Voting and Information Aggregation

Theories and Experiments in the Tradition of Condorcet

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for my parents

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Contents

Introduction

This thesis consists of three independent papers that offer a justification for the use of plurality rule as an optimal way to aggregate information for societies composed of individuals with common interests but diverse information.

The motivation of this thesis follows a line of research in social choice that dates back to the French mathematician and political philosopher Jean-Antoine-Nicolas de Caritat, Marquis de Condorcet (1743-1794). In his attempt to provide support for the existence of a rationally determined order in the human world, Condorcet argued that all events in human life and behavior could be actually predicted using the calculus of probabilities. In his *Essai sur l'application de l'analyse à la probabilité des decisions rendues à la pluralité des voix* (1785), Condorcet applied this idea to the area of political decision making. He posited that social justice would be secured if nations would adopt political constitutions that facilitate accurate group judgments, and argued that the majority rule would be the most likely constitutional tool to achieve this goal. Condorcet also analyzed the probabilities of a tribunal's reaching a correct verdict, and used this to discuss the qualifications of judges and the best form of a jury for a fair trial.

Condorcet's results have been rediscovered by Black (1958), who used them to formulate what is now known in the social choice literature as the Condorcet Jury Theorem, and later by Young (1988, 1995). However, Young provided a deeper analysis by reinterpreting, correcting and extending Condorcet's results. Since then, there have been many extensions, generalizations and versions of the Condorcet Jury Theorem. Piketty (1999) wrote a survey that summarizes the most important results of these works.

Following this line of research, the first paper of this thesis discusses the conditions under which plurality rule provides the society with the most likely method to reach accurate group judgments, when more than two choices are possible[1]. In this paper, as in Condorcet's work, it is assumed that voters act honestly.

[1] Some time after finishing the first paper, I become aware of the work of Paroush and Ben-Yashar ("Optimal Decision Rules for Fixed-Size Committees in Polychotomous Choice Situations", *Social Choice and Welfare*, Vol. 18, 4, 2001, 737-746), who studied a similar model with the one proposed here and with whom I coincide on some of the results.

However, natural developments in the theory of voting, that brought in the issues of incentives and strategic interaction in group decision making, were used to challenge this assumption. Austen-Smith and Banks (1996) were the first to notice that the combination of private information and common interests in the framework proposed by Condorcet might create an incentive for voters to act strategically. This observation led them to ask the question of whether honest voting is compatible with the Nash equilibrium behavior in the game induced by majority rule. This issue was also taken up by other authors. The work of Feddersen and Pesendorfer (1997) was especially relevant for this thesis. One of the conclusions which emerged from this literature was that optimal information aggregation is sensitive to the voters' behavior. The second paper of this thesis advances this problematic by studying voters' behavior in the game induced by plurality rule.

The interest in real-world institutions, for which voting is an important element, raised for some time the question of whether voters behave as predicted by the theoretical models. Another question was of how to deal with the complexity of the strategic environment. The second paper of this thesis calls for answers to these types of questions. Since the literature on voting experiments seems to provide reasonable answers to these questions, the third paper of this thesis uses laboratory experiments to test the implications of the second paper. In doing so, it extends the work of Ladha, Miller and Oppenheimer (1996) and Guarnaschelli, McKelvey and Palfrey (2000), who looked for empirical evidence of strategic behavior in Condorcet-type models.

Chapter 1

Optimal Group Decision Rules: Plurality and Its Extensions

Abstract

This paper offers a justification for the use of plurality rule as an optimal way to aggregate information for societies composed of individuals with common interests but with private information. The paper follows and extends a line of research that was introduced by Condorcet (1785), and which is known in the social choice literature as the Condorcet Jury Theorem. This allows for a full characterization of plurality rule and extensions of this rule, such as the generalized weighted plurality rule, as optimal ways to make decisions in uncertain situations. Further, it offers Condorcet's result as a special case.

1. Introduction

One argument for the use of voting in making decisions in the presence of uncertainty is that the society is more likely to give correct answers than any individual is. More precisely, if individuals in the society have a common interest but they have private information, then voting may provide the society with mechanisms of aggregating information that lead to optimal decisions.

This idea inspired one of the earliest models of voting that dates back to Condorcet (1743-1794). In his *Essai sur l'application de l'analyse à la probabilité des decisions rendues à la pluralité des voix* (1785), Condorcet argued that if the objective of the society is to find the "truth" or the "best" choice in a certain situation, but individuals in the society make sometimes errors in their judgments about what is the "truth" or what is "best", then voting by majority is the most likely method to give optimal results.

Condorcet provided the following examples of situations for which his claim would work:

- A ruling assembly deciding about establishing a new law. The objective of the assembly is to make decisions that are in the best interest of the society. However, the members of the assembly may judge differently the consequences of the new law for the society.
- A trial by jury, where the truth is either the innocence or the guilt of a defendant. All members of the jury want to discover the truth, but each of them may interpret differently the evidence during the trial.

To justify his assertion, Condorcet explored the practical use of the calculus of probabilities. More precise, his argument was as follows. A group of voters has to choose between two states of affairs, one of which is objectively correct (for example, innocence in a trial by jury). Voters have a common interest, namely, they all want to determine the correct state. To make a decision, voters register their opinions that reflect their judgments about which of the two states is more likely to be correct. Each voter is more probable to vote for the correct state than for the incorrect one. Further, voters have the same probability of making the correct choice. Under these conditions, Condorcet proved that if voters make their choices independently, then the choice with most votes is the one most likely to be correct. In other words, the majority rule is a statistically optimal method for aggregating voters' opinions about a fact. Condorcet also proved a precursor of the statistical fact now known as the law of large numbers, showing that the decisions made by majority voting tend to become a complete certainty as the number of voters becomes large. Condorcet's findings have been rediscovered by Black (1958), who used them to formulate what is now known in the social choice literature as the Condorcet Jury Theorem.

Following Condorcet's analysis, this paper addresses a more difficult question: which rule is most likely to produce a correct or best result when there are more than two states or choices? When extending Condorcet's reasoning to the case of more than two choices, matters become more complicated. This is because when there are more than two choices one can address two types of questions. One is to determine which is the most likely rule to give the correct choice; the second one is to find the ranking of choices that is most likely to be correct, assuming that one exists. When there are only two choices, the answer to the second question provides an answer to the first one: the choice that is most likely to be correct is the top-ranked choice in the ranking most likely to be correct. One may think that this also holds for the case of more than two choices. However, Young (1988, 1995)[1] argued that this does not hold in most cases. He provided an example where the most likely to be the correct ranking is given by Condorcet's pairwise comparisons method, whereas the most likely to be the correct choice is given by the Borda scoring method.

The aim of this paper is to provide an answer to the first question: which rule is most likely to select the correct or best choice from a set of choices containing more than two elements? However, even in this case, matters are not simple. This is because when there are more than two choices there is a variety of voting rules that may recommend different choices. Just to give some examples, consider plurality rule, the Condorcet's rule, the Borda's rule, approval voting, etc. For the case of two choices, these rules will all predict the same outcome. The problems arise when there are more than two choices, as different rules may predict different outcomes. Since a natural extension of majority

[1] In the above-mentioned papers, Young provided an excellent review and interpretation of the results in Condorcet's *Essai*.

rule is plurality rule, the concern here is to find out when is it that the use of plurality rule is optimal to aggregate information.

The framework used in this paper is one in which a decision maker has to find the optimal decision rule to aggregate the information provided by a group of individuals with common interests, but, with different opinions about the correct or the best choice and possibly with different expertise or abilities to identify this choice. Some examples[2] are as follows:

- The head of an academic department asks its members' opinions on which of several possible candidates to employ. All members in the department agree that the best candidate would be the one that will publish most in the future. However, they have different opinions about the potential of each candidate.
- The president of a company believes that demand for one of its products is going to increase. He asks the executive directors of the company to estimate which of the available policies is best at meeting this new level of demand. Although they are on the president's side, the executive directors disagree about the policy that is most likely to meet this goal.

It is assumed that the decision maker is a Bayesian and so has beliefs on the set of possible choices. Before he needs to make a decision, the decision maker learns that he can get more information from a group of experts. It is assumed that each expert receives a private signal about the true state of the world and that the signals are independent draws from a state-dependent distribution. The probability with which an expert receives the correct signal, i.e., the signal corresponding to the true state of the world, will give his ability or expertise. In other words, these probabilities parameterize the informativeness of the experts' signals. The probabilities are subjectively provided by the decision maker and based on his beliefs about the experts' abilities. The decision maker requires the experts to vote for one of the available choices and then uses these votes as additional information to update his prior beliefs. Finally, the choice that is most likely to be correct will be the one that maximizes his posterior probability. Under certain conditions, this choice is the one that has more votes, that is the outcome chosen by plurality rule. This finding allows for a full characterization of plurality rule as an

[2] The examples proposed by Condorcet could also be extended to more than two choices. For instance, in the jury context, one could consider a guilty party among several innocent ones.

optimal way to aggregate information. Specifically, Proposition 1 says that plurality rule is the optimal way to aggregate information if and only if the experts' ability parameter is higher than an expression that depends on the number of choices and on the prior beliefs of the decision maker. When the decision maker's prior belief is well biased in one direction, the experts' ability parameter needs to be at least as high as this belief in order for plurality rule to aggregate information optimally. An interesting feature of the condition of Proposition 1 is that it does not depend on the number of experts. Thus, whenever the condition of Proposition 1 holds, plurality rule is optimal even when the number of experts becomes large. In addition, the Condorcet Jury Theorem is obtained as a Corollary of the Proposition 1. These results hold for the case where all experts have the same probabilities to identify the correct choice. When this assumption is relaxed, the optimal way to aggregate information is given by the generalized weighted plurality rule. This rule assigns to each choice a score consisting of the sum of two terms. The first term depends on the decision maker's beliefs and on the product between the plurality score of that particular choice. The second term depends on the experts' probabilities of identifying the correct choice. The generalized weighted plurality rule selects the choice with the highest score. Theorem 2 gives a full characterization of the generalized weighted plurality rule as an optimal way to aggregate information. It would be interesting to find the conditions under which plurality rule is optimal in this context. This is left for further research. However, an example is given suggesting that plurality rule will be optimal if the probabilities of different experts' to make the correct choice are similar.

Related literature

There have been many extensions, generalizations and versions of the Condorcet Jury Theorem. Some examples include Nitzan and Paroush (1982, 1984), Shapley and Grofman (1984), Young (1986, 1988, 1995). However, while the problem introduced in this paper is similar in spirit with the one in the literature mentioned above, there is a difference in the approach as well as in the interpretation of the concepts. Apart from the assumption of a Bayesian decision maker, that distinguishes the present analysis from the classical one, three other aspects deserve attention. First, the analysis in this paper extends the previous results by allowing for more than two choices. Condorcet himself allowed for more than two choices and argued that the most likely to be the correct choice would be given by the majority choice in pairwise comparisons.

However, the information (data) required to derive the optimal rule in this paper is different from the one used by Condorcet. In his model, individuals had to compare the choices paiwisely, whereas here, the experts have to vote only for one choice. In addition, Young proved that Condorcet was actually mistaken and argued that if the data used consisted in pairwise votes, then the most likely to be the correct choice would be given by the Borda rule. Second, the analysis in this paper allows asymmetry among alternatives. Although Nitzan and Paroush (1982, 1984), and Shapley and Grofman (1984), had an attempt to model this, their analysis was for two choices only. Finally, the concept of the optimal decision rule is different. For instance, Nitzan and Paroush (1982, 1984) defined the optimal decision rule as the rule that maximizes the expected utility of the group. Young (1985) proposed that the optimal decision rule[3] should select that choice that maximizes a likelihood function. In the present paper, the optimal decision rule is defined as the rule that selects the choice that maximizes the decision maker's posterior probability of selecting the correct choice.

Another line of research analyzed whether Condorcet's result continues to hold when the game induced by the majority rule is played by rational individuals. Research in this direction includes Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1998), McLennan (1998), Myerson (1994), Wit (1998) and Coughlan (2000). These works studied strategic aspects of information aggregation for the 2-choice case, corresponding to Condorcet's result. A similar approach is used in Rata (2001) to analyze the strategic aspects of information aggregation for the framework proposed in the present paper. Finally, Ladha, Miller and Oppenheimer (1996), Guarnaschelli, McKelvey and Palfrey (2000) and Brandts and Rata (2002) offered experimental results justifying some of the game theoretical predictions.

The rest of the paper is organized as follows. Section 2 introduces the uncertain multiple choice model. Section 3 gives the main result of the paper, namely, a characterization of plurality rule as an optimal way to aggregate information. Section 4 gives the conditions under which the generalized weighted plurality rule is optimal. The last section concludes.

[3] For the case of equal priors, the concept used here coincides with the one proposed in Young; this is because his approach is similar to Bayesian inference with uniform priors.

2. The Uncertain Multiple Choice Model

Choices and States of the World

Consider *a finite set of choices* $A = \{a_1, a_2, ..., a_m\}$. Among these choices there is one that is *correct*; all the other choices are incorrect. In the examples provided in the Introduction, this would correspond to: a good candidate or a good policy among bad ones, a guilty party among innocents, etc. There is a set of possible *states of the world* denoted with $S = \{s_1, s_2, ..., s_m\}$, where the state s_i corresponds to the choice a_i being correct.

Decision Maker

Now there is an individual, the *decision maker* in what follows, whose purpose is to identify the correct choice or the true state of the world. In the examples given in the Introduction the role of the decision maker was played by the head of the academic department and the president of the company.

Beliefs

Although the decision maker does not know the true state of the world for certain, he is a Bayesian and, therefore, has *beliefs* about its identity. These beliefs are given by the decision maker's prior probability: $p_s(\cdot)$[4]. For simplicity, the decision maker's prior that the system will be in the state s_i, $p_s(s_i)$, is denoted by p_i, i.e., $p_s(s_i) = p_i$.

Without loss of generality, it is assumed that $1 > p_1 > p_2 > ... > p_m > 0$.

Experts

Before he needs to make a decision, the decision maker learns that he can get more information by asking the advice of *a group of experts* $N = \{1,2,...,n\}$. In the examples, the experts were the members of the department and the executive directors of the company.

The experts are characterized by their *expertise* or *abilities* to identify the correct choice. A more formal definition of these abilities is provided shortly.

[4] According to the subjectivist school of probability, a rational man should organize his beliefs in a way that it is possible to represent those beliefs by probabilities.

The decision maker asks each expert in the group N to register his opinion on the choices in the set A. More precise, each expert has to vote for (abstentions are ruled out) one choice in the set A.

The *vote* of an expert t is denoted by x^t, where $x^t \in \{a_1, a_2, ..., a_m\}$.

If the decision maker would not have additional information, he would select the choice that has the highest prior probability. However, given the additional information provided by the experts, he can do better. The decision maker performs an informative experiment to observe the experts' opinions.

Data-generation Process

The *data-generation process* can be represented in this case by a multinomial distribution. To see this, consider the following random variable:

$$X^t = i \text{ if expert } t \text{ votes for the choice } a_i.$$

For each $t \in N$, denote by $q_{ik}^t = \Pr(X^t = i / s_k)$ with $i, k \in \{1,2,...,m\}$, the probability with which an expert t votes for the choice a_i when the true state of the world is s_k. It should be noted that $\sum_{i=\overline{1,m}} q_{ik}^t = 1$ for any individual i and given the state s_k. These probabilities are interpreted as the expertise or abilities of the experts; they are subjectively provided by the decision maker and based on his beliefs about the experts' abilities. The decision maker assumes that the values of the q_{ij}^t's are statistically independent and that are the same for all experts, that is, $q_{ij}^t = q_{ij}^l = q_{ij}$ for any t, $l \in N$, with $t \neq l$. He also assumes that $q_{ij} = q$ whenever $i = j$ and $q_{ij} = \frac{1-q}{m-1}$ for any i and j such that $i \neq j$. Finally, the decision maker assumes that $q > \frac{1}{m}$.

The following holds, $\sum_{i=\overline{1,m}} q_{ij} = 1$ for any j.

Consider now the random variables X_1, X_2,..., X_m, where:

$$X_k = \left|\{t \in N / X^t = k\}\right|^5.$$

[5] $|S|$ denotes the number of elements in the set S.

Now if it is to interpret the independent votes of the n experts in the group N as a sequence of n independent trials of an experiment that has m outcomes, $a_1,...,a_k,...,a_m$, with corresponding probabilities given by $\frac{1-q}{m-1}$,..., q,..., $\frac{1-q}{m-1}$ and frequencies X_1,..., X_k,..., X_m, then the m-dimensional random variable X = $(X_1, X_2,..., X_m)$ will be m-nomial distributed. Furthermore, for a particular sequence of n trials with $X_1 = x_1$, $X_2 = x_2$,...., $X_m = x_m$, the probability function of the m-nomial distribution function will be given by:

$$p_X(x/s_k) = \frac{n!}{\prod\limits_{i=1,m} x_i!}\left(\frac{1-q}{m-1}\right)^{x_1}\left(\frac{1-q}{m-1}\right)^{x_2}...q^{x_k}...\left(\frac{1-q}{m-1}\right)^{x_m} = \frac{n!}{\prod\limits_{i=1,m} x_i!}q^{x_k}\left(\frac{1-q}{m-1}\right)^{n-x_k},$$

where $x = (x_1, x_2,...., x_m)$.

The probability $p_X(\cdot/s_i)$ is interpreted as describing the decision maker's belief in the quantity X before the experiment is performed given that he imagines that he knows which is the true state of the world.

Revision of Beliefs

If he then observes X = x, the decision maker simply updates his prior belief through Bayes' theorem:

$$p_s(s_k\ /x) = \frac{\frac{n!}{\prod\limits_{i=1,m} x_i!}q^{x_k}\left(\frac{1-q}{m-1}\right)^{n-x_k}\cdot p_k}{\sum\limits_{l=1,m}\frac{n!}{\prod\limits_{i\in 1,m} x_i!}q^{x_l}\left(\frac{1-q}{m-1}\right)^{n-x_l}\cdot p_l} = \frac{q^{x_k}\left(\frac{1-q}{m-1}\right)^{n-x_k}\cdot p_k}{\sum\limits_{l=1,m} q^{x_l}\left(\frac{1-q}{m-1}\right)^{n-x_l}\cdot p_l}.$$

The *posterior distribution* $p_s(\cdot/x)$ represents the decision maker's belief about s after observing X = x.

Optimal Decision Rules

A *decision rule* is given by *d:* $X \rightarrow 2^A \backslash Ø$, where X is the set of voting vectors $x = (x_1, x_2,...., x_m)$.

An *optimal decision rule* is

$$d^*(x) = \{a_k \in A : p(s_k\ /x) \geq p(s_i/x), \mathit{for\ all}\ s_i \in S\}.$$

In other words, the optimal decision rule selects the choice(s) corresponding to the most likely to be the true state(s) of the world, i.e., the state(s) maximizing the decision maker's posterior probability.

The purpose in what follows is to give a natural interpretation and characterization of plurality rule as an optimal decision rule.

3. Plurality Rule as an Optimal Decision Rule

The following result provides the optimal decision rule for the model proposed here. This result is further used to find the conditions under which the optimal decision rule is plurality rule.

Theorem 1: The optimal decision rule is the ***weighted plurality rule***, that is, the rule that selects the choice(s) a_k with:

$$\ln p_k + x_k \ln\left(\frac{(m-1)q}{1-q}\right) \geq \ln p_j + x_j \ln\left(\frac{(m-1)q}{1-q}\right) \text{ for all } j \neq k .$$

Proof. See Appendix.

The score assigned to a choice by the weighted plurality rule is given by the sum of a *constant* (the logarithm of the decision maker's priors about each choice being correct) and *the plurality score* corresponding to that choice multiplied with some *weights* (the logarithm of the decision maker's priors about individual odds of identifying the correct choice). Given this, and for certain parameters of the model, the weighted plurality rule can be interpreted as ***plurality rule*** with ties broken in favor of the choice with highest prior. Specifically, if the decision maker's priors are almost uniformly distributed on the set of the possible states of the world, then the score associated to a choice by the weighted plurality rule does not depend on the probability *q*, as long as $q > \frac{1}{m}$, and as a result the weighted plurality rule will select the choice with highest number of votes. This is an immediate consequence of the fact that in comparing the scores of two choices that have almost equal priors the dominant factor will be the plurality score and not the priors. Moreover, in case of a tie, the weighted plurality rule will select the choice with highest prior. In other words, if the priors are not too different and $q > \frac{1}{m}$, then the optimal voting rule will be given by plurality rule with ties broken in favor of the choice with the highest prior. The intuition is simple: if two choices (or more) get the same number of votes, then the designer of the decision rule should take into account only his priors. The next result states precisely when plurality rule is optimal.

Proposition 1: Given a vector of priors $(p_1, p_2, ..., p_m)$ with $p_1 > p_2 > .... > p_m$, then ***plurality rule with ties broken in favor of the choice with highest prior*** is the optimal decision rule for the above problem iff $q > \frac{p_1}{(m-1)p_m + p_1}$.

Proof. See Appendix.

Remark 1: The condition of Proposition 1 has a very interesting feature, namely, it depends only on the extreme priors p_1 and p_m, not on the other p_i's. This is because the only relevant situations in deriving this condition are those in which the choice a_m receives the highest number of votes and the choice a_1 receives one vote less than the choice a_m. In what follows, such a situation will be called a situation of “almost ties”. The condition of Proposition 1 insures that in a situation of almost ties, plurality rule selects the choice a_m despite of its possible very low prior. This argument is made more precise in the proof of Proposition 1.

Remark 2: The condition of Proposition 1 does not depend on the number of experts n in the voting process. This can be explained by using a similar argument with the one in Remark 1. The condition does not depend on the number of experts because the only relevant situations are those in which there are almost ties. However, the total number of the experts is irrelevant for these situations. This, in turn, implies that Proposition 1 holds as the number of experts become large.

Remark 3: If $p_1 \cong p_m$ (almost equal priors), then the condition in Proposition 1 becomes $q > \frac{1}{m}$, that is, plurality rule will be generally optimal. This is a consequence of the fact that, when the priors are almost equal, the only relevant information to the decision maker is the number of votes for a choice. Further, if the prior gets stronger, namely, $\frac{p_1}{p_m}$ increases, then the expertise or ability parameter q must also increase for plurality rule to stay optimal. This says that for plurality rule to be optimal, the quality of the information q available to the experts should increase with the difference between the highest prior p_1 and the smallest prior p_m.

Corollary 1: For $m = 2$, the majority rule with ties broken in favor of the choice with higher prior is optimal iff $q > p_1$.

Proof. See Appendix.

Remark 4: For $m = 2$ and for the case in which the priors are almost equal, $p_1 \cong p_2$, the condition in Proposition 1 becomes $q > \frac{1}{2}$ and, thus it gives Condorcet's result as a particular case.

The following example helps to understand the issues discussed in this section.

Example 1: Consider 3 choices. Assume that the decision maker's priors are given by: .34, .335 and .325. Then, the optimal decision rule will be given by plurality rule with ties broken in favor of the choice with highest prior, provided that $q > .34$. To see this, notice that plurality rule is optimal if the following inequalities hold:

$$\ln .34 + x_k \ln\left(\frac{2q}{1-q}\right) < \ln .335 + (x_k + 1)\ln\left(\frac{2q}{1-q}\right)$$

$$\ln .34 + x_k \ln\left(\frac{2q}{1-q}\right) < \ln .325 + (x_k + 1)\ln\left(\frac{2q}{1-q}\right)$$

$$\ln .335 + x_k \ln\left(\frac{2q}{1-q}\right) < \ln .325 + (x_k + 1)\ln\left(\frac{2q}{1-q}\right).$$

In other words, the optimal decision rule will not select a choice with higher prior if there is another choice with higher plurality score, or number of votes, available.

The above inequalities are equivalent to:

$$\frac{.34}{.335} < \frac{2q}{1-q}, \quad \frac{.34}{.325} < \frac{2q}{1-q} \text{ and } \frac{.335}{.325} < \frac{2q}{1-q}, \text{ or } q > .34.$$

Furthermore, if the priors are given by .5, .3 and .2, then plurality rule with ties broken in favor of the choice with highest prior will be optimal only if $q > .55$. At last, for plurality rule with ties broken in favor of the choice with highest prior to be optimal when the priors are .6, .25 and .15, the ability parameter has to be $q > .66$.

The results of this section provided a formal justification for the use of plurality rule as an optimal way to make decisions for situations in which individuals have common interests. For any number of individuals in the group, the probability with which plurality rule is optimal is not necessarily high. However, whenever it is optimal, it makes the correct decision with very high probability.

4. Generalized Weighted Plurality Rule as an Optimal Decision Rule

This section relaxes the assumption that experts have equal probabilities of identifying the correct choice or true state of the world. In what follows, experts in the group N will be organized into subgroups. More precisely, the experts with equal probabilities of identifying the true state of the world, q^t, will be assigned to the subgroup N_t. It is assumed that $\tilde{n}$ such subgroups are possible, with $\tilde{n} \leq n$. Denote these subgroups with $N_1,\ N_2,...,N_{\tilde{n}}$ and the size of the subgroup N_l with $n_l = |N_l|$. It is assumed that the values of the q^t's are statistically independent and that $q^t > \dfrac{1}{m}$.

Using a similar reasoning with the one in the previous section, it is possible to construct for each $l \in \{1,...,\tilde{n}\}$ the m-nomial distributed random variable $X^l = (X_1^l, X_2^l,..., X_m^l)$, where:

$$X_k^l = \left|\left\{t \in N_l \,/\, X^t = k\right\}\right|.$$

Then, for a particular sequence of n_l trials with $X_1^l = x_1^l$, $X_2^l = x_2^l$,..., $X_m^l = x_m^l$, the probability function of the m-nomial distribution function will be given by:

$$p_{X^l}(\ x^l / s_k\ ) = \frac{n_l!}{\prod\limits_{i \in \{1,...m\}} x_i^l!}\left(q^l\right)^{x_k^l}\left(\frac{1-q^l}{m-1}\right)^{n_l - x_k^l},$$

where $x^l = (x_1^l, x_2^l,....,x_m^l)$.

Then from the independence assumption, it follows that:

$$p_X(x/s_k) = \prod_{l=1,\tilde{n}} \frac{n_l!}{\prod_{i=1,m} x_i^l!} \left(q^l\right)^{x_k^l} \left(\frac{1-q^l}{m-1}\right)^{n_l - x_k^l} = \frac{\prod_{l=1,\tilde{n}} n_l!}{\prod_{l=1,\tilde{n}} \prod_{i=1,m} x_i^l!} \prod_{l=1,\tilde{n}} \left(q^l\right)^{x_k^l} \left(\frac{1-q^l}{m-1}\right)^{n_l - x_k^l} .$$

The posterior probability becomes:

$$p_s(s_k/x) = \frac{\frac{\prod_{l=1,\tilde{n}} n_l!}{\prod_{l=1,\tilde{n}} \prod_{i=1,m} x_i^l!} \prod_{l=1,\tilde{n}} \left(q^l\right)^{x_k^l} \left(\frac{1-q^l}{m-1}\right)^{n_l - x_k^l} p_k}{\sum_{j=1,m} \frac{\prod_{l=1,\tilde{n}} n_l!}{\prod_{l=1,\tilde{n}} \prod_{i=1,m} x_i^l!} \prod_{l=1,\tilde{n}} \left(q^l\right)^{x_j^l} \left(\frac{1-q^l}{m-1}\right)^{n_l - x_j^l} p_j} .$$

The optimal decision rule for this problem will select the choice(s) a_k corresponding to the most probable state(s) of the world, i.e., the state(s) for which the above expression is maximal[6]:

$$\prod_{l=1,\tilde{n}} \left(q^l\right)^{x_k^l} \left(\frac{1-q^l}{m-1}\right)^{n_l - x_k^l} p_k .$$

Theorem 2: The decision optimal decision rule is the ***generalized weighted plurality rule***, that is, the rule that selects the choice(s) a_k that satisfies:

$$\ln p_k + \sum_{l=1,\tilde{n}} x_k^l \ln\left(\frac{(m-1)q^l}{1-q^l}\right) \geq \ln p_j + \sum_{l=1,\tilde{n}} x_j^l \ln\left(\frac{(m-1)q^l}{1-q^l}\right) \text{ for all } j \neq k .$$

Proof: See Appendix.

Further, it would be interesting to find the conditions under which plurality rule is the optimal decision rule in this more complex framework. However, given the high number of parameters in this section, this is a difficult task. Nonetheless, the following example provides some hints in this direction.

Example 2: Consider *8* experts and m choices. The experts are organized in three groups according to their ability parameter, which is given by one of the following three probabilities: q^1, q^2 and q^3. Without loss of generality, it is assumed that these

[6] This expression is proportional to the decision maker's posterior probability.

probabilities can be ordered as follows: $q^1 \geq q^2 \geq q^3$. Then, the score associated to a choice a_i by the generalized weighted plurality rule is:

$$\ln p_i + x_i^1 \ln\left(\frac{(m-1)q^1}{1-q^1}\right) + x_i^2 \ln\left(\frac{(m-1)q^2}{1-q^2}\right) + x_i^3 \ln\left(\frac{(m-1)q^3}{1-q^3}\right).$$

For plurality rule to be optimal, the selected choice needs to have the highest number of votes. As in the previous section, this will require a comparison of the scores of the "extreme" choices - a_1 and a_m - in situations of almost ties. In the example considered here, such a situation would arise for $x_m^3 = 4$ and $x_1^1 = 3$, meaning that there are four experts with (the lowest) ability q^3 voting for the least probable choices a_m and three experts with (the highest) ability q^1 voting for the most probable choice a_1. Thus, plurality rule is optimal iff:

$$\ln p_m + 4\ln\left(\frac{(m-1)q^3}{1-q^3}\right) > \ln p_1 + 3\ln\left(\frac{(m-1)q^1}{1-q^1}\right).$$

After rearranging terms, the above inequality becomes:

$$\frac{p_1}{p_m} < (m-1)\left(\frac{q^3}{1-q^3}\cdot\frac{1-q^1}{q^1}\right)^3 \cdot \left(\frac{q^3}{1-q^3}\right),$$

or, alternatively,

$$q^3 > \frac{p_1}{(m-1)\left(\frac{q^3}{1-q^3}\cdot\frac{1-q^1}{q^1}\right)^3 \cdot p_m + p_1}.$$

Since $q^1 \geq q^3$, it follows that $\frac{q^3}{1-q^3}\cdot\frac{1-q^1}{q^1} \leq 1$. Further, since $1 < \frac{p_1}{p_m}$, for the above inequality to be satisfied, the expression $\frac{q^3}{1-q^3}\cdot\frac{1-q^1}{q^1}$ must be very close to 1. However, this happens only if $q^1 \cong q^3$, namely, the abilities of the individuals are very similar to each other. In this case, the above inequality will take a similar form to the one found in Proposition 1. In conclusion, Example 2 suggests that plurality rule will be optimal if the ability parameters will be very similar to each other and if the smallest ability parameter is not too small.

5. Conclusions and Further Research

The paper has followed and extended a line of research that has been introduced by Condorcet (1785). Specifically, the framework used in this paper was one in which a Bayesian decision maker had to find the optimal decision rule to aggregate the information provided by a group of individuals with common interests, but, with different opinions about the correct or the best choice and possibly with different expertise or abilities to identify this choice. This has led to full characterizations of plurality rule and extensions of this rule, such as, the generalized weighted plurality rule, as optimal rules to aggregate experts' opinions.

One assumption in this model has been that the experts vote in an informative and objective manner. This is equivalent to assuming that an expert' choice depends only on his private information, and does not depend upon the choices of the other experts present in the decision making process. In game theoretical language, experts are taken to vote non-strategically. The incentive to vote strategically may arise because an expert's vote only matters when his vote is pivotal and because the information available to the other experts is relevant for the expert's decision. However, as already mentioned in the Introduction, there is some theoretical and experimental evidence that strategic voting will occur in the presence of private information and common interests. Austen-Smith and Banks (1996) were the first to argue that non-strategic voting in the Condorcet's Jury Theorem may be inconsistent with Nash equilibrium under general conditions. A similar result was proved by Feddersen and Pesendorfer (1998) for the specific case of jury procedures in criminal trials. In response to these results, Myerson (1994), Wit (1998) and Coughlan (2000) have attempted to find conditions under which non-strategic voting is compatible with Nash equilibrium behavior. A similar approach is used in Rata (2001) to identify conditions under which non-strategic voting is consistent with Nash equilibrium behavior for the problem considered here.

APPENDIX

Theorem 1: The optimal decision rule is the ***weighted plurality rule***, that is, the rule that selects the choices(s) a_k with:

$$\ln p_k + x_k \ln\left(\frac{(m-1)q}{1-q}\right) \geq \ln p_j + x_j \ln\left(\frac{(m-1)q}{1-q}\right) \text{ for all } j \neq k. \qquad (1)$$

Proof. Assume that the optimal decision rule selects the choice a_k. The purpose is to show that the choice a_k satisfies (1). By definition, the optimal decision rule selects the choice(s) a_k corresponding to the state(s) s_k such that:

$$p(s_k / x) \geq p(s_i / x) \text{ for all } s_i \in S .$$

Alternatively, the optimal decision rule selects the choice(s) with x_k such that:

$$q^{x_k}\left(\frac{1-q}{m-1}\right)^{n-x_k} p_k \;\geq\; q^{x_j}\left(\frac{1-q}{m-1}\right)^{n-x_j} p_j \text{ for any } j \neq k .$$

However, since the values of the variables that maximize a function also maximize monotonic transformations of it, and since the function to be maximized contains exponential terms, it is more convenient to work with the natural logarithm of this function. Therefore, taking logarithms in the above inequality and then using the properties of the logarithms, we get that:

$$x_k \ln q \;+\; (n-x_k)\ln\left(\frac{1-q}{m-1}\right) + \ln p_k \;\geq\; x_j \ln q \;+\; (n-x_j)\ln\left(\frac{1-q}{m-1}\right) + \ln p_j \text{ for any } j \neq k .$$

Notice that the term $n\ln\left(\frac{1-q}{m-1}\right)$ can be deleted since it appears in both sides of the inequality. Thus, the above inequality becomes:

$$x_k \ln q - x_k \ln\left(\frac{1-q}{m-1}\right) + \ln p_k \;\geq\; x_j \ln q - x_j \ln\left(\frac{1-q}{m-1}\right) + \ln p_j \text{ for any } j \neq k .$$

Rearranging the terms in the inequality and using the properties of the logarithms, it follows that:

$$\ln p_k + x_k \ln\left(\frac{(m-1)q}{1-q}\right) \;\geq\; \ln p_j + x_j \ln\left(\frac{(m-1)q}{1-q}\right) \text{ for any } j \neq k ,$$

which proves the result. QED

Proposition 1: Given a vector of priors $(p_1, p_2, ..., p_m)$ with $p_1 > p_2 > .... > p_m$, then ***plurality rule*** with ties broken in favor of the choice with highest prior is the optimal decision rule for the above problem iff $q > \frac{p_1}{(m-1)p_m + p_1}$.

Proof. To prove the statement of the Proposition 1, it is enough to find the values of q for which weighted plurality rule coincides with plurality rule with ties broken in favor of the choice with higher prior. The proof is done in two steps. First, the result of Theorem 1 is used to determine the conditions under which weighted plurality coincides with plurality rule with ties broken in favor of the choice with higher prior. Second, the values of q for which these conditions are satisfied will be determined.

First step For weighted plurality rule to coincide with plurality rule with ties broken in favor of the choice with higher prior, the following conditions must be satisfied:

1) if $x_j = x_k$ and $p_j > p_k$ for some j and k, and $x_j > x_i$ for all i with $i \neq j,k$, then

$$\ln p_j + x_j \ln\left(\frac{(m-1)q}{1-q}\right) > \ln p_k + x_k \ln\left(\frac{(m-1)q}{1-q}\right);$$

2) if $x_j = x_k + 1$ and $p_k > p_j$ for some j and k, and $x_j > x_i$ for all i with $i \neq j,k$, then

$$\ln p_j + x_j \ln\left(\frac{(m-1)q}{1-q}\right) > \ln p_k + x_k \ln\left(\frac{(m-1)q}{1-q}\right);$$

3) if $x_j = x_k + x$, with $x \in Z_+^*$ [7], $x > 1$, and $p_k > p_j$ for some j and k, and $x_j > x_i$ for all i with $i \neq j$, then

$$\ln p_j + x_j \ln\left(\frac{(m-1)q}{1-q}\right) > \ln p_k + x_k \ln\left(\frac{(m-1)q}{1-q}\right).$$

The first condition says that, if two choices, a_k and a_j, have the same highest number of votes, then the choice with higher prior is selected. This gives the tie-braking rule. A similar reasoning applies if there are more than two choices with the same highest number of votes. The second condition says that the choice with the highest number of votes is selected, and that a choice a_k is not selected only because it has a higher prior. To make the point clear, the priors are used only to break ties. The third condition is a generalization of the second one and, as it will be shown, it is satisfied whenever condition 2 is satisfied. In other words, conditions 2 and 3 state that the choice with highest number of votes must be selected.

Second step It consists in finding the values of q for which the above conditions are satisfied. It is easy to see that the first condition will be satisfied for any value of q. To find the values of q for which the second condition is satisfied, consider a situation in

[7] Z_+^* is the set of integers.

which there are two choices, say a_j and a_k, such that $x_j = x_k + 1$ and $p_k > p_j$, and $x_j > x_i$ for all *i* with $i \neq j,k$, then the second condition can be written as:

$$\ln p_j + (x_k + 1)\ln\left(\frac{(m-1)q}{1-q}\right) > \ln p_k + x_k \ln\left(\frac{(m-1)q}{1-q}\right).$$

After rearranging the terms, the above inequality becomes:

$$\ln\left(\frac{(m-1)q}{1-q}\right) > \ln\frac{p_k}{p_j}.$$

Since $\frac{(m-1)q}{1-q} > 1$ and $\frac{p_k}{p_j} > 1$, the inequality is equivalent to $q > \frac{p_k}{(m-1)p_j + p_k}$. But, for plurality rule to be optimal, this condition must be true for any *j* and *k* with $p_k > p_j$.

Thus, $q > \max\limits_{k,j:p_k > p_j}\left\{\frac{p_k}{(m-1)p_j + p_k}\right\} = \frac{p_1}{(m-1)p_m + p_1}$.

Finally, the third condition is satisfied whenever the second one does. To see this, it is enough to notice that:

$\ln p_j + (x_k + x)\ln\left(\frac{(m-1)q}{1-q}\right) > \ln p_j + (x_k + 1)\ln\left(\frac{(m-1)q}{1-q}\right)$ for any $x > 1$ and any two choices satisfying condition 3. This proves the result. QED

Corollary 1: For $m = 2$, the majority rule with ties broken in favor of the choice with higher prior is optimal iff $q > p_1$.

Proof. It follows directly from Proposition 1 and the remark that $q > \frac{p_1}{p_2 + p_1} = p_1$.

Theorem 2: The decision optimal decision rule is the ***generalized weighted plurality rule***, that is, the rule that selects the choice(s) a_k that satisfies:

$$\ln p_k + \sum_{l=1,\bar{n}} x_k^l \ln\left(\frac{(m-1)q^l}{1-q^l}\right) \geq \ln p_j + \sum_{l=1,\bar{n}} x_j^l \ln\left(\frac{(m-1)q^l}{1-q^l}\right) \text{ for all } j \neq k.$$

Proof. Assume that the optimal decision rule selects the choice a_k. Then it should be the case that:

$$\prod_{l=1,\tilde{n}} \left(q^l\right)^{x_k^l} \left(\frac{1-q^l}{m-1}\right)^{n_l - x_k^l} p_k \ \geq \prod_{l=1,\tilde{n}} \left(q^l\right)^{x_j^l} \left(\frac{1-q^l}{m-1}\right)^{n_l - x_j^l} p_j \text{ for all } j \neq k .$$

Using the same argument with the one in Theorem 1, one can take logarithms in the above inequality and then use the proprieties of the logarithms, to get that:

$$\sum_{l=1,\tilde{n}} x_k^l \ln(q^l) + \sum_{l=1,\tilde{n}} (n_l - x_k^l) \ln\left(\frac{1-q^l}{m-1}\right) + \ln(p_k) \geq \sum_{l=1,\tilde{n}} x_j^l \ln(q^l) + \sum_{l=1,\tilde{n}} (n_l - x_j^l) \ln\left(\frac{1-q^l}{m-1}\right)$$

$+ \ln(p_j)$ for all $j \neq k$.

Rearranging the terms in the previous inequality, it follows that:

$$\sum_{l=1,\tilde{n}} x_k^l \ln\left(\frac{(m-1)q^l}{1-q^l}\right) + \ln(p_k) \geq \sum_{l=1,\tilde{n}} x_j^l \ln\left(\frac{(m-1)q^l}{1-q^l}\right) + \ln(p_j) \text{ for all } j \neq k ,$$

which gives the result. QED

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Chapter 2

Strategic Aspects of Information Aggregation by Plurality Rule

Abstract

This paper is concerned with rules that aggregate information among individuals with common interests but private information. Specifically, it studies optimal information aggregation and individuals' voting behavior in this context. A rule aggregates information optimally if it selects the choice that individuals would unanimously agree upon if all the signals describing the private information were common knowledge. An individual votes non-strategically if he reveals his private information during the voting process. It is shown that when plurality rule is optimal, non-strategic voting constitutes a Bayesian Nash equilibrium of the voting game induced by this rule. However, this equilibrium coexists with other interesting equilibria, where revelation does not occur. It is also shown that individuals may find in their interest to vote non-strategically even when plurality rule is not optimal.

1. Introduction

This paper analyzes information aggregation and voting behavior in environments in which individuals have private information about which of several available choices best meets their common interests. Examples of such environments are as follows:

- The members of an academic department have to decide whom of several possible candidates to employ. All members in the department agree that the best candidate would be the one that will publish most in the future. However, they have different opinions about the potential of each candidate.
- The executive directors of a company believe that demand for one of its products is going to increase. They have to estimate which of the available policies is best at meeting this new level of demand. However, the executive directors disagree about the policy that is most likely to meet this goal.
- The members of a jury have to identify the guilty party among several parties. All members of the jury want to discover the guilty party, but each of them may interpret differently the evidence during the trial.

As in the above examples, the paper considers a group of individuals that have common interests: they all want to select the choice that is correct (best candidate, best policy and guilty party, in the examples). Individuals do not know which is the correct choice, but they have common priors on the set of choices. In addition, before they need to make a decision, individuals see a private signal that provides them additional information about the correct choice. The final decision is reached by taking a simultaneous vote. Each individual must vote (abstentions are ruled out) for a single choice. The outcome of the voting process is given by plurality rule with ties broken in favor of the choice with higher prior. The use of this tie-breaking rule is very natural. Given their common interests, if individuals would not have additional information then they would unanimously agree that the choice with the highest prior should be selected. A situation of ties is similar in the sense that does not provide much information either.

In this setting, the paper analyses first the conditions under which plurality rule is the optimal way to aggregate information. The criterion for the optimality is close to the one of the first-best optimum in Informational Economics. Namely, the optimal voting rule selects the choice that would have been selected if all the private information were common knowledge. However, this definition assumes that all individuals reveal truthfully their private information or signals during the voting process. In other words, individuals are assumed to vote non-strategically. To see why this may not be the case, consider an individual that sees a signal supporting a choice with very small prior. Assume further that the signals do not provide the individuals with much information, and that he knows that. Under these assumptions, if this individual thinks that his vote can change the outcome of the voting process, i.e., his vote is pivotal, then he might consider voting for a choice that has higher prior. This suggests that individuals may have incentives to vote strategically and motivates the second question of this paper: Do individuals have incentives to reveal their information during the voting process? To answer this question, the paper analyses whether non-strategic voting is a Bayesian Nash equilibrium of the common voting game induced by plurality rule with ties broken in favor of the choice with highest prior. This also allows for an analysis of optimal information aggregation and voting behavior in a simple, but realistic, environment.

The paper offers three types of results. First, it provides a full characterization of plurality rule as an optimal way to aggregate information. In order for plurality rule to aggregate information optimally, the quality of the information (which measures the informativness of the signals) available to the individuals must be high enough. In particular, it has to be higher than an expression that depends on the number of the choices and the extreme priors. If the quality of the information is not high enough, an individual that sees a signal corresponding to a high prior choice may not be willing to agree on the selection of a choice with smaller prior but supported by more signals. This is further enhanced if the priors are well biased in one direction.

The second result proves that there is a tension between optimal information aggregation and non-strategic voting. The non-strategic voting behavior takes the form of informative voting, in which individuals vote according to the signal received. In this case, individuals reveal their private information through their votes. It is shown that if plurality rule aggregates information optimally, then informative voting is a Bayesian Nash equilibrium of the voting game induced by this rule.

However, the opposite is not true. Although this seems like a surprising result, the reason behind it is simple. For plurality rule to be optimal and informative voting to be an equilibrium, the quality of the information available to the individuals has to be high. Otherwise, in the first case, an individual that sees a signal corresponding to a high prior choice is not willing to agree on a small prior choice even if it is supported by more signals and, in the latter, an individual that sees a signal corresponding to a small prior choice would vote for the highest prior choice. However, in the latter case, the quality of the information does not need to be as high as in the first case since the individual that created problems for the optimal information aggregation cannot do better than voting informatively.

The third part illustrates some other interesting equilibria that coexist with the informative equilibrium. These are semi-pooling equilibria in which individuals that see signals different from the one corresponding to the highest prior choice prefer to vote for this particular choice. These equilibria are calculated in an example with three choices and three individuals. The existence of semi-pooling equilibria reinforces the idea of a tension between optimal information aggregation and informative voting. It shows that even when optimal information aggregation requires informative voting, individuals may prefer to use semi-pooling strategies instead. However, the informative voting equilibrium outperforms the semi-pooling equilibria even in the interval in which plurality rule is not optimal. More precisely, if plurality rule is used to aggregate votes then, among all possible equilibria, the informative one maximizes the probability of selecting the correct choice. This softens the above-mentioned tension. Unfortunately, the computational complexity of the equilibria in games with private information makes it difficult to generalize these findings.

Related Literature

The questions addressed here follow an old literature on information aggregation in groups that dates back to Condorcet. In his *Essai sur l'application de l'analyse à la probabilité des decisions rendues à la pluralité des voix (1785)*, Condorcet argued that, in a two-alternative election in which individuals are equally competent, have common interests and vote independently, the majority rule is the mechanism of aggregating information that leads to optimal decisions[1]. Young (1988, 1995) provides an excellent review, interpretation and further analysis of Condorcet's work. Other variations and extensions include: Berg (1993), Grofman and Feld (1988), Ladha (1992)[2]. The results of these papers are statistical in nature.

Another line of research studies the extent to which Condorect's result continues to hold when strategic voting is allowed. Austen-Smith and Banks (1996) were the first to point out that Condorcet's result might not hold for rational individuals. They showed that for the majority rule to aggregate optimally the information, non-strategic voting has to be an equilibrium in the voting game induced by the majority rule, and vice versa. The issue of strategic voting was taken also by: Myerson (1994), Feddersen and Pesendorfer (1998), McLennan (1998), Wit (1998) and Coughlan (2000).

The approach taken here is related to the approach taken by Austen-Smith and Banks (1996). There are three points of departure from their paper and, in general, from the literature presented above. First, the paper extends this literature to the case of more than two choices. This is a very natural extension since there are many real life situations that entail a selection from more than two choices. Second, the paper explains the use of plurality rule as an optimal way to aggregate information. A third departure consists in the use of a rule for breaking ties. In the case of two choices, ties can be avoided by assuming an odd number of individuals. When there are more than two choices, the ties become a real issue. To see this, take an example with five choices and three individuals. In this paper, the issue of ties is solved in an original, but natural, way. Ties are broken in favor of the choice with higher prior. The use of this rule was justified at the beginning of this Introduction.

The motivation of this paper is also close to the one in Feddersen and Pesendorfer (1997). Their concern was with voting behavior and information aggregation in environments where individuals are uncertain about a state variable. However, their

[1] Condorcet's findings have been rediscovered by Black (1958), who used them to formulate what is now known in the social choice literature as the Condorcet Jury Theorem.
[2] See Piketty (1999) for the most recent survey of extensions of Condorcet's result.

paper is different from the present one in several ways. One of them, but not the only one, is that individuals have different preferences.

Finally, Ladha, Miller and Oppenheimer (1996) and Guarnaschelli, McKelvey and Palfrey (2000) offer experimental results justifying the game theoretical predictions of some of the above-mentioned papers. An experimental study for the results presented here is done in Brandts and Rata (2002).

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 provides a characterization of plurality rule as an optimal way to aggregate information. Section 4 allows for strategic behavior and analyses the tension between information aggregation and informative voting. It also illustrates other voting equilibria. The last section provides some final remarks.

2. The Model

Consider a *set of choices* $A = \{a_1, a_2, ..., a_m\}$. Among these choices there is one that is *correct*; all the others are incorrect. In the examples provided in the Introduction, this would correspond to: a good candidate or a good policy among bad ones, a guilty party among innocents, etc. There is a *set of possible states of the world* S = $\{a_1, a_2, ..., a_m\}$, where the state a_i corresponds to the choice a_i being correct.

A *group of individuals* $N = \{1, 2, \ldots n\}$ has to decide on the choice to be selected from the set A. Individuals have common interests. They all want to identify the correct choice or the true state of the world.

The true state of the world is unknown, but individuals have *common priors* over the set of possible states of the world, given by p_1, $p_2, ..., p_m$, where p_i is the prior probability that a_i is the true state of the world. The priors satisfy $\sum_{j=1}^{m} p_j = 1$. With no loss of generality, it is assumed that $1 > p_1 > p_2 > .... > p_m > 0$.

Before any decision is made, an individual i receives *a private signal* $s_i \in \{1, 2, ..., m\}$ about the true state of the world. A *profile of signals* is denoted with

$s = (s_1, s_2, ..., s_n)$. The signal that an individual i receives is correlated with the true state of the world:

$$\Pr(s_i = k / a_j) = q^i_{kj},$$

where q^i_{kj} denotes the probability that the individual i receives the signal k when a_j is the true state of the world. Conditional on the state of the world, the signals received by different individuals are statistically independent.

It is assumed that $q^i_{kk} = q$ and $q^i_{kj} = \frac{1-q}{m-1}$, for any $j \neq k$ and all $i \in N$. Thus, the parameter q gives the probability that individuals receive the "correct" signal and $\frac{1-q}{m-1}$ is the probability individuals receive any of the remaining *m-1* "incorrect" signals. It is further assumed that $q \in (1/m, 1)$, meaning that each individual is more likely to receive the correct signal than any of the incorrect ones. The last two observations suggest that the probability q measures the *quality of information* received by the individuals, higher q implying higher informativness of the signals.

The final decision is reached by taking a *simultaneous vote.* Each individual must vote (abstentions are ruled out) for a single choice in the set A. The choice that receives the most number of votes is selected and, in case of a tie, the ties are broken in favor of the choice with higher prior probability. In other words, *plurality rule* with ties broken in favor of the choice with higher prior is used to aggregate individuals' votes. It should be noted that if individuals would not have additional information (the signals), then they would unanimously vote for the choice with the highest prior. However, those situations in which two or more choices receive the same number of votes are similar with those of no private information in the sense that do not provide much information about the correct choice. This suggests that breaking ties in favor of the choice with higher prior has a natural interpretation in this particular environment.

There are m possible plurality outcomes and, as said before, each individual prefers the outcome that matches the true state of the world. In other words, individuals all prefer choice a_k in state a_k. Specifically, if $u_i(a_k, a_j)$ represents individual *i*'s utility given the plurality outcome a_k and state a_j, then:

$$u_i(a_k, a_k) = 1$$

$$u_i(a_k, a_j) = 0, \text{ for all } i \in N.$$

Since the individuals all have the same utility, the subscripts are dropped from this function.

All the aspects of the environment described above other than the signals and the votes are common knowledge.

The purpose in what follows is twofold. The first is to determine the conditions under which plurality rule with ties broken in favor of the choice with higher prior is the optimal way to aggregate information. The second is to investigate whether individuals have incentives to reveal their information during the voting process. This, in turn, requires an analysis of the strategic aspects of individuals' behavior in the common value voting game induced by plurality rule with ties broken in favor of the choice with higher prior.

3. Plurality Rule as an Optimal Voting Rule

The concern here is with identifying the conditions under which plurality rule is the optimal way to aggregate information. The starting point is to notice that, in this particular environment, if individuals would have complete information about the true state of the world, then they would unanimously agree on the choice to be selected. Namely, they would select the choice that matches the true state of the world. Following this idea, a natural way to define an optimal voting rule arises.

The *optimal voting rule* is defined as the rule selecting the choice that every individual would select if he were told the signals received by the other individuals[3]. In other words, the optimal voting rule selects the choice a_k that maximizes the individuals expected utility when all the signals are perfectly observed:

$$a_k \in \arg\max_{a \in A} Eu(a, a_j)/(s_1, s_2, ..., s_n)],$$

where $E[u(a_k, a_j)/(s_1, s_2, ..., s_n)] = \sum_{a_j \in S} u(a_k / a_j) \Pr(a_j /(s_1, s_2, ..., s_n)) =$

[3] This definition of an optimal voting rule, introduced in Austen-Smith and Banks (1996), is close to the concept of first-best optimum in the Informational Economics literature. There, as here, the first-best optimum is one that maximizes an expected utility when all the information is perfectly observed.

$$= \Pr ob(a_k /(s_1, s_2,..., s_n)) = \frac{\Pr ob((s_1, s_2,..., s_n)/a_k)\Pr ob(a_k)}{\Pr ob(s_1, s_2,..., s_n)} =$$

$$= \frac{\Pr ob((s_1, s_2,..., s_n)/a_k)\, p_k}{\sum_{k=1}^{m} \Pr ob((s_1, s_2,..., s_n)/a_k)\, p_k} = \frac{q^{n_k}\left(\frac{1-q}{m-1}\right)^{n-n_k} p_k}{\sum_{k=1}^{m} q^{n_k}\left(\frac{1-q}{m-1}\right)^{n-n_k} p_k},\; n_k = \left|i \in N : s_i = k\right|.$$

The last equality follows from the independence of the individuals' signals given the true state of the world.

The following result gives the condition under which plurality rule is optimal.

Proposition 1: Plurality rule with ties broken in favor of the choice with higher prior is optimal iff $q > \frac{p_1}{(m-1)p_m + p_1}$.

Proof: See Appendix 1.

Proposition 1 says that plurality rule is the optimal way to aggregate information if and only if the quality of information q available to the individuals is higher than an expression that depends on the number of choices and the extreme priors. In particular, when the prior beliefs are well biased in one direction (p_1 is very high), the quality of the information needs to increase accordingly in order for plurality rule to aggregate information optimally. This means that, if p_1 is high, then it makes sense to add up votes that go in the same direction only if they bring additional information, i.e., they are backed by a high q. When the priors are almost uniformly distributed ($p_1 \cong p_m$), the only valuable information is provided by the individuals' votes and, since $q > \frac{1}{m}$, the more votes the better. In this case, plurality rule is optimal most of the time.

The condition of Proposition 1 has some interesting features. First, it does not depend on the number of the individuals, n. This is because the only relevant situations for optimal information aggregation are those in which one has to decide whether signals or priors are more important. Such situations are those of "almost ties", when the choice a_m, with the smallest prior, is supported by the highest number of signals and the choice a_1, with the highest prior, is supported by one signal less than the choice a_m. In a situation of almost ties, the total number of individuals is irrelevant. The condition of Proposition 1 insures that in a situation of almost ties, plurality rule selects the choice a_m despite of its possible very low prior. This argument is made more precise in the proof of Proposition 1.

The second interesting feature of the condition in Proposition 1 is that it depends only on the extreme priors p_1 and p_m, not on the other p_i's. To provide some intuition for this result, consider an example with three choices and eight individuals. In this case, the condition in Proposition 1 becomes $q > \frac{p_1}{2p_3 + p_1}$. Assume the following distribution of signals: two individuals see signal 1, another two see signal 2, and four individuals see signal 3. This information is available to everyone. Now, for an individual receiving signal 1 to agree on the selection of the choice a_3 (the choice supported by most signals), it should be the case that *q*, the quality of the signal, is high in general and, in particular, is higher than the prior p_1. Otherwise, since he has observed a signal corresponding to a choice that has the highest prior, he would be inclined to believe his signal more than the signals received by the others. In other words, when a choice with small prior is supported by many signals, an individual has to be given reasons to put more weight on these signals rather than on his own signal and the priors. Further, if an individual receiving a signal 1 is willing to agree on the selection of the choice a_3, supported by more signals but with the smallest prior, then he will be willing to agree on the selection of a_2, if this choice would be supported by more signals. This is the reason why the condition in Proposition 1 depends only on the extreme priors and not on the intermediate ones.

An important consequence of the Proposition 1 is that if individuals reveal their private information (signals) truthfully, then plurality rule maximizes the likelihood of making correct decisions whenever the quality of this information is high. However, it should be noticed that in this case individuals act as if they would observe perfectly all the signals. This leads to the following question: Would an individual act the same way (reveal his signal) if he assumes that all the others will reveal honestly their signals, but knowing that he cannot observe these signals? To see that the answer to this question is not that simple, consider a situation in which p_1 (the highest prior) is very high (thus p_m is very small) and that the quality of the information q is a bit higher than p_1. Imagine that an individual sees the signal m corresponding to the choice with the smallest prior p_m. Would he still reveal his signal by voting for the choice a_m? If he believes his vote to be pivotal, i.e., can change the outcome of the voting process, then he may very well vote for the choice a_1. Yet, this is not clear and thus motivates the analysis of the strategic aspects of individuals' behavior in an environment where plurality rule aggregates optimally the information. This is the purpose of the following section.

4. Voting Behavior

This section analyses individuals' voting behavior by looking at the Bayesian Nash equilibria of the common voting game induced by plurality rule with ties broken in favor of the choice with higher prior.

The voting game can be described as follows. An individual i observes his type, which is given by the signal received and is denoted with $t_i \in \{1,2,...,m\}$. The type is privately observed by each individual. Then, the individuals in the group N simultaneously vote for a choice from the set A. The outcome is given by plurality rule with ties broken in favor of the choice with higher prior probability. Individuals' beliefs over the types and expected utilities over the decisions and types are described in Appendix 2.

The behavior of an individual i is given by *a strategy mapping* $v_i : \{1,2,...,m\} \to A$, so that the choice to be selected by an individual i depends on the signal seen or type t_i. *A voting profile* is a mapping $v : \{1,2,...m\}^n \to A^n$, with $v(t) = (v_1(t_1), v_2(t_2),..., v_n(t_n))$.

Two types of strategies are of interest here: strategic voting and non-strategic voting. The *non-strategic voting strategy* takes the form of informative voting. A voting strategy is *informative* if $v_i(j) = a_j$ for any $j \in \{1,2,...,m\}$. Thus, informative voting is defined as voting according to the signal received. It should be noted that informative voting implicitly assumes that the individuals behave as their vote alone determines the outcome. In addition, when all the individuals adopt informative strategies all of their private information is revealed during the voting process.

Some remarks are in order now. First, the concern here is with symmetric Bayesian Nash equilibria, where individuals' strategies are identical. Thus, the subscripts from the equilibrium strategies are dropped. Second, as shown in Austin-Smith and Banks (1990), the equilibrium condition will be trivially satisfied if others use strategies $v^*(t_{-i})$ that never make i pivotal, that is, his vote can change the outcome of the voting process. Therefore, in computing the decision that is the best response to other individuals' strategies, an individual should be concerned only with those situations (type profiles) where his vote is pivotal. Third, the interest here is with equilibria in pure strategies.

Informative Voting Equilibrium

As previously said, an important issue here is of whether individuals have incentives to reveal their private information during the voting process. In other words, is informative voting a Bayesian Nash equilibrium of the voting game introduced before? The following result gives a sufficient condition for the informative voting to be a Bayesian Nash equilibrium.

Proposition 2: Plurality rule with ties broken in favor of the choice with higher prior is optimal implies that informative voting is a Bayesian Nash equilibrium in the voting game induced by this rule.

Proof: See Appendix 2.

Apart from showing the existence of the informative equilibrium, Proposition 2 points to a surprising aspect of this particular environment, namely, that there is a tension between optimal information aggregation and informative behavior.

To see that the opposite of Proposition 2 does not hold and to understand what actually drives the above-mentioned tension, consider the following example.

Example 1: Assume three choices and eight individuals (m = 3 and n = 8 in the model) and the priors $p_1 = .6$, $p_2 = .25$ and $p_3 = .15$. By Proposition 1, plurality rule is optimal iff $q > .66$. However, informative voting is an equilibrium for all $q > .54$. Thus, for any $.54 < q \leq .66$, informative voting is an equilibrium but plurality rule is not optimal. This is actually true for any values of the priors. For example, when $p_1 = .38$, $p_2 = .33$ and $p_3 = .29$ (almost equal priors), plurality rule is optimal for any $q > .39$, whereas informative voting is an equilibrium for any $q > .37$. This suggests that for equal priors, Proposition 2 may hold in the other direction too: plurality rule is optimal becoming equivalent with informative voting being a Bayesian Nash equilibrium in the voting game induced by this rule. The conditions on q for arbitrary values of p_1, p_2 and p_3 are provided in Appendix 2.

The example points to an interesting aspect of individuals' behavior in the game introduced here. Namely, given that all the other types vote informatively, a type 1 individual will prefer to vote informatively for any q, p_1, p_2 and p_3. The reason for this is simple: if all individuals receiving signals 2 and 3 vote informatively, then an individual receiving a signal 1 cannot do better than voting informatively since his signal is also supported by a high prior. However, for a type 2 individual matters are different. Assuming that the others vote informatively, he faces the trade-off between: voting for the choice a_1, which is supported by a higher prior, or voting for the choice a_2, which is supported by his signal. These considerations exclude a vote for the choice a_3. Similarly, a type 3 individual will face the trade-off between voting for a_1 or a_3. This reasoning turns out to be more general and leads, as it will be seen next, to interesting results.

Also, the example suggests that the values of q for which plurality rule is optimal and for which informative voting is an equilibrium are not that different. In both cases q needs to be high enough. If q is not high enough then, in the first case, individuals that see a high prior choice cannot agree on a small prior choice even if it is supported by a majority of signals and, in the second case, individuals that see a small prior choice would be tempted to vote for the highest prior choice. This is what brings together, though for different reasoning, the optimal aggregation of information and the informative voting.

Illustrating some Strategic Voting Equilibria

It should be noticed that there might exist other Bayesian Nash equilibria for the voting game proposed here. For example, the strategy profile where all individuals are voting for the same choice constitutes a Bayesian Nash equilibrium. To see this, assume that all individuals other than individual i vote for the choice a_k, say. Then individual i's vote is irrelevant, since the choice a_k will be selected by plurality rule. There are m such *pooling equilibria*. However, since at these equilibria an individual is never pivotal, they all involve weakly dominated strategies and, as argued in Feddersen and Pesendorfer (1995), elimination of weakly dominated strategies is a natural refinement. Therefore, applying this argument eliminates all the pooling equilibria. Nevertheless, there exist other equilibria (in pure strategies) that do not involve weakly dominated strategies and that turn out to be very interesting and to have nice interpretation. The following example illustrates such equilibria.

Example 2: Consider three choices and three individuals (m = 3 and n = 3). Fix some priors $p_1 > p_2 > p_3$. In this case, informative voting is an equilibrium iff q satisfies:

$$\frac{q\left(\frac{1-q}{2}\right)+\left(\frac{1-q}{2}\right)^2}{2q^2} \leq \frac{p_2}{p_1},\quad \frac{q\left(\frac{1-q}{2}\right)+\left(\frac{1-q}{2}\right)^2}{2q^2} \leq \frac{p_3}{p_1} \text{ and } \frac{\left(\frac{1-q}{2}\right)}{q} \leq \frac{p_3}{p_2}.$$

The first remark is that informative voting is not the unique equilibrium. There exist two other equilibria. These are semi-pooling equilibria, in which an individual that has a low type (or, sees a small prior choice) votes for the choice with the highest prior. The first semi-pooling equilibrium is one where type 1 and type 2 individuals vote for the choice a_1 and type 3 individuals vote for the choice a_3:

$$v^*(t=1) = v^*(t=2) = a_1,\ v^*(t=3) = a_3. \qquad \text{Semi-pooling (1)}$$

These strategies constitute an equilibrium for all q such that:

$$\frac{q\left(\frac{1-q}{2}\right)+\left(\frac{1-q}{2}\right)^2}{2q^2} \leq \frac{p_3}{p_1} \leq \frac{q+\left(\frac{1-q}{2}\right)}{2q}.$$

The other semi-pooling equilibrium is given by the voting strategies:

$$v^*(t=1) = v^*(t=3) = a_1,\ v^*(t=2) = a_2, \qquad \text{Semi-pooling (2)}$$

meaning that types 1 and 3 vote for the choice a_1, whereas type 2 votes for a_2. This is an equilibrium for all q such that:

$$\frac{q\left(\frac{1-q}{2}\right)+\left(\frac{1-q}{2}\right)^2}{2q^2} \le \frac{p_2}{p_1} \le \frac{q+\left(\frac{1-q}{2}\right)}{2q}.$$

Since the above inequalities are hard to interpret, assume the priors are given by $p_1 = .6$, $p_2 = .25$ and $p_3 = .15$. In this case:

- Informative voting is an equilibrium for all $q \ge .57$
- Semi-pooling (1) is an equilibrium for all $q \ge .57$
- Semi-pooling (2) is an equilibrium for all $q \ge .48$.

To see that this is not only a casual happening, consider the priors $p_1 = .5$, $p_2 = .3$ and $p_3 = .2$. Then:

- Informative voting is an equilibrium for all $q \ge .48$
- Semi-pooling (1) is an equilibrium for all $q \ge .48$
- Semi-pooling (2) is an equilibrium for all $.41 \le q \le .71$.

The second remark is that these equilibria coexist in the domain of q for which plurality rule is optimal. When $p_1 = .6$, $p_2 = .25$ and $p_3 = .15$:

- plurality rule is optimal for all $q > \dfrac{.6}{2 * .15 + .6} = .66$.

When $p_1 = .5$, $p_2 = .3$ and $p_3 = .2$:

- plurality rule is optimal for all $q > \dfrac{.5}{2 * .2 + .5} = .55$.

Further, it is an easy exercise to show that even in the interval $.57 < q < .66$ (where plurality rule is not optimal), the informative voting equilibrium performs better than the other two equilibria, in the sense that the probability of making correct decisions at the informative voting equilibrium is higher than at the other two equilibria. This alleviates the tension between optimal aggregation information and voting behavior in common interest environments.

The equilibria for the game introduced here have been calculated also for: m = 3 and n = 5 and, m = 4 and n = 3. Similar patterns of results with the ones for m = 3 and n

= 3 have been observed. Specifically, in each case, there exist semi-pooling equilibria of the type described above. These equilibria typically coexist with the informative voting in the domain of q for which plurality rule is optimal. Interestingly, for the case of four choices (m =4), the semi-pooling equilibria all had the same structure: types 1 and 2 and/or 3 and/or 4 vote for the choice a_1 and all the others vote informatively. In other words, individuals consider only deviations for the highest prior choice a_1. This makes sense in this common interests setting: the deviation has to be in the most plausible direction. Also, these semi-pooling equilibria and the informative one were the only[4] equilibria in pure strategies in these examples.

5. Final Remarks

This paper has explored the implications of private information for environments in which individuals have common interests. Specifically, its purpose has been to identify those aspects of the informational and decisional making environment that influence whether individuals reveal their information during the voting process. It has been shown that there is a tension between optimal information aggregation and informative voting. This is a combination of two factors. One is that, individuals may find in their interest to vote informatively even when information is not aggregated optimally. In this direction, it has been shown that plurality rule is optimal implies that individuals vote informatively, but the opposite does not hold in general. The other is that, even when optimal information aggregation requires informative voting, individuals may prefer to use semi-pooling strategies instead. This is supported by the fact that informative voting equilibrium coexists with semi-pooling equilibria in the range of q's for which plurality aggregates information optimally.

Since games with private information are complex, the existence of semi-pooling equilibria is shown by means of examples. Although these examples are only a beginning for a more general theoretical analysis, they are enough to support the above predictions. In that case, the multiplicity of equilibria in this environment calls for an analysis of the equilibrium selection. One way to examine this would be to study the stability of the equilibria with respect to some stability notion, such as

[4] Of course, apart from the pooling equilibria that were discarded from the beginning.

information-proofness (Kalai, 2000). Another way would be to use laboratory experiments. The latter approach has been taken in Brandts and Rata (2002), where voting behavior and optimal information aggregation is analyzed for a three-choice and three-individual environment.

There are other interesting issues that were not considered in this paper. One of them is the analysis of voting behavior and information aggregation in the domain of q for which plurality rule is not optimal. Another one is the study of optimal information aggregation for those situations where individuals vote strategically. These extensions are left for further research.

Appendix 1

Proposition 1: Plurality rule with ties broken in favor of the choice with higher prior is optimal iff $q > \frac{p_1}{(m-1)p_m + p_1}$.

Proof: The proof is done in two steps.

Step 1: It is shown that the optimal voting rule selects the choice(s) a_k for which:

$$\ln p_k + n_k \ln\left(\frac{(m-1)q}{1-q}\right) \geq \ln p_j + n_j \ln\left(\frac{(m-1)q}{1-q}\right) \text{for all } j \neq k .$$

Step 2: It is shown that, for plurality rule with ties broken in favor of the choice with higher prior to be optimal, the conditions derived in Step 1 must reduce to: $q > \frac{p_1}{(m-1)p_m + p_1}$.

Step 1: As stated before, the optimal voting rule selects the choice(s) that maximizes the expected utility of the individuals given the vector of signals $s = (s_1, s_2, ..., s_n)$. In other words, the optimal voting rule selects the choice(s) a_k for which the expression below is maximal:

$$\frac{q^{n_k}\left(\frac{1-q}{m-1}\right)^{n-n_k} p_k}{\sum_{i=1}^{m} q^{n_i}\left(\frac{1-q}{m-1}\right)^{n-n_i} p_i}.$$

or:

$$q^{n_k}\left(\frac{1-q}{m-1}\right)^{n-n_k} p_k \;\geq\; q^{n_i}\left(\frac{1-q}{m-1}\right)^{n-n_i} p_i \text{ for any } i \neq k.$$

After taking logarithms in the above inequality and rearranging its terms, the inequality becomes:

$$\ln p_k + n_k \ln\left(\frac{(m-1)q}{1-q}\right) \geq \ln p_i + n_i \ln\left(\frac{(m-1)q}{1-q}\right) \text{ for any } i \neq k.$$

Thus, the optimal voting rule will select the choice(s) a_k whose *score*:

$$\ln p_k + n_k \ln\left(\frac{(m-1)q}{1-q}\right) \text{ is maximal.}$$

Step 2: Plurality rule with ties broken in favor of the choice with higher prior is optimal if the score attached by this rule to a choice a_j is such that:

1) $\ln p_j + n_j \ln\left(\frac{(m-1)q}{1-q}\right) > \ln p_k + n_k \ln\left(\frac{(m-1)q}{1-q}\right)$ for any a_j and a_k such that $p_j > p_k$ and $n_j = n_k > n_i$ for all *i* with $i \neq j,k$;

2) $\ln p_j + n_j \ln\left(\frac{(m-1)q}{1-q}\right) > \ln p_k + n_k \ln\left(\frac{(m-1)q}{1-q}\right)$ for any a_j and a_k such that $n_j = n_k + x > n_i$ for all *i* with $i \neq j$ and $x \in Z_+^*$ [5], and $p_k > p_j$ (this condition is obviously satisfied when $p_j > p_k$).

The first condition gives the tie-breaking rule, that is, if two or more choices have the same number of votes, then the choice with higher prior should be selected. The second condition states that the choice with the highest number of votes will always be selected. Hence, a choice a_k will not be selected only because it has a higher prior.

Further, since $\frac{(m-1)q}{1-q} > 1$, condition 2) needs to be satisfied only for $x = 1$. It is easy to see that if condition 2) is satisfied for $x = 1$, then it will be satisfied for any $x > 1$. In that case, given two choices a_j and a_k with $n_j = n_k + 1$, $p_k > p_j$ and $n_j > n_i$ for all *i*, $i \neq j$, condition 2) becomes:

[5] Z_+^* is the set of strictly positive integers.

$$\ln p_j + (n_k + 1)\ln\left(\frac{(m-1)q}{1-q}\right) > \ln p_k + n_k \ln\left(\frac{(m-1)q}{1-q}\right).$$

Furthermore, since $\frac{(m-1)q}{1-q} > 1$ and $\frac{p_k}{p_j} > 1$, the above inequality can be rewritten as $q > \frac{p_k}{(m-1)p_j + p_k}$. But, for plurality rule to be optimal, this condition must be true for any two choices a_j and a_k that satisfy condition 2), or alternatively, for any j and k with $p_k > p_j$.

Thus, $q > \max_{k,j:p_k > p_j} \left\{ \frac{p_k}{(m-1)p_j + p_k} \right\} = \frac{p_1}{(m-1)p_m + p_1}$. The last equality follows from the assumption that $p_1 > p_2 > .... > p_m$. This proves the result. QED

APPENDIX 2

The voting game is described by:

- The set of *type profiles* (or vectors) $T = \times_i T_i$, with each set $T_i = \{1,2,...m\}$ describing the types of individual *i*; a type profile will be denoted with $t = (t_1,...,t_n)$ or $t = (t_i, t_{-i})$.

- Individuals' *common preference* over the voting profile $v(t) = (v_1(t_1),...,v_n(t_n))$ and the type profile t:

$$U((v_1(t_1),...,v_n(t_n)),t) := E[u(PL((v_1(t_1),...,v_n(t_n)),a)/t] =$$
$$= \sum_{a \in S} u(PL(v(t)),a) \Pr ob(a/t) = \Pr ob(a = PL((v_1(t_1),...,v_n(t_n))/t),$$

where $PL((v_1(t_1),...,v_n(t_n))$ is plurality outcome at $(v_1(t_1),...,v_n(t_n))$.

- *i's belief* describing *i*'s uncertainty about the *n-1* other players' possible types, t_{-i}, given *i*'s own type, t_i: $p(t_{-i}/t_i)$. This can be done using Bayes rule:

$$p(t_{-i}/t_i) = \frac{p(t_{-i},t_i)}{p(t_i)} = \frac{p(t)}{\sum_{i \in N} p(t_{-i},t_i)},$$

where

$$p(t) = \sum_{j=\overline{1,m}} p_j \Pr ob(t/a_j) = \sum_{j=\overline{1,m}} p_j \prod_{i \in N} \Pr ob(t_i / a_j)$$

is the *common prior probability* of a type profile $t \in T$.

The last equality follows from the independence of the individuals' signals given the true state of the world.

The *expected utility* from voting for $v_i(t_i)$, given the updated belief $p(t_{-i}/t_i)$ and the strategies of the other players $v_{-i}(t_{-i})$ is given by:

$$EU(v_i(t_i);t_i,v_{-i}(t_{-i})) = \frac{1}{\sum_{t_{-i}} p(t_{-i},t_i)} \sum_{t_{-i}} \Pr ob[PL(v(t)) \,\&\, t].$$

All this is taken to be common knowledge between the individuals.

A *Bayesian Nash equilibrium* (in pure strategies) of the above game is a strategy profile $v^*(\cdot)$ such that for all $i \in N$ and all $t_i \in T_i$, $v^*(t_i) = a_k$ only if

$$EU(a_k;t_i,v^*) \geq EU(a_j;t_i,v^*) \text{ for any } j \neq k.$$

Proposition 2: Plurality rule with ties broken in favor of the choice with higher prior is optimal implies that informative voting is a Bayesian Nash equilibrium in the voting game induced by this rule.

Proof: Recall that at a Bayesian equilibrium, every individual makes a decision that is optimal relative to the distribution of the individuals' types and their decision-selection rules. Then, at the informative voting equilibrium, an individual makes the decision that is best relative to the individuals' types and given that all the other individuals vote informatively.

The conditions for an individual i of type k to vote informatively given that all the other individuals vote informatively are given by:

$$E[U(a_k;t_i = k, v_{-i}^{\inf}(t_{-i}))] \geq E[U(a_j,t_i = k, v_{-i}^{\inf}(t_{-i}))] \text{ for all } j \neq k,$$

where $v_{-i}^{\inf}(t_{-i})$ is the vector of decisions of all individuals except i according to the informative strategy profile $v^{\inf}(\cdot)$.

Further, using the remark of Section 4 that at the equilibrium the only relevant type profiles for individual i are those where he is pivotal, the above conditions can be rewritten as:

$$\sum_{t_{-i} \in Piv(T_{-i})} \Pr ob[PL(a_k,v_{-i}^{\inf}(t_{-i})) \,\&\, (k,t_{-i})] \geq \sum_{t_{-i} \in Piv(T_{-i})} \Pr ob[PL(a_j,v_{-i}^{\inf}(t_{-i})) \,\&\, (k,t_{-i})]$$

(1)

for any $j \neq k$, where $Piv(T_{-i})$ is the set of profiles at which an individual i is pivotal and (k, t_{-i}) is the type profile with $t_i = k$ and $t_{-i} \in Piv(T_{-i})$.

The objective is to show that if plurality rule[6] is optimal, then (1) holds for all $t_i = k \in \{1,2,...,m\}$.

To prove this, consider an individual i with $t_i = k$, and fix an arbitrary profile $\bar{t}_{-i} \in Piv(T_{-i})$.

First, it is proved that if plurality rule is optimal, then the following should hold:

$$\Pr ob\left[PL(a_k, v_{-i}^{\inf}(\bar{t}_{-i})) \,\&\, (k, \bar{t}_{-i})\right] \geq \Pr ob\left[PL(a_j, v_{-i}^{\inf}(\bar{t}_{-i})) \,\&\, (k, \bar{t}_{-i})\right] \qquad (2)$$

for all $j \neq k$.

Since the profile $\bar{t}_{-i}$ was arbitrary chosen, if (2) holds at the type profile $(k, \bar{t}_{-i})$, then it must hold for all type profiles (k, t_{-i}) with $t_{-i} \in Piv(T_{-i})$. For the same reason, (2) should hold for all $k \in \{1,2,...,m\}$. Further, one can take the sum over all $t_{-i} \in Piv(T_{-i})$ in (2) to obtain (1) for all $k \in \{1,2,...,m\}$.

To prove (2), consider three choices a_1, a_2 and a_3. As it will become clear at the end of the exposition, a similar reasoning applies for more than three choices.

Denote with $n_k = \left| i \in N : t_i = k \right|$ the number of individuals of type k at a type profile t. Thus $\sum_{k \in \{1,...,m\}} n_k = n$. The same notation was used in Proposition 1 to denote the number of individuals receiving a specific signal. There is no loss of generality in using this notation here too, since the types coincide with the signals.

The purpose now is to show that (2) holds for all $t_i \in \{1,2,3\}$ and all $t_{-i} \in Piv(T_{-i})$.

Consider each possibility for t_i:

I) $\underline{t_i = 1}$

In this case (2) becomes:

$$\Pr ob\left[PL(a_1, v_{-i}^{\inf}(t_{-i})) \,\&\, (1, t_{-i})\right] \geq \Pr ob\left[PL(a_2, v_{-i}^{\inf}(t_{-i})) \,\&\, (1, t_{-i})\right] \qquad (Ia)$$

$$\Pr ob\left[PL(a_1, v_{-i}^{\inf}(t_{-i})) \,\&\, (1, t_{-i})\right] \geq \Pr ob\left[PL(a_3, v_{-i}^{\inf}(t_{-i})) \,\&\, (1, t_{-i})\right]. \qquad (Ib)$$

[6] It should be kept in mind that it is plurality rule with ties broken in favor of the choice with higher prior.

A profile $(1, t_{-i})$ with $t_{-i} \in Piv(T_{-i})$ is characterized by:

- *i being pivotal and having to decide between* a_1 *and* a_2 *(Ia)*

In this case, the profile $(1, t_{-i})$ is such that one of the following is true:

1a) $n_1 = n_2 \geq n_3$ (that is, there is a tie between a_1 and a_2, and individual i's vote determines which of the two choices will be selected)

1a') $n_1 = n_2 + 1 \geq n_3$ (the same as before)

2a) $n_3 = n_2 + 1 > n_1$ (if i votes for a_1, then a_3 is selected; if i votes for a_2, then a_2 is selected)

2a') $n_3 = n_1 > n_2 + 1$ (if i votes for a_1, then a_1 is selected; if i votes for a_2, then a_3 is selected).

In the cases 1a) and 2a'), if plurality rule is optimal (i.e., q is such that $q > \dfrac{p_1}{2p_3 + p_1}$), then (Ia) is trivially satisfied. For 1a'), (Ia) becomes:

$$p_1 q^{n_1} \left(\frac{1-q}{2} \right)^{n-n_1} \geq p_2 q^{n_2} \left(\frac{1-q}{2} \right)^{n-n_2} \text{ or } p_1 q \geq p_2 \left(\frac{1-q}{2} \right).$$

However, it is easy to see that the last inequality holds for all q's such that plurality rule is optimal.

In the case 2a), (Ia) becomes:

$$p_3 q^{n_3} \left(\frac{1-q}{2} \right)^{n-n_3} \geq p_2 q^{n_2} \left(\frac{1-q}{2} \right)^{n-n_2} \text{ or } p_3 q \geq p_2 \left(\frac{1-q}{2} \right).$$

This last inequality holds whenever plurality rule is optimal.

Thus, if plurality rule is optimal, then (Ia) is satisfied.

- *i being pivotal and having to decide between* a_1 *and* a_3 *(Ib)*

In this case, the profile $(1, t_{-i})$ is such that one of the following is true:

1b) $n_1 = n_3 \geq n_2$

1b') $n_1 = n_3 + 1 > n_2$

In the above situations, a_1 is selected by plurality rule (with ties broken in favor of the choice with higher prior) whenever i votes for a_1 and, similarly, a_3 is selected whenever i votes for it.

2b) $n_1 = n_2 > n_3$ (if i votes for a_1, then a_1 is selected; if i votes for a_3, then a_2 is selected)

2b’) $n_3 = n_2 > n_1$ (if i votes for a_1, then a_2 is selected; if i votes for a_3, then a_3 is selected).

In the cases 1b) and 2b), (Ib) is trivially satisfied. For 1b’), (Ib) becomes:

$$p_1\, q^{n_1}\left(\frac{1-q}{2}\right)^{n-n_1} \geq p_3 q^{n_3}\left(\frac{1-q}{2}\right)^{n-n_3} \text{ or } p_1\, q \geq p_3\left(\frac{1-q}{2}\right),$$

which holds whenever plurality rule is optimal.

In the case 2b’), (Ib) becomes:

$$p_2\, q^{n_2}\left(\frac{1-q}{2}\right)^{n-n_2} \geq p_3 q^{n_3}\left(\frac{1-q}{2}\right)^{n-n_3} \text{ or } p_2\, q \geq p_3\left(\frac{1-q}{2}\right).$$

This last inequality holds whenever plurality rule is optimal. Thus (Ib) is satisfied.

II) $\underline{t_i = 2}$

In this case (2) becomes:

$$\Pr ob\left[PL(a_2, v_{-i}^{\inf}(t_{-i}))\ \&\ (2, t_{-i})\right] \geq \Pr ob\left[PL(a_1, v_{-i}^{\inf}(t_{-i}))\ \&\ (2, t_{-i})\right] \qquad \text{(IIa)}$$

$$\Pr ob\left[PL(a_2, v_{-i}^{\inf}(t_{-i}))\ \&\ (2, t_{-i})\right] \geq \Pr ob\left[PL(a_3, v_{-i}^{\inf}(t_{-i}))\ \&\ (2, t_{-i})\right]. \qquad \text{(IIb)}$$

A profile $(2, t_{-i})$ with $t_{-i} \in Piv(T_{-i})$ is characterized by:

- *i being pivotal and having to decide between a_2 and a_1 (IIa)*

In this case, the profile $(2, t_{-i})$ is such that one of the following is true:

1a) $n_2 = n_1 + 1 \geq n_3$

1a’) $n_2 = n_1 + 2 \geq n_3$

In these situations, individual i’s vote gives the outcome of the voting process.

2a) $n_2 = n_3 > n_1 + 1$ (if i votes for a_2, then a_2 is selected; if i votes for a_1, then a_3 is selected)

2a’) $n_3 = n_1 + 1 > n_2$ (if i votes for a_2, then a_3 is selected; if i votes for a_1, then a_1 is selected).

For 2a), (IIa) is trivially satisfied.

For 1a), (IIa) becomes: $p_2\, q \geq p_1\left(\frac{1-q}{2}\right)$.

For 1a’), (IIa) becomes: $p_2\, q^2 \geq p_1\left(\frac{1-q}{2}\right)^2$.

For 2a’), (IIa) becomes: $p_3\, q \geq p_1\left(\frac{1-q}{2}\right)$.

All the above inequalities are satisfied when plurality rule is optimal.

- *i being pivotal and having to decide between* a_2 *and* a_3 *(IIb)*

1b) $n_2 = n_3 > n_1$

1b') $n_2 = n_3 + 1 > n_1$

2b) $n_2 = n_1 + 1 \geq n_3 + 2$ (if i votes for a_2, then a_2 is selected; if i votes for a_3, then a_1 is selected)

2b') $n_3 = n_1 \geq n_2$ (if i votes for a_2, then a_1 is selected; if i votes for a_3, then a_3 is selected).

For 1b) and 2b'), (IIb) is trivially satisfied.

For 1b') and 2b), (IIb) becomes: $p_2\, q \geq p_3\left(\frac{1-q}{2}\right)$ and $p_2\, q \geq p_1\left(\frac{1-q}{2}\right)$, respectively. Both inequalities are satisfied when plurality rule is optimal.

III) $\underline{t_i = 3}$

In this case (2) becomes:

$$\Pr ob\left[PL(a_3, v_{-i}^{\inf}(t_{-i}))\ \&\ (3, t_{-i})\right] \geq \Pr ob\left[PL(a_1, v_{-i}^{\inf}(t_{-i}))\ \&\ (3, t_{-i})\right] \qquad \text{(IIIa)}$$

$$\Pr ob\left[PL(a_3, v_{-i}^{\inf}(t_{-i}))\ \&\ (3, t_{-i})\right] \geq \Pr ob\left[PL(a_2, v_{-i}^{\inf}(t_{-i}))\ \&\ (3, t_{-i})\right]. \qquad \text{(IIIb)}$$

A profile $(3, t_{-i})$ with $t_{-i} \in Piv(T_{-i})$ is characterized by:

- *i being pivotal and having to decide between* a_3 *and* a_1 *(IIIa)*

1a) $n_3 = n_1 + 1 > n_2$

1a') $n_3 = n_1 + 2 > n_2$

2a) $n_1 = n_2 + 1 \geq n_3$ (if i votes for a_3, then a_2 is selected; if i votes for a_1, then a_1 is selected)

2a') $n_3 = n_2 + 1 > n_1 + 2$ (if i votes for a_3, then a_3 is selected; if i votes for a_1, then a_2 is selected).

For 1a), (IIa) becomes: $p_3\, q \geq p_1\left(\frac{1-q}{2}\right)$.

For 1a'), (IIa) becomes: $p_3\, q^2 \geq p_1\left(\frac{1-q}{2}\right)^2$.

For 2a), (IIa) becomes: $p_2\, q \geq p_1\left(\frac{1-q}{2}\right)$.

For 2a'), (IIa) becomes: $p_3 q \geq p_2\left(\frac{1-q}{2}\right)$.

All the above inequalities are satisfied when plurality rule is optimal.

- *i being pivotal and having to decide between* a_3 *and* a_2 *(IIIb)*

1b) $n_3 = n_2 + 1 > n_1$

1b') $n_3 = n_2 + 2 > n_1$

2b) $n_1 = n_2 \geq n_3$ (if i votes for a_3, then a_1 is selected; if i votes for a_2, then a_2 is selected)

2b') $n_3 = n_1 + 1 > n_2 + 2$ (if i votes for a_3, then a_3 is selected; if i votes for a_2, then a_1 is selected).

For 2b), (IIb) is trivially satisfied.

For 1b), 1b') and 2b'), (IIb) becomes:

$p_3 q \geq p_2\left(\frac{1-q}{2}\right)$, $p_3 q^2 \geq p_2\left(\frac{1-q}{2}\right)^2$, $p_3 q \geq p_1\left(\frac{1-q}{2}\right)$, respectively.

All the above inequalities are satisfied when plurality rule is optimal. Thus (IIIb) holds.

The above reasoning can be extended to the case of more than three choices. A type profile (k, t_{-i}), with $t_{-i} \in Piv(T_{-i})$, will be characterized by either 1a), 1a'), 2a) or 2a'). In these situations, an individual i can influence "directly" (situations 1a), 1a')) or "indirectly" (situations 2a), 2a')) the outcome of the voting process. Of course, for more than three choices, the situations of type 2a) and 2a') will be many more. For *m* choices there will be *m-1* situations of the type 2).

More importantly, all these situations will result in conditions of the type $p_j q \geq p_i\left(\frac{1-q}{m-1}\right)$ ($p_j q^2 \geq p_i\left(\frac{1-q}{m-1}\right)^2$ are actually equivalent to $p_j q \geq p_i\left(\frac{1-q}{m-1}\right)$).

These conditions can be rewritten as:

$$q \geq \frac{p_i}{(m-1)p_j + p_i}.$$

But, $\dfrac{p_1}{(m-1)p_m + p_1} = \max\left\{\dfrac{p_i}{(m-1)p_j + p_i}\right\}$. This implies that whenever plurality rule is optimal these conditions are satisfied. Thus, condition (2) is satisfied for all (k, t_{-i}), with $t_{-i} \in Piv(T_{-i})$ and, finally, (1) is satisfied. QED

Example 1 Let m = 3 and n = 8 in the framework proposed above. Given p_1, p_2 and p_3, informative voting is a Bayesian Nash equilibrium of the voting game introduced here if and only if the following three inequalities hold:

$$\frac{6q\left(\frac{1-q}{2}\right)^2 + 12\left(\frac{1-q}{2}\right)^3}{q^3 + 11q^2\left(\frac{1-q}{2}\right) + 6q\left(\frac{1-q}{2}\right)^2} \leq \frac{p_2}{p_1} \tag{3}$$

$$\frac{6q\left(\frac{1-q}{2}\right)^2 + 12\left(\frac{1-q}{2}\right)^3}{q^3 + 14q^2\left(\frac{1-q}{2}\right)} - \frac{p_2}{p_1}\,\frac{q\left(\frac{1-q}{2}\right) + 2\left(\frac{1-q}{2}\right)^2}{14q\left(\frac{1-q}{2}\right) + q^2} \leq \frac{p_3}{p_1} \tag{4}$$

and

$$\frac{9q\left(\frac{1-q}{2}\right)^2 + 6\left(\frac{1-q}{2}\right)^3}{q^3 + 14q^2\left(\frac{1-q}{2}\right)} \leq \frac{p_3}{p_2}. \tag{5}$$

Given the priors p_1, p_2 and p_3, a type 2 individual will vote informatively, provided that everyone else is voting informatively, if and only if q satisfies (3). Similarly, (4) and (5) give the conditions for which a type 3 individual would find it optimal to vote informatively, when the other types vote informatively. A type 1 individual votes informatively for any q, p_1, p_2 and p_3, provided that the other types vote informatively. If p_1, p_2 and p_3 are given then the conditions (3), (4) and (5) impose restrictions on the fourth parameter q.

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Chapter 3

Information Aggregation by Plurality Rule: An Experiment[1]

Abstract

The paper reports on a voting experiment in which groups of three subjects select one of three choices by plurality vote. One of the three choices is the best for all three subjects, but subjects have different signals about what the best choice is. In this game, there are several equilibria in pure strategies. The aggregate data from the experiment exhibit a clear and reasonable pattern, but are not consistent with any of the equilibria.

[1] This is a joint work with Jordi Brandts, Institut d'Anàlisi Econòmica, CSIC, Universitat Autònoma de Barcelona.

1. Introduction

In uncertain environments, individuals may fail to reach optimal results despite their common interests. Some voting situations provide examples in this direction. Consider the members of an academic department who have to decide whom of several possible candidates to employ. All members of the department agree that the best candidate would be the one that will publish most in the future. However, they may have different opinions, due to different private information, about the potential of each candidate. Assume that the final decision is made by taking a vote: every member has to vote for one candidate. If the members of the department are similar in their abilities to interpret the private information, then the most likely to be the best candidate is the one that is supported by most number of votes, provided that the individuals reveal their private information during the voting process. However, will the members of the department reveal their information? One may argue that, given their common interests, there is no point in voting strategically.

There are two interrelated reasons for which strategic voting may arise in this environment. One is that an individual must condition his beliefs about the performance of a candidate on the event that his vote can change the outcome of the voting process, i.e., his vote is pivotal. The other is that an individual's private information may not be enough to prevail over the prior belief. For example, if seven individuals have to decide by plurality rule on three candidates, then an individual's vote is pivotal when the other six individuals divide equally their votes between two candidates (this is one possibility among several). If the pivotal individual assumes that the other individuals reveal their private information, then he can infer that the third candidate, and the one supported by his private information, is not the best. If, in addition, the third candidate is a priori the least likely to be the best, then his deduction is reinforced. This, in turn, may make the third individual change his mind and vote for one of these two candidates despite of his initial inclination to vote for the third candidate.

The above example suggests that the private information is responsible for the failure to optimally aggregate information. If individuals knew all the private information, then they would unanimously agree that the candidate or choice with most votes (or the plurality outcome) is the most likely to be the best. The general argument is made in Rata (2002), where it is shown that there is a tension between optimal information aggregation and information revelation, or informative voting, even for common interests environments. One reason for this tension is that the informative voting is not the unique Bayesian Nash equilibrium in the voting game induced by plurality rule. It coexists with semi-pooling equilibria. These are equilibria in which individuals whose private information supports low prior choices prefer to vote for the highest prior choice.

The purpose of this paper is to test individuals' voting behavior in the setting proposed in Rata (2002), for the case of three choices and three individuals. This requires both theoretical and experimental analysis. Theoretically, the paper provides a description of the Bayesian Nash equilibria for the voting game induced by plurality rule. Experimentally, the paper tries to answer the following questions. Do the data support the informative voting behavior? This is desirable because informative voting leads to optimal information aggregation. Further, if informative voting is not confirmed by the data, do they support any of the remaining Bayesian Nash equilibria? If not, is there any possible explanation for these findings?

The results are outlined as follows. First, the data rejects informative voting behavior as well as the other Bayesian Nash equilibria. Whereas those individuals seeing the signal supporting the choice that is a priori most likely to be correct are voting most of the time informatively, those seeing the other signals are voting strategically a large proportion of the time. Second, the data exhibits a clear pattern that supports a mixed strategy behavior, where the individuals seeing the signal supporting the most probable to be the correct choice a priori are voting informatively and the others are mixing between voting informatively and voting for the choice that is a priori most likely to be correct. However, these mixed strategies do not constitute a Bayesian Nash equilibrium. Third, the probability of making the correct choice evaluated at the fractions provided by the data is smaller than the one under the informative voting. Thus, individuals' decisions do not lead to optimal information aggregation.

The problem addressed in this paper is not new. Ladha, Miller and Oppenheimer (1996) and Guarnaschelli, McKelvey and Palfrey (2000) offered experimental results on binary choice models of decision making in elections and juries, respectively. Ladha&al tested individuals' behavior in the context proposed by Condorcet (1785). Their results confirmed Condorcet's finding that majority rule yields decisions that are better than the one of an individual alone and, that this holds even when voters act strategically. Guarnaschelli&al tested the implications of a jury model introduced in Feddersen and Pesendorfer (1998). They found evidence of strategic voting under the unanimity rule, which is consistent with the theoretical predictions.

This paper departs from the above literature by allowing for more than two choices. This is a very natural extension since there are many real life situations that entail a selection from more than two choices. Further, unlike the above papers, in the present model with three choices there exist interesting equilibria that coexist with the informative one. Thus, an experimental analysis is more meaningful, as it might help in equilibrium selection.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 describes the equilibria and discusses the purpose of the experiment. Section 4 presents the experimental design. The results are given in Section 5. The last section provides some final remarks.

2. The Model

A *group of individuals* $N = \{1,2,3\}$ has to decide on the choice to be selected from the set $A = \{a_1, a_2, a_3\}$[2]. Among these choices there is one that is *correct*, all the others are incorrect.

There is a *set of possible states of the world* $S = \{a_1, a_2, a_3\}$, where the state a_i corresponds to the choice a_i being correct. In the example provided in the Introduction, the correct choice or the true state of the world was the candidate that would publish most in the future.

[2] For a general treatment, i.e., *m* choices and *n* individuals, see Rata (2002).

The true state of the world is unknown, but individuals have *common priors* over the set of possible states of the world, given by p_1, p_2, p_3, where p_i is the prior probability that a_i is the true state of the world. The priors satisfy $p_1 + p_2 + p_3 = 1$. With no loss of generality, it is assumed that $1 > p_1 > p_2 > p_3 > 0$.

Before any decision is made, an individual i receives *a private signal* $s_i \in \{1, 2, 3\}$ about the true state of the world. The signal that an individual i receives is correlated with the true state of the world:

$$\Pr(s_i = k / a_j) = q_{kj}^i,$$

where q_{kj}^i denotes the probability that the individual i receives the signal k when a_j is the true state of the world. Conditional on the state of the world, the signals received by different individuals are statistically independent.

It is assumed that $q_{kk}^i = q$ and $q_{kj}^i = \frac{1-q}{2}$, for any $j \neq k$ and all $i \in N$. Thus, the parameter q gives the probability that individuals receive the "correct" signal and $\frac{1-q}{2}$ is the probability individuals receive any of the remaining *2* "incorrect" signals. It is further assumed that $\frac{1}{3} < q < 1$, meaning that each individual is more likely to receive the correct signal than any of the incorrect ones.

The final decision is reached by taking a *simultaneous vote*. Each individual must vote (abstentions are ruled out) for a single choice in the set A. *Plurality rule*[3] with ties broken in favor of the choice with higher prior is used to aggregate individuals' votes.

Individuals have *common interests*. They all want to identify the correct choice or the true state of the world. Alternatively, individuals have a common utility on the plurality

[3] Plurality rule selects the choice with the highest number of votes.

outcome and the true state of the world, given by: $u_i(a_k, a_k)$ = 1 and $u_i(a_k, a_j)$ = 0, for any $i \in N$ and $j \neq k$.

The voting behavior of an individual i is given by a *strategy mapping* $v_i : \{1,2,3\} \to A$, so that the choice to be selected by an individual i depends on his signal s_i. The interest in what follows is with those strategies that constitute a Bayesian Nash equilibria in the above game[4]. In particular, the concern is with informative voting strategies. A voting strategy is *informative* if $v_i(j) = a_j$ for any $j \in \{1,2,3\}$. Thus, informative voting assumes that individuals reveal all their information during the voting process. The following section describes the Bayesian Nash equilibria in pure strategies of this game.

3. Equilibria

The concern here is with symmetric Bayesian Nash equilibria, that is, equilibria in which individuals who receive the same signal use the same strategies. For this reason, the subscripts are dropped from the equilibrium strategies. The following result provides the conditions on q, given p_1, p_2 and p_3, for which informative voting is an equilibrium.

Proposition 1: *Informative voting* is a Bayesian Nash equilibrium of the voting game induced by plurality rule if and only if the following three inequalities hold:

$$\frac{q\left(\frac{1-q}{2}\right)+\left(\frac{1-q}{2}\right)^2}{2q^2} \leq \frac{p_2}{p_1} \qquad (1)$$

$$\frac{q\left(\frac{1-q}{2}\right)+\left(\frac{1-q}{2}\right)^2}{2q^2} \leq \frac{p_3}{p_1} \qquad (2)$$

and

[4] For a more detailed description of the Bayesian voting game, see Appendix 1.

$$\frac{\left(\frac{1-q}{2}\right)}{q} \le \frac{p_3}{p_2}. \tag{3}$$

Proof: See Appendix 1.

The first inequality gives the condition for which a type 2 individual prefers to vote informatively, when the other types vote informatively. Similarly, (2) and (3) give the conditions for which a type 3 individual would find it optimal to vote informatively, when the other types vote informatively. A type 1 individual votes informatively for any q, p_1, p_2 and p_3, provided the other types vote informatively. The intuition is simple: if q is small, in particular, smaller than p_1, then his signal is not very informative and therefore the best thing a type 1 individual can do is to follow the prior beliefs. If q is higher than p_1, then since his signal supports the highest prior choice he should vote for it.

In addition to the informative voting equilibrium, there exist *semi-pooling equilibria*. More precisely, there are two types of semi-pooling equilibria, one in which individuals that receive signal 2 vote for the choice a_1 and individuals that receive signals 1 or 3 vote informatively, and another in which individuals that receive signal 3 vote for the choice a_1 and individuals that receive signals 1 and 2 vote informatively. The following two propositions describe these equilibria.

Proposition 2: The strategies $v(s=1) = v(s=3) = a_1$, $v(s=2) = a_2$ constitute a Bayesian Nash equilibrium iff:

$$\frac{q\left(\frac{1-q}{2}\right)+\left(\frac{1-q}{2}\right)^2}{2q^2} \le \frac{p_2}{p_1} \le \frac{q+\left(\frac{1-q}{2}\right)}{2q}.$$

Proof: See Appendix 1.

The condition of Proposition 2 does not depend on the prior p_3. This is because the choice a_3 is not selected at any voting profile. Since in this equilibrium a type 3 individual does not vote for a_3, then a type 2 individual does not vote either for it. Thus, the real decision involves only choices a_1 and a_2. A similar reasoning applies for the following result.

Proposition 3: The strategies $v(s=1) = v(s=2) = a_1$, $v(s=3) = a_3$ constitute a Bayesian Nash equilibrium iff:

$$\frac{q\left(\frac{1-q}{2}\right)+\left(\frac{1-q}{2}\right)^2}{2q^2} \leq \frac{p_3}{p_1} \leq \frac{q+\left(\frac{1-q}{2}\right)}{2q}.$$

Proof: See Appendix 1.

Remark: The strategy profile where all individuals are voting for the same choice constitutes a Bayesian Nash equilibrium for any $q > \frac{1}{3}$. There are three such *pooling equilibria*. To see this, assume that all individuals other than individual i vote for choice a_k, say. Then individual i's vote is irrelevant, since choice a_k is selected by plurality rule anyway.

The multiplicity of the equilibria in this common interest environment motivates the experimental study. More precisely, one purpose is to answer the following question. Does the data support the informative voting equilibrium? Informative equilibrium is desirable because it aggregates information optimally. This means that, if plurality rule is used to aggregate votes then, among all possible equilibria in pure strategies, the informative one maximizes the probability of selecting the correct choice. The following examples, which are also used in the experiment, illustrate this idea.

The table below gives the equilibria in function of q for two treatments: one in which the prior beliefs are almost equal $p_1 = .38$, $p_2 = .33$, $p_3 = .29$, and another in which the priors are different $p_1 = .6$, $p_2 = .25$, $p_3 = .15$. The experiment uses these examples with q fixed at .75, for the first case, and .6, for the second case. Actually, the first case served as a benchmark and does not play an important role in the analysis.

		$p_1 = .38 \ p_2 = .33 \ p_3 = .29$	$p_1 = .6 \ p_2 = .25 \ p_3 = .15$
informative equilibrium	$v(s=1) = a_1 \ \ v(s=2) = a_2$ $v(s=3) = a_3$	$q \in (.37,\ 1)$	$q \in (.57,\ 1)$
semi-pooling equilibria	$v(s=1) = v(s=3) = a_1$ $v(s=2) = a_2$	$q \in (.37,\ .44)$	$q \in (.56,\ 1)$
	$v(s=1) = v(s=2) = a_1$ $v(s=3) = a_3$	$q \in (.35,\ .4)$	$q \in (.54,\ .1)$
pooling equilibria	$v(s=1) = v(s=2) =$ $= v(s=3) = a_i$	$q \in (.33,\ 1)$	$q \in (.33,\ 1)$

The following calculations prove that the informative equilibrium leads to optimal information aggregation. Consider the second treatment. The probability that the plurality outcome gives the correct choice under informative voting is:

$$(.6)\left[(.6)^3 + 6(.6)^2(.2) + 6(.6)(.2)^2\right] + (.25)\left[(.6)^3 + 6(.6)^2(.2)\right] + (.15)\left[(.6)^3 + 6(.6)^2(.2)\right] = .734$$

which is higher than the corresponding probability under:

- the first semi-pooling equilibrium

$$(.6)\left[(.6)^3 + 6(.6)^2(.2) + 9(.6)(.2)^2 + 4(.2)^3\right] + (.25)\left[(.6)^3 + 6(.6)^2(.2)\right] = .699$$

- the second semi-pooling equilibrium

$$(.6)\left[(.6)^3 + 6(.6)^2(.2) + 9(.6)(.2)^2 + 4(.2)^3\right] + (.15)\left[(.6)^3 + 6(.6)^2(.2)\right] = .634.$$

- the pooling equilibrium (consider here the pooling on a_1, which gives the highest probability)

$$(.6)\left[(.6)^3 + 8(.2)^3 + 6(.6)^2(.2) + 12(.6)(.2)^2\right] = .6$$

These calculations suggest that, informative voting aggregates information optimally. This means that, given their common interest, individuals should reveal all their private information during the voting process. Thus, going back to the purposes of the experiment, it would be interesting to see whether the data supports this hypothesis.

Another question that this experimental study asks is: if the data do not support the informative equilibrium, do they support any of the remaining Bayesian Nash equilibria? If this is not the case, is it possible to provide an explanation for the findings?

4. Experimental Design

The experiment consisted of 4 sessions, each consisting in 6 subjects participating in 15 rounds[5]. No subject participated in more than one session. Subjects were paid a show up fee of 500 Pesetas plus whatever they earned during the session. At the beginning of the experiment, the instructions (Appendix 2) were handed out and read aloud.

The experiment used colored urns, called the Red Urn, the White Urn and the Black Urn, to denote the choices a_1, a_2 and a_3, respectively. Each urn contained a number of red, white and black balls, which represented the three possible signals. Subjects were told that in each round they would be divided in two groups of three. After that, one of the urns would be selected for each group, but they would not be shown the colors of the selected urns. Given this, the members of each group would have to determine the color of the urn selected for their group.

To help them assess how likely was that the Urn Red, Urn White or Urn Black had been selected, the subjects were given some additional information. More precisely, the subjects were given two types of information: common and private information.

Common information: First, subjects were given information about the way in which the urn would be selected. They were told that the urn would be selected by rolling two ten-sided dies as follows. The two dies to be rolled would give a number between 1 and 100, with the first die determining the first ("tens") digit, and the second die determining the second ("ones") digit of the number (00 stands for 100).

[5] As mentioned in the previous section, there was an additional session that served as a benchmark. The results of this session were straightforward, as one can easily imagine for the parameters considered. The results will be very briefly described in the next section.

If the number determined by the two dies was between 1-60, then Red Urn would be selected; if the number was between 61-85, then the White Urn would be selected and if the number was between 86-100, the Black Urn would be selected. Thus, the subjects were provided with a set of common priors over the possible states of the world given by: p(Red Urn) = .6, p(White Urn) = .25, p(Black Urn) = .15.

Second, the subjects were informed that the Red Urn contained six red balls, one white ball and one black ball, the White contained six white balls, two red balls and two black balls, and the Black Urn contained six black balls, two red balls and two black balls.

Private information After selecting the urn by rolling the two dies, its contents were emptied into an unmarked container, so that the subjects could not learn its color. Then, the subjects were asked to extract a ball at random from this container. The color of the ball extracted by each subject constituted his private information. Given the number of red, white and black balls in each urn, this implies that each subject saw a private signal that has a 60% chance of being correct. Using the notation of the previous section, this is equivalent to $q = .6$.

After extracting a ball from the container, subjects were told that they would have to vote for one of the Red, White or Black Urn and the decision of the group would be given by plurality rule with ties broken in the following order: Red Urn, White Urn and Black Urn. The subjects were asked to record this information on record sheets. Each vote was made privately to the experimenter and was unknown to the other subjects. At the end of each round, the subjects were told the outcome of the voting process of their group, but they were not told neither the colors of the balls extracted by the other subjects nor their votes.

Finally, the subjects were rewarded based on whether the group decision was correct or incorrect: they all received 250 Pesetas if the plurality outcome was the urn selected at the beginning of the experiment, and 25 Pesetas otherwise. The resulting game was played for 15 rounds.

5. Results

Table 1 shows the frequencies of the individuals' decisions broken down by session and at the aggregate level. Tables 2 to 5 give the results of the four sessions of the experiment.

The aggregate figures in Table 1 suggest that there is a clear pattern in the individuals' behavior. More precisely, the individuals receiving the highest prior signal, red, voted most of the time (94%) informatively. However, those individuals receiving the white signal voted 17% of the time for the Red Urn and only 82% of the time informatively. The individuals receiving the black signal voted 49% of the time informatively and 45% of the time for the Red Urn. Thus, it seems that individuals receiving the signals corresponding to the higher priors urns, Red and White, would vote most of the time informatively, whereas the individuals receiving the signal supporting the lowest prior urn, Black, would trade-off their signal against the prior belief. The intuition behind these findings is simple: those individuals seeing the red ball cannot do better than voting for the Red Urn, since their vote is supported by both signal and prior belief. Sometimes, they make mistakes. However, there is some logic behind their mistakes, as they err more in the direction of the White Urn (5%) rather than the Black Urn (1%). Individuals seeing the black ball, corresponding to the smallest prior, face a clear trade-off between voting the Black Urn, the least likely to realize a priori, but supported by their signal and voting the Red Urn, the most likely a priori. Their erring rate is of 5%. The decision process for the individuals seeing the white ball is more complicated. They are in some sense in the middle, and the trade-off is not very clear. They seem to follow their signal rather than the prior belief and thus vote more often for the White Urn than for the Red Urn.

The data broken by session confirm these findings. For example, in session 1, the individuals seeing red and white are voting informatively in a proportion of 97% and 87%, respectively. Those individuals seeing black vote 46% of the time informatively and another 46% of the time for Red.

These findings suggest that the individuals' decisions are inconsistent with the pure strategy Bayesian Nash equilibrium behavior described in the model. Actually, the aggregate data indicate that individuals decisions follow a mixed strategy consisting in voting Red if red ball is seen (informative behavior), voting 20% of the time for Red and 80% of the time for White if white ball is seen and voting 50% of the time for Red and 50% of the time if black is seen. This seems reasonable taking into account that for each type (except black in sessions 1 and 3, and red in session 2), there is an error in the individuals' decisions of at most 5%. We have checked whether these mixed strategies constitute a Bayesian Nash equilibrium in the voting game introduced here, and the answer was negative. We have also checked whether the strategies in which individuals seeing red and white vote informatively and those seeing black vote for Red (50%) and Black (50%) constitute a Bayesian Nash equilibrium. It turned out that it was not. The reason for which we checked the last one is that in sessions 1, 3 and 4, the individuals' decisions indicate high percentages of informative voting for those seeing red and white.

Since one of our main concerns was with the optimal information aggregation, we have checked the performance of the data with respect to this issue. It turned out that the aggregate data performed even worse than the pooling behavior. The probability of getting the correct choice at the frequencies induced by the data at aggregate level was of .55.

In conclusion, the experimental results reject the Bayesian Nash equilibrium behavior and with this the informative voting equilibrium. In particular, they reject the optimal aggregation of information. However, given the clear pattern in the data, it would be interesting to explain it. A possible way would be to incorporate an error in the individuals' decisions and use the Quantal Response Equilibrium (QRE) introduced by McKelvey and Palfrey (1995, 1998). The QRE assumes that, in equilibrium, individuals choose better responses more often than worse responses. The present data seem to support such a hypothesis, as far as the informative voting behavior is concerned.

6. Final Remarks

Optimal information aggregation in a common interest environment, with private information, requires informative voting. However, the data rejects the informative voting hypothesis. Although, a high percentage of informative voting can be seen for those types supporting choices that are a priori more likely, this is not enough to compensate for the behavior of the type supporting the choice that is a priori the least likely. The probability of the group making the correct choice under the behavior given by the data is smaller even than the one under the pooling equilibrium.

The data also rejects the other Bayesian Nash equilibria. However, the figures seem to point to a mixed strategy behavior given by: 80% votes for White and 20% votes for Red if white was seen, 50% votes for Black and 50% votes for Red if black was seen and 100% votes for Red if red was seen. These mixed strategies assume an error rate of at most 5%. However, these mixed strategies does not constitute a Bayesian Nash equilibrium. Thus, there is work to be done in explaining the pattern in the data. In particular, The Quantal Response Equilibrium provides us with the tools for a good start in this direction.

APPENDIX 1

The voting game is described by:

- The set of *type profiles* $T = \times_i T_i$, with $T_i = \{1,2,3\}$ describing the types of individual i; a type profile is denoted with $t = (t_1, t_2, t_3)$ or $t = (t_i, t_{-i})$.

- Individuals' *common preference* over the voting profile $v(t) = (v_1(t_1), v_2(t_2), v_3(t_3))$ and the type profile t:

$$U((v_1(t_1), v_2(t_2), v_3(t_3)), t) := E\left[u(PL((v_1(t_1), v_2(t_2), v_3(t_3)), a)/t\right] =$$

$$\sum_{a \in S} u(PL(v(t)), a)\Pr ob(a/t) = \Pr ob(a = PL((v_1(t_1), v_2(t_2), v_3(t_3)/t),$$

where $PL((v_1(t_1), v_2(t_2), v_3(t_3))$ is the plurality outcome at $(v_1(t_1), v_2(t_2), v_3(t_3))$.

- *i's belief* describing i's uncertainty about the *n-1* other players' possible types, t_{-i}, given i's own type, t_i: $p(t_{-i}/t_i)$. This can be done using Bayes rule:

$$p(t_{-i}/t_i) = \frac{p(t_{-i}, t_i)}{p(t_i)} = \frac{p(t)}{\sum_{i \in \{1,2,3\}} p(t_{-i}, t_i)},$$

where

$$p(t) = \sum_{j=1,3} p_j \Pr ob(t/a_j) = \sum_{j=1,3} p_j \prod_{i \in \{1,2,3\}} \Pr ob(t_i/a_j)$$

is the *common prior probability* of a type profile $t \in T$.

The last equality follows from the independence of the individuals' signals given the true state of the world.

All this is taken to be common knowledge between the individuals.

The *expected utility* from voting for $v_i(t_i)$, given the updated belief $p(t_{-i} / t_i)$ and the strategies of the other players $v_{-i}(t_{-i})$ is given by:

$$EU(v_i(t_i); t_i, v_{-i}(t_{-i})) = \frac{1}{\sum_{t_{-i}} p(t_{-i}, t_i)} \sum_{t_{-i}} \Pr ob[PL(v(t)) \,\&\, t]. \tag{A}$$

A *Bayesian Nash equilibrium* (in pure strategies) of the above game is a strategy profile $v^*(\cdot)$ such that for all $i \in \{1,2,3\}$ and all $t_i \in T_i$, $v^*(t_i) = a_k$ only if:

$$EU(a_k; t_i, v^*) \geq EU(a_j; t_i, v^*) \text{ for any } j \neq k. \tag{B}$$

Further, as shown in Austin-Smith and Banks (1990), at the equilibrium the only relevant type profiles for individual i are those where he is pivotal. Thus, condition (B) can be rewritten as:

$$\sum_{t_{-i} \in Piv(T_{-i})} \Pr ob\left[PL(a_k, v^*(t_{-i})) \,\&\, t\right] \geq \sum_{t_{-i} \in Piv(T_{-i})} \Pr ob\left[PL(a_j, v^*(t_{-i})) \,\&\, t\right] \tag{C}$$

for any $j \neq k$, where $Piv(T_{-i})$ is the set of profiles at which an individual i is pivotal.

Proof of Proposition 1:

Fix p_1, p_2 and p_3. Then, informative voting is a Bayesian Nash equilibrium, if (C) is satisfied for all possible types, or:

$$\sum_{t_{-i} \in Piv(T_{-i})} \Pr ob\left[PL(a_1, v^{\inf}(t_{-i})) \,\&\, (t = 1, t_{-i})\right] \geq \sum_{t_{-i} \in Piv(T_{-i})} \Pr ob\left[PL(a_2, v^{\inf}(t_{-i})) \,\&\, (t = 1, t_{-i})\right]$$

$$\sum_{t_{-i} \in Piv(T_{-i})} \Pr ob\left[PL(a_1, v^{\inf}(t_{-i})) \,\&\, (t = 1, t_{-i})\right] \geq \sum_{t_{-i} \in Piv(T_{-i})} \Pr ob\left[PL(a_3, v^{\inf}(t_{-i})) \,\&\, (t = 1, t_{-i})\right]$$

$$\sum_{t_{-i} \in Piv(T_{-i})} \Pr ob\left[PL(a_2, v^{\inf}(t_{-i})) \,\&\, (t = 2, t_{-i})\right] \geq \sum_{t_{-i} \in Piv(T_{-i})} \Pr ob\left[PL(a_1, v^{\inf}(t_{-i})) \,\&\, (t = 2, t_{-i})\right]$$

$$\sum_{t_{-i} \in Piv(T_{-i})} \Pr ob\left[PL(a_2, v^{\inf}(t_{-i})) \,\&\, (t = 2, t_{-i})\right] \geq \sum_{t_{-i} \in Piv(T_{-i})} \Pr ob\left[PL(a_3, v^{\inf}(t_{-i})) \,\&\, (t = 2, t_{-i})\right]$$

$$\sum_{t_{-i} \in Piv(T_{-i})} \Pr ob\left[PL(a_3, v^{\inf}(t_{-i})) \,\&\, (t = 3, t_{-i})\right] \geq \sum_{t_{-i} \in Piv(T_{-i})} \Pr ob\left[PL(a_1, v^{\inf}(t_{-i})) \,\&\, (t = 3, t_{-i})\right]$$

$$\sum_{t_{-i}\in Piv(T_{-i})} \Pr ob\left[PL(a_3, v^{\inf}(t_{-i}))\ \&\ (t=3, t_{-i})\right] \geq \sum_{t_{-i}\in Piv(T_{-i})} \Pr ob\left[PL(a_2, v^{\inf}(t_{-i}))\ \&\ (t=3, t_{-i})\right],$$

where $v^{\inf}(t_{-i})$ is the informative strategy profile of all the individuals except i.

For this case, the set of profiles at which an individual i is pivotal is given by:

$$Piv(T_{-i}) = \{(1,1,0), (0,1,1), (1,0,1)\}.$$

After calculating the sums over these profiles, the above inequalities become:

$$p_1\left[2q^2\left(\frac{1-q}{2}\right)+q\left(\frac{1-q}{2}\right)^2\right] \geq p_2\left[2q\left(\frac{1-q}{2}\right)^2\right]+p_1\left[q^2\left(\frac{1-q}{2}\right)\right]$$

$$p_1\left[2q^2\left(\frac{1-q}{2}\right)+q\left(\frac{1-q}{2}\right)^2\right] \geq p_3\left[2q\left(\frac{1-q}{2}\right)^2\right]+p_1\left[q^2\left(\frac{1-q}{2}\right)\right]$$

$$p_2\left[2q^2\left(\frac{1-q}{2}\right)\right]+p_1\left[q\left(\frac{1-q}{2}\right)^2\right] \geq p_1\left[2q\left(\frac{1-q}{2}\right)^2+\left(\frac{1-q}{2}\right)^3\right]$$

$$p_2\left[2q^2\left(\frac{1-q}{2}\right)\right]+p_1\left[q\left(\frac{1-q}{2}\right)^2\right] \geq p_1\left[q\left(\frac{1-q}{2}\right)^2\right]+p_3\left[2q\left(\frac{1-q}{2}\right)^2\right]$$

$$p_3\left[2q^2\left(\frac{1-q}{2}\right)\right]+p_1\left[q\left(\frac{1-q}{2}\right)^2\right] \geq p_1\left[2q\left(\frac{1-q}{2}\right)^2+\left(\frac{1-q}{2}\right)^3\right]$$

$$p_3\left[2q^2\left(\frac{1-q}{2}\right)\right]+p_1\left[q\left(\frac{1-q}{2}\right)^2\right] \geq p_2\left[2q\left(\frac{1-q}{2}\right)^2\right]+p_1\left[q\left(\frac{1-q}{2}\right)^2\right].$$

After doing some calculations and after rearranging the terms of the above inequalities, the first two inequalities and the forth one become:

$$p_1(1+q) \geq p_2[2(1-q)]$$

$$p_1(1+q) \geq p_3[2(1-q)]$$

$$p_2(2q) \geq p_3[(1-q)].$$

It is a trivial exercise to show that the above inequalities hold for any $q > \frac{1}{3}$ and $p_1 > p_2 > p_3$, and thus to see that they do not constrain in any way the informative equilibrium.

The remaining three inequalities can be rewritten to match the inequalities (1), (2) and (3) in Proposition 1. QED

Proof of Proposition 2:

Given p_1, p_2 and p_3, the strategies[6] $v(t=1)=v(t=3)=a_1$, $v(t=2)=a_2$ constitute a Bayesian Nash equilibrium, if the first four equilibrium conditions (for types 1 and 2, who vote informatively) in the proof of Proposition 1 plus the following two conditions[7] for type 3 individuals are satisfied:

$$\sum_{t_{-i}\in Piv(T_{-i})} \Pr ob\left[PL(a_1,v(t_{-i}))\ \&\ (t=3,t_{-i})\right] \geq \sum_{t_{-i}\in Piv(T_{-i})} \Pr ob\left[PL(a_2,v(t_{-i}))\ \&\ (t=3,t_{-i})\right]$$

$$\sum_{t_{-i}\in Piv(T_{-i})} \Pr ob\left[PL(a_1,v(t_{-i}))\ \&\ (t=3,t_{-i})\right] \geq \sum_{t_{-i}\in Piv(T_{-i})} \Pr ob\left[PL(a_2,v(t_{-i}))\ \&\ (t=3,t_{-i})\right].$$

The set of profiles at which an individual i is pivotal is given by:

$$Piv(T_{-i}) = \{(1,1,0),(0,1,1)\}.$$

It should be noticed that for these strategies, an individual i is not pivotal at the type profile (1,0,1). At this profile and given that individuals use the above strategies, the choice a_1 is selected for any possible type of individual i.

The equilibrium conditions at these profiles are:

$$p_1\left[q^2\left(\frac{1-q}{2}\right)+q\left(\frac{1-q}{2}\right)^2\right] \geq p_2\left[2q\left(\frac{1-q}{2}\right)^2\right]$$

$$p_1\left[q^2\left(\frac{1-q}{2}\right)+q\left(\frac{1-q}{2}\right)^2\right] \geq p_1\left[q^2\left(\frac{1-q}{2}\right)\right]+p_3\left[q\left(\frac{1-q}{2}\right)^2\right]$$

$$p_2\left[2q^2\left(\frac{1-q}{2}\right)\right] \geq p_1\left[q\left(\frac{1-q}{2}\right)^2+\left(\frac{1-q}{2}\right)^3\right]$$

$$p_2\left[2q^2\left(\frac{1-q}{2}\right)\right] \geq p_1\left[q\left(\frac{1-q}{2}\right)^2+\left(\frac{1-q}{2}\right)^3\right]$$

$$p_1\left[q\left(\frac{1-q}{2}\right)^2+\left(\frac{1-q}{2}\right)^3\right] \geq p_1\left[q\left(\frac{1-q}{2}\right)^2+\left(\frac{1-q}{2}\right)^3\right]$$

[6] In the statement of Proposition 2, the strategies were written as functions of signals received. Here they are written as functions of the types. The two are equivalent since the types are given by the signals.

[7] However, the equilibrium conditions have to be evaluated at the (semi-informative or semi-pooling) strategy profile proposed in Proposition 2 and not at the informative strategy profile. Also, as it is explained further in the proof, the pivotal profiles for the semi-informative strategy profile are different from the informative ones. It follows that the resulting equilibrium conditions for the types 1 and 2 of Proposition 2 are different from those of Proposition 1.

$$p_1\left[q\left(\frac{1-q}{2}\right)^2+\left(\frac{1-q}{2}\right)^3\right]\geq p_2\left[2q\left(\frac{1-q}{2}\right)^2\right].$$

The only inequalities that impose restrictions on q are the first and the third (or fourth, as they are the same) inequalities. The two inequalities can be rewritten to obtain the condition of the Proposition 2. The remaining inequalities are satisfied for any $q>\frac{1}{3}$ and $p_1>p_2>p_3$. After dividing with $q\left(\frac{1-q}{2}\right)$ and rearranging its terms, the second inequality becomes $p_1\geq p_3$, which is always satisfied. The fifth inequality is satisfied with equality. Since $q\geq\left(\frac{1-q}{2}\right)$, the sixth inequality is satisfied whenever the third inequality is satisfied. QED

Proof of Proposition 3:

Given p_1, p_2 and p_3, the strategies $v(t=1)=v(t=3)=a_1$, $v(t=2)=a_2$ constitute a Bayesian Nash equilibrium, if the equilibrium conditions for types 1 and 3 (those who vote informatively) in the proof of Proposition 1 plus the following two conditions[8] for type 2 individuals are satisfied:

$$\sum_{t_{-i}\in Piv(T_{-i})}\Pr ob\left[PL(a_1,v(t_{-i}))\ \&\ (t=2,t_{-i})\right]\geq\sum_{t_{-i}\in Piv(T_{-i})}\Pr ob\left[PL(a_2,v(t_{-i}))\ \&\ (t=2,t_{-i})\right]$$

$$\sum_{t_{-i}\in Piv(T_{-i})}\Pr ob\left[PL(a_1,v(t_{-i}))\ \&\ (t=2,t_{-i})\right]\geq\sum_{t_{-i}\in Piv(T_{-i})}\Pr ob\left[PL(a_3,v(t_{-i}))\ \&\ (t=2,t_{-i})\right].$$

The set of pivotal profiles in this case is given by:

$$Piv(T_{-i})=\{(1,0,1),(0,1,1)\}.$$

Then, the equilibrium conditions are:

$$p_1\left[q^2\left(\frac{1-q}{2}\right)+\left(\frac{1-q}{2}\right)^3\right]\geq p_1\left[q^2\left(\frac{1-q}{2}\right)+\left(\frac{1-q}{2}\right)^3\right]$$

[8] However, the equilibrium conditions have to be evaluated at the (semi-informative or semi-pooling) strategy profile proposed in Proposition 2 and not at the informative strategy profile. Also, as it is explained further in the proof, the pivotal profiles for the semi-informative strategy profile are different from the informative ones. It follows that the resulting equilibrium conditions for the types 1 and 2 of Proposition 2 are different from those of Proposition 1.

$$p_1\left[q^2\left(\frac{1-q}{2}\right)+\left(\frac{1-q}{2}\right)^3\right]\geq p_1\left[q^2\left(\frac{1-q}{2}\right)+\left(\frac{1-q}{2}\right)^3\right]$$

$$p_1\left[q\left(\frac{1-q}{2}\right)^2+\left(\frac{1-q}{2}\right)^3\right]\geq p_1\left[q\left(\frac{1-q}{2}\right)^2+\left(\frac{1-q}{2}\right)^3\right]$$

$$p_1\left[q\left(\frac{1-q}{2}\right)^2+\left(\frac{1-q}{2}\right)^3\right]\geq p_3\left[2q\left(\frac{1-q}{2}\right)^2\right]$$

$$p_3\left[2q^2\left(\frac{1-q}{2}\right)\right]\geq p_1\left[q\left(\frac{1-q}{2}\right)^2+\left(\frac{1-q}{2}\right)^3\right]$$

$$p_3\left[2q^2\left(\frac{1-q}{2}\right)\right]\geq p_1\left[q\left(\frac{1-q}{2}\right)^2+\left(\frac{1-q}{2}\right)^3\right].$$

The inequalities that determine the equilibrium condition in Proposition 3 are the forth and the fifth ones. The second inequality is true whenever the fourth one is satisfied. To see this, consider the forth inequality rewritten as:

$$\frac{p_3}{p_1}\leq\frac{q+\left(\frac{1-q}{2}\right)}{2q}.$$

Similarly, the second inequality can be rewritten as:

$$\frac{p_3}{p_1}\leq\frac{q^2+\left(\frac{1-q}{2}\right)^2}{2q\left(\frac{1-q}{2}\right)}.$$

Simple calculations show that $\dfrac{q+\left(\frac{1-q}{2}\right)}{2q}<\dfrac{q^2+\left(\frac{1-q}{2}\right)^2}{2q\left(\frac{1-q}{2}\right)}$ and thus that the second inequality is satisfied for all q that verify the fourth inequality.

The remaining inequalities are either equivalent with the forth or fifth are they are satisfied with equality. QED

APPENDIX 2

Experiment Instructions

This is an experiment of group decision making. *Your earnings will depend partly on your decisions, partly on the decisions of others, and partly on chance*. Different subjects may earn different amount of money. At this time, you will be given 500 Pts. This payment will compensate you for showing up today.

General Description

In this experiment, *you, as a group*, will be asked to predict from which randomly chosen urn you draw a ball. There will be three colored urns, which will be called the Red Urn, the White Urn and the Black Urn. Each urn contains red, white and black balls. One of the urns will be selected by rolling two ten-sided dies; the selection process (by rolling the dies) will be made precise in a short while. *You will not be told the color of the urn selected by rolling the two dies!* However, you will be given some additional information that will help you assess which of the three urns has been selected. Namely, you will be told the number of red, white and black balls in each urn and, then each of you will be allowed to draw a ball from the selected urn (without knowing the color of the urn). The color of the ball will be your private information, and you will not be allowed to communicate with the other participants. Finally, you will be required to vote for one of the urns without communicating with the other participants. Your group decision will be given by the urn that receives most number of votes. If you, as a group, will vote for the correct urn, that is, *the urn that has been selected by rolling the dies at the beginning of the experiment*, you will receive 250 Pts. each. If not, you will get 25 Pts. each. Notice that your success does not depend only on your vote alone, but on the final decision made by your group. In other words, *even if you vote for the correct urn, you will not win (get 250 Pts.) unless the decision of your group is the correct urn*.

It is very important that you do not communicate with each other during the experiment. If you have any question during the experiment, raise your hand and an experimenter will come and assist you.

Each subject will be given a record sheet and an identification number. This is written at the top of your record sheet; this is how you will identify yourself during the experiment. Since you are 12, you will receive identification numbers from 1 to 12. The experiment will consist of 15 matches. Each match corresponds to one entry in the Column 1 of the record sheet.

In each match, you will be associated with two other subjects as follows. Subjects that are assigned identification numbers between 1-6 will be matched within themselves. In each match you will be randomly matched in a different group consisting of 3 subjects. Similarly, those subjects that are assigned identification numbers between 7-12, will be randomly matched in different groups of 3 subjects. Notice that there will always be 4 groups of 3 subjects. At the end of the last match you will be paid the total amount you have accumulated during the experiment. Let us begin with the instructions of the experiment.

Instructions

Here are the three urns: the Red Urn, the White Urn and the Black Urn. Each urn contains a number of red, white and black balls. The number of red, white or black balls contained by each urn will be specified to you soon. Now we will randomly select an urn for each group. The urns are selected independent from each other. The only relevant urn in making your decision is the urn selected for your group. Each urn is selected by rolling two ten-sided dies. The result will be a number in the range 1 - 100. The first die to be rolled determines the first ("tens") digit, and the second die to be rolled determines the second ("ones") digit; 00 will stand for 100. For example, if the first die shows 7 and the second die shows 5, then the resulting number will be 75.

READ **P1)** OR P2)

P1) Almost equal priors' instructions: If the number shown by the dies is between 1-38 (including 1 and 38), then the urn will be Red; if the number shown by the dies is between 39-71, then the urn will be White; if the number shown by the dies is between 72-100, then the urn will be Black. In other words, the Urn Red has a chance 38 in 100 to be selected, the White Urn has a chance of 33 in 100 to be selected and the Black Urn has a chance of 29 in 100 to be selected. This information appears in the following table.

Red Urn	White Urn	Black Urn
1 – 38	39 – 71	72 - 100

P2) Different priors' instructions: If the number shown by the dies is between 1-60 (including 1 and 38), then the urn will be Red; if the number shown by the dies is between 61-85, then the urn will be White; if the number shown by the dies is between 86-100, then the urn will be Black.

Red Urn	White Urn	Black Urn
1 – 60	61 – 85	86 - 100

You will not see the outcome of the rolling of the two dies, so you will not know in advance the color of the urn selected. However, to help you determine which of the urn has been selected for your group, you will be given some additional information. More precise, you will be told the number of red balls, white balls and black balls in each of the urns. This information will be known by everybody. In addition, once an urn is determined for each group, the contents of the selected urns will be emptied in two containers, and each member of your group will be allowed to draw one ball at random from the container corresponding to his or her group. This will constitute your private information and it should not be shared with the other subjects.

Information known by everybody:

READ **q1)** OR q2)

q1) *q* s.t. plurality rule is optimal instructions: There are in total 8 balls of red, white and black colors. The Red Urn contains 6 red balls, 1 white ball and 1 black ball. The White Urn contains 6 white balls, 1 red ball and 1 black ball. The Black Urn contains 6 black balls, 1 red ball and 1 white ball.

Red Urn	White Urn	Black Urn
Balls: 6 red, 1 white, 1 black	6 white, 1 red, 1 black	6 black, 1 red, 1 white

q2) *q* s.t. plurality rule is not optimal instructions: There are in total 10 balls of red, white and black colors. The Red Urn contains 6 red balls, 2 white balls and 2 black balls. The White Urn contains 6 white balls, 2 red ball and 2 black ball. The Black Urn contains 6 black balls, 2 red ball and 2 white ball.

Red Urn	White Urn	Black Urn
Balls: 6 red, 2 white, 2 black	6 white, 2 red, 2 black	6 black, 2 red, 2 white

Private information

At the beginning of each round, we will come around with the containers, one for each group. Please draw a ball now, and write its color in Column 2, entitled "Ball seen", of your record sheet. For example, if the ball drawn has color red, you should write *red* in Column 2. All the members of a group will draw a ball from the same urn. After you draw a ball, you have to return the ball to the container before the next member of the group draws a ball. Thus, every subject will have one private draw, *with the ball being replaced after each draw*. Recall that you are not allowed communicating among yourselves.

Voting

After each subject has seen his or her own draw, you will be asked to vote for one of the urns: Red Urn, White Urn or Black Urn. Please write your vote in Column 3, entitled "Urn voted", of your record sheet. For example, if you want to vote for the Red Urn, you should write *Red* in Column 3. Your *group's decision* will be that urn which receives more number of votes. If each of the three urns receives one vote, your *group's decision* will be the Red Urn.

Rewards

Your earnings will be determined in the following manner. If your *group's decision* is the same as the urn selected at the beginning of the experiment, then the decision is correct, and each of your group members earns 250 Pesetas. If your group's decision is different from the true urn, then each member of your group will get 25 Pesetas.

Now we will come around and record your votes. Then we will tell you your group decision. Register your group decision in Column 4. For example, if you are told that your group decision is the Red Urn, you should write *Red* in Column 4. Then we will tell you which was the true urn for each group. Record this in Column 5. Then, if the colors in Column 4 and 5 coincide, you will get 250 Pts; if not, you will get 25 Pts. Now write your payoff in Column 6.

Each round will proceed in the same way.

If there are any questions or problems, please raise your hand and an experimenter will come and assist you. Everyone should remain silent until the end of the last match.

Sessions 2, 3, 4, 5

This session will also last 20 matches. The rules are the same as before, with one exception.

READ **P1)**

READ **q2)**

Table 1: Proportions of voting strategies by session/ aggregate

Session	R-R	R-W	R-B	W-W	W-R	W-B	B-B	B-R	B-W
1	0,97	0,03	0,00	0,87	0,13	0,00	0,46	0,46	0,08
2	0,89	0,11	0,00	0,69	0,29	0,03	0,45	0,50	0,05
3	0,97	0,03	0,00	0,87	0,13	0,00	0,55	0,35	0,10
4	0,91	0,04	0,04	0,89	0,11	0,00	0,52	0,48	0,00
aggregate	0,94	0,05	0,01	0,82	0,17	0,01	0,49	0,45	0,05

Table 2: Results of session 1

		Group	Extracted Ball			Voted Urn			Urn selected	Group Decision
1	1	1-2-3	B	W	B	B	W	R	R	R
	2	4-5-6	W	R	W	W	R	W	R	W
2	1	1-2-4	W	W	W	W	W	R	W	W
	2	3-5-6	W	W	W	W	W	W	W	W
3	1	1-2-5	B	R	W	R	R	W	W	R
	2	3-4-6	B	B	B	R	R	R	B	R
4	1	1-2-6	R	R	B	R	R	R	R	R
	2	3-4-5	R	B	B	R	R	B	B	R
5	1	1-3-4	R	R	W	R	R	W	R	R
	2	2-5-6	W	W	B	W	W	B	W	W
6	1	1-3-5	R	R	R	R	R	R	R	R
	2	2-4-6	B	B	W	B	R	W	W	R
7	1	1-3-6	R	R	R	R	R	R	R	R
	2	2-4-5	W	W	W	W	R	W	W	W
8	1	1-4-5	W	W	W	R	W	W	B	W
	2	2-3-6	R	R	B	R	W	R	R	R
9	1	1-4-6	R	R	R	R	R	R	R	R
	2	2-3-5	B	W	B	B	W	B	B	B
10	1	1-5-6	R	R	B	R	R	B	R	R
	2	2-3-4	B	B	B	B	R	B	B	B
11	1	1-2-3	B	W	W	W	W	W	W	W
	2	4-5-6	R	W	B	R	W	W	W	W
12	1	1-2-6	R	R	R	R	R	R	R	R
	2	3-4-5	W	W	B	R	W	B	W	R
13	1	1-3-4	W	R	R	W	R	R	B	R
	2	2-5-6	R	R	R	R	R	W	R	R
14	1	1-4-5	B	R	B	R	R	B	B	R
	2	2-3-6	W	R	W	W	R	W	R	W
15	1	1-5-6	R	R	B	R	R	R	R	R
	2	2-3-4	W	R	B	W	R	B	R	R

Table 3: Results of session 2

		Group	Extracted Ball			Voted Urn			Urn selected	Group Decision
1	1	1-2-3	R	R	W	W	R	W	R	W
	2	4-5-6	W	W	R	W	W	R	R	W
2	1	1-2-4	R	R	R	R	R	R	R	R
	2	3-5-6	R	R	R	R	R	R	R	R
3	1	1-2-5	W	B	W	W	B	W	R	W
	2	3-4-6	R	W	W	W	W	R	W	W
4	1	1-2-6	R	B	R	R	B	R	R	R
	2	3-4-5	R	B	W	R	R	R	R	R
5	1	1-3-4	W	W	R	W	R	R	B	R
	2	2-5-6	B	B	R	B	B	R	B	B
6	1	1-3-5	W	R	R	W	R	R	W	R
	2	2-4-6	B	W	W	B	W	R	W	R
7	1	1-3-6	W	W	W	B	R	R	W	R
	2	2-4-5	W	R	W	W	W	R	R	W
8	1	1-4-5	W	W	W	W	W	W	W	W
	2	2-3-6	B	B	R	R	B	R	W	R
9	1	1-4-6	W	W	W	W	W	W	R	W
	2	2-3-5	B	W	R	R	W	R	B	R
10	1	1-5-6	R	W	B	R	R	R	B	R
	2	2-3-4	B	B	B	W	R	R	B	R
11	1	1-2-3	R	R	R	R	R	R	R	R
	2	4-5-6	B	W	R	R	W	R	R	R
12	1	1-2-6	W	B	R	W	B	R	B	R
	2	3-4-5	R	R	B	R	R	R	R	R
13	1	1-3-4	B	R	W	R	W	W	B	W
	2	2-5-6	R	R	R	R	R	R	R	R
14	1	1-4-5	B	W	W	B	W	R	R	R
	2	2-3-6	W	W	R	W	W	R	B	W
15	1	1-5-6	B	W	B	B	W	R	B	R
	2	2-3-4	R	R	W	R	R	R	R	R

Table 4: Results of session 3

		Group	Extracted Ball			Voted Urn			Urn selected	Group Decision
1	1	1-2-3	W	W	R	W	W	R	B	W
	2	4-5-6	B	W	W	R	W	W	W	W
2	1	1-2-4	R	R	W	R	R	W	W	R
	2	3-5-6	R	W	R	R	W	R	R	R
3	1	1-2-5	R	W	R	R	W	R	R	R
	2	3-4-6	W	B	R	R	R	R	R	R
4	1	1-2-6	R	R	R	R	R	R	R	R
	2	3-4-5	B	W	W	R	W	W	B	W
5	1	1-3-4	R	W	R	W	W	R	R	W
	2	2-5-6	W	W	W	W	W	W	W	W
6	1	1-3-5	R	R	B	R	R	B	W	R
	2	2-4-6	W	B	W	W	R	W	W	W
7	1	1-3-6	B	B	W	W	B	W	W	W
	2	2-4-5	W	R	W	R	R	W	R	R
8	1	1-4-5	W	W	W	R	R	W	W	R
	2	2-3-6	B	B	R	B	B	R	B	B
9	1	1-4-6	R	R	R	R	R	R	R	R
	2	2-3-5	R	B	R	R	R	R	R	R
10	1	1-5-6	R	W	B	R	W	R	R	R
	2	2-3-4	W	R	R	W	R	R	R	R
11	1	1-2-3	B	B	R	W	B	R	B	R
	2	4-5-6	B	R	W	B	R	W	W	R
12	1	1-2-6	B	B	R	B	B	R	B	B
	2	3-4-5	R	R	W	R	R	W	R	R
13	1	1-3-4	W	B	W	W	B	W	W	W
	2	2-5-6	W	R	R	W	R	R	R	R
14	1	1-4-5	B	B	W	B	B	W	W	B
	2	2-3-6	R	W	R	R	W	R	W	R
15	1	1-5-6	R	R	B	R	R	R	R	R
	2	2-3-4	R	R	R	R	R	R	R	R

Table 5: Results of session 4

		Group	Extracted Ball			Voted Urn			Urn selected	Group Decision
1	1	1-2-3	R	B	W	R	B	W	W	R
	2	4-5-6	R	R	R	R	R	R	R	R
2	1	1-2-4	B	R	W	R	R	W	R	R
	2	3-5-6	R	R	R	R	R	R	R	R
3	1	1-2-5	R	R	B	W	R	B	R	R
	2	3-4-6	R	W	R	R	W	R	R	R
4	1	1-2-6	R	R	R	R	R	R	R	R
	2	3-4-5	R	B	R	R	B	R	R	R
5	1	1-3-4	R	R	R	R	R	R	R	R
	2	2-5-6	B	B	B	R	B	R	B	R
6	1	1-3-5	R	W	B	B	W	R	R	R
	2	2-4-6	B	B	R	R	B	R	R	R
7	1	1-3-6	R	R	W	W	R	W	R	W
	2	2-4-5	R	R	R	R	R	R	R	R
8	1	1-4-5	R	W	W	R	W	W	W	W
	2	2-3-6	B	B	B	R	B	R	B	R
9	1	1-4-6	R	B	B	R	B	R	W	R
	2	2-3-5	B	R	R	R	R	R	R	R
10	1	1-5-6	R	B	R	R	B	R	R	R
	2	2-3-4	W	W	B	W	W	B	R	W
11	1	1-2-3	R	W	W	R	R	W	W	R
	2	4-5-6	R	B	B	R	B	B	R	B
12	1	1-2-6	R	R	W	B	R	W	R	R
	2	3-4-5	W	B	R	W	B	R	B	R
13	1	1-3-4	R	W	R	R	W	R	R	R
	2	2-5-6	B	W	B	R	W	R	B	R
14	1	1-4-5	R	W	R	R	W	R	R	R
	2	2-3-6	W	W	W	W	W	R	W	W
15	1	1-5-6	B	R	B	B	R	R	B	R
	2	2-3-4	R	R	R	R	R	R	R	R

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