

# A FORTIORI LOGIC: Innovations, History and Assessments

By **Avi Sion** PH.D.

© Copyright **Avi Sion, 2013; 2014. All rights reserved.**

Protected by international copyright conventions. No part of this book may be reproduced in any manner whatsoever, or stored in a retrieval system or transmitted, without express permission of the Author-publisher, except in case of brief quotations with due acknowledgement.

Self-published through Lulu; CreateSpace & Kindle.

Website: [www.TheLogician.net](http://www.TheLogician.net). E-mail: [avi-sion@thelogician.net](mailto:avi-sion@thelogician.net).

**A Fortiori Logic** can be freely read online at [The Logician.net](http://TheLogician.net) and in various other locations. It can be purchased, in print and e-book editions, in [Amazon.com](http://Amazon.com), [Lulu.com](http://Lulu.com) and many other online booksellers.

The present document contains **excerpts** from this book, namely: The Abstract; the Contents in brief; the Foreword; Sample text (Chapter 1, Sections 1-3); and the Main references.

**Avi Sion** (Ph.D. Philosophy) is a researcher and writer in logic, philosophy, and spirituality. He has, since 1990, published original writings on the theory and practice of inductive and deductive logic, phenomenology, epistemology, aetiology, psychology, meditation, ethics, and much more. Over a period of some 28 years, he has published 27 books. He resides in Geneva, Switzerland.

It is very difficult to briefly summarize Avi Sion's philosophy, because it is so wide-ranging. He has labeled it '**Logical Philosophy**', because it is firmly grounded in formal logic, inductive as well as deductive. This original philosophy is dedicated to demonstrating the efficacy of human reason by detailing its actual means; and to show that the epistemological and ethical skepticism which has been increasingly fashionable and destructive since the Enlightenment was (contrary to appearances) quite illogical – the product of ignorant, incompetent and dishonest thinking.

# Abstract

*A Fortiori Logic: Innovations, History and Assessments*, by Avi Sion, is a wide-ranging and in-depth study of a fortiori reasoning, comprising a great many *new theoretical insights* into such argument, *a history* of its use and discussion from antiquity to the present day, and *critical analyses* of the main attempts at its elucidation. Its purpose is nothing less than to lay the foundations for a new branch of logic and greatly develop it; and thus, to once and for all dispel the many fallacious ideas circulating regarding the nature of a fortiori reasoning.

The work is divided into three parts. The first part, **Formalities**, presents the author's largely original theory of a fortiori argument, in all its forms and varieties. Its four (or eight) principal moods are analyzed in great detail and formally validated, and secondary moods are derived from them. A crescendo argument is distinguished from purely a fortiori argument, and similarly analyzed and validated. These argument forms are clearly distinguished from the pro rata and analogical forms of argument. Moreover, we examine the wide range of a fortiori argument; the possibilities of quantifying it; the formal interrelationships of its various moods; and their relationships to syllogistic and analogical reasoning. Although a fortiori argument is shown to be deductive, inductive forms of it are acknowledged and explained. Although a fortiori argument is essentially ontical in character, more specifically logical-epistemic and ethical-legal variants of it are acknowledged.

The second part of the work, **Ancient and Medieval History**, looks into use and discussion of a fortiori argument in Greece and Rome, in the Talmud, among post-Talmudic rabbis, and in Christian, Moslem, Chinese and Indian sources. Aristotle's approach to a fortiori argument is described and evaluated. There is a thorough analysis of the Mishnaic *qal vachomer* argument, and a reassessment of the *dayo* principle relating to it, as well as of the Gemara's later take on these topics. The valuable contribution, much later, by Moshe Chaim Luzzatto is duly acknowledged. Lists are drawn up of the use of a fortiori argument in the Jewish Bible, the Mishna, the works of Plato and Aristotle, the Christian Bible and the Koran; and the specific moods used are identified. Moreover, there is a pilot study of the use of a fortiori argument in the Gemara, with reference to Rodkinson's partial edition of the Babylonian Talmud, setting detailed methodological guidelines for a fuller study. There is also a novel, detailed study of logic in general in the Torah.

The third part of the present work, **Modern and Contemporary Authors**, describes and evaluates the work of numerous (some thirty) recent contributors to a fortiori logic, as well as the articles on the subject in certain lexicons. Here, we discover that whereas a few authors in the last century or so made some significant contributions to the field, most of them shot woefully off-target in various ways. The work of each author, whether famous or unknown, is examined in detail in a dedicated chapter, or at least in a section; and his ideas on the subject are carefully weighed. The variety of theories that have been proposed is impressive; and stands witness to the complexity and elusiveness of the subject, and to the crying need for the present critical and integrative study. But whatever the intrinsic value of each work, it must be realized that even errors and lacunae are interesting because they teach us how *not* to proceed.

This book also contains, in a final appendix, some valuable contributions to general logic, including new analyses of symbolization and axiomatization, existential import, the tetralemma, the Liar paradox and the Russell paradox.

## Contents in brief

Abstract 1	
Foreword .....	9
<b>PART I - FORMALITIES 11</b>	
1. The standard forms.....	11
2. More formalities.....	33
3. Still more formalities .....	49
4. Apparently variant forms .....	71
5. Comparisons and correlations .....	82
<b>PART II – ANCIENT AND MEDIEVAL HISTORY 98</b>	
6. A fortiori in Greece and Rome .....	98
7. A fortiori in the Talmud .....	114
8. In the Talmud, continued .....	147
9. Post-Talmudic rabbis .....	186
10. A fortiori in the Christian Bible .....	214
11. Islamic ‘logic’ .....	235
12. A fortiori in China and India.....	276
<b>PART III – MODERN AND CONTEMPORARY AUTHORS 285</b>	
13. Moses Mielziner .....	285
14. Adolf Schwarz .....	293
15. Saul Lieberman .....	298
16. Louis Jacobs .....	305
17. Heinrich Guggenheimer.....	324
18. Adin Steinsaltz.....	329
19. Jonathan Cohen.....	339
20. Michael Avraham .....	342
21. Gabriel Abitbol .....	347
22. Hyam Maccoby.....	360
23. Alexander Samely.....	368
24. Lenartowicz and Koszteyn.....	388
25. Abraham, Gabbay and Schild .....	394
26. Stefan Goltzberg .....	407
27. Andrew Schumann.....	414
28. Allen Wiseman .....	432
29. Yisrael Ury .....	464
30. Hubert Marraud .....	479
31. Various other commentaries .....	493
32. A fortiori in various lexicons .....	509
33. Conclusions and prospects.....	522
<b>APPENDICES 529</b>	
1. A fortiori discourse in the Jewish Bible .....	529
2. A fortiori discourse in the Mishna.....	534
3. A fortiori discourse in the two Talmuds.....	553
4. A fortiori discourse by Plato and Aristotle.....	560
5. A fortiori discourse in other world literature.....	570
6. Logic in the Torah.....	576
7. Some logic topics of general interest .....	591
Main References	620

# Foreword

When I started writing the present work, in late 2010, I thought it would take a dozen pages and a couple of weeks at most to say what I felt the need to say. I had, I believed, said most of what needed to be said in my previous foray in the field of a fortiori logic, in my 1995 study of Judaic logic. But having noticed that some people were still writing on the subject without reference to my work, and to boot were making serious mistakes, I felt the need to show them the errors of their ways. However, as I proceeded in this set task, I found myself more and more involved in its intricacies.

For a start, to be fair the critiques had to be detailed, and show exactly what had been said and where lay the errors and lacunae. Secondly, I kept discovering more and more commentaries which needed to be similarly reviewed and evaluated. Thirdly, it became obvious that I needed to expand my theoretical investigations, to be able to answer various questions these commentaries brought up, consciously or unconsciously. Eventually, I realized that I had to aim for a history of the subject and a survey of more recent contributions to it, to be able to demonstrate precisely who said what first.

Thus, the work ended up taking me three years to complete. Three parts emerged. The first presented my new, much more detailed theory of a fortiori argument. The second part traced the early history of use and discussion of such argument, so far as I could make it out with the resources available to me. The third focused on modern commentaries on the subject. However, *these parts did not emerge separately, but repeatedly impinged on each other*, so that many chapters or sections had to be written more than once to be adapted to new findings. For this reason, it was impossible to publish any part of the work before it was all done.

It should be stressed that the work did not proceed in the order that the chapters are now set out. Whereas now all commentators are ordered chronologically, I did not comment on their work in their order of appearance in history. It was all a matter of chance encounter and personal mood. Moreover, my theoretical baggage at each stage was different. For this reason, some earlier chapters may appear more perspicacious or analytically cutting than some later ones. I tried, of course, to harmonize things as much as I could; but as the book grew in size, it became more and more unwieldy. No doubt my memory in these later years is not what it was once; so I may have missed some things.

## 1. Innovations

The present work is replete with valuable innovations in the field of a fortiori logic, and in other, related subjects. The present, wider ranging research confirms that my past work in this field, in my 1995 book, *Judaic Logic*, was novel and important. But moreover, the present work corrects some inaccuracies in that past work, and greatly enlarges and sharpens our theory of a fortiori argument, so that it may be said to address almost every nook and cranny of the subject. There is not a single topic that I worked on here that did not yield some new insight or new theoretical development in a fortiori logic. This means that the research was certainly worthwhile and interesting; it is not a mere collection and rehashing of old material.

‘Formalities’, part one of the present volume, presents the author’s largely original theory of a fortiori argument, in all its forms and varieties. Its four (or eight) principal moods are analyzed in

great detail and formally validated, and secondary moods are derived from them. A crescendo argument is distinguished from purely a fortiori argument, and similarly analyzed and validated. These argument forms are clearly distinguished from the pro rata and analogical forms of argument. Moreover, we examine the wide range of a fortiori argument; the possibilities of quantifying it; the formal interrelationships of its various moods; and their relationships to syllogistic and analogical reasoning. Although a fortiori argument is shown to be deductive, inductive forms of it are acknowledged and explained. Although a fortiori argument is essentially ontical in character, more specifically logical-epistemic and ethical-legal variants of it are acknowledged.

The present work also contains, in a final appendix, valuable innovations relating to certain topics in general logic; namely, symbolization and axiomatization, existential import, the tetralemma, the Liar paradox and the Russell paradox.

## 2. History

Logic science, properly conceived, is not just a theoretical enterprise, but also an investigation into the historical roots of the forms of human discourse. The present work on a fortiori logic constitutes an excellent case study of how a particular form of thought is rooted deep in antiquity (in history), and probably much earlier, in language itself (in prehistory), and then gradually develops as awareness of it dawns, expands and intensifies. There is ample evidence that a fortiori discourse existed in very ancient times and in very diverse cultures. A fortiori reasoning was present in early Greek literature (Homer, Aesop), long before Aristotle first discussed it (in his *Rhetoric* and *Topics*); and it was present before that in Jewish literature (the Torah and other Biblical books). Aristotle did not invent the a fortiori argument, any more than he invented the syllogism; he ‘merely’ observed, described and explained them, as a botanist might notice and catalogue interesting plants.

‘Ancient and Medieval History’, part two of the present volume, looks into use and discussion of a fortiori argument in Greece and Rome, in the Talmud, among post-Talmudic rabbis, and in Christian, Moslem, Chinese and Indian sources. Aristotle’s approach to a fortiori argument is described and evaluated. There is a thorough analysis of the Mishnaic *qal vachomer* argument, and a reassessment of the *dayo* principle relating to it, as well as of the Gemara’s later take on these topics. The valuable contribution, much later, by Moshe Chaim Luzzatto is duly acknowledged. Lists are drawn up of the use of a fortiori argument in the Jewish Bible, the Mishna, the works of Plato and Aristotle, the Christian Bible and the Koran; and the specific moods used are identified. Moreover, there is a pilot study of the use of a fortiori argument in the Gemara, with reference to Rodkinson’s partial edition of the Babylonian Talmud, setting detailed methodological guidelines for a fuller study. There is also a novel, detailed study of logic in general in the Torah.

## 3. Assessments

When I started to study a fortiori logic, I was little aware of the number of people who have since the late 19<sup>th</sup> century attempted to describe and explain this common form of reasoning. The field seemed nearly empty of contributors, a desert yet to be explored. Only little by little did I realize that many people have indeed tried their hand at solving the enigma of a fortiori argument – some, to be sure, more competently than others. It gradually became clear that a survey of existing contributors needed to be made, and their work had to be carefully studied and assessed. Such assessment depended, of course, on the theoretical and historiographical work undertaken earlier.

It was interesting to see how many of the contributors studied past work very little before proposing their own ideas. Each apparently thought he was one of the first explorers.

'Modern and Contemporary Authors', part three of the present work, describes and evaluates the work of numerous (some thirty) recent contributors to a fortiori logic, as well as the articles on the subject in certain lexicons. Here, we discover that whereas a few authors in the last century or so made some significant contributions to the field, most of them shot woefully off-target in various ways. The work of each author, whether famous or unknown, is examined in detail in a dedicated chapter, or at least in a section; and his ideas on the subject are carefully weighed. The variety of theories that have been proposed is astonishing; and stands witness to the complexity and elusiveness of the subject, and to the crying need for the present critical and integrative study. But whatever the intrinsic value of each work, it must be realized that even errors and lacunae are interesting because they teach us how *not* to proceed.

# Sample text (chapter 1, sections 1-3)

## 1. Copulative a fortiori arguments

Based on close analysis of a large number of Biblical and Talmudic examples (some known to Jewish tradition and some newly identified by me), as well as examples from everyday discourse, I discovered and proposed in my book *Judaic Logic* the four valid moods of copulative a fortiori argument listed below.

An a fortiori argument consists of three propositions called the major premise, the minor premise and the conclusion. A copulative such argument is one involving terms. It comprises four terms, which are always symbolized in the same way. The four terms are called the major, the minor, the middle and the subsidiary; and the symbols for them are respectively P, Q, R and S<sup>1</sup>. Other terminology used will be clarified as we proceed.

a. The **positive subjectal** {+s} mood:

P is more R than (or as much R as) Q (is R),  
and Q is R enough to be S;  
therefore, all the more (or equally), P is R enough to be S.

Notice that the valid inference goes ‘from minor to major’; that is, from the minor term (Q) to the major one (P); meaning: from the minor term as subject of ‘R enough to be S’ in the minor premise, to the major term as subject of same in the conclusion. Any attempt to go from major to minor in the same way (i.e. positively) would be invalid inference.

b. The **negative subjectal** {–s} mood:

P is more R than (or as much R as) Q (is R),  
yet P is R *not* enough to be S;  
therefore, all the more (or equally), Q is R *not* enough to be S.

Notice that the valid inference goes ‘from major to minor’; that is, from the major term (P) to the minor one (Q); meaning: from the major term as subject of ‘R not enough to be S’ in the minor premise, to the minor term as subject of same in the conclusion. Any attempt to go from minor to major in the same way (i.e. negatively) would be invalid inference.

We can summarize all information about subjectal argument as follows: “Given that P is more R than or as much R as Q is R, it follows that: if Q is R enough to be S, then P is R enough to be S; and if P is R not enough to be S, then Q is R not enough to be S; on the other hand, if Q is R not enough to be S, it does not follow that P is R not enough to be S; and if P is R enough to be S, it

---

<sup>1</sup> Notice that the symbols R and S, respectively, happen to match the words “Range” (the middle item always refers to a range) and Subsidiary.

does not follow that Q is R enough to be S.” In this summary format, we resort to nesting: the major premise serves as primary antecedent, and the valid minor premises and conclusions appear as consequent conditions and outcomes, while the invalid moods are expressed as non-sequiturs.

For example: granted Jack (P) can run faster (R) than Jill (Q), it follows that: if Jill can run (at a speed of) one mile in under 15 minutes (S), then surely so can Jack; and if he can’t, then neither can she. Needless to say, the conditions are presumed identical in both cases; we are talking of the same course, in the same weather, and so on. If different conditions are intended, the argument may not function correctly. The a fortiori argument is stated categorically only if there are no underlying conditions. Obviously, if there are conditions they ought to be specified, or at least we must ensure they are the same throughout the argument.

c. The **positive predicatal** {+p} mood:

More (or as much) R is required to be P than (as) to be Q,  
and S is R enough to be P;  
therefore, all the more (or equally), S is R enough to be Q.

Notice that the valid inference goes ‘from major to minor’; that is, from the major term (P) to the minor one (Q); meaning: from the major term as predicate of ‘S is R enough to be’ in the minor premise, to the minor term as predicate of same in the conclusion. Any attempt to go from minor to major in the same way (i.e. positively) would be invalid inference.

d. The **negative predicatal** {−p} mood:

More (or as much) R is required to be P than (as) to be Q,  
yet S is R *not* enough to be Q;  
therefore, all the more (or equally), S is R *not* enough to be P.

Notice that the valid inference goes ‘from minor to major’; that is, from the minor term (Q) to the major one (P); meaning: from the minor term as predicate of ‘S is R not enough to be’ in the minor premise, to the major term as predicate of same in the conclusion. Any attempt to go from major to minor in the same way (i.e. negatively) would be invalid inference.

We can summarize all information about predicatal argument as follows: “Given that more or as much R is required to be P than to be Q, it follows that: if S is R enough to be P, then S is R enough to be Q; and if S is R not enough to be Q, then S is R not enough to be P; on the other hand, if S is R not enough to be P, it does not follow that S is R not enough to be Q; and if S is R enough to be Q, it does not follow that S is R enough to be P.” In this summary format, we resort to nesting: the major premise serves as primary antecedent, and the valid minor premises and conclusions appear as consequent conditions and outcomes, while the invalid moods are expressed as non-sequiturs.

For example: granted that it takes more strength (R) to lift 50 kilos (P) than 30 (Q): if someone (S) can lift 50 kilos, then surely he can lift 30; and if he can’t lift 30, then he can’t lift 50. Needless to say, the conditions are presumed identical in both cases; we are talking of the same handle, on the same day, and so on. If different conditions are intended, the argument may not function correctly. The a fortiori argument is stated categorically only if there are no underlying conditions. Obviously,



if there are conditions they ought to be specified, or at least we must ensure they are the same throughout the argument.

Thus, to summarize, there are four valid moods of copulative a fortiori argument: two subjectal moods, in which the major and minor terms (P and Q) are the logical subjects of the three propositions concerned; and two predicatal moods, in which the major and minor terms (P and Q) are the logical predicates of the three propositions concerned. The major premise is always positive, though it differs in form in subjectal and predicatal arguments. In each of these types, there are two variants: in one, the minor premise and conclusion are positive; and in the other, they are negative. The positive and negative versions in each case are obviously closely related – the minor premise of the one is the negation of the conclusion of the other, and vice versa; that is, each can be used as a *reductio ad absurdum* for the other.

Note well the order in which the major and minor terms (P and Q) appear in the four moods: in the subjectal moods they are subjects; and in the predicatal ones they are predicates. It follows that in the two subjectal moods, the subsidiary term (S) is a predicate; and in the two predicatal moods, it (S) is a subject. The middle term (R), however, is a predicate in both premises and the conclusion of *all* the moods, note well. In subjectal moods it is a predicate of the major and minor terms (P and Q); in the predicatal moods it is a predicate of unspecified subjects in the major premise and a predicate of the subsidiary term (S) in the minor premise and conclusion, the subsidiary term being one instance of the unspecified subject-matter of the major premise.

The difference between subjectal and predicatal moods is called a difference of structure. The difference between positive and negative moods is called a difference of polarity. The difference between moods that go “from minor to major” and those that go “from major to minor” is called a difference of orientation. Sometimes this difference of direction is stated in Latin, as “*a minori ad majus*” and “*a majori ad minus*”<sup>2</sup>. Note that the “from” term may be the minor or major and occurs in the minor premise; and the “to” term is accordingly the major or minor, respectively, and occurs in the conclusion. Notice the variations in orientation in accord with the structure and polarity involved.

In sum, these four valid moods are effectively four distinct figures (and not merely moods) of a fortiori argument, since the placement of their terms differs significantly in each case. This is clearly seen in the following table:

---

<sup>2</sup> I notice that that the Soncino Talmud does not apparently use the term *a fortiori* as a general term, but distinguishes between *a minori* and *a fortiori* (instead of *a majori*). Maybe this was an error. In any case, in my opinion, such usage should be avoided as it would leave us with no general term. The term *a fortiori* is needed as a common label for all forms of the argument. Whereas *a majori* means from the major (term to the minor term), *a fortiori* means with stronger (reason); so these expressions are not equivalent.

Figure/mood	+s	-s	+p	-p
major premise	PQR	PQR	RPQ	RPQ
minor premise	QRS	PRS	SRP	SRQ
conclusion	PRS	QRS	SRQ	SRP

Table 1.1

We shall deal with the validation of all these arguments further on. Meanwhile, the following clarifications should also be kept in mind:

- The expression “all the more,” and others like it (such as “a fortiori,” “how much more,” and so on), are often used in practice to signal an intention of a fortiori argument. This is useful specifically when the argument is only partly explicit; but when the argument is fully explicit, as shown above, such expression is in fact redundant, and (as we shall see) can even be misleading (suggestive of ‘proportionality’). When the argument is stated in full, it is sufficient to say “therefore” to signal the conclusion; nothing is added by saying “all the more.”

Incidentally, in practice people sometimes reserve “all the more” for argument that goes from minor to major and “all the less” for argument that goes from major to minor; but it is also true that the expression “all the more” (and others like it) is also often used indiscriminately, and this is the way we usually intend it here.

- The four arguments function just as well if the major term is greater (in respect of the middle term) than the minor term, or if they are equal. Whence, I have inserted in brackets in each mood: an “as much as” alternative clause to “more than” in the major premise, and an “equally” alternative to the traditional expression “all the more” in the conclusion. So though we have four figures, we may say that they contain two moods each, a ‘superior’ and an ‘egalitarian’ one, making a total of eight moods. Egalitarian a fortiori argument is also sometimes called ‘*a pari*’.
- Note that for subjectal moods, I have specified the major premise as “P is more R than Q (is R)” – this is done to avoid confusion with a proposition of the form “P is more R than (P is) Q.” If we try using the latter with “P is Q enough to be S” to conclude “P is R enough to be S,” we would have an argument vaguely resembling a fortiori but which is in fact invalid<sup>3</sup>. In the valid form,  $R_p > R_q$ ; whereas in the fake form  $R_p > Q_p$ . Watch out for occurrences of this fallacy in common discourse.

The major premise of predicatal argument, i.e. “More R is required to be P than to be Q,” does not have the same potential for ambiguity. Note, however, that it could alternatively be formulated as “To be P requires more R than to be Q (requires R)” – in which form it might be

---

<sup>3</sup> For example, “This screw is longer than it is wide; and it is wide enough to fit into this hole; therefore it is long enough to do so.” Clearly, this would be fallacious reasoning; the conclusion does not follow from the premises.

confused with the major premise of subjectal argument, viz. “What is P is more R than what is Q (is R).”<sup>4</sup>

- The major premise may occasionally in practice be converted – i.e. it may be stated, in subjectal argument, as “Q is less R than P” instead of as “P is more R than Q;” and in predicatal argument, as “Less R is required to be Q than to be P” instead of as “More R is required to be P than to be Q.” The validity of the argument in such cases is not affected, *provided the minor premise and conclusion remain the same*. Note this proviso well. Very often, such conversion of the major premise confuses people and they erroneously transpose the minor premise and conclusion<sup>5</sup>. Arguments involving such converted major premises, which may be labeled ‘inferior’, should not be counted as distinct moods.
- In practice, the major premise is very often simply left out. The proponent of a given argument may have it explicitly or tacitly in mind. But he may also be quite unaware of it, in which case it is only we logicians who tell him it is logically present in the background and playing an active role in the inference. This is not something peculiar to a fortiori argument, but is likewise often encountered in syllogism and other forms of argument. It is called enthymemic argument (a mere technical term); you can call it abridged or abbreviated argument, if you like.
- Concerning the minor premise and conclusion, the phrase “R enough to be” is often left out in practice. This may occur with the major premise absent, so that the middle term (R) is completely unstated (though of course still logically implicit); or it may occur with the major premise present, in which case the mention of the middle term in it is deemed sufficient for the whole argument. When the said phrase is left out, the minor premise and conclusion are usually stated in one if–then proposition: e.g. “If Q is S, then P is S,” which (to repeat) may be combined with an explicit major premise or presented alone.

The fact that often in practice the middle term R is left tacit should not blind us to the fact that it is a *sine qua non* for successful a fortiori argument. The proposition “P is more R than Q” combined with “Q is S” is logically quite compatible with “P is not S;” or combined with “P is not S” is logically quite compatible with “Q is S.” Similarly, The proposition “More R is required to be P than to be Q” combined with “P is S” is logically quite compatible with “Q is not S;” or combined with “Q is not S” is logically quite compatible with “P is S.” Note this well. Many commentators fail to realize this, or having learned it quickly forget it. Without the relation “R enough to be” in the minor premise, the a fortiori conclusion cannot be drawn and the argument is fallacious.

- Evidently, the clause “R enough to be” in positive moods, or “R not enough to be,” in negative moods, even if it is not explicitly stated in the minor premise and conclusion, is absolutely essential to a fortiori argument. If there is no intended threshold of R to be attained or surpassed in order for S to be predicated of or to be subject to the major and minor terms, there is no operative a fortiori argument (though there might be some other thought-process, such as mere

---

<sup>4</sup> We might also put the major premise of subjectal argument in the form: “More R is found in P than in Q.” However, the most natural form for the subjectal major premise is active and that for the predicatal major premise is passive.

<sup>5</sup> To avoid confusion always simply reflect on the question: which term ‘is more R’ or ‘requires more R’ than the other?

analogy). This is evident from the fact that, without this crucial clause, we simply cannot validate the argument. Keep that well in mind.

Note that the expression “R not enough to be” can also be stated as “not enough R to be” or “not R enough to be,” without change of meaning. The form “X is not R enough to be Y,” which is used in the minor premise and conclusion of negative subjectal or predicatal arguments, is the most ambiguous, being used for cases where *X is not R at all*, as well as more obviously to cases where it *is R to some insufficient extent*. More will be said about this further on.

- Moreover, the middle term R must remain constant throughout the argument. That is, the middle term R specified in the minor premise must be identical with the one specified in the major premise. This can be seen by an example: although humans are more intelligent than horses, it does not follow that they can run faster than horses! Obviously, we can only speak of the superiority of humans over horses with respect to what was intended, viz. ‘intelligence’ in this case; this does not exclude the possibility that with respect to other attributes, such as leg muscles, horses are superior.

On a formal level, what this means is that if we do not specify or keep in mind the middle term R intended in the major premise, we might easily intend *another* middle term, say R', in the minor premise and conclusion; in which case, our reasoning (whether unconsciously or deliberately done) would of course be faulty. This often happens in practice, and is one reason some people doubt the validity of a fortiori argument in general. But the problem here is not with the argument as such, but with the use of *two middle terms*. If we use, explicitly or implicitly, two middle terms, the argument is of course invalid, for it cannot be validated any longer. We could label such practice ‘the fallacy of two middle terms’ so as to remember to avoid it and not be taken in by it.

- Any or all of the four terms, P, Q, R, S, may be a compound, i.e. a conjunction of two or more terms. This of course happens in practice often enough.

It should be stressed that, albeit their various formal differences, the four principal forms of copulative a fortiori argument above enumerated truly deserve to be called by one and the same name; they constitute a family of arguments. The positive and negative moods of a given orientation (subjectal or predicatal) are obviously two facets of the same coin. But moreover, notice the similarity between the positive subjectal and negative predicatal moods, and also between the negative subjectal and positive predicatal moods. Note that the former two moods may be characterized as going “from minor to major,” and the latter two as going “from major to minor.” More will be said about this further on.

The positive subjectal mood may be viewed as the prototype of all a fortiori argument, because of its relative simplicity. Many accounts of a fortiori argument tend to mention only this mood; or rather, examples thereof. Nevertheless, this does not mean that the other three copulative moods, or indeed their implicational analogues, can be ignored. They are distinct movements of thought that merit separate attention.

I should also draw your attention to the possibility that the whole subjectal or predicatal a fortiori argument concerns only one subject, as shown next:

When this thing (say, X) is P, it is more R than when it is Q,  
and when it is Q, it is enough R to be S;  
therefore, when it is P, it is enough R to be S.

More R is required for this thing (say, X) to be P than for it to be Q,  
and when it is S, it is R enough to be P;  
therefore, when it is S, is R enough to be Q.

We can construct similar negative moods, of course. Notice that I have specified the subject as ‘this thing’ (or X) in both major premises, but these could equally be generalities, i.e. have ‘something, anything’ as their subject. Such single-subject a fortiori arguments are not mere theoretical possibilities, but often occur in practice. Note the conditional form the sentences take; these are really, therefore, cases of implicational argument (see next section). The conditioning may obviously be based on any type of modality – extensional, natural, temporal or spatial.

## 2. Implicational a fortiori arguments

In addition to the above four valid copulative moods, I identified in *Judaic Logic* four comparable ‘implicational’ moods. The first two I called antecedental (instead of subjectal) and the last two I called consequential (instead of predicatal). These four moods have the same figures as the preceding four; but they differ in involving the relation of implication instead of the copulative one, and therefore theses instead of terms as the items under consideration. I list them for you anyway, just to make sure there is no misunderstanding:

e. The **positive antecedental** (+a) mood:

P implies more R than (or as much R as) Q (implies R),  
and, Q implies enough R to imply S;  
therefore, all the more (or equally), P implies enough R to imply S.

f. The **negative antecedental** (–a) mood:

P implies more R than (or as much R as) Q (implies R),  
yet, P does *not* imply enough R to imply S;  
therefore, all the more (or equally), Q does *not* imply enough R to imply S.

g. The **positive consequential** (+c) mood:

More (or as much) R is required to imply P than to imply Q,  
and, S implies enough R to imply P;  
therefore, all the more (or equally), S implies enough R to imply Q.

h. The **negative consequential** (-c) mood:

More (as much) R is required to imply P than to imply Q,  
yet, S does *not* imply enough R to imply Q;  
therefore, all the more (or equally), S does *not* imply enough R to imply P.

Clearly, mostly similar comments can be made regarding the structures of these additional four valid moods (or eight, if we distinguish between superior and egalitarian moods) as for those preceding them.

In particular note well the fact that the middle thesis (R) is always a consequent (or non-consequent), whereas the other three theses (P, Q and S) have varied roles as antecedents (or non-antecedents) or consequents (or non-consequents) depending on the figure concerned. In antecedental argument, R is (or is not) a consequent of P and Q; while in consequential argument, R is (or is not) a consequent of S. Do not be misled by the fact that R is placed to the left of P and Q in the major premise of consequential a fortiori arguments. The thesis R does not there play the role of antecedent of P and Q (i.e. it does not imply them). The theses P, Q and R are there all consequents of some unstated antecedents; and thesis S is a specified instance of such unstated antecedent (in the positive case) or not so (in the negative case).

**Variation of the middle thesis.** Concerning the middle thesis R, the sense in which it is quantitatively variable (i.e. that more or less of it can be implied) needs to be clarified. A *proposition as such does not have degrees*; so it would be incorrect to imagine that the proposition R as a whole has degrees. A thesis (e.g. Rp) is not a quantity, and so cannot be “greater” than another thesis (e.g. Rq). Therefore, when in the major premises of implicational a fortiori argument we say that “more of thesis R” is implied or required, we must refer to a variation in the predicate and/or in the subject *within* thesis R. This insight can be better understood if we formulate an implicational a fortiori argument in such a way that the categorical propositions inherent in it are made explicit. This can be done with antecedental and consequential arguments of whatever polarity. Consider for instance the following case, which is doubtless the most frequent:

P (= A is p) implies more R (= C is r) than Q (= B is q) does, and  
Q (= B is q) implies enough R (= C is r) to imply S (= D is s).  
So, P (= A is p) implies enough R (= C is r) to imply S (= D is s).

Here, I have shown each of the four categorical propositions as involving four different subjects (A, B, C, D) with four different predicates (p, q, r, s). The middle thesis R is here taken to mean that ‘C is r’. The variation of R may in this light be understood in various ways. In the most frequent

case, the subject  $C$  is constant and it is the predicate  $r$  within  $R$  that is variable,  $C$  being  $r_p$  in thesis  $R_p$  and  $C$  being  $r_q$  in thesis  $R_q$  ( $r_p > r_q$ ). Comparatively rarely, the predicate  $r$  is constant and it is the subject  $C$  within  $R$  that is variable,  $C_p$  being  $r$  in thesis  $R_p$  and  $C_q$  being  $r$  in thesis  $R_q$  ( $C_p > C_q$ )<sup>6</sup>. In more complex cases, both the subject  $C$  and the predicate  $r$  might conceivably vary,  $C_p$  being  $r_p$  in thesis  $R_p$  and  $C_q$  being  $r_q$  in thesis  $R_q$ . The important point is that the resultant  $R$  theses can reasonably be said to satisfy the condition that  $R_p > R_q$ .

As regards language, the major and minor theses might in practice be stated in gerundive form, as ‘ $A$  being  $p$ ’ and ‘ $B$  being  $q$ ’, while the subsidiary term might more naturally be stated in the infinitive form, as ‘ $D$  to be  $S$ ’. For the middle thesis, we might say ‘more  $r$  in  $C$ ’ to signify that it is the predicate that varies, or ‘more  $C$  to be  $r$ ’ to signify that it is the subject that varies. Quite often in practice, people do not state the whole middle thesis, but only the most relevant term in it – i.e. the variable predicate (usually) or subject (rarely). Thus, instead of saying in the major premise “implies more  $R$ ,” they might say “implies more  $r$ ” or “implies more  $C$ ”; and likewise, instead of saying in the minor premise and conclusion “implies  $R$  enough,” they might say “implies  $r$  enough” or “implies  $C$  enough.”

Strictly speaking, of course, this is inaccurate, because *a lone term cannot be implied (or imply)*. The logical relation of implication concerns whole theses, never mere terms. But since this confusion occurs in everyday discourse, it is well to be aware of it and to take it into consideration. Thus, when in practice we encounter an a fortiori argument with whole theses as major and minor items, and a lone term as middle item, we should not think that this exemplifies a ‘hybrid’ type of argument which is partly copulative and partly implicational. Formally, such a construct is still implicational argument, except that the middle thesis is not entirely spoken out loud; i.e. either its subject or its predicate is left tacit. In the same way, the subsidiary thesis is sometimes incompletely stated. To validate such partly formulated arguments, we of course need to specify the intended unspoken term(s).

We could in fact say that *all a fortiori arguments are tacitly implicational*. The thin line between copulative and implicational argument becomes evident when we reword a typical copulative argument in implicational form, as follows:

$P$  (= something being  $p$ ) implies more  $R$  (=  $r$  in it) than  $Q$  (= something being  $q$ ) does, and  
 $Q$  (= something being  $q$ ) implies enough  $R$  (=  $r$  in it) to imply  $S$  (= it to be  $s$ );  
 therefore,  $P$  (= something being  $p$ ) implies enough  $R$  (=  $r$  in it) to imply  $S$  (= it to be  $s$ ).

This argument is obviously a special case of the preceding one. Here, instead of four subjects ( $A$ ,  $B$ ,  $C$ ,  $D$ ), we only have two (or even just one). They are unspecific (i.e. not labeled  $A$  and  $B$ , as earlier done), in the sense that they each refer to ‘something’ (i.e. anything – the intent is general, not particular) that is solely defined by the predicate initially attached to it (viz.  $p$ ,  $q$ , respectively).

---

<sup>6</sup> That we have to acknowledge the possibility that the subject varies in magnitude will be evident further on, when we consider predicatal a crescendo (i.e. proportional a fortiori) argument. There it is manifest that this is logically possible and occurs in practice. As regards the assumption that  $R_p$  ( $C_p$  is  $r$ )  $>$   $R_q$  ( $C_q$  is  $r$ ) is implied when  $C_p > C_q$  (rather than when  $C_p < C_q$ ) – this seems reasonable to me at this time, though some uncertainty persists.

The ‘something’ that is intended in P and the ‘something (else)’ intended in Q are here distinct objects, note (although, as we have already seen, they could well in some cases be one and the same subject). Each of them is subject to a different measure or degree of the middle predicate ‘r’ (whence r is ‘in it’). And each of them is or turns out to be subject to the subsidiary predicate ‘s’. The case shown (here again) is the positive antecedental mood; the same can obviously be done with the positive predicatal mood, and with the negative forms of both of these.

Looking back at the way I came upon these various argument forms when I wrote *Judaic Logic*, I remember first discovering the copulative forms and later, finding them insufficient to account for all examples of a fortiori argument I came across, I developed the implicational forms. In a sense, they were conceived as generalizations of the corresponding copulative forms. Indeed, I overgeneralized a bit, because I did not realize at the time that the notion that a thesis may “imply more” of another thesis is logically untenable. Much later, I started wondering whether ‘hybrid’ arguments signified additional types, besides the copulative and implicational. It is only recently that I better understood the relationships between the various forms of argument as above described. So the present account amends past errors and uncertainties.

I should also here mention the following special case, where the major premise “P implies more A to be B than Q does” means “P implies that a number  $x$  of A are B, and Q implies that a number  $y$  of A are B, and  $x > y$ .” The change in magnitude involved in this case is not in the subject A or the predicate B inherent in the middle thesis, but in the quantifiers of A. So the middle thesis is not, as might be thought, about “how much A is B,” or even “how much B A is,” but about *the frequency* of occurrence of ‘A being B’. In such case, the proposition could be stated less ambiguously as “P implies more instances of A to be B than Q does.” The frequency involved may be extensional, as here; or it could have to do with another mode of modality, i.e. more often in time or place, or in more circumstances or contexts.

Moreover, though I have here presented the middle thesis R as a single categorical proposition, it should be kept in mind that R could contain a compound thesis, i.e. it could involve a complex set of variable factors.

In conclusion, when in formulating implicational a fortiori argument we refer to the middle thesis ‘R’, the intention is more precisely ‘*something in R*’, meaning ‘some term(s) in thesis R’ or even ‘some modal qualifier in thesis R’. That is, when we say: ‘implies more R’ or ‘more R is required to imply’ or ‘implies enough R’ – we must be understood to mean: ‘implies more of something in R’ or ‘more of something in R is required to imply’ or ‘implies enough of something in R’, respectively. Though I will continue to use the abridged formulae, these more elaborate formulae will be tacitly intended.

More will, of course, be said about implicational a fortiori argument as we proceed.

### 3. Validations

Validation of an argument means to demonstrate its validity. An argument is ‘valid’ if, given its premises, its conclusion logically follows. Otherwise, if the putative conclusion does not follow from the given premises or if its denial follows from them, the argument is ‘invalid’. If the putative conclusion is merely not implied by the given premises, it is called a *non sequitur* (Latin for ‘it does not follow’); in such case, the contradictory of the putative conclusion is logically as compatible with the given premises as the putative conclusion is. If a contrary or the contradictory of the putative conclusion is positively implied by the given premises, the putative conclusion is



called an absurdity (lit. ‘unsound’) or more precisely an antinomy (lit. ‘against the laws’ of thought).

The validity of an argument does not guarantee that its conclusion is true, note well. An argument may be valid even if its premises and conclusion are in fact false. Likewise, the invalidity of an argument does not guarantee that its conclusion is false. An argument may be invalid even if its premises and conclusion are in fact true. The validity (or invalidity) of an argument refers to the logical *process*, i.e. to the claim that a set of premises of this kind formally implies (or does not imply) a conclusion of that kind.

A material a fortiori argument may be validated simply by showing that it can be credibly cast into any one of the valid moods listed above. If it cannot be fitted into one of these forms, it is invalid – or at least, it is not an a fortiori argument. The validations of the forms of a fortiori argument may be carried out as we will now expound. Invalid forms are forms that cannot be similarly validated. Obviously, material arguments can also be so validated; but the quick way is as just stated to credibly cast them into one of the valid forms. Once the forms are validated by logical science, the material cases that fit into them are universally and forever thereafter also validated.

One way to prove the validity of a new form of deduction is through the intermediary of another, better known, form of deduction. Such derivation is called ‘reduction’. ‘Direct’ reduction is achieved by means of conversions or similar immediate inferences. If the premises of the tested argument imply those of an argument already accepted as valid, and the conclusion of the latter implies that of the former, then the tested argument is shown to be equally valid. ‘Indirect’ reduction, also known as reduction *ad absurdum*, on the other hand, proceeds by demonstrating that denial of the tested conclusion is inconsistent with some already validated process of reasoning.

It works like this: Suppose A and B are the two (or more) premises of a proposed argument, and C is its putative conclusion. If the C conclusion is correct, this would mean that (A + B) implies C; which means that the conjunction (A + B + not-C) is logically impossible. Let us now hypothetically suppose that C is *not* a necessary implication in the context of A + B; i.e. that not-C is not impossible in it. In that case, we could combine not-C with one of the premises A or B, without denying the other. But we already know from previous research that, say, (A + not-C) implies not-B; which means that the conjunction (A + not-C + B) is logically impossible. Therefore, we must admit the validity of the newly proposed argument. Note that the two stated conjunctions of three items are identical except for the relative positions (which are logically irrelevant) of the items conjoined.

### **Analysis of constituents**

The validation procedures<sup>7</sup> are accordingly uniform for copulative and implicational a fortiori arguments. They are based on analysis of the meanings of the propositions involved in such argument, i.e. on *reduction* of these more complex forms to simpler forms more studied and better understood by logicians.

The following are the two main reductions needed for validation of the earlier listed **copulative** arguments. The major premises (characterized as “*commensurative*” because they compare measures or degrees) of subjectal and predicatal arguments are always positive and have the following components:

---

<sup>7</sup> See my *Judaic Logic* chapter 3, section 2 – ‘Validation Procedures’ – for additional details on this topic. However, note well, there are significant changes in the present treatment.

The subjunctal major premise, “P is more R than (or as much R as) Q is,” means:

- P is R, i.e. P is to a certain measure or degree R (say, Rp);
- Q is R, i.e. Q is to a certain measure or degree R (say, Rq);
- and Rp is greater than (or equal to) Rq (whence: Rp implies Rq).

The predicatal major premise, “More (or as much) R is required to be P than to be Q,” means:

- Only what is at least to a certain measure or degree R (say, Rp) is P;
- only what is at least to a certain measure or degree R (say, Rq) is Q;
- and Rp is greater than (or equal to) Rq (whence: Rp implies Rq).

We could more briefly write the first two components of the predicatal major premise as exclusive implications: ‘If and only if something is Rp, then it is P’ and ‘If and only if something is Rq, then it is Q’; or more briefly still, as: ‘Iff Rp, then P’ and ‘Iff Rq, then Q’<sup>8</sup>. Note that in my past treatment of the predicatal major premise, in my book *Judaic Logic*, I did not specify the exclusiveness of these two implications; but their exclusiveness is clearly implied by the word “required.”

The positive minor premises and conclusions (labeled “*suffective*” because they concern sufficiency) of copulative arguments have the following four components in common. The symbols X and Y here stand for the symbols P or Q and S as appropriate in each mood; that is, we may have “P is R enough to be S,” “Q is R enough to be S,” “S is R enough to be P,” or “S is R enough to be Q.”

A proposition of the form “X is R enough to be Y” means:

- X is R, i.e. X is to a certain measure or degree R (say, Rx);
- whatever is at least to a certain measure or degree R (say, Ry), is Y, and
- whatever is *not* at least to that measure or degree R (i.e. is not Ry), is *not* Y;
- and Rx is greater than (or equal to) Ry<sup>9</sup> (whence: “Rx implies Ry”<sup>10</sup>).

All this implies that X is Y, of course. We could more briefly write the two middle components of a suffective proposition as: ‘If something is Ry or more, then it is Y’ and ‘If something is not Ry or more, then it is not Y’; and these can be put together in a single proposition: ‘If and only if something is Ry or more, then it is Y’, which can be expressed still more briefly as: ‘Iff  $\geq$  Ry, then Y’.

Note that in my past treatment of suffective propositions, in my book *Judaic Logic*, I did not specify the third component, which is the inverse of the second component. I did not at the time realize the significance for a fortiori argument of this negative component, i.e. how essential it is to such argument; so this is an important new finding here. Note that since Ry implies Y and not-Ry implies

<sup>8</sup> Note that the form “If X, then Y” (or “X implies Y”) can only strictly speaking be used if X and Y are *propositions*. If, as here, X and Y are *terms*, then we must strictly speaking say: “If something is X, then it is Y.” However, it is all right to use the abridged form in practice if one is well aware of this caveat.

<sup>9</sup> Note that in *Judaic Logic*, I here have “Ry includes Rx” – but it is evident that “ $Rx \geq Ry$ ” is a more meaningful and accurate statement, Ry being the threshold for Y and Rx being sufficient to pass that threshold and therefore equal to it or greater than it. The reason I complicated things in *Judaic Logic* is that I wanted to take into consideration all conceivable ranges (including discontinuous ones), whereas now I realize that the matter is simpler, because in relation to a fortiori argument specifically we need only consider continuous ranges.

<sup>10</sup> This implication is intended in the sense that a larger number implies every smaller number. For example, if I have \$5, then I obviously have \$3.

not-Y, we may say that there is a causal relation – more precisely, a necessary and complete causation – between these two items.

It is this feature that gives meaning to the word “**enough**” (or “sufficiently”) in such propositions. This tells us that X has whatever amount of R it takes to be Y; i.e. that X has at least the amount of R *required for Y*. It informs us that there is a **threshold of R** (viz.  $R_y$ ) *as of and above which X is indeed Y*, and anywhere *before which X is not Y*;  $R_x$  is then specified as falling on the required side of the known threshold. In some cases, of course,  $R_x$  is exactly equal to  $R_y$ ; in such cases, the proposition would be stated more precisely as: “X is R **just enough** to be Y.” If it is known that  $R_x$  is (not equal to but) greater than  $R_y$ , we would say: “X is R **more than enough** to be Y.” Thus, “enough” means “either just enough or more than enough.” It is also clear from the above definition that another way to say “X is R enough to be Y” is: “X is **too much R** to be not-Y” (note the negation of the predicate in the latter form).

Although a proposition of the form ‘X is R enough to be Y’ implies that ‘X is R’ and ‘X is Y’ and ‘ $R_x \geq R_y$ ’, it does not follow that the latter propositions together imply the former, for it is not always true that there is a threshold value of R ( $R_y$ ) as of which a subject (such as X) gains access to the predicate Y. Thus, we must know (or at least inductively assume) that ‘Iff  $\geq R_y$ , then Y’ before we can construct a suffactive proposition; without that threshold condition for predication, we do not have such a proposition.

The threshold ( $R_y$ ), though in principle an exact quantity, need not be precisely specified in practice, but can be vaguely intended by saying “the minimum value of R corresponding to Y, whatever it happen to be.” But in any case, note well, if there is a threshold, there has to be a negative as well as a positive side of it. We shall see the full significance of this insight further on, when we examine negative suffactive propositions more closely. As regards the negative moods of copulative arguments (which involve such propositions), they can, as already mentioned, be validated by *reductio ad absurdum* to the corresponding positive moods, without pressing need to interpret their negative propositions.

It should be emphasized that the kind of thinking that makes a fortiori argument possible depends on there being a regular increase or decrease of the middle term, i.e. along the range R. If we came across a subject (X) whose predicate (Y) varies with respect to R in complex ways – unevenly rising and then falling and/or vice versa, or fluctuating from positive to negative and/or vice versa – we would just not use a fortiori argument. Such argument form is too simple to deal with more complex variables. We would normally only use it for continuous ranges; for discontinuous ones, we would resort to more detailed descriptions and perhaps to mathematical formulas.

Note also that ‘X is R enough to be Y’ implies ‘X is Y’ *provided R is indeed by itself enough for Y*. If R is in fact only *part of* a set of conditions necessary for Y, then factor R cannot be truthfully said to be ‘enough’ for Y – or, if it happens to be proposed as ‘enough’ for Y, the remaining required factors must at least be *tacitly* intended. This would mean, effectively, that the proposition ‘X is R enough to be Y’ is not as it appears categorical but in fact conditioned on the tacit factors, or alternatively that the outcome of R is not yet Y but some earlier stage of development than Y. To give an example of this important issue: suppose membership in an exclusive club depends on one’s age, level of income and maybe other criteria. In that event, one might well say, “this man is old enough but not rich enough to be admitted” – and here, obviously, the man being old ‘enough’ does not imply he will be admitted, although he may be put on a waiting list till he gets rich ‘enough’ too. Thus, in common discourse, the word ‘enough’ may not signify full sufficiency but merely a tendency towards it. But in the present treatise, we intend the word ‘enough’ in its strict sense.

The above general form of suffective proposition will of course concretize in different ways according to the orientation of the copulative a fortiori argument under consideration:

In positive subjectal arguments (where P, Q are subjects), it will have the forms “P or Q is R enough to be S,” which mean:

P or Q (as the case may be) is to a certain measure or degree R (say, Rp or Rq, as appropriate);

whatever is at least to a certain measure or degree R (say, Rs) is S and

whatever is not at least to that measure or degree R (i.e. is not Rs) is not S;

and Rp or Rq is greater than or equal to Rs.

In positive predicatal arguments (where P, Q are predicates), it will have the forms “S is R enough to be P or Q,” which mean:

S is to a certain measure or degree R (say, Rs);

whatever is at least to a certain measure or degree R (say, Rp or Rq, as appropriate) is P or Q (as the case may be), and

whatever is not at least to that measure or degree R (i.e. is not Rp or Rq) is not P or Q;

and Rs is greater than or equal to Rp or Rq.

The formal difference between commensurative and suffective propositions ought to be clarified here, as I did not do this in my previous writings on this topic. Although their components are very similar in form, namely comparative and hypothetical propositions, what distinguishes them is that in commensurative forms the terms compared, viz. P and Q, are either both subjects or both predicates, whereas in suffective forms the terms compared, viz. X and Y, are one a subject and the other a predicate. For this reason, we cannot reduce commensuratives to suffectives or vice versa.

Even so, it is well to notice that the major premise of *predicatal* a fortiori argument, i.e. the commensurative proposition “More (or as much) R is required to be P than to be Q,” is essentially about sufficiency. The word “required” tells us that there is an unstated quantity of R sufficient for P, whereas *lacking* that quantity, whatever it happen to be, being R does *not* entail being P; similarly with regard to Q, of course<sup>11</sup>. Thus, this major premise is a comparison between the thresholds for P and Q, telling us that amounts of R enough for Q are not all enough for P. On the other hand, the major premise of *subjectal* a fortiori arguments makes no mention of sufficiency, merely informing us that P is R and Q is R, and that these two quantities of R are one greater than (or equal to) the other.

All the above comments can be repeated with regard to the propositions involved in **implicational** a fortiori argument, *mutatis mutandis*. Briefly put, we can interpret the commensurative major premises of a fortiori arguments as follows.

The antecedental major premise “P implies more R than (or as much R as) Q does” means:

P implies a certain measure or degree of R (say, Rp);

---

<sup>11</sup> A requirement is a *sine qua non*. On this basis, we may add two components to the above definition of the predicatal major premise, namely: “what is *not* to the required measure or degree R (i.e. Rp), is *not* P” and “what is *not* to the required measure or degree R (i.e. Rq), is *not* Q.” I have done this simply by making the positive premise exclusive – i.e. adding “only” at the beginning of the clauses concerned.

Q implies a certain measure or degree of R (say, Rq);  
and Rp is greater than (or equal to) Rq (whence: Rp implies Rq).

The consequential major premise "More (or as much) R is required to imply P than to imply Q" means:

Only what implies at least a certain measure or degree of R (say, Rp) implies P;  
only what implies at least a certain measure or degree of R (say, Rq) implies Q;  
and Rp is greater than (or equal to) Rq (whence: Rp implies Rq).

The suffective propositions which are used as minor premises and conclusions of a fortiori arguments can be interpreted as follows. Let us first look at the general positive form, "X implies R enough to imply Y;" this means:

X implies to a certain measure or degree R (say, Rx);  
whatever implies at least to a certain measure or degree R (say, Ry) implies Y, and  
whatever does *not* imply at least to that measure or degree R (i.e. does not imply Ry) does *not* imply Y;  
and Rx is greater than or equal to Ry.

Notice that in the negative third clause of this definition, I have opted for the minimalist supposition. This choice seems sufficient to make the intended point, viz. that "without the power to imply at least Ry, Y does not follow." I could of course have opted for the more emphatic interpretation, viz. "whatever implies less than that measure or degree R (i.e. implies *not*-Ry), implies *not*-Y," but this would limit the application of the form considerably and unnecessarily. It could be that someone, or myself at a later date, considers the more emphatic option more appropriate; but until some specific reason is found to do so, we are wise to opt for the minimalist position. From the point of view of validation of a fortiori argument, both options are acceptable, because in both cases (as we shall presently see) the third and fourth clauses of the minor premise pass over intact into the conclusion.<sup>12</sup>

The above general form of suffective proposition will of course concretize in different ways according to the orientation of the implicational a fortiori argument under consideration:

In positive antecedental arguments (where P, Q are antecedents), it will have the forms "P or Q implies R enough to imply S," which mean:

---

<sup>12</sup> This question does not arise in the case of copulative suffectives, since "is" is negated solely by "is not;" in implicational suffectives, however, though the strict negation of "implies" is "does not imply," there is additionally a stronger form "implies not."

P or Q (as the case may be) implies to a certain measure or degree R (say,  $R_p$  or  $R_q$ , as appropriate);

whatever implies at least to a certain measure or degree R (say,  $R_s$ ) implies S and

whatever does not imply at least to that measure or degree R (i.e. does not imply  $R_s$ ) does not imply S;

and  $R_p$  or  $R_q$  is greater than or equal to  $R_s$ .

In positive consequential arguments (where P, Q are consequents), it will have the forms “S implies R enough to imply P or Q,” which mean:

S implies to a certain measure or degree R (say,  $R_s$ );

whatever implies at least to a certain measure or degree R (say,  $R_p$  or  $R_q$ , as appropriate) implies P or Q (as the case may be), and

whatever does not imply at least to that measure or degree R (i.e. does not imply  $R_p$  or  $R_q$ ) does not imply P or Q;

and  $R_s$  is greater than or equal to  $R_p$  or  $R_q$ .

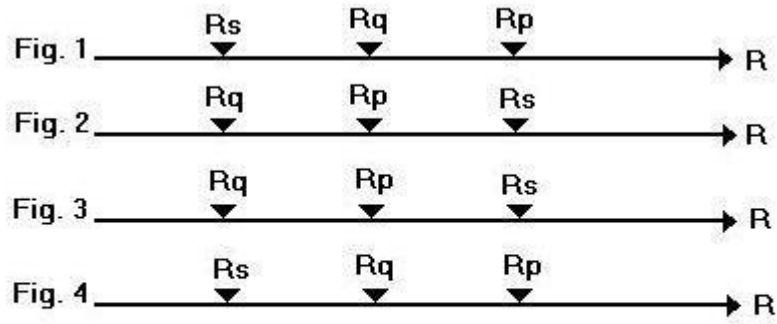
For the rest, what was said earlier for copulatives may be adapted to implicationals.

As regards the **production** of commensurative and suffective propositions, the following should be said. How are they produced, one might ask? That is, how do we get to know them in the first place? The answer is very simple and obvious. The above stated components of commensurative or suffective propositions may be viewed as the premises of the productive arguments giving rise to them. That is to say, the simpler forms, which we have above identified as implied in and together defining these more complex forms, may be presented as premises of arguments whose conclusions are commensurative or suffective propositions. Note this well, for here we have numerous new arguments for formal logic to list as such. There is, to be sure, a bit of circularity in claiming such arguments. However, though that may be true at the most formal level, at more concrete levels such arguments are quite useful.

### Validation procedures

We are now in a position to examine a fortiori argument for purposes of validation. What must be understood is that the middle term (R) of copulative argument is *its essential element*. Being the subject or predicate of the three other terms (the major term P, the minor term Q, and the subsidiary term S), the middle term underlies, is present in, all of them. Similarly, of course, implicational argument hinges on the middle thesis. We can say that a fortiori argument is principally about the middle item, and only incidentally about the other three items; it is the core or center of gravity of the whole argument; it is the common ground and intermediary of the three other items.

What a fortiori argument does is to relate together *three values of the middle item R* (here symbolized by  $R_p$ ,  $R_q$  and  $R_s$ ) found in relation to the other three items and thus representing them. The middle item of a fortiori argument is always something that varies quantitatively, in measure or degree – and the argument constitutes a comparison and hierarchical ordering of its different values (which are given in relation to the three other items). The truth of all this can be easily seen with reference to the following diagram, where quantities of R on the right are greater than quantities of R on the left.



The a fortiori argument orders items P, Q, S, according to their position in a common continuum R

Diagram 1.1

That, then, is the essence of a fortiori argument: it is a comparison between the various quantities (measures or degrees) of the middle item (term or thesis) that are copulatively or implicationally involved in the other three items (as subjects or predicates, or antecedents or consequents, of it, as the case may be). We can thus present the quantitative core of the validations very simply as follows, with reference to the comparative propositions implied in the premises and conclusions. Here, as always,  $\geq$  means ‘is greater than or equal to’ and  $<$  means ‘is less than’<sup>13</sup>:

Structure	Subjectal or antecedental		Predicatal or consequential	
Polarity	positive	negative	positive	negative
Major premise	$Rp \geq Rq$	$Rp \geq Rq$	$Rp \geq Rq$	$Rp \geq Rq$
Minor premise	$Rq \geq Rs$	$Rp < Rs$	$Rs \geq Rp$	$Rs < Rq$
Conclusion	So, $Rp \geq Rs$	So, $Rq < Rs$	So, $Rs \geq Rq$	So, $Rs < Rp$

Table 1.2

Note that the egalitarian positive subjectal (or antecedental) conclusion  $Rp = Rs$  can only be drawn from the premises  $Rp = Rq$  and  $Rq = Rs$ . Likewise, the egalitarian positive predicatal (or consequential) conclusion  $Rs = Rq$  can only be drawn from the premises  $Rs = Rp$  and  $Rp = Rq$ . In all other positive arguments, the conclusions would be  $Rp > Rs$  or  $Rs > Rq$  (as the case may be), even if one of the premises concerned involves an equation. It follows that the egalitarian negative argument of subjectal form has premises  $Rp \geq Rq$  and  $Rp \neq Rs$  and conclusion  $Rq \neq Rs$ ; while that of predicatal form has premises  $Rp \geq Rq$  and  $Rs \neq Rq$  and conclusion  $Rs \neq Rp$ .

Another way to illustrate the quantitative aspect of a fortiori argument is by means of bar charts, as in the diagram below. Given that  $Rp$  is greater than (or equal to)  $Rq$ , there are three possible

<sup>13</sup> As regards use here of the ‘is less than’ relation in the negative moods, see the justification for this in the next section.

positions for  $R_s$ : in (a)  $R_s$  is greater than (or equal to)  $R_p$  and therefore than (or to)  $R_q$ ; in (b)  $R_s$  is smaller than (or equal to)  $R_q$  and therefore than (or to)  $R_p$ ; and in (c)  $R_s$  is in between  $R_p$  and  $R_q$ , in which case no conclusion can be drawn. Chart (a) can be used to illustrate the positive predicatal and negative subjectal moods, and chart (b) the positive subjectal and negative predicatal moods, while chart (c) can be used to explain invalid arguments.

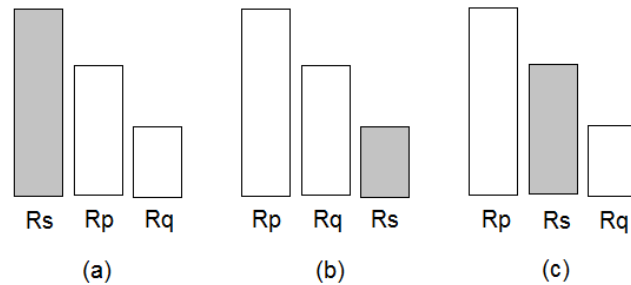


Diagram 1.2

In addition to the quantitative arguments above tabulated<sup>14</sup>, we only need to select certain clauses from our premises to derive our conclusions, as follows (check and see for yourself):

- The conclusion of a positive subjectal argument, namely the positive suffective proposition “P is R enough to be S,” is composed of four clauses:
  - P is to a certain measure or degree R (say,  $R_p$ );
  - whatever is at least to a certain measure or degree R (say,  $R_s$ ), is S;
  - whatever is not at least to that measure or degree R (i.e. is not  $R_s$ ), is not S;
  - and  $R_p$  is greater than (or equal to)  $R_s$ .

In this case, the four components are obtained as follows: *the first from the major premise, the second and third from the minor premise, and the fourth from the tabulated quantitative argument which is drawn from both premises.* Here, note well, the “enough R” condition of the conclusion (implied in its second and third components) comes from the minor premise, because it concerns the subsidiary term (S). Here, then, the crucial threshold value of R is  $R_s$ , i.e. the minimum value of R needed to be S; knowing that  $R_q$  equals or exceeds  $R_s$ , we can predict that  $R_p$  does so too.

- The conclusion of a positive predicatal argument, namely the positive suffective proposition “S is R enough to be Q,” is composed of four clauses:
  - S is to a certain measure or degree R (say,  $R_s$ );
  - whatever is at least to a certain measure or degree R (say,  $R_q$ ), is Q;
  - whatever is not at least to that measure or degree R (i.e. is not  $R_q$ ), is not Q;

<sup>14</sup> Looking at the tabulated quantitative arguments, we are tempted to say that there is ‘something of syllogistic reasoning’ in a fortiori argument, insofar as they all involve movement of thought from one item to another via an intermediary. But, note well, this is not really syllogistic inference from one class to another, but a more mathematical inference based on comparison of magnitudes. Clearly, we cannot say that a fortiori argument *is* syllogism; it is manifestly a distinct form of reasoning.



and  $R_s$  is greater than (or equal to)  $R_q$ .

In this case, the four components are obtained as follows: *the first from the minor premise, the second and third from the major premise, and the fourth from the tabulated quantitative argument which is drawn from both premises*. Here, note well, the “enough R” condition of the conclusion (implied in its second and third components) comes from the major premise, because it concerns the minor term (Q). Here, then, the crucial threshold value of R is  $R_q$ , i.e. the minimum value of R needed to be Q; knowing that  $R_p$  equals or exceeds  $R_q$ , we can predict that  $R_s$  does so too.

Note that in both the above moods, the conclusion of the a fortiori argument comes solely and entirely from the two premises together (not separately). It is true that the premises contain more information than the conclusion does; but that only means that not all the information in them is used. This does not signify redundancies in the premises, because their form is essential to intuitive human understanding of the argument, whose conclusion has similar form to the minor premise.

The corresponding negative moods are most easily validated by *reductio ad absurdum*. We say: suppose the putative conclusion is denied, then combining such denial with the same major premise we would obtain a denial of the given minor premise; this being absurd, the putative conclusion must be valid.

More briefly put, the positive conclusions are composed of the following elements drawn from the respective premises: in subjectal argument, “P is  $R_p$ , what is  $R_s$  is S and what is not  $R_s$  is not S, and  $R_p \geq R_s$ ,” and in predicatal argument, “S is  $R_s$ , what is  $R_q$  is Q and what is not  $R_q$  is not Q, and  $R_s \geq R_q$ .” The corresponding negative conclusions imply that one or more of these four elements is denied.

It is worth here stressing the utility of the threshold condition, i.e. the implication of the minor premise that there is a threshold value of R (say,  $R_t$ ), which has to be reached or surpassed before a subject X can accede to a predicate Y (i.e.  $R_x$  must be  $\geq R_t$  which is  $\geq R_y$ ).

- In positive subjectal argument, the threshold of the minor premise and thence of the conclusion means that *not all* R are S (since some things are not  $R_s$ ). Clearly, if all R were S, then we could from the major premise ‘P is more R than Q’ (which implies that ‘P is R’ and ‘Q is R’), without recourse to the simplified minor premise ‘Q is S’, obtain the conclusion that ‘P is S’ (and even that ‘Q is S’)!
- In positive predicatal argument, one of the thresholds of the major premise and thence of the conclusion means that *not all* R are Q (since some things are not  $R_q$ ). Clearly, if all R were Q, then we could from the major premise ‘More R is required to be P than to be Q’ (which implies that ‘R is required to be P’, and thence that ‘all P are R’<sup>15</sup>), together with the simplified minor premise ‘S is P’, obtain (via the intermediate conclusion ‘S is R’) the conclusion that ‘S is Q’!

In both these eventualities, the argument would be *merely syllogistic*, and not function like an a fortiori argument. Thus, the threshold condition is *essential* to the formation of a truly a fortiori

---

<sup>15</sup> Of course, ‘more R is required to be P than to be Q’ first implies that ‘*specifically*  $R_p$  is required to be P’ (as well as ‘*specifically*  $R_q$  is required to be Q’, and  $R_p > R_q$ ), which means: ‘if not  $R_p$ , then not P’. But here, ‘ $R_p$ ’ refers to the value of R required for P, *whatever it happens to be*; so it is no different (except symbolically) than the mere, indefinite ‘R’. In other words, ‘all P are R’ does not refer to just any or all values of R, but some appropriate value, whatever it happens to be. And clearly, this predicate R (meaning  $R_p$ ) fits under the wider generality ‘all R’ in the syllogistic major premise ‘all R are Q’. Thus, we can indeed infer, from ‘S is P’ and ‘all things P are R’ that ‘S is R’, and from the latter and the supposed ‘all things R are Q’ that ‘S is Q’.

argument; it is not something that can be ignored or discarded. Many people think that a fortiori argument can be formulated without this crucial condition, but that is a grave error on their part.

The same validation work can be easily done with implicational arguments, *mutatis mutandis*. We have thus formally and indubitably demonstrated all the said moods of a fortiori argument to be valid. As regards *invalid* a fortiori arguments, the following can be said. If the major item P is not identical in the major premise and in the minor premise or conclusion (so that there are effectively two major items), and/or if the minor item Q is not identical in the major premise and in the minor premise or conclusion (so that there are effectively two minor items), and/or if the middle item R is not identical in the major premise, the minor premise and the conclusion (so that there are effectively two or three middle items), and/or if the subsidiary item S is not identical in the minor premise and the conclusion (so that there are effectively two subsidiary items) – in any such cases, there is illicit process. Needless to say, “identical” here refers to identity *not only in the words used, but also in their intentions*; we are sometimes able to formulate two terms in such a way as to make them seem the same superficially, although in fact they are not the same deeper down<sup>16</sup>.

Likewise, if an item or a proposition is negative where it should be positive or vice versa – here again, we have fallacious reasoning. Although all such deviations from the established norms are obviously invalid, since we cannot formally validate them, they are often tried by people in practice, so it is worth keeping them in mind.

**Identification in practice.** We have so far theoretically described and validated a fortiori arguments. But the reader should also develop the ability *to recognize* such arguments when they occur in practice, in written text or oral discourse. The following are a few useful pointers. A fortiori argument is usually signaled by some distinctive word or phrase like “a fortiori” or “all the more/less,” or “so much (the) more/less,” or more rhetorically: “how much (the) more/less?!” Such signals are of course helpful, though they do not always occur (and moreover, they are sometimes used misleadingly, when there is no a fortiori argument in fact). Sometimes, we can guess that an a fortiori argument is involved, by noticing the use of an expression like “enough” or “sufficiently.” But sometimes, there is no verbal indicator at all, and we can only determine the a fortiori form of the argument at hand by examining its content.

Very often, the major premise remains unstated, though it can be readily formulated in the light of the minor premise and conclusion. Very often, too, the middle term is left tacit, in the major premise or in the minor premise or in the conclusion, or even throughout the argument; in such cases, we have to guess at the underlying intent of the argument’s author. All we are given, in very many cases, is an if–then statement with three terms; and often the ‘if’ and ‘then’ operators are missing too! There is nevertheless usually enough information for us to reconstruct the intended a fortiori argument, assuming some such argument is indeed intended (i.e. we must be careful not to artificially ‘read in’ the argument for our own purposes).

The following indices permit us to determine the exact mood of copulative argument. Find the term (S) common to both propositions (the premise and conclusion), and see whether it stands as subject or predicate. The positive subjectal form appears as: “Q is S; therefore, P is S;” and the negative subjectal form appears as: “P is not S; therefore, Q is not S.” Notice here that S (the common term) is a predicate, and P and Q (the other two terms) are subjects. The positive predicatal form appears as: “S is P; therefore, S is Q;” and the negative predicatal form appears as: “S is not Q; therefore,

---

<sup>16</sup> See further on the discussion concerning ‘Species and Genera’.

S is not P.” Notice that here S (the common term) is a subject, and P and Q (the other two terms) are predicates. Similarly for implicational arguments, except that “implies” appears instead of “is.” Of course, not even all the details given in the preceding paragraph may appear. For example, instead of “Q is S; therefore, P is S,” the speaker may say “Q is S: all the more P!” But we can easily add the missing clause “is S” that makes the consequent (conclusion) a mirror image of the antecedent (minor premise). We must then look for a middle term R, such that “P is more R than Q” is true (or at least somewhat credible), and also such that “Q is R enough to be S” is true, and therefore “P is R enough to be S” is likewise true – and we have reconstructed the intended a fortiori argument.

Obviously, a proposition of the form “X is Y” does not, strictly speaking, imply one of the form “X is R enough to be Y” – that is, the mere fact that X is Y does not indicate that there is a threshold of R that needs to be crossed for X to be Y. Nevertheless, we often *inductively* infer the latter from the former by reasoning that if there indeed is an a fortiori argument there must indeed be such a threshold condition for the predication. Thus, we construct the more complex premise from the simpler given, thinking “well, if X is Y, it must have been R enough to be Y!” This concerns the minor premise; as regards the conclusion, we deduce the simpler proposition from the more complex.

It should be stressed that the term common to the two given propositions is in some cases the middle term (R), rather than the subsidiary term (S). An example of that would be the sentence: “Q is bad enough; imagine what P would be!” Here, the common term “bad” is of course the middle term (as the expression “enough” indicates); and no subsidiary term is mentioned, though one can guess what it might be. A fuller statement of the minor premise and conclusion would thus be: “if Q is bad (R) enough to be avoided (S), then all the more P is bad (R) enough to be avoided (S).”

Of course, though we may manage to fully reconstruct the intended a fortiori argument, it may yet be found invalid – e.g. if, as sometimes happens, the roles of P and Q are reversed; but this is another issue, of course. That is: first find out what form the author’s intended argument has; then judge whether it is objectively valid or not. Also, do not confuse the issues of validity and truth: the argument may be well-formed, and yet be wrong due to its reliance on a false premise or other.

**© Copyright Avi Sion, 2013; 2014. All rights reserved.**

Protected by international copyright conventions. No part of this book may be reproduced in any manner whatsoever, or stored in a retrieval system or transmitted, without express permission of the Author-publisher, except in case of brief quotations with due acknowledgement.

**A Fortiori Logic** can be freely read online at [The Logician.net](http://TheLogician.net) and in various other locations.

It can be purchased, in print and e-book editions, in [Amazon.com](http://Amazon.com), [Lulu.com](http://Lulu.com) and many other online booksellers.

## Main references

*The following are the main references made in the present work. There are, in fact, many more references scattered throughout the present volume, in footnotes. The selection of some under the heading of ‘main references’ is somewhat subjective. References made through Internet links, even if important are usually excluded here – as are references to works quoted rather incidentally. Note that this listing is not intended as a bibliography, at least not as an exhaustive one.*

Abitbol, Gabriel. *Logique du droit talmudique*. Paris: Sciences Hébraïques, 1993.

M. Abraham, D. Gabbay and U. Schild.

- *Analysis of the Talmudic Argumentum A Fortiori Inference Rule (Kal-Vachomer) using Matrix Abduction*. *Studia Logica* 92(3): 281-364 (2009).
- “מידות הדרש ההגיוניות כאבני הבסיס להיסקים לא דדוקטיביים” (Hebrew paper on Kal-Vachomer). In *BDD Journal*. Ramat Gan: Bar Ilan University, 2009.

Al-Ghazali, Abu Hamid. *Al-Qistas al-mustaqim* (The Correct Balance). Trans. Richard J. McCarthy. Boston: Twayne, 1980.

Aristotle. *The Works of Aristotle*. Ed. William David Ross. Chicago: Encyclopædia Britannica, 1952.

Arnaldez, Roger. *Fakhr al-Din al-Razi: Commentateur du Coran et philosophe*. Paris: Vrin, 2002.

Avraham, Michael. “The ‘Kal Vachomer’ as a Syllogism – Arithmetic Model” (in Hebrew). In: *Higayon: Studies in Rabbinic Logic* (vol. 2, pp. 29-46). Ed. Moshe Koppel and Ely Merzbach. Jerusalem: Aluma, 1992. Pp. 29-46.

Bar-Zeev, Haï. *Une lecture juive du Coran*. Paris: Berg, 2005.

Bergman, Rabbi Meir Zvi. *Gateway to the Talmud*. New York: Mesorah, 1985.

Blau, Lajos. “*Die hermeneutische Analogie in der talmudischen Litteratur* par Adolf Schwarz” (Vienna, 1897). In: *Revue des Études Juives*, Tome 36<sup>e</sup>. Paris: Durlacher, 1898. Pp. 150-159.

Brachfeld, Meir. “A Formal Analysis of Kal VaChomer and Tsad Hashaveh (Common Element).” In: *Higayon: Studies in Rabbinic Logic* (vol. 2, pp. 47-55). Ed. Moshe Koppel and Ely Merzbach. Jerusalem: Aluma, 1992.

Chavel, Charles B. *Encyclopedia of Torah Thoughts*. New York: Shilo, 1980.

Cicero, Marcus Tullius. *Topica*. Trans. H. M. Hubble. Cambridge, Mass. Harvard UP, 1949.

Daube, David. "Rabbinic Methods of Interpretation and Hellenistic Rhetoric." In: *HUCA*, Vol. XXII, pp. 239-264.

*Encyclopaedia Judaica*. Ed. Michael Berenbaum and Fred Skolnik. 2<sup>nd</sup> ed. Detroit: Macmillan Reference USA, 2007.

*Encyclopédie Philosophique Universelle*. Paris: PUF, 1990.

*Encyclopaedia Talmudit* (vol. 7, in Hebrew). Jerusalem: Yad HaRav Herzog Torah Institute, 1990.

Feigenbaum, R. Yitzchak. *Understanding the Talmud: A Systematic Guide to Talmudic Structure and Methodology*. 2nd rev. ed. Jerusalem: Darche Noam, 1988.

Freely, John. *Aladdin's Lamp: How Greek Science Came to Europe Through the Islamic World*. New York: Knopf, 2009.

Goltzberg, Stefan. "The A Fortiori Argument In The Talmud." In: *Judaic Logic*, ed. Andrew Schumann. Piscataway, N.J.: Gorgias, 2010.

Guggenheimer, Heinrich. "Logical Problems in Jewish Tradition." In: *The Great Society: Confrontations with Judaism. A Symposium*. Ed. Philip Longworth. London: Blond, 1966.

Hallaq, Wael B. *A History of Islamic Legal Theories: An Introduction to Sunni usul al-fiqh*. Cambridge: UP, 1997.

Hirschfeld, H. S. *Halachische Exegese*. Berlin: Athenaemum, 1840.

Jacobs, Louis.

- *Studies in Talmudic Logic and Methodology*. London: Vallentine Mitchell, 1961. (Republished as paperback, 2006.)
- "The Talmudic Argument." In: *Essential Papers on the Talmud*. Ed. Michael Chernick. New York: NYU Press, 1994. Pp. 429-460. (Orig. ed. 1984.)
- *The Jewish Religion: A Companion*. Oxford: OUP, 1995.
- *Rabbinic Thought in the Talmud*. London: Vallentine Mitchell, 2005.

*Jewish Encyclopedia, The*. 2<sup>nd</sup> ed. New York: Funk & Wagnalls, 1901-1906. (Available Online.)

Joseph, Horace William Brindley. *Introduction to Logic*. 2nd ed. rev. Oxford: Clarendon, 1916.

Kneale, William and Martha. *The Development of Logic*. Oxford, London: Clarendon, 1962.

*Koran, The*. Trans. by John Medows Rodwell (1861, 2nd ed. 1876). Seattle, Wash.: Kindle, 2011.

Kunst, Arnold. "An Overlooked Type of Inference." In: *Bulletin of the School of Oriental and African Studies*, Vol. X, Part 4, pp. 976-991 (1942).

Lalande, Pierre André. *Vocabulaire technique et critique de la philosophie*. Paris: Alcan, 1926. Paris: PUF, 1972.

Lenartowicz, Piotr SJ, and Jolanta Koszteyn. "On Paley, Epagogé, Technical Mind and A Fortiori Argumentation." In: *Forum Philosophicum Facultas Philosophica Ignatianum Cracovia – Cracow*, 7: 2002, 49-83.

Lieberman, Saul. "Rabbinic Interpretation of Scripture." In: *Essential Papers on the Talmud*. Ed. Michael Chernick. New York: NYU Press, 1994. Pp. 429-460. (Orig. ed. 1950.)

Lifshitz, Avraham. "Kal VaChomer and its Disprovings" (in Hebrew). In: *BDD (Bekhol Derakhekha Daehu)* journal No. 24 (2011), pp. 73-88.

Luzzatto, R. Moshe Chaim.

- *The Ways of Reason* (orig. *Derech Tevunot*). Trans. Rabbis D. Sackton and Ch. Tscholkowski. Jerusalem: Feldheim, 1989.
- *The Book of Logic* (orig. *Sepher haHigayon*). Trans. Rabbis D. Sackton and Ch. Tscholkowski. Jerusalem: Feldheim, 1995.

Maccoby, Hyam. *The Philosophy of the Talmud*. London: Routledge Curzon, 2002.

Maïmonide. *Terminologie Logique (Makala fi sana'at al mantik – Milot haHigayon)*. Trans. Moise Ventura. Paris: Librairie Philosophique, 1935. Reprinted, 1982.

Maimonides, Moses.

- *Guide for the Perplexed*. Trans. M. Friedlander. 2nd ed. New York: Pardes, 1946.
- *Commentary on the Mishnah*. Trans. Fred Rosner. Northvale, N.J.: Aronson, 1995.

Marraud, Hubert. “From a Stronger Reason.” Draft paper (14p), posted online at The Logic Forum (2012) and possibly later published in the Spanish journal *Theoria*.

Mielziner, Moses. *Introduction to the Talmud*. 3<sup>rd</sup> ed. New York: Bloch, 1925. (1<sup>st</sup> ed. 1894.)

Neusner, Jacob. *Rabbinic Literature: An Essential Guide*. Nashville, Tenn.: Abingdon, 2005.

*New Testament, The Revised King James*. Ed. Brad Haugaard. Seattle, Wash.: Kindle, 2008.

Ostrovsky, Moshe. *Hamidot She-haTora Nidreshet Bahem (The Rules that the Torah Requires)*. Jerusalem: K. I. Milman, 1924/5.

Philon d’Alexandrie. *De Fuga et Inventione*. Trans. Esther Starobinski-Safran. Paris: Cerf, 1970.

Plato. *The Dialogues of Plato*. Trans. Benjamin Jowett. 3rd ed. Oxford: Oxford UP, 1892.

Porton, Gary G.

- “Rabbinic Midrash.” In: *Judaism in Late Antiquity*. Ed. Jacob Neusner. Leiden, Netherlands: Brill, 1995. Pp. 217-238.
- “Rabbi Ishmael and his Thirteen Middot.” In: *New Perspectives on Ancient Judaism Volume One: Religion, Literature, and Society in Ancient Israel Formative Christianity and Judaism*. Ed. J. Neusner, P. Borgen, E. S. Frerichs, and R. Horsley. Lanham: University Press of America, 1987. Pp. 3-18.

Radhakrishnan, Sarvepalli and Charles A. Moore, eds. *A Sourcebook of Indian Philosophy*. Princeton, N.J.: Princeton UP, 1957.

Ravitsky, Aviram. “Aristotelian Logic and Talmudic Methodology: The Commentaries On The 13 Hermeneutic Principles And Their Application Of Logic.” In: *Judaic Logic*. Ed. Andrew Schumann. Piscataway, N.J.: Gorgias, 2010.

Rodkinson, Michael L. *The Rodkinson Edition of the Babylonian Talmud (1903)*. Seattle, Wash.: Kindle, 2003.

Saadia Gaon.

- *Œuvres Complètes de R. Saadia ben Iosef al-Fayyumi* (vol. 9). Ed. Joseph Derenbourg, Hartwig Derenbourg and Mayer Lambert. Paris : Leroux, 1897.
- *The Book of Doctrines and Beliefs*. Abridged ed. Trans. Alexander Altmann. Oxford: Phaidon, date not specified.
- *The Book of Beliefs and Opinions* (Vol. I). Trans. Samuel Rosenblatt. New Haven: Yale, 1948.

Samely, Alexander. *Rabbinic Interpretation of Scripture in the Mishnah*. Oxford: Oxford UP, 2002.

Sartor, Giovanni. *Legal Reasoning: A Cognitive Approach to the Law*. (Volume 5 of *A Treatise of Legal Philosophy and General Jurisprudence*. Chief ed. Enrico Pattaro.) Dordrecht, Netherlands: Springer, 2005.

Schumann, Andrew (Ed.).

- *Logic in Religious Discourse*. Heusenstamm: Ontos Verlag, 2010.
- *Judaic Logic*. Piscataway, N.J.: Gorgias, 2010.
- *History and Philosophy of Logic*. London: Taylor & Francis, 2011.

Schwarz, Adolf. *Der Hermeneutische Syllogismus in der Talmudischen, Litteratur*. Vienna: Israelitisch-Theologischen Lehranstalt, 1901.

Sion, Avi. *Judaic Logic: A Formal Analysis of Biblical, Talmudic and Rabbinic Logic*. Geneva: Slatkine, 1997. (1st ed. 1995.)

*Soncino Babylonian Talmud*. Ed. Rabbi Dr. I. Epstein. London: Soncino, 1952.

*Soncino Chumash: The Five Books of Moses with Haphtaroth*. Ed. Cohen, Rev. Dr. A. 2nd ed. London: Soncino, 1983.

Steinsaltz, R. Adin.

- *The Talmud. The Steinsaltz Edition. A Reference Guide*. Trans. and ed. R. Israel V. Berman. New York: Random, 1989.
- *The Essential Talmud*. Trans. Chayah Galai. New York: Basic, 2006.

Strack, H. L. et Stemberger, G. *Introduction au Talmud et au Midrash*. Trans. et adapt. Maurice-Ruben Hayoun. Paris: Cerf, 1986.



*Talmud Bavli: Tractate Bava Kamma.* Vol. I. Ed. R. Yisroel Simcha Schorr. Brooklyn, N.Y.: Mesorah, 2010.

*Talmud Bavli: Tractate Kiddushin.* Vol. I. Ed. R. Hersh Goldwurm. Brooklyn, N.Y.: Mesorah, 2010.

Ury, Yisrael. *Charting the Sea of Talmud: A Visual Method for Understanding the Talmud.* Jerusalem: Mosaica, 2012.

Villanova, Ron. *Legal Methods: A Guide For Paralegals And Law Students.* Coral Springs: Llumina, 1999.

Wiseman, Allen Conan. *A Contemporary Examination of the A Fortiori Argument Involving Jewish Traditions.* Waterloo, Ont.: University of Waterloo, 2010.