambda_0)\theta+5(\delta_1-2\Delta e\lambda_0)^2-9\delta_1^2-36K\beta\lambda_0]$. We get a root $\theta_2=\frac{4(\delta_1^2+9K\beta\lambda_0+5\Delta e\delta_1\lambda_0-5\Delta e^2\lambda_0^2)}{9(\delta_1^2+9K\beta\lambda_0)}$. Firstly, $\theta_2-\frac{4}{9}=\frac{20\Delta e\lambda_0(\delta_1-\Delta e\lambda_0)}{9(\delta_1^2+9K\beta\lambda_0)}>0$. Secondly, $\theta_2-\theta_0=-\frac{5(\delta_1-2\Delta e\lambda_0)^2}{9(\delta_1^2+9K\beta\lambda_0)}<0$. Thus, if $\frac{4}{9}<\theta<\theta_2$, then $q_1^r<q_1^n$; if $\theta_2<\theta<\theta_0$, then $q_1^r>q_1^n$.

Case 2: $0<\Delta e<\Delta e^n$ and $\theta_0\le\theta\le 1$. $q_1^r-q_1^n=\frac{\delta_1-2\Delta e\lambda_0}{6\beta}>0$.

In summary, if $\frac{4}{9}<\theta<\theta_2$, then $q_1^r<q_1^n$ and $p^r<p^n$; if $\theta_2<\theta\le 1$, then $q_1^r>q_1^n$ and $p^r>p^n$.

Proof of Corollary 4.1

From Table 4.2, we get $T^n=\frac{2e_1\delta_1-2\Delta e^2\lambda_0+\Delta e(\delta_1-e_1\lambda_0)}{3\beta}$ and $T^r(\lambda_1^r)=\frac{e_1}{10\beta}[5\delta_1+\sqrt{5}\sqrt{(1-\theta)\delta_1^2+(4-9\theta)K\beta\lambda_0}]$ and $T^r(\lambda_2^r)=\frac{e_1\delta_1}{2\beta}$. Now, we will discuss this from the following two cases.

Case 1: $0<\Delta e<\Delta e^n$ and $\frac{4}{9}<\theta<\theta_0$. $T^r(\lambda_1^r)-T^n=\frac{1}{30\beta}[5(2\Delta e+e_1)(2\Delta e\lambda_0-\delta_1)+3\sqrt{5}e_1\sqrt{(1-\theta)\delta_1^2+(4-9\theta)K\beta\lambda_0}]$. Since $[3\sqrt{5}e_1\sqrt{(1-\theta)\delta_1^2+(4-9\theta)K\beta\lambda_0}]^2-[5(2\Delta e+e_1)(\delta_1-2\Delta e\lambda_0)]^2=-5\theta(9e_1^2\delta_1^2+81K\beta e_1^2\lambda_0)-20[20\Delta e^4\lambda_0^2+20\Delta e^3\lambda_0(e_1\lambda_0-\delta_1)+5\Delta e^2(\delta_1^2-4e_1\delta_1\lambda_0+e_1^2\lambda_0^2)+5(\delta_1-e_1\lambda_0)e_1\delta_1\Delta e-e_1^2(\delta_1^2+9K\beta\lambda_0)]$. We get a root $\theta_5=-\frac{4}{9e_1^2(\delta_1^2+9K\beta\lambda_0)}[20\Delta e^4\lambda_0^2+20\Delta e^3\lambda_0(e_1\lambda_0-\delta_1)+5\Delta e^2(\delta_1^2-4e_1\delta_1\lambda_0+e_1^2\lambda_0^2)+5(\delta_1-e_1\lambda_0)e_1\delta_1\Delta e-e_1^2(\delta_1^2+9K\beta\lambda_0)]$. Firstly, $\theta_5-\theta_0=-\frac{5(2\Delta e+e_1)^2(\delta_1-2\Delta e\lambda_0)^2}{9e_1^2(\delta_1^2+9K\beta\lambda_0)}<0$. Secondly, $\theta_5-\frac{4}{9}=\frac{20\Delta e(-\delta_1+2\Delta e\lambda_0+e_1\lambda_0)(\Delta e\delta_1+e_1\delta_1-2\Delta e^2\lambda_0-\Delta e e_1\lambda_0)}{9e_1^2(\delta_1^2+9K\beta\lambda_0)}$. Let $\phi(\Delta e)=(-\delta_1+2\Delta e\lambda_0+e_1\lambda_0)(\Delta e\delta_1+e_1\delta_1-2\Delta e^2\lambda_0-\Delta e e_1\lambda_0)$ and we get three roots, $\Delta e^a=\frac{\delta_1-e_1\lambda_0}{2\lambda_0}$, $\Delta e^b=\frac{\delta_1-e_1\lambda_0-\sqrt{\delta_1^2+6e_1\delta_1\lambda_0+e_1^2\lambda_0^2}}{4\lambda_0}$ and $\Delta e^c=\frac{\delta_1-e_1\lambda_0+\sqrt{\delta_1^2+6e_1\delta_1\lambda_0+e_1^2\lambda_0^2}}{4\lambda_0}$.

If $\delta_1>e_1\lambda_0$, then $\Delta e^b<0<\Delta e^a<\Delta e^c$. $\Delta e^c-\Delta e^n=-\frac{\delta_1+e_1\lambda_0-\sqrt{\delta_1^2+6e_1\delta_1\lambda_0+e_1^2\lambda_0^2}}{4\lambda_0}>0$. Thus, if $0<\Delta e<\Delta e^a$, then $\theta_5<\frac{4}{9}$; if $\Delta e^a<\Delta e<\Delta e^n$, then $\theta_5>\frac{4}{9}$. That means if $0<\Delta e<\Delta e^a$ and $\frac{4}{9}<\theta<\theta_0$, then $T^r(\lambda_1^r)<T^n$; if $\Delta e^a<\Delta e<\Delta e^n$ and $\theta_5<\theta<\theta_0$, then $T^r(\lambda_1^r)<T^n$; if $\Delta e^a<\Delta e<\Delta e^n$ and $\frac{4}{9}<\theta<\theta_5$, then $T^r(\lambda_1^r)>T^n$.

If $0<\delta_1<e_1\lambda_0$, then $\Delta e^b<\Delta e^a<0<\Delta e^c$. $\Delta e^c>\Delta e^n$, so $\theta_5>\frac{4}{9}$ for $0<\Delta e<\Delta e^n$. Thus, if $0<\Delta e<\Delta e^n$ and $\theta_5<\theta<\theta_0$, then $T^r(\lambda_1^r)<T^n$; if $0<\Delta e<\Delta e^n$ and $\frac{4}{9}<\theta<\theta_5$, then $T^r(\lambda_1^r)>T^n$.

Case 2: $0<\Delta e<\Delta e^n$ and $\theta_0\le\theta\le 1$. $T^r(\lambda_2^r)-T^n=\frac{(2\Delta e+e_1)(2\Delta e\lambda_0-\delta_1)}{6\beta}<0$.

In summary, we get when $\delta_1 > e_1\lambda_0$: if $0 < \Delta e < \Delta e^a$ and $\frac{4}{9} < \theta \leq 1$, then $T^r < T^n$; if $\Delta e^a < \Delta e < \Delta e^n$ and $\theta_5 < \theta \leq 1$, then $T^r < T^n$; if $\Delta e^a < \Delta e < \Delta e^n$ and $\frac{4}{9} < \theta \leq \theta_5$, then $T^r > T^n$. When $0 < \delta_1 < e_1\lambda_0$: if $0 < \Delta e < \Delta e^n$ and $\theta_5 < \theta \leq 1$, then $T^r < T^n$; if $0 < \Delta e < \Delta e^n$ and $\frac{4}{9} < \theta \leq \theta_5$, then $T^r > T^n$. That means if $0 < \Delta e < \Delta e^n$ and $max\{\frac{4}{9}, \theta_5\} < \theta \leq 1$, then $T^r < T^n$; otherwise, $T^r > T^n$.

Proof of Proposition 4.2

(1) From Table 4.2, for FL coopetition model, we get $q_1^f = q_2^f = \frac{\delta_1}{3\beta} > 0$, $p^f > c + \lambda_0 e_1$ and $M^f = \frac{(\delta_1^2 + 9K\beta\lambda_0)(2\theta - 1)}{9\beta} > 0$, which indicates $\frac{1}{2} < \theta \leq 1$. At this time, we can easily get $\pi_1^f(q_1^f) > 0$ and $\pi_2^f(q_2^f) > 0$. For competition model, we get $q_2^n = \frac{\delta_1 - 2\Delta e\lambda_0}{3\beta} > 0$, which means $0 < \Delta e < \Delta e^n$, where $\Delta e^n = \frac{\delta_1}{2\lambda_0}$.

According to the above analysis, we get if $0 < \Delta e < \Delta e^n$ and $0 \leq \theta \leq \frac{1}{2}$, then FL coopetition strategy does not exist, so competition is the better strategy. If $0 < \Delta e < \Delta e^n$ and $\frac{1}{2} < \theta \leq 1$, then FL coopetition strategy and competition coexist, so we should choose the strategy with more profit. Next, we will compare the profit of these strategies.

$\pi^f - \pi^n = \frac{\Delta e\lambda_0(2\delta_1 - 5\Delta e\lambda_0)}{9\beta}$. Let $\Delta e^m = \frac{2\delta_1}{5\lambda_0}$ and we obtain if $0 < \Delta e < \Delta e^m$ and $\frac{1}{2} < \theta \leq 1$, then $\pi^f > \pi^n$; if $\Delta e^m < \Delta e < \Delta e^n$ and $\frac{1}{2} < \theta \leq 1$, then $\pi^f < \pi^n$.

In summary, if $0 < \Delta e < \Delta e^m$ and $\frac{1}{2} < \theta \leq 1$, then FL coopetition is the better strategy; if $0 < \Delta e < \Delta e^n$ and $0 \leq \theta \leq \frac{1}{2}$; or $\Delta e^m < \Delta e < \Delta e^n$ and $\frac{1}{2} < \theta \leq 1$, then competition is the better strategy.

(2) From Table 4.2, $\pi_1^f(q_1^f) - \pi_1^n(q_1^n) = \frac{-\delta_1^2 - 9K\beta\lambda_0 - 2\Delta e\delta_1\lambda_0 - \Delta e^2\lambda_0^2 + \theta(2\delta_1^2 + 18K\beta\lambda_0)}{9\beta}$. Defining $F_4(\theta) = -\delta_1^2 - 9K\beta\lambda_0 - 2\Delta e\delta_1\lambda_0 - \Delta e^2\lambda_0^2 + \theta(2\delta_1^2 + 18K\beta\lambda_0)$, we get a root $\theta_6 = \frac{\delta_1^2 + 9K\beta\lambda_0 + 2\Delta e\delta_1\lambda_0 + \Delta e^2\lambda_0^2}{2(\delta_1^2 + 9K\beta\lambda_0)}$ for $F_4(\theta) = 0$. Firstly, $\theta_6 - \frac{1}{2} = \frac{\Delta e\lambda_0(2\delta_1 + \Delta e\lambda_0)}{2(\delta_1^2 + 9K\beta\lambda_0)} > 0$. Secondly, $1 - \theta_6 = \frac{\delta_1^2 + 9K\beta\lambda_0 - 2\Delta e\delta_1\lambda_0 - \Delta e^2\lambda_0^2}{2(\delta_1^2 + 9K\beta\lambda_0)}$. Set $\psi_1(\Delta e) = \delta_1^2 + 9K\beta\lambda_0 - 2\Delta e\delta_1\lambda_0 - \Delta e^2\lambda_0^2$, we get $\psi_1(\Delta e)|_{\Delta e = 0} = \delta_1^2 + 9K\beta\lambda_0 > 0$ and $\psi_1(\Delta e)|_{\Delta e = \Delta e^m} = \frac{1}{25}(\delta_1^2 + 225K\beta\lambda_0) > 0$, which suggests if $0 < \Delta e < \Delta e^m$, then $\theta_6 < 1$. Therefore, in FL coopetition strategy region, if $0 < \Delta e < \Delta e^m$ and $\theta_6 < \theta \leq 1$, then $\pi_1^f(q_1^f) > \pi_1^n(q_1^n)$; if $0 < \Delta e < \Delta e^m$ and $\frac{1}{2} < \theta < \theta_6$, then $\pi_1^f(q_1^f) < \pi_1^n(q_1^n)$.

$\pi_2^f(q_2^f) - \pi_2^n(q_2^n) = -\frac{-\delta_1^2 - 9K\beta\lambda_0 - 4\Delta e\delta_1\lambda_0 + 4\Delta e^2\lambda_0^2 + \theta(2\delta_1^2 + 18K\beta\lambda_0)}{9\beta}$. Define $F_5(\theta) = -\delta_1^2 - 9K\beta\lambda_0 -$

$4\Delta e\delta_1\lambda_0 + 4\Delta e^2\lambda_0^2 + \theta(2\delta_1^2 + 18K\beta\lambda_0)$ and we get a root $\theta_7 = \frac{\delta_1^2+9K\beta\lambda_0+4\Delta e\delta_1\lambda_0-4\Delta e^2\lambda_0^2}{2(\delta_1^2+9K\beta\lambda_0)}$ for $F_5(\theta) = 0$. Firstly, $\theta_7 - \frac{1}{2} = \frac{2\Delta e\lambda_0(\delta_1-\Delta e\lambda_0)}{\delta_1^2+9K\beta\lambda_0} > 0$. Secondly, $1 - \theta_7 = \frac{\delta_1^2+9K\beta\lambda_0-4\Delta e\delta_1\lambda_0+4\Delta e^2\lambda_0^2}{2(\delta_1^2+9K\beta\lambda_0)}$. Set $\psi_2(\Delta e) = \delta_1^2 + 9K\beta\lambda_0 - 4\Delta e\delta_1\lambda_0 + 4\Delta e^2\lambda_0^2$ and its discriminant $\Delta = -144K\beta\lambda_0^3 < 0$, so $\theta_7 < 1$. Therefore, in FL coopetition strategy zone, if $0 < \Delta e < \Delta e^m$ and $\frac{1}{2} < \theta < \theta_7$, then $\pi_2^f(q_2^f) > \pi_2^n(q_2^n)$; if $0 < \Delta e < \Delta e^m$ and $\theta_7 < \theta \le 1$, then $\pi_2^f(q_2^f) < \pi_2^n(q_2^n)$. At this time, $\theta_7 - \theta_6 = \frac{\Delta e\lambda_0(2\delta_1-5\Delta e\lambda_0)}{2(\delta_1^2+9K\beta\lambda_0)} > 0$, so we get if $0 < \Delta e < \Delta e^m$ and $\theta_6 < \theta < \theta_7$, then $\pi_1^f(q_1^f) > \pi_1^n(q_1^n)$ and $\pi_2^f(q_2^f) > \pi_2^n(q_2^n)$, which suggests there is a Pareto improvement for FL coopetition strategy.

(3) From Lemma 4.2, it is easy to get $p^f < p^n$ in stable FL coopetition strategy.

Proof of Lemma 4.2

From Table 4.2, we should meet $0 < \Delta e < \Delta e^n$ and $\frac{1}{2} < \theta \le 1$ for competition and FL coopetition strategy respectively. Recall $p^n = c + \lambda_0 e_1 + \beta q_1^n$ and $p^f = c + \lambda_0 e_1 + \beta q_1^f$, so $p^f - p^n = \beta(q_1^f - q_1^n)$. From Table 4.2, we can easily get $q_1^f < q_1^n$. Therefore, if $0 < \Delta e < \Delta e^n$ and $\frac{1}{2} < \theta \le 1$, then $p^f < p^n$.

Proof of Corollary 4.2

From Table 4.2, we get $T^f = \frac{2e_1\delta_1}{3\beta}$ and $T^n = \frac{2e_1\delta_1-2\Delta e^2\lambda_0+\Delta e(\delta_1-e_1\lambda_0)}{3\beta}$. $T^f - T^n = \frac{\Delta e(2\Delta e\lambda_0+e_1\lambda_0-\delta_1)}{3\beta}$ and we get a root $\Delta e^a = \frac{\delta_1-e_1\lambda_0}{2\lambda_0}$. If $0 < \delta_1 < e_1\lambda_0$, then $\Delta e^a < 0$; if $\delta_1 > e_1\lambda_0$, then $0 < \Delta e^a < \Delta e^n$.

Therefore, if $0 < \delta_1 < e_1\lambda_0$ and $0 < \Delta e < \Delta e^n$ then $T^f > T^n$; if $\delta_1 > e_1\lambda_0$ and $\Delta e^a < \Delta e < \Delta e^n$, then $T^f > T^n$; if $\delta_1 > e_1\lambda_0$ and $0 < \Delta e < \Delta e^a$, then $T^f < T^n$. That means if $max\{0, \Delta e^a\} < \Delta e < \Delta e^n$ and $\frac{1}{2} < \theta \le 1$, then $T^f > T^n$; otherwise $T^f < T^n$.

Proof of Proposition 4.3

(1) From Table 4.2, we know for competition model, we should meet $0 < \Delta e < \Delta e^n$, where $\Delta e^n = \frac{\delta_1}{2\lambda_0}$. For ML coopetition model, $M^l = \frac{(8K\beta\lambda_0+\delta_1^2)\theta-\delta_1^2-4K\beta\lambda_0}{4\beta} > 0$ suggests $\theta_1 < \theta \le 1$, where $\theta_1 = \frac{\delta_1^2+4K\beta\lambda_0}{\delta_1^2+8K\beta\lambda_0}$.

If $0 < \Delta e < \Delta e^n$ and $\theta_1 < \theta \le 1$, then $\pi^l - \pi^n = \frac{(\delta_1-2\Delta e\lambda_0)(\delta_1+10\Delta e\lambda_0)}{36\beta} > 0$, so ML coopetition is the better strategy. If $0 < \Delta e < \Delta e^n$ and $0 < \theta \leq \theta_1$, which means ML coopetition strategy does not exist, then competition is the better strategy.

(2) $\pi_1^l(q_1^l) - \pi_1^n(q_1^n) = \frac{-4\delta_1^2 - 36K\beta\lambda_0 - 8\Delta e\delta_1\lambda_0 - 4\Delta e^2\lambda_0^2 + \theta(9\delta_1^2 + 72K\beta\lambda_0)}{36\beta}$, Let $F(\theta) = -4\delta_1^2 - 36K\beta\lambda_0 - 8\Delta e\delta_1\lambda_0 - 4\Delta e^2\lambda_0^2 + \theta(9\delta_1^2 + 72K\beta\lambda_0)$ and we get a root $\theta_8 = \frac{4(\delta_1^2 + 9K\beta\lambda_0 + 2\Delta e\delta_1\lambda_0 + \Delta e^2\lambda_0^2)}{9(\delta_1^2 + 8K\beta\lambda_0)}$. Since $\theta_8 - \theta_1 = \frac{(2\Delta e\lambda_0 - \delta_1)(5\delta_1 + 2\Delta e\lambda_0)}{9(\delta_1^2 + 8K\beta\lambda_0)} < 0$, then $\pi_1^l(q_1^l) > \pi_1^n(q_1^n)$ for all ML coopetition strategy region.

$\pi_2^l(q_2^l) - \pi_2^n(q_2^n) = \frac{5\delta_1^2 + 36K\beta\lambda_0 + 16\Delta e\delta_1\lambda_0 - 16\Delta e^2\lambda_0^2 - \theta(9\delta_1^2 + 72K\beta\lambda_0)}{36\beta}$. Let $F(\theta) = 5\delta_1^2 + 36K\beta\lambda_0 + 16\Delta e\delta_1\lambda_0 - 16\Delta e^2\lambda_0^2 - \theta(9\delta_1^2 + 72K\beta\lambda_0)$ and we get a root $\theta_9 = \frac{5\delta_1^2 + 36K\beta\lambda_0 + 16\Delta e\delta_1\lambda_0 - 16\Delta e^2\lambda_0^2}{9(\delta_1^2 + 8K\beta\lambda_0)}$. Similarly, because $\theta_9 - \theta_1 = -\frac{4(\delta_1 - 2\Delta e\lambda_0)^2}{9(\delta_1^2 + 8K\beta\lambda_0)} < 0$, then $\pi_2^l(q_2^l) < \pi_2^n(q_2^n)$ for all ML coopetition strategy region.

In summary, ML coopetition strategy cannot realize a Pareto improvement.

Proof of Lemma 4.3

From Table 4.2, we get if $0 < \Delta e < \Delta e^n$ and $\theta_1 < \theta \leq 1$, then $q_1^l - q_1^n = \frac{\delta_1 - 2\Delta e\lambda_0}{6\beta} > 0$. Since $p^l = c + \lambda_0 e_1 + \beta q_1^l$ and $p^n = c + \lambda_0 e_1 + \beta q_1^n$, then $p^l - p^n = \beta(q_1^l - q_1^n)$. Thus, if $0 < \Delta e < \Delta e^n$ and $\theta_1 < \theta \leq 1$, then $p^l > p^n$.

Proof of Corollary 4.3

From Table 4.2, we know $T^l = \frac{e_1\delta_1}{2\beta}$ and $T^n = \frac{2e_1\delta_1 - 2\Delta e^2\lambda_0 + \Delta e(\delta_1 - e_1\lambda_0)}{3\beta}$, so $T^l - T^n = \frac{(2\Delta e + e_1)(2\Delta e\lambda_0 - \delta_1)}{6\beta}$. Therefore, if $0 < \Delta e < \Delta e^n$ and $\theta_1 < \theta \leq 1$, then $T^l < T^n$.

Proof of Proposition 4.4

(1) Firstly, we choose the optimal strategy among RL, FL coopetition and competition strategy, then we compare the optimal strategy (RL or FL or competition) with ML strategy. From Table 4.2, we know for RL coopetition model, we should satisfy $\frac{4}{9} < \theta < \theta_0$ for $\lambda^r = \lambda_1^r$ and $\theta_0 \leq \theta \leq 1$ for $\lambda^r = \lambda_2^r$. That means if $0 \leq \theta \leq \frac{4}{9}$, then RL coopetition strategy does not exist. For FL coopetition model, we should meet $\frac{1}{2} < \theta \leq 1$, which indicates if $0 \leq \theta \leq \frac{1}{2}$, then FL coopetition strategy does not exist. Notice that if $0 \leq \theta \leq \frac{4}{9}$, then RL and FL coopetition strategy do not exist, so we should omit this case at this time.

Therefore, according to the above analysis, we get if $\frac{4}{9} < \theta \leq \frac{1}{2}$, then RL coopetition is the better strategy than FL coopetition strategy (no FL coopetition); if $\frac{1}{2} < \theta \leq 1$, then FL and RL coopetition strategies coexist, so we compare the profit of these strategies under this condition.

$\theta_0 - \frac{1}{2} = \frac{\delta_1^2 - K\beta\lambda_0}{2(\delta_1^2 + 9K\beta\lambda_0)}$, which means if $K > \frac{\delta_1^2}{\beta\lambda_0}$, then $\theta_0 < \frac{1}{2}$; if $0 < K < \frac{\delta_1^2}{\beta\lambda_0}$, then $\theta_0 > \frac{1}{2}$. Thus, when $K > \frac{\delta_1^2}{\beta\lambda_0}$, we get $\pi^r(\lambda_2^r) - \pi^f = \frac{\delta_1^2}{36\beta} > 0$ for $\frac{1}{2} < \theta \leq 1$; when $0 < K < \frac{\delta_1^2}{\beta\lambda_0}$, we have $\pi^r(\lambda_1^r) - \pi^f = \frac{(9\theta-4)(\delta_1^2+9K\beta\lambda_0)}{180\beta} > 0$ for $\frac{1}{2} < \theta < \theta_0$ and $\pi^r(\lambda_2^r) - \pi^f = \frac{\delta_1^2}{36\beta} > 0$ for $\theta_0 \leq \theta \leq 1$. That indicates if $\frac{1}{2} < \theta \leq 1$, then RL coopetition is the better strategy.

In summary, if $\frac{4}{9} < \theta \leq 1$, then RL coopetition is the better strategy than FL coopetition. Next, according to the Proposition 4.1, we get $\theta_3 - \frac{4}{9} = \frac{20\Delta e\lambda_0(5\Delta e\lambda_0 - 2\delta_1)}{9(\delta_1^2+9K\beta\lambda_0)} > 0$ for $\Delta e^m < \Delta e < \Delta e^n$, where $\Delta e^m = \frac{2\delta_1}{5\lambda_0}$. Therefore, if $0 < \Delta e < \Delta e^m$ and $\frac{4}{9} < \theta \leq 1$; or $\Delta e^m < \Delta e < \Delta e^n$ and $\theta_3 < \theta \leq 1$, then RL coopetition is the optimal strategy; otherwise, competition is the optimal strategy.

(2) Secondly, from (4-1), we know it is impossible for FL coopetition to be the optimal strategy, thus we will choose the optimal strategy among RL, ML and competition strategy.

From Table 4.2, we know $\theta_1 < \theta \leq 1$ should be satisfied for ML coopetition model. Thus, we should compare RL and ML strategy in the region which RL is the optimal strategy than FL and competition (i.e., $0 < \Delta e < \Delta e^m$ and $\frac{4}{9} < \theta \leq 1$; or $\Delta e^m < \Delta e < \Delta e^n$ and $\theta_3 < \theta \leq 1$).

$\theta_1 - \frac{4}{9} = \frac{5\delta_1^2+4K\beta\lambda_0}{9(\delta_1^2+8K\beta\lambda_0)} > 0$ and $\theta_1 - \theta_3 = \frac{1}{9(\delta_1^2+8K\beta\lambda_0)(\delta_1^2+9K\beta\lambda_0)}[5\delta_1^4 + 49K\beta\delta_1^2\lambda_0 + 36K^2\beta^2\lambda_0^2 + \Delta e(40\delta_1^3\lambda_0 + 320K\beta\delta_1\lambda_0^2) - \Delta e^2(100\delta_1^2\lambda_0^2 + 800K\beta\lambda_0^3)] > 0$. Thus, if $0 < \Delta e < \Delta e^m$ and $\frac{4}{9} < \theta \leq \theta_1$; or $\Delta e^m < \Delta e < \Delta e^n$ and $\theta_3 < \theta \leq \theta_1$, then ML strategy does not exist and we choose RL coopetition strategy in this region. If $0 < \Delta e < \Delta e^n$ and $\theta_1 < \theta \leq 1$, then RL and ML coopetition strategy coexist. Since $\theta_1 - \theta_0 = \frac{K\beta\lambda_0(\delta_1^2+4K\beta\lambda_0)}{(\delta_1^2+8K\beta\lambda_0)(\delta_1^2+9K\beta\lambda_0)} > 0$, then we get $\pi^r(\lambda_2^r) = \pi^l = \frac{\delta_1^2+8K\beta\lambda_0}{4\beta}$. Although there is no difference between RL coopetition and ML coopetition strategy from the total profit of manufacturer 1 and manufacturer 2, manufacturer 1 decides whether it will license its technology to manufacturer 2 or which licensing type is considered. Thus we should also the profit of manufacturer 1 under RL coopetition and ML coopetition strategy. Since $\pi_1^l - \pi_1^r(\lambda_2^r) = \frac{-\delta_1^2+\theta\delta_1^2-4K\beta\lambda_0+8K\beta\theta\lambda_0}{4\beta} > 0$ for $\theta_1 < \theta \leq 1$, where $\theta_1 = \frac{\delta_1^2+4K\beta\lambda_0}{\delta_1^2+8K\beta\lambda_0}$, manufacturer 1 prefers the ML strategy, which means ML coopetition is the optimal strategy at this region.

In summary, if $0 < \Delta e < \Delta e^m$ and $\frac{4}{9} < \theta \leq \theta_1$; or $\Delta e^m < \Delta e < \Delta e^n$ and $\theta_3 < \theta \leq \theta_1$, then RL coopetition is the optimal strategy; if $0 < \Delta e < \Delta e^n$ and $\theta_1 < \theta \leq 1$, then ML coopetition is the optimal strategy.

Derivation of Table 4.3

(1) Competition model

From (4-1) and (4-2), we obtain $\frac{d^2\pi_1^n(q_1)}{dq_1^2} = \frac{d^2\pi_2^n(q_2)}{dq_2^2} = -2\beta$, so $\pi_1^n(q_1)$ and $\pi_2^n(q_2)$ are concave function of q_1 and q_2 respectively. Thus, we obtain $q_1^n = \frac{\delta_1+\Delta e\lambda_0-\Delta\alpha}{3\beta}$ and $q_2^n = \frac{\delta_1-2\Delta e\lambda_0+2\Delta\alpha}{3\beta}$ by $\frac{d\pi_1^n(q_1)}{dq_1} = \frac{d\pi_2^n(q_2)}{dq_2} = 0$. Recall that $p_i = \alpha_i - \beta(q_1+q_2)$, we obtain $p_1^n = c + \lambda_0 e_1 + \beta q_1^n$ and $p_2^n = c + \lambda_0 e_2 + \beta q_2^n$. To have $q_1^n = \frac{\delta_1+\Delta e\lambda_0-\Delta\alpha}{3\beta} > 0$ and $q_2^n = \frac{\delta_1-2\Delta e\lambda_0+2\Delta\alpha}{3\beta} > 0$, we should satisfy $max\{0, \Delta e^d\} < \Delta e < \Delta e^n$, where $\Delta e^d = \frac{\Delta\alpha-\delta_1}{\lambda_0}$ and $\Delta e^n = \frac{\delta_1+2\Delta\alpha}{2\lambda_0}$.

(2) RL coopetition model

From (4-3) and (4-4), we get $\frac{d^2\pi_1^r(q_1)}{dq_1^2} = \frac{d^2\pi_2^r(q_2)}{dq_2^2} = -2\beta$, so $\pi_1^r(q_1)$ and $\pi_2^r(q_2)$ are concave function of q_1 and q_2 respectively. Let $\frac{d\pi_1^r(q_1)}{dq_1} = \frac{d\pi_2^r(q_2)}{dq_2} = 0$, we obtain $q_1(\lambda) = \frac{\delta_1+\lambda-\Delta\alpha}{3\beta}$ and $q_2(\lambda) = \frac{\delta_1-2\lambda+2\Delta\alpha}{3\beta}$. Replacing $q_1(\lambda)$ and $q_2(\lambda)$ in (4-5), we can get $\frac{d\pi^r(\lambda)}{d\lambda} = \pi^r(\lambda)[\theta\frac{1}{\pi_1^r(q_1(\lambda))}\frac{d\pi_1^r(q_1(\lambda))}{d\lambda} + (1-\theta)\frac{1}{\pi_2^r(q_2(\lambda))}\frac{d\pi_2^r(q_2(\lambda))}{d\lambda}]$. $\frac{d\pi^r(\lambda)}{d\lambda} = 0$ is equivalent to $\Phi(\lambda) = 0$, where $\Phi(\lambda) = -40\lambda^3 + \lambda^2(72\Delta\alpha + 24\Delta\alpha\theta + 60\delta_1) - \lambda[(48\Delta\alpha^2 + 24\Delta\alpha\delta_1 + 18\delta_1^2 + 162K\beta\lambda_0)\theta + 24\Delta\alpha^2 + 72\Delta\alpha\delta_1 + 12\delta_1^2 - 72K\beta\lambda_0] + (24\Delta\alpha^3 + 24\Delta\alpha^2\delta_1 + 24\Delta\alpha\delta_1^2 + 9\delta_1^3 + 108K\beta\Delta\alpha\lambda_0 + 81K\beta\delta_1\lambda_0)\theta - 8\Delta\alpha^3 + 12\Delta\alpha^2\delta_1 - 4\delta_1^3 - 72K\beta\Delta\alpha\lambda_0 - 36K\beta\delta_1\lambda_0$, so we can get three roots $\lambda_1 = \frac{1}{10}f_1 - \frac{(1-i\sqrt{3})(f_2+g_1)}{120*2^{2/3}(f_3+\sqrt{4(f_2+g_1)^3+(f_3)^2})^{\frac{1}{3}}} + \frac{(1+i\sqrt{3})}{240*2^{1/3}}(f_3+\sqrt{4(f_2+g_1)^3+(f_3)^2})^{\frac{1}{3}}$; $\lambda_2 = \frac{1}{10}f_1 + \frac{f_2+g_1}{60*2^{2/3}(f_3+\sqrt{4(f_2+g_1)^3+(f_3)^2})^{\frac{1}{3}}} - \frac{1}{120*2^{1/3}}(f_3+\sqrt{4(f_2+g_1)^3+(f_3)^2})^{\frac{1}{3}}$; $\lambda_3 = \frac{1}{10}f_1 - \frac{(1+i\sqrt{3})(f_2+g_1)}{120*2^{2/3}(f_3+\sqrt{4(f_2+g_1)^3+(f_3)^2})^{\frac{1}{3}}} + \frac{(1-i\sqrt{3})}{240*2^{1/3}}(f_3+\sqrt{4(f_2+g_1)^3+(f_3)^2})^{\frac{1}{3}}$, where $f_1 = 6\Delta\alpha + 2\Delta\alpha\theta + 5\delta_1$; $f_2 = -2304\Delta\alpha^2 + 2304\Delta\alpha^2\theta - 576\Delta\alpha^2\theta^2$; $f_3 = 221184\Delta\alpha^3 - 331776\Delta\alpha^3\theta + 165888\Delta\alpha^3\theta^2 - 27648\Delta\alpha^3\theta^3 + 311040\Delta\alpha\delta_1^2 - 466560\Delta\alpha\theta\delta_1^2 + 155520\Delta\alpha\theta^2\delta_1^2 + 1244160K\beta\Delta\alpha\lambda_0 - 1088640K\beta\Delta\alpha\theta\lambda_0 + 1399680K\beta\Delta\alpha\theta^2\lambda_0$ and $g_1 = -2160\delta_1^2 + 2160\theta\delta_1^2 - 8640K\beta\lambda_0 + 19440K\beta\theta\lambda_0$. Recall $q_1(\lambda) = \frac{\delta_1+\lambda-\Delta\alpha}{3\beta} \geq 0$ and $q_2(\lambda) = \frac{\delta_1-2\lambda+2\Delta\alpha}{3\beta} \geq 0$, we get $max\{0, \Delta\alpha - \delta_1\} \leq \lambda \leq \Delta\alpha + \frac{1}{2}\delta_1$.

(3) FL coopetition model

From (4-6) and (4-7), we get $\frac{d^2\pi_1^f(q_1)}{dq_1^2} = \frac{d^2\pi_2^f(q_2)}{dq_2^2} = -2\beta$, so $\pi_1^f(q_1)$ and $\pi_2^f(q_2)$ are concave function of q_1 and q_2 respectively. Let $\frac{d\pi_1^f(q_1)}{dq_1} = \frac{d\pi_2^f(q_2)}{dq_2} = 0$, we obtain $q_1^f = \frac{\delta_1-\Delta\alpha}{3\beta}$ and $q_2^f = \frac{\delta_1+2\Delta\alpha}{3\beta}$, so we get $p_1^f = c + \lambda_0 e_1 + \beta q_1^f$ and $p_2^f = c + \lambda_0 e_1 + \beta q_2^f$. Replacing q_1^f and q_2^f in (4-8), we obtain $\frac{d\pi^f(M)}{dM} = \pi^f(M)[\theta \frac{1}{\pi_1^f(M)} \frac{d\pi_1^f(M)}{dM} + (1-\theta)\frac{1}{\pi_2^f(M)}\frac{d\pi_2^f(M)}{dM}]$. Recall $q_1^f = \frac{\delta_1-\Delta\alpha}{3\beta} \geq 0$ and $q_2^f = \frac{\delta_1+2\Delta\alpha}{3\beta} \geq 0$, we get $-\frac{1}{2}\delta_1 \leq \Delta\alpha \leq \delta_1$. $\frac{d\pi^f(M)}{dM} = 0$ derives a root $M^f = \frac{(5\Delta\alpha^2+2\Delta\alpha\delta_1+2\delta_1^2+18K\beta\lambda_0)\theta-\Delta\alpha^2+2\Delta\alpha\delta_1-\delta_1^2-9K\beta\lambda_0}{9\beta}$. $M^f > 0$ means $\theta_{11} < \theta \leq 1$, where $\theta_{11} = \frac{(\Delta\alpha-\delta_1)^2+9K\beta\lambda_0}{5\Delta\alpha^2+2\delta_1(\Delta\alpha+\delta_1)+18K\beta\lambda_0}$.

(4) ML coopetition model

From (4-9) and (4-10), we get $\frac{d^2\pi_1^l(q_1)}{dq_1^2} = \frac{d^2\pi_2^l(q_2)}{dq_2^2} = -2\beta$, so $\pi_1^l(q_1)$ and $\pi_2^l(q_2)$ are concave functions of q_1 and q_2 respectively. Let $\frac{d\pi_1^l(q_1)}{dq_1} = \frac{d\pi_2^l(q_2)}{dq_2} = 0$, then we obtain $q_1(\lambda) = \frac{\delta_1+\lambda-\Delta\alpha}{3\beta}$ and $q_2(\lambda) = \frac{\delta_1-2\lambda+2\Delta\alpha}{3\beta}$. Replacing $q_1(\lambda)$ and $q_2(\lambda)$ in (11), we can get $ln\pi^l(\lambda,M) = \theta\, ln\pi_1^l(\lambda,M) + (1-\theta)ln\pi_2^l(\lambda,M)$. $\frac{\partial\pi^l(\lambda,M)}{\partial\lambda} = \pi^l(\lambda,M)[\theta\frac{1}{\pi_1^l(\lambda,M)}\frac{\partial\pi_1^l(\lambda,M)}{\partial\lambda} + (1-\theta)\frac{1}{\pi_2^l(\lambda,M)}\frac{\partial\pi_2^l(\lambda,M)}{\partial\lambda}]$ and $\frac{\partial\pi^l(\lambda,M)}{\partial M} = \pi^l(\lambda,M)[\theta\frac{1}{\pi_1^l(\lambda,M)}\frac{\partial\pi_1^l(\lambda,M)}{\partial M} + (1-\theta)\frac{1}{\pi_2^l(\lambda,M)}\frac{\partial\pi_2^l(\lambda,M)}{\partial M}]$, we get three roots by $\frac{\partial\pi^l(\lambda,M)}{\partial\lambda} = \frac{\partial\pi^l(\lambda,M)}{\partial M} = 0$. $\lambda_1 = \frac{1}{2}\delta_1 - 2\Delta\alpha$ and $M_1 = \frac{12\Delta\alpha^2-\delta_1^2-4K\beta\lambda_0+\theta(\delta_1^2+8K\beta\lambda_0+4\Delta\alpha^2)}{4\beta}$; $\lambda_2 = \lambda_1 - \frac{3}{2}\sqrt{4\Delta\alpha^2+\delta_1^2+8K\beta\lambda_0}$ and $M_2 = \frac{8\Delta\alpha^2+\delta_1^2+9K\beta\lambda_0+4\Delta\alpha\sqrt{4\Delta\alpha^2+\delta_1^2+8K\beta\lambda_0}}{\beta}$; $\lambda_3 = \lambda_1 + \frac{3}{2}\sqrt{4\Delta\alpha^2+\delta_1^2+8K\beta\lambda_0}$ and $M_3 = \frac{8\Delta\alpha^2+\delta_1^2+9K\beta\lambda_0-4\Delta\alpha\sqrt{4\Delta\alpha^2+\delta_1^2+8K\beta\lambda_0}}{\beta}$. Obviously, $\lambda_2 < \lambda_1 < \lambda_3$. Recall $q_1(\lambda) = \frac{\delta_1+\lambda-\Delta\alpha}{3\beta} \geq 0$ and $q_2(\lambda) = \frac{\delta_1-2\lambda+2\Delta\alpha}{3\beta} \geq 0$, we get $max\{0,(\Delta\alpha-\delta_1)\} < \lambda \leq \frac{1}{2}(\delta_1+2\Delta\alpha)$. Since $\lambda_3 - \frac{1}{2}(\delta_1+2\Delta\alpha) = \frac{3}{2}\left(\sqrt{4\Delta\alpha^2+\delta_1^2+8K\beta\lambda_0} - 2\Delta\alpha\right) > 0$, then we omit (λ_3, M_3).

Now, considering (λ_1, M_1) and (λ_2, M_2). If $\lambda_1 = \frac{1}{2}\delta_1 - 2\Delta\alpha \leq 0$, then $\lambda_2 < \lambda_1 \leq 0$. Thus, that means if $\Delta\alpha \geq \frac{1}{4}\delta_1$, then we should omit (λ_1, M_1) and (λ_2, M_2).

If $\lambda_1 > 0$(i.e., $\Delta\alpha < \frac{1}{4}\delta_1$), then $(\delta_1 - 4\Delta\alpha)^2 - \left(3\sqrt{4\Delta\alpha^2+\delta_1^2+8K\beta\lambda_0}\right)^2 = -4\left(5\Delta\alpha^2 + 2\Delta\alpha\delta_1 + 2\delta_1^2 + 18K\beta\lambda_0\right) < 0$ indicates $\lambda_2 < 0$, so we omit (λ_2, M_2) at this condition. Next, we will continue to compare λ_1 with $\frac{1}{2}(\delta_1+2\Delta\alpha)$. $\lambda_1 - \frac{1}{2}(\delta_1+2\Delta\alpha) = -3\Delta\alpha$. If $\Delta\alpha < 0$, then $\lambda_1 > \frac{1}{2}(\delta_1+2\Delta\alpha)$, so we should omit (λ_1, M_1) at this time; if $\Delta\alpha > 0$, then we obtain $0 < \lambda_1 < \frac{1}{2}(\delta_1+2\Delta\alpha)$. At this time, we have to satisfy $M_1 =$

$\frac{(\delta_1^2+8K\beta\lambda_0+4\Delta\alpha^2)\theta+12\Delta\alpha^2-\delta_1^2-4K\beta\lambda_0}{4\beta}>0$. Let $g(\theta)=(\delta_1^2+8K\beta\lambda_0+4\Delta\alpha^2)\theta+12\Delta\alpha^2-\delta_1^2-4K\beta\lambda_0$ and we derive a root $\theta_1=\frac{\delta_1^2+4K\beta\lambda_0-12\Delta\alpha^2}{\delta_1^2+8K\beta\lambda_0+4\Delta\alpha^2}$. According to $0<\Delta\alpha<\frac{1}{4}\delta_1$, we have $0<\theta_1<1$. Thus, if $\theta_1<\theta\leq 1$ then $M_1>0$.

According to the second partial derivative, we have $A\overset{\Delta}{=}\frac{\partial^2(\pi^l(\lambda,M))}{\partial\lambda^2}|_{\lambda=\lambda_1,M=M_1}=\frac{8\{4\Delta\alpha^2[-1+\frac{32}{(-1+\theta)\theta}]-(\delta_1^2+8K\beta\lambda_0)\}}{9(4\Delta\alpha^2+\delta_1^2+8K\beta\lambda_0)^2}\pi^l(\lambda_1,M_1)<0$; $B\overset{\Delta}{=}\frac{\partial^2(\pi^l(\lambda,M))}{\partial\lambda\partial M}|_{\lambda=\lambda_1,M=M_1}=\frac{128\beta\Delta\alpha}{3(-1+\theta)\theta(4\Delta\alpha^2+\delta_1^2+8K\beta\lambda_0)^2}\pi^l(\lambda_1,M_1)$ and $C\overset{\Delta}{=}\frac{\partial^2(\pi^l(\lambda,M))}{\partial M^2}|_{\lambda=\lambda_1,M=M_1}=\frac{16\beta^2}{(-1+\theta)\theta(4\Delta\alpha^2+\delta_1^2+8K\beta\lambda_0)^2}\pi^l(\lambda_1,M_1)$. Since $AC-B^2=\frac{128\beta^2}{9(1-\theta)\theta(4\Delta\alpha^2+\delta_1^2+8K\beta\lambda_0)^3}\pi^l(\lambda_1,M_1)^2>0$, then (λ_1,M_1) is the optimal value of $\pi^l(\lambda,M)$.

In summary, if $0<\Delta\alpha<\frac{1}{4}\delta_1$ and $\theta_1<\theta\leq 1$, then (λ_1,M_1) is the optimal value of $\pi^l(\lambda,M)$. Replacing (λ_1,M_1) in $q_1(\lambda)$ and $q_2(\lambda)$, so we get $q_1^l=\frac{\delta_1-2\Delta\alpha}{2\beta}$ and $q_2^l=\frac{2\Delta\alpha}{\beta}$. Recall that $p_i=\alpha_i-\beta(q_1+q_2)$, we obtain $p_1^l=c+\lambda_0e_1+\beta q_1^l$ and $p_2^l=c+\lambda_0e_1+\frac{1}{2}\delta_1$.

Derivation of Table 4.4

(1) Competition model

From (4-1) and (4-2), we obtain $\frac{d^2\pi_1^n(q_1)}{dq_1^2}=\frac{d^2\pi_2^n(q_2)}{dq_2^2}=-2\beta$, so $\pi_1^n(q_1)$ and $\pi_2^n(q_2)$ are concave functions of q_1 and q_2 respectively. Thus, we obtain $q_1^n=\frac{2\beta\delta_1-\gamma(\delta_1-\Delta e\lambda_0)}{4\beta^2-\gamma^2}$ and $q_2^n=\frac{2\beta(\delta_1-\Delta e\lambda_0)-\gamma\delta_1}{4\beta^2-\gamma^2}$ by $\frac{d\pi_1^n(q_1)}{dq_1}=\frac{d\pi_2^n(q_2)}{dq_2}=0$. Recall that $p_i=\alpha-\beta q_i-\gamma q_j$, we obtain $p_1^n=c+\lambda_0e_1+\beta q_1^n$ and $p_2^n=c+\lambda_0e_2+\beta q_2^n$. Since $0<\gamma\leq\beta$, we get $q_1^n>0$. To have $q_2^n=\frac{2\beta(\delta_1-\Delta e\lambda_0)-\gamma\delta_1}{4\beta^2-\gamma^2}>0$, we should satisfy $0<\Delta e<\Delta e^n$, where $\Delta e^n=\frac{(2\beta-\gamma)\delta_1}{2\beta\lambda_0}$.

(2) RL coopetition model

From (4-3) and (4-4), we get $\frac{d^2\pi_1^r(q_1)}{dq_1^2}=\frac{d^2\pi_2^r(q_2)}{dq_2^2}=-2\beta$, so $\pi_1^r(q_1)$ and $\pi_2^r(q_2)$ are concave functions of q_1 and q_2 respectively. Let $\frac{d\pi_1^r(q_1)}{dq_1}=\frac{d\pi_2^r(q_2)}{dq_2}=0$, we obtain $q_1(\lambda)=\frac{(2\beta-\gamma)\delta_1+\gamma\lambda}{4\beta^2-\gamma^2}$ and $q_2(\lambda)=\frac{(2\beta-\gamma)\delta_1-2\beta\lambda}{4\beta^2-\gamma^2}$. Replacing $q_1(\lambda)$ and $q_2(\lambda)$ in (4-5), we can get $\frac{d\pi^r(\lambda)}{d\lambda}=\pi^r(\lambda)[\theta\frac{1}{\pi_1^r(q_1(\lambda))}\frac{d\pi_1^r(q_1(\lambda))}{d\lambda}+(1-\theta)\frac{1}{\pi_2^r(q_2(\lambda))}\frac{d\pi_2^r(q_2(\lambda))}{d\lambda}]$. $\frac{d\pi^r(\lambda)}{d\lambda}=0$ is equivalent to $\Phi(\lambda)=0$, where $\Phi(\lambda)=8\beta^4(8\beta^2-3\gamma^2)\lambda^3+4\delta_1\lambda^2\beta^3(2\beta-\gamma)[2\theta(\gamma^2+\beta\gamma-2\beta^2)+5\gamma^2-16\beta^2-4\beta\gamma]+2\beta\lambda(\gamma-2\beta)^2[3\theta(4\beta^2-\gamma^2)(\beta\delta_1^2+$

$4K\beta^2\lambda_0 + 4K\beta\gamma\lambda_0 + K\gamma^2\lambda_0) + 2\beta(2\beta^2\delta_1^2 + 2\beta\gamma\delta_1^2 - \gamma^2\delta_1^2 - 2K\lambda_0\beta(2\beta+\gamma)^2)] + \delta_1(\gamma - 2\beta)^3[(8\beta^2 + 2\beta\gamma - \gamma^2)\theta - 4\beta^2][K\lambda_0(\gamma+2\beta)^2 + \beta\delta_1^2]$, so we can get three roots and $\Phi(\lambda^r) = 0$. Recall $q_2(\lambda) = \frac{(2\beta-\gamma)\delta_1 - 2\beta\lambda}{4\beta^2-\gamma^2} \geq 0$, we get $0 < \lambda \leq \frac{(2\beta-\gamma)\delta_1}{2\beta}$.

(3) FL coopetition model

From (4-6) and (4-7), we get $\frac{d^2\pi_1^f(q_1)}{dq_1^2} = \frac{d^2\pi_2^f(q_2)}{dq_2^2} = -2\beta$, so $\pi_1^f(q_1)$ and $\pi_2^f(q_2)$ are concave functions of q_1 and q_2 respectively. Let $\frac{d\pi_1^f(q_1)}{dq_1} = \frac{d\pi_2^f(q_2)}{dq_2} = 0$, we obtain $q_1^f = \frac{\delta_1}{2\beta+\gamma}$ and $q_2^f = \frac{\delta_1}{2\beta+\gamma}$, so we get $p_1^f = c + \lambda_0 e_1 + \beta q_1^f$ and $p_2^f = c + \lambda_0 e_1 + \beta q_2^f$. Replacing q_1^f and q_2^f in (4-8), we obtain $\frac{d\pi^f(M)}{dM} = \pi^f(M)[\theta\frac{1}{\pi_1^f(M)}\frac{d\pi_1^f(M)}{dM} + (1-\theta)\frac{1}{\pi_2^f(M)}\frac{d\pi_2^f(M)}{dM}]$. $\frac{d\pi^f(M)}{dM} = 0$ derives a root $M^f = \frac{(2\theta-1)[\beta\delta_1^2 + (4K\beta^2+4K\beta\gamma+K\gamma^2)\lambda_0]}{(2\beta+\gamma)^2}$. $M^f > 0$ means $\frac{1}{2} < \theta \leq 1$.

(4) ML coopetition model

From (4-9) and (4-10), we get $\frac{d^2\pi_1^l(q_1)}{dq_1^2} = \frac{d^2\pi_2^l(q_2)}{dq_2^2} = -2\beta$, so $\pi_1^l(q_1)$ and $\pi_2^l(q_2)$ are concave functions of q_1 and q_2 respectively. Let $\frac{d\pi_1^l(q_1)}{dq_1} = \frac{d\pi_2^l(q_2)}{dq_2} = 0$, then we obtain $q_1(\lambda) = \frac{(2\beta-\gamma)\delta_1 + \gamma\lambda}{4\beta^2-\gamma^2}$ and $q_2(\lambda) = \frac{(2\beta-\gamma)\delta_1 - 2\beta\lambda}{4\beta^2-\gamma^2}$. Replacing $q_1(\lambda)$ and $q_2(\lambda)$ in (11), we can get $ln\pi^l(\lambda, M) = \theta\, ln\pi_1^l(\lambda, M) + (1-\theta)ln\pi_2^l(\lambda, M)$. $\frac{\partial\pi^l(\lambda,M)}{\partial\lambda} = \pi^l(\lambda, M)[\theta\frac{1}{\pi_1^l(\lambda,M)}\frac{\partial\pi_1^l(\lambda,M)}{\partial\lambda} + (1-\theta)\frac{1}{\pi_2^l(\lambda,M)}\frac{\partial\pi_2^l(\lambda,M)}{\partial\lambda}]$ and $\frac{\partial\pi^l(\lambda,M)}{\partial M} = \pi^l(\lambda, M)[\theta\frac{1}{\pi_1^l(\lambda,M)}\frac{\partial\pi_1^l(\lambda,M)}{\partial M} + (1-\theta)\frac{1}{\pi_2^l(\lambda,M)}\frac{\partial\pi_2^l(\lambda,M)}{\partial M}]$, we get three roots by $\frac{\partial\pi^l(\lambda,M)}{\partial\lambda} = \frac{\partial\pi^l(\lambda,M)}{\partial M} = 0$. $\lambda_1 = \frac{(2\beta-\gamma)^2\gamma\delta_1}{2\beta(4\beta^2-3\gamma^2)}$ and $M_1 = K\lambda_0(2\theta - 1) + \frac{f(\theta)}{4\beta(4\beta^2-3\gamma^2)^2}$; $\lambda_2 = \lambda_1 + \frac{(4\beta^2-\gamma^2)}{2\beta(4\beta^2-3\gamma^2)}\sqrt{h(\gamma)}]$ and $M_2 = \frac{1}{(4\beta^2-3\gamma^2)^2}[12\beta^3\delta_1^2 - 16\beta^2\gamma\delta_1^2 + 5\beta\gamma^2\delta_1^2 + 48K\beta^4\lambda_0 - 48K\beta^2\gamma^2\lambda_0 + 9K\gamma^4\lambda_0 + (4\beta\gamma\delta_1 - 4\beta^2\delta_1)\sqrt{h(\gamma)}]$; $\lambda_3 = \lambda_1 - \frac{(4\beta^2-\gamma^2)}{2\beta(4\beta^2-3\gamma^2)}\sqrt{h(\gamma)}$ and $M_3 = \frac{1}{(4\beta^2-3\gamma^2)^2}[12\beta^3\delta_1^2 - 16\beta^2\gamma\delta_1^2 + 5\beta\gamma^2\delta_1^2 + 48K\beta^4\lambda_0 - 48K\beta^2\gamma^2\lambda_0 + 9K\gamma^4\lambda_0 + (4\beta^2\delta_1 - 4\beta\gamma\delta_1)\sqrt{h(\gamma)}]$. Where $f(\theta) = \delta_1^2\{4\beta^2\gamma^2(9-5\theta) + 3\gamma^3(\gamma - 8\beta)(1-\theta) - 16\beta^3[2\gamma\theta - \beta(2\theta-1)]\}$ and $h(\gamma) = 8\beta^2\delta_1^2 - 8\beta\gamma\delta_1^2 + \gamma^2\delta_1^2 + 32K\beta^3\lambda_0 - 24K\beta\gamma^2\lambda_0$. Recall $q_2(\lambda) = \frac{(2\beta-\gamma)\delta_1 - 2\beta\lambda}{4\beta^2-\gamma^2} \geq 0$, we get $0 < \lambda \leq \frac{(2\beta-\gamma)\delta_1}{2\beta}$. $\lambda_2 - \frac{(2\beta-\gamma)\delta_1}{2\beta} = \frac{(4\beta^2-\gamma^2)[2(\gamma-\beta)\delta_1 + \sqrt{(8\beta^2-8\beta\gamma+\gamma^2)\delta_1^2 + 8K\beta(4\beta^2-3\gamma^2)\lambda_0}]}{8\beta^3-6\beta\gamma^2} > 0$, so we omit (λ_2, M_2). $\lambda_3 = \frac{(2\beta-\gamma)[\gamma(2\beta-\gamma)\delta_1 - (2\beta+\gamma)\sqrt{(8\beta^2-8\beta\gamma+\gamma^2)\delta_1^2 + 8K\beta(4\beta^2-3\gamma^2)\lambda_0}]}{8\beta^3-6\beta\gamma^2} < 0$, so we omit (λ_3, M_3). For (λ_1, M_1), we can easily get $0 < \lambda_1 < \frac{(2\beta-\gamma)\delta_1}{2\beta}$. $M_1 = K\lambda_0(2\theta-1) + \frac{f(\theta)}{4\beta(4\beta^2-3\gamma^2)^2} = -16\beta^4\delta_1^2 + 36\beta^2\gamma^2\delta_1^2 - 24\beta\gamma^3\delta_1^2 + 3\gamma^4\delta_1^2 - 64K\beta^5\lambda_0 + 96K\beta^3\gamma^2\lambda_0 - 36K\beta\gamma^4\lambda_0 + \theta(4\beta^2 - 3\gamma^2)[(8\beta^2 - 8\beta\gamma + \gamma^2)\delta_1^2 + 8K\beta\lambda_0(4\beta^2 - 3\gamma^2)]$, which increases in θ, so we get a root $\theta_1 = \frac{(16\beta^4-36\beta^2\gamma^2+24\beta\gamma^3-3\gamma^4)\delta_1^2+4K\beta(4\beta^2-3\gamma^2)^2\lambda_0}{(4\beta^2-3\gamma^2)((8\beta^2-8\beta\gamma+\gamma^2)\delta_1^2+8K\beta(4\beta^2-3\gamma^2)\lambda_0)}$. If $\theta_1 < \theta \leq 1$, then $M_1 > 0$.

Chapter 5

Derivation of Table 5.2

(1) Competition model: From (5-1), we obtain $\frac{d^2\pi_1^n(p_1)}{d{p_1}^2} = -2(1+\beta) < 0$, then $\pi_1^n(p_1)$ is a concave function of p_1. Similarly, From (5-2), we obtain $\frac{d^2\pi_2^n(p_2)}{d{p_2}^2} = -2(1+\beta) < 0$, then $\pi_2^n(q_2)$ is a concave function of q_2. Let $\frac{d\pi_1^n(p_1)}{dp_1} = \frac{d\pi_2^n(p_2)}{dp_2} = 0$, we obtain $p_1^n = c + w + \frac{\tau\Delta s\beta(1+\beta)+\delta(2+3\beta)}{(2+\beta)(2+3\beta)}$ and $p_2^n = c + w + \frac{\delta(2+3\beta)-\tau\Delta s(2+4\beta+\beta^2)}{(2+\beta)(2+3\beta)}$. Recall that $q_i = \alpha - p_i + \beta(p_j - p_i) + \tau\left[s_i - \beta(s_j - s_i)\right]$, then $q_1^n = \frac{(1+\beta)[\delta(2+3\beta)+\beta(1+\beta)\tau\Delta s]}{(2+\beta)(2+3\beta)} = (1+\beta)(p_1^n - w - c)$ and $q_2^n = \frac{(1+\beta)[\delta(2+3\beta)-(2+4\beta+\beta^2)\tau\Delta s]}{(2+\beta)(2+3\beta)} = (1+\beta)(p_2^n - w - c)$.

(2) OC model: For given m, from (5-3) we obtain $\frac{d^2\pi_1^o(p_1)}{d{p_1}^2} = -2(1+\beta) < 0$, then $\pi_1^o(p_1)$ is a concave function of p_1. Similarly, from (5-4), we obtain $\frac{d^2\pi_2^o(p_2)}{d{p_2}^2} = -2(1+\beta) < 0$, then $\pi_2^o(q_2)$ is a concave function of q_2. Let $\frac{d\pi_1^o(p_1)}{dp_1} = \frac{d\pi_2^o(p_2)}{dp_2} = 0$, we obtain $p_1 = \frac{2s_1\tau+2\alpha+3m\beta+3s_1\tau\beta+3\alpha\beta+3m\beta^2+2c(1+\beta)+w(2+5\beta+3\beta^2)}{4+8\beta+3\beta^2}$ and $p_2 = \frac{2s_1\tau+2\alpha+c\beta+3s_1\tau\beta+3\alpha\beta+m(2+4\beta+3\beta^2)+w(2+5\beta+3\beta^2)}{4+8\beta+3\beta^2}$. Replace $p_1 = \frac{2s_1\tau+2\alpha+3m\beta+3s_1\tau\beta+3\alpha\beta+3m\beta^2+2c(1+\beta)+w(2+5\beta+3\beta^2)}{4+8\beta+3\beta^2}$ and $p_2 = \frac{2s_1\tau+2\alpha+c\beta+3s_1\tau\beta+3\alpha\beta+m(2+4\beta+3\beta^2)+w(2+5\beta+3\beta^2)}{4+8\beta+3\beta^2}$ in (5-5), we obtain $\pi^{om}(m) = (\pi_1^o)^\theta(\pi_2^o)^{1-\theta} = \left\{\frac{1}{(2+\beta)^2(2+3\beta)^2}[\delta^2(1+\beta)(2+3\beta)^2 + \delta m(8+40\beta+72\beta^2+57\beta^3+18\beta^4) - c^2(8+40\beta+73\beta^2+59\beta^3+18\beta^4) - m^2(8+40\beta+73\beta^2+59\beta^3+18\beta^4) - c(1+2\beta)(\delta(8+24\beta+24\beta^2+9\beta^3) - 2m(8+24\beta+25\beta^2+9\beta^3))]\right\}^\theta \left\{\frac{1}{(2+\beta)^2(2+3\beta)^2}[(1+\beta)(\delta(2+3\beta) + c(2+4\beta) - 2(m+2m\beta))^2]\right\}^{1-\theta}$. Let $\pi^{om}(m) = (\pi_1^o)^\theta(\pi_2^o)^{1-\theta} = G(m)$, then $\ln G(m) = \theta \ln \pi_1^o + (1-\theta) \ln \pi_2^o$ and $\frac{1}{G(m)}\frac{dG(m)}{dm} = \theta\frac{1}{\pi_1^o}\frac{d\pi_1^o}{dm} + (1-\theta)\frac{1}{\pi_2^o}\frac{d\pi_2^o}{dm}$, so $\frac{dG(m)}{dm} = G(m)\left[\theta\frac{1}{\pi_1^o}\frac{d\pi_1^o}{dm} + (1-\theta)\frac{1}{\pi_2^o}\frac{d\pi_2^o}{dm}\right]$. Let $\frac{dG(m)}{dm} = 0$ and recalling $G(m) > 0$, then $\theta\frac{1}{\pi_1^o}\frac{d\pi_1^o}{dm} + (1-\theta)\frac{1}{\pi_2^o}\frac{d\pi_2^o}{dm} = \{(1+2\beta)[4c^2(8+40\beta+73\beta^2+59\beta^3+18\beta^4) + 4m^2(8+40\beta+73\beta^2+59\beta^3+18\beta^4) + \delta^2(2+3\beta)^2(-4+8\theta+3\beta^2\theta+2\beta(-2+5\theta)) - 2\delta m(2+3\beta)(4(2+\theta)+10\beta^2(3+\theta)+3\beta^3(4+\theta)+2\beta(14+5\theta)) + 2c(-4m(8+40\beta+73\beta^2+59\beta^3+18\beta^4) + \delta(2+3\beta)(4(2+\theta)+10\beta^2(3+\theta)+3\beta^3(4+\theta)+2\beta(14+5\theta)))]\}\{[\delta(2+3\beta) + c(2+4\beta) - 2(m+2m\beta)][\delta^2(1+\beta)(2+3\beta)^2 + \delta m(8+40\beta+72\beta^2+57\beta^3+18\beta^4) - c^2(8+40\beta+73\beta^2+59\beta^3+18\beta^4) - m^2(8+40\beta+73\beta^2+59\beta^3+18\beta^4) - c(1+2\beta)(\delta(8+24\beta+24\beta^2+9\beta^3) + 2m(8+

$24\beta+25\beta^2+9\beta^3))]\}^{-1}=0$, and we obtain two real roots: $m_1=c+\frac{1}{4(8+40\beta+73\beta^2+59\beta^3+18\beta^4)}\{16\delta+80\delta\beta+144\delta\beta^2+114\delta\beta^3+36\delta\beta^4+8\delta\theta+32\delta\beta\theta+50\delta\beta^2\theta+36\delta\beta^3\theta+9\delta\beta^4\theta-\delta(4+8\beta+3\beta^2)A\}$ and $m_2=c+\frac{1}{4(8+40\beta+73\beta^2+59\beta^3+18\beta^4)}\{16\delta+80\delta\beta+144\delta\beta^2+114\delta\beta^3+36\delta\beta^4+8\delta\theta+32\delta\beta\theta+50\delta\beta^2\theta+36\delta\beta^3\theta+9\delta\beta^4\theta+\delta(4+8\beta+3\beta^2)A\}>0$, where $A=[(48+256\beta+500\beta^2+432\beta^3+144\beta^4)(1-\theta)+(4+16\beta+28\beta^2+24\beta^3+9\beta^4)\theta^2]^{\frac{1}{2}}$.

Recall that $m>c$, then we check if $m_1>c$ and $m_2>c$. $m_2-c=\frac{1}{4(8+40\beta+73\beta^2+59\beta^3+18\beta^4)}\{16\delta+80\delta\beta+144\delta\beta^2+114\delta\beta^3+36\delta\beta^4+8\delta\theta+32\delta\beta\theta+50\delta\beta^2\theta+36\delta\beta^3\theta+9\delta\beta^4\theta+\delta(4+8\beta+3\beta^2)A\}>0$ implies $m_2>c$. Similarly, $m_1-c=\frac{1}{4(8+40\beta+73\beta^2+59\beta^3+18\beta^4)}\{16\delta+80\delta\beta+144\delta\beta^2+114\delta\beta^3+36\delta\beta^4+8\delta\theta+32\delta\beta\theta+50\delta\beta^2\theta+36\delta\beta^3\theta+9\delta\beta^4\theta-\delta(4+8\beta+3\beta^2)A\}$ and $(16\delta+80\delta\beta+144\delta\beta^2+114\delta\beta^3+36\delta\beta^4+8\delta\theta+32\delta\beta\theta+50\delta\beta^2\theta+36\delta\beta^3\theta+9\delta\beta^4\theta)^2-\delta^2(4+8\beta+3\beta^2)^2A^2=4\delta^2(2+3\beta)^2(8+40\beta+73\beta^2+59\beta^3+18\beta^4)(-4-4\beta+8\theta+10\beta\theta+3\beta^2\theta)$. Let $-4-4\beta+8\theta+10\beta\theta+3\beta^2\theta=0$ and we obtain $\theta_3=\frac{4(1+\beta)}{8+10\beta+3\beta^2}$ and $0<\theta_3<1$. Therefore, if $0<\theta<\theta_3$, then $-4-4\beta+8\theta+10\beta\theta+3\beta^2\theta<0$ and $m_1<c$; if $\theta_3<\theta<1$, then $-4-4\beta+8\theta+10\beta\theta+3\beta^2\theta>0$ and $m_1>c$.

Replace m_2 in $p_1=m+w+\frac{c-3m+2s_1-2w+2\alpha}{4+2\beta}+\frac{c-m}{4+6\beta}$ and $p_2=\frac{2(m+s_1+w+\alpha)+(c+4m+3s_1+5w+3\alpha)\beta+3(m+w)\beta^2}{(2+\beta)(2+3\beta)}$, we obtain $p_1^o|_{m=m_2}=c+w+\frac{\delta\{(4+8\beta+3\beta^2)(16+68\beta+90\beta^2+36\beta^3+6\beta\theta+12\beta^2\theta+9\beta^3\theta)+3\beta(4+8\beta+3\beta^2)A\}}{4(2+\beta)(1+2\beta)(2+3\beta)(8+16\beta+9\beta^2)}>0$ and $p_2^o|_{m=m_2}=c+w+\frac{\delta\{(4+8\beta+3\beta^2)(24+112\beta+188\beta^2+138\beta^3+36\beta^4+4\theta+16\beta\theta+28\beta^2\theta+24\beta^3\theta+9\beta^4\theta)+(2+4\beta+3\beta^2)(4+8\beta+3\beta^2)A\}}{4(2+\beta)(1+2\beta)(2+3\beta)(8+24\beta+25\beta^2+9\beta^3)}>0$. Recall that $q_i=\alpha-p_i+\beta(p_j-p_i)+\tau s_1$, then $q_2^o|_{m=m_2}=\frac{\delta\{(8+32\beta+50\beta^2+36\beta^3+9\beta^4)(2-\theta)-(4+8\beta+3\beta^2)A\}}{2(2+\beta)(2+3\beta)(8+16\beta+9\beta^2)}$. $[(8+32\beta+50\beta^2+36\beta^3+9\beta^4)(2-\theta)]^2-(4+8\beta+3\beta^2)^2A^2=4\delta^2(4+8\beta+3\beta^2)^2(8+48\beta+97\beta^2+84\beta^3+27\beta^4)(\theta-1)<0$ implies that $q_2^o|_{m=m_2}<0$.

$$\pi_1^o|_{m=m_1}=\frac{\delta\theta\{-\delta(4+8\beta+3\beta^2)[-24-128\beta-250\beta^2-216\beta^3-72\beta^4+(4+16\beta+28\beta^2+24\beta^3+9\beta^4)\theta]+(2+4\beta+3\beta^2)\delta(4+8\beta+3\beta^2)A\}}{8(2+\beta)(1+2\beta)(2+3\beta)(8+24\beta+25\beta^2+9\beta^3)}$$

and for $\beta>0$ and $0<\theta<1$, we obtain $-24-128\beta-250\beta^2-216\beta^3-72\beta^4+(4+16\beta+28\beta^2+24\beta^3+9\beta^4)\theta<0$. Therefore, $\pi_1^o|_{m=m_1}>0$. Similarly, we can obtain $\pi_2^o|_{m=m_1}>0$.

$\frac{d^2G(m)}{dm^2}|_{m=m_1}=G(m_1)\frac{d\left[\theta\frac{1}{\pi_1^o}\frac{d\pi_1^o}{dm}+(1-\theta)\frac{1}{\pi_2^o}\frac{d\pi_2^o}{dm}\right]}{dm}|_{m=m_1}$ and since $\pi_1^o|_{m=m_1}>0$ and $\pi_2^o|_{m=m_1}>0$, then $G(m_1)>0$. $\frac{d\left[\theta\frac{1}{\pi_1^o}\frac{d\pi_1^o}{dm}+(1-\theta)\frac{1}{\pi_2^o}\frac{d\pi_2^o}{dm}\right]}{dm}|_{m=m_1}=\{64\delta(1+2\beta)^2(4+8\beta+3\beta^2)(8+24\beta+25\beta^2+9\beta^3)^2[-(2+4\beta+3\beta^2)\delta(4+8\beta+3\beta^2)\,A(16\beta(16-13\theta+\theta^2)+9\beta^4(16-13\theta+\theta^2)+4(12-10\theta+\theta^2)+$

$12\beta^3(36-29\theta+2\theta^2)+\beta^2(500-403\theta+28\theta^2))+\delta(4+8\beta+3\beta^2)(81\beta^8(-80+96\theta-21\theta^2+\theta^3)+16(-48+60\theta-16\theta^2+\theta^3)+64\beta(-124+152\theta-37\theta^2+2\theta^3)+108\beta^7(-356+424\theta-89\theta^2+4\theta^3)+32\beta^3(-2888+34452\theta-739\theta^2+34\theta^3)+24\beta^5(-6464+76660\theta-1563\theta^2+68\theta^3)+9\beta^6(-11284+133384\theta-2745\theta^2+120\theta^3)+4\beta^2(-8956+108166\theta-2445\theta^2+120\theta^3)+4\beta^4(-37329+443345\theta-9176\theta^2+406\theta^3))]\}/\{\theta[-\delta(8+32\beta+50\beta^2+36\beta^3+9\beta^4)(-2+\theta)+\delta(4+8\beta+3\beta^2)A]^2[\delta(4+8\beta+3\beta^2)(24\beta^3(-9+\theta)+16\beta(-8+\theta)+9\beta^4(-8+\theta)+4(-6+\theta)+\beta^2(-250+28\theta))-(2+4\beta+3\beta^2)\delta(4+8\beta+3\beta^2)A]^2\}$ and we obtain that for $\beta>0$ and $0<\theta<1$, $\frac{d^2G(m)}{dm^2}|_{m=m_1}<0$.

Replace m_1 in $p_1=m+w+\frac{c-3m+2s_1-2w+2\alpha}{4+2\beta}+\frac{c-m}{4+6\beta}$ and $p_2=\frac{2(m+s_1+w+\alpha)+(c+4m+3s_1+5w+3\alpha)\beta+3(m+w)\beta^2}{(2+\beta)(2+3\beta)}$, we obtain $p_1^o|_{m=m_1}=c+w+\frac{\delta(4+8\beta+3\beta^2)\{(16+68\beta+90\beta^2+36\beta^3+6\beta\theta+12\beta^2\theta+9\beta^3\theta)-3\beta A\}}{4(2+\beta)(1+2\beta)(2+3\beta)(8+16\beta+9\beta^2)}>0$ and $p_2^o|_{m=m_1}=c+w+\frac{\delta(4+8\beta+3\beta^2)\{(24+112\beta+188\beta^2+138\beta^3+36\beta^4+4\theta+16\beta\theta+28\beta^2\theta+24\beta^3\theta+9\beta^4\theta)-(2+4\beta+3\beta^2)A\}}{4(2+\beta)(1+2\beta)(2+3\beta)(8+24\beta+25\beta^2+9\beta^3)}>0$. Recall that $q_i=\alpha-p_i+\beta(p_j-p_i)+\tau s_1$, then $q_1^o|_{m=m_1}=\frac{\delta(4+8\beta+3\beta^2)\{(16+52\beta+58\beta^2+24\beta^3-2\beta\theta-4\beta^2\theta-3\beta^3\theta)+\beta A\}}{4(2+\beta)(2+3\beta)(8+24\beta+25\beta^2+9\beta^3)}>0$ and $q_2^o|_{m=m_1}=\frac{\delta\{(8+32\beta+50\beta^2+36\beta^3+9\beta^4)(2-\theta)+(4+8\beta+3\beta^2)A\}}{2(2+\beta)(2+3\beta)(8+16\beta+9\beta^2)}>0$. Therefore, if $\theta_3=\frac{4(1+\beta)}{8+10\beta+3\beta^2}<\theta<1$, then $m^o=m_1=c+\frac{1}{4(8+40\beta+73\beta^2+59\beta^3+18\beta^4)}\{16\delta+80\delta\beta+144\delta\beta^2+114\delta\beta^3+36\delta\beta^4+8\delta\theta+32\delta\beta\theta+50\delta\beta^2\theta+36\delta\beta^3\theta+9\delta\beta^4\theta-\delta(4+8\beta+3\beta^2)A\}$. $p_1^o=c+w+\frac{\delta}{4(8+32\beta+41\beta^2+18\beta^3)}\{16+9\beta^3(4+\theta)+6\beta^2(15+2\theta)+\beta(68+6\theta)+3\beta A\}$, $p_2^o=c+w+\frac{\delta}{4(1+2\beta)(8+24\beta+25\beta^2+9\beta^3)}\{9\beta^4(4+\theta)+4(6+\theta)+16\beta(7+\theta)+6\beta^3(23+4\theta)+4\beta^2(47+7\theta)-(2+4\beta+3\beta^2)A\}$, $q_1^o=\frac{\delta(4+8\beta+3\beta^2)}{4(2+\beta)(2+3\beta)(8+24\beta+25\beta^2+9\beta^3)}\{(16+52\beta+58\beta^2+24\beta^3-2\beta\theta-4\beta^2\theta-3\beta^3\theta)+\beta A\}$ and $q_2^o=\frac{\delta}{2(2+\beta)(2+3\beta)(8+16\beta+9\beta^2)}\{(8+32\beta+50\beta^2+36\beta^3+9\beta^4)(2-\theta)+(4+8\beta+3\beta^2)A\}$, where $A=[(48+256\beta+500\beta^2+432\beta^3+144\beta^4)(1-\theta)+(4+16\beta+28\beta^2+24\beta^3+9\beta^4)\theta^2]^{\frac{1}{2}}$.

(3) MC model: From (5-5) we obtain $\frac{d^2\pi_1^m(p_1)}{dp_1{}^2}=-2(1+\beta)<0$, then $\pi_1^m(p_1)$ is a concave function of p_1. Similarly, from (5-6), we obtain $\frac{d^2\pi_2^m(p_2)}{dp_2{}^2}=-2(1+\beta)<0$, then $\pi_2^m(p_2)$ is a concave function of p_2. Let $\frac{d\pi_1^m(p_1)}{dp_1}=\frac{d\pi_2^m(p_2)}{dp_2}=0$ and we obtain $p_1=\frac{2w+2\alpha+3u\beta+5w\beta+3\alpha\beta+3u\beta^2+3w\beta^2+\tau s_1(2+3\beta)+c(2+5\beta+3\beta^2)}{4+8\beta+3\beta^2}$ and $p_2=\frac{2u+2w+2\alpha+4u\beta+5w\beta+3\alpha\beta+3u\beta^2+3w\beta^2+\tau s_1(2+3\beta)+c(2+5\beta+3\beta^2)}{4+8\beta+3\beta^2}$.

Replace p_1 and p_2 in (5-8), and let $\pi^{mT}(u,T)=(\pi_1^m)^\theta(\pi_2^m)^{1-\theta}=H(u,T)$, then $ln\,H(u,T)=$

$\theta\ln\pi_1^m+(1-\theta)\ln\pi_2^m$. By derivation we obtain $\frac{1}{H(u,T)}\frac{\partial H(u,T)}{\partial u}=\theta\frac{1}{\pi_1^m}\frac{\partial\pi_1^m}{\partial u}+(1-\theta)\frac{1}{\pi_2^m}\frac{\partial\pi_2^m}{\partial u}$ and $\frac{1}{H(u,T)}\frac{\partial H(u,T)}{\partial T}=\theta\frac{1}{\pi_1^m}\frac{\partial\pi_1^m}{\partial T}+(1-\theta)\frac{1}{\pi_2^m}\frac{\partial\pi_2^m}{\partial T}$, then $\frac{\partial H(u,T)}{\partial u}=H(u,T)\left[\theta\frac{1}{\pi_1^m}\frac{\partial\pi_1^m}{\partial u}+(1-\theta)\frac{1}{\pi_2^m}\frac{\partial\pi_2^m}{\partial u}\right]$ and $\frac{\partial H(u,T)}{\partial T}=H(u,T)\left[\theta\frac{1}{\pi_1^m}\frac{\partial\pi_1^m}{\partial T}+(1-\theta)\frac{1}{\pi_2^m}\frac{\partial\pi_2^m}{\partial T}\right]$. Let $\frac{\partial H(u,T)}{\partial u}=\frac{\partial H(u,T)}{\partial T}=0$, we obtain $u^m=\frac{\delta\beta(2+3\beta)^2}{8+24\beta+34\beta^2+18\beta^3}>0$ and $T^m=\frac{\delta^2[-16-96\beta-284\beta^2-456\beta^3-405\beta^4-162\beta^5+(32+160\beta+396\beta^2+552\beta^3+441\beta^4+162\beta^5)\theta]}{4(1+\beta)(4+8\beta+9\beta^2)^2}$. Let $-16-96\beta-284\beta^2-456\beta^3-405\beta^4-162\beta^5+(32+160\beta+396\beta^2+552\beta^3+441\beta^4+162\beta^5)\theta=0$, we obtain $\theta_0=\frac{16+96\beta+284\beta^2+456\beta^3+405\beta^4+162\beta^5}{32+160\beta+396\beta^2+552\beta^3+441\beta^4+162\beta^5}<1$. So, if $\theta_0<\theta<1$, then $T^m>0$. $\frac{\partial^2H(u,T)}{\partial T^2}|_{u=u^m,T=T^m}=-H(u^m,T^m)\frac{16(1+\beta)^2(4+8\beta+9\beta^2)^2}{\delta^4(8+24\beta+33\beta^2+18\beta^3)^2(1-\theta)\theta}<0$ and $\begin{vmatrix}\frac{\partial^2H(u,T)}{\partial T^2} & \frac{\partial^2H(u,T)}{\partial T\partial u}\\ \frac{\partial^2H(u,T)}{\partial u\partial T} & \frac{\partial^2H(u,T)}{\partial u^2}\end{vmatrix}|_{u=u^m,T=T^m}=\frac{128(1+\beta)^4(1+2\beta)(4+8\beta+9\beta^2)^4}{\delta^6(2+\beta)^2(2+3\beta)^2(8+24\beta+33\beta^2+18\beta^3)^3(1-\theta)\theta}>0$, then replace u^m in p_1 and p_2, we obtain that if $\theta_0=\frac{16+96\beta+284\beta^2+456\beta^3+405\beta^4+162\beta^5}{32+160\beta+396\beta^2+552\beta^3+441\beta^4+162\beta^5}<\theta<1$, $p_1^m=c+w+\frac{\delta(4+6\beta+9\beta^2)}{2(4+8\beta+9\beta^2)}$ and $p_2^m=c+w+\frac{\delta(4+12\beta+18\beta^2+9\beta^3)}{2(4+12\beta+17\beta^2+9\beta^3)}$. Recall that $q_i=\alpha-p_i+\beta(p_j-p_i)+\tau s_1$, then $q_1^m=\frac{\delta(4+14\beta+21\beta^2+12\beta^3)}{2(1+\beta)(4+8\beta+9\beta^2)}$ and $q_2^m=\frac{\delta(2+4\beta+3\beta^2)}{4+8\beta+9\beta^2}$.

Proof of Proposition 5.1

$q_1^n>0$ and $q_2^n>0$ imply that $0<\Delta s<\Delta s^K=\frac{2+3\beta}{\tau(2+4\beta+\beta^2)}\delta$. If $0<\theta<\theta_3=\frac{4(1+\beta)}{8+10\beta+3\beta^2}$, then $m^o<c$, which means OC strategy is not feasible and competition is the better strategy.

(1) From Table 5.2, (5-1), (5-2), (5-3) and (5-4), we obtain $\pi^o-\pi^n=\frac{M(\Delta s)}{8(1+\beta)(2+\beta)^2(1+2\beta)(2+3\beta)^2(8+16\beta+9\beta^2)^2}$, where $A=[(48+256\beta+500\beta^2+432\beta^3+144\beta^4)(1-\theta)+(4+16\beta+28\beta^2+24\beta^3+9\beta^4)\theta^2]^{\frac{1}{2}}$ and $M(\Delta s)=-8\Delta s^2\tau^2(8+24\beta+25\beta^2+9\beta^3)^2(4+24\beta+53\beta^2+52\beta^3+22\beta^4+4\beta^5)+16\delta\tau\Delta s(2+3\beta)(2+7\beta+6\beta^2)(8+24\beta+25\beta^2+9\beta^3)^2+\delta^2(2+3\beta)(4+8\beta+3\beta^2)(4+10\beta+10\beta^2+3\beta^3)(9\beta^2\theta+4(2+\theta)+8\beta(2+\theta))A-\delta^2(2+3\beta)^2(512+4096\beta+14176\beta^2+27808\beta^3+33512\beta^4+24752\beta^5+10344\beta^6+1872\beta^7-256\theta-2304\beta\theta-9152\beta^2\theta-21056\beta^3\theta-30504\beta^4\theta-28120\beta^5\theta-15906\beta^6\theta-4968\beta^7\theta-648\beta^8\theta+64\theta^2+448\beta\theta^2+1504\beta^2\theta^2+3056\beta^3\theta^2+4052\beta^4\theta^2+3536\beta^5\theta^2+1956\beta^6\theta^2+612\beta^7\theta^2+81\beta^8\theta^2)$, $M(\Delta s)$ is a quadratic function of Δs whose quadratic coefficient is negative. $M(0)=\delta^2(2+3\beta)(4+8\beta+3\beta^2)(4+10\beta+10\beta^2+3\beta^3)(9\beta^2\theta+4(2+\theta)+8\beta(2+\theta))A-\delta^2(2+3\beta)^2(512+4096\beta+14176\beta^2+27808\beta^3+33512\beta^4+24752\beta^5+10344\beta^6+1872\beta^7-256\theta-2304\beta\theta-9152\beta^2\theta-21056\beta^3\theta-30504\beta^4\theta-28120\beta^5\theta-15906\beta^6\theta-4968\beta^7\theta-648\beta^8\theta+64\theta^2+448\beta\theta^2+1504\beta^2\theta^2+3056\beta^3\theta^2+4052\beta^4\theta^2+3536\beta^5\theta^2+1956\beta^6\theta^2+612\beta^7\theta^2+81\beta^8\theta^2)$ and we

obtain if $0 < \beta < \beta_1 \approx 0.6996$ and $\theta_3 < \theta < \theta_4 = \frac{4(16+128\beta+472\beta^2+936\beta^3+1049\beta^4+630\beta^5+153\beta^6)}{128+736\beta+2064\beta^2+3424\beta^3+3536\beta^4+2190\beta^5+693\beta^6+81\beta^7}$, or $\beta > \beta_1$ and $\theta_3 < \theta < 1$, then $M(0) > 0$. If $0 < \beta < \beta_1$ and $\theta_4 < \theta < 1$, then $M(0) < 0$.

(a) When $0 < \beta < \beta_1$ and $\theta_3 < \theta < \theta_4$, or $\beta > \beta_1$ and $\theta_3 < \theta < 1$, $M(0) > 0$ and there is only one positive real root for $M(\Delta s) = 0$, then $\Delta s^T = \frac{\delta(B+U)}{16\tau(8+24\beta+25\beta^2+9\beta^3)^2(4+24\beta+53\beta^2+52\beta^3+22\beta^4+4\beta^5)}$, where $B = 16(2+3\beta)(2+7\beta+6\beta^2)(8+24\beta+25\beta^2+9\beta^3)^2$ and $U = \big[256(2+3\beta)^2(2+7\beta+6\beta^2)^2(8+24\beta+25\beta^2+9\beta^3)^4 + 32(8+24\beta+25\beta^2+9\beta^3)^2(4+24\beta+53\beta^2+52\beta^3+22\beta^4+4\beta^5)[(2+3\beta)(4+8\beta+3\beta^2)(4+10\beta+10\beta^2+3\beta^3)(9\beta^2\theta+4(2+\theta)+8\beta(2+\theta))A - (2+3\beta)^2(512+4096\beta+14176\beta^2+27808\beta^3+33512\beta^4+24752\beta^5+10344\beta^6+1872\beta^7-256\theta-2304\beta\theta-9152\beta^2\theta-21056\beta^3\theta-30504\beta^4\theta-28120\beta^5\theta-15906\beta^6\theta-4968\beta^7\theta-648\beta^8\theta+64\theta^2+448\beta\theta^2+1504\beta^2\theta^2+3056\beta^3\theta^2+4052\beta^4\theta^2+3536\beta^5\theta^2+1956\beta^6\theta^2+612\beta^7\theta^2+81\beta^8\theta^2)]\big]^{\frac{1}{2}}$.

Recall that $0 < \Delta s < \Delta s^K$, so we compare Δs^K and Δs^T. $\Delta s^K - \Delta s^T = \frac{\delta}{16\tau(2+4\beta+\beta^2)(8+24\beta+25\beta^2+9\beta^3)^2(4+24\beta+53\beta^2+52\beta^3+22\beta^4+4\beta^5)}\{16\beta(1+\beta)^3(4+8\beta+3\beta^2)(8+32\beta+41\beta^2+18\beta^3)^2 - (2+4\beta+\beta^2)U\}$, and $[16\beta(1+\beta)^3(4+8\beta+3\beta^2)(8+32\beta+41\beta^2+18\beta^3)^2]^2 - \{(2+4\beta+\beta^2)U\}^2 = 32\big\{8\beta^2(1+\beta)^6(4+8\beta+3\beta^2)^2(8+32\beta+41\beta^2+18\beta^3)^4 - (2+3\beta)(2+4\beta+\beta^2)^2(8+24\beta+25\beta^2+9\beta^3)^2\big[8(2+3\beta)(2+7\beta+6\beta^2)^2(8+24\beta+25\beta^2+9\beta^3)^2 + (4+24\beta+53\beta^2+52\beta^3+22\beta^4+4\beta^5)[A(2+\beta)^2(4+14\beta+18\beta^2+9\beta^3)(9\beta^2\theta+4(2+\theta)+8\beta(2+\theta)) - (2+3\beta)(81\beta^8(-8+\theta)\theta+64(8-4\theta+\theta^2)+64\beta(64-36\theta+7\theta^2)+36\beta^7(52-138\theta+17\theta^2)+32\beta^2(443-286\theta+47\theta^2)+16\beta^3(1738-1316\theta+191\theta^2)+6\beta^6(1724-2651\theta+326\theta^2)+8\beta^5(3094-3515\theta+442\theta^2)+4\beta^4(8378-7626\theta+1013\theta^2))]\big]\big\}$ and we obtain that if $\beta > \beta_2 \approx 1.8754$ and $\theta_3 < \theta < \theta_5 = \frac{4}{(2+4\beta+\beta^2)(96+976\beta+4424\beta^2+11892\beta^3+20860\beta^4+24496\beta^5+18910\beta^6+8889\beta^7+2115\beta^8+162\beta^9)}\big[32\beta+444\beta^2+2602\beta^3+8588\beta^4+17771\beta^5+24007\beta^6+21087\beta^7+11443\beta^8+3384\beta^9+396\beta^{10}-(12+90\beta+272\beta^2+433\beta^3+389\beta^4+187\beta^5+39\beta^6)\sqrt{16+128\beta+432\beta^2+800\beta^3+872\beta^4+552\beta^5+209\beta^6+84\beta^7+36\beta^8}\big]$, then $\Delta s^K - \Delta s^T > 0$ and $\Delta s^K > \Delta s^T$. So, if $0 < \beta < \beta_1$ and $\theta_3 < \theta < \theta_4$, or $\beta_1 < \beta < \beta_2$ and $\theta_3 < \theta < 1$, or $\beta > \beta_2$ and $\theta_5 < \theta < 1$, then $\Delta s^K < \Delta s^T$. If $\beta > \beta_2$ and $\theta_3 < \theta < \theta_5$, then $\Delta s^K > \Delta s^T$.

(b) When $0 < \beta < \beta_1$ and $\theta_4 < \theta < 1$, $M(0) < 0$ and we check if $\Delta(\Delta s) > 0$. $\Delta(\Delta s) = 256\delta^2\tau^2(2+3\beta)^2(2+7\beta+6\beta^2)^2(8+24\beta+25\beta^2+9\beta^3)^4 + 32(8+24\beta+25\beta^2+9\beta^3)^2(4+24\beta+53\beta^2+52\beta^3+22\beta^4+4\beta^5)\{\delta^2(2+3\beta)(4+8\beta+3\beta^2)(4+10\beta+10\beta^2+3\beta^3)(9\beta^2\theta+4(2+\theta)+8\beta(2+\theta))A -$

$\delta^2(2+3\beta)^2[81\beta^8(-8+\theta)\theta+64(8-4\theta+\theta^2)+64\beta(64-36\theta+7\theta^2)+36\beta^7(52-138\theta+17\theta^2)+32\beta^2(443-286\theta+47\theta^2)+16\beta^3(1738-1316\theta+191\theta^2)+6\beta^6(1724-2651\theta+326\theta^2)+8\beta^5(3094-3515\theta+442\theta^2)+4\beta^4(8378-7626\theta+1013\theta^2)]\}$, and for $\beta>0$ and $\theta_3<\theta<1$, $\Delta(\Delta s)>0$. When $0<\beta<\beta_1$ and $\theta_4<\theta<1$, the symmetric axis is $\Delta s=\frac{\delta(2+3\beta)^2}{\tau(4+16\beta+21\beta^2+10\beta^3+2\beta^4)}>0$, then there are two positive real roots for $M(\Delta s)=0$, then $\Delta s^T>\Delta s^K>0$ and $\Delta s^R=\frac{\delta(B-U)}{16\tau(8+24\beta+25\beta^2+9\beta^3)^2(4+24\beta+53\beta^2+52\beta^3+22\beta^4+4\beta^5)}$, where $B=16(2+3\beta)(2+7\beta+6\beta^2)(8+24\beta+25\beta^2+9\beta^3)^2$ and $U=\big[256(2+3\beta)^2(2+7\beta+6\beta^2)^2(8+24\beta+25\beta^2+9\beta^3)^4+32(8+24\beta+25\beta^2+9\beta^3)^2(4+24\beta+53\beta^2+52\beta^3+22\beta^4+4\beta^5)[(2+3\beta)(4+8\beta+3\beta^2)(4+10\beta+10\beta^2+3\beta^3)(9\beta^2\theta+4(2+\theta)+8\beta(2+\theta))A-(2+3\beta)^2(512+4096\beta+14176\beta^2+27808\beta^3+33512\beta^4+24752\beta^5+10344\beta^6+1872\beta^7-256\theta-2304\beta\theta-9152\beta^2\theta-21056\beta^3\theta-30504\beta^4\theta-28120\beta^5\theta-15906\beta^6\theta-4968\beta^7\theta-648\beta^8\theta+64\theta^2+448\beta\theta^2+1504\beta^2\theta^2+3056\beta^3\theta^2+4052\beta^4\theta^2+3536\beta^5\theta^2+1956\beta^6\theta^2+612\beta^7\theta^2+81\beta^8\theta^2)]\big]^{\frac{1}{2}}$.

Recall that $0<\Delta s<\Delta s^K$, so we compare Δs^K and Δs^R. $\Delta s^K-\Delta s^R=\frac{\delta\{16\beta(1+\beta)^3(4+8\beta+3\beta^2)(8+32\beta+41\beta^2+18\beta^3)^2+(2+4\beta+\beta^2)U\}}{16\tau(2+4\beta+\beta^2)(8+24\beta+25\beta^2+9\beta^3)^2(4+24\beta+53\beta^2+52\beta^3+22\beta^4+4\beta^5)}>0$ implies $\Delta s^K>\Delta s^R$. Therefore, if $0<\beta<\beta_1$, $\theta_3<\theta<\theta_4$ and $0<\Delta s<\Delta s^K$, or $0<\beta<\beta_1$, $\theta_4<\theta<1$ and $\Delta s^R<\Delta s<\Delta s^K$, or $\beta_1<\beta<\beta_2$, $\theta_3<\theta<1$ and $0<\Delta s<\Delta s^K$, or $\beta>\beta_2$, $\theta_3<\theta<\theta_5$ and $0<\Delta s<\Delta s^T$, or $\beta>\beta_2$, $\theta_5<\theta<1$ and $0<\Delta s<\Delta s^K$, then $\pi^o>\pi^n$; if $0<\beta<\beta_1$, $\theta_4<\theta<1$ and $0<\Delta s<\Delta s^R$, or $\beta>\beta_2$, $\theta_3<\theta<\theta_5$ and $\Delta s^T<\Delta s<\Delta s^K$, then $\pi^o<\pi^n$.

Recall that $\delta=\alpha-c-w+\tau s_1$, therefore, $\Delta s<\Delta s^K=\frac{2+3\beta}{\tau(2+4\beta+\beta^2)}\delta$ equals $\tau<\tau^K=\frac{(2+3\beta)(\alpha-c-w)}{(2+4\beta+\beta^2)\Delta s-(2+3\beta)s_1}$, $\Delta s<\Delta s^R$ equals $\tau<\tau^R=\frac{(B-U)(\alpha-c-w)}{16(8+24\beta+25\beta^2+9\beta^3)^2(4+24\beta+53\beta^2+52\beta^3+22\beta^4+4\beta^5)\Delta s-(B-U)s_1}$ and $\Delta s<\Delta s^T$ equals $\tau<\tau^T=\frac{(B+U)(\alpha-c-w)}{16(8+24\beta+25\beta^2+9\beta^3)^2(4+24\beta+53\beta^2+52\beta^3+22\beta^4+4\beta^5)\Delta s-(B+U)s_1}$, where $B=16(2+3\beta)(2+7\beta+6\beta^2)(8+24\beta+25\beta^2+9\beta^3)^2$ and $U=\big[256(2+3\beta)^2(2+7\beta+6\beta^2)^2(8+24\beta+25\beta^2+9\beta^3)^4+32(8+24\beta+25\beta^2+9\beta^3)^2(4+24\beta+53\beta^2+52\beta^3+22\beta^4+4\beta^5)[(2+3\beta)(4+8\beta+3\beta^2)(4+10\beta+10\beta^2+3\beta^3)(9\beta^2\theta+4(2+\theta)+8\beta(2+\theta))A-(2+3\beta)^2(512+4096\beta+14176\beta^2+27808\beta^3+33512\beta^4+24752\beta^5+10344\beta^6+1872\beta^7-256\theta-2304\beta\theta-9152\beta^2\theta-21056\beta^3\theta-30504\beta^4\theta-28120\beta^5\theta-15906\beta^6\theta-4968\beta^7\theta-648\beta^8\theta+64\theta^2+448\beta\theta^2+1504\beta^2\theta^2+3056\beta^3\theta^2+4052\beta^4\theta^2+3536\beta^5\theta^2+1956\beta^6\theta^2+612\beta^7\theta^2+81\beta^8\theta^2)]\big]^{\frac{1}{2}}$. Therefore, when $0<\beta<\beta_1$, if $\theta_3<\theta<\theta_4$ and $0<\tau<\tau^K$, or $\theta_4<\theta<1$ and $\tau^R<\tau<\tau^K$; or when $\beta_1<\beta<\beta_2$, if $\theta_3<\theta<1$ and $0<\tau<\tau^K$; or when $\beta>\beta_2$, if $\theta_3 < \theta < \theta_5$ and $0 < \tau < \tau^T$, or $\theta_5 < \theta < 1$ and $0 < \tau < \tau^K$, then OC is the better strategy; otherwise, competition is the better strategy.

(2) Blow we compare the profits of each firm under OC and competition model.

(a) From Table 5.2, (5-1) and (5-3), we obtain $\pi_1^o(p_1^o) - \pi_1^n(p_1^n) = \frac{H(\Delta s)}{8(1+2\beta)(4+8\beta+3\beta^2)^2(8+24\beta+25\beta^2+9\beta^3)}$, where $A = [(48 + 256\beta + 500\beta^2 + 432\beta^3 + 144\beta^4)(1-\theta) + (4 + 16\beta + 28\beta^2 + 24\beta^3 + 9\beta^4)\theta^2]^{\frac{1}{2}}$ and $H(\Delta s) = -8\Delta s^2\tau^2\beta^2(1+\beta)^4(8 + 32\beta + 41\beta^2 + 18\beta^3) - 16\delta\tau\Delta s\beta(1+\beta)^3(16 + 88\beta + 178\beta^2 + 159\beta^3 + 54\beta^4) - \delta^2(2+3\beta)^2[64 + 384\beta + 904\beta^2 + 1056\beta^3 + 616\beta^4 + 144\beta^5 + (-8(12+A) - 8(76+3A)\beta - 6(256+5A)\beta^2 - 8(249+2A)\beta^3 + (-1402-3A)\beta^4 - 504\beta^5 - 72\beta^6)\theta + (16 + 80\beta + 180\beta^2 + 224\beta^3 + 160\beta^4 + 60\beta^5 + 9\beta^6)\theta^2]$. $H(\Delta s)$ is a quadratic function of Δs whose quadratic coefficient is negative. $\Delta(\Delta s) = 32\delta^2\beta^2(1+\beta)^4\tau^2[8(1+\beta)^2(16 + 88\beta + 178\beta^2 + 159\beta^3 + 54\beta^4)^2 + (2+3\beta)^2(8 + 32\beta + 41\beta^2 + 18\beta^3)(9\beta^6(-8+\theta)\theta + 8(8 - (12+A)\theta + 2\theta^2) + 12\beta^5(12 - 42\theta + 5\theta^2) + 8\beta(48 - (76 + 3A)\theta + 10\theta^2) + 8\beta^3(132 - (249+2A)\theta + 28\theta^2) + 2\beta^2(452 - 3(256+5A)\theta + 90\theta^2) + \beta^4(616 - (1402+3A)\theta + 160\theta^2))]$, and for $\beta > 0$ and $\theta_3 < \theta < 1$, we obtain $H(0) = -\delta^2(2+3\beta)^2[64 + 384\beta + 904\beta^2 + 1056\beta^3 + 616\beta^4 + 144\beta^5 + (-8(12+A) - 8(76+3A)\beta - 6(256+5A)\beta^2 - 8(249+2A)\beta^3 + (-1402-3A)\beta^4 - 504\beta^5 - 72\beta^6)\theta + (16 + 80\beta + 180\beta^2 + 224\beta^3 + 160\beta^4 + 60\beta^5 + 9\beta^6)\theta^2] > 0$. Therefore, only one positive real root exists for $H(\Delta s) = 0$, then $\Delta s^H = \frac{\delta L}{4\tau\beta^2(1+\beta)^4(8+32\beta+41\beta^2+18\beta^3)}$, where $L = -4\beta(1+\beta)^3(16 + 88\beta + 178\beta^2 + 159\beta^3 + 54\beta^4) + \sqrt{2}\beta(1+\beta)^2(4 + 8\beta + 3\beta^2)[(-8 - 32\beta - 41\beta^2 - 18\beta^3)\theta(-A(2 + 4\beta + 3\beta^2) + 24\beta^3(-9+\theta) + 16\beta(-8+\theta) + 9\beta^4(-8+\theta) + 4(-6+\theta) + \beta^2(-250 + 28\theta))]^{\frac{1}{2}}$. If $0 < \Delta s < \Delta s^H$, then $H(\Delta s) > 0$ and $\pi_1^o(p_1^o) > \pi_1^n(p_1^n)$; if $\Delta s > \Delta s^H$, then $H(\Delta s) < 0$ and $\pi_1^o(p_1^o) < \pi_1^n(p_1^n)$.

Recall that $0 < \Delta s < \Delta s^K$, therefore we compare Δs^K and Δs^H. $\Delta s^K - \Delta s^H = \frac{\delta(4+8\beta+3\beta^2)}{4\tau\beta(1+\beta)^2(2+4\beta+\beta^2)(8+32\beta+41\beta^2+18\beta^3)}\left\{4(1+2\beta)^2(8 + 24\beta + 25\beta^2 + 9\beta^3) - \sqrt{2}(2 + 4\beta + \beta^2)[(-8 - 32\beta - 41\beta^2 - 18\beta^3)\theta(-A(2 + 4\beta + 3\beta^2) + 24\beta^3(-9+\theta) + 16\beta(-8+\theta) + 9\beta^4(-8+\theta) + 4(-6+\theta) + \beta^2(-250 + 28\theta))]^{\frac{1}{2}}\right\}$. $[4(1+2\beta)^2(8 + 24\beta + 25\beta^2 + 9\beta^3)]^2 - \left\{\sqrt{2}(2 + 4\beta + \beta^2)\left[(-8 - 32\beta - 41\beta^2 - 18\beta^3)\theta\left(-A(2 + 4\beta + 3\beta^2) + 24\beta^3(-9+\theta) + 16\beta(-8+\theta) + 9\beta^4(-8+\theta) + 4(-6+\theta) + \beta^2(-250 + 28\theta)\right)\right]^{\frac{1}{2}}\right\}^2 = 2(1+2\beta)(8 + 16\beta + 9\beta^2)(64 + 640\beta + 2696\beta^2 + 6208\beta^3 + 8456\beta^4 + 6832\beta^5 + 3040\beta^6 + 576\beta^7 - 96\theta - 8A\theta - 896\beta\theta - 48A\beta\theta - 3528\beta^2\theta - 116A\beta^2\theta - 7616\beta^3\theta - 144A\beta^3\theta - 9792\beta^4\theta - 94A\beta^4\theta - 7600\beta^5\theta - 28A\beta^5\theta - 3418\beta^6\theta - 3A\beta^6\theta - 792\beta^7\theta - 72\beta^8\theta + 16\theta^2 + 128\beta\theta^2 + 448\beta^2\theta^2 +

$896\beta^3\theta^2 + 1112\beta^4\theta^2 + 864\beta^5\theta^2 + 400\beta^6\theta^2 + 96\beta^7\theta^2 + 9\beta^8\theta^2)$ and if $\beta > 0$ and $\theta_3 < \theta < \theta_{10} = \frac{4(12+124\beta+541\beta^2+1293\beta^3+1832\beta^4+1544\beta^5+720\beta^6+144\beta^7)}{64+640\beta+2676\beta^2+6100\beta^3+8252\beta^4+6716\beta^5+3141\beta^6+729\beta^7+54\beta^8} - \frac{4}{(2+4\beta+\beta^2)(64+640\beta+2676\beta^2+6100\beta^3+8252\beta^4+6716\beta^5+3141\beta^6+729\beta^7+54\beta^8)}[64 + 1408\beta + 14464\beta^2 + 92160\beta^3 + 408000\beta^4 + 1332160\beta^5 + 3323300\beta^6 + 6475656\beta^7 + 9996412\beta^8 + 12342328\beta^9 + 12284765\beta^{10} + 9953286\beta^{11} + 6665657\beta^{12} + 3768876\beta^{13} + 1826316\beta^{14} + 744192\beta^{15} + 236016\beta^{16} + 50112\beta^{17} + 5184\beta^{18}]^{\frac{1}{2}}$, then $0 < \Delta s^H < \Delta s^K$. If $\beta > 0$ and $\theta_{10} < \theta < 1$, then $0 < \Delta s^K < \Delta s^H$. Therefore, if $\beta > 0$, $\theta_3 < \theta < \theta_{10}$ and $0 < \Delta s < \Delta s^H$ or $\beta > 0$, $\theta_{10} < \theta < 1$ and $0 < \Delta s < \Delta s^K$, then $H(\Delta s) > 0$ and $\pi_1^o(p_1^o) > \pi_1^n(p_1^n)$. If $\beta > 0$, $\theta_3 < \theta < \theta_{10}$ and $\Delta s^H < \Delta s < \Delta s^K$, then $H(\Delta s) < 0$ and $\pi_1^o(p_1^o) < \pi_1^n(p_1^n)$.

(b) From Table 5.2, (5-2) and (5-4), we obtain $\pi_2^o(p_2^o) - \pi_2^n(p_2^n) = \frac{K(\Delta s)}{2(1+\beta)(2+\beta)^2(2+3\beta)^2(8+16\beta+9\beta^2)^2}$, where $A = [(48 + 256\beta + 500\beta^2 + 432\beta^3 + 144\beta^4)(1-\theta) + (4 + 16\beta + 28\beta^2 + 24\beta^3 + 9\beta^4)\theta^2]^{\frac{1}{2}}$ and $K(\Delta s) = 4\delta\tau\Delta s(4 + 14\beta + 14\beta^2 + 3\beta^3)(8 + 24\beta + 25\beta^2 + 9\beta^3)^2 - 2\tau^2\Delta s^2(16 + 80\beta + 154\beta^2 + 142\beta^3 + 61\beta^4 + 9\beta^5)^2 - \delta^2(2+3\beta)^2[A(2+\beta)^2(2 + 4\beta + 3\beta^2)(-2+\theta) + 16\beta(48 - 5\theta)\theta - 16(-8+\theta)\theta + \beta^2(56 + 1896\theta - 180\theta^2) + \beta^4(392 + 1722\theta - 160\theta^2) + \beta^5(276 + 624\theta - 60\theta^2) - 9\beta^6(-8 - 10\theta + \theta^2) - 8\beta^3(-31 - 305\theta + 28\theta^2)]$. $K(\Delta s)$ is a quadratic function of Δs whose quadratic coefficient is negative, and for $\beta > 0$ and $\theta_3 < \theta < 1$, $K(0) = -\delta^2(2+3\beta)^2[A(2+\beta)^2(2 + 4\beta + 3\beta^2)(-2+\theta) + 16\beta(48 - 5\theta)\theta - 16(-8+\theta)\theta + \beta^2(56 + 1896\theta - 180\theta^2) + \beta^4(392 + 1722\theta - 160\theta^2) + \beta^5(276 + 624\theta - 60\theta^2) - 9\beta^6(-8 - 10\theta + \theta^2) - 8\beta^3(-31 - 305\theta + 28\theta^2)] < 0$ and the symmetry axis is $\Delta s = \frac{\delta(2+3\beta)}{\tau(2+4\beta+\beta^2)} = \Delta s^K$. Therefore, there is only one positive real root for $K(\Delta s) = 0$, then $\Delta s^J = \frac{\delta N}{2\tau(16+80\beta+154\beta^2+142\beta^3+61\beta^4+9\beta^5)^2}$, where $N = 2(4 + 14\beta + 14\beta^2 + 3\beta^3)(8 + 24\beta + 25\beta^2 + 9\beta^3)^2 - \sqrt{2}(64 + 448\beta + 1304\beta^2 + 2040\beta^3 + 1842\beta^4 + 950\beta^5 + 255\beta^6 + 27\beta^7)[-A(2 + 4\beta + 3\beta^2)(-2+\theta) + 24\beta^3(11 - 11\theta + \theta^2) + 16\beta(10 - 10\theta + \theta^2) + 9\beta^4(10 - 10\theta + \theta^2) + 4(8 - 8\theta + \theta^2) + \beta^2(306 - 306\theta + 28\theta^2)]^{\frac{1}{2}}$. So, if $0 < \Delta s < \Delta s^J$, then $K(\Delta s) < 0$ and $\pi_2^o(p_2^o) < \pi_2^n(p_2^n)$; if $\Delta s^J < \Delta s < \Delta s^K$, then $K(\Delta s) > 0$ and $\pi_2^o(p_2^o) > \pi_2^n(p_2^n)$. Therefore, if $\beta > 0$, $\theta_3 < \theta < \theta_{10}$ and $0 < \Delta s < \Delta s^H$ or $\beta > 0$, $\theta_{10} < \theta < 1$ and $0 < \Delta s < \Delta s^K$, then $\pi_1^o(p_1^o) > \pi_1^n(p_1^n)$. If $\beta > 0$, $\theta_3 < \theta < \theta_{10}$ and $\Delta s^H < \Delta s < \Delta s^K$, then $\pi_1^o(p_1^o) < \pi_1^n(p_1^n)$; if $0 < \Delta s < \Delta s^J$, then $\pi_2^o(p_2^o) < \pi_2^n(p_2^n)$; if $\Delta s^J < \Delta s < \Delta s^K$, then $\pi_2^o(p_2^o) > \pi_2^n(p_2^n)$.

In summary, if $0 < \beta < \beta_1$, $\theta_3 < \theta < \theta_4$ and $0 < \Delta s < \Delta s^K$, or $0 < \beta < \beta_1$, $\theta_4 < \theta < 1$ and $\Delta s^R < \Delta s < \Delta s^K$, then $\pi^o > \pi^n$, where if $\Delta s^J < \Delta s < \Delta s^H$, then both $\pi_1^o(p_1^o) > \pi_1^n(p_1^n)$ and $\pi_2^o(p_2^o) > \pi_2^n(p_2^n)$; if $\beta_1 < \beta < \beta_2$, $\theta_3 < \theta < 1$ and $0 < \Delta s < \Delta s^K$, then $\pi^o > \pi^n$, where if $\Delta s^J < \Delta s < \Delta s^H$, then both $\pi_1^o(p_1^o) >$

$\pi_1^n(p_1^n)$ and $\pi_2^o(p_2^o) > \pi_2^n(p_2^n)$; if $\beta > \beta_2$, $\theta_3 < \theta < \theta_5$ and $0 < \Delta s < \Delta s^T$, or $\beta > \beta_2$, $\theta_5 < \theta < 1$ and $0 < \Delta s < \Delta s^K$, then $\pi^o > \pi^n$, where if $\Delta s^J < \Delta s < \Delta s^H$, then both $\pi_1^o(p_1^o) > \pi_1^n(p_1^n)$ and $\pi_2^o(p_2^o) > \pi_2^n(p_2^n)$.

Recall that $\delta = \alpha - c - w + \tau s_1$, therefore, $\Delta s < \Delta s^K = \frac{2+3\beta}{\tau(2+4\beta+\beta^2)}\delta$ equals $\tau < \tau^K = \frac{(2+3\beta)(\alpha-c-w)}{(2+4\beta+\beta^2)\Delta s-(2+3\beta)s_1}$, $\Delta s < \Delta s^H$ equals $\tau < \tau^H = \frac{(\alpha-c-w)L}{4\beta^2(1+\beta)^4(8+32\beta+41\beta^2+18\beta^3)\Delta s-Ls_1}$ and $\Delta s < \Delta s^J$ equals $\tau < \tau^J = \frac{(\alpha-c-w)N}{2(16+80\beta+154\beta^2+142\beta^3+61\beta^4+9\beta^5)^2\Delta s-Ns_1}$, where $L = -4\beta(1+\beta)^3(16+88\beta+178\beta^2+159\beta^3+54\beta^4)+\sqrt{2}\beta(1+\beta)^2(4+8\beta+3\beta^2)[(-8-32\beta-41\beta^2-18\beta^3)\theta(-A(2+4\beta+3\beta^2)+24\beta^3(-9+\theta)+16\beta(-8+\theta)+9\beta^4(-8+\theta)+4(-6+\theta)+\beta^2(-250+28\theta))]^{\frac{1}{2}}$ and $N = 2(4+14\beta+14\beta^2+3\beta^3)(8+24\beta+25\beta^2+9\beta^3)^2-\sqrt{2}(64+448\beta+1304\beta^2+2040\beta^3+1842\beta^4+950\beta^5+255\beta^6+27\beta^7)[-A(2+4\beta+3\beta^2)(-2+\theta)+24\beta^3(11-11\theta+\theta^2)+16\beta(10-10\theta+\theta^2)+9\beta^4(10-10\theta+\theta^2)+4(8-8\theta+\theta^2)+\beta^2(306-306\theta+28\theta^2)]^{\frac{1}{2}}$.

Therefore, (1)when $0 < \beta < \beta_1$, if $\theta_3 < \theta < \theta_4$ and $0 < \tau < \tau^K$, or $\theta_4 < \theta < 1$ and $\tau^R < \tau < \tau^K$; or when $\beta_1 < \beta < \beta_2$, if $\theta_3 < \theta < 1$ and $0 < \tau < \tau^K$; or when $\beta > \beta_2$, if $\theta_3 < \theta < \theta_5$ and $0 < \tau < \tau^T$, or $\theta_5 < \theta < 1$ and $0 < \tau < \tau^K$, then OC is the better strategy; otherwise, competition is the better strategy. (2) When OC is a better strategy than competition is, if $\tau^J < \tau < \tau^H$, then OC delivers Pareto improvement.

Proof of Lemma 5.1

$q_1^n > 0$ and $q_2^n > 0$ imply that $0 < \Delta s < \Delta s^K = \frac{2+3\beta}{\tau(2+4\beta+\beta^2)}\delta$. $m^o > c$ implies $\theta_3 = \frac{4(1+\beta)}{8+10\beta+3\beta^2} < \theta < 1$. Set $V = c + w$.

(1) From Table 5.2, we obtain $CS_1^o - CS_1^n = \frac{\beta(1+\beta)G(\Delta s)}{32(2+\beta)^2(1+2\beta)^2(2+3\beta)^2(8+16\beta+9\beta^2)^2}$, where $A = [(48+256\beta+500\beta^2+432\beta^3+144\beta^4)(1-\theta)+(4+16\beta+28\beta^2+24\beta^3+9\beta^4)\theta^2]^{\frac{1}{2}}$ and $G(\Delta s) = [-4\tau\Delta s(8+40\beta+73\beta^2+59\beta^3+18\beta^4)+3\delta(2+3\beta)(-A(2+\beta)+4(2+\theta)+10\beta^2(3+\theta)+3\beta^3(4+\theta)+2\beta(14+5\theta))][4(8+32\beta+41\beta^2+18\beta^3)(\tau\Delta s\beta(1+\beta)+2V(4+8\beta+3\beta^2))+\delta(2+3\beta)(64+9\beta^4(4+\theta)+6\beta^3(39+5\theta)+\beta(280-6A+12\theta)+\beta^2(412-3A+30\theta))]$, then $G(\Delta s)$ is a quadratic function of Δs whose quadratic coefficient is negative. $G(0) = 3\delta(2+3\beta)^2[-A(2+\beta)+4(2+\theta)+10\beta^2(3+\theta)+3\beta^3(4+\theta)+2\beta(14+5\theta)]\{8V(16+72\beta+114\beta^2+77\beta^3+18\beta^4)+\delta[64+9\beta^4(4+\theta)+6\beta^3(39+5\theta)+\beta(280-6A+12\theta)+\beta^2(412-3A+30\theta)]\}$ and for $\theta_3 < \theta < 1$, $G(0) > 0$, which implies there is only one positive real root for $G(\Delta s) = 0$, then $\Delta s^C = \frac{3\delta(2+3\beta)(-A(2+\beta)+4(2+\theta)+10\beta^2(3+\theta)+3\beta^3(4+\theta)+2\beta(14+5\theta))}{4\tau(8+40\beta+73\beta^2+59\beta^3+18\beta^4)}$. If

$0 < \Delta s < \Delta s^C$, then $G(\Delta s) > 0$ and $CS_1^o > CS_1^n$; if $\Delta s > \Delta s^C$, then $G(\Delta s) < 0$ and $CS_1^o < CS_1^n$.

Recall that $0 < \Delta s < \Delta s^K$, then we compare Δs^K and Δs^C. $\Delta s^K - \Delta s^C = -\frac{\delta(2+\beta)(2+3\beta)[8-3A(2+4\beta+\beta^2)+12\theta+48\beta(1+\theta)+9\beta^4(4+\theta)+6\beta^3(15+8\theta)+4\beta^2(25+18\theta)]}{4\tau(1+\beta)(1+2\beta)(2+4\beta+\beta^2)(8+16\beta+9\beta^2)}$ and $-[3A(2+4\beta+\beta^2)]^2 + [8 + 12\theta + 48\beta(1+\theta) + 9\beta^4(4+\theta) + 6\beta^3(15+8\theta) + 4\beta^2(25+18\theta)]^2 = 4(8+40\beta+73\beta^2+59\beta^3+18\beta^4)[-52+60\theta+27\beta^4\theta+36\beta^3(-3+5\theta)+44\beta(-5+6\theta)+12\beta^2(-24+31\theta)]$. Let $-52+60\theta+27\beta^4\theta+36\beta^3(-3+5\theta)+44\beta(-5+6\theta)+12\beta^2(-24+31\theta) = 0$, we obtain $\theta_6 = \frac{52+220\beta+288\beta^2+108\beta^3}{60+264\beta+372\beta^2+180\beta^3+27\beta^4}$ and $\theta_6 - \theta_3 = \frac{4(44+246\beta+529\beta^2+549\beta^3+279\beta^4+54\beta^5)}{3(2+\beta)(4+3\beta)(2+4\beta+\beta^2)(10+24\beta+9\beta^2)} > 0$ implies $\theta_6 > \theta_3$.

If $\theta_3 < \theta < \theta_6$, then $8 - 3A(2+4\beta+\beta^2) + 12\theta + 48\beta(1+\theta) + 9\beta^4(4+\theta) + 6\beta^3(15+8\theta) + 4\beta^2(25+18\theta) < 0$, that is, $\Delta s^K > \Delta s^C$. If $\theta_6 < \theta < 1$, then $8 - 3A(2+4\beta+\beta^2) + 12\theta + 48\beta(1+\theta) + 9\beta^4(4+\theta) + 6\beta^3(15+8\theta) + 4\beta^2(25+18\theta) > 0$, that is, $\Delta s^K < \Delta s^C$. Therefore, if $\theta_3 < \theta < \theta_6$ and $0 < \Delta s < \Delta s^C$, or $\theta_6 < \theta < 1$ and $0 < \Delta s < \Delta s^K$, then $CS_1^o > CS_1^n$; if $\theta_3 < \theta < \theta_6$ and $\Delta s^C < \Delta s < \Delta s^K$, then $CS_1^o < CS_1^n$.

(2) From Table 5.2, we obtain $CS_2^o - CS_2^n = \frac{H(\Delta s)}{32(1+\beta)(2+\beta)^2(1+2\beta)^2(2+3\beta)^2(8+16\beta+9\beta^2)^2}$, where $A = [(48 + 256\beta + 500\beta^2 + 432\beta^3 + 144\beta^4)(1-\theta) + (4+16\beta+28\beta^2+24\beta^3+9\beta^4)\theta^2]^{\frac{1}{2}}$ and $H(\Delta s) = [4(8 + 40\beta + 73\beta^2 + 59\beta^3 + 18\beta^4)(-\Delta s\tau(2+4\beta+\beta^2) + 2V(4+8\beta+3\beta^2)) + \delta(2+3\beta)(-A(4+10\beta + 10\beta^2 + 3\beta^3) + 9\beta^5(4+\theta) + 8(10+\theta) + 12\beta(34+3\theta) + 12\beta^2(65+6\theta) + 6\beta^4(47+7\theta) + \beta^3(700 + 76\theta))][4\tau\Delta s(16+112\beta+314\beta^2+450\beta^3+345\beta^4+131\beta^5+18\beta^6) + \delta(4+14\beta+18\beta^2 + 9\beta^3)(-A(2+\beta) + 4(2+\theta) + 10\beta^2(3+\theta) + 3\beta^3(4+\theta) + 2\beta(14+5\theta))]$, then $H(\Delta s)$ is a quadratic function of Δs whose quadratic coefficient is negative, and $H(0) = \delta(2+3\beta)(4+14\beta+18\beta^2 + 9\beta^3)(-A(2+\beta) + 4(2+\theta) + 10\beta^2(3+\theta) + 3\beta^3(4+\theta) + 2\beta(14+5\theta))(8V(16+88\beta+186\beta^2 + 191\beta^3 + 95\beta^4 + 18\beta^5) + \delta(-A(4+10\beta+10\beta^2+3\beta^3) + 9\beta^5(4+\theta) + 8(10+\theta) + 12\beta(34+3\theta) + 12\beta^2(65+6\theta) + 6\beta^4(47+7\theta) + \beta^3(700+76\theta)))$. For $\beta > 0$ and $\theta_3 < \theta < 1$, $(-A(2+\beta) + 4(2+\theta) + 10\beta^2(3+\theta) + 3\beta^3(4+\theta) + 2\beta(14+5\theta))(8V(16+88\beta+186\beta^2+191\beta^3+95\beta^4+18\beta^5) + \delta(-A(4 + 10\beta + 10\beta^2 + 3\beta^3) + 9\beta^5(4+\theta) + 8(10+\theta) + 12\beta(34+3\theta) + 12\beta^2(65+6\theta) + 6\beta^4(47+7\theta) + \beta^3(700+76\theta))) > 0$, so there is only one positive real root for $H(\Delta s) = 0$, then $\Delta s^P = \frac{2+3\beta}{4\tau(16+112\beta+314\beta^2+450\beta^3+345\beta^4+131\beta^5+18\beta^6)}[8V(16+88\beta+186\beta^2+191\beta^3+95\beta^4+18\beta^5) + \delta(-A(4 + 10\beta + 10\beta^2 + 3\beta^3) + 9\beta^5(4+\theta) + 8(10+\theta) + 12\beta(34+3\theta) + 12\beta^2(65+6\theta) + 6\beta^4(47+7\theta) + \beta^3(700+76\theta))]$. If $0 < \Delta s < \Delta s^P$, then $H(\Delta s) > 0$ and $CS_2^o > CS_2^n$; if $\Delta s > \Delta s^P$, then $H(\Delta s) < 0$ and $CS_2^o < CS_2^n$.

Recall that $0 < \Delta s < \Delta s^K$, then we should compare Δs^K and Δs^P. $\Delta s^K - \Delta s^P = \frac{(2+\beta)(2+3\beta)}{4\tau(2+4\beta+\beta^2)(8+40\beta+73\beta^2+59\beta^3+18\beta^4)}[8V(8+40\beta+73\beta^2+59\beta^3+18\beta^4)+\delta(-A(2+4\beta+3\beta^2)+9\beta^4(4+\theta)+4(6+\theta)+16\beta(7+\theta)+6\beta^3(23+4\theta)+4\beta^2(47+7\theta))]$ and for $\theta_3 < \theta < 1$, we obtain that $8V(8+40\beta+73\beta^2+59\beta^3+18\beta^4)+\delta\big(-A(2+4\beta+3\beta^2)+9\beta^4(4+\theta)+4(6+\theta)+16\beta(7+\theta)+6\beta^3(23+4\theta)+4\beta^2(47+7\theta)\big) < 0$, which implies $\Delta s^K < \Delta s^P$. Therefore, when $0 < \Delta s < \Delta s^K$, then $CS_2^o > CS_2^n$.

In summary, if $\theta_3 < \theta < \theta_6$ and $0 < \Delta s < \Delta s^C$, or $\theta_6 < \theta < 1$ and $0 < \Delta s < \Delta s^K$, then $CS_1^o > CS_1^n$ and $CS_2^o > CS_2^n$; if $\theta_3 < \theta < \theta_6$ and $\Delta s^C < \Delta s < \Delta s^K$, then $CS_1^o < CS_1^n$ and $CS_2^o > CS_2^n$.

Recall that $\delta = \alpha - c - w + \tau s_1$, therefore, $\Delta s < \Delta s^K = \frac{2+3\beta}{\tau(2+4\beta+\beta^2)}\delta$ equals $\tau < \tau^K = \frac{(2+3\beta)(\alpha-c-w)}{(2+4\beta+\beta^2)\Delta s-(2+3\beta)s_1}$, $\Delta s < \Delta s^C$ equals $\tau < \tau^C = \frac{(\alpha-c-w)G}{4(8+40\beta+73\beta^2+59\beta^3+18\beta^4)\Delta s - G s_1}$, where $G = 3(2+3\beta)(-A(2+\beta)+4(2+\theta)+10\beta^2(3+\theta)+3\beta^3(4+\theta)+2\beta(14+5\theta))$ and $A = [(48+256\beta+500\beta^2+432\beta^3+144\beta^4)(1-\theta)+(4+16\beta+28\beta^2+24\beta^3+9\beta^4)\theta^2]^{\frac{1}{2}}$. Therefore, if $\theta_3 < \theta < \theta_6$ and $0 < \tau < \tau^C$, or $\theta_6 < \theta < 1$ and $0 < \tau < \tau^K$, then $CS_1^o > CS_1^n$; if $\theta_3 < \theta < \theta_6$ and $\tau^C < \tau < \tau^K$, then $CS_1^o < CS_1^n$; if $\theta_3 < \theta < 1$ and $0 < \tau < \tau^K$, then $CS_2^o > CS_2^n$.

Proof of Proposition 5.2

$q_1^n > 0$ and $q_2^n > 0$ imply that $0 < \Delta s < \Delta s^K = \frac{(2+3\beta)\delta}{\tau(2+4\beta+\beta^2)}$. If $0 < \theta < \theta_0 = \frac{16+96\beta+284\beta^2+456\beta^3+405\beta^4+162\beta^5}{32+160\beta+396\beta^2+552\beta^3+441\beta^4+162\beta^5}$, then $T^m < 0$, which means MC strategy is not feasible and competition is the better strategy.

(1) From Table 5.2, (5-1), (5-2), (5-5) and (5-6), we obtain $\pi^m - \pi^n = \frac{Y(\Delta s)}{4(1+\beta)(2+\beta)^2(2+3\beta)^2(4+8\beta+9\beta^2)}$, where $Y(\Delta s) = \delta^2\beta^2(1+2\beta)(2+3\beta)^4 + 8\delta\tau\Delta s(2+5\beta+3\beta^2)^2(4+8\beta+9\beta^2) - 4\tau^2\Delta s^2(1+\beta)^2(16+96\beta+248\beta^2+352\beta^3+277\beta^4+106\beta^5+18\beta^6)$, then $Y(\Delta s)$ is a quadratic function of Δs whose quadratic coefficient is negative, and $Y(0) = \delta^2\beta^2(1+2\beta)(2+3\beta)^4 > 0$ implies that there is only positive real root for $Y(\Delta s) = 0$, then $\Delta s^Q = \frac{\delta\left\{2(2+5\beta+3\beta^2)^2(4+8\beta+9\beta^2)+(2+3\beta)^2(2+3\beta+\beta^2)\sqrt{16+80\beta+200\beta^2+292\beta^3+273\beta^4+172\beta^5+86\beta^6+36\beta^7}\right\}}{2\tau(1+\beta)^2(16+96\beta+248\beta^2+352\beta^3+277\beta^4+106\beta^5+18\beta^6)}$. If $0 < \Delta s < \Delta s^Q$, then $Y(\Delta s) > 0$ and $\pi^m > \pi^n$; if $\Delta s > \Delta s^Q$, then $Y(\Delta s) < 0$ and $\pi^m < \pi^n$.

Recall that $0 < \Delta s < \Delta s^K$, therefore we compare Δs^K and Δs^Q. $\Delta s^K - \Delta s^Q = -\frac{\delta(4+8\beta+3\beta^2)(4+8\beta+9\beta^2)(4+16\beta+21\beta^2+10\beta^3+2\beta^4)\big(16+96\beta+240\beta^2+320\beta^3+232\beta^4+88\beta^5+33\beta^6+18\beta^7\big)}{2\tau(1+\beta)(2+4\beta+\beta^2)(16+96\beta+248\beta^2+352\beta^3+277\beta^4+106\beta^5+18\beta^6)} < 0$ implies that $\Delta s^K < \Delta s^Q$. Therefore, if $\theta_0 < \theta < 1$ and $0 < \Delta s < \Delta s^K$, then $\pi^m > \pi^n$. Recall that $\delta = \alpha - c - w + \tau s_1$, therefore, $\Delta s < \Delta s^K = \frac{2+3\beta}{\tau(2+4\beta+\beta^2)}\delta$ equals $\tau < \tau^K = \frac{(2+3\beta)(\alpha-c-w)}{(2+4\beta+\beta^2)\Delta s-(2+3\beta)s_1}$. Therefore, if $\theta_0 < \theta < 1$ and $0 < \tau < \tau^K$, then MC is the better strategy; otherwise, competition is the better strategy.

(2) Blow we compare the profits of each firm under MC and competition model.

(a) From Table 5.2, (5-1) and (5-6), we obtain $\pi_1^m(p_1^m) - \pi_1^n(p_1^n) = \frac{P(\Delta s)}{4(1+\beta)(4+8\beta+3\beta^2)^2(4+8\beta+9\beta^2)}$, where $P(\Delta s) = -4\tau^2\Delta s^2\beta^2(1+\beta)^4(4+8\beta+9\beta^2) - 8\tau\delta\Delta s\beta(1+\beta)^3(8+28\beta+42\beta^2+27\beta^3) + \delta^2(2+3\beta)^2(-16-64\beta-116\beta^2-104\beta^3-36\beta^4+(32+128\beta+236\beta^2+228\beta^3+105\beta^4+18\beta^5)\theta)$, then $P(\Delta s)$ is a quadratic function of Δs whose quadratic coefficient is negative, and we obtain that if $\frac{4(4+16\beta+29\beta^2+26\beta^3+9\beta^4)}{(2+\beta)^2(8+24\beta+33\beta^2+18\beta^3)} < \theta < 1$, then $-16-64\beta-116\beta^2-104\beta^3-36\beta^4+(32+128\beta+236\beta^2+228\beta^3+105\beta^4+18\beta^5)\theta > 0$. Recall that $\theta_0 < \theta < 1$ and $\theta_0 - \frac{4(4+16\beta+29\beta^2+26\beta^3+9\beta^4)}{(2+\beta)^2(8+24\beta+33\beta^2+18\beta^3)} = \frac{\beta(64+416\beta+1136\beta^2+1708\beta^3+1500\beta^4+729\beta^5+162\beta^6)}{(2+\beta)^2(4+8\beta+9\beta^2)(8+24\beta+33\beta^2+18\beta^3)} > 0$ implies that $\theta_0 > \frac{4(4+16\beta+29\beta^2+26\beta^3+9\beta^4)}{(2+\beta)^2(8+24\beta+33\beta^2+18\beta^3)}$, so for $\theta_0 < \theta < 1$, $P(0) = \delta^2(2+3\beta)^2(-16-64\beta-116\beta^2-104\beta^3-36\beta^4+(32+128\beta+236\beta^2+228\beta^3+105\beta^4+18\beta^5)\theta) > 0$ and there is only one positive real root for $P(\Delta s) = 0$, then $\Delta s^E = \frac{\delta}{2\tau\beta(1+\beta)^2(4+8\beta+9\beta^2)}\left[-2(1+\beta)(8+28\beta+42\beta^2+27\beta^3) + (4+8\beta+3\beta^2)\sqrt{(32+160\beta+396\beta^2+552\beta^3+441\beta^4+162\beta^5)\theta}\right]$. If $0 < \Delta s < \Delta s^E$, then $P(\Delta s) > 0$. If $\Delta s > \Delta s^E$, then $P(\Delta s) < 0$.

Recall that $0 < \Delta s < \Delta s^K$, then we compare Δs^K and Δs^E. $\Delta s^K - \Delta s^E = \frac{\delta(4+8\beta+3\beta^2)}{2\tau\beta(1+\beta)^2(2+4\beta+\beta^2)(4+8\beta+9\beta^2)}\left[8+40\beta+82\beta^2+86\beta^3+36\beta^4-(2+4\beta+\beta^2)\sqrt{(32+160\beta+396\beta^2+552\beta^3+441\beta^4+162\beta^5)\theta}\right]$. $(8+40\beta+82\beta^2+86\beta^3+36\beta^4)^2 - \left[(2+4\beta+\beta^2)\sqrt{(32+160\beta+396\beta^2+552\beta^3+441\beta^4+162\beta^5)\theta}\right]^2 = -(4+8\beta+9\beta^2)(-16-128\beta-436\beta^2-824\beta^3-916\beta^4-560\beta^5-144\beta^6+32\theta+224\beta\theta+676\beta^2\theta+1144\beta^3\theta+1148\beta^4\theta+648\beta^5\theta+177\beta^6\theta+18\beta^7\theta)$ and if $\beta > 0$ and $\theta_0 < \theta < 1$, then $-16-128\beta-436\beta^2-824\beta^3-916\beta^4-560\beta^5-144\beta^6+32\theta+224\beta\theta+676\beta^2\theta+1144\beta^3\theta+1148\beta^4\theta+648\beta^5\theta+177\beta^6\theta+18\beta^7\theta > 0$ and then $\Delta s^K < \Delta s^E$. Therefore, if $\theta_0 < \theta < 1$ and $0 < \Delta s < \Delta s^K$, then $\pi_1^m(p_1^m) > \pi_1^n(p_1^n)$.

(b) From Table 5.2, (5-2) and (5-7), we obtain $\pi_2^m(p_2^m) - \pi_2^n(p_2^n) = \frac{-Q(\Delta s)}{4(1+\beta)(4+8\beta+3\beta^2)^2(4+8\beta+9\beta^2)}$, where $Q(\Delta s) = 4\tau^2\Delta s^2(4+8\beta+9\beta^2)(2+6\beta+5\beta^2+\beta^3)^2 - 8\tau\delta\Delta s(1+\beta)^2(16+88\beta+204\beta^2+250\beta^3+150\beta^4+27\beta^5) + \delta^2(2+3\beta)^2(-16+18\beta^5(-1+\theta)+32\theta+64\beta(-1+2\theta)+3\beta^4(-23+35\theta)+4\beta^3(-31+57\theta)+4\beta^2(-30+59\theta))$, then $Q(\Delta s)$ is a quadratic function of Δs whose quadratic coefficient is positive, and $Q(0) = \delta^2(2+3\beta)^2(-16+18\beta^5(-1+\theta)+32\theta+64\beta(-1+2\theta)+3\beta^4(-23+35\theta)+$

$4\beta^3(-31+57\theta)+4\beta^2(-30+59\theta))$. If $\frac{16+64\beta+120\beta^2+124\beta^3+69\beta^4+18\beta^5}{(2+\beta)^2(8+24\beta+33\beta^2+18\beta^3)}<\theta<1$, then $Q(0)>0$. Recall that $\theta_0<\theta<1$ and $\theta_0-\frac{16+64\beta+120\beta^2+124\beta^3+69\beta^4+18\beta^5}{(2+\beta)^2(8+24\beta+33\beta^2+18\beta^3)}=\frac{4\beta(16+100\beta+256\beta^2+345\beta^3+246\beta^4+72\beta^5)}{(2+\beta)^2(4+8\beta+9\beta^2)(8+24\beta+33\beta^2+18\beta^3)}>0$, so for $\theta_0<\theta<1$, $Q(0)>0$. The symmetric axis of $Q(\Delta s)$ is $\Delta s=\Delta s^K=\frac{2+3\beta}{\tau(2+4\beta+\beta^2)}\delta$ and $\Delta(\Delta s)=16\tau^2\delta^2(1+\beta)^2(8+40\beta+74\beta^2+62\beta^3+23\beta^4+3\beta^5)^2(32+160\beta+396\beta^2+552\beta^3+441\beta^4+162\beta^5)(1-\theta)>0$, so there is one positive real root for $Q(\Delta s)=0$ in the range of $0<\Delta s<\Delta s^K$, then $\Delta s^M=\frac{\delta(2+3\beta)F}{2\tau(4+8\beta+9\beta^2)(2+6\beta+5\beta^2+\beta^3)}$, where $F=8+24\beta+34\beta^2+18\beta^3-(2+\beta)\sqrt{(32+160\beta+396\beta^2+552\beta^3+441\beta^4+162\beta^5)(1-\theta)}$ If $0<\Delta s<\Delta s^M$, then $Q(\Delta s)>0$, and $\pi_2^m(p_2^m)<\pi_2^n(p_2^n)$. If $\Delta s^M<\Delta s<\Delta s^K$, then $P(\Delta s)<0$, and $\pi_2^m(p_2^m)>\pi_2^n(p_2^n)$. Therefore, if $\theta_0<\theta<1$ and $0<\Delta s<\Delta s^M$, then $\pi_1^m(p_1^m)>\pi_1^n(p_1^n)$ and $\pi_2^m(p_2^m)<\pi_2^n(p_2^n)$; if $\theta_0<\theta<1$ and $\Delta s^M<\Delta s<\Delta s^K$, then $\pi_1^m(p_1^m)>\pi_1^n(p_1^n)$ and $\pi_2^m(p_2^m)>\pi_2^n(p_2^n)$.

Combining with Proposition 5.2, we obtain if $\theta_0<\theta<1$ and $0<\Delta s<\Delta s^M$, then $\pi_1^m(p_1^m)>\pi_1^n(p_1^n)$, $\pi_2^m(p_2^m)<\pi_2^n(p_2^n)$ and $\pi^m>\pi^n$; if $\theta_0<\theta<1$ and $\Delta s^M<\Delta s<\Delta s^K$, then $\pi_1^m(p_1^m)>\pi_1^n(p_1^n)$, $\pi_2^m(p_2^m)>\pi_2^n(p_2^n)$ and $\pi^m>\pi^n$.

Recall that $\delta=\alpha-c-w+\tau s_1$, therefore, $\Delta s<\Delta s^K=\frac{2+3\beta}{\tau(2+4\beta+\beta^2)}\delta$ equals $\tau<\tau^K=\frac{(2+3\beta)(\alpha-c-w)}{(2+4\beta+\beta^2)\Delta s-(2+3\beta)s_1}$, $\Delta s<\Delta s^M$ equals $\tau<\tau^M=\frac{F(\alpha-c-w)}{2(4+8\beta+9\beta^2)(2+6\beta+5\beta^2+\beta^3)\Delta s-Fs_1}$, where $F=8+24\beta+34\beta^2+18\beta^3-(2+\beta)\sqrt{(32+160\beta+396\beta^2+552\beta^3+441\beta^4+162\beta^5)(1-\theta)}$. Therefore, (1) if $\theta_0<\theta<1$ and $0<\tau<\tau^K$, then MC is the better strategy; otherwise, competition is the better strategy. (2) when MC is the better strategy, if $\tau^M<\tau<\tau^K$, then MC delivers Pareto improvement.

Proof of Lemma 5.2

$q_1^n>0$ and $q_2^n>0$ imply that $0<\Delta s<\Delta s^K=\frac{2+3\beta}{\tau(2+4\beta+\beta^2)}\delta$. $T^m>0$ implies that $\theta_0=\frac{16+96\beta+284\beta^2+456\beta^3+405\beta^4+162\beta^5}{32+160\beta+396\beta^2+552\beta^3+441\beta^4+162\beta^5}<\theta<1$. Set $V=c+w$.

(1) From Table 5.2, we obtain $CS_1^m-CS_1^n=\frac{\beta(1+\beta)}{8(2+\beta)^2(2+3\beta)^2(4+8\beta+9\beta^2)^2}[3\delta\beta(2+3\beta)^2-2\tau\Delta s(4+12\beta+17\beta^2+9\beta^3)][\delta(32+112\beta+180\beta^2+144\beta^3+27\beta^4)+2(4+8\beta+9\beta^2)(\Delta s\tau\beta(1+\beta)+2V(4+8\beta+3\beta^2))]$ and $\delta(32+112\beta+180\beta^2+144\beta^3+27\beta^4)+2(4+8\beta+9\beta^2)\big(\Delta s\tau\beta(1+\beta)+2V(4+8\beta+3\beta^2)\big)>0$. Let $3\delta\beta(2+3\beta)^2-2\tau\Delta s(4+12\beta+17\beta^2+9\beta^3)=0$, we obtain $\Delta s^D=\frac{3\delta\beta(2+3\beta)^2}{\tau(8+24\beta+34\beta^2+18\beta^3)}$. If $0<\Delta s<\Delta s^D$, then $3\delta\beta(2+3\beta)^2-2\tau\Delta s(4+12\beta+17\beta^2+9\beta^3)>0$ and $CS_1^m>CS_1^n$; if $\Delta s>\Delta s^D$, then

$3\delta\beta(2+3\beta)^2 - 2\tau\Delta s(4+12\beta+17\beta^2+9\beta^3) < 0$ and $CS_1^m < CS_1^n$.

Recall that $0 < \Delta s < \Delta s^K$, then we should compare Δs^K and Δs^D. $\Delta s^D - \Delta s^K = \frac{\delta(2+\beta)(2+3\beta)(-4-4\beta+6\beta^2+9\beta^3)}{2\tau(1+\beta)(2+4\beta+\beta^2)(4+8\beta+9\beta^2)}$, and let $-4-4\beta+6\beta^2+9\beta^3 = 0$, we obtain $\beta_3 \approx 0.7413$. If $0 < \beta < \beta_3$, then $-4-4\beta+6\beta^2+9\beta^3 < 0$ and $\Delta s^D < \Delta s^K$; if $\beta > \beta_3$, then $-4-4\beta+6\beta^2+9\beta^3 > 0$ and $\Delta s^D > \Delta s^K$. Therefore, if $0 < \beta < \beta_3$, $\theta_0 < \theta < 1$ and $0 < \Delta s < \Delta s^D$, or $\beta > \beta_3$, $\theta_0 < \theta < 1$ and $0 < \Delta s < \Delta s^K$, then $CS_1^m > CS_1^n$; if $0 < \beta < \beta_3$, $\theta_0 < \theta < 1$ and $\Delta s^D < \Delta s < \Delta s^K$, then $CS_1^m < CS_1^n$.

(2) From Table 5.2, we obtain $CS_2^m - CS_2^n = \frac{1}{8(1+\beta)(2+\beta)^2(2+3\beta)^2(4+8\beta+9\beta^2)^2}[\delta\beta(2+3\beta)^2(2+4\beta+3\beta^2) + 2\tau\Delta s(8+40\beta+86\beta^2+98\beta^3+53\beta^4+9\beta^5)][\delta(32+152\beta+320\beta^2+354\beta^3+180\beta^4+27\beta^5) + 2(4+12\beta+17\beta^2+9\beta^3)(-\tau\Delta s(2+4\beta+\beta^2)+2V(4+8\beta+3\beta^2))]$ and $\delta\beta(2+3\beta)^2(2+4\beta+3\beta^2) + 2\tau\Delta s(8+40\beta+86\beta^2+98\beta^3+53\beta^4+9\beta^5) > 0$. Let $\delta(32+152\beta+320\beta^2+354\beta^3+180\beta^4+27\beta^5) + 2(4+12\beta+17\beta^2+9\beta^3)\left(-\tau\Delta s(2+4\beta+\beta^2)+2V(4+8\beta+3\beta^2)\right) = 0$, we obtain $\Delta s^Y = \frac{(2+3\beta)(4V(8+28\beta+46\beta^2+35\beta^3+9\beta^4)+\delta(16+52\beta+82\beta^2+54\beta^3+9\beta^4))}{2\tau(2+4\beta+\beta^2)(4+12\beta+17\beta^2+9\beta^3)}$. If $0 < \Delta s < \Delta s^Y$, then $\delta(32+152\beta+320\beta^2+354\beta^3+180\beta^4+27\beta^5) + 2(4+12\beta+17\beta^2+9\beta^3)\left(-\tau\Delta s(2+4\beta+\beta^2)+2V(4+8\beta+3\beta^2)\right) > 0$ and $CS_2^m > CS_2^n$; if $\Delta s > \Delta s^Y$, then $\delta(32+152\beta+320\beta^2+354\beta^3+180\beta^4+27\beta^5) + 2(4+12\beta+17\beta^2+9\beta^3)\left(-\tau\Delta s(2+4\beta+\beta^2)+2V(4+8\beta+3\beta^2)\right) < 0$ and $CS_2^m < CS_2^n$.

Recall that $0 < \Delta s < \Delta s^K$, then we should compare Δs^K and Δs^Y. $\Delta s^Y - \Delta s^K = \frac{(2+\beta)(2+3\beta)\left(4V(4+12\beta+17\beta^2+9\beta^3)+\delta(4+12\beta+18\beta^2+9\beta^3)\right)}{2\tau(2+4\beta+\beta^2)(4+12\beta+17\beta^2+9\beta^3)} > 0$ implies that $\Delta s^Y > \Delta s^K$. Therefore, if $\theta_0 < \theta < 1$ and $0 < \Delta s < \Delta s^K$, then $CS_2^m > CS_2^n$. In summary, if $0 < \beta < \beta_3$, $\theta_0 < \theta < 1$ and $0 < \Delta s < \Delta s^D$, or $\beta > \beta_3$, $\theta_0 < \theta < 1$ and $0 < \Delta s < \Delta s^K$, then $CS_1^m > CS_1^n$ and $CS_2^m > CS_2^n$; if $0 < \beta < \beta_3$, $\theta_0 < \theta < 1$ and $\Delta s^D < \Delta s < \Delta s^K$, then $CS_1^m < CS_1^n$ and $CS_2^m > CS_2^n$.

Recall that $\delta = \alpha - c - w + \tau s_1$, therefore, $\Delta s < \Delta s^K = \frac{2+3\beta}{\tau(2+4\beta+\beta^2)}\delta$ equals $\tau < \tau^K = \frac{(2+3\beta)(\alpha-c-w)}{(2+4\beta+\beta^2)\Delta s-(2+3\beta)s_1}$, $\Delta s < \Delta s^D$ equals $\tau < \tau^D = \frac{3\beta(\alpha-c-w)(2+3\beta)^2}{(8+24\beta+34\beta^2+18\beta^3)\Delta s-3\beta(2+3\beta)^2 s_1}$. Therefore, (1) if $0 < \beta < \beta_3$, $\theta_0 < \theta < 1$ and $0 < \tau < \tau^D$, or $\beta > \beta_3$, $\theta_0 < \theta < 1$ and $0 < \tau < \tau^K$, then $CS_1^m > CS_1^n$; if $0 < \beta < \beta_3$, $\theta_0 < \theta < 1$ and $\tau^D < \tau < \tau^K$, then $CS_1^m < CS_1^n$. (2) $\theta_0 < \theta < 1$ and $0 < \tau < \tau^K$, then $CS_2^m > CS_2^n$.

Proof of Proposition 5.3

$q_1^n > 0$ and $q_2^n > 0$ imply that $0 < \Delta s < \Delta s^K = \frac{2+3\beta}{\tau(2+4\beta+\beta^2)}\delta$. $T^m > 0$ implies that $\theta_0 = \frac{16+96\beta+284\beta^2+456\beta^3+405\beta^4+162\beta^5}{32+160\beta+396\beta^2+552\beta^3+441\beta^4+162\beta^5} < \theta < 1$. $m^o > c$ implies $\theta_3 = \frac{4(1+\beta)}{8+10\beta+3\beta^2} < \theta < 1$. $\theta_3 - \theta_0 =$

$\frac{-160\beta-1056\beta^2-2984\beta^3-4680\beta^4-4302\beta^5-2187\beta^6-486\beta^7}{(8+10\beta+3\beta^2)(32+160\beta+396\beta^2+552\beta^3+441\beta^4+162\beta^5)} < 0$ implies that $\theta_3 < \theta_0$.

From Table 5.2, (5-3), (5-4), (5-6) and (5-7), we obtain $\pi^o - \pi^m = \frac{\delta^2}{8(1+\beta)(2+\beta)(1+2\beta)(2+3\beta)(4+8\beta+9\beta^2)(8+16\beta+9\beta^2)^2}\{(8+32\beta+62\beta^2+60\beta^3+27\beta^4)(9\beta^2\theta+4(2+\theta)+8\beta(2+\theta))(4+8\beta+3\beta^2)A-(4+8\beta+3\beta^2)[729\beta^8(8-8\theta+\theta^2)+512\beta(9-5\theta+\theta^2)+64(8-4\theta+\theta^2)+648\beta^7(43-41\theta+5\theta^2)+96\beta^2(197-122\theta+21\theta^2)+64\beta^3(730-503\theta+77\theta^2)+48\beta^5(1762-1466\theta+191\theta^2)+18\beta^6(3461-3105\theta+386\theta^2)+\beta^4(76296-58088\theta+8116\theta^2)]\}$, where $A=[(48+256\beta+500\beta^2+432\beta^3+144\beta^4)(1-\theta)+(4+16\beta+28\beta^2+24\beta^3+9\beta^4)\theta^2]^{\frac{1}{2}}$. $\{(8+32\beta+62\beta^2+60\beta^3+27\beta^4)(9\beta^2\theta+4(2+\theta)+8\beta(2+\theta))(4+8\beta+3\beta^2)A\}^2-\{(4+8\beta+3\beta^2)[729\beta^8(8-8\theta+\theta^2)+512\beta(9-5\theta+\theta^2)+64(8-4\theta+\theta^2)+648\beta^7(43-41\theta+5\theta^2)+96\beta^2(197-122\theta+21\theta^2)+64\beta^3(730-503\theta+77\theta^2)+48\beta^5(1762-1466\theta+191\theta^2)+18\beta^6(3461-3105\theta+386\theta^2)+\beta^4(76296-58088\theta+8116\theta^2)]\}^2=-4\delta^2(4+8\beta+3\beta^2)^2(8+32\beta+41\beta^2+18\beta^3)^2(-16+162\beta^5(-1+\theta)+32\theta+32\beta(-3+5\theta)+24\beta^3(-19+23\theta)+9\beta^4(-45+49\theta)+4\beta^2(-71+99\theta))^2<0$. Therefore, combined with Proposition 5.2, we obtain that if $\theta_0 < \theta < 1$ and $0 < \Delta s < \Delta s^K$, then $\pi^m > \{\pi^n, \pi^o\}$.

$\theta_0 - \theta_4 = \frac{-640\beta-5632\beta^2-22208\beta^3-51136\beta^4-74272\beta^5-67548\beta^6-34218\beta^7-4671\beta^8+3807\beta^9+1458\beta^{10}}{(4+8\beta+9\beta^2)(8+24\beta+33\beta^2+18\beta^3)(32+120\beta+204\beta^2+178\beta^3+69\beta^4+9\beta^5)}$ and let $-640\beta-5632\beta^2-22208\beta^3-51136\beta^4-74272\beta^5-67548\beta^6-34218\beta^7-4671\beta^8+3807\beta^9+1458\beta^{10}=0$, we obtain $\beta \approx 3.0123 > \beta_1$. Therefore, if $0<\beta<\beta_1$, then $\theta_0 < \theta_4$. $\theta_0 - \theta_5 = \frac{1}{(2+4\beta+\beta^2)(4+8\beta+9\beta^2)(8+24\beta+33\beta^2+18\beta^3)(24+196\beta+660\beta^2+1212\beta^3+1306\beta^4+785\beta^5+219\beta^6+18\beta^7)}\Big[768+11392\beta+77248\beta^2+318816\beta^3+895680\beta^4+1808688\beta^5+2697056\beta^6+3003912\beta^7+2507116\beta^8+1576342\beta^9+766920\beta^{10}+307701\beta^{11}+105057\beta^{12}+25920\beta^{13}+2916\beta^{14}+4\sqrt{16+128\beta+432\beta^2+800\beta^3+872\beta^4+552\beta^5+209\beta^6+84\beta^7+36\beta^8}(96+1008\beta+4732\beta^2+13178\beta^3+24100\beta^4+30017\beta^5+25431\beta^6+14109\beta^7+4653\beta^8+702\beta^9)\Big]>0$ implies $\theta_0 > \theta_5$. Therefore, combined with Proposition 5.1, we obtain that if $0<\beta<\beta_1$, $\theta_3<\theta<\theta_0$ and $0<\Delta s<\Delta s^K$, or $\beta_1<\beta<\beta_2$, $\theta_3<\theta<\theta_0$ and $0<\Delta s<\Delta s^K$, or $\beta>\beta_2$, $\theta_3<\theta<\theta_5$ and $0<\Delta s<\Delta s^T$, or $\beta>\beta_2$, $\theta_5<\theta<\theta_0$ and $0<\Delta s<\Delta s^K$, then $\pi^o > \{\pi^n, \pi^m\}$; if $\beta>\beta_2$, $\theta_3<\theta<\theta_5$ and $\Delta s^T<\Delta s<\Delta s^K$, then $\pi^n > \{\pi^o, \pi^m\}$.

From Proposition 5.1, we obtain that $\theta_5 = \frac{4}{(2+4\beta+\beta^2)(96+976\beta+4424\beta^2+11892\beta^3+20860\beta^4+24496\beta^5+18910\beta^6+8889\beta^7+2115\beta^8+162\beta^9)}\Big[32\beta+444\beta^2+2602\beta^3+8588\beta^4+17771\beta^5+24007\beta^6+21087\beta^7+11443\beta^8+3384\beta^9+396\beta^{10}-(12+90\beta+272\beta^2+$

$433\beta^3 + 389\beta^4 + 187\beta^5 + 39\beta^6)\sqrt{16 + 128\beta + 432\beta^2 + 800\beta^3 + 872\beta^4 + 552\beta^5 + 209\beta^6 + 84\beta^7 + 36\beta^8}\Big]$ and $\Delta s^T = \frac{\delta(B+U)}{16\tau(8+24\beta+25\beta^2+9\beta^3)^2(4+24\beta+53\beta^2+52\beta^3+22\beta^4+4\beta^5)}$, where $B = 16(2+3\beta)(2+7\beta+6\beta^2)(8+24\beta+25\beta^2+9\beta^3)^2$ and $U = \big[256(2+3\beta)^2(2+7\beta+6\beta^2)^2(8+24\beta+25\beta^2+9\beta^3)^4 + 32(8+24\beta+25\beta^2+9\beta^3)^2(4+24\beta+53\beta^2+52\beta^3+22\beta^4+4\beta^5)[(2+3\beta)(4+8\beta+3\beta^2)(4+10\beta+10\beta^2+3\beta^3)(9\beta^2\theta+4(2+\theta)+8\beta(2+\theta))A - (2+3\beta)^2(512+4096\beta+14176\beta^2+27808\beta^3+33512\beta^4+24752\beta^5+10344\beta^6+1872\beta^7-256\theta-2304\beta\theta-9152\beta^2\theta-21056\beta^3\theta-30504\beta^4\theta-28120\beta^5\theta-15906\beta^6\theta-4968\beta^7\theta-648\beta^8\theta+64\theta^2+448\beta\theta^2+1504\beta^2\theta^2+3056\beta^3\theta^2+4052\beta^4\theta^2+3536\beta^5\theta^2+1956\beta^6\theta^2+612\beta^7\theta^2+81\beta^8\theta^2)]\big]^{\frac{1}{2}}$. In summary, if $\theta_0 < \theta < 1$ and $0 < \Delta s < \Delta s^K$, then $\pi^m > \{\pi^n, \pi^o\}$; if $0 < \beta < \beta_1$, $\theta_3 < \theta < \theta_0$ and $0 < \Delta s < \Delta s^K$, or $\beta_1 < \beta < \beta_2$, $\theta_3 < \theta < \theta_0$ and $0 < \Delta s < \Delta s^K$, or $\beta > \beta_2$, $\theta_3 < \theta < \theta_5$ and $0 < \Delta s < \Delta s^T$, or $\beta > \beta_2$, $\theta_5 < \theta < \theta_0$ and $0 < \Delta s < \Delta s^K$, then $\pi^o > \{\pi^n, \pi^m\}$; if $\beta > \beta_2$, $\theta_3 < \theta < \theta_5$ and $\Delta s^T < \Delta s < \Delta s^K$, then $\pi^n > \{\pi^o, \pi^m\}$.

Recall that $\delta = \alpha - c - w + \tau s_1$, therefore, $\Delta s < \Delta s^K = \frac{2+3\beta}{\tau(2+4\beta+\beta^2)}\delta$ equals $\tau < \tau^K = \frac{(2+3\beta)(\alpha-c-w)}{(2+4\beta+\beta^2)\Delta s-(2+3\beta)s_1}$ and $\Delta s < \Delta s^T$ equals $\tau < \tau^T = \frac{(B+U)(\alpha-c-w)}{16(8+24\beta+25\beta^2+9\beta^3)^2(4+24\beta+53\beta^2+52\beta^3+22\beta^4+4\beta^5)\Delta s-(B+U)s_1}$, where $B = 16(2+3\beta)(2+7\beta+6\beta^2)(8+24\beta+25\beta^2+9\beta^3)^2$ and $U = \big[256(2+3\beta)^2(2+7\beta+6\beta^2)^2(8+24\beta+25\beta^2+9\beta^3)^4 + 32(8+24\beta+25\beta^2+9\beta^3)^2(4+24\beta+53\beta^2+52\beta^3+22\beta^4+4\beta^5)[(2+3\beta)(4+8\beta+3\beta^2)(4+10\beta+10\beta^2+3\beta^3)(9\beta^2\theta+4(2+\theta)+8\beta(2+\theta))A - (2+3\beta)^2(512+4096\beta+14176\beta^2+27808\beta^3+33512\beta^4+24752\beta^5+10344\beta^6+1872\beta^7-256\theta-2304\beta\theta-9152\beta^2\theta-21056\beta^3\theta-30504\beta^4\theta-28120\beta^5\theta-15906\beta^6\theta-4968\beta^7\theta-648\beta^8\theta+64\theta^2+448\beta\theta^2+1504\beta^2\theta^2+3056\beta^3\theta^2+4052\beta^4\theta^2+3536\beta^5\theta^2+1956\beta^6\theta^2+612\beta^7\theta^2+81\beta^8\theta^2)]\big]^{\frac{1}{2}}$. Therefore, (1) when $0 < \beta < \beta_2$, if $\theta_3 < \theta < \theta_0$ and $0 < \tau < \tau^K$, or when $\beta > \beta_2$, if $\theta_3 < \theta < \theta_5$ and $0 < \tau < \tau^T$, or $\theta_5 < \theta < \theta_0$ and $0 < \tau < \tau^K$, then OC is the optimal strategy; (2) when $\theta_0 < \theta < 1$ and $0 < \tau < \tau^K$, then MC is the optimal strategy; (3) otherwise, competition is the optimal strategy.

Proof of Lemma 5.3

From Proposition 5.1 we obtain $\theta_3 = \frac{4(1+\beta)}{8+10\beta+3\beta^2}$ and $\frac{d\theta_3}{d\beta} = -\frac{4(2+6\beta+3\beta^2)}{(2+\beta)^2(4+3\beta)^2} < 0$ implies θ_3 decreases in β. From Proposition 5.2 we obtain that $\theta_0 = \frac{16+96\beta+284\beta^2+456\beta^3+405\beta^4+162\beta^5}{32+160\beta+396\beta^2+552\beta^3+441\beta^4+162\beta^5}$ and $\frac{d\theta_0}{d\beta} =$

$\frac{8(2+4\beta+3\beta^2)(32+280\beta+936\beta^2+1680\beta^3+1746\beta^4+972\beta^5+243\beta^6)}{(4+8\beta+9\beta^2)^2(8+24\beta+33\beta^2+18\beta^3)^2} > 0$ implies θ_0 increases in β. Therefore, θ_3 decreases in β and θ_0 increases in β.

Proof of Lemma 5.4

From Proposition 5.1, we obtain $\Delta s^T = \frac{\delta(B+U)}{16\tau(8+24\beta+25\beta^2+9\beta^3)^2(4+24\beta+53\beta^2+52\beta^3+22\beta^4+4\beta^5)} > 0$ and $\tau^T = \frac{(B+U)(\alpha-c-w)}{16(8+24\beta+25\beta^2+9\beta^3)^2(4+24\beta+53\beta^2+52\beta^3+22\beta^4+4\beta^5)\Delta s-(B+U)s_1} > 0$. $(B+U)(\alpha-c-w) > 0$ and $16(8+24\beta+25\beta^2+9\beta^3)^2(4+24\beta+53\beta^2+52\beta^3+22\beta^4+4\beta^5) > 0$ implies τ^T decreases in Δs. Similarly, from Proposition 5.1, we obtain that $\Delta s^H = \frac{\delta L}{4\tau\beta^2(1+\beta)^4(8+32\beta+41\beta^2+18\beta^3)} > 0$ and $\tau^H = \frac{(\alpha-c-w)L}{4\beta^2(1+\beta)^4(8+32\beta+41\beta^2+18\beta^3)\Delta s-Ls_1} > 0$, where $L = -4\beta(1+\beta)^3(16+88\beta+178\beta^2+159\beta^3+54\beta^4) + \sqrt{2}\beta(1+\beta)^2(4+8\beta+3\beta^2)[(-8-32\beta-41\beta^2-18\beta^3)\theta(-A(2+4\beta+3\beta^2)+24\beta^3(-9+\theta)+16\beta(-8+\theta)+9\beta^4(-8+\theta)+4(-6+\theta)+\beta^2(-250+28\theta))]^{\frac{1}{2}}$. $(\alpha-c-w)L > 0$ and $4\beta^2(1+\beta)^4(8+32\beta+41\beta^2+18\beta^3) > 0$ implies τ^H decreases in Δs. From Proposition 5.1, we obtain that $\Delta s^J = \frac{\delta N}{2\tau(16+80\beta+154\beta^2+142\beta^3+61\beta^4+9\beta^5)^2} > 0$ and $\tau^J = \frac{(\alpha-c-w)N}{2(16+80\beta+154\beta^2+142\beta^3+61\beta^4+9\beta^5)^2\Delta s-Ns_1} > 0$, where $N = 2(4+14\beta+14\beta^2+3\beta^3)(8+24\beta+25\beta^2+9\beta^3)^2 - \sqrt{2}(64+448\beta+1304\beta^2+2040\beta^3+1842\beta^4+950\beta^5+255\beta^6+27\beta^7)[-A(2+4\beta+3\beta^2)(-2+\theta)+24\beta^3(11-11\theta+\theta^2)+16\beta(10-10\theta+\theta^2)+9\beta^4(10-10\theta+\theta^2)+4(8-8\theta+\theta^2)+\beta^2(306-306\theta+28\theta^2)]^{\frac{1}{2}}$. $(\alpha-c-w)N > 0$ and $2(16+80\beta+154\beta^2+142\beta^3+61\beta^4+9\beta^5)^2 > 0$ implies τ^J decreases in Δs. From Proposition 5.2, we obtain that $\Delta s^M = \frac{\delta(2+3\beta)F}{2\tau(4+8\beta+9\beta^2)(2+6\beta+5\beta^2+\beta^3)}$ and $\tau^M = \frac{F(\alpha-c-w)}{2(4+8\beta+9\beta^2)(2+6\beta+5\beta^2+\beta^3)\Delta s-Fs_1} > 0$, where $F = 8+24\beta+34\beta^2+18\beta^3-(2+\beta)\sqrt{(32+160\beta+396\beta^2+552\beta^3+441\beta^4+162\beta^5)(1-\theta)}$. $F(\alpha-c-w) > 0$ and $2(4+8\beta+9\beta^2)(2+6\beta+5\beta^2+\beta^3) > 0$ implies τ^M decreases in Δs. Therefore, τ^T, τ^H, τ^J and τ^M decrease in Δs.

Derivation of Table 5.3

(1) Competition model: From (5-1), we obtain $\frac{d^2\pi_1^n(p_1)}{dp_1{}^2} = -2(1+\beta) < 0$, then $\pi_1^n(p_1)$ is a concave function of p_1. Similarly, from (5-2), we obtain $\frac{d^2\pi_2^n(p_2)}{dp_2{}^2} = -2(1+\beta) < 0$, then $\pi_2^n(q_2)$ is a concave function of q_2. Let $\frac{d\pi_1^n(p_1)}{dp_1} = \frac{d\pi_2^n(p_2)}{dp_2} = 0$ and we obtain $p_1^n = c + w + \frac{\tau\Delta s\beta(1+\beta)+\delta_1(2+3\beta)-\beta\Delta\alpha}{(2+\beta)(2+3\beta)}$ and $p_2^n = c + w +$

$\frac{\delta_1(2+3\beta)-\tau\Delta s(2+4\beta+\beta^2)-2(1+\beta)\Delta\alpha}{(2+\beta)(2+3\beta)}$. Recall that $q_i=\alpha_i-p_i+\beta(p_j-p_i)+\tau[s_i-\beta(s_j-s_i)]$, then $q_1^n=\frac{(1+\beta)[\delta_1(2+3\beta)+\beta(1+\beta)\tau\Delta s-\beta\Delta\alpha]}{(2+\beta)(2+3\beta)}=(1+\beta)(p_1^n-w-c)$ and $q_2^n=\frac{(1+\beta)[\delta_1(2+3\beta)-(2+4\beta+\beta^2)\tau\Delta s-2(1+\beta)\Delta\alpha]}{(2+\beta)(2+3\beta)}=(1+\beta)(p_2^n-w-c)$.

(2) OC model: For given m, from (5-3) we obtain $\frac{d^2\pi_1^o(p_1)}{d{p_1}^2}=-2(1+\beta)<0$, then $\pi_1^o(p_1)$ is a concave function of p_1. Similarly, from (5-4), we obtain $\frac{d^2\pi_2^o(p_2)}{d{p_2}^2}=-2(1+\beta)<0$, then $\pi_2^o(q_2)$ is a concave function of q_2. Let $\frac{d\pi_1^o(p_1)}{dp_1}=\frac{d\pi_2^o(p_2)}{dp_2}=0$ and we obtain $p_1=m+w+\frac{3c+2\delta_1-3m-\Delta\alpha}{4+2\beta}+\frac{c-m+\Delta\alpha}{4+6\beta}$ and $p_2=m+w+\frac{3c+2\delta_1-3m-\Delta\alpha}{4+2\beta}-\frac{c-m+\Delta\alpha}{4+6\beta}$. Replace $p_1=m+w+\frac{3c+2\delta_1-3m-\Delta\alpha}{4+2\beta}+\frac{c-m+\Delta\alpha}{4+6\beta}$ and $p_2=m+w+\frac{3c+2\delta_1-3m-\Delta\alpha}{4+2\beta}-\frac{c-m+\Delta\alpha}{4+6\beta}$ in (5-5), we obtain $\pi^{om}(m)=(\pi_1^o)^\theta(\pi_2^o)^{1-\theta}=\Big\{\frac{1}{(2+\beta)^2(2+3\beta)^2}[-(c-m)(1+\beta)(2+\beta)(2+3\beta)(\delta_1(2+3\beta)+c(2+4\beta)-2(m+\Delta\alpha+2m\beta+\Delta\alpha\beta))+(\delta_1(2+3\beta)-\beta(\Delta\alpha+3c(1+\beta)-3m(1+\beta)))(\delta_1(2+5\beta+3\beta^2)+\beta(c+2c\beta-\Delta\alpha(1+\beta)-m(1+2\beta)))]\Big\}^\theta\Big\{\frac{1}{(2+\beta)^2(2+3\beta)^2}[(1+\beta)(\delta_1(2+3\beta)+c(2+4\beta)-2(m+\Delta\alpha+2m\beta+\Delta\alpha\beta))^2]\Big\}^{1-\theta}$. Let $\pi^{om}(m)=(\pi_1^o)^\theta(\pi_2^o)^{1-\theta}=G(m)$, then $\ln G(m)=\theta\ln\pi_1^o+(1-\theta)\ln\pi_2^o$. By derivation we obtain $\frac{1}{G(m)}\frac{dG(m)}{dm}=\theta\frac{1}{\pi_1^o}\frac{d\pi_1^o}{dm}+(1-\theta)\frac{1}{\pi_2^o}\frac{d\pi_2^o}{dm}$, that is, $\frac{dG(m)}{dm}=G(m)\left[\theta\frac{1}{\pi_1^o}\frac{d\pi_1^o}{dm}+(1-\theta)\frac{1}{\pi_2^o}\frac{d\pi_2^o}{dm}\right]$. Let $\frac{dG(m)}{dm}=0$ and recalling $G(m)>0$, then $\theta\frac{1}{\pi_1^o}\frac{d\pi_1^o}{dm}+(1-\theta)\frac{1}{\pi_2^o}\frac{d\pi_2^o}{dm}=\{4(m+2m\beta)^2(8+24\beta+25\beta^2+9\beta^3)-2m(1+2\beta)[4c(8+40\beta+73\beta^2+59\beta^3+18\beta^4)+\delta_1(2+3\beta)\big(4(2+\theta)+10\beta^2(3+\theta)+3\beta^3(4+\theta)+2\beta(14+5\theta)\big)-\Delta\alpha\big(8(2+\theta)+32\beta(2+\theta)+9\beta^4(2+\theta)+4\beta^3(16+9\theta)+\beta^2(96+50\theta)\big)]+4(c+2c\beta)^2(8+24\beta+25\beta^2+9\beta^3)+{\delta_1}^2(1+2\beta)(2+3\beta)^2(-4+8\theta+3\beta^2\theta+2\beta(-2+5\theta))+2\Delta\alpha^2(1+\beta)(8\theta+32\beta\theta+9\beta^4\theta+4\beta^3(-1+9\theta)+\beta^2(-2+50\theta))-\delta_1\Delta\alpha(2+3\beta)(16\theta+21\beta^4\theta+2\beta^3(-8+45\theta)+2\beta^2(-12+65\theta)+\beta(-8+76\theta))+2c(1+2\beta)[\delta_1(2+3\beta)(4(2+\theta)+10\beta^2(3+\theta)+3\beta^3(4+\theta)+2\beta(14+5\theta))-\Delta\alpha(8(2+\theta)+32\beta(2+\theta)+9\beta^4(2+\theta)+4\beta^3(16+9\theta)+\beta^2(96+50\theta))]\}\{(\delta_1(2+3\beta)+c(2+4\beta)-2(m+\Delta\alpha+2m\beta+\Delta\alpha\beta))(-8m^2-8m\Delta\alpha-40m^2\beta-32m\Delta\alpha\beta-73m^2\beta^2-48m\Delta\alpha\beta^2+\Delta\alpha^2\beta^2-59m^2\beta^3-32m\Delta\alpha\beta^3+\Delta\alpha^2\beta^3-18m^2\beta^4-9m\Delta\alpha\beta^4+{\delta_1}^2(1+\beta)(2+3\beta)^2-c^2(8+40\beta+73\beta^2+59\beta^3+18\beta^4)+\delta_1(2+3\beta)(-2\Delta\alpha\beta(1+\beta)+m(4+14\beta+15\beta^2+6\beta^3))+c(\Delta\alpha(8+32\beta+48\beta^2+32\beta^3+9\beta^4)-\delta_1(8+40\beta+72\beta^2+57\beta^3+18\beta^4)+2m(8+40\beta+73\beta^2+59\beta^3+18\beta^4)))\}^{-1}=0$, $\Delta(m)=4(4+16\beta+19\beta^2+6\beta^3)^2[\Delta\alpha^2(2+4\beta+3\beta^2)^2(-2+\theta)^2+{\delta_1}^2(24\beta^3(18-18\theta+\theta^2)+16\beta(16-16\theta+\theta^2)+9\beta^4(16-16\theta+\theta^2)+4(12-12\theta+\theta^2)+4\beta^2(125-125\theta+7\theta^2))-2\delta_1\Delta\alpha(4(-2+\theta)^2+9\beta^4(8-8\theta+\theta^2)+16\beta(6-6\theta+\theta^2)+$

$4\beta^3(49-49\theta+6\theta^2)+4\beta^2(52-52\theta+7\theta^2))\big]$, and for $\beta>0$ and $0<\theta<1$, there is no real root for $\Delta\alpha^2(2+4\beta+3\beta^2)^2(-2+\theta)^2+{\delta_1}^2\big(24\beta^3(18-18\theta+\theta^2)+16\beta(16-16\theta+\theta^2)+9\beta^4(16-16\theta+\theta^2)+4(12-12\theta+\theta^2)+4\beta^2(125-125\theta+7\theta^2)\big)-2\delta_1\Delta\alpha\big(4(-2+\theta)^2+9\beta^4(8-8\theta+\theta^2)+16\beta(6-6\theta+\theta^2)+4\beta^3(49-49\theta+6\theta^2)+4\beta^2(52-52\theta+7\theta^2)\big)=0$, then $\Delta(m)>0$, so we obtain two real roots: $m_1=c+\frac{1}{4(1+2\beta)^2(8+24\beta+25\beta^2+9\beta^3)}\{\Delta\alpha(-16-96\beta-224\beta^2-256\beta^3-146\beta^4-36\beta^5-8\theta-48\beta\theta-114\beta^2\theta-136\beta^3\theta-81\beta^4\theta-18\beta^5\theta)+\delta_1(16+112\beta+304\beta^2+402\beta^3+264\beta^4+72\beta^5+8\theta+48\beta\theta+114\beta^2\theta+136\beta^3\theta+81\beta^4\theta+18\beta^5\theta)-(4+16\beta+19\beta^2+6\beta^3)A_1\}$ and $m_2=c+\frac{1}{4(1+2\beta)^2(8+24\beta+25\beta^2+9\beta^3)}\{\Delta\alpha(-16-96\beta-224\beta^2-256\beta^3-146\beta^4-36\beta^5-8\theta-48\beta\theta-114\beta^2\theta-136\beta^3\theta-81\beta^4\theta-18\beta^5\theta)+\delta_1(16+112\beta+304\beta^2+402\beta^3+264\beta^4+72\beta^5+8\theta+48\beta\theta+114\beta^2\theta+136\beta^3\theta+81\beta^4\theta+18\beta^5\theta)+(4+16\beta+19\beta^2+6\beta^3)A_1\}$, where $A_1=\big[\Delta\alpha^2(2+4\beta+3\beta^2)^2(-2+\theta)^2+{\delta_1}^2\big((48+256\beta+500\beta^2+432\beta^3+144\beta^4)(1-\theta)+(4+16\beta+28\beta^2+24\beta^3+9\beta^4)\theta^2\big)+2\delta_1\Delta\alpha\big((16+96\beta+208\beta^2+196\beta^3+72\beta^4)(1-\theta)+(4+16\beta+28\beta^2+24\beta^3+9\beta^4)\theta^2\big)\big]^{\frac{1}{2}}$.

Replace m_2 in $p_1=m+w+\frac{3c+2\delta_1-3m-\Delta\alpha}{4+2\beta}+\frac{c-m+\Delta\alpha}{4+6\beta}$ and $p_2=m+w+\frac{3c+2\delta_1-3m-\Delta\alpha}{4+2\beta}-\frac{c-m+\Delta\alpha}{4+6\beta}$, and recall that $q_i=\alpha_i-p_i+\beta(p_j-p_i)+\tau s_1$, we obtain that $q_2^o|_{m=m_2}=-\frac{1}{2(8+16\beta+9\beta^2)}[4\Delta\alpha+8\Delta\alpha\beta+6\Delta\alpha\beta^2+\delta_1(2+4\beta+3\beta^2)(-2+\theta)-2\Delta\alpha\theta-4\Delta\alpha\beta\theta-3\Delta\alpha\beta^2\theta++A_1]$. For $\beta>0$ and $0<\theta<1$, $4\Delta\alpha+8\Delta\alpha\beta+6\Delta\alpha\beta^2+\delta_1(2+4\beta+3\beta^2)(-2+\theta)-2\Delta\alpha\theta-4\Delta\alpha\beta\theta-3\Delta\alpha\beta^2\theta++A_1>0$, which implies $q_2^o|_{m=m_2}<0$. Then we check if $m_1>c$. $m_1-c=\frac{1}{4(1+2\beta)^2(8+24\beta+25\beta^2+9\beta^3)}\{\Delta\alpha(-16-96\beta-224\beta^2-256\beta^3-146\beta^4-36\beta^5-8\theta-48\beta\theta-114\beta^2\theta-136\beta^3\theta-81\beta^4\theta-18\beta^5\theta)+\delta_1(16+112\beta+304\beta^2+402\beta^3+264\beta^4+72\beta^5+8\theta+48\beta\theta+114\beta^2\theta+136\beta^3\theta+81\beta^4\theta+18\beta^5\theta)-(4+16\beta+19\beta^2+6\beta^3)A_1\}$ and we obtain that if $\theta_3=\frac{4(1+3\beta+2\beta^2)(\Delta\alpha\beta-\delta_1(2+3\beta))^2}{(4+8\beta+3\beta^2)(2\Delta\alpha^2(2+6\beta+7\beta^2+3\beta^3)+{\delta_1}^2(8+34\beta+45\beta^2+18\beta^3)-\delta_1\Delta\alpha(8+34\beta+48\beta^2+21\beta^3))}<\theta<1$ and $\Delta\alpha<\Delta\alpha^D=\frac{\delta_1(2+3\beta)}{2(1+\beta)}$, then $m_1>c$.

$\pi_1^o|_{m=m_1}=\frac{1}{16(1+\beta)(1+2\beta)(8+16\beta+9\beta^2)}\big[-{A_1}^2+2A_1(\delta_1-\Delta\alpha)(2+4\beta+3\beta^2)\theta-\Delta\alpha^2(2+4\beta+3\beta^2)^2(-4+\theta^2)+{\delta_1}^2(48-4\theta^2+\beta^2(500-28\theta^2)-24\beta^3(-18+\theta^2)-16\beta(-16+\theta^2)-9\beta^4(-16+\theta^2))+2\delta_1\Delta\alpha(9\beta^4(-8+\theta^2)+16\beta(-6+\theta^2)+4(-4+\theta^2)+4\beta^3(-49+6\theta^2)+4\beta^2(-52+7\theta^2))\big]$ and we obtain that if $\theta_3<\theta<1$ and $\Delta\alpha<\Delta\alpha^D$, then $\pi_1^o|_{m=m_1}>0$. Similarly, we obtain that $\pi_2^o|_{m=m_1}=\frac{(A_1-(\delta_1-\Delta\alpha)(2+4\beta+3\beta^2)(-2+\theta))^2}{4(1+\beta)(8+16\beta+9\beta^2)^2}>0$. $\frac{d^2G(m)}{dm^2}|_{m=m_1}=G(m_1)\frac{d\left[\theta\frac{1}{\pi_1^o}\frac{d\pi_1^o}{dm}+(1-\theta)\frac{1}{\pi_2^o}\frac{d\pi_2^o}{dm}\right]}{dm}|_{m=m_1}$ and since $\pi_1^o|_{m=m_1}>0$

and $\pi_2^o|_{m=m_1} > 0$, then $G(m_1) > 0$. $\frac{d\left[\theta\frac{1}{\pi_1^o}\frac{d\pi_1^o}{dm}+(1-\theta)\frac{1}{\pi_2^o}\frac{d\pi_2^o}{dm}\right]}{dm}|_{m=m_1} = \frac{32(1+\beta)^2(1+2\beta)^2(8+\beta(16+9\beta))^2}{(4+8\beta+3\beta^2)^2}\left\{\frac{\theta-1}{(A_1-(\delta_1-\Delta\alpha)(2+\beta(4+3\beta))(-2+\theta))^2} - \frac{2\theta\left(A_1-(\delta_1-\Delta\alpha)\left(2+\beta(4+3\beta)\right)\theta\right)^2}{{B_1}^2} + \frac{\theta}{B_1}\right\}$, where $B_1 = -4\Delta\alpha^2(2+\beta(4+3\beta))^2 - 4{\delta_1}^2(1+2\beta)(2+3\beta)(6+\beta(11+6\beta)) + 8\delta_1\Delta\alpha(1+2\beta)(4+\beta(16+\beta(20+9\beta))) + (A_1-(\delta_1-\Delta\alpha)(2+\beta(4+3\beta))\theta)^2$. For $\theta_3 < \theta < 1$ and $\Delta\alpha < \Delta\alpha^D$, $\frac{\theta-1}{(A_1-(\delta_1-\Delta\alpha)(2+\beta(4+3\beta))(-2+\theta))^2} - \frac{2\theta\left(A_1-(\delta_1-\Delta\alpha)\left(2+\beta(4+3\beta)\right)\theta\right)^2}{{B_1}^2} + \frac{\theta}{B_1} < 0$, then $\frac{d\left[\theta\frac{1}{\pi_1^o}\frac{d\pi_1^o}{dm}+(1-\theta)\frac{1}{\pi_2^o}\frac{d\pi_2^o}{dm}\right]}{dm}|_{m=m_1} < 0$. Therefore, $\frac{d^2G(m)}{dm^2}|_{m=m_1} < 0$.

Replace m_1 in $p_1 = m + w + \frac{3c+2\delta_1-3m-\Delta\alpha}{4+2\beta} + \frac{c-m+\Delta\alpha}{4+6\beta}$ and $p_2 = m + w + \frac{3c+2\delta_1-3m-\Delta\alpha}{4+2\beta} - \frac{c-m+\Delta\alpha}{4+6\beta}$, we obtain that $p_1^o|_{m=m_1} = c + w + \frac{\delta(4+8\beta+3\beta^2)}{4(1+2\beta)(8+16\beta+9\beta^2)}\{\delta_1(16+9\beta^3(4+\theta)+6\beta^2(15+2\theta)+\beta(68+6\theta)) - \beta[3A_1+\Delta\alpha(20+6\theta+9\beta^2(2+\theta)+4\beta(10+3\theta))]\} > 0$ and $p_2^o|_{m=m_1} = c + w - \frac{1}{4(1+2\beta)(8+24\beta+25\beta^2+9\beta^3)}\{A_1(2+4\beta+3\beta^2) + \Delta\alpha(9\beta^4(2+\theta)+28\beta^2(5+\theta)+4(6+\theta)+16\beta(6+\theta)+8\beta^3(11+3\theta)) - \delta_1(9\beta^4(4+\theta)+4(6+\theta)+16\beta(7+\theta)+6\beta^3(23+4\theta)+4\beta^2(47+7\theta))\} > 0$. Recall that $q_i = \alpha_i - p_i + \beta(p_j - p_i) + \tau s_1$, then $q_1^o|_{m=m_1} = \frac{\delta_1(16+\beta^2(58-4\theta)-2\beta(-26+\theta)-3\beta^3(-8+\theta))+\beta(A_1+\Delta\alpha(2+4\beta+3\beta^2)(-2+\theta))}{4(8+24\beta+25\beta^2+9\beta^3)} > 0$ and $q_2^o|_{m=m_1} = \frac{A_1-(\delta_1-\Delta\alpha)(2+4\beta+3\beta^2)(-2+\theta)}{2(8+16\beta+9\beta^2)} > 0$.

Therefore, if $\theta_3 < \theta < 1$ and $\Delta\alpha < \Delta\alpha^D$, then $p_1^o = c + w + \frac{\delta(4+8\beta+3\beta^2)}{4(1+2\beta)(8+16\beta+9\beta^2)}\{\delta_1(16+9\beta^3(4+\theta)+6\beta^2(15+2\theta)+\beta(68+6\theta)) - \beta[3A_1+\Delta\alpha(20+6\theta+9\beta^2(2+\theta)+4\beta(10+3\theta))]\}$ and $p_2^o = c + w - \frac{1}{4(1+2\beta)(8+24\beta+25\beta^2+9\beta^3)}\{A_1(2+4\beta+3\beta^2) + \Delta\alpha(9\beta^4(2+\theta)+28\beta^2(5+\theta)+4(6+\theta)+16\beta(6+\theta)+8\beta^3(11+3\theta)) - \delta_1(9\beta^4(4+\theta)+4(6+\theta)+16\beta(7+\theta)+6\beta^3(23+4\theta)+4\beta^2(47+7\theta))\}$ and $m^o = c + \frac{1}{4(1+2\beta)^2(8+24\beta+25\beta^2+9\beta^3)}\{\Delta\alpha(-16-96\beta-224\beta^2-256\beta^3-146\beta^4-36\beta^5-8\theta-48\beta\theta-114\beta^2\theta-136\beta^3\theta-81\beta^4\theta-18\beta^5\theta) + \delta_1(16+112\beta+304\beta^2+402\beta^3+264\beta^4+72\beta^5+8\theta+48\beta\theta+114\beta^2\theta+136\beta^3\theta+81\beta^4\theta+18\beta^5\theta) - (4+16\beta+19\beta^2+6\beta^3)A_1\}$, where $A_1 = \left[\Delta\alpha^2(2+4\beta+3\beta^2)^2(-2+\theta)^2 + {\delta_1}^2\left((48+256\beta+500\beta^2+432\beta^3+144\beta^4)(1-\theta)+(4+16\beta+28\beta^2+24\beta^3+9\beta^4)\theta^2\right) + 2\delta_1\Delta\alpha\left((16+96\beta+208\beta^2+196\beta^3+72\beta^4)(1-\theta)+(4+16\beta+28\beta^2+24\beta^3+9\beta^4)\theta^2\right)\right]^{\frac{1}{2}}$.

(3) MC model: From (5-5) we obtain $\frac{d^2\pi_1^m(p_1)}{d{p_1}^2} = -2(1+\beta) < 0$, then $\pi_1^m(p_1)$ is a concave function of p_1. Similarly, from (5-6), we obtain $\frac{d^2\pi_2^m(p_2)}{d{p_2}^2} = -2(1+\beta) < 0$, then $\pi_2^m(p_2)$ is a concave function of p_2. Let

$\frac{d\pi_1^m(p_1)}{dp_1} = \frac{d\pi_2^m(p_2)}{dp_2} = 0$ and we obtain $p_1 = c + \frac{1}{4+8\beta+3\beta^2}[4w + 3u\beta + 8w\beta - \Delta\alpha\beta + 3u\beta^2 + 3w\beta^2 + \delta_1(2 + 3\beta)]$ and $p_2 = c + \frac{1}{4+8\beta+3\beta^2}[2u + 4w - 2\Delta\alpha + 4u\beta + 8w\beta - 2\Delta\alpha\beta + 3u\beta^2 + 3w\beta^2 + \delta_1(2 + 3\beta)]$.

Replace p_1 and p_2 in (5-8), and let $\pi^{mT}(u,T) = (\pi_1^m)^\theta(\pi_2^m)^{1-\theta} = H(u,T)$, then $\ln H(u,T) = \theta \ln \pi_1^m + (1-\theta)\ln \pi_2^m$. By derivation we obtain $\frac{1}{H(u,T)}\frac{\partial H(u,T)}{\partial u} = \theta\frac{1}{\pi_1^m}\frac{\partial \pi_1^m}{\partial u} + (1-\theta)\frac{1}{\pi_2^m}\frac{\partial \pi_2^m}{\partial u}$ and $\frac{1}{H(u,T)}\frac{\partial H(u,T)}{\partial T} = \theta\frac{1}{\pi_1^m}\frac{\partial \pi_1^m}{\partial T} + (1-\theta)\frac{1}{\pi_2^m}\frac{\partial \pi_2^m}{\partial T}$, then $\frac{\partial H(u,T)}{\partial u} = H(u,T)\left[\theta\frac{1}{\pi_1^m}\frac{\partial \pi_1^m}{\partial u} + (1-\theta)\frac{1}{\pi_2^m}\frac{\partial \pi_2^m}{\partial u}\right]$ and $\frac{\partial H(u,T)}{\partial T} = H(u,T)\left[\theta\frac{1}{\pi_1^m}\frac{\partial \pi_1^m}{\partial T} + (1-\theta)\frac{1}{\pi_2^m}\frac{\partial \pi_2^m}{\partial T}\right]$. Let $\frac{\partial H(u,T)}{\partial u} = \frac{\partial H(u,T)}{\partial T} = 0$, we obtain $u^m = \frac{\beta[\delta_1(1+2\beta)(2+3\beta)^2 - \Delta\alpha\beta(8+16\beta+9\beta^2)]}{8+40\beta+82\beta^2+86\beta^3+36\beta^4}$ and $T^m = \frac{P(\Delta\alpha)}{4(1+3\beta+2\beta^2)(4+8\beta+9\beta^2)^2}$, where $P(\Delta\alpha) = \Delta\alpha^2(2 + 4\beta + 3\beta^2)^2(9\beta^2(-1+\theta) + 4\theta + 8\beta\theta) - 2\delta_1\Delta\alpha(1+2\beta)(81\beta^5(-1+\theta) + 16\theta + 16\beta(-1+5\theta) + 36\beta^4(-5 + 6\theta) + 28\beta^2(-3+7\theta) + 16\beta^3(-11+17\theta)) + {\delta_1}^2(1+2\beta)(-16 + 162\beta^5(-1+\theta) + 32\theta + 32\beta(-3 + 5\theta) + 24\beta^3(-19+23\theta) + 9\beta^4(-45+49\theta) + 4\beta^2(-71+99\theta))$. If $\Delta\alpha < \Delta\alpha^P = \frac{\delta_1(1+2\beta)(2+3\beta)^2}{\beta(8+16\beta+9\beta^2)}$, then $u^m > 0$. As for T^m, we obtain that if $\theta_0 = \frac{9\Delta\alpha^2\beta^2(2+4\beta+3\beta^2)^2 - 2\delta_1\Delta\alpha\beta(16+116\beta+344\beta^2+532\beta^3+441\beta^4+162\beta^5) + {\delta_1}^2(16+128\beta+476\beta^2+1024\beta^3+1317\beta^4+972\beta^5+324\beta^6)}{(4+8\beta+9\beta^2)(\Delta\alpha^2(2+4\beta+3\beta^2)^2 - 2\delta_1\Delta\alpha(4+20\beta+40\beta^2+41\beta^3+18\beta^4) + {\delta_1}^2(8+40\beta+81\beta^2+84\beta^3+36\beta^4))} < \theta < 1$ and $\Delta\alpha < \Delta\alpha^P$, then $T^m > 0$. Replace u^m in p_1 and p_2, we obtain $p_1^m = c + w + \frac{-\Delta\alpha\beta(2+4\beta+9\beta^2) + \delta_1(4+14\beta+21\beta^2+18\beta^3)}{8+32\beta+50\beta^2+36\beta^3}$ and $p_2^m = c + w + \frac{-\Delta\alpha(4+16\beta+30\beta^2+28\beta^3+9\beta^4) + \delta_1(4+20\beta+42\beta^2+45\beta^3+18\beta^4)}{8+40\beta+82\beta^2+86\beta^3+36\beta^4}$. Recall that $q_i = \alpha_i - p_i + \beta(p_j - p_i) + \tau s_1$, then $q_1^m = \frac{-\Delta\alpha\beta(2+4\beta+3\beta^2) + \delta_1(4+14\beta+21\beta^2+12\beta^3)}{2(1+\beta)(4+8\beta+9\beta^2)}$ and $q_2^m = \frac{(\delta_1 - \Delta\alpha)(2+4\beta+3\beta^2)}{4+8\beta+9\beta^2}$. $q_2^m > 0$ implies $\Delta\alpha < \delta_1 < \Delta\alpha^P$. For $\theta_0 < \theta < 1$ and $\Delta\alpha < \delta_1$, we obtain p_1^m, p_2^m, q_1^m and $q_2^m > 0$. $\frac{\partial^2 H(u,T)}{\partial T^2}|_{u=u^m,T=T^m} = -\frac{16(1+\beta)^2(1+2\beta)^2(4+8\beta+9\beta^2)^2 H(u^m,T^m)}{\left(\Delta\alpha^2(2+4\beta+3\beta^2)^2 - 2\delta_1\Delta\alpha(4+20\beta+40\beta^2+41\beta^3+18\beta^4) + {\delta_1}^2(8+40\beta+81\beta^2+84\beta^3+36\beta^4)\right)^2(1-\theta)\theta} < 0$ and $\begin{vmatrix} \frac{\partial^2 H(u,T)}{\partial T^2} & \frac{\partial^2 H(u,T)}{\partial T\partial u} \\ \frac{\partial^2 H(u,T)}{\partial u\partial T} & \frac{\partial^2 H(u,T)}{\partial u^2} \end{vmatrix}|_{u=u^m,T=T^m} = \frac{128(1+\beta)^4(1+2\beta)^4(4+8\beta+9\beta^2)^4}{(2+\beta)^2(2+3\beta)^2\left(\Delta\alpha^2(2+4\beta+3\beta^2)^2 - 2\delta_1\Delta\alpha(4+20\beta+40\beta^2+41\beta^3+18\beta^4) + {\delta_1}^2(8+40\beta+81\beta^2+84\beta^3+36\beta^4)\right)^3(1-\theta)\theta} > 0$. So, if $\theta_0 < \theta < 1$ and $\Delta\alpha < \delta_1$, $p_1^m = c + w + \frac{-\Delta\alpha\beta(2+4\beta+9\beta^2) + \delta_1(4+14\beta+21\beta^2+18\beta^3)}{8+32\beta+50\beta^2+36\beta^3}$, $p_2^m = c + w + \frac{-\Delta\alpha(4+16\beta+30\beta^2+28\beta^3+9\beta^4) + \delta_1(4+20\beta+42\beta^2+45\beta^3+18\beta^4)}{8+40\beta+82\beta^2+86\beta^3+36\beta^4}$, $u^m = \frac{\beta[\delta_1(1+2\beta)(2+3\beta)^2 - \Delta\alpha\beta(8+16\beta+9\beta^2)]}{8+40\beta+82\beta^2+86\beta^3+36\beta^4}$ and $T^m = \frac{P(\Delta\alpha)}{4(1+3\beta+2\beta^2)(4+8\beta+9\beta^2)^2}$, where $P(\Delta\alpha) = \Delta\alpha^2(2 + 4\beta + 3\beta^2)^2(9\beta^2(-1+\theta) + 4\theta + 8\beta\theta) - 2\delta_1\Delta\alpha(1 + 2\beta)(81\beta^5(-1+\theta) + 16\theta + 16\beta(-1+5\theta) + 36\beta^4(-5+6\theta) + 28\beta^2(-3+7\theta) + 16\beta^3(-11+17\theta)) +$

$\delta_1{}^2(1+2\beta)[-16+162\beta^5(-1+\theta)+32\theta+32\beta(-3+5\theta)+24\beta^3(-19+23\theta)+9\beta^4(-45+49\theta)+4\beta^2(-71+99\theta)]$.