View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by PhilPapers

EXPOSING FAKE LOGIC

By **Avi Sion** PH.D.

© Copyright Avi Sion, 2018, 2019. All rights reserved.
Protected by international copyright conventions. No part of this book may be reproduced in any manner whatsoever, or stored in a retrieval system or transmitted, without express permission of the Author-publisher, except in case of brief quotations with due acknowledgement.

Self-published through Lulu; CreateSpace & Kindle.
Website: www.TheLogician.net. E-mail: avi-sion@thelogician.net.

Exposing Fake Logic can be freely read online at avisionpolemics and in various other locations. It can be purchased, in print and e-book editions, in Amazon.com, Lulu.com and many other online booksellers.

The present document contains **excerpts** from this book, namely: The Abstract; the Contents; Sample text (Chapter 2); and the Main references.

Avi Sion (Ph.D. Philosophy) is a researcher and writer in logic, philosophy, and spirituality. He has, since 1990, published original writings on the theory and practice of inductive and deductive logic, phenomenology, epistemology, aetiology, psychology, meditation, ethics, and much more. Over a period of some 28 years, he has published 27 books. He resides in Geneva, Switzerland.

It is very difficult to briefly summarize Avi Sion's philosophy, because it is so wide-ranging. He has labeled it '**Logical Philosophy**', because it is firmly grounded in formal logic, inductive as well as deductive. This original philosophy is dedicated to demonstrating the efficacy of human reason by detailing its actual means; and to show that the epistemological and ethical skepticism which has been increasingly fashionable and destructive since the Enlightenment was (contrary to appearances) quite illogical – the product of ignorant, incompetent and dishonest thinking.

Abstract

Exposing Fake Logic by Avi Sion is a collection of essays written after publication of his book *A Fortiori Logic*, in which he critically responds to derivative work by other authors who claim to know better. This is more than just polemics; but allows further clarifications of a fortiori logic and of general logic.

This collection includes essays on: a fortiori argument (in general and in Judaism); Luis Duarte D'Almeida; Mahmoud Zeraatpishe; Michael Avraham (et al.); an anonymous reviewer of *BDD* (a Bar Ilan University journal); and self-publishing. None of these essays were previously published in print, although most of them were posted online.

Contents

Foreword 6
1. A Fortiori Argument, in General and in Judaism 12
   1. Formalization of a fortiori argument 12
   2. Validation of a fortiori argument 18
   3. Arguments involving proportionality 32
   4. A few words on the history 44
   5. Mishna Baba Qama 2:5 46
   6. Gemara Baba Qama 25a-b 54
   7. Some rabbinical hermeneutic principles 60
2. Luis Duarte D'Almeida 63
   1. A second-hand conception 63
   2. A second-rate conception 89
   3. Examples used 109
   4. With stronger reason, literally? 119
   5. A Peter Keating performance 145
3. Mahmoud Zeraatpishe 159
   8. Introduction 159
   9. Cases proposed by Zeraatphishe 163
   10. Summary of results 192
   11. Fake general claims 198
   12. Apologetic nonsense 204
4. Michael Avraham, et al. 213
   1. About "translation" 214
   2. About deduction 219
   3. Past attempts 225
   4. A fortiori argument? 230
   5. In denial 236

5. Bar Ilan's Journal, BDD 244
1. Submission and rejection of an article..... 244
2. First retort to the anonymous referee..... 250
3. Second retort to the same referee 271
4. Torah and science 277
6. Self-Publishing and Other-Publishing 283
1. A bit of history..... 283
2. Weaknesses and abuses of the system..... 285
3. On academe 290
4. Publishing attempts 293
5. On librarians 297
6. On historians..... 299
Main References 305

Sample text (chapter 2)

Luis Duarte D'Almeida

1. A second-hand conception

The subject of a fortiori argument is treated, mainly with legal perspectives, in a 2017 paper by Luis Duarte D'Almeida (henceforth, Duarte)[1] called *Arguing a fortiori*[2]. This is a pretentious essay, with few if any novel thoughts, and many egregious displays of ignorance and fallacious reasoning. Precisely for that reason, it is interesting to examine, as a case study in logical hubris. One might think that an essay replete with dishonesty, errors and omissions, and sophistry, is not worth writing about and reading about; but the fact is that much of value can learned from such a study.

As we shall see, the author, Duarte, draws a great deal of his ideas and terminology from my 2013 work *A Fortiori Logic* (2013) (henceforth, AFL), without duly acknowledging his intellectual debts; such dishonesty is of course morally reprehensible, and needs to be publicly exposed. Moreover, in his treatment of a fortiori argument, he makes serious errors and ignores some important aspects of the subject, due to lack of understanding and inattention to details. Furthermore, the author shows outstanding logical incompetence, when he eventually attempts to formulate and prove ideas which are truly his own. The present critique patiently details the essay's many deficiencies.

Starting his analysis of a fortiori argument by adopting the example "*He does not touch cider; he will certainly refuse whisky*" (p. 204)[3] as representative, Duarte's proposed formal description of such argument is as follows[4]:

> "(1) There is a point T in the scale of P such that, for every x, if x meets T, then x is Q.
>
> (2) a meets T.
>
> (3) b ranks higher than a on the scale of P.[5]

---

[1] A Professor of Jurisprudence (and Director of Equality & Diversity, whatever that entails) at the U. of Edinburgh Law School. I was amazed to discover this; I thought, reading his essay, he might be a novice lecturer trying desperately to make a name for himself.

[2] Published in The Modern Law Review (MLR) 2017, 80(2), pp. 202–237. Can be purchased online at: http://onlinelibrary.wiley.com/doi/10.1111/1468-2230.12252/full. The author is a Reader in Jurisprudence at the Edinburgh School of Law, University of Edinburgh. The Modern Law Review seems to be closely connected to the London School of Economics.

[3] Which he takes, after a small modification, from David Daube in (his citation) 'Rabbinic Methods of Interpretation and Hellenistic Rhetoric' (1949) 22 Hebrew Union College Annual 239, 254.

[4] I have left out his italics on symbols, and have changed his numbering from Roman to Arabic numerals.

Therefore (from (2) and (3)),
(4) b meets T.
Therefore (from (1) and (4)),
(5) b is Q" (p. 208).

The illustration he gives for it is[6]:

"(1) There is a point T in the scale of alcohol content such that if a beverage meets T, then our friend will refuse it.
(2) Cider meets T.
(3) Whisky ranks higher than cider on the scale of alcohol content.
Therefore (from (2) and (3)),
(4) Whisky meets T.
Therefore (from (1) and (4)),
(5) Our friend will refuse whisky" (p. 205).[7]

He declares that: "Any instance of this pattern will be a deductively valid argument" (p. 207); and I largely agree, since it contains two known deductive processes, namely a quantitative inference (a quantity greater than a second quantity which is greater than a third quantity must be greater than that third quantity) and a positive apodosis (*modus ponens*: given that an antecedent implies a consequent, if the antecedent is realized, then the consequent is realized). This formula can be put more succinctly as follows, if we interpret "meets" as meaning "is equal to or greater than":

If a ≥ T (2) and b > a (3), then b > T (4);
and x ≥T implies x is Q (1),
and b is an x and so fits x ≥T;
therefore, b is Q (5).

Note that I have added a clarifying sentence, "and b is an x and so fits x ≥T," which is left tacit in Duarte's formulation. The above described argument-form corresponds to what I have called

---

[5] Note that although Duarte does not at this stage mention the possibility of egalitarian or *a pari* a fortiori argument, he does so later, on pp. 236-7. There, premise (3) takes the form: "a and b are equally ranked on the scale of P."

[6] Notice that the way he formulates his example does not exactly match the way he formulates his form. Granting the form to be the correct formulation, his conclusion should have been: "then whisky will be refused by our friend" (b is Q). Such sloppiness can result in error. See next footnote.

[7] Actually, as I will show further on, this interpretation by Duarte of the example drawn from Daube that he chose at the outset as representative is formally inaccurate. For the time being, I deal with his account at face value, and leave this issue of appropriateness for later treatment. Suffice to say that such an error by Duarte, at the very start of his analysis of a fortiori argument, puts his logical knowhow in doubt.

positive subjectal a fortiori argument[8]. Duarte himself 'kindly' admits it in a footnote in the 5th-6th pages of his essay (n. 8, pp. 206-7):

> "This intermediate inference—the inference from (ii) and (iii) to (iv)—bears some structural similarity to what Sion isolates as one (complete) valid pattern of *a fortiori* argument: he calls it the 'positive subjectal mood' of the 'copulative' *a fortiori* argument—he also calls it the 'paradigm of a fortiori argument'—and renders it as 'P is more R than (or as much R as) Q (is R); and Q is R enough to be S; therefore, all the more (or equally), P is R enough to be S.' See A. Sion, *A Fortiori Logic: Innovations, History and Assessments* (Geneva: Avi Sion, 2013) 10-11, 117."[9]

This is a hint as to where he got his insight into a fortiori argument from: from my work. However, this statement is inaccurate, if not a conscious lie: there is no mere, vague, partial "structural similarity" with my work. As I will presently demonstrate, ***all* the constituents of his argument-form are derived from my validation procedure for positive subjectal a fortiori**. Duarte has *only changed the symbols used for the terms, and reshuffled the propositions* involved (leaving out one important clause without explanation). There is nothing wrong with such *rewording and rearranging*; what is wrong is *not giving full credit where credit is due*. Although he tries to project himself as an original thinker, gradually making new discoveries, the truth is he has not innovated in this matter, but merely retransmitted information already written and published by me years before.

I will perform the analysis of Duarte's rendition using my own symbols, which I established as standard in my past works on the subject because of their broad utility. These are: P (for the major term), Q (for the minor term), R (for the middle term) and S (for the subsidiary term). These four symbols correspond (in the present context) respectively to his symbols *b*, *a*, *P* and *Q*; his fifth symbol *T* corresponds to my symbol Rs, but he has no symbols for my Rp and Rq. In AFL 1.3, I present the positive subjectal a fortiori argument as having the following form on the surface, based on everyday use:

P is more R than (or as much R as) Q is (major premise);

and Q is R enough to be S (minor premise);

therefore, P is R enough to be S (conclusion).[10]

Notice the simplicity and directness of this standard format, how it conveys all the needed information in three propositions: a major premise, a minor premise and a conclusion, and how it reflects common a fortiori discourse. Compare this to Duarte's five propositions (plus two explanatory sentences). Apparently, Duarte found himself incapable of wrapping his mind around the standard three propositions. Finding the wording too conceptual and direct for his liking, he sought for a more graphic, and more verbose, means of expression – a rationale that he could

---

[8] Already in my 1995 book, *Judaic Logic*.

[9] It is not clear from the page numbers Duarte gives just which edition of AFL he is using. To avoid all ambiguity in this respect, I prefer to refer to my work by means of the chapter and section; e.g. here, chapter 1, section 3.

[10] In Duarte's terminology, this would read: 'b' is more 'P' than 'a'; and 'a' is 'P' enough to be 'Q'; therefore, 'b' is 'P' enough to be 'Q'.

grasp. But as I will be showing in much detail, this rationale is inadequate and detrimental. Now, looking deeper into the above three propositions, we have:

- *for the major premise*:
  P is R, i.e. P is to a certain measure or degree R (say, Rp);
  Q is R, i.e. Q is to a certain measure or degree R (say, Rq);
  and Rp is greater than (or equal to) Rq (whence: Rp implies Rq[11]).
- and *for the minor premise*:
  Q is R, i.e. Q is to a certain measure or degree R (say, Rq);
  whatever is at least to a certain measure or degree R (say, Rs), is S, and
  whatever is *not* at least to that measure or degree R (i.e. is not Rs), is *not* S;
  and Rq is greater than (or equal to) Rs (whence: Rq implies Rs).
- yielding *the conclusion*:
  P is R, i.e. P is to a certain measure or degree R (say, Rp);
  whatever is at least to a certain measure or degree R (say, Rs), is S, and
  whatever is *not* at least to that measure or degree R (i.e. is not Rs), is *not* S;
  and Rp is greater than (or equal to) Rs (whence: Rp implies Rs).

This is validation by direct reduction to more widely studied and understood propositional forms, note. The conclusion has four components; its first component comes from the major premise; the second and third come from the minor premise; and the fourth comes from both premises. Whence, the conclusion logically follows from the given premises.

If we rewrite Duarte's formula with my symbols, we obtain the following result, in which **the sources of his formula** in my prior work are made quite evident (shown in bold):

(1) *There is a point Rs in the scale of R such that what meets Rs is S.*

This corresponds to the second component of my minor premise: **Whatever is Rs or more, is S.**

But note that Duarte has ignored third component, the negative clause (which is the exact inverse of the said positive clause): whatever is *not* at least Rs, is *not* S, perhaps due to his wish to avoid using the summary form: Q is R *enough to* be S (see further down).

(2) *Q meets Rs.*

This corresponds to the first and fourth components of my minor premise: **Q is R (Rq), and Rq is greater than (or equal to) Rs.**

(3) *P ranks higher than Q on the scale of R.*

This corresponds to my whole major premise (its three components), viz.: **P is R (Rp)**, and **Q is R (Rq)**, and **Rp is greater (i.e. more R) than Rq.**

Therefore (from (2) and (3)),

(4) *P meets Rs.*

---

[11] This implication is intended in the sense that a larger number implies every smaller number. For example, if I have $5, then I obviously have $3.

This corresponds to the fourth component of my conclusion, viz.: **Rp is greater than (or equal to) Rs**, which is inferred from the third component of the major premise and the fourth component of the minor premise.

Therefore (from (1) and (4)),

(5) *P is S.*

This follows from the first, second and fourth components of my conclusion, **P is R (Rp)**, and **Rp is greater than (or equal to) Rs**, and **whatever is Rs or more, is S**.

Here again, Duarte has ignored the negative clause (the inverse): whatever is *not* at least Rs, is *not* S, and avoided use of the summary form: P is R *enough to* be S, which implies that P is S (see further down).

It is evident from the above detailed analysis that *not only* his propositions (2), (3) and (4) are derived from my treatment, as he kind-of admits, *but also* propositions (1) and (5) are so! His statement (1) "There is a point Rs in the scale of R such that what meets Rs is S" is not semantically different from my "Whatever is at least to a certain measure or degree R (say, Rs) is S;" and his statement (5) "P is S" is explicitly included in my "P is R enough to be S." as well as implied in the conclusion of my validation procedure. If Duarte had not changed the symbols, all this would have been glaring. He changed the symbols, and the language somewhat, I submit, so as to camouflage the derivation of his proposal from my earlier work and avoid having to acknowledge it.

Now, Duarte's formulation may superficially seem more succinct than my validation procedure[12], mainly because he makes use of a brief but vague expression, "meets." Let us consider this term, which he does not define anywhere. At first sight, it seems to simply mean "is equal to or greater than." However, upon reflection one realizes that this is not exactly true. If one examines Duarte's formula more closely, one notices that he muddle-headedly conflates two distinct ideas. Using his symbols, we can say that the terms a and b (or any x) are subjects in relation to the predicates P and Q; but in that case, we cannot say that a and b (or x) are comparable to T on the scale P; we can only say that the values *of P* corresponding to a and b (or x) are comparable to T. In other words, it is not a and b that are ≥ T, but Pa and Pb that are ≥ T. This fine distinction is missed by Duarte, revealing a mind functioning in an approximate manner.

In case this distinction is not clear to some readers, let me repeat it using Duarte's main example. It is not the same to say that cider (a) and whisky (b) contain more or as much alcohol (P) than the measure of alcohol (P) designated by T, and to say that the alcohol-content of cider (Pa) and that of whisky (Pb) are greater or equal to T. This is not hair-splitting, but a significant logical difference that a logician must be careful to reflect in his formula to ensure no mistakes later take place. Thus, the term Duarte used, "meets," was designed to gloss over an ambiguity that he was unable to notice and sort out. To repeat, the expression a or b "meets T" does not mean, as it seems to at first, that a or b is ≥ T, but only that a or b has *a value of P* that is ≥ than the value of P signified by T. In my formulation, note well, the major and minor terms P and Q (my symbols)

---

[12] In other parts of the text I also resort to briefer language, e.g. in Table 1.2, I have: Rp ≥ Rq, Rq ≥ Rs, so, Rp ≥ Rs; and in Diagram 1.1, I show these relations by way of an illustration. In any case, my standard form is much briefer, and more accurately reflects the way we all in practice express our a fortiori reasoning.

are clearly distinguished from the values of the middle term (R) corresponding to them, labeled Rp and Rq respectively.

Now, consider the order in which Duarte has placed the premises in his formula: (1) and (2) from my minor premise, and (3) from my major premise. That is, he has put the minor premise before the major. Chronologically, he claims, (2) and (3) should come first, yielding (4); then (4) together with (1) yield (5). Yet, he has placed (1) in very first place, apparently to stress its importance. Why? If he is thinking in terms of logical priority, why has he not accordingly placed (3) before (2)? Surely, in the first leg of his argument, (3) is the major premise and (2) is the minor premise, and (4) is their conclusion. So, Duarte cannot claim to be ordering the premises either chronologically or logically. It seems obvious, then, that he placed (1) at the start of his formula in order to conceal its origin in my minor premise; i.e. he moved it to first place to make the argument as a whole *look* different from mine.

Notice also that Duarte's formula does not explicitly tell us that (using his symbols) "a is Q," although he does realize this later, numbering this proposition (2a) and pointing out that it is inferable from (1) and (2). He is evidently a bit surprised by this finding, saying: "interestingly, the claim in (2a) was the single one that the arguer had actually made explicit;" and he goes on to argue that:

> "That single claim … provided the basis for our interpretative reconstruction of the further premises we take the arguer to be implicitly relying on … but once we have unpacked these premises into the conjunction of (1), (2), and (3), the claim in (2a) no longer needs to be spelled out as a part of the argument in order for the inference to run. In a sense, then, the argument as originally stated included none of its crucial premises" (p. 206).

He is here referring to his main example, the cider-and-whisky argument. What he is saying is, of course, balderdash – mere ex post facto *rationalization*. The fact remains that his model of a fortiori argument lacks the very premise that it set off to explain, which is unheard of in formal logic. According to him, although most people who engage in a fortiori argument use the proposition "a is Q" as their foremost premise (very often mentioning it alone), it is merely an *incidental* implication of the 'true' a fortiori argument, and not an effective part of the argument. It is a historical *accident* that people speak thus, not something inherent to a fortiori reasoning. This may be characterized as adapting reality to theory, instead of theory to reality.

In my corresponding standard form, which is built on extensive empirical studies, on the other hand, this proposition, viz. (using my symbols) 'Q is S', is explicitly embedded in the minor premise 'Q is R enough to be S', and is implied in the validation procedure in the first and second components of that premise.

Moreover, Duarte makes a big show (p. 205 and on) of discovering and introducing the concept of a *threshold*, but this concept is clearly presented and repeatedly emphasized in my earlier work. For instance, in AFL 1.1, I write:

> Evidently, the clause "R enough to be" in positive moods, or "R not enough to be," in negative moods, even if it is not explicitly stated in the minor premise and conclusion, is absolutely essential to a fortiori argument. If there is no intended threshold of R to be attained or surpassed in order for S to be predicated of or to be subject to the major and minor terms, there is no operative a fortiori argument (though there might be some other thought-process, such as

mere analogy). This is evident from the fact that, without this crucial clause, we simply cannot validate the argument. Keep that well in mind.

As regards the negative clause, "whatever is *not* at least Rs, is *not* S," Duarte consciously or unconsciously dispenses with it. He certainly does not acknowledge it or explain why he ignores it[13]. Let us here consider what the logical utility of this missing clause is and how important it is. The negative clause complements the positive clause "whatever is at least Rs, is S;" it is its full inverse. The positive clause tells us that all items that are equal to or greater than Rs are implied to be S; the proposed general negative clause informs us additionally that all items that are lesser than Rs are implied not to be S.

a) Given only the positive clause, we could not exclude the possibility that *all* items lesser than Rs are also S. Logically, the positive clause would remain true even if no items lesser than Rs were not S. In that event, the conclusion drawn would be syllogistic instead of a fortiori, merely the application of a generality to a particular subsumed under it. The argument would be: since all items are S, it follows that this item (the subject of the conclusion) is S. For this reason, at least, if Rs is to be a *threshold*, i.e. a boundary between opposites, at least the negative clause "*Some* things that are R, but less than Rs, are not S" must be specified. However, this is not the whole story.
b) The concept of a threshold is not essentially exclusive. There may be ups and downs along the same continuum, and therefore *more than one* threshold in it. In that event, the positive clause "whatever is at least Rs, is S" would remain true, and the putative conclusion from it (this item is S) would remain valid, but the concept of *sufficiency* inherent to a fortiori argument would be missing. The argument would be logically valid, but not truly of an a fortiori type. Sufficiency suggest here that nothing less[14] than the specified threshold will do; i.e. it admits of *only one* threshold. Thus, whereas sufficiency implies threshold, threshold does not imply sufficiency. For this reason, we must specify, not only that "Some things that are R, but less than Rs, are not S" (as above shown), but more generally that "*All* things that are R, but less than Rs, are not S."

Thus, genuine a fortiori argument requires a *general* negative clause (a full inverse) to complement the positive clause. By itself, the positive clause cannot guarantee that a fortiori reasoning is involved. Furthermore, inserting the concept of threshold does not guarantee this; the concept of sufficiency must also be involved. If there are values of R below Rs that are also S, can we still say that something is R *enough to* be S? No, this phrase loses its force. Yet, as I show repeatedly in AFL, to qualify as a fortiori, the argument must include the idea of sufficiency. An inference of sorts would be possible without it, but it would not constitute an a fortiori inference in the common sense of the term.[15]

---

[13] As I do clearly in AFL 1.3, I have to confess that: "in my book *Judaic Logic*, I did not specify the third component, which is the inverse of the second component. I did not at the time realize the significance for a fortiori argument of this negative component, i.e. how essential it is to such argument; so this is an important new finding here." However, since Duarte only refers to my AFL, he cannot claim my earlier work as an excuse for his leaving this clause out.

[14] Or 'more', as the case may be. In positive subjectal argument, where we argue from minor to major, 'less' than the threshold is applicable; but in positive predicatal argument, where we go from major to minor, 'more' than the threshold applies.

[15] In truth, the idea of sufficiency is conveyed not just by the negative clause, but by the positive and negative clauses *both together*. This will become evident in the next section, when we examine Duarte's second form.

Duarte was obviously unaware of the logical significance of this clause, since he chose to discard it (assuming he noticed it in AFL – but how could he miss it? it was present from the beginning). Had he been aware of it, and of its importance, he would have complemented his premise (1) with the *a contrario* hypothetical: "but if x does not meet T, then x is not Q." Note well that there is no redundancy in stating this information; it is logically required to give T the status of a threshold, and moreover to impose the more demanding idea of sufficiency. To withhold this information is not only sloppy, but inaccurate formalization.

We could draw the conclusion Duarte draws (b is Q) without it, but such inference might not constitute a fortiori argument. Suppose that 'x is Q' were *categorically* true, then the conditional 'if x meets T, then x is Q' would still be true, and the conclusion 'b is Q' would be valid, but it would not be a fortiori but mere syllogistic application (from b is x and x is Q). Suppose now that the *compound conditional* 'if x meets T, or x is less than T and between T1 and T2, then x is Q' were true, then the conditional 'if x meets T, then x is Q' would still be true, and the conclusion 'b is Q' would be valid, but it would not be genuinely a fortiori because it would lack the idea of a single threshold for applicability (i.e. of sufficiency) which such reasoning involves.

When Duarte discusses actual cases, he seems somewhat aware of this issue. For example, referring to Daube's original example, where the subject is identified as a teetotaller, Duarte rightly points out: "if we know that he is a teetotaller, then the fact that he does not touch cider plays no role in the argument: if he is a teetotaller, then it already follows that he will certainly refuse whisky" (p. 204); similarly, with reference to a legal example about ownership, he rightly points out that it is "not a fortiori" (p. 203). Also, his discussion of invalid arguments seems to acknowledge the possibility of ups and downs in the data (pp. 209-10). However, he does not integrate these insights in his formalization; i.e. he fails to include the needed general negative clause.

In his formal treatment, Duarte does not realize that in cases where "x is Q" *unconditionally* (as in the teetotaler and ownership examples mentioned above), the propositions "if x meets T, then x is Q" *and* "if x *does not* meet T, then x *is* Q" are both true. If he had known this logical implication, he would have realized that his premise "if x meets T, then x is Q" is not sufficient to guarantee that the argument is truly a fortiori. This is a *self-contradiction* on his part, since he has admitted (using the example on p. 203) that argument with an unconditional "x is Q" is "not a fortiori."

Even though Duarte does show awareness that the proposition "if x meets T, then x is Q" does not imply the complement "if x does not meet T, then x is not Q" – or, as he puts it, "x's meeting the threshold [T] is a sufficient condition of x being Q, not a necessary one" (p. 210), he does not adjust his formula accordingly. He is aware that, given only the positive clause, without meeting T an x might logically still be Q (or might not), but he does not add a preventive clause to his formulation. In other words, he should have made "x's meeting the threshold [T]" *not only* "a sufficient condition," *but also* "a necessary one."

In sum, Duarte's alleged formalization of a fortiori argument *fails to formally exclude* from it, as it should have, eventual cases where all x, whether meeting T or not, are Q. Similarly, it fails to formally exclude, as it should have, occasional cases where an x meeting some other threshold(s), below T, might well be Q. Even though he had access to my formulation of a fortiori argument, which contains the needed negative premise, i.e. (using his symbols) "if x does not meet T, then x is not Q," he ignores it. For this reason, he is led to claim (see below) that sufficiency is not essential to a fortiori argument. Alternatively, he ignores the needed negative premise *in order to*

reject the idea that sufficiency is inherent to a fortiori argument, so as to make his formula distinct from mine.

Note also that some of Duarte's own legal illustrations contain language pointing to sufficiency. Of his thirteen legal examples, four have such linguistic markers: "deemed sufficiently important" (p. 203), "not sufficiently specific" (p. 212), "an insufficient reason for" (p. 212), "appropriate in the case of" (p. 217). Indeed, in *his own* discussions of some legal and non-legal examples, as well as in some more formal contexts, he freely uses the expressions "enough," "suffice," and "sufficient" to clarify things. It is difficult to see how a fortiori argument could be made fully explicit and explained without resort to such language.

For instance: "And the thought behind the argument as expressed would seem to be this: if cider, lower in alcohol content as it is, is nevertheless already so high in alcohol content that our friend would refuse it—if it is already too high in alcohol content for our friend to accept it—then surely whisky too is high *enough* in alcohol content that our friend would refuse it" (p- 204). Or again: "but in the law's view even that stronger reason is not strong or important or weighty *enough* to justify allowing a taxpayer to have a new decision made on the point" (p. 214). (Emphases mine.)[16]

These examples show that Duarte resorts to language of sufficiency in practice. Even though he is careful to exclude such language from his formula for a fortiori argument, it does not mean that the thought of sufficiency is not intended by him in the background. Nevertheless, it seems clear he does not realize the absolute necessity of the idea of sufficiency, i.e. the idea that a fortiori argument involves the thought "enough to" in the minor premise (and thence in the conclusion). He certainly does not try to explicitly include it in his formula. Although he does explicitly acknowledge the idea of a threshold, through his term T, more is needed to formally acknowledge the idea of sufficiency. To do that, he would have to add the above-mentioned negative clause to his formula.

Adopting a position *inconsistent* with the above-given examples from his own essay, Duarte explicitly eschews and disputes the requirement of sufficiency, saying (in footnote 8, again):

> "Sion's formalisations also fail to reflect the fact that *a fortiori* arguments are not—certainly not necessarily—arguments for conclusions of the form '… is (is not/implies/does not imply) … enough to be (to imply) …' Moreover, the semantics of '… is ... enough (or not enough) to be *x*' does not always licence inferences to '… is *x*'; I may be tall enough to be a basketball player, and yet not be one."

Here again, Duarte shows his lack of thorough understanding of the ways of formal logic. The conclusion "P is R enough to be S" is *the maximum inferable from the given premises*. In this conclusion, the sufficiency specification ('enough to') can eventually be left unstated, if we happen not to need it in a given context; but if we *do* happen to need it for a subsequent round of reasoning, it is here made available to us by inference. We can conclude more simply "P is S" as he does, and most of us usually do so in common discourse; but we thereby lose some potentially useful information.

---

[16] See also examples on pp. 209, 212-3, 216, 220, 221, 225.

On the other hand, in the minor premise the sufficiency clause is a *sine qua non*, if we are to draw the said conclusion (fully, with the 'enough' phrasing, or partly, without it); it just cannot be left out. Even if we do not explicitly acknowledge it in our wording, it is an essential factor in the argument at a subconscious level; it makes the argument understandable and credible to us. A fortiori reasoning (in the positive subjectal mood) *must* reflect the pattern: P is more R than (or as much R as) Q, and Q is R *enough to* be S; therefore, P is (R enough to be) S. This pattern is the very essence of such reasoning.

To repeat, Duarte's premise (1), with its positive reference to a threshold, does partly acknowledge sufficiency. Looking at the above analysis of his formula, we can see that this positive hypothetical originated in the minor premise. He has simply moved it to another location. Nevertheless, he has not kept the above-mentioned negative component of the minor premise, i.e. the inverse of the positive hypothetical. To be sure, the conclusion that he draws, "b is Q," does logically follow from his premises; but that does not prove that the argument is a fortiori in character. The character of a fortiori is bound up with the notion 'enough'. Possibly, he did not realize the crucial role it plays[17].

As regards his objection that "I may be tall enough to be a basketball player, and yet not be one," he adduces it as proof that if we accept that the form "P is R enough to be S" implies "P is S," we are wrongly suggesting that a potential (to be S) is an actuality (being S)[18]. But this is not what this implication (from 'is R enough to be' to 'is') is about. The issue here is what the minor premise states *in a given case*. If it states that the predicate is an actuality, then the conclusion must do the same; and if it states that the predicate is a potentiality, then the conclusion must do the same. Certainly, a fortiori argument as I have presented it does not advocate the inference of an actuality (in the conclusion) from a potentiality (in the minor premise), as he wrongly suggests. Taking Duarte's example, it is certainly true that one cannot infer that a man is a basketball player from the fact that he is tall enough to qualify as one. But no truthful and valid a fortiori argument does that.

If (as in his proposed example) the minor premise is that someone 180 cm tall is tall enough to *actually be* a basketball player (which is unlikely in fact, and so *would not be our minor premise*), then the valid conclusion would be that someone 185 cm tall is tall enough to *actually be* a basketball player (which is unlikely in fact, and so looks like a wrong conclusion). But if the minor premise is that someone 180 cm tall is tall enough to *potentially be* a basketball player (without implying he is one), then the valid conclusion would be that someone 185 cm tall is tall enough to *potentially be* a basketball player (*without implying he is one*). The argument may be formulated either way (i.e. with the general symbol 'S' here meaning more specifically either

---

[17] Possibly, too, he was influenced by the desire to distinguish his formula from mine and make his appear original and independent, little realizing that he thus rendered his formula logically inadequate as a representation of a fortiori argument.

[18] Notice his attempt at pedantry, when, instead of just saying "does not imply," he says "the semantics of '… is… enough (or not enough) to be *x*' does not always licence inferences to '… is *x*'." This snobbish language is designed to project that he has deeply studied the semantics of 'enough'. But he has surely nowhere done so, whereas I have done it in detail in AFL. No wonder his statement is wrong.

'actually S' or 'potentially S'), *as factually appropriate in the case at hand*. The conclusion follows the minor premise; it cannot add more information.[19]

In short, this comment by Duarte had in fact nothing to do with my presentation of a fortiori argument! It just reveals his own ignorance of logic. As Duarte should have reflected, the symbol 'S' here used for the predicate (of the minor premise and conclusion) is very broadly intended. It includes any sort of term, descriptive or normative, of any polarity or modality, simple or compound – it is not limited to simple predications of actuality. I say this clearly, and it is anyway standard knowledge in logic (for example, in texts on syllogism). In fact, further on (pp. 207-8), Duarte himself insists on this generality of a symbol:

> "we can adopt the following simplified formulation as a way of highlighting the fact that the consequent of the conditional in the first premise of our argument—and so too the conclusion of the argument—is any claim, descriptive or normative, involving the threshold-meeting item: '… then x is Q.' To be clear, the point of my proviso is not merely that the consequent can be either a descriptive or a normative claim. The point, more generally, is that there is no reason to think that the consequent must be a claim in which something is predicated of the threshold-meeting item. All that is necessary is that the consequent involve the threshold-meeting item in some way."

Thus, not only is his said objection invalid, but he later explicitly disowns it. From this episode, I am forced to conclude that Duarte was looking for some contrived way to put down my formulation of a fortiori argument, so as to give readers the impression that he has surpassed it. Furthermore, it should be noted that even here, where he makes a show of being aware of a variety of possible predicates, "descriptive or normative," he is merely vaguely reiterating something that I have long before treated in much greater detail, notably in AFL 4. There, I clearly distinguish between ontical[20], logical-epistemic, and ethical-legal a fortiori arguments, and take into consideration all sorts of possible contents for such forms. Nowhere does he mention this, so as to make it appear to be his own observation. These are not honest ways of doing business.

Adding insult to injury, Duarte even has the gall to say (again in footnote 8):

> "But Sion's formalisations are, I think, too crude to do justice to his insights. They are also potentially confusing. In his explanations he sometimes uses a scheme like 'Rx' to represent the point on a given continuum R at which a certain *item* x stands—which suggests that 'x' is to be taken to be an individual constant—but sometimes he also uses it to represent a relevant threshold on a continuum (for example, a point that *any* item *x* needs to meet in order to have a certain property)—which would make '*x*' a variable instead; and as a result he is led to say

---

[19] One can infer a 'can be' from an 'is', but not vice versa. Similarly, in his main example, the conclusion might either be that the friend would refuse or that he will refuse to drink, depending on the formulation of the minor premise. One can infer a 'would' from a 'will', but not vice versa.

[20] The word 'ontical' is commonly used nowadays; the equivalent words 'ontal' (found. e.g., in the Enc. Brit.) and 'ontic' (found, e.g., in a W. Windelband text) are also sometimes used.

> that an *a fortiori* argument orders *three items* (P, Q, and S), rather than just two, 'according to their position in a common continuum' (*ibid*, 21)."

Here, the pretentious novice Duarte struts about, daring to call my formalization work "crude" and "potentially confusing." This is phony criticism by someone who is desperately looking for a pretext for negative judgment, not having found any actual fault in my work. All his criticism is smoke and mirrors to conceal his intellectual debts. He needs to project a critical attitude, so as to make it seem (to others, and maybe also to himself) that he operated by himself, without reference to my work. But as we have seen, his formalizations are directly and exclusively (even if imperfectly) derived from mine; there is nothing novel in them. If my formalizations are 'crude', how come he has found it worthwhile to 'borrow' them, shamelessly, without so much as a word of thanks, claiming them as his own findings? If they are potentially confusing, how come he found them so instructive that he learned all that he claims as his own findings from them?

Moreover, Duarte has the *chutzpah* to suggest that I am unable to tell the difference between a variable and a constant! Judging by his mention of the symbol Rx, Duarte seems to be referring to the following passage in AFL 1.3:

> The positive minor premises and conclusions (labeled "*suffective*" because they concern sufficiency) of copulative arguments have the following four components in common. The symbols X and Y here stand for the symbols P or Q and S as appropriate in each mood; that is, we may have "P is R enough to be S," "Q is R enough to be S," "S is R enough to be P," or "S is R enough to be Q." A proposition of the form "<u>X is R enough to be Y</u>" means: X is R, i.e. X is to a certain measure or degree R (say, Rx); whatever is at least to a certain measure or degree R (say, Ry), is Y, and whatever is not at least to that measure or degree R (i.e. is not Ry), is not Y; and Rx is greater than (or equal to) Ry (whence: "Rx implies Ry").

I am surprised that Duarte found such an easy passage intellectually challenging. Apparently, he is unaware of the idea of a variable of variables, i.e. of *a variable whose values are variables, whose own values (in turn) are constants*.[21]

In the quoted passage, I wish to analyze the propositional form I call 'suffective' into its simpler constituents, in order to validate a fortiori arguments involving it. I start by pointing out that, in valid a fortiori argument, such propositions arise only in the minor premises and conclusions of positive arguments. This means that we have to analyze four *specific* (copulative) forms, viz.: 'Q is R enough to be S' and 'P is R enough to be S' (in subjectal argument), and 'S is R enough to be P' and 'S is R enough to be Q' (in predicatal argument). However, being a lazy guy and not wanting to bore readers to death, I propose to analyze all four forms in one go, by using as *generic* symbols, X for the subject and Y for the predicate. Thus, X stands for Q, P or S, as the case may be, while Y stands for S, P or Q, as the case may be.

Thus, X and Y are here convenient placeholders for the variables P, Q, S, whose values are (obviously) various constants. No misunderstanding is possible as to what I was doing there, since a bit further in the same section I write: "The above general form of suffective proposition

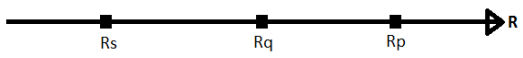
---

[21] Funnily enough, in his footnote 9, he seems to resort to variables of variables. He speaks of the possibility of "more complex forms," giving as example: "'For every x, if x is a D1, then x meets T'; 'For every x and every y, if x is a D1, and y is a D2, then y ranks higher than x on the scale of P'; and so on." However, he does not develop this thought further, so it is difficult to guess exactly what he meant by it.

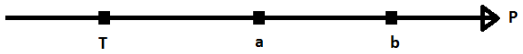
will of course concretize in different ways according to the orientation of the copulative a fortiori argument under consideration," and I go on to present the forms implied in terms of the underlying variables (P, Q, S), instead of the earlier variables of variables (X, Y). Obviously, Duarte did not have the patience to read on and learn.

Although it is conceivable that this simple artifice went over Duarte's head, I suspect rather that he was just trying hard to find some flaw in my work, in order to support his fake contention that it is "crude" and "potentially confusing;" I let the reader judge the matter. Moreover, his statement that "as a result" of this alleged confusion between constants and variables I am "led to say" that "an *a fortiori* argument orders *three items* (P, Q and S)" is bizarre, as is his suggestion that a fortiori argument in fact orders "just two items" on the common continuum. How would my three-item claim follow from the said confusion? He does not explain. As for his two-item claim, it again shows surprising incomprehension on his part of quite simple matters.

First, it is not exactly accurate to say that I order the items P, Q, and S, along R; rather, I order *the values of* R corresponding to them, viz. Rp, Rq, and Rs, respectively, along R. This is evident from the diagram that I draw in AFL 1.3, which I reproduce below. This diagram tells us that, in the positive subjectal mood, any item with a value of R at or to the right of threshold Rs is S; this explains why P, like Q, is subject to the predicate S. It should be added that any item with a value of R to the left of Rs is not subject to S.



Second, since these three values of the middle term are *equally present* in his formulation of a fortiori argument, as T (for Rs), a (for Rq) and b (for Rp), along the scale P (i/o R)[22], there is no basis for his rant. As the following illustration shows, there are in fact (as there should be) three items in his own formulation! This diagram tells us that any item with a value of P at or to the right of threshold T is Q; this explains why b, like a, is Q.



Presumably, for he does not clarify his statement, Duarte does not consider threshold T (or perhaps the predicate Q that it opens the door to) to be part of the argument. But the truth is, if T were placed in between the terms a and b, the argument would be invalid (as Duarte readily admits on pp. 209-10). And if T were placed higher along P than the other two terms, then the argument would be major-to-minor instead of minor-to-major (as Duarte readily admits on pp. 210-11). So, the position of T along P is very significant. The illustration cannot be understood without including T in it. And this has nothing to do with interpretation of symbols as variables or constants – it applies either way. So here again, I suspect that Duarte was just *making up* something negative about my work, so as to give naïve readers the impression that he is a critical, independent – and let us put it bluntly, altogether superior – thinker.

---

[22] No need to repeat here what I pointed out in detail earlier, that Duarte conflates the ideas (using his symbols) of a and Pa, and likewise of b and Pb. That is, he does not distinguish like I do (using my symbols) P from Rp, and Q from Rq.

I have noticed this is a role commonly played by people new to a subject who want to appear in-the-know and authoritative: they invent unflattering comments about their predecessors' work, so as to place themselves in readers' eyes on relatively higher ground looking down. They hope that this will keep readers from going to the source and judging it for themselves, and of course from seeing the errors in their own shabby presentation by comparison. This is more akin to deliberate calumny than to scientific discourse. In Duarte's case, it was very important to keep readers away from my book AFL, because so much of his 'work' is manifestly directly derived from it, and he has glossed over a great deal of important material. He might have gotten away with it if I was already dead; but I am still alive and still quite able to bite back.

To summarize what we have seen thus far: Duarte proposes a theory of a fortiori argument in a way that makes him seem like its originator, indeed like a (or the) pioneer in the field. But as it turns out, his theory is wholly and exclusively derived from my much larger-scale anterior work. Moreover, his proposed formalization is incomplete – he has missed a logically important element (the inverse of the initial hypothetical). He tries to mask the source of his proposal by failing to mention my work prominently and to admit his having learned from it; and by spinning a fantasy critique of my work, which only further reveals the limits of his understanding.

2. A second-rate conception

Duarte's conception of a fortiori argument is not only second-hand, but second rate (or third-rate or fourth). Further on (pp. 209-11), Duarte tries to broaden his theory by taking into consideration negative terms and relations, and indicating invalid forms. In this context, he presents a "second type" of a fortiori argument, as follows (italics mine):

> "(1) There is a point T in the scale of P such that, for every x, if x *does not* meet T, then x is Q.
> (2) a *does not* meet T.
> (3) b ranks *lower* than a on the scale of P.
> Therefore (from (2) and (3)),
> (4) b *does not* meet T.
> Therefore (from (1) and (4)),
> (5) b is Q."

He declares this "a valid inference" (and I largely agree). Having introduced this second form, Duarte concludes: "There are therefore two forms of the a fortiori." He does not say whether there are more possible forms; so, he seems to be saying that there are *only* two forms. Indeed, he names a section "*The* two forms of a fortiori" (my emphasis). Notice, anyway, what distinguishes this form from the preceding one: in propositions (1), (2) and (4), "x meets T" is replaced by "x does not meet T;" and in (3) b is lower than a (instead of higher) on the scale of P. This formula can be put more succinctly as follows, if we interpret "does not meet" as meaning "is less than":

> If a < T (2) and b < a (3), then b < T (4);
> and x < T implies x is Q (1),
> and b is an x and so fits x < T;

therefore, b is Q (5).

What is this second argument-form? Since it proceeds from the major term (a) to the minor (b), while these terms are both subjects, it can be taken to correspond to what I have called **negative subjectal** a fortiori argument. That is, argument of the standard form:

P is more R than (or as much R as) Q (major premise),

yet P is R *not* enough to be S (minor premise);

therefore, Q is R *not* enough to be S (conclusion).[23]

Duarte does not point this out, and here again does not admit outright his debt to my work, although he does point out in his footnote 8 that:

> "Sion distinguishes between 'copulative' and 'implicational' a fortiori arguments, each of which comes in four moods, in a total of eight different valid patterns. All eight patterns are patterns of two-premise arguments combining four terms (or theses, as the case may be) P, Q, R, S, and all have conclusions of the form '… is (is not/implies/does not imply) … enough to be (to imply)."

To which he adds, ingenuously, as we have seen, "But Sion's formalisations are, I think, too crude to do justice to his insights," implying that his rewriting of my discoveries is more accurate. Of my list of four (or eight) moods of a fortiori argument, then, Duarte effectively lays claim to only two, the positive subjectal and the negative subjectal. It should also be said in passing that even his observation that the comparative premise is sometimes expressed in reverse form, i.e. with 'ranks lower' instead of 'ranks higher', is by no means new. I point this out clearly in AFL 1.1: "The major premise may occasionally in practice be converted."

As I will now demonstrate, it is Duarte's formalizations (not mine) which are too crude to do justice to my insights. In my work, insight and technical progress always go hand in hand, mutually feeding on each other; neither is possible without the other. As we saw in the previous section, I validate the positive subjectal argument by direct reduction to a series of conditional propositions and quantitative comparisons: the constituents found in the premises are used to wholly construct the conclusion, which stamps it as valid. As we also saw, Duarte's first formula is entirely and exclusively derived from this validation process, even if he does not admit it or realize it. We also pointed out there that he failed to mention an important general negative proposition involved in the minor premise (and thence in the conclusion), and showed some confusion regarding the terms involved.

What has happened here, in Duarte's second formula, is the opposite: he does effectively mention the said negative clause, but this time fails to mention the complementary positive clause (i.e. the positive inverse of the negative hypothetical). As a result, the unity of the two moods is not formally evident in his scheme, although one can intuitively sense it. Nevertheless, we can say that Duarte's second form could be validated using the exact same set of propositions (listed in

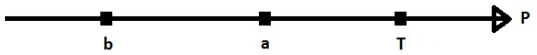
---

[23] In Duarte's terminology, this would read: 'a' is more 'P' than 'b'; and 'a' is 'P' not enough to be 'Q'; therefore, 'b' is 'P' not enough to be 'Q'. Note well that in this context Duarte defines 'a' as greater than 'b', whereas in my treatment my P remains greater than my Q.

the previous section) – which means that his second form is as wholly and exclusively based on my validation process, just as his first form was (even if in this second case, I suspect, he proceeded by analogy from the first formula).

The said ease of validation is due to the fact that positive and negative subjectal arguments are *one and the same argument expressed in two ways and validated in the same single way*. This is the reason that we can (using the standard form) reduce either one *ad absurdum* to the other, note well. Given that P is more R than Q, it follows that (a) if Q is R enough to be S, then P is R enough to be S; *and* that (b) if P is not R enough to be S, then Q is not R enough to be S. The positive and negative arguments are two sides of the same coin. This is evident in the two arguments having the exact same validation process, to repeat. It is for this reason that Duarte's second argument appears as valid *prima facie*.

Now, Duarte's second form can be illustrated as shown below (compare this to the earlier illustration for the first form), since it tells us that b<a<T along continuum P, and also tells us that anything less than T is Q[24].



Put in standard form, this argument would be expressed as follows:

a is more P than b;

and a is *not* P enough to be *not*-Q;

therefore, b is *not* P enough to be *not*-Q.

Notice the double negation involved; instead of the positive form 'is P enough to be Q', we here find the form 'is not P enough to be not-Q'. The latter is the true meaning of Duarte's positive consequent and conclusion 'is Q', although he is not aware of it. If he was aware of it, he would have formulated the consequent and conclusion as 'is not Q'[25]. In that case, his second form would have been the proper negative parallel of his positive form; i.e. its minor premise and conclusion would have had the form 'is not P enough to be Q'. Furthermore, there was no point in Duarte switching the roles of terms a and b, making b the lesser item (instead of the greater, as in the first form). Even if these choices are not per se harmful, they complicate validation because they obscure the unity of the second form with the first form (i.e. they force us to restate the set of constituents with these arbitrary changes).

Thus, to conclude this issue of validation, Duarte's second form can be validated in the exact same way as we validated his first form, but for that we would have to make small changes on one side or the other (to follow or cancel the changes he introduced due to ignorance of the consequences). If his first form had been formulated accurately, with the missing inverse clause "if x does not meet T, then x is not Q," he would have immediately seen in it the second form

---

[24] Here again, I remind readers that Duarte's symbols a and b are equivocal, referring both to the subjects a and b themselves, and to the values of P corresponding to them (i.e. Pa and Pb).

[25] In his footnote 10, he writes: "It makes no difference that the consequent of the conditional … and therefore also the conclusion … is the claim that b is Q. We would have an inference of the exact same kind if the consequent of the conditional in the first premise was instead the claim that b is not Q." This shows explicitly that he did not realize the formal connection between his two forms.

(with predicate Q). Alternatively, if his second form had been formulated accurately, with the missing inverse clause "if x meets T, then x is not Q," he would have immediately seen in it the first form (with predicate not-Q). And he should have allowed the relative meanings of symbols a and b to remain fixed, either as b > a or as b < a, to emphasize the parallelism between the two forms.

However, as we can see, Duarte developed his second form intuitively, by analogy to his first form, changing the polarity of some of its elements, without bringing out their exact formal ties. In both forms, he makes the same mistake of leaving out the (negative or positive) inverse of his premise (1); and for this reason, the formal ties between the two forms are invisible to him. Consequently also, his second form is *not* reducible *ad absurdum* to his first form (or vice versa), as it should have been if he had formalized them both properly. Clearly, Duarte's understanding of formalization and validation is very limited. That Duarte made the choices he made shows that he was not fully aware of what he was doing. He is a tyro, in no way qualified to speak condescendingly of my work. My logical knowhow and methodology are way ahead of his.

Moreover, it should be emphasized that even while Duarte's two formulas are valid (but for one significant omission each, to repeat) as validation processes, they are not *per se* a fortiori arguments. They are, at best (in view of the said flaw), formulas that a fortiori arguments *incidentally imply*; but they do not have the distinctive 'flavor' of a fortiori argument. The two usual forms of subjectal argument are (to repeat, briefly put): "P is more R than Q; and Q is R enough to be S, therefore, P is R enough to be S" and "P is more R than Q; and P is not R enough to be S, therefore, Q is not R enough to be S." These are the forms used (in full or in part) in our everyday discourse – not the more intricate arguments which I articulated for the purpose of logical validation. Duarte's account focuses primarily on the latter subtext, largely ignoring or dismissing the main thrust.

The truth of this contention can readily be seen if we formulate his main (non-legal) example, used by him to illustrate and develop his first form; as a standard positive subjectal a fortiori argument, it would run as follows:

Whisky is more alcoholic than cider;

and cider is alcoholic enough to be undesirable to this man;

therefore, whisky is alcoholic enough to be undesirable to this man.

My point is that this argument, more akin to everyday discourse, is quite capable of being *credible by itself*. This is *the true form* of a fortiori argument of this sort. Normally, we do not need to unpack the more complicated reasoning that I have identified as mere *validation procedure* and that Duarte has (for the most part) made his own The a fortiori argument of practice is not to be confused with the theoretical validation process that buttresses it. The validation process is a necessity mainly for logicians, and students of logic, to reduce newer and more complex propositions to simpler ones that are already well known. But the form used in practice has a life of its own.

The validation procedure is *not a substitute* for the phenomenological argument; it merely seeks to ground the latter in more widely known and better understood logical and mathematical propositional forms. The validation discourse may operate more or less consciously in people's minds, when they think, speak or hear a fortiori arguments; but this background thinking does not replace the primary thought. The validation does not make the commonplace argument it validates redundant; the latter remains the principal and true expression of a fortiori reasoning.

This is scientifically evident from examination of hundreds of examples in world literature across history.

Duarte did not understand this distinction, which I make quite clear in my work; as a result, he tried to ignore the primary a fortiori argument and equate such argument with the validation procedure. For him, the discursive form, as e.g. in "He does not touch cider; he will certainly refuse whisky," is just an incidental derivative of his more complex form, which he regards as 'true' a fortiori argument. But the reverse is true – the phenomenological form (when fully expressed, as "He is not *so alcohol-loving as to* touch cider; he will certainly refuse whisky") is the 'true' form of a fortiori argument, whereas the validation procedure is only an underlying, subconscious thought, which (to repeat) the logician fishes out to justify it scientifically. As we have seen earlier, Duarte reverses the order of things, imagining that we draw the commonplace form from his model form.

Furthermore, if we look at his example, "He does not touch cider; he will certainly refuse whisky," we see that Duarte mistakenly interprets this as a *positive subjectal* argument, whereas strictly-speaking it is a *negative predicatal* argument, since the subject common to the minor premise and conclusion is "he," while "cider" and "whisky" are (part of) the predicates. Properly interpreted, the argument should run (say): "***This man is not alcoholic enough to drink cider; all the more, he is not alcoholic enough to drink whisky***" (this being the minor premise and conclusion, the unstated major premise being: "One needs to be more alcoholic to drink whisky than to drink cider"). From this we see that Duarte is so ignorant of a fortiori logic that he has *misjudged the form of his main example*! He presents himself as an expert; but he makes the most elementary mistake from the very start of his investigation. (Yes, I'm laughing out loud; or at least, chuckling.)

This draws our attention to *a very important omission* in Duarte's theoretical treatment of a fortiori argument. After presenting his second form, Duarte rightly remarks (p. 211):

> "That there are two forms of a fortiori arguments in law is not exactly a new point. In Continental jurisprudence lawyers and scholars draw a distinction between arguments *a maiore ad minus* (literally, 'from the greater to the lesser') and arguments *a minore ad maius* ('from the lesser to the greater'), which they identify as two species of a fortiori arguments (even though 'a fortiori' means, literally, 'from the stronger')."

And in footnote 11 he adds, among other references:

> "see also, outside the jurisprudential context, Sion's discussion of the difference of orientation between what he identifies as the positive (from major to minor term or thesis) and the negative (from minor to major term or thesis) moods of a fortiori arguments."[26]

---

[26] Here again, note the negative slant in Duarte's mention of my work: he alleges that my discussion is "outside the jurisprudential context," deliberately ignoring my analysis of numerous legal principles and examples from the Talmud and from Roman law, and other legal systems. See AFL, Part II and relevant Appendices.

He thus places his two forms in a larger historical context[27], his first form being argument 'from the lesser to the greater,' and his second 'from the greater to the lesser'. This is all very well, but it is only part of the truth. Furthermore, he misrepresents my work, and again reveals his ignorance of a fortiori logic, by suggesting that I equate positive moods of a fortiori argument to major-to-minor reasoning and negative ones to minor-to-major reasoning. In fact, this only applies to predicatal arguments; as regards subjectal ones, the reverse is true!

Although most commentators are unaware of it, the expressions minor-to-major and major-to-minor are applicable not only to positive and negative subjectal arguments, respectively, but also to negative and positive **predicatal argument**, respectively (note the reversal in polarities). As I have shown with multiple examples throughout AFL, predicatal argument *occurs often in all discourse*, including legal discourse, and cannot simply be ignored in any theory of a fortiori argument. But it is a fact that most people are only aware of subjectal a fortiori reasoning, and not (or hardly at all) of predicatal a fortiori reasoning (even as they use it).

Duarte apparently also has this blind spot, because, although he presumably has come across this form of argument in my work, he totally ignores it in his work. There is not even a brief mention of predicatal argument in his essay; and he accounts for only two forms of a fortiori argument (the positive and negative subjectal forms) instead of four as I do. Why this important omission, I can only guess. Possibly, he was too lazy to read what I wrote about it. More probably, he read a bit, but did not manage to understand the distinctive workings of predicatal argument, and so found it too difficult to transcribe into his terms; so, he just completely ignored it, not even mentioning it. In any case, as we have seen above, he misinterprets the form of his main example as subjectal, whereas it is in fact predicatal.

It should additionally be mentioned that *two* of the thirteen legal examples that Duarte cites in the course of his paper are not subjectal. One (on p. 232-3) is positive predicatal, and one (on p. 212) is negative consequental (i.e. the implicational equivalent of negative predicatal); I analyze these two examples in detail in the next section. He does not notice the different structure of these arguments; notably that the positive predicatal though positive goes from major to minor, and the negative consequental though negative goes from minor to major. These two arguments could conceivably be recast in subjectal form, with judicious informal manipulations; but this would not reflect the ways the arguments were originally stated and intended. This shows that Duarte was scientifically rather negligent; to test his hypothesis that the two forms he concocted covered the legal field exhaustively, he should have at least analyzed all the empirical data at hand[28].

In AFL, I engage in very detailed studies of this sort, listing, analyzing and classifying *hundreds* of examples from very diverse literary sources. Duarte was satisfied with only four applications, a very small sampling. On p. 204 he promises that "the preceding conclusions will be tested against several examples from judicial decisions," and on p. 212 he boasts that he will be "looking at some examples from actual judicial decisions—not just for illustration purposes, but also as a

---

[27] In fact, my AFL includes the most thorough history of a fortiori argument ever attempted. Duarte refers to "Continental jurisprudence lawyers and scholars" without even naming them, and ignores more ancient roots. There is not much evidence of in-depth historical study in his essay. What is evident is that he has not taken the trouble to read AFL, but only skimmed through a bit of it.

[28] I should add that just because only 2 out of 13 arguments that Duarte selected are not subjectal, but predicatal or consequental, it does not follow that this proportion is true of all legal a fortiori arguments. A much larger sample of cases would be needed to estimate the probable proportion in the world at large.

means of testing whether my proposed schemes do actually capture the arguments" – all this to give his thesis an appearance of scientific rigor. But in reality, he does not conscientiously and systematically test his conceptions. That is why his theory of a fortiori argument remains narrow and superficial.

Duarte's omission of positive and negative predicatal a fortiori argument is a serious matter. Predicatal arguments cannot formally be reduced to subjectal ones, even though positive subjectal and negative predicatal arguments have some features in common (they are both minor-to-major, notably) and negative subjectal and positive predicatal arguments have some features in common (they are both major-to-minor, notably). Duarte may have ignored the predicatal forms because he imagined that they were implicit in the subjectal ones; but if so, he was wrong. I examine this issue in great detail in AFL 3.5, under the heading of 'traduction', and conclude that "mixed traductions (from predicatal forms to subjectal ones, or vice versa) are rather verbal than truly logical."[29]

I will now, in a concise manner, so as to further demonstrate Duarte's incompetence, show how he *could have* represented predicatal arguments in his characteristic way (though he did not do so himself, to repeat). This will help make their distinctions from subjectal arguments stand out for all to see. The main distinction to note is that whereas in subjectal arguments the minor premise and conclusion have the major and minor terms (P, Q) as *subject* and the subsidiary term (S) as predicate, in predicatal arguments the minor premise and conclusion have the subsidiary term (S) as subject and the major and minor terms (P, Q) as *predicate* (whence the different names I have given to these forms).

The standard positive predicatal a fortiori argument is (with my symbols):

More R is required to be P than to be Q,

and S is R enough to be P;

therefore, S is R enough to be Q.

This would in Duarte's perspective be rephrased as (with his symbols):

(1) There is a point T in the scale of P such that, for every x, if T meets (the level of P needed for) x, then Q is x.

(2) T meets b.

(3) b ranks higher than a on the scale of P.

Therefore (from (2) and (3)),

(4) T meets a.

Therefore (from (1) and (4)),

(5) Q is a.

The standard negative predicatal a fortiori argument is (with my symbols):

---

[29] This is true so long as we only consider whole terms. If terms are subdivided into parts, a formal theory off such traductions is conceivable. This is evident from the fact that in material cases, we are able to informally traduce many arguments. Closer scrutiny shows that this is possible due to subdivision of the terms initially involved. However, even though it has importance, I have personally not gone down that long road, so far.

More R is required to be P than to be Q,
yet S is R *not* enough to be Q;
therefore, S is R *not* enough to be P.

This would in Duarte's perspective be rephrased as (with his symbols):

(1) There is a point T in the scale of P such that, for every x, if T *does not* meet (the level of P needed for) x, then Q is x.
(2) T *does not* meet b.
(3) b ranks *lower* than a on the scale of P.
Therefore (from (2) and (3)),
(4) T *does not* meet a.
Therefore (from (1) and (4)),
(5) Q is a.

Thus, we learn from the latter form, Duarte should have formulated his main example as follows:

(1) There is a point T in the scale of alcoholism such that, for every x, if T is below the level of alcoholism needed to drink x, then this man will refuse x.
(2) T is below the alcoholism level needed to drink cider.
(3) Drinking cider ranks lower than drinking whisky on the scale of alcoholism.
Therefore (from (2) and (3)),
(4) T is below the alcoholism level needed to drink whisky.
Therefore (from (1) and (4)),
(5) This man will refuse whisky.

Note that I have deliberately left out the necessary complementary (inverse) clauses, as he would have done. We have thus demonstrated that Duarte could well have formulated positive and negative predicatal arguments in the same way as he did positive and negative subjectal arguments, provided he concluded his arguments with "Q is a" instead of "b is Q" – and, of course, adjusted all other features in them as exposed above. That Duarte did not do this is, I suggest, due to the comparative difficulties involved in formulating predicatal arguments. He probably found the job too difficult, and so avoided all mention of this topic.

Duarte had difficulty assimilating these forms of reasoning, I submit, because his way of expressing a fortiori argument is so artificial and awkward. Compare his representations to my standard formulations. Clearly, mine are far more immediately comprehensible and credible, because they better reflect the way people naturally reason a fortiori.

The following table lists the four forms of (copulative) a fortiori argument very briefly, in a manner akin to Duarte's, for purposes of summary and mutual comparison:

Positive subjectal (+s):
If $a \geq T$ (2) and $b > a$ (3), then $b > T$ (4);
and $x \geq T$ implies x is Q (1),
and b is an x and so fits $x \geq T$;
therefore, b is Q (5).
Minor to major, $b>a>T$.

Negative subjectal (-s):
If $a < T$ (2) and $b < a$ (3), then $b < T$ (4);
and $x < T$ implies x is Q (1),
and b is an x and so fits $x < T$;
therefore, b is Q (5).
Major to minor, $b<a<T$.

Positive predicatal (+p):
If $T \geq b$ (2) and $b > a$ (3), then $T > a$ (4);
and $T > x$ implies Q is x (1),
and a is an x and so fits $T > x$;
therefore, Q is a (5).
Major to minor, $T>b>a$.

Negative predicatal (-p):
If $T < b$ (2) and $b < a$ (3), then $T < a$ (4);
and $T < x$ implies Q is x (1),
and a is an x and so fits $T < x$;
therefore, Q is a (5).
Minor to major, $T<b<a$.

Note that my here listing the two predicatal arguments in forms akin to those used by Duarte to describe his two subjectal arguments is not intended to give credence to Duarte's manner of description (which, as already explained, is incomplete and does not reflect actual usage), but merely to show the limited scope of his understanding. That he did not become aware of and try to formalize predicatal a fortiori argument shows that his grasp of the subject was lacking in depth and breadth. He did not have the patience and intelligence needed to study the matter earnestly and thoroughly, but was content with putting on a show.

Note well, I am not here faulting Duarte for not discussing the differences between copulative and **implicational argument**. This is comparatively not so important, and he does at least (as we have seen, in his footnote 8) draw attention to my distinction between these forms. Furthermore, we have already quoted him as saying (on p. 208): "there is no reason to think that the consequent must be a claim in which something is predicated of the threshold-meeting item. All that is necessary is that the consequent involve the threshold-meeting item in some way." We may take this as signifying his awareness and admission that the "consequent" (i.e. the subsidiary item) need not be copulative but may (in certain cases) be implicational.

Even so, we should note that Duarte's awareness of this further variety of a fortiori argument was quite informal, a mere observation that he does not attempt to follow up in a formal manner. He could, in fact, have rather easily proposed four implicational arguments, analogous to the above described four copulative arguments, by means of the following changes in wording.

Instead of the clause in his premise (1) that "if x meets T, then x is Q" or "if x does not meet T, then x is Q" (for positive and negative subjectals, respectively), he would have had: "if x implies something that meets T, then x implies Q" or "if x implies something that does not meet T, then x implies Q" (for positive and negative antecedentals, respectively). Likewise, instead of the clause in the premise (1) that "if T meets x, then Q is x" or "if T does not meet x, then Q is x" (for positive and negative predicatals, respectively), he would have had: "if T implies something that meets x, then Q implies x" or "if T implies something that does not meet x, then Q implies x" (for positive and negative consequentals, respectively). The conclusions would have been modified

accordingly, *mutatis mutandis*. That is, instead of "b is Q" or "Q is a," they would have been "b implies Q" and "Q implies a," respectively.

Here again, please do not interpret my wording the implicational a fortiori arguments in Duarte's characteristic manner as implying my approval of his approach; to repeat, I regard it as inadequate for various reasons. My purpose here is only to show that Duarte's theory of a fortiori argument is even more limited in scope than it might have been. My purpose is to draw attention to and emphasize Duarte's lack of logical skill and vision in formulating his theory. One would expect such approximation and stunting from a dilettante.

Another glaring and important omission in Duarte's theory of a fortiori argument is **a crescendo argument**. As I show in AFL, many commentators on a fortiori argument have noticed that sometimes people conclude such argument with the exact same predicate, while other times they conclude it with *a quantitatively greater or lesser predicate*. Most researchers have not known how to deal with this issue formally. I solve this problem formally and in great detail (in AFL 2.2-2.3); so, Duarte should have known about it if he had read it. However, he seems to have skipped reading this investigation, perhaps out of laziness.

No study of a fortiori argument, which does not examine the issue of proportionality, and determine precisely when and why we can draw a proportional conclusion, is credible. Consequently, this omission is a very serious flaw in Duarte's treatment of a fortiori argument. Duarte does raise the issue incidentally (on pp. 217-8), by means of a legal example in which the predicate of the conclusion is quantitatively greater than that of the minor premise it apparently follows from. He comments on this unusual inference as follows:

> "… the a fortiori argument itself is an argument that justifies applying the same conclusion to the target that applies to the source. … this does also not rule out, of course, the possibility that the plaintiff was indeed entitled to more than that, even to £100,000 more; it is simply that she would need a separate argument to establish that."

This comment means that he does not regard a quantitatively increased (or presumably, decreased) conclusion as part of the a fortiori argument, but as something to be established by means of a "separate argument," without specifying what that additional reasoning might be. His comment is partly right, insofar as he does allow for an augmented conclusion in certain cases (in contrast to many other commentators who tend to reject such a result offhand); but it is merely intuitive, made without formal consideration and follow-up. Instead, he gets sidetracked into the unrelated issue of the strength of an a fortiori conclusion compared to its premises (we shall consider this other issue later on).

Thus, it is evident that Duarte did not take the trouble to investigate a crescendo argument. As I show in my study of this topic, a crescendo argument is a special case of a fortiori argument, in which there is an *additional premise* about proportionality. In other words, a crescendo argument is a fortiori argument combined with pro rata argument. People who think 'a crescendo' are intending to think 'a fortiori' with a flourish. The additional premise takes the purely a fortiori conclusion and adjusts it to fit the demand of proportionality, which is in some cases factually justified. Indeed, where such adjustment is justifiable, and useful, it logically ought to be applied; it is not a matter of choice, not a last resort. This adjustment could be identified with the "separate argument" that Duarte intuitively assumes occurs; but he does not look further into the matter. In

any case, see my reading of the example at hand in the next section, when I interpret all of Duarte's legal examples.

Duarte also briefly raises the issue of **analogical argument**, close to the end of his essay (on p. 237); but here again he fails to mention my formal study of the subject **(**in AFL 5.1). Here is what he writes:

> "But there also seem to be important dissimilarities between arguments a fortiori and the kinds of argument that both lawyers and theorists normally refer to as 'analogical' arguments. One salient difference is that even in fully reconstructed arguments by analogy, the comparison between items—between the source and the target case—does not rely on the identification of any unifying rule specifying their relevant common features. In a fully reconstructed a fortiori inference, on the other hand, the relevant unifying scale does have to be specified in the premises in order for the argument to run. A broader exploration of the contrast, however, I must leave for another day."

This shows that Duarte did not formally investigate the matter as he ought to have, so as to clearly distinguish a fortiori argument from other, yet somewhat comparable, forms of argument. In this case, additionally, his intuitive assessment is wrong. As I show, in properly formulated analogical argument, capable of formal validation, there indeed is a "unifying rule," and it does need to be identified. As often, he speaks without knowledge, out of hand; and on top of that conceitedly presents himself as the potential savior in this matter, which feat he, however, "must leave for another day"![30]

Let us now summarize what we have uncovered in this second section: Duarte presents a second form of a fortiori argument resembling the first, except that it contains certain negative elements. When we examine this additional form, we find that it was constructed intuitively, by mere analogy, without any deep understanding of its exact formal relation to the preceding form. This is evident from the formal and symbolic choices made in it. Moreover, the second form, like the first form, lacks important information (namely, the full inverse of the initial hypothetical proposition), and indulges in some ambiguity. Indeed, this omission in both cases explains why Duarte made wrong choices in constructing the second form by analogy to the first, and why he failed to see the deeper relation between his two forms.

Furthermore, Duarte apparently considers his two-form theory of a fortiori argument to be exhaustive. This is naïve on his part. Even his own selection of legal examples should have empirically alerted him to the fact that there are other forms. He thus fails to realize that a fortiori argument is not always subjectal, but may be predicatal. He does not apparently even notice this form of argument, even though it is treated in detail in my works. Similarly, he does not pay attention to and assimilate my formal research on implicational arguments, a crescendo

---

[30] In his footnote 9, Duarte writes: "I discuss analogical arguments at length in L. Duarte d'Almeida and C. Michelon, 'The Structure of Arguments by Analogy in Law' Argumentation (2016) at http://link.springer.com/article/10.1007/s10503-016-9409-3 (last accessed 26 November 2016)." I have not consulted this article, because he does not refer to my formal work on this topic. I assume it is not a conclusive formal logic study, since in the present essay which he wrote later he implies the issues are still open.

arguments and analogical arguments. Consequently, Duarte's awareness and understanding of a fortiori argument remain very limited.

3. Examples used

I have been through all thirteen *legal examples* of a fortiori argument that Duarte cites, and found them to have the following forms[31]: seven are positive subjectal (pp. 203, 217, 225b, 226a, 226b, 227-8, 229), one of these being a crescendo (p. 217); three are negative subjectal (213-4, 216, 225a); one is subjectal but not fully quoted, so I could not tell whether it is intended as positive or negative (p. 220); one is positive predicatal (p. 232-3); and one is negative consequental (p. 212). Duarte explicitly interprets only four of these arguments in terms of his first and second forms (pp. 203, 212, 213-4, 227-8). The following are my interpretations of all thirteen cited arguments, in standard form:

The seven *positive subjectal* (minor to major) a fortiori arguments are:

- Ex. on p. 203: Voting for the form of government (P) is more important (R) than voting for a representative in a legislature (Q). If voting for a representative in a legislature (Q) is important (R) enough that it should be guaranteed to all (S), then voting for the form of government (P) is important (R) enough that it should be guaranteed to all (S). Duarte rightly interprets this argument by means of his first form (on p. 230-1).
- Ex. on p. 217: A well-behaving woman whose marriage was happy and contented (P) has more merit than a misbehaving woman whose marriage ended in divorce and dissension (Q). If a misbehaving woman etc. (Q) has merit (R) enough to be awarded a provision of £350K, then a well-behaving woman etc. (P) has merit (R) enough to be awarded a larger provision of (say) £450K. Note that this argument is not purely a fortiori, but *a crescendo*, so that it relies on an additional, though tacit, premise of proportionality, viz.: awards should be proportional to degree of loyalty.
- Ex. on p. 225b: Removing a card index of the employer's customers (P) is more (say) damaging (R) than making or copying a list of his customers (Q). If making or copying a list of the employer's customers (Q) is damaging (R) enough to be a breach of the duty of good faith by the employee (S), then removing a card index of the employer's customers (P) is damaging (R) enough to be a breach of the duty of good faith by the employee (S)[32].
- Ex. on p. 226a: A steamer has more power to get out of the way than a sailing vessel. If a sailing vessel (Q) has power (R) enough to be legally required to get out of the way of a trawler which is denoting by her lights that she has her trawl down (S), then a steamer (P) has power (R) enough to be legally required to get out of the way of a trawler etc. (S).
- Ex. on p. 226b: The name 'gramophone' by which the article is popularly known (P) is more specific about the article's nature (R) than a laudatory word such as 'perfection' that has become distinctive of the goods of a particular manufacturer (Q). If a laudatory word such as 'perfection' (Q) is specific (R) enough to be disallowed for registration (S), then the name

---

[31] I use the suffixes a and b to a page number when there are two examples on the same page.

[32] Notice that although the original wording here, viz. "if it is a breach of the duty of good faith for the employee to make or copy a list of the employer's customers," seems to place "breach of duty" before "the employee" (suggesting predicatal reasoning), it in fact has "it" as the subject and "breach of duty etc." as the predicate, and this "it" stands for "the employee" making or copying, etc. (so it is subjectal reasoning).

'gramophone' by which the article is popularly known is specific (R) enough to be disallowed for registration (S).

- Ex. on pp. 227-8: A wooden ship (P) is more endangered by flammable material than a metal ship (Q). If a metal ship (Q) is endangered by flammable material (R) enough by having gasoline drums in its hold (S), then a wooden ship (P) is endangered by flammable material (R) enough by having gasoline drums in its hold (S)[33]. Note that the speaker goes on to put in doubt the argument, by claiming that the ship would be destroyed well before an eventual fire reached the hold; this is effectively a denial of the major premise. But the argument initially stated remains formally valid anyway; and this is what concerns us here. Duarte rightly interprets this argument by means of his first form, albeit with certain modifications (on p. 228-9).
- Ex. on p. 229: Larger vehicles (P) are more obstructive standing on the highway (R) than smaller vehicles (Q). If smaller vehicles (Q) are obstructive (R) enough to be restricted by certain regulations (S), then larger vehicles (P) are obstructive (R) enough to be restricted by these same regulations (S)[34]. Note that here again the speaker goes on to put in doubt the argument, by claiming that the regulations referred to were only meant to apply to smaller vehicles; this is again effectively a denial of the major premise. But the argument initially stated remains formally valid anyway; and this is what concerns us here.

The three *negative subjectal* (major to minor) a fortiori arguments are:

- Ex. on pp. 213-4: A point on which there is an issue (P) is more significant (R) than a point on which there is no issue (Q). If a point on which there is an issue (P) is not significant (R) enough to reopen the case (S), then a point on which there is no issue (Q) is not significant (R) enough to reopen the case (S). Duarte rightly interprets this argument by means of his second form.
- Ex. on p. 216: A prisoner charged with an offence (P) is in more legal trouble (R) than a prisoner not charged with an offence (Q). If a prisoner charged with an offence (P) is in legal trouble (R) not enough to have his temporary license revoked (S), then a prisoner not charged with an offence (Q) is in legal trouble (R) not enough to have his temporary license revoked (S).
- Ex. on p. 225a: The Court of Session (P) is more high-ranking in the judiciary (R) than civil action in the Sheriff Court (Q). If the Court of Session (P) is not high-ranking (R) enough to review acts and decrees of the Court of Justiciary (S), then a civil action in the Sheriff Court is not high-ranking (R) enough to review such acts and decrees (S)[35].

The one *subjectal* a fortiori argument of undetermined polarity is:

---

[33] Notice that the original wording in this example, "Everything that could be said against these gasolene drums on a steel or iron ship would a fortiori apply to them on a wooden ship," has the metal and wooden ships as subjects and what is "said" or "applied" against the gasoline drums as predicate.

[34] Here again, although the original wording mentions the restrictions before it does the vehicles, the argument is clearly intended as subjectal, because it speaks of the former being "applicable to" (i.e. predicated of) the latter.

[35] Here, although the original wording mentions the "acts and decrees of the Court of Justiciary" before "the Court of Session" and "civil action in the Sheriff Court," this is only because the speaker is using a grammatically passive voice; in active voice, the latter courts are the subjects of the reviewing of the former acts and decrees. So, the argument is subjectal as shown, and not predicatal.

- Ex. on p. 220: this example is incompletely detailed, the ruling not being quoted by Duarte. The major premise is given: An alien resident who came to the host country as a child or was born there (P) is more connected to it (R) than an alien resident who came to the host country as an adult (Q). This reveals the argument to have been subjectal. But it is not clear from the rest of the citation what the minor premise and conclusion were originally.

The one *positive predicatal* (major to minor) a fortiori argument is:

- Ex. on pp. 232-3: More permissiveness (R) is required to allow for heavy penalties (P) than for light penalties. If the Convention (S) is permissive (R) enough to allow for more stringent penalties (P), then it (S) is permissive (R) enough to allow for more lenient penalties (Q). Note that here again the speaker goes on to put the argument in doubt, by pointing out that the Convention referred to does not in fact necessarily allow for more lenient penalties; this is again effectively a denial of the major premise. But the argument initially stated remains formally valid anyway; and this is what concerns us here.
  Duarte takes for granted that this example can be put into one of his two forms. I could, by manipulating the terms somewhat, recast this argument as positive subjectal (minor to major): Lenient penalties (P) are more permissible (R) than stringent penalties (Q). If stringent penalties (Q) are permissible (R) enough to be allowed by the Convention (S), then lenient penalties (P) are permissible (R) enough to be allowed by it (S). But the way the argument is originally articulated places the Convention as the subject and the penalties are the predicates:

  > "*[I]n the area of human rights he who can do more cannot necessarily do less. The [European] Convention [of Human Rights] permits under certain conditions some very serious forms of treatments, such as the death penalty (article 2(1), second sentence), whilst at the same time prohibiting others which by comparison can be regarded as rather mild, for example 'unlawful' detention for a brief period (Article 5(1)) or the expulsion of a national (Article 3(1) of Protocol No. 4). The fact that it is possible to inflict on a person one of the first-mentioned forms of treatment cannot authorise his being subjected to one of the second-mentioned, even if he agrees or acquiesces…*"

  The opening sentence "he who can do more cannot necessarily do less" is the denial of the a fortiori argument under scrutiny, with "he" standing for the Convention, and "do more" and "do less" standing for the more stringent and more lenient penalties, respectively. Moreover, the last sentence, "The fact that it is… etc." places the more stringent penalties in the minor premise and the more lenient ones in the conclusion. All this implies that the a fortiori argument in question, in the speaker's mind, is predicatal. Even if we can in this case reshuffle the argument and fit it into Duarte's first form (i.e. in positive subjectal form, as shown), this would be an artificial interpolation. Interpretation of examples should, as much as possible, stick to the given parameters.

The one *negative consequental*[36] (minor to major) a fortiori argument is:

- Ex. on p. 212: More descriptive precision (R) is required to imply that horses satisfy a certain statute (P) than to imply that cows satisfy the same statute (Q). If merely specifying the

[36] Negative consequental is to implicational a fortiori argument what negative predicatal is to copulative such argument.

number of animals (S) is descriptively precise (R) not enough to imply that cows satisfy the statute (Q), then merely specifying the number of animals (S) is descriptively precise (R) not enough to imply that horses satisfy it (P).

Note that Duarte inaccurately interprets this argument by means of his second form. I could, by manipulating the terms somewhat, recast it as negative subjectal (major to minor): Since more precision in description is required of horses than of cows, and both were in this case described in the same way, by mere number, it follows that: the description of the cows (P) was relatively more precise (R) than that of the horses (Q); whence, if the description of the cows (P) was not precise (R) enough to satisfy the statute (S), then the description of the horses Q was not precise (R) enough to satisfy the statute (S). However, so casting the argument loses the original mood in which it was expressed:

> "*The schedule describing the 'stock' as '2 horses, 4 cows' is not sufficiently specific to satisfy the statute [section 4 of the Bills of Sale Act, 1882]. In Carpenter v Deen it was held that '21 milch cows' was an insufficient description. By that decision we are bound. Moreover, the cows here are not even described as milch cows. The description is therefore even less specific than in that case. As to the two horses, it follows a fortiori that their description is insufficient, for even Lopes LJ, the dissentient judge in Carpenter v Deen, was of the opinion that, as it was usual to describe horses by their colour, a greater degree of particularity was required in the case of horses than in that of cows. The bill of sale was therefore bad as to the horses and cows.*"

Notice the form of the major premise "a greater degree of particularity was *required* in the case of horses than in that of cows" (my emphasis), which is clearly predicatal (or more precisely, upon reflection, consequental) rather than subjectal (or antecedental); and notice that horses are placed *above* cows in it. This tells us in what mood the judge who uttered the judgment was actually thinking. In the proposed recast version, the cows are (on the contrary) placed above the horses. So, if we want to accurately reflect the judge's thinking, we must use the consequental form.

The reason I cast all of Duarte's examples in standard forms was to verify if they all fit into his first and second forms, even if he only tested four examples (as already mentioned). As it turned out, this effort was worthwhile, since it revealed that two of the examples he selected in fact do not fit into his two-form scheme. Another reason I formulated the examples one by one in standard forms was to show how easy it is to follow and believe the arguments in question when they are in such forms. Compare the complications involved in Duarte's formulations: his two forms are awkward contraptions that do not match ordinary human a fortiori reasoning.

Duarte proposes only one non-legal example of a fortiori argument, namely the cider and whisky example on p. 204, which he got from Daube, then modified. He uses this as his main illustration, to develop his first form, and again (as we shall see further on) to develop a more elaborate version of it incorporating a literal interpretation of the expression 'a fortiori'. Although he returns to this example in many contexts, he nevertheless (as we have seen already) makes significant errors and omissions in his analysis of it. The legal examples he provides for his first form on p. 203 and on pp. 227-8 are apt. He offers no non-legal example of his second form, choosing instead to illustrate it by means of the legal example on p. 212-3, relating to cows and horses, which (as we have just seen) he misinterprets too, and the one on pp. 213-4, which he

reads correctly. He provides no illustration for the more elaborate, literally 'a fortiori', version of his second form.

One other example Duarte mentions, in passing (in footnote 35), is a Talmudic argument mentioned by Hyam Maccobi, described as a 'parodic' a fortiori argument. Duarte makes no attempt to examine this absurd argument in terms of his own analytical tools, and tell us why it is wrong. He does comment that it "illustrates what can go wrong when there is no (true) premise to be found identifying a scale (and a relevant threshold) against which to compare both source and target;" but he does not pinpoint precisely where the errors lie as he should have. In any event, this rough guess of his is wrong.

He shows no curiosity, and does no further research on the matter, even though the example is shaking – so very shaking, in fact, that the individual who proposed it is said to have been excommunicated for doing so. I deal with this argument in detail in AFL 13.4. I there formulate it as[37]: "Marriage with a married woman's daughter (P) is more unlawful (R) than marriage with the married woman herself (Q); and marriage with another man's wife (Q) is unlawful (R) enough to be prohibited (S); therefore, marriage with another man's wife's daughter (P) is unlawful (R) enough to be prohibited (S)." I then explain that the argument is indeed technically valid, but still has to be wrong since its conclusion contradicts one of its premises.

The fault lies with the major premise, which has been constructed by generalization from the givens regarding *one's own* wife (whom one can marry) and daughter (whom one cannot marry) to *all* wives and daughters (i.e. including those other than one's own). Although the proposition is indeed true for one's own wife and daughter, it is not true for any other wives or daughters. Duarte could not have known this explanation, because he has not studied the ways the premises of a fortiori argument are generated.

4. With stronger reason, literally?

Let us now move on and see where Duarte's paper wishes to take us after presenting his two forms of a fortiori argument and analyzing some concrete examples with them.

Duarte rightly points out many details concerning a fortiori argument. For instance, that "arguers may leave many crucial elements of their inferences unstated," and that "some of those elements will occasionally be opaque… even to the arguers themselves," adding "and that certainly includes the relevant scale" (p. 225). And he rightly argues that theoretical knowledge of a fortiori argument makes it easier to confirm or refute a given a fortiori argument we might encounter (p. 227). While he should be congratulated for such observations, he should also be reproved for giving readers the impression that they are his own, when they are not.

He tries to project himself as an original researcher; but as we shall now show in great detail, when he tries to be genuinely original he fails abysmally. It is like a person who has been painting by numbers (and badly at that) and thinks that this qualifies him for original artwork. Duarte asks (as of p. 214) why people use expressions like "a fortiori" or "with stronger reason," which seem

---

[37] See further details there. My account was based on Moses Mielziner's citation of the argument (given in *Derech Eretz Rabba*, chapter I) as: "If the marriage with one's own daughter is prohibited, although the marriage with her mother is permitted, how much more unlawful must it be to marry another married woman's daughter, since the marriage with her mother, a married woman, is positively prohibited?"

to suggest that the conclusion has literally "greater force" than the premises it is drawn from. This is a good question, which needs to be asked and answered.

He points out that in some cases the argument seems to suggest the possibility that "something be less—or more—allowed than something else," but rightly rejects this as "nonsensical" and "not to be taken literally," arguing that "something either is or is not allowed; being allowed is not a matter of degree" (p. 215). I already make this point clearly in AFL 26.3, in response to Stefan Goltzberg's use of the expression "even more forbidden." I there say: "Very often, the subsidiary term allows of no measures or degrees… so that what is said of it in the minor premise is bound to remain the same in the conclusion. For examples, something 'imperative' or 'black' cannot be more or less so – either it is so or it is not." Duarte here again lays claim to an insight that is not originally his, but mine.

A bit further, quoting Arnold Kunst using the expressions "more illicit" and "less illicit" in his description of a Talmudic principle, Duarte tries to look clever by saying sarcastically: "*More* illicit? *Less* illicit?" – as if until now no one realized the absurdity of such statements. Note that I deal in detail, in AFL 8.5, with the Talmudic principle paraphrased by Kunst. The latter seems to be referring to the Mishna *Beitzah*, 5:2, which compares Sabbaths and Festivals, rather than major and minor festivals as he suggests[38]. Furthermore, the original wording is not as Kunst puts it "more illicit" and "less illicit," but *chayav* (meaning "culpable," in the Soncino translation) and *kal vachomer* (meaning "how much more" so). And its intent is clearly that what is forbidden on Festivals is forbidden on the Sabbath; and by contraposition, what is permitted on the Sabbath is permitted on Festivals: no difference in degree is implied[39].

Duarte also rightly (at first, anyway) rejects the idea that the premise might be less "solidly established" than the conclusion drawn from it (p. 216). He also briefly considers a legal example in which the quantity in the putative conclusion is greater than that given in a premise; but in his view such argument, though it includes a fortiori reasoning, additionally involves a "separate argument" (p. 217). I have already (in the previous two sections) exposed this example as a crescendo. It is evident that Duarte has not studied my past work on a crescendo argument, nor made any attempt to develop the formalities involved by himself; he remains essentially unconscious of this important aspect of a fortiori logic throughout his paper.

Nevertheless, Duarte does not entirely discount use of suggestive expressions like "a fortiori" or "with stronger reason," as mere "rhetorical flourish" (p. 218). He thinks there is a "sense in which it is perfectly meaningful," and tries to develop more elaborate argument forms in support of this claim. This is in truth *the climax* of his thesis on a fortiori, his one attempt at original thought. As we shall see, it is *bunk*. He does not manage to prove what he set out to prove, nor even make it seem somewhat credible. Worse still, his attempt to do so is on logical principle vain; and moreover, it is developed in a very sophistical manner.

---

[38] To my knowledge, there is no principle such as Kunst describes, which infers prohibitions in "major festivals" from those in "minor festivals," or permissions in the latter from those in the former. The laws of Purim and Hanukkah are unrelated to those of Pesach, Shavuot and Succot. The former, minor festivals were added on to Judaism long after the latter, major festivals; and the laws relating to them are very different.

[39] Duarte does not mention my analysis of this Talmudic passage, evidently not having bothered to read it. He prefers to quote Kunst, whom he refers to as a scholar, even though I have shown, in AFL 14.3, that Kunst's understanding of a fortiori argument was very superficial. This was just name-dropping on Duarte's part, trying to look intellectual; but he bet on the wrong horse.

But before we start our detailed analysis of his proposals, the following **preparatory remarks** are worth making.

I do not know who first coined the Latin expression "*a fortiori ratione*" (with stronger reason). Obviously, someone must have, and the name given such argument stuck and became widely used till this day. Obviously, whoever coined this name consciously or at least subconsciously imagined that the conclusion of such argument was somehow rationally "stronger" (more forceful, more reliable, more cogent) than the premise(s) it was derived from. This idea, that the argument effected what we might characterize as an '***epistemic profit***', was in fact logically impossible, an error; but he did not realize it, or maybe he was aware of it but just had a taste for hyperbole. Nevertheless, since the *a fortiori ratione* label was coined, many people have taken it literally and assumed it to mean just what it says. Every so often, amateur logicians try to defend the idea and make fools of themselves trying.

Let us briefly look at some of the history of studies on a fortiori argument. **Aristotle** (Greece, 384-322 BCE)[40], in all probability the first person ever to reflect on such argument, refers to it descriptively, in his *Rhetoric* 2:23(4) as concerning "the more and less", and in his *Topics* 2:10 as "from greater and less degrees;" in practice, he favors similar neutral wording. Aristotle does discuss and occasionally use a crescendo arguments, and he does sometimes engage in logical-epistemic a fortiori arguments. But he cannot be said to have anywhere suggested that the conclusion of an a fortiori argument can be epistemically "stronger" than the premise(s) it is based on; his wording always suggests epistemic equality if not weakening.

The renowned Roman jurist **Cicero** (106-43 BCE)[41], in his *Topics* §23, likewise speaks of "the greater" (*maiore*) and "the lesser" (*minori*); and he uses ontical rather than logical-epistemic illustrations of such argument. Interestingly, he nowhere there or elsewhere (so far as I know) uses the Latin expression "*a fortiori ratione*;" and more significantly, he nowhere suggests the said idea of an epistemic gain. Much the same can be said concerning the 3rd cent. CE Hellenistic Peripatetic Alexander of Aphrodisias[42]. Similarly, in Judaic logic; although Biblical and Talmudic statements intending a fortiori and even a crescendo reasoning are numerous, none of them involve language or commentary indubitably implying epistemic increase[43].

So, it is not clear who coined the expression "*a fortiori ratione*" or first suggested it implies an epistemic gain. It is possible that this idea antedates the expression and led to it; but it is more likely, to my mind, that the expression was first coined rather unconsciously, and later elaborated on by someone. These events may both date from before Cicero, or have arisen in between him and the said Alexander; or again, they may have arisen later, in Christian times. I do not know the exact time line, and it may be too late to determine it. Not that it matters greatly – there is no doubt that the said expression is inaccurate and the idea it suggests logically untenable.

What is sure, also, is that Duarte is not the first to advocate, inspired by the said expression, the idea of epistemic gain. For instance, not so long ago (in 2002), Piotr Lenartowicz and Jolanta

---

[40] See my detailed treatment of Aristotle's theory and practice in AFL 6.1 and Appendix 4.2. I there give full quotations and exact statistics.

[41] See AFL 6.5.

[42] See AFL 6.6.

[43] As regards the Talmudic doctrine of *pirka* (objection) to a fortiori arguments, see AFL 13.4. See also, in AFL appendix 2, the rival arguments in Pesahim 6.2 and in Sotah 6.3.

Koszteyn proposed the following as the "true" a fortiori argument: "if it is irrational and non-empirical to doubt that [the minor] Q is S, then it is even more irrational and non-empirical to doubt that [the major] P is S;" and I later showed why their proposal is specious[44]. More recently (in 2012), Hubert Marraud tried to justify a similar idea, and I showed him too to be dead wrong[45]. Evidently, Duarte did not take the trouble to read these chapters and others of AFL, which might have dissuaded him from trying the same tomfoolery.

Let us now look more closely at Duarte's specific suggestions. He imagines, using again the cider-and-whisky illustration, a more complex scenario in which: "the claim in (5)" (i.e. the conclusion that the man will refuse whisky) "may be more strongly supported than the claim in (2a)" (i.e. the earlier side-inference that the man will refuse cider). He imagines, to begin with, the following simple scenario (p. 218):

> "For it may be the case that our friend, despite his principled stance of refusing beverages that exceed a certain degree of alcohol content, might be willing, on some occasions, to make exceptions to the principle. He will have his reason or reasons for not drinking at least some alcoholic beverages: the reason is, suppose, that beverages with an alcohol content above a certain degree will give him a terrible headache. But he may find himself in a situation in which he will also have reasons for having a drink, and these reasons may outweigh his reasons for not doing it: imagine, for example, that our friend is presented with a newly produced wine by his son, a winemaker, who would really like to know his father's opinion on how it tastes."

Let us first look at this simple scenario, before considering the further complications Duarte brings to it and the solutions he proposes. In the initial cider-and-whisky illustration, we argue that since cider, which has a lesser alcohol content, was refused (by a certain man), it follows a fortiori that whisky, which has a greater alcohol content, will be refused (by that man). This argument did not tell us whether or not the subject (the man) is a teetotaler; nor did it inform us as to how much alcohol content he would tolerate, assuming he is not a teetotaler.

The man's "principled stance" against alcoholic beverages, we are now told, is probably due to the headaches they can give him as of a certain degree of alcohol content[46]. This added detail by itself does not affect the a fortiori inference from refusal of cider to refusal of whisky, note well. But it does suggest that if an alcoholic beverage did *not* cause him headaches, the man might well accept to drink it. So, it looks like the man is not a teetotaler, i.e. not against alcohol per se; but merely opposed to a certain level of alcohol intake, because of its deleterious effects. This means that there are possible exceptions to his refusal of alcohol: he may well *not*-refuse (i.e. accept) beverages with alcohol levels below the threshold where headaches begin.

Next, additionally, Duarte introduces the idea that the man's son might pressure him to taste some of the wine he (the son) produced. If that wine has alcohol content below the threshold as of

---

44 See AFL 24.3.

45 See AFL 30.3 and 30.5.

46 Note in passing that Duarte mentions "degree" of alcohol content, but not the quantity of beverage with such alcohol content that would cause a headache. It is unlikely, however, that in a real case the quantity absorbed would be irrelevant.

which the man gets headaches, such indulgence would of course be unproblematic; i.e. the original a fortiori argument would be unaffected[47]. But if the wine is strong enough to cause him a headache, the scenario is radically changed, because he may (say, out of love for his son) nevertheless accept to drink some of it, even if it is more alcoholic than the cider which he refuses.

In that event, we can no longer automatically argue from refusal of a less alcoholic beverage (say, cider) to refusal of a more alcoholic beverage (namely, strong wine). We can still do so *conditionally*, granting that the man is under no pressure taste the strong wine; but if this condition is not met, i.e. if the son *does* ask his father to drink some of his strong wine, the said a fortiori argument can no longer be relied on *at all*. It would perforce result in a wrong conclusion; namely, that he will refuse to drink the strong wine when in fact we are told that he won't refuse to do so. In this situation, the a fortiori argument would need to be completely revised, somehow. How?

Obviously, the middle term (alcohol content or headaches) and the subsidiary term (the man will refuse to drink the beverage) of the original a fortiori argument need to be changed. The issue now *determining* the man's refusal or acceptance of strong wine is no longer alcohol content, or headaches, but the son's wish. We can build a new a fortiori argument as follows, but clearly this argument is very different from the preceding. It has a new middle term (R), viz.: 'importance to the son', and a new subsidiary term (S), viz. 'the man will accept to drink the beverage'.

Strong wine (P) is more important to the son (R) than cider (Q).

If cider (Q) is important to the son (R) enough that his father will accept it (S);

then strong wine (P) is important to the son (R) enough that his father will accept it (S).

In truth, this new a fortiori argument is quite contrived (by me), because it continues to refer to cider. We do not in fact need to *infer* that the father will accept to drink his son's strong wine, because we already know this as *a given* of the new scenario. Thus, all that this new scenario has done is make the original a fortiori argument (inferring refusal of strong wine from refusal of cider) applicable *if and only if* the son does not ask his father to taste the wine. In the event that the son *does* ask that of his father, there is *no* a fortiori argument, but merely a given proposition, viz. that the father will accept to taste it.

This is the "exception" mentioned in the above scenario. The initially broad "principled stance" that the man will refuse beverages that give him headaches is made more limited, applicable only if the man's "reasons for having a drink" (namely, his son's wishes) do not "outweigh" his "reasons for not doing it" (namely, his fears of headaches). In the event that the former reasons do outweigh the latter, he will accept to drink. So far, then, Duarte's attempt to make the logic of a fortiori argument more complicated has failed. Nothing in the scenarios we have seen thus far have yielded a modified form of a fortiori argument.

Vaguely sensing this failure, Duarte tries to project a more complex situation (pp. 218-9):

---

[47] Wine normally has alcohol content at 8-14%, cider at 4-8% and whisky at 40-60% (according to information found in the Internet). Let us here, for the sake of argument, first assume that the wine in question is very dilute (under 8%) and the cider very strong (over 8%).

> "Now suppose, further, that the higher the alcohol content of the beverage, the more intense our friend's headache would be, ranging all the way up to almost paralyzing pain. In that case, it seems clear that the higher the degree of alcohol content of a certain beverage, the harder it will be for his reason against drinking it to be outweighed. In other words, the range of reasons that might outweigh his reason for not drinking whisky is narrower than the range of reasons that might outweigh his reason for not drinking cider. But what that means is that our friend has a stronger reason against drinking whisky than against drinking wine; and thus that the conclusion that he would refuse whisky is also stronger, in a sense—harder to defeat—than the conclusion that he would refuse cider."

In this new scenario, Duarte proposes that the headache caused by alcohol is variable; that is, it varies with alcohol content of beverages, so that the more alcohol, the more pain. Before, headaches were flagged by a single threshold along the horizontal line tracking alcohol content. Now, there are two variables, varying concomitantly. We can represent this by means of a two-variable graph, with the vertical dimension tracking pain and the horizontal one tracking alcohol content. The graph itself, however, is still a line, even if now inclined or maybe curved; and the reasoning involved is in fact the same as if the graph were horizontal. What is changed now is simply the position of the threshold. Whereas before the threshold was *any amount of* headache pain (symbol T) now the threshold is *a certain higher degree of* headache pain (symbol U, say). Formally, nothing is different; what is different is the precise cut-off point.

The man's behavior is, we are told, motivated by conflicting interests: he wants to avoid headaches as much as possible, but he also wants to please his son *if* he can stand the pain. So, the issue for him is: *until what point* along the curve is compromise possible? While the man might accept to drink strong wine if his son asks him to, there may be a threshold as of which his son's request will be ignored, because the pain would be "paralyzing." Whisky would seem to be too strong for him to bear, but the son's wine might not be. So, the proposed complication is not as great as it first seems. Thus, contrary to appearances, this new scenario is not radically different from the preceding.

To wit: there is a continuum of values of alcohol content and concomitantly of pain; along this line, there is a new threshold U (unbearable pain) higher than the preceding threshold T (any amount of pain). In the upper range, as of and above point U, the man would *categorically* refuse any beverage, even if the son wishes otherwise; an a fortiori argument from refusal of one strong beverage to refusal of a still stronger beverage could validly be constructed in this range. In the medium range, between U and T, a similar a fortiori argument could (as before) be formulated *conditionally*, provided the son does not ask his father to drink; but if the son does ask his father to drink, the man would likely accept to do so, albeit the pain involved (and the a fortiori argument would be inapplicable). Finally, in the lower range, below point T, there being no pain, the man is free to accept his son's eventual request without qualms (and no a fortiori argument is needed).

As regards a fortiori logic, then, Duarte's new, more complex scenario, is as irrelevant as the preceding, simpler scenario. That is, nothing in either scenario makes a new, more complicated form of a fortiori argument possible or needed. We are still dealing with the same old standard a fortiori argument forms, but applying them selectively, as called for by the projected situations. As above explicated, in some situations, the a fortiori argument is applicable unconditionally; in others, it is applicable only conditionally; in others still, it is inapplicable; and in some, it is not at all relevant.

Notice now that, whereas in the former scenario Duarte refers to "reasons" for having a drink or refusing it, in the latter he refers to "range[s] of reasons," so as to make matters look still more complicated.

It seems that Duarte's is here trying to develop a calculus of "reasons" for doing or not doing something, but has difficulty formulating it. Let's try and help him do that. For a start, we are told that there are two beverages x and y; x and y both "meet" a threshold T; and y contains more alcohol than x. Next, it appears that the subject ("our friend") has reasons for drinking y (call them Pro-y), and reasons against drinking y (call them Con-y); and Pro-y may be more or less persuasive than (or equal to) Con-y. Similarly, the subject has reasons for drinking x (call them Pro-x), and reasons against drinking x (call them Con-x); and Pro-x may be more or less persuasive than (or equal to) Con-x. Furthermore, let us call Ky the net result of Con-y minus Pro-y (or vice versa), and Kx the net result of Con-x minus Pro-x (or vice versa); and these two need to be compared.

If, following such comparison, it is found that Ky "is narrower" (meaning, presumably, overall less persuasive) than Kx, then – according to Duarte – the conclusion (5) may be taken to be "more justified" than the inference (2a). This calculus seems to be what is floating vaguely in Duarte's pedestrian mind. The thought intended is far from clear; but even if the argument can be clarified and formulated in a more scientific manner, does the putative conclusion that this inference is logically "stronger" than that one really follow? Is there here any basis for Duarte's claim that "the conclusion that he would refuse whisky is also stronger, in a sense—harder to defeat—than the conclusion that he would refuse cider." Surely not.

When dealing with logical inference, however short or long the argument(s) involved, an established principle is that the conclusion can never be more reliable than the premises[48]. In deduction, the conclusion's reliability is the same as that of the least reliable premise, if not less. In induction, the outcome is necessarily less reliable, since a hypothesis is always involved in generalization or adduction. Of course, propositions arrived at by induction can over time increase in reliability, due to more and more evidence (compared to before) being brought to bear on their behalf; but such propositions always remain less reliable compared to the reliability of the evidence (small or large) supporting them.

Of course, if the premises of two distinct arguments have different levels of cogency, whether due to one being inductive and the other deductive, or due to their having premises with different degrees of reliability, then their respective conclusions will also do so, and we can say that one is "stronger" than the other. But this is not the case here. Remember that (2a) is originally deduced by positive apodosis from (1) and (2), while (5) is deduced in the same way from (1) and (4), the latter being deduced from (2) and (3) by quantitative comparison. This means that (2a) and (5) have the premises (1) and (2) *in common*, and are distinguished only by premise (3) which (2a) lacks but (5) has.

---

[48] This is an application of what I have called the principle of deduction, or the fifth law of thought. Simply put, it says that we cannot get more information or certainty out of a deductive argument than we put into it. A more extreme statement would be 'garbage in, garbage out': if our premises are uninformative or weak, we cannot expect our deductive conclusions to be more informative or stronger. This law of logic can be compared to the law of physics called the second law of thermodynamics, which predicts that disorder (entropy) in the material world is bound to increase, overall.

Therefore, since all inferential processes involved were deductive (i.e. equally valid, 100% sure), the only possible logical difference that can be postulated in the hope of epistemic gain is that (3) somehow adds some cogency to (5) that is not added to (2a). But this is contrary to reason: an additional premise can diminish cogency (if it carries some inductive doubt) or leave it unaffected (if it is absolutely sure), but it can never increase cogency! No scenario, however complex, can conceivably bypass or mollify this elementary principle of logic. It can safely be predicted, since (2a) *requires for its deduction less information* than (5) does, that (2a) will always be either more certain than or at least as certain as (5), never less.

Clearly, Duarte managed to confuse himself, in his vain pursuit of something logically impossible. The above calculus of "reasons" concerns, rather than logic (as he supposes), psychology and ethics. Its content is not epistemic, but ontical and teleological. Such a calculus would aim to determine what course of action the subject should prefer in given circumstances. We might, of course, construct an a fortiori argument using a comparative major premise like: "the reasons relating to x (P) are more persuasive (R) than the reasons relating to y (Q); the reasons relating to y (Q) are persuasive (R) enough to encourage this man to do so and so (S); therefore, the reasons relating to x (P) are persuasive (R) enough to encourage this man to do so and so (S)." This would be a valid a fortiori argument involving the sort of *terms* that Duarte seems to have in mind. But is this sort of a fortiori argument (or maybe another mood) involved in the present case? I do not see it.

Or we might attempt to concoct an a crescendo argument, in which the pro rata additional premise concerns epistemic status[49]. Maybe: "Inference P is based on more reasons (R) than inference Q, and, Q is based on enough reasons (R) to be reliable (S); therefore, P is based on enough reasons (R) to be reliable (S). Given, moreover, that reliability (S) varies in proportion to number of reasons (R), then, since the reasons for P are more numerous than those for Q, the reliability of P is greater than that of Q." But of course, this is made-to-order, circular reasoning – in fact, the minor premise (and therefore the conclusion), relating reliability (S) to number of reasons (R), would be impossible to establish in practice.

It should be emphasized that the use of the word "reasons" in this context is very misleading. As we have seen, it does not here refer, as Duarte evidently thinks it does and wants us to think it does, to items with *objective epistemic significance* in the a fortiori argument at hand, but merely to items perceived by the subject (the man under discussion) as *more or less subjectively significant to his personal decision* to drink or not drink. These are two very different senses of the word "reasons," and Duarte is quite deluded, and evidently trying to also delude us, by this banal *equivocation*.

The term "reasons," in its primary, epistemological sense, refers to *empirical and/or logical grounds to believe a factual or theoretical claim*: the observations which inductively strengthen or weaken a thesis are "reasons" for believing or disbelieving it; the fact that the thesis is deductively consistent with (if not derived from) one's wider knowledge context is a "reason" in its favor, while the fact that it is self-contradictory or contradictory to some observation or to one's current scientific beliefs is a "reason" against it. The "reasons" that Duarte here invokes are rational only in the secondary sense used in practical philosophy: here the term refers to values

---

[49] See AFL 2.3 and 4.2.

and motives, to *judgments that influence the subject's choices and will, towards or against some object or course of action*.

"Reasons" in the primary sense refers to *de dicto* causes of a doctrinal conclusion, whereas in the secondary sense it refers to *de re* causes of a physical or mental outcome. The former refers to procedural issues: it concerns knowing, the knowledge-acquisition process; the latter refers to resulting substantive issues: it concerns being, having, doing. These are quite distinct cognitive phenomena, albeit use of the same word for both. Duarte's attempt to prove epistemic gain conflates these two senses of the term "reasons," whether out of ignorance and unintelligence or out of dishonesty.

Note also Duarte's use of ambiguous relational terms like "range of reasons," "capable of countervailing," "in favour of," "narrower," which show how vaguely he conceives even the calculus of "reasons" that his thesis depends on. What is a "range of reasons"? Does this simply mean a set of reasons, or is some sort of more complex quantitative comparison between the reasons intended? In what sense is one such range of reasons "narrower" than another? Is it simply an issue of the number of reasons, or are the reasons to be weighted somehow? What does "countervailing… in favor of" mean? Does this refer to a mere quantitative superiority, or to a causal relation of some sort? These unasked and unanswered questions testify to the fuzziness of Duarte's thinking processes.

In conclusion, then, the scenarios projected by Duarte in defense of his epistemic profit hypothesis do not logically support it. He thinks that by spinning a sufficiently complicated tale, he can make us believe in his punch line – but the whole thing is manifestly a figment of his imagination. If we carefully dissect each element of his thinking, we can clearly see that none of his attempts work. And indeed, to repeat, it is inconceivable that any attempt would work.

Duarte's attempt at proof is nothing less than comical. It consists in adding on a premise (6) to his earlier scheme (comprising propositions (1) to (5), thanks to which a new conclusion (7) can be drawn (pp. 219-20):

> "(6) For any two beverages x and y, if both x and y meet T, and if y ranks higher on the scale of alcohol content than x, then the range of reasons capable of countervailing the reason(s)-in-favour-of-our-friend-refusing-y given by the fact that y-has-the-degree-of-alcohol-content-that-it-does is narrower than the range of reasons capable of countervailing the reason(s)-in favour- of-our-friend-refusing-x given by the fact that x-has-the-degree of- alcohol-content-that-it-does.
>
> Therefore … (7) The range of reasons capable of countervailing the reason(s)-in-favour-of our-friend-refusing-whisky given by the fact that whisky-has-the-degree of-alcohol-content-that-it-does is narrower than the range of reasons capable of countervailing the reason(s)-in-favour-of-our-friend-refusing-cider given by the fact that cider-has-the-degree-of-alcohol-content-that-it does."

I do not want to ridicule Duarte (not too much, anyway), but surely the above formulation can be characterized as very vaguely put and quite confusing! This is like talking with one's mouth full. He is lumping too many things together into a single proposition. No one reasons that way, with propositions as convoluted and opaque, for the simple reason that such cognitive behavior is bound to result in errors. He resorts to multiple hyphenation because he does not know how to

articulate his thought with clarity and precision: how to cut it up into smaller units and then bind them together logically.

He tries to defend his presentation by saying: "despite its complex-looking formulation [it] is stating a simple point." His purpose is stated explicitly as: "to establish not merely the conclusion that our friend would refuse whisky—that is the claim in (5)—but also the further claim that that conclusion is 'all the more' justified, or that it follows 'even more strongly' from the relevant premises." But his means to that end are far from explicit and far from credible. He certainly does not manage to demonstrate that inference (5) can conceivably be epistemically "stronger" than inference (2a).

His claim of epistemic gain is too roughly conceived. He does not manage to fully concretize what he has in mind and effectively convince others. A lesson one quickly learns when engaged in formal logic research is that something that seems conceivable or reasonable at first glance may turn out, upon much closer scrutiny, to be inconceivable or unreasonable. Experience teaches that the mere fact that one has a thought, does not guarantee that the thought is worth something. One has to keep digging into the thought until it is absolutely clear and certain before granting it cogency. One should not be fooled by tantalizing appearances. One should avoid vague or approximate thinking. It is no use hoping that no one will notice the deficiencies and one will get away with it. Someone is sure to eventually see through an incompetent attempt.

Duarte does offer a validation process of sorts for his argument (by means of successive intermediary inferences: "Therefore (from (1) and (2))" … "Therefore (from (2) and (3))" … "Therefore (from (1) and (4))" … "Therefore (from (2), (3)—which together imply (4)—and (6))). Note that logically missing here, in between (6) and (7), is a statement that cider and whisky are beverages, and therefore qualify as x and y; but this is not so important. What matters is that Duarte *nowhere actually validates* the idea of epistemic gain which is his actual thesis. All he validates, at best (in truth, as shown below, not even that), is the outcome of the calculus of so-called "reasons" that he vaguely projects – the rest is his gross *misinterpretation* of what that means or implies. Therefore, Duarte's validation process is not logic but crass sophistry.

It should be seen and stressed that the addons (6) and (7) constitute *a setup*. That is, Duarte has deliberately designed the additional premise (6) in such a way that conclusion (7) is bound to follow when the variables x and y are replaced with the terms cider and whisky. There is no additional reasoning involved; no reasoning that is tied to the preceding a fortiori argument (i.e. to propositions (1) to (5)) and yet amplifies it in a significant way. Premise (6) is a stand-alone thesis with variables x and y, and (7) is an application of this same stand-alone thesis with the specific terms cider and whisky. The only reasoning involved is simply, then, applying a given general rule to particular terms given as subsumed under it by the clause "if both x and y meet T, and if y ranks higher on the scale of alcohol content than x, then…."

To be sure, the antecedent of (6) does contain elements of the preceding a fortiori argument, namely (2) "x meets T," and (3) "y ranks higher than x on the scale of alcohol content," and their joint implication (4) "y meets T;" but the antecedent of (6) does not contain premise (1) and therefore does not imply conclusion (5). The consequent of (6) also contains indirect references to the a fortiori argument, implying that the subject may refuse to drink y ("our friend refusing y") and that y has some alcohol content ("y has the degree of alcohol content that it does"); this also makes it look connected to the a fortiori argument, since the latter has similar implications from (5) ("Our friend will refuse y") and from (3) ("Whisky ranks … on the scale of alcohol content").

Nevertheless, despite these partial references to the a fortiori argument, proposition (6) cannot be said to have any significant logical tie to it. The crucial elements about two "ranges of reasons," with one "narrower than" the other – these are totally without connection to the a fortiori argument. They are just his say-so. The only link they have is to be sought in Duarte's imagination and his will that they be linked. He obviously thinks that his vague "reasons" calculus is relevant, but he does not in any way demonstrate this claim. Making a claim does not constitute proof that the claim is justified.

It follows that the if–then proposition in (6) is a sham; it is custom-made to rig the desired result. It is, obviously, retro-engineered from (7), so as to create the illusion that (7) is a conclusion, a thesis with proof. But (7) is clearly a fabrication, a thesis without proof. The antecedent clause in (6) is inserted to point us to the preceding a fortiori argument, to make it seem relevant to the present attempted expansion (i.e. propositions (6) and (7)), but the consequent clause in (6), viz. "then the range of reasons [etc.]," bears no logical relation to it – it is just, to repeat, his arbitrary say-so. The added premise (6) injects Duarte's vaguely thought out "reasons" calculus out of nowhere; it is not demonstrated in any serious manner. Therefore, (7) is *a foregone conclusion*; i.e. not the conclusion of an argumentative process, but a manipulated and quite spurious result. It begs the question.

In other words, ***one could put any thesis one wants to in its place, and similarly obtain the programmed result!*** E.g. one could equally well say "is wider" instead of "is narrower" in propositions (6) and (7). Or even: (6) For any two beverages x and y, if both x and y meet T, and if y ranks higher on the scale of alcohol content than x, then Mickey Mouse's addiction to y makes him fatter than Donald Duck's addiction to x; therefore, (7) Mickey Mouse's addiction to whisky makes him fatter than Donald Duck's addiction to cider. This shows how utterly contrived and laughable Duarte's argument is. Anything goes, since the hypothetical in premise (6) is a *non-sequitur*, i.e. since its antecedent and consequent are not logically connected.

If propositions (6) and (7) are in this way carefully deconstructed, it becomes evident that they cannot be claimed to prove the main thesis that conclusion (5) is, or even just can be, literally stronger than (2a), even if we accepted his claim that the "reasons" involved are of epistemic significance, which (for reasons already put forward) we of course do not and cannot accept. The use of long hyphenated terms in his additional propositions is not the main problem, note well; even with simpler and clearer terms, these propositions would be useless and misleading. The main problem is their logical disconnection from the a fortiori argument (i.e. (1) to (5)). Duarte's development of the idea of epistemic gain is manifestly riddled with fallacies, amazingly numerous fallacies.

Nevertheless, the long, hyphenated terms do play an important *psychological* role in making Duarte's arguments pass, in that people reading propositions (6) and (7) get mentally caught up in trying to make sense of those terms, and finding them too complicated soon give up trying to understand them, and then naïvely assume that the propositions (6) and (7) containing them must mean something intelligent and must be logically credible, since their author believed in them and the editors and publishers gave them a pass. I do not suppose that Duarte consciously willed to deceive; but I do think that he was at least subconsciously willing to deceive. Such intellectual negligence cannot be entirely innocent; the author must have been at some level aware of his deceit, but he assumed he would get away with it.

It should be mentioned that Duarte does not consider that a fortiori argument is always intended by speakers to entail an epistemic gain. As he puts it: "the claim that the conclusion about the target of an a fortiori inference is 'more strongly' supported than the parallel conclusion about its

source, appears to be a claim only typically rather than necessarily made by whoever gives an a fortiori argument"[50]. The difference, he claims, is due to "elements that are characteristic but not essential components of a fortiori inferences" (p. 222). What are these elements? Apparently, the caveat "and there are no defeating considerations" inserted in his premises (1) and (4), and the additional premise (6) (p. 223).

This reflection arises when Duarte belatedly becomes aware of a contradiction in his scenario, between the initial given that the man wants to avoid overly alcoholic drinks and his later supposition that the man might accept such drinks anyway, and tries to fix the problem by rewriting his initial premise in conditional form, with the proviso that "there are no defeating considerations" (p. 221). This takes him into a discussion of "defeasible conditionals," and he affirms the need for "a rider of some sort specifying that no exceptions are present" if we are "to draw deductively valid inferences" (p. 222). But this assessment by Duarte is inappropriate for two reasons.

First, as I explained earlier, when the man is actually influenced by his son's request to taste his wine, the a fortiori argument is in fact nullified, and instead what is operative is the given proposition that he will accept to drink. This is true both in the simpler scenario where the man's fear is any headache pain and in the more complex scenario where the man's fear is unbearable headache pain, as we have seen. So, the a fortiori argument as such cannot be claimed to contain a modifying caveat within it – the caveat is logically outside it[51]. An a fortiori argument properly formulated always has deductive force; it cannot rightly be qualified as "defeasible," assuming its premises are true. The appeal by Duarte to the notion of "defeasible" arguments is merely an attempt to gloss over the inadequacies in his project by pseudo-intellectual double-talk.

Second, one cannot in formal logic research construct an argument using such vague riders as "and there are no defeating considerations" or "all other things being equal." That is an admission from the start that one has not managed to formulate a complete argument capable of formal validation. It is inexactitude and uncertainty masquerading as precision and decision. The situations that Duarte describes hypothetically can be assimilated under formal logic, but only in the way I have detailed earlier on, in my preparatory remarks. His attempt to do it in another way just reveals his limited understanding of the issues at hand.

Despite Duarte's above moderating remark about epistemic gain being typical but not necessary, which suggests his addons are in practice applicable to most but not all concrete cases, my contention remains in force that epistemic gain is logically inapplicable, ever, no matter what he or other people imagine. Note also that he not only fails to properly formulate and validate his

---

[50] This claim by Duarte, viz. that a fortiori argument is not always intended to entail epistemic gain, creates an ambiguity. Is he saying that his alleged "reasons" calculus is (a) always applicable but not always applied? Or is he saying that it is (b) not always applicable? If he opts for (a), why does he not say so openly, and instead refers to customary behavior? If he opts for (b), he should clarify under what precise conditions it is not applicable. In either case, the reference to unspecified "defeating considerations" does not resolve this issue. Presumably, if such considerations come into play, they defeat not only the a fortiori argument itself, but also the reasons calculus applied to it. But can the reasons calculus be defeated independently of the a fortiori argument? This is a question that Duarte leaves unanswered.

[51] Duarte does, on p. 222, observe that some people would raise this objection, and he admits that "there is some truth to this view;" but he opts (without here giving precise reasons) to ignore the objection and to insert the said vague caveat into his argument anyway. I explain in the next paragraph why this is a bad choice.

thesis, but he tries to defend it by mendacious means. He imagines that it suffices for him to say the following, to make his proposal credible (p. 220):

> "I think this third inferential step, or something like it, is what we need to bring out in order to make sense of the idea that the conclusion that is drawn in an a fortiori argument about the target of the inference is a conclusion that follows 'even more so' from the relevant premises than does the parallel conclusion about the source."

Notice the escape hatch: "or something like it" – meaning, he knows deep inside that his introduction of propositions (6) and (7) is vague and incredible, but he hopes somebody else will come up with a more precise and convincing proposal in support of his charade.

Thus far, Duarte has functioned on a *largely informal* level, developing his thesis in terms of the cider-and-whisky example (plus some symbols, namely T, x and y). But further on, he tries to get *more formal*, rewording the above-mentioned two additional propositions in more general terms (with added symbols P and Q), as follows (p. 223):

> "(6) For every x and every y, if both x and y meet T, and if y ranks higher than x on the scale of P, then the range of reasons capable of countervailing the reason(s)-in-favour-of-y-being-Q given by the fact that y-has-P-to the- degree-that-it-does is narrower than the range of reasons capable of countervailing the reason(s)-in-favour-of-x-being-Q given by the fact that x-has-P-to-the-degree-that-it-does.
>
> Therefore… (7) The range of reasons capable of countervailing the reason(s)-in-favour-of b-being-Q given by the fact-that-b-has-P-to-the-degree-that-it-does is narrower than the range of reasons capable of countervailing the reason(s)- in-favour-of-a-being-Q given by the fact that a-has-P-to-the-degree that-it-does."

What this alleged "formalization" does, practically, is *further conceal* the non-epistemic real meaning of the term "reasons" used in the initial scenario. In the more informal presentation, the real meaning is more discernible, because we still have in mind the narrative on which it is based, i.e. the story of a man in relation to alcoholic drinks. But in this relatively formal presentation, there is no longer a background narrative that reveals the actual meaning of the term. Effectively, and without any logical warrant, Duarte has generalized his presentation from non-epistemic "reasons" to epistemic ones. This extrapolation is fallacious because these two sets of "reasons" are fundamentally different, as we have seen.

Note in passing that he further on (on pp. 223-4) develops by analogy a similar set of addons for his second form of a fortiori argument, i.e. for the form concerned with "not meeting" the threshold. This, needless to say, is as much a faux argument as the previous, positive form.

In his summing up, Duarte pleads (p. 233):

> "My goal in this section has been to illustrate the illuminating power of the argument schemes articulated and explained in the previous sections; and to deflate the possible objection that to engage with those schemes is to bring in a degree of complexity that hinders rather than helps our understanding and assessment of real instances of a fortiori arguments as they are

> deployed in judicial decisions. I hope to have shown that the schemes are not merely very helpful, but indeed necessary for the proper analysis of such real arguments."

I would retort, without malicious intent, that his approach to a fortiori argument is indeed more befuddling than necessary. My standard forms are much simpler and more accurate representations of a fortiori argument. Why use a bad imitation when you can use the real thing? Why travel with a sick horse when a healthy one is available?

I earlier contended, on the basis of comparisons of Duarte's formulas for a fortiori argument to my earlier findings, that his treatment was entirely and exclusively derived from mine, even if in an imperfect manner. It could not be a fortuitous independent discovery, since he admits having access to my work. If anyone still had any doubts as to whether he independently developed propositions (1) to (5), it should now be clear to them, from the above analysis of propositions (6) and (7), that he lacked the logical understanding and skills to do so.

Evidently, though he can express thoughts he learned from others, he cannot formulate and validate thoughts of his own. The moment he tries to branch off on his own into unexplored territory, he gets woefully lost. His flaky treatment of the idea of epistemic profit is absolute proof of his logical incompetence. Duarte is indubitably not an original logician, not someone who has mastered the fine art of formal logic research. Certainly, he deserves some praise for showing interest in a fortiori logic and trying to break new ground; but trying is not the same as succeeding. He has, plainly, contributed exactly zilch to this field.

5. A Peter Keating performance

Duarte's overall performance reminds me of the character of Peter Keating in Ayn Rand's celebrated novel, *The Fountainhead*, which I read in my teens and found quite inspiring. This fictional character typifies the "second-hander," a person without much creative capacity of his own, who snitches ideas and products (in his case, in the field of architecture) from the genuine innovator, the "first-hander" (in the novel, embodied by Howard Roark). Keating is a second-rate architect, whose building plans are cut-and-paste jobs from the work of other architects, who nevertheless has much success and adulation from his peers, simply because his peers are just as lacking in vision and judgment as he is.

As we have seen, in his 2017 paper *Arguing a fortiori* Duarte shamelessly pretends to discover and expose two of the forms of a fortiori argument, even though all that he presents is already to be found explicitly in my 2013 work *A Fortiori Logic* (and in fact, even long before that, in my 1995 work *Judaic Logic*), and even while he reluctantly admits, in a mere footnote, being peripherally aware of my work. Throughout his essay, he comes on as an original and independent logic researcher, who slowly but surely works his way to new and important findings. He presents his work as a pioneering effort in the field of a fortiori logic, even though there is almost nothing new in it, and many errors and omissions besides. He never acknowledges his intellectual debts to my work.

In this way, Duarte steals and takes credit for the ideas and products emerging my many years of study, research and reflection on the subject. Even if he does not use the exact same wording and symbols, so as to avoid textual plagiarism, this still constitutes plagiarism of ideas. Possibly, what we have here is not someone engaged in conscious, systematic misappropriation, but someone who has skimmed through one or two of chapters of my work – which is very broad in scope,

detailed and thorough – and gleaned a few bits and pieces of it, comprehensible to his limited intelligence, then put them in wording he feels more comfortable with, and fantasied himself as their originator. Maybe it was just an ego-trip; but this is still reprehensible.

The one idea that Duarte did not snitch from me, but can claim full authorship for (for all I care), is his notion that the conclusion of an a fortiori argument is (or can be) literally 'stronger' epistemically than its premises (or more precisely, than an earlier side-effect of them). This notion, which I have labeled (for brevity's sake) 'epistemic gain', is contrary to a universal logical principle. Duarte blithely ignores this established principle and attempts to prove his position by introducing various complications into his a fortiori scheme. As I have shown in great detail (in the previous section), his attempt to demonstrate epistemic gain is filled with sophistry of various sorts, and therefore totally incredible. If proof was needed that Duarte was intellectually incapable of producing his two basic forms of a fortiori argument without reference to my work, his multi-fallacious attempt to demonstrate epistemic gain provides that proof.

Let me make clear: I consider that there is nothing wrong with passing on the findings of other people. This is what I write for – to increase and spread knowledge. I want the knowledge passed on, and make it freely available. My motive is entirely benevolent – I am not in this business for fame or fortune. It is out of sincere compassion for the surprising ignorance of many people in the fields of logic and philosophy that I have devoted many years of my life to research and writing on these subjects, and a lot of my money and time propagating my findings. But I insist on being correctly cited, so as to ensure that people get acquainted with the original doctrine, rather than an ersatz derivative of it. What I resent here is not Duarte using my work, but his glaring failure to mention that it is mine. I am not flattered by his imitation, because it is done in a dishonest and disrespectful manner.

All he needed to do was to clearly say, at the very start of his essay, something like: "I am indebted to Avi Sion's work on the formalization of a fortiori argument, which inspired me to apply it to the contemporary legal field." This is what honest people do. This is the honorable thing to do. In that case, I would simply have examined, and criticized or praised, his work with regard to accuracy and completeness, or eventual truly new ideas. But he chose the ignoble path, the Peter Keating path. It appears that he thought he would get away with it, because my work is self-published. This is suggested by his strange statement in footnote 8: "In this striking (and self-published) book, which includes, among other things, a minute study of the topic, Sion…." Why "and self-published," I ask?

It seems that Duarte is one of those people who think that if a book is "self-published" – no matter how "striking" it is and how "minute" a study it constitutes, it has no social or legal standing. To him, self-published implies '*not given a stamp of approval by some authority*' and therefore '*not really existing, not needing to be duly acknowledged*'. Although an alleged legal expert, he evidently thinks that a self-published author is one from whom one can freely 'borrow' without earning public disapproval or breaching copyright laws. This is, of course, an attitude typical of the second-hander. Duarte evidently thinks that the reliability and authority of a book comes not from its inner credibility and value, but from the approval of some anonymous 'reviewers' hired by some known 'editor' or 'publisher'[52].

---

[52] For more on this topic, see my essay 'Self-publishing and Other-publishing', published as chapter 6 in the present volume.

But to my mind, "peer review" is a massive scam: if the "peers" themselves happen to know no better, or be no more intelligent, how can they spot his sources or errors? If they can be fooled by fake work, of what use are they? That is why I have generally avoided resort to 'other-publishing'. How can people who happen to know less, or to be less intelligent, judge the work of people who in fact know more?

In the present case, if the reviewers hired by The Modern Law Review were knowledgeable and skillful in the field of a fortiori logic, they would surely have spotted Duarte's dishonesty, errors and omissions, and sophistry. Since they did not, but okayed publication of the article *Arguing a fortiori* as is, we must infer that they were not knowledgeable or skillful, but themselves fakes putting on an act. Yet, they were trusted and charged with the task of review and selection by The Modern Law Review's editors and publishers! Over the years, I have found this to be often the case, that the blind lead the blind.

At the foot of Duarte's paper's first page, he grandiloquently writes: "For helpful comments and discussion, I am grateful to…," listing the names of eight people (not me among them), plus "the Edinburgh Legal Theory Research Group," "audiences in Oxford and Lisbon," and "the two anonymous reviewers for The Modern Law Review." From this we are supposed to infer his great humility and openness, and at the same time the stamp of approval of many presumably prestigious persons and institutions. Again, this is typical second-hander behavior. Such people live in a bubble, divorced from reality.

To further emphasize his independence and originality, Duarte belittles my work by mentioning it only late in his essay, and only briefly and in passing in a mere footnote, in the way of an afterthought of minor significance. And even then, as we have seen, he has the gall to engage in fake criticism and in condescension of my work, which only serve to highlight the limits of his logical acumen. It is worth noting that he never once wrote to me with humble questions or even arrogant criticisms, before publication; and he never announced the completion and publishing of his essay to me (someone whom he does, after all, mention and criticize). All his writing and criticism was done surreptitiously, behind my back, as is to be expected of someone engaged in intellectual theft and calumny.

I am not, of course, saying that there should be no criticism. Criticism is, of course, welcome – indeed, it is *the condition precedent* of intellectual progress. But criticism must be aimed at real technical or doctrinal faults, and be the outcome of more intense research and insight than that which is being criticized. To fabricate imaginary faults, to engage in spin and innuendo, so as to seem superior – that is what is reprehensible. The true intellectual has a solemn respect for reality; for him or her, thought is not an instrument of manipulation.

Throughout Duarte's essay, we find the use of language suggesting new discovery by him. At the very start, in his Abstract and again in his Introduction, he claims that a fortiori arguments are till now "not well understood" and "have not drawn much attention," and he postures as the one who will "make some progress" and "bring out the form" of such arguments:

> "But how exactly are they distinct, and why are they important? That is less clear. Despite their popularity, a fortiori arguments are not well understood and have not drawn much attention from legal theorists. I try in this paper to make some progress on the topic. I will be pursuing two goals. The first is to bring out the form of a fortiori arguments, articulating those assumptions that, though typically left unstated, are necessary elements of arguments of this kind" (p. 202).

This is *imposture*, since there is already a published work (my AFL), some 700 A4 pages long, treating the subject of a fortiori argument in painstaking detail, both on the formal level and in a historical-critical perspective (including scholarly account and assessment of some 30 theories on the subject). He knows about it, even if he has obviously not read it all, but maybe only a few pages here and there of it; and as we have seen, he heavily draws from it, even if incompetently; yet, he does not duly acknowledge it, so as to project a false image of his work as needed and innovative.

I can cite many more phrases or sentences designed to project an image of innovation in process: "So let us start by trying to identify its elements and structure;" "In order to begin to make sense of the argument" (p. 204). "Now in identifying this assumption… we have singled out what I will call a 'scalar' property: a property… that something can have either more or less of. And we have also made clear that there is a relevant threshold in the scale;" "What seems to be going on in the argument is that…;" "Here is a first attempt at reconstructing…;" "This will need to be refined, but it puts us on the right path…" (p. 205). "I said that our reconstructive work is not yet finished—there may be more to the … argument than we have uncovered so far;" "we can try to begin to isolate the form of this argument: the common form, that is, of arguments like this. What should we say? Here is a first, half-way attempt" (p. 206). "But as I said, we have more to uncover" (p.208). "Our discussion so far has revealed three simple but important features of the a fortiori" (p. 209). "a means of testing whether my proposed schemes do actually capture the arguments" (p. 212).

And to top it all, in his conclusion (p. 237), Duarte boasts, misleadingly:

> "… very little scholarly attention has been paid so far to inferences of this kind, which have remained considerably obscure to legal practitioners and theorists alike. This paper was an attempt to remedy this state of affairs. I have sought to bring out the distinctive form of a fortiori inferences, and to show how an awareness of their structural features can assist us in assessing real instances of the argument for both logical validity and substantive soundness."

The claims that there has been "very little scholarly attention paid so far" to a fortiori arguments, and that they have "remained considerably obscure to … theorists," is of course a barefaced lie. It is intended to suggest that Duarte has broken new theoretical ground. But as we have seen, he has not. It is also intended to suggest that he is a scholar, capable of telling scholarship from its absence. But his behavior throughout his essay has demonstrated that he is the very antithesis of a scholar; he has no idea what real scholarship is: his appropriation of someone else's ideas (mine) without due acknowledgment; his failure to fully read and assimilate a major work (AFL) before he tries to criticize it; his use of phony criticism to keep readers at bay from the source of his ideas – these are all so much evidence of his lack of scholarship.

As already mentioned, in AFL I analyze in excruciating detail, sentence by sentence, word by word, the work of just about everyone who has written anything about a fortiori argument: now, *that* is scholarship. And it is not very little: some of these analyses are longer than the texts they analyze, and took months of daily, conscientious work to achieve. The present essay regarding Duarte's alleged contributions is written with the same conscientious attention to detail, and is longer than the paper it assesses. Clearly, Duarte is not merely ignorant, but dishonest. He did not carefully study the literature before writing his piece, yet pretends to know it.

In all fairness, Duarte's paper is not badly written. He develops the subject in a manner that shows some potential, if he only took the trouble to study more and paid more attention to personal ethics. As a teacher myself, I clearly see all the things about a fortiori argument that he has learned from me, and feel some satisfaction that he has. What is enervating, is his pretentiousness, and of course all the errors, omissions and sophistry in his work that we have here pointed out.

I speak here especially of the logical aspects of his essay; I am not so much concerned with the legal applications or principles that he delves into. I make no effort to assess his knowledge and understanding of contemporary law. My only concern here, really, is with issues of formal logic. Duarte could very well have acknowledged my formalizations of a fortiori argument and gone on from there into his analysis of legal discourse to his heart's content. What excites my ire, to repeat, is that he effectively lays illegitimate claim to my prior formal findings.

Some readers might rightly wonder why I have wasted so much of my time debunking the work of a very minor player in the field of logic and philosophy. I have asked myself the same question: if I regard this guy as rather ignorant and unintelligent, why bother with him at all? Surely, I would do better to pass my time criticizing Kant or Wittgenstein, and other famous writers who have done much greater harm to logic and philosophy. There are hundreds if not thousands of second-rate (or third- or fourth- or fifth- rate) writers and lecturers like Duarte out there, saying all sorts of stupid things. It is impossible to criticize them all in full detail as would be ideal.

Frankly, I do not care two hoots what this particular specimen says or fails to say. Nevertheless, since I have done very important new work in the field of a fortiori logic, I consider myself its guardian. I regard it as my duty, so long as I am alive and well, to protect this specific field from incompetent intruders. Duarte's attempt to have an impact on the field needed to be assessed. The fact that this assessment exposed his many failings does not diminish the value of the assessment. We can learn from the faults of others as well as from their good work. We can learn much logic by observing the shenanigans of sophists.

Allow me now some social commentary, here. Dishonorable behavior like that displayed by Duarte appears to be rather common nowadays in some academic circles. It seems that rectitude is not highly prized in that milieu, or its absence sufficiently looked down on. There is much pressure to produce, but the low quality of the product is not considered too problematic. The trouble with this cultural context is that it does a great disservice to science. Serious work is disparaged by people who do not know what they are talking about; and at the end of the day, it becomes difficult for third parties to tell the wheat from the chaff. It would be wiser for such people to keep quiet than to speak nonsense and muddy the waters. Egged on by conceit, desiring to be noticed and admired, they just make fools of themselves by displaying their ignorance in public, and they mislead many people. And few people ultimately care: the stamp of approval of some publisher or institution or celebrity suffices to convince them.

I also want to take this case as an example, and draw attention to the sorry state of academia and academic publishing, at least in the realm of philosophy and logic (though, I gather, it is true in other fields too). It is evident that Duarte's faculty friends or colleagues (those he mentions by name, at least), as well as the two *anonymous* Modern Law Review reviewers, were in fact all lacking in the logical knowledge and skills needed to properly judge the article in question and tell him his mistakes. *We do not know what their qualifications and past intellectual achievements were, yet they are effectively treated and presented as authorities*. And the sad thing is that this is not exceptional. The 'prestige' of the institution and some of staff involved

does not make reviewers immune to ignorance and incompetence. Their approval is evidently, often enough, in reality, of little or even without objective value. Position does not guarantee knowledge or skill – it only signifies position.

Writers of papers on logic and philosophy cannot produce good material if their motivation is merely personal ambition – the desire to stand out and impress their peers, and maybe get or keep certain academic positions or even just some financial benefits. Such motives can only produce 'fake news'. Logic is not a game, or an ego-trip, but a serious human endeavor, aimed at human welfare. Good reasoning sometimes saves or improves lives, and bad reasoning sometimes puts them in danger or affects them negatively. A fit argument in a court of law can produce justice, as an unfit one can produce injustice.

Logic and philosophy researchers need to be motivated by truth, idealistically and fanatically so. They must view scientific truth as *a sacred goal* of all research. They must view research as *a spiritual act*, an act that calls on their power of virtue. Only thus will they work carefully and relentlessly till they achieve truly credible results. There is no room in the field of logic for fake logicians, who manipulate or approximate logic theory to make believe they are authentic logicians. Logic is a scientific discipline, not a playing field or a theatrical stage.

In conclusion, I would like to reflect on the usually deafening silence, of authors on papers relating to a fortiori argument, in reaction to my detailed critiques of their work. Usually, they do not respond; and when they do, it is only to indignantly deny any intellectual wrongdoing. None of them write back to me saying: 'Oh, thanks, I am really grateful that you took the trouble to review my work in so much detail and pointed out its flaws to me!' As the *Dhammapada* (v. 76) puts it, in a more spiritual context:

> "*Look upon the man who tells thee thy faults as if he told thee of a hidden treasure.*"

And this ancient work of wisdom adds that the man who tells people their faults "will be loved by those who are good and hated by those who are not" (§77). It seems to me that a sincere searcher after truth would always rejoice at being corrected. Therefore, I conclude that those who do not show gratitude – either sullenly refusing to reply to criticism or being loudly in denial, no matter what evidence is put before them – such people cannot be truly scientific researchers. I wait to see how Duarte reacts to the present scathing critique of his article.

Prescript and postscript

When I came across Duarte's essay a few months ago, I had the following e-mail exchange with him (it was in March 2017). Having now completed my analysis of it, I am glad to say that my initial reactions, expressed my e-mails, were quite appropriate.

- From Avi Sion: "Mr D'Almeida. I am the author of A Fortiori Logic. I have just found your article Arguing A Fortiori on the Internet, and briefly perused it. I must tell you my first reaction – it is indignation. Indignation that you display so little respect for my work, which I suspect you have read only a chapter or two of, and that in in a cursory manner, while at the same time using a great deal of the terminology and ideas contained in it without due acknowledgment. You have the chutzpah, in a mere footnote, to call my formulations "too crude to do justice to his insights" and make other unfair remarks concerning them! Apart from the dishonesty involved, I can tell you that there [are] things you have not understood

and have treated incorrectly. I am too busy right now to write an article showing this up, but I hope to do it at a future date. And it will be self-published."

- From Luis Duarte D'Almeida: "Dear Mr Sion. Having read some of the texts on your website, I can't say I'm surprised by either the tone or the content of your email. I look forward to reading your article. Best wishes."
- From Avi Sion: "No shame on your part."
- From Luis Duarte D'Almeida: "Dear Mr Sion. Now I am surprised (at the unnecessary insult); and sorry you feel that way. I know that you have more than once felt that people doing work on a fortiori arguments have dishonestly relied on your work, or paid insufficient attention to it, or learned from it without quoting it; and that you express such views in strong terms on your website (and indeed in your book). But I meant what I said – I do look forward to reading your article. I'm imagining you will have had access to a pre-print pdf of my paper. I'm attaching the MLR file, which is the final publication (although the text is the same). Best wishes."
- From Avi Sion: "Hello Mr. Duarte. Do you not think that your calling my formalizations of afa, the work of years of careful study, "too crude" was insulting on your part? You can be sure that, when I write the article, I will be strictly fair, even so – as I have been strictly fair with others, even when angered. What I dislike is people who are debonair towards others' work in order to give an impression of superiority. I have noticed this to be a common trait nowadays, and have resolved not to tolerate it. That said, I appreciate your last e-mail. I do have your article, which I obtained through ReadCube."

After writing the above essay, I posted it (in Sept. 2017) in a blog, and informed Duarte of this as follows:

- From Avi Sion: "Hello, Mr. Duarte D'Almeida. I have completed my examination of your paper "Arguing a fortiori" published earlier this year in the Modern Law Review. Below are the first two paragraphs, which summarize my judgment. I hope you will have the patience and attention span needed to read it all carefully, and learn a thing or two from it. Knowing how your mind works perhaps better than you do, I know that you are rather lazy and not very bright. But I ask you to make an extra effort in this instance."

I simultaneously sent copies of this e-mail to the journal concerned and to his university. I was not surprised that Duarte never replied, either to thank me or to defend his thesis. He did not, either, as a repentant man would have, retract his fake article and publicly apologize for it. Having already demonstrated his dishonesty and pretentiousness, I expected no better behavior from him. I also received no acknowledgment or comment from the journal, showing that they do not mind publishing plagiarized and invalid material! The university, too, did not display any concern regarding this matter.

© Copyright Avi Sion, 2018, 2019. All rights reserved.
Protected by international copyright conventions. No part of this book may be reproduced in any manner whatsoever, or stored in a retrieval system or transmitted, without express permission of the Author-publisher, except in case of brief quotations with due acknowledgement.

Exposing Fake Logic can be freely read online at avisionpolemics and in various other locations. It can be purchased, in print and e-book editions, in Amazon.com, Lulu.com and many other online booksellers.

Main references

The following are the main references made in the present work. There are, in fact, some more references scattered throughout the present volume, in footnotes. The selection of some under the heading of 'main references' is somewhat subjective. References made through Internet links, even if important are usually excluded here – as are references to works quoted rather incidentally or second-hand. Note that this listing is not intended as a bibliography, at least not as an exhaustive one.

Abraham, Michael. "The 'Kal Vachomer' as a Syllogism – Arithmetic Model" (in Hebrew). In: *Higayon: Studies in Rabbinic Logic* (vol. 2, pp. 29-46). Ed. Moshe Koppel and Ely Merzbach. Jerusalem: Aluma, 1992. Pp. 29-46.

Abraham, M., D. Gabbay & U. Schild.

Analysis of the Talmudic Argumentum A Fortiori Inference Rule (Kal-Vachomer) using Matrix Abduction. Studia Logica 92(3): 281-364 (2009).

"מידות הדרש ההגיוניות כאבני הבסיס להיסקים לא דדוקטיביים" (Hebrew paper on Kal-Vachomer). In *BDD Journal*. Ramat Gan: Bar Ilan University, 2009.

Aristotle. *The Works of Aristotle*. Ed. William David Ross. Chicago: Encyclopædia Britannica, 1952.

Cicero, Marcus Tullius. *Topica*. Trans. H. M. Hubble. Cambridge, Mass. Harvard UP, 1949.

Dhammapada, The. Juan Mascaro, trans. Harmondsworth, Mx.: Penguin, 1973.

Encyclopaedia Talmudit (vol. 7, in Hebrew). Jerusalem: Yad HaRav Herzog Torah Institute, 1990.

Everyman's Talmud. A. Cohen. London: Phoenix, 1949.

Luzzatto, R. Moshe Chaim. *The Book of Logic* (orig. *Sepher haHigayon*). Trans. Rabbis D. Sackton and Ch. Tscholkowski. Jerusalem: Feldheim, 1995.

Hyam Maccoby. *The Philosophy of the Talmud*. London: Routledge Curzon, 2002.

Plato. *The Dialogues of Plato*. Trans. Benjamin Jowett. 3rd ed. Oxford: Oxford UP, 1892.

Sion, Avi.

- *Judaic Logic: A Formal Analysis of Biblical, Talmudic and Rabbinic Logic*. Geneva: Slatkine, 1997 (1st ed. 1995).
- *A Fortiori Logic: Innovations, History and Assessments*. Geneva: CreateSpace, 2013.

Soncino Babylonian Talmud. Ed. Rabbi Dr. I. Epstein. London: Soncino, 1952.

Soncino Chumash: The Five Books of Moses with Haphtaroth. Ed. Cohen, Rev. Dr. A. 2nd ed. London: Soncino, 1983.

Windelband, Wilhelm. *History of Ancient Philosophy.* Trans. H. E. Cushman, from 2nd German ed. New York: Dover, 1956.