

Tarski Undefinability Theorem Terse Refutation

Both Tarski and Gödel “prove” that provability can diverge from Truth. When we boil their claim down to its simplest possible essence it is really claiming that valid inference from true premises might not always derive a true consequence. This is obviously impossible.

Formalizing this simple English:

A connected set of known truths necessarily always derives truth.

We derive the sound deductive inference model with:

[a connected sequence of valid deductions from true premises to a true conclusion]

This equivalent end result is achieved in mathematical symbolic logic:

[a connected sequence of valid inference from axioms to a true consequence]

When Axioms are construed as expressions of language having the semantic property of Boolean true.

This last part connects the semantic notion of Boolean values to the syntax of formal expressions. Rudolf Carnap proposed this same idea in his (1952) Meaning Postulates.

The axiom of: [Deductively Sound Formal Proofs] -- $\text{True}(x) \leftrightarrow (\vdash x)$

True Premises Necessarily derive a True Consequence: $\Box(\text{True}(P) \vdash \text{True}(C))$

Expressions of language that are not decided to be True are decided to be $\neg\text{True}$, thus undecidable sentences cannot be expressed.

The above explicitly redefines the notion of formal system by providing the axiom schema / meta-mathematics to eliminate undecidability, incompleteness and inconsistency in formal systems.

When-so-ever C and $\neg C$ both evaluate to $\neg\text{True}$ we now have an additional semantic criteria that can be added to the syntactic criteria of well-formedness. Although this goes directly against the grain of conventional wisdom that semantics can only be specified using model theory, (challenges to this conventional wisdom have Haskell Curry precedents) this semantic axiom schema / meta-mathematics can be construed as accepting semantically well-formed formula and thus rejecting others.

The third step of Tarski's proof:

the symbol 'Pr' which denotes the class of all provable sentences

we denote the class of all true sentences by the symbol 'Tr'

(3) $x \notin \text{Pr}$ if and only if $x \in \text{Tr}$ // $\sim\text{Provable}(x) \leftrightarrow \text{True}(x)$

simply assumes that Provability diverges from Truth.

The third step of Tarski's proof is refuted by contradicting the axiom: $\text{True}(x) \leftrightarrow (\vdash x)$ of [Deductively Sound Formal Proofs] (DSFP) causing the whole proof to fail.

The 1936 Tarski Undefinability Proof

http://liarparadox.org/Tarski_Proof_275_276.pdf