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Investigation of Stored Energy Distribution in Filters Using K-Means Clustering Algorithm

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***Abstract*— The k-means clustering algorithm has been implemented to find patterns in the time-averaged stored energy distribution in various filter networks. A large data set comprising of numerous topologies for 50 different single band specifications has been investigated. By finding key characteristics within this data set, general guidelines for predicting the optimum topology for power handling have been established.**

***Keywords*— Stored energy distribution, RF filters, k-means algorithm, pattern recognition**

I. Introduction

A. Motivation

In order to avoid interchannel interference, the typical filtering specifications for multiplexers in communications systems demand low passband insertion loss (typically < 0.1 dB), very high rejection levels on one side of the passband and comparatively low rejection levels on the other side of the passband [1]. Such high performance filters require estimation of their power handling capacity to avoid multipaction (for space applications) or ionisation breakdown (for terrestrial communications).

An accurate estimation of the power handling capacity of a microwave filter involves modelling and fine-tuning of 3-dimensional resonant cavities in an electromagnetic (EM) software. Even with the available efficient software packages, the entire process is cumbersome and inefficient to make initial predictions about the power handling capacity of the filter. An alternative approach for predicting the power handling capacity by investigating the stored energy distribution in the equivalent circuit model has been explored previously [2],[3],[4]. Ernst [5] used a stochastical search method to find the optimum topology for a given transfer function. However, this method can result in topologies that are practically unrealisable.

In this work, an unsupervised pattern recognition algorithm, called the k-means clustering algorithm, has been implemented to discover patterns in the stored energy distribution of bandpass filter networks. With the aid of this algorithm, key characteristics of the stored energy distribution within various network topologies realising the same power transfer function can be discovered. Therefore, by finding regularities in large stored energy data sets comprising of numerous transfer functions and their network topologies, predictions about the optimum topology for power handling can be made. A large set of specifications have been investigated to provide generalized "rules" to predict the optimum topology. The study has been limited to practically realisable and widely used topologies comprising of cascaded triplets (CT) and cascaded quartets (CQ).

B. Stored Energy Calculation

For an input power of 1W, the complex nodal voltages for a lossless bandpass filter network were obtained using the following system of equations [6]:

$$[V] = [Y]^{-1} \cdot [I] \tag{1}$$

where, the bandpass admittance matrix is the sum of the capacitive admittance matrix and the inductive admittance matrix respectively, $[Y] = j\omega[M_C] + [M_L]/(j\omega)$ as in [7]. The time-averaged stored energy (t.a.s.e) at the i^{th}-node is then calculated using the following formula:

$$W_i(\omega) = \frac{|V_i(\omega)|^2}{4}\left(C_i + \frac{1}{\omega^2 L_i}\right) \tag{2}$$

II. K-Means Clustering Algorithm

A. Method

The k-means algorithm is an iterative algorithm that partitions data into K clusters using the squared Euclidean distance measure. The algorithm commences by selecting K initial centroids and assigning objects to the nearest centroids. In each iteration, the centroids are updated by taking the mean of the objects assigned to that cluster. The objects are re-allocated to the nearest cluster and the algorithm continues to iterate until either the cost function is minimized or the assignment of data points stops changing. For n objects described by m variables, that are to be partitioned into K groups, the cost function to be minimized is given by [8]:

$$E = \sum_{l=1}^{K}\sum_{i=1}^{n} y_{il} \cdot d(X_i, C_l) \tag{3}$$

where, $y_{il} = 1$ if the object i belongs to the cluster l and $y_{il} = 0$ otherwise. $d(X_i, C_l)$ is the square Euclidean distance between object i in the input vector matrix $\mathbf{X}$ and centroid l in the centroid vector matrix $\mathbf{C}$. A detailed description of the algorithm can be found in the series of papers by Huang [8],[9].

III. Modelling Stored Energy Input Data

The clustering algorithm was applied to network topologies generated for 50 different bandpass specifications. In order to simplify the analysis process, the degree of the filter network, the passband frequencies and the number of finite transmission zeros were fixed to 9, 1805 MHz - 1880 MHz and 5 respectively. The position of the finite transmission zeros, however, was chosen at random from a reasonable range of values. The total t.a.s.e of the power transfer function is strongly influenced by its selectivity, as proven in [5]. Although the degree of the filter and passband frequencies were fixed, varying the finite transmission zero positions effectively alters the selectivity of the power transfer function.

For a 9^{th}-order bandpass filter network with 5 finite transmission zeros, three different n-tuplet arrangements have been considered: (a) quartet, quartet and triplet, (b) quartet, triplet and quartet, (c) triplet, quartet and quartet (Fig. 1). Each pair of consecutive n-tuplets share a common resonator. For each of the n-tuplet arrangements, by changing the direction of cross-couplings for the two quartets and the sequence of transmission zero extraction, 360 topologies were obtained for each specification. In total, 18000 topologies were obtained.

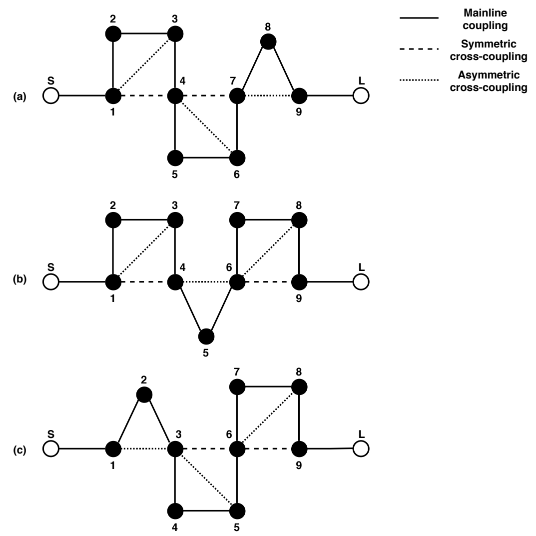

Fig. 1. Schematics of n-tuplet arrangements possible with a 9-5 filter. (Each quartet is drawn with a forward cross-coupling direction).

As the objective is to discover patterns that aid in predicting the optimum topology for power handling, 10 topologies giving the lowest peak t.a.s.e and the highest peak t.a.s.e were extracted. Thus, a total of 1000 topologies were to be clustered. The input vector matrix was composed of rows representing the objects and columns respresenting the attributes that describe each object. For the case in consideration, the attributes were narrowed down to the peak t.a.s.e, the resonator demonstrating the peak t.a.s.e, the relative position of the transmission zeros (from the closest band-edge) in the sequence of extraction and the cross-coupling direction of the first quartet.

IV. Clustering Results

A. Choosing the value of K

The second input to the clustering algorithm is the number of clusters that the data has to be partitioned into. One of the techniques used to determine the optimal number of clusters is the silhouette method. The silhouette coefficients lie in the range $[-1, +1]$ and provide a measure of the closeness of an object to their neighbouring clusters. A high average value of silhouette coefficient indicates that the objects in a cluster are well-matched and a negative value indicates otherwise. Table 1 provides silhouette coefficients for different values of K. It can be seen that the maximum value occurs for $K = 11$.

B. Cluster Analysis

Graphical representation of the number of objects with various n-tuplet arrangements (refer to a,b and c in Fig. 1) within each cluster is presented in Fig. 2. It also represents the statistics for the cross-coupling direction of the first quartet in each of these topologies. Visual representation of the mean values for the peak t.a.s.e and the relative position of finite transmission zeros (in the order of extraction) for each cluster is presented in Fig. 3.

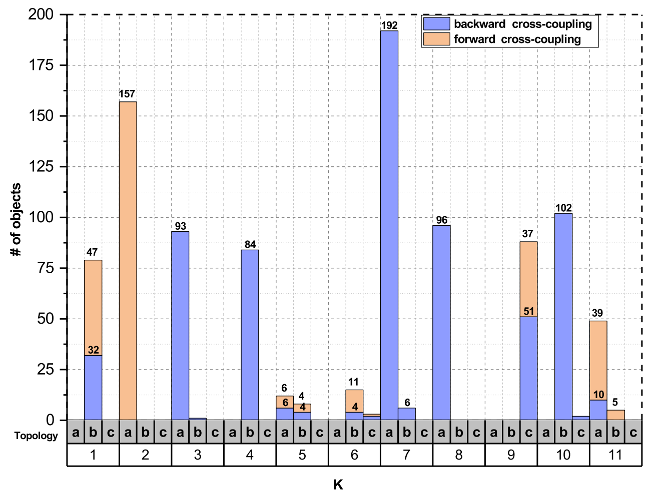

Fig. 2. Statistics for the cross-coupling directions and n-tuplet arrangements (as labelled in Fig. 1) for each cluster.

1) Worst Peak Stored Energy Clusters

Clusters 4, 5, 7, 8 and 10 comprise of objects with the highest peak t.a.s.e. It can be seen that 99.5% of the objects in these clusters have n-tuplet arrangements (a) and (b). Both of these arrangements have a quartet placed closest to the source. From Fig. 3, it can be concluded that the finite transmission zeros produced by the first quartet in these groups lie in the same stopband. Values of the peak t.a.s.e were observed to be the highest when these finite transmission zeros were located close to the respective passband edge and/or had a small separation gap between them. Choosing a quartet to produce transmission zeros located in the same stopband increases the

Table 1. Silhouette coefficients (S.C) for various values of K

K	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
S.C	0.42772	0.5252	0.4943	0.5565	0.5942	0.5748	0.6362	0.6209	0.6209	0.6689	0.5575	0.5435	0.5548	0.5684	0.5567	0.5557

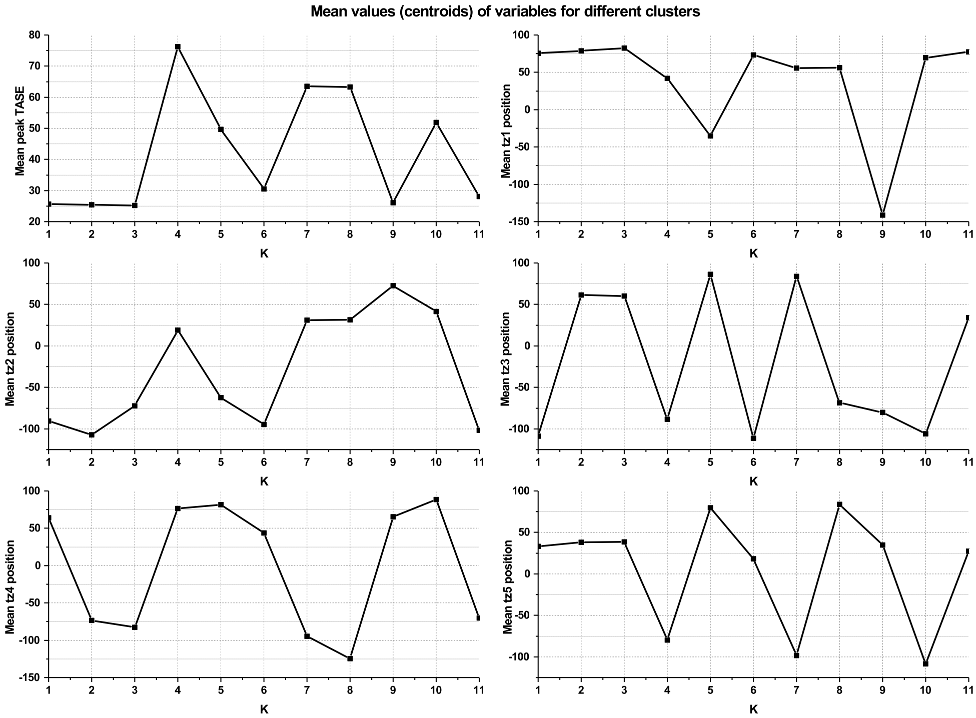

Fig. 3. Mean values for the peak t.a.s.e and relative finite transmission zero positions for each cluster.

asymmetry of its network element values, thus, resulting in high peak t.a.s.e values for one of its resonators. This can be illustrated with the aid of an example specification. Consider a 9-5 bandpass filter response with finite transmission zeros located at 1745 MHz, 1760 MHz, 1780 MHz, 1925 MHz and 1970 MHz (Fig. 4), realised using n-tuplet arrangement (a).

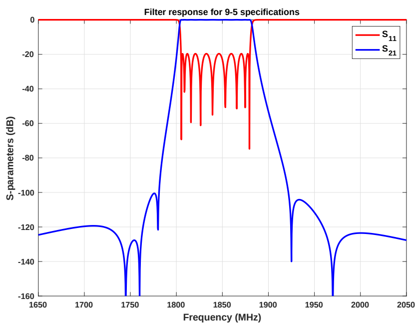

Fig. 4. An example specification of a 9-5 bandpass filter

The t.a.s.e distribution plots for the first 4 resonators forming a quartet with backward cross-coupling are displayed in Fig. 5. It can be seen that the t.a.s.e is distributed evenly in all resonators except resonator 3, where a sharp peak is observed at a frequency just below the lower cut-off frequency.

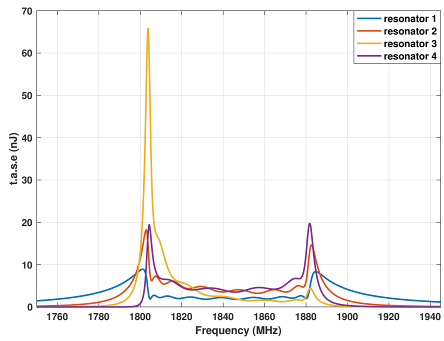

Fig. 5. T.a.s.e plots for resonators in a quartet producing transmission zeros at frequencies 1780 MHz and 1760 MHz. The remaining network consists of a CT producing a transmission zero at 1925 MHz followed by a CQ producing transmission zeros at 1745 MHz and 1970 MHz.

From Fig. 2, it can be seen that about 98% of the objects clustered in groups 4, 5, 7, 8 and 10 have backward cross-coupling. In other words, the asymmetric cross-coupling was directed towards the shared resonator. In such a case, energy from a larger number of paths goes into the common node, thus, increasing the peak t.a.s.e observed at that resonator. It is interesting to note that the resonator demonstrating the highest peak t.a.s.e for these clusters was always resonator 3. For quartets with backward cross-coupling, this is the resonator opposite to the asymmetric cross-coupling.

2) *Lowest Peak Stored Energy Clusters*

Clusters 1, 2, 3, 6, 9 and 11 comprise of objects demonstrating the lowest peak t.a.s.e. The first quartets in all of these topologies are used to produce transmission zeros located in separate stopbands - upper stopband (USB) and lower stopband (LSB). The position of these finite transmission zeros is either furthest away or at similar distances from their respective band-edges. If the latter is the case, the nodal admittances in the quartet are similar in value. This results in a more evenly distributed t.a.s.e. Consider the same specifications as given in section IV-B1 realised using n-tuplet arrangement (a) and (b). The plots for t.a.s.e distribution in the first four resonators forming a quartet with forward cross-coupling are displayed in Fig. 6. By comparing Figures 5 and 6, it can be seen that the peak t.a.s.e can be reduced to less than half the value by extracting appropriate transmission zeros with the first quartet. The remaining n-tuplets in the optimum topologies were used to extract transmission zeros in the order of: furthest from the band-edge to closest to the band-edge.

For n-tuplet arrangement (c), the optimum t.a.s.e distribution was obtained when the triplet was used to generate the transmission zeros located furthest away from the closest band-edge and the quartet was used to generate transmission zeros lying at similar values from the respective band-edges. It can also be seen from Fig. 2 that the optimum cross-coupling direction for the first quartet in arrangements (a) and (b) is forward and that for (c) is backward. Thus, it can be concluded that lowering the number of energy paths to the shared resonator results in a lower peak t.a.s.e for the filter network.

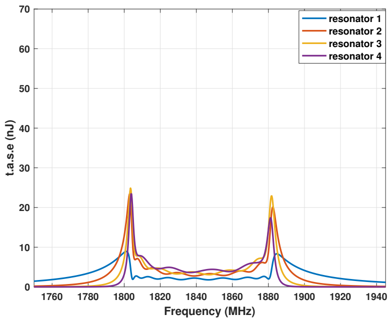

Fig. 6. T.a.s.e plots for resonators in a quartet producing transmission zeros at frequencies 1745 MHz and 1925 MHz. The remaining network consists of a quartet producing transmission zeros at 1760 MHz and 1970 MHz followed by a triplet producing a transmission zero at 1780 MHz.

C. *General Patterns*

To emphasize, an n-tuplet placed closest to the source should be used to produce transmission zero/s located furthest away from the closest band-edge. In order to generate a finite transmission zero using an n-tuplet, a cross-coupling (an admittance) between non-adjacent resonators has to be introduced. To shift the finite transmission zero closer to the band-edge, a stronger coupling (higher admittance) is required. The addition of a larger cross-coupling admittance at a node increases the nodal admittance and consequently, increases the peak t.a.s.e observed at the node.

In order to lower the peak t.a.s.e, a quartet should be used to generate finite transmission zeros on either side of the passband. The finite transmission zeros produced by the quartets should either lie furthest away from the respective band-edges or at similar distances from the respective band-edges. Choosing transmssion zeros furthest from the band-edges lowers the nodal admittance and hence, the peak t.a.s.e. On the other hand, choosing finite transmission zeros lying at similar distances from the band-edges results in a more even t.a.s.e distribution.

If two n-tuplets share a common resonator between them, the direction of the asymmetric cross-coupling should be such that the number of energy paths going into the shared resonator is a minimum. It is interesting to note that the resonator demonstrating the highest peak t.a.s.e for a triplet is the one opposite to the asymmetric cross-coupling. For a quartet, this resonator could be either of the ones opposite to the symmetric cross-coupling. This is dependent on the difference in positions of transmission zeros generated by the quartet. If the response at hand is symmetric, the peak t.a.s.e can be lowered by synthesising a network topology that is symmetric.

V. Conclusion

An unsupervised pattern recognition algorithm has been implemented to find regularities in a stored energy data set of n-tuplet topologies for 50 different specifications. By analysing the clustering results, generalized "rules" have been presented to aid the prediction of optimum filter topology for a given specification. This eliminates the need to investigate different filter topologies individually.

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