NATURALNESS AND CONVEX CLASS NOMINALISM

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ABSTRACT. In this paper I argue that the analysis of natural properties as convex subsets of a metric space in which the distances are degrees of dissimilarity is incompatible with both the definition of degree of dissimilarity as number of natural properties not in common and the definition of degree of dissimilarity as proportion of natural properties not in common, since in combination with either of these definitions it entails that every property is a natural property, which is absurd. I suggest it follows that we should think of the convex class analysis of natural properties as a variety of resemblance nominalism.

1.

According to a simple variety of class nominalism, the properties are all and only the sets of (possible) individuals (for discussion see, for example, Armstrong (1978b, pp. 28-42) and Lewis (1986, pp. 50-69)). The property of being white, for example, is simply the set of white things. But according to simple class nominalism *every* set of (possible) individuals, no matter how heterogenous, is a property. So simple class nominalism is incompatible

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with a simple analysis of similarity as sharing properties (Lewis 1983, p. 346; Lewis 1986, p. 59).[1]

A standard solution to this problem is to distinguish natural from unnatural properties, and argue that only the former are relevant to resemblance. As David Lewis, for example, writes "... properties do nothing to capture facts of resemblance ... It would be otherwise if we had not only the countless throng of all properties, but also an elite minority of special properties. Call these the *natural* properties ... Natural properties, would be the ones whose sharing makes for resemblance" (1983, pp. 346-7).

But which properties are the natural properties?

In this paper I consider an analysis proposed by Peter Gardenfors (2000, p. 71) and embraced by Graham Oddie (2005, pp. 152-8), according to

[1]Note that some sparse conceptions of properties are equally incompatible with the simple analysis of similarity as sharing properties. According to David Armstrong's theory of properties, for example, there are no disjunctive properties. So if being orange is a property and being red is a property, then being red or orange is not a property. Nevertheless, red or orange things do resemble each other. To avoid denying this, Armstrong argues for an analysis of similarity according to which "... a particular a resembles a particular b if and only if: There exists a property, P, such that a has P, and there exists a property, Q, such that b has Q and *either* $P = Q$ *or* P resembles Q" (1978, p. 96). This is supposed to avoid the problem because the property of being red is supposed to resemble the property of being orange. Since the analysis of natural properties discussed in this paper accepts that some disjunctive properties are natural, it might be thought to be able to avoid this problem, without having to appeal to resemblances between properties. But the results of this paper show this hope is vain.

which the natural properties are the convex subsets of a metric space in which the distances are degrees of dissimilarity. For a simple illustration consider the set of coloured particulars, with their distance given by their degree of dissimilarity with respect to hue. Then the natural hue properties, according to the analysis, are just the convex subsets of the colour circle.[2]

The convex class analysis of natural properties presupposes that degree of dissimilarity is distance in a metric space. But what justifies this presupposition? A simple answer would be a definition of degree of dissimilarity as a function of number of natural properties in common and not in common. The degree of dissimilarity between two particulars could be defined, for example, as their total number of natural properties not in common or, more plausibly, as their proportion of natural properties not in common (Blumson 2018b, pp. 11-4).[3]

[2]Apart from the explication of similarity, naturalness is supposed to have many other roles in philosophy (see especially Lewis 1983). But the various roles of naturalness are arguably in tension with each other, and it is doubtful whether a single notion can fulfill them all (for a detailed discussion of this issue, see Dorr and Hawthorne 2013). For this reason, I focus exclusively in this paper on the connections between naturalness and similarity.

[3]Degree of similarity is defined as number of (sparse or natural) properties in common by Gonzales Rordiguez-Pereyra (2002, p. 65) and as proportion of (sparse or natural) properties in common by Alex Oliver (1996, p. 55). These definitions are of course extremely simple. But simplicity is a virtue, and so if the convex class analysis of natural properties were compatible with these simple definitions, then that would be a point in

Gardenfors himself would not accept this simple answer. According to him, "Seeing properties ... as more primitive than similarity leads to grim problems. A fundamental task for such an approach is to determine exactly what counts as a property. ... I know of no theory of properties that furnishes a satisfactory solution to this problem. Consequently, I see no way of defining similarity as the number of shared properties" (2000, p. 111). But why isn't the convex class analysis of natural properties itself a satisfactory solution to this problem?

There are three reasons. The first reason is that if the number of (possible) particulars is infinite, then it follows from simple class nominalism that the number of properties any two distinct particulars have in common is infinite too. As Gardenfors writes, "... any two objects can be shown to share an infinite number of properties ... If it is the number of shared properties that determines the similarity of objects, then any two objects will be arbitrarily similar. Restricting the problem to natural properties ... does not help – there are still arbitrarily many" (2000, p. 111).

But whether distinguishing between natural and non-natural properties can avoid this problem is disputable. As David Armstrong, for example, writes "We should perhaps take quick and unfavourable notice of the view sometimes encountered that degrees of resemblance are quite arbitrary because with respect to any two things at all we can find an indefinite number

its favour. Unfortunately the convex class analysis, as I will argue below, is incompatible with both definitions.

of resemblances and an indefinite number of differences and that as a result, no two things are in themselves more, or less, alike than any other two. To take this view is really to go back to the view that there are no objective natural classes" (1989, p. 40).

In any case, I want to put this problem aside in this paper and focus on the finite case. Simple class nominalism entails that every two particulars have the same number of properties in common and not in common even in the finite case (Goodman 1972, pp. 443-4). Moreover, the presupposition that degree of dissimilarity is distance in a metric space has its own problems with infinity (Lewis 1973, p. 51; Williamson 1988, pp. 458-9; Blumson 2018a). I want to know whether the analyses work at least in the finite case, before considering problems with infinity.[4]

The second reason is that simultaneously analysing natural properties in terms of degree of dissimilarity, as the convex class analysis does, and degree of dissimilarity in terms of natural properties is obviously circular. As Gardenfors writes "There is ... a contrary view of the relationship between concepts and similarity. According to this position, two objects falling under different concepts are similar because they have many *properties* in common .... According to this view, concepts can be used to define similarity. Hence, we face an apparent circularity" (Gardenfors 2000, p. 111).

[4]For the infinite case see, for example, Rodriguez-Pereyra (2002, pp. 173-4), Paseau (2012, pp. 372-4), Paseau (2015, pp. 107-10), Blumson (2018b, pp. 14-6), Yi (2018, pp. 797, 802-3), and Blumson (n.d.).

But the fact that accepting both analyses would lead to a circularity does not show that the analyses are not both true, but merely that they do not both succeed as reductive analyses. Gardenfors, for example, could argue that both analyses are true, but that only the convex class analysis is reductive, by arguing that similarity is prior to naturalness. A philosopher of a different temperament could also argue that both analyses are true, but argue that only the definition of degree of dissimilarity as number of properties not in common is reductive, by arguing that naturalness is prior to similarity.

Likewise, a third philosopher could argue that both analyses are true, but neither analysis is reductive, by arguing that neither similarity nor naturalness is prior to the other. In this case, we would have reciprocal analyses, which would clarify the relationship between naturalness and similarity, without reducing either to anything more fundamental. So as long as both the convex class analysis and the definition of degree of dissimilarity were both true, the question of whether naturalness is prior to similarity or vice versa would be left completely open.

I will argue for a third, and more surprising, reason that a proponent of the convex class analysis of natural properties cannot accept the definition of degree of dissimilarity in terms of number of properties not in common – namely, that they are mutually incompatible. The underlying problem is that they entail, in combination with the convex class analysis of natural properties, that every subset, no matter how heterogeneous, is a natural property. It follows that everything is dissimilar to everything different to the same degree, which is absurd.[5]

So proponents of the convex class analysis of natural properties *must* reject the analyses of degree of dissimilarity as a function of number of properties in common and not in common. In this case, we have an analysis of naturalness in terms of degree of dissimilarity, but no analysis of degree of dissimilarity in terms of naturalness. If this order of analysis reflects the underlying order of metaphysical priority, it follows that the convex class analysis embodies a variety of resemblance nominalism, according to which similarity is prior to naturalness, rather than naturalness to similarity.

2.

The definition of natural properties as convex subsets presupposes that degree of dissimilarity is distance in a metric space, where:

[5]In a previous paper, I argued that the conceptions of dissimilarity in terms of number of properties not in common and in terms of distance in a metric space are "logically independent" but "philosophically inconsonant" (Blumson 2018b). But the claim that two conceptions are logically independent is not very surprising, and the claim that they are philosophically inconsonant is extremely nebulous. In contrast, the claim in this paper that the convex class analysis and the definition of degree of dissimilarity in terms of number or proportion of properties not in common entail that everything different is dissimilar to the same degree is exact and surprising.

Definition 1. A *metric space* is an ordered pair $\langle A, \delta \rangle$ of a set A and a distance function $\delta : A \times A \rightarrow \mathbb{R}$ from pairs to real numbers such that for all $a, b, c \in A$:

(1) $\delta(a, a) = 0$ (minimality)

(2) $\delta(a, b) > 0$ if $a \neq b$ (identity of indistants)

(3) $\delta(a, b) = \delta(b, a)$ (symmetry)

(4) $\delta(a, b) + \delta(b, c) \geq \delta(a, c)$ (the triangle inequality).

In the current application, the set A is the set of (possible) particulars, and the distance function δ takes each pair of (possible) particulars to their degree of dissimilarity, in such a way that (1) no particular is at all dissimilar to itself, (2) all distinct particulars are dissimilar, (3) a is as dissimilar to b as b is to a, and (4) the degree of dissimilarity between a and b plus the degree of dissimilarity between b and c is greater than or equal to the degree of dissimilarity between a and c.

As a simpler example, we could think of A as the set of coloured particulars, with their distance δ given by their degree of dissimilarity with respect to hue. The resulting space is the familiar colour circle, with the dissimilarity in hue between different particulars being represented by the distance between them around the circle. A little more realistically, we could think of the distance δ as degree of dissimilarity in respect of hue, brightness and saturation, in which case the resulting space is the familiar colour spindle (Gardenfors 2000, pp. 9-11).

In this example, the respect of dissimilarity concerned is subjective, since similarity between red and purple particulars, for example, depends on how they look to us. But we may also consider spaces in which the respect of dissimilarity concerned is objective. For example, A could be the visible lightwaves with the distance between them given by the difference in their wavelengths. The resulting space is a spectrum instead of a circle, with red and purple particulars at opposite ends, rather than side-by-side, as they are in the colour circle.

Because they are drawn mostly from psychology, nearly all Gardenfors' examples concern subjective respects of similarity. As he writes "In this book, I focus on cognitive phenomena. The question of whether things are inherently similar independent of any cognising subject will, in general, be irrelevant to my concerns. Realism is thus put within brackets" (2000, p. 110). Although whether similarity is subjective or objective is closer to my concerns, I too intend to stay neutral on this question here.

Note that interpreted as the thesis that all distinct particulars are dissimilar, condition (2) is closely related to the identity of indiscernibles, which is extremely controversial (Williamson 1988, p. 463). And interpreted as the thesis that all distinct particulars of a certain kind are dissimilar in a certain respect – for example, that all distinct coloured particulars are dissimilar in respect of hue – it is certainly false. In this case, condition (2) should be weakened to $\delta(a, b) \geq 0$ if $a \neq b$, and the resulting space is known as a pseudometric.

However, in the presence of the other conditions of a pseudometric, being zero distance apart or exactly alike (in a certain respect) is an equivalence relation, which divides A into equivalence classes of exactly alike particulars, and the function from each pair of equivalence classes to the distance between their elements defines a genuine metric space, so it is possible to treat particulars which are exactly similar (in some respect) as if they are identical (Suppes et al. 1989, p. 47). I return to this point in section (5).

3.

Two further definitions are required to understand the convex class analysis of natural properties. First, an element b is *between* elements a and c in a metric space $\langle A, \delta \rangle$ if and only if $\delta(a, b) + \delta(b, c) = \delta(a, c)$ – in other words, if and only if the distance from a to c via b is equal to the distance from point a directly to point c. Another way to say this – which will be helpful in sections (7) and (8) below – is that an element b is between elements a and c in the special case in which condition (4), the triangle inequality, is an exact equality.

Secondly, a subset $B \subset A$ of a metric space $\langle A, \delta \rangle$ is *convex* if and only if for all $a, c \in B$ and $b \in A$ such that b is between a and c, $b \in B$ – in other words, if and only if every point between points in the subset is in the subset. The set of red particulars, for example, is a convex subset of the colour circle, because every particular which is between two red particulars in respect of hue is itself a red particular.

The natural properties, according to Gardenfors and Oddie, are the convex subsets of the metric space consisting of the set of (possible) particulars, with the distances between them given by their degree of dissimilarity (Gardenfors 2000, p. 71; Oddie 2005, pp. 152-8). For a simple example, reconsider the colour circle or, in other words, the set of coloured particulars with distance given by their degree of dissimilarity in respect of hue. The natural properties of the colour circle, according to the analysis, are just its convex subsets.

The analysis counts the familiar hues – red, orange, yellow, green, blue and purple – as natural properties. Since any particular which is between two red particulars is itself red, for example, the set of red things is convex, and so the property of being red is a natural property. But the analysis also includes other determinable hues – since, any particular between two cyan particulars is itself cyan – as well as determinate shades – since any particular between two scarlet particulars, for example, is itself scarlet.

Two clarifications. First, whereas I am interpreting the analysis as requiring that convexity is both necessary and sufficient for naturalness, Gardenfors writes "It should be emphasised, however, that I only view the criterion as a *necessary* and perhaps not sufficient condition on a natural property" (2000, p. 76). I will ignore Gardenfors' caveat about this because I think that one of the main virtues of the convex class analysis is that it does try to give explicit necessary and sufficient conditions for naturalness – in other words, that it is an *analysis*.

Second, since convex classes are classes, the convex class analysis of natural properties is a sophisticated version of class nominalism.[6] But this does not preclude the convex class analysis from also being a variety of resemblance nominalism, according to which similarity is prior to naturalness, and so a class is natural *in virtue of* the resemblance of its members (compare Rodriguez-Pereyra 2002, pp. 56-62). I will argue in section (9) that the convex class analysis is committed to the priority of similarity over naturalness, and so is also a sophisticated version of resemblance nominalism.

4.

How well the analysis of natural properties as convex subsets captures the characterisation of natural properties as making for resemblance depends in part on the nature of the underlying metric space.[7] But there are three consequences of the analysis which don't depend on the nature

[6]It follows that the convex class analysis inherits some other well-known problems of class nominalism. For example, in order to distinguish between coextensive properties – whether natural or not – the classes in question must be classes of *possible* particulars (Armstrong 1978b, pp. 35-6; Lewis 1986, p. 51). In addition, possible particulars are required to prevent a class counting as convex simply because individuals which would be between its elements happen not to exist.

[7]Gardenfors and others also argue for the convex class analysis via a vast array of applications (Gardenfors 2000; Gardenfors 2014; Zenker and Gardenfors 2015). But just as it is doubtful whether the distinction between natural and non-natural properties can fulfil its many roles, it is doubtful whether the convex class analysis can deliver on all its promises (for criticism in this vein see, for example, Gauker 2007 and Hernandez-Conde

of the underlying metric space, and still support the claim that it captures the properties which make for resemblance. In particular, the analysis entails that the conjunctions, but not the disjunctions or negations, of natural properties are also natural properties (Oddie 2005, pp. 152-7).

Firstly, all intersections of convex subsets are also convex.[8] So because the set of individuals instantiating a conjunction of natural properties is the intersection of the sets which instantiate the conjuncts, the analysis of natural properties as convex subsets entails that conjunctions of natural properties are also natural properties (Oddie 2005, p. 157). If being bright and being red, for example, are both natural properties, then the analysis entails that being bright red is also a natural property.

This is intuitive, since if two distinct properties make for resemblance, possession of both properties should make for even more resemblance. As Armstrong writes "... we noted that it is implausible to say that in the case of disjunctive and negative "properties", there is something identical in virtue of which the corresponding predicates apply. By contrast, if a number of particulars each have two properties, P and Q, it is perfectly natural to say that this constitutes a respect in which they are identical" (1978, p. 34).

2016). For this reason, I continue to focus exclusively on the connection between natural properties and similarity.

[8]For suppose B and C are convex. And suppose a and c are in $B \cap C$, and b is between a and c. Then since B is convex, $b \in B$. And since C is convex $b \in C$. It follows that $b \in B \cap C$ – or in other words that that the intersection of B and C is convex.

Secondly, some but not all unions of convex subsets are also convex. Being red and being orange, for example, are convex subsets of the colour circle, and so is their disjunction being red or orange, since everything on the circle between something red or orange is itself red or orange. But although being red and being green, for example, are convex subsets of the colour circle, being red or green is obviously not, since yellow things are between red and green things, but yellow things are not red or green (Oddie 2005, pp. 157-158).

It's sometimes alleged that no disjunctions of natural properties are natural properties. Armstrong, for example, writes "Suppose a has a property P but lacks Q, while b has Q but lacks P. It seems laughable to conclude from these premises that a and b are identical in some respect. Yet both have the "property", P *or* Q" (1978, p. 20). But it's too extreme to draw from this that no disjunctions of natural properties are natural properties, since some disjunctions of natural properties do make for resemblance.

The property of being red, for example, is the disjunction of all the determinate shades of red. So if no disjunctions of natural properties are natural, and if the determinate shades of red are natural properties – which they plausibly are, since being the same determinate shade of colour makes for resemblance – then being red is not a natural property (Armstrong 1978a, p. 118). But intuitively, being red is also a property which makes for resemblance, and so being red is a natural property.

So according to the characterisation of natural properties as those which make for resemblance, some but not all disjunctions of natural properties are natural properties. It's an advantage of the analysis of natural properties as convex subsets of a metric space in which the distances are degrees of dissimilarity that it predicts which disjunctions of natural properties are natural properties – namely, those which correspond to convex subsets of the space – and which are not (Oddie 2005, p. 158).

Secondly, some but not all complements of convex subsets are also convex. So because the set instantiating the negation of a property is the complement of the set instantiating that property, the definition of natural properties as convex subsets allows that some but not all negations of natural properties are also natural properties (Oddie 2005, p. 157).

For example, the property of being red or orange or yellow corresponds to a convex subset of the colour circle, since every particular in between two red, orange or yellow particulars is itself a red, orange or yellow particular (no purple, blue or green particular is between any two red, orange or yellow particulars, since the distance between the two red, orange or yellow particulars is less than half the circle, whereas the distance via the purple, blue or green particular is more than half the circle).

But the negation of being red or orange or yellow – being neither red nor orange nor yellow, or, in other words, being green or blue or purple – also corresponds to a convex subset of the colour circle, since every particular in between two purple, blue or green particulars is itself purple, blue or green.

This consequence is intuitive, since both being red or orange or yellow – or being orangish – and being green or blue or purple – or being bluish – are properties that make for resemblance.

But although being orange, for example, is a natural property, its negation, being not orange, does not correspond to a convex subset, since red particulars and yellow particulars are both not orange, but orange particulars are between red and yellow particulars (the distance around the colour circle from a tomato to a banana, for example, is the sum of the distance from the tomato to a mandarin, and the distance from the mandarin to the banana). This is intuitive, since red and green particulars, for example, do not resemble each other in respect of hue, even though they are not orange.

So just as the analysis of properties as convex subsets can predict which disjunctions of natural properties are properties, it can also predict which negations of natural properties are properties (Oddie 2005, p. 158). Compare again Armstrong, who writes "when $\neg P$ applies to a number of particulars, it is implausible to suggest that the predicate applies because the particulars are identical in a respect. If particulars are identical in a respect, then they resemble each other. But it is surely implausible to suggest that *not being* P is a point in which a, b, c ... etc. resemble each other" 1978, p. 23.[9]

[9]According to Armstrong's whole theory, every (instantiated) conjunction of natural properties is a natural property, but no disjunction of natural properties is a natural property, and no negation of a natural property is a natural property. John Bacon has proven that it follows from Armstrong's theory that each individual instantiates exactly one natural property, and so if degree of resemblance is a function of number of properties

5.

I argued in the last section that three intuitive consequences of the convex class analysis are evidence that it captures the spirit of the characterisation of natural properties as those which make for resemblance. But in this section I will point out that in three degenerate cases – namely, in the case of tautologous, inconsistent and haecceitistic properties – the analysis has consequences which are less intuitive. The last case is most important, since it's the source of the inconsistencies proved in sections (7) and (8).

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in common, everything resembles everything different to the same degree (see Bacon 1986 and Blumson n.d. for discussion).

At the other end of the extreme, according to abundant conceptions of properties, every conjunction, disjunction or negation of a property is a property (in other words, the set of properties is a complete and complemented boolean lattice). In his "theorem of the ugly duckling", Satoshi Watanabe proved this conception entails that everything different has the same number of properties in common with everything else, so if degree of resemblance is a function of number of properties in common, everything resembles everything different to the same degree (see Watanabe 1969, pp. 376-9 and Blumson n.d. for discussion).

By allowing that some but not all disjunctions and negations of natural properties are natural properties, the convex class analysis might be thought to sail just between the Scylla of the abundant conception on the one hand and the Charybdis of the extremely sparse conception on the other. Unfortunately, the proofs in sections (7) and (8) show that the convex class analysis entails an extremely similar result – namely, that if degree of resemblance is number or proportion of properties not in common, then everything resembles everything different to the same degree.

Firstly, the entirety of a metric space is a convex subset of itself, because if a and c are in the space, and b is between a and c then b is in the space as well – vacuously, since b is in the space regardless of whether it is between a and c. So since the set of individuals instantiating the property of existence or the property of being self-identical is the set of individuals in the entire space, it follows that the property of existence or the property of being self-identical is a natural property.

But this is not intuitive, since the property of existence or being self-identical is not intuitively a property which makes for resemblance between the things which exist or are identical to themselves – existence or being self-identical is a property all things have, regardless of how alike or unalike they are (compare the remarks in Armstrong 1978a, pp. 10-1). On the other hand it's not *too* counterintuitive, since as properties that everything has, the properties of existence and being self-identical don't make for more resemblance amongst some things than others, and thus may be ignored.

Secondly, the empty set is a convex subset, since if a and c are in the empty set, and b is between a and c, then b is in the empty set too – but vacuously so, since of course a and c are not in the empty set. Since the set of individuals which instantiates an inconsistent property is the empty set, it follows that inconsistent properties are natural. It's not intuitive that inconsistent properties make for resemblance. But it's not *too* counterintuitive, since as properties that nothing has, they don't make for more resemblance amongst some things than others, and may also be ignored.

Finally, all singleton sets are convex. The set $\{a\}$ is convex, for example, since every point between points in $\{a\}$ – namely, just a, which is between a and a itself – is also in $\{a\}$. Since the set of individuals which instantiate a haecceity – the property of being a specific individual – is the singleton set of that individual, it seems to follow that haecceities are natural properties. The set of individuals which instantiates the property of being Socrates, for example, is just {Socrates} and {Socrates} is convex, so it follows that being Socrates is a natural property.

However, recall from section (2) that if some particulars are exactly alike, then in order to satisfy condition (2) of the definition of a metric space, A should be interpreted as a set of equivalence classes of exactly alike particulars, rather than as a set of particulars simpliciter. In this case, a singleton set of the space corresponds not to the property of being a particular individual, but to the property of being one of a class of exactly alike individuals.[10] And since all individuals which have this property are exactly alike, this is a property that makes for resemblance.

I've emphasised this point because it's the fact that singleton sets correspond to natural properties that is responsible for the inconsistencies proved

[10]Note that this move entails that strictly speaking the convex classes of the metric space in question are now classes of classes, rather than classes of individuals. However, we can sidestep this issue by defining the natural properties as the unions of the classes in those convex classes, so that the natural properties will still be identified with classes of individuals simpliciter, in accordance with class nominalism as usually stated.

in sections (7) and (8). If the consequence that singleton sets correspond to natural properties were too counterintuitive, then a proponent of the convex class analysis of natural properties might argue that their account should be altered in order to exclude this case. But since it's intuitive that properties corresponding to classes of exactly alike individuals are natural, this move would be unacceptably *ad hoc*.

6.

What justifies the presupposition that degree of dissimilarity is distance in a metric space in the first place? A simple answer would be to define the degree of dissimilarity between two individuals as their total number of natural properties not in common or, in other words, the number of natural properties the first has not in common with the second, plus the number of natural properties the second has not in common with the first (Blumson 2018b, pp. 11-12). In this section, I explain how this definition meets the four conditions of definition (1).

Firstly, the definition of degree of dissimilarity as number of properties not in common obviously entails condition (1), minimality, since no particular has any natural properties not in common with itself. It also satisfies condition (3), symmetry, since addition is commutative.

However, the definition may not strictly speaking satisfy condition (2), because there may be distinct perfect duplicates, which share all their natural properties (Lewis 1983, p. 356). But if two particulars share all their natural properties, then their number of natural properties not in common is zero, and so the definition as number of natural properties not in common satisfies the weaker condition $\delta(a,b) \geq 0$ if $a \neq b$. It follows that having zero natural properties not in common is an equivalence relation, which divides (possible) particulars into equivalence classes of perfect duplicates.

Finally and most importantly, the definition satisfies condition (4), the triangle inequality – in other words, the total number of natural properties a and b have not in common plus the total number of natural properties b and c have not in common is at least as great as the total number of natural properties a and c have not in common. The proof of this is very simple, but I will dwell on it because it is important in the sections that follow, and it prepares the way for the more complicated proof in section (8).

Let α be the number of natural properties a has not in common with b or c, β the number b has not in common with a or c, and γ the number c has not in common with a or b, as illustrated in figure (1). Likewise, let ϵ be the number of properties a and b have not in common with c, ζ the number b and c have not in common with a, and η the number a and c have not in common with b. Finally, let θ be the number of natural properties in common between a, b and c.[11]

[11]More formally, let A be the set of the natural properties of a, B be the set of the natural properties of b, and C be the set of the natural properties of c. Then let $\alpha = |A - (B \cup C)|$, $\beta = |B - (A \cup C)|$ and $\gamma = |C - (A \cup B)|$. Let $\epsilon = |(A \cap B) - C|$, $\zeta = |(B \cap C) - A|$ and $\eta = |(A \cap C) - B|$. And let $\theta = |A \cap (B \cap C)|$.

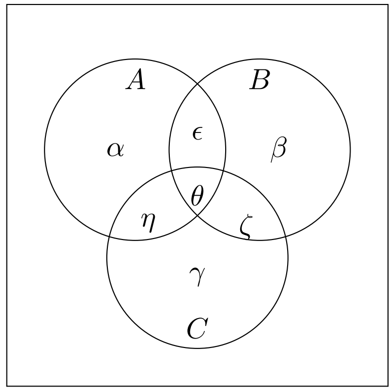

FIGURE 1.

Then the triangle inequality for total number of natural properties not in common can be rewritten as:

$$(\alpha + \eta + \beta + \zeta) + (\epsilon + \beta + \gamma + \eta) \geq (\alpha + \epsilon + \gamma + \zeta) \tag{1}$$

where the terms between the first pair of parentheses sum to the number of natural properties a and b have not in common, the terms between the second pair to the number b and c have not in common, and the terms between the third pair to the number a and c have not in common.

After substracting the right hand side from the left (which we can do since we are restricting our attention to the finite case) and gathering common terms together, equation (1) simplifies to:

$$2\beta + 2\eta \geq 0 \tag{2}$$

which says that twice the number of properties of b alone plus twice the number of natural properties a and c have not in common with b is greater than or equal to zero. Since both these quantities must be at least zero, this is obviously true, and the triangle inequality is proven.

7.

In this section I will argue that the definition of degree of dissimilarity as total number of natural properties not in common and the convex class analysis are incompatible with each other, because they jointly entail that *every* property is a natural property, which is absurd. The problem is that the definition of degree of dissimilarity as number of properties not in common and the convex class analysis jointly entail that in that space, no two particulars have any other particular between them, and so every subset is vacuously convex.

To see this, suppose for *reductio* that a particular b is between two different particulars a and c. Then, as I emphasised in section (3), it follows that the triangle inequality is an exact equality. So it follows that the total number of natural properties a and b have not in common, plus the total number of natural properties b and c have not in common is *exactly* equal to the total number of natural properties that a and c have not in common.

But recall from equation (2) in section (6) that the total number of natural properties a and b have not in common plus the total number of natural properties b and c have not in common exceeds the total number of natural properties a and c have not in common by exactly twice η – the number of natural properties a and c have not in common with b – plus twice β – the number of natural properties b has not in common with a or c.

So η and β must both be zero. Happily, the convex class analysis entails that η – the number of natural properties a and c have not in common with b – *is* zero, for since b is between a and c, every convex subset containing a and c must also contain b. But it also follows that β is *not* zero. Recall from section (2) that singleton sets – and so $\{b\}$ in particular – are vacuously convex. It follows that $\{b\}$ is a natural property, and so b has at least one natural property not in common with a or c.

Moreover, although it is not intuitive that b's haecceity is a natural property, recall from the last section that $\{b\}$ corresponds to a property shared by all and only members of an equivalence class of exactly alike particulars, or perfect duplicates, which intuitively does make for resemblance. So it would be *ad hoc* for proponents of the convex class analysis of natural properties to disagree at this point.

8.

Another way to justify the presupposition that degree of dissimilarity is distance in a metric space would be to define degree of dissimilarity as proportion of natural properties not in common. In other words, the degree of dissimilarity between a and b can be defined as $\frac{|A-B|+|B-A|}{|A \cup B|}$, where A

and B are the properties of a and b respectively. (This formula for the distance between sets A and B is sometimes known as "Jaccard dissimilarity" (Levandowsky and Winter 1971), after Jaccard 1912.)

Assuming that every particular has at least one natural property, and so the denominator of the proportion of properties not in common is always positive (and continuing to focus on the finite case, so both the numerator and denominator are finite), then this definition satisfies the first three conditions of the definition of a metric space (1) for the same reason as its predecessor (Blumson 2018b, pp. 12-4).

Given the labelling scheme described in section (6) and illustrated in figure (1), the triangle inequality for proportion of natural properties not in common can be rewritten as:

(3)

$$\frac{\alpha+\eta+\beta+\zeta}{\alpha+\eta+\beta+\zeta+\epsilon+\theta}+\frac{\epsilon+\beta+\gamma+\eta}{\epsilon+\beta+\gamma+\eta+\zeta+\theta}\geq\frac{\alpha+\epsilon+\gamma+\zeta}{\alpha+\epsilon+\gamma+\zeta+\eta+\theta}$$

which, after subtracting the right hand side from the left, putting the fractions in common terms and cancelling the positive denominator (all of which we can do since we are restricting our attention to the finite case) "simplifies" to:

(4)
$$\begin{aligned}
&\alpha^2\beta+\alpha^2\epsilon+\alpha^2\gamma+\alpha^2\eta+\alpha\beta^2+2\alpha\beta\epsilon+2\alpha\beta\gamma+2\alpha\beta\theta\\
&\quad+4\alpha\beta\eta+2\zeta\alpha\beta+\alpha\epsilon^2+2\alpha\epsilon\gamma+\alpha\epsilon\theta+4\alpha\epsilon\eta+\zeta\alpha\epsilon+\alpha\gamma^2\\
&\quad+2\alpha\gamma\theta+4\alpha\gamma\eta+2\zeta\alpha\gamma+3\alpha\theta\eta+3\alpha\eta^2+3\zeta\alpha\eta+\beta^2\epsilon\\
&\quad+\beta^2\gamma+2\beta^2\theta+2\beta^2\eta+\zeta\beta^2+\beta\epsilon^2+2\beta\epsilon\gamma+3\beta\epsilon\theta+5\beta\epsilon\eta\\
&\quad+2\zeta\beta\epsilon+\beta\gamma^2+2\beta\gamma\theta+4\beta\gamma\eta+2\zeta\beta\gamma+2\beta\theta^2+6\beta\theta\eta\\
&\quad+3\zeta\beta\theta+4\beta\eta^2+5\zeta\beta\eta+\zeta^2\beta+2\epsilon^2\eta+3\epsilon\gamma\eta+\zeta\epsilon\gamma+4\epsilon\theta\eta\\
&\quad+4\epsilon\eta^2+4\zeta\epsilon\eta+\gamma^2\eta+\zeta\gamma^2+3\gamma\theta\eta+\zeta\gamma\theta+3\gamma\eta^2+4\zeta\gamma\eta\\
&\quad+\zeta^2\gamma+2\theta^2\eta+4\theta\eta^2+4\zeta\theta\eta+2\eta^3+4\zeta\eta^2+2\zeta^2\eta\geq 0\,.
\end{aligned}$$

Since all the terms on the left hand side are nonnegative, the left hand side is at least as great as the right hand side, and so the triangle inequality is proven (the proof is from Marczewski and Steinhaus 1958, p. 321 and Levandowsky and Winter 1971; see also Grygorian and Iacob 2018).

If b is between a and c, the triangle inequality must be an exact equality, and so the left hand side of equation (4) must be equal to zero. But the first summand on the left hand side is $\alpha^2\beta$. Since $\{a\}$ and $\{b\}$ are both vacuously convex, it follows that α – the number of natural properties a has not in common with b or c – and β – the number b has not in common with a or c – are at least one, so $\alpha^2\beta$ is also at least one.[12] So the left hand side is strictly greater than zero, contradicting that b is between a and c.

In other words, it follows from the convex class analysis and the definition of degree of dissimilarity as proportion of properties not in common that the triangle inequality is never an exact equality, and so no particular is between any other two in the metric space in which distances are degree of dissimilarity. But if no points in that space are between any other two, then every subset of that space is vacuously convex. But then every property is a natural property, which is absurd.

9.

I have argued that the analysis of natural properties as the convex subsets of the metric space in which the distances are degrees of dissimilarity is

[12]The same point could be made using $\alpha\beta^2$, $\alpha^2\gamma$, $\alpha\beta\gamma$, $\alpha\gamma^2$, $\beta^2\gamma$ and $\beta\gamma^2$.

incompatible with the definition of degree of dissimilarity as number or proportion of properties not in common, because they jointly entail that every subset of that space is convex, and so entail that every property is a natural property, which is absurd. How should proponents of the convex class analysis of natural properties respond?

The simplest response would be to look for an alternative definition of degree of dissimilarity in terms of natural properties. But it seems unlikely that there is another function of natural properties in common or not in common which both entails that degree of dissimilarity is distance in a metric space, and avoids the incompatibility raised in this paper. So a proponent of the convex class analysis of natural properties may have to deny that degree of dissimilarity is definable in terms of natural properties.

Another response would be to concede that convexity is only a necessary and not a sufficient condition for a property to be natural, thus escaping the consequence that equivalence classes of particulars which share all their natural properties themselves correspond to natural properties, and so escaping the proofs which rely on that consequence.

A cost of this solution is to lose the consequence that conjunctions of natural properties are themselves natural. For suppose that one of the equivalence classes of particulars which share all their natural properties does not correspond to a natural property. Then there is a conjunction of natural properties which is not itself a natural property – viz., the conjunction of natural properties shared by the particulars in that class.[13]

A better response would be to look for an alternative justification of the presupposition that degree of dissimilarity is distance in a metric space. In particular, one may seek to prove a representation theorem, according to which if comparative dissimilarity meets certain qualitative conditions, then it is representable by distance in a metric space, and then to argue that comparative dissimilarity does in fact meet those conditions.[14]

In this case, the relation of comparative dissimilarity is adopted as a primitive. So we have an analysis of natural properties in terms of degree of dissimilarity, but no analysis of degree of dissimilarity in terms of natural properties. If this order of analysis reflects the underlying order of metaphysical priority, then it follows that the convex class analysis of natural properties embodies a variety of resemblance nominalism, according to which similarity is prior to naturalness.[15]

[13]Yet another response would be to attempt to develop the intuition behind the convex class analysis, but without the presupposition that degree of dissimilarity is distance in a metric space. Thomas Mormann, for example, argues for a generalisation according to which the natural properties are the *connected* subsets of a *topological* space, which need not be characterised in terms of numerical degrees (1993). Because my primary interest is in numerical degrees of resemblance, I didn't consider Mormann's approach here.

[14]For details of this approach see Suppes et al. 1989, pp. 159-225 and Blumson 2018a.

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