Deductively sound formal proofs of mathematical logic

Could the intersection of [formal proofs of mathematical logic] and [sound deductive inference] specify formal systems having [deductively sound formal proofs of mathematical logic]?

All that we have to do to provide [deductively sound formal proofs of mathematical logic] is select the subset of conventional [formal proofs of mathematical logic] having true premises and now we have [deductively sound formal proofs of mathematical logic].

Validity and Soundness https://www.iep.utm.edu/val-snd/
A deductive argument is said to be valid if and only if it takes a form that makes it impossible for the premises to be true and the conclusion nevertheless to be false. Otherwise, a deductive argument is said to be invalid.

A deductive argument is sound if and only if it is both valid, and all of its premises are actually true. Otherwise, a deductive argument is unsound.

In other words in sound deduction there is a:
[connected sequence of valid deductions from true premises to a true conclusion]

> **Introduction to Mathematical logic Sixth edition Elliott Mendelson (2015) Page 28**
> A wf C is said to be a consequence in S of a set Γ of wfs if and only if there is a sequence B1, …, Bk of wfs such that C is Bk and, for each i, either Bi is an axiom or Bi is in Γ, or Bi is a direct consequence by some rule of inference of some of the preceding wfs in the sequence. Such a sequence is called a proof (or deduction) of C from Γ. The members of Γ are called the hypotheses or premises of the proof.
> **We use Γ ⊢ C as an abbreviation for “C is a consequence of Γ”...**

To explain the notion of [sound deduction] using the terms of the art and symbols of [formal proofs] requires totally understanding this one aspect of [formal proofs]:
Γ ⊢ C means that "C is a consequence of premises Γ"

"Γ" ----- Specifies the premises of a formal proof.
"C" ----- Specifies the consequence of a formal proofs.
"⊢" ------ Specifies valid deduction from premises to consequence of formal proofs.
"Γ ⊢ C" Specifies that C is provable from Γ, in other words "Γ ⊢ C" a valid deductive argument.

To convert a valid deductive argument into a sound deductive argument only requires that all the premises are true. We only have to find some way to select the subset of conventional formal proofs having entirely true premises.

Curry, Harkell B. 1977. Foundations of Mathematical Logic. Page:45

> The statements of F are called elementary statements to distinguish them from other statements which we may form from them …
> A theory (over F is defined as a conceptual class of these elementary statements. Let T be such a theory. **Then the elementary statements which belong to T we shall call the elementary theorems of T; we also say that these elementary statements are true for T. Thus, given T, an elementary theorem is an elementary statement which is true.** A theory is thus a way of picking out from the statements of F a certain subclass of true statements.

As Haskell Curry stipulates we simply stipulate that Axioms are true. This means that Axioms are expressions of language having been defined to have the semantic value of Boolean True. By this single stipulation it becomes very easy to define a universal truth predicate.

Within the stipulated definition of Axiom and the conventional notation of formal systems we specify this predicate: $\exists \Gamma \subseteq \text{Axioms}(F)\ \exists C \in \text{WFF}(F)\ (\text{Deductively\_Sound}(\Gamma \vdash C))$

From the above $\text{Deductively\_Sound}(\Gamma \vdash C)$ we can derive a universal truth predicate: It is common knowledge in the sound deductive inference model that true premises combined with valid deduction necessitates a true conclusion.

Thus we know that $\text{Deductively\_Sound}(\Gamma \vdash C) \rightarrow \text{True}(C)$. It is also common notation convention to not indicate an empty set of premises, (which means the proof is based on axioms), thus this $(\Gamma \vdash C)$ becomes this $(\vdash C)$ Because of this we can define $\text{True}(C) := (\vdash C)$.

We are not simply specifying that $\text{True}(C)$ is $\text{Provable}(C)$. We are specifying the common idea from sound deduction that a true conclusion necessarily follows from true premises and valid inference. Within the stipulation that Axioms are True then any formal proof to theorem consequences necessarily derives a true consequence.

$\exists F \in \text{Formal\_System}\ \exists G \in \text{Closed\_WFF}(F)\ (G \leftrightarrow ((F \nvdash G) \land (F \nvdash \neg G)))$
Then it becomes quite obvious that Gödel's G is not true.

When True(G) requires provability from axioms $(\vdash G)$ and no provability from axioms or anything else exists $\therefore \neg\text{True}(G)$.

When True(¬G) requires provability from axioms $(\vdash \neg G)$ and no provability from axioms or anything else exists $\therefore \neg\text{True}(\neg G)$.